

Type II Report

Rutherford's Scattering Experiment

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Abstract. The relationship between the direct count rate of alpha particles (N_d) and angle (θ) at which they are detected is explored using a rotating Am-241 source in a vacuum chamber. This includes calculation of a correction factor which allows the determination of the corrected count rate (N) over the cone shape spacial distribution. The direct count rate as a function of scattering angle has the shape of a normal distribution. The validation of 'Rutherford's scattering formula' is presented by dimensional analysis. At $\theta = 15^\circ$, the atomic number of aluminum was found to be $Z_{Al} = 14 \pm 2$ (*unitless*) and at $\theta = 5^\circ$, $Z_{Al} = 52 \pm 8$ (*unitless*).

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1. Introduction

Until this experiment Thompson's model was widely accepted as how atoms were structured. Briefly speaking, the model consisted of an evenly spread distribution of positive and negative charges in the atom - the 'plum pudding model'. The subsequent evidence eliminates the possibility of Thompson's plum pudding model by showing that very high angles of deflections (up to 180 degrees) of alpha particles can occur from collisions with the nuclei of atoms.

The main resultant principle behind this experiment is that when alpha particles pass nearby a nucleus that their path will be altered due to the interactions between the two particles. Most are barely deflected, if at all, whilst others can be deflected by very large angles - the extreme case being 180 degrees where the alpha particle will head straight back to the radiation source. The observations of this experiment led directly to a new and still currently accepted model which is known as the Rutherford model. This is based around the nucleus being much smaller, very dense and positively charged with orbiting relatively light ($\approx \frac{1}{2000}$ nucleus mass) negatively charged particles (electrons).

This led to Rutherford developing calculating the angular distribution of the scattering rate $N(\theta)$. This is the number of alpha particles scattered per second in a small angle $d\theta$ at an angle of θ from the source to the detector, figure 1 shows this information. $d\Omega$ is the resulting area of the projected cone shape which forms a ring with a thickness of $\theta + \delta\theta$. In turn, this results in Rutherford's scattering formula which will be used and verified in this report - see the 3rd objective.

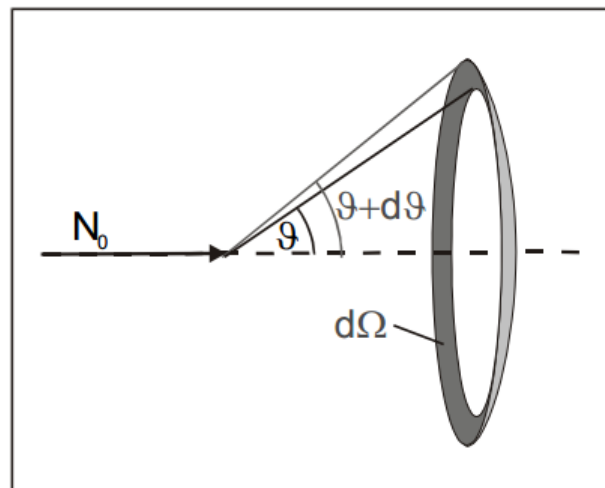


Figure 1.

[2] A number, N_0 of alpha particles being scattered into a ring of thickness, $\theta + \delta\theta$ which forms a ring of area, $d\Omega$.

The influences on values which lead to errors are as follows: purely statistical errors due to the randomness of radioactive decay, a negligible error in the total time over which the decay events were measured, the precision of the angular scale on vacuum chamber, the presence of particles in the vacuum chamber, the size of the slit aperture which is proportional to angular error of incoming alpha particles, the thickness of the foils (gold and aluminium), background noise from external natural radiation events and finally the distance between the source and the detector.

The measured quantities are the angle from the source to the detector using a scale on the scattering chamber θ , the counting rate (N_d) of α particles scattered as a function of angle (θ), the total time T over which the decay events were detected and the distance between the source and the detector R .

The main equipment in use for this experiment were: scattering chamber, vacuum pump, discriminator preamplifier, cassy module with timer box, gold foil (thickness, $d = 2\mu m$), aluminium foil (thickness, $d = 7\mu m$) and both a 5mm and 1mm slit aperture within the scattering chamber. Figure 2 shows the a diagram of the scattering chamber. A complete list of equipment and their relevant settings which can be used for this experiment can be found in both the laboratory script Version 1.0 and the Didactic report [2].

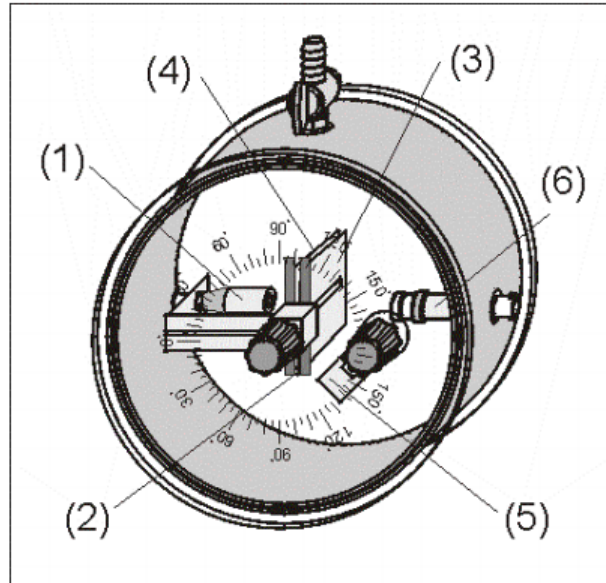


Figure 2.

[2] - The scattering chamber which is evacuated, it holds the foil (3), the holder of the foil (2), the radioactive alpha source (1), the slit (4), the detector (6). The other component is the swivel arm (5) which rotates all internal components except the detector which is fixed and wall mounted. It also has a printed scale of the angle from the detector to the source with a precision of 5 degrees.

2. Objectives

- (i) Record counting rate (N_d) of α particles scattered as a function of angle (θ)
- (ii) Correct measured rates measured in one place to compensate for how the scattering occurs over a 3D cone shape in order to match the theoretical model
- (iii) Validate Rutherford's scattering formula - $N(\theta) = N_0 \cdot c_F \cdot d_F \frac{Z^2 \cdot e^4}{(8\pi \cdot \epsilon_0 \cdot E_\alpha)^2 \cdot \sin^4(\frac{\theta}{2})}$
- (iv) Determine the atomic number of Aluminium (${}^{26}_{13}\text{Al}$) experimentally

3. Method

There was one significant deviation made from the method in the lab script. Initially the counting rate (N_d) of α particles scattered as a function of angle (θ) was recorded without the protective plastic bag over the apparatus. After that deviation, the method in the lab script was followed exactly. Please refer to the lab script for further details where necessary.

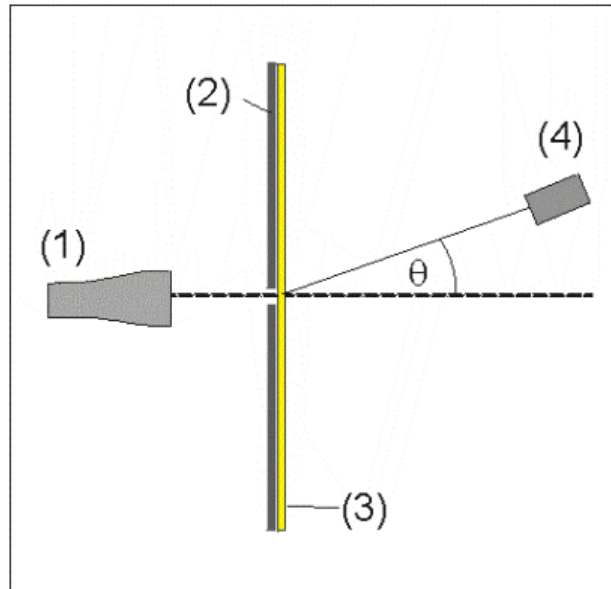


Figure 3.

[2] Top down view of the layout of components and scattering geometry within the vacuum chamber.

1 - Alpha radiation source. 2 - Slit of aperture 5mm or 1mm. 3 - Thin foil - Al or Au. 4 - Radiation detector.

Out of the fundamental forces, electromagnetism is by far the most relevant force for this experiment due to its relative strength. Within that, electrostatics is the area of interest as this potential $\left(V(r) = \frac{q_1 q_2}{4\pi\epsilon_0 r}\right)$ defines the minimum separation distance between the alpha particle and any nuclei that it encounters. Non quantum mechanically, this scattering is elastic scattering as the incoming alpha particle leaves with the same energy as when it entered and this is related to the potential - there is no energy exchange between the alpha particle and the interacting nucleus. There is in reality an energy transfer due to spin and magnetic moment along with relativistic effects, this is called Mott scattering.

Very briefly, when an alpha particle approaches a nucleus, it feels an electrostatic repulsion and the minimum separation distance is determined by the potential described above. The recoil angle is determined by the potential along with how close to a 'head-on' inelastic collision occurs. Most of the alpha particle pass straight through the foil and are not deflected because they never end up being near any nuclei, but as they get closer to a 'head-on' collision, the scattering angle tends towards 180 degrees - straight back to the source. This is graphically shown in 4.2, above about 30 degrees however using this experimental setup, it would not be practical over the desired timescale to record data as it would either take an extremely long time or background events would overwhelm the scattered alpha particles.

When an alpha particle encounters a less charged nucleus (aluminium instead of gold), the potential decreases and so the alpha particle is scattered less. This alters the scattering geometry in such a way that there is a lower count with larger angles. This can be quantitatively seen in 4.4.

In order to achieve the required environment required for this experiment, several issues had to be addressed. Firstly, the scattering chamber in which the interactions and detection occurred had to be vacuum pumped in order to reduce interactions with air molecules. Secondly, that the scattering chamber had to be covered in order to reduce noise from light to the semiconductor based detector, in this case a black plastic sheet was used. Although it was measured that there was not a significant difference between the covered and uncovered chamber, the precaution was made due to the sensitivity of the radiation detector to visible light. Standard vacuum precautions were taken by the lab demonstrator, such as carefully venting the chamber when changing the slits or ensuring that there is a good seal before turning on the vacuum pump.

The radiation detector was connected to the discriminator preamplifier which effectively controls the level of noise signal allowed to pass through the device. This is done by applying a 'discriminator voltage' of approximately -0.3V, this is the voltage at which the noise radiation counts is as small as possible, ideally 0

counts per second.

The cassy module was connected to the detector and to the computer where the data was recorded by the accompanying cassy software. The software was set to record a number of events over a certain gate time (which was changed based on how many events occurred over the selected gate time), the counts per second over the total time was calculated from this data.

The measurements contained within 4.1 to record the count rate of alpha particles as a function of angle were taken as follows, an alternative but very similar method is documented in the manufacturer's report [2]. The angle of the sample to the detector was varied from -30 to 30 degrees in increments of 5 degrees, hence the direct count rate was measured as a function of angle. At least 100 events were recorded in order to reduce statistical errors, therefore as the angle increased, a larger amount of time was required to get 100 events. At large angles, it is advisable to repeat measurements because the background noise is now a significant factor compared to the expected count rate. The slit with an aperture of 5mm was used for this part.

Similarly to the method used for 4.1, there is a nearly identical method described in the manufacturer's report [2] for 4.4 where the atomic number of aluminium is measured. The slit with an aperture of 1mm was used for this part. An angle at which a 'reasonable' count rate was chosen, in this case both 5 and 15 degrees. The count rate was then measured at these angles with both gold and aluminium foils with the same restrictions as in 4.1 - at least 100 events were measured in order to reduce the statistical error.

The radioactive source used in the whole experiment was Am-241 which emits alpha particles of energies most commonly ranging from 5.391MeV to 5.486MeV and has a half life of 458 years. The first level of the alpha decay chain all lead to excited states of Np-237. [3]

4. Results

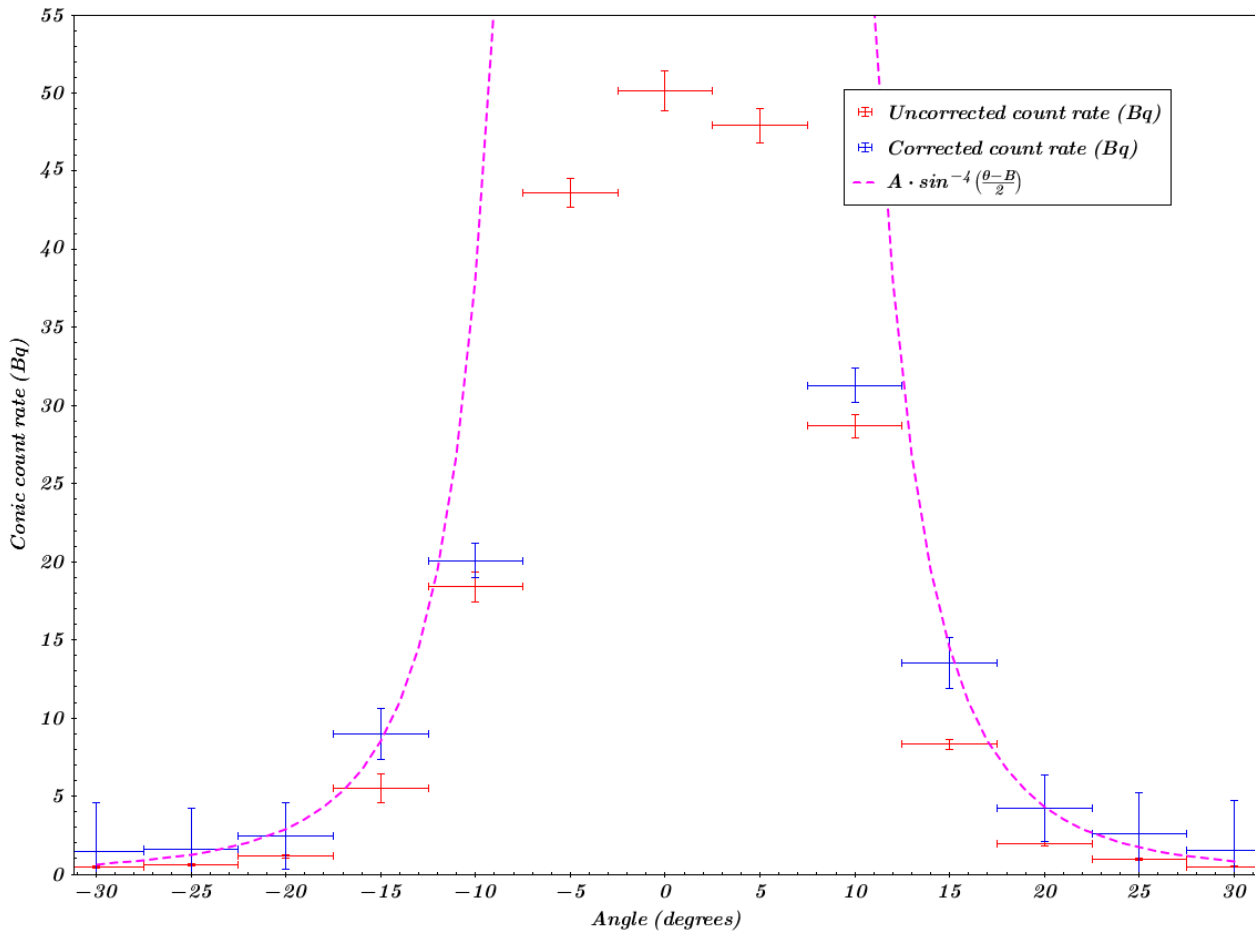
All symbols used in this report match the convention of the lab report unless otherwise explicitly stated.

4.1. Record counting rate (N_d) of α particles scattered as a function of angle (θ)

Angle - $\theta(^{\circ})$	Total events - $N_{Total}(-)$	Total time - $T(s)$	Count rate - $N_d(s^{-1})$
+30	104	210	0.495
+25	116	120	0.967
+20	235	120	1.96
+15	665	80	8.31
+10	1434	50	28.68
+5	1917	40	47.93
0	1504	30	50.13
-5	2181	50	43.62
-10	368	20	18.4
-15	331	20	5.52
-20	137	120	1.14
-25	137	180	0.588
-30	110	240	0.458

Table 1: The distribution of count rate as a function of the scattering angle using gold foil ($d_{Au} = 2\mu m$) with a slit aperture of $d = 5mm$. The $\theta(^{\circ})$ uncertainty is a constant and is 2.5° due to the markings on the apparatus being at every 5 degrees. The uncertainties in N_d and N_{Total} have both been sourced from [1] and are as follows: $\delta N_d = \frac{\sqrt{N}}{T} \delta N_{Total} = \pm \sqrt{N}$ where N is the total number of events and T is the time for which those events were measured over. The relative uncertainty for T would be incredibly small as it was measured through a device which was connected to a computer.

4.2. Correct measured rates measured in one place to compensate for how the scattering occurs over a 3D cone shape in order to match the theoretical model



Graph 1 - Distribution of corrected & uncorrected count rate on a linear scale with a fitted $A \cdot \sin^{-4} \left(\frac{\theta-B}{2} \right)$ line where $A=0.0032$ $B=0.01745$ and are manually fitted parameters. Although the predicted number of radiation events tends towards infinity towards 0 degrees, the measured count rate does not and therefore the theory deviates from the measurements very significantly and is not valid at small angles (approximately less than 15 degrees). **Vertical error bars for uncorrected count rate decrease with bigger angles so that they are not visible at this scale but are still present.**

For angles of at least 10 degrees, it can be seen that there is a very good agreement with the fitted line when taking the error bars (boxes) into account. At angles of less than 10 degrees, there is an increasingly worse agreement because the theoretical model approaches infinity at 0 degrees which does not happen in nature. An accurate model would require a correction factor which makes the count rate smaller as it approaches 0 degrees so that the graph would certainly intercept at a finite number. Perhaps a Gaussian distribution would approximate the results better.

There is a systematic error involved in this experiment which can be observed from all counting rates, both corrected and uncorrected, which are measured at positive angles are noticeably higher than those at their respective negative angles. The suggested reason for this is “unwanted misalignments in the setup” [2], this is further elaborated later on “A small inaccuracy of the collimator-slit adjustment or non centric distribution of the radiation, coming from the preparation in the holder, may cause a shift of the curve along the horizontal axis (angle shift $< 3^\circ$). Due to such effects it is useful to record scattering rates as well in the positive as in the negative angular range, to get information of both branches with respect to an accurate determination of the symmetry-axis displacement” [2].

Another systematic error which has not been corrected for is the background radiation which would have increased the number of events as there are always radioactive decays happening randomly. This error would have had far more of an effect with lower counts, becoming significant at approximately 20 degrees either way.

A way to correct for this would be subtract an average background radiation rate from the results, this was not actually measured at the time and so will not be taken into account. This correction would have the visual effect of shifting all the data down slightly on graph 1. At 25 degrees or more, the count rate is in the order of the expected background radiation and is thus a major unaccounted error.

The overwhelming source of random error in this experiment is the radioactive decay events themselves and it is an intrinsic part of how they work. There is no known way of predicting when a particular nucleus will decay, only a probability distribution which works increasingly well with more nuclei involved (decay events per time unit). This uncertainty has been accounted for by stating that the actual counts have an uncertainty of \sqrt{N} because "If the number of counts recorded in a time T is N, then the uncertainty in the number of counts is \sqrt{N} . That is, if you repeat the experiment many times, you will get a Gaussian distribution of N's, centered on \bar{N} , the average value of N, with a standard deviation of $\sqrt{\bar{N}}$. If you do the experiment just once, then the best estimate of the count is $N \pm \sqrt{N}$, and the best estimate of the count rate is $R = \frac{N}{T} \pm \frac{\sqrt{N}}{T}$ " [1]. Another random error would have been introduced from the fact that the chamber was not fully evacuated of air particles which would have interacted with the alpha particles which were being measured.

The 3 dimensional correction factor $\frac{d\Omega}{d\theta}$ can be obtained by a trigonometric analysis of Figure 1 and a more detailed is in the laboratory book. The result of this analysis is that the correction factor is unitless and is $\frac{d\Omega}{d\theta} = 2\pi \cdot \sin(\theta)$. Therefore the combined corrected formula for the $N(\theta) = 2\pi \cdot \sin(\theta) \cdot N_d(\theta)$.

There is a suggested function from [3] (bottom of page 4) which gives a curve which apparently fits the data from this type of experiment much more closely than Rutherford's formula at small angles especially. Also [3] offers a different experimental method based on a different expression of Rutherford's formula which is based on the cross section interaction and can be seen on that document.

4.3. Validate Rutherford's scattering formula - question from lab script

Show that the ratio $\frac{N(\theta)}{N_0}$ from $N(\theta) = N_0 \cdot c_F \cdot d_F \cdot \frac{Z^2 \cdot e^4}{(8\pi\epsilon_0 E_\alpha)^2 \cdot \sin^4(\frac{\theta}{2})}$ is dimensionless Rutherford's scattering formula to be validated:

$$\frac{N(\theta)}{N_0} = c_F \cdot d_F \cdot \frac{Z^2 \cdot e^4}{(8\pi\epsilon_0 E_\alpha)^2 \cdot \sin^4(\frac{\theta}{2})}$$

The following quantities are unitless and are therefore irrelevant with this dimensional analysis : Z^2, π and $\sin^4(\frac{\theta}{2})$

Quantity	c_F	d_F	E_α	e	ϵ_0
Units	m^{-3}	m	J	C	$C^2 J^{-1} m^{-1}$

Substituting these units into their respective variables:

$$\frac{N(\theta)}{N_0} = m^{-3} \cdot m \cdot \frac{C^4}{(C^2 J^{-1} m^{-1} \cdot J)^2}$$

$$\frac{N(\theta)}{N_0} = \frac{m^{-2} \cdot C^4}{C^4 J^{-2} m^{-2} \cdot J^2} = \text{Unitless}$$

This shows that Rutherford's scattering formula is dimensionally correct and is thus validated in that sense.

4.4. Determine the atomic number of Aluminium (${}^{26}_{13}\text{Al}_{13}$) experimentally

The equation below can be rearranged to be solved for Z_{Al} as that is the quantity of interest - the atomic number of Aluminium which is known to be 13. Substituting the measured values in yields the answers at a scattering angle of 5 and 15 degrees.

$$\frac{N_{Au}}{N_{Al}} = \frac{C_{Au} d_{Au} Z_{Au}^2}{C_{Al} d_{Al} Z_{Al}^2}$$

$$\frac{C_{Au}}{C_{Al}} = 1$$

Rearranges to the final equation:

$$Z_{Al} = \sqrt{\frac{d_{Au}}{d_{Al}} \frac{Z_{Au}^2}{N_{Au}} \cdot \frac{N_{Al}}{N_{Au}}}$$

$$Z_{Al} = \sqrt{\frac{7 \pm 0.5\mu m \times 79^2}{2 \pm 0.5\mu m} \times \frac{N_{Al}}{N_{Au}}}$$

At $\theta = 15^\circ$, $N_{Al} = 0.075s^{-1}$ $N_{Au} = 0.667s^{-1}$

$$Z_{Al} = 14 \pm 2 \text{ (Unitless)}$$

At $\theta = 5^\circ$, $N_{Al} = 20.01s^{-1}$ $N_{Au} = 13.30s^{-1}$

$$Z_{Al} = 52 \pm 8 \text{ (Unitless)}$$

The data from which the atomic numbers above has been calculated is presented below

Material	Aluminium	Gold	Aluminium	Gold
Angle θ (degrees)	15	15	5	5
Count rate $N(\theta) s^{-1}$	0.075	0.667	20.01	13.30
Uncertainty in $N(\theta) \left(\pm \frac{\sqrt{N(\theta)}}{T(s)} \right) [1]$	0.003	0.041	0.04	0.03

Table 2: The count rates as a function of the angles with their respective error calculations. The relative count rate between gold and aluminium at 15 degrees (approximately 10) is the same as in the manufacturer's report [2] - see 'Evaluation and results part b'. NB: uncertainty calculated for Aluminium at 15 degrees is a combination of 2 sessions with different gate times.

As graph 1 shows, at 5 degrees, the theoretical model has broken down as so, at such an angle it would not be expected to obtain a correct answer for the atomic number for aluminium and does not give a reasonable answer at all. At 15 degrees however, the model still is closely matched to the observations and so an answer which lies within the error bounds has been obtained - $Z_{Al} = 14 \pm 2$ where the known answer is $Z_{Al} = 13$ exactly.

5. Error calculations

5.1. Uncertainty for $\frac{A}{\sin^4\left(\frac{\theta-B}{2}\right)}$ - fitted \sin^{-4} curve

The function which the uncertainty will be calculated for following the 'standard' method using partial differentiation. $f(\theta, A, B)$ is abbreviated to f for the sake of simplicity and space.

$$f(\theta, A, B) = \frac{A}{\sin^4\left(\frac{\theta-B}{2}\right)}$$

Where $\theta = 5^\circ \pm 2.5^\circ$ or $15^\circ \pm 2.5^\circ$

$A = 0.03(-)$; $B = 0.01(\text{radians}) \approx 0.57^\circ$

$$\Delta f(\theta, A, B)^2 = \left| \frac{\partial f}{\partial \theta} \right|^2 \times \delta \theta^2$$

$$\frac{\partial f}{\partial \theta} = \frac{\partial}{\partial \theta} \left(\frac{A}{\sin^4\left(\frac{\theta-B}{2}\right)} \right)$$

Factor out A because the differentiation is with respect to θ and not A

$$= A \cdot \frac{\partial}{\partial \theta} \left(\frac{1}{\sin^4 \left(\frac{\theta-B}{2} \right)} \right)$$

Apply reciprocal rule

$$= A \cdot \frac{-\frac{\partial}{\partial \theta} \left(\sin^4 \left(\frac{\theta-B}{2} \right) \right)}{\left(\sin^4 \left(\frac{\theta-B}{2} \right) \right)^2}$$

Apply power rule then chain rule

$$= \frac{-4A \cdot \left(\sin^3 \left(\frac{\theta-B}{2} \right) \right) \cdot \frac{\partial}{\partial \theta} \left(\sin \left(\frac{\theta-B}{2} \right) \right)}{\sin^8 \left(\frac{\theta-B}{2} \right)}$$

Take derivative of the sin term and apply chain rule

$$= \frac{-4A \cdot \cos \left(\frac{\theta-B}{2} \right) \cdot \frac{\partial}{\partial \theta} \left(\frac{\theta-B}{2} \right)}{\sin^5 \left(\frac{\theta-B}{2} \right)}$$

Differential of the argument is $\frac{1}{2}$, B is not being differentiated

$$= \frac{-4A \cdot \cos \left(\frac{\theta-B}{2} \right) \cdot \frac{1}{2}}{\sin^5 \left(\frac{\theta-B}{2} \right)}$$

Simplification step - $\frac{1}{2} \times -4 = -2$

$$\frac{\partial f}{\partial \theta} = \frac{-2A \cdot \cos \left(\frac{\theta-B}{2} \right)}{\sin^5 \left(\frac{\theta-B}{2} \right)}$$

$$\left(\frac{\partial f}{\partial \theta} \right)^2 = \frac{4A^2 \cos^2 \left(\frac{\theta-B}{2} \right)}{\sin^{10} \left(\frac{\theta-B}{2} \right)}$$

$$\Delta f^2 = \frac{4A^2 \cos^2 \left(\frac{\theta-B}{2} \right)}{\sin^{10} \left(\frac{\theta-B}{2} \right)} \times \delta \theta^2$$

$$f^2 = \frac{A}{\sin^8 \left(\frac{\theta-B}{2} \right)}$$

Divide Δf^2 by f^2

$$\frac{\Delta f^2}{f^2} = \frac{4 \cos^2 \left(\frac{\theta-B}{2} \right)}{\sin^2 \left(\frac{\theta-B}{2} \right)} \times \delta \theta^2 = 4 \cot^2 \left(\frac{\theta-B}{2} \right) \times \delta \theta^2$$

Rearrange to get

$$\Delta f^2 = f^2 \cdot 4 \cot^2 \left(\frac{\theta-B}{2} \right) \times \delta \theta^2$$

Final result for the uncertainty equation of the function f .

$$\Delta f = f \sqrt{4 \cot^2 \left(\frac{\theta-B}{2} \right) \times \delta \theta^2} = 2f \sqrt{\cot^2 \left(\frac{\theta-B}{2} \right) \times \delta \theta^2}$$

5.2. Uncertainty for $\frac{d\Omega}{d\theta}$ - correction factor

As with the previous method, the 'standard' way for calculating uncertainties involving the partial differential of the desired function will be calculated here. Another method was used to compare uncertainties, this will be displayed immediately after the first method.

$$\frac{d\Omega}{d\theta} = C = 2\pi R^2 \cdot \sin\theta$$

Where $R = 1$ and $\delta\theta = \pm 2.5^\circ$

For the general case

$$\Delta A^2 = \sum_{i=1} \left| \frac{\delta f(x_i)}{\delta x_i} \right|^2 \times \Delta x_i$$

$$C = f(x_i); \quad \sum_{i=1}^N \rightarrow N = 1$$

$$\Delta C^2 = \left| \frac{\delta C}{\delta R} \right|^2 \times \delta R^2 + \left| \frac{\delta C}{\delta \theta} \right|^2 \times \delta \theta^2$$

$$\frac{\delta C}{\delta R} = 2R \quad \frac{\delta C}{\delta \theta} = \cos\theta$$

$$\Delta C^2 = 4R^2 \cdot \delta R^2 + \cos\theta \cdot \delta \theta^2$$

$$C^2 = 4\pi^2 R^4 \sin^2\theta$$

$$\frac{\Delta C^2}{C^2} = \frac{4R^2 \cdot \delta R^2 + \cos\theta \cdot \delta \theta^2}{4\pi^2 R^4 \sin^2\theta} = \frac{4R^2 \cdot \delta R^2}{4\pi^2 R^4 \sin^2\theta} + \frac{\cos\theta \cdot \delta \theta^2}{4\pi^2 R^4 \sin^2\theta}$$

$$\Delta C = C \sqrt{\frac{\delta R^2}{\pi^2 R^2 \sin^2\theta} + \frac{\cos\theta \cdot \delta \theta^2}{4\pi^2 R^4 \sin^2\theta}}$$

Final equation for the uncertainty for $\frac{d\Omega}{d\theta}$. $\delta R^2 = 0$, $\delta\theta = 2.5^\circ$ and $R^2 = 1$.

$$\Delta C = C \sqrt{\left(\frac{\cos\theta \times 2.5^{\circ 2}}{4\pi^2 \sin^2\theta} \right)}$$

Substituting in θ , $\delta\theta$ and $C = \frac{d\Omega}{d\theta}$

$$\Delta \frac{d\Omega}{d\theta} = 2\pi \cdot \sin\theta \sqrt{\left(\frac{\cos\theta \times 2.5^{\circ 2}}{4\pi^2 \sin^2\theta} \right)}$$

Factor out $\sqrt{1/4\pi^2 \sin^2\theta}$ which cancels out $2\pi \cdot \sin\theta$ and $\sqrt{2.5^2} = 2.5$

$$\Delta \frac{d\Omega}{d\theta} = 2.5 \sqrt{\cos\theta}$$

6. Conclusion

The first part of this experiment (4.1), the recording of the count rate of alpha particles scattered as a function of angle shows that there is a sharp increase between 10 and 15 degrees. The general shape of the distribution could be said to appear like a normal distribution, this is reflected in the theoretical model of the fitted \sin^{-4} line at above 15 degrees. This has a visually identical shape to the manufacturer's report [2].

A correction factor was required to account for the fact that the model assumes the knowledge of the count rate over a projected 2 dimensional circle from the radiation source (cone shaped) and the measurement is only taken at a small part of the overall circle. Therefore the measured count rate is always smaller than the overall count rate at that same angle because only a small part of the circle will be measured. 2 parameters within the correction factor had to be manually adjusted to match the corrected count rate curve. One (A) controlled the vertical scaling in a logarithmic way and the other (B) controls a horizontal shift.

With the correction factor and the fitted sine curve, it can be seen that there is very good agreement beyond ± 15 degrees. This is entirely due to the inaccuracy of the model that it predicts the count rate will be at infinity when the angle is 0 degrees which is physically impossible. Rutherford's scattering formula was verified that it was dimensionally correct by rearranging the equation and substituting in the respective units. The manufacturer's report does not measure the count rate at 0 degrees but the rest of the data visually matches that report. Some of the unaccounted reasons as to why Rutherford's may be explained by the nature of scattering cross sections, multiple scattering of alpha particles and quantum mechanical effects. These quantum mechanical effects would absolutely be both relevant and expected at this size range.

Finally, the atomic number of aluminium at 15 degrees was found to be 14 ± 2 where the known value is exactly 13. At 5 degrees, it was found to be 52 ± 8 because of the exact same inaccuracy of Rutherford's scattering formula at small angles that has just been discussed. The most significant error sources in this experiment are the random nature of radioactive decay $\left(\pm \frac{\sqrt{N}}{T}\right)$ and the equipment limitations of angle measurement of ± 2.5 degrees.

7. References

Laboratory script for Rutherford's scattering experiment PH500 - Version 1.0 - 06/08/08 - University of Kent

[1] - Part O1.3 - Accessed on 10/03/14 - http://www.colorado.edu/physics/phys1140/phys1140_sp05/Experiments/O1Fall04.

[2] - Didactic report - Accessed on 10/03/14 - http://www.ld-didactic.de/literatur/hb/e/p6/p6521_e.pdf

[3] - Rutherford Scattering - Published 01/09/13 - <http://web.mit.edu/8.13/www/JLEperiments/JLExp15.pdf>

8. Appendix

Angle $\theta(^{\circ})$	Count rate - $N_d (s^{-1})$	Corrected count rate - $N(\theta) (s^{-1})$
+30	0.495	1.56
+25	0.967	2.57
+20	1.96	4.21
+15	8.31	13.52
+10	28.68	31.29
+5	47.93	26.24
0	50.13	∞
-5	43.62	23.89
-10	18.4	20.08
-15	5.52	8.97
-20	1.14	2.45
-25	0.588	1.56
-30	0.458	1.44

Table 3: Holds the data used to plot graph 1 (4.2). The corrected count rates for angles -5, 0 and +5 were not plotted on the graph because they are either infinity or do not show relevant information and the reasons for this have already been discussed.