Cosmic muon half-life

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Abstract

We recorded a cosmic muon energy spectrum which showed that the the vast majority of events came from below 7 MeV, with an expected calculated peak at 52.8 MeV and a maximum muon energy of 85 MeV. We calculated the muon half life using a timing based electronics solution, with the associated uncertainties we confirmed the expected half life of $t_{0.5} = 1.4\mu s + 0.8 - 0.5\mu s$ - with the known muon half life as $2.2\mu s$. Finally, we measured an energy spectrum from the decay electrons/positrons from the muons which showed that most of the events had an energy of $10 \pm 2.5MeV$ with a maximum energy of $75.5MeV$.

Note: British English spelling is adhered to within this document.

1 Theory

Pions (pi mesons), a set of particles composed of 2 quarks (1 quark and 1 anti-quark) are generated by collisions between high energy protons and various nuclei. This frequently happens in the upper atmosphere of the Earth, thus generating a cascade (shower) of various subsequent particles. These particles have very short half-lives and some of which decay to muons. The following are the equations for both types of pion decay which lead to muons.

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

$$\pi^- \rightarrow \mu^- + \bar{\nu_\mu}$$

Muons have a relatively long mean half life of $2.2\mu s$ and experience significant relativistic (time dilation) effects from their near light speed velocities.

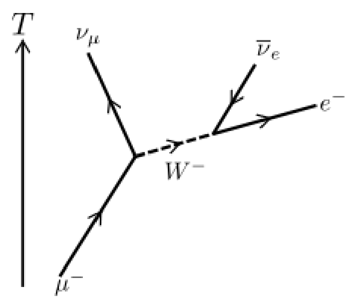

Figure 1: Most probable Feynman diagram of muon decay scheme via a W^- boson.

The following are the specific muon decay equations, both the muons (particle and anti-particle) with charge $= \pm e$.

$$\mu^- \rightarrow e^- + \bar{\nu_e} + \nu_\mu$$

$$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu_\mu}$$

Both types of muons have the mass of $105.7 MeV/c^2$ approximately $207 m_e$ (electron mass - $0.511 MeV/c^2$).

The result of these interactions and decays leads to an average muon flux at sea level of roughly $1\ cm^{-2} min^{-1}$. This varies by $cos^2\theta$ due to geometric constraints of propagation through the Earth's atmosphere. Typically by the time they have reached the surface they still have 4 GeV with a linear energy deposition rate of $2MeV/(g\ cm^{-2})$.

2 Experiment

The combination of properties of cosmic muons at sea level allows the detection of cosmic muons within a reasonably sized detector. In the $27000 cm^3$ liquid scintillator detector used in this experiment, the muons that reach the detector will occasionally decay within the detector. This results in an energetic electron which deposits most of the muon rest-mass energy promptly after the muon decay. Using the electronics as detailed in subsequent figures, we were able to select only the muon decay events with an associated electron event.

2.1 Method and Procedure

1. Calibrate by measuring muon energy spectrum from the detector and calculating expected energy deposition rate

2. Calculate muon half life using measurements based on the time difference between the 2 energetic pulses (muon and electron event)

3. Calculate electron energy spectrum by measuring the energies of only the electron events (second of 2 events)

2.1.1 Electronics, experimental diagrams

1. Liquid scintillator 30x30x30 cm (EJ-305) - eljentechnology.com

2. Integrated PMT (Photomultiplier tube)

3. 4 ORTEC Delay boxes (0-63.5ns per box)

4. ORTEC TAC (Time to amplitude converter)

5. ORTEC Easy MCA (Multiple Channel Analyser)

6. ORTEC Maestro - Computer

7. High Voltage (HV) power supply (PSU) at -1200V for detector bias voltage

8. Cosmic muons $\mu^+ \ and \ \mu^-$

9. 2 ORTEC Timing SCA (single channel analyser)

10. ORTEC Delay linear amplifier (μs scale)

11. ORTEC Linear gate stretcher

12. ORTEC Dual spectroscopy amplifier (dual spec. amp.)

13. Coaxial cables and one coaxial splitter

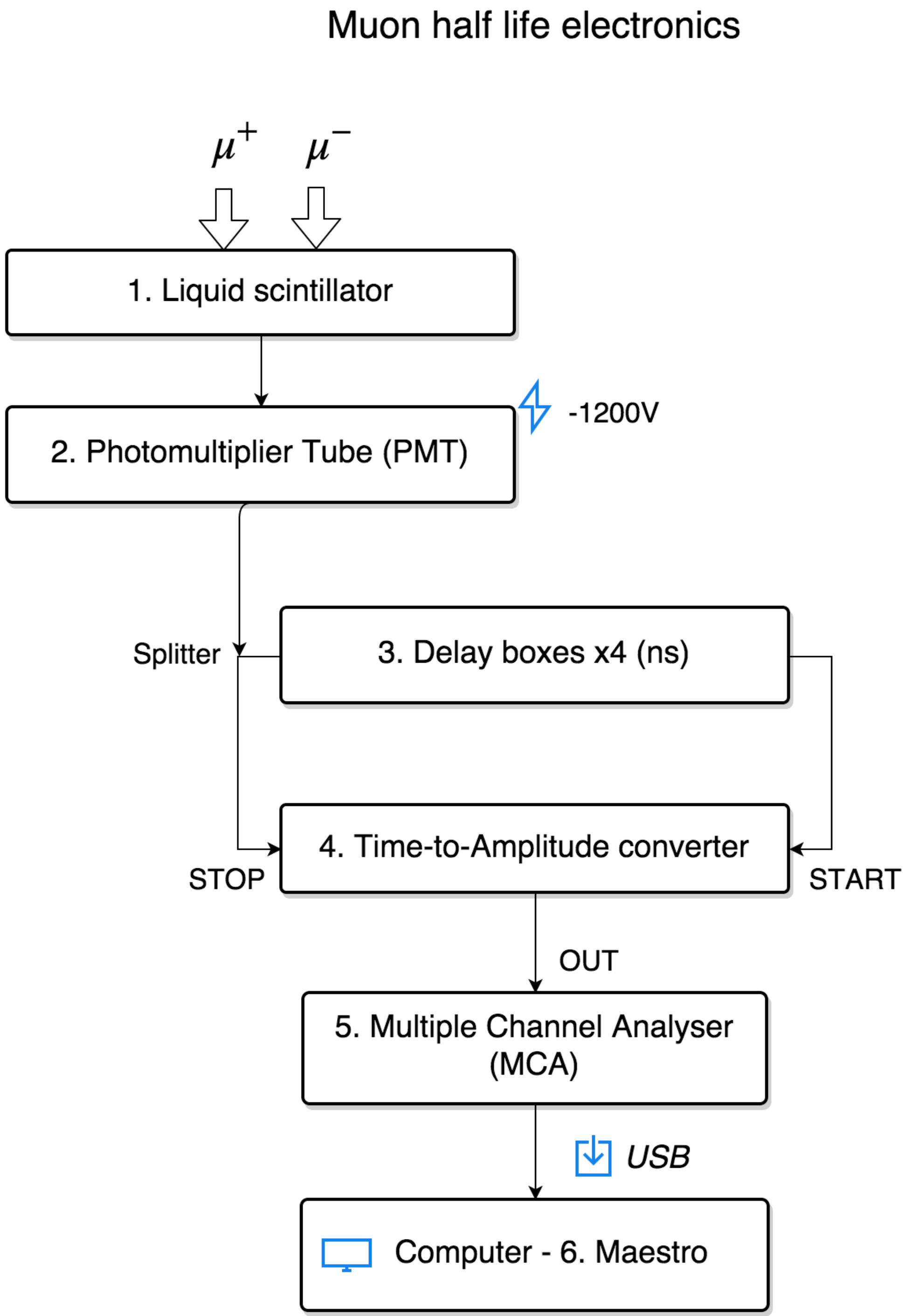

Figure 2: Experimental electronics logical overview diagram from 2.1.2. Other electronics diagrams have not been included because they are both adequately shown in the lab script.

2.2 Data

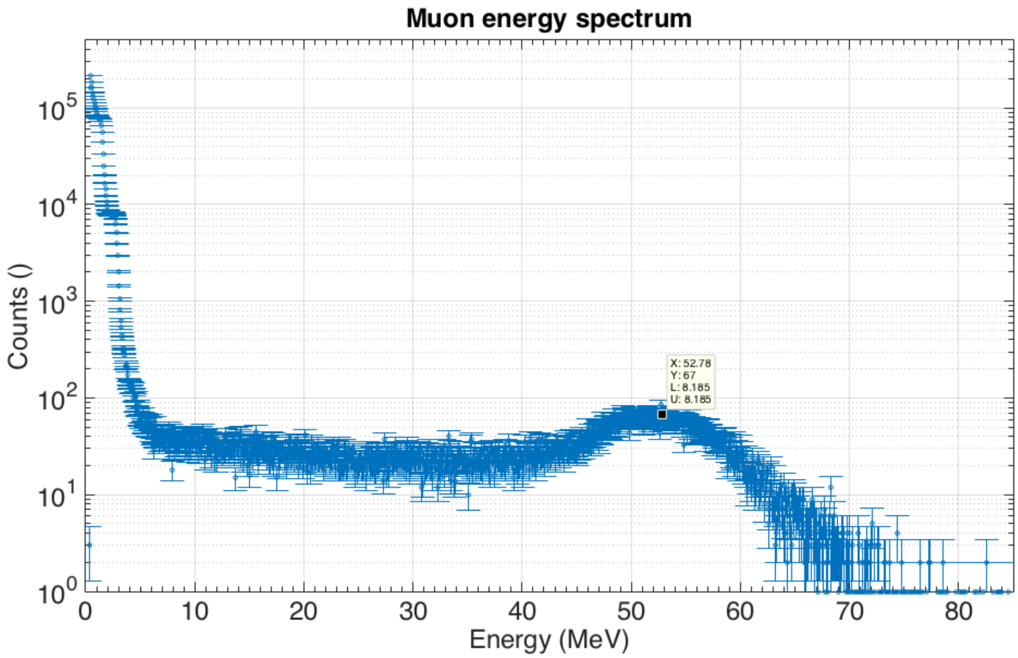

Figure 3: Initial muon energy spectrum

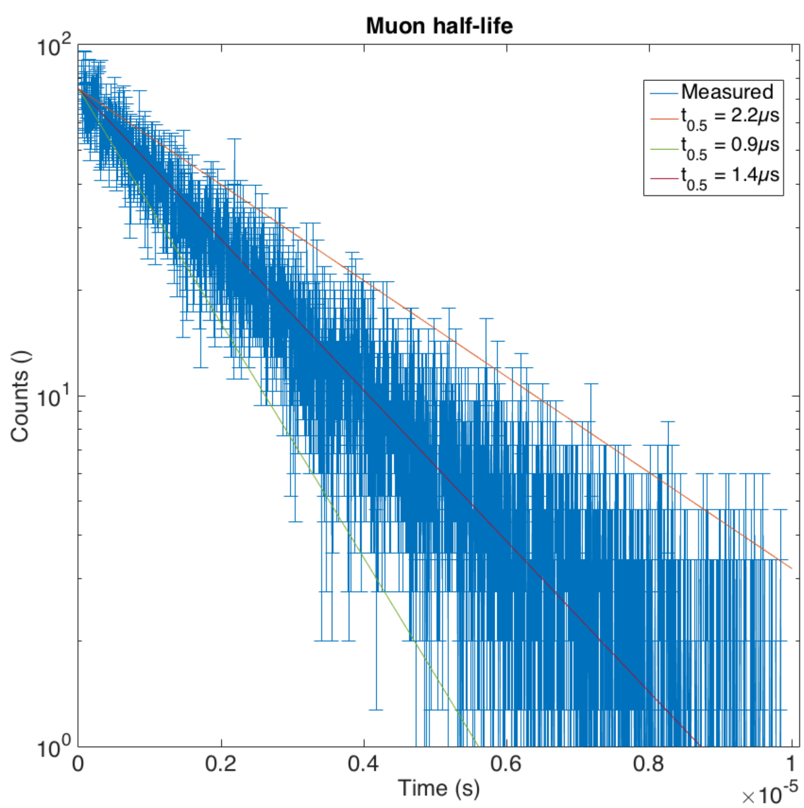

Figure 4: Muon exponential decay with 3 fitted half lives.

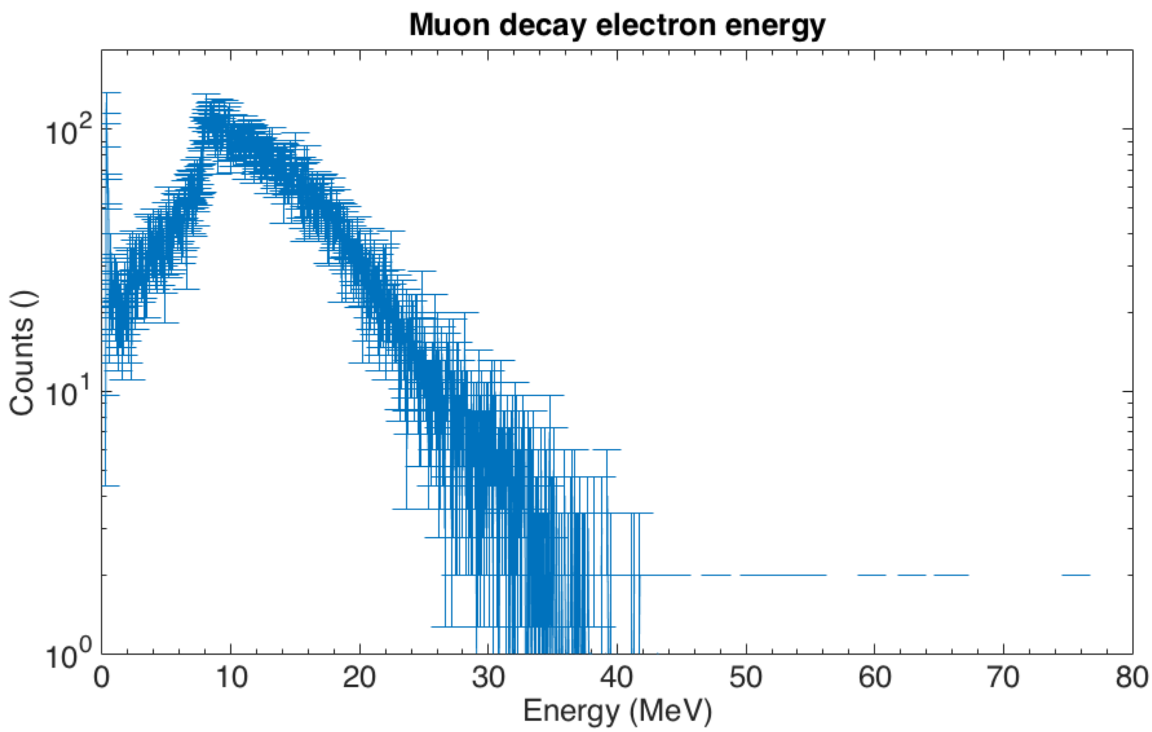

Figure 5: Electron energy spectrum exclusively from muon decay events.

2.3 Analysis and Results

Using the known muon flux rate at sea level ($\approx 1\ muon\ cm^{-2}\ min^{-1}$), we have calculated the expected count rate with angular dependence.

$$Count\ rate_{\mu} = 900cm^2 \times 1\ muon\ cm^{-2}\ min^{-1} \times cos^2(\theta)$$

With an angle range of $-\frac{\pi}{4}\ to\ \frac{\pi}{4}$ (a 90 degree cone) and a detector top cross sectional area of $900cm^2$ ($30cm \times 30cm$), the expected count rate range is

$$Count\ rate_{\mu} = 1 \times cos^2(\pm\frac{\pi}{4}) = 7.5 - 15\ counts\ second^{-1} = 450 - 900\ counts\ minute^{-1}$$

Using the energy deposited by minimally ionising radiation (average energy loss rate for singularly charged particles near light speed $= 2MeV/(g\ cm^{-2})$), we have determined the average energy deposited by a muon going through the scintillator (assuming top to bottom) with angular dependence. This also assumes the majority of the muons come from a solid angle of π (a 90 degree cone) as in the previous calculation.

$$\Delta E = 2MeV/(g\ cm^{-2}) \times pathlength \times scintillator\ density \times cos^2(\pm\frac{\pi}{4})$$

$$= 2MeV/(g\ cm^{-2}) \times (30cm\ to\ 42.4cm) \times 0.8761g\ cm^{-3}(scintillator\ material\ -\ pseudocumene)$$

$$= 52.6MeV\ to\ 74.3MeV\ (through\ the\ detector)$$

The extra distance (12.4 cm) is due to muons that could enter at a 45 degree angle, hence using Pythagoras the maximum linear muon path length within the detector is $pathlength_{max} = \sqrt{(30cm)^2 + (30cm)^2} = \sqrt{1800} = 42.4cm$. The expected muon energy range is therefore $0\ to\ 74.3MeV$

with a peak around $52.6MeV$, this range defines the electronics scale with the amplifier gains setting. The calibration that we achieved was based on the range of the amplifier along with the number of bits (11 bit - 2048 channels) that the MCA (multi channel analyser) was able to sample at.

Figure 3 shows a peak at the expected maximum muon energy of approximately $52.6MeV$. The energies above that level are due to multiple collisions, this necessarily means that the muon did not take a straight path. Theoretically multiple collisions lead to the potential for all of the muons $4GeV$ energy to be deposited. We do not see this because the probability of a collision is not sufficiently high and the detector is not large enough.

Muon half life has been measured by triggering a TAC (time to amplitude converter) based on a muon stopping and then decaying in the detector. With this sequence of events, 2 signals (normally shaped overlapping pulses) were produced per event. One was for the typical energy deposition from the muon stopping in the scintillator. The other is from the decay electron/positron carrying most of the muon rest-mass energy ($105.7MeV$). The decay electron/positron signal arrives promptly after the muon signal - almost no other events would occur between the muon and the electron/positron signals.

Figure 4 shows muon decay times against the number of counts (natural logarithm scale) - fitting exponential decay equations allows upper and lower bounds of the muon half life to be estimated. This assumes a starting count number of 75 - this value was chosen because it appears to be the most probable value from the graph. The fitted exponential equations as plotted in figure 4 were

$$Counts(t) = 75\ counts\ \times 2^{-\frac{t}{t_{0.5}}}$$

With the half life $\rightarrow t_{0.5} = 0.9,\ 1.4\ and\ 2.2\mu s$.

The energy deposition by the muon and its decay inside the liquid scintillator are significantly above the natural radioactive background at 2.6 MeV (from lab script). The TAC starts when the first (muons primarily) signal arrives and stops when the next (electron/positron primarily) signal arrives. The voltage output of the TAC is then linearly proportional to the time difference between the signals. If the allotted timing range for the TAC is set beyond the time between events then the TAC will not output a signal. This is an important use of the TAC because if there is stop a stop signal in time then no false readings will be given. This method yields an effective muon half life of $\mu\ t_{0.5} = 1.4\mu s + 0.8 - 0.5\mu s$. This value covers the well established literature value of $t_{0.5} = 2.19703\mu s \pm 0.00004\mu s$.

The muon decay electron energy spectrum has an expected minimum of $0.511MeV$ due to the rest-mass energy of electrons/positrons and an expected maximum of approximately $74.3MeV$.

Figure 5 taken over 600,000 seconds, reflects these properties mostly. There is an unaccounted peak at $0.30 \pm 0.06KeV$ which appears to be a Compton edge contribution from the $0.511MeV$ electron/positron signal. The peak electron/positron energy is mostly within $10.0 \pm 2.5\ MeV$. There is a maximum electron/positron energy in this data of $75.5MeV$ which is almost exactly the expected upper energy value of $74.3MeV$.

This maximum electron/positron energy comes from a specific kinetic condition. In this case, the decay tau and electron neutrinos would be emitted in exactly the opposite direction to the electron/positron, thus the electron/positron would have the highest energy (half of the total kinetic energy) due to the kinetics of the decay. The electron/positron then loses energy extremely quickly via bremsstrahlung radiation.

For a muon decay at rest, the maximum energy that could be imparted to the decay electron/positron is $\approx 74.3\ MeV$ as previously mentioned. More channels from the MCA and a detector with a larger cross sectional area would have improved the measurement of the decay energy spectrum.

The time dilation effect on muons is calculated by finding the gamma factor and using previous estimates of the average muon kinetic energy of 2 GeV at sea level.

$$K = (\gamma - 1)mc^2$$

$$where\ \gamma = \frac{1}{\sqrt{1 - \frac{v_\mu^2}{c^2}}}$$

$$\gamma = \frac{K + mc^2}{mc^2} = \frac{(2 + 0.1057)GeV}{0.1057GeV} = 19.9$$

$$\Delta t = \gamma \Delta t$$

$$\Delta t = 19.9 \times 2.2\mu s = 43.78\mu s$$

$$Range \rightarrow x = ct$$

$$x = 2.9989 \times 10^8 ms^{-1} \times 43.78\mu s = 13129m$$

2.4 Error Analysis

As usual, there is an associated statistical uncertainty with all data recorded here which is $\sqrt{N}$, with N being the number of counts in this document. All of the uncertainties for the x axes of all graphs are based on the 2048 channels available. They have not been plotted due to their negligible size. Background radiation sources should not have been a concern with these data due to the energy discrimination along with the timing electronics used in all stages of the experiment. Evidence of this is how there are no unaccounted peaks and the fitted lines all appear to be within statistical uncertainties with no compensation.

3 Conclusions

Our muon energy spectrum measurement shows that the the vast majority of events came from below 7 MeV, with an expected calculated peak at 52.8 MeV and a maximum muon energy of 85 MeV recorded. The muon half life measurement with the associated uncertainties confirms the expected half life of $t_{0.5} = 1.4\mu s + 0.8 - 0.5\mu s$ - known half life $2.2\mu s$. The energy spectrum from the decay electrons/positrons shows that most of the events had an energy of $10 \pm 2.5MeV$ with a maximum energy of $75.5MeV$.

Possible improvements to this experiment include a detector with a larger cross sectional surface area so that less particles enter through the side; a detector with a higher volume; longer measurement times for better statistics; more bins on the MCA and finally a scintillator with a higher linear attenuation coefficient.

4 References

Time dilation calculations in 2.3 - $http://1.usa.gov/1K2DFra$ - accessed on April 21, 2015