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CHAOTIC SYNCHRONIZATION AND NEURAL NETWORKS

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A method of developing chaotically synchronized systems when the structure of the system that generates the driving signal is unknown is proposed in this paper. This method is based on the approximation of the driving system structure by neural networks incorporated in the response system. This makes the method different from those proposed previously, which are based on the analysis of the presumedly known equations of motion of the driving system. The method is illustrated by an example of developing synchronized systems for the dynamic Lorentz system.

1. INTRODUCTION

The last decade has seen an increase in the understanding of the role played by complicated dynamic events in the processing of information. On the one hand, experimental evidence of the fundamental role of oscillations in the neural system has been reported [1-4]. In the brain, the coupled systems that have nontrivial dynamics ("phasotone" in the motor system [3], and chaotically interacting sections of the olfactory system [1, 2]) are singled out and described in sufficient detail. The authors of these studies interpret their results using the following terms: self-oscillations, attractors, response chaos, synchronization, and dynamic logic. Special attention is paid to the possible role of chaos in the data processing and data transmission among various structures. On the other hand, in the studies of modeling the dynamics of systems of coupled active elements, nontrivial modes appear that can be interpreted as pattern recognition and bring about certain analogies with the experimental evidence about how the neural system functions [4].

The phenomenon of chaotic synchronization of dynamic systems [7-9] attracts our attention as a possible mechanism of data transmission in biological systems. Chaotic synchronization manifests itself in a strong correlation between the states of dynamic systems related by a common chaotic signal, the systems' structures being correlated in a certain way. However, if we consider this phenomenon as a method of synchronization of biological oscillators, difficulties arise in connection with the fact that the transmitting and receiving systems can enter different biological structures, which can result in an uncorrelated variation in the parameters of any one of the above systems. Therefore, to insure a reliable data transfer, a mechanism of the system structure correlation is required. One of the possible solutions of this problem is to enable the receiving system to self-adjust its structure using additional information about the transmitting system.

In this paper, we propose a method of developing a synchronized system based on the neural networks that are adjusted by the state variables of the synchronizing system which are only observed at the time of learning; the system structure is assumed explicitly unknown. The choice of the neural system as the structure of the synchronized system itself is determined by the fact that the transmitters and the receivers of information in a neural system are sufficiently complicated neural networks which can "learn the dynamics" of systems and predict their behavior [12-14].

2. CHAOTIC SYNCHRONIZATION OF DYNAMIC SYSTEMS

The most well-known method of constructing chaotically synchronizing systems [7], as applied to the discrete-time systems, is as follows. Let us consider an n-dimensional autonomous dynamic system with the state variables $\mathbf{u} = (u_1, u_2, \ldots, u_n)$ and parameters μ:

$$\mathbf{u}(k+1) = \mathbf{f}(\mathbf{u}(k), \mu). \tag{1}$$

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Let us divide the system into two subsystems: the driving one with the state variables $\mathbf{u}_d = (u_1, \ldots, u_m)$, and the response one with the state variables $\mathbf{u}_r = (u_{m+1}, \ldots, u_n)$: $\mathbf{u} = (\mathbf{u}_d, \mathbf{u}_r)$. Equations (1) can then be rewritten in the form

$$\mathbf{u}_d(k+1) = \mathbf{f}_d\left(\mathbf{u}_d(k), \mathbf{u}_r(k), \mu\right), \tag{2}$$

$$\mathbf{u}_r(k+1) = \mathbf{f}_r\left(\mathbf{u}_d(k), \mathbf{u}_r(k), \mu\right). \tag{3}$$

Let us develop a "copy" of the subsystem $\mathbf{u}_r$ with the state variables $\mathbf{u}'_r$:

$$\mathbf{u}'_r(k+1) = \mathbf{f}_r\left(\mathbf{u}_d(k), \mathbf{u}'_r(k), \mu'\right). \tag{3$'$}$$

This method of constructing the response subsystem $\mathbf{u}'_r$, as well as later methods [10, 11], is based on the knowledge of dynamic equations of the system $\mathbf{u}$. Equations (2), (3), and (3$'$) determine an autonomous dynamic system that consists of three interrelated subsystems: the driving system $\mathbf{u}_d$ and two response systems, $\mathbf{u}_r$ and $\mathbf{u}'_r$. For a certain choice of breaking the system $\mathbf{u}$ into the driving and the response parts, synchronization of the subsystems $\mathbf{u}_r$ and $\mathbf{u}'_r$ is possible, i.e., a regime under which $|\mathbf{u}_r - \mathbf{u}'_r| \to 0$ for $t \to \infty$, including the case where the system $\mathbf{u}$ is in the chaotic regime, and the subsystems $\mathbf{u}_r$ and $\mathbf{u}'_r$ have different initial conditions. For a small difference in the subsystem parameters ($\mu \neq \mu'$) their states can still be correlated, whereas synchronization degrades for a large discrepancy in the parameters. In information transmission systems [5, 6] the system $\mathbf{u}$ is a basis of the information transmitter, while the system $\mathbf{u}'_r$ is one of the receiver's constituent components.

Depending on the context, the autonomous system $\mathbf{u}$ can be called master, driving, synchronizing, or transmitting, while the nonautonomous system $\mathbf{u}'_r$ can be called slave, response, synchronized, or receiving, respectively.

3. NEURAL NETWORKS USED FOR MODELING DYNAMIC SYSTEMS

Neural networks are a promising way of constructing implicit models of dynamic systems with an unknown structure [12-14] for which all the information is limited by the time sequences of the quantities observed (realizations) $\{\hat{\mathbf{u}}(k)\}_{k=1}^{N}$ (N is the realization length). For the data available, the nonlinear self-regression model has the form

$$\hat{\mathbf{u}}(k+1) = \mathbf{h}\left(\hat{\mathbf{u}}(k), \eta\right) + \xi(k), \tag{4}$$

where $\mathbf{h}(\mathbf{u}, \eta) = (h_1(\mathbf{u}, \eta), \ldots, h_n(\mathbf{u}, \eta))$, h_i is the "predictor" of the component $\hat{u}_i$ of the vector $\hat{\mathbf{u}}$, and ξ is an "error" that characterizes the model imperfection.

The values of the adjusted parameters η are determined from the condition of minimization of the normalized root-mean-square prediction errors

$$\mathbf{err}_i = \frac{1}{N\sigma_i^2} \sum_{k=1}^{N} |\xi_i(k)|^2 \tag{5}$$

of the individual realization components $\{\hat{u}_i(k)\}_{k=1}^{N}$ (σ_i^2 is the dispersion of the corresponding component).

In the case of a small residual error (5), we think it is reasonable to consider a hypothesis of the determinate generation of the data available by the dynamic system (1) and to rely on the fact that the predictors $\mathbf{h}$ reflect the structure of this system. However, this gives no reason for interpreting the predictors $h_i(\mathbf{u}, \eta)$ as approximations of the corresponding functions $f_i(\mathbf{u}, \mu)$. An option is possible when the adjusted predictors only reflect the specific properties of the realization available, and the prediction error will be significant for the independent control realizations from the same system. Another case is the modeling of strongly dissipative systems in realizations from which there is no information about the transition processes, and the predictors obtained can predict correctly only the behavior at the attractor.

The multi-step prediction is realized by a sequential iteration using the single-step predictors $\mathbf{h}$:

$$\mathbf{u}(k+1) = \mathbf{h}\left(\mathbf{u}(k), \eta\right). \tag{6}$$

Equations (6) determine an autonomous dynamic modeling system with the state variables $\mathbf{u}$. The system properties the reflect the characteristics of the initial system (1) manifested in the realization.

The neural-network predictor used in this paper is determined as follows:

$$h_i(\mathbf{u}, W, \beta) = \beta^{(out)} + \sum_{j=1}^{N^{(h_2)}} W_j^{(out)} g \left(\beta_j^{(h_2)} + \sum_{k=1}^{N^{(h_1)}} W_{jk}^{(h_2)} \times \right.$$

$$\left. \times \; g \left(\beta_k^{(h_1)} + \sum_{l=1}^{N^{(in)}} W_{kl}^{(h_1)} u_l \right) \right), \tag{7}$$

where

$$g(x) = \frac{1}{1+\exp(-x)} - \frac{1}{2}.$$

The structure of Eq. (7) can be understood based on the interpretation of a neural network as a set of several layers of connected active elements (neurons), each of which can perform the nonlinear transformation from the weighted and biased sum of activities of the neurons of the previous layer. The adjusted parameters of the neural-network predictor are the values of the link thresholds β and weights W.

4. DEVELOPMENT OF CHAOTICALLY SYNCHRONIZING SYSTEMS BASED ON NEURAL NETWORKS

The problem of parameter correlation in synchronized systems can be considered in a more general statement. Let it be necessary to develop a response nonautonomous system that can synchronize chaotically with a driving system whose structure is unknown. In the developing process, both the driving signal $\mathbf{u}_d$ and the synchronized variables $\mathbf{u}_r$ of the driving system are observed. If the observed variables $\mathbf{u} = (\mathbf{u}_d, \mathbf{u}_r)$ provide a one-valued definition of the driving system state, the following modeling system can be developed for it:

$$\mathbf{u}_d(k+1) = \mathbf{h}_d \left(\mathbf{u}_d(k), \, \mathbf{u}_r(k), \, \eta \right), \tag{8}$$

$$\mathbf{u}_r(k+1) = \mathbf{h}_r \left(\mathbf{u}_d(k), \, \mathbf{u}_r(k), \, \eta \right), \tag{9}$$

where $\mathbf{h}_d$ and $\mathbf{h}_r$ are the predictors adjusted by the observed minimizations of the prediction error (5).

If the predictors are correct approximations of the real mappings which determine the driving system, the system

$$\mathbf{u}'_r(k+1) = \mathbf{h}_r \left(\mathbf{u}_d(k), \mathbf{u}'_r(k), \eta \right) \tag{9'}$$

will possess the same synchronizability properties as (3′) and, therefore, will be the desired nonautonomous driving dynamic system. However, if the predictors $\mathbf{h}_r$ are adjusted to predict the behavior only at the attractor, the possibilities of system (9′) to establish, maintain, and recover synchronization are not obvious beforehand.

The proposed approach was tested using as an example the development of systems synchronized by a signal from the dynamic Lorentz system ($\sigma = 10$, $\rho = 60$, $\beta = \frac{8}{3}$) working in a chaotic regime. The neural-network predictors (7): ($N^{(in)} = 3$, $N^{(h_2)} = 10$, and $N^{(h_1)} = 12$) $h_x(x, y, z)$, $h_y(x, y, z)$, and $h_z(x, y, z)$ were adjusted by the realization of the variables x, y, and z from an attractor; the realization included 3000 values with a time step 0.01. Minimization of the average prediction error (5) was performed by the quasi-Newtonian method, and the residual error was below 10^{-4} for each component. The independent control realization $\{(x(k), y(k), z(k))\}_{k=1}^{10000}$ was also obtained from the Lorentz attractor.

The three nonautonomous systems with driving variables (inputs) x, y, z and response variables (y', z'), (x', z'), (x', y'), respectively, are determined by the equations

$$y'(k+1) = h_y\big(x(k), \, y'(k), \, z'(k)\big); \quad z'(k+1) = h_y\big(x(k), \, y'(k), \, z'(k)\big) \tag{10}$$

$$x'(k+1) = h_y\big(x'(k), \, y(k), \, z'(k)\big); \quad z'(k+1) = h_y\big(x'(k), \, y(k), \, z'(k)\big) \tag{11}$$

$$x'(k+1) = h_y\big(x'(k), \, y'(k), \, z(k)\big); \quad y'(k+1) = h_y\big(x'(k), \, y'(k), \, z(k)\big). \tag{12}$$

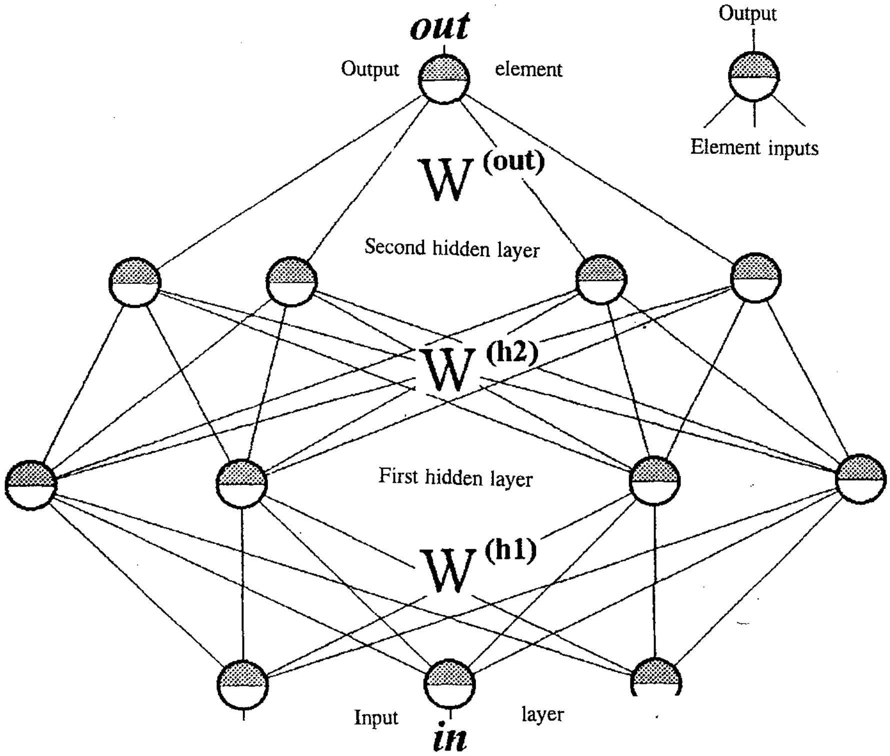

Fig. 1. Schematic representation of a neural network, using (7) as the predictor.

TABLE 1. Correlation Coefficients between the Synchronized Variables (original driving system)

	$\rho(x, x')$	$\rho(y, y')$	$\rho(z, z')$
One-step prediction	1.000	1.000	1.000
Driving variable x	0.999	0.914	0.901
Driving variable y	1.000	1.000	0.996
Driving variable z	–0.056	–0.053	0.988

The behavior of each of the above systems was studied in the regime of input coupling with the driving system. Table 1 incorporates the obtained values of the correlation coefficients ρ between the synchronized variables of the driving and response systems. Based on these values, one can conclude that synchronization is achieved using x or y ($\rho > 0.9$) as the driving variable, while it is not achieved when z ($\rho < 0.06$) is used. Therefore, the neural-network systems developed could be synchronized with the chaotic dynamic Lorentz system, using the same driving signals as the systems obtained by "copying" [7].

This conclusion is also valid when an autonomous neural-network modeling system (Table 2) is used as a driving system, though the correlations are less perfect in this case. The trajectories that correspond to the steady-state regimes for different methods of coupling between the driving and response systems are shown in Fig. 2.

It is worth noting that the trajectory images in the phase space that correspond to the presence or absence of synchronization (Fig. 2f and 2i) have few visual discrepancies.

TABLE 2. Correlation Coefficients between the Synchronized Variables (modeling driving system)

	$\rho(x, x')$	$\rho(y, y')$	$\rho(z, z')$
One-step prediction	1.000	1.000	1.000
Driving variable x	0.998	0.855	0.896
Driving variable y	0.999	1.000	0.995
Driving variable z	–0.285	–0.282	0.991

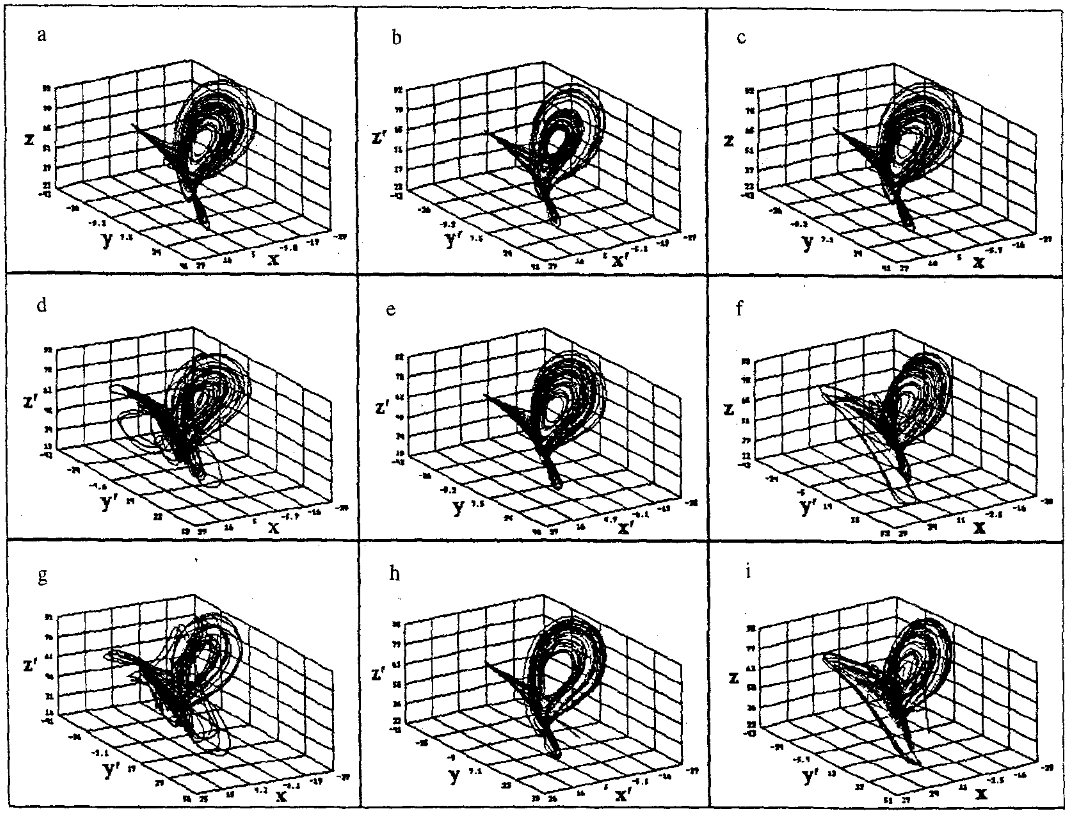

Fig. 2. Trajectories and phase space.

5. CONCLUSION

One of the difficulties that arises in the analysis of the natural realizability of chaotic synchronization as a method of information transmission in biological systems is the necessity of an additional system maintaining the correspondence of the parameters of dynamic systems that exchange information. When the proposed method of developing the synchronized response system is used, this problem is solved owing to the fact that the receiving system has a neural-network structure that can be adjusted to the transmitting system structure. The mechanism that provides the information transmission reliability with the help of chaotic synchronization is as follows. When the receiving system finds out the nonreliability of the transmission channel (for example, by the meaning of the information received), it attempts to acquire additional data about the transmitting system in the form of current values of the synchronized variables. If the information which reflects the "self-interpretation" of the driving signal by the transmitting system is obtained, the receiving system recovers the transmitting system structure (predictor developing stage) and then decides whether synchronization by the current driving variable is possible (the stage of development from the synchronized system predictors).

After this paper was ready for publication, the author learned about a paper [15], dedicated to the derivation of ordinary differential equations by the chaotic realizations from a dynamic system with unknown structure. The chaotic synchronization with initial realizations was used as a nontrivial test of the modeling system developed. Of special interest are the suggestions set forth in the conclusion of this paper about the use of the chaotic synchronization phenomenon in medical and technical diagnostics.

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