4.8.3

QDA, Normal Dist

$p = 1$ K classes

$X \sim N(\mu_K, \sigma_K^2)$

$$f_K(x) = \frac{1}{\sqrt{2\pi}\,\sigma_K} \exp\left(\frac{-1}{2\sigma_K^2}(x-\mu_K)^2\right)$$

We know that $p_K(x) = \dfrac{\pi_K f_K(x)}{\sum_{l=1}^{K} \pi_l f_l(x)}$

$k^* \rightarrow$ max numerator of $p_k(x)$

$$= \operatorname{argmax}\left\{ \log \pi_k - \left(\log\left(\sqrt{2\pi}\,\sigma_k\right)\right) - \frac{x^2}{2\sigma_k^2} + \frac{\mu_k \cdot x}{\sigma_k^2} - \frac{\mu_k^2}{2\sigma_k^2} \right.$$

Since σ_k depends on k, cannot ignore this term like we did in LDA where it was a constant

Above argmax is a quadratic function of x_k.

4.8.7 $1 \to$ dividends, $0 \to$ no dividend

$\bar{X}_1 = 10$; $\bar{X}_0 = 0$

$\hat{\sigma}^2 = 36$; Prior Prob $\pi_1 = 0.8$

Need to calculate $P(Y = 1 \text{ or } 0 \mid X = 4)$

$$P(Y=k \mid X=4) \propto \pi_k \cdot f_k(x)$$

$$0.8 \cdot \cancel{\frac{1}{\sqrt{2\pi}\cdot 6}} \exp\left(-\frac{(4-10)^2}{2\cdot 36}\right) \lesseqgtr 0.2 \cancel{\frac{1}{\sqrt{2\pi}\cdot 6}} \exp\left(\frac{-4^2}{2\cdot 36}\right)$$

$$= 0.8 \cdot \frac{1}{e^{0.5}} \lesseqgtr 0.2 \cdot \frac{1}{e^{0.22}}$$

$$\Rightarrow \underbrace{0.8 \times 0.60}_{\text{higher.}} \lesseqgtr 0.2 \times 0.8025$$

Class : Dividend = Yes

To calc probability.

$$P(Y=1 \mid X=4) = \frac{\pi_1 f_1(x)}{\pi_1 f_1(x) + \pi_0 f_0(x)}$$

$$\frac{0.8 \cancel{\times \frac{1}{\sqrt{2\pi} \cdot 6}} \cdot \exp\left(\frac{-(4-10)^2}{2 \times 36}\right)}{0.8 \cancel{\times \frac{1}{\sqrt{2\pi} \cdot 6}} \exp\left(\frac{-(4-10)^2}{2 \cdot 36}\right) + 0.2 \cancel{\times \frac{1}{\sqrt{2\pi} \cdot 6}} \exp\left(\frac{-(4-0)^2}{2 \cdot 36}\right)}$$

$$= \frac{0.8\, e^{-0.5}}{0.8 \cdot e^{-0.5} + 0.2\, e^{-0.22}} = \frac{0.8 \times 0.606}{(0.8 \times 0.606) + (0.2 \times 8025)}$$

$$= \frac{0.48}{0.48 + 0.1605} = \underline{\underline{74\%}}$$

74% prob that company will issue dividend.