

### Ratio and Proportion

Ratio: comparison of 2 individuals  $\rightarrow A:B$

Proportion: comparison of 2 ratios  $\rightarrow \underline{A:B::C:D}$

Q

1. If  $A:B = 1:2$ ,  $B:C = 7:9$ ,  $A:C = ?$

$\Rightarrow$  Solu<sup>n</sup>: A B C

$$\begin{array}{ccccccc} & & & \nearrow 2 & & & \\ & & & \times & & & \\ 1 & : & 2 & : & 9 & & \\ \downarrow & \times & \downarrow & & \downarrow & & \\ 7 & : & 7 & : & 9 & & \\ \swarrow & & & & & & \\ 7 & : & 14 & : & 18 & & \end{array} \quad A:C = 7:18$$

2. If  $A:B = 1:2$ ,  $B:C = 3:4$ ,  $C:D = 5:7$ ,  $A:D = ?$

$\Rightarrow$  Solu<sup>n</sup>:

$$\begin{array}{ccccccc} & A & B & C & D & & \\ & \swarrow 2 & \searrow 2 & \swarrow 2 & & & \\ 1 & : & 2 & : & 2 & : & 2 \\ \times & \swarrow & \times & \searrow & \times & \searrow & \\ 3 & : & 3 & : & 4 & : & 4 \\ \times & \swarrow & \times & \searrow & \times & \searrow & \\ 5 & : & 5 & : & 5 & : & 7 \\ \times & \swarrow & \times & \searrow & \times & \searrow & \\ 15 & : & 30 & : & 40 & : & 56 \end{array} \quad A:D = 15:56$$

\*\*\* 3. Two numbers are in ratio of 2:5 & their LCM is 180. The smallest number is.

$\Rightarrow$  Solu<sup>n</sup>:

Method 1:

$$\text{LCM of } 2, 5 = 20$$

$$\text{LCM of } 2x, 5x = 20x$$

$$\text{given LCM} = 180.$$

$$20x = 180$$

$$x = 9$$

$$2 \overline{) 180} = 20$$

$$x \overline{) 20x} = 20x$$

$$\therefore \text{smallest number} = 2x = 2 \times 9 = 36$$

Method 2:

$$180 \& 4:5$$

$$\frac{180}{4} = 45, \frac{180}{5} = 36 \quad \text{smallest}$$

4. Three numbers are in the ratio of 1:2:3  
& their HCF is 12. find the numbers.

$\Rightarrow$  Soln:

ratio 1 : 2 : 3

HCF = 12  $\uparrow$   $\uparrow$   $\uparrow$

12 : 24 : 36 numbers are 12, 24, 36

5. What must be added to each term of the ratio 7:11,  
so as to make it equal to 3:4?

$\Rightarrow$  Soln:

$$\frac{7+x}{9+x} = \frac{3}{4} \Rightarrow 28+4x = 33+3x$$

$$x = 33 - 28 = 5$$

$$x = 5$$

6. calculate

a) Mean proportion of 16 & 25

b) Third proportion to 7, 21

$\Rightarrow$  Soln:

a) Mean proportion =  $\sqrt{ab}$

$$= \sqrt{16 \times 25}$$

$$= 4 \times 5 = 20$$

b) third proportion =  $\frac{b^2}{a} = \frac{21^2}{7} = \frac{441}{7} = 63$

Not Done

c) Fourth proportion of 7, 14, 21, —

7 : 14 : 21 : x  
 $\therefore$

$$x = \frac{21 \times 21}{7} = 42 \quad \boxed{x = 42}$$

\*\*\* 7) The income of A and B are in the ratio of 3:5 and their expenditure are in the ratio of 5:3. If each saves RS 1000, then the income of A is ?

$\Rightarrow$  Soln:

$$\frac{\text{Income} - \text{Savings}}{(\text{Salary})} = \frac{\text{Expenditure}}{(\text{Spending})}$$

$$\begin{array}{rcl} A & \frac{3x - 1000}{2x - 1000} & \cancel{x} \quad \frac{5}{3} \\ B & \end{array}$$

$$3(3x - 1000) = 5(2x - 1000)$$

$$9x - 3000 = 10x - 5000$$

$$5000 - 3000 = x$$

$$2000 = x$$

$$\text{Income of } A = 3x = 3 \times 2000$$

$$\boxed{\text{Income of } A = 6000}$$

8) The present age of Dolu & Bala are in the ratio of 7:8 respectively. Four years hence this ratio will become 9:10. Present age of Dolu is — ?

A) 22 years B) 14 years C) 23 years D) 39 years

$\Rightarrow$  Soln:

Method 1: Present Hence Future

$$\begin{array}{llll} \text{Dolu} & 7x & +4 & 9 \\ 8x & 8x & +9 & 10 \end{array}$$

$$\frac{7x+4}{8x+4} = \frac{9}{10} \Rightarrow 70x + 40 = 72x + 36$$

$$2x = 4$$

$$7x = 7 \times 2 = 14$$

$$\boxed{x = 2}$$

Present age of Dolu is 14.

Method 2: see the options & which is divisible by age ratio (7) and that is the answer (14).

\*\*\* 9) A bag contains an equal number of 1T, 50 paisa & 25 paisa coins if the total value is RS 35, how many coins of each type are there?

$\Rightarrow$  coins are 1T, 50 paisa & 25 paisa  
total amount 35T

$$1x + 0.5x + 0.25x = 35 \text{ method}$$

$$1x + \frac{1}{2}x + \frac{1}{4}x = 35$$

$$\frac{7x}{4} = 35$$

$$7x = 140$$

$$\underline{\underline{x = 20}}$$

one more method for easy from my side -

1 coin is 0.25 so to round up multiply by 4

$$1 : 0.5 : 0.25 \times 4 = 35$$

$$x : 4 : 2 : 1 = 140$$

$$4x + 2x + 1x = 140$$

$$7x = 140$$

$$x = 140/7 = \underline{\underline{20}}$$

\*\*\* 10) In a college parking space there are bikes & cars.

Total vehicles = 175, Total wheels = 580

$$\Rightarrow 2x + 4y = 580$$

$$2x + 2y = 175 - 350$$

$$2x + 4y = 580$$

$$2x + y = 175$$

$$x + 3x = 2x + 5$$

$$2y = 170$$

$$14 = 85$$

$$x = 175 - 85$$

$$\boxed{x = 90}$$

simple method

$$\Rightarrow y = 175 - x$$

$$\Rightarrow 2x + 4(175 - x) = 580$$

$$\Rightarrow 2x + 700 - 4x = 580$$

$$2x = 180$$

$$x = 90 = \text{Bike}$$

$$\frac{85 \times 4 - 180}{580} = 180$$

$$x = 90 = \text{Bike}$$

$$\frac{180}{580} = 180$$

$$\begin{array}{r} 18 \\ 85 \cdot 90 \\ \hline 17 \cdot 18 = 85 \cdot 18 \end{array}$$

$$\text{Bike} = 90$$

$$\text{Car} = 85$$

$$y = 175 - 90$$

$$y = 85 = \text{car}$$

11) The sides of a  $\Delta$  are in the ratio of  $1/2 : 1/3 : 1/4$  & its perimeter is 104 cm. Then length of longest side is - ? LCM of  $2:3:4 = 12$

$$= \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = 104$$

$6:4:3$

$$\frac{6x+4x+3x}{12} = 104$$

$13x$

$\cancel{12}$

$\cancel{x}$

$\cancel{x}$

$$6x + 4x + 3x = 104$$

$$13x = 104$$

$$x = \frac{104}{13} = 8$$

$$6x = 48 \text{ cm}$$

$$\begin{array}{r} 6x+4x+3x \\ \hline 12x \\ \hline 104 \end{array}$$

$\cancel{12}$

$\cancel{x}$

$\cancel{x}$

2  
1

\*\*\*

12) Income of  $x$  and  $y$  are in the ratio of  $4:3$ . Their expenditures are in the ratio of  $12:7$ . Both saves 3200₹, income of  $x = ?$

$\Rightarrow$

$$\Rightarrow \text{Income} - \text{Saves} = \text{Expenditure}$$

$$\frac{x}{y} = \frac{4x - 3200}{3x - 3200} = \frac{12}{7}$$

$$7(4x - 3200) = 12(3x - 3200)$$

$$28x - 22400 = 36x - 38400$$

$$8x = 16000$$

$$x = 2000$$

Balances of  $x = 4x$

$$= 4 \times 2000$$

$$\boxed{\text{Income of } x = 8000}$$

$$\begin{array}{r} 32x \\ \hline 64 \\ 32 \\ \hline 384 \\ 224 \\ \hline 160 \end{array}$$

2)

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### Time and Work

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Type 1: Based on chain rule

$$\text{with work} \quad P_1 H_1 D_1 = W_1$$

$$P_2 H_2 D_2 = W_2$$

P - Person

H - Hours

D - Days

$$\text{without work} \quad P_1 H_1 D_1 = P_2 H_2 D_2$$

a

1) 15 men can type 3240 pages in 6 days working thus a day. How many men needed to type 5400 page working thus a day for 3 day.

⇒

$$P_1 = 15 \quad H_1 = 2 \quad D_1 = 6 \quad W_1 = 3240$$

$$P_2 = \quad H_2 = 4 \quad D_2 = 3 \quad W_2 = 5400$$

$$\frac{15 \times 2 \times 6}{x \times 4 \times 3} = \frac{3240}{5400}$$

$$\frac{135 \times 12}{162} = \frac{135}{135} \\ 81 \sqrt{2025} \quad 162 \\ 162 \\ 405 \\ 405 \\ 0$$

$$x = \frac{2025}{81} = 25$$

2) If 72 man can build a wall 287 m length in 21 days how many man needed for building 100 meter wall in 18 days.

$$\Rightarrow P_1 = 72, \quad W_1 = 287, \quad D_1 = 21$$

$$P_2 = \quad W_2 = 100, \quad D_2 = 18$$

$$\frac{72 \times 21}{x \times 18} = 287 \Rightarrow 287 P_2 = 8900$$

$$\frac{72 \times 21}{6} = 100 \quad P_2 = 8900 \quad \frac{287 \times 3}{287} = 861$$

$$287 \sqrt{8900}$$

$$\frac{43}{10} \times \frac{3}{5} = 12.9$$

3. 39 persons can repair their road in 12 days working 5 hrs per day. In how many days 30 persons working 6 hrs per day to complete work.

$$P_1 = 39 \quad D_1 = 12 \quad h_1 = 5 \\ P_2 = 30 \quad D_2 = ? \quad h_2 = 6 \\ 39 \times 12 \times 5 = 30 \times 6 \times D_2 \\ D_2 = \frac{39 \times 5}{30 \times 6} \\ D_2 = \frac{39}{36}$$

$$\underline{\underline{D_2 = 13}}$$

4. 15 labours to complete work in 10 days working 6 hrs per day if 18 labours to complete work in 5 days how many hours work?

$$P_1 = 15, \quad D_1 = 10, \quad h_1 = 6 \\ P_2 = 18, \quad D_2 = 5, \quad h_2 = ? \\ 15 \times 10 \times 6 = 18 \times 5 \times h_2 \\ h_2 = \frac{15 \times 10 \times 6}{18 \times 5}$$

$$\underline{\underline{h_2 = 10}}$$

\*\*\* type 2 :- Pipes & Cisterns

Q 1. A pipe A & B can fill a tank 5 hrs & 4 hrs respectively if both pipes are opening on alternate hours, pipe A is open first hours tank is full

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### Method 1

$$A = 5 \text{ hr} \quad B = 4 \text{ hr}$$

$$\text{tank capacity} = \text{LCM of } A \text{ & } B = 20 \text{ units}$$

$$\frac{20}{4} = A = 5 \text{ hr} \quad |$$

$$\frac{20}{5} = B = 4 \text{ hr} \quad |$$

A will fill after  
in hours

B will fill 5 hr  
in hours.

SO	A	2	30 min	
	B	5	hrs 4	
	A	4	hrs 3	
	B	5	hrs 2	
	A	4	hrs 1	

} 18 SO Ans 4 hrs

1 hrs 30 min

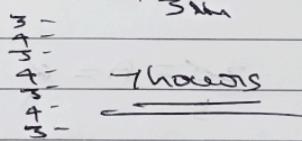
Q2. 2 pipe A & B, 6 hrs & 8 hrs respectively pipe B is opened first for alternate hours.

$$\Rightarrow A = 6 \text{ hr} \quad B = 8 \text{ hr.}$$

$$\text{tank capacity} = 24 \text{ lhr.}$$

$$\frac{24}{4} = A = 6 \quad |$$

$$\frac{24}{3} = B = 8 \quad |$$



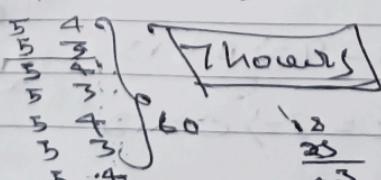
Q3. 3 pipe A, B, C need 12, 15, 20 hrs respectively  
pipe A is opening on all the time B & C  
opening on alternate hours pipe B opening  
first how many hrs need?

$$\text{LCM} = 60 = \text{tank capacity}$$

$$\frac{60}{5} = A = 12, \quad \frac{60}{3} = B = 15, \quad \frac{60}{4} = C = 20$$

$$1 = 8$$

$$\frac{960}{8} = 8$$



$$2 \left| 12 \quad 15 \quad 20 \right.$$

$$5 \left| 6 \quad 15 \quad 10 \right.$$

$$2 \left| 6 \quad 3 \quad 2 \right.$$

$$3 \left| 3 \quad 3 \quad 1 \right.$$

$$1 \quad 1 \quad 1$$

$$5 \quad 7$$

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Type 3: Fractions and work

\*  $\frac{2}{3}$  work completed by A in 10 days

$$A = \frac{10 \times 3}{2} = 15 //$$

$$\Rightarrow \frac{2}{3} A = 10 \Rightarrow A = \frac{10 \times 3}{2} = 15 //$$

\* A completes a work in 20 days  $\frac{2}{5}$  work

$$\Rightarrow 1A = 20 \text{ days}$$

$$\frac{2}{5} A =$$

$$\frac{2 \times 20}{5} = 8 //$$

Q3: P  
40%  
work  
 $\frac{1}{2}$   
7  
8  
 $\Rightarrow$

Q1: A can complete  $\frac{2}{3}$  of work in 4 day & B can complete  $\frac{3}{5}$  of work in 6

$$\frac{2}{3} A = 4 \Rightarrow A = \frac{2 \times 3}{2} = 6 //$$

$$\frac{3}{5} B = 6 \Rightarrow B = \frac{3 \times 5}{3} = 10 //$$

$$\Rightarrow \frac{30}{5} = A = 6 \quad \frac{30}{3} = B = 10$$

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Q2: A can do one bgs  $\frac{1}{2}$  of work in 5 day  
B can do  $\frac{3}{5}$  of work in a days  
C can do  $\frac{2}{3}$  of work in 8 day all together how many days,

$$\Rightarrow \frac{1}{2} A = 5 \Rightarrow A = 10 \quad \text{LCM} = 60.$$

$$\frac{3}{5} B = 9 \Rightarrow B = 15$$

$$\frac{2}{3} C = 8 \Rightarrow C = 12$$

$$\begin{array}{rcl} 60 = A = 10 & & 4 \text{ days} \\ 60 = B = 15 & & \\ 60 = C = 12 & & \\ 15 \times 4 = 60 & & \end{array}$$

Q1:  $\frac{1}{2}$  in

A

$\Rightarrow$

days

$$\text{Method} = A=10 \quad B=15 \quad C=12$$

$$\frac{1}{\text{day work}} = \frac{1}{10} + \frac{1}{15} + \frac{1}{12} = \frac{6+4+5}{60} = \frac{15}{60} = \frac{1}{4}$$

4 days

work

Q3: P can complete work  $\frac{1}{4}$  in 10 days, Q can complete 40% work in 15 days, R can complete the work  $\frac{1}{3}$  in 13 days, S can complete work  $\frac{1}{6}$  in 7 days who will complete the work first.

$$\Rightarrow \frac{1}{4}P = 10 \Rightarrow P = 40$$

B can

$$\frac{3}{5}Q = 15 \quad Q = 37.5 \quad 40\% = \frac{3}{5}$$

$$\frac{1}{3}R = 15 \Rightarrow R = 39$$

$$\frac{1}{6}S = 7 \Rightarrow S = 42$$

#### Type 4: Work and Wages

Step 1: Find one day work

Step 2: Find ratio

Step 3: Find share

days

Q1: A can do a work in 3 days, B can do work in 2 days, Both of them finish the work together to get ₹150 what is share amount of A?

$$\Rightarrow \frac{A+B}{A+B} = \frac{1}{3} + \frac{1}{2} = \frac{5+6}{6} = \frac{11}{6} \rightarrow \text{one day work}$$

$$A:B = \frac{1}{3} : \frac{1}{2} = \frac{2:3}{6:6} = 2:3 \text{ ratio. } \checkmark$$

$$\frac{2}{5} \times 150 = 60 \text{ ₹ share of A } \checkmark$$

Q2. A and B can do the work for ₹ 9500  
 A alone can do the work in 8 days, B alone  
 can do the work in 12 days, with the help  
 of C they can do the work in 4 days then  
 what is the share amount of C.

$$\Rightarrow A = 8 \text{ days}, B = 12 \text{ days} \Rightarrow C =$$

$$\text{one day work} = \frac{1}{8} + \frac{1}{12} + \frac{1}{C} = \frac{1}{4} \Rightarrow \frac{1}{C} = \frac{1}{12}$$

$$\frac{1}{C} = \frac{1}{4} - \frac{1}{8} = \frac{1}{12} \Rightarrow \frac{6-3-2}{24} = \frac{1}{24}$$

$$= \frac{1}{8} : \frac{1}{12} : \frac{1}{24} = \boxed{3:2:1} \text{ ratio.}$$

$$\boxed{3+2+1=6} \quad \frac{1}{6} \times 9500 = \underline{\underline{750}} \text{ ₹}$$

Q3. 3 persons can do the work for ₹ 1200. 1st  
 person can do the work in 8 days, 2nd person  
 in 12 days, 3rd person in 16 days, they  
 can do the work with the help of 4th person  
 in 3 days what is the share amount of 4th  
 person.

$$\Rightarrow A = 8, B = 12, C = 16, D = ?$$

$$= \frac{1}{8} + \frac{1}{12} + \frac{1}{16} + D = \frac{1}{3}$$

$$D = \frac{1}{3} - \frac{1}{8} - \frac{1}{12} - \frac{1}{16}$$

$$D = \frac{48 - 6 - 4 - 3}{48} = \frac{31}{48}$$

$$\text{Ratio} = \frac{1}{8} : \frac{1}{12} : \frac{1}{16} : \frac{1}{48}$$

$$\begin{array}{r} 6+4+3+3=16 \\ \hline 23 \end{array} \quad \begin{array}{r} 6:4:3:3 \\ \hline 848 \end{array} \quad \begin{array}{r} 3 \times 1200 \Rightarrow 225 \\ \hline 16 \end{array}$$

$$\begin{array}{r} 1200 \\ \hline 48 \\ \hline 240 \\ \hline 240 \\ \hline 0 \end{array} \quad \begin{array}{r} 48 \times 23 \\ \hline 96 \\ \hline 96 \\ \hline 0 \end{array} \quad \begin{array}{r} 225 \\ \hline 1200 \end{array}$$

\*\*\* Type 5: Based on Efficiency

\* A is  $\alpha$  times efficient than B

assumption - A =  $x$  & B =  $\alpha x$

if A can do work in  $x$  days then B need  $\alpha x$  days

\* A is 50% more than efficient than B

assumption - A is 100% & B is 150%

$10 \text{ day}$        $15 \text{ day}$

\* A takes  $50\%$  <sup>more</sup> time than B

assumption :- if B takes  $x$  time then A takes

$\frac{3}{2}x$  or  $1.5x$  time,

Q1 A is twice as B, B is twice as C, A and B together complete the work in  $x$  days, C alone complete the work ?

$$\Rightarrow A = x \text{ days}, B = 2x \text{ days}, C = 4x \text{ days.}$$

$$= \frac{1}{x} + \frac{1}{2x} = \frac{1}{4}$$

$$\frac{3}{2x} = \frac{1}{4}$$

$$3x = 8 \Rightarrow x = \frac{8}{3} = \frac{3x}{2} = 6$$

$$4x = 24 \text{ days}$$

24 days

Q2. A & B together can complete the work in 12 days  
 B & C together can complete the work in 15 days  
 A efficiency is twice as C, B alone completes in.

$$\Rightarrow A = xc, B = 2xc, C = xc$$

$$\frac{1}{x} + B = \frac{1}{12} \quad \frac{1}{2x} + B = \frac{1}{15}$$

$$= B = \frac{1}{12} - \frac{1}{x} = \frac{1}{15} - \frac{1}{2x} \Rightarrow \frac{1}{12} - \frac{1}{15} = \frac{1}{x} - \frac{1}{2x}$$

$$\frac{5-4}{60} = \frac{1}{2x} \Rightarrow \frac{1}{60} = \frac{1}{30}$$

$$x = 30$$

equ @

$$\frac{1}{30} + B = \frac{1}{12} \quad B = \frac{1}{12} - \frac{1}{30} = \frac{5-2}{60} = \frac{3}{60} = \frac{1}{20}$$

$$B = 20$$

B alone completes work in 20 days

Q3. A can do a work in 40 days, B is 40% more efficient than A. the number of days to taken by B to complete work.

$$\Rightarrow A = 100\%, \quad B = 100 + 40\% = 140\%$$

$$140 = 70 \quad \Rightarrow \quad x = \frac{50}{100 \times 70}$$

$$x = 50$$

B completes work in 50 days.

Q4. To complete a work A takes 30% more time than B if together they completed the work in 18 days how much B alone need.

$$\Rightarrow A = 100\%, \quad B = 100\% \\ A = \frac{3}{2}x \quad B = x$$

$$\frac{1}{\frac{3x}{2}} + \frac{1}{x} = \frac{1}{18} \Rightarrow \frac{2}{3x} + \frac{1}{x} = \frac{1}{18}$$

$$\Rightarrow \frac{2+3}{3x} = \frac{1}{18} \Rightarrow \frac{5}{3x} = \frac{1}{18} \Rightarrow x = \frac{5 \times 18}{3} \\ x = 30$$