Differentail embedding

What is differentail embedding?

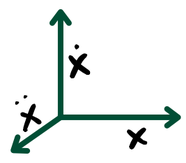
Similar to time delay embedding the goal of differential embeddding is to reconstruct the state space dynamics of a dynamical system using the time series data. Here we analyze the time series data using derivatives of the observed signal.

How to construct the embedding vector

We use thesuccessive derivatives. For the observed variable x(t) the embedding vector is the following

$$\mathbf{x}(t) = [x(t), \dot{x}(t), \ddot{x}(t), \ldots, x^{(m-1)}(t)]$$

here *m* is the embedding dimension



When to use Differential embedding ?

- When system evolves smoothly and derivatives are well behaved.
- To reduce impact of low-frequency noise in data

How to calculate derivatives ?

We calculate numerical derivatives Using finite difference methods and different choices exist such as Forward difference, Backword difference, Central difference methods.

Forwad difference method
using current point and hte next point.

For the first derivative:
$f'(x_i) \approx \frac{f(x_{i+1})-f(x_i)}{\Delta x}$

For the second derivative:
$f''(x_i) \approx \frac{f(x_{i+2})-2f(x_{i+1})+f(x_i)}{\Delta x^2}$

Backword difference method
using current point and the previous point.

For the first derivative:
$f'(x_i) \approx \frac{f(x_i)-f(x_{i-1})}{\Delta x}$

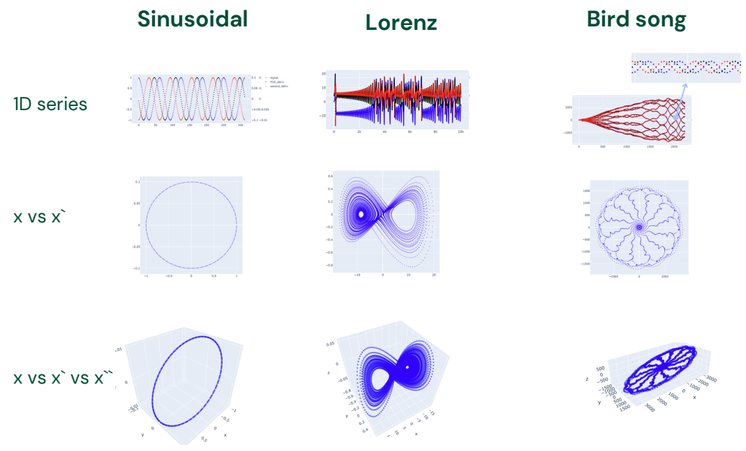
For the second derivative:
$f''(x_i) \approx \frac{f(x_i)-2f(x_{i-1})+f(x_{i-2})}{\Delta x^2}$

Central difference method
using Previous point and the next point.

For the first derivative:
$f'(x_i) \approx \frac{f(x_{i+1})-f(x_{i-1})}{2\Delta x}$

For the second derivative:
$f''(x_i) \approx \frac{f(x_{i+1})-2f(x_i)+f(x_{i-1})}{\Delta x^2}$

phase space reconstruction for dynamial system using Differential embedding



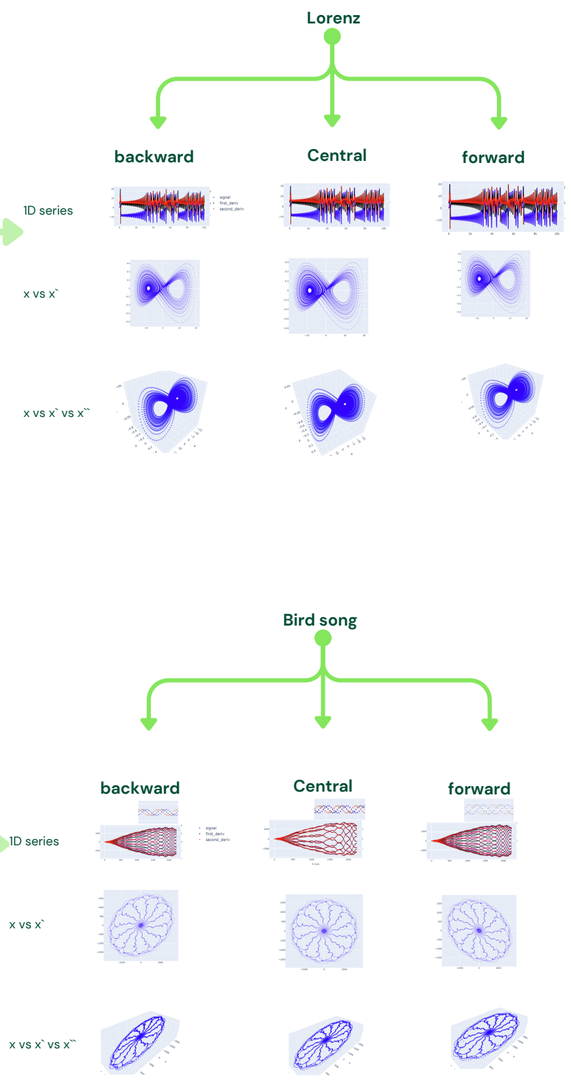
Does the choice of finite difference scheme to calculate the derivatives has any effect on the reconstructed attractor?

No, it doesnt seem much difference. We used the finte difference scheme to first and second derivatives of the time series data for the lorenz attractor and bird songs data. Here we have choice of selecting the finite difference scheme as central, forward, backward difference schemes.

Lorenz attractor: All the three schems resulted in similar reconstrcuted attractors.

Bird song attractor:

1. For central difference scheme we can see maximum opening of the attractor
2. For the forward and backward difference scheme the attactor is little distorted and looks like a mirror image of each other, simlar to the influence of the delay time tau in time delay embedding
3. Neverthless the reconstructed attractors in all the three schemes share similar characteristics



comparing the reconstructed attractor from Time delay embedding and Differential embedding for the bird songs

We dont see any difference. Infact it seems like one to one matching. But we need to analyze more data to conclude this.