1. Show that the Set of all integers is a Countable Set?

Ans:- We construct a bijection $f: \mathbb{Z} \to \mathbb{N}$ as follow's. For $x \in \mathbb{Z}$,

Let
$$f(x) = \begin{cases} -2x+1 & \text{if } x \leq 0 \\ 2x & \text{if } x > 0 \end{cases}$$

To show that f is onto, Let k be any positive integer. If k is even, then clearly $f(k/2) = k$. If k is odd, then $f(x) = k$ for $x = (k-1)/-2$.

To show that f is one to one, we need to Show that if $x_1 \neq x_2$ then $f(x_1) \neq f(x_2)$. In fact, if $f(x_1) = f(x_2)$ and is ~~the~~ an odd integer, that would mean $-2x_1+1 = -2x_2+1$, not possible if $x_1 \neq x_2$. Similarly in the case $f(x_1) = f(x_2)$ is an even integer. Hence the function is One by one and onto and thus a bijection. This process that $\mathbb{Z}$ is a Countable Set.

2. Show that mod_7 is an equivalence relation?

Show that mod_7 is an equivalence relation on the natural numbers, written $\equiv_7$.

Ans:- Define an equivalence relation on the natural numbers, written $\equiv_7$. For $i, j \in \mathbb{N}$, Say that $i \equiv_7 j$, if $i-j$ is a multiples of 7. This is an equivalence relation because it satisfies the three Conditions.

First, it is reflexive, as $i-i = 0$, which is a multiple of 7.

Second, it is Symmetric, as $i-j$ is a multiple of 7 if $j-i$ is a multiple of 7.

Third, it is transitive, as whenever $i-j$ is a multiple of 7 and $j-k$ is a multiple of 7, then $i-k = (i-j)+(j-k)$ is the sum of two multiples of 7 and hence a multiple of 7, too.