4(a) Use pumping lemma that shows each of the following language is not regular.

$A = \{a^{2^n} \mid n \geq 0\}$.

Ans:- Assume that $A = \{a^{2^n} \mid n \geq 0\}$ is regular.

Let 'p' be the pumping length given by pumping lemma

Choose 's' to be the string a^{2^p}. Because 's' is a member of A and 's' is longer than p, the pumping lemma guarantees that 's' can be split into three pieces $s = xyz$, satisfying the three conditions of pumping lemma..

The third condition tells us that $|xy| \leq p$. Furthermore, $p < 2^p$ and so $|y| < 2^p$.

Therefore $|xyyz| = |xyz| + |y| < 2^p + 2^p = 2^{p+1}$

The second condition requires $|y| > 0$. So, $2^p < |xyyz| < 2^{p+1}$

The length of $|xyyz|$ cannot be a power of 2. Hence $xyyz$ is not a member of A, a contradiction.

Therefore A is not regular.

4(b) Use pumping lemma to show that each of the following language is not regular.

$B = \{0^i 1^j \mid i \neq j\}$.

Ans:- Assume $B = \{0^i 1^j \mid i \neq j\}$ is regular.

Let 'p' be the pumping length given by the pumping lemma. Observe that $P!$ is divisible by all integers from 1 to p, Where $P! = p(p-1)(p-2) \dots 1$. The string $s = 0^p 1^{p+p!} \in B$ and $|s| \geq p$. Thus the pumping lemma implies that s can be divided as xyz with $x = 0^a$, $y = 0^b$, $z = 0^c 1^{p+p!}$, Where $b \geq 1$ and $a+b+c = p$.

Let 's' be the string $xy^{i+1}z$, where $i = P!/b$. Then $y^i = 0^{P!}$ so $y^{i+1} = 0^{b+p!}$, and so $s' = 0^{a+b+c+p!} 1^{p+p!}$

That gives $s' = 0^{p+p!} 1^{p+p!} \notin B$, a contradiction.

There fore B is not regular.