* Show that if A and B are countably infinite sets, then so is $A \times B$.

Ans:- Let $A = \{a_1, a_2, a_3, a_4 \cdots\}$

$B = \{b_1, b_2, b_3, b_4 \cdots\}$

Then there cartesian product of A and B is

$$A \times B = \{a_1, a_2, a_3, a_4, \cdots\} \times \{b_1, b_2, b_3, b_4, \cdots\}$$

$$= \{(a_1 \times b_1), (a_2 \times b_1), (a_3 \times b_1), (a_4 \times b_1), \cdots,$$
$$(a_1 \times b_2), (a_2 \times b_2), (a_3 \times b_2), (a_4 \times b_2), \cdots,$$
$$(a_1 \times b_3), (a_2 \times b_3), (a_3 \times b_3), (a_4 \times b_3), \cdots,$$
$$(a_1 \times b_4), (a_2 \times b_4), (a_3 \times b_4), (a_4 \times b_4), \cdots,$$
$$\cdots\}$$

for n number's, where $n \in \mathbb{N}$.

$$A_n = A \times \{b_n\}.$$

$$= \{(a \times b_n) : a \in A\}.$$

$$A \times B = \bigcup_{n=1}^{\infty} A_n = A_1 \cup A_2 \cup A_3 \cup A_4 \cup \cdots$$

$$= (a \times b_1) \cup (a \times b_2) \cup (a \times b_3) \cup (a \times b_4) \cup \cdots$$

$\therefore$ $A \times B$ is countable. Since, union of countable sets is countable