Commutative Property)

- For $a, b, c \in R, (ab)c = a(bc)$
  ($\therefore$ *Associative Property*)
- $1 \in R, a \times 1 = 1 \times a = a$
  i.e. 1 is multiplicative identity
- For each $a \in R$ there exists an element $a^{-1} \in R$ Such that, $a.a^{-1} = a^{-1}.a = 1$Where a and a^{-1} are multiplicative inverse of each other.

Symbols	Meaning
$\Rightarrow$	Implies
$\Leftarrow$	Is implied by
$\Leftrightarrow$	Biconditional i.e. implies and is implied by
iif	If and only if
$\in$	Belongs to
$\notin$	Doesn't belong to
$\forall$	For all
$\subseteq$	Is subset of
$\emptyset$	Null set
$\subset$	Is proper subset of
$\exists$	There exists / at least one

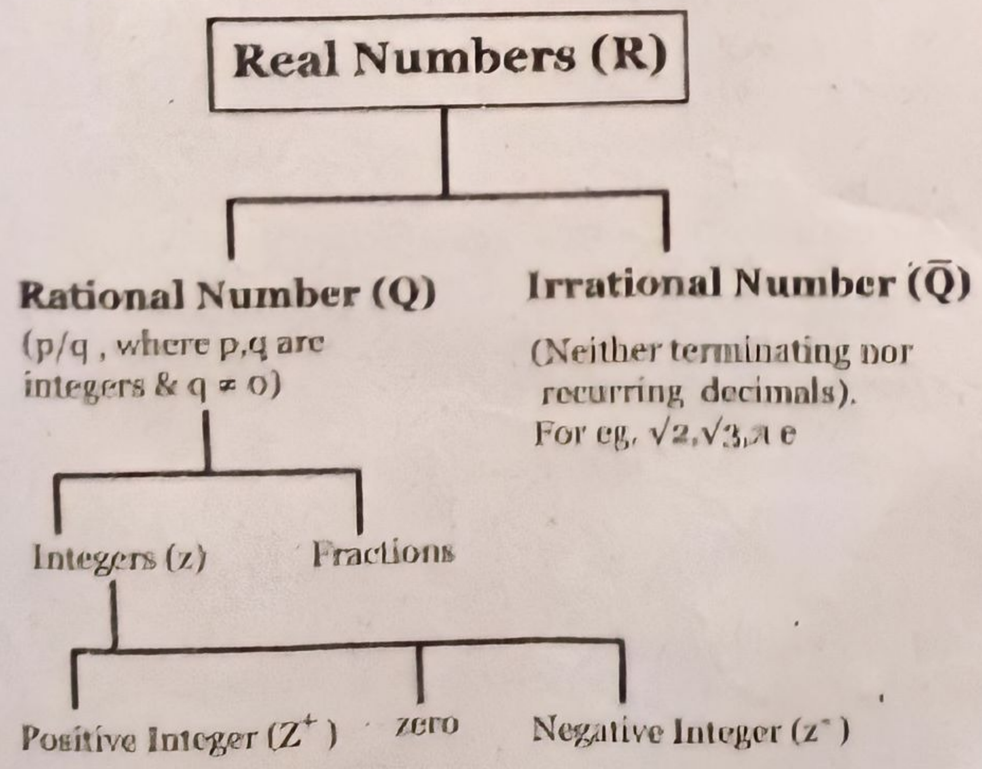
Remarks: $|x| < a \Leftrightarrow -a < x < a$

$$|x| \leq a \Leftrightarrow -a \leq x \leq a$$

$$|x| > a \Leftrightarrow x < -a \cup x > a$$

$$|x| \geq a \Leftrightarrow x \leq -a \cup x \geq a$$

Real Number ®



Where,

- set of integers $(z) = \{\ldots\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$
- Set of all positive integers or natural numbers $(Z^+ / N) = \{1, 2, 3, \ldots\ldots\}$
- Set of all negative integers $(Z^- / N^-) = \{\ldots\ldots, -4, -3, -2, -1\}$

Absolute value: - Absolute value of 'x' is a non-negative real number denoted by $|x|$ and given by

$$|x| = \begin{cases} x & if\ x \geq 0 \\ -x & if\ x < 0 \end{cases}$$

Note: For any two real number x, y:

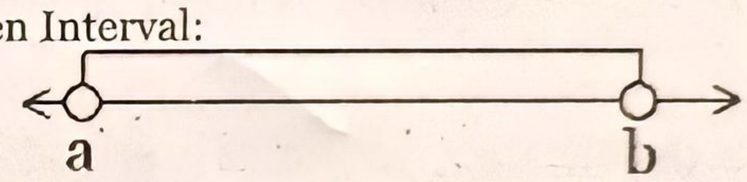
$|x + y| \leq |x| + |y|$ [$\therefore$ *traingle in equality*]

$$|x - y| \geq |x| - |y|$$

$$|x|^2 = x^2$$

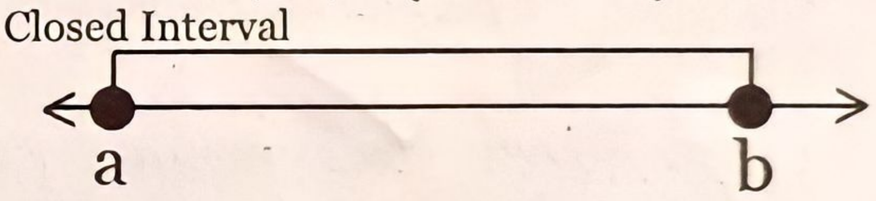
Intervals:- The set all the real numbers between a and b $(a, b \in R, a < b)$ is called an interval with a & b as end points. The length of this interval is $b - a$ and mid-point is $\frac{a+b}{2}$

a. Open Interval:



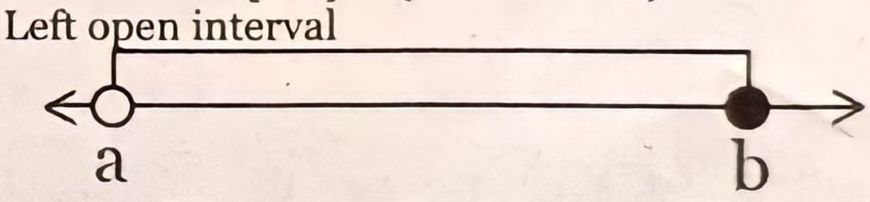
$$(a, b) = \{x: a < x < b\}$$

b. Closed Interval



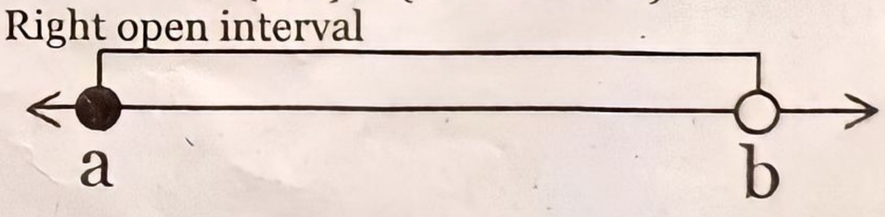
$$[a, b] = \{x: a \leq x \leq b\}$$

c. Left open interval



$$(a, b] = \{x: a < x \leq b\}$$

d. Right open interval



$$[a, b) = \{x: a \leq x < b\}$$

Also note:

- $\{x \in R : x \leq a\} \Rightarrow (-\infty, a]$
- $\{x \in R : x < a\} \Rightarrow (-\infty, a)$
- $\{x \in R : x > a\} \Rightarrow (a, \infty)$
- $\{x \in R : x \geq a\} \Rightarrow [a, \infty)$
- $R = (\infty, -\infty)$

Q. (-∞, ∞)= **[Entrance 2072]**

a) {x: -∞ ≤ x ≤ ∞} b) {x: -∞ < x < ∞}
c) {x: ∞ < x < ∞} d) {x: x ≥ 0}

Answer: (b.

Q. [a, b] = **[Entrance 2072]**

a) {x: a ≤ x ≤ b} b) {x: a < x < b}
c) {x: ∞ < x ≤ ∞} d) {x: a ≤ x < b}

Answer: (a.

Multiple Choice Questions

1. If A, B and C are the sets such that $A \subset B, B \subset C$ and $C \subset A$, then,

a. A = B ≠ C b. A ≠ B ≠ C
c. A = B = C d. A = B

2. If A, B and C are non-empty sets and $A \subset B$, then $A \cap B$ is [IOST 2069]
a. B b. A c. Empty set d. C

3. If A and B are any two sets, then $A - (A - B)$ is equal to [IOST 2070]
a. B – A b. A∩B c. $A \cup B$ d. Φ

4. Let A = [-3, 2] and B = [-2, 3], then A-B is equal to [IOST 2070]
a. [-3, 3) b. [-3, -2) c. (-2, 2) d. (-2, 1)

5. Two sets A = {1, 2, 3} and B = $\{x: x^3 - 6x^2 + 11x - 6 = 0\}$ are:
a. Disjoint sets b. Proper sets
c. Equal sets d. Complement of each other

6. The collection of intelligent students in a class is:
a. A null set b. A singleton set
c. A finite set d. Not a well-defined collection

7. Symmetric difference between any two sets A and B is:
a. (A – B) + (B – A) b. $(A \cup B) - (A \cap B)$
c. (A∩B) d. $(A - B) - (B - A)$

8. If A and B are any two non-empty sets, then which one is correct?
a. (A∪B) ⊂ (A∩B) b. (A∩B) ⊂ (A∪B)
c. (A∪B) = (A∩B) d. (A∪B) - (A∩B) = Φ

9. If A and B are two sets and (A∪B. = (A∩B., then,
a. A = Φ b. B = Φ
c. A = B d. (A∪B. = Φ

10. If X and Y are two sets then $X \cap (Y \cup X)^c$ is equals to:
a. X b. Y c. Φ d. X – Y

11. If A ⊆ B then B' – A' is equals to:
a. A' b. B' c. A – B d. Φ

12. $(A \cup B) - B =$
a. A = B b. A – B c. B - A d. 2B

13. Let U be the universal set and $A \cup B \cup C = U$, then, $[(A - B) \cup (B - C) \cup (C - A)]' =$
a. A ∪ B ∪ C b. A ∪ (B ∪ C.
c. (A ∩ B ∩ C. d. A ∩ (B ∪ C.

14. A – B = [IOST 2073]
a. $B - A$ b. $A^c - B^c$
c. $B^c - A^c$ d. $B^c - A$

15. If |x| = 2, then the solution set is [IOST 2069]
a. {2} b. {-2, 2}
c. {3,2} d. None of above

16. If |x – 2| = 5, then the solution set is [IOST 2073]
a. {3, 5} b. {1, 2}
c. {2, 5} d. {-3, 7}

17. If |x| = 1, then x is in [IOST 2074]
a. {1} b. {-1} c. {-1, 1} d. {-1, -1}

18. If 5 ≤ x ≤ 8, then,
a. (x - 5) (x - 8) > 0 b. (x - 5) (x - 8) > 0
c. (x - 5) (x - 8) ≤ 0 d. (x - 5) (x - 8) < 0

19. If x ∈ R, then |x| > x is when,
a. x ≥ 0 b. x < 0
c. x = 0 d. None of above

20. Let A = [-3, 2] and B = (-2, 3), then $A \cup B$ is equal to
a. [-3, 3) b. [-3, 2] c. (-2,2) d. (-2, 1)

21. The statement is: [IOST 2072]
a. Bagmati is a river. b. x > 0
c. How old are you? d. y > 0

22. Which one of the following is not a logical statement? [IOST 2073]
a. If you work hard, you will pass.
b. I am sick.
c. The sun rises in the west.
d. 30 is a number.

23. $(A \cup B) \cap (A \cup B)^c =$
a. A b. B c. U d. Φ

24. If a, b ∈ R, then $\frac{a}{b}$ is indeterminate if [2072]
a. a = 0, b ≠ 0 b. a = 0, b = 0
c. a = 0, b ≠ 0 d. a ≠ 0, b = 0

25. p/q, q ∈ R is undefined if [IOST 2074]
a. p = 0, q = 0 b. p = 0, q = 0
c. p = 0, q ≠ 0 d. p = 0, q ≠ 0

26. For three sets A,B and C, $A - (BUC)$ is equal to
a. $(A - B) \cap (A - C)$ b. $(A - B) \cup (A - C)$
c. $(A - B) \cup C$ d. $(A - B) \cap C$

27. If $n(U) = 120, n(P \cup Q) = 70$ then $n\overline{(P \cup Q)^C} =$
a. 120 b. 70
c. 60 d. 50

28. Which one is true?
a. $A \subseteq B \Rightarrow \bar{A} \subseteq \bar{B}$ b. $(A - B) \Rightarrow \bar{A} - \bar{B}$
c. $A - B \Rightarrow A \cap \bar{B}$ a. $A - B \Rightarrow \bar{A} - \bar{B}$

29. If $n(U) = 100, n(A) = 70$ and $n(B) = 40$ then the minimum and maximum value of $n(A \cup B)$ and $n(A \cup B)$ are
a. 10,70 b. 10,60
c. 50,60 d. 70,100

30. Sum of two irrational number is
a. always irrational
b. never irrational
c. may also be rational
d. can't be said.
Hint:
let $a = 2 + \sqrt{5}$, b =2-$\sqrt{5}$

31. If A = (-3,2) and B = (1,3] then $\overline{A \cap B} =$
a. [1,2] b. $(-\infty, -3) \cup (3, \infty)$
c. $(-\infty, 1] \cup [2, \infty)$ d. $(-\infty, 1) \cup (2, \infty)$

32. Solution set of $x^2 + 5x + 6 \leq 0$ is
a. (-3, -2) b. [-3, -2]
c. [-2, -3] d. (2,3)

33. $-3 \leq x \leq -1$ can be written as
a. $|x + 2| < 1$ b. $|x + 3| \leq 1$
c. $|x + 2| \leq 1$ d. $|x + 2| \leq 1$

34. e and π are
a. both rational
b. both irrational
c. one rational and one irrational
d. natural numbers

35. $|5 - 2x| < 1$ imple
a. $-1 < x < \frac{-1}{5}$ b. $1 < x < \frac{1}{5}$
c. $2 < x < 3$ d. $\frac{-1}{2} < x < \frac{1}{5}$

36. Each prime number has
a. no factor b. only one factor
c. only two factors d. more than two factors

37. (∅) is
a. null set b. singleton set
c. void set d. finite set

38. The set $\{x: x^2 + 9 = 0\}$ where $x \epsilon R$ is :
a. null set b. finite set
c. infinite set d. singleton set

39. In real number system correct order is:
a. $R \subseteq QZ \subseteq N \subseteq P$
b. $R \subseteq Z \subseteq Q \subseteq N \subseteq P$
c. $P \subseteq N \subseteq Z \subseteq Q \subseteq R$
d. $P \subseteq N \subseteq Q \subseteq Z \subseteq R$

40. $p \rightarrow q$ can't be read as:
a. p implies q b. if p then q
c. p only if q d. q only if p

41. $p \Leftrightarrow q$ is true iff
a. both statements p and q are true
b. both statements p and q are false
c. p is true and q is false
d. both a and b

42. Negation of converse of statement is termed as:
a. Inverse b. Reciprocal
c. Contrapositive d. Contradiction

43. The contraposition of "3 is odd and $2^2 = 5$" is
a.3 is not odd or $2^2 \neq 5$
b. 3 is odd and $2^2 \neq 5$
c. if $2^2 \neq 5$ then 3 is not odd
d. if 3 is not odd then $2^2 \neq 5$

44. Solution set for $x^2 - 2x - 3 > 0$ is
a. $(-1,3)$ b. $(-\infty, -1) \cup (3, \infty)$
c. $(-\infty, 3)$ d. $(-\infty, -1) \cup [\sqrt{3}, \infty)$

45. The no of Proper subset of $A = \{1,2,3,4,5\}$ is
a. 64 b. 63
c. 32 d. 31

Answer Key

1. c	2. b	3. b	4. b	5. c
6. d	7. b	8. b	9. c	10. c
11. d	12. b	13. c	14. c	15. b
16. d	17. c	18. c	19. b	20. a
21. a	22. b	23. d	24. b	25. a
26. a	27. b	28. c	29. d	30. c
31. c	32. b	33. c	34. b	35. c
36. c	37. b	38. a	39. c	40. d
41. d	42. c	43. c	44. b	45. d

Chapter: 2
Relation and Functions

1. **Ordered Pair:** - A pair of elements (a, b) is said to be an ordered pair if 'a' is fixed as the first element and 'b' as the second element. It is denoted by (a, b)
It is to be noted that: $(a, b) \neq (b, a)$
Any two ordered pairs are said to be equal if their corresponding elements are equal i.e. $(a, b) = (c, d)$ iff $a = c$ & $b = d$

2. **Cartesian product:** - Let A and B be two given non-empty sets. Then, the set of all ordered pairs (a, b. such that $a \in A$ and $b \in B$ is termed as Cartesian products of sets A & B. It is denoted by $A \times B$. Thus $A \times B = \{(a, b): a \in A \text{ and } b \in B\}$
For eg. If $A = \{1,2\}$ & $B = \{2,3,6\}$ then;
$A \times B = \{(1,2), (1,3), (1,6), (2,2), (2,3), (2,6)\}$
$B \times A = \{(2,1), (2,2), (3,1), (3,2), (6,1), (6,2)\}$

Note:

a. If there are 'n' no. of elements in A and 'm' no. of elements in B then, the no. of elements in $A \times B$ is mn. i.e. $n(A \times B) = n(A) \times n(B)$.
b. $A \times (B \cup C) = (A \times B) \cup (A \times C)$
c. $A \times (B \cap C) = (A \times B) \cap (A \times C)$
d. $A \times (B - C) = (A \times B) - (A \times C)$
e. $A \times (\overline{B \cup C}) = (A \times \bar{B}) \cap (A \times \bar{C})$
f. $A \times (\overline{B \cap C}) = (A \times \bar{B}) \cup (A \times \bar{C})$
g. Number of possible relations from set A to set $B = 2^{mn}$ where, m= no of element of set A
h. n= no of elements of set B
i. If $n(A \cap B) = n$ then $n((A \times B) \cap (B \times A)) = n^2$
j. The number of function from a finite set A into finite set $B = (n(B))^{n(A)}$

Example
The number of functions from (3,4,5) into (1,2) are
a. 4 b. 12 c. 8 d. 16

Hints
Let, $n(A) = 3, n(B) = 2$
No of functions from A to B = $(n(B))^{n(A)}$
$= (2)^3 = 8$

Relation
Let A and B be two non-empty sets. Then any subset of $A \times B$ is a relation from A to B, denoted by R. The relation from A to A is called relation on A.

Domain and Range
If R is a relation from A to B then the set of all first element of all ordered pair in R is called domain of R and the set of all second element is called range of R.

Hence, domain $(R) = \{a: (a, b) \in R\}$

Range $(R) = \{b: (a, b) \in R\}$

Inverse Relation
Let R be a relation from A and B. Then inverse of R from B set to set A, denoted by R^{-1}, is the set of all ordered pairs obtained by interchanging the first and second element in the ordered pairs of given relation

$\cos^{-1} x$	$[-1,1]$	$[0,\pi]$
$\tan^{-1} x$	$(-\infty,\infty)$	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$
$\cot^{-1} x$	R	$[0,\pi]$
$\sec^{-1} x$	$R-(-1,1)$	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]-\{0\}$
$cosec^{-1} x$	$R-(-1,1)$	$[0,\pi]-\left\{\frac{\pi}{2}\right\}$

Types of Functions:

a. **One to one function / Injective mapping**

A function $f: A \to B$ is said to be one to one function if different element in A have different images in B.

In symbol, for $x_1, x_2 \epsilon A$

$x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$

or $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

b. **Onto function surjective mapping**

A function $f: A \to B$ is said to be onto if range of the function is equal to co-domain. i.e $f(A) = B$. Also, for into function $f(A) \subset B$ $(proper)$.

c. **Bijective function**

A function which is both injective and surjective is called bijective function.

Note: only bijective functions have inverse.

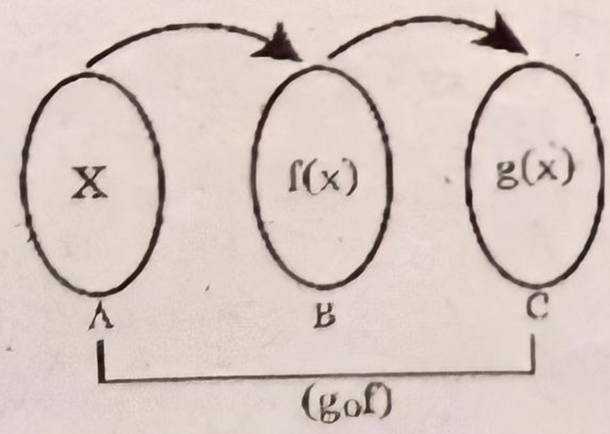
$f: A \to B$ is a bijective function then $n(A) = n(B)$

d. **Inverse function:**

If $f: A \to B$ is bijective then there exists a function $f^{-1}: B \to A$ called inverse function i.e. $f^{-1}(y) = \{x \in x : f(x) = y\}$

e. **Composite function**

If $f: A \to B$ and $g: B \to C$ then composite of f and g, denoted by gof is called composite function and defined as $gof: A \to C$ such that $(gof)x = g[f(x)]$



Example Question

If $f(x) = x^3 - 1, g(x) = 2x - 3$, find $(f^{-1}og)(2)$

Hints

$(f^{-1}og)(2) = f^{-1}[g(2)]$

$= f^{-1}[2 \times 2 - 3]$

$= f^{-1}[1] \ldots\ldots\ldots 1$

To find $f^{-1}(x)$,

$y = f(x) = x^3 - 1$

Interchanging place of 'x' & 'y'

$x = y^3 - 1$

$\therefore f^{-1}(x) = y = (x+1)^{\frac{1}{3}}$

Then equation 1 become, $(f^{-1}og)(2) = f^{-1}[1]$

$= (1+1)^{1/3}$

$= 2^{\frac{1}{3}}$

f. **Odd and even function**

i. A function $f(x)$ is said to be an odd function if $f(-x) = -f(x)$. for eg. X, $\sin x$, $cosec\ x$, $\sin x + x^5, x^3 - 3, x + \tan x$ etc

ii. A function $f(x)$ is said to be an event function if $f(-x) = f(x)$. For eg : $x^2, \sin^2 x + \cos x - 5, x^6 + \tan^2 x, \cos x$, $\sec x$ etc

g. **Periodic function**

A function $f(x)$ is perodic if there exits a positive number T independent of x such that $f(T+x) = f(x)$. Smallest such value of T is called period.

S.N	Periodic functions	Period
1	$\sin x, \cos x, \sec x, cosec\ x$	2π
2	$\tan x, \cot x$	π
3	$\lvert\sin x\rvert, \lvert\cos x\rvert, \lvert\tan x\rvert, \lvert\cot x\rvert$ $\lvert\sec x\rvert, \lvert cosec\ x\rvert$	π
4	$\sin^n x, \cos^n x, \sec^n x, cosec$	$2\pi(if\ n\ is\ odd)$ $\pi(if\ n\ is\ even)$
5	$\tan^n x, \cot^n x$	π

i. If period of $f(x)$ is T then function $\frac{1}{f(x)}, \sqrt{f(x)}$ are also function of same period.

ii. If period of f(x) is T then Period of $f(ax+b)$ is $\frac{T}{|a|}$

Example Question

Period of function $\sin 2x$ is

Hints

Period of $\sin 2x = \frac{2\pi}{2} = \pi$

Multiple Choice Questions

1. If (2x − 1, 3) = (3, y+3), then, [IOST 2072]
   a. x = 1, y = 0 b. x = -1, y = -3
   c. x = 2, y = 0 d. x = 0, y = -1

2. If A = {x: x² − 5x + 6 = 0}, B = {2, 4} and C = {4, 5}, then A × (B ∩ C. = [Model Question]
   a. {(2, 4), (3, 4)} b. {(4, 2), (4, 3)}
   c. {4} d. Empty set

3. If A is a set of even natural numbers less than 8 and B is the set of prime number less than 7 then the number of relations from A to B is:
   a. 2^9 b. 9^2 c. $2^9 - 1$ d. $9^2 - 1$

4. The range of $y = \sqrt{4 - x^2}$ is [Model Question]
a. $[-2, 2]$ b. $[-2, 0]$ c. $[0, 2]$ d. $(-\infty, \infty)$

5. If fog: R → R be defined by $f(x) = x^2 + 1$ and $g(x) = x^5$, then $fog(x) =$
a. $(x^2 + 1)^5$ b. $x^{10} + 1$
c. $(x^{10} + 1)^5$ d. $x^5 + x^2 + 1$

6. If $f(x) = \sqrt{4 - x^2}$, then f(2) is [IOST 2069]
a. 0 b. 4 c. 2 d. -2

7. The range of the function $y = \sqrt{a^2 - x^2}$, a>0 is equal to [IOST 2070]
a. [-a, a] b. (-a, a c. [0, a] d. (0, a

8. If f: R → R be defined by $f(x) = x^2 + 3$, then $f^{-1}(4)$ is [IOST 2071]
a. Null set b. {-1, 1} c. {1, 0} d. {4, 3}

9. If f: R → R is defined by $f(x) = x$ and g: R → R is defined by $g(x) = x$, then $fog(x) =$ [IOST 2072]
a. x b. x^2 c. $\frac{1}{x^2}$ d. $\frac{1}{x}$

10. If $f(x) = 2x - 3$, then $f^{-1}(x) =$
a. $\frac{x+3}{2}$ b. $\frac{1}{2x-3}$ c. $\frac{1}{3-2x}$ d. $\frac{1}{2x+3}$

11. If f: R → R be defined by $f(x) = \sqrt{4 - x^2}$, then $f(-3)$ is [IOST 2075]
a. Undefined b. $\sqrt{13}$ c. $i\sqrt{5}$ d. $-\sqrt{13}$

12. The range of the relation R = {(1, 2), (2,3), (3, 4)} is [IOST 2075]
a. {1, 2, 3} b. {1, 2, 3, 4}
c. {2, 3, 4} d. R

13. If A = {1, 3, 5} and B = {1, 2, 7, 8}, then n(A×B) =
a. 6 b. 3 c. 9 d. 12

14. If R = {(1, 2), (3, 4), (5, 6)}, then domain of R is
a. {1, 3, 5} b. {2, 4, 6}
c. {1, 4, 6} d. {2, 3, 5}

15. Let f: R → R and g: R → R be defined by $f(x) = x^3 - 1$ and $g(x) = x^2$, then $fog(x) =$
a. $(x^3 - 1)^2$ b. $x^6 - 1$
c. $x^3 + x^2 + 1$ d. $(x^6 - 1)^2$

16. Let $f(x) = 3x + 10$, $g(x) = x^2 - 1$, then $(fog)^{-1}(x) =$
a. $\left(\frac{x+7}{3}\right)^{\frac{1}{2}}$ b. $\left(\frac{x+3}{7}\right)^{\frac{1}{2}}$
c. $\left(\frac{x-7}{3}\right)^{\frac{1}{2}}$ d. $\left(\frac{x-3}{7}\right)^{\frac{1}{2}}$

17. If $f(x) = \frac{3x+2}{5x-3}$, then,
a. $f^{-1}(x) = f(x)$ b. $f^{-1}(x) = -f(x)$
c. $f(f(x)) = -x$ d. $f^{-1}(x) = -\frac{1}{3}f(x)$

18. The range of the function $y = x^3$ is
a. $[0, \infty)$ b. $(-\infty, 0]$ c. R d. R − {0}

19. The range of the function $y = \sqrt{x - 2}$ is
a. $[2, \infty)$ b. $[0, \infty)$
c. R d. $(-\infty, 0]$

20. The range of the function $y = x^2 - 1$ is
a. $[-1, \infty)$ b. $[0, \infty)$ c. R d. $(-\infty, 0]$

21. If a set A contains 4 elements and a set B contains 8 elements; the maximum number of elements in A∪B is
a. 8 b. 12 c. 16 d. 4

22. If A ={x: x is the set of all the points in x-axis}, B = {x: x is the set of all the points in y-axis}, then A and B are
a. Disjoint sets b. Subsets
c. Intersecting set d. Null sets

23. $-21 \le x \le 12$ can be written as
a. $|2x - 9| \le 33$ b. $|x + 9| \le 33$
c. $|x| \le 12$ d. $|2x + 9| \le 33$

24. The domain of the function $f(x) = \frac{1}{\sqrt{|x|-x}}$ is
a. $(0, \infty)$ b. $(-\infty, 0)$
c. $(-\infty, \infty)$ d. *none*

25. The periodicity of the function $f(x) = \sin ax$ is
a. 2a b. $2a\pi$ c. πa d. $\frac{2\pi}{a}$

26. If a function $f: R \longrightarrow R$ be defined by $f(x) = 2x + 1$, The pre-image of 17 is:
a. 8 b. 10 c. 9 d. 18

27. let $f: R \to f$ be defined by $f(x) = 3x - 2$, Then $f^{-1}(4) =$
a. 1 b. 2 c. 3 d. 4

28. Range of $f(1) = \sqrt{2 - x - x^2}$ is
a. $(-\infty, \infty)$ b. $\left[\frac{3}{2}, 0\right]$
c. $\left[0, \frac{3}{2}\right]$ d. $\left[-2, \frac{3}{2}\right]$

29. Range of $f(x) = \sqrt{1 - x^2}$ is
a. [0,1] b. [−1,1]
c. [−2,3] d. [−1,3]

30. If $f(x - 1) = x + 2$ then $f(x^3) =$
a. $x^2 + 3$ b. $x^3 + 3$
c. $x^2 + x + 1$ d. $x^3 - 1$

31. Range of $f(x) = (x - 2)$ is
a. $(-\infty, 0)$ b. $[0, \infty]$
c. $(0, \infty)$ d. $(-\infty, \infty)$

32. Domain of $\cot^{-1} x$ is
a. $[0, \pi]$ b. $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
c. $(0, \pi)$ d. $\left(-\frac{\pi}{2}, 0\right)$

33. If $f = \{(1,2), (3,5), (4,1)\}$ and $g = \{(2,3), (5,1), (1,1)\}$ then $g_o f(1) =$
a. 2 b. 3 c. 4 d. 5

34. If $f(x) = \frac{x-1}{x+1}$ then $f\left(\frac{1}{x}\right) =$
a. $f(x)$ b. x c. $-f(x)$ d. $\frac{2x}{1+x}$

35. Period of function $\sin x + \tan x$ is
a. $\frac{1}{\sqrt{2}}$ b. 2π c. $\frac{\pi}{2}$ d. $\frac{\sqrt{2}}{\pi}$

36. $f(x) = |x - 5|$ is
a. odd function b. even function
c. neither odd nor even d. both a and b

37. Function $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ is
a. symmetry about x- axis
b. symmetry about y- axis
c. symmetry about origin
d. none

38. Period of $\tan\left(\frac{3x}{5}\right)$ is
a. $\frac{3\pi}{5}$ b. $\frac{5\pi}{3}$ c. $\frac{3\pi}{5\pi}$ d. $\frac{\pi}{2}$

39. Domain of $f(x) = \sqrt{-x}$ is
a. $[0, \infty)$ b. $[1, \infty)$
c. $(-\infty, 0]$ d. $(-\infty, 0)$

40. The function $f: R \longrightarrow R$ defined by $f(x) = 2x + 1$ is
a. Injective b. surjective
c. Bijective d. none

41. The function $f: [-2,3] \to R$ given by $f(x) = x^3$ is
a. one-one b. onto
c. both d. none

42. $f(x) = \tan x - x$ is
a. odd b. even
c. both
d. neither even nor odd

43. Domain of function $\frac{1}{(x^2 - 5x + 6)}$
a. {0} b. $R - \{2,3\}$
c. $(-\infty, \infty)$ d. $R - \{3\}$

44. Domain of function $f(x) = \frac{(x-2)}{|x-2|}$ is :
a. $R - \{0\}$ b. $R - \{2\}$
c. $R - \{0,2\}$ d. $\{-1,1\}$

45. Range of $\tan x$ is :
a. $[-1,1]$ b. $(-\infty, \infty)$
c. $[0, \infty)$ d. $[-\infty, -1][1, \infty)$

46. Number of relations that can be defined on set $A = \{5,6,7,8\}$ is
a. 24 b. 16 c. 2^{16} d. 4^4

47. If R is a relation on A then R is called symmetric relation if
a. $R = R^T$ b. $R = R^{-1}$
c. A=R d. $RR^{-1} = 0$

48. If $f(x) = \sqrt{1 + x + x^2} - \sqrt{1 - x - x^2}$ then it is
a. even function b. odd function
c. neither and nor even d. both odd and even

49. If $f(0) = 1$, $f(1) = 2$, $f(2) = 5$, $f(3) = 10$ then $f(-1) =$
a. 0 b. 1 c. 11 d. 2

50. Domain of function $f(x) = \frac{1}{\sqrt{x^2 - 3x + 2}}$ is
a. $(-\infty, 1)$ b. $(-\infty, 1) \cup (2, \infty)$
c. $(-\infty, 1] \cup [2, \infty)$ d. $(2, \infty)$

Answer Key:

1. c	2. a	3. a	4. c	5. b
6. a	7. c	8. b	9. a	10. a
11. a	12. c	13. d	14. a	15. b
16. c	17. a	18. c	19. b	20. a
21. b	22. c	23. d	24. b	25. d
26. a	27. c	28. c	29. a	30. b
31. d	32. c	33. b	34. c	35. b
36. c	37. c	38. b	39. c	40. c
41. a	42. a	43. b	44. b	45. b
46. c	47. a	48. b	49. d	50. d

Multiple Choice Questions

1. What is the value of $\log_a \sqrt{a^3\sqrt{a^2}}$? [IOST 2070]
   a. 4 b. 3 c. 2 d. 1
2. The value of $\log_2 \sqrt{2} + \log_3 \sqrt{3} + \log_4 \sqrt{4}$ is [IOST 2071]
   a. 0 b. $\frac{3}{2}$ c. $-\frac{3}{2}$ d. 1
3. If $\log_2(x-7) = 1$, then x is
   a. Not known b. 8 c. 6 d. 9
4. If $y = a^x$ is the exponential function passing through (4, 81), then the base a is [IOST 2075]
   a. 4 b. 3 c. 81 d. a
5. The value of $\log_8 128 =$
   a. $\frac{2}{5}$ b. $\frac{3}{4}$ c. $\frac{8}{3}$ d. $\frac{7}{3}$
6. If $\frac{\log x}{2} = \frac{\log y}{3} = \frac{\log z}{5}$ then $x^2 =$
   a. 1 b. $\sqrt{xy}$ c. $\sqrt{yz}$ d. $\sqrt{zx}$
7. The value of $\log_2 \sqrt{2\sqrt{2\sqrt{2}}} =$
   a. $\frac{7}{8}$ b. $\frac{1}{8}$ c. $\frac{1}{4}$ d. $\frac{5}{8}$
8. $\log_{10} tan\, 1^0 + \log_{10} tan\, 2^0 + \cdots + \log_{10} tan\, 89^0$
   a. 0 b. 1 c. 2 d. 3
9. The solution of $\log_4(x-1) = \log_2(x-3)$ is
   a. 3 b. 1 c. 2 d. 5
10. If a, b, c are three consecutive integers then $\log(1+ac) =$
    a. $\log b$ b. $2\log b$
    c. $\log a + \log c$ d. 0
11. Value of $a^{\log_a x} =$
    a. a b. 0 c. x d 1
12. If $a^2 + b^2 = 7ab$ then $\ln\left(\frac{a+b}{3}\right) =$
    a. $\ln a - \ln b$ b. $\frac{\ln a + \ln b}{2}$
    c. $\frac{\ln a - \ln b}{5}$ d. $\frac{\ln(ab)}{2}$
13. if $\frac{\ln x}{y-z} = \frac{\ln y}{z-x} = \frac{\ln z}{x-y}$ then xyz=
    a. -1 b. 1 c. 2 d. 0
14. The value of x for $\log_2(x^2+7) = 3$ is
    a. 1 b. ± 2 c. ± 1 d. ± 3
15. $\frac{1}{2}\log_{10}25 - 2\log_{10}3 + \log_{10}18 =$
    a. 1 b. 18 c. $\frac{2}{3}$ d. $\frac{1}{5}$
16. Value of $\log_2\sqrt{2} + \log_3\sqrt[3]{3} - \log_4\sqrt[4]{4}$
    a. 1 b. -1 c. $\frac{3}{4}$ d. $\frac{7}{12}$
17. Value of x for $\log_a(3x+2) - \log_a(2x+3) = \log_a 2$
    a. -1 b. -6 c. -4 d. 2
18. Value of $(\log_2 5).(\log_{25} 8)$ is equal to.
    a. $\frac{1}{2}$ b. $\frac{1}{4}$ c. 1 d. $\frac{3}{2}$
19. if $\log_7 2 = c$ then $\log_{49} 28 =$
    a. $2(1+3c)$ b. $(1+2c)/2$
    c. $2/1+2c$ d. $1+c$
20. If a,b,c are in G.P. then $\log_q m, \log_b m, \log_c m$ are in
    a. A.P b. G.P c. H.P d. none
21. $\log\left(\frac{p}{q}\right) + \log\left(\frac{q}{r}\right) + \log\left(\frac{r}{p}\right)$
    a. 0 b. 1
    c. 2 d. 3
22. If $\frac{\log x}{2} = \frac{\log y}{3} = \frac{\log 2}{5}, x^2 =$
    a. 2xy b. $\sqrt{yz}$ c. $\sqrt{xy}$ d. $\sqrt{zx}$
23. Value of $\log_{2\sqrt{3}} 144 =$
    a. 2 b. 3 c. 6 d. 4
24. If $x = 20!$ then the value of $\frac{1}{\log_1 x} + \frac{1}{\log_2 x} + \frac{1}{\log_3 x} + \cdots + \frac{1}{\log_x 20}$
    a. -1 b. 0 c. 1 d. 2
25. If $a^m = b^n$ then $n \log_a x =$
    a. $n \log_a x$ b. $\frac{n}{m} \log_b x$
    c. $m \log_b x$ d. $\frac{m}{n} \log_a x$
26. $7\ln\frac{15}{16} + 6\ln\frac{8}{3} + 5\ln\frac{2}{5} + \ln\frac{32}{25} =$
    a.1 b. $ln2$ c. $\ln(-1)$ d. $\ln 3$
27. If $x = \log_a bc, y = \log_b ca, z = \log_c ab,$ then $\frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1} =$
    a.2 b. 1 c. -1 d. 0
28. The number of solutions of $\log_2(5x^2 - x - 2) - 2\log_2 x = 2$ is
    a. 1 b. 2 c. 3 d. no solution
29. The base of an exponential function if it passes through $\left(-2, \frac{1}{9}\right)$ is
    a. 5 b. 2 c. 3 d. $\frac{1}{5}$
30. If $\log_{64} 4 = \frac{1}{3}$ then $64^{\frac{1}{3}} =$
    a. 4 b. 2 c. 3 d. $\frac{1}{4}$

Answer Key:

1. c	2. b	3. d	4. b	5. d
6. c	7. a	8. a	9. d	10. b
11. c	12. b	13. b	14. c	15. a
16. d	17. c	18. d	19. b	20. c
21. a	22. b	23. d	24. c	25. c
26. d	27. b	28. a	29. c	30. a

- Any five number in A.P are $(a-2d), (a-d), a, (a+d), (a+2d)$

Geometric Sequence /G. P/G.S)

A sequence is said to be a geometric sequence if three is a uniform common ratio between any two consecutive terms in the series . i.e. $r = \frac{a_{k+1}}{a_k}; k \geq 1$

If 'a' is the first term and 'r' is the common ration then, n^{th} *term of series* $(t_n) = ar^{n-1}$

Sum up to first n terms of G.P.

$$s_n = \frac{a(1-r^n)}{(1-r)}, r \neq 1, r < 1$$

Also $s_n = \frac{a(r^n-1)}{r-1}, r > 1$

Geometric Means (G.M.)

Geomatics mean between any two numbers 'a' and 'b' is given by, $G.M. = \sqrt{ab}$

If a, b, c are any three numbers in G.P then, $\frac{a}{b} = \frac{c}{b}$

$\Rightarrow b^2 = ac, \Rightarrow b = \sqrt{ac}$

If $G_1, G_2, G_3, \ldots\ldots G_n$ are the n G.M's inserted between a and b then, $a, G_1, G_2, G_3, \ldots\ldots, G_n, b$ in G.P. where;

$$r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$$

Some Properties of Geometric Sequence

1. The reciprocals of a terms of a G.P results G.P.
2. If each term of G.S. is multiplied or directed by a non-Zero constant number, then result is also G.P.
3. If each term of a G.S. is raised to a constant power, then resulting sequence is also G.P.

   i.e. if a, b, c, d,..... are in G.P then a^k, b^k, c^k, d^k .... are also in G.P.

For Practical purpose we choose

- Three numbers in G.P. are $\frac{a}{r}, a, ar$
- for numbers in G.P. are $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$
- five number in G.P. are $\frac{a}{a^2}, \frac{a}{r}, a, ar, ar^2$

Sum to infinity (C_∞)

- $S_\infty = \frac{a}{1-r}$ such that $|r| < 1$
- for Artho- geometric (AG) series with first terms 'a' common difference 'd' of A.P and common ration of G.P. such that $|r| < 1$ then,

$$S_\infty = \frac{a}{1-r} + \frac{d.r}{(1-r)^2}$$

Harmonic Sequence (H.P./ H.S)

The reciprocals of the terms of an arithmetic sequence from a Harmonic sequence.

For eg. If a, b, c are in A.P. then $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in H.P.

Harmonic Mean (H.M/H)

Let H be the harmonic mean between a and b then a, H, b are in H.S if $\frac{1}{a}, \frac{1}{H}, \frac{1}{b}$ are in A.S

Then, $\frac{1}{H} - \frac{1}{a} = \frac{1}{b} - \frac{1}{H}$

$\therefore H = \frac{2ab}{a+b}$ When $a \neq -b$

Relation between AM, GM, HM

1. $A.M \times H.M = G.M^2$
2. $A.M > G.M > H.M$

Note:

1. $1+2+3+\cdots+n = \sum n - \frac{n(n+1)}{2}$
2. $2+4+6+\cdots+2n = n(n+1)$
3. $1+3+5+7+\cdots+2n-1 = n^2$
4. $1^2+2^2+3^2+\cdots+n^2 = \sum n^2 = \frac{n(n+1)(2n+1)}{6}$
5. $1^3+2^3+3^3+\cdots.+n^3 = \sum n^3 = \left[\frac{n(n+1)}{2}\right]^2$

Multiple Choice Questions

1. The sum of n terms of the series $a + ar + ar^2 + \ldots\ldots$ is [Model Question]
   a. $a^{rn} - 1$ b. $\frac{a(r^n-1)}{r-1}$
   c. $\frac{ar^n-1}{r-1}$ d. $\frac{ar^{n-1}-1}{r-1}$
2. If $a^x = b^y = c^z$ and a, b, c are in G.P, then x, y, z are in [IOST 2070]
   a. G.P b. A.P c. H.P d. None
3. If a^2, b^2, c^2 are in H.P, then [IOST 2071]
   a. $2a^2c^2 = b^2(a^2+c^2)$
   b. $a^2c^2 = b^2(a^2+c^2)$
   c. $2b^2c^2 = a^2(b^2+c^2)$
   d. $c^2a^2 = 2b^2(a^2+c^2)$
4. If $2p, p+8, 3p+1$ are in A.P, then p is [IOST 2072]
   a. 5 b. 3 c. 7 d. -5
5. A, G and H are arithmetic mean, geometric mean and harmonic means respectively, then [IOST 2072]
   a. $A^2 = GH$ b. $H^2 = AG$
   c. $G = AH$ d. $G^2 = AH$
6. The fourth, seventh and tenth terms of a G.P are l, m ,n respectively, then [IOST 2073]
   a. $ln = m^2$ b. $l^2 = m^2 + n^2$
   c. $l^2 = mn$ d. $n^2 = lm$
7. The nth term of the series: $1.3 + 3.5 + 5.7 + \ldots.$ is
   a. $(n+1)(n+2)$ b. $(n+1)(n+3)$
   c. $4n^2 - 1$ d. $n(n-1)$

8. The sum to infinity of $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} \ldots$ is [IOST 2074]
   a. 0 b. $\frac{2}{3}$ c. $\frac{1}{3}$ d. 1
9. The sum to infinity of the G.P $-\frac{1}{4}, \frac{1}{16}, -\frac{1}{64}, \ldots$ is [IOST 2075]
   a. 5 b. −5 c. $\frac{-1}{5}$ d. 0
10. The nth term of the series 2.3 + 3.4 + 4.5 + ... is
    a. $4n^2-1$ b. (n+1) (n+3)
    c. (n+1) (n+2) d. n(n-1)
11. The nth term of an A.P is (4n+1), then the common difference is
    a. 3 b. 4 c. 5 d. 6
12. If (x+3), 3x, (4x+1) are in A.P, then x =
    a. 3 b. 4 c. 5 d. 1
13. If a, b, c are in A.P, b, c, a are in H.P, then c, a, b are in
    a. A.P b. H.P c. G.P d. None
14. If $t_n = (2n + 3)$, then the value of s_{10} =
    a. 120 b. 140 c. 240 d. 110
15. The sum of any 3 numbers of A.P is 36, then the 2nd number is
    a. 14 b. 18 c. 15 d. 12
16. The general term of G.S is given by $t_n = 3(2^n)$, $n \in N$, then the common ratio is
    a. 4 b. 3 c. 2 d. 1
17. If (4x - 3), 2x, x+1 are in G.P then x =
    a. 4 b. 5 c. 6 d. 3
18. The G.M between two numbers 2 and k is 8, then the value of k =
    a. 8 b. 16 c. 32 d. $4\sqrt{2}$
19. The value of $x^{\frac{1}{2}}.x^{\frac{1}{4}}.x^{\frac{1}{8}}\ldots.\infty$ is
    a. x b. x^2 c. $\frac{1}{x}$ d. x^3
20. The common ratio of a G.P is 3 and its last term is 486. If the sum of all the terms of this G.P is 728, then the 1st term is:
    a. $a = 4$ b. $a = 3$
    c. $a = \frac{1}{2}$ d. $a = 2$
21. If 3, x, 7 are in H.P, then x =
    a. $\frac{15}{2}$ b. $\frac{7}{3}$ c. $\frac{21}{5}$ d. $\frac{25}{4}$
22. If a, b, c are in H.P, then bc, ca, ab are in
    a. A.P b. G.P c. H.P d. None
23. The A.M between any two numbers is 9 and the G.M is 3, then the H.M is
    a. $\frac{1}{3}$ b. 27 c. 1 d. 4
24. If a, 4, b are in A.P; a, 2, b are in G.P, then a, 1, b are in
    a. A.P b. G.P c. H.P d. None
25. The sum of the series $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \cdots\infty$ is
    a. $\log_e 2 - 1$ b. $\log_e 2$ c. 1 d. 0
26. If a, b, c are in A.P, then $3^a, 3^b, 3^c$ are in
    a. A.P b. G.P c. H.P d. none
27. $9^{\frac{1}{3}}.9^{\frac{1}{9}}.9^{\frac{1}{27}}\ldots.\infty =$
    a. 81 b. 9 c. 27 d. 3
28. If x, y, z are in G.P, then log x, log y, log z are in
    a. A.P b. G.P c. H.P d. None
29. If y = 1 + x + x^2 + ...∞, then x = ?
    a. $\frac{y-1}{y}$ b. $\frac{1-y}{y}$ c. $\frac{y}{1-y}$ d. $\frac{y+1}{y}$
30. If 1 + 6 + 11 + ...+ x =148, then x =
    a. 36 b. 8 c. 30 d. 20
31. If $s_{n1} = 1 + 2 + 3 + \ldots\ldots + n = 21$, then $s_{n2} = 1^2 + 2^2 + 3^2 + \ldots\ldots + n^2 =$
    a. 455 b. 250 c. 375 d. 200
32. Sum of all the two digits number is
    a. 4900 b. 4700 c. 4905 d. 4000
33. If the first term of an A.P. is 2 and the common difference is 4 then the sum of its 40 terms is
    a. 3200 b. 1600 c. 200 d. 2800
    Hint:
    $$S_n = \frac{n}{2}[2a + [n-1]d]$$
    $$= 20[4 + 39 \times 4] = 3200$$
34. if 2x, (x+8), (3x+1) are in A.P. then x=
    a. 3 b. 7 c. 5 d. -2
    Hint: Here, as they are in A.P.
    $(x+8) - 2x = (3x+1) - (x+8)$
    Or, $8 - x = 2x - 7$
    Or, 3x=15 , x=5
35. if A,G, H denote respectively the arithmetic mean, Geometric mean and Harmonic mean respectively then
    a. $A > G > H$ b. $A < G < H$
    c. $A < H < G$ d. $G > H > A$
    Hint: it is a obvious relation
    $A.M > G.M > H.M.$ i.e $A > G > H$
36. If the sum of geometric progression is 'S', their product is 'p' and the sum of their reciprocals is 'R' then $\left(\frac{S}{R}\right)^n$
    a. P b. P^2 c. $P^{\frac{1}{2}}$ d. P^3
37. If p^{th}, q^{th} *and* r^{th} terms of both A.P and G.P be a, b and c then $a^{b-c}\, b^{c-a}\, c^{a-b} =$
    a. pqr b. $p^2q^2r^2$ c. $\frac{p+q}{2}$ d. 1
38. If a^2, b^2, c^2 are in AP then $b+c, c+a, a+b$ are in
    a. A.P b. G.P c. H.P d. none
39. If $(b-c)^2, (c-a)^2, (a-b)^2$ are in A.P then $\frac{1}{b-c} + \frac{1}{c-a} + \frac{1}{a-b}$ *are* in
    a. A.P b. G.P c. H.P d. none
40. If a, b, c, d are in G.P then $a^2 - b^2, b^2 - c^2, c^2 - d^2$ are in
    a. A.P b. G.P c. H.P d. none

41. The sum of an infinite number of terms of G.P is 23 and sum of their squares is 69. Then 'r' =

a. $\frac{69}{13}$ b. $\frac{3}{17}$ c. $\frac{10}{13}$ d. 1

42. Sum to n^{th} terms of series 1+3+6+10+...

a. $\frac{n(n+1)(2n+1)}{3}$ b. $\frac{n(n+1)(n+2)}{6}$

c. $\frac{n(n+1)(2n+1)}{3}$ d. $\frac{n(n+1)(n+2)}{3}$

43. Sum to n terms of series 0.7+0.77+0.777+...

a. $\frac{9}{7}\left[n-\frac{1}{9}\left(1-\frac{1}{10^n}\right)\right]$ b. $\frac{7}{9}\left[n-\frac{7}{9}\left(n-\frac{1}{10^n}\right)\right]$

c. $\frac{7}{9}\left[n-\frac{1}{9}\left(1-\frac{1}{10^n}\right)\right]$ d. $\frac{9}{7}\left[n-\frac{9}{7}\left(1-\frac{1}{10^n}\right)\right]$

44. $1+(1+a)r+(1+a+a^2)r^2+(1+a+a^2+a^3)r^3+\cdots,\infty$

a. $\frac{1}{(1-r)(a-r)}$ b. $\frac{1-ar}{1-r}$

c. $\frac{1-r}{1-ar}$ d. $\frac{1}{(1-r)(1-ar)}$

45. $1-2x+3x^2-4x^3+\cdots..\infty\ (|x|<1)$

a. $\frac{1+x}{(1-x)^2}$ b. $\frac{(1-x)^2}{1+x}$

c. $\frac{1}{(1+x)^2}$ d. $\frac{-1}{(1+x)^2}$

46. Sum to n terms : $1+2x+3x^2+4x^3+\cdots|x|<1$

a. $\frac{1-x^n}{1-x^2}-\frac{nx^n}{1-x}$ b. $\frac{x^n-1}{1-x^2}-\frac{nx^n}{1-x}$

c. $\frac{(1-x^n)nx^n}{(1-x^2)^2}$ d. $\frac{(x^n-1)nx^n}{(1-x)^2}$

47. Sum to n terms $1+\frac{1}{1+2}+\frac{1}{1+2+3}+\cdots$

a. $\frac{2n}{n+1}$ b. $\frac{n}{n+2}$

c. $\frac{n^2}{n+2}$ d. $\frac{3n-1}{n+2}$

Hint: $t_n=\frac{2}{n(n+1)}=2\left(\frac{1}{n}-\frac{1}{n+1}\right)$

48. $2+2^2+2^3+\cdots+2^n=$

a. 2^n-1 b. $2(2^n-1)$

c. $2^n(2^{n+1}-1)$ c. 0

49. Value of $2^{\frac{1}{4}}.4^{\frac{1}{8}}.8^{\frac{1}{16}}.16^{\frac{1}{32}}....\infty$ is

a. $\sqrt{2}$ b. $\frac{1}{2}$ c. -1 d. $\frac{3}{2}$

50. If $x=1+a+a^2+\cdots+\infty, a<1,$

$y=1+b+b^2+\cdots+\infty, b<1$

Then, value of $1+ab+a^2b^2+\cdots+\infty=$

a. $\frac{xy}{x+y+1}$ b. $\frac{x+y+1}{zxy}$

c. $\frac{xy}{x+y-1}$ d. $\frac{xy}{x-y-1}$

51. If m^{th} term of AP is n and n^{th} term is m then r^{th} is

a. $(m-n-r)$ b. $(m+n-r)$

c. $(m+n-r)$ d. $n-r-m$

52. If a, b, c are in $G.P$ and $a^{\frac{1}{x}}=b^{\frac{1}{y}}=c^{\frac{1}{z}}$ then x,y,z are in

a. A.P b. G.P c. H.P d. none

53. If x, y, z > 1 are in G.P then $\frac{1}{1+\ln x},\frac{1}{1+\ln y},\frac{1}{1+\ln z}$ are in

a. A.P b. G.P c. H.P d. none

54. General terms of series

$$1+\frac{4}{5}+\frac{7}{5^2}+\frac{10}{5^3}+\cdots+n\ terms$$

a. $\frac{2n-1}{5^n}$ b. $\frac{3^{n+1}}{5^{n-1}}$ c. $\frac{3n-2}{5^{n-2}}$ d. $\frac{4n+1}{5^n}$

55. $\frac{1}{1.2}+\frac{1}{3.4}+\frac{1}{5.6}+\cdots\infty$ is

a. $\log_e 2$ b. $1-\log_e 2$ c. e^n d. a^x

56. If p^{th} term of a G.P is 'a' and q^{th} term is b [illegible] n^{th} term

a. $\left(\frac{a^{n-q}}{b^{n-p}}\right)^{\frac{1}{p-q}}$ b. $\left(\frac{b^{n-q}}{a^{n-p}}\right)^{\frac{1}{p-q}}$

c. $\left(\frac{a}{b}\right)^{\frac{n-q}{p-q}}$ d. $\left(\frac{a^{n-p}}{b^{n-q}}\right)^{\frac{pq}{p-q}}$

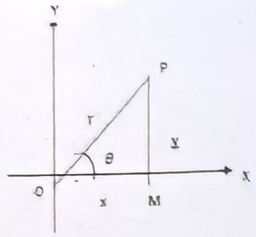
57. The sum of first 'n' terms of an A.P is $3n^2$ [illegible] Then r^{th} term is:

a. $3r-1$ b. $6r-1$

c. $4r-1$ d. r^2+2r

Answer Key

1. b	2. c	3. a	4. a	5. [illegible]
6. a	7. c	8. b	9. c	10. [illegible]
11. b	12. b	13. c	14. b	15. [illegible]
16. c	17. d	18. c	19. a	20. [illegible]
21. c	22. a	23. c	24. c	25. [illegible]
26. b	27. d	28. a	29. a	30. [illegible]
31. a	32. c	33. a	34. c	35. [illegible]
36. b	37. d	38. c	39. a	40. b
41. c	42. b	43. c	44. d	45. [illegible]
46. a	47. a	48. b	49. a	50. [illegible]
51. b	52. a	53. c	54. c	55. [illegible]
56. a	57. b			



Properties:

i. Arg (z_1z_2)= Argz_1+ Argz_2

ii. Arg $\left(\frac{z_1}{z_2}\right)$= Arg$z_1$- Arg$z_2$

iii. Arg $(\bar{z})$=2π-Arg(z)

iv. Arg (z^n)=nArg(z)

iii. Exponential form:

$$\bar{z} = e^{i\theta}$$

where $e^{i\theta} = \cos\theta + i\sin\theta$

Product and Quotient of Complex Number:

Let,

$z_1=r_1(\cos\theta_1 + i\sin\theta_1)$

$z_2=r_2(\cos\theta_2 + i\sin\theta_2)$

$z_1z_2= r_1r_2[\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)$

$\frac{z_1}{z_2}=\frac{r_1}{r_2}[\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2)$

De-Moivre's Theorem:

For any real number n (positive or negative or zero).

$[r(\cos\theta + i\sin\theta)]^n = r^n[\cos n\theta + i\sin n\theta]$

Note:

i) $(\cos\theta + i\sin\theta)^n=\cos n\theta - i\sin n\theta$

ii) $(\cos\theta - i\sin\theta)^n=\cos n\theta - i\sin n\theta$

Multiple Choice Questions

1. The sum of three cube roots of unity is [IOST Model]
a. 0 b. 1 c. i d. −i

2. The value of $\frac{(\cos\theta+i\sin\theta)^n}{(\cos\theta-i\sin\theta)^n}$ is
a. $1-i\sqrt{3}$ b. $1+i\sqrt{3}$ c. $i\sqrt{3}$ d. 1 [IOST Model]

3. The value of $(1-w)(1-w^2)$, where w is the cube root of in complex number is [IOST 2066]
a. 0 b. 1 c. 2 d. 3

4. The magnitude of $(2-3i)$ is [IOST 2069]
a. $\sqrt{13}$ b. $\sqrt{5}$ c. 1 d. 13

5. The absolute value of the complex number $(1-i)^{-1}$ is [IOST 2070]
a. 1 b. $\sqrt{2}$ c. $\frac{1}{\sqrt{2}}$ d. 2

6. If $z=\cos\theta + i\sin\theta$, then $z^n+\frac{1}{z^n}$ equal to [IOST 2070]
a. $2\cos n\theta$ b. $2\sin n\theta$
c. $2\cos(n\theta)$ d. $2\sin(n\theta)$

7. The value of $i^7+\frac{1}{i^4}$ is [IOST 2071]
a. 1 b. -1 c. −i d. −i+1

8. The value of $\frac{x+wy+w^2z}{y+zw+w^2x}$, where w is the cube root of unity is [IOST 2071]
a. w b. x+y+z c. w^2 d. 1

9. The real part of $\frac{(1+i)^2}{2-i}$ is [IOST 2072]
a. $-2/5$ b. $2/5$ c. $1/5$ d. 1

10. $(1+w+w^2)(1+w-w^2) =$ [IOST 2072]
a. 0 b. 2 c. 4 d. 3

11. $(1+i)^6 + (1-i)^6 =$ [IOST 2073]
a. 0 b. 1 c. i d. -1

12. $\left(\frac{2i}{1+i}\right)^2 =$ [IOST 2073]
a. −I b. 1+i c. 1 d. -2i

13. $[2(\cos 15° + i\sin 15°)]^6=$ [IOST 2074]
a. 64i b.64 c. 0 d. -64i

14. $(1+w+w^2)^3 + (1+w-w^2)^3 =$ [IOST 2074]
a. 16 b. 16w c. -8 d. $16w^2$

15. If w is the cube root of unity, then $(1-w+w^2)^3 =$ [IOST 2075]
a. 0 b. -8 c. 1 d. 8

16. $\left|\frac{1}{1+i}\right|=$ [IOST 2075]
a. $\frac{1}{2}$ b. $\sqrt{2}$ c. $\frac{1}{\sqrt{2}}$ d. 2

17. $(0,1)^2 =$
a. i b. −i c. 2i d. 2i

18. If $(x, y)=\frac{(2-i)}{(-1+i)}$, then the values of x and y are
a. $\frac{1}{2}, -\frac{3}{2}$ b. $\frac{3}{2}, -\frac{1}{2}$ c. $\frac{1}{2}$, 0 d. 0, $\frac{1}{2}$

19. The multiplicative inverse of 2+ 3i is
a. $\frac{9}{13}+\frac{21}{13}i$ b. $\frac{3}{13}-\frac{11}{13}i$
c. $\frac{2}{13}-\frac{3}{13}i$ d. None

20. The value of $w^7 + w^8 + w^9$ is
a. 1 b. -1 c. 0 d. w^2

21. The value of $(2+5w+2w^2)^6 =$
a. 729 b. 512 c. 256 d. 128

22. The cube roots of -8 are
a. 1, w, w^2 b. 2, 2w, $2w^2$
c. 2, 2w, $2w^2$ d. 2, −2w, $2w^2$

23. The roots of the equation $(x-2)^3-8=0$ are
a. 1, 3+2w, 3+2w^2 b. −5, −3−2w, −3−2w^2
c. 0, 2w, 2w^2 d. 5, 5w, $5w^2$

24. The fourth roots of unity are
a. ±1, ±i b. 1, −1, i, −i
c. −1, i, −i, ±1 d. None

25. If $(x+ia)(x+ib)(x+ic)=x^3+x^2+x+1$, then a+b+c=
a. 1 b. −1 c. 6 d. −6

26. The value of $i+i^2+i^3+i^4+i^5+i^6=$
a. 1 b. 0 c. −1 d. −i

27. The complex number $\frac{1+2i}{1-i}$ lies in .....quadrant
a. 1st b. 2nd c. 3rd d. 4th

28. The value of $(1+i)^{20}=$
a. 2^{10} b. 2^{15} c. 2^{20} d. -2^{10}

29. The square roots of 2i are
a. $\pm(2+3i)$ b. $\pm(1+i)$
c. $\pm(1-i)$ d. $\pm(3-2i)$

30. Im (z) is equal to
a. $\frac{1}{2}(z+\bar{z})$I b. $\frac{1}{2}(z-\bar{z})$
c. $\frac{1}{2i}(z-\bar{z})$ d. $\frac{1}{2i}(z+\bar{z})$

31. i^{n+4}
a. 1 b. i^n c. i^{-n} d. i^2

32. The value of i^{-71}
a. 1 b. −i c. i d. 1

33. The value of $(1+i)^4\left(1+\frac{1}{i}\right)^4$
a. 16 b. 4 c. 81 d. 64

34. $i^n + i^{n+1} + i^{n+2} + i^{n+3} = \forall, n \in z$
a.0 b. 1 c. 4 d. 9

35. Additive inverse of complex number −3 + 2i is
a. 3 − 2i b. −3 − 2i
c. $\frac{1}{-3+2i}$ d. none

36. Multiplicative inverse of 5i + 2 is
a. $\frac{3}{19}-\frac{5i}{19}$ b. $\frac{-5}{29}+\frac{2i}{29}$
c. $\frac{-5i}{29}+\frac{2}{29}$ d. $\frac{-5i}{29}-\frac{2}{29}$

37. The modulus of $\frac{1}{3+i}$
a. $\frac{3}{\sqrt{2}}$ b. $\frac{-1}{\sqrt{2}}$ c. $\frac{\sqrt{3}}{4}$ d. $\frac{1}{\sqrt{2}}$

38. If $x+iy=\frac{(a+1)^2}{2a-i}$, then x^2+y^2
a. $\frac{(a+1)^2}{4a^2-1}$ b. $\frac{(a+1)^2}{4a^2+1}$
c. a. $\frac{(a+1)^4}{4a^2+1}$ d. a. $\frac{(a+1)^6}{4a^2-1}$

39. $\left|\frac{2+i}{4i(1+i)^2}\right|$
a. $\frac{\sqrt{5}}{8}$ b. $\frac{\sqrt{5}}{2}$ c. $\frac{\sqrt{5}}{4}$ d. $\frac{1}{\sqrt{2}}$

40. Square root of −8 − 6i
a. ±(1 + 2i) b. ±(1 − 3i)
c. ±(2 − 2√3i) d. ±(1 − i)

41. $(x-y)(xw-y)(xw^2-y) =$
a. x^2-y^2 b. $x-y$
c. x^2+y^2 d. x^3-y^3

42. $(1-\omega+\omega^2)^4+(1+\omega-\omega^2)^4 =$
a. −2 b. −16 c. 16 d. 4

43. The value of $\left(\frac{-1+\sqrt{-3}}{2}\right)^{27}+\left(\frac{-1-\sqrt{-3}}{2}\right)^{27} =$
a. 1 b. 2 c. −1 d. i

44. If 'a' and 'b' be the imaginary cube roots of unity then $a^2+b^2-ab =$
a. −2 b. 2 c. −3 d. 3

45. $-1+\sqrt{-3}$ can be written as:
a. $\sqrt{2}\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)$
b. $\sqrt{3}\left(\cos\frac{2\pi}{3}-i\sin\frac{2\pi}{3}\right)$
c. $\frac{1}{\sqrt{2}}\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)$
d. $2\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)$

46. $4e^{i\pi}$ in standard form:
a. −4 b. -3 c. i^2 d. −i
Hints: $e^{i\theta} = (\cos\theta + i\sin\theta)$

47. $(-1+i)^{14} =$
a.−2i b. $2^3 i$ c. $2^7 i$ d. $\frac{-1}{\sqrt{2}}i$

48. The value of $\frac{(\cos\theta-i\sin\theta)^{10}}{(\cos\theta+i\sin\theta)^{12}}$
a. $\cos 12\theta - i\sin 10\theta$ b. $\cos 2\theta + i\sin 22\theta$
c. $\sin 22\theta - i\cos 22\theta$ d. $\cos 22\theta - i\sin 22\theta$

49. If $z = e^{i\theta}$ then $z^n - \frac{1}{z^n}$
a. $2\cos\theta$ b. $2i\cos\theta$ c. $2i\sin\theta$ d. 0
Hints: $[e^{i\theta} = \cos\theta + i\sin\theta]$

50. $(-2\sqrt{3}+2i)^3 =$
a. 32 i b. −32 i c. −64 i d. 64 i

51. $\left|\frac{z-1}{z+1}\right| = 1$ represents
a. circle b. ellipse
c. st. line d. pair of st. line

52. If $(1-i)^n = 2^n$ then n=
a. 1 b. 0 c. -1 d. 2

53. Amplitude of $z = \left(\frac{1+i\sqrt{3}}{\sqrt{3}+i}\right)$ is
a. $\frac{\pi}{3}$ b. $\frac{\pi}{4}$ c. $\frac{\pi}{6}$ d. $\frac{\pi}{2}$

54. if n is not a multiple of 3 then
$$\left(\frac{-1-\sqrt{3}i}{2}\right)^n+\left(\frac{-1+\sqrt{3}i}{2}\right)^n =$$
a. 1 b. -1 c. 2 d. 0

55. The value of $\sqrt{-1-\sqrt{-1-\sqrt{-1\ldots\ldots\infty}}}$
a. $\omega \neq \omega^2$ b. -1 c. ω, ω^2 d. 0

56. Square root of ω is
a. ω^2 b $-\omega^2$ c. $\pm\omega^2$ d. $\pm\omega^2$

57. If $\arg(z) = \frac{\pi}{2}$ then z is
a. purely real
b. purely imaginary
c. sometimes real & sometime irregularly
d. none

58. $Arg(\bar{Z})$
a. $2\pi - Arg(\bar{Z})$ b. $-Arg(\bar{Z})$
c. $\pi - Arg(\bar{Z})$ d. $\frac{\pi}{2} - Arg(\bar{Z})$

59. $Z - \bar{Z}$ is equal to
a. $2Re(Z)$ b. $2i\,Im(Z)$
c. $2\,Re(\bar{Z})$ d. $-2Re(Z)$

60. $|Z|^2$ Implies

a. $z.z$ b. $z.|z|$ c. $z.\bar{z}$ d. none

61. If $\left|\frac{z-4}{z-8}\right| = 1$ Then Re(z)=

a. -6 b. 6 c. 8 d. 17

62. Additive inverse of The fourth root of unity are:

a. $\pm 2, \pm\sqrt{2}\, i$ b. $\pm 1, \pm i\sqrt{3}$

c. $\pm 1, \pm i$ d. $\pm\sqrt{2}, \pm\sqrt{2}\, i$

63. Cube root of unity on Argand plane forms

a. circle b. st. line

c. Equilateral triangle d. parabola

64. if $\left(\frac{1-i}{1+i}\right)^{1000} = x + iy\,; x, y \in R$ then,

a. $x = 1, y = 0$ b. $x = -2, y = -1$

c. $x = -\frac{1}{2}, y = \frac{1}{2}$ d. $x = -1, y = 0$

65. $\left|\frac{z-2}{z+2}\right| = 1$ is a

a. ellipse b. parabola

c. St. line d. circle

Answer Key:

1. a	2. b	3. d	4. a	5. c
6. b	7. d	8. a	9. a	10. a
11. a	12. c	13. a	14. c	15. b
16. c	17. b	18. b	19. c	20. c
21. a	22. c	23. a	24. a	25. d
26. b	27. b	28. d	29. b	30. c
31. b	32. c	33. a	34. a	35. a
36. c	37. d	38. c	39. a	40. b
41. d	42. b	43. c	44. a	45. d
46. a	47. c	48. d	49. c	50. d
51. c	52. b	53. a	54. b	55. c
56. c	57. b	58. a	59. b	60.
61. b	62. c	63. c	64.	

i. If one root (α)=p+$\sqrt{q}$, then other root (β)= p-$\sqrt{q}$

and

ii. If one root(α)=p+iq, then other root (β)=p-iq

8. If α, β, γ be the roots of a cubic equation $ax^3 + bx^2 + cx + d = 0; a \neq 0$, then

a. $\alpha + \beta + \gamma = \frac{-b}{a}$

b. $\alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a}$

c. $\alpha\beta\gamma = \frac{-d}{a}$

9. Let $a_1x^2+b_1x+c_1=0$ and $a_2x^2+b_2x+c_2=0$ be two quadratic equations having one root common.

Then, $(b_1c_2 - b_2c_1)(a_1b_2 - a_2b_1) = (c_1a_2 - c_2a_1)^2$

10. Let $a_1x^2+b_1x+c_1=0$ and $a_2x^2+b_2x+c_2=0$ be two quadratic equations having both roots common. Then,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Multiple Choice Questions

1. If α=-3, β=2 be the two roots of an equation $ax^2 + bx + c = 0$, then the equation is [IOST Model]
   a. $x^2 + x + 6 = 0$ b. $x^2 + x - 6 = 0$
   c. $x^2 - x - 6 = 0$ d. $x^2 - x + 6 = 0$

2. 1 and 2 are the roots of [IOST 2069]
   a. $x^2 - 3x + 2 = 0$ b. $x^2 + 3x + 2 = 0$
   c. $x^2 + x + 2 = 0$ d. $x^2 - 3x + 1 = 0$

3. For what value of k will be the equation $5x^2 - kx + 45 = 0$ have equal roots? [IOST 2070]
   a. 0 b. ±30 c. ±90 d. ±33

4. The quadratic equation whose one root is 2+$\sqrt{3}$ is [IOST 2070]
   a. $x^2 + 4x + 1 = 0$ b. $x^2 - 4x + 1 = 0$
   c. $x^2 - 4x - 1 = 0$ d. $-x^2 - 4x - 1 = 0$

5. If the roots of $x^2 + 4x + k = 0$ are equal, then k is [IOST 2071]
   a. 0 b. 4 c. 16 d. -4

6. The value of k for which the equation $x^2 + 2(k + 2)x + 9k = 0$ has equal roots are [IOST 2072]
   a. 4, 1 b. -4,-1 c. -4, 1 d. 4,-1

7. If one root of $5x^2 + 2k - k = 0$ is the reciprocal of the other, then k= [IOST 2073]
   a. -5 b. 5 c. $\frac{2}{5}$ d. $-\frac{2}{5}$

8. The quadratic equation whose roots are -3 and 2 is [IOST 2074]
   a. $x^2 + x - 6 = 0$ b. $x^2 + x + 6 = 0$
   c. $x^2 - x - 6 = 0$ d. $x^2 - x + 6 = 0$

9. The quadratic equation whose roots are 3 and 4 is [IOST 2075]
   a. $x^2 - 7x - 12 = 0$ b. $x^2 - 7x + 12 = 0$
   c. $x^2 + 7x + 12 = 0$ d. $x^2 + 7x - 12 = 0$

10. The number of zeros of the equation $4x^4 + 3x^3 - 12x^2 + 8x + 2 = 0$ are
    a. 2 b. 3 c. 4 d. 5

11. When $x^3 + 3x^2 - kx - 4$ is divided by (x-2), then the remainder is k, then the value of k is
    a. 5 b. 8 c. 3 d. 2

12. The remainder when $x^3 - 2x^2 + 1$ is divided by $(x - 1)$ is
    a. 4 b. -2 c. 0 d. 1

13. If p and q are the roots of the equation $x^2 + px + q = 0$, then the values of p and q are
    a. 1, -1 b. 1, 0 c. 1, -2 d. -1, 0

14. The A.M of the roots is A and G.M is the G, then the equation is
    a. $x^2 - 2Ax + G^2 = 0$ b. $x^2 + 2Ax + G^2 = 0$
    c. $x^2 - 2Gx + A^2 = 0$ d. $x^2 + 2Gx + A^2 = 0$

15. If one root of the equation $3x^2 - 5x + (k - 4) = 0$ is zero, then k=
    a. 3 b. -4 c. 4 d. 5

16. The roots of the equation $4x^2 - (k - 7)x + 8 = 0$ are equal in magnitude but opposite in sign, then k=?
    a. 7 b. -4 c. 8 d. $1/2$

17. The roots of the equation $5x^2 - px + 30 = 0$ are equal, then p=
    a. ±15 b. ±30 c. ±10$\sqrt{6}$ d. ±20

18. If the equation $x^2 + px + q = 0$ and $x^2 + qx + p = 0$ have a common root, then
    a. $p + q = 0$ b. $p + q - 1 = 0$
    c. $p - q - 1 = 0$ d. $p - q + 3 = 0$

19. If the roots of the equation $ax^2 + bx + c = 0$ be the ratio 3:4, then
    a. $12b^2 = 49ac$ b. $12a^2 = 49bc$
    c. $12c^2 = 49ab$ d. $3b^2 = 49ac$

20. The equation where roots are the squares of the roots of the equation $x^2 - 3x + 2 = 0$ is
    a. $x^2 - 5x + 4 = 0$ b. $3x^2 - 4x + 12 = 0$
    c. $2x^2 - 9x + 7 = 0$ d. $x^2 + 5x - 3 = 0$

21. $\sqrt{2 + \sqrt{2 + \sqrt{2 + \ldots \infty}}} =$
    a. $\sqrt{2}$ b. $1/2$ c. 2 d. None

22. if the sum of the root of the equation $x^2 + px + q = 0$ is equals to sum of their square then
    a. $p^2 + p = 2q$ b. $p^2 - p = 2q$
    c. $p^2 + 2q = p$ c. $p^2 + 4p = q$

23. if the roots of $ax^2 + b = 0$ are real and distinct then
    a. $ab > 0$ b. $ab < 0$
    d. a>0, b>0

24. For what value of p the difference of the roots of the equation $x^2 - px + 8 = 0$ is 2?
a. 4 b. 8 c. 6 d. 9

25. Both the roots of the equation $x^2 + ax + b = 0$ & $x^2 + dx + e = 0$ are common then
a. ab=de b. $d^2e = a^2b$
c. ae=bd d. ad=be

26. Quadratic equation with rational coefficient with one root $5 + 3i$
a. $24x^2 - 10x + 1 = 0$
b. $x^2 - 10x + 24 = 0$
c. $12x^2 - 10x + 1 = 0$
d. $34x^2 - 5x + 1 = 0$

27. The quadratic equation whose roots are the square of roots of $3x^2 - 5x - 2 = 0$
a. $9x^2 - 37x + 4 = 0$
b. $2x^2 - 5x - 3 = 0$
c. $9x^2 - 10x - 4 = 0$
d. $x^2 - 12x + 9 = 0$

28. If the roots of equation $x^2 - mx + (m + 1) = 0$ may have its roots in ratio 2:3 then m=
a. 2 b. $\frac{1}{5}$ c. 5 d. 3

29. If 'r' be the ratio of roots of $ax^2 + bx + c = 0$ then
a. $\frac{a^2}{bc}$ b. $\frac{b^2}{ac}$ c. $\frac{c^2}{ab}$ d. $abc = 1$

30. If x is real ,the maximum value of $(5 + 4x - x^2)$ is
a. 5 b. 9
c. 1 d. 2

31. If the roots of equation $x^2 + ax + c = 0$ differ by 1 then
a. $a^2 = 4c + 1$ b. $a^2 = 4c - 1$
c. $c^2 = 4a + 1$ d. $a^2 = 2c + 1$

32. A polynomial where all coefficients 0 is
a. real polynomial b. complex polynomial
c. Zero polynomial d. no polynomial

33. Degree of $ax^2 + bx + c = 0$ is
a. 2 b. 3 c. 1 d. undefined

34. The number of roots of equation $3x^4 - 7x^2 + 6 = 0$ is
a. 6 b. 1 c. 4 d. 2

35. The quotient and remainder when $(x^3 + 5x^2 - 18)$ is divided by $x + 3$ are
a. $2, (x^2 + 5x + 3)$ b. $0, (x^2 + 2x - 6)$
c. $1, (x - 4)$ d. $4, (x^2 + x - 4)$

36. If one root of the equation $ax^2 + bx + c = 0$ is $\sqrt{5} + 2$ then the other root is
a. $\sqrt{5} - 2$ b. $\sqrt{5} + 2$
c. $2 - \sqrt{5}$ d. $-2 - \sqrt{5}$

37. The roots of equation $x^2 + px + q = 0$ are in same ratio as those of $x^2 + lm + m = 0$ then
a. $pm^2 = lq^2$ b. $mp^2 = lq^2$
c. $pl^2 = qm^2$ c. $p^2m = l^2q$

38. If the sum of roots of equation $(m + 1)x^2 + (2m + 3)x + (3m + 4) = 0$ is zero then product is
a. 1 b. -1 c. 0 d. $\frac{1}{2}$

39. If α and β are the roots of equation $3x^2 - 5x + 19 = 0$ then the value of $\frac{1}{\alpha} + \frac{1}{\beta}$ is
a. $\frac{7}{5}$ b. $\frac{5}{19}$
c. $\frac{19}{7}$ d. $\frac{19}{3}$

40. For what value of 'k' the equation $4x^2 + kx + 1 = 0$ have equal roots?
a. ± 3 b. ± 6 c. ± 4 d. ± 2

41. The roots of the equation $x^2 - (2k - 1)x + k^2 = 0$ are real if
a. $k \le \frac{1}{2}$ b. $k \le \frac{1}{4}$ c. $k \ge 1/2$ d. $k \ge \frac{1}{4}$

42. For what value of 'm' will he quadratic eq^n $x^2 - 2(5 + 2m)x + 3(7 + 10m) = 0$ have reciprocal roots?
a. $-\frac{2}{3}$ c. $\frac{2}{3}$ c. $\frac{3}{2}$ d. $-\frac{3}{2}$

43. If roots of equation $(x^2 + nx + n = 0)$ be in the roots $p:q$ then $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} =$
a. $\sqrt{\frac{l}{n}}$ b. $-\sqrt{\frac{l}{n}}$ c. $-\sqrt{\frac{n}{l}}$ d. 0

44. If one roots equation $ax^2 + bx + c = 0$ be four times the other then
a. $4a^2 = 25ab$ b. $4b^2 = 25ac$
c. $4a^2 = 25bc$ d. $4b^2 = 16ac$

45. The equation whose one roots is $\sqrt{3} + 2$
a. $x^2 - 4x + 1 = 0$ b. $x^2 - 2\sqrt{3}x - 1 = 0$
c. $x^2 + 2\sqrt{3}x - 1 = 0$ d. $x^2 - 2\sqrt{3}x + 1 = 0$

46. If both the roots of the equation are negative then
a. $ac > 0$ b. $ac = 0$
c. $ac < 0$ d. $b^2 = ac$

47. Roots of $2x^2 - 6x + 7 = 0$ are
a. real b. real and rational
c. real, rational & unequal d. imaginary

48. The condition for polynomial equation $ax^2 + bx + c = 0$ to be quadratic
a. $a > 0$ b. $a < 0$
c. $a \ne 0, c \ne 0$ d. $a \ne 0$

49. Both the roots of the equation $3x^2 - (k - 5)x + \lambda = 0$ then value of k and λ are
a. 3,5 b. 3,0
c. $-3, -5$ d. 5,0

50. The roots of equation $x^2 + (2k - 1)x + k^2 = 0$ are real if
a. $k = 0$ b. $k \ge \frac{1}{2}$
c. $k \le \frac{1}{4}$ d. $k \ge 0$

51. The number of real solutions of the equation $(x - 1)^2 + (x - 2)^2 + (x - 3)^2 = 0$ are
a. 1 b. 2 d. 0

52. The value of k so that the equation $3x^2 + kx - 2 = 0$ roots sum equal to 6
a. 12 b. -16
c. 20 d. -18

53. The number of real roots of $x^2 - 4|x| + 3 = 0$
a. 0 b. 1 c. 2 d. 4

54. The minimum value of $x^2 + 8x + 17$ is
a. 0 b. -1 c. 1 d. 17

Answer Key:

1. b	2. a	3. c	4. b	5. b
6. a	7. a	8. a	9. b	10. c
11. b	12. c	13. c	14. a	15. c
16. a	17. c	18. b	19. a	20. a
21. c	22. a	23. b	24. c	25. c
26. b	27. a	28. c	29. b	30. b
31. a	32. c	33. d	34. c	35. b
36. c	37. d	38. a	39. b	40. c
41. b	42. a	43. c	44. b	45. a
46. b	47. d	48. d	49. b	50. c
51. d	52. d	53. c	54. c	

Chapter: 8

System of Linear Equa

Equation: An equity relation is said equation if its is time for certain valu indeterminate.

Linear Equation: An equation of the $by = c$ is a linear equation where a, b, c constants and x, y are the unknown variab

Two or more than linear equations co system of linear equation.

Example: $a_1x + b_1y = c_1 \ldots\ldots\ldots.1$

$a_2x + b_2y = c_2 \ldots\ldots\ldots.2$

is a system of linear equations.

If the system of equations have a common then equations are called simultaneous equa

Roots of Equation: The value of the set which satisfies the given equation are know roots of equations.

For eg: $x^2 + 5x + 6 = 0$

$\Rightarrow x^2 + 3x + 2x + 6 = 0$

$\Rightarrow x(x + 3) + 2(x + 3) = 0$

$\Rightarrow (x + 2)(x + 3) = 0$

Either, $x + 2 = 0$ *or* $x + 3 = 0$

$\therefore x = -2 \quad \therefore x = -3$

Hence, -2 and-3 are the roots of the equatio $5x + 6 = 0$

Consistent and Inconsistent Systems

Consistent: System of equations is said consistent if it has a solution whether uni infinite no of solution.

Inconsistent: If the system has no solution, is in consistent.

Trivial and non-trivial solution:

A solution in which every variable has value is trivial solution *i.e.* $x = y = z = 0$

A non-zero solution is called non-trivial solutio

Note:

1. For equation of the type $sin\theta = k$ or $cos\theta = k$, we must check $|k| \leq 1$.
2. Do not cancel the common variable factor from the two sides of the equation because we may lose some solutions.
3. To solve the equation of the type $asin\theta + bcos\theta = c$, divide both sides of the equation by $\sqrt{a^2 + b^2}$. The equation possesses the solution only if $|c| \leq \sqrt{a^2 + b^2}$; otherwise not.

Multiple Choice Questions

1. The solution of $cosx = \frac{1}{2}$ is [Entrance 2069]

   a. $2n\pi$ b. $\frac{\pi}{3}$

   c. $2n\pi \pm \frac{\pi}{3}$ d. $2n\pi + \frac{\pi}{3}$

2. Solution of $tanax = cotbx$ is [Entrance 2070]

   a. $\frac{2n+1}{a+b}\frac{\pi}{4}$ b. $\frac{2n-1}{a-b}\frac{\pi}{2}$

   c. $\frac{2n+1}{a+b}\frac{\pi}{2}$ d. $\frac{\pi}{2}$

3. If $\cos^2\theta = \frac{1}{2}$, then θ is [Entrance 2071]

   a. $n\pi \pm \frac{\pi}{4}$ b. $n\pi + \frac{\pi}{4}$

   c. $n\pi \pm \frac{\pi}{3}$ d. $2n\pi$

4. If $\sin^2 x = 1$, then x is [Entrance 2072]

   a. $\frac{\pi}{2}$ b. ± 1 c. $-\frac{\pi}{2}$ d. $n\pi \pm \frac{\pi}{2}$

5. If $cosec^2 x - 2 = 0$, then x is [Entrance 2073]
a. $n\pi \pm \frac{\pi}{6}$ b. $\frac{\pi}{4}$
c. $n\pi \pm \frac{\pi}{4}$ d. $n\pi + \frac{\pi}{2}$

6. If $\tan^2 x = 3$, then x is [Entrance 2074]
a. $n\pi + \frac{\pi}{3}$ b. $n\pi - \frac{\pi}{3}$
c. $n\pi \pm \frac{\pi}{3}$ d. $n\pi$

7. If $\tan^2 x = 1$, then x is [Entrance 2075]
a. $n\pi \pm \frac{\pi}{4}$ b. $n\pi + \frac{\pi}{4}$
c. $n\pi - \frac{\pi}{4}$ d. $\pm\frac{\pi}{4}$

8. If $2\cos^2 x - 5\cos x + 2 = 0$ $(0 \le x \le 360°)$, then $x =$
a. 60°, 180° b. 30°, 360°
c. 60°, 300° d. 120°, 300°

9. If $\cot x + \tan x = 2$, then $x =$
a. $n\pi + \frac{\pi}{4}$ b. $2n\pi + \frac{\pi}{4}$
c. $2n\pi - \frac{\pi}{4}$ d. $2n\pi$

10. If $\sqrt{3}\sin x - \cos x = \sqrt{3}$ for $0 \le x \le 2\pi$, then $x =$
a. 90°, 120° b. 30°, 120°
c. 60°, 150° d. 90°, 150°

11. The general solution of $\tan 3x = 1$ is
a. $n\pi + \frac{\pi}{4}$ b. $\frac{n\pi}{3} + \frac{\pi}{12}$
c. $n\pi$ d. $n\pi \pm \frac{\pi}{4}$

12. The general solution of $\tan 2x \tan x = 1$ is
a. $\frac{\pi}{3}$ b. $(6n \pm 1)\frac{\pi}{6}$
c. $(4n \pm 1)\frac{\pi}{4}$ d. $(2n \pm 1)\frac{\pi}{6}$

13. If $\sin^2\theta = \frac{3}{4}$, then the general value of θ is
a. $n\pi \pm \frac{\pi}{3}$ b. $n\pi + (-1)^n\frac{\pi}{3}$
c. $2n\pi \pm \frac{\pi}{3}$ d. None

14. The solution for $(2\cos x - 1)(2\sin x - \sqrt{3}) = 0$ are:
a. 30°, 150°, 330° b. 60°, 120°, 300°
c. 45°, 135°, 315° d. None of these

15. If $\cos 2A = \sin 3A$, then the value of A is
a. 36° b. 54° c. 18° d. 72°

16. If $\tan^2 x = \frac{1}{3}$, then x
a. $n\pi + \frac{\pi}{3}$ b. $n\pi \pm \frac{\pi}{3}$
c. $n\pi \pm \frac{\pi}{6}$ d. $n\pi \pm \frac{\pi}{2}$

17. The general value of 'x' for $4\cos^2 x + 6\sin^2 x = 5$
a. $n\pi \pm \frac{\pi}{4}$ b. $2n\pi + \frac{\pi}{4}$
c. $2n\pi - \frac{\pi}{4}$ d. $(2n+1)\frac{\pi}{2}$

18. The general value of 'x' which satisfies $\sin x = \frac{1}{\sqrt{2}}$ and $\cos x = \frac{1}{2}$ are
a. $2n\pi + \frac{5\pi}{4}$ b. $2n\pi + \frac{\pi}{4}$
c. $2n\pi + \frac{5\pi}{6}$ d. $2n\pi - \frac{\pi}{6}$

19. The value of x for $\sin x + \sqrt{3}\cos x = \sqrt{2}$
a. $n\pi + \frac{\pi}{6} \pm \frac{\pi}{4}$ b. $2n\pi \pm \frac{\pi}{6} - \frac{\pi}{4}$
c. $2n\pi + \frac{\pi}{6} \pm \frac{\pi}{4}$ d. $n\pi + \frac{\pi}{6} - \frac{\pi}{4}$

20. Period of $\cot 2x$
a. π b. $\frac{\pi}{2}$ c. $\frac{\pi}{3}$ d. 2π

21. The value of x for $2\sin x - 1 = 0$
a. $n\pi + \frac{2\pi}{3}$ b. $n\pi \pm \frac{\pi}{3}$
c. $n\pi + \frac{\pi}{6}$ d. $n\pi + (-1)^n\frac{7\pi}{6}$

22. The general values of $\sin\theta - \cos\theta = \sqrt{2}$
a. $n\pi + \frac{\pi}{4} + (-1)^n\frac{\pi}{2}$ b. $2n\pi \pm \frac{\pi}{4} + \frac{\pi}{3}$
c. $(4n+1)\frac{\pi}{2} + \frac{\pi}{3}$ d. none

23. The general value of $\sec x \tan x = \sqrt{2}$
a. $(6n \pm 1)\frac{\pi}{6}$ b. $n\pi \pm \frac{\pi}{4}$
c. $(2n+1)\frac{\pi}{8}$ d. $n\pi + (-1)^n\frac{\pi}{4}$

24. $\sin x + \cos x = 2$ has
a. unique solution b. no solution
c. finite solution d. infinite solution

25. If $\tan\frac{\theta}{2} = x$ then $\frac{1-x^2}{1+x^2} =$
a. $\cos\theta$ b. $\sin\theta$
c. $\tan\theta$ d. $\sec\theta$

26. If $(\cot A + \tan A) = 4$ then $\cot^4 A + \tan^4 A =$
a. 190 b. 194 c. 188 d. 180
Hint: $\cot A + \tan A = 4$
Squaring on both sides
$\cot^2 A + \tan^2 A + 2\tan A \cot A = 16$
i.e $\cot^2 A + \tan^2 A = 14$
Again, squaring on both sides to get, 194.

27. Value of $\cos 52^0 + \cos 68^0 + \cos 172^0 =$
a. $2\cos 30^0$ b. 1 c. 0 d. -1

28. Value of $\tan 75^0$
a. $\frac{\sqrt{3}-1}{2\sqrt{2}}$ b. $\frac{\sqrt{3}+1}{4}$
c. $\frac{\sqrt{3}+1}{2\sqrt{2}}$ d. $2+\sqrt{3}$

29. The value of $\cos 36^0$.
a. $\frac{(\sqrt{5}-1)}{4}$ b. $\frac{\sqrt{5}+1}{4}$
c. $\frac{\sqrt{10+2\sqrt{5}}}{4}$ d. $\frac{\sqrt{3}-1}{2\sqrt{2}}$

30. Value of $\cot\frac{15}{2}$.
a. $\sqrt{6}+\sqrt{2}+\sqrt{3}+\sqrt{4}$
b. $\sqrt{6}+\sqrt{2}-\sqrt{3}-\sqrt{4}$
c. $\sqrt{6}-\sqrt{2}-\sqrt{3}-\sqrt{4}$
d. $\sqrt{6}+\sqrt{2}+\sqrt{3}-\sqrt{4}$

31. The maximum value of $3\sin x - 4\cos x + 7$
a. 5 b. 12 c. 13 d. 16

32. If $f(x) = \log\left(\frac{1+x}{1-x}\right)$ then $f(x)$ is
a. even function b. odd function
c. constant function d. none

33. The minimum and maximum values of $\sin^4 x + \cos^4 x$.
a. 2, 1 b. 1, $\frac{1}{2}$
c. 3, $\frac{1}{2}$ d. $\frac{1}{2}$, 1

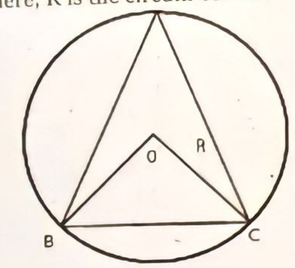
Answer Key:

1. c	2. c	3. a	4. d	5. c
6. c	7. a	8. c	9. a	10. d
11. b	12. b	13. a	14. b	15. c
16. c	17. a	18. c	19. c	20. b
21. d	22. a	23. d	24. b	25. a
26. b	27. c	28. d	29. b	30. a
31. b	32. b	33. d		

Chapter- 13
Properties of Triangles

Properties of Triangle:

1. **Sine Law:**
In any triangle ABC, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$,
Where, R is the circum-radius.



2. **Cosine Law:**
In any triangle ABC,
$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$
$$\cos B = \frac{c^2+a^2-b^2}{2ca}$$
$$\cos C = \frac{a^2+b^2-c^2}{2ab}$$

3. **Projection Law:**
In any triangle ABC,
$$a\cos B + b\cos A = c$$
$$c\cos A + a\cos C = b$$
$$b\cos C + c\cos B = a$$

4. **Tangent Law:**
In any triangle ABC,
$$\tan\left(\frac{A-B}{2}\right) = \frac{a-b}{a+b}\cot\frac{C}{2}$$
$$\tan\left(\frac{B-C}{2}\right) = \frac{b-c}{b+c}\cot\frac{A}{2}$$
$$\tan\left(\frac{C-A}{2}\right) = \frac{c-a}{c+a}\cot\frac{B}{2}$$

5. **Area of a Triangle:**
If Δ is the area of the triangle ABC, then,
(i) $\Delta = \frac{1}{2}bc\sin A = \frac{1}{2}ca\sin B = \frac{1}{2}ab\sin C$
(ii) $\Delta = \frac{abc}{4R}$
(iii) $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{a+b+c}{2}$
(iv) $\Delta = \frac{1}{2} \times base \times height$
(v) $\Delta = \frac{\sqrt{3}}{4}a^2$ (for equilateral triangle only)
(vi) $\Delta =$
$$\frac{1}{4}\sqrt{2a^2b^2+2b^2c^2+2c^2a^2-a^4-b^4-c^4}$$

6. **Half-angle Formula:**
(i) $\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$
(ii) $\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$
(iii) $\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$

Multiple Choice Questions

1. If the sides of a Δ are in the ratio 13 : 7 : 8 then the greatest angle is
a. $\frac{\pi}{3}$ b. $\frac{\pi}{6}$ c. $\frac{\pi}{4}$ d. $\frac{2\pi}{3}$
Hint: Angle opposite to the greatest side is the greatest angle a=13, b=7, c=8
$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

2. If any Δ ABC, if $a = 2c$ and $b = 3c$, then $\cos B$ equals to
a. -1 b. $\frac{1}{2}$ c. $\frac{1}{3}$ d. 1
Hint:
$$\cos B = \frac{c^2+a^2-b^2}{2ca}$$
i. $= \frac{c^2+4c^2-9c^2}{2c(2c)}$
$= -\frac{4c^2}{4c^2}$
$= -1$

3. If $\sin A, \sin B, \sin C$ are in A.P, then a, b, c are in
a. A.P b. G.P c. H.P d. None
Hint:
$\sin A, \sin B, \sin C$ are in A.P
$\sin B - \sin A = \sin C - \sin B$
$\frac{b}{2R} - \frac{a}{2R} = \frac{c}{2R} - \frac{b}{2R}$
$b - a = c - b$
a, b, c are in A.P

4. In Δ ABC, $a = 2, b = 3, c = 4$, then the value of $\angle A =$
a. $\cos^{-1}\frac{1}{7}$ b. $\cos^{-1}\frac{7}{8}$
c. $\cos^{-1}\frac{3}{4}$ d. $\cos^{-1}\left(-\frac{1}{4}\right)$

5. Centroid divides the median of a triangle in the ratio
a. 2 : 1 b. 3 : 4 c. 1 : 1 d. 3 : 1

6. If $\cot A, \cot B, \cot C$ are in A.P a^2, b^2, c^2 are in
a. A.P b. G.P c. H.P d. none
Hint:
$$\cot A = \frac{\cos A}{\sin A}$$
so, $\frac{b^2+c^2-a^2}{a}, \frac{c^2+a^2-b^2}{b}$ & $\frac{a^2+b^2-c^2}{c}$ are in A.P
$2bc.\frac{a}{2R}$ $2ca\frac{b}{2R}$ And $2ab\frac{c}{2R}$
$\frac{b^2+c^2-a^2}{2abc}, \frac{c^2+a^2-b^2}{2abc}, \frac{a^2+b^2-c^2}{2abc}$ are in A.P
$(b^2+c^2-a^2), (c^2+c^2-a^2), (a^2+b^2-c^2)$
Are in A.P
Then,
$$(c^2+a^2-b^2) - (c^2+c^2-a^2)$$
$$= (a^2+b^2-c^2) - (c^2+a^2-b^2)$$
Or, $2a^2 - 2b^2 = 2b^2 - 2c^2$
Or, $a^2 + c^2 = 2b^2$
$\therefore a^2 + c^2 = 2b^2$

7. In $\Delta ABC, a = 2, b = 3, c = 4$ then the value of $\angle A =$

a. $\cos^{-1}\left(\frac{1}{7}\right)$ b. $\cos^{-1}\left(\frac{7}{8}\right)$
c. $\cos^{-1}\left(\frac{3}{4}\right)$ d. $\cos^{-1}\left(-\frac{1}{4}\right)$

Hint

$A = 2, b = 3, c = 4$, then

$$\angle A = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2bc}\right)$$

$$\angle A = \cos^{-1}\left(\frac{7}{8}\right)$$

8. The area of the triangle is $80cm^2$ and its perimeter is 8 cm then the radius of a inscribed circle is

a. 40 cm b. 20cm c. 15 cm d. 12cm

Hint

$$\Delta = 80cm^2$$
$$2s = 8cm \Rightarrow s = 4cm$$

Then,

$$r = \frac{\Delta}{s} = \frac{80}{4} = 20cm$$

9. The sides of a Δ are 1, $\sin\theta$, $\cos\theta$ then the greatest angle is

a. 30^0 b. 60^0 c. 90^0 d. 120^0

Hint:

$$h^2 = p^2 + b^2$$
$$1^2 = \sin^2\theta + \cos^2\theta$$
$$1 = 1$$

10. The value of $\frac{1}{ab} + \frac{1}{bc} + \frac{1}{ca} =$

a. $\frac{1}{2Rr}$ b. $\frac{1}{2r_1}$ c. $\frac{1}{2R_1}$ d. $\frac{1}{2R}$

Hint: $\frac{1}{ab} + \frac{1}{bc} + \frac{1}{ca}$

$= \frac{a+b+c}{abc}$

$= \frac{2s}{4R\Delta}$ $\left[\because R = \frac{abc}{4\Delta}\right]$

$= \frac{s}{2R\Delta} = \frac{1}{2R\Delta/s} = \frac{1}{2Rr}$ $\left[\because r = \frac{\Delta}{s}\right]$

11. $\sin\left(\frac{C}{2}\right) =$

a. $\sqrt{\frac{(s-b)(s-c)}{bc}}$ b. $\sqrt{\frac{(s-c)(s-a)}{ca}}$
c. $\sqrt{\frac{(s-a)(s-b)}{ab}}$ d. $\sqrt{\frac{s(s-c)}{ab}}$

12. In any $\Delta ABC, \Delta =$

a. $\frac{abc}{4R}$ b. $\frac{abc}{2R}$
c. $2R^2 \sin A \sin B \cos C$
d. $s\sqrt{(s-a)(s-b)(s-c)}$

13. In any ΔABC, $\tan\left(\frac{A-B}{2}\right) =$

a. $\left(\frac{a+b}{a-b}\right)\tan\frac{A}{2}$ b. $\left(\frac{a-b}{a+b}\right)\cot\frac{C}{2}$
c. $\left(\frac{a-b}{a+b}\right)\tan\frac{C}{2}$ d. $\left(\frac{a+b}{a-b}\right)\tan\frac{C}{2}$

14. In any ΔABC, $2\left(a\sin^2\frac{C}{2} + c\sin^2\frac{A}{2}\right) =$

a. $a + b + c$ b. $2a + b - c$
c. $a + c - b$ d. $a - b - 2c$

15. $\left(\frac{b-c}{a}\right)\cos\left(\frac{A}{2}\right) =$

a. $\cos\left(\frac{A-B}{2}\right)$ b. $\cos\left(\frac{B-C}{2}\right)$
c. $\cot\left(\frac{B-C}{2}\right)$ d. $\sin\left(\frac{B-C}{2}\right)$

16. If $b - a = mc$ then $\cot\left(\frac{B-A}{2}\right)$

a. $\frac{1+m\cos B}{m\sin B}$ b. $\frac{1+m\sin B}{m\cos B}$
c. $\frac{1-m\cos B}{m^2\sin B}$ d. 0

17. In any ΔAbC, $1 - \tan\frac{A}{2}\tan\frac{B}{2} =$

a. $\frac{2a}{a+b+c}$ b. $\frac{2b}{a+b+c}$
c. $\frac{2c}{a+b+c}$ d. $\frac{abc}{a^2+b^2+c^2}$

18. If $(a + b + c)(b + c - a) = 3bc$ then A =

a. 30^0 b. 45^0
c. 75^0 d. 60^0

19. $bc\cos^2\frac{A}{2} + ca\cos^2\frac{B}{2} + ab\cos^2\frac{C}{2} =$

a. s b. $\frac{1}{s^2}$
c. s^3 c. s^2

20. If $a^2 + b^2 + c^2 = 8R^2$ then it is

a. isosceles Δ b. *right angled* Δ
c. *scalene* Δ d. none

21. In any ΔABC, if $a = 13, b = 14, c = 15$ then $\tan\frac{A}{2}$

a. $\frac{1}{\sqrt{5}}$ b. $\frac{2}{\sqrt{5}}$
c. 1 d. $\frac{1}{2}$

22. In any triangle, ABC, $\Delta =$

a. $2R\sin A\sin B\sin C$ b. $4R\sin A\ \sin B\sin c$
c. $2R^2\sin A\ \sin B\sin c$ d. none

23. In ΔABC, if $b = 1$, $c = \sqrt{3}$-1 and $A = 60$ then $a =$

a. $\sqrt{6 - 3\sqrt{3}}$ b. $\frac{\sqrt{5}+1}{4}$
c. $\sqrt{3} - \sqrt{2}$ d. 2

24. If the sides of triangle's are a, b and $\sqrt{a^2 + b^2 + ab}$ then the greatest angles is

a. 72^0 b. 36^0
c. 120^0 d. 70^0

25. The angles of triangle are 105^0 and 15^0. Then, ratio of its sides are,

a. $\sqrt{2}+1 : \sqrt{2}-1 : \sqrt{6}$
b. $\sqrt{3}+1 : \sqrt{3}-1 : \sqrt{6}$
c. $2\sqrt{2}+1 : 2\sqrt{2}-1 : 5$
d. $\sqrt{5}+1 : \sqrt{5}-1 : \sqrt{3}$

26. In any ΔABC, if $A = 45^0, B = 60^0$ *then* $a : c$

a. $\sqrt{3}+1 : 2$ b. $\sqrt{3}-2 : \sqrt{2}$
c. $\sqrt{3}+1 : \sqrt{3}-1$ d. $2 : \sqrt{3}+1$

27. In any ΔABC, if $\cos A = \frac{4}{5}, \cos B = \frac{3}{5}$ then $a : b : c$

a. 1: 2: 3 b. 3: 4: 5
c. 6: 5: 7 d. 1: 3: 3

28. In any ΔABC, $r_1r_2 + rr_3 =$

a. $a^2 + b^2$ b. $\sqrt{a^2 + b^2}$
c. $\frac{2ab}{a+b}$ d. ab

29. In any ΔABC, $r_1r_2 + r_2r_3 + r_3r_1 =$

a. s^2 b. $\frac{1}{s^2}$ c. $s\sqrt{s}$ d. s^3

30. ΔABC is right angled if $r =$

a. $r_1 + r_2 + r_3$ b. $r_1 - (r_2 - r_3)$
c. $r_1 - r_2 + r_3$ d. 90

31. In any $\Delta ABC, a = 13, b = 14, c = 15$ then R

a. 4 b. 21/2
c. $\frac{15}{2}$ d. $\frac{\sqrt{34}}{2}$

32. In ΔABC, then value of $\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos c}{c}$

a. $\frac{a+b+c}{2}$ b. $\frac{a^2+b^2+c^2}{2abc}$
c. $\frac{s}{abc}$ d. $\frac{\Delta}{R}$

33. In $\Delta ABC, \sin A + \sin B + \sin C =$

a. $\frac{s}{r}$ b. $\frac{R}{s}$ c. $\frac{r}{s}$ d. $\frac{s}{R}$

34. The value of $\frac{r_1r_2r_3}{r}$ is

a. s^2 b. R^2 c. r^2 d. Δ^2

35. In ΔABC, $\operatorname{cosec} A =$

a. $\frac{abc}{R}$ b. $\frac{bc}{\Delta}$ c. $\frac{bc}{2\Delta}$ d. $\frac{abc}{2s}$

36. If $a\cos A = b\cos B$ then triangle is

a. equilateral b. isosceles
c. rt. angled d. rt. Angled isosceles

37. If $\cos^2 A + \cos^2 B + \cos^2 C = 1$ then Δ is

a. rt. Angled Δ b. equilateral Δ
c. isosceles Δ d. scene Δ

38. In an equilateral triangle, side $= 2\sqrt{3}$, then circumradius.

a. 1 b. $\sqrt{3}$ c. 2 d. $2\sqrt{3}$

39. The value of $\frac{r_1 - r}{a\tan\frac{A}{2}}$ is

a. R b. r_2 c. r_3 d. 1

40. In ΔABC, $\frac{c - a\cos B}{b - a\cos C} =$

a. $\frac{\sin B}{\sin C}$ b. $\frac{\cos B}{\cos C}$
c. $\frac{\cos A}{\cos B}$ d. $\frac{\sin A}{\sin B}$

Answer Key:

1. d	2. a	3. a	4. b	5. a
6. a	7. b	8. b	9. c	10. a
11. c	12. a	13. b	14. c	15. d
16. a	17. c	18. d	19. d	20. b
21. d	22. c	23. a	24. c	25. b
26. d	27. b	28. d	29. a	30. b
31. c	32. b	33. d	34. a	35. c
36. d	37. a	38. c	39. d	40. a

Unit: 3 Co - ordinate Geometry

Chapter: 14
Straight Lines

1. **Distance formula:**
If $A(x_1, y_1)$ and $B(x_2, y_2)$ be any two points, then
$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

2. **Section formula:**
If $C(x, y)$ divides the join of $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio $m_1 : m_2$,
Then,
$$C(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$
If $C(x, y)$ divides the join of $A(x_1, y_1)$ and $B(x_2, y_2)$ externally in the ratio$m_1 : m_2$,
Then,
$$C(x, y) = \left(\frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \frac{m_1y_2 - m_2y_1}{m_1 - m_2}\right)$$

3. **Mid-point formula:**
If C divides AB in the ratio 1:1 , then
$$C(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

4. **Centroid formula:**
If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of a triangle, then
$$Centroid = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

5. **Area of triangle:**
If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the three vertices of a triangle, then
$$A = \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix}$$
$$= \frac{1}{2}[x_1y_2 - x_2y_1 + x_2y_3 - x_3y_2 + x_3y_1 - x_1y_3]$$

6. **Area of quadrilateral:**
If $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ and $D(x_4, y_4)$be the four vertices of a quadrilateral, then
$$A = \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{vmatrix}$$
$$= \frac{1}{2}[x_1y_2 - x_2y_1 + x_2y_3 - x_3y_2 + x_3y_4 - x_4y_3 + x_4y_1 - x_1y_4]$$

7. **Polar coordinates:**
The coordinate $P(r, \theta)$ is called the polar coordinate if r is the distance of point P from the origin and θ is the angle included between the radius vector and the initial line in counter clockwise (CCW) direction.

Multiple Choice Questions

1. The points (a, 0), (0, b. and (1, 1) are collinear if
a. a+b=ab b. a-b=ab
c. b-a=ab d. a+b+ab=0

2. If p is the length of perpendicular form origin on the line where intercepts on the axes are a and b
a. $p^2 = a^2 + b^2$ b. $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$
c. $a^2 - b^2 = p^2$ d. $\frac{1}{p^2} = \frac{1}{a^2}$

3. The distance between the parallel lines $3x + 5y = 11$ and $3x + 5y = -23$ is
a. 6 b. $\sqrt{34}$ c. $\sqrt{43}$ d. $\sqrt{33}$

4. The Cartesian form of the parametric equation $x = a\cos^3\theta$ and $y = b\sin^3\theta$ is
a. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ b. $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$
c. $\left(\frac{x}{a}\right)^{3/2} + \left(\frac{y}{b}\right)^{3/2} = 1$ d. $\left(\frac{x}{a}\right)^{3} + \left(\frac{y}{b}\right)^{3} = 1$

5. If the lines ax+2y+1=0, bx+3y+1=0 and cx+4y+1=0 are concurrent, then a, b, c are in
a. G.P b. H.P c. A.P d. None

6. The coordinates of foot of perpendicular from (6, 8) to the line through (1,5) and (9, 3) is
a. (2, 1) b. (3, 2) c. (4, 3) d. (5, 4)

7. The equation of the line which has Y-intercept 4 units and is parallel to the line $2x - 3y - 7 = 0$ is
a. $4x + 3y - 15 = 0$ b. $x + y - 7 = 0$
c. $3x - 4y + 5 = 0$ d. $2x - 3y + 12 = 0$

8. The value of k for which the lines x-2y+1=0, 2x-5y+3=0 and 5x-3y+k=0 are concurrent is,
a. 1 b. -2 c. 3 d. -5

9. If 5x-12y-65=0 and 5x-12y+26=0 are equations of a pair of opposite sides of a square, then its area is
a. 25 b. 36 c. 49 d. 72

10. The vertices of a triangle are A(1, 3), B(1,-1) and C(5, 1), then the length of median through vertex A is
a. $3\sqrt{5}$ b. 10 c. $\sqrt{13}$
d. 7

11. The equation of the line passing through the point of intersection of straight lines 3x-y+2=0 and $5x - 2y + 7 = 0$ and having infinite slope is
a. x=3 b. y=2 c. x=y-3 d. 3y=x+1

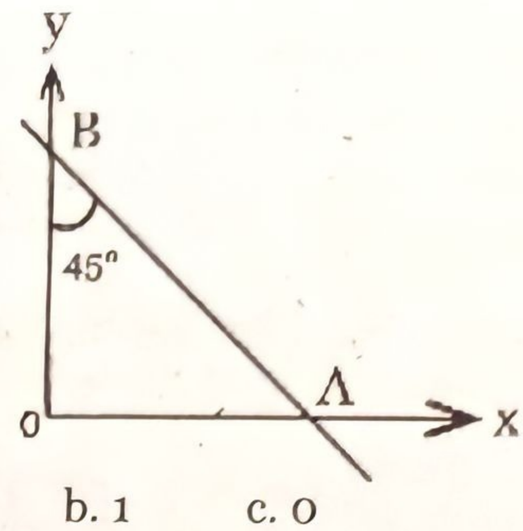
12. If P and Q are the points (-3, 4) and (2, 1) then the coordinates of the point R on PQ produced such that PR=2QR are
a. (1, 2) b. (0, -2) c. (7, -2) d. (3, 4)

13. The points A(6, 6), B(2, 3) and C(4, 7) forms
a. Straight line b. Right angle triangle
c. Isosceles triangle d. Scalene triangle

14. The area of triangle with vertices (p, q+r), (q, r+p), (r, p+q) is
a. pqr b. 2pq c. ½ pq d. 0

15. The ratio in which the point P(-7, 3) divides the join of A(-2, 4) and B(3, 5) is
a. -1:2 b. 2:3 c. 3:4 d. -1:4

16. The slope of the line AB is



a. -1 b. 1 c. 0 d. ½

17. The slope of the line joining (2, 5) and (x, 3) is 2 then the value of x is
a. 2 b. 4 c. -3 d. 1

18. The distance of a point (x, y) from y-axis is
a. x b. y c. $|x|$ d. $|y|$

19. Slope of the line is not defined if
a. Parallel to x-axis b. Parallel to the line y=x
c. Parallel to the line y=-x d. Parallel to y-axis

20. The line $7x - 3y = 5$ and $14x - 6y - 3 = 0$ are
a. Perpendicular b. Parallel
c. Intersecting but not perpendicular
d. Coincident

21. The lines $x + 2y - 3 = 0, 2x + y - 3 = 0$ and the line L are concurrent. If the line L passes through the origin, then its equation is,
a. $x - y = 0$ b. $x + y = 0$
c. $x + 2y = 0$ d. $2x + y = 0$

22. The polar coordinates of $(2, 2\sqrt{3})$ is
a. (2, 30°) b. (4, 60°) c. (2, 55°) d. (2, 90°)

23. The Cartesian coordinates (x, y) of $(2, \frac{\pi}{3})$ is
a. $(2\sqrt{2}, 2\sqrt{2})$ b. (1, 2)
c. $(\sqrt{3}, \sqrt{2})$ d. $(1, \sqrt{3})$

24. The coordinates of 3 consecutive vertices of a parallelogram are (1, 3), (-1, 2) and (2, 5). The coordinate of the fourth vertex are:
a. (2, 3) b. (4, 6) c. (-2, 0) d. (1, 1)

25. If the points (2,3) and (-6, -5) are equidistant from the point (x, y), then
a. $x + y + 3 = 0$ b. $x - y - 5 = 0$
c. $2x + 3y = 1$ d. $3x - 4y + 1 = 0$

26. The equation $r^2 \sin 2\theta = 2c^2$ into Cartesian coordinates is:
a. $xy = c^2$ b. $\frac{x}{y} = 2c^2$
c. $x^2y = 2c^2$ c. *none*

27. The point of concurrency is
a. Curcumin-center b. Ortho-center
c. Median d. Perpendicular bisector

28. Area of triangle whose vertices are $(a, a + c), (a, c)$ and $(a, c - a)$ is
a. $2ac$

29. For what value of k are the points $(1,-1)$, $(k,-1)$ and $(2,-1)$ are collinear?
a. $\frac{1}{2}$ b. 5
c. $\frac{3}{2}$ d. For any value of k

30. Area of quadrilateral whose vertices are (0,0), (3,1), (2,4) & (1,2) is
a. 3 $sq.unit$ b. $2\sqrt{3}$ $sq.unit$
c. 5 $sq.unit$ d. 18 $sq.unit$

31. In what ratio is the line joining the points $(1,-1)$ and (5,6) divides by the line whose equation is $2x - y = 1$?
a. 1:2 b. 3:1 c. 1:3 d. -2:3

32. The centroid of triangle whose vertices are formed by St. Lines $x - 2y = 0$, $x + y = 3$ and $2x + y + 4 = 0$
a. $\left(-\frac{11}{5}, \frac{17}{5}\right)$ b. $\left(\frac{6}{7}, \frac{5}{4}\right)$
c. $\left(\frac{13}{9}, \frac{17}{5}\right)$ d. $\left(3, \frac{15}{26}\right)$

33. Gradient of any vertical line is
a. 0 b. 1 c. 30 d. undefined

34. The line $ax + 4y = 5$ makes an intercept of 3 on the x-axis then length of line segment.
a. $\frac{5}{3}$ b. $\frac{25}{12}$ c. $\frac{17}{5}$ d. 5

35. The equation of st. line that passes thought points $(-5,4)$ and is such that portion of it intercepted between axes is divided by point in 1:2 is
a. $4x - 3y + 15 = 0$ b. $8x - 5y + 60 = 0$
c. $15x - 17y + 33 = 0$ d. $x - 3y + 6 = 0$

36. The equation of line that is at a distance of 4 units form origin and is inclined at angle of 120^0 with x-axis is:
a. $2x - \sqrt{3}y = 8$ b. $x - 2\sqrt{2}y = 0$
c. $\sqrt{3}x + 2y = 6$ d. $\sqrt{3}x + y = 8$

37. Point $p(\alpha, \beta)$ lies on the line $6x - y = 1$ and $Q(\beta, \alpha)$ lies on the line $2x - 5y = 5$. Then eqn of PQ
a. $x + y = 6$ b. $2x - 3y = 8$
c. $x - 2y = 3$ d. none

38. Where do the lines $\frac{x}{a} + \frac{y}{b} = 1$ and $\frac{x}{b} + \frac{y}{a} = 1$ intersect?
a. (1,1) b. $\left(\frac{a}{b}, \frac{b}{a}\right)$
c. $\left(\frac{ab}{a+b}, \frac{ab}{a+b}\right)$ d. $\left(\frac{\sqrt{2}a}{2b}, \frac{\sqrt{2}a}{2b}\right)$

39. For what value of 'c' so that the line $2x + 3y + c = 0$ will turn a triangle with coordinate axes whose area is 27sq unit?
a. ± 12 b. ± 6
c. ± 18 d. ± 24

40. Equation of any lines perpendicular to $ax + by + c = 0$ is
a. $ay - bx + c = 0$ b. $bx + ay - c = 0$
c. $ax - by + c = 0$ d. $bx - ay + k = 0$

41. obtuse angle between the lines $y = 2x + 5$ and $3y = x + 2$ is
a. 45^0 b. 120^0
c. 135^0 d. 180^0

42. Equation of line drawn at right angles to the line $\frac{x}{a} - \frac{y}{b} = 1$ through the point where it meets the x-axis is
a. $ax + by = ab$ b. $ax - by = ab$
c. $bx - ay = ab$ d. $ax + by = ab$

43. for what value of 'k' so that the lines $kx + 3y = 4$ and $3x - 4y = 7$ are perpendicular?
a. 3 b. 4 c. 2 d. -1

44. for what value of 'a' the lines $3x + y - 2 = 0$, $ax + 2y - 3 = 0$ and $2x - y - 3 = 0$ are concurrent.
a. 3 b. 5 c. 3 d. $\frac{5}{2}$

45. The product of perpendicular drawn from two points $(C \pm \sqrt{a^2 - b^2}, 0)$ upon the line $\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$ is
a. $\frac{a^2-b^2}{2ab}$ b. $\frac{2\sqrt{a^2-b^2}}{3b^2}$
c. $\frac{2ab}{a^2-b^2}$ d. b^2

46. The distance of lines $3x - 4y + 7 = 0$ form point $(a, 2)$ is 2 then $a =$
a. $\frac{11}{3}, -3$ b. $\frac{10}{3}, 3$
c. $\frac{5}{3}, 6$ d. $1, \frac{2}{3}$

47. locus is
a. st. line b. point
c. curve traced by variable points
d. equation of circle

48. The bisector of acute angle between the lines $2x + y = 4$ and $4y + 2x = 5$ is
a. $6x + 6y = 13$ b. $4x + 5y = 18$
c. $2x - 3y = 7$ d. $x - 6y = 14$

49. Find the centroid of triangle which is formed by joining the mid points of the triangle whose vertices are (1,2),(0,4) and (5,3)
a. (2,3) b. $(-2,3)$
c. $(2,-3)$ d. $(-2,-3)$

50. Find the orthocenter of the triangle formed by intersecting line 3x+4y=12 with the coordinate axes
a. (4,0) b. (0,3)
c. (4,3) d. (0,0)

Answer key

1. a	2. b	3. b	4. b	5. c
6. d	7. d	8. b	9. c	10. c
11. a	12. c	13. b	14. d	15. a
16. a	17. d	18. c	19. d	20. b
21. a	22. b	23. d	24. b	25. a
26. a	27. b	28. b	29. d	30. c
31. d	32. a	33. d	34. b	35. b
36. d	37. a	38. c	39. c	40. d
41. c	42. d	43. b	44. b	45. d
46. a	47. c	48. a	49. a	50. d

Chapter - 15

Pair of Straight lines

1. The equation
$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0 - -(i)$
is the general equation of 2nd degree in x and y.
Conversely,
If $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are the equation of two straight lines, then their combined equation is
$(a_1x + b_1y + c_1 = 0).(a_2x + b_2y + c_2) = 0$
gives equation (i)

2. Various cases of $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$
i. If $h^2<ab$, conic is an ellipse ($\Delta \neq 0$)
ii. If $h^2=ab$, conic is a parabola ($\Delta \neq 0$)
iii. If $h^2>ab$, conic is a hyperbola ($\Delta \neq 0$)
iv. If a=b, h=o conic is a circle ($\Delta \neq 0$)
v. Rectangular hyperbola if a+b=o
vi. If
$$\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$$
$= abc + 2fgh - af^2 - bg^2 - ch^2 = 0$, then it represents a pair of straight lines.
if $\Delta = 0$ and $h^2 > ab$, then two lines are real and distinct.
if $\Delta = 0$ and $h^2 = ab$, then two lines are coincident.
if $\Delta = 0$ and $h^2 < ab$, then two lines are imaginary.

3. The equation of the form $ax^2 + 2hxy + by^2 = 0$ is called homogeneous equation of 2nd degree in x and y. It always represents a pair of straight lines through the origin. If the lines are $y = m_1x$ and $y = m_2x$, then
$(y - m_1x)(y - m_2x) = ax^2 + 2hxy + by^2 = 0$
where,
$m_1 + m_2 = -\frac{2h}{b}$ and $m_1m_2 = \frac{a}{b}$.

4. Angle between the lines represented by
$ax^2 + 2hxy + by^2 = 0$
Let, θ be the angle between them, then,
$\tan\theta = \pm\frac{2\sqrt{h^2-ab}}{a+b}$
$\therefore \theta = \tan^{-1}\left(\pm\frac{2\sqrt{h^2-ab}}{a+b}\right)$
i) If two lines are coincident, then $\theta = 0°$
$\therefore h^2 = ab$
ii) If two lines are perpendicular, then $\theta = 90°$
$\therefore a + b = 0$

5. The bisector of the angles between the pair of lines represented by $ax^2 + 2hxy + by^2 = 0$ is
$h(x^2 - y^2) = (a - b)xy$

6. Distance between parallel lines represented by general equation of 2nd degree is
$$2\sqrt{\frac{g^2 - ac}{a^2 + ab}} \text{ or } 2\sqrt{\frac{f^2 - bc}{b^2 + ab}}$$
And point of intersection is $= \left(\frac{bg-fh}{h^2-ab}, \frac{af-gh}{h^2-ab}\right)$

Multiple Choice Questions

1. If pair of lines $x^2 - 2pxy - y^2 = 0$ and $x^2 - 2qxy - y^2 = 0$ be such that each pair bisects angle bwteen the other pair then pq=
a. 1 b. -1 c. 0 d. $\frac{1}{2}$

2. The equation $xy + a^2 = a(x + y)$ represents
a. a parabola b. a pair of st.lines
c. an ellipse d. st.line

3. Angle between the pair of lines $y^2\sin^2\theta - xy\sin^2\theta + x^2(\cos^2 - 1) = 0$ is
a. π b. $\pi/2$ c. $\pi/3$ d. $\pi/4$

4. The value of k for which the line pair $x^2 + kxy + 2y^2 + 3x + 5y + 2 = 0$ represents a straight line is,
a. k=3, 9 b. -4 c. 6 d. $7/2$

5. If $2x^2 + 3xy + my^2 = 0$ represents two real and mutually perpendicular lines, then the value of m is,
a. Any –ve real number b. Any +ve real number
c. -2 d. 0

6. The equation $xy + a^2 = a(x + y)$ represnts
a. A parabola b. A pair of straight lines
c. An ellipse d. Hyperbola

7. If the sum of the slopes of the lines $x^2 + kxy - 3y^2 = 0$ is twice the product of the slopes, then k=
a. 1 b. 2 c. -2 d. 0

8. The angle between the lines represented by $2x^2 + 7xy + 3y^2 = 0$ is
a. $\pi/4$ b. $\pi/6$ c. $\pi/2$ d. $\pi/3$

9. The equation of bisectors of the angle between the line represented by $x^2 - 9xy + 18y^2 = 0$ is
a. $x^2 - y^2 = 14xy$ b. $9(x^2 - y^2) = 34xy$
c. $9(x^2 + y^2) = 14xy$ d. $14(x^2 - y^2) = 9xy$

10. The single equation representing the pair of lines y=x and y=-x is
a. $y^2 - x^2 = 0$ b. $y - x = 0$
c. $x - y = 0$ d. $x^2 + y^2 = 0$

11. Which one is non-homogeneous equation?
a. $ax^2 + 2hxy + by^2 = 0$
b. $x^2 + 2xy + y^2 = 0$
c. $ay^3 + by^2x + cyx^2 + dx^3 = 0$
d. $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

12. The general equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents parabola if
a. $b^2 < ab, \Delta \neq 0$ b. $h^2 = ab, \Delta \neq 0$
c. $h^2 > ab, \Delta \neq 0$ d. $a = b, h = 0, \Delta \neq 0$

13. If pair of lies $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ intersect on $y-$ axis then.
a. $2fgh = bg^2 + ch^2$ b. $g^2 = ac$
c. $f^2 + g^2 = 1$ d. $af + bg = c$

14. If $a = b, h = 0$ and $\Delta = 0$ then conic is
a. ellipse b. st.line
c. circle d. hyperbola

15. If equations $x^2 + y^2 + 2gx + 2fy + 1 = 0$ represents a pair of st.lines then
a. $g^2 - f^2 = 1$ b. $f^2 + g^2 = \frac{1}{2}$
c. $g^2 + f^2 = 1$ d. $f^2 - g^2 = \frac{1}{2}$

16. The value of k for which the Equation $2x^2 + 7xy + 3y^2 + kx + 14y + 8 = 0$ represents a pair of straight lines
a. 4 b. −6 c. 8 d. −2

17. General equation of 2nd degree in 'x' and 'y' represents a pair of parallel lines if
a. $af^2 = bg^2$ b. $bf^2 = cg^2$
c. $\frac{a}{f} = \frac{b}{g}$ d. none

18. Distance between the pair of lines represents by $x^2 - 6xy + 9y^2 + 3x - 9y - 4 = 0$ is
a. $\frac{4}{\sqrt{5}}$ b. $\frac{\sqrt{5}}{4}$
c. $\frac{\sqrt{5}}{2}$ d. $\frac{3}{2}$

19. The Length of intercept on x-axis cut off by $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is
a. $\frac{2\sqrt{g^2-ac}}{a}$ b. $\frac{2\sqrt{g^2-ac}}{ac}$
c. $g^2 = ac$ d. none

20. The lines represents by $ax^2 + 2hxy + by^2 = 2gx + 2fy + c = 0$ intersect x-axis if
a. $2(gh - af^2 + ch^2$ b. $g^2 = ac$
c. $fg = abc$ d. $a + b = 0$

21. The angles between lines $xy = 0$ is
a. 45° b. 60°
c. 90° d. 180°

22. The bisector of angles between the lines $y^2 - 2xy + 2x^2 = 0$ are
a. $3x^2 + xy + 3y^2 = 0$
b. $2x^2 + xy - 4y^2 = 0$
c. $4x^2 + 2xy - y^2 = 0$
d. $3x^2 + 2xy - 3y^2 = 0$

23. Point of intersection of two lines represented by $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is
a. $\left(\frac{hf-bg}{ab-h^2}, \frac{hg-af}{ab-h^2}\right)$ b. $\left(\frac{gh-bf}{ab-h^2}, \frac{fh-ag}{ab-h^2}\right)$
c. $\left(\frac{hf-bg}{ab-h^2}, \frac{fh-ag}{ab-h^2}\right)$ d. none

24. The product of perpendicular drawn from point (1,2) on lines $x^2 + 4xy + y^2 = 0$ is
a. $\frac{3}{2}$ b. $\frac{3}{5}$ c. $\frac{5}{2}$ d. $\frac{2}{5}$

25. The gradient of one of lines $ax^2 + 2hxy + by^2 = 0$ is twice of the other then
a. $h^2 = ab$ b. $h = (a + b)$
c. $9h^2 = 8ab$ d. $8h^2 = 9ab$

26. The st.lines joining the origin to point of intersection of lines $\frac{x}{a} + \frac{y}{b} = 1$ and curves $x^2 - y^2 = c^2$ are at right angles if
a. $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$ b. $\frac{1}{a^2} + \frac{1}{c^2} = \frac{1}{b^2}$
c. $\frac{1}{b^2} + \frac{1}{b^2} = \frac{2}{c^2}$ d. $a^2 + b^2 = c^2$

27. The seperate equations represented by $x^2 - y^2 - x + 3y - 2 = 0$ is
a. $x - 2y + 2 = 0, x - 2y = 0$
b. $x + 2y - 4 = 0, x - 2y + 2 = 0$
c. $x - 2y = 0, x - 3y = 6$
d. $3x - y + 2 = 0, x + 3y + 6 = 0$

28. The st. lines joining the origin at point of intersection of line $y = mc + c$ and curve $x^2 + y^2 = a^2$ are a right angles if
a. $2c^2 = a^2(1 + m^2)$ b. $c^2 = a^2(1 + m^2)$
c. $a^2 = c^2(1 + m^2)$ d. $a^2c^2 = 1 - m^2$

29. The difference of the slopes of the lines represented by $ax^2 + 2hxy + by^2 = 0, (a = 0)$
a. $-\frac{2h}{b}$ b. $\frac{2\sqrt{h^2-ab}}{b}$
c. $\frac{2}{b}$ d. $\sqrt{\frac{h^2-ab}{(a+b)}}$

30. Angle between pair of lines $x^2 + 6xy + 9y^2 + 4x + 12y - 5 = 0$ is
a. 0° b. 30°
c. 60° d. 90°

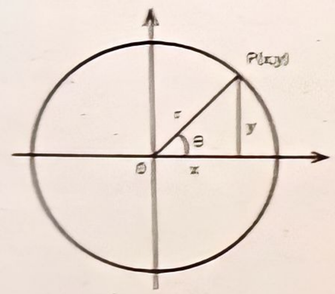
Answer Key:

1. b	2. b	3. b	4. a	5. c
6. c	7. c	8. a	9. b	10. a
11. d	12. b	13. a	14. c	15. c
16. d	17. a	18. c	19. b	20. a
21. c	22. d	23. a	24. a	25. d
26. c	27. b	28. a	29. b	30. a

Chapter - 16
Circle

A circle is the locus of a point which moves in a plane such that its distance from a fixed point (called the centre) is constant. i.e eccentricity (e)=0

1. Equation of circle:
   (i) Standard form : $x^2 + y^2 = r^2$
   (ii) Central form: $(x - h)^2 + (y - k)^2 = r^2$
   (iii) General form: $x^2 + y^2 + 2gx + 2fy + c = 0$;
   Where, centre (h, k) =(-g,-f)
   Radius (r)= $\sqrt{g^2 + f^2 - c}$
   If $g^2 + f^2 - c > 0$, then the circle is real circle.
   If $g^2 + f^2 - c = 0$, then the circle is point circle.
   If $g^2 + f^2 - c < 0$, then the circle is imaginary circle.
   (iv) Diameter form: $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$
   (v) Polar or parametric form:
   Consider a circle $x^2 + y^2 = r^2$ with the centre at origin and radius r. Let P(x,y) be any point on the circle.



Then,

$$\cos\theta = \frac{x}{r} \Rightarrow x = r\cos\theta$$

$$\sin\theta = \frac{y}{r} \Rightarrow y = r\sin\theta$$

$\therefore x = r\cos\theta$ and $y = r\sin\theta (0 \le \theta \le 2\pi)$ are required polar equations whose cartesian form is $x^2 + y^2 = r^2$.

Similarly, for the circle of the form $(x - h)^2 + (y - k)^2 = r^2$

$$x - h = r\cos\theta \Rightarrow x = h + r\cos\theta$$

$$y - k = r\sin\theta \Rightarrow y = k + r\sin\theta$$

2. Point and a circle:
Let, $S = x^2 + y^2 + 2gx + 2fy + c = 0$ be a circle and $P(x_1, y_1)$ be any point, then,

$$S_1 = x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$$

(i) P lies outside the circle if $S_1 > 0$
(ii) P lies on the circle if $S_1 = 0$
(iii) P lies inside the circle if $S_1 < 0$

3. Condition for tangency:
The line y=mx+c will be tangent to $x^2+y^2=a^2$ if $c=\pm a\sqrt{1 + m^2}$

4. Equation of the tangent to the circle $x^2 + y^2 = a^2$ at any point (x_1, y_1) is,

$$xx_1 + yy_1 = a^2$$

5. Equation of tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at point (x_1, y_1) on the circle is

$$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$$

6. Equation of normal to the circle $x^2 + y^2 = a^2$ at any point (x_1, y_1) is

$$xy_1 = x_1y$$

7. Equation of normal to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at point (x_1, y_1) on the circle is

$$y - y_1 = \frac{y_1 + f}{x_1 + g}(x - x_1)$$

8. Length of tangent from an external point (x_1, y_1) to a circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is,

$$\sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c}$$

9. Length of tangent from an external point (x_1, y_1) to a circle $x^2 + y^2 - a^2$ is

$$\sqrt{x_1^2 + y_1^2 - a^2}$$

10. Concentric circles:
Given, equation of circle $x^2 + y^2 + 2gx + 2fy + c = 0$. Then, the equation of family of concentric circles with the given circle is , $x^2 + y^2 + 2gx + 2fy + k = 0$; where k is any real.

11. If two circles touch each other externally then, $r_1 + r_2 = |c_1c_2|$; where r_1 and r_2 are the radii of the circles and $|c_1c_2|$ is the distance between their circles.

12. If two circles touch each other internally then, $r_1 - r_2 = |c_1c_2|; r_1 > r_2$

13. If two circles do not touch each other then, Either, $|c_1c_2| < r_1 - r_2$ or $|c_1c_2| > r_1 + r_2$

Multiple Choice Questions

1. The equation of a circle having centre at (3, 4) and touches the x-axis is,
   a. $x^2 + y^2 - 2x - 6y + 1 = 0$
   b. $2x^2 + 2y^2 - 7y + 3x + 5 = 0$
   c. $x^2 + y^2 - 6x - 8y + 9 = 0$
   d. $x^2 + y^2 - x - 3y + 10 = 0$

2. The equation $x^2 + y^2 = 0$ represents
   a. A circle b. A regenerate circle
   c. An empty circle d. A straight line

3. The equation of the circle is $x^2 + y^2 - 6x + 8y - 11 = 0$, then the points (0, 0) and (1, 8) lie
   a. Both inside
   b. Both outside
   c. One inside and one outside
   d. On the circle

4. The equation of the circle with end points of diameter (3, 4) and (-3, -4) is:
   a. $x^2 + y^2 - x + 3 = 0$

29. Find the length of tangent of circle $2x^2 + 2y^2 = 5$ form point $(4, -1)$ is
a. -5 b. $4\sqrt{3}$
c. $2\sqrt{3}$ d. $\sqrt{\frac{29}{2}}$

30. The point of contact if the st. line $y = x + c\sqrt{2}$ touches the circle $x^2 + y^2 = c^2$ is
a. $\left(\frac{c}{\sqrt{2}}, \frac{-c}{\sqrt{2}}\right)$ b. $\left(\frac{-c}{\sqrt{2}}, \frac{c}{\sqrt{2}}\right)$
c. $\left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$ d. $\left(c, \frac{c}{2}\right)$

31. The parametric form of circle $x^2 + y^2 - 2x - 4y - 4 = 0$ is
a. $(1 - 3\cos\theta, 2 - 3\sin\theta)$
b. $(1 + 3\cos\theta, 2 + 3\sin\theta)$
c. $(2\cos\theta, 2\sin\theta + 1)$
d. $(2 + 3\cos\theta, 3 + 2\sin\theta)$

32. If $f^2 = c$ for a circle $x^2 + y^2 + 2gx + 2fy + c = 0$ it touches
a. origin b. x-axis
c. y-axis d. none

33. The length of y-intercept made by circle $x^2 + y^2 - 8x + y - 20 = 0$ is
a. 5 b. 7 c. 6 d. 9

34. If two circles $(x-1)^2 + (y-3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersect in two distinct points then.
a. $2 \le r < 8$ b. $2 \le r \le 8$
c. $2 < r < 8$ d. $-2 < r \le 8$

35. Then length of common chord of circles $x^2 + y^2 = 12$ and $x^2 + y^2 - 4x + 3y - 2 = 0$ is
a. 4 b. $4\sqrt{2}$ c. 8 d. $16\sqrt{3}$

36. Equation $x^2 + y^2 + 4x + 6y + 13 = 0$ represents
a. circle b. pair of two distinct lines
c. pair of coincident lines d. point circle

37. The number of circle that can be drawn from three non-collinear points is
a. 1 b. 2 c. infinite d. 0

38. Number of tangents that can be drawn from (4,0) to circle $x^2 + y^2 = 4$ is
a. 1 b. 3 c. 2 d. infinite

39. Parametric equations $x = at^2, y = 4at$ represents
a. parabola b. hyperbola
c. ellipse d. circle

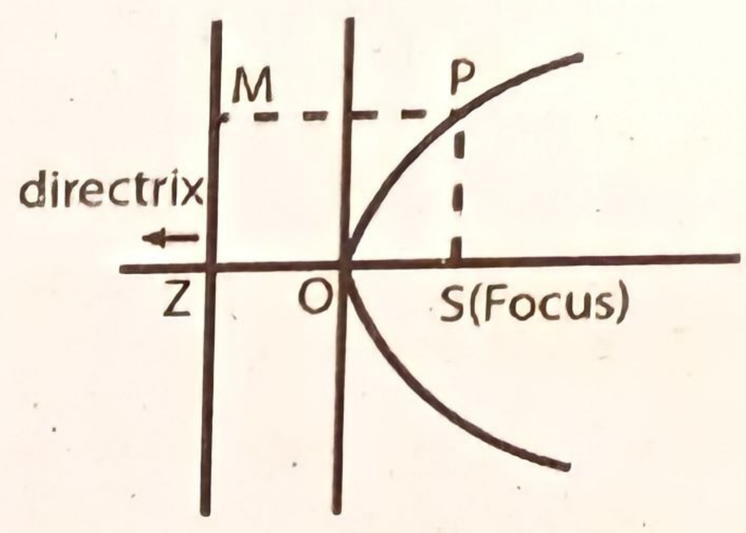
40. In a conic section, if $e = 0$ then curve is
a. parabola b. ellipse
c. circle d. hyperbole

Answer key:

1. c	2. b	3. c	4. c	5. a
6. a	7. b	8. a	9. a	10. c
11. a	12. a	13. c	14. d	15. d
16. c	17. d	18. b	19. c	20. a
21. a	22. c	23. b	24. c	25. a
26. b	27. a	28. c	29. d	30. b
31. b	32. c	33. d	34. c	35. b
36. d	37. a	38. c	39. a	40. c

Chapter - 17
Parabola

Parabola is the locus of a point which is equidistance from a fixed point (called focus) and fixed line (called directrix).



And the ratio pm/ps is known as eccentricity.

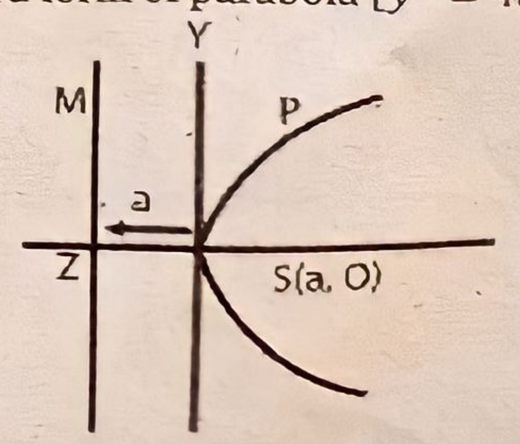
Note – eccentricity of parabola is always equal to 1.

Result of parabola

	$y^2 = 4ax$	$x^2 = 4ay$	$(y-k)^2 = 4a(x+h)$	$(x-h)^2 = 4a(y-k)$
Focus	(a, 0)	(0, a)	(h+a, k)	(h, k+a)
Vertex	(0, 0)	(0, 0)	(h, k)	(h, k)
Axis	y = 0	x = 0	y = k	x = h
Equation of directrix	x = -a	y = -a	x = h – a	y = k - a
Length of lactus reaction	|4a|	|4a|	|4a|	|4a|
Extremities of latus rectum	(a,±2a)	(±2a,a)	(h+2a, k±a)	(h ±2a, k+a)

1. Any chord passing through the focus is called focal chord.
2. Latus rectum (focal width) – The double coordinate passing through the focal and perpendicular to the axis is known as L.r.
– All latus rectum are focal chord but all focal chords are not latusreetum

Standard form of parabola $[y^2 = 4ax]$



Focus = (a, 0)
Vertex = (0, a)
Axis of parabola = $X - axis\ (y = 0)$

Unit: 4 Calculus

Chapter: 21
Limit and Continuity

Limit of a function:
A function f(x) is said to have the limit A at x = a, if, for small positive number ϵ, there exists another positive number δ, such that $|f(x) - A| < \epsilon$ whenever $0 < |x - a| < \delta$.
Symbolically, $\lim_{x\to a} f(x) = A$

Right-hand Limit:
If x approaches 'a' from right i.e. from larger value of x than a, then the limit of f(x) is called right hand limit and is written as $\lim_{x\to a^+} f(x)$.

3. Left-hand Limit:
If x approaches 'a' from left i.e. from smaller values of x than a, then limit of f(x) is called the left hand limit and is written as $\lim_{x\to a^-} f(x)$.

4. If $\lim_{x\to a^+} f(x)$ and $\lim_{x\to a^-} f(x)$ both exist and are equal to A, then $\lim_{x\to a} f(x)$ exits and $\lim_{x\to a} f(x) = A$.

Some important relations:

i. $\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1 = \lim_{\theta\to 0}\frac{\tan\theta}{\theta}$; θ is measured in radian.

ii. $\lim_{\theta\to 0}\cos\theta = 1$

iii. $\lim_{\theta\to\infty}\frac{\sin\theta}{\theta} = 0$

iv. $\lim_{\theta\to a}\frac{x^n - a^n}{x-a} = na^{n-1}$

v. $\lim_{x\to 0}(1+x)^{\frac{1}{x}} = e = \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$

vi. $\lim_{x\to\infty}\left(1+\frac{a}{x}\right)^x = e^a$

vii. $\lim_{x\to 0}\frac{a^x-1}{x} = 1$

viii. $\lim_{x\to 0}\frac{a^x-1}{x} = \log_e a$

ix. $\lim_{x\to 0}\frac{\log_e(1+x)}{x} = 1$

x. $\lim_{x\to 0}\frac{\log_a(1+x)}{x} = \log_a e$

xi. $\lim_{x\to 0}\frac{\sin^{-1}x}{x} = 1 = \lim_{x\to 0}\frac{\tan^{-1}x}{x}$

5. L – Hospital Rule:
If $\frac{f(a)}{g(a)}$ is of the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$, then $\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$ where $f'(x)$ and $g'(x)$ represents differentiation of f(x) and g(x) respectively.

6. Continuity of a function:
A function $f:(a,b)\to R$ is said to be continuous at a point c in (a, b) if
i. f is defined at x = c i.e. f(c. exists.
ii. $\lim_{x\to c}$ f(x) exists.
iii. $\lim_{x\to c}$ f(x) = f(c..
Otherwise, the function is said to be discontinuous at point c.

7. Basic Types of Discontinuities:
i. Removable Discontinuity:
If $\lim_{x\to c} f(x) \neq f(c)$, then, the function is said to have removable discontinuity.
ii. Simple discontinuity or discontinuity of the first kind or jump:
If $\lim_{x\to c^-} f(x) \neq \lim_{x\to c^+} f(x)$ i.e. $\lim_{x\to c}$ f(x) does not exist, then the function f(x) is said to have simple discontinuity.
iii. Infinite discontinuity or discontinuity of 2nd kind:
If $\lim_{x\to c} f(x) \to \infty$ or $-\infty$, then the function is said to have infinite discontinuity.

Multiple Choice Questions

1. $\lim_{x\to 0}\frac{1-\cos 3x}{3x^2}$ equals [Model Question]
a. $\frac{2}{3}$ b. $\frac{1}{3}$ c. $\frac{3}{2}$ d. 0

2. $\lim_{x\to 0}\frac{x^3-2^3}{x-2}$ is [Entrance 2069]
a. 4 b. 12 c. 9d. 0

3. $\lim_{x\to 0}\frac{\tan x-5x}{x}$ is equal to [Entrance 2069]
a. 6 b. -5 c. 0 d. -4

4. The function $f(x) = \begin{cases} x+2; & x\geq 2 \\ 2x; & x<2 \end{cases}$ is continuous at
a. x = 0 b. x = 2 c. x = -2 d. x = 1

5. If $\lim_{x\to a^-} f(x) \neq \lim_{x\to a^+} f(x)$, then f(x) is said to have [Entrance 2070]
a. Infinite Discontinuity
b. Removable Discontinuity
c. Jump Discontinuity
d. None

6. The value of $\lim_{x\to 2}\frac{x^5-2^5}{x-2}$ is [Entrance 2071]
a. 80 b. 40 c. 10 d. 0

7. The value of $\lim_{x\to 0}\frac{3x+4\tan 2x}{x}$ is [Entrance 2071]
a. 7 b. 3 c. 11 d. 4

8. $\lim_{x\to 2}\frac{x-2}{\sqrt{x^2-4}} =$ [Entrance 2072]
a. $\frac{0}{0}$ b. 0 c. 1 d. 4

9. $\lim_{n\to\infty}\frac{1+2+3+\cdots+n}{n^2} =$
a. $\frac{1}{2}$ b. 0 c. $\frac{1}{n}$ d. n

10. $\lim_{x\to 0}\frac{5x^2+4x\tan x}{x^2} =$
a. 9 b. 0 c. 5 d. 4

11. $\lim_{x\to\infty}\frac{3x^2+2x-5}{6x^2+4x-7} =$
a. 2 b. 3 c. $\frac{5}{7}$ d. $\frac{1}{2}$

12. $\lim_{x\to 0}\frac{1-\cos ax}{x^2} =$
a. $\frac{a^2}{2}$ b. a^2 c. $2a^2$ d. $4a^2$

13. $\lim_{x\to e}\frac{\log x-1}{x-e} =$
a. e b. $\frac{1}{e}$ c. e^2 d. $\frac{1}{e^2}$

14. $\lim_{x\to 0}\frac{2x-\sin^{-1}x}{2x+\tan^{-1}x} =$
a. 2 b. $\frac{1}{3}$ c. $\frac{2}{3}$ d. $-\frac{1}{3}$

15. $\lim_{x\to 0}\frac{3^x-2^x}{x} =$
a. $\log\frac{2}{3}$ b. $\left(\frac{3}{2}\right)$
c. $(\log 2)(\log 3)$ d. $\log 6$

16. If $\lim_{x\to 5}\frac{x^k-5^k}{x-5} = 500$, then the positive integral value of k is
a. 3 b. 4 c. 5 d. 6

17. The value of $\lim_{x\to 0}\frac{\log\cos x}{x}$ is equal to
a. 0 b. 1 c. ∞ d. -1

18. $\lim_{x\to 0}\frac{a^x-1}{\sqrt{1+x}-1} =$
a. $2\log_e a$ b. $\log a$ c. $2\log x$ d. $\frac{2}{\log a}$

19. $\lim_{x\to\frac{3\pi}{4}}\frac{1+\tan x}{\cos 2x} =$
a. $\frac{1}{2}$ b. 1 c. -1 d. 0

20. $\lim_{n\to\infty}\frac{4^{\frac{1}{n}}-1}{3^{\frac{1}{n}}-1} =$
a. $\log 4$ b. $\log_4 3$ c. $\log_3 4$ d. 1

21. $\lim_{x\to 0}\left(\frac{x^0}{\sin x^0}\right) =$
a. -1 b. $\frac{100}{\pi}$ c. $\frac{\pi}{100}$ d. 1

22. If $f(x) = \frac{ax+b}{x+1}$, $\lim_{x\to 0} f(x) = 2$ and $\lim_{x\to\infty} f(x) = 1$ then the value of a and b are:
a. $a = 2, b = -1$ b. $a = 1, b = 2$
c. $a = 0, b = 1$ d. $a = \frac{1}{2}, b = 3$

23. $\lim_{x\to\infty}\frac{ax^2+bx+c}{dx^2+ex+f} =$
a. $\frac{b}{e}$ b. $\frac{a}{d}$ c. $\frac{abc}{def}$ d. $\frac{ae}{bd}$

24. If $\lim_{x\to 3}\frac{x^n-3^n}{x-3} = 27$ and n is a positive integer then n =
a. 6 b. 2 c. 3 d. 4

25. $\lim_{x\to 0}\frac{\sin x^0}{x} =$
a. -1 b. 1 c. $\frac{\pi}{180^\circ}$ d. $\frac{180^\circ}{\pi}$

26. $\lim_{x\to 0}\frac{3^x-2^x}{x} =$
a. $\frac{3}{2}$ b. $\log\frac{3}{2}$ c. $\log\frac{2}{3}$ d. 0

27. Every continuous function is:
a. Increasing b. Decreasing
c. Not differentiable d. differentiable

28. A function f(x) defined by $f(x) = \begin{cases} kx^2 & if\ x\leq 2 \\ 3 & if\ x>2 \end{cases}$ is continuous at x = 2, then the value of k =
a. $\frac{1}{4}$ b. $\frac{3}{4}$ c. $\sqrt{3}$ d. $\frac{1}{3}$

29. $\lim_{n\to\infty}\frac{1}{n^2}+\frac{2}{n^2}+\frac{3}{n^2}+\cdots+\frac{n}{n^2} =$
a. $\frac{1}{2}$ b. 1 c. $\frac{1}{4}$ d. $\frac{1}{3}$

30. $\lim_{x\to 0}\left(\frac{\sin x-x}{x^3}\right)$ is equal to:
a. $\frac{1}{6}$ b. $-\frac{1}{6}$ c. $\frac{1}{2}$ d. $\frac{1}{3}$

31. $\lim_{x\to a}\frac{x-a}{\sqrt{x^2-a^2}}$
a. 1 b. 0 c. $\frac{1}{2a}$ d. $\frac{x-a}{2a}$

32. $\lim_{x\to 3}\left(\frac{1}{x-3}-\frac{9}{x^3-3x^2}\right)$
a. $\frac{5}{6}$ b. $\frac{x^2}{3}$ c. $\frac{2}{3}$ d. $-\frac{1}{2}$

33. Every continuous function is
a. increasing b. decreasing
c. differentiable d. Not differentiable

34. $\lim_{x\to\infty}\sqrt{x}\left(\sqrt{x}-\sqrt{x-a}\right)$
a. $\frac{5a}{4}$ b. $\frac{2}{3}a^2$ c. $\frac{a}{2}$ d. $\frac{-1}{\sqrt{2a}}$

35. $\lim_{x\to 1}\frac{x^2-\sqrt{x}}{\sqrt{x}-1}$
a. 3 b. $\frac{2}{3}$ c. 1 d. $\frac{-1}{\sqrt{2}}$

36. If $f(x) = \frac{(x+2)^2-4}{x}$ then $\lim_{x\to 0} f(x)$.
a. 2 b. 0 c. $\frac{1}{2}$ d. 4

kind.

If $\lim_{x \to c} f(x) \to \infty$ or $-\infty$, then the function is said to have infinite discontinuity.

Multiple Choice Questions

1. $\lim_{x \to 0} \frac{1-\cos 3x}{3x^2}$ equals [Model Question]

   a. $\frac{2}{3}$ b. $\frac{1}{3}$ c. $\frac{3}{2}$ d. 0

2. $\lim_{x \to 0} \frac{x^3-2^3}{x-2}$ is [Entrance 2069]

   a. 4 b. 12 c. 9 d. 0

3. $\lim_{x \to 0} \frac{\tan x-5x}{x}$ is equal to [Entrance 2069]

   a. 6 b. -5 c. 0 d. -4

4. The function $f(x) = \begin{cases} x+2; & x \geq 2 \\ 2x; & x<2 \end{cases}$ is continuous at

   a. x = 0 b. x = 2 c. x = -2 d. x = 1

5. If $\lim_{x \to a^-} f(x) \neq \lim_{x \to a^+} f(x)$, then f(x) is said to have [Entrance 2070]

   a. Infinite Discontinuity
   b. Removable Discontinuity
   c. Jump Discontinuity
   d. None

6. The value of $\lim_{x \to 2} \frac{x^5-2^5}{x-2}$ is [Entrance 2071]

   a. 80 b. 40 c. 10 d. 0

7. The value of $\lim_{x\to 0} \frac{3x+4\tan 2x}{x}$ is [Entrance 2071]

a. 7 b. 3 c. 11 d. 4

8. $\lim_{x\to 2} \frac{x-2}{\sqrt{x^2-4}} =$ [Entrance 2072]

a. $\frac{0}{0}$ b. 0 c. 1 d. 4

9. $\lim_{n\to\infty} \frac{1+2+3+\cdots+n}{n^2} =$

a. $\frac{1}{2}$ b. 0 c. $\frac{1}{n}$ d. n

10. $\lim_{x\to 0} \frac{5x^2+4x\tan x}{x^2} =$

a. 9 b. 0 c. 5 d. 4

11. $\lim_{x\to\infty} \frac{3x^2+2x-5}{6x^2+4x-7} =$

a. 2 b. 3 c. $\frac{5}{7}$ d. $\frac{1}{2}$

12. $\lim_{x\to 0} \frac{1-\cos ax}{x^2} =$

a. $\frac{a^2}{2}$ b. a^2 c. $2a^2$ d. $4a^2$

13. $\lim_{x\to e} \frac{\log x-1}{x-e} =$

a. e b. $\frac{1}{e}$ c. e^2 d. $\frac{1}{e^2}$

14. $\lim_{x\to 0} \frac{2x-\sin^{-1}x}{2x+\tan^{-1}x} =$

a. 2 b. $\frac{1}{3}$ c. $\frac{2}{3}$ d. $-\frac{1}{3}$

15. $\lim_{x\to 0} \frac{3^x-2^x}{x} =$

a. $\log\frac{2}{3}$ b. $\left(\frac{3}{2}\right)$
c. $(\log 2)(\log 3)$ d. $\log 6$

16. If $\lim_{x\to 5} \frac{x^k-5^k}{x-5} = 500$, then the positive integral value of k is

a. 3 b. 4 c. 5 d. 6

17. The value of $\lim_{x\to 0} \frac{\log\cos x}{x}$ is equal to

a. 0 b. 1 c. ∞ d. -1

18. $\lim_{x\to 0} \frac{a^x-1}{\sqrt{1+x}-1} =$

a. $2\log_e a$ b. $\log a$ c. $2\log x$ d. $\frac{2}{\log a}$

19. $\lim_{x\to\frac{3\pi}{4}} \frac{1+\tan x}{\cos 2x} =$

a. $\frac{1}{2}$ b. 1 c. -1 d. 0

20. $\lim_{n\to\infty} \frac{4^{\frac{1}{n}}-1}{3^{\frac{1}{n}}-1} =$

a. $\log 4$ b. $\log_4 3$ c. $\log_3 4$ d. 1

21. $\lim_{x\to 0} \left(\frac{x^0}{\sin x^0}\right) =$

a. -1 b. $\frac{180}{\pi}$ c. $\frac{\pi}{180}$ d. 1

22. If $f(x) = \frac{ax+b}{x+1}$, $\lim_{x\to 0} f(x) = 2$ and $\lim_{x\to\infty} f(x) = 1$ then the value of a and b are:

a. $a = 2, b = -1$ b. $a = 1, b = 2$
c. $a = 0, b = 1$ d. $a = \frac{1}{2}, b = 3$

23. $\lim_{x\to\infty} \frac{ax^2+bx+c}{dx^2+ex+f} =$

a. $\frac{b}{e}$ b. $\frac{a}{d}$ c. $\frac{abc}{def}$ d. $\frac{ae}{bd}$

24. If $\lim_{x\to 3} \frac{x^n-3^n}{x-3} = 27$ and n is a positive integer then n =

a. 6 b. 2 c. 3 d. 4

25. $\lim_{x\to 0} \frac{\sin x^0}{x} =$

a. -1 b. 1 c. $\frac{\pi}{180°}$ d. $\frac{180°}{\pi}$

26. $\lim_{x\to 0} \frac{3^x-2^x}{x} =$

a. $\frac{3}{2}$ b. $\log\frac{3}{2}$ c. $\log\frac{2}{3}$ d. 0

27. Every continuous function is:

a. Increasing b. Decreasing
c. Not differentiable d. differentiable

28. A function f(x) defined by $f(x) = \begin{cases} kx^2 & if\, x \le 2 \\ 3 & if\, x > 2 \end{cases}$ is continuous at x = 2, then the value of k =

a. $\frac{1}{4}$ b. $\frac{3}{4}$ c. $\sqrt{3}$ d. $\frac{1}{3}$

29. $\lim_{n\to\infty} \frac{1}{n^2}+\frac{2}{n^2}+\frac{3}{n^2}+\cdots+\frac{n}{n^2} =$

a. $\frac{1}{2}$ b. 1 c. $\frac{1}{4}$ d. $\frac{1}{3}$

30. $\lim_{x\to 0} \left(\frac{\sin x-x}{x^3}\right)$ is equal to:

a. $\frac{1}{6}$ b. $-\frac{1}{6}$ c. $\frac{1}{2}$ d. $\frac{1}{3}$

31. $\lim_{x\to a} \frac{x-a}{\sqrt{x^2-a^2}}$

a. 1 b. 0 c. $\frac{1}{2a}$ d. $\frac{x-a}{2a}$

32. $\lim_{x\to 3} \left(\frac{1}{x-3}-\frac{9}{x^3-3x^2}\right)$

a. $\frac{5}{6}$ b. $\frac{x^2}{3}$ c. $\frac{2}{3}$ d. $-\frac{1}{2}$

33. Every continuous function is

a. increasing b. decreasing
c. differentiable d. Not differentiable

34. $\lim_{x\to\infty} \sqrt{x}\left(\sqrt{x}-\sqrt{x-a}\right)$

a. $\frac{5a}{4}$ b. $\frac{2}{3}a^2$ c. $\frac{a}{2}$ d. $\frac{-1}{\sqrt{2a}}$

35. $\lim_{x\to 1} \frac{x^2-\sqrt{x}}{\sqrt{x}-1}$

a. 3 b. $\frac{2}{3}$ c. 1 d. $\frac{-1}{\sqrt{2}}$

36. If $f(x) = \frac{(x+2)^2-4}{x}$ then $\lim_{x\to 0} f(x)$.

a. 2 b. 0 c. $\frac{1}{2}$ d. 4

37. The limit of $f(x) = (1-x)^{-1}$ at $x = 1$ is

a. 1 b. -1

c. 0 d. doesn't exit

38. $\lim_{x\to a} \frac{(x+2)^{\frac{5}{3}} - (a+2)^{\frac{5}{3}}}{x-a}$

a. $\frac{-1}{2x^{\frac{3}{2}}}$ b. $\frac{5}{3}(a+2)^{\frac{2}{3}}$

c. $\frac{2}{3}a^{\frac{1}{6}}$ d. 0

39. $\lim_{x\to 0} \frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{2x^2}$

a. $\frac{1}{2}$ b. $-\frac{1}{2}$

c. $\frac{5}{2\sqrt{2}}$ d. $\frac{3}{2}$

40. $\lim_{x\to 0} \frac{\sqrt{1+x}-1}{(1+x)^{\frac{1}{3}}-1}$

a. $\frac{1}{4}$ b. $\frac{3}{2}$ c. $\frac{1}{2}$ d. $\frac{8}{3}$

41. If $\lim_{x\to 0} \frac{x^4-1}{x-1} = \lim_{x\to k} \frac{x^3-K^3}{x^2-k^2}$ then $k =$

a. $\frac{4}{3}$ b. $\frac{16}{3}$ c. $\frac{8}{3}$ d. 0

42. $\lim_{x\to a} \frac{|x-a|}{x-a}$ is equal to

a. -1 b. 1

c. 0 d. doesnot exist

43. $\lim_{x\to 3} \frac{\sqrt{x^2+7}-4}{(x-3)}$ iss equal to

a. $-\frac{3}{4}$ b. $\frac{3}{4}$

c. $\frac{7}{2}$ d. $\frac{1}{4}$

44. $\lim_{x\to 0} \frac{\tan x}{x}$ is equal to

a. $\sqrt{2}$ b. 1

c. $\frac{1}{2}$ d. $\frac{3}{2}$

45. The value of a if limit of $f(x)$ exists where f is defined by

$$f(x) = \begin{cases} ax - 1 \text{ if } x < 2 \\ x + 1 \text{ if } x > 2 \end{cases}, \text{at } x = 2$$

a. -7 b. 2

c. $-\frac{2}{3}$ d. $\frac{-1}{2}$

46. $\lim_{x\to \frac{\pi}{4}} \frac{\sec^2 x - 2}{\tan x - 1}$

a. $\frac{\pi}{2}$ b. $\frac{\pi}{2}$ c. 2 d. $\frac{\pi}{4}$

47. $\lim_{x\to\theta} \frac{x\sin\theta - \theta\sin x}{x-\theta}$

a. $\sin\theta + \cos\theta$ b. $\sin\theta - \theta\cos\theta$

c. $\cos\theta - \theta\sin\theta$ d. $\cos\theta + \theta\sin\theta$

48. $\lim_{\theta\to\frac{\pi}{4}} \frac{\cos\theta - \sin\theta}{\theta - \frac{\pi}{4}}$

a. $\frac{3}{2}$ b. 1 c. $-\sqrt{2}$ d. $\frac{3}{2}$

49. $\lim_{x\to 1} \frac{1+\cos\pi x}{\tan^2 \pi x}$

a. $\frac{3}{2}$ b. 8 c. $\frac{1}{4}$ d. 0

50. $\lim_{x\to\theta} \frac{x\tan\theta - \theta\tan x}{x-\theta}$

a. $\tan\theta - \theta\sec\theta$ b. $\tan\theta - \theta\sec^2\theta$

c. $\sec^2\theta - \theta\tan\theta$ d. none

51. $\lim_{x\to 0} \frac{1-\cos px}{1-\cos qx}$

a. $\frac{p}{q}$ b. $p-q$

c. $\frac{p^2}{q^2}$ d. 0

52. $\lim_{x\to 0} \frac{\tan\alpha x}{\tan\beta x}$

a. $\frac{\alpha}{\beta}$ b. $\frac{\sec^2\alpha}{\sec^2\beta}$

c. $\frac{\sec^2\beta}{\sec^2\alpha}$ d. $\tan\alpha\beta$

53. $\lim_{x\to 0} \frac{a^x-1}{b^x-1}$

a. $\log_e 1$ b. $\log_e\left(\frac{a}{b}\right)$

c. $\log_b a$ d. $\left(\frac{a}{b}\right)^x$

54. $\lim_{x\to 0} \frac{x(e^x-1)}{1-\cos x}$

a. 2 b. 0 c. $-\frac{1}{2}$

55. $\lim_{x\to 0} \frac{e^x - \sin x - 1}{x}$

a. 0 b. 1

c. e d. undefined

56. The point of discontinuities for $\frac{2}{x^2-x}$ is

a. 0 b. {0,1}

c. {0, −1} d. {0,1, −1}

57. The value of k show that

$$f(x) = \begin{cases} \frac{x^2-2x-3}{x+1}, x \neq -1 \\ k \quad , x = -1 \end{cases}$$ is continuous at

−1

a. (3.2) b. −4 c. 3

58. $\lim_{x\to 0} \frac{x(2^x-1)}{1-\cos x}$

a. $\frac{3}{5}$ b. 2 ln 2

c. 1 d. $\log_e 2$

59. $\lim_{x\to\infty} \frac{(3x-1)(2x+5)}{(x-3)(3x+7)}$

a. $\frac{1}{8}$ b. 1 c. 2

60. $\lim_{x\to 0} \sin\frac{1}{x}$

a. 1 b. $\frac{\pi}{180}$

c. 0 d. doesn't exist

61. $\lim_{x\to\infty} \left(\frac{x+2}{x+1}\right)^{x+3} =$

a. e b. $\frac{1}{e}$ c. 2e

62. $\lim_{x\to 0^+} x^x =$

a. 0 b. e

c. 1 d. not defined

63. $\lim_{x\to 5^-} \frac{|x-5|}{x-5} =$

a. 1 b. −1

c. 0 d. doesn't exist

64. $\lim_{x\to 1} (1-x)\tan\left(\frac{\pi x}{2}\right) =$

a. 0 b. π c. $\frac{2}{\pi}$

65. $\lim_{x\to 0} \frac{5^x - 5^{-x}}{2x} =$

a. log 5 b. 0

c. 1 d. 2 log 5

66. if $f(x) + g(x)$ is discontinuous then,

a. $f(x)$ is discontinuous

b. $g(x)$ is discontinuous

c. both $f(x)$& $g(x)$ must be discontinuous

d. any one of these is discontinuous

67. Which one is not indeterminate form?

a. $\frac{\infty}{\infty}$ b. $\frac{0}{1^0}$ c. 0^∞ d. 0^0

68. $\lim_{x\to 0}(1+x)^{\frac{1}{x}} =$

a. 1 b. $\log_a e$

c. e d. 0

69. $\lim_{x\to\infty} \frac{1-n^2}{n} =$

a. 1 b. -2 c. $\frac{1}{2}$ d. -1

70. $\lim_{x\to a^-} f(x) \neq \lim_{x\to a^+} f(x)$ then $f(x)$ is said to have

a. removable discontinuity at x=a

b. ordinary discontinuity at $x = a$

c. not – continuous at $x = a$

d. can't be said

Answer Key:

1. C	2. b	3. d	4. b	5. c
6. a	7. c	8. b	9. a	10. a
11. d	12. a	13. b	14. b	15. c
16. b	17. a	18. a	19. b	20. c
21. d	22. b	23. b	24. c	25. c
26. b	27. c	28. b	29. a	30. b
31. b	32. c	33. d	34. c	35. a
36. d	37. d	38. b	39. a	40. b
41. c	42. d	43. b	44. b	45. b
46. c	47. b	48. c	49. a	50. b
51. c	52. a	53. c	54. a	55. a
56. b	57. b	58. b	59. c	60. d
61. a	62. c	63. b	64. c	65. a
66. d	67. b	68. c	69. b	70. b

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Some Form

1. Product

2. Quotient

3. Power Ru

4. Chain Rul

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function, t

8. Derivative of inverse trigonometric functions:

a. $\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$

b. $\frac{d}{dx}\cos^{-1}x = -\frac{1}{\sqrt{1-x^2}}$

c. $\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$

d. $\frac{d}{dx}\cot^{-1}x = -\frac{1}{1+x^2}$

e. $\frac{d}{dx}\sec^{-1}x = \frac{1}{x(\sqrt{x^2-1})}$

f. $\frac{d}{dx}\text{cosec}^{-1}x = -\frac{1}{x\sqrt{x^2-1}}$

9. Derivative of exponential and logarithmic functions:

a. $\frac{d}{dx}e^x = e^x$

b. $\frac{d}{dx}a^x = a^x \log_e a$

c. $\frac{d}{dx}\ln x = \frac{1}{x}$

d. $\frac{d}{dx}(\log_a x) = \frac{1}{x\log_e a} = \frac{\log_a e}{x}$

e. $\frac{d}{dx}|x| = \frac{x}{|x|}, x \neq 0$

10. Derivative of hyperbolic functions:

a. $\frac{d}{dx}\sinh x = \cosh x$

b. $\frac{d}{dx}\cosh x = \sinh x$

c. $\frac{d}{dx}\tanh x = \text{sech}^2 x$

d. $\frac{d}{dx}\coth x = -\text{cosech}^2 x$

e. $\frac{d}{dx}\text{cosech}\, x = -\text{cosech}\, x . \coth x$

f. $\frac{d}{dx}\text{sech}\, x = -\text{sech}\, x . \tanh x$

11. Derivative of inverse hyperbolic functions:

a. $\frac{d}{dx}\sinh^{-1}x = \frac{1}{\sqrt{1+x^2}}$

b. $\frac{d}{dx}\cosh^{-1}x = \frac{1}{\sqrt{x^2-1}}$

c. $\frac{d}{dx}\tanh^{-1}x = \frac{1}{1-x^2}$

d. $\frac{d}{dx}\text{sech}^{-1}x = -\frac{1}{x\sqrt{1-x^2}}$

e. $\frac{d}{dx}\coth^{-1}x = -\frac{1}{x^2-1}$

f. $\frac{d}{dx}\text{cosech}^{-1}x = -\frac{1}{x\sqrt{x^2+1}}$

Multiple Choice Questions

1. If $y = \sqrt{x+\sqrt{x+\sqrt{x+\ldots}}}$, then $\frac{dy}{dx}$ is [Entrance 2073]

a. $\frac{1}{2y-1}$ b. $\frac{\sqrt{x}}{2y-1}$ c. $2\sqrt{x}$ d. $\sqrt{x}$

2. If $y = \tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right]$ and $\theta = \tan^{-1}x$, then $\frac{dy}{d\theta} =$

a. $\frac{x}{2}$ b. $\frac{1}{2}$ c. $\frac{3}{2}$ d. $-\frac{1}{2}$

3. If $x^p y^q = (x+y)^{p+q}$, then $\frac{dy}{dx} =$

a. $\frac{y}{x}$ b. $\frac{py}{qx}$ c. $\frac{x}{y}$ d. $\frac{x}{y^2}$

4. If $x^n + y^n = a^n$, then $\frac{dy}{dx}$ is equal to:

a. $\left(\frac{x}{y}\right)^{n-1}$ b. $\left(\frac{y}{x}\right)^{n-1}$ c. $-\left(\frac{x}{y}\right)^{n-1}$ d. $-\left(\frac{y}{x}\right)^{n-1}$

5. $\frac{d}{dx}\cosh^{-1}(\sec x) =$

a. $\sec x$ b. $\sin x$ c. $\tan x$ d. $\text{cosec}\, x$

6. The derivative of $\tan^{-1}\left(\frac{a+x}{1-ax}\right) =$

a. 0 b. $\frac{x}{a+x}$ c. $\frac{1}{1+x^2}$ d. $\frac{2a}{1+x^2}$

7. If $3\sin(xy) + 4\cos(xy) = 5$, then $\frac{dy}{dx} =$

a. $\frac{2y}{x}$ b. -1 c. $\frac{x}{y}$ d. $-\frac{y}{x}$

8. If $y = \log(\log \log x)$ then $\frac{dy}{dx} =$

a. $\frac{1}{x\log x.\log(\log x)}$ b. $\frac{1}{\log(\log x)}$

c. $\frac{1}{x\log x}$ d. $\frac{1}{x^2\log(\log x)}$

9. If $y^2 = 4ax$, then $\frac{dy}{dx} =$

a. $\frac{y}{x}$ b. $\frac{2a}{y}$ c. $-\frac{x}{y}$ d. $\frac{2x}{a}$

10. The differential coefficient of x^6 w.r.to x^3 is

a. $6x^5$ b. $3x^2$ c. $2x^3$ d. 4x

11. The derivative of an odd function $y = f(x)$ is

a. Odd function b. Neither even nor odd

c. Even function d. Constant function

12. If $x = at^2, y = 2at$, then $\frac{d^2y}{dx^2} =$

a. $-2at$ b. $-\frac{1}{2at^3}$ c. $\frac{t^3}{2a}$ d. $-\frac{1}{t^3}$

13. If $f(x) = mx + c$ and $f(0) = f'(0) = 1$, then the value of $f(2) =$

a. 1 b. -1 c. 0 d. 3

14. The differential coefficient of $y = 5^x$ is [BSc. CSIT 2070]

a. $5^x \log_e 5$ b. $\frac{5^x}{\log_e 5}$ c. $\frac{5e^x}{\log x}$ d. $\frac{e^{5x}}{\log 5}$

15. If $y = \log_{\sqrt{x}} x$, then $\frac{dy}{dx} =$

a. $\frac{1}{x}$ b. 1 c. $\frac{1}{x\log\sqrt{x}}$ d. 0

16. If $x^y = y^x$, then $\frac{dy}{dx} =$

a. $\frac{y}{x}$ b. $-\frac{y(y+x\log y)}{x(y\log x+x)}$

c. $\frac{y(x\log y-y)}{x(y\log x-x)}$ d. $-\frac{x}{y}$

17. $\frac{d}{dx}(\cot x)$ equals [Model Question]

a. $\text{cosec}^2 x$ b. $\cot x . \text{cosec}\, x$

c. $-\cot x . \text{cosec}\, x$ d. $-\text{cosec}^2 x$

18. If the derivative of $x^2 + 3x$ is 1, the value of x is [BSc. CSIT 2069]

a. 0 b. -1 c. 2 d. 5

19. If $x = 2t + 1$ and $y = t^2$, then $\frac{dy}{dx} =$ [BSc. CSIT069]

a. t b. $\frac{1}{t}$ c. t^2 d. 0

20. The derivative of $y = b^x$ is [BSc. CSIT 2071]

a. xb^{x+1} b. [illegible]

21. The derivative of $y = \cos(\cos\sqrt{x})$ is [BSc. CSIT 2071]

a. $\frac{\sin(\cos\sqrt{x})\sin\sqrt{x}}{2\sqrt{x}}$ b. $\sin(\cos\sqrt{x})$

c. $\sin(\sin\sqrt{x})$ d. $\frac{\sin(\cos\sqrt{x})}{2\sqrt{x}}$

22. The value of x for which the derivative of $x^2 + 2x + 2$ is zero is [BSc. CSIT 2071]

a. 1 b. 2 c. 0 d. -1

23. The values of x for which the function $y = x^3 - x^2$ has $\frac{dy}{dx} = 0$ [BSc. CSIT 2072]

a. $0, \frac{2}{3}$ b. 0 c. $\frac{2}{3}$ d. $-\frac{2}{3}$

24. If $y = e^{\sqrt{ax^2+b}}$ then $\frac{dy}{dx} =$ [BSc. CSIT 2073]

a. $\frac{axy^2}{\sqrt{ax^2+b}}$ b. $\frac{ax}{\sqrt{ax^2+b}}\, e^{\sqrt{ax^2+b}}$

c. $\frac{x}{\sqrt{ax^2+b}}$ d. $e^{\sqrt{ax^2+b}}$

25. If $x = t + \sin t, y = 1 - \cos t$, then $\frac{dy}{dx}$ is [BSc. CSIT 2074]

a. $\tan t$ b. $\frac{\tan t}{2}$ c. $-\tan t$ d. $\tan\frac{t}{2}$

26. The derivative of $\cos x$ w.r.to $\sin x$ is [BSc. CSIT 2074]

a. $-\tan x$ b. $\tan x$ c. $\cot x$ d. $-\cot x$

27. The derivative of x^2 w.r.to $-x^2$ is [BSc. CSIT 2075]

a. 0 b. 1 c. 2x d. -1

28. If $y = e^{x+e^{x+e^{x+\cdots\infty}}}$, then $\frac{dy}{dx} =$

a. $\frac{2y}{1+y}$ b. $\frac{1}{1-y}$ c. $\frac{y+1}{2y}$ d. $\frac{y}{1-y}$

29. The differential coefficient of x^x is

a. $x^2 \log x$ b. $x^x\left(\log x + \frac{1}{x}\right)$

c. $x^x(1 + \log x)$ d. $x^x - 1$

30. If $y = (a + \sqrt{x})(a - \sqrt{x})$ then $\frac{dy}{dx} =$

a. -1 b. 1

c. $\frac{1}{2\sqrt{x}}$ d. $-a\sqrt{x}$

31. $\frac{dy}{dx}$ of $y = 1/\sqrt{a^n - x^n}$

a. $\frac{nx^{n-1}}{2a^{\frac{n}{2}}}$ b. $\frac{nx^{n-1}}{2(a^n-x^n)^{\frac{3}{2}}}$

c. $\frac{ax^{n-1}}{2(x^n-a^n)^{\frac{3}{2}}}$ d. $\frac{nx^{n-1}}{2(x^n-a^n)^{\frac{3}{2}}}$

32. Find $\frac{dy}{dx}$ of $y = \sqrt{2u^2+3}, u = \sqrt{2x+1}$

a. $\frac{2}{\sqrt{4x+5}}$ b. $-\frac{2x}{(x^2+3)^2}$

c. $\frac{x}{\sqrt{x^2+1}}$ d. $-\frac{5}{2\sqrt{2x+1}}$

33. Derivative of $\sqrt{x} + \sqrt{y} = \sqrt{a}$

a. $\frac{2a}{y}$ b. $\frac{2bx}{a^2}$

c. $-\sqrt{\frac{y}{x}}$ c. $\frac{x}{y}$

34. $\frac{dy}{dx}$ of $x\sqrt{y} + y\sqrt{x} = a$

a. $\frac{-y(2x+y)}{x(x+2y)}$ b. $\frac{-y^{\frac{3}{2}}+2y\sqrt{x}}{x^{\frac{3}{2}}+2x\sqrt{y}}$

c. $\sqrt{\frac{x}{y}}$ d. $-\sqrt{\frac{y}{x}}$

35. If $x = \sin\theta$ and $y = \cos\theta$ find $\frac{d^2y}{dx^2}$

a. $\sin^3\theta$ b. $\cos^3\theta$

c. $cosec^3\theta$ d. $\sec^3\theta$

36. If $x = a(\tan t - t\sec^2 t), y = a\sec^2 t$, $\frac{dy}{dx} =$

a. $-\frac{1}{t}$ b. $-\frac{t^2}{2t}$

c. $\frac{1}{t}$ d. $\frac{2t^2}{\tan t}$

37. If $x = \cos^3 2\theta, y = \sin^3 2\theta$ then $f'(x) =$

a. $\cot 2\theta$ b. $\sin 6\theta$

c. $-\tan 2\theta$ d. $\frac{\tan 3\theta}{6}$

38. Differentiation of $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right) =$

a. $\frac{1}{2}$ b. $\frac{2}{1+x^2}$

c. $2\left(\frac{1-2x^2}{\sqrt{1-x^2}}\right)$ d. $-\frac{1}{2}$

39. Derivative of $\frac{1+\tan x}{1-\tan x}$

a. $\cot x$ b. $\cot\left(\frac{\pi}{4}+x\right)$

c. $\sec^2\left(\frac{\pi}{4}+x\right)$ d. $\tan\left(x+\frac{\pi}{4}\right)$

40. $\frac{dy}{dx}$ of $\frac{1}{cosec\, x - \cot x} =$

a. $-\, cosec\, x\,(cosec\, x + \cot x)$

b. $\cot x\,(cosec\, x - \cot x)$

c. $\sin x - \cos x$

d. $cosec\, x\,(\cot x - cosec\, x)$

41. $\frac{dy}{dx}$ when $x^2y^2 = \cos x^2y^2$ is

a. $\frac{2xy^2 - \sin x^2y^2}{2y + x\cos x^2y^2}$ b. $\frac{y}{x}$

c. -1 d. $-\frac{y}{x}$

42. If $x = a\cos^3\theta, y = b\sin^3\theta$ then $\frac{dy}{dx} =$

a. $\frac{b}{a}\sin\theta$ b. $\frac{b}{a}\cos\theta$

c. $-\frac{b}{a}\cot\theta$ d. $-\frac{b}{a}\tan\theta$

43. $\frac{dy}{dx}$ of $\sin^{-1}(1 - 2x^2) =$

a. $\frac{2}{1+x^2}$ b. $\frac{2}{\sqrt{x^2-1}}$

c. $\frac{-2}{\sqrt{1-x^2}}$ d. $\frac{1}{\sqrt{1-2x^2}}$

44. $\frac{dy}{dx}$ of $\tan^{-1}\left(\frac{\sin 2x}{1+\cos 2x}\right)$ Is

a. $\frac{1}{2(1+x^2)}$ b. $\cos 2x$

c. $-2\sin 2x$ d. 1

45. $\frac{d^2y}{dx^2}$ of $y = \frac{1-x}{1+x}$

a. $\frac{-2}{(1+x)^2}$ b. $\frac{2}{(1+x)^3}$

c. $\left(\frac{1+x}{1-x}\right)^2$ d. $\frac{-x}{1-x^2}$

46. Differentiation of $\tan^2 x$ w.r.t $\sin^2 x$ is

a. $-\tan x$ b. $-\cot x$

c. $\sec^4 x$ d. -1

47. Derivative of a^{x^2} is
a. $2 \ln a \; a^{x^2}$
b. $x^2 \ln 2 . a\, x^2$
c. $2\, x \ln a . a^{x^2}$
d. $\frac{2x}{\ln a^{x^2}}$

48. $\frac{dy}{dx}$ of $2^x + 2^y = 2^{x+y}$ when $x = y = 1$ is
a. 2^{x-y}
b. -1
c. 1
d. $\frac{1}{2}$

49. If $\sqrt{x} + \sqrt{y} = 4$ then $\frac{dy}{dx}$ at $x = 1$ is
a. 2
b. $-\frac{3}{2}$
c. $\frac{1}{2}$
d. -3

50. $\frac{dy}{dx}$ of $\sqrt{\frac{1-\cos x}{1+\cos x}}$ is
a. $\sec^2 \frac{x}{2}$
b. $\tan^2 \frac{x}{2}$
c. $\frac{1}{2} \sec^2 \frac{x}{2}$
d. $\frac{\sec^2 \sqrt{x}}{2\sqrt{x}}$

51. $\frac{d \operatorname{cosec}^{-1} x}{dx} =$
a. $\frac{1}{x\sqrt{x^2-1}}$
b. $\frac{-1}{\sqrt{1-x^2}}$
c. $\frac{-1}{x\sqrt{x^2-1}}$
d. $\frac{1}{x\sqrt{1-x^2}}$

52. $\frac{d}{dx}\left(\sec^{-1} \frac{1}{\sqrt{1-x^2}}\right) =$
a. $\frac{1}{1+x^2}$
b. $\frac{1}{\sqrt{1-x^2}}$
c. $-\frac{1}{\sqrt{1-x^2}}$
d. $\frac{1-x^2}{1+x^2}$

53. $\frac{d}{dx} \cosh^{-1}(\sec x) =$
a. $\sin x$
b. $\tan x$
c. $\sec x$
d. $\operatorname{cosec} x$

54. n^{th} derivative of $\sin x$ is
a. $\cos\left(x + \frac{n\pi}{2}\right)$
b. $\sin\left(x + \frac{n\pi}{2}\right)$
c. $\sin\left(x - \frac{n\pi}{2}\right)$
d. $\cos\left(x - \frac{n\pi}{2}\right)$

55. If $y = a \sin mx + b \cos mx$ then $f''(x) =$
a. $m^2 y$
b. my
c. $-mx^2$
d. $-m^2 y$

56. If $y = \log_{\sqrt{e}} \sin x$ then $y_1 =$
a. $\sqrt{e} \cot x$
b. $\frac{1}{2} \cot x$
c. $2 \cot x$
d. $\frac{1}{\sqrt{e}} \cot x$

57. If $x^n + y^n = a^n$ then $\frac{dy}{dx}$
a. $\frac{-x^{n-1}}{y^{n-1}}$
b. $\frac{-1}{y}$
c. $\frac{-x^n}{y^n}$
d. $\frac{y}{x}$

58. If $y = e^{ax}$ then $y_n =$
a. ne^{ax}
b. $a^n y$
c. ae^{nx}
d. $a^{n-1} y$

59. If $y = \tan^{-1}\left(\frac{4x}{1-4x^2}\right)$ then $y' =$
a. $\frac{1}{1+4x^2}$
b. $\frac{16}{1+4x^2}$
c. $\frac{4}{1+4x^2}$
d. $\frac{2x}{1-4x^2}$

60. $\frac{d}{dx}(\log|x|) =$
a. $\frac{1}{x}$
b. x
c. $\frac{|x|}{x}$
d. 0

61. If $y = x^x$ then $f'(x) =$
a. $x^x \log_e x$
b. $x^x (1 + \ln x)$
c. $x^x . e^x$
d. $\log_e (x-1) x^x$

62. Identify the fact:
a. if $f(x)$ is not continuous at x=a then it is differentiable at x=a
b. if f(x) is continuous at x=a then it differentiable at x=a
c. If f(x) and g(x) are differentiable at x=a, $f(x) + g(x)$ is also differentiable at x=a
d. If f(x) is continuous at x=a then $\lim_{x \to a}$ exists.

63. Derivative of $(5 + 3x)^4$ w.t.r $(3x + 5)$ is
a. $5(3x+5)^3$
b. $4(3x+5)^3$
c. $3x^2 + 5x + 4$
d. $12\,(3x+5)^3$

64. Differential coefficient of a^x w.r.t 'x' is
a. $a^x \log_e a$
b. $\frac{a^x}{\log a}$
c. $\frac{-a^x}{\log a}$
d. $\frac{\log_e a}{x}$

65. If $y = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$ then $\frac{dy}{dx} =$
a. $\frac{1}{\sqrt{1-x^2}}$
b. $\frac{3}{1+x^2}$
c. $\frac{1}{1+x^2}$
d. $\sec^2 3x$

66. If $e^{xy} = xy$ then $f'(x) =$
a. $\frac{y}{x}$
b. $\frac{y+x}{y}$
c. $\frac{-y}{x}$
d. $\frac{x}{y}$

67. If $f(x) = |x|$ then it is
a. continuous and derivable at x=0
b. neither continuous nor derivable at x=0
c. derivable but not continuous at x=0
d. continuous but not derivable at x=0

68. If $y = \log \sinh\left(\frac{x}{a}\right)$ then $\frac{dy}{dx} =$
a. $\tanh \frac{x}{a}$
b. $\frac{1}{a} \tanh \frac{x}{a}$
c. $\frac{1}{a} \coth \frac{x}{a}$
d. $\coth \frac{x}{a}$

69. Derivative of odd function is
a. odd function
b. even function
c. neither odd nor even
d. constant function

70. If $y = \tan|x|$ then $\frac{dy}{dx} =$
a. $\sec^2|x| . \frac{x}{|x|}; x \neq 0$
b. $\sec^2|x|$
c. $2 \sec^2 x$
d. $2 \cot|x| . \operatorname{cosec}|x|$

71. If $y = \tan^{-1}\left(\frac{a \sin x + b \cos x}{a \cos x - b \sin x}\right)$ then $\frac{dy}{dx}$
a. $\frac{a}{b}$
b. $-\frac{b}{a}$
c. $\frac{1}{ab} \tan x$
d. 1
Hint: put $a = r \cos\theta, b = r \sin\theta$)

72. If $y = \frac{e^x + e^{-x}}{e^x - e^{-x}}$ then $\frac{dy}{dx} =$
a. $\operatorname{sech}^2 x$
b. $\operatorname{cosech}^2 x$
c. $-\operatorname{sech}^2 x$
d. $-\operatorname{cosech}^2 x$

73. If $x = \frac{1-t^2}{1+t^2}$ and $y = \frac{2t}{1+t^2}$ then $\frac{dy}{dx} =$

a. $-\frac{x}{y}$ b. $-\frac{y}{x}$

c. $\frac{y}{x}$ d. $\frac{x}{y}$

74. If $x = a(\cos t + t\sin t)$; $y = a(\sin t - t\cos t)$ then $\frac{d^2y}{dx^2} =$

a. $\frac{\sec^3 t}{at}$ b. $\tan t$

c. $\sec^2 t$ d. $\frac{\tan t.\sec t}{at}$

75. $\frac{d}{dx}(\tan^{-1}(\sec x + \tan x)) =$

a. 1 b. $\frac{1}{2}$

c. $\cos x$ d. $\sec x$

Answer Key:

1. a	2. b	3. a	4. c	5. a
6. c	7. d	8. a	9. b	10. c
11. c	12. b	13. d	14. a	15. d
16. c	17. d	18. b	19. a	20. d
21. a	22. d	23. a	24. b	25. d
26. a	27. d	28. d	29. c	30. a
31. b	32. a	33. c	34. b	35. c
36. a	37. c	38. d	39. c	40. a
41. d	42. d	43. c	44. d	45. a
46. c	47. c	48. b	49. d	50. c
51. c	52. b	53. c	54. b	55. d
56. c	57. a	58. b	59. c	60. a
61. b	62. b	63. b	64. a	65. b
66. c	67. d	68. c	69. b	70. a
71. d	72. d	73. a	74. a	75. b

A

1. $\frac{dy}{dx}$
$f(x$

2. The
$f(x$

3. If y

4. **Incr**
A fu
if $f'($

5. **Decr**
A fun
if $f'(x$

6. **Criti**
The p
extren
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points

7. **Steps**
minin
a. Find
b. For n

Then,
called
c. Find
$f''(x)$.

Multiple Choice Questions

1. The instantaneous rate of change at $t = 0$ for the function $f(t) = te^{-t} + 9$ is
   a. -1 b. 9 c. 0 d. 2

2. The acceleration of a moving particle whose space time equation is given by $s = 3t^2 + 2t - 5$ is
   a. $0\ m/s^2$ b. $1m/s^2$
   c. $5m/s^2$ d. $6m/s^2$

3. For the curve $x = t^2 - 1, y = t^2 - t$, tangent is parallel to x-axis where,
   a. $t = 0$ b. $t = \frac{1}{\sqrt{3}}$
   c. $t = 0.5$ d. $t = -\frac{1}{\sqrt{3}}$

4. The function $f(x) = x^2 - 2x$ is increasing in the interval
   a. $x \leq -1$ b. $x \geq -1$
   c. $x \leq 1$ d. $x > 1$

5. The minimum value of the function $y = 2x^3 - 21x^2 + 36x - 20$ is
   a. -128 b. -126 c. -120 d. None

6. If $y = 8x^3 - 60x^2 + 144x + 27$ is a decreasing function in the interval (a, b. then (a, b. is
   a. (-3, 2) b. (2, 3) c. (5, 6) d. (-3, -2)

7. x^x has a stationary point at
   a. $x = e$ b. $x = \frac{1}{e}$ c. $x = 1$ d. $x = \sqrt{e}$

8. The point of inflection of the curve $x^3 + 4x$ is [Entrance 2069]
   a. 0 b. 1 c. 6 d. -1

9. The point of inflection of the curve $x^3 + 3x^2 - 9x + 2$ is [Entrance 2071]
   a. -1 b. 1 c. 6 d. 0

10. The point of inflection for $y = x^2 - 2x$ is [Entrance 2072]
    a. 0 b. doesn't exist c. 2 d. 1

11. The critical points for $f(x) = x^3 - 3x$ are [Entrance 2073]
    a. ± 3 b. $\pm\sqrt{3}$ c. ± 1 d. $0, \pm\sqrt{3}$

12. Find the interval in which the function $f(x) = 2x^3 - 15x^2 + 36x + 1$ is increasing
    a. $(-\infty, 2) \cup (3, \infty)$ b. $(-\infty, 2)$
    c. $(3, \infty)$ d. $(-\infty, 2] \cup [3, \infty)$

13. The stationary point for the curve $f(x) = x^2 - 2x$ is
    a. (1, -1) b. (1, 1) c. (-1, 0) d. (0, 2)

14. The curve given by $x + y = e^{xy}$ has a tangent parallel to y-axis at the point.
    a. (0,1) b. (1,0) c. (1,1) d. none

15. The function $f(x) = 5x^3 - 135x + 22$ is decreasing in
    a. $(3, \infty)$ b. $(-\infty, -3)$
    c. $(-\infty, 3)$ d. $(-\infty, \infty)$

16. Function $y = x^3 - 3x^2 - 24x + 5$ is increasing at the interval.
    a. $(-2,4)$ b. $(-\infty, -2) \cup (4, \infty)$
    c. $(-\infty, -2] \cup [4, \infty)$ d. $[-2,4]$

17. The function $f(x) = x^4 - 8x^3 + 18x^2 - 24$ is concave downward at
    a. (1,3) b. $(3, \infty)$
    c. $(-\infty, 1) \cup (3, \infty)$ d. $(-\infty, 3)$

18. The point of inflection of $x^3 - 6x^2 + 9x + 12$ is
    a. 1 b. 2 c. 3 d. $-\frac{1}{2}$

19. A function has
    a. Only one local point
    b. Two local points
    c. three local points
    d. can have many local points

20. Maximum value of $x + \frac{1600}{x}$ is
    a. +40 b. −40 c. +80 d. −80

21. The curve $y = x^2$ and $y = 4 - x^2$ intersect at the point $(\sqrt{2}, 2)$ at an angle of
    a. $\frac{\pi}{4}$ b. $\frac{\pi}{3}$ c. $\frac{\pi}{2}$ d. zero

22. A man who has 144m of tending material wishes to enclose a rectangular garden then the maximum area he can enclose is
    a. $1296\ sq.m$ b. $400\ sq.m$
    c. $600\ m^2$ d. $2025\ sq.m$

23. $f(x) = x^2$ is concave upward in
    a. (−1,1) b. $(-\infty, \infty)$
    c. $[0, \infty)$ d. $[-\infty, 0) \cup [1, \infty)$

24. The function $f(x) = -3x^2 + 12x + 8$ is decreasing at the interval.
    a. $x > 2$ b. $x < 2$
    c. $x > 1$ d. $x < 0$

25. The Velocity of a rectilinear motion given by the equation $s = 5t^2 - 2t + 1$ at any instant $t = 2$ is
    a. 14 b. 16 c. 18 d. 12

26. The radius of sphere is increasing at a constant rate of $1cm/sec$, When the radius is 3 cm. The rate of change in volume at this time is
    a. $42\ \pi\ cm^3/sec$ b. $9\ \pi\ cm^3/sec$
    c. $36\ \pi\ cm^3/sec$ d. $108\ \pi\ cm^3/sec$

27. The distance 's', travelled in 't' seconds by a particle moving in a st.line is given by $s = t^4 - 18t^2$ then, acceleration at $t = 10\ sec$ is
    a. $964\ m/sec^2$ b. $1164\ m/sec^2$
    c. $1120\ m/sec^2$ d. $400\ m/sec^2$

28. The radius of an air bubble is increasing at rate of $\frac{1}{2} cm/sec$. At what rate is volume of bubble increasing when radius is 1 cm?
    a. $2\pi\ cm^3/sec$ b. $\frac{1}{2}\pi\ cm^3/sec$
    c. $\pi^2\ cm^3/sec$ d. $\frac{3\pi}{4}\ cm^3/sec$

29. A spherical snowball is melting such that its volume is decreasing at rate of $1\ cm^3/min$. At

what rate is diameter decreasing when diameter is 10 cm ?

a. $\frac{1}{25\pi}$ cm/min b. $\frac{1}{48\pi}$ cm/min

c. $\frac{1}{50\pi}$ cm/min d. $\frac{5}{\pi}$ cm/min

30. Water is poured into a right circular cylinder of radius 2m at the rate 20 m^3/min. At what rate the level of water is rising the cylinder?

a. $\frac{25}{\pi}$ m/min b. $\frac{108}{\pi}$ π/min

c. $\frac{1}{48}$ π/min d. $\frac{5}{\pi}$ m/min

31. The maximum value of $\left(\frac{x}{a} - \frac{x^2}{2a}\right)$ for $a > 0$ is

a. $\frac{1}{a}$ b. $\frac{1}{2a}$ c. 1 d. 0

32. The distance of a particle in time t' sec is given by $s = t^3 - 6t^2 - 4t - 8$. Then, its acceleration vanishs at t=

a. 1 b. 3 c. 2 d. 4

33. The min value of $\frac{2}{\sin x + \cos x}$ is

a. 1 b. 2 c. $\frac{1}{\sqrt{2}}$ d. $\sqrt{2}$

34. Angle of interaction of curve $y = x^2$ & $x = y^2$ at (1,1) is

a. 90^0 b. 45^0

c. $\tan^{-1}\left(\frac{4}{3}\right)$ d. $\tan^{-1}\left(\frac{3}{4}\right)$

35. The point on the curve $4y = x^2$ where tangent drawn makes angle 45^0. With x-axis is

a. (2,1) b. (−2,1)

c. (3,2) d. (1,3)

36. The equation of tangent to curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at point (a, b) is

a. $\frac{x}{a} + \frac{y}{b} = 1$

b. $\frac{x}{a} + \frac{y}{b} = 2$

c. $x^2 + y^2 = a^2b^2$

d. $x^2a + y^2b = (a^2 + b^2)xy$

37. At what angle dues curve $y((1 + x)) = x$ cut x − axis?

a. $\frac{3\pi}{4}$ b. $\frac{\pi}{3}$ c. $\frac{\pi}{4}$ d. $\frac{3\pi}{2}$

38. The maximum value of $\sin x + \cos x$ is

a. 1 b. 2 c. $\frac{1}{2}$ d. $\sqrt{2}$

39. Let the function $f: R \to R$ be defined by $f(x) = 2x + \cos x$ then f

a. has a minimum value of $x = \pi$

b. has a maximum value at x=0

c. is a decreasing function

d. is an increasing function

40. $f(x) = \tan x - x$

a. always increase

b. always decreases

c. never decreases

d. sometimes increases and sometimes decreases

41. The slope of tangent to curve $2y = 2 - x^2$ at $x = 1$ is

a. −2 b. 1 b. −1 d. $\frac{1}{2}$

42. $f(x) = \tan^{-1} x$ is

a. strictly increasing

b. strictly decreasing

c. neither increasing nor decreasing

d. differentiable everywhere

43. Polynomial $x^3 - 3x^2 + 6x + 3$ has

a. a maximum value

b. a minimum value

c. neither maximum nor minimum value

d. both maximum and minimum

44. The normal to curve $y = f(x)$ is parallel to x − axis if

a. $\frac{dy}{dx} = 0$ b. $\frac{dx}{dy} = 0$

c. $\frac{dy}{dx} = 1$ d. $\frac{dx}{dy} = 1$

45. Maximum area of rectangle which is inscribed in a circle $x^2 + y^2 = a^2$ is

a. πa^2 b. $\pi^2 a^2$

c. $2a^2$ d. $\frac{16\pi a}{3}$

Answer Key:

1. c	2. d	3. c	4. d	5. a
6. b	7. b	8. a	9. a	10. b
11. c	12. a	13. a	14. b	15. c
16. b	17. a	18. b	19. d	20. d
21. d	22. a	23. b	24. a	25. c
26. c	27. b	28. a	29. c	30. d
31. b	32. c	33. d	34. d	35. a
36. b	37. c	38. d	39. d	40. a
41. b	42. a	43. c	44. b	45. c