Математические задачи мобильной робототехники: навигация, автономность и управление движением при коммуникационных ограничениях

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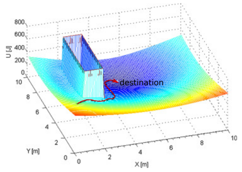

Методы автономной навигации, опирающиеся на дискретизацию сцены

Общие определения и примеры

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Sample-based representation of the scene

- The continuum of points in the free space is represented by finitely many “delegates” (samples)
- Edges signal that there is a known way to transfer the robot from the “tail” sample to the “head” one

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- Single query: a single and a priori known pair "(start sample)-(destination sample)" should be served. These samples are among the nodes of the graph. The overall search process is terminated as soon as the way between these samples is found
- Multi-query: many and a priori unknown pairs should be served. 1 is run until the scene is comprehensively described. Search of the way to the graph (roadmap) and from it is needed.

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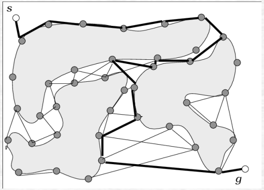

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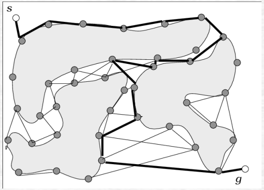

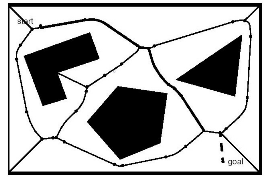

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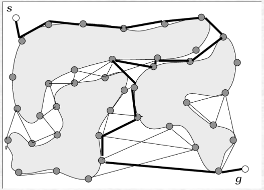

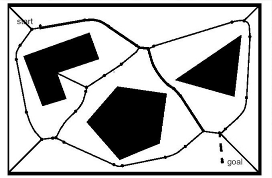

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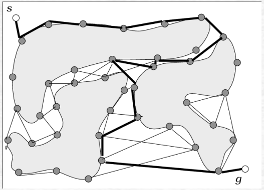

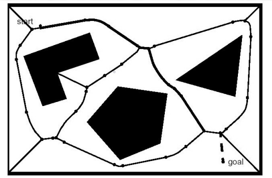

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Criteria of completeness

- Deterministic connectivity of the result
- With probability 1, eventually finds a way
- Finds a way provided that the resolution of some basic underlying sampling structures is high enough

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Sampling set and sampling sequence

Sampling sequence $\Rightarrow$ sampling set
Algorithm of building a sampling sequence assumes an endless process

Dense sequence (in fact algorithm) $\overset{\text{def}}{\Leftrightarrow}$ this sequence is everywhere dense in the sampled set

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- The Riemannian measure on a Riemannian manifold is complete
- Suppose that the probability measure $\mathcal{P}(dx)$ has a density $\rho(\cdot)$ with respect to a complete probability measure $P(dx)$ and $\rho(x) > 0$ for P-almost all $x \in X$. Then the measure $\mathcal{P}(dx)$ is complete.
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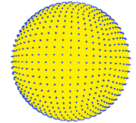

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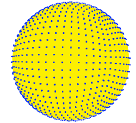

Pseudorandom number generators

- Linear congruential generator

$x_k := m^{-1} X_k, X_{k+1} = (aX_k + c) \mod m$

- Lagged Fibonacci generator

$$x_k := \begin{cases} x_{k-a} - x_{k-b} & \text{if } x_{k-a} \geq x_{k-b} \\ x_{k-a} - x_{k-b} + 1 & \text{otherwise} \end{cases}$$

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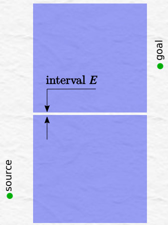

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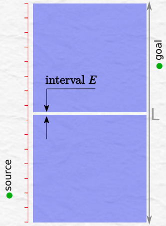

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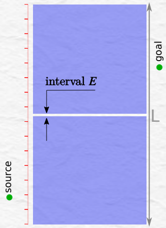

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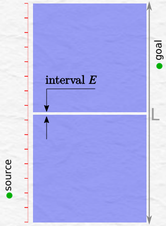

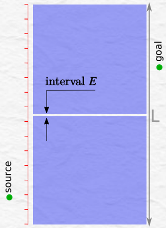

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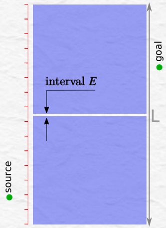

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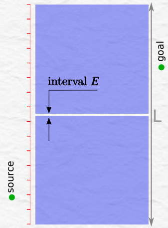

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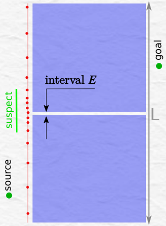

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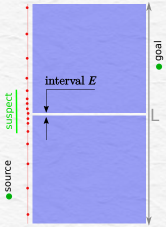

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$$\boldsymbol{P}[x_1 \notin E, \ldots, x_s \notin E] = \prod_{i=1}^{s} \boldsymbol{P}[x_i \notin E] = (1 - \boldsymbol{P}(E))^s$$

$$= \left(1 - \frac{|E|}{L}\right)^s \approx 1 - s\frac{|E|}{L}$$

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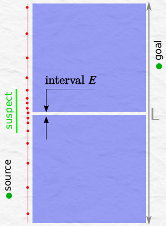
$$= 1 - (1 - \boldsymbol{P}(E))^s \approx \frac{|E|}{L/s}$$

Рандомность и плотность



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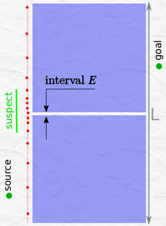

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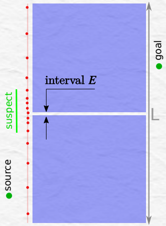

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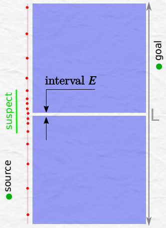

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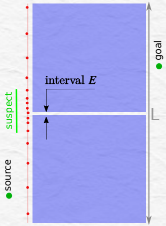

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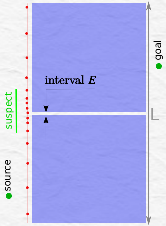

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Test

$$\chi_1^2 \le \chi^2(X) \le \chi_2^2$$

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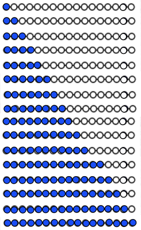

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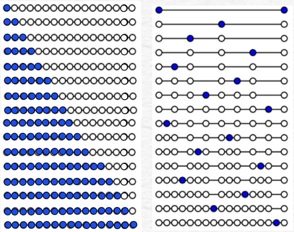

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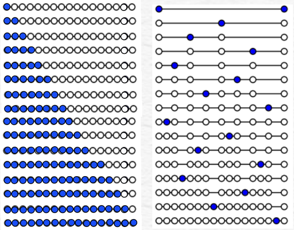

Van der Corput sequence

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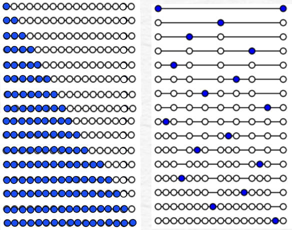

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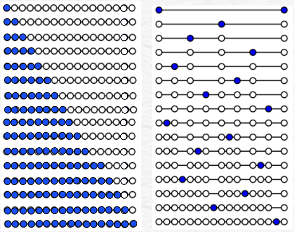

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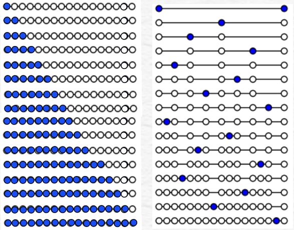

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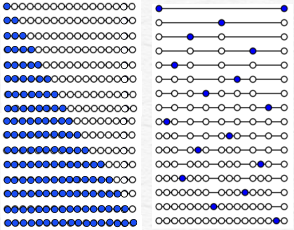

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Discrepancy with the probability distribution with respect to a class of sets

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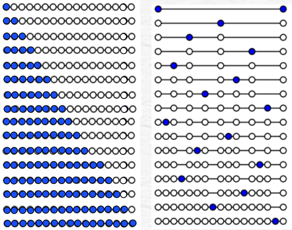

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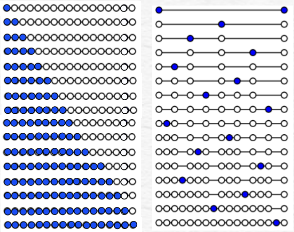

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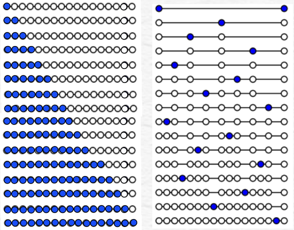

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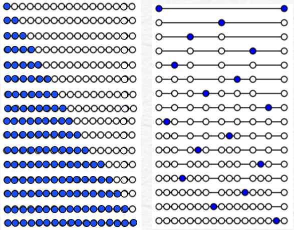

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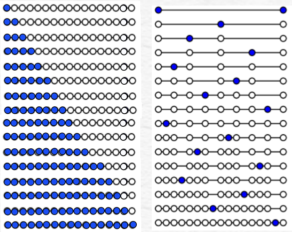

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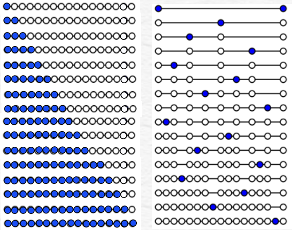

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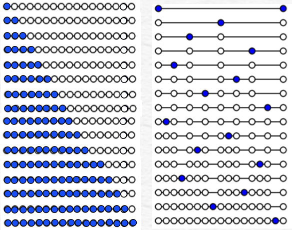

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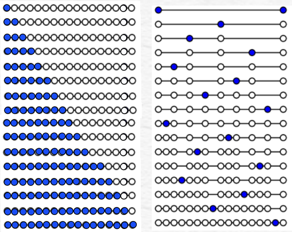

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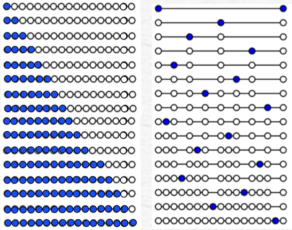

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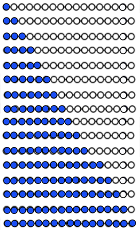

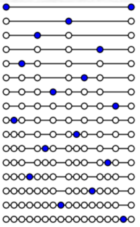

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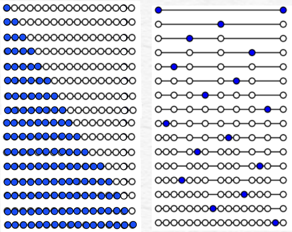
Последовательность ван дер Корпута



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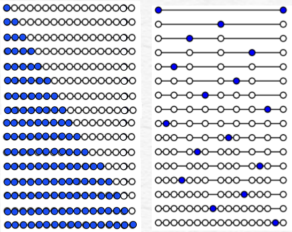

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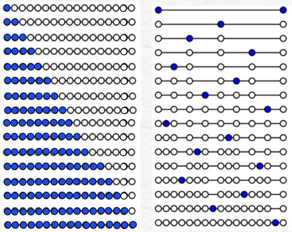

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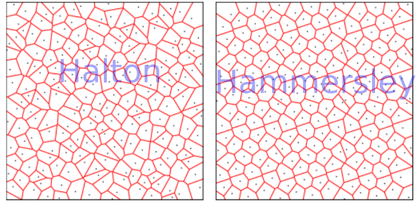

Сэмплирование с низкой дисперсией; сетки, решетки.

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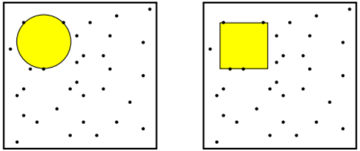

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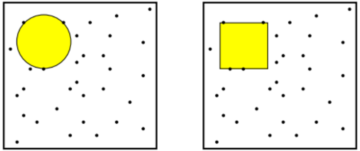

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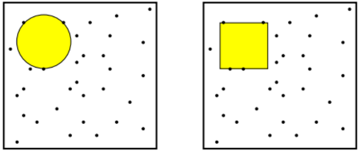

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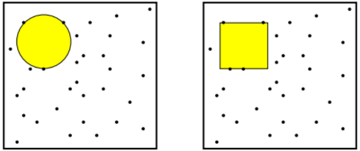

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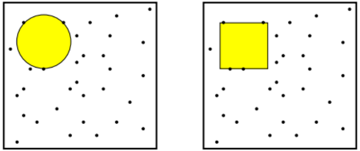

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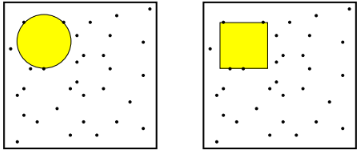

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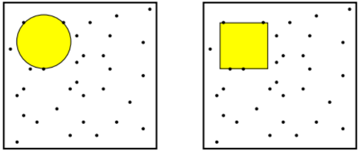

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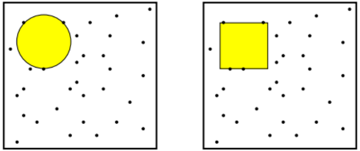

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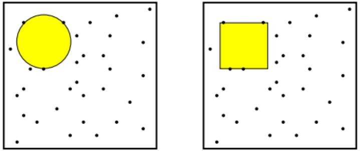

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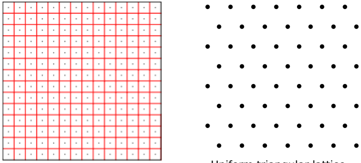

(a) 196-point Sukharev grid

Uniform triangular lattice

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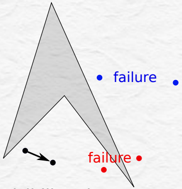

visibility planner

Политика по отношению точкам из запретных областей

- Ignoring (neglecting, skipping, missing)
- Using to build a sample in the free space

Local planner for a given free space

A device (algorithm) that is fed by a pair of samples (s_0, s_+) and outputs the following:

- Either a path $\mathfrak{P}$ from s_0 to s_+ that goes through the given free space and can be traced by the robot (robots)
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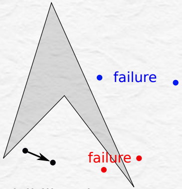

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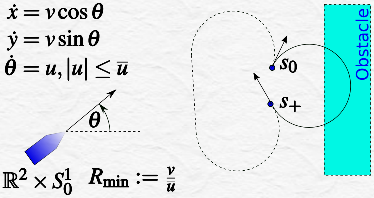

non-holonomic local planner

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Using local planner $\mathfrak{LP}$ for the ideal obstacle-free space

- Form the set S_{true} of "true" samples step-by-step
- If a new sample s_+ lies in the free space, add this sample to S_{true}
- Otherwise
  - Find a sample $s_0 \in S_{\mathrm{true}}$ most beneficial for $\mathfrak{LP}$ to build a path $\mathcal{P}$ from s_0 to s_+ in the ideal, obstacle-free space (e.g., the point s_0 nearest to s_+)
  - Run $\mathfrak{LP}$ to build $\mathcal{P}$
  - Truncate $\mathcal{P}$ by leaving some its initial portion $\mathcal{P}_{\mathrm{in}}$ that does not collide with the obstacles
  - Enrich S_{true} with the end of $\mathcal{P}_{\mathrm{in}}$ different from s_0

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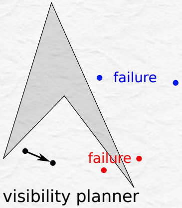

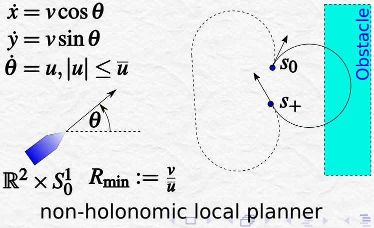

Метод случайной дорожной карты

The basic algorithm

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    - run over all these neighbors
    - for any of them s_+, apply the local planner to (s, s_+)
    - draw an edge from s to s_+, if the local planner is able to construct a path

Some ideas of finer sampling near obstacles

- If a sample is within an obstacle, draw a random direction from the uniform distribution, find a free sample in this direction, a finally find the closest free sample in this direction (via e.g., a dichotomy)
- Step aside any new sample according to a Gaussian distribution, only if one of these samples is free and the other is within an obstacle, the free sample from this pair is recorded

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Find a path between two particular locations ⇒ END

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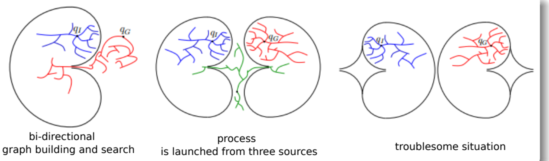

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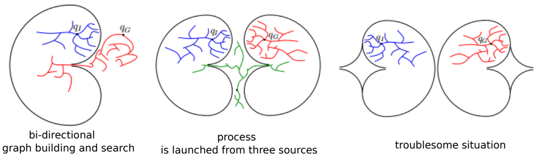

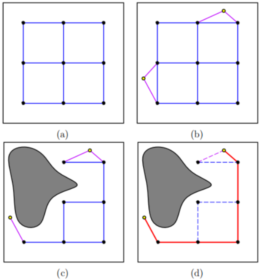

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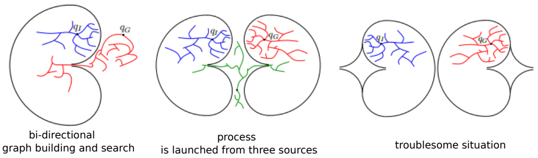

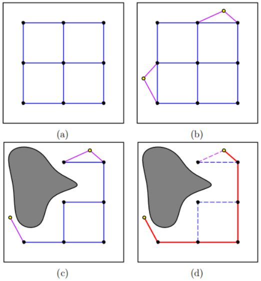

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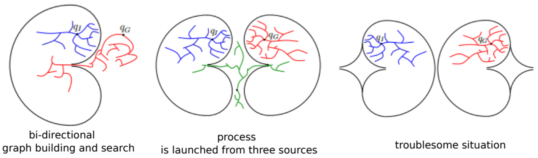

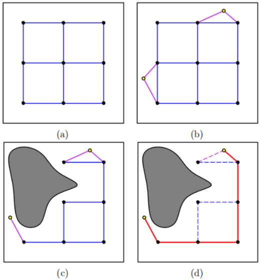
(a) (b) (c) (d)

- Multi-step sessions of graph building interspersed with multi-step sessions of graph searching
- Random walks with creating nodes and edges

Typical algorithm

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- Two trees T_{from} and T_{to} are built, starting from the initial and destination location, respectively

Расширяющиеся деревья

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- This distribution typically depends on T and varies from step to step.
- It is designed so that the nodes with lesser number of neighbors be drawn with a higher probability

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Remark on $P_T(dx)$

- This distribution typically depends on T and varies from step to step.
- It is designed so that the nodes with lesser number of neighbors be drawn with a higher probability

$N(p)$ — the number of nodes in the vicinity of p, including itself

$$\text{Probability to pick } p := \frac{N(p)^{-1}}{\sum_{\text{all nodes } p'} N(p')^{-1}}$$

Быстро растущее случайное (плотное) дерево

“Rapidly expanding tree” algorithm

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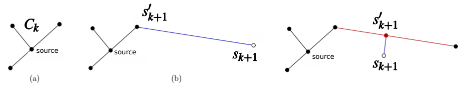

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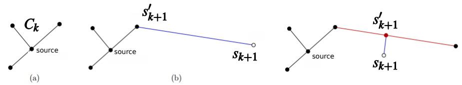

45 iterations

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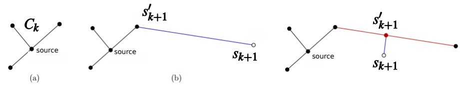

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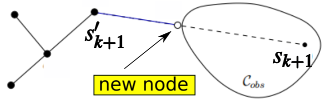

Некоторые вопросы практической реализации

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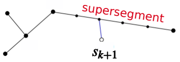

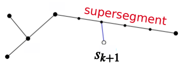

Kd-tree algorithm

The point of S nearest to s_+

1. Find the median x_* of the projection $\mathrm{Pr}_x(S)$
2. Divide S into two parts: $x \le x_*$ and $x > x_*$
3. Redefine S as the part with the same position w.r.t x_* as s_+
4. Find the median y_* of the projection $\mathrm{Pr}_y(S)$
5. Divide S into two parts: $y \le y_*$ and $y > y_*$
6. Redefine S as the part with the same position w.r.t y_* as s_+
7. go to 1