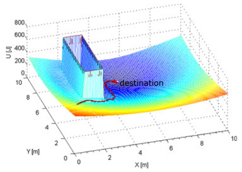
Математические задачи мобильной робототехники: навигация, автономность и управление движением при коммуникационных ограничениях

A. Matveev

Saint Petersburg state University,
Scientific and Technological University “Sirius”
almat1712@yahoo.com



Тема 2: Метод искусственного потенциального поля и навигационных функций

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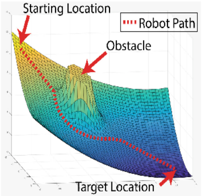

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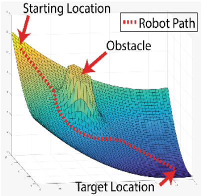

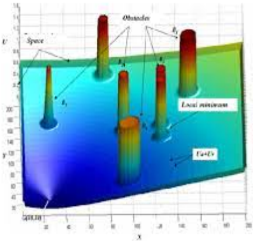

Figure 3. Common artificial potential field.

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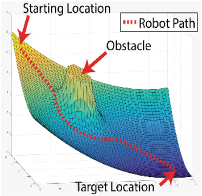

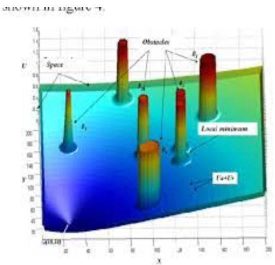

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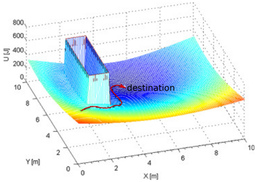

Базовый сценарий и основные идеи метода

What is given and fully known

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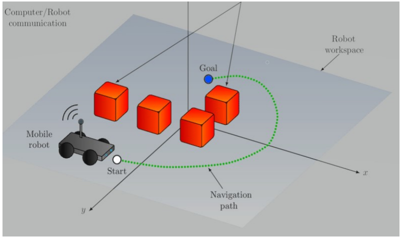

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- Robotic system with a configuration space X whose state $x = x(t)$ evolves over time and is driven by controls $u(t)$
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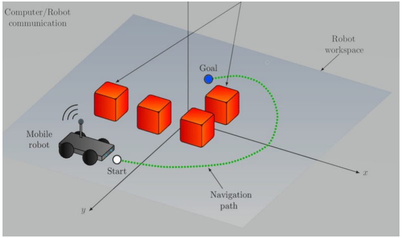

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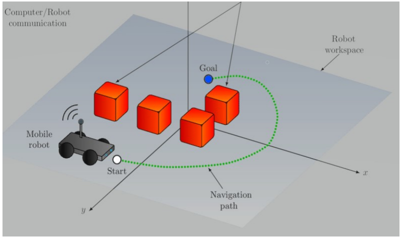

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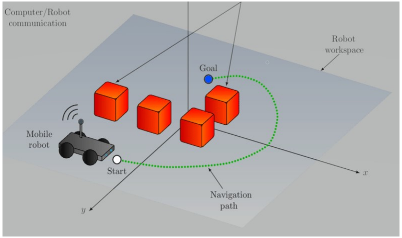

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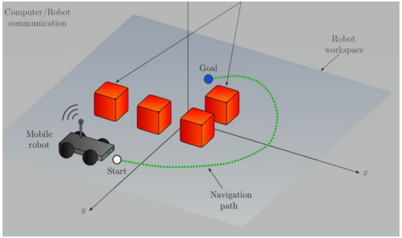

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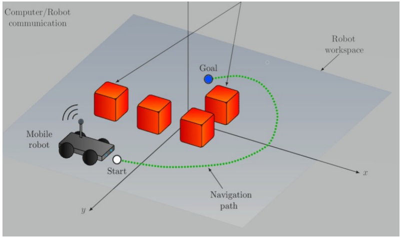

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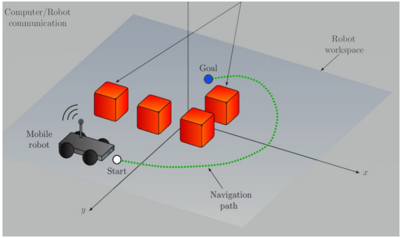

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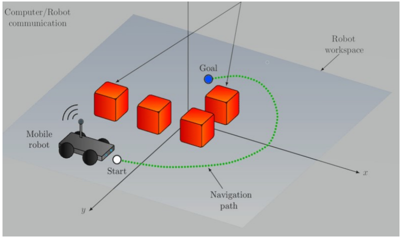

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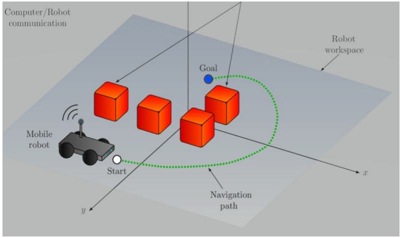

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What is given and fully known in a basic scenario

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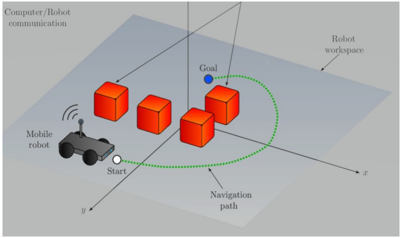
- optional: and ensures that the robot stops at x_{goal}



Базовый сценарий и основные идеи метода

What is given and fully known in a basic scenario

- Robotic system with a configuration space $X = \mathbb{R}^n$ whose state $x = x(t)$ evolves over time and can be driven in any direction
- Working zone $W \subset \mathbb{R}^n$
- Several disjoint obstacles $O_i \subset W$, each simply connected (without holes) and with a smooth boundary ∂O_i (in the case of a plane, bounded by a Jordan curve)
- Destination location $x_{\text{goal}} \in F := W \setminus \bigcup_i O_i$
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Objective

Design a control algorithm such that

- it is of a positional feedback type: $x(t) \mapsto$ direction of motion
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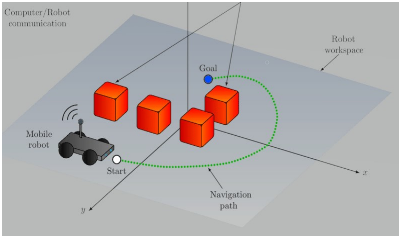

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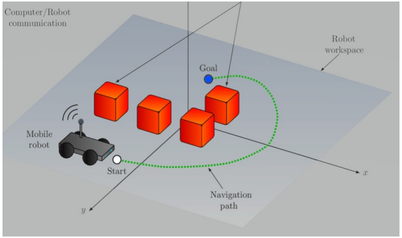

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The task

Path planning

Базовый сценарий и основные идеи метода

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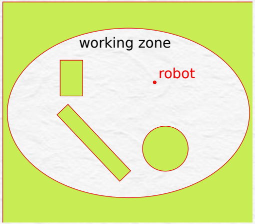

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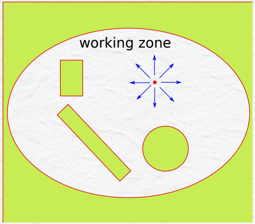

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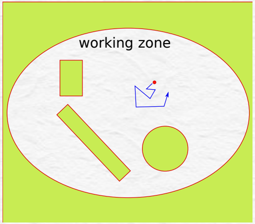

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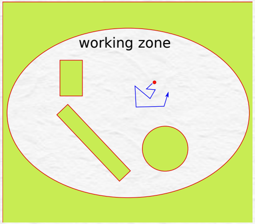

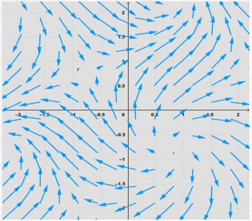

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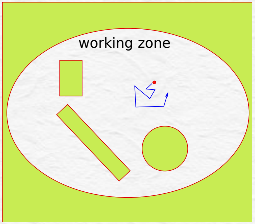

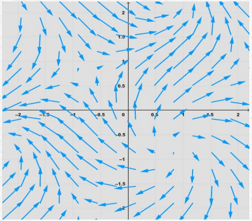

Vector field

Maps the position x into a vector $\vec{v}(x)$ of the same dimension. This vector has the meaning of velocity; in effect, it indicates the direction of motion when being in the position x.

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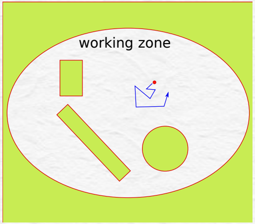

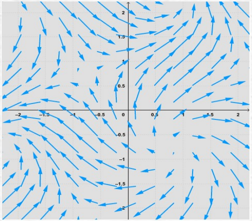

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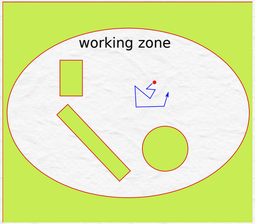

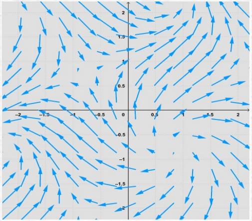

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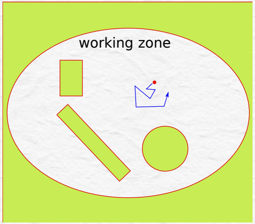

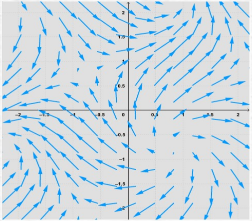

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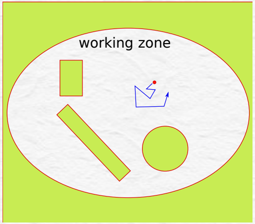

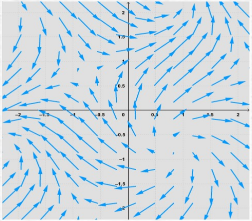

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Particular case: $X_{\mathrm{in}} = F$ the initial position may be at any point of the free space F

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Objective

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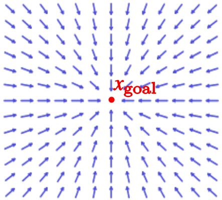
Deciphering the requirements

- Any integral curve that starts in F goes to x_{goal}



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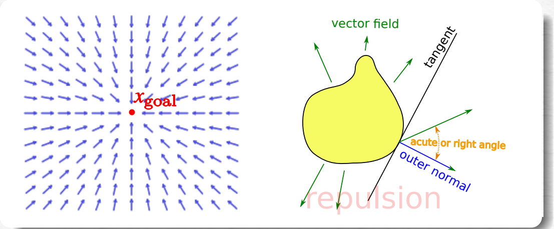
Objective

- drives the robotic system from any initial state in F to the goal $x_{\rm goal}$ through the free space F

Deciphering the requirements

- Any integral curve that starts in F goes to $x_{\rm goal}$
- Any such curve does not intersect the obstacles



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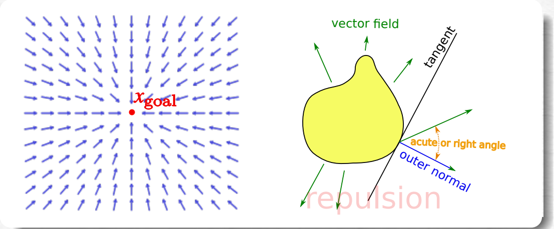
- Any integral curve that starts in F goes to $x_{\rm goal}$
- Any such curve does not intersect the obstacles
- The vector field is smooth (and defined) in F (and its vicinity)



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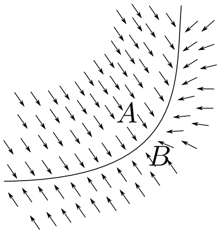

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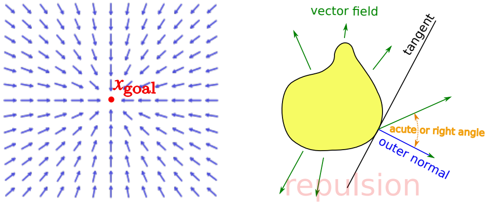

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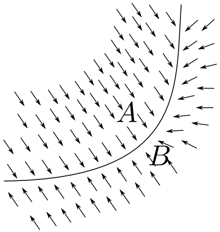
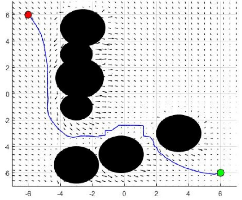
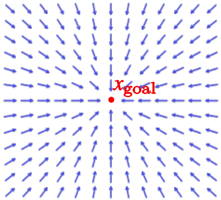
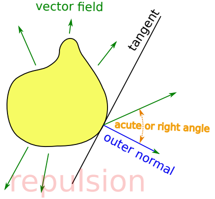
- Any such curve does not intersect the obstacles
- The vector field is smooth (and defined) in F (and its vicinity)









Потенциальное векторное поле

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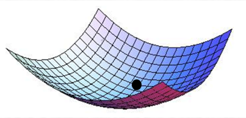
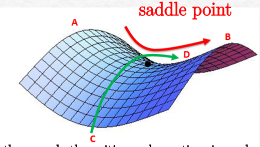
local minimum



the eigenvalues of φ'' are positive

saddle point



there are both positive and negative eigenvalues

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  - For Morse potential, any integral curve goes either to a local minimizer or to a saddle point

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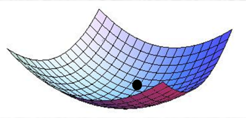

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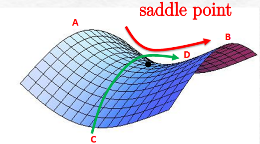

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Выводы: идеальный искусственный потенциал, решающий задачу

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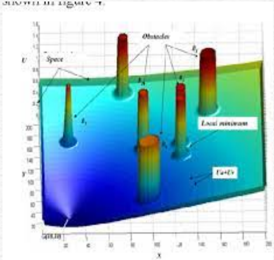

Figure 3. Common artificial potential field.

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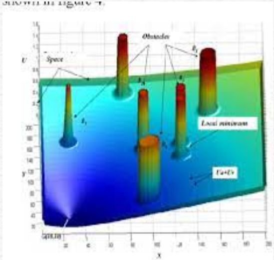

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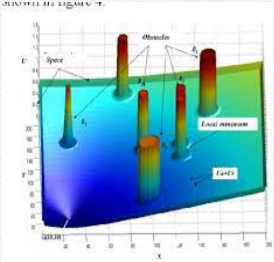

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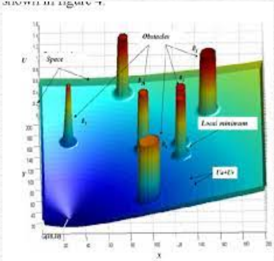

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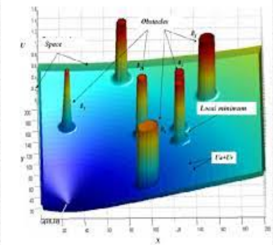

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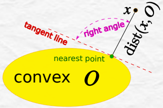

Выводы: идеальный искусственный потенциал, решающий задачу

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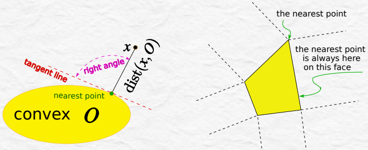

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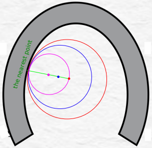

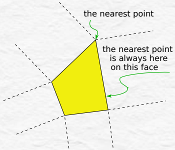

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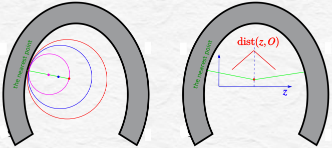

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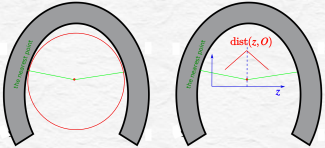

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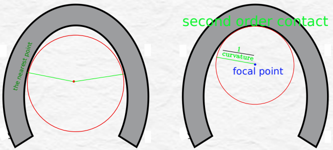

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- For any obstacle O with smooth boundary, there exists $\varepsilon > 0$ such that the function $x \notin O \mapsto \operatorname{dist}(x, O)$ is smooth and has no critical points in the ε-neighborhood N_ε of the obstacle defined by $N_\varepsilon := \{x \notin O : \operatorname{dist}(x, O_i) < \varepsilon\}$.
- If the above O is convex, the distance function is smooth and has no critical points in the entire exterior of the obstacle

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Popular solutions

$$R_i(x) = \frac{1}{[\mathrm{dist}(x, O_i)]^{\varkappa}}, \; R_i = f_{\varkappa} \circ \mathrm{dist}(\cdot, O_i)$$

where $\mathrm{dist}(x, O_i) := \inf_{y \in O_i} \rho(x, y)$ and $\varkappa > 0$

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Popular solutions

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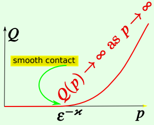

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$$A(x) = [\rho(x, x_{\text{goal}})]^r,$$
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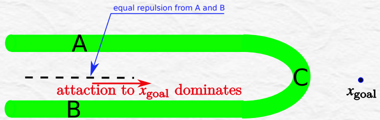
Иллюстрация проблемы построения искуственного потенциала



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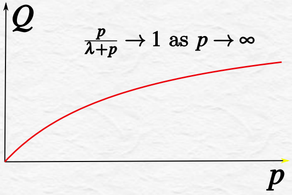
potential φ with the infinite boundary value

$$\Downarrow$$

potential $Q \circ \varphi$ with the finite boundary value 1



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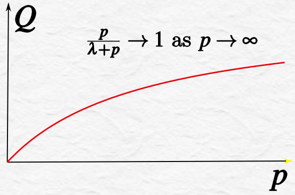

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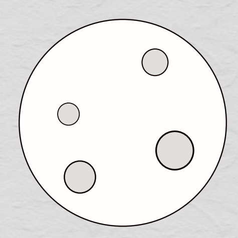

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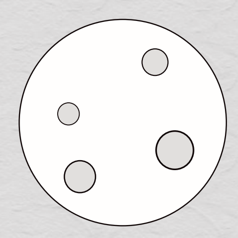

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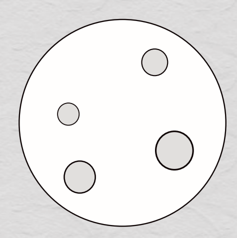

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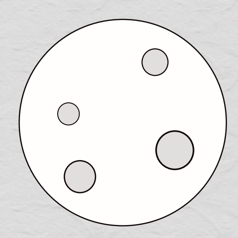

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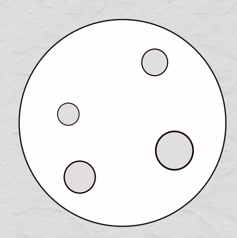

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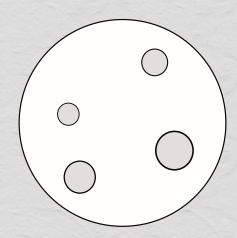

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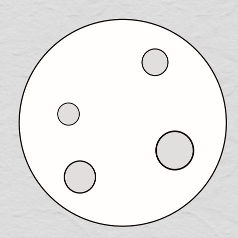

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For any navigation function, the integral curve of the related navigation vector field (i.e., the solution of $\dot{x} = -\nabla\varphi(x)$) goes to x_{goal} through the free space F from almost all initial states in this space.

Навигационная функция

Definition of the navigation function

- It is a Morse function
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An extra assumption

The working zone W is bounded

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For any sphere world with at least one obstacle

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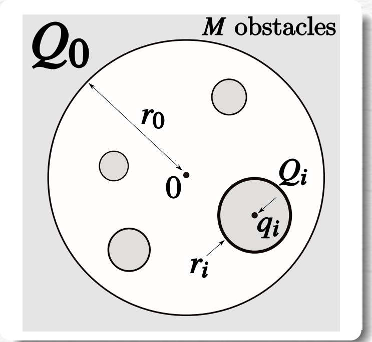
- This set E has the zero volume (Lebesgue measure): for any $\varepsilon > 0$, this set can be covered by balls so that the summary volume of these balls is less than ε

Навигационная функция для сферического мира



Навигационная функция для сферического мира

Analytical representation of the obstacles

$$Q_i = \{x : \beta_i(x) \le 0 \text{ or } \beta_i(\cdot) \text{ is undefined}\}, \quad \partial Q_i = \{x : \beta_i(x) = 0\},$$
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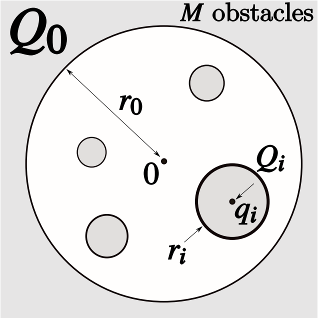

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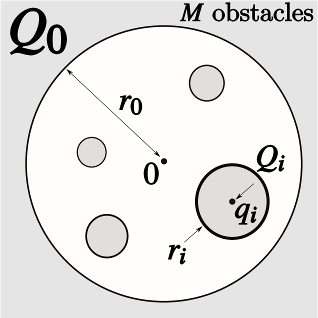

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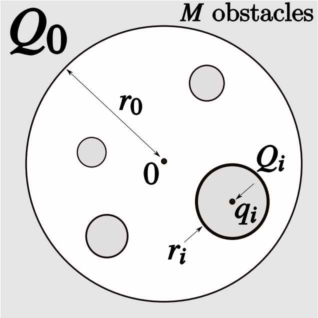

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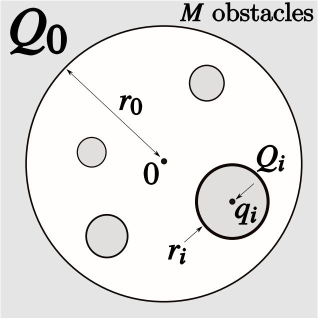

Theorem

For all large enough k, the following formula defines a navigation function of the spherical world

$$\varphi(x) := \left(\frac{\|x - x_{\mathrm{goal}}\|^{2k}}{\beta(x) + \|x - x_{\mathrm{goal}}\|^{2k}} \right)^{\frac{1}{k}}$$

Переход между мирами

Definition of the navigation function

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Observations

- The transformation maps the boundary of the working zone W' into that of W''
- The transformation maps the boundary of any obstacle O_i' into that of the eponymous obstacle O_i''
- The transformation maps the free space of the first world F' into that of the second world F''
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Two worlds with the same number of obstacles

	World 1	World 2
Working zone	W'	W''
Destination	x'_{goal}	x''_{goal}
Obstacles	$O'_1, \ldots, O'_M$	$O''_1, \ldots, O''_M$

These two worlds are isomorphic if there exists a transformation T with the following properties

- It is defined, smooth and one-to-one in a vicinity of F'
- Its Jacobian matrix is everywhere nonsingular
- It maps F' into F''
- It maps O'_i into O''_i for any $i = 1, \ldots, M$
- It maps x'_{goal} into x''_{goal}

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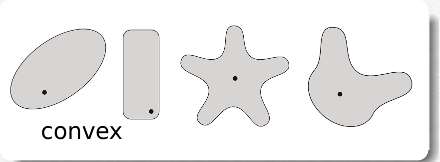

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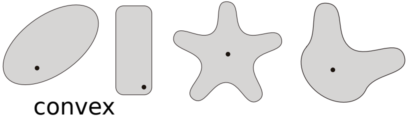

Boundary function of the star

$e \in S_0^1 := \{e \in \mathbb{R}^n : \|e\| = 1\} \mapsto b(e)$ the length of the segment that starts at the center, goes in the direction of e, and ends on the boundary of S

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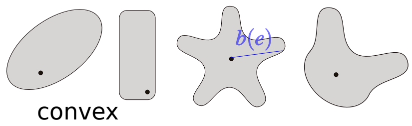

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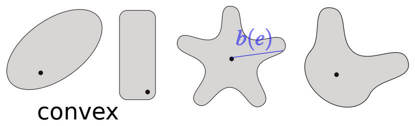

Isomorphism "exterior of a star O_i" ↔ "exterior of a ball"

$$x \mapsto T_i^{\text{ext}}(x) := \rho_i \frac{x-c_i}{b_i\left(\frac{x-c_i}{\|x-c_i\|}\right)} + p_i,$$

p_i the center of the ball

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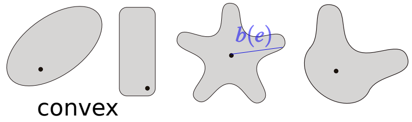

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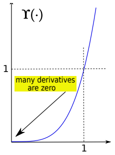
$$x \mapsto T_i^{\text{int}}(x) := \rho_i \Upsilon\left[\frac{\|x - c_i\|}{b_i\left(\frac{x-c_i}{\|x-c_i\|}\right)}\right] \frac{x - c_i}{\|x - c_i\|} + p_i$$



$\Upsilon'(z) > 0 \; \forall z > 0$

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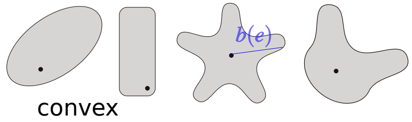

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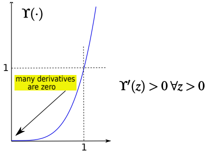

Analytical representation of the star

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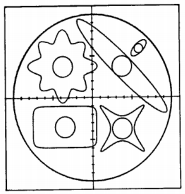

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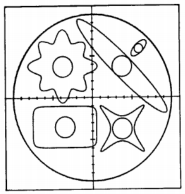

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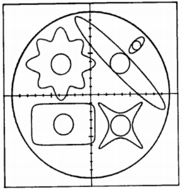

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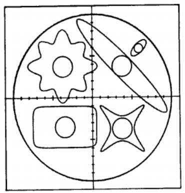

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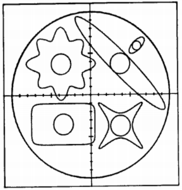

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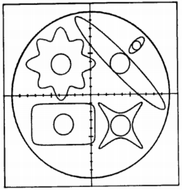


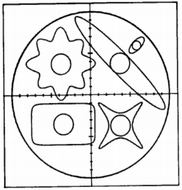
Theorem

For any large enough parameter λ, the map T_λ isomorphically transforms the star world onto some sphere world

Навигационная функция для звездного мира

A subset $S \subset \mathbb{R}^n$ is called star with the center $c \in \mathbb{R}^n$ if

- This set has a smooth boundary and is bounded and closed
- Along with any point $x \in S$, this set contains the segment of the straight line with the end-points x and c (inclusively)
- Any ray emitted from c intersects the boundary of this set and the point of intersection is unique
- At this point, the ray is not tangential to the boundary



Theorem

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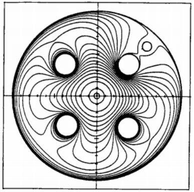
Transformation

- $\overline{\beta}_i := \prod_{j=0, j\neq i}^{M} \beta_j$
- $\gamma_\varkappa(x) := \|x - x_{\text{goal}}\|^{2\varkappa}, \quad \varkappa \geq 1$
- $s_i(x, \lambda) := \frac{\gamma_\varkappa \overline{\beta}_i}{\gamma_\varkappa \overline{\beta}_i + \lambda \beta_i}$, where $\lambda > 0$ is a parameter
- $T_\lambda(x) := \sum_{i=0}^{M} s_i(x, \lambda) T_i(x) + \left[1 - \sum_{i=0}^{M} s_i(x, \lambda)\right] (x - q_0) + p_0$



Навигация в звездных лесах

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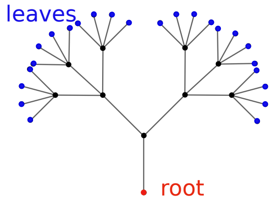

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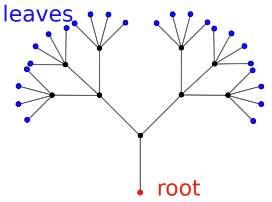

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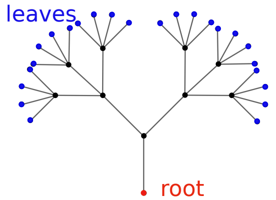

Star-tree shaped obstacle O

The obstacle is the union of a finite set $\mathfrak{S}$ of starts S such that the undirected graph whose nodes are associated with $S \in \mathfrak{S}$ and two nodes are linked if and only if the respective starts are not disjoint is a tree and also the following statements are true under some choice of the root and the center point of every star:

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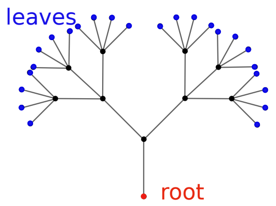

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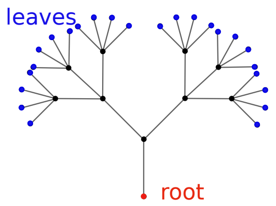

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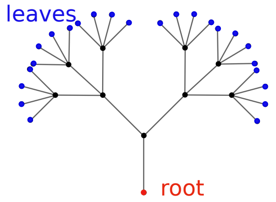

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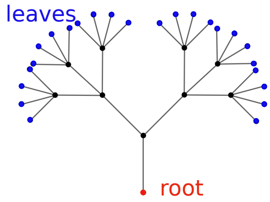

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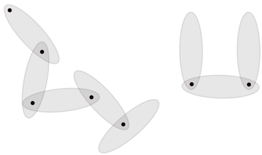

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- Forest of stars = all obstacles are star-tree shaped

Star-tree shaped obstacle O

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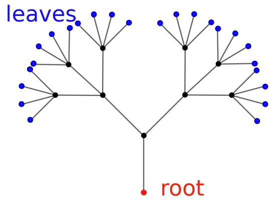
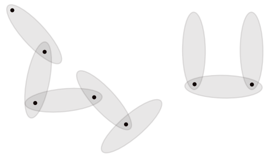




Навигация в звездных лесах

Basic idea

- Isomorphically transform the star forest of depth d into another star forest of depth $d-1$
- Do this iteratively until $d=0$
- Note that star forest of depth $d=0$ is a star world
- Apply the known solution to it and then “reverse” the iterations

Basic step of iteration

- Represent any obstacle as the tree-shaped union of stars. Let $\mathcal{S}$ stand for the united set of these stars.
- Choose a root for any tree
- Identify the set of all leaves $\mathcal{L}$ of the forest
- $T_\lambda(x) :=$ $\sum_{i=0}^{M} s_i(x,\lambda) T_i(x) + \left[1 - \sum_{i=0}^{M} s_i(x,\lambda)\right](x - q_0) + p_0$, where

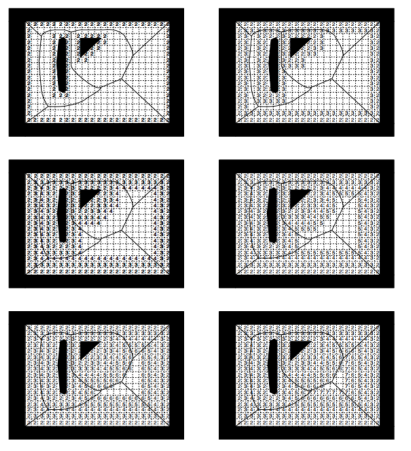
$$s_i(x,\lambda) := \frac{\gamma_\varkappa \overline{\beta}_i}{\gamma_\varkappa \overline{\beta}_i + \lambda \beta_i}$$

$$\gamma_\varkappa(x) := \|x - x_{\mathrm{goal}}\|^{2\varkappa}, \quad \varkappa \geq 1$$

$$\overline{\beta}_i := \left[\prod_{j\in\mathcal{S},\, j\neq i,\, p(i)} \beta_j\right] \times \left[\prod_{j\in\mathcal{L},\, j\neq i} \beta_j\right] \times \widehat{\beta}_{p(i)}$$

$$\widehat{\beta}_{p(i)} := \beta_{p(i)} + (2E_i - \beta_i) + \sqrt{\beta_{p(i)}^2 + (2E_i - \beta_i)^2}$$

Приближенное вычисление расстояния до препятствий: метод лесного пожара



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