Автономная навигация мобильных роботов

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University “Sirius”

Планирование с учетом динамических и кинематических ограничений

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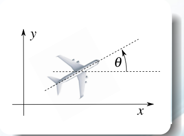

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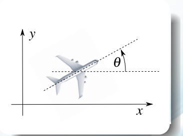

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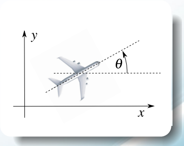

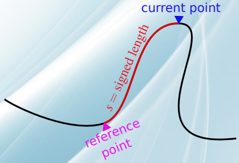

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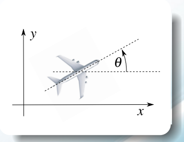

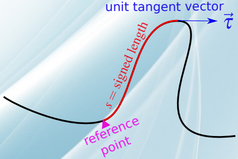

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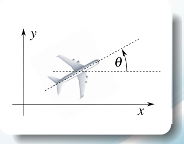

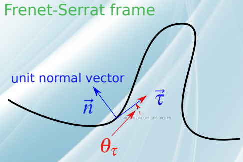

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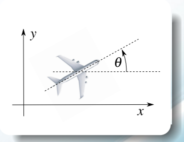
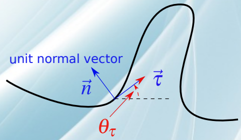


Frenet-Serret frame



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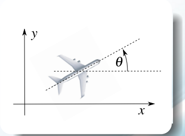

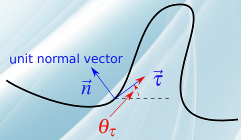

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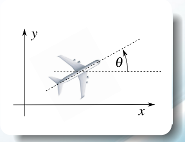

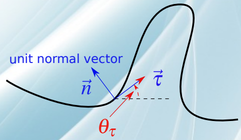

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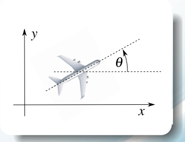

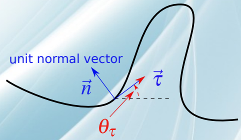

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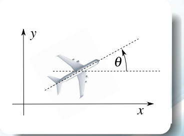

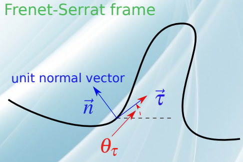

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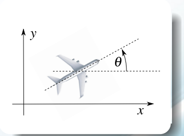

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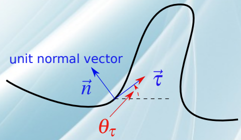

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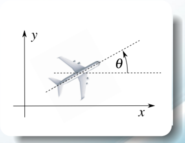

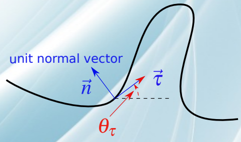

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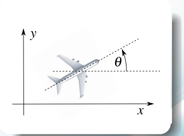

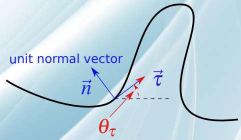

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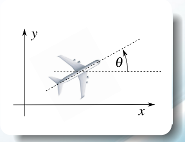

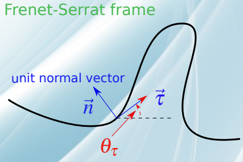

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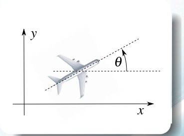

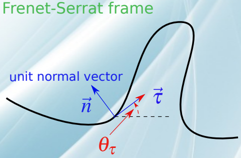

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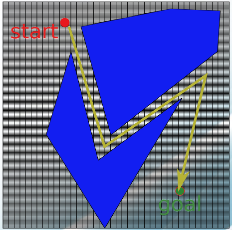

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$$\dot{r}(t) = \begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \end{bmatrix} = v\begin{bmatrix} \cos\theta(t) \\ \sin\theta(t) \end{bmatrix} \qquad \frac{d}{dt}p[s(t)] = \frac{dp}{ds}\dot{s} = \pm\vec{\tau}v = \pm v\begin{bmatrix} \cos\theta_\tau[s(t)] \\ \sin\theta_\tau[s(t)] \end{bmatrix}$$

trackability $\Leftrightarrow \exists s(\cdot)$ s.t. $\theta(t) := \pm\theta_\tau[s(t)]$ meets the limits on the angular velocity

$$\Leftrightarrow |\dot{\theta}(t)| = \left|\frac{d\,\theta_\tau(s)}{ds}\right| |\dot{s}| = |\varkappa| v \leq \overline{\omega}$$

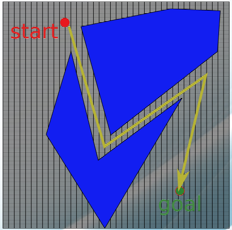
$$\Leftrightarrow |\varkappa| \leq \frac{\overline{\omega}}{v} \Leftrightarrow \varkappa|^{-1} \geq R_{\min} := \frac{v}{\overline{\omega}} \quad \text{curvature radius}$$

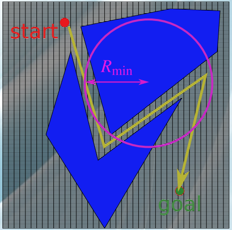
Планирование с учетом динамических и кинематических ограничений

Primary constraints

- Constant speed $v > 0$
- Control by a rudder: sets up the angular velocity of rotation ω
- Constraints on this velocity $|\omega| \leq \overline{\omega}$

$$\dot{x} = v\cos\theta, \;\; \dot{y} = v\sin\theta, \;\; \dot{\theta} = \omega$$





Description of the path

- Path $p = (x, y); p = p(s) \in \mathbb{R}^2$, where s is the natural parameter (arc length)
- $\vec{\tau}(s) = \frac{d\,p(s)}{ds}$ – unit tangent vector
- $\frac{d\,\theta_\tau(s)}{ds} = \varkappa(s)$ – signed curvature

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Path planning for Dubins-car-like robot

Given two points S and G in the plane, along with the attached orientations θ_s and θ_g. Find a smooth path between them such its curvature radius at any point is no less than a given constant $R_{\min} > 0$

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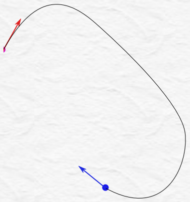

Планирование пути и оптимизация

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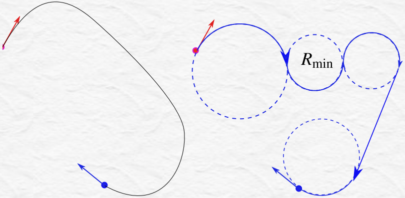

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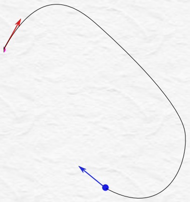
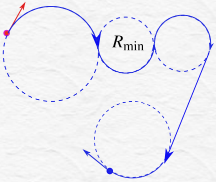
Find the shortest path
The fastest transition





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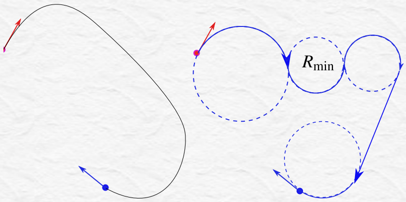

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$J(\text{option}) \to \min$ subject to $\boxed{\text{option} \in X}$

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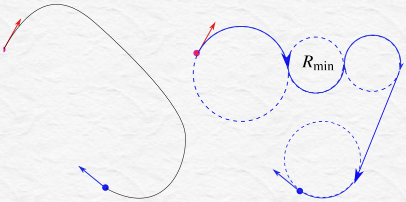
Example

option $\leftrightarrow x \in \mathbb{R}^n$, $\quad X = \mathbb{R}^n$
x is optimal $\Rightarrow \nabla J(x) = 0 \quad$ Fermat equation



Принцип максимума Понтрягина

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Parameters and variables

- t, time
- $t \in [0, T]$, where $T > 0$ is given
- $x = \{x_i\}_{i=1}^n = x(t) \in \mathbb{R}^n$, state at time t
- $u = u(t) \in \mathbb{R}^m$, control at time t

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Problem statement

$$\dot{x}(t) = f[x(t), u(t)], \quad u(t) \in \Omega \qquad \forall t \in [0, T], \tag{1}$$

$$x(0) = x_0, \quad x(T) = x_1, \qquad x_0, x_1 \in \mathbb{R}^m \text{ are given} \tag{2}$$

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Conjugate variable $\psi = \psi(t) \in \mathbb{R}^n$

$$\psi \sim \dot{x} = f[x, u, t] \leftrightarrow x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \psi = \begin{bmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_n \end{bmatrix}, \quad \begin{matrix} \psi_1 \sim \dot{x}_1 = f_1(\dots) \\ \psi_2 \sim \dot{x}_2 = f_2(\dots) \\ \vdots \\ \psi_n \sim \dot{x}_n = f_n(\dots) \end{matrix}$$

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Hamiltonian function

$$H[\psi, x, u, t] := \psi^\top f[x, u] - \lambda_0 \varphi[x, u]$$
$$= \sum_{j=1}^n \psi_j f_j[x, u] - \lambda_0 \varphi[x, u]$$

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Theorem (Pontryagin's maximum principle)

Let $[T^0, x^0(\cdot), u^0(\cdot)]$ be an optimal process. Then there exist a Lagrange multipliers λ_0 and smooth conjugate function $\psi(\cdot) : [0, T^0] \to \mathbb{R}^n$ such that

$$\dot{\psi}_i(t)^\top = -\frac{\partial H}{\partial x_i}[\psi(t), x^0(t), u^0(t)] \quad \forall i = 1, \dots, n, \tag{4}$$

$$u^0(t\pm) = \arg\max_{v \in \overline{\Omega}} H[\psi(t), x^0(t), v] \qquad \forall t \in [0, T^0], \tag{5}$$

$$\lambda_0 \geq 0, \qquad H[\psi(t), x^0(t), u^0(t)] = 0 \qquad \forall t, \tag{6}$$

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Ω is a polygon
the functions $f(x, u)$ and $\varphi(x, u)$ are linear in the control u

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$H(\psi, x, u) = \psi^\top f(x, u) - \lambda_0 \varphi(x, u)$ is linear in u as well
Any linear function attains maximum on a polygon in the set of its vertices $\mathrm{Vert}(\Omega)$

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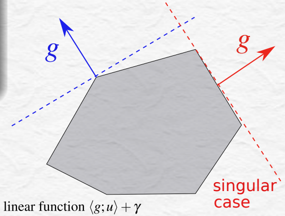

Применение принципа максимума в машине Дубинса

Применение принципа максимума в машине Дубинса

Problem statement

$$\dot{x} = v\cos\theta, \quad \dot{y} = v\sin\theta, \quad \dot{\theta} = u \qquad u \in [-\overline{\omega}, \overline{\omega}], \qquad \begin{array}{ll} x(0) = x_0, & x(T) = x_1, \\ y(0) = y_0, & y(T) = y_1, \\ \theta(0) = \theta_0 & \theta(T) = \theta_1 \end{array} \qquad T = \int_0^T \underbrace{1}_{\varphi} dt \to \min_{T, u(\cdot)}$$

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Maximum principle

Hamiltonian function: $H = v\psi_x \cos\theta + v\psi_y \sin\theta + \psi_\theta u - \lambda_0$

$$\dot{\psi}_x = -\frac{\partial H}{\partial x} = 0, \quad \dot{\psi}_y = -\frac{\partial H}{\partial y} = 0, \quad \dot{\psi}_\theta = -\frac{\partial H}{\partial \theta} = v(\psi_y \cos\theta - \psi_x \sin\theta)$$

$$u(t\pm) = \arg\max_{\mathrm{u} \in [-\overline{\omega}, \overline{\omega}]} [v\psi_x \cos\theta(t) + v\psi_y \sin\theta(t) + \psi_\theta(t)\mathrm{u} - \lambda_0]$$

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$\lambda_0 \geq 0$ and either $\lambda_0 > 0$ or some of the functions $\psi_x, \psi_y, \psi_\theta$ is not identically zero

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$$u(t\pm) = \begin{cases} \overline{\omega} & \text{if } \psi_\theta(t) > 0 \\ -\overline{\omega} & \text{if } \psi_\theta(t) < 0 \\ ? & \text{if } \psi_\theta(t) = 0 \end{cases}$$

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Применение принципа максимума в машине Дубинса

Problem statement

$$\begin{array}{lll} \psi_x \sim & \dot{x} = v\cos\theta, & \\ \psi_y \sim & \dot{y} = v\sin\theta, & u \in [-\overline{\omega}, \overline{\omega}], \\ \psi_\theta \sim & \dot{\theta} = u & \end{array} \quad \begin{array}{ll} x(0) = x_0, & x(T) = x_1, \\ y(0) = y_0, & y(T) = y_1, \\ \theta(0) = \theta_0 & \theta(T) = \theta_1 \end{array} \quad \lambda_0 \sim T = \int_0^T \underbrace{1}_{\varphi}\, dt \to \min_{T, u(\cdot)}$$

Maximum principle

Hamiltonian function: $H = v\psi_x \cos\theta + v\psi_y \sin\theta + \psi_\theta u - \lambda_0$

$$\psi_x = \mathrm{const}, \;\; \psi_y = \mathrm{const}, \quad \dot{\psi}_\theta = v(\psi_y \cos\theta - \psi_x \sin\theta)$$

$$u(t\pm) = \begin{cases} \overline{\omega} & \text{if } \psi_\theta(t) > 0 \\ -\overline{\omega} & \text{if } \psi_\theta(t) < 0 \\ ? & \text{if } \psi_\theta(t) = 0 \end{cases}$$

$$v\psi_x \cos\theta(t) + v\psi_y \sin\theta(t) + \overline{\omega}|\psi_\theta(t)| - \lambda_0 \equiv 0 \quad \forall t$$

$\lambda_0 \geq 0$ and either $\lambda_0 > 0$ or some of the functions $\psi_x, \psi_y, \psi_\theta$ is not identically zero

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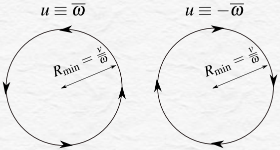

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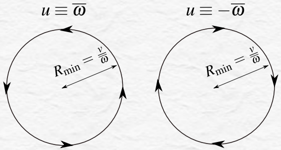

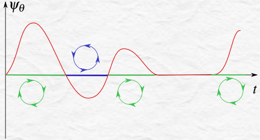

Применение принципа максимума в машине Дубинса

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\color{blue}{\psi_x \sim} & \dot{x} = v\cos\theta, & \\
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\color{blue}{\psi_\theta \sim} & \dot{\theta} = u &
\end{array}
\quad
\begin{array}{ll}
x(0) = x_0, & x(T) = x_1, \\
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\theta(0) = \theta_0 & \theta(T) = \theta_1
\end{array}
\quad
\color{blue}{\lambda_0} \sim T = \int_0^T \underbrace{1}_{\varphi}\, dt \to \min_{T, u(\cdot)}
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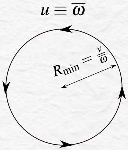

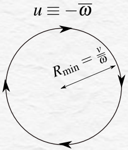

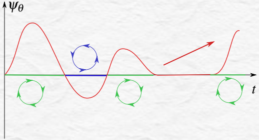

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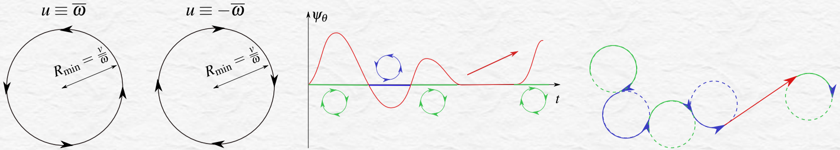

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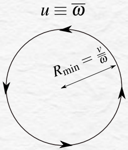

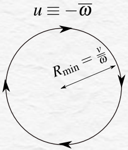

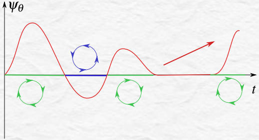

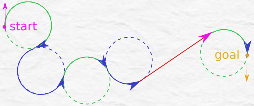

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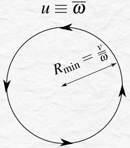

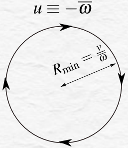

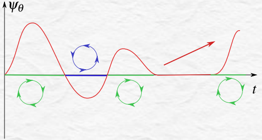

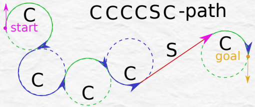

Применение принципа максимума в машине Дубинса

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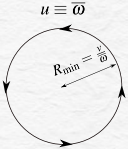
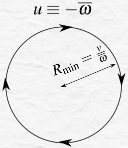
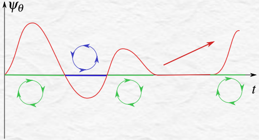
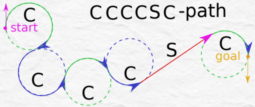
Theorem

(Dubins) Any optimal path between two locations and orientations is either of type CSC, or of type CCC with the arc of the middle circle $> \pi$, or a degenerate of these.









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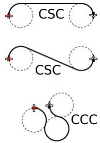

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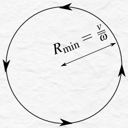

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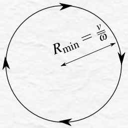

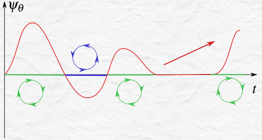

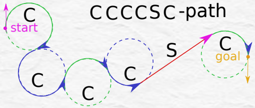

Кинодинамическое планирование для сцен с препятствиями

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Кинодинамическое планирование для сцен с препятствиями

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- Planar robot modelled as a “dynamical particle”
  $\dot{p} = v, \quad \dot{v} = u,$ the acceleration u is the control input
  $p \sim (x, y)$ is the robot’s location

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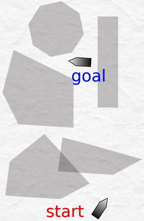

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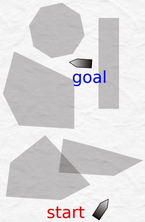
- Static obstacles $O_1, \dots, O_N$, possibly overlapping. Each of them is a convex polygon. The complexity of the scene is assessed via the total number n of the faces of all obstacles
- Goal: pass from the given initial configuration $p(0) = p_0, v(0) = v_0$ to the given desired configuration $p(T) = p_+, v(T) = v_+$ through the obstacle-free part of the plane. The time T of transition is not pre-specified.



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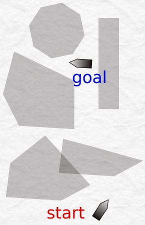
- Optimal transition: do the same in the minimal possible time



Кинодинамическое планирование для сцен с препятствиями

Problem statement

- Planar robot modelled as a “dynamical particle”
  $\dot{p} = v, \quad \dot{v} = u,$ the acceleration u is the control input
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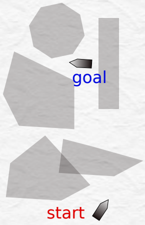

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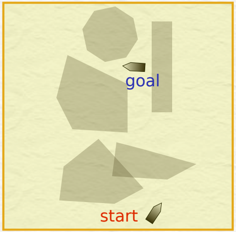

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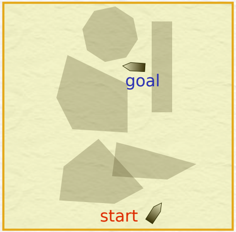

Problem statement: enhancement

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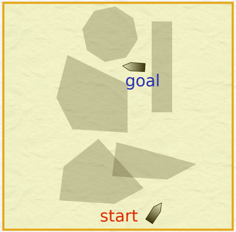

Problem statement: enhancement

- Safe trajectory: distance to the obstacles $\geq c_0 + c_1\|v(t)\| \ \forall t$

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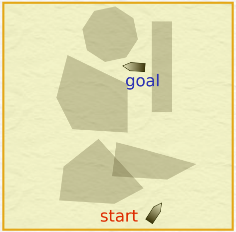
- Find a time-optimal safe trajectory $p^0(t), t \in [0, T^0]$



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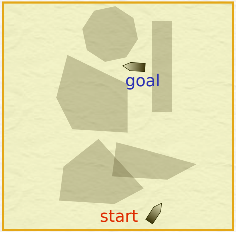

Problem statement: enhancement

- Safe trajectory: distance to the obstacles $\geq c_0 + c_1\|v(t)\| \ \forall t$
- Find a time-optimal safe trajectory $p^0(t), t \in [0, T^0]$

Problem statement: relaxation

- ε-suboptimal safe trajectory

$$\text{distance} \geq (1-\varepsilon)[c_0 + c_1\|v(t)\|] \ \forall t,$$

$$T \leq (1+\varepsilon)T^0,$$

$$\|p(0) - p_0\| \leq C_1\varepsilon, \left\|\frac{v(0)}{1+\varepsilon} - v_0\right\| \leq C_2\varepsilon$$

$$\|p(T) - p_+\| \leq C_3\varepsilon, \left\|\frac{v(T)}{1+\varepsilon} - v_+\right\| \leq C_3\varepsilon$$

Кинодинамическое планирование для сцен с препятствиями

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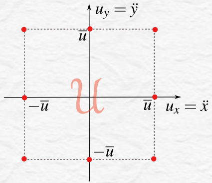

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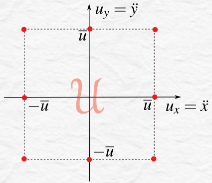

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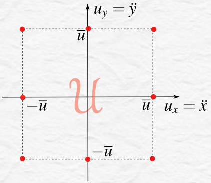

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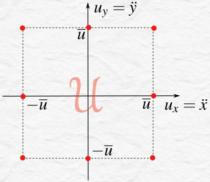

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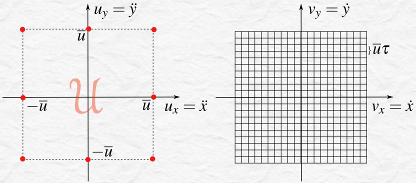

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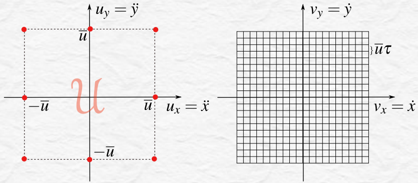

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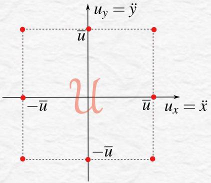

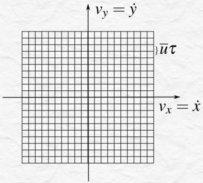

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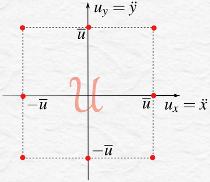

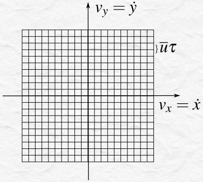

When starting from a vertex of the τ-grid, the run ends at another vertex of this grid

Кинодинамическое планирование для сцен с препятствиями

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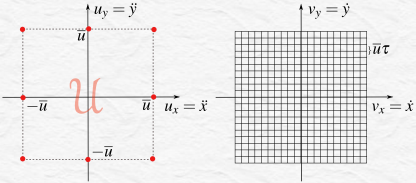

The vertices (p_x, p_y, v_x, v_y) of the grid:

$$p_x = p_x^* + n_x \frac{\overline{u}}{2}\tau^2, \quad p_y = p_y^* + n_y \frac{\overline{u}}{2}\tau^2$$
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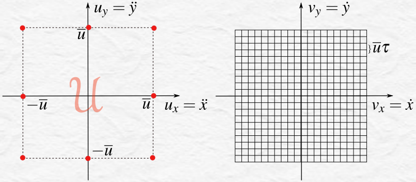

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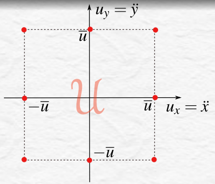

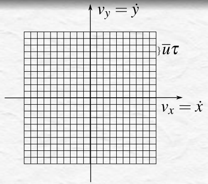

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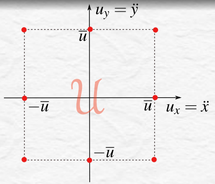

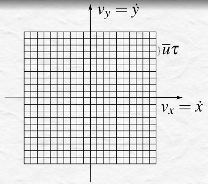

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$$u \in \mathcal{U} \Leftrightarrow u_x = k_x\overline{u}, u_y = k_y\overline{u}, \quad k_x, k_y \in \{-1, 0, 1\}$$

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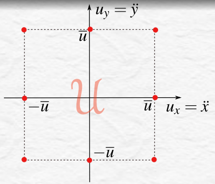
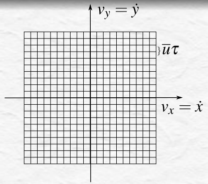
When starting from a vertex of the τ-grid, the run ends at another vertex of this grid





The vertices (p_x, p_y, v_x, v_y) of the grid:

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where n_x, n_y, m_x, m_y are integers Let

$$(p_x, p_y, v_x, v_y) \xrightarrow{u=(u_x,u_y)\in\mathcal{U}} (p_x^+, p_y^+, v_x^+, v_y^+)$$

$$\begin{aligned} v_x^+ &= v_x + \tau u_x \\ v_y^+ &= v_y + \tau u_y \end{aligned}$$

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Кинодинамическое планирование для сцен с препятствиями

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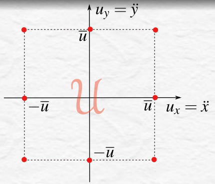

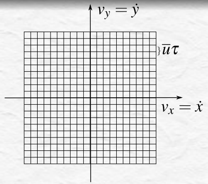

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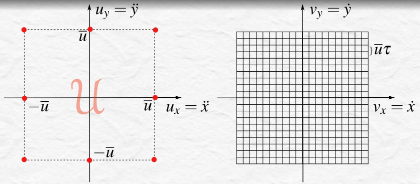

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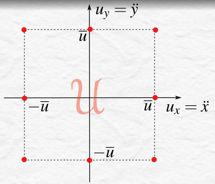

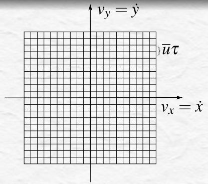

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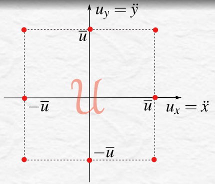

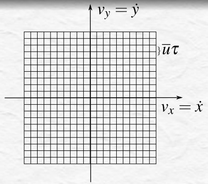

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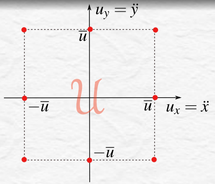

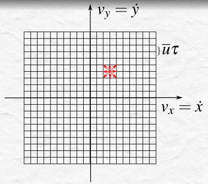

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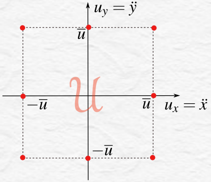

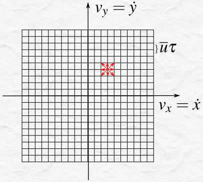

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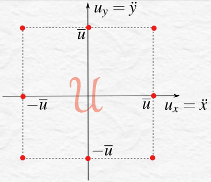

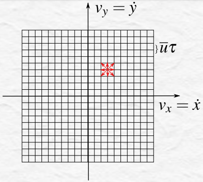

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- Take $\tau = O(\varepsilon^{3/2})$ (explicit formula)

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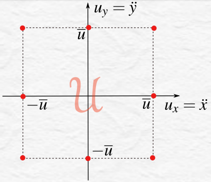

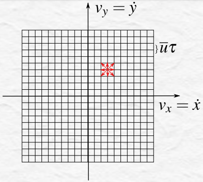

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- Consider the node of the graph closest to $[p_0, \frac{v_0}{1+\varepsilon}]$
- By using the breath-first method, find a shortest path in the graph Γ from this node to a node that is within $(\overline{u}\tau^2/2, \overline{u}\tau/2)$ of $[p_1, \frac{v_1}{1+\varepsilon}]$

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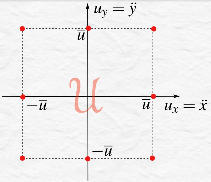

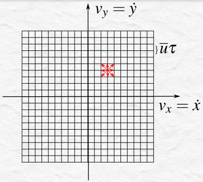

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If there is a safe trajectory, the algorithm finds an ε-suboptimal trajectory. Its complexity (the number of operations) = $O\left(\frac{n}{\varepsilon^{12}}\right)$, where n is the number of the faces in the totality of all obstacles