Control and estimation under communication constraints

Alexander Pogromsky

Lecture 4

Technische Universiteit
Eindhoven
University of Technology

September 10, 2021

(τ, Q)-spanning set

$$x(t+1) = \varphi(x,u) := \varphi_u(x), \quad x \in \mathbb{R}^n, \ \ u \in \mathcal{U}, \ \ t \in \mathbb{N}$$

$$\underline{u} := \{u_0, u_1, u_2, \dots, \} - \text{sequence of controls}$$

$$F(j, x, \underline{u}) = \varphi_{u_{j-1}} \circ \varphi_{u_{j-2}} \circ \cdots \circ \varphi_{u_0}(x) := x_u(j, x)$$

Let $Q \in \mathbb{R}^n$ be a compact set with nonvoid interior.
For a natural τ a set $\mathcal{S}$ is called (τ, Q)-spanning, if for every $x \in Q$ there is a control sequence $\underline{u}$ of length τ such that

$$x_u(j, x) \in \text{int} Q, \ \ \forall j = 1, \dots, \tau$$

Invariance entropy

The minimal cardinality of the (τ, Q)-spanning set is denoted by $r_{\text{inv}}(\tau, Q)$.

Definition

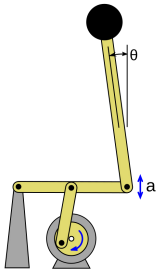
The invariance entropy is

$$H_{\text{inv}}(Q) = \limsup_{\tau \to \infty} \frac{1}{\tau} \log_2 r_{\text{inv}}(\tau, Q)$$

A limited number of examples.

An excited pendulum (Kapitza pendulum)

$$\ddot{\psi} + \delta\dot{\psi} - (u(t) + \omega^2)\sin\psi = 0$$



https://youtu.be/rwGAzy0noU0

https://www.youtube.com/watch?v=R_XH9IaDFtU (start watching from 21:20)

8. Stabilization over communication channels

$$x(t+1) = Ax(t) + Bu, \quad y = Cx$$

Problem

What is the smallest data rate so that there is a coder and controller which stabilize the system?

Theorem W. Wong and R. Brockett (1997,1998), G. Nair and R. Evans (1998, 2000, 2002, 2004), S. Tatikonda (2000)

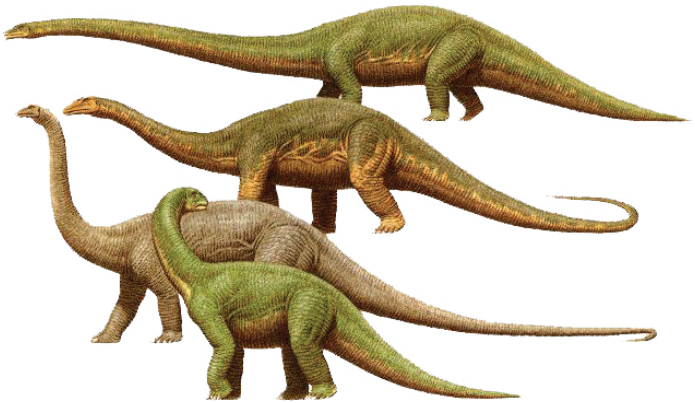
$$c_* = \sum_{\text{unstable}\lambda_i} \log_2 |\lambda_i|$$

Instead of conclusion

Milton Erikson, used to tell a story, that a centipede was asked how it was able to move all the hundred legs in such a synchronous way. After this question had been put to the poor creature, it had been unable to make a step ever since.

J. Haley, *Jay Haley On Milton H. Erikson*, Brunner/Mazel, Bristol, 1993

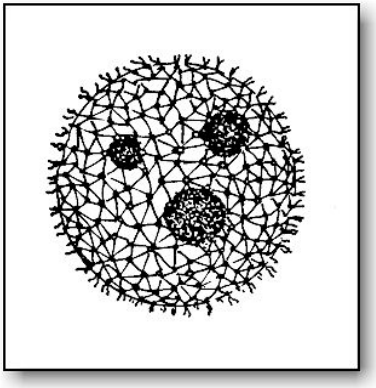
Communication in biology



A moving centipede

borneo.avi
Lego-Centipede.avi
centipede4-1.mp4

Communication in a living colony



Volvox: a colony or an organism?