Математические задачи мобильной робототехники: навигация, автономность и управление движением при коммуникационных ограничениях

A. Matveev

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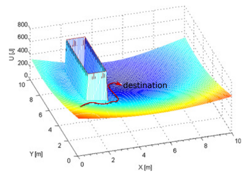

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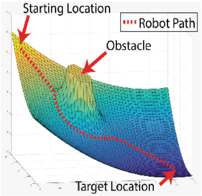

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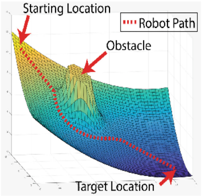

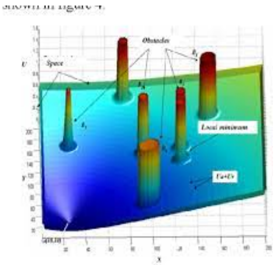

Figure 3. Common artificial potential field.

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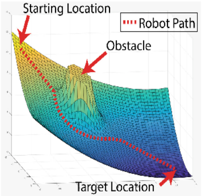

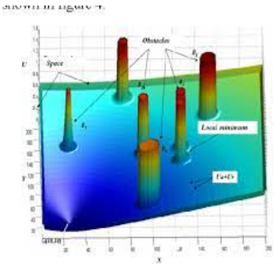

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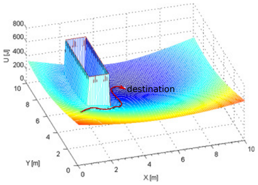

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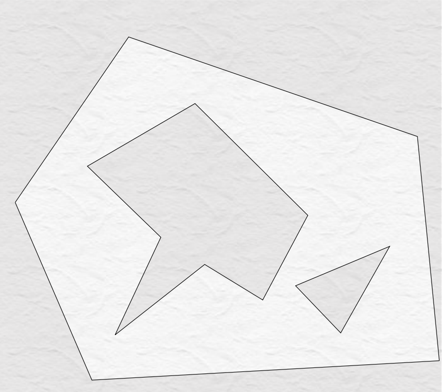

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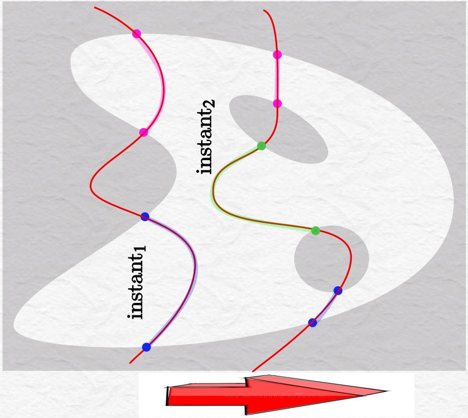

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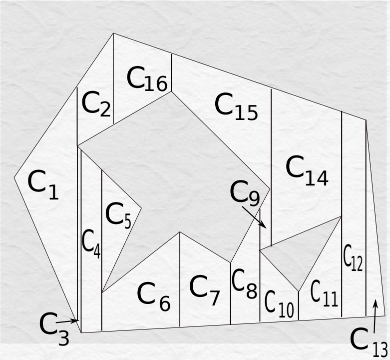

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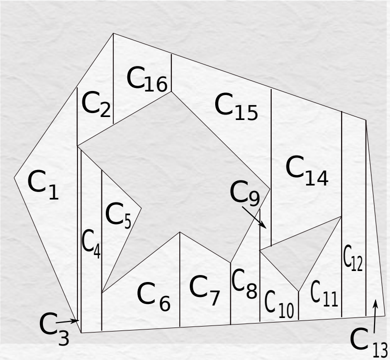

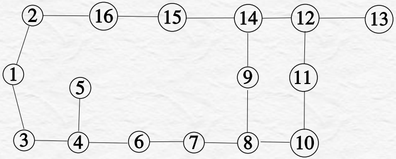

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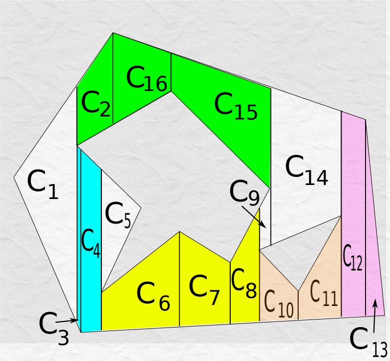

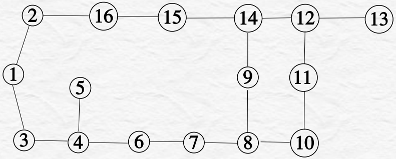

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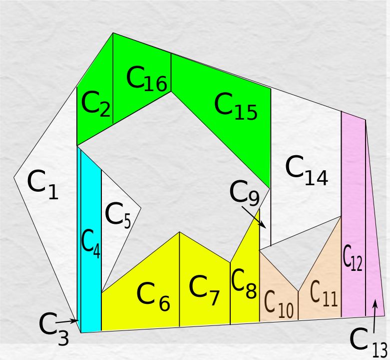

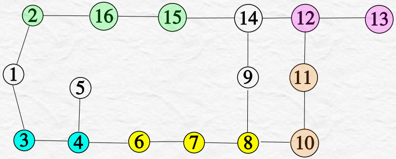

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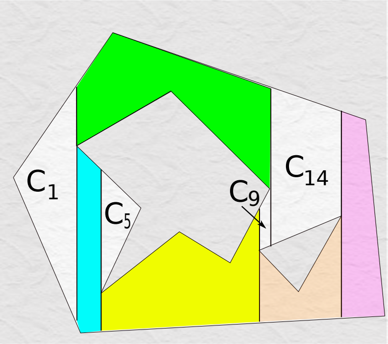

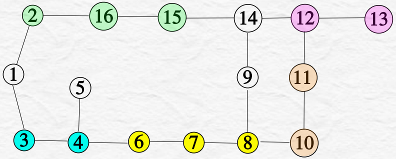

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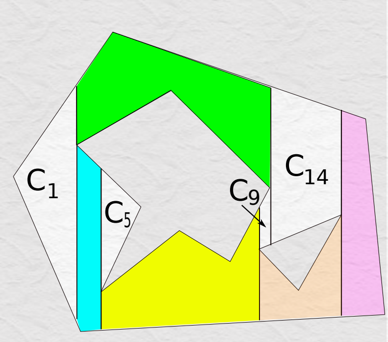

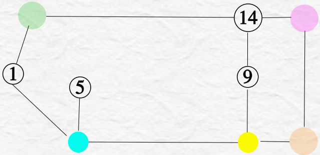

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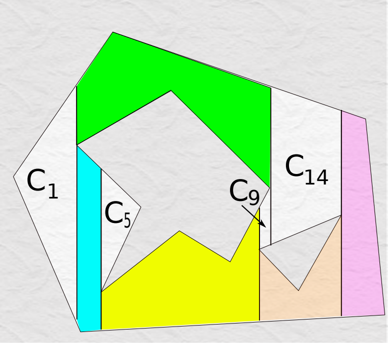
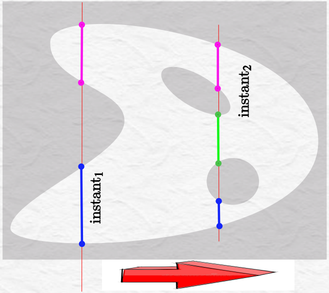
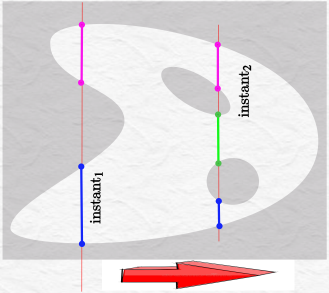


$$\oint_{\partial D} \frac{(x - x_0)dy - (y - y_0)dx}{r^2} = \begin{cases} 2\pi & \text{if } \left| \begin{array}{l} \text{the point } p_0 \\ \text{with the coordinates } x_0, y_0 \\ \text{lies inside } D \end{array} \right. \\ 0 & \text{if this point lies outside } D \end{cases}$$

r is the distance from p_0 to the current point


instant$_1$
instant$_2$

The sweeping line L depends of a real parameter λ, i.e., $L = L_\lambda$.



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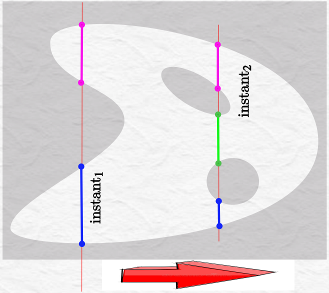

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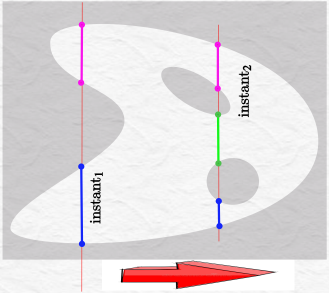

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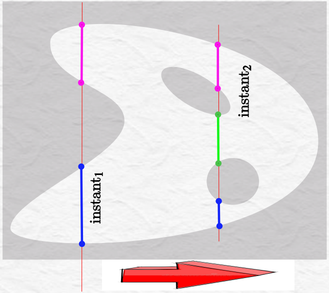

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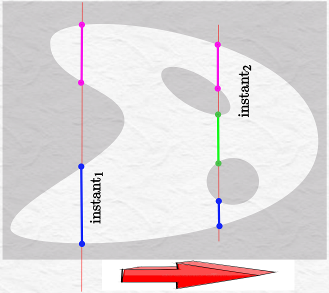

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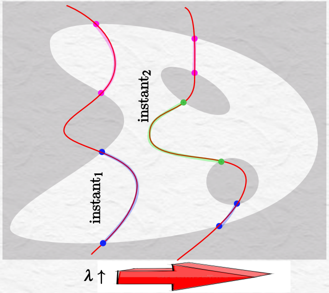

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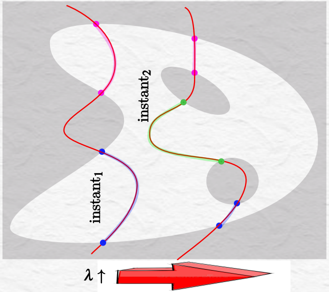

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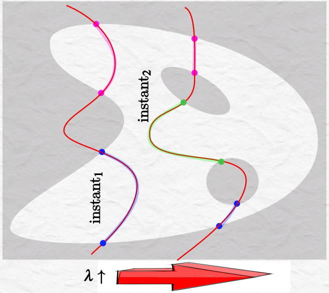

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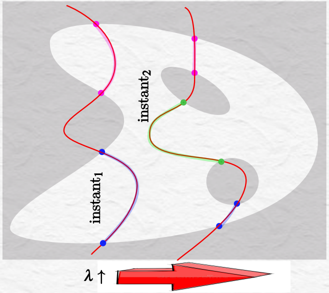

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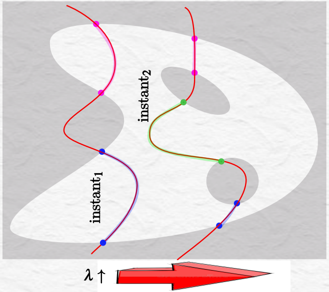

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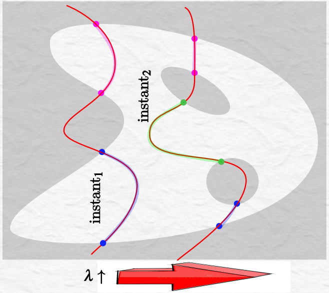

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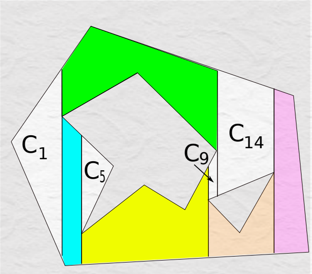

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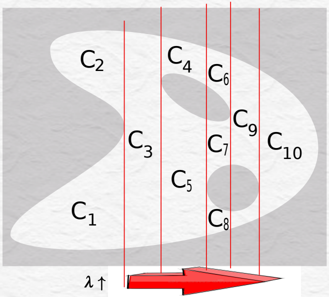

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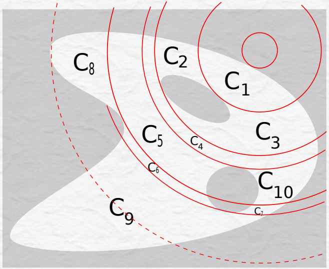

g is the distance to a certain point

The sweeping line L depends of a real parameter λ, i.e., $L = L_\lambda$.
Specifically, $L_\lambda := \{x = (x_1, x_2) : x_1 = \lambda\} = \{x : g(x) = \lambda\}$, where $g(x) := x_1$

Prerequisite

A smooth function $g(\cdot)$ of the state that is defined in a vicinity of the free zone and has no critical points in the free zone: $\nabla g(x) \neq 0 \ \forall x \in \overline{F}$

Sweeping curve $\Gamma_\lambda = \{x : g(x) = \lambda\}$

Basic procedure

- Let λ continuously increase, starting with a value that ensures subsequent full coverage of the free zone
- Register the "bifurcation" points: the values of λ such that passing through them is accompanied with alteration of the number of connected components (segments) in the intersection of the sweeping curve Γ_λ with the free zone. A indication: the gradient $\nabla g(x)$ is normal to the boundary ∂O of some obstacle O at some point $x \in \Gamma_\lambda \cap \partial O$
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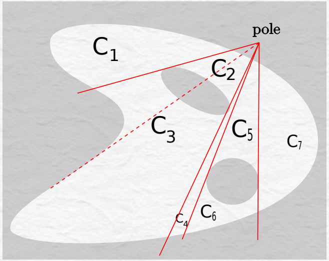

g is the polar angle

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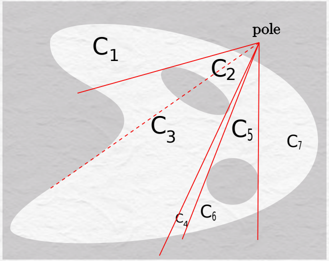

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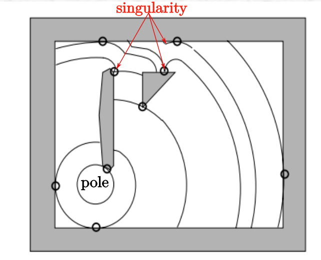

g is the length of the shortest path to the pole in the free zone

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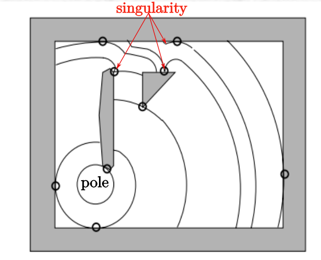

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Voronoi partition

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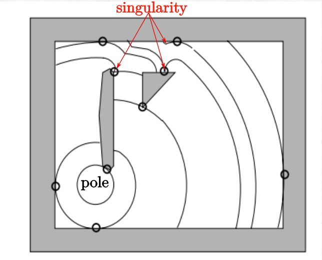

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Voronoi partition

- Generalizations on higher dimensions

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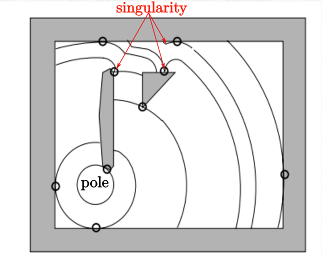

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Voronoi partition

- Generalizations on higher dimensions
- Approximate cell decompositions

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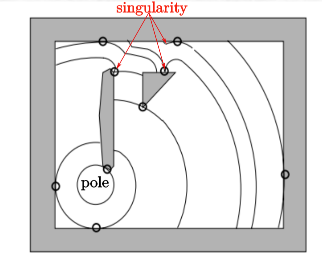

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Voronoi partition

- Generalizations on higher dimensions
- Approximate cell decompositions
- Probabilistic cell decompositions