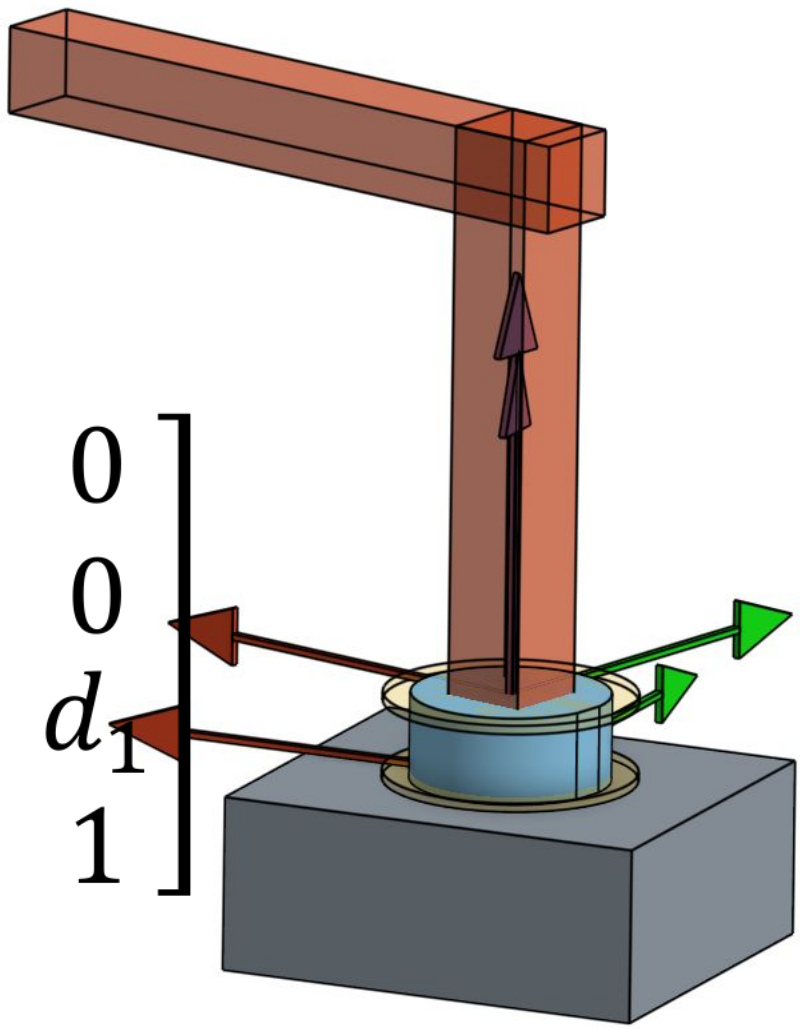
Link 1: revolute joint

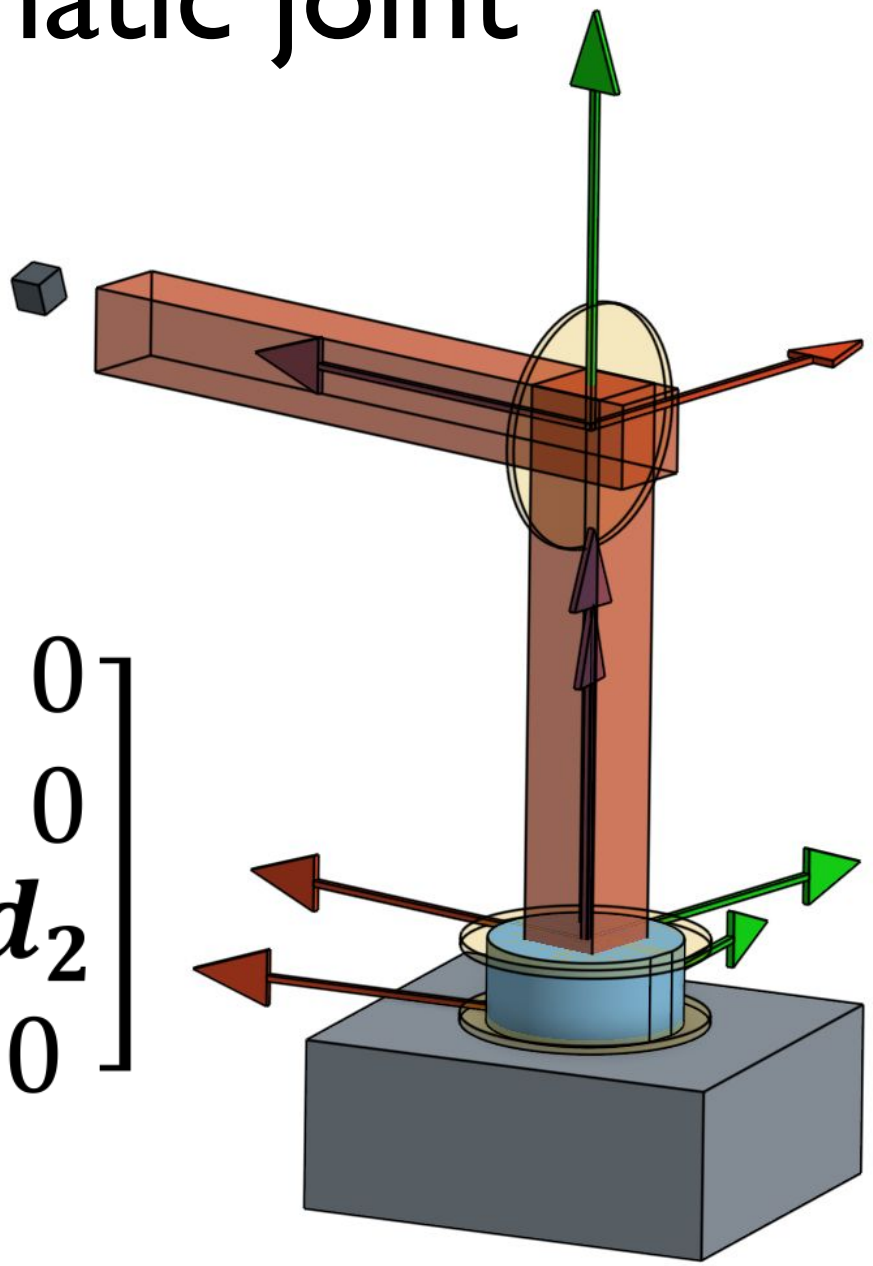
Link	a_i	α_i	d_i	θ_i
1	0	0	d_1	$\underline{\theta_1}$
2	0	-90	$\underline{d_2}$	0
3	0	0	$\underline{d_3}$	0

$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Link 2: prismatic joint

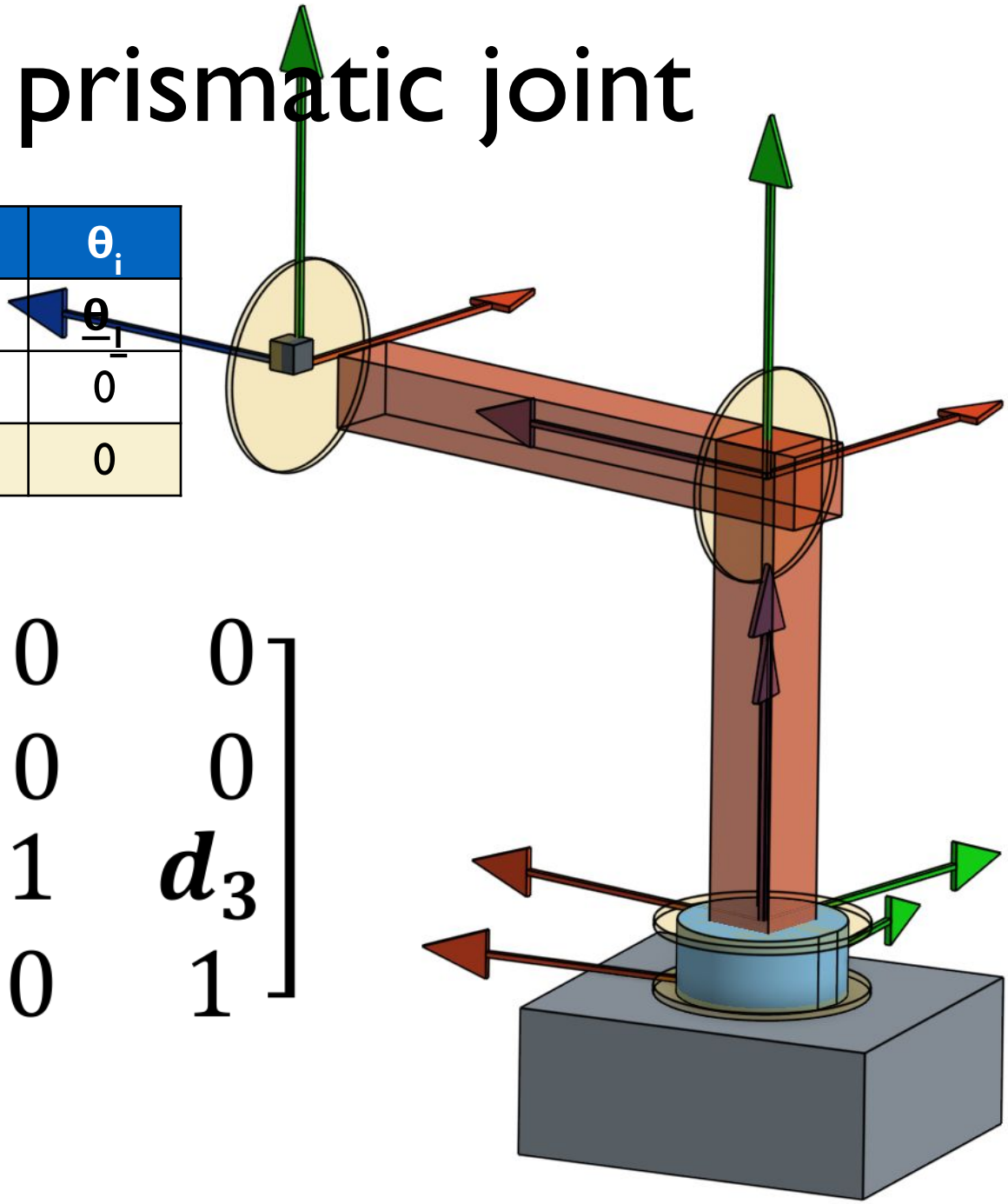
Link	a_i	α_i	d_i	θ_i
1	0	0	d_1	$\underline{\theta_1}$
2	0	-90	$\underline{d_2}$	0
3	0	0	$\underline{d_3}$	0



$$A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Link 3: prismatic joint

Link	a_i	α_i	d_i	θ_i
1	0	0	d_1	θ_1
2	0	-90	d_2	0
3	0	0	d_3	0



$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

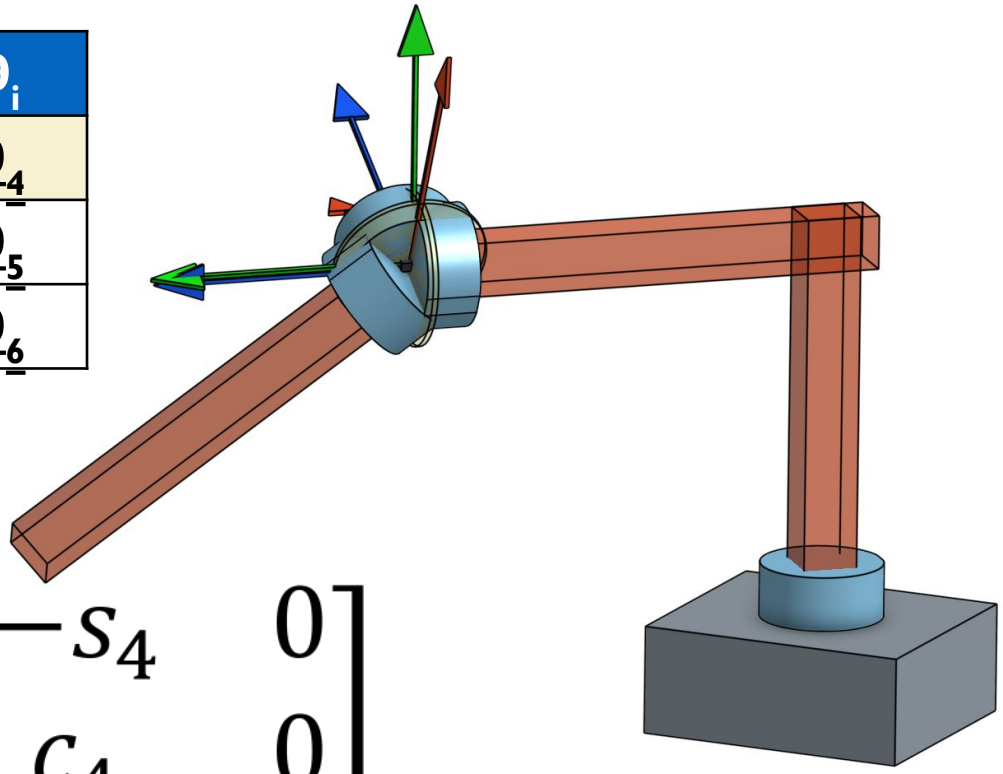
End-effector Transform

$$T_{03} = A_1 A_2 A_3$$

$$T_{03} = \begin{bmatrix} c_1 & 0 & -s_1 & -s_1 d_3 \\ s_1 & 0 & c_1 & c_1 d_3 \\ 0 & -1 & 0 & d_1 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Link 4: revolute joint

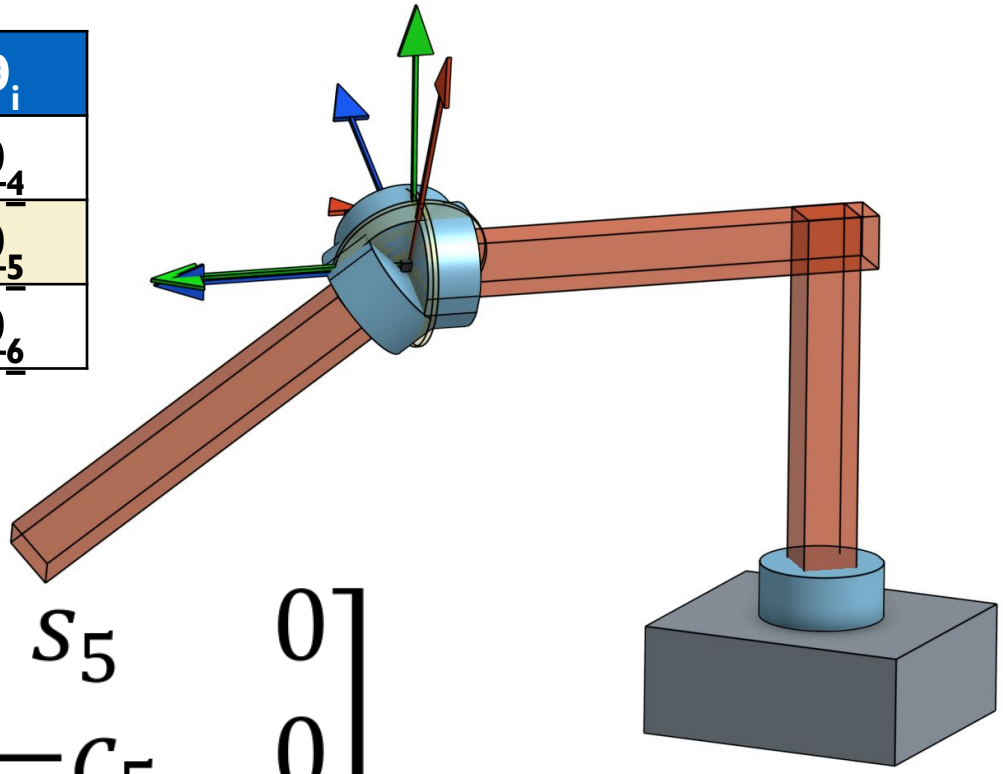
Link	a_i	α_i	d_i	θ_i
4	0	-90	0	$\underline{\theta_4}$
5	0	90	0	$\underline{\theta_5}$
6	0	0	d_6	$\underline{\theta_6}$



$$A_4 = \begin{bmatrix} c_4 & 0 & -s_4 & 0 \\ s_4 & 0 & c_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Link 5: revolute joint

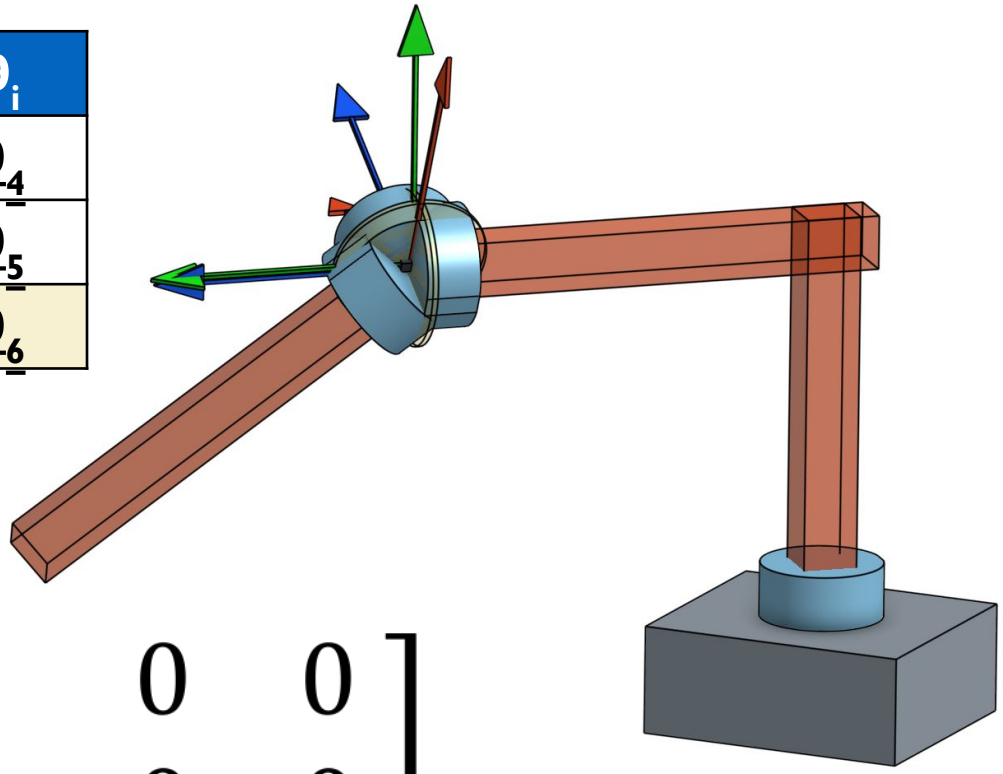
Link	a_i	α_i	d_i	θ_i
4	0	-90	0	$\underline{\theta_4}$
5	0	90	0	$\underline{\theta_5}$
6	0	0	d_6	$\underline{\theta_6}$



$$A_5 = \begin{bmatrix} c_5 & 0 & s_5 & 0 \\ s_5 & 0 & -c_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Link 6: revolute joint

Link	a_i	α_i	d_i	θ_i
4	0	-90	0	$\underline{\theta_4}$
5	0	90	0	$\underline{\theta_5}$
6	0	0	d_6	$\underline{\theta_6}$



$$A_6 = \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

End-effector Transform

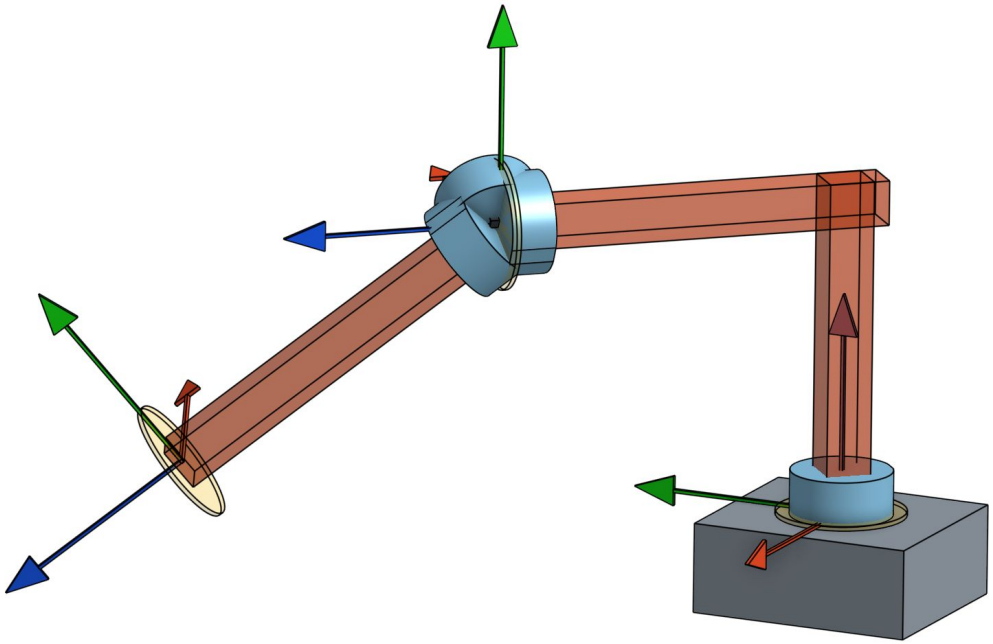
$$T_{36} = A_4 A_5 A_6$$

$$T_{36} = \begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_5c_6 & c_4s_5 & c_4s_5d_6 \\ s_4c_5c_6 + c_4s_6 & -s_4c_5s_6 + c_4c_6 & s_4s_5 & s_4s_5d_6 \\ -s_5c_6 & s_5s_6 & c_5 & c_5d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

End-effector Transform

$$T_{06} = T_{03}T_{36}$$

$$= \begin{bmatrix} c_1 & 0 & -s_1 & -s_1 d_3 \\ s_1 & 0 & c_1 & c_1 d_3 \\ 0 & -1 & 0 & d_1 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_4 c_5 c_6 - s_4 s_6 & -c_4 c_5 s_6 - s_5 c_6 & c_4 s_5 & c_4 s_5 d_6 \\ s_4 c_5 c_6 + c_4 s_6 & -s_4 c_5 s_6 + c_4 c_6 & s_4 s_5 & s_4 s_5 d_6 \\ -s_5 c_6 & s_5 s_6 & c_5 & c_5 d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



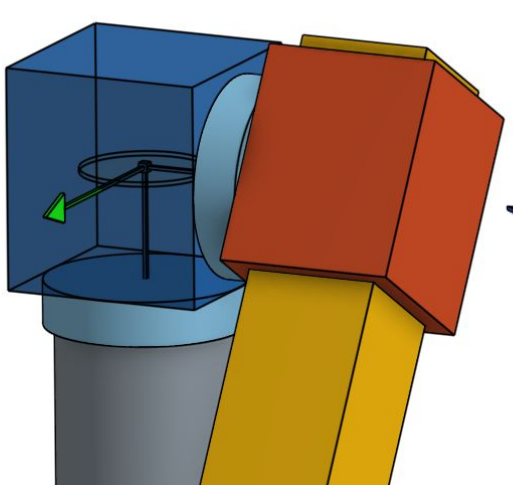
Final Transform

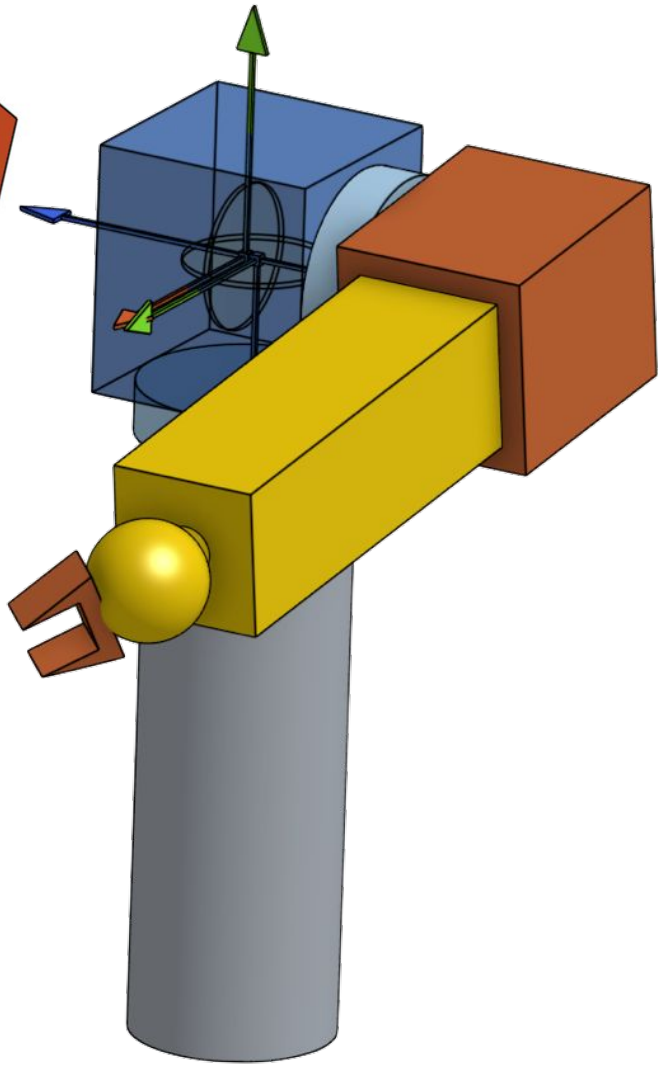
$$T_{06} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & d_x \\ r_{21} & r_{22} & r_{23} & d_y \\ r_{31} & r_{32} & r_{33} & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$r_{11} = c_1c_4c_5c_6 - c_1s_4s_6 + s_1\, s_5c_6$$
$$r_{21} = s_1c_4c_5c_6 - s_1s_4s_6 - c_1s_5c_6$$
$$r_{31} = -s_4c_5c_6 - c_4s_6$$
$$r_{12} = -c_1c_4c_5s_6 - c_1s_4s_6 - s_1s_5s_6$$
$$r_{22} = -s_1c_4c_5s_6 - s_1s_4c_6 + c_1s_5s_6$$
$$r_{32} = s_4c_5s_6 - c_4c_6$$
$$r_{13} = c_1c_4s_5 + s_1c_5$$
$$r_{23} = s_1c_4s_5 + c_1c_5$$
$$r_{33} = -s_4s_5$$
$$d_x = c_1c_4s_5d_6 - s_1c_5d_6 - s_1d_3$$
$$d_y = s_1c_4s_5d_6 + c_1c_5d_6 + c_1d_3$$
$$d_z = -s_4s_5d_6 + d_1 + d_2$$

Link 1: revolute joint

Link	a_i	α_i	d_i	θ_i
1	0	-90	0	$\underline{\theta_1}$
2	0	90	d_2	$\underline{\theta_2}$
3	0	0	$\underline{d_3}$	0
4	0	-90	0	$\underline{\theta_4}$
5	0	90	0	$\underline{\theta_5}$
6	0	0	d_6	$\underline{\theta_6}$



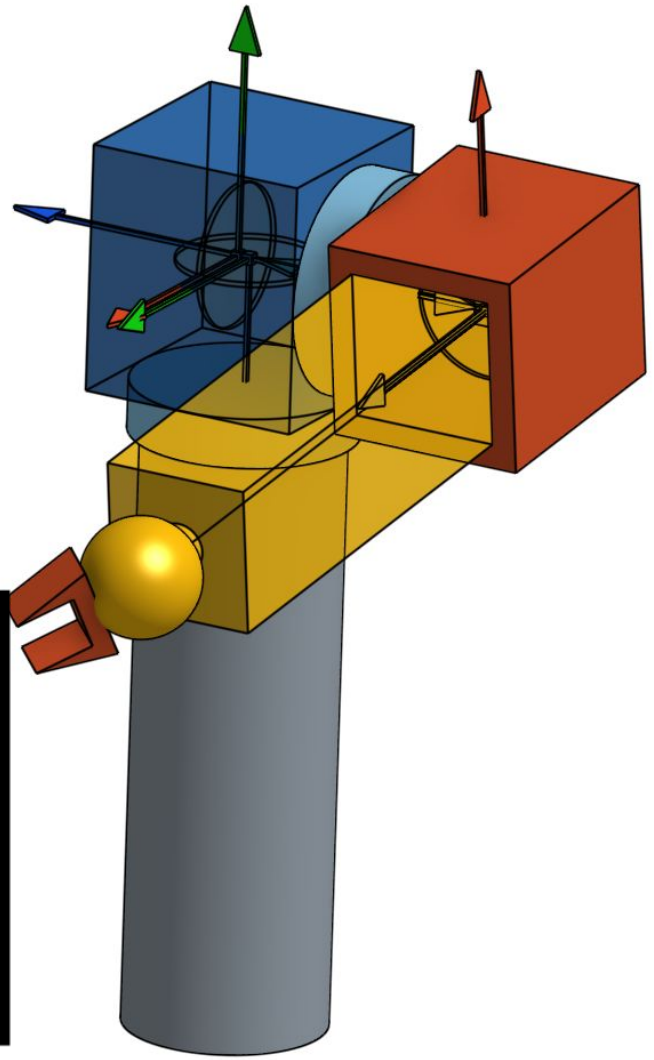


$$A_1 = \begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Link 2: revolute joint

Link	a_i	α_i	d_i	θ_i
1	0	-90	0	$\underline{\theta_1}$
2	0	90	d_2	$\underline{\theta_2}$
3	0	0	$\underline{d_3}$	0
4	0	-90	0	$\underline{\theta_4}$
5	0	90	0	$\underline{\theta_5}$
6	0	0	d_6	$\underline{\theta_6}$

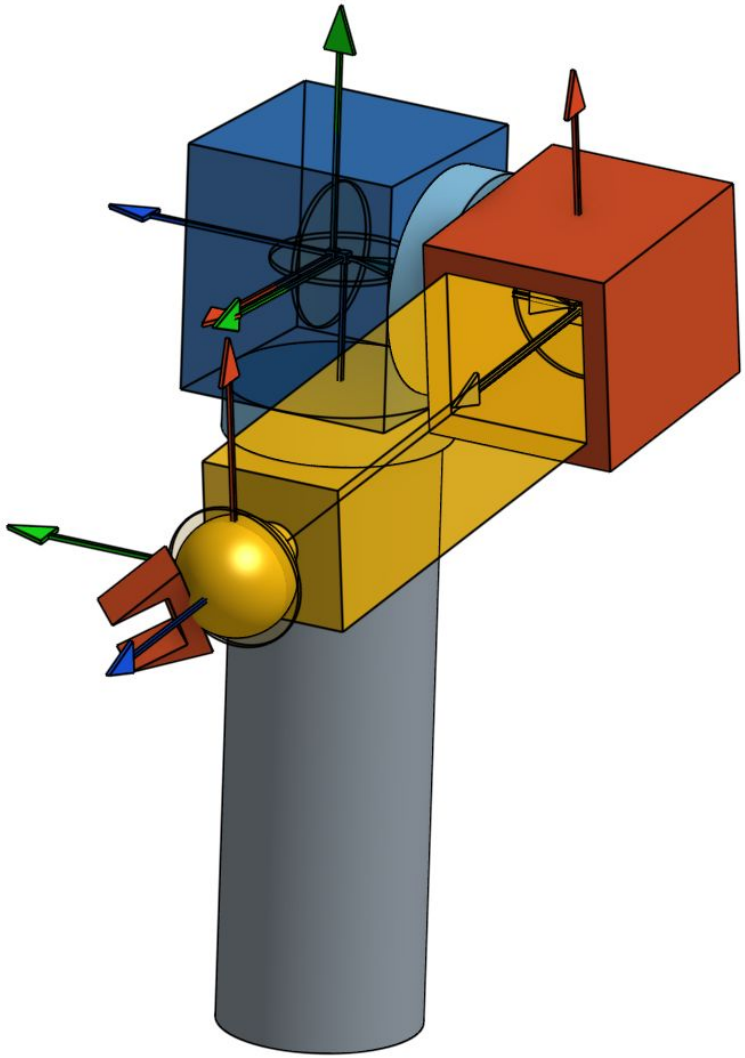
$$A_2 = \begin{bmatrix} c_2 & 0 & s_2 & 0 \\ s_2 & 0 & -c_2 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Link 3: prismatic joint

Link	a_i	α_i	d_i	θ_i
1	0	-90	0	$\underline{\theta_1}$
2	0	90	d_2	$\underline{\theta_2}$
3	0	0	$\underline{d_3}$	0
4	0	-90	0	$\underline{\theta_4}$
5	0	90	0	$\underline{\theta_5}$
6	0	0	d_6	$\underline{\theta_6}$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

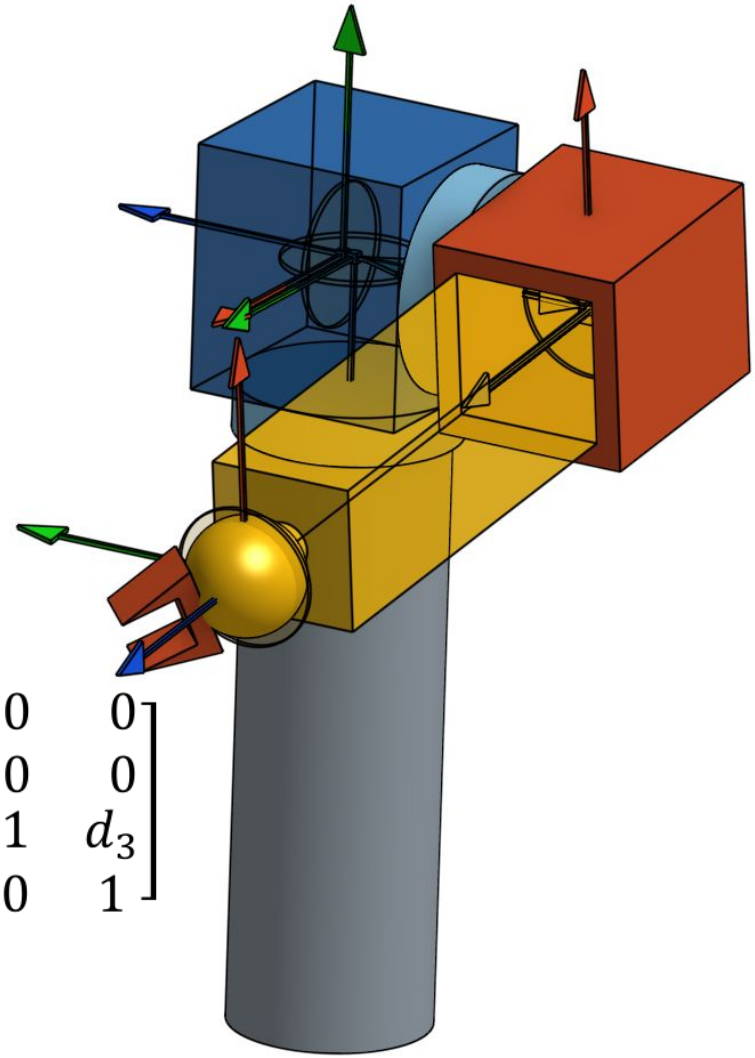


Link 1-3: prismatic joint

Link	a_i	α_i	d_i	θ_i
1	0	-90	0	$\underline{\theta_1}$
2	0	90	d_2	$\underline{\theta_2}$
3	0	0	$\underline{d_3}$	0
4	0	-90	0	$\underline{\theta_4}$
5	0	90	0	$\underline{\theta_5}$
6	0	0	d_6	$\underline{\theta_6}$

$$T_{03} = A_1 A_2 A_3$$

$$\begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & 0 & s_2 & 0 \\ s_2 & 0 & -c_2 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

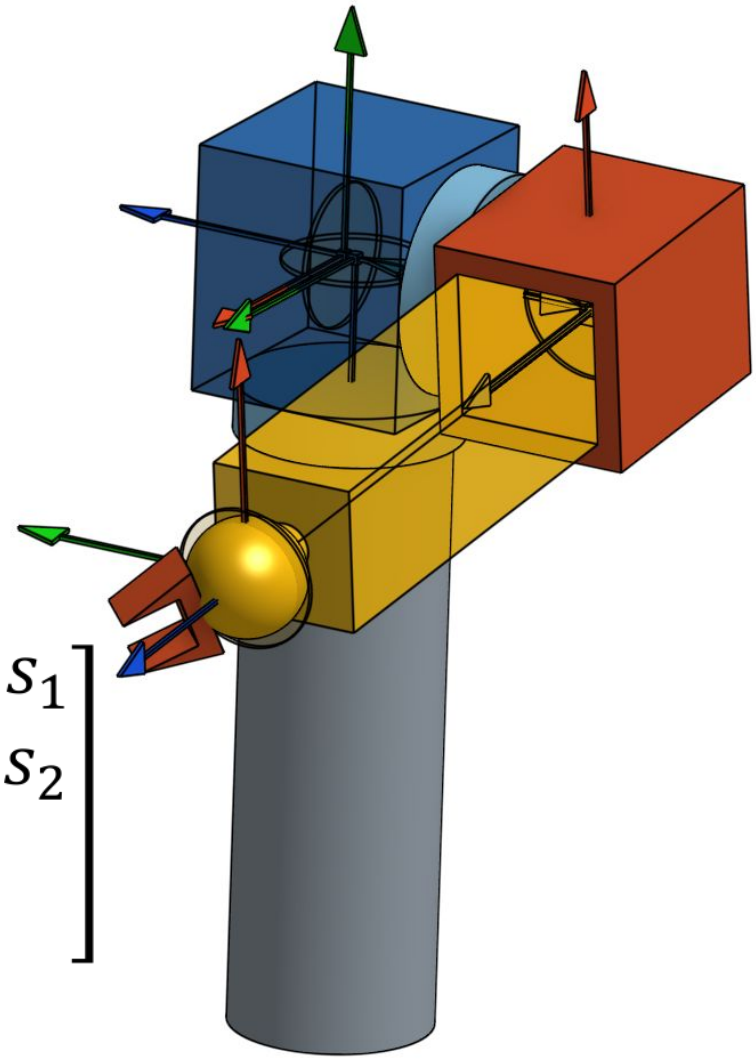


Link 1-3: prismatic joint

Link	a_i	α_i	d_i	θ_i
1	0	-90	0	$\underline{\theta_1}$
2	0	90	d_2	$\underline{\theta_2}$
3	0	0	$\underline{d_3}$	0
4	0	-90	0	$\underline{\theta_4}$
5	0	90	0	$\underline{\theta_5}$
6	0	0	d_6	$\underline{\theta_6}$

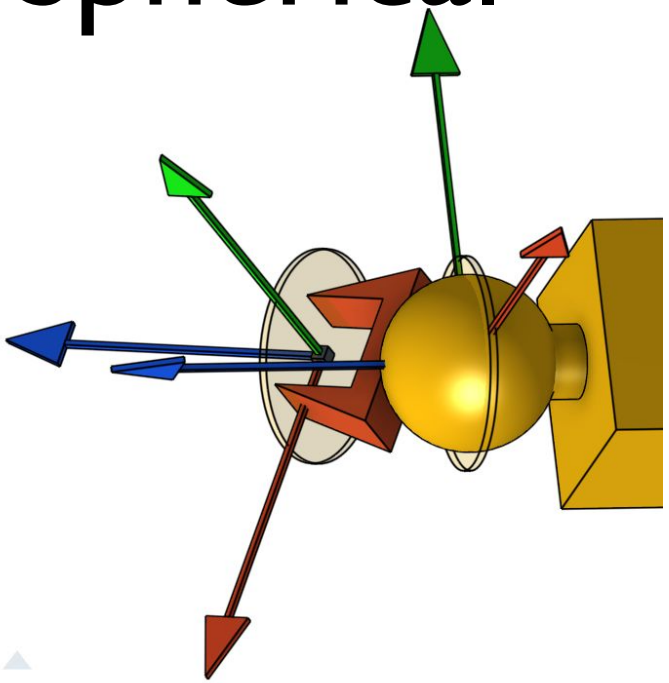
$$T_{03} = A_1 A_2 A_3$$

$$\begin{bmatrix} c_1c_2 & -s_1 & c_1s_2 & c_1d_3s_2 - d_2s_1 \\ c_2s_1 & c_1 & s_1s_2 & c_1d_2 + d_3s_1s_2 \\ -s_2 & 0 & c_2 & c_2d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Links 4-6: 3-axis Spherical Joint

Link	a_i	α_i	d_i	θ_i
1	0	-90	0	$\underline{\theta_1}$
2	0	90	d_2	$\underline{\theta_2}$
3	0	0	$\underline{d_3}$	0
4	0	-90	0	$\underline{\theta_4}$
5	0	90	0	$\underline{\theta_5}$
6	0	0	d_6	$\underline{\theta_6}$



$$T_{36} = \begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_5c_6 & c_4s_5 & c_4s_5d_6 \\ s_4c_5c_6 + c_4s_6 & -s_4c_5s_6 + c_4c_6 & s_4s_5 & s_4s_5d_6 \\ -s_5c_6 & s_5s_6 & c_5 & c_5d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

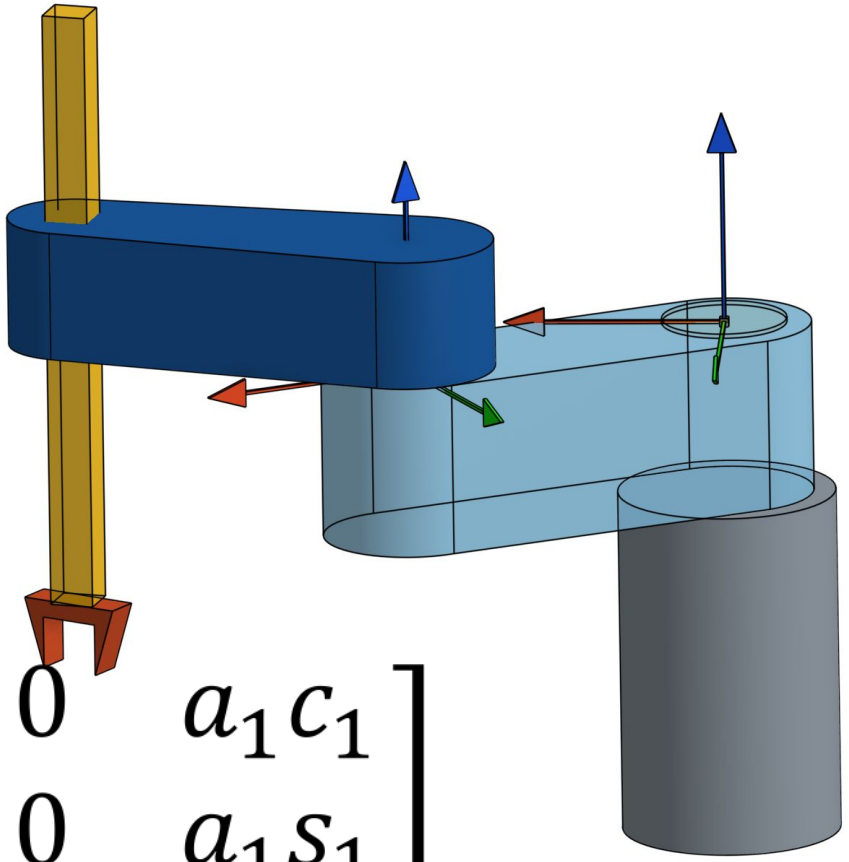
Stanford Arm Transform

$$T_{06} = T_{03}T_{36} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & P_x \\ r_{21} & r_{22} & r_{23} & P_y \\ r_{31} & r_{32} & r_{33} & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$r_{11} = c_1[c_2(c_4c_5c_6 - s_4c_6) - s_2s_5c_6] - s_1(s_4c_5c_6 + c_4s_6)$$
$$r_{21} = s_1[c_2(c_4c_5c_6 - s_4c_6) - s_2s_5c_6] + c_1(s_4c_5c_6 + c_4s_6)$$
$$r_{31} = -s_2(c_4c_5c_6 - s_4s_6) - c_2s_5c_6$$
$$r_{12} = c_1[-c_2(c_4c_5s_6 + s_4c_6) + s_2s_5s_6] - s_1(-s_4c_5c_6 + c_4s_6)$$
$$r_{22} = s_1[-c_2(c_4c_5s_6 + s_4c_6) + s_2s_5s_6] + c_1(-s_4c_5c_6 + c_4s_6)$$
$$r_{32} = s_2(c_4c_5c_6 + s_4s_6) + c_2s_5c_6$$
$$r_{13} = c_1(c_2c_4s_5 + s_2c_5) - s_1s_4s_5$$
$$r_{23} = s_1(c_2c_4s_5 + s_2c_5) + c_1s_4s_5$$
$$r_{33} = -s_2c_4s_5 + c_2c_5$$
$$P_x = c_1s_2d_3 - s_1d_2 + d_6(c_1c_2c_4s_5 + c_1s_2c_5 - s_1s_4s_5)$$
$$P_y = s_1s_2d_3 + c_1d_2 + d_6(s_1c_2c_4s_5 + s_1s_2c_5 + c_1s_4s_5)$$
$$P_z = c_2d_3 + d_6(c_2c_5 - s_2c_4s_5)$$

Link 1: *z*-axis revolute joint

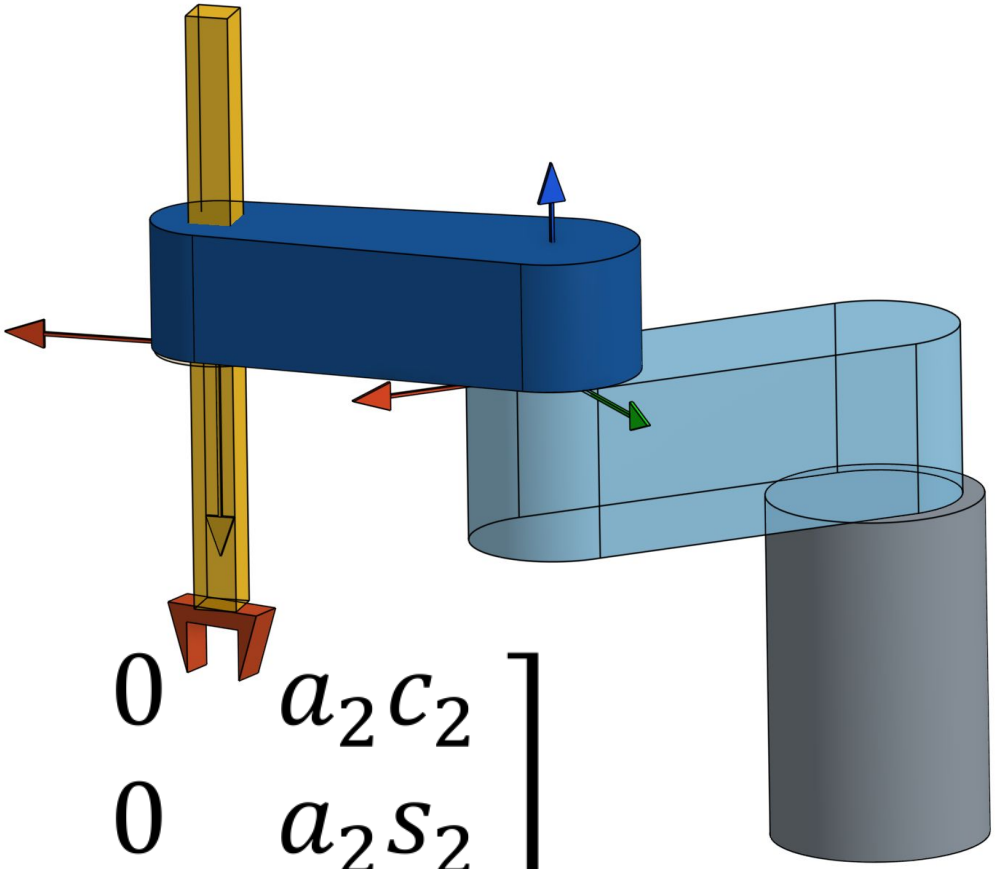
Link	a_i	α_i	d_i	θ_i
1	a_1	0	0	$\underline{\theta_1}$
2	a_2	180	0	$\underline{\theta_2}$
3	0	0	$\underline{d_3}$	0
4	0	0	d_4	$\underline{\theta_4}$



$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1 c_1 \\ s_1 & c_1 & 0 & a_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Link 2: *z*-axis revolute joint

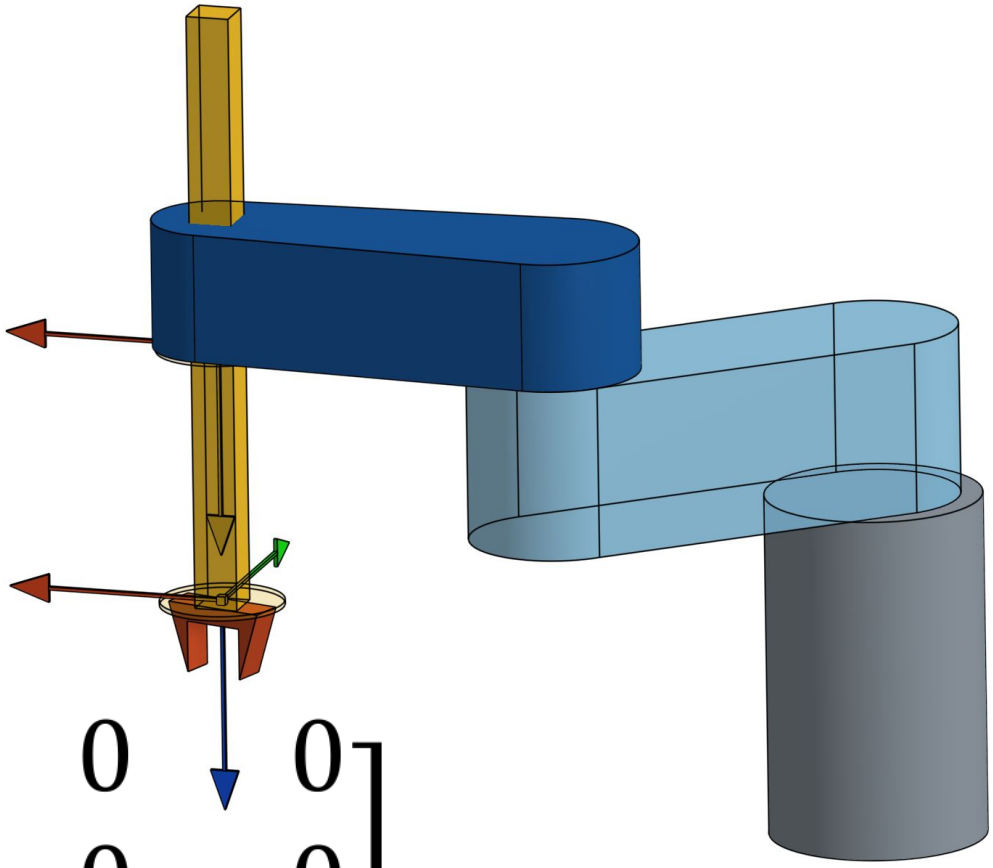
Link	a_i	α_i	d_i	θ_i
1	a_1	0	0	$\underline{\theta_1}$
2	a_2	180	0	$\underline{\theta_2}$
3	0	0	$\underline{d_3}$	0
4	0	0	d_4	$\underline{\theta_4}$



$$A_2 = \begin{bmatrix} c_2 & s_2 & 0 & a_2 c_2 \\ s_2 & -c_2 & 0 & a_2 s_2 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Link 3: prismatic joint

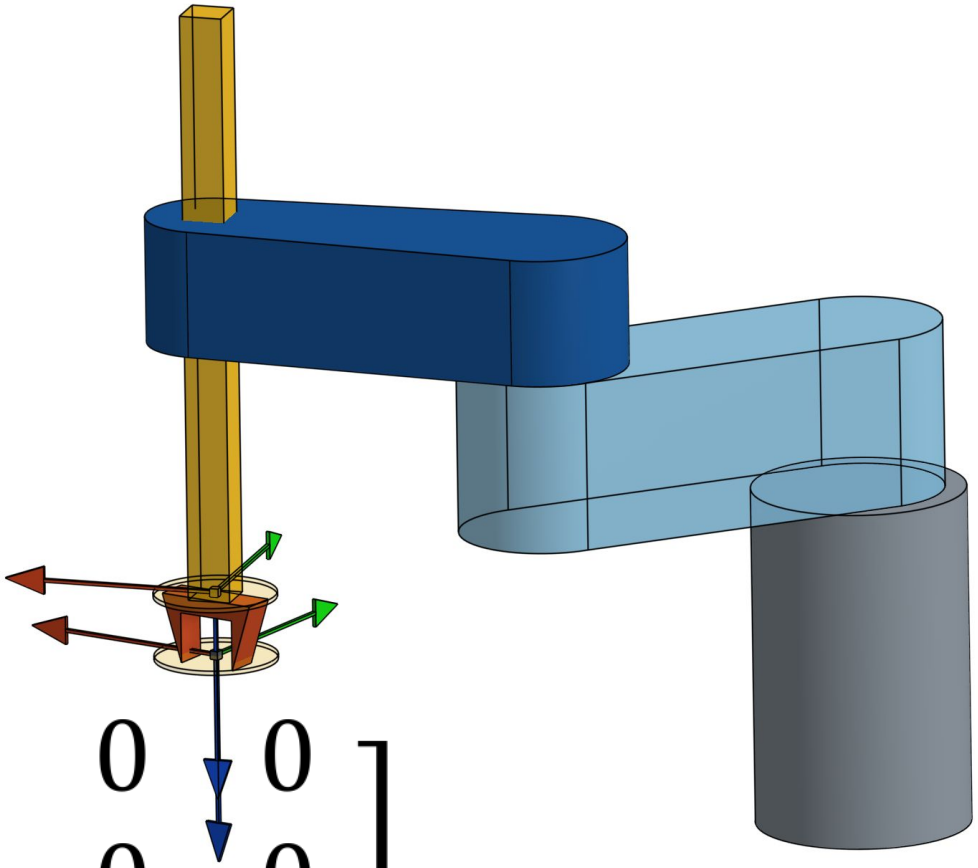
Link	a_i	α_i	d_i	θ_i
1	a_1	0	0	$\underline{\theta_1}$
2	a_2	180	0	$\underline{\theta_2}$
3	0	0	$\underline{d_3}$	0
4	0	0	d_4	$\underline{\theta_4}$



$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Link 4: *z*-axis revolute joint

Link	a_i	α_i	d_i	θ_i
1	a_1	0	0	$\underline{\theta_1}$
2	a_2	180	0	$\underline{\theta_2}$
3	0	0	$\underline{d_3}$	0
4	0	0	d_4	$\underline{\theta_4}$



$$A_4 = \begin{bmatrix} c_4 & -s_4 & 0 & 0 \\ s_4 & c_4 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

End-effector Transform

$$T_{04} = A_1 A_2 A_3 A_4$$

$$T_{04} = \begin{bmatrix} c_{12}c_4 + s_{12}s_4 & -c_{12}s_4 + s_{12}c_4 & 0 & a_1c_1 + a_2c_{12} \\ s_{12}c_4 - c_{12}s_4 & -s_{12}s_4 - c_{12}c_4 & 0 & a_1s_1 + a_2s_{12} \\ 0 & 0 & -1 & -d_3 - d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$