Nonlinear Approximation Algorithm for Object Trajectory Estimation in MLAT Systems

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*Abstract*— The paper presents an algorithm for estimating the parameters of radio-emission source trajectories for multi-position passive radar complexes. The algorithm is based on a nonlinear approximation of a sample of observations on the time interval by the criterion of the minimum of the root-mean-square error of the state vector estimation. The peculiarity of the algorithm is more complete use of the information contained in the observations, due to the use of incomplete sets of simultaneous measurements. The performance and quality of the proposed algorithm have been tested by computer simulation. This paper considers the algorithm under the assumption of linear uniform motion of the radio radiation sourse over the observation interval (first-order dynamics model). However, the proposed algorithm can be extended to higher-order models in order to estimate the parameters of the trajectories of maneuvering objects. Thus, based on the proposed algorithm, it is possible to construct multi-channel adaptive algorithms for constructing trajectories with random dynamics.

Keywords— radiolocation, passive location, multiposition systems, multilateration, non-linear approximation, least square method

# Introduction

The main task of multiposition radar systems (MPS) also callde MLAT systems (multilateration) is to determine the coordinates of radio radiation sources (RRS) and to build their trajectories. By construction of trajectory is understood the determination of trajectory parameters of RRS (coordinates, velocity components) at given moments of time throughout the tracking interval.

The possibility to determine RRS coordinates is provided by reception of radio emission of own onboard RRS equipment at separated synchronized receiving stations (RS) with known coordinates. Information about the location of the RRS is contained in the differences in the time moments of the reception of the RRS radio signal at the RS. Time differences are proportional to geometric distance differences between RRS and RS. In this regard, such MPS implement the difference-distance method of determining the coordinates of RRS, and the MPS are also called hyperbolic [1].

However, there are situations when the RRS radio pulse is not received by all RS. This occurs for a number of reasons: lack of reception at some RS due to the peculiarities of radio signals propagation (reflections, no line of sight), lack of reception due to low energy of radio signals (at large distances, when RS is outside the main beam of the directional diagram of narrow beam antennas of onboard radio equipment), errors in operation of cross-station identification algorithms of the received radio signals.

For determining three-dimensional RRS coordinates, the minimum number of RS equals four. The mark is understood as a vector of values of times of reception of one RRS radio pulse on all RS recorded in a single MPS time scale. Cross-station identification of the received radio pulses at different stations, corresponding to the same radiated radio pulse, is a separate task, solved at the stage of primary processing, and is not considered in this work. Three independent range differences (or six dependent ones) can be calculated for marks that include four reception times. For such marks, a one-step estimate of the RS coordinates can be calculated by iterative [2] or analytical methods [3].

Such marks, containing an incomplete set of values of RRS radio pulse reception times, are called incomplete marks. Analysis of real data from an operating MPS with four RSs shows that the number of complete marks (so-called "fours") is less than 30% of all marks coming to the input of the secondary processing algorithms. Marks containing only two reception time values ("twos") are not suitable for one-step calculation of RRS coordinates, since they are equivalent to a single hyperbola. Marks containing three reception time values ("triples") can be used for one-step calculation of two-dimensional coordinates with substitution of known altitude, which can be determined on the basis of previous coordinate estimates or using a priori information about RRS, its type, typical values of flight heights. However, when calculating two-dimensional coordinates with known altitude by "triples", one has to deal with ambiguity. In this case, the RRS coordinates are determined by the intersection points of two hyperbolas. There are two such points in the general case. To resolve the ambiguity, it is necessary to resort to the a priori information about the location of the RRS, obtained on the basis of previous coordinate estimates.

Thus, most of the input data for the secondary processing algorithms, are unsuitable for one-step calculation of coordinates.

The most common trajectory filtering algorithms are based on smoothing the coordinate estimates obtained in a one-step manner [4,5]. Kalman Filter, trajectory interpolation, and polynomial approximation algorithms are used for this purpose [6,7]. The use of such algorithms in MPS completely excludes the use of "twos" and limits the use of "triples" in the absence of a priori information about the location of RRS.

To use all the information contained in the observations, we use algorithms whose inputs use primary measurements (reception times or range differences) without a preliminary one-step calculation of coordinates, and which take into account the nonlinear relation between the observations and the RRS coordinates. In such an approach, the algorithms of the extended Kalman filter, the anscent Kalman filter, and the particle filter are used [8,9]. However, these algorithms are based on the description of the state vector dynamics model in the form of drifting Markov processes, which is not adequate for aircraft-type RRS, which are characterized by rectilinear uniform motion with constant velocity over almost the entire tracking interval. Therefore, for such RRS, it is reasonable to search for the estimation of the state vector in the form of a deterministic function.

The purpose of this work is to develop and study an algorithm for solving the problem of determining the parameters of RRS motion, taking into account the assumption of the deterministic nature of the trajectory on a fixed time interval of observation. In this case, the state vector is estimated not at a single moment, but by the entire set of marks on the considered time interval. This approach allows us to extract the information from incomplete marks from RRS radio pulses with a non-permanent emission period to the fullest extent.

Similar approaches based on the polynomial approximation of the RRS coordinates already calculated by the one-step method are known. The error of one-step estimates of RRS coordinates in MPS is not constant and depends on the DOP value [2]. Fig.1.a shows the scatter of the coordinate estimates obtained for each mark (red points) against the background of the true RRS trajectory (black line). Fig. 1.b shows the realizations of the range differences corresponding to this trajectory. It can be seen that the statistical characteristics of the range differences (standard deviation) are constant over the entire observation interval, in contrast to the one-step coordinate estimates calculated from them.

|  |
| --- |
|  |
| a) |
|  |
| b) |
| 1. Trajectory estimate (a) and its respective range differences (b). |

In this regard, the algorithm considered in this paper is based on the approximation of range differences under the condition that the RRS motion model is deterministic at a given interval.

# Problem statement

Consider a MPS consisting of *M* RS with known coordinates in the local topocentric coordinate system (TCS) and with a single time scale in which all processes are further described. An RRS is present in the working area of a given MPS. At a given time interval the RRS emits *N* radio pulses with an arbitrary period.

On the time interval under consideration, we assume that the RRS motion is described by a polynomial model:

|  |  |
| --- | --- |
|  | (1) |

where are the current RRS coordinates; are polynomial parameters, constant at a given time interval; *t* is the time variable.

Thus, on a given time interval, the trajectory of RRS is described by *L*x3 constant parameters .

At time moments marks are observed. The values of the time moments are fixed in the MPS time scale and are multiples of the MPS observation period . At that, and more than one mark may fall into one review period due to an arbitrary emission period of RRS. Each time moment corresponds to a vector of measured range differences containing dependent range differences. The rules for calculating the range differences are determined by the matrix of size *K*x2:

|  |  |
| --- | --- |
|  | (2) |

The elements of the matrix take a discrete set of values 1...*M*, corresponding to the order numbers of RS. The matrix is defined by the developer at the design stage of the MPS secondary processing unit.

Thus, the observations are a matrix of size *K*x*N* of the following form

|  |  |
| --- | --- |
|  | (3) |

where is the range difference matrix, is the matrix of discrete white Gaussian observational noise with zero mean and dispersion . Each column of the matrix corresponds to a separate mark.

The matrix element corresponding to the *i*-th mark and the *j*-th range difference is written as follows:

|  |  |
| --- | --- |
|  | (4) |

where

|  |  |
| --- | --- |
|  | (5) |

is the geometric distance between the RRS and the *k*-th RS (*k*=1...*M*) at time .

In addition, a matrix of size *K*x*N* is known, showing the absence or presence of range differences in the mark and consisting of zeros and ones. The formation of this matrix is carried out at the stage of range difference calculation from the values of radio pulse reception times at RS. The use of this matrix is due to the need to account for incomplete marks.

**It is required** to estimate the state vector from the observation sample at the observation period with the minimum mean square error.

# Algorithm synthesis

Let's develop the algorithm for the special case of RRS, including four RS, range differences for which are calculated by the following rule ()

|  |  |
| --- | --- |
|  | (6) |

To describe the model of RRS motion on the observation interval, we restrict ourselves to the first order of polynomials, since for the aircraft-type RRS, a rectilinear uniform motion with constant velocity is the most characteristic:

|  |  |
| --- | --- |
|  | (7) |

where are the RRS coordinates at time ; are the velocity vector components constant over the observation interval. Thus, six trajectory parameters are subject to evaluation.

To find an estimate based on the criterion of the minimum mean square of the errors of the primary observations (range differences), we minimize the following functional

|  |  |
| --- | --- |
|  | (8) |

We need to solve a system of non-linear equations of the following form

|  |  |
| --- | --- |
|  | (9) |

where the derivative of the functional (8) by an arbitrary parameter

|  |  |
| --- | --- |
|  | (10) |

To solve system (9) we will use Newton's numerical method. We linearize the nonlinear functions included in the system of equations in the vicinity of the state vector. To do this, we need to calculate the second derivatives of the functional (8) over the state vector. The estimate of the state vector is found by the iterative method

|  |  |
| --- | --- |
|  | (11) |

where the maximum number of iterations or a given accuracy is chosen as the stopping criterion. Here *p* is the number of iteration.

The general expression for the second derivative by an arbitrary parameter

|  |  |
| --- | --- |
|  | (13) |

and for the mixed second derivative on the parameters

|  |  |
| --- | --- |
|  | (14) |

Consider the derivatives in (8,9,13,14)

|  |  |
| --- | --- |
|  | (15) |
|  | (16) |
|  | (17) |

Next, we present in detail some derivatives

|  |  |
| --- | --- |
|  | (18) |
|  | (19) |
|  | (20) |

The derivatives for the other components of the state vector are written similarly to (18-20), with

|  |  |
| --- | --- |
|  | (21) |

All other first- and second-order derivatives of the functions are zero.

# Modeling

To test the performance and study the features of the algorithm functioning, a software package was created in Matlab environment, including the following functional modules: RRS trajectory generator; RRS radio emission model; generator of primary MPS measurements, corresponding to the trajectories; module of the proposed processing algorithm.

Initial modeling parameters: observation intervals and ; standard deviation of primary measurements ; RRS velocity module - 200 m/s; height - 10000 m; Initial RRS coordinates and direction of velocity vector are determined randomly. The radiation period is not constant and is formed randomly.

RS Coordinates at TCSC:

Thus, the RRS trajectory was modeled, for which and marks are observed at the given time intervals, respectively. The generated true state vector is written in Tables 1 and 2 in column . The realization of the trajectory and its location relative to the MPS are shown in Fig. 2.

|  |
| --- |
|  |
| 1. Modeled trajectory. |

Three variants of input data for the algorithm are considered:

1. All marks are complete (all "fours," all elements of the matrix are equal to one);
2. There are incomplete marks in the input data. The percentage of " fours"/"triples"/"twos" is 25%/35%/40%;
3. Worst case: all marks are "twos".

Samples II and III are simulated by artificially degrading sample I by equating the corresponding elements of matrix to zero. Experiments II and III are aimed at testing the performance of the proposed algorithm and the quality of its performance on incomplete marks. The true value of the state vector is used as the initial approximation for the iterative algorithm, the given accuracy is , the maximum number of iterations is 10.

The simulation results are presented in Tables 1 and 2. In addition to the errors of the state vector estimates the table also presents estimates of the accuracy of the estimated parameters, obtained as follows:

|  |  |
| --- | --- |
|  | (22) |

Modeling has shown that the proposed algorithm gives a solution in all configurations considered, especially noteworthy is the ability to estimate the state vector for experiment III. The results show that adding incomplete marks leads to a decrease in the accuracy of state vector estimation. Increasing the accumulation period, on the contrary, increases the estimation accuracy.

# Conclusion

This paper presents an algorithm for estimating the parameters of RRS trajectories in the MPS by nonlinear approximation using Newton's iterative method. The peculiarity of the proposed algorithm is the estimation of the RRS coordinates not at a single moment, but at a time interval by the set of accumulated marks. This approach allows the so-called incomplete marks to be used in the solution, thereby providing a more complete use of the information contained in the observations. In addition, in addition to the coordinates, the velocity vector of the RRS is also estimated.

This paper considers the algorithm under the assumption of linear uniform motion of the RRS over the observation interval (first-order dynamics model). However, the proposed algorithm can be extended to higher-order models in order to estimate the parameters of the trajectories of maneuvering RRS. Thus, based on the proposed algorithm, it is possible to construct multi-channel adaptive algorithms for constructing RRS trajectories with random dynamics.

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| --- | --- | --- | --- | --- | --- | --- | --- |
| 1. Modeling results for | | | | | | | |
| **Parameters** |  | **I** | | **II** | | **III** | |
|  |  |  |  |  |  |
|  | 36728.0 | 33.0 | 134.4 | 167.9 | 199.2 | 124.8 | 318.6 |
|  | 28.0 | 14.8 | 7.3 | 9.4 | 10.8 | 15.7 | 19.5 |
|  | -66091.0 | 43.0 | 231.4 | 341.1 | 343.2 | 244.3 | 548.3 |
|  | 198.0 | 24.3 | 12.2 | 7.2 | 18.2 | 26.1 | 32.6 |
|  | 10000.0 | 160.4 | 89.2 | 353.3 | 132.5 | 11.7 | 200.8 |
|  | 0.0 | 0.4 | 4.8 | 11.1 | 7.1 | 3.6 | 12.0 |
| 1. Modeling results for | | | | | | | |
| **Parameters** |  | **I** | | **II** | | **III** | |
|  |  |  |  |  |  |
|  | 36728.0 | 298.5 | 75.3 | 312.0 | 107.2 | 583.1 | 190.8 |
|  | 28.0 | 2.8 | 1.7 | 2.5 | 2.5 | 8.9 | 4.1 |
|  | -66091.0 | 500.3 | 127.5 | 527.3 | 181.2 | 979.8 | 321.1 |
|  | 198.0 | 5.5 | 2.6 | 5.3 | 3.9 | 16.1 | 6.5 |
|  | 10000.0 | 241.7 | 53.0 | 276.7 | 76.2 | 435.3 | 130.5 |
|  | 0.0 | 3.0 | 1.2 | 3.2 | 1.8 | 3.9 | 3.1 |
|  | | | | | | | |

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