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EXTREME LEARNING MACHINE'S THEORY & APPLICATION

QAM equalization and symbol detection in OFDM systems using extreme learning machine

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Abstract This paper presents a new learning-based framework to jointly solve equalization and symbol detection problems in orthogonal frequency division multiplexing systems with quadrature amplitude modulation. The framework utilizes extreme learning machine (ELM), a recent addition to the class of supervised learning algorithms, to achieve fast training, high performance, and low error rates. The proposed ELM scheme employs infinitely differentiable nonlinear activation functions in least-square solution to learn the channel response, which is the equalization part. In addition to equalization, ELM performs symbol detection. Existing learning-based schemes require an additional symbol slicer for the symbol detection. The proposed framework does not experience training bottleneck imposed by gradient descent–based approaches. Simulation results show that the proposed framework outperforms other learning-based equalizers in terms of symbol error rate and training speeds.

Keywords Equalization · QAM slicer · Symbol detection · Analytical training · OFDM

1 Introduction

Orthogonal frequency division multiplexing (OFDM) is deployed in high data rate applications such as digital video broadcasting (DVB), digital audio broadcasting (DAB), wireless local area network (WLAN), digital subscriber line (DSL), and power line communications (PLC) [1–5].

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An OFDM system transforms the frequency selective channel to a flat fading channel in order to simplify the process of equalization at the receiver. However, time-varying multipath fading, Doppler frequency shift, and local oscillator frequency drift break up carrier orthogonality and introduce inter-symbol interference (ISI) and inter-carrier interference (ICI). Channel estimation and equalization techniques are employed to mitigate the effects of ISI and ICI in an OFDM receiver. Those techniques can be divided into three categories based on how they operate: blind, semi-blind, and trained. Blind techniques use the statistical information of the transmitted signals to estimate the channel response, hence they do not use training sequence. Using training sequence decreases overall throughput but reduces the receiver complexity. However, semi-blind techniques first use training sequence to estimate the channel response then reverts to blind adaptation [6–14]. The majority of those techniques employ algorithms based on linear equalization using least-square (LS), zero-forcing (ZF), or minimum mean square error (MMSE) criterion [9, 13].

Nonlinear classifications, such as Bayesian decision theory, offer promising solution for equalization and symbol detection problems [15, 16]. Neural networks (NN), such as NNs based on multilayer feed-forward neural networks and radial basis function (RBF), have been used for system identification and noise cancelation problems to recover transmitted data [17]. Learning-based techniques that employ NNs can process a complex signal utilizing two independent real-valued multilayer perceptrons (MLP) or a split-complex activation function [18]. MLP have been successfully used in channel estimation/equalization without symbol detection [19–23], in symbol detection [24] and in QAM demodulation [25]. Complex-valued radial basis function (CRBF) network in [21] and complex-valued minimal resource allocation network (CMRAN) developed in [20] use stochastic gradient approach for parameter adjustments in channel equalization. CMRAN is a complex-valued version of the real-valued minimal resource allocation network (MRAN), which is based on sequential learning with ability to prune and grow hidden neurons in order to achieve superior performance. CMRAN requires shorter training time and data for model learning than RBF [22]. On the other hand, a fully complex activation function was deployed in complex back propagation (CBP) in [19] as an extension to the traditional back propagation learning. Both CRBF and CMRAN can work with complex signals by deploying a split-complex activation function, which comprises of two real-valued activation functions, one for real and one for imaginary part of the input signal. Learning-based equalization schemes require manual tuning of the learning rate and epochs and have limited success due to slow convergence to local minima. A single hidden layer feed-forward neural network, called extreme learning machine (ELM), transforms learning paradigm into a simple linear solution [26]. ELM and its variants such as fully complex-valued ELM (C-ELM), online sequential ELM, incremental ELM, and ensemble of ELM are surveyed in [27]. Most of these techniques are used for real value processing except C-ELM which is a true complex-valued technique. C-ELM was proposed in [22] for channel equalization with complex input data. Computational complexity become higher in fully complex-valued neural networks like C-ELM and CBP. A receiver structure that combines a decision feedback equalizer and a self organizing map (SOM) as symbol slicer was proposed in [25]. Later, a receiver structure that combines recurrent neural network (RNN) equalizer with SOM detector to estimate QAM symbols was proposed in [28]. Those structures require two separate systems, one for equalization and one for symbol detection, and converge slowly due to the use of traditional neural networks that require manual tuning of control parameters, such as epochs and learning rate.

In this paper, a framework that uses real-valued ELM that solves the combined problem of equalization and symbol detection is proposed. This framework converges much faster than traditional NNs and jointly solves equalization and detection problems. In the joint solution, the framework does not need an additional QAM slicer circuit, which is required in C-ELM, CMRAN, CRBF, and CBP equalizers. In addition to this, the proposed framework employs a fully real-valued processing scheme where QAM constellation points are transformed to a real-valued vector of 2-tuple, which are labeled with integer values (i.e., class number). Using real-valued vectors eliminates complex-valued processing and reduces computational complexity of the receiver. This leads to faster convergence. Another advantage of the proposed framework is that it achieves higher accuracy by associating a 2-tuple pattern to one of the M possible classes, which corresponds to an individual QAM constellation point. The advantages of the proposed framework are demonstrated via simulation studies. The rest of the paper is organized as follows. The system model for the OFDM system and review of ELM is discussed in Sect. 2. Section 3 describes the proposed framework for joint equalization and symbol detection. Simulation results are discussed in Sect. 4 followed by concluding remarks in Sect. 5.

2 Preliminaries

2.1 OFDM system model

OFDM system model that is tested in this paper is illustrated in Fig. 1. In this model, data bits are mapped to one of the symbols of QAM and then N-point fast inverse Fourier transform (IFFT) is applied to produce orthogonal subcarriers. In order to simplify the simulations, the broadband carrier modulation scheme is omitted in the model. The output of IFFT block is transmitted in time-domain where N base-band symbols, $x(n)$ are uncorrelated to each other and are obtained by the relation

$$x(n) = \sum_{k=0}^{N-1} X(k) e^{j(2\pi kn/N)}, \quad n = 0, 1, 2, \ldots, N-1. \tag{1}$$

A guard interval called cyclic prefix (CP) of length v is inserted at the start of each OFDM symbol which is a copy of the last v samples of the OFDM symbols to mitigate inter-symbol interference (ISI). The resulted signal with guard interval $x_g(n)$ is transmitted over a frequency selective time-varying fading channel. The signal, $x_g(n)$, which is white and wide-sense stationary (W.S.S), is transmitted through the linear time-varying finite-impulse response (FIR) channel whose impulse response is $h(n, p)$. The output of the channel (i.e., received signal), $y_g(n)$, is given by

$$y_g(n) = \sum_{l=0}^{p-1} h(n,p) x_g(l) + v(n), \tag{2}$$

where n is the sample index and $v(n)$ is a zero-mean additive white Gaussian noise (AWGN) sample with variance of σ_v^2; $h(n, p)$ is the impulse response of the sampled time-varying channel, and p is the number of propagation paths. Each path has an amplitude of a complex Gaussian distribution and the power spectrum. The power spectrum of the channel is determined by Doppler frequency shift of f_D. The guard interval, at the receiver, is removed from $y_g(n)$ and DFT is applied on $y(n)$, which is given by

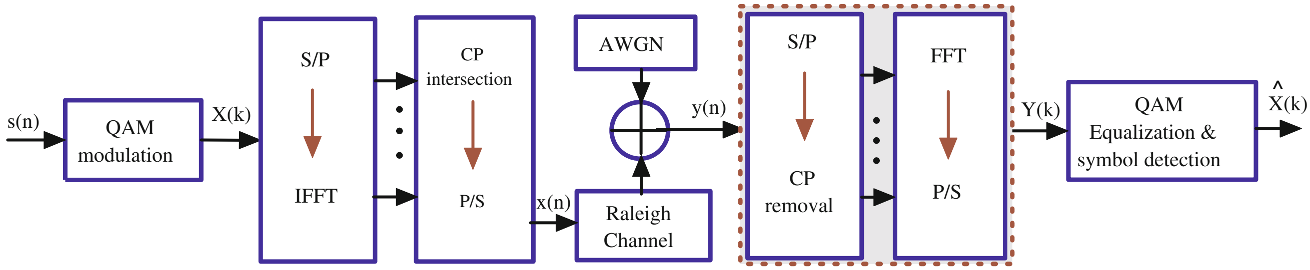

Fig. 1 OFDM system model

$$Y(k) = \sum_{n=0}^{N-1} y(n)e^{-j(2\pi kn/N)}, \quad k = 0, 1, 2, \ldots, N-1. \tag{3}$$

In a conventional OFDM system, the channel estimation is performed using pilot symbols. The channel transfer function after extraction of pilot symbols and estimation is denoted by $H(k)$. The transmitted QAM symbols can be recovered as

$$\hat{X}(k) = \frac{Y(k)}{\hat{H}(k)}, \tag{4}$$

where $\hat{H}(k)$ is an estimate of $H(k)$. The signal $\hat{X}(k)$ is fed through QAM symbol slicer to detect the actual transmitted data which is transformed into binary sequence using a demapper.

2.1.1 *Extreme learning machine*

Fast single layer feedforward neural network (SLFNN) called ELM was proposed by Huang et al. [26] which is shown in Fig. 2. Given S arbitrary distinct samples of $(\mathbf{x}_i, \mathbf{d}_i)$, where $\mathbf{x}_i = [x_{i1}, x_{i2}, \ldots, x_{ip}]^T \in \mathbf{R}^p$ and $\mathbf{d}_i = [d_{i1}, d_{i2}, \ldots, d_{im}]^T \in \mathbf{R}^m$, where both column vectors are of length p input neurons and m output neurons, respectively. In ELM, the input weights and hidden layer biases are generated randomly instead of tuned and the nonlinear training problem is transformed into a linear system and is formulated as $\mathbf{H}\boldsymbol{\beta} = \mathbf{D}$ where

$$\mathbf{H} = \begin{bmatrix} f(\mathbf{w}_1.\mathbf{x}_1 + b_1) & \ldots & f(\mathbf{w}_L.\mathbf{x}_1 + b_L) \\ \vdots & \ldots & \vdots \\ f(\mathbf{w}_1.\mathbf{x}_S + b_1) & \ldots & f(\mathbf{w}_L.\mathbf{x}_S + b_L) \end{bmatrix}_{S\times L}, \tag{5}$$

$$\boldsymbol{\beta} = \left[\beta_1^T, \beta_2^T, \ldots, \beta_L^T\right]^T_{L\times m}, \quad \text{and} \quad \mathbf{D} = \left[\mathbf{d}_1^T, \mathbf{d}_2^T, \ldots, \mathbf{d}_S^T\right]^T_{S\times m}.$$

$\mathbf{H}$ is the hidden layer output matrix of ELM. Huang et al. [26] proposed minimum least-square (LS) solution for ELM which is defined as

$$\hat{\boldsymbol{\beta}} = \mathbf{H}^* \mathbf{D} \tag{6}$$

$\mathbf{H}$ can be a non-square matrix for a number of hidden nodes $L \ll S$, where $\mathbf{H}^*$ is the Moore-Penrose generalized inverse of a matrix $\mathbf{H}$ for non-square matrix when $L \ll S$ where as the solution is straightforward for $S = L$. It has been theoretically analyzed and experimentally demonstrated by Huang et al. [26] that ELM can obtain good generalization performance with extremely fast learning speed.

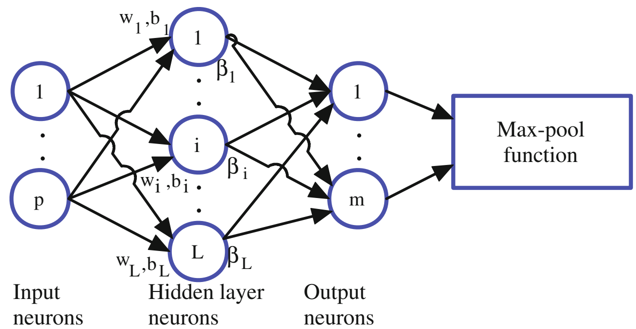

Fig. 2 ELM architecture

3 Proposed algorithm

This section explains the proposed framework that jointly solves the problem of equalization and symbol detection using fully real-valued ELM. The block diagram of the proposed framework is shown in Fig. 3. OFDM signals are transmitted over a frequency selective fading channel and AWGN noise is added. The received OFDM signal is disturbed due to several factors such as multipath fading, Doppler frequency shift, and local oscillator frequency drift. The OFDM signal is reconstructed using *FFT*, which re-generates M-QAM symbols transmitted using N subcarriers. Reconstructed OFDM signals are fed into the proposed joint equalization and symbol detection module after splitting them into training and testing data. The training part of the data is used to learn a generalized network model. After analytically adjusting ELM parameters, the classifier processes the real data. The number of output neurons is analogous to the value of M representing QAM mode, whereas input layer consists of two neurons, one assigned to the real part and the other for the imaginary part of the complex input signal.

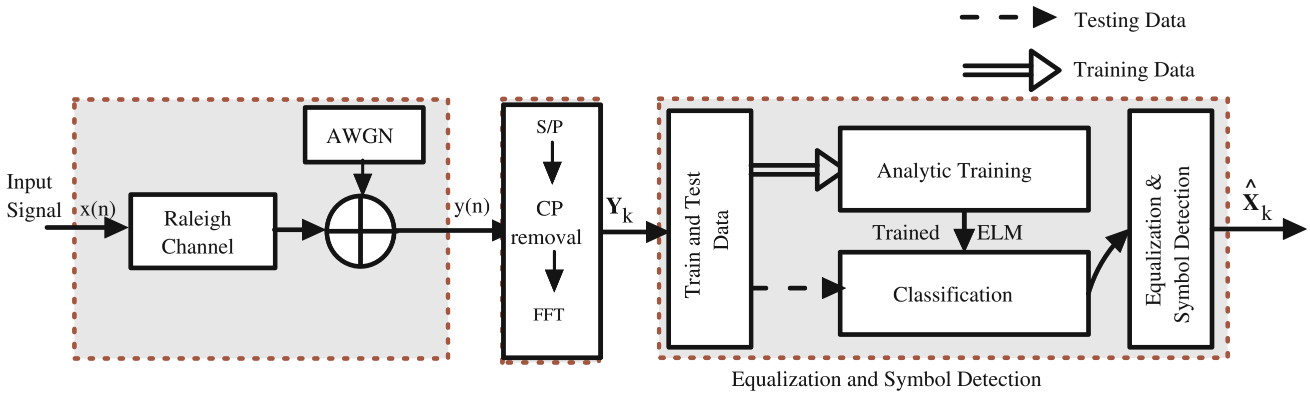

Fig. 3 Proposed block diagram of OFDM system for joint equalization and symbol detection using ELM

Equalization and symbol detection problem is solved using real-valued ELM by establishing a mapping between 2-tuple real input vector to a single complex symbol that corresponds to an integer. However, existing approaches perform equalization using regression through a single mapping between a pair of input and output values [22], which compromises accuracy and requires additional QAM slicer. Figure 4 shows the process of 4-QAM constellation where the received QAM signal $Y(k)$ is transformed to 2-tuple real data vector, which is used as an input into a fully real-valued ELM, $\mathbf{x}_i = [re(Y_k), Im(Y_k)]^T$, and the target or desired output is the level that corresponds to one of the known QAM constellation points shown in Fig. 5. The equalization is performed analytically and the symbols are decided based on ELM Max-pool decision rule that determines the winner output neuron [26]. As shown in right side of Fig. 4, the index of a winner output neuron represents an integer value (also called 'level'); it is important to state that each level corresponds to one QAM symbol of the equivalent 4-QAM constellations in Fig. 4. The relationship between output levels of the proposed framework and 4- and 16-QAM are shown in Fig. 5. The number of levels is always equal to the number of classes or distinct symbols of the QAM being equalized. In addition, the proposed scheme can automatically identify the QAM mode/size based on training information, therefore, it builds up knowledge about the transmitted data for any QAM-based receiver.

The CBP and C-ELM are based on complex inputs, weights and activation functions that cause additional computational complexity. In contrast, the proposed framework utilizes real-valued classifier, i.e., ELM [26] whose training is performed analytically using least-square solution. Using inputs and activation function belongs to real domain reduces computational cost of the proposed framework and allows us to use large number of nonlinear infinitely differentiable activation functions. The proposed algorithm can be described as follows:

1. Using the training data, the QAM mode is identified and the QAM symbols are mapped to different levels according to Fig. 5.
2. Transformation of complex values to a 2-tuple vector of real values. This step is performed as follows: Given a set of complex training data ($\mathbf{Y}_i$, $\mathbf{X}_i$) where $\mathbf{Y}_i$ is the received QAM symbol (complex value) and $\mathbf{X}_i$ is the expected QAM symbol (complex value). Note that i here corresponds to kth QAM symbol index. The corresponding input–output of the ELM is formed by $\mathbf{x}_i = [Re(Y_i), Im(Y_i)]^T$ and $\mathbf{d}_i = [d_{i1}, d_{i2}, \ldots, d_{im}]^T, i = 1, 2, 3, \ldots, S$, and m represents the number of classes (number of output neurons) formed according to

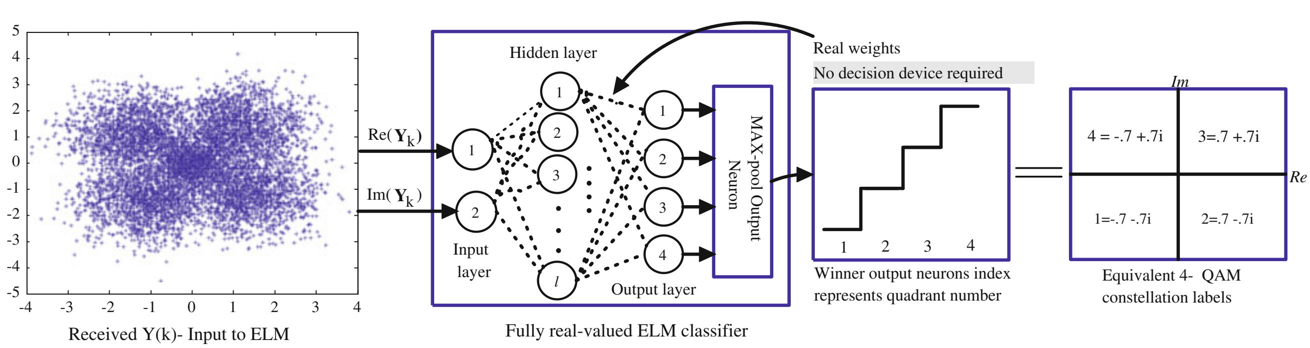

Fig. 4 Equalization and symbol detection using ELM for 4-QAM

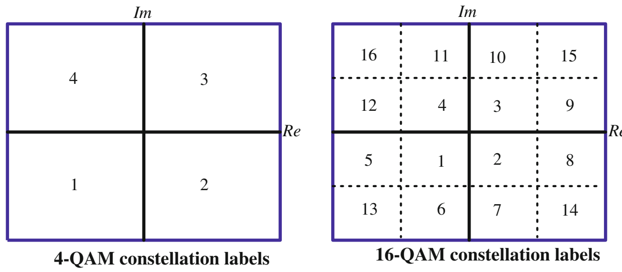

Fig. 5 4 and 16 QAM quadrant distribution

quadrature distribution of complex plane (as shown in Fig. 5).

3. Randomly assign real values input weight w_i and bias b_i, $i = 1, 2, \ldots, L$.
4. Calculate the hidden layer matrix $\mathbf{H}$.
5. Calculate the output weight matrix $\hat{\boldsymbol{\beta}} = \mathbf{H}^{*}\mathbf{D}$.
6. Equalization is performed by calculating the hidden layer matrix, $\mathbf{H}$, for the actual received data and using the estimated output weight $\hat{\boldsymbol{\beta}}$, the equalized data are found by $\mathbf{O} = \mathbf{H}\hat{\boldsymbol{\beta}}$. A correct class/level is chosen from the equalized output, $\mathbf{O}$, using the Max-pool decision rule of ELM.
7. The selected class/output neuron(which is one of the quadrant or subquadrant of Fig. 5) is mapped back to the QAM symbol (complex value).

In the proposed framework, the complex values are split for processing as real values in a similar fashion like CMRAN and CRBF. However, the proposed framework works complectly in the real domain as the classification problem is treated as mapping between a pair of real data to single class that requires only single activation function whereas CMRAN and CRBF require two activation functions [22]. The proposed scheme offers improved scalability through automated QAM mode selection, and use of variety of activation functions from real as well as complex domain. This adds the advantage of lowering the complexity due to real-valued processing, scalability in terms of automated identification of QAM mode, and use of most of activation function of our proposed method over the above mentioned techniques.

4 Results and discussion

Wireless OFDM systems with time-varying Raleigh fading channel and three different QAM modes are constructed in the experimental setup in order to test the performance of the proposed framework. Simulated OFDM system parameters and assumptions of the experiments are provided in Table 1. The time-varying Raleigh fading channel simulator described in [29] is used as a channel model.

Table 1 Simulation parameters

Modulation	4QAM, 16QAM, 64QAM
Symbol rate	250 kSymbol/s
Number of subcarriers	52
IFFT and FFT points	64
Guard interval	1/4 of OFDM symbol
Signal-to-noise ratio	1 to 15 dB
Doppler frequency	[200, 250, 300, 350, 400, 450, 500]
OFDM packet length (QAM symbols/packet)	[64, 100, 150, 200, 250, 325]
Channel model	Raleigh fading channel [29]

It is assumed that the system does not provide any *a priori* knowledge of the channel being estimated. The presented results are averaged values of 10 runs of the same experiment with random selection of training data. The proposed scheme is trained and tested using OFDM signal of same SNR, which is analogous to a real-life scenario, where as in C-ELM [22], the system is trained at higher SNRs to improve the accuracy, and the trained system is tested with data transmitted at lower SNRs. Furthermore, all experiments are executed in Matlab environment on an Intel Core 2 Duo processor at 2.0 GHz clock speed and 3GB RAM. The proposed framework is compared with other learning-based equalization schemes, namely C-ELM, CRBF, CMRAN, k-nearest neighbor (k-NN) [30], back propagation (BP) neural network [23], and stochastic gradient boosting (SG-Boosting) [31]. In the following subsection, experiments and their results are explained in detail.

4.1 Optimal parameter selection

There are mainly three parameters that require consideration to effectively deploy the proposed framework. These parameters are as follows: (1) activation functions, (2) amount of training data, and (3) the number of hidden neurons. The variations in normalization factor of an input signal do not severely degrade the performance of the proposed scheme, therefore, their analysis are not included in the experiments.

- *Activation functions*: Activation functions play a pivotal role in correct classification and are mainly divided into real and complex domain based on their operability. We split the complex numbers into two real values that are considered the inputs to an ELM. This approach has two advantages, namely: 1) the ability to apply real-valued activation functions and 2) the ability to reduce computational complexity. Representation of complex signal in real domain allows us to exploit variety of activation functions in the proposed scheme which is not possible in the complex domain such as in C-ELM. It is important to state that the proposed framework presents equalization and symbol detection problem as a classification task. Traditionally, classification results are presented in terms of accuracy where a higher accuracy corresponds to a lower symbol error rate (SER). The maximum accuracy is represented as 1 which is equal to 0 SER, (i.e., $accuracy = 1 - SER$). For 4-QAM, an accuracy analysis is presented using 8 different activation functions namely sine, sigmoid, hard limit, tribas, radial basis, asinh, and atanh. Figure 6a provides results for accuracy vs. different activation functions. These activation functions are operable on real and complex-valued signals as well. In this experiment, the number of hidden neurons, amount of training data, and normalization factor are kept constant for all activation functions in order to compare them precisely. The accuracy comparison for various activation functions using the proposed scheme and C-ELM are presented in Fig. 6a. From the figure, the proposed scheme using given activation functions for real-valued inputs outperforms C-ELM. Among the given activation functions, the performance of *Hardlim*(.) was the worst with accuracy around 90% whereas *sig*(.) generated the best accuracy followed by *sin*(.) and *asinh*(.).

- *Amount of training data*: In another set of trials, the performance of the proposed framework is tested in terms of correct classification, accuracy, using data at different SNRs (dB). A learner is rated based on its ability to learn a model using minimum training time and amount of data. The data are divided into training and testing parts. For varying SNRs, the percentage of training data is increased at equal intervals between 1% and 19%. The correct classification of the proposed scheme for varying size of training data is presented in Fig. 6b, it is noticeable that higher accuracy is obtained for data with rising SNRs. Except for training data of size 1%, the proposed framework generates accuracy starting from 99.8 % and more for increasing SNRs. High accuracy is noticeable for training data of size 10% and more to learn the channel being investigated. As observed from Fig. 6b, there is no significant improvement for exploiting higher percentage of information during model learning. Therefore, in all of experiments, 10% of the data are utilized for training and rest of the data for testing purpose.
- *Number of hidden neurons*: In learning-based frameworks, nonlinearity of an equalization scheme is mainly dependent on the number of hidden layers and number of neurons in each layer. Using only one hidden layer of neurons in the proposed framework results in an efficient way of processing QAM signals to attain higher accuracy. Efficiency of the proposed framework is attributed to analytic training in real domain where a single complex number is represented by a 2-tuple vector for improved separability and correct classification. Training in an analytic fashion has a limitation of growing computational complexities with increasing number of neurons during computation of Moore-Penrose generalized inverse of a matrix. For that reason, the performance analysis of the framework tested with changing number of neurons in the hidden layer of ELM using data of two different modes of QAM, namely 4-QAM and 16-QAM. For both modes, data are generated using four different SNRs ranging from 2 to 8 dB. Generally, equalization and symbol detection becomes more complicated for higher mode QAM where the QAM constellation points become

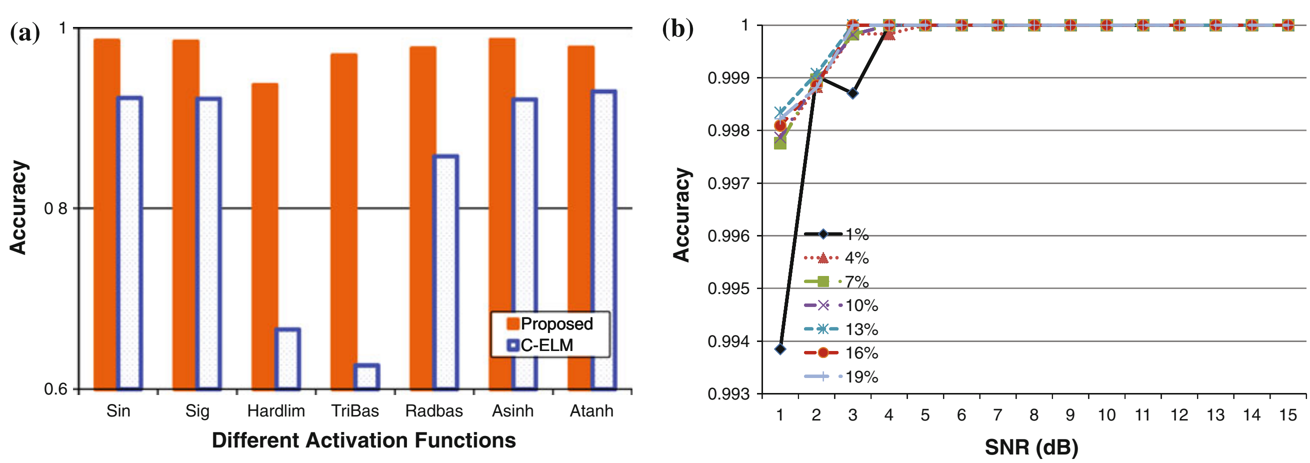

Fig. 6 Performance analysis for 4-QAM (**a**) for different activation functions using the proposed scheme and C-ELM (**b**) for varying percentage of training data using proposed framework

closer to each others. Therefore, an algorithm that performs well with high order QAM is needed, and the proposed framework can effectively equalize and detect the symbols, which is considered as an enhancement over the existing schemes. ELM with only one hidden layer of neurons restricts the options of improving the equalization capacity of the classifier by only changing the number of hidden neurons. It is not recommended to increase the number of neurons since large number of hidden neurons contributes to a higher computational complexity with an increase in approximation errors. Additional approximation errors can occur due to the random selection of input weights in ELM with large number of hidden neurons [32]. Therefore, this leads to ill-conditioned matrix **H** that may cause the system to be sensitive to perturbation in data. In [32], the authors proposed an input weight selection algorithm to improve the system robustness when number of hidden neurons are large. In this paper, it was observed that increasing the number of neurons does not guarantee proportional improvement in accuracy. Therefore, to limit the complexity of a system and to avoid the sensitivity of random selection of weights, a sufficient number of neurons were chosen for each QAM mode accordingly which is explained later in this subsection. For example, Fig. 7a–b presents performance analysis for changing number of hidden neurons using data acquired for two different modes. The number of hidden neurons are varied from 6–20 and 10–70 for 4-QAM and 16-QAM, respectively. It is observed that the performance of the proposed framework is gradually improving with increasing SNR, furthermore, higher number of hidden neurons are required for 16-QAM channel equalization compared to 4-QAM channel equalization due to the increase in QAM constellation points. The proposed framework results in smaller variations and stable behavior for data with higher SNRs and for $L \geq 2M$ where L and M represent the number of hidden neurons and the mode of investigated QAM, respectively. Besides, a rippling behavior of the graph lines for various SNRs and modes of QAM is also spotted for small number of hidden neurons. A similar trend in accuracy is found for 64-QAM data; based on evidence from the experiments, it is recommended for hidden layer of an ELM to satisfy $L \geq 2M$ criterion for a minimum number of neurons to achieve improved equalization and symbol detection.

4.2 Varying packet length and Doppler frequency

For time-varying channels, equalization schemes are heavily dependent on the change in Doppler frequency and OFDM packet length. SER is directly proportional to the length of its input packets and Doppler frequency. For better understanding, a set of experiments in time-varying channels for increasing Doppler frequency and packet length are performed for 4-QAM. In these experiments, SNR is fixed to 4 dB. The channels with Doppler frequencies ranging from 64 to 325 Hz are used to test the performance of SER in seven different schemes and the results of these tests are shown in Fig. 8a. In this figure, the SER using the proposed framework at 100 Hz Doppler frequency is approximately 100 times better than kNN, SG-Boosting, and CRBF. The proposed framework outperforms all other equalizers by generating minimum SER, and 5 times on average better than C- ELEM, CMRAN, and BP. Figure 8b illustrates the SER versus varying packet length that ranges from 200 OFDM symbols to 500 OFDM symbols. The figure shows that at packet length of 300 OFDM symbols, the SER using kNN, SG-boosting,

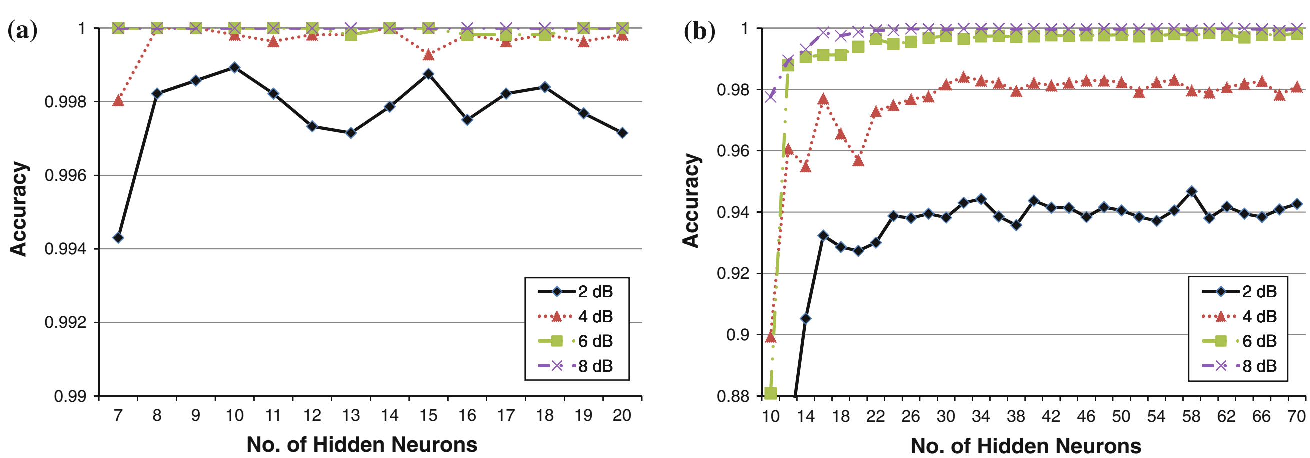

Fig. 7 Performance analysis for varying number of hidden neurons of ELM in the proposed framework (**a**) using 4-QAM (**b**) using 16-QAM

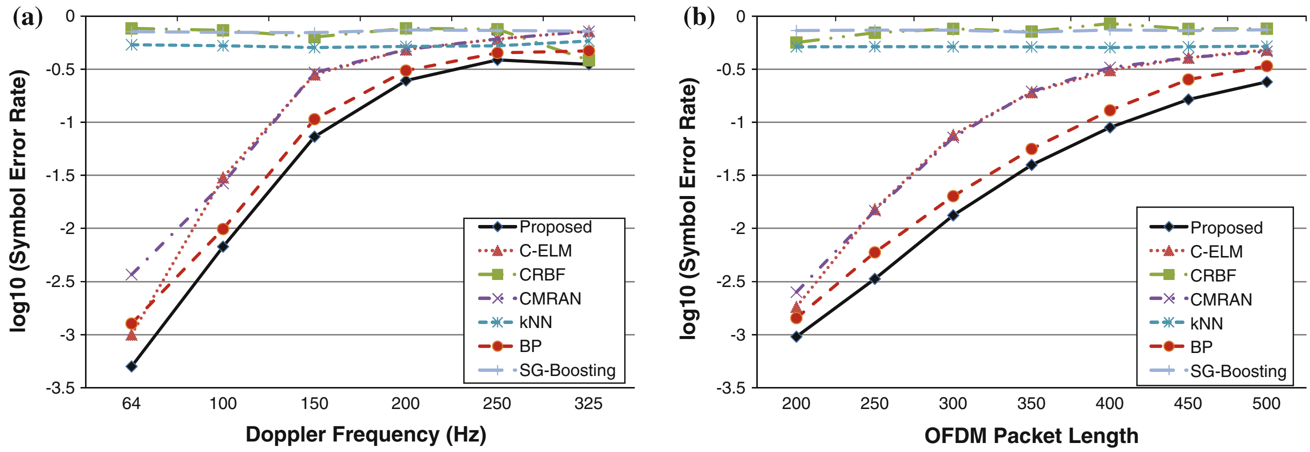

Fig. 8 Equalization and symbol detection in time-varying channel for 4-QAM (**a**) SER versus Doppler frequency using packet length of 200 symbols (**b**) SER versus packet length using 500 Hz Doppler frequency

and CRBF is worse than the proposed by approximately 100 times where as using CMRAN, C-ELM and BP worse than the proposed by approximately 5 times on average. Overall, the proposed framework outperforms all other equalizers in achieving minimum SER under varying Doppler frequencies and packet lengths.

4.3 Performance comparison of 4-, 16-, and 64-QAM

The performance comparison of the proposed framework against existing equalization schemes is presented in Figs. 9 and 10 for M-QAM signals, namely 4-, 16-, and 64-QAM. For all three experiments, QAM with varying SNR ranging from 1 to 15 dB is used where each experiment utilizes equal amount of training samples. SER for 4-QAM is presented in Fig. 9 where seven equalization schemes are divided into three noticeable groups based on SER performances. The best group among the three is the proposed framework and BP. The group consisted of SG-Boosting, kNN, and CRBF did not perform well. SER performances of C-ELM and CMRAN lie in the middle where a slow decrease in SER is observed for increasing SNR.

The performance comparison for 16-QAM is presented in Fig. 10a where it is clear that all schemes show poor performance except the proposed framework and C-ELM. For the proposed framework, a steep and consistent drop of SER ascertains superior performance whereas C-ELM is the second best equalizer with widening gap in performance against the proposed framework. No improvement in SER are observed for the rest of the schemes that perform consistently high SERs as observed in Fig. 10.

For 64-QAM, Fig. 10b presents results that establishes superior realization of the proposed framework with consistently improved performance. The performance of C-ELM is significantly degraded in current set of experiments with a

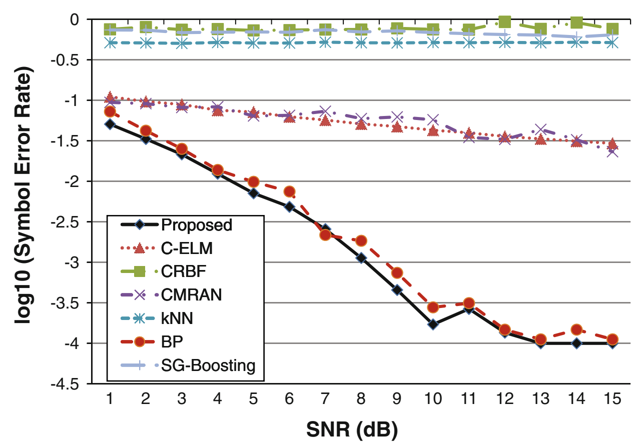

Fig. 9 Error probability of various scheme for 4-QAM

very small improvement for increasing SNR. As observed from Fig. 10, increase in the SER for the rest of the equalizers is attributed toward rising of the QAM constellation points as in 16- and 64-QAM. Clearly, the lowest SER for the proposed framework is consistent throughout the set of experiments on different settings.

4.4 Computational comparison

The architecture of the proposed framework is very much alike to C-ELM and other learning-based equalization schemes. However, presentation of equalization and symbol detection problem in a classification domain removes inherent limitation to use a decision slicer to identify a symbol. On the other hand, C-ELM offers regression-based equalization in complex domain where an additional decision slicer is required for symbol detection; whereas the proposed framework operates in a real domain by splitting complex signal into two real-valued inputs.

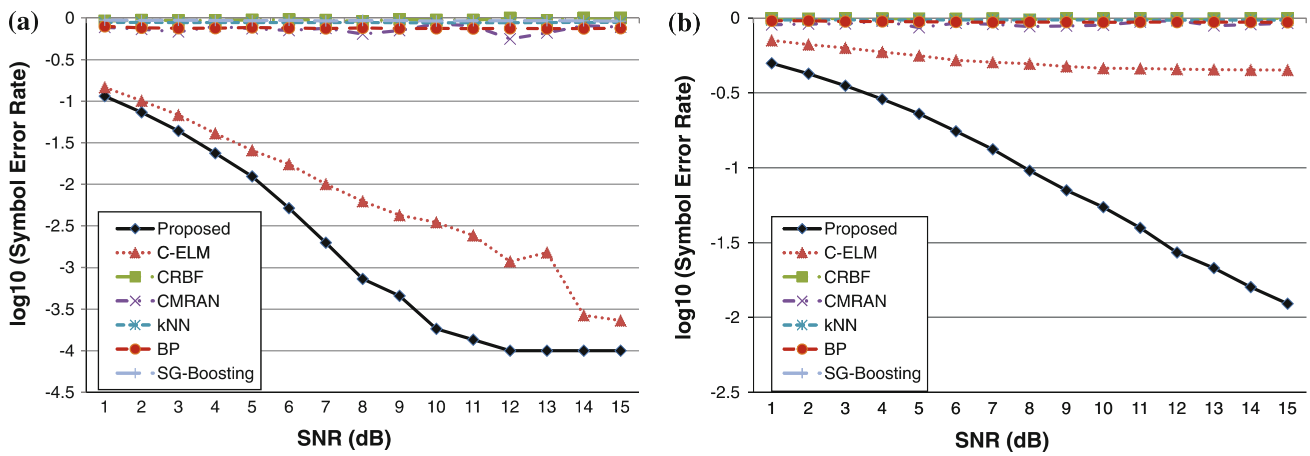

Fig. 10 Error probability of various scheme for **a** 16-QAM, and **b** 64-QAM

Moreover, analytic training and equalization in complex domain is computationally intensive compared to the real domain operations. In order to check the computational cost of various methods, a set of experiments are performed using data of 16-QAM and 64-QAM. The higher mode QAM are being used to test the performance of various methods for nonlinear channel equalization in terms of computational time (in seconds) during training and testing phases. The computational time for channel estimation and testing is presented in Table 2. Deploying seven different techniques named proposed framework using ELM, kNN, BP, SG-Boosting, C-ELM, CRBF, and CMRAN. Naturally, the computational time for training is much higher than the testing phase due to optimization constraints to learn a model or channel under investigation. Note that the training time for kNN scheme is not applicable (N/A) since the centers of sub-quadrants as the means of clusters are provided, to associate an incoming input to a symbol using Euclidean distance. The size of training and testing data is set to be equal for all schemes for a fair comparison. The computational time for learning phase in 16-QAM and 64-QAM are presented in Table 2, where the training time for SG-Boosting and CMRAN are the highest among all techniques whereas the proposed framework has the minimal training time. In addition, schemes like BP require a careful selection of number of epochs and layers, and learning parameters since these are implicit characteristics of gradient-descent methods. The training using proposed framework runs at least 6 times faster than C-ELM and the rest of the algorithms. Moreover, the computational time for various methods during testing phase for equalization and symbol detection is shown in Table 2 for both 16-QAM and 64-QAM. The proposed framework consumes minimum resources, whereas kNN is the second inline of the most prudent methods in the trials. On other hand, CRBF and CMRAN schemes consumes the highest amount of CPU time for equalization and symbol identification. Based on computational analysis, it is affirmed that the proposed framework requires minimum computational time among different learning-based equalization schemes.

Table 2 Complexity time comparison of training and testing for different algorithms

Algorithms	16-QAM		64-QAM	
	Training time (s)	Testing time (s)	Training time (s)	Testing time (s)
Proposed	0.0156	0.2031	0.0625	0.2656
kNN	N/A	0.4062	N/A	0.4375
BP	2.530	0.7308	2.112	0.6702
SG-Boost	241.6	0.65625	419.0	0.8125
C-ELM	0.0937	0.6875	0.4218	1.140
CRBF	0.7656	2.625	0.7656	2.578
CMRAN	44.60	7.125	40.73	7.5

5 Conclusions

The problem of equalization and symbol detection has been presented as an optimum classification task. The use of a classification framework removed the inherent limitation in symbol detection schemes to use an additional decision slicer followed by equalization step. In channel estimation, analytic learning approach finishes training faster than traditional gradient-descent schemes. The proposed framework performed joint equalization and symbol detection in real domain by transforming a complex signal into a single 2-tuple real-valued vectors. Such transformation offers equalization in real domain with minimum computational load and high accuracy. Simulation results of the proposed framework showed significant improvement in terms of

SERs and has lower computations than existing algorithms. One of the benefits of operating in real domain is to use large number of nonlinear and infinitely differentiable activation functions. Some aspects of this work require future investigation, such as implementation issues involving matrix inverse, and channel equalization issues for fast time-varying channels. Also, pilot symbols that exist in the OFDM systems can be exploited to further enhance the performance of the proposed framework.

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