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ORIGINAL ARTICLE

Developing correlations by extreme learning machine for calculating higher heating values of waste frying oils from their physical properties

Ömer F. Ertuğrul[1] · Şehmus Altun[2]

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Abstract In this study, a novel approach was proposed based on extreme learning machine (ELM) for developing correlations in order to calculate higher heating values (HHVs, kj/kg) of waste frying oils from their physical properties such as density (ρ, kg/m^3) and kinematic viscosity (v, mm^2/s) values. These values can easily be determined by using laboratory equipment. For developing the correlations, an experimental dataset from the literature covering 35 samples was collected to be employed in the training and validation steps. The obtained optimum parameters of artificial neural network in the training stage by ELM were employed to develop new correlations. The HHVs calculated by using density-based correlation ($HHV = 50823.183 - 12.34095\rho$) showed the mean absolute and relative errors of 145.8048 kJ/kg and 0.3695 %, respectively. In the case of the viscosity-based correlation ($HHV = 40172.85 - 17.93615v$), they were found as 129.04 kJ/kg and 0.327 %, respectively. Additionally, new correlations were performed better than those available in the literature and those obtained by other machine learning methods; therefore, it is highly suggested that the proposed approach can be used for developing new correlations.

Keywords Extreme learning machine · Mathematical modeling · Higher heating value · Waste frying oils

✉ Ömer F. Ertuğrul
omerfarukertugrul@gmail.com

[1] Department of Electrical and Electronic Engineering, Batman University, 72100 Batman, Turkey

[2] Department of Automotive Engineering, Batman University, 72100 Batman, Turkey

1 Introduction

Biodiesel is a renewable diesel replacement fuel that is produced from oils such as soybean oil, recycled cooking oils or animal fats, and it can be used alone or blended with petroleum diesel in any proportion [1]. Biodiesel contains no aromatics, approximately no sulfur and 10–12 % oxygen by weight [2]. The main advantage in its utilization is attributed to lesser exhaust emissions with regard to carbon monoxide (CO), unburned HCs and PM emissions [3] with an increase in NO_x emissions [4]. However, biodiesel is not competitive with fossil diesel fuels, basically due to the high price of the feedstock [5], which greatly obstructs its widespread application. In this regard, the use of waste oil instead of food-grade oils to produce biodiesel is an influential way to lessen the cost since waste oil is available at a relatively low price [6]. Also, using waste oil contributes to solve the problem of waste oil disposal [7]. On the other hand, the feedstock source affects the biodiesel properties; hence, it must be controlled in detail prior to transesterification reaction to obtain a quality biodiesel [8]. One of the most crucial characteristics of a fuel is its higher heating value (HHV), which is the amount of heat released during the combustion of one gram of fuel to produce CO_2 and H_2O at its initial temperature and pressure [9]. It is important to estimate fuel consumption as the grater the heating value, the lower the fuel consumption (if the thermal efficiency, as expected, is not affected) [10].

The HHV of a fuel can be determined with a bomb calorimeter; however, it is time-consuming and has high costs as it requires exclusive laboratory equipment. It can also be estimated from some fuel properties and fatty acid composition. For instance, the HHV of vegetable oils, methyl esters and liquid fuels was suggested to be a function of their viscosities (v) and densities (ρ) in the

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study of Demirbaş [11, 12] with the following expressions: $HHV = 38.053 + 0.0317v$, $HHV = 39.450 + 0.4625v$, $HHV = 37.945 + 0.049\rho$ and $HHV = 79.014 - 43.126\rho$. Although these models give precise results, it may be useful to develop new HHV correlations specifically for waste frying oils as their importance for biodiesel production is progressively increasing [13]. Therefore, in this study, the new correlations were developed to estimate the HHV of waste frying oils from their density and kinematic viscosity values by using ELM as a novel approach. This is a newly employed model in this area.

A system or a process can be modeled by developing mathematical correlations or machine learning (ML) methods (such as an artificial neural network (ANN) and a fuzzy system) by using the collection of experimental data [14]. The mathematical ones may be deficient in descriptive capability, but it has an excellent manipulation capability [15]. On the other hand, the descriptive capability of ML methods is generally high, but unfortunately, their manipulation capability is less than mathematical models. Furthermore, developing a correlation mathematically is harder than modeling by a ML method, but once a correlation is developed it can be employed easily for problem solution and optimization.

The aim of this paper is to propose a novel approach that can bring together the high modeling capability of ML methods with manipulation capability of mathematical modeling. Generally, linear regression is employed to develop mathematical models since the ML methods are generally black or gray box approach such as ANN. In this study, ELM, which is a training method of single hidden layer feed-forward ANNs (SLFNs), was employed to develop novel correlations. In ELM, the weights in the input layer and biases are assigned arbitrary and the weights in the output layer are calculated mathematically by the Moore–Penrose generalized inverse matrix [16, 17]. Therefore, a high generalization capability and extremely fast learning stage were shown. To evaluate and validate the proposed approach, new correlations between the higher heating value (HHV, kJ/kg) of waste frying oils and their density (ρ, kg/m^3) and kinematic viscosity (v, mm^2/s) values were developed as a case study.

2 Materials and methods

2.1 HHV dataset

Thirty-five waste frying oils (WFOs), whose properties are listed in Table 1, were taken from the study of Şanlı et al. [18]. First 25 samples were used as a training dataset, while others (26–35) were used as a test or validation dataset. The origin of samples 17–30 (from fish restaurants) and 31–35 (from hospital restaurants) was sunflower oil. The first 16 samples (from fast foods) were of hydrogenated soybean oil origin except for samples 9 and 13, which were also of sunflower oil origin. The frying process causes various chemical reactions, which result in the changes in the fatty acid composition mainly due to cracked double bonds [19, 20], and the frying conditions (such as temperature, duration and type of food) affect the fuel properties. The measurement of density and kinematic viscosity of a fuel is relatively simple, and the HHV of an oil sample is a function of its density and viscosity; therefore, both properties are significant inputs for the HHV estimation methods [21, 22].

2.2 Extreme learning machine (ELM)

The extreme learning machine (ELM) was proposed by Huang et al. [23], and it is a training method for SLFNs. ELM not only needs less training parameter but also optimizes less network features than traditional ANN. In ELM, the weights and biases in the input layer are assigned randomly. Later, the weights in the output layer are calculated analytically by Moore–Penrose generalized inverse method if and only if the employed transfer function is infinitely differentiable; and the number of neurons in the hidden layer is less than the number of observations [23]. Therefore, it has a high accuracy and generalization capacity [24] with extremely fast train stage and speed in comparison with conventional neural networks [23–25]. Although some studies have recently been done on ELM for modeling the performance of engines [25], the estimation of fuel properties is a new concept.

The inputs of the network were varied from 1 to 3 depending on the parameters of the obtained equations, representing the density and viscosity values of 25 samples for training and 10 samples for validation. The topology of ELM used in this work is shown in Fig. 1. In this figure, x_i denotes input, n indicates number of neurons in the input layer, m represents the number of neurons in the hidden layer, k is the number of neurons in the output layer and y symbolizes output which is mathematically corresponds to [26]:

$$y = \sum_{j=1}^{m} \beta_{j,k} g\left(\sum_{i=1}^{n} w_{i,j} x_i + b_j \right) \quad (1)$$

where $w_{i,j}$ denotes weights of input neurons, $\beta_{j,k}$ symbolizes weights of output neurons, b_j symbolizes threshold values (biases) of the neurons in hidden layer and $g(\cdot)$ stands for the activation function [27]. Equation (1) can be rewritten as;

$$H\beta = y \quad (2)$$

Table 1 Fuel properties

Sample no	C14:0	C16:0	C16:1	C18:0	C18:1	C18:2	C18:3	C20:0	C22:0	ρ (g/cm^3)	v (mm^2/s)	HHV (kJ/kg)
1	0.638	25.043	0.246	4.283	37.942	30.032	0.190	0.151	0.324	0.9237	42.28	39,223
2	0.774	29.117	0.226	4.375	37.455	26.233	0.210	0.152	0.259	0.9194	39.81	39,259
3	0.701	25.645	0.202	3.863	43.228	24.306	0.271	0.095	0.308	0.9191	40.14	39,441
4	0.704	25.950	0.180	3.899	43.574	23.637	0.265	0.178	0.297	0.9183	39.39	39,833
5	0.738	27.614	0.205	3.930	42.754	22.805	0.281	0.180	0.266	0.9196	41.76	39,312
6	1.151	41.438	0.171	4.775	40.363	10.293	0.182	0.140	–	0.9207	44.47	39,741
7	1.051	40.637	0.205	4.369	42.104	9.958	0.173	0.148	0.062	0.9190	43.65	39,336
8	0.974	33.359	0.194	4.240	41.921	17.639	0.187	0.160	0.143	0.9183	42.37	39,090
9	0.495	16.244	0.294	5.081	37.617	38.020	0.129	0.160	0.562	0.9273	51.44	39,007
10	0.992	34.478	0.261	4.926	40.005	17.281	0.148	0.150	0.167	0.9214	44.37	39,372
11	0.980	32.707	0.211	4.326	38.685	21.164	0.154	0.142	0.204	0.9201	40.79	39,421
12	1.041	36.928	0.179	4.713	41.453	13.787	0.141	0.157	0.119	0.9190	42.30	39,966
13	0.271	11.204	0.224	3.752	35.835	47.360	–	0.155	0.576	0.9274	46.66	39,450
14	0.693	26.283	0.167	3.999	36.302	30.852	0.152	0.145	0.307	0.9209	39.99	39,442
15	1.017	40.203	0.207	4.492	40.912	11.614	0.154	0.144	0.062	0.9197	42.92	39,311
16	1.033	40.100	0.188	4.433	41.180	11.473	0.152	0.146	0.062	0.9165	42.78	39,502
17	0.195	7.411	0.446	4.073	29.527	56.201	0.125	0.203	0.680	0.9248	36.94	39,492
18	0.170	8.856	0.142	3.943	28.379	56.685	0.228	0.174	0.601	0.9252	37.12	39,449
19	0.151	7.674	0.289	4.271	31.999	53.941	0.062	0.163	0.721	0.9238	35.99	39,519
20	0.087	7.158	0.095	4.409	25.259	60.316	0.303	0.159	0.727	0.9269	39.86	39,404
21	0.109	6.415	0.204	4.005	31.865	55.768	0.070	0.161	0.700	0.9236	35.61	39,556
22	0.215	6.687	0.328	4.103	30.805	56.049	0.112	0.176	0.721	0.9241	34.89	39,367
23	0.131	6.812	0.227	4.048	33.856	53.239	0.272	0.151	0.675	0.9243	37.74	39,518
24	0.087	6.660	0.143	3.881	32.473	55.125	0.099	0.159	0.674	0.9238	35.58	39,449
25	0.231	6.663	0.305	4.120	36.582	49.657	0.302	0.185	0.764	0.9238	36.37	39,652
26	0.176	7.334	0.270	4.100	33.039	52.309	0.458	0.273	0.704	0.9264	40.52	39,344
27	0.118	7.191	0.211	3.606	29.841	57.227	0.203	0.175	0.614	0.9233	33.74	39,624
28	0.174	7.040	0.380	4.381	28.512	57.749	0.076	0.183	0.720	0.9239	34.85	39,562
29	0.088	6.674	0.139	4.170	34.104	53.545	0.293	0.153	0.743	0.9249	38.14	39,465
30	0.155	6.572	0.216	4.267	37.798	49.123	0.319	0.176	0.827	0.9262	41.60	39,474
31	0.325	8.646	0.463	4.896	28.398	55.555	0.344	0.174	0.545	0.9231	34.80	39,650
32	0.126	7.998	0.206	3.639	26.469	60.123	0.072	0.140	0.598	0.9273	39.91	39,322
33	0.083	6.453	0.145	3.819	30.461	57.668	0.058	0.126	0.673	0.9242	34.81	39,498
34	0.135	6.722	0.229	3.596	36.751	48.606	0.078	0.468	0.552	0.9231	35.18	39,689
35	0.243	9.060	0.156	4.046	34.185	50.600	–	0.144	0.611	0.9311	47.24	38,925

where H, hidden layer output matrix, stands for [23]:

$$H\left(w_{i,j}, b_j, x_i\right) = \begin{bmatrix} g\left(w_{1,1}x_1 + b_1\right) & \cdots & g\left(w_{1,m}x_m + b_m\right) \\ \vdots & \ddots & \vdots \\ g\left(w_{n,1}x_n + b_1\right) & \cdots & g\left(w_{n,m}x_m + b_m\right) \end{bmatrix} \tag{3}$$

The weights of output neurons, $\beta_{1,\ldots,m,\,1,\ldots,k}$, can be analytically computed by Moore–Penrose generalized inverse method. As a result:

$$\widehat{\beta} = H^{+}y \tag{4}$$

where H^{+} denotes the Moore–Penrose generalized inverse matrix of H. Any transfer function, which is infinitely differentiable, can be preferred in ELM. A linear transfer function was chosen as a transfer function of the hidden layer for determining equations. Equation (1) can be redefined as:

$$y = \sum_{j=1}^{m} \beta_{j,k} \left(\sum_{i=1}^{n} w_{i,j}x_i + b_j \right) \tag{5}$$

Then,

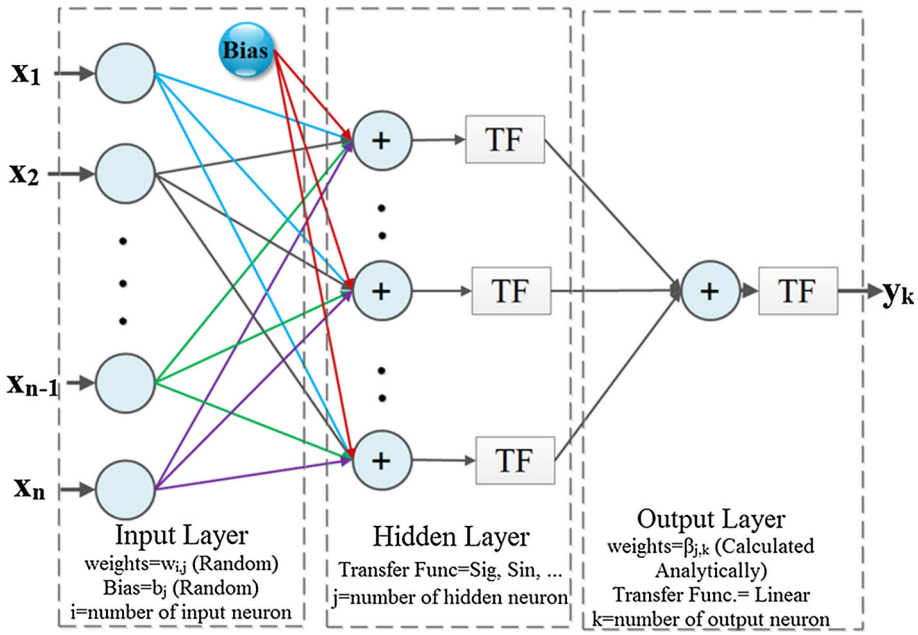

Fig. 1 ELM network architecture

$$y = \beta_1(w_{1,1}x_1 + w_{2,1}x_2 + \cdots + w_{n,1}x_n) + \beta_2(w_{1,2}x_1 + w_{2,2}x_2 + \cdots + w_{n,2}x_n) + \cdots + \beta_m(w_{1,m}x_1 + w_{2,m}x_2 + \cdots + w_{n,m}x_n) + \sum_{j=1}^{m}\beta_j\{b_1 + b_2 + \cdots + b_n\} \quad (6)$$

The constant terms can be redefined as $\psi = \sum_{j=1}^{m}\beta_j\{b_1 + b_2 + \cdots + b_n\}$, for simplicity, so the Eq. 6 can be reformed as;

$$y = (\beta_1 w_{1,1} + \beta_2 w_{1,2} + \cdots + \beta_m w_{1,m})x_1 + (\beta_1 w_{2,1} + \beta_2 w_{2,2} + \cdots + \beta_m w_{2,m})x_2 + \cdots + (\beta_1 w_{n,1} + \beta_2 w_{n,2} + \cdots + \beta_m w_{n,m})x_n + \psi. \quad (7)$$

Equation 7 can be rewritten as:

$$y_k = \alpha_1 x_1 + \alpha_2 x_2 + \cdots + \alpha_n x_n + \psi \quad (8)$$

where the coefficients of features are: $\alpha_n = \beta_1 w_{n,1} + \beta_2 w_{n,2} + \cdots + \beta_m w_{n,m}$.

2.3 Performance indices and model validation

To elucidate the performance of suggested models, the previous suggested correlations in the literature and simulations by the employed ML methods are presented by mean absolute error (MAE), root mean square error (RMSE), mean absolute percentage error (MAPE) and relative error (RE) by using Eqs. (9–12).

$$\text{MAE} = \frac{1}{n}\sum_{i=1}^{n}|f_i - y_i| \quad (9)$$

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(f_i - y_i)^2} \quad (10)$$

$$\text{MAPE} = \frac{100\,\%}{n}\sum_{i=1}^{n}\left|\frac{f_i - y_i}{y_i}\right| \quad (11)$$

$$\text{RE} = \left|\frac{f_i - y_i}{y_i}\right| \quad (12)$$

where f is estimated and y is the true value. The validation of the correlations developed was done by using experimental data (samples 26–35), which was not used in developing the model process. The best fit was expressed as the lowest error terms of MAE, RMSE and MAPE.

3 Results and discussion

The fundamental issue in ML is to determine the optimum parameters of the employed artificial intelligence method. In ELM, both network structure (number of neurons in the hidden layer) and learning parameters (transfer function) must be optimized. This optimization can be performed by both testing the network for different parameters and simply comparing accuracies obtained or by an expert view. Achieved MAPE in trials (in predicting HHV by using density data) is shown in Fig. 2. The accuracies summarized in Fig. 2 were calculated by tenfold cross-validation scheme in which 30 data were formed training dataset and the other 5 data were made the validation dataset, and this process continued 7 times until each datum was employed as a member of test dataset. The

reported accuracies are mean of these 7 trials. The lowest MAPE was achieved by sigmoid, hard limit and linear transfer functions as 0.3484, 0.3854 and 0.3144, respectively. Addition to these, sine and radial basis transfer functions were also employed. However, they were not reported here due to their high MAPEs. Based on the findings in Fig. 2, linear transfer functions with three neurons in the hidden layer were used in the structure of SLFN that is employed for developing correlations.

First- to third-degree polynomial equations that depend on physical variables were developed. The errors obtained by using these equations are given in Table 2. Depending on the increased degree of the polynomial for each degree of the variable, a separate input (exponential degree of these variables) was added to SLFN. As it can be seen in Table 2, in general, error rates for training and validation data are very similar. Moreover, the higher-degree equations (the upper than third degree) were not taken into consideration due to insignificant decrease in the error rate and the increase in complexity.

The comparison of the recent and the other estimation methods is given in Table 3 where the calculated HHV values are compared with measured ones. Each developed equation was found to have acceptable accuracy as their RE was not exceeding 0.014, which is less than the measurement instrument error. As seen in Table 3, the RE obtained for the test data is slightly lower than the RE of the train data, and this may be due to the data distribution in the dataset. Also, the MAE and MAPE are better for viscosity-based correlations than that of density-based correlations; they are 129.04 kJ/kg and 0.3270 % and 145.8048 kJ/kg and 0.3695 %, respectively. Furthermore, in general, HHVs calculated by the recent methods are closer to measured values compared with the results of

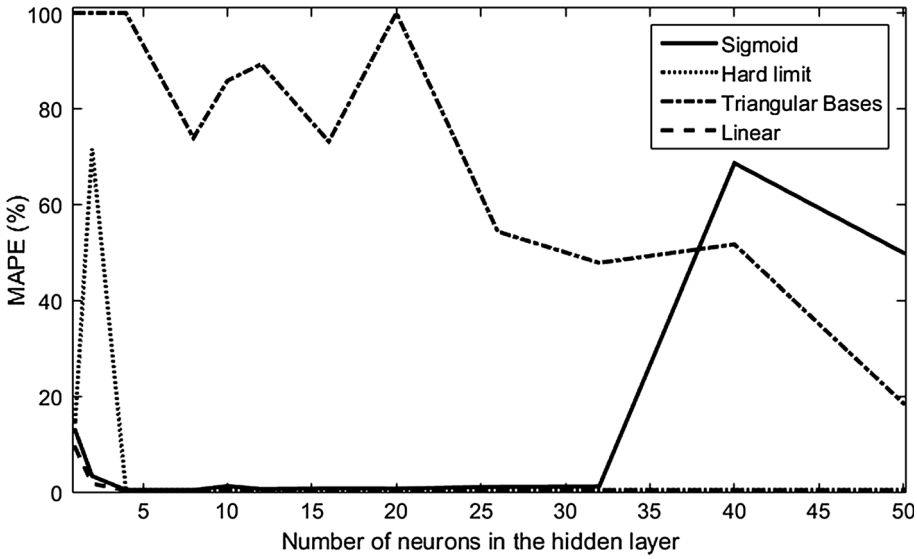

Fig. 2 Obtained accuracies for different number of neurons in the hidden layer

Table 2 Correlations with different degrees

	Degree	Data	MAE	RMSE	MAPE	Correlations
Density	1	Train	152.66	200.86	0.3868	$HHV = 50823.183 - 12.34095\rho$
		Test	154.11	189.70	0.3911	
	2	Train	156.64	198.24	0.3969	$HHV = -3107495.1225 + 6836.83857\rho - 3.713269\rho^2$
		Test	107.42	128.84	0.2719	
	3	Train	156.82	198.32	0.3974	$HHV = 5032.9266 - 3301.6667\rho + 7.29476\rho^2 - 0.003984\rho^3$
		Test	104.12	124.99	0.2636	
Viscosity	1	Train	133.62	192.29	0.3382	$HHV = 40172.85 - 17.93615v$
		Test	107.82	151.57	0.2744	
	2	Train	134.70	186.30	0.3409	$HHV = 36281.52 + 169.70299v - 2.24074v^2$
		Test	137.76	172.24	0.3499	
	3	Train	130.46	184.62	0.3301	$HHV = 59458.07 - 1478.495v + 36.48333v^2 - 0.30051v^3$
		Test	112.39	163.00	0.2860	

Table 3 Comparison of performance of the recent and the other estimation methods

Sample no	Measured HHV (kJ/kg)	This study[a]	RE	This study[b]	RE	[11, 12][a]	RE	[11, 12][b]	RE	[18][c]	RE
1	39,223	39,423.847	0.005	39,414.510	0.005	39,178.514	0.001	40,016.720	0.020	39,447	0.006
2	39,259	39,476.914	0.006	39,458.812	0.005	39,363.956	0.003	39,895.690	0.016	39,460	0.005
3	39,441	39,480.616	0.001	39,452.893	0.000	39,376.893	0.002	39,911.860	0.012	39,387	0.001
4	39,833	39,490.489	0.009	39,466.345	0.009	39,411.394	0.011	39,875.110	0.001	39,481	0.009
5	39,312	39,474.445	0.004	39,423.836	0.003	39,355.330	0.001	39,991.240	0.017	39,508	0.005
6	39,741	39,460.870	0.007	39,375.229	0.009	39,307.892	0.011	40,124.030	0.010	39,434	0.008
7	39,336	39,481.850	0.004	39,389.937	0.001	39,381.206	0.001	40,083.850	0.019	39,462	0.003
8	39,090	39,490.489	0.010	39,412.895	0.008	39,411.394	0.008	40,021.130	0.024	39,471	0.010
9	39,007	39,379.420	0.010	39,250.214	0.006	39,023.260	0.000	40,465.560	0.037	39,333	0.008
10	39,372	39,452.232	0.002	39,377.023	0.000	39,277.704	0.002	40,119.130	0.019	39,397	0.001
11	39,421	39,468.275	0.001	39,441.234	0.001	39,333.767	0.002	39,943.710	0.013	39,421	0.000
12	39,966	39,481.850	0.012	39,414.151	0.014	39,381.206	0.015	40,017.700	0.001	39,414	0.014
13	39,450	39,378.186	0.002	39,335.949	0.003	39,018.948	0.011	40,231.340	0.020	39,402	0.001
14	39,442	39,458.402	0.000	39,455.583	0.000	39,299.267	0.004	39,904.510	0.012	39,445	0.000
15	39,311	39,473.211	0.004	39,403.030	0.002	39,351.018	0.001	40,048.080	0.019	39,457	0.004
16	39,502	39,512.702	0.000	39,405.542	0.002	39,489.021	0.000	40,041.220	0.014	39,455	0.001
17	39,492	39,410.272	0.002	39,510.289	0.000	39,131.075	0.009	39,755.060	0.007	39,483	0.000
18	39,449	39,405.336	0.001	39,507.060	0.001	39,113.825	0.008	39,763.880	0.008	39,484	0.001
19	39,519	39,422.613	0.002	39,527.328	0.000	39,174.201	0.009	39,708.510	0.005	39,392	0.003
20	39,404	39,384.356	0.000	39,457.915	0.001	39,040.511	0.009	39,898.140	0.013	39,400	0.000
21	39,556	39,425.082	0.003	39,534.144	0.001	39,182.826	0.009	39,689.890	0.003	39,398	0.004
22	39,367	39,418.911	0.001	39,547.058	0.005	39,161.263	0.005	39,654.610	0.007	39,436	0.002
23	39,518	39,416.443	0.003	39,495.940	0.001	39,152.638	0.009	39,794.260	0.007	39,457	0.002
24	39,449	39,422.613	0.001	39,534.682	0.002	39,174.201	0.007	39,688.420	0.006	39,405	0.001
25	39,652	39,422.613	0.006	39,520.512	0.003	39,174.201	0.012	39,727.130	0.002	39,442	0.005

[a] Obtained from the density data

[b] Obtained from the viscosity data

[c] Obtained from the fatty acid composition data

Şanlı et al. [18]. The values calculated by equations given in the study of Demirbaş [11, 12] that is based on density are lower while those based on kinematic viscosity data are higher than the measured values. The highest RE for the correlations developed using density and viscosity data is 0.012 and 0.014, respectively, which are lower than those of Demirbaş [11, 12] (0.015 and 0.037, respectively) and same with Şanlı et al. [18] (0.014).

In order to validate the accuracy of developed correlations, a separate dataset, which was not utilized for developing the correlations, is also used, and the results are listed in Table 4 in comparison with other correlations. The MAPE is found as 0.3089, 0.3034, 0.8804, 0.9298 and 0.3786 % for the average of recent correlations, correlations proposed by Demirbaş [11, 12] and by Şanlı et al. [18], respectively. When the results of validation data are taken into consideration, the overall accuracy is decreased as given in Table 4. This result is not consistent with the fact that ELM produces coefficients to give the lowest error during the training stage. In training of ELM, generally, it does not only tend to reach the smallest training error but also the smallest norm of the weights that means the network tends to have better generalization performance [23, 24]. Thus, it is expected that ELM shows higher success for training rather than validation data. This is because the equations obtained are fitted to train data. A comparison of error parameters obtained by using the all of 35 samples is also given in Table 5. It is clearly seen in Table 5 that the error rates obtained by ELM are consistently lower than those of other correlations in the literature.

The performance of the proposed method is also compared with the predictions made by some popular machine learning (ML) methods such as linear regression (LR), artificial neural network (ANN), k nearest neighbor regression (kNNR), ridge regression (Ridger), least absolute shrinkage and selection operator regression (LASSOR), kernel smoother (kSR), partial least squares regression (PLSR) and Gaussian process regression (GPR).

Table 4 Results of validation dataset

Sample no	Measured HHV (kJ/kg)	This study[a]	RE	This study[b]	RE	[11, 12][a]	RE	[11, 12][b]	RE	[18][c]	RE
26	39,344	39,390.527	0.001	39,446.077	0.003	39,062.074	0.007	39,930.480	0.015	39,607.000	0.007
27	39,624	39,428.784	0.005	39,567.684	0.001	39,195.764	0.011	39,598.260	0.001	39,497.000	0.003
28	39,562	39,421.379	0.004	39,547.775	0.000	39,169.889	0.010	39,652.650	0.002	39,439.000	0.003
29	39,465	39,409.038	0.001	39,488.765	0.001	39,126.763	0.009	39,813.860	0.009	39,453.000	0.000
30	39,474	39,392.995	0.002	39,426.706	0.001	39,070.699	0.010	39,983.400	0.013	39,445.000	0.001
31	39,650	39,431.252	0.006	39,548.672	0.003	39,204.389	0.011	39,650.200	0.000	39,541.000	0.003
32	39,322	39,379.420	0.001	39,457.018	0.003	39,023.260	0.008	39,900.590	0.015	39,443.000	0.003
33	39,498	39,417.677	0.002	39,548.493	0.001	39,156.951	0.009	39,650.690	0.004	39,739.000	0.006
34	39,689	39,431.252	0.006	39,541.856	0.004	39,204.389	0.012	39,668.820	0.001	39,637.000	0.001
35	38,925	39,332.524	0.010	39,325.546	0.010	38,859.381	0.002	40,259.760	0.034	39,337.000	0.011

[a] Obtained from the density data

[b] Obtained from the viscosity data

[c] Obtained from the fatty acid composition data

Table 5 Comparison of error parameters

Reference	MAE	RMSE	MAPE
This study[a]	145.8048	184.4838	0.369462
This study[b]	129.04	180.4605	0.326981
[11, 12][a]	270.5216	316.4902	0.684236
[11, 12][b]	454.4666	572.6039	1.156418
[18][c]	148.4	200.2811	0.376259

[a] Obtained from the density data

[b] Obtained from the viscosity data

[c] Obtained from the fatty acid composition data

Table 6 Comparison of error parameters

Method	MAE	RMSE	MAPE
ANN	151.9504	210.4378	0.385363
LR	170.3388	209.4339	0.432108
kNNR	198.8286	293.5295	0.504201
Ridger	151.9854	210.4405	0.38545
LASSOR	151.9854	210.4405	0.38545
kSR	151.9504	210.4377	0.385363
PLSR	196.6948	288.3882	0.498774
GPR	154.1196	209.9376	0.390742

The accuracy-based sevenfold cross-validation is reported in Table 6. When Tables 5 and 6 are analyzed together, it can be seen that the performance of the proposed method is better than the performance of other ML methods and the other correlations in the literature in terms of accuracies obtained. These results show that the proposed approach has a high ability to develop new correlations for calculating HHV of waste frying oils, and it brings together the modeling ability of ML with the reusability ability of mathematical modeling.

4 Conclusions

In this study, a novel approach using ELM was proposed to develop new empirical correlations for calculating the HHV of waste frying oils from their density and kinematic viscosity data as a case study. Due to the simplicity of the measuring these parameters, they were selected as input parameters for developing models. The results showed that ELM gave more precise result than conventional ways when the determined and calculated values were compared. For all samples considered, the HHVs calculated using densities showed the mean absolute and relative errors of 145.8048 kJ/kg and 0.3695 %, while for using viscosities, they were 129.04 kJ/kg and 0.327 %, respectively. The correlation that uses viscosity data was more successful than the correlation that uses density data. The success rate did not drop when the samples, which were not employed in developing the models, were used; this is a strong indicator of the success of the correlations. As a summary, the new correlations performed better than those available in the literature and other ML methods because of high generalization capacity of ELM.

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