PTZ Transformation Models for 2D Pixel Tracking

Abstract

This contains the mathematical formulation for the PTZ Transformations between two images taken at different pan values. The pan values are assumed to be rotations of the x-axis with no translations of any of the axes and zero rotations along the other two axes.

Keywords: PTZ cameras, transformations, computer vision

1. Introduction

This report contains a mathematical formulation for configuring the PTZ cameras. The objective is to take a pixel coordinate from the current image and map it to the camera's centre by rotating the camera along pan and tilt parameters. This will allow us to zoom in on a specific point in the image.

2. Basic Camera Model

The basic camera model which maps a 3D world coordinates to 2D pixel coordinates is described in the following equation:

$$x' = K(R|t)X \tag{1}$$

2.1. Estimating the real world coordinates

By taking the inverse, equation 1 can be rewritten as:

$$X' = (R|t)^{-1}K^{-1}x \tag{2}$$

Equation 2 can be used to estimate the real world coordinates using the pixel coordinates from an image.
Since this is a transformation from 2D to 3D, there will be some loss in data and hence an error.

3. Camera Parameters

3.1. Intrinsic Parameters

The following matrix defines the intrinsic parameters of the camera:

$$K = \begin{bmatrix} m_x f & 0 & p_x \\ 0 & m_y f & p_y \\ 0 & 0 & 1 \end{bmatrix} \tag{3}$$

Describe each parameter. (TBD)

3.2. Intrinsic Matrix Inverse

To find the determinant of K, we refer to the last row of the matrix in Equation 3, and their cofactors.

$$det(K) = 0 * \begin{vmatrix} 0 & p_x \\ m_y f & p_y \end{vmatrix} - 0 * \begin{vmatrix} m_x f & p_x \\ 0 & p_y \end{vmatrix} + 1 * \begin{vmatrix} m_x f & 0 \\ 0 & m_y f \end{vmatrix} \tag{4}$$

$$det(K) = m_x m_y f^2 \tag{5}$$

Next, we compute the adjoint of K by computing the cofactor of each element:

$$adj(K) = \begin{bmatrix} m_y f & 0 & 0 \\ 0 & m_x f & 0 \\ -p_x m_y f & -p_y m_x f & m_x m_y f^2 \end{bmatrix} \tag{6}$$

Using the determinant from equation 5 and inverse of K from equation 6 we can compute the inverse K^{-1}:

$$K^{-1} = \frac{1}{det(K)} adj(K) \tag{7}$$

$$K^{-1} = \frac{1}{m_x m_y f^2} \begin{bmatrix} m_y f & 0 & 0 \\ 0 & m_x f & 0 \\ -p_x m_y f & -p_y m_x f & m_x m_y f^2 \end{bmatrix} \tag{8}$$

$$K^{-1} = \begin{bmatrix} \frac{1}{m_x f} & 0 & 0 \\ 0 & \frac{1}{m_y f} & 0 \\ \frac{-p_x}{m_x f} & \frac{-p_y}{m_y f} & 1 \end{bmatrix} \tag{9}$$

3.3. Extrinsic Parameters

The extrinsic parameter matrix of a camera defined as $(R|t)$ in equation 1 includes rotation and translation along the three dimensions x,y and z.
The matrix can be defined as:

$$(R|t) = \begin{bmatrix} R_{3x3} & t_{3x1} \\ 0_{1x3} & 1 \end{bmatrix} \tag{10}$$

The matrix R in equation 10 is the product of the three rotation matrices along each axis.
The vector t in equation 10 is the translation along each axis.

Given our problem at hand, we are only rotating horizontally along one axis by modifying the pan parameter between 0.0 and 1.0. We assume this to be the x axis in our case. There is no translation along any of the axis in our case. This allows us to omit the translation vector. So, for our experiment the extrinsic parameter matrix can be written as:

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\theta_x & -sin\theta_x \\ 0 & sin\theta_x & cos\theta_x \end{bmatrix} \tag{11}$$

3.4. Extrinsic Matrix Inverse

The Rotation Matrix is an orthogonal matrix. This allows us to use the following property of orthogonal matrices:

$$R^{-1} = R^T \tag{12}$$

Using this property the inverse of R_x is given as:

$$R_x^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\theta_x & sin\theta_x \\ 0 & -sin\theta_x & cos\theta_x \end{bmatrix} \tag{13}$$

4. Pixel Coordinate Transformation along 1 Dimension

By replacing the estimated world coordinates X' from equation 2 into X in equation 1, we get the following equation:

$$x' = K(R_2|t)(R_1|t)^{-1}K^{-1}x \tag{14}$$

For our case, the equation can be rewritten as:

$$x' = KR_2R_1^{-1}K^{-1}x \tag{15}$$

R_1 is the rotation matrix which maps the world coordinates X onto the first image, and R_2 is the rotation matrix which maps the world coordinates onto the second image.

This transformation will allow us to map the pixel coordinates x from the first image to the pixel coordinates x' from the second image.

4.1. Rotation Matrix

The rotation matrix R is defined as:

$$R = R_2R_1^{-1} \tag{16}$$

The rotation about the x-axis is defined in equation 11 and the inverse is defined in 13. Therefore, we can rewrite R as:

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\theta_2 & -sin\theta_2 \\ 0 & sin\theta_2 & cos\theta_2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\theta_1 & sin\theta_1 \\ 0 & -sin\theta_1 & cos\theta_1 \end{bmatrix} \tag{17}$$

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\theta_1 cos\theta_2 + sin\theta_1 sin\theta_2 & sin\theta_1 cos\theta_2 - cos\theta_1 sin\theta_2 \\ 0 & cos\theta_1 sin\theta_2 - sin\theta_1 cos\theta_2 & sin\theta_1 sin\theta_2 + cos\theta_1 cos\theta_2 \end{bmatrix} \tag{18}$$

Using trignometric identities equation 18 can be rewritten as:

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos(\theta_1 - \theta_2) & sin(\theta_1 - \theta_2) \\ 0 & -sin(\theta_1 - \theta_2) & cos(\theta_1 - \theta_2) \end{bmatrix} \tag{19}$$

We define the angle of difference as:

$$\alpha = \theta_1 - \theta_2 \tag{20}$$

R can be written as:

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\alpha & sin\alpha \\ 0 & -sin\alpha & cos\alpha \end{bmatrix} \tag{21}$$

4.2. Transformation Matrix

Given equation 15, we can substitute K^{-1} from equation 9:

$$x' = K \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\alpha & sin\alpha \\ 0 & -sin\alpha & cos\alpha \end{bmatrix} \begin{bmatrix} \frac{1}{m_x f} & 0 & 0 \\ 0 & \frac{1}{m_y f} & 0 \\ \frac{-p_x}{m_x f} & \frac{-p_y}{m_y f} & 1 \end{bmatrix} x \tag{22}$$

$$x' = K \begin{bmatrix} \frac{1}{m_x f} & 0 & 0 \\ -\frac{p_x}{m_x f} sin\alpha & \frac{1}{m_y f} cos\alpha - \frac{p_y}{m_y f} sin\alpha & sin\alpha \\ -\frac{p_x}{m_x f} cos\alpha & -\frac{1}{m_y f} sin\alpha - \frac{p_y}{m_y f} cos\alpha & cos\alpha \end{bmatrix} x \tag{23}$$

Substitute K from equation 3 into the above matrix:

$$x' = \begin{bmatrix} m_x f & 0 & p_x \\ 0 & m_y f & p_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{m_x f} & 0 & 0 \\ -\frac{p_x}{m_x f} sin\alpha & \frac{1}{m_y f} cos\alpha - \frac{p_y}{m_y f} sin\alpha & sin\alpha \\ -\frac{p_x}{m_x f} cos\alpha & -\frac{1}{m_y f} sin\alpha - \frac{p_y}{m_y f} cos\alpha & cos\alpha \end{bmatrix} x \tag{24}$$

$$x' = \begin{bmatrix} 1 - \frac{p_x^2}{m_x f} c_\alpha & -\frac{p_x}{m_y f} s_\alpha - \frac{p_x p_y}{m_y f} c_\alpha & p_x c_\alpha \\ -\frac{m_y p_x}{m_x} s_\alpha - \frac{p_x p_y}{m_x f} c_\alpha & (1 - \frac{p_y^2}{m_y f}) c_\alpha - (p_y + \frac{p_y}{m_y f}) s_\alpha & m_y f s_\alpha + p_y c_\alpha \\ -\frac{p_x}{m_x f} c_\alpha & -\frac{1}{m_y f} s_\alpha - \frac{p_y}{m_y f} c_\alpha & c_\alpha \end{bmatrix} x \tag{25}$$

5. Pixel Coordinate Transformation along 2 Dimensions

For this case, equation 14 can be rewritten as:

$$x' = KR_{x2}R_{y2}R_{y1}^{-1}R_{x1}^{-1}K^{-1}x \tag{26}$$

R_{x1} and R_{y1} are the rotation matrices for pan and tilt respectively, which map the world coordinates X onto the first image. R_{x2} and R_{y2} are the rotation matrices for pan and tilt respectively, which maps the world coordinates onto the second image.

This transformation will allow us to map the pixel coordinates x from the first image to the pixel coordinates x' from the second image.

5.1. Rotation Matrix

The rotation matrix R is defined as:

$$R = R_{x2} R_y R_{x1}^{-1} \tag{27}$$

Over here R_y is defined as:

$$R_y = R_{y2} R_{y1}^{-1} \tag{28}$$

We define the angle of rotation about the x-axis(pan) as α and the y-axis(tilt) as β.

$$\alpha = \alpha_1 - \alpha_2 \tag{29}$$

$$\beta = \beta_1 - \beta_2 \tag{30}$$

R_{x1}, R_{x2}, R_{y1} and R_{y2} are defined by the angles $\alpha_1, \alpha_2, \beta_1$ and β_2 respectively.

The rotation matrix R_y can be written as:

$$R_y = \begin{bmatrix} cos\beta & 0 & -sin\beta \\ 0 & 1 & 0 \\ sin\beta & 0 & cos\beta \end{bmatrix} \tag{31}$$

The rotation about the two dimensions is defined in equation 27. Therefore, we can rewrite R as:

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\alpha_2 & -sin\alpha_2 \\ 0 & sin\alpha_2 & cos\alpha_2 \end{bmatrix} \begin{bmatrix} cos\beta & 0 & -sin\beta \\ 0 & 1 & 0 \\ sin\beta & 0 & cos\beta \end{bmatrix} R_{x1}^{-1} \tag{32}$$

$$R = \begin{bmatrix} cos\beta & 0 & -sin\beta \\ -sin\alpha_2 sin\beta & cos\alpha_2 & -sin\alpha_2 cos\beta \\ cos\alpha_2 sin\beta & sin\alpha_2 & cos\alpha_2 cos\beta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\alpha_1 & sin\alpha_1 \\ 0 & -sin\alpha_1 & cos\alpha_1 \end{bmatrix} \tag{33}$$

$$R = \begin{bmatrix} c_\beta & s_{\alpha_1} s_\beta & -c_{\alpha_1} s_\beta \\ -s_{\alpha_2} s_\beta & c_{\alpha_1} c_{\alpha_2} + s_{\alpha_1} s_{\alpha_2} c_\beta & s_{\alpha_1} c_{\alpha_2} - c_{\alpha_1} s_{\alpha_2} c_\beta \\ c_{\alpha_2} s_\beta & c_{\alpha_1} s_{\alpha_2} - s_{\alpha_1} c_{\alpha_2} c_\beta & s_{\alpha_1} s_{\alpha_2} + c_{\alpha_1} c_{\alpha_2} c_\beta \end{bmatrix} \tag{34}$$

We can assume that the initial angle α_1 is 0. Referring to equation 29, we can derive the following relation:

$$\alpha = -\alpha_2 \tag{35}$$

Substituting this in the matrix given in equation 34, we can write R as:

$$R = \begin{bmatrix} cos\beta & 0 & -sin\beta \\ sin\alpha sin\beta & cos\alpha & sin\alpha cos\beta \\ cos\alpha sin\beta & -sin\alpha & cos\alpha cos\beta \end{bmatrix} \tag{36}$$

5.2. Transformation Matrix

Given equation 26, we can substitute R from equation 27:

$$x' = KRK^{-1}x \tag{37}$$

Substituting K from 3, we can rewrite the transformation as:

$$KRK^{-1} = \begin{bmatrix} m_x f & 0 & p_x \\ 0 & m_y f & p_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos\beta & 0 & -sin\beta \\ sin\alpha sin\beta & cos\alpha & sin\alpha cos\beta \\ cos\alpha sin\beta & -sin\alpha & cos\alpha cos\beta \end{bmatrix} K^{-1} \tag{38}$$

$$\begin{bmatrix} m_x f c_\beta + p_x c_\alpha s_\beta & -p_x s_\alpha & -m_x f s_\beta + p_x c_\alpha c_\beta \\ m_y f s_\alpha s_\beta + p_y c_\alpha s_\beta & m_y f c_\alpha - p_y s_\alpha & m_y f s_\alpha c_\beta + p_y c_\alpha c_\beta \\ c_\alpha s_\beta & -s_\alpha & c_\alpha c_\beta \end{bmatrix} K^{-1} \tag{39}$$

Substituting K^{-1} from equation 9, equation 37 can be rewritten as:

$$x' = \begin{bmatrix} c_\beta(1 - \frac{p_x^2}{m_x f} c_\alpha) - s_\beta(\frac{p_x}{m_x f} c_\alpha + p_x) & -\frac{p_x}{m_y f} s_\alpha + \frac{m_x p_y}{m_y} s_\beta - \frac{p_x p_y}{m_y f} c_\alpha c_\beta & -m_x f s_\beta + p_x c_\alpha c_\beta \\ s_\beta(\frac{m_y}{m_x} s_\alpha + \frac{p_y}{m_x f} c_\alpha) - c_\beta(\frac{m_y p_x}{m_x} s_\alpha + \frac{p_x p_y}{m_x f} c_\alpha) & c_\alpha(1 - \frac{p_y^2}{m_y f} c_\beta) - s_\alpha(\frac{p_y}{m_y f} + p_y c_\beta) & c_\beta(m_y f s_\alpha + p_y c_\alpha) \\ c_\alpha(\frac{1}{m_x f} s_\beta - \frac{p_x}{m_x f} c_\beta) & -\frac{1}{m_y f} s_\alpha - \frac{p_y}{m_y f} c_\alpha c_\beta & c_\beta c_\alpha \end{bmatrix} x \tag{40}$$

5.3. Principal Point Approximation

We can substitute the principal points p_x and p_y as 0 in equation 40. Our transformation is reduced to the following:

$$x' = \begin{bmatrix} c_\beta & 0 & -m_x f s_\beta \\ s_\beta(\frac{m_y}{m_x} s_\alpha) & c_\alpha & c_\beta(m_y f s_\alpha) \\ c_\alpha(\frac{1}{m_x f} s_\beta) & -\frac{1}{m_y f} s_\alpha & c_\beta c_\alpha \end{bmatrix} x \tag{41}$$

6. Summary and conclusions

We have discussed two cases in the above methodology.

The first one restricts the PTZ camera's movement to rotation along the x-axis. This is the pan parameter. We have derived the transformation matrix for this particular approach in equation 25.

The second one extends this idea to 2D rotation along the x and y axes. This includes our pan and tilt parameters. We have derived the transformation matrix for this particular approach in equation 41.

Using the transformation matrices, we have to find the unknown angles of rotation when the coordinates x and x' are provided.