

Laws

- 0: the definition of temperature, objects in equilibrium have the same temperature.
- 1: Energy is conserved. energy can transfer from one type to another, that transfer has direction.
- 2: heat moves from hot to cold. energy has a "grade" (useful, less useful), systems tend towards disorder.
- 3: temperature has a lower limit. it is 0 K or -273.15°C .

Systems & States

Systems: in thermodynamics, we want to study specific objects or regions of space. we call these "systems", and there are a few types. systems are defined by their type and their boundary.

- ↳ Open: both mass and energy may cross the system boundary.
- ↳ Closed: only energy may cross the system boundary.
- ↳ Isolated: nothing can cross the system boundary.

States: states describe the properties of materials (gas or fluid) inside our system. the state of a system is characterized by the properties temperature, pressure, and volume, although there are many other properties that can describe a state.
The Stat Postulate says that if we know two of the (intensive) properties, we can find the others.

Properties

Properties describe the state of materials in our system. they tell us about the energy of our system and the space it takes up. there are two types of properties:

Intensive: intensive properties describe the intrinsic aspects of a material.

diamond is hard, and copper is conductive - regardless of how much you have of them. intensive properties do NOT depend on the size of the system.

- ↳ Specific heat, pressure, temperature, thermal conductivity, hardness, viscosity, density, resistivity, Young's modulus, etc

Extensive: extensive properties are actual quantities, like mass. they depend on the size (often mass) of a system.

- ↳ Mass, Resistance (NOT resistivity), Volume, energy

Specific: a type of intensive property. More specifically, they are extensive properties turned into intensive ones by dividing by mass (sort of).

they tell you how much of "something" (like energy or volume) you have, given an amount of mass.

$$v = \text{Specific Volume} = \frac{V}{m} \quad (\text{NOTE: } \frac{1}{v} = \rho, \text{ so if you know specific volume, you know density})$$

↳ exceptions: "Specific gravity" is actually how many times more dense something is than water at 4°C ($SG = \frac{\rho}{\rho_{\text{H}_2\text{O}}}$)

"Specific heat"... See later section

"Specific weight" is like density but with weight instead of mass

Temp & Pressure

Temperature is a measure of the kinetic energy in particles, although we treat it like its own kind of energy.

$$\hookrightarrow T(\text{K}) = T(^{\circ}\text{C}) + 273.15, \quad T(\text{R}) = T(^{\circ}\text{F}) + 459.67, \quad T(\text{R}) = 1.8 T(\text{K}), \quad T(^{\circ}\text{F}) = 1.8 T(^{\circ}\text{C}) + 32$$

Pressure is a measure of how much particles bump into (push against) container walls.

$$\hookrightarrow \text{Metric: } 1 \text{ Pa} = 1 \frac{\text{N}}{\text{m}^2}, \quad 1 \text{ bar} = 10^5 \text{ Pa} = 100 \text{ kPa} = 0.1 \text{ MPa}, \quad 1 \text{ atm} = 760 \text{ Torr} = 101.325 \text{ kPa} = 1.01325 \text{ bar}$$

$$\hookrightarrow \text{English: } 1 \text{ Psi} = 1 \text{ lbf/in}^2, \quad 1 \text{ atm} = 14.696 \text{ Psi}$$

Pressure Measuring Devices

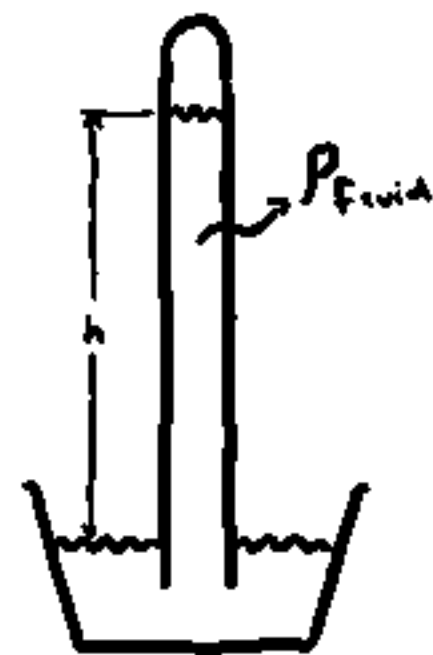
Gauges: gauges tell you the pressure relative to the surroundings. For absolute pressure, use these equations:

$$\left. \begin{aligned} P_{\text{abs}} &= P_{\text{gauge}} + P_{\text{atm}}, & P_{\text{abs}} &= P_{\text{atm}} - P_{\text{vac}} \end{aligned} \right\} \times \text{Note that Vacuum gauges measure how much LESS pressurized}$$

Barometers: measure atmospheric pressure.

$$P_{\text{atm}} = \rho \cdot g \cdot h, \text{ where } \rho \text{ is the density of the fluid (usually } 13.5 \text{ g/cm}^3 \text{ or } 13,500 \text{ kg/m}^3 \text{ for mercury)}$$

g is the acceleration due to gravity, and h is the height of fluid (mercury) in the vessel.
if you use kg/m^3 for ρ , m/s^2 for g , and m for h , your answer is in Pascals.

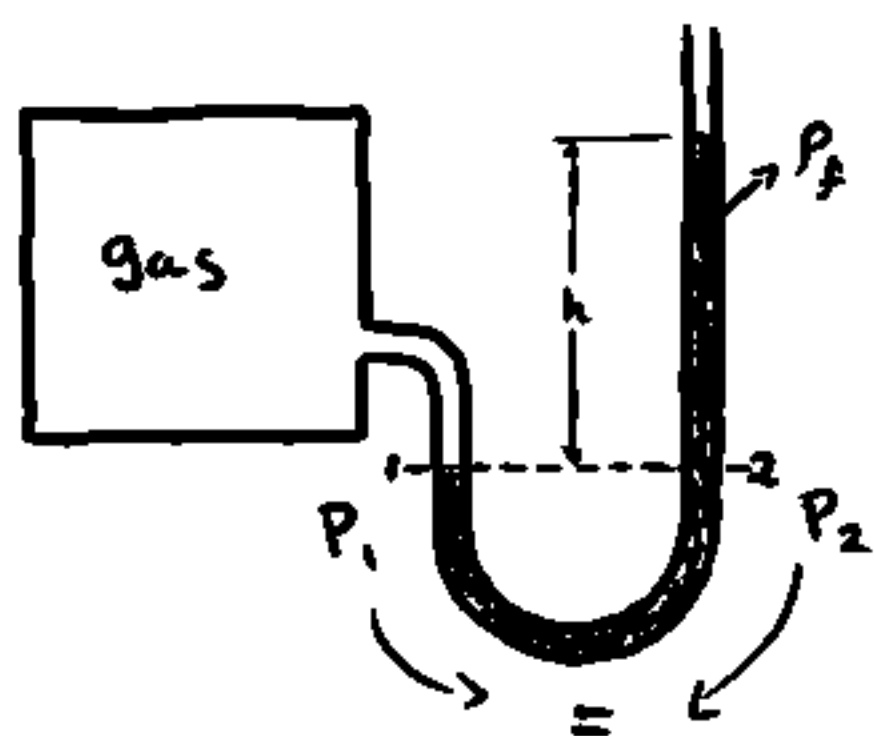


If you're asked to measure the height of something with one of these:

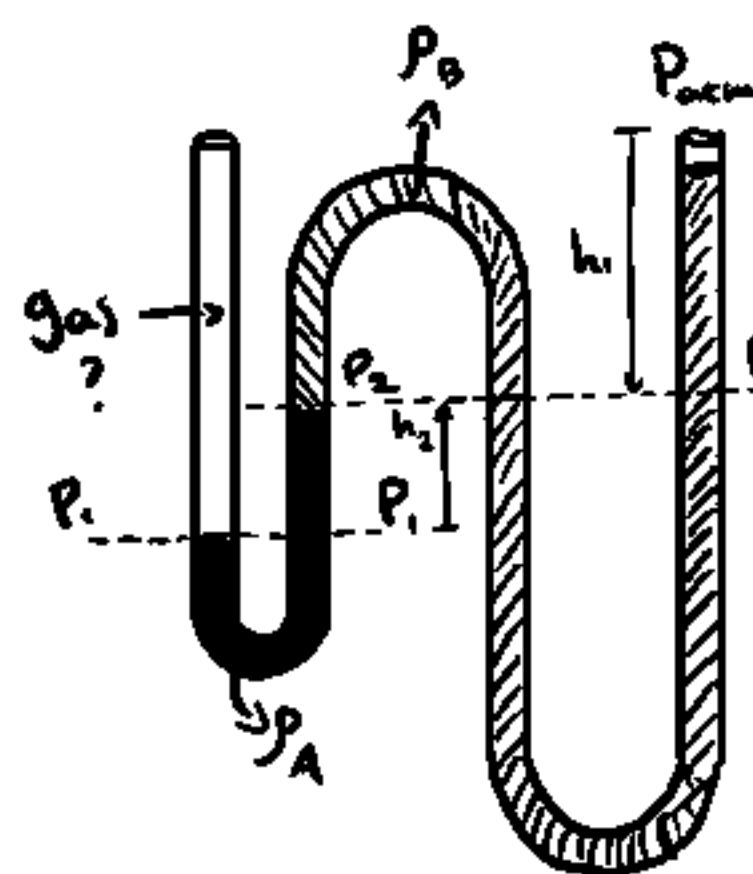
$$\text{height} = \frac{\Delta P_{\text{atm}}}{\rho_{\text{air}} \cdot g}; \quad \Delta P_{\text{atm}} = (\rho_{\text{Hg}} \cdot g \cdot h_{\text{Hg}})_{\text{top}} - (\rho_{\text{Hg}} \cdot g \cdot h_{\text{Hg}})_{\text{bottom}}$$

Manometers: These measure GAUGE PRESSURE, not absolute or atmospheric.

$$P_{\text{gas(gauge)}} = \rho_f \cdot g \cdot h, \text{ if you use } \text{kg/m}^3 \text{ for } \rho, \text{ m/s}^2 \text{ for } g, \text{ and m for } h, \text{ your answer is in Pascals.}$$



Problems involving these often have more complex tubes with multiple fluids. The main idea is that the pressure in one fluid is the same at the same height.



$$\begin{aligned} P_2 &= P_{\text{atm}} + \rho_B g h_1 \\ P_1 &= P_2 + \rho_A g h_2 \\ P_{\text{gas}} &= P_1 \end{aligned}$$

First Law ++

the first law tells us that energy is conserved, however, it can change from one type to another. the obvious types are kinetic and potential. if a given problem feels like a physics problem, it's probably about these. otherwise, it's probably about heat or work.

Heat: heat energy is NOT temperature! it doesn't even necessarily raise the temperature of a system! i like to think of heat as "pure" energy. don't feel bad if the concept of heat isn't intuitive to you. it's actually quite unintuitive. heat can escape a system, like a box of leftovers going cold, or it can enter a system, like ice cream melting in a poorly insulated cooler. both of those examples describe a change in temperature. But remember, while heat might change the temperature of something, it doesn't have to. adding more heat to boiling water doesn't make it boil hotter. Instead, that heat energy helps change the phase of the water.

Work: Work can be a lot of different things. Work can be pushing a boulder up a hill every day for the rest of your life. it can also be compressing a piston or using electricity to heat something up. But, while you can push a boulder and compress a cylinder, that boulder can push you back down the hill, and that piston can push back on you. that is where the whole W_{in} or W_{out} thing comes from. anything that has energy can do work — and when it does, it loses some of that energy.

First Law +++

For this exam, only heat and work can change the energy of a system. The energy stored in a system, sort of like a battery, is expressed by E . We can't write $E = \dots$ something (unless you're Einstein) because in our context, we only consider changes of energy. Here comes the most confusing equation of your life:

$$\Delta E = Q_{in} - W_{out}$$

↓ ↓ ↓
gain The The
in heat work
energy you that
 add ...
 comes out?

i don't know whose idea it was to adopt that stupid sign convention.
this is what i do:

$$\Delta E = \sum Q + \sum W \quad \left. \vphantom{\sum Q + \sum W} \right\} \text{don't think about the signs of "ins" or "outs" ... Yet.}$$

if a problem says you add heat, that Q is positive. if heat "escapes", it's negative.
if it says you compress a piston, you're physically doing work to it, so that W is positive.
if a piston "expands", it's doing work, not you, so that one is negative.

$$\Delta E = -Q_{escapes} + Q_{added} + W_{on\ piston} \dots \text{etc}$$

This enables us to find the change in energy of a system, or, a particular value, like how much heat energy escapes.

We can also relate ΔE to ΔU or ΔH (more on that later).

Processes

a process is a change in state of a system. for example, if i increase the temperature of a gas while holding its pressure constant, that is a process. The state of the gas changed. Certain special types of processes have names (usually when a property is held constant).

isobaric: Pressure remains the same, $P_1 = P_2$
↓ ↓
same pressure

isochoric: Volume remains the same, $V_1 = V_2$
↓ ↓
same volume

isothermal: Temperature remains the same, $T_1 = T_2$
↓ ↓
same temperature

adiabatic: No heat in or out, $Q_{in} = Q_{out} = 0$
↓ ↓ ↓
not Passable
 through

Work ++

there are lots of types of Work, so it needs its own section, i'm not including the intuitive ones, like $W = \Delta E_p = mg\Delta h$ or $W = \Delta E_k = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_o^2$. Well i suppose i just did...

Boundary Work: When the boundary of a system changes, resulting in a transfer of energy. for example, a piston is compressed or expands. generally, $W_b = \int_1^2 P(V) dV$

Constant Pressure (isobaric)... Ideal gas only

$$W_b = P\Delta V$$

...if all of the work goes to raising T , $W_b = nR\Delta T$ (more later on)

Constant Temperature (isothermal)... Ideal gas only

$$W_b = P_1 V_1 \ln\left(\frac{V_2}{V_1}\right) \quad V \text{ is Volume. I'm pretty sure both big } V \text{ and little } v \text{ should work, but if you use little } v, W_b \text{ will be Work-Per-Unit-Mass.}$$

Electric Work: Work done by electricity (usually a resistor that heats up)

$$W_{elec} = P_{elec} \cdot t = I \cdot V \cdot t = I^2 R \cdot t = \frac{V^2}{R} \cdot t, \text{ any Work. } I = \text{Current, } V = \text{Volts, } R = \text{resistance.}$$

Shaft Work (lol): for Paddles/rotating things.

$$W_{sh} = 2\pi n T, \text{ for } n = \frac{\text{Num rotations}}{\text{time}}, T = \text{torque} = F \cdot r$$

Spring Work: for springs

$$W_s = \frac{1}{2}K(x_2^2 - x_1^2), \text{ for } K = \text{Spring const, } x_2 = \text{final dist from equilibrium, } x_1 = \text{initial dist from equilibrium}$$

Elastic Work: for solid bars. The following equation was not provided in class, but it should work.

$$W = \frac{1}{2} \sigma_u \cdot E \cdot V = \frac{1}{2} E E E V = \frac{1}{2} E V E^2, \text{ for } E = \text{Youngs modulus, } V = \text{Volume, } \epsilon = \frac{\Delta L}{L_o}$$

$$\text{alternatively, compute this integral: } W_{elastic} = \int_{x_1}^{x_2} \sigma_u A dx$$

Work to stretch a thin film: You probably won't need this one, but just in case...

$$W_{film} = \int_1^2 \sigma_s dA \quad (\text{i don't even know what } \sigma_s \text{ is})$$

Road Work: When Utah spends our taxes on the slowest construction workers they can find

Just kidding

* Spontaneous expansion to vacuum: When gas expands into a vacuum, no work is done.

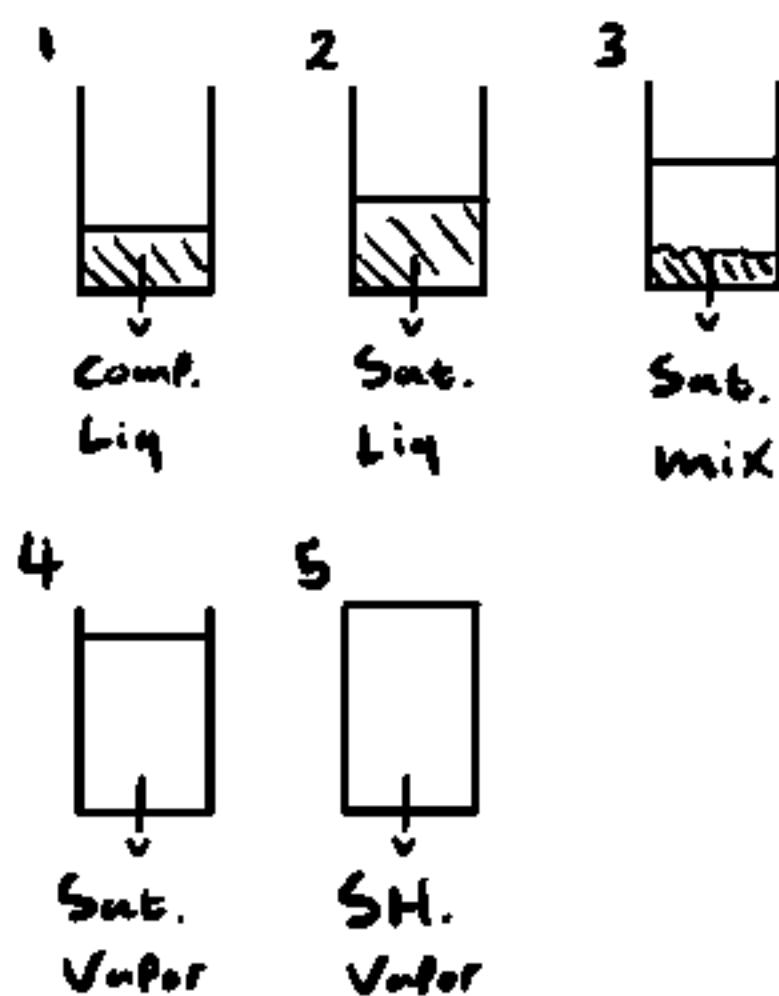
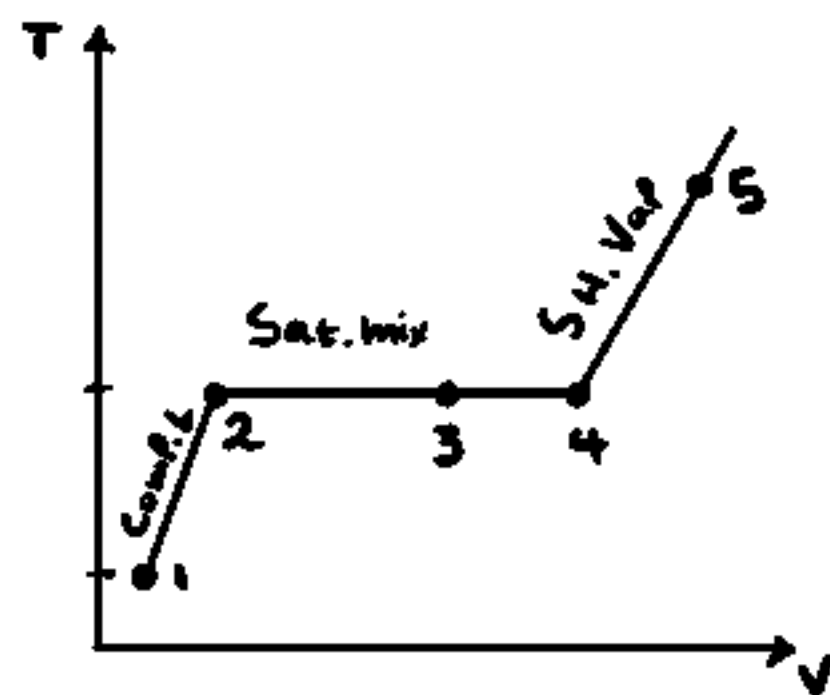
Phases + Diagrams

We will be considering Pure Substances (one type of molecule). Prerequisite Knowledge: What liquid and gas is. first, some terminology:

- ↳ Compressed Liquid: it's literally just a liquid. Usually under certain conditions (like high pressure) that make it so. → and low temp
- ↳ Saturated (liq or gas or mix): a liquid or gas that's on the edge of changing phases.
- ↳ Superheated Vapor: it's literally just a vapor. Usually under certain conditions (like low pressure) that make it so. → and high temp

T-V diagrams (for constant pressure processes)

these diagrams show the changes of phase through a const. pressure heating process.



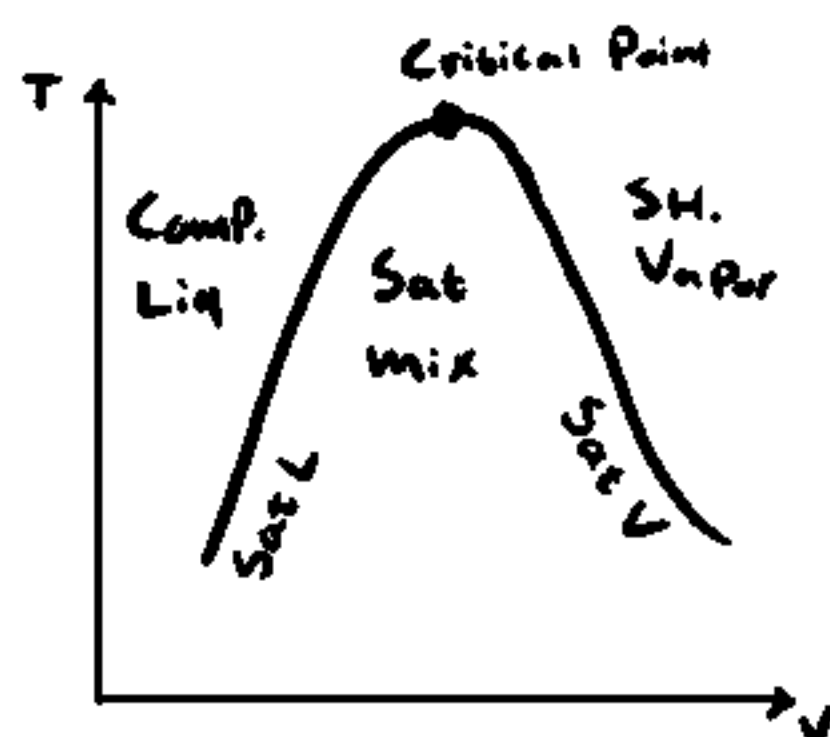
"1→2" shows temperature of a liquid increasing until it's ready to boil. during "1→2", the liquid expands a bit.

at "2", it's saturated, and ready to vaporize.

from "2→4", all of the heat added to the system goes into making the liquid change into vapor, so no temp increase occurs. but, the volume increases, since vapor has a higher specific volume than liquid.

between "2→4", there is a mix of liquid and vapor, both on the edge of changing phases (saturated).

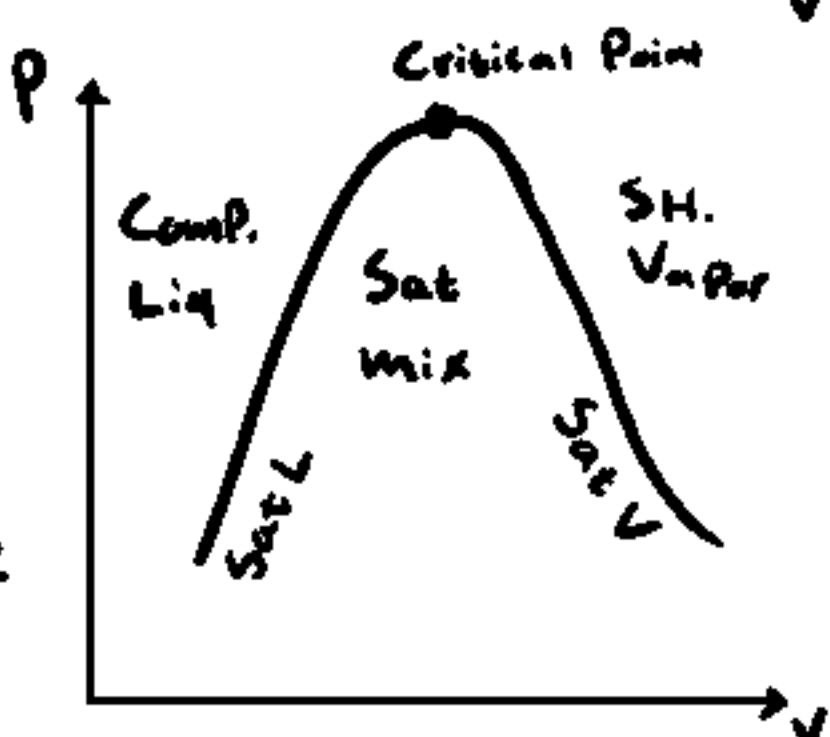
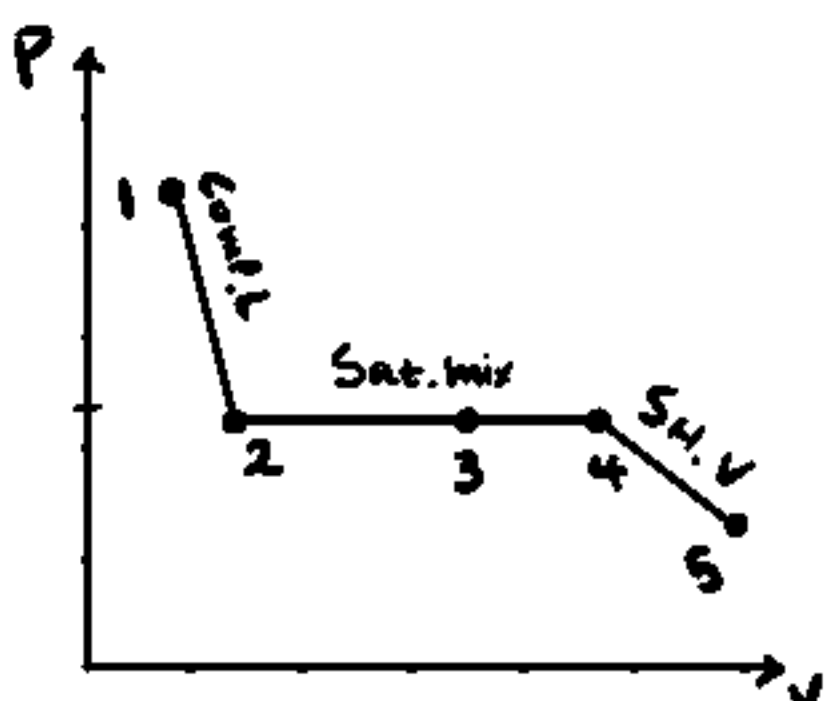
at "4", all of the liquid is now vapor. Now, any heat added raises the temperature of that vapor and lets it expand.



This curve is the saturation curve. It just shows the regions where certain phases occur.

The critical point is where you have saturated liquid and vapor at the same time. Not like a mixture — it really is both at once for the entire sample.

P-V diagrams (for constant temp processes)



The same as the T-V curve

These diagrams are very similar to T-V ones.

"1→2" shows a liquid's pressure reducing and volume increasing until it's ready to boil. at "2", the liquid is entirely on the edge of boiling.

"2→4" shows it slowly vaporizing. between 2 and 4, it is a mixture of liquid and vapor, both of which are saturated.

at "4", it's entirely a saturated vapor. from "4→5", its pressure slowly decreases, pushing it into the "superheated vapor" region. unintuitively, it's "superheated state" is characterized by its low pressure, rather than a high temperature, an intuitive way to understand it is that it's above its saturation temperature at that pressure.

Property Table

When you know at least two things about the state of your sample, you can figure the rest out. The Property table is one way of doing this.

First, a brief appendix:

	A1 → Molar masses, gas constant
	A2 → ideal gas stuff
	A3 → Specific heats of Solids and Fluids
H ₂ O	A4 → Saturated H ₂ O (given Temp)
	A5 → Saturated H ₂ O (given Pressure)
	A6 → Superheated H ₂ O
	A7 → Compressed Liq. H ₂ O
R134a	A11 → Saturated R134-a (given Temp)
	A12 → Saturated R134-a (given Pressure)
	A13 → Superheated R134-a
	A17 → ideal gas Properties of air

The table you need to use to find unknown Properties depends on the state of your sample.

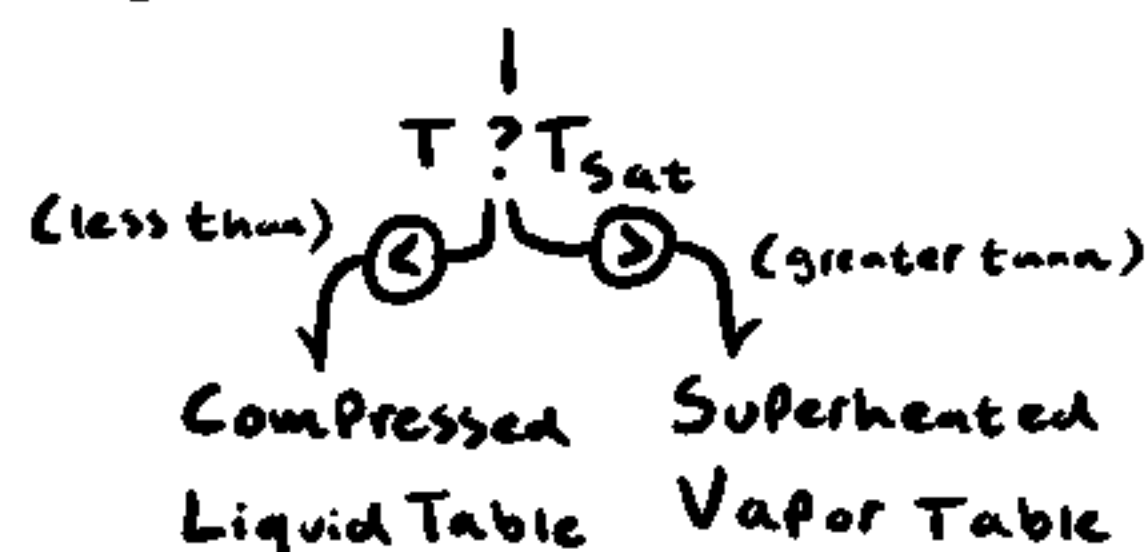
If you're told the sample is Saturated, use the Saturated table. Easy enough.

Otherwise, use The flowchart below. ☺

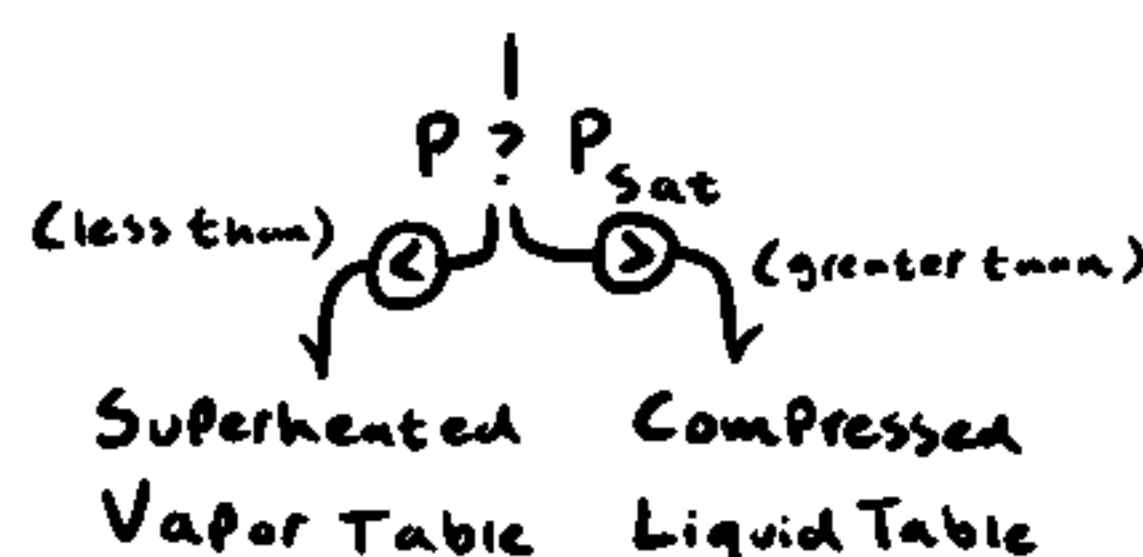
"I have no idea what table to use" flowchart

Δ told it's Saturated → Saturated Table

Δ Given Pressure first



Δ Given temperature first



Δ Given Some Specific Property (lets call it b to represent any)

$b < b_{\text{fluid}} \rightarrow$ Compressed Liquid Table

$b_{\text{fluid}} < b < b_{\text{gas}} \rightarrow$ Saturated Table... BUT, it's a mixture.

$b > b_{\text{gas}} \rightarrow$ Superheated Vapor Table

$b = b_{\text{fluid}} \text{ or } b_{\text{gas}} \rightarrow$ Saturated Table - Pure, not mixture.

I know this works if b is v (specific volume), but I'm not 100% sure it does if b is anything else. Only 98% sure.

* if the compressed liquid table doesn't have the information you need, use the Saturated table (fluid values) as an approximation.

How to Calculate Properties →

Property Calculation (Tables)

Linear interpolation: Since the table may not have your exact given properties, we find a middle value that works pretty well.

Consider properties "a" and "b" — you know an exact value for "a", and want the corresponding exact value for "b":

$$\frac{b_{\text{exact}} - b_{\text{lower}}}{b_{\text{upper}} - b_{\text{lower}}} = \frac{a_{\text{exact}} - a_{\text{lower}}}{a_{\text{upper}} - a_{\text{lower}}}$$

$$b_{\text{exact}} = \left(\frac{a_{\text{exact}} - a_{\text{lower}}}{a_{\text{upper}} - a_{\text{lower}}} \right) (b_{\text{upper}} - b_{\text{lower}}) + b_{\text{lower}}$$

Saturated Mixtures: When you have both saturated liquid and saturated vapor, their properties are blended.

$$\text{let } x = \frac{m_{\text{gas}}}{m_{\text{gas}} + m_{\text{liquid}}}, \text{ let } b_{fg} = b_g - b_f$$

$$b_{\text{mix}} = b_f + x \cdot b_{fg} \text{ for } b \text{ is any intensive property}$$

* it's just a weighted average...

WTF is Enthalpy?

in thermodynamics, the quantity " $U + PV$ " shows up often, so we gave it a name.

first, let's look at U : U is internal energy. I'm pretty sure it's just the energy stored as temperature, for us. its units are kJ , or kJ/m if it's specific.

So, since U has units of energy, $P \cdot V$ should too, and it does: $\frac{\text{kJ}}{\text{m}^2} \cdot \text{m}^3 = \text{kJ} \cdot \text{m} = \text{kJ}$
but what does that mean, physically? basically, nothing. it's sort of like $E_p = mgh$.
without comparing it to a reference, it means very little.

One way to interpret it is the " PV " quantity is the energy it takes to take up physical space.
Since U is internal energy,

$$H = U + PV = \text{the energy it requires to exist}.$$

Ideal Gasses

Gas molecules sometimes interact with each other, which complicates the math.
So, we pretend they don't. This is best at low pressures or high temperatures.

different gasses behave differently, so we introduce R , the gas constant.

$$R = \frac{R_u}{M}, \text{ for } R_u = 8.31 \frac{\text{J}}{\text{mol} \cdot \text{K}} \text{ and } M = \text{molar mass of gas} \left. \vphantom{R = \frac{R_u}{M}} \right\} R_u \text{ is the universal gas constant}$$

$\hookrightarrow = 1.99 \frac{\text{Btu}}{\text{lbmol} \cdot \text{R}}$

R can be found in table A1. Ideal Gas Properties of Air in A17.

Ideal gas eqn \rightarrow

Ideal Gas Equation

We use it to find unknown Properties — either directly or after a Process.
in this course, it has a few forms. The two you need are below (✓)

$$PV = mRT \quad \checkmark$$

$\hookrightarrow \div m \rightarrow P \frac{V}{m} = \frac{R}{m} T \rightarrow P v = R_u T \quad \checkmark$

\nearrow Specific Volume
 \hookrightarrow universal gas constant (See Prev Page)

Theres a Cool and Very Useful trick. if You are dealing with a Change of State,
You can move all the constants to one side.

$$\frac{PV}{T} = mR = \text{Constant, so: } \downarrow$$

if mass doesn't change (it probably doesn't),

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \quad \rightarrow V \text{ can be specific or normal, just use whichever you have}$$

now, if temperature doesn't change ($T_1 = T_2$, isothermal), it cancels out:

$$P_1 V_1 = P_2 V_2$$

Similarly, if Volume doesn't change ($V_1 = V_2$, isochoric)

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

finally, if Pressure doesn't change ($P_1 = P_2$, isobaric)

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

just use whichever you need, depending on the process!

~ Measurement of how ideal a gas is

"Z" is a measure of this. it's called the Compressibility factor.

$$Z = \frac{V_{\text{actual}}}{V_{\text{ideal}}} = \frac{P V_{\text{actual}}}{RT}$$

~ Correcting the ideal gas approximation — Van Der Waals eqn of State

$$\left(\underbrace{P + \frac{a}{v^2}}_{P_{\text{adjusted}}} \right) \left(\underbrace{v - b}_{V_{\text{adjusted}}} \right) = RT$$

\rightarrow i think this is supposed to be R_u , but not according to lecture notes

for

$$a = \frac{27 R^2 T_{cr}^2}{64 P_{cr}} \quad \begin{matrix} \nearrow \text{Table A1} \\ \nearrow \text{(right side)} \end{matrix}$$
$$b = \frac{R T_{cr}}{8 P_{cr}} \quad \begin{matrix} \nearrow * \text{i have no idea} \\ \nearrow \text{which } R \text{ it's} \\ \nearrow \text{supposed to be,} \end{matrix}$$

Lecture notes implies NOT R_u

Specific Heat for Ideal Gases

Some materials require more energy to increase their temperature.
The constant "c" accounts for this. However, c changes with both T and P.

Under the assumption of an ideal gas, though, c only changes with T.
Table A2 will tell it to you. BUT... there are two of them..?

C_v - We use this one when volume doesn't change in a process.

C_p - We use this one when pressure doesn't change in a process.

* Note: $C_p = C_v + R$
↳ for specific gas (A1)

Which temperature do you look for in the table?

Option 1: find the average of T_1 and T_2 , $T_{ave} = \left(\frac{T_1 + T_2}{2}\right)$
↳ Then find c at T_{ave}

Option 2: find c at T_1 , and c at T_2 , then average them

Changes in internal energy and enthalpy

$$\Delta E = mC\Delta T \text{ or } \Delta e = c\Delta T$$

Remember to use the correct c,
 C_v for const volume and C_p for const pressure!

Const V: $mC_v\Delta T = \Delta U$ or $C_v\Delta T = \Delta u$ * Change in internal NRU

Const P: $mC_p\Delta T = \Delta H$ or $C_p\Delta T = \Delta h$ * Change in enthalpy

Specific Heat for Solids and (incompressible) Fluids

For solids and incompressible fluids, constant temp or pressure doesn't matter.
For any process, $C_p = C_v = c$. (c is still averaged over the temperature range)
↳ table A3

For change in internal energy,

$$\Delta U = c\Delta T \text{ or } \Delta U = mC\Delta T$$

For change in enthalpy, it depends on if it's a fluid or solid, and it depends on the process.

Solid: $\Delta h = \Delta u$ or $\Delta H = \Delta U$

Liquid → Const. P: $\Delta h = \Delta u$ or $\Delta H = \Delta U$

↳ Const. T: $\Delta h = v\Delta P$ or $\Delta H = V\Delta P$

} These come from $\Delta h = \Delta u + (\Delta P)v + v(\Delta P)$
where some terms are zero in each case

* Correction equation: $h@P,T \approx h_{f@T} + v_{f@T}(P - P_{sat@T})$

Mass Conservation in open Systems

in open systems, mass can flow in and out of the control volume. Since mass is conserved,

$$\left. \begin{array}{c} \Sigma \dot{m}_{in} \rightarrow \text{CV} \rightarrow \Sigma \dot{m}_{out} \end{array} \right\} \Sigma \dot{m}_{in} = \Sigma \dot{m}_{out}, \text{ so } \Sigma \dot{m}_{in} = \Sigma \dot{m}_{out}$$

We can rewrite mass:

$$\left. \begin{array}{l} m = \rho V \quad \begin{array}{l} \text{Volume} \\ \text{density} \end{array} \\ \dot{m} = \rho \dot{V} = \rho V A \quad \begin{array}{l} \text{Velocity} \\ \text{Cross sectional area of flow} \\ \text{Volumetric flow rate} \\ \text{density} \end{array} \end{array} \right\} \Sigma (\rho V A)_{in} = \Sigma (\rho V A)_{out}$$

Special Case: incompressible steady flow (ρ const)

$$\Sigma (VA)_{in} = \Sigma (VA)_{out} \therefore \Sigma (\dot{V})_{in} = \Sigma (\dot{V})_{out}$$

energy in a flowing fluid

$$\theta = \overbrace{Pv}^h + u + \frac{v^2}{2} + gz$$

↳ multiply by \dot{m} for rate of energy flowing in/out of CV

First law for open systems

conservation of energy applied to a CV: energy can change in a CV by means of heat, work, and flowing fluids

$$\dot{E}_{cv,2} - \dot{E}_{cv,1} = \Delta \dot{E}_{cv} = \dot{Q}_{in} - \dot{W}_{out} + \left[\Sigma \dot{m}_{in} \theta_{in} - \Sigma \dot{m}_{out} \theta_{out} \right]$$

When neglecting KE and PE,

$$\dot{E}_{cv,2} - \dot{E}_{cv,1} = \Delta \dot{E}_{cv} = \dot{Q}_{in} - \dot{W}_{out} + \left[\Sigma \dot{m}_{in} h_{in} - \Sigma \dot{m}_{out} h_{out} \right]$$

Note: \dot{m} can also be just m , or the whole equation can be written per-unit-mass:
 $\Delta e = q_{in} - w_{out} + \Delta h$

Steady State: energy in CV remains constant with time

$$0 = \dot{Q}_{in} - \dot{W}_{out} + \left[\Sigma \dot{m}_{in} h_{in} - \Sigma \dot{m}_{out} h_{out} \right]$$

usually, just use

$$0 = Q - W + m(h_2 - h_1) \text{ or } 0 = \dot{Q} - \dot{W} + \dot{m}(h_2 - h_1) \text{ or } 0 = q - w + \Delta h$$

* Note: When the fluid is not flowing, Δh is just Δu , which gives us our old familiar first law equation

FINDING H (extra relevant for next page):

① Property table - MUST change h into J instead of kJ if v is in $\frac{m}{s}$

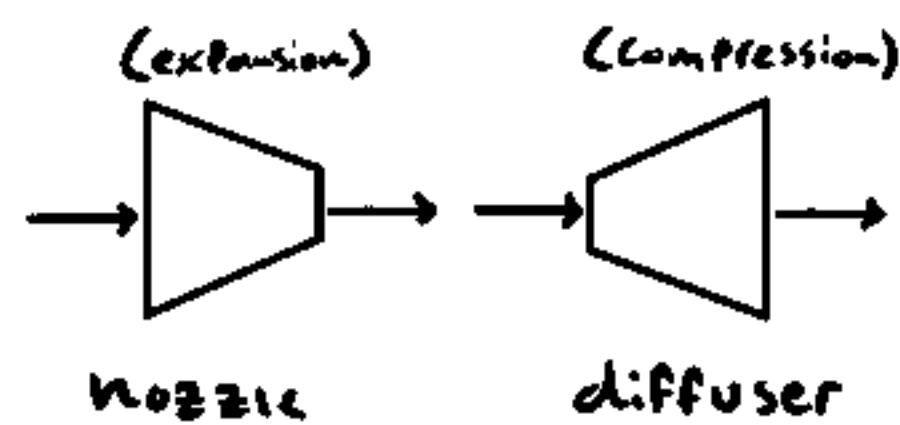
② Ideal gas, const C_p ONLY: $\Delta h = C_p \Delta T$, eqn above becomes $C_p \Delta T = \frac{v_i^2}{2} - \frac{v_e^2}{2}$

③ Ideal gas, const C_p ONLY: $h = C_p T$ * hypothetically not allowed, but I do it all the time and it works fine

Steady flow Systems

These machines take in a fluid or gas and output the fluid or gas after altering its state. You can think of it like a state function.

Nozzles and Diffusers



nozzles speed up fluids, decreasing h , T , and P^* .

diffusers slow down fluids, increasing h , T , and P^* .

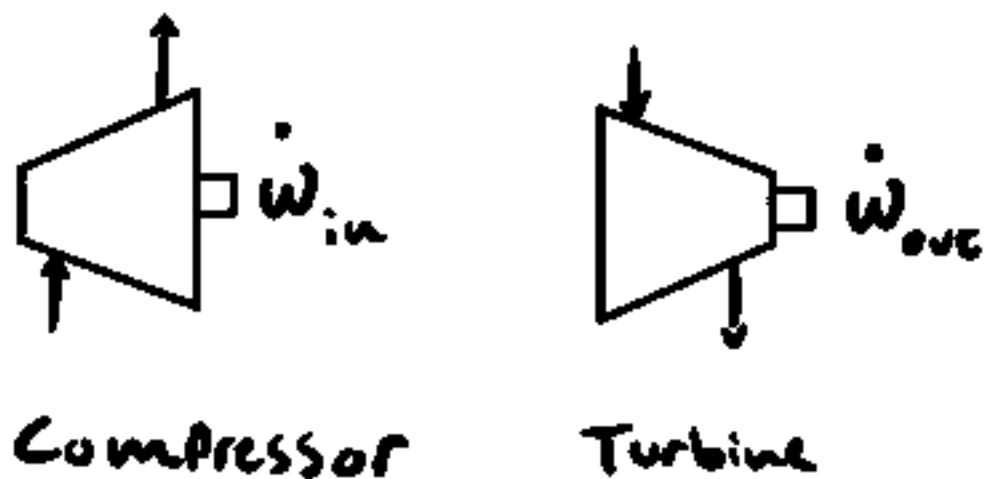
* Counter-intuitive, I know.

We usually neglect heat (adiabatic) and nozzles don't do work, so:

$$h_i + \frac{V_i^2}{2} = h_e + \frac{V_e^2}{2} \quad \text{* This comes from the general eqn on the previous page, which covers all edge cases if they arise.}$$

What happens is enthalpy is exchanged for kinetic energy.

Turbines and Compressors



compressors use an input power (\dot{W}_{in}) to increase the enthalpy of its input substance.

Turbines harness the enthalpy of its input substance to generate power (\dot{W}_{out})

We usually neglect heat (adiabatic) and kinetic/potential energy, so:

$$\text{Turbine: } \dot{W}_{out} = \dot{m}(h_i - h_e) \rightarrow \text{Power harnessed from } h$$

$$\text{Compressor: } \dot{W}_{in} = \dot{m}(h_e - h_i) \rightarrow \text{Power put in to } h$$

Throttling Valves

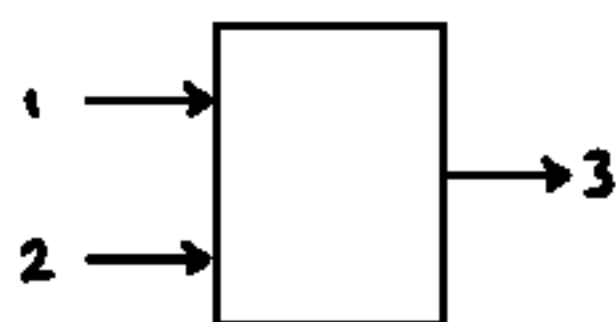


They restrict flow... that's it

$$h_i \approx h_e$$

$$u_i + Pv_i = u_e + Pv_e$$

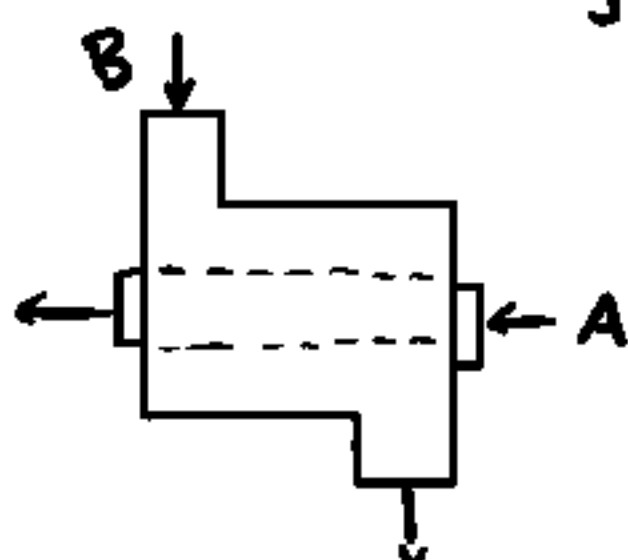
Mixing Chamber



These just combine two inputs.

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$$

Heat exchanger



$$\dot{m}_A (h_{A2} - h_{A1}) = \dot{m}_B (h_{B1} - h_{B2})$$

These just allow energy to be exchanged between two substances without mixing.

Second law and entropy

entropy is the Property of disorder. The Second law States that the entropy of our universe always increases. This means entropy is NOT a Conserved Property - IE, it can be generated, unlike mass and energy.

Reversible Processes are changes to the state of something that don't require an input of energy to "undo".

to find changes in entropy, we always use reversible Processes - even if they are imaginary.

$$\Delta S_{sys} = \left(\frac{Q_{in}}{T_{sys}} \right)_{rev} \quad \Delta S_{surr} = \left(\frac{Q_{in}}{T_{surr}} \right)_{rev} \quad S_{gen} = \Delta S_{sys} + \Delta S_{surr} \quad \left. \vphantom{\begin{matrix} \Delta S_{sys} \\ \Delta S_{surr} \\ S_{gen} \end{matrix}} \right\} \begin{matrix} \text{usually negative} \\ \text{in } \frac{KJ}{kgK} \text{ or } \frac{KJ}{K} \\ \downarrow \quad \downarrow \\ \text{Specific} \quad \text{not Specific} \end{matrix}$$

if a Process is reversible ($T_{sys} = T_{surr}$), then

$$\Delta S_{sys} = -\Delta S_{surr}.$$

IE, entropy was conserved for this particular Process, and $S_{gen} = \Delta S_{sys} + \Delta S_{surr} = 0$.

Note that one of the signs ^{in S_{gen}} must be negative, ^{which one} depends on direction of heat flow.

entropy changes (general)

Special Case: entropy generated in Spontaneous expansion to Vacuum (ideal gas, I think)

$$S_{gen} = m R \ln\left(\frac{V_2}{V_1}\right)$$

Tds relations (for any Process, reversible or irreversible)

these are good for relating other Properties to entropy:

$$\begin{aligned} dh &= Tds + v dp, & du &= Tds - p dv \\ \hookrightarrow ds &= \frac{dh}{T} + \frac{v dp}{T} & \hookrightarrow ds &= \frac{du}{T} + \frac{p dv}{T} \end{aligned}$$

You probably won't use these in this form (see next section).

entropy changes of Solids and liquids

by integrating the eqn from the last section and $dv=0$ (because incompressible),

$$\Delta S = S_2 - S_1 = C_{ave} \ln\left(\frac{T_2}{T_1}\right)$$

\hookrightarrow Specific form, multiply by mass if asked for "Total" change

* C_{ave} is C_p I think because v can't change and thus is constant

Entropy Changes for ideal gasses

if assuming constant C_v, C_p :

$$\Delta S = S_2 - S_1 = C_{v,avg} \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{V_2}{V_1}\right) \quad : \text{ Use if Volume Changes}$$

$$\Delta S = S_2 - S_1 = C_{p,avg} \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right) \quad : \text{ Use if Pressure Changes}$$

if not assuming constant C_v, C_p :

$$\Delta S = S_2 - S_1 = \underbrace{S_2^0 - S_1^0}_{\text{from table}} - R \ln\left(\frac{P_2}{P_1}\right) \quad : \text{ ONLY for } C_p \text{ Processes, not } C_v$$

Entropy balance analysis

this topic covers the flow and generation of entropy

generally,

$$\Delta S_{sys} = S_{in} - S_{out} + S_{gen}$$

$$\dot{S}_{sys} = \dot{S}_{in} - \dot{S}_{out} + \dot{S}_{gen}$$

if multiple sources/sinks:

$$\left. \begin{aligned} \Delta S_{sys} &= \sum_k \frac{Q_k}{T_k} + S_{gen} \\ \dot{S}_{sys} &= \sum_k \frac{\dot{Q}_k}{T_k} + \dot{S}_{gen} \end{aligned} \right\} \begin{aligned} &Q_k \text{ is Positive if flowing in and negative if flowing out} \\ &T_k \text{ is temperature of exterior reservoir} \end{aligned}$$

for open systems (mass flow):

$$\left. \begin{aligned} \Delta S_{cv} &= \sum_k \frac{Q_k}{T_k} + \sum \dot{m}_i s_i - \sum \dot{m}_e s_e \\ \dot{S}_{cv} &= \sum_k \frac{\dot{Q}_k}{T_k} + \sum \dot{m}_i s_i - \sum \dot{m}_e s_e \end{aligned} \right\} \begin{aligned} &Q_k \text{ is Positive if flowing in and negative if flowing out} \\ &T_k \text{ is temperature of where the heat is} \\ &\text{coming from (if } Q \text{ pos) or going to (if } Q \text{ neg)} \\ &* \text{ like the wall problem, } T_k \text{ may be the boundary or} \\ &\text{surroundings.} \end{aligned}$$

Steady flow (no S accumulation inside Control Volume):

the above two equations, but $\dot{S}_{cv} = 0$ or $\Delta S_{cv} = 0$

Second law on heat and work

just like a heat transfer Process is reversible if the System and Surroundings are at the same temperature, a boundary work interaction (Piston/Cylinder) is reversible if the System and Surroundings are at the same Pressure.

$$W_{\text{boundary, rev}} = \int_1^2 P dV$$

for Steady flow,

$$W_{\text{boundary, rev}} = - \int_1^2 V dP$$

} for a Steady state, reversible interaction in an open System

↳ This eqn applies to turbines and compressors, but V is a function of P and we usually don't know it. But,

for an incompressible fluid (V const)

$$\text{Turbine: } W_{\text{rev, out}} = -V(P_2 - P_1)$$

$$\text{Compressor: } W_{\text{rev, in}} = V(P_2 - P_1)$$

} Can multiply by \dot{m} to get \dot{W}_{rev}

how does reversible work compare to actual work?

$$W_{\text{rev}} > W_{\text{irrev}}$$

$$\Delta W_{\text{act}} = \Delta W_{\text{rev}} - \underbrace{T \Delta S_{\text{gen}}}$$

W_{lost} , but Prof said don't calculate W_{lost} as $W_{\text{rev}} - W_{\text{act}}$

isentropic efficiencies

turbines, compressors, and nozzles aren't perfectly efficient in reality. Sometimes, energy is lost to entropy.

losses are minimized for reversible adiabatic Processes ($S_{\text{gen}} = 0$, $Q = 0$)

↳ aka, when the Process is isentropic: $S_1 = S_2$

Turbines

$$\eta_T = \frac{h_1 - h_{2a}}{h_1 - h_{2s}} = \frac{W_{\text{act}}}{W_{\text{isen}}}$$

Compressors/Pumps

$$\eta_c = \frac{h_{2s} - h_1}{h_{2a} - h_1} = \frac{W_{\text{isen}}}{W_{\text{act}}}$$

$$\eta_p = \frac{V(P_2 - P_1)}{h_{2a} - h_1} = \frac{W_{\text{isen}}}{W_{\text{act}}}$$

Nozzles

$$\eta_N = \frac{h_1 - h_{2a}}{h_1 - h_{2s}} = \frac{V_{2a}^2}{V_{2s}^2}$$

$$\times V_2^2 = 2(h_1 - h_2) \text{ if } V_1 \text{ small}$$

* The key for these is to find h_{2a} and h_{2s} , for h_{2s} , use final P NOT T .

Find h_{2a} using given final Properties or other given info and η eqn.

* For ideal gasses: You can replace ' h ' with ' T '.

Isentropic Processes for ideal gasses

if C_p, C_v Constant:

$$\left. \begin{aligned} T_1 v_1^{(\kappa-1)} &= T_2 v_2^{(\kappa-1)} \\ T_1 P_1^{(1-\kappa)/\kappa} &= T_2 P_2^{(1-\kappa)/\kappa} \\ P_1 v_1^\kappa &= P_2 v_2^\kappa \end{aligned} \right\} \text{Use to get info about final isentropic state}$$

if C_p, C_v Not Constant:

$$\frac{P_2}{P_1} = \frac{P_{r2}}{P_{r1}} = \frac{P_{n@T_{2s}}}{P_{n@T_{1s}}} \quad \frac{v_2}{v_1} = \frac{v_{r2}}{v_{r1}} = \frac{v_{n@T_{2s}}}{v_{n@T_{1s}}}$$

* You probably don't need this, but:

$$P_r = \frac{P}{P_{cr}} \quad T_r = \frac{T}{T_{cr}} \quad v_r = \frac{v}{R T_{cr} / P_{cr}}$$

Isentropic efficiencies for ideal gas (i made these up but they should work):

Turbines

$$\eta_T = \frac{T_1 - T_{2a}}{T_1 - T_{2s}} = \frac{W_{act}}{W_{isen}}$$

Compressors/Pumps \rightarrow incompressible fluid

$$\eta_c = \frac{T_{2s} - T_1}{T_{2a} - T_1} = \frac{W_{isen}}{W_{act}}$$

$$\eta_p = \frac{v(P_2 - P_1)}{T_{2a} - T_1} = \frac{W_{isen}}{W_{act}}$$

Nozzles

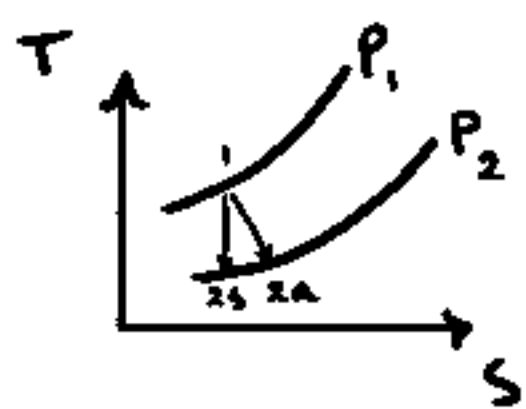
$$\eta_N = \frac{T_1 - T_{2a}}{T_1 - T_{2s}} = \frac{v_{2a}^2}{v_{2s}^2}$$

$$\times v_2^2 = 2(T_1 - T_2) \text{ if } v_1 \text{ small}$$

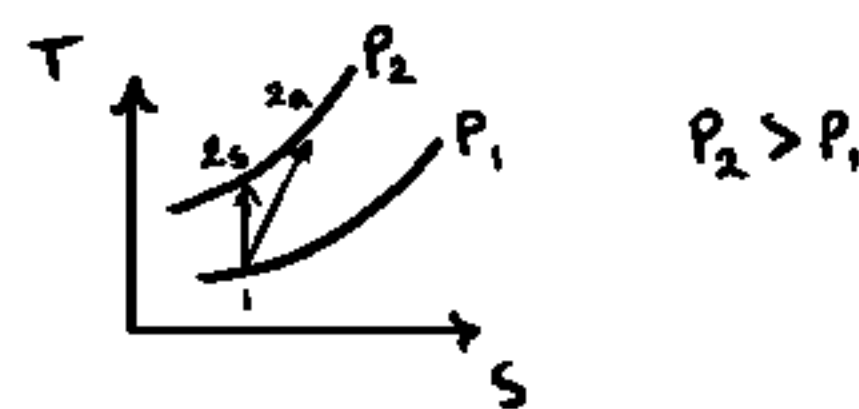
T-S diagrams (these ~should~ be all you need)

Ideal gas

Turbine

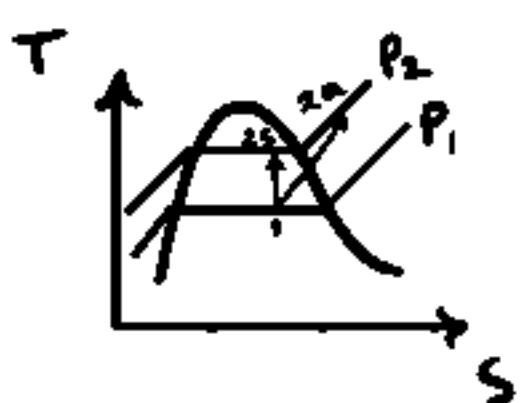


Compressor

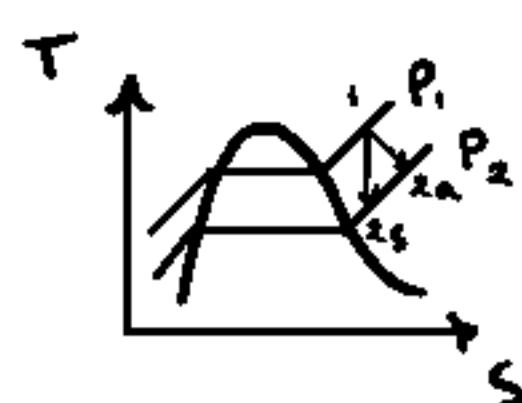


Not ideal gas (make sure to draw start and end points in correct region)

Turbine



Compressor



Thermodynamic Cycles - Overview

the goal of heat engines is to turn heat into usable energy.

all heat engines involve the absorption of heat from a high temperature reservoir and the rejection of heat to a low temperature. heat engines cannot generate any net work without rejecting heat to a low temperature reservoir because of the second law.

Cycles aren't perfectly efficient. there are two types of efficiency for cycles.

First law efficiency

heat engine (basic)

$$\eta_{th} = \frac{\text{desired}}{\text{required}} = \frac{W_{net}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{T_L}{T_H}$$

Refrigerator

$$COP_R = \frac{Q_L}{W_{net}} = \frac{Q_L}{Q_H - Q_L}$$

\downarrow
in

heat pump

$$COP_{HP} = \frac{Q_H}{W_{net}} = \frac{Q_H}{Q_H - Q_L} = COP_R + 1$$

\downarrow
in

Second law efficiency

the second law efficiency for a heat engine is the comparison of that heat engine to the best possible heat engine.

$$\eta_{II} = \frac{\eta_{TH}}{\eta_{Carnot}}$$

$$W_{net} = W_{Carnot} - W_{lost} = \underbrace{\left(1 - \frac{T_L}{T_H}\right) Q_H}_{W_{Carnot}} - T_L S_{gen}$$

$$COP_{II} = \frac{COP}{COP_{Carnot}}$$

* a system delivers the maximum possible work when it undergoes a reversible process from the specified initial state to the state of its environment, which is the dead state

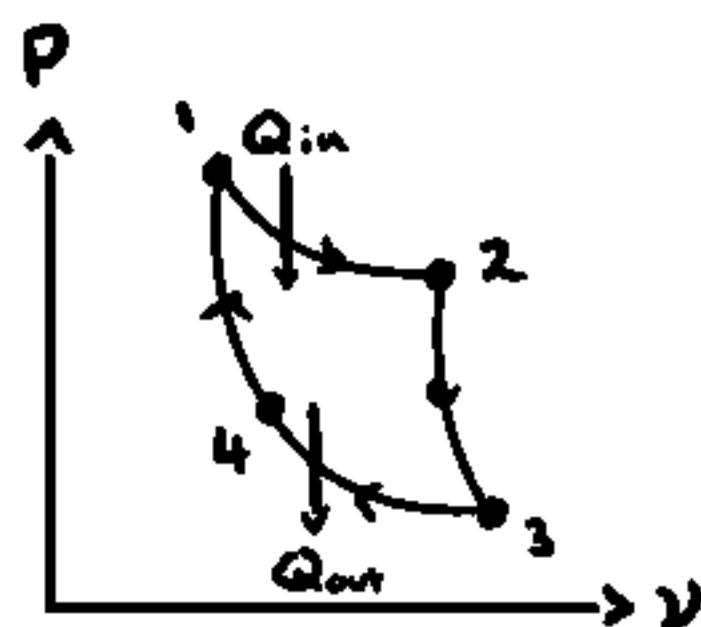
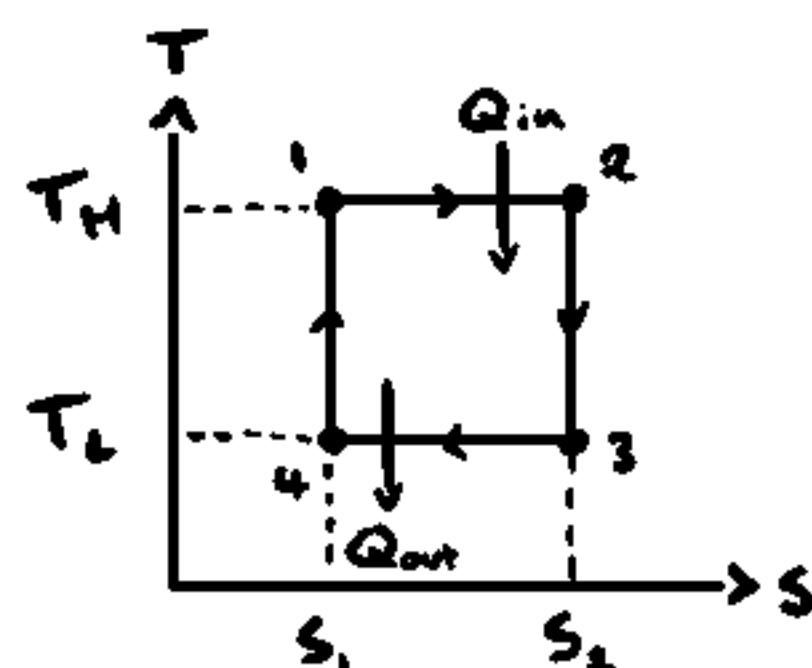
for our cycles, under standard cold air assumption, use:

$$K = 1.4 \quad R = 0.287 \frac{kJ}{kgK} \quad C_v = 0.718 \frac{kJ}{kgK} \quad C_p = 1.005 \frac{kJ}{kgK}$$

Carnot Cycle

the best possible heat engine! it is completely fictitious, as all processes are reversible. even so, the Carnot cycle isn't perfectly efficient.

Heat Engine



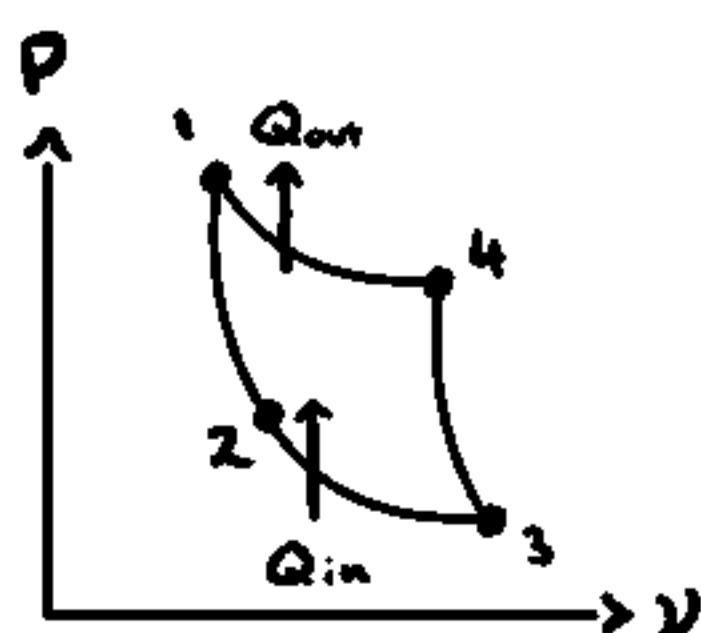
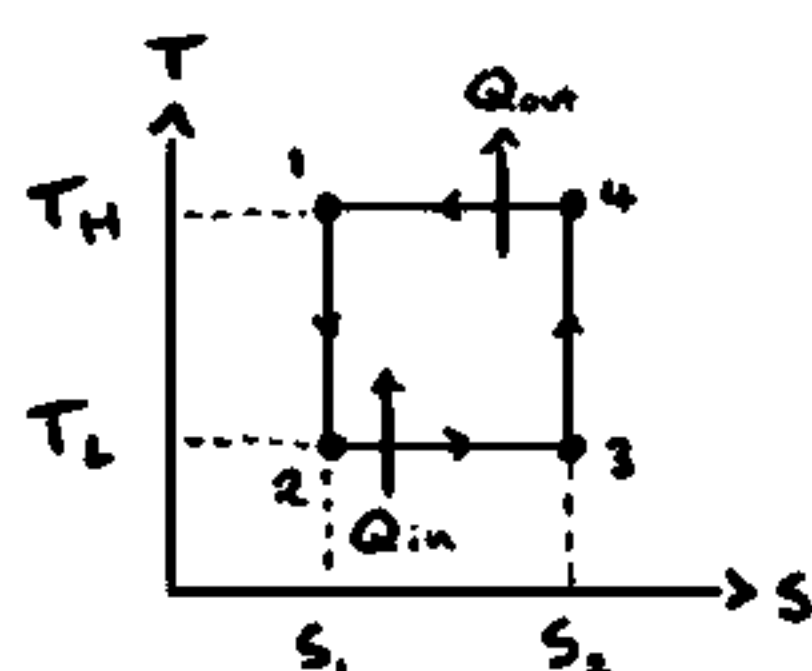
$$Q_{in} = T_H (s_2 - s_1) \text{ Positive in}$$

$$Q_{out} = T_L (s_2 - s_1) \text{ Positive out}$$

$$W_{net} = Q_{in} - Q_{out} = (T_H - T_L)(s_2 - s_1)$$

$$\eta_{Carnot} = 1 - \frac{Q_L}{Q_H} = 1 - \frac{T_L}{T_H}$$

Refrigeration Cycle / Heat Pump



$$Q_{in} = T_L (s_2 - s_1) \text{ Positive in}$$

$$Q_{out} = T_H (s_2 - s_1) \text{ Positive out}$$

$$W_{in} = Q_{in} - Q_{out} = (T_H - T_L)(s_2 - s_1)$$

$$COP_{R, Carn} = \frac{Q_L}{Q_H - Q_L} = \frac{T_L}{T_H - T_L}$$

$$COP_{HP, Carn} = \frac{Q_H}{Q_H - Q_L} = \frac{T_H}{T_H - T_L}$$

Solving (Heat engine)

$$p v = R T \left. \begin{array}{l} \textcircled{1} \xrightarrow[T_{const} (T_H)]{} \textcircled{2} \xrightarrow[S_{const} (s_2)]{} \textcircled{3} \xrightarrow[T_{const} (T_L)]{} \textcircled{4} \xrightarrow[S_{const} (s_1)]{} \textcircled{1} \end{array} \right\} \begin{array}{l} S_{const}: T_1 v_1^{(\kappa-1)} = \text{const} \\ T_1 p_1^{(1-\kappa)/\kappa} = \text{const} \\ p_1 v_1^\kappa = \text{const} \end{array}$$

$$W_{out} = (T_H - T_L) \left(C_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} \right)$$

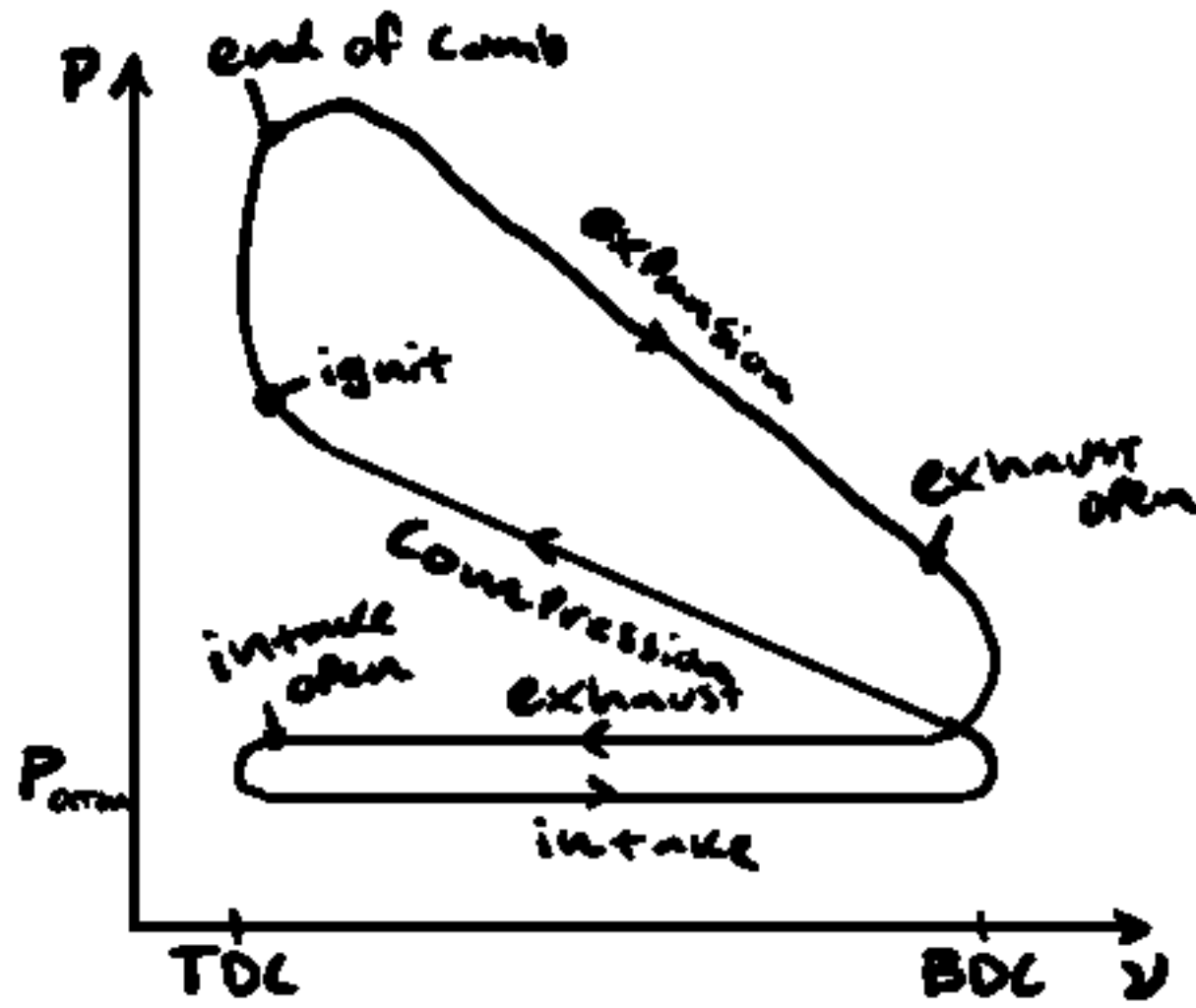
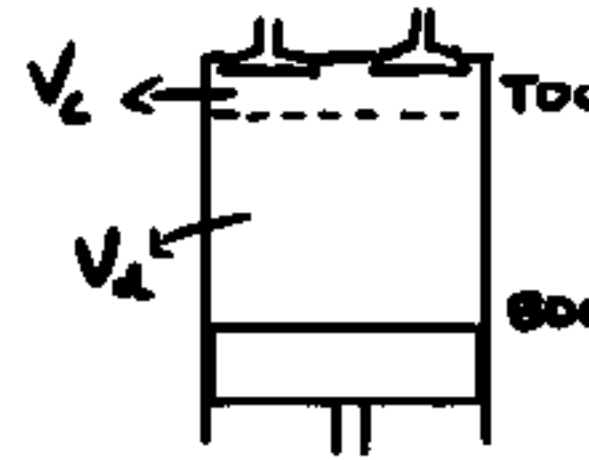
Not T_H and T_L

Otto Cycle

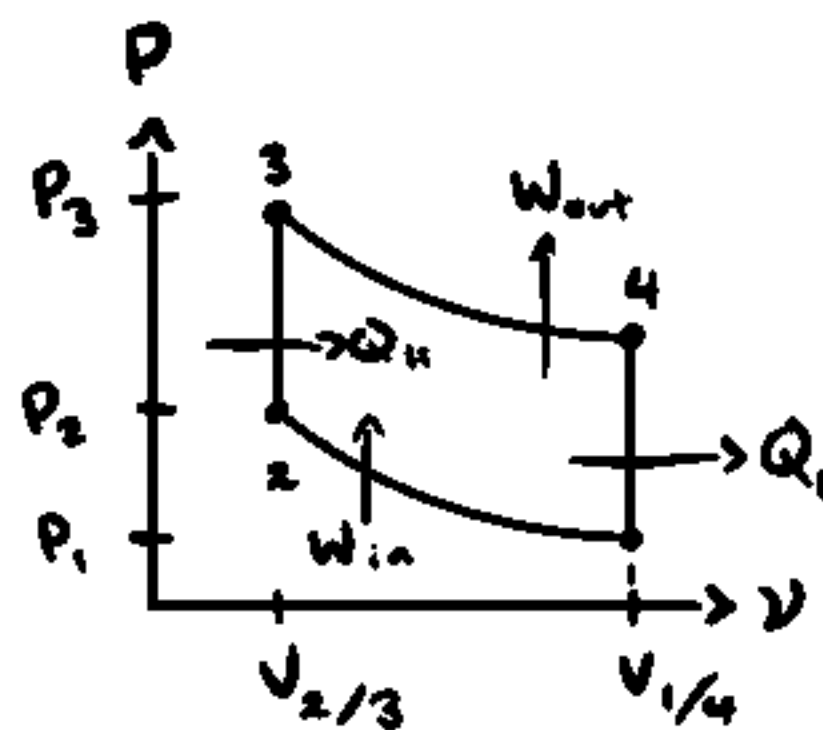
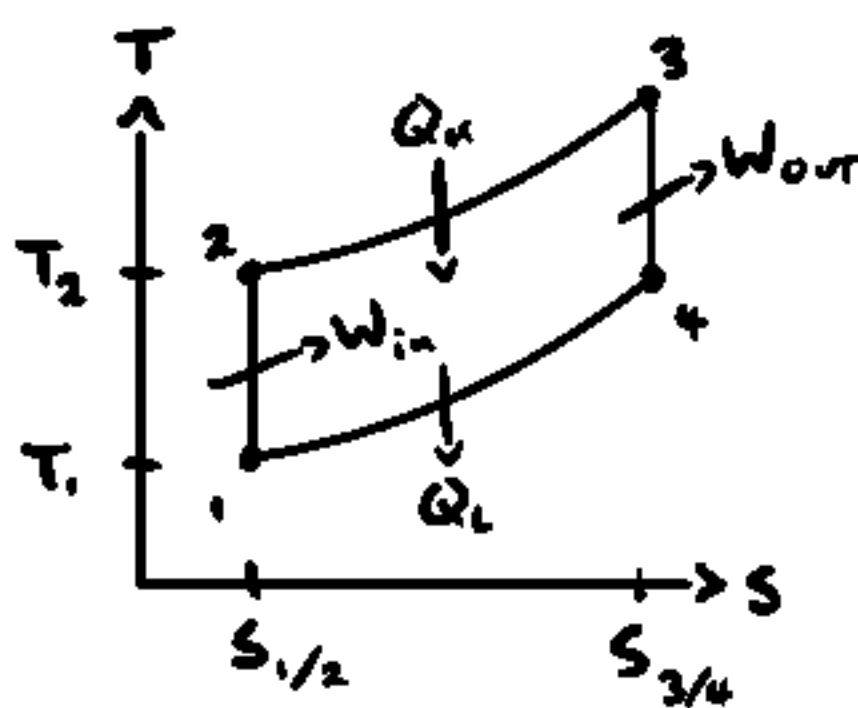
Often used to Model the Spark ignition engine.

ignition engine

Compression ratio $r = \frac{V_{BDC}}{V_{TDC}} = \frac{V_d + V_c}{V_c}$



Otto



$$q_H = C_v(T_3 - T_2)$$

$$q_L = C_v(T_4 - T_1)$$

$$W_{in} = C_v(T_2 - T_1)$$

$$W_{out} = C_v(T_3 - T_4)$$

$$W_{net} = Q_H - Q_L = W_{out} - W_{in} = Q_H \eta$$

$$\eta_{Otto} = 1 - \frac{1}{r^{k-1}} = \frac{W_{net}}{Q_H}$$

* $r = \frac{V_1}{V_2}$, displacement $= V_d = (V_1 - V_2) m$

* 2 RPM \rightarrow 1 cycle/m

* remember mass!

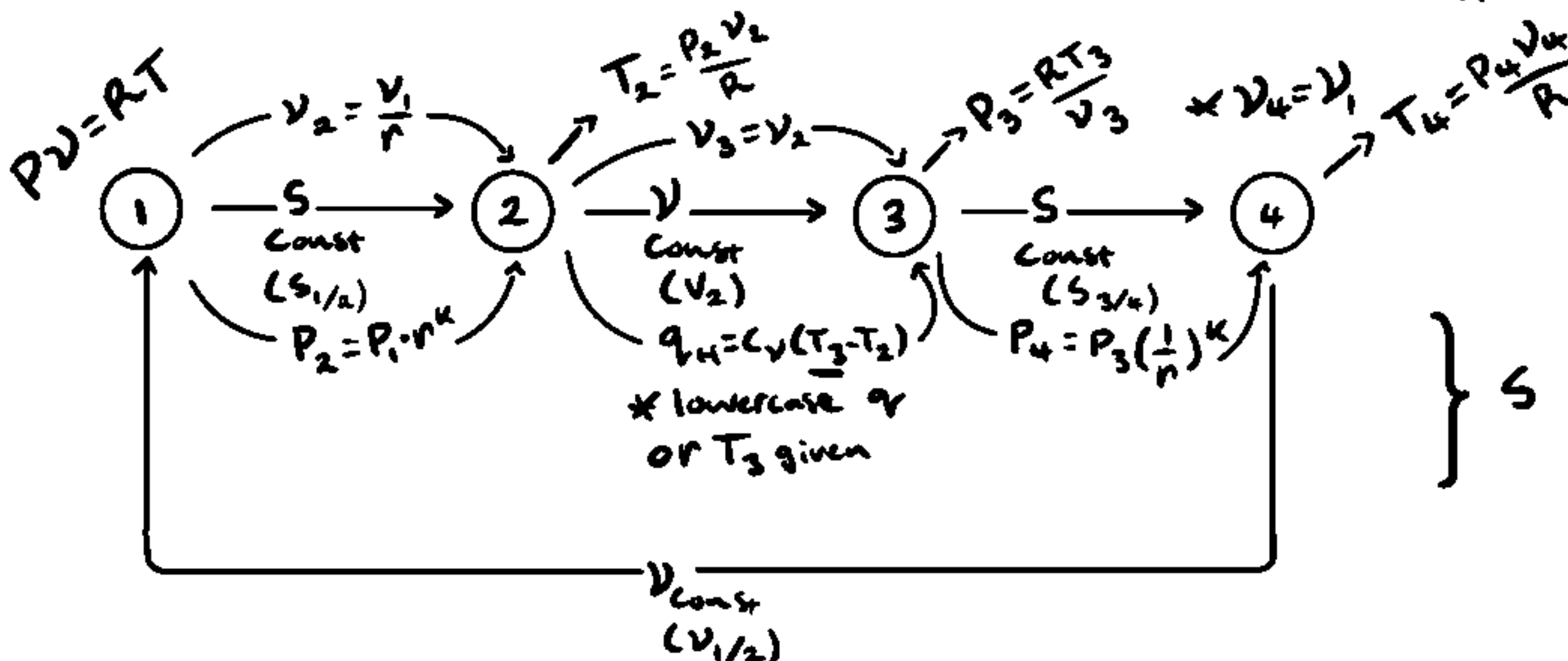
$$MEP = \frac{W_{net}}{V_{1/4} - V_{2/3}} \text{ (kPa)}$$

indicated Power $= \dot{W}_{net} = W_{net} \left(\frac{k}{s} \right) N \left(\frac{cycle}{sec} \right)$

brake work $= W_{net} - W_{friction}$

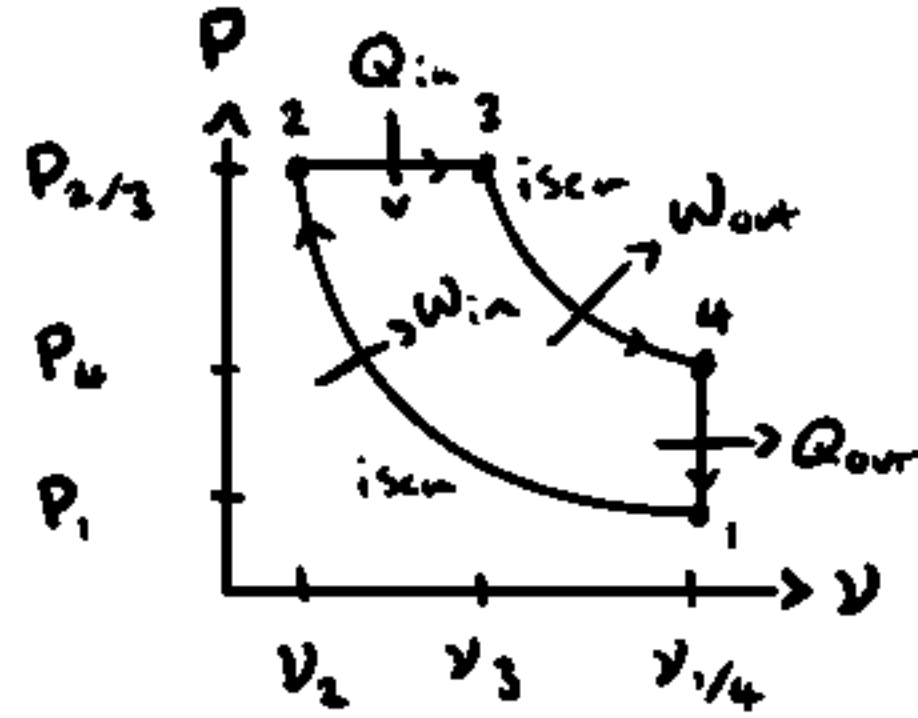
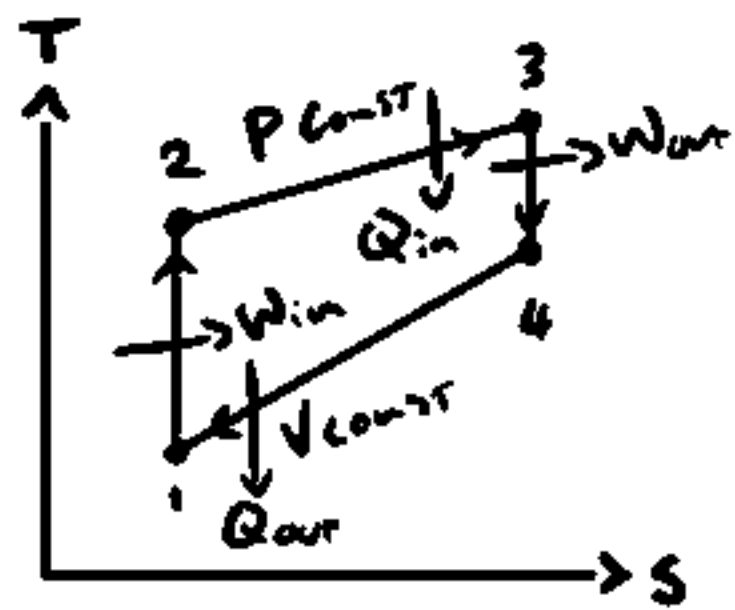
$$\eta_{mech} = \frac{W_b}{W_{net}}$$

Typically given P, T, r



$$\left. \begin{aligned} S \text{ const: } T_1 V_1^{(k-1)} &= \text{const} \\ T_1 P_1^{(1-k)/k} &= \text{const} \\ P_1 V_1^k &= \text{const} \end{aligned} \right\}$$

diesel cycle



$$r = \frac{v_1}{v_2} \quad r_c = \frac{v_3}{v_2}$$

$$q_H = C_p(T_3 - T_2) \quad \text{*** } C_p$$

$$q_L = C_v(T_4 - T_1) \quad \text{*** } C_v$$

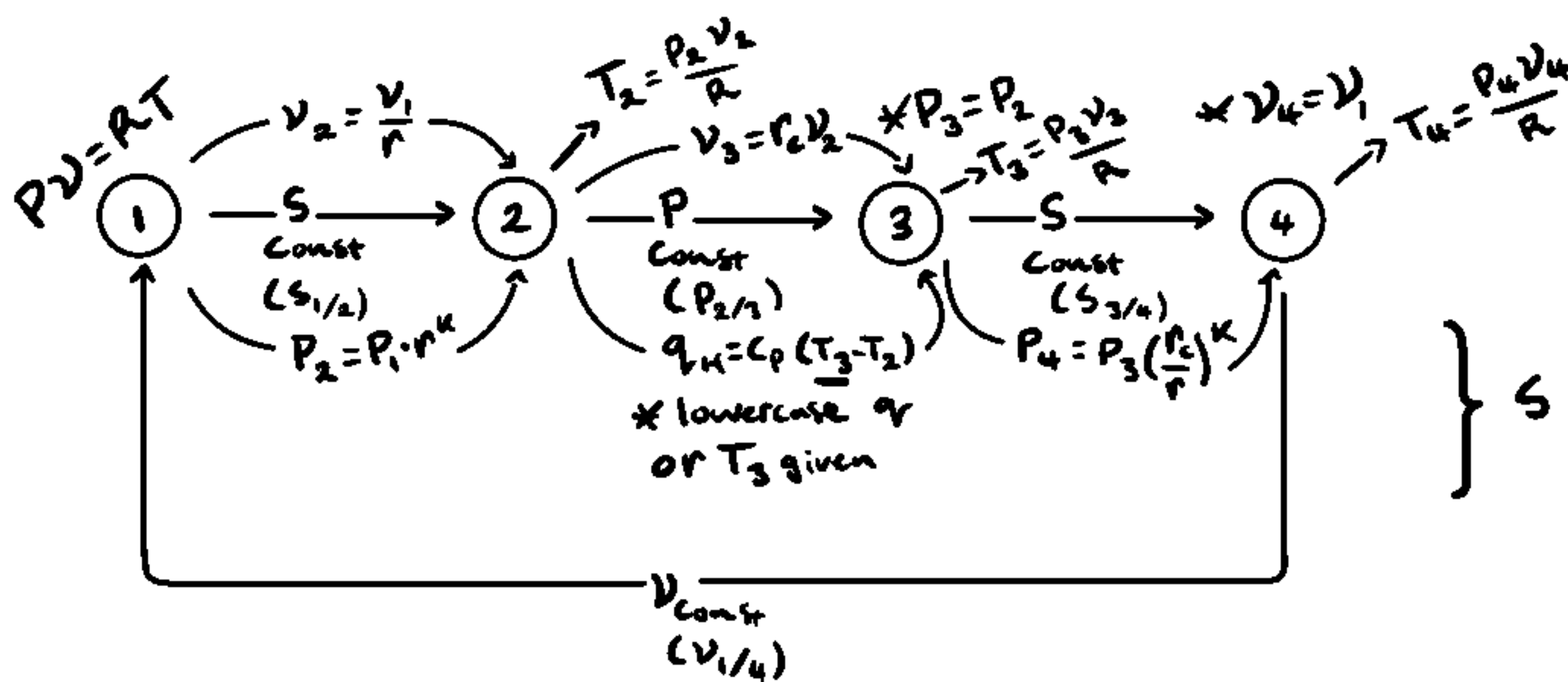
$$\left. \begin{aligned} W_{in} &= C_v(T_2 - T_1) \quad \text{*** } C_v \\ W_{out} &= C_v(T_4 - T_1) \quad \text{*** } C_v \end{aligned} \right\} \text{Prof specified in review}$$

$$W_{net} = Q_{in} - Q_{out} = W_c - W_e, \quad \eta_{th} = \frac{W_{net}}{Q_{in}} = 1 - \frac{1}{r^{k-1}} \left(\frac{r_c^k - 1}{k(r_c - 1)} \right)$$

$$MEP = \frac{W_{net}}{v_1 - v_2}$$

$$T_{max} = T_3 = T_1 r^{k-1} r_c$$

Typically given P_1, T_1, r, r_c



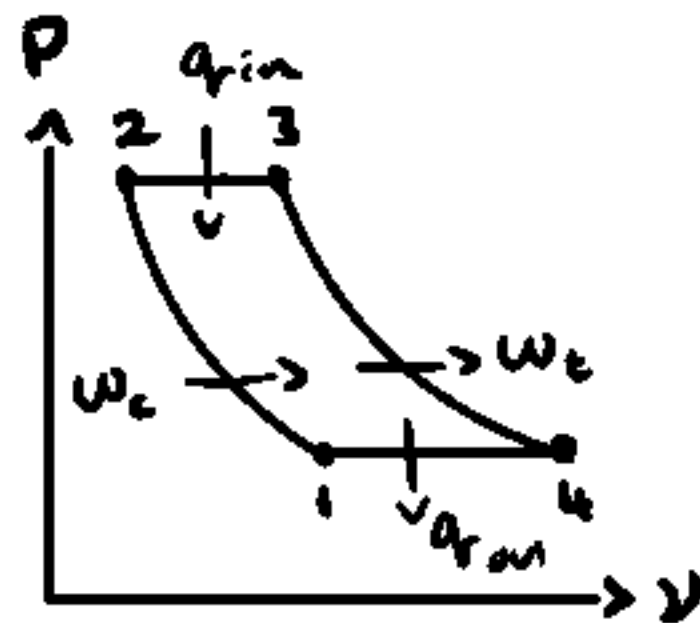
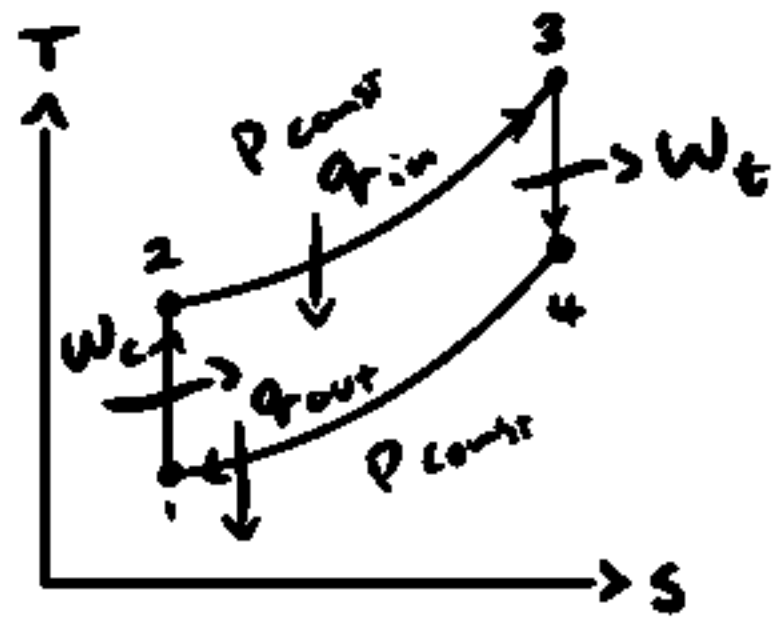
$$\left. \begin{aligned} S \text{ const: } T_1 v_1^{(k-1)} &= \text{const} \\ T_1 P_1^{(1-k)/k} &= \text{const} \\ P_1 v_1^k &= \text{const} \end{aligned} \right\}$$

diesel vs otto

① for the same compression ratio, $\eta_{diesel} < \eta_{otto}$

② for the same P_{max} , $\eta_{diesel} > \eta_{otto}$

brayton cycle

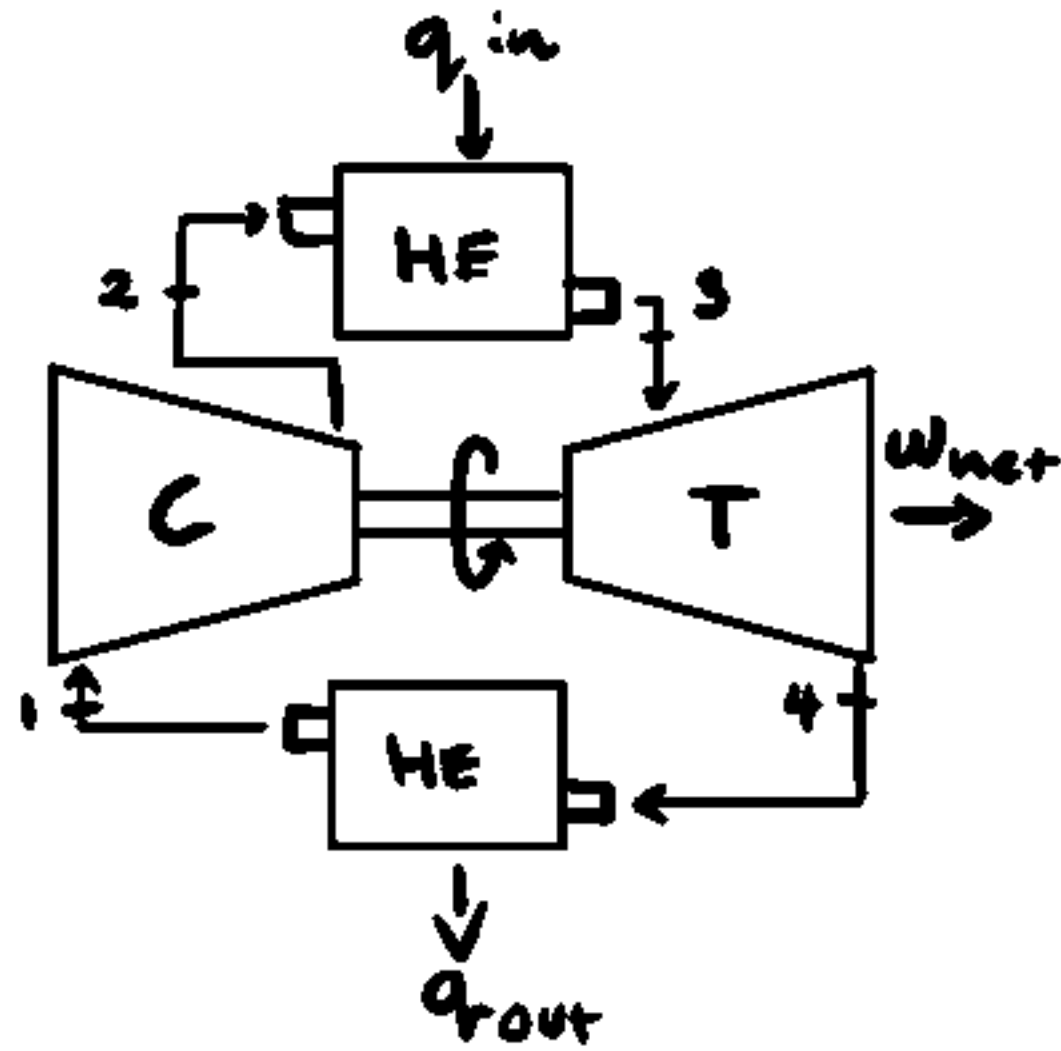


$$r_p = \frac{P_2}{P_1} \quad r_{bw} = \frac{W_{comp}}{W_{turb}} \quad \eta_{th} = \frac{W_t - W_c}{q_{in}} = 1 - \left(\frac{1}{r_p}\right)^{\frac{k-1}{k}} \quad \rightarrow = W_{net}$$

$$r_{p,max} = \left(\frac{T_3}{T_1}\right)^{k/2(k-1)}$$

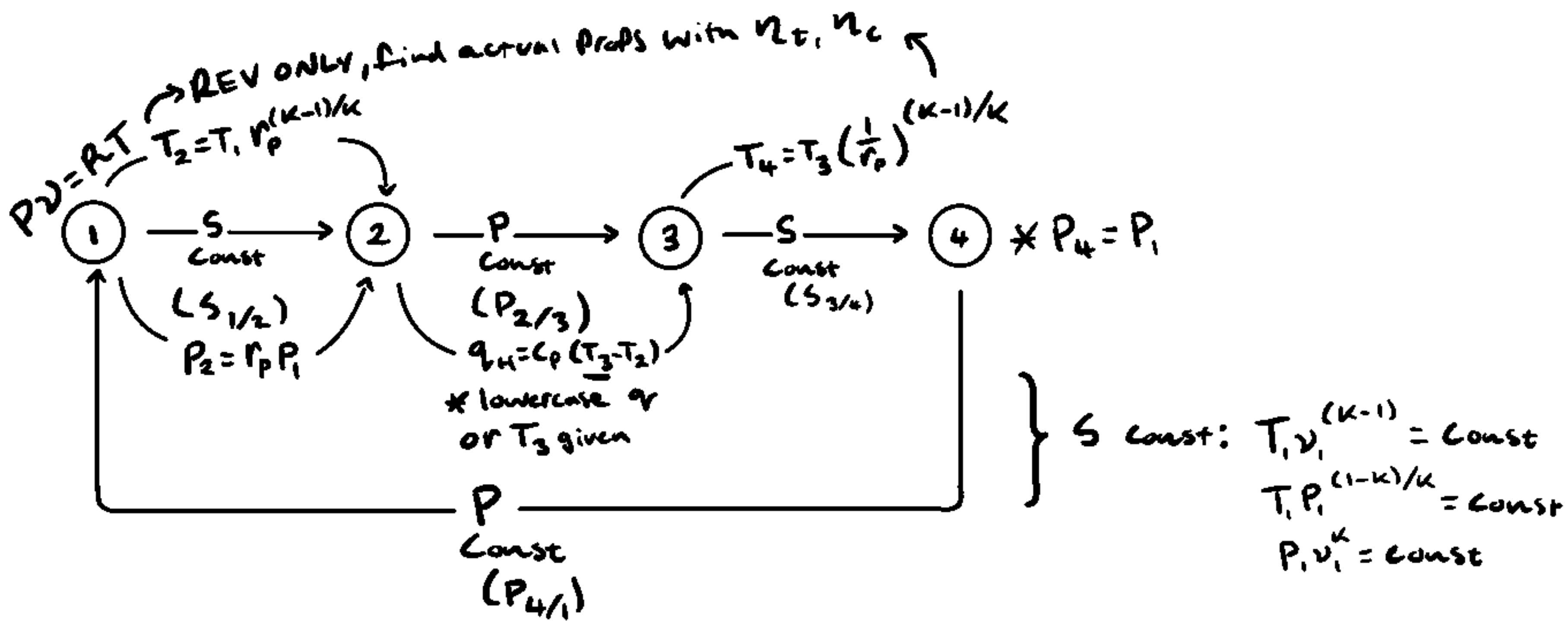
↳ ideal compression ratio (maximizes output)

$$\eta_c = \frac{T_{2s} - T_1}{T_2 - T_1} \quad \eta_T = \frac{T_3 - T_4}{T_3 - T_{4s}}$$

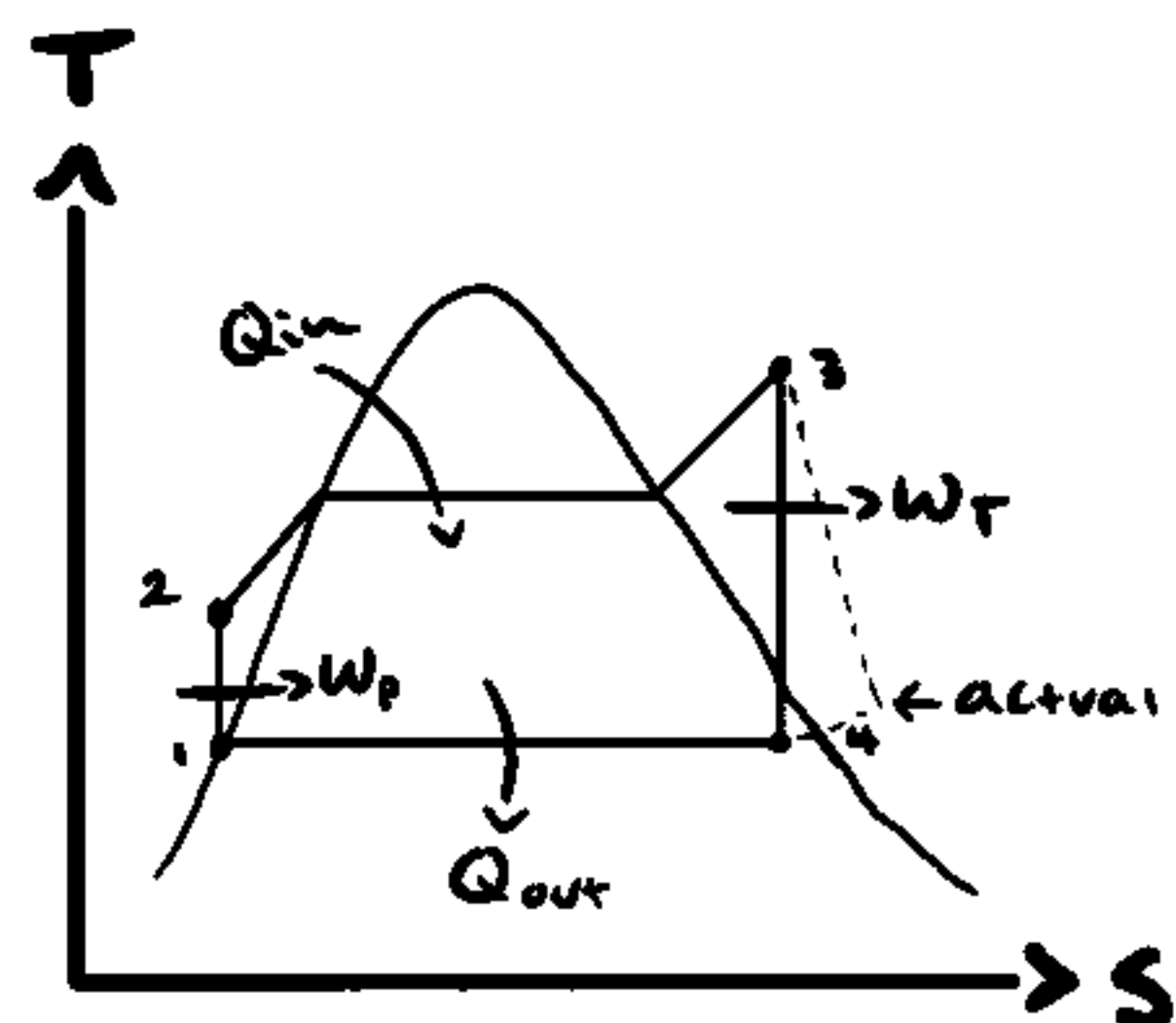


$$\left. \begin{aligned} q_H &= C_p(T_3 - T_2) \\ q_L &= C_p(T_4 - T_1) \\ W_c &= C_p(T_2 - T_1) \\ W_t &= C_p(T_3 - T_4) \end{aligned} \right\} \dot{W}_{out} = \dot{m}(W_t - W_c)$$

Typically given P_1, T_1, r_p, T_3



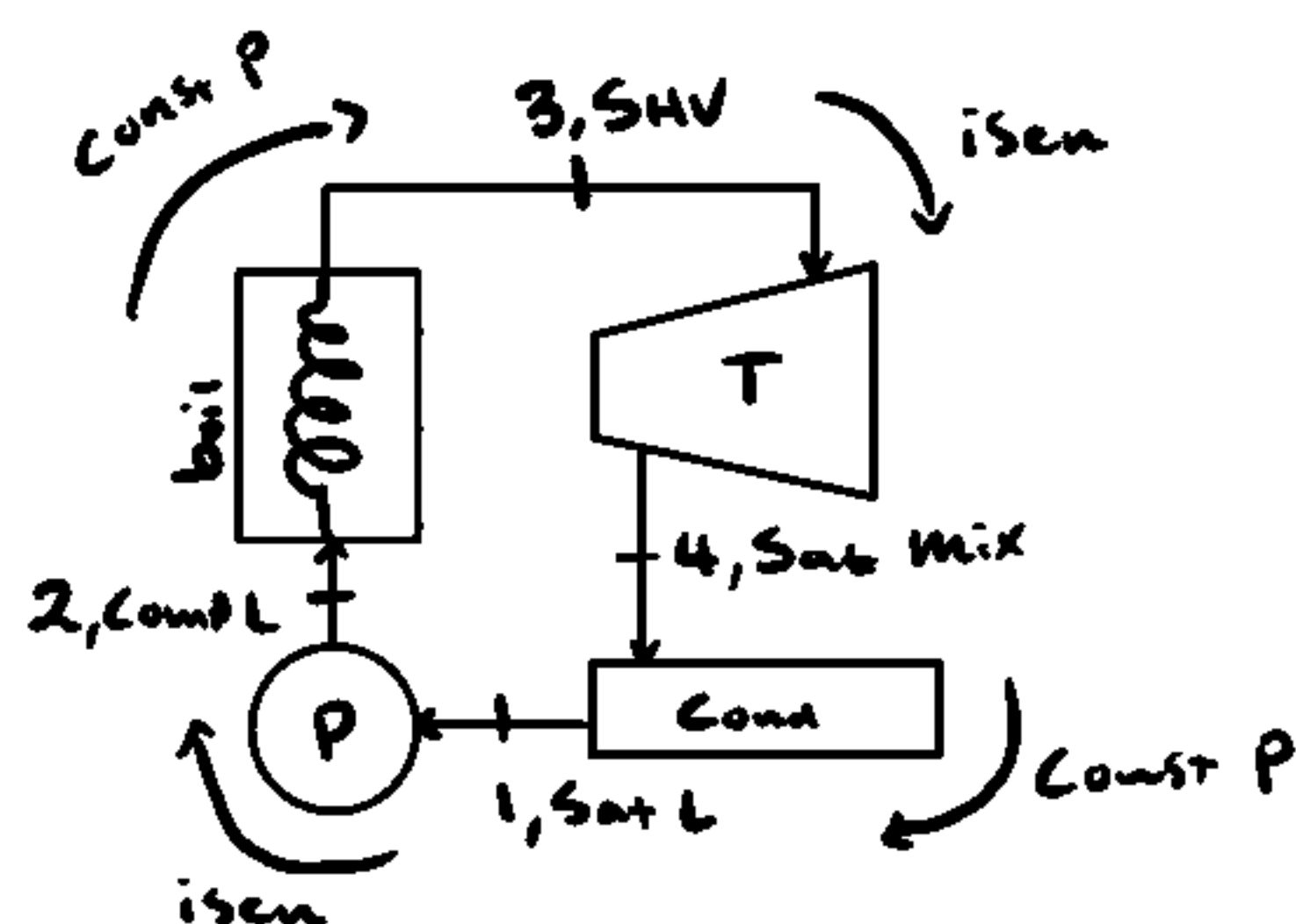
RANKINE CYCLE



* ALL H FROM TABLE, NEVER T
 ↳ Water!

$$\eta_P = \frac{v_1(P_2 - P_1)}{h_{2a} - h_1}$$

$$\eta_T = \frac{h_3 - h_4}{h_3 - h_{4s}}$$



$$W_{turb} = h_3 - h_4$$

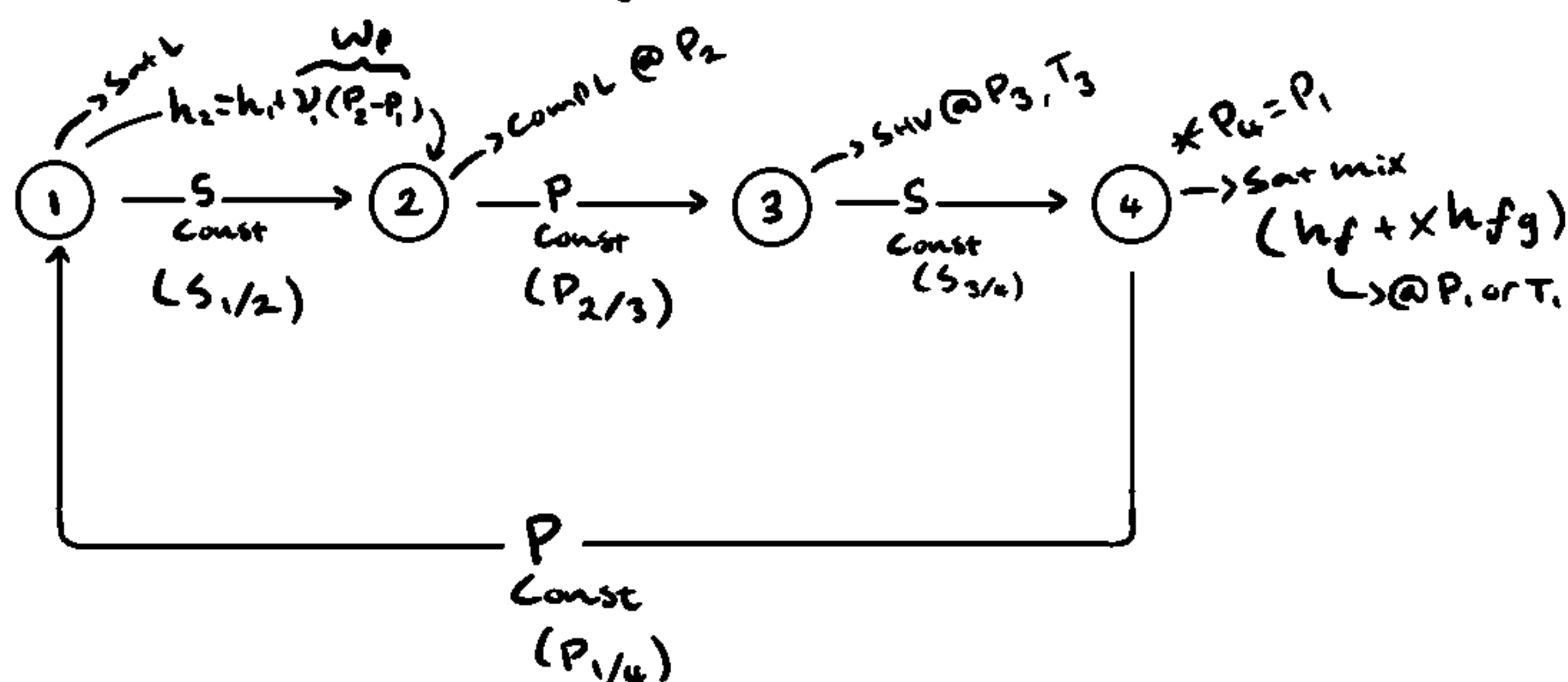
$$W_{pump} = h_2 - h_1 = v_1(P_2 - P_1)$$

$$q_{in} = h_3 - h_2 \quad \left(\begin{array}{l} \approx v_f @ T_1 \\ \approx h_f @ T_1 \end{array} \right)$$

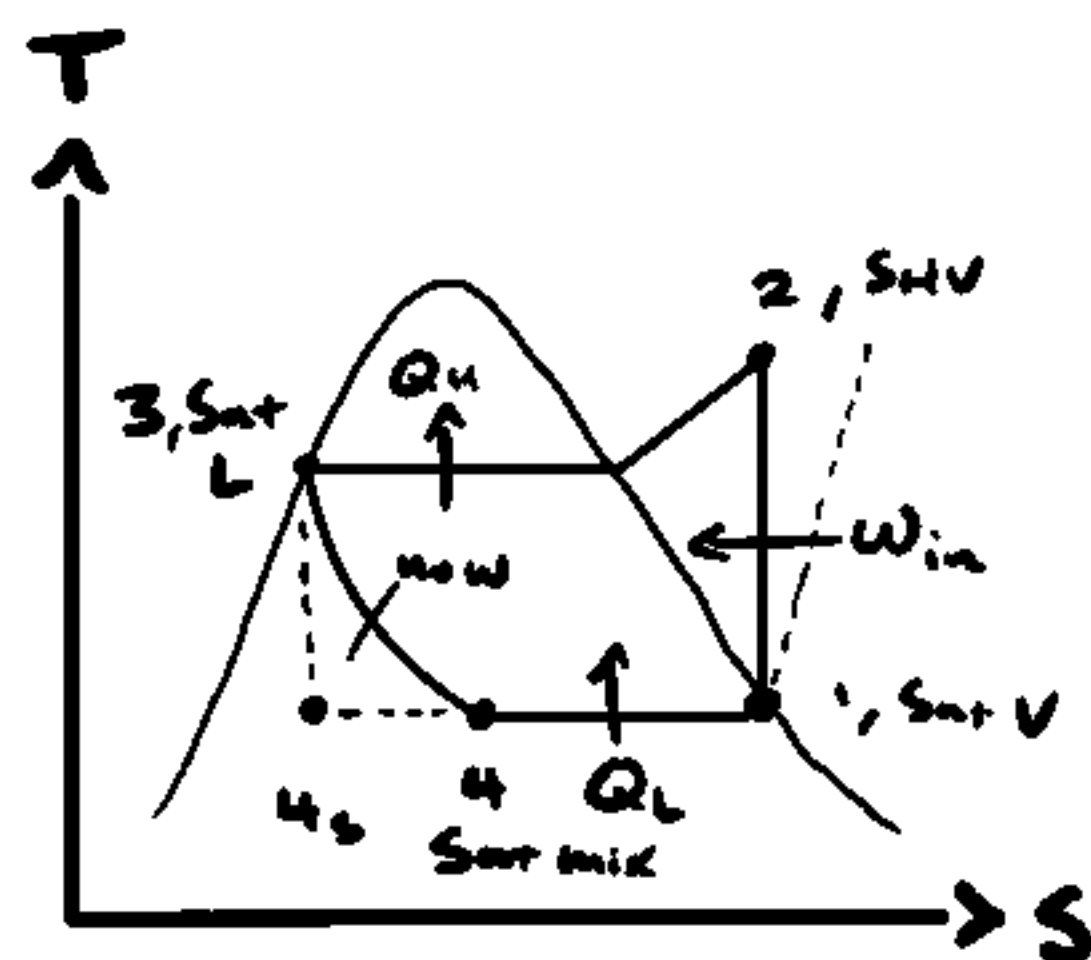
$$q_{out} = h_4 - h_1$$

$$W_{net} = W_t - W_p \quad \left(\begin{array}{l} \text{only } v_1(P_2 - P_1) \text{ if isen} \end{array} \right)$$

Typically given P_1, P_2, T_3

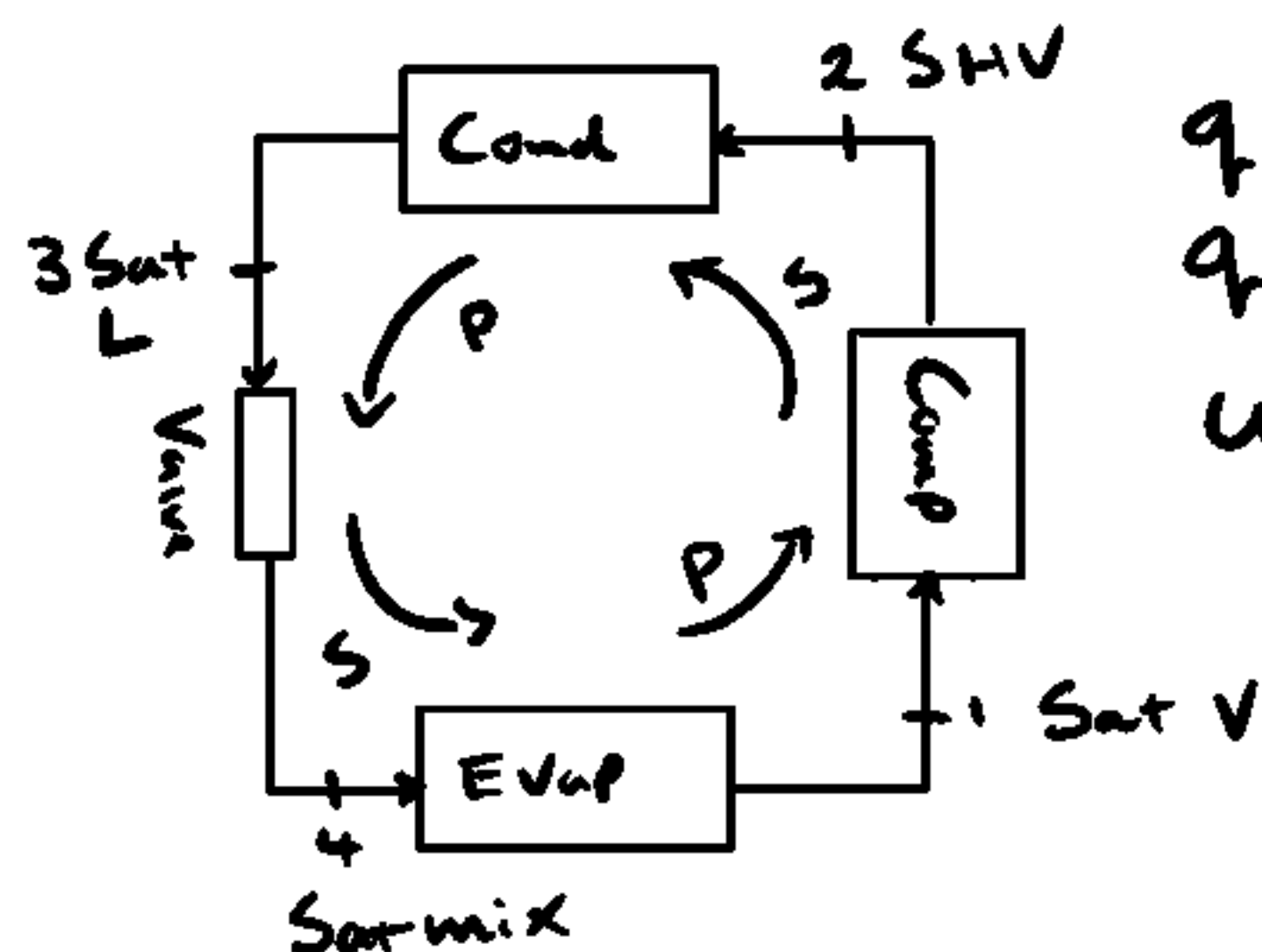


Refrigeration Cycle



* ALL H FROM TABLE, NEVER T
L > R 134-a!

1 actual: Slightly Superheated
3 actual: Slightly Subcooled



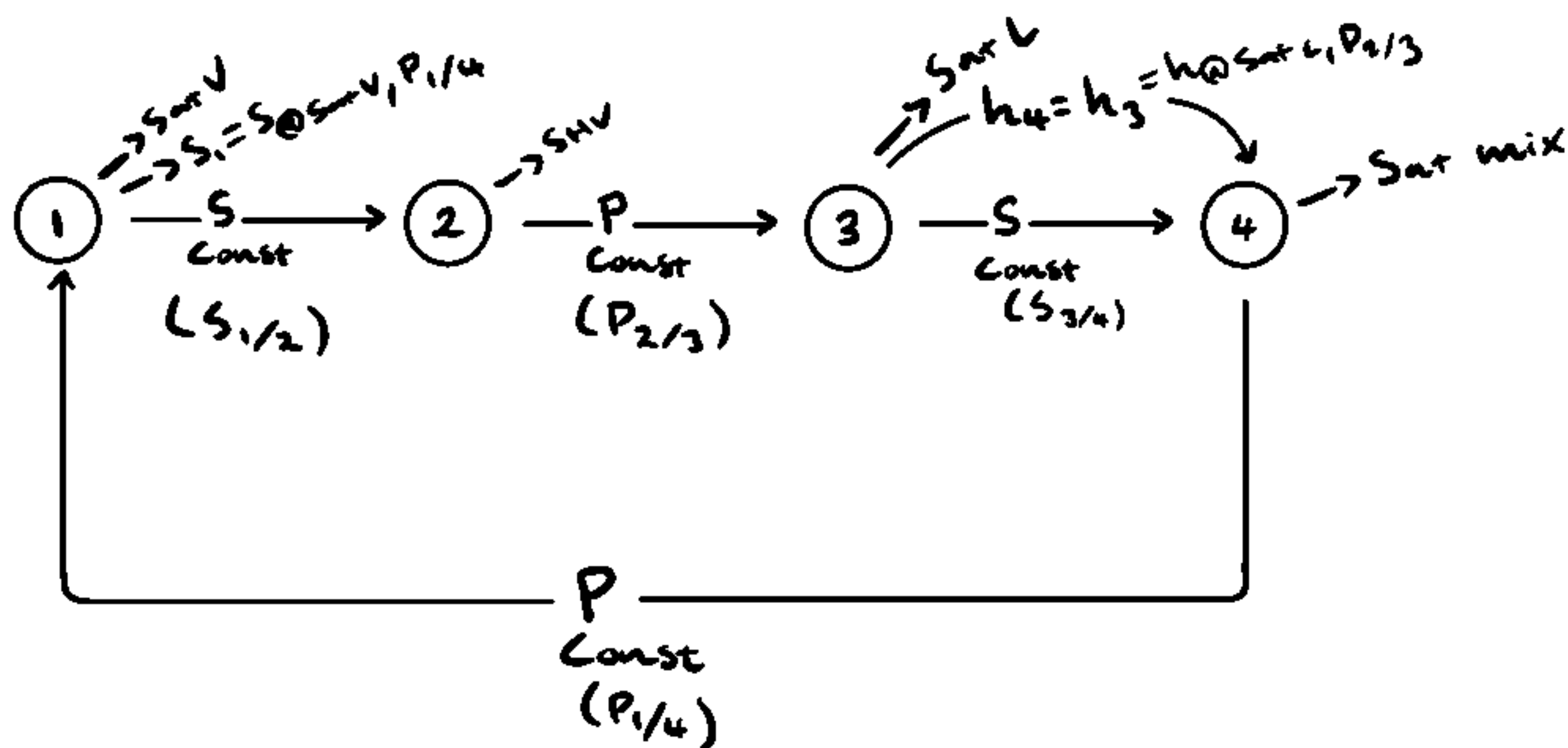
$$q_c = h_1 - h_4$$

$$q_H = h_2 - h_3$$

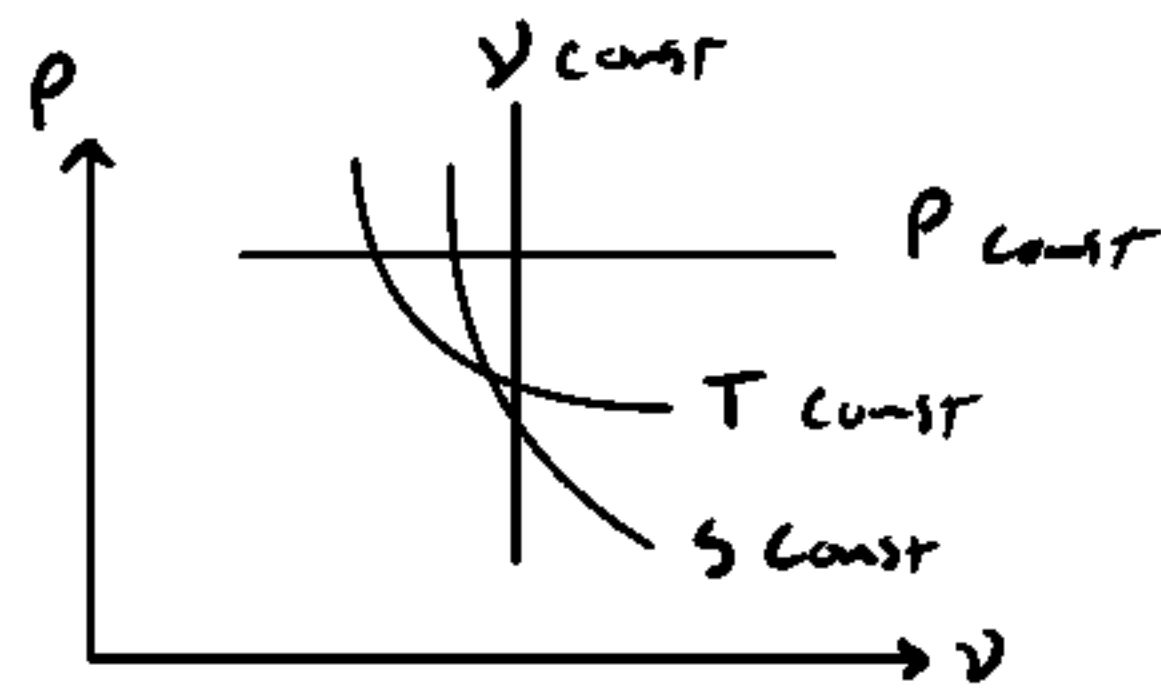
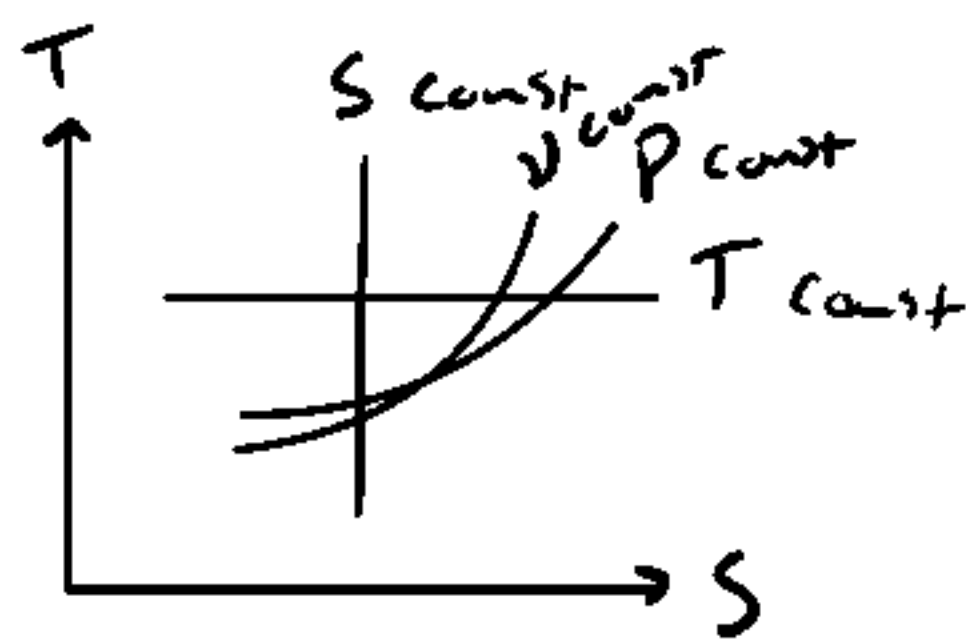
$$W_{in} = q_H - q_c = h_2 - h_1$$

$$COP_R = \frac{q_c}{W_{in}}$$

$$COP_{HP} = \frac{q_H}{W_{in}}$$



Shapes of S, V, P, T Const



Units

Force

$$N = \frac{\text{kg} \cdot \text{m}}{\text{s}^2}$$

$$\text{lbf} = \frac{\text{lbm} \cdot \text{ft}}{\text{s}^2}$$

Pressure

$$P_a = \frac{N}{\text{m}^2}$$

Energy

$$J = N \cdot \text{m}$$

$$\text{cal} = 4.1868 \text{ J}$$

$$\text{kWh} = 3600 \text{ kJ}$$

$$\text{BTU} = 1055.06 \text{ J}$$

Power

$$W = \frac{J}{s}$$

$$\text{hp} = 0.7457 \text{ kW}$$