Mathematical methods for classification of state-of-charge time series for cycle lifetime prediction

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Abstract

In this note we give a precise mathematical definition of a cycle, which can be used independently of the application. Thus we derive an algorithm for detecting cycles in any state-of-charge (SoC) time series by geometric interpretation. This leads to a transformation of any SoC profile in a cycle-depth-of-cycle(DoC) histogram, which hence can be used for cycle lifetime prediction by means of battery manufacturer data sheet or laboratory measurement series. Finally, we show a relation of our cycle definition with the well known rainflow cycle (RFC) counting method from mechanical engineering and prove the equivalence of both counting methods.

1 Introduction

Lifetime prediction of the battery is one of the most important challenges in modern battery-powered applications like electric vehicles (EVs) or renewable energy systems. One ingredient to overcome the lifetime prediction problem, is a suitable *cycle - depth-of-discharge* counter. Due the operation conditions of the application, a certain stress factor, in particular an application specific state-of-charge time series as shown in Fig.1, is imposed to the battery storage system. Often the data sheet of a battery manufacturer is given the number of cycles to failure N at a fixed depth-of-discharge (DoD) level. That means a new battery is cycled at fixed DoD until e.g. 80% of the nominal capacity C_N is reached; often per definition called *end-of-life* (EoL), Fig.2. Neither in EVs nor in renewable energy systems such specific operation conditions are realistic and thus there is no direct use of diagrams of type Fig.2. This leads to the following questions: How can one detect all cycles including their depth-of-cycles (DoC) for a given arbitrary SoC time series? Suppose this can be done, how can the result be classified via cycle-DoC histograms by choosing an appropriate ΔDoD bin?

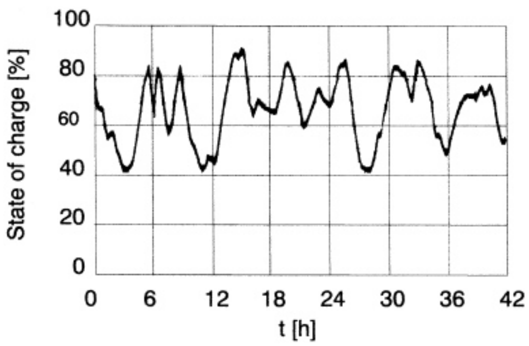


Figure 1: SoC time series of the battery during frequency regulation at BEWAG in Berlin [Wa07]

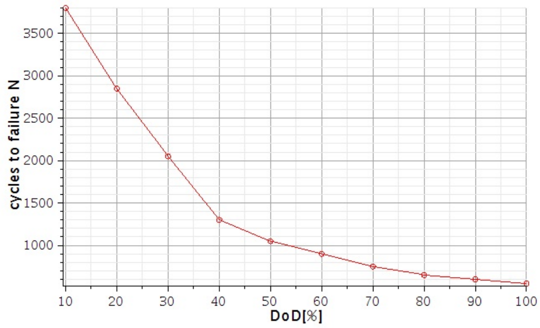


Figure 2: Cycle lifetime of flooded lead-acid battery in a renewable energy system; data supplied by manufacturer from [Bi05]

In this note we give a precise mathematical definition of a cycle, which we have not found in the literature, yet. By geometric interpretation, we thus derive an algorithm for detecting cycles in any SoC-time series. This leads to a transformation of any SoC-profile in a cycle-DoC histogram, which hence can be used in combination with Fig.2 for cycle lifetime prediction by means of relative frequency of occurred cycles n divided by N for each ΔDoD bin. Finally we show a relation of our cycle definition with the well known rainflow cycle counting method (RFC) from mechanical engineering. The notion of a cycle should be trivial, but a formal clear definition seems to be difficult. The encyclopedia of electrochemical power sources [Ku09] defines a cycle as *sequence of charge and discharge*. Similar statements can be found in many other literature. However, such definitions are problematic for cycle detection. Indeed a formal definition of a cycle will support the design of an appropriate cycle detection algorithm.

2 Theory

In this section we first define a cycle in formal way, followed by geometric interpretation and some basically remarks. Secondly we give a construction of a cycle detection algorithm of which input is the SoC time series, and a predefined ΔDOD (or equivalent ΔDOC) and its output is the cycle-DoD histogram. Thirdly we show how this can help to predict cycle lifetime. Fourthly we give a short explanation of a existing cycle detection algorithm from mechanical engineering, and fifthly and finally we show a relation to our cycle definition.

Definition of a cycle: Without loss of generality, we may assume, that the SoC profile is never local constant, that is, there is no $t_0 \in \mathbb{R}^+$ and no $\varepsilon > 0$ such that $SOC_{|(t_0-\varepsilon,t_0+\varepsilon)} = c, c \in \mathbb{R}_0^+$. Furthermore all results stated for SoC profiles are fully applicable to energy profiles, which is in practical application more important than SoC.

Definition 1. Let us denote $SOC(t)$ the SoC as a function of time $t \in \mathbb{R}_0^+$. Let $SOC(0) = 1$ and $0 \leq t_1 \leq t_2$. An Ah-throughput within a battery (or more general in a reversible energy storage device) at the interval $[t_1,t_2] \subset \mathbb{R}_0^+$ is called pre-cycle with respect to detph-of-cycle $DOC := SOC(t_1) - \min\{SOC(t) \,|\, t_1 \leq t \leq t_2\}$, if the following properties are fulfilled:

(i) $SOC(t_1) = SOC(t_2)$

(ii) $SOC(t) \leq SOC(t_1)$ for all $t_1 \leq t \leq t_2$

(iii) $(t_1, SOC(t_1))$ or $(t_2,SOC(t_2))$ is a local maximum point

Definition 2. A pre-cycle at the interval $[t_1,t_2]$ with respect to a given DoC is called cycle, if there is no pre-cycle with a proper greater interval such that it has the same local minimum point, that is, there is no interval $[t_3,t_4] \supsetneq [t_1,t_2]$ such that $\min\{SOC(t) \,|\, t_1 \leq t \leq t_2\} = \min\{SOC(t) \,|\, t_3 \leq t \leq t_4\}$ with respect to the same time of some $t_{\min}$.

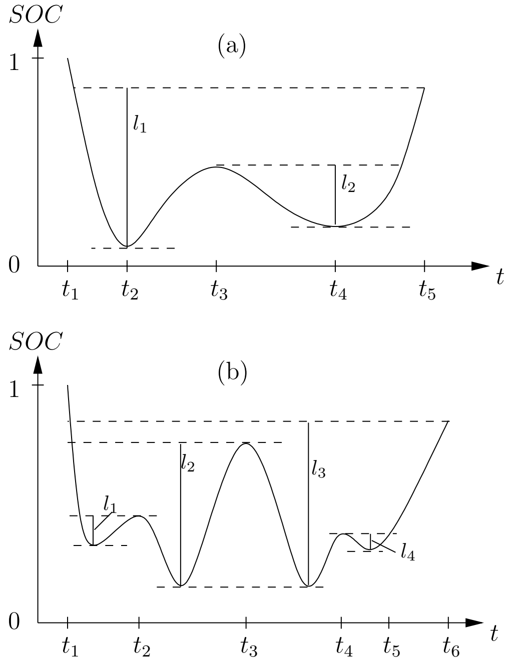


Figure 3: SoC-profiles for geometric interpretation of the (pre-)cycle definition

Notice: The maximum condition in Def. 1 (iii) includes also boundaries of the considered interval too. A cycle is obviously a pre-cycle but not vice versa. In principle a pre-cycle is a potential candidate for a cycle, but in a cycle detection algorithm the condition in Def. 2 ensures no overlapping and hence multiple counted cycles. A (pre-)cycle is uniquely determined by its data as given in Def. 1,2, which means, two (pre-)cycles are identical if and only if they are defined on the same interval with respect to the same DoC.

Geometric interpretation: A pre-cycle and its interval is determined by horizontal lines in the maximum and minimum points. The height l, its DoC, is the difference between the max- and min-value of SoC.

For example: Fig. 3 (a) shows 3 pre-cycles, but 2 cycles, (b) shows 7 pre-cycles, but only 4 cycles. The hight l_n's represent the related DoCs.

Remark 3. (i) For each *isolated* maximum point in the SoC time series there are precisely two pre-cycles, but at least one, if necessary with the smaller DoC, will be considered as cycle (see Fig. 3(a)).
(ii) Is there more than one minimum point at the same SOC, then the cycle-determination is independent of their choice. In Fig. 3 (b) there is a cycle at $DOC = l_2$ in $[t_1,t_3]$, or equivalent in $[t_3,t_6]$ of the same DoC. In consequence

the cycle of $DOC = l_3$ exchanges its minimum point.

Construction of cycle detection: With our cycle definition and its geometric interpretation in mind an algorithm (e.g. in matlab) for detecting cycles and its DoCs in arbitrary SoC profiles is easily constructed. The flow chart of the implemented matlab algorithm is shown in Fig. 4. The cycle-DoC histogram is parametrizes by choosing a certain DoD bin ΔDOD.

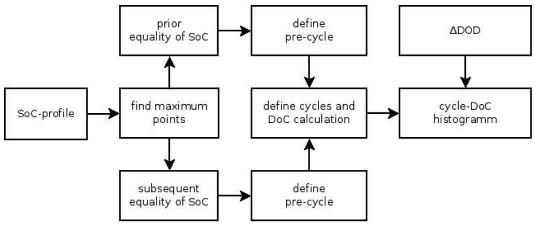


Figure 4: Operational principle of new cycle detecting method

From histogram to lifetime prediction For cycle lifetime prediction the so-called *event oriented aging model* is used[1] which is mathematically trivial. In our case an event is defined by a cycle at specifed DoC, normalized by the cycle $N(DOC)$ to failure with respect to a given DoC bin, that is $1/N(DOC)$. This corresponds to loss of cycle lifetime if a battery is cycled precisely one times with respect to this DoC. For practical reasons we specify a certain ΔDOC bin, which leads to a discretization of Fig.2 in ΔDOC bins and mean values of cycles to failure $\bar{N}$ at the boundaries of each DoC bin. Thus, there is $m \in \mathbb{N}$ such that $0 = DOC_0 < DOC_1 < \cdots < DOC_m = 1$ with $DOC_k = k \cdot \Delta DOC$, $k = 1,2,\ldots,m$ and hence $\Delta DOC = 1/m$, which means an equidistant subdivision of DoC-interval [0,1]. A *discrete* event in the above sense is $a^k_{\text{cyc}} := 1/\bar{N}_k$ whereas k specify the $DOC_k - DOC_{k-1}$ bin. Let $SOC(t)$ an arbitrary SoC time series. A cycle detection algorithm counts cycles with their DoCs $n_k(t)$ at time t and maps it to the related k'th DoC bin. Thus we obtain

$$A_{\text{cyc}}(t) = \sum_{k=1}^{m} n_k(t) a^k_{\text{cyc}} = \sum_{k=1}^{m} \frac{n_k(t)}{\bar{N}_k}, \qquad (1)$$

whereas $\frac{n_k(t)}{\bar{N}_k} =: h_k(t)$ is the relative frequency occuring the event relative loss of lifetime with respect to the k'th DoC bin until the time t. Hence $A_{\text{cyc}}(t)$ ist total relative loss of cycle lifetime until the time t of the battery. Of course Eq. (1) implies implicitly: first it assumes that the order of aging events is independent or, more precisely, the a^k_{cyc}'s are independent in the sense that a^k_{cyc} is not a function of a^l_{cyc} for all $k \neq l$. Secondly it assumes that the magnitude of a single aging event a^k_{cyc} does not depend on the total accumulated relative loss of lifetime, that is the a^k_{cyc} is not a function of n_k for all $k = 1,2,\ldots,m$. Notice, we haven't take calendar aging phenomena into account whereby e.g. the mean value of SoC around the cycling take place is also regarded. In consequence cycle-counting can provide only a part of full lifetime prediction.

[1] further details we refer to the review [SW08]

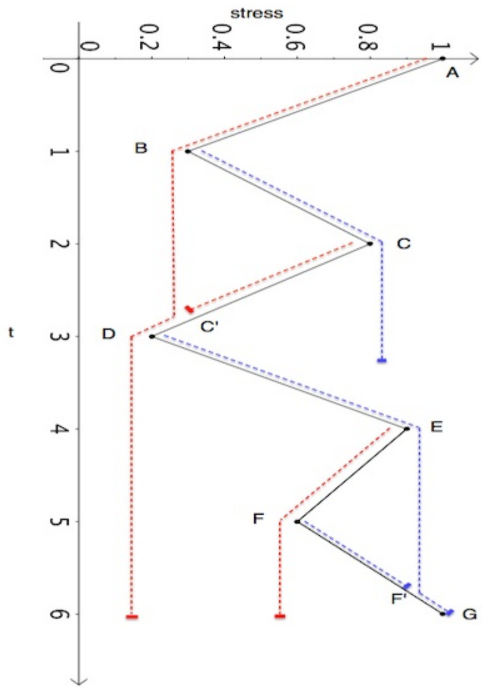


Figure 5: Operational principle of the RFC method

The rainflow cycle counting method: The algorithm was first introduced by Matsuishi and Endo in 1968 [ME68].
Imagine the time history is turned clockwise 90° as it can be seen in Fig. 5. If the right side of the curve is concerned, a blue line arises at every minimum, flows to the next maximum and drops down. If the down falling water rised from a lower minimum than the flow beneath, it stops the flow. Referred to Fig. 5 this can be seen at line segment $\overline{FG}$. Otherwise it will be stopped by the flow beneath, e.g. line segment $\overline{DE}$. A half-cycle is counted between a minimum and a maximum of one line. In our

case, there are three half-cycles made by blue lines, $\overline{BC}$, $\overline{DG}$ and $\overline{FF'}$. This has to be done analogously on the other side of the curve with the red lines, which arise at maximum points and drop down at minimum points.
A resulting cycle is a combination of two half cycles on the left and the right side of the curve with the same maximum and minimum values. There are three cycles in Fig. 5: ADG, BCC' and EFF'.
Several algorithms have been proposed in the literature for implementing the RFC. One of them is the three-point cycle counting technique which has been introduced by Downing and Socie in the year of 1982 [DS82].

Relation to our cycle definition: Both cycle detection methods produce cycles, but the results are obviously not the same. This is because our cycle is characterized by DoC *and* by its time stamps occurring in the history. The last condition is superfluous for the RFC, although it is used for computation. But its time stamps do not characterize the begin and the end of a cycle. In that way RFC gives no full definition of a cycle and hence is not equivalent to our definition. Regarding to the battery aging model point of view, the time stamps of a cycle are assumed to be irrelevant. In real applications these assumptions are only a rough approximation and not necessarily fulfilled [SW08]. We now state a trivial

Remark 4. (i) The RFC half cycle does not correspond to pre-cycle definition.
(ii) The water in the RFC method runs always in one direction, but the horizontal lines at the maximum points in our cycle counting method can also define a cycle backwards.
(iii) As we have seen in geometric interpretation, our references are the horizontal lines in the maximum points, but in the RFC this situation is symmetrical. This is motivated by the practical application of reversible energy storage systems.

Proposition 5. *Let* $\Delta DOC \in [0,1]$ *fixed. For given SoC time series both RFC counting algorithm and our method produce the same cycle-DoC histogram.*

Proof. First we notice, that Fig. 5 is obtained by connecting lines between maximum and minimum stress values of the underlying stress time series. This can also be done by any SoC profiles which lead to same type diagram for both methods. The assertion is a consequence of the following statements: 1. the condition in RFC section that 'down falling water rised from a lower minimum than the flow beneath, it stops the flow' is equivalent to Rem. 3 (i), and 2. 'otherwise the flow itself will be stopped' follows from Def. 2. □

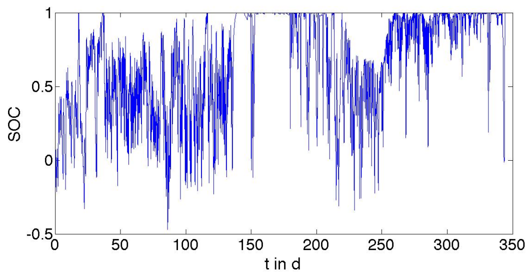
It's an easy exercise to prove the proposition in the case of Fig. 5.

3 Application to experimental data

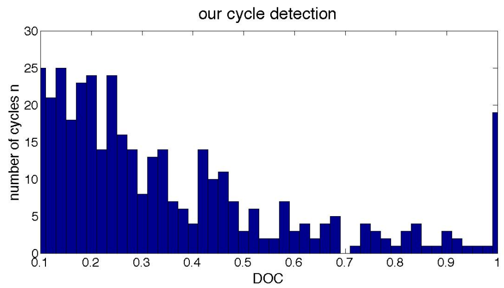
The cycle-DoC counting algorithm is applied on various test SoC time series, and, on real life SoC profiles as shown in Fig. 6. This SoC time series is taken from photovoltaic-wind-diesel hybrid system containing a lead-acid battery as shown in Table 1.

chemistry	type	capacity	voltage
lead-acid	flooded	250Ah	360V

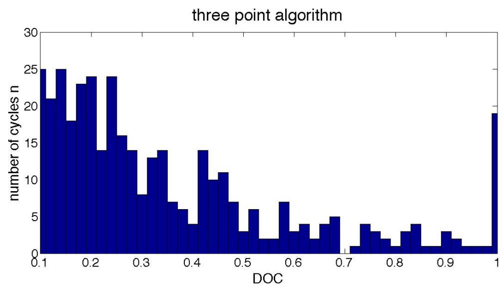
Table 1: Technical data of the battery from Fig. 6



(a) SoC time series



(b) DoC-cycle histogram



(c) DoC-cycle histogram

Figure 6: Validation on real life SoC time series of PV-wind-hybrid system with diesel generator and lead-acid battery [IZ99]

Both three point method and our cycle detection algorithm produce fully congruent DoC-cycle histograms as shown in Fig. 6. In this example a $\Delta DOC = 2\%$ was used. According Eq. (1) and the data in Fig. 2 the remaining cycle lifetime of the lead-acid battery after one year in the field is about 75%.

4 Conclusion

In a first view, the meaning of a cycle seems to be trivial, but a second view shows, this is not. Indeed for cycle detection in an arbitrary e.g. SoC time series a precise definition of a cycle is indispensable. Thus, first a formal precisely definition of cycle with notion to reversible energy storage systems is given and then use this definition for constructing an appropriate cycle detection and hence cycle counting algorithm. This is done in this publication in that manner, that any SoC profile can be transformed to a DoC-cycle histogram provided a certain DoC bin is specified. By means of data of battery manufacturer this histogram is used for a rough calculation of the remaining cycle lifetime. This was also demonstrated on data sets of a PV-wind-diesel hybrid system. Finally the relation of our algorithm is compared to the well known RFC counting method and it is proved that both methods produce the same DoC-cycle histogram (by the same DoC bin) in spite of the difference in cycle interpretation.

Further studies have to discuss the influence of the boundaries in SoC time series on cycle detection quality. In large and correct data sets the influence can be neglected, but often SoC profile data sets are corrupted which thus can be lead to problems.

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