UTM
UNIVERSITI TEKNOLOGI MALAYSIA

SECI 1O13 - DISCRETE STRUCTURE

SEMESTER 1 2023/2024

SECTION 02

ASSIGNMENT 2 (CHAPTER 2)

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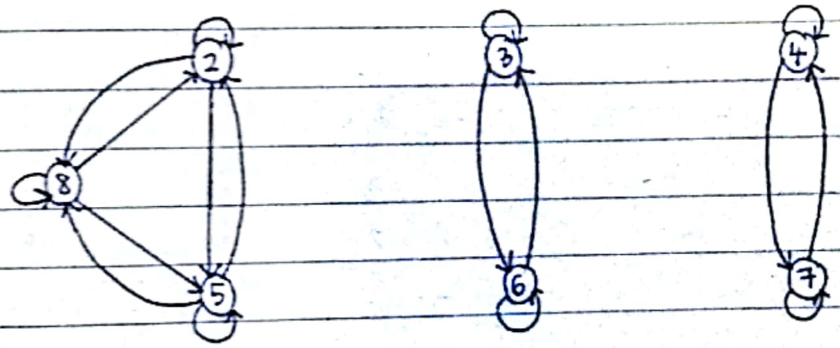
Q: Relation

1- $A = \{2, 3, 4, 5, 6, 7, 8\}$ xRy if $x - y = 3n$ $n \in \mathbb{Z}$

$R = \{(2,2), (3,3), (4,4), (5,5), (6,6), (7,7), (8,8), (8,5), (8,2), (7,4),$
$(6,3), (5,2), (5,8), (2,8), (4,7), (3,6), (2,5)\}$

directed graph:



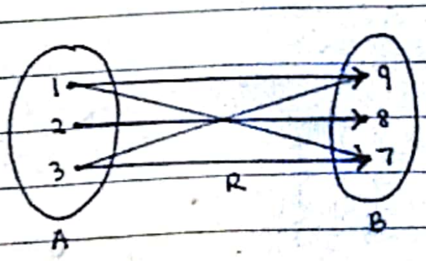
2. $A = \{1, 2, 3\}$ $B = \{9, 8, 7\}$

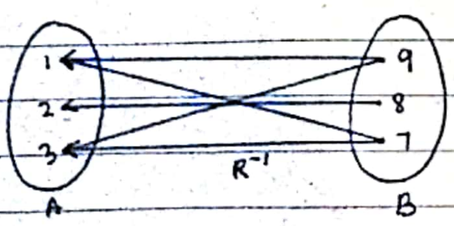
Let R: A to B. For ~~All~~ all ~~(a,x~~ $(a,b) \in A \times B$, and given $aRb \leftrightarrow a+b$ is an even number.

a. $R = \{(1,9), (1,7), (2,8), (3,9), (3,7)\}$

$R^{-1} = \{(9,1), (7,1), (8,2), (9,3), (7,3)\}$

b.





c. Let R^{-1} : B to A. For all $(b,a) \in B \times A$, and given $bR^{-1}a$ ~~$aR^{-1}b$~~ $\leftrightarrow b+a$ is an even number.

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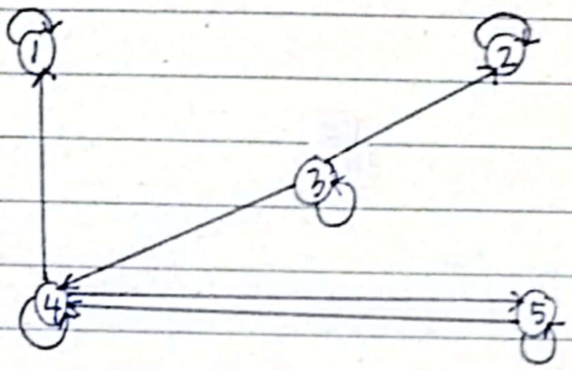
3. $A = \{1, 2, 3, 4, 5\}$

$R = \{(1,1), (2,2), (3,3), (4,4), (5,5), (3,2), (3,4), (4,1), (4,5), (5,4)\}$

digraph of R:

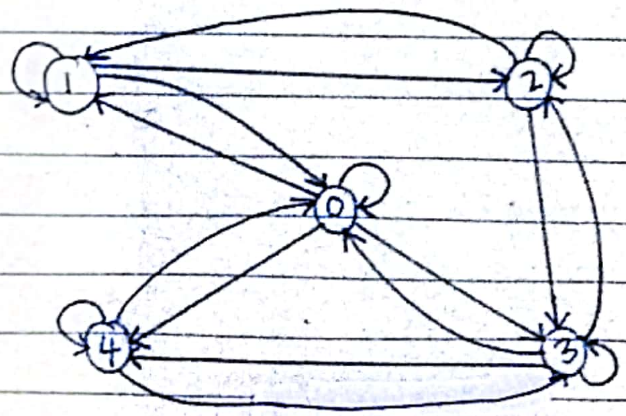


	1	2	3	4	5
in-degrees	2	2	1	3	2
out-degrees	1	1	3	3	2

4. $A = \{0, 1, 2, 3, 4\}$

$R = \{(0,0), (0,1), (0,3), (0,4), (1,0), (1,1), (1,2), (2,1), (2,2), (2,3), (3,0), (3,2), (3,3), (3,4), (4,0), (4,3), (4,4)\}$

digraph of R:



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$\therefore$ R is reflexive because $\{(0,0),(1,1),(2,2),(3,3),(4,4),(5,5)\} \in R$

$$M_r = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix} = M_r^T$$

$\therefore$ R is symmetric because $M_r = M_r^T$

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$\therefore$ R is transitive because $\forall i \forall j$, if $(n_{ij} = 1)$ then $(m_{ij} = 1)$

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5. $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14\}$

$R = \{(x, y) : 3x - y = 0\}$

$3x - y = 0$

$3x = y \qquad R = \{(1,3), (2,6), (3,9), (4,12)\}$

$\therefore$ The relation is irreflexive because $(1,1), (2,2), (3,3), (4,4) \notin R$.

It is antisymmetric because $(1,3) \in R$ but $(3,1) \notin R$

It is not transitive because $(1,3)$ and $(3,9) \in R$ but $(1,9) \notin R$.

6. $R = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad S = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

a) $RS = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

b) $SR = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

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7. Relation is a relationship between the variables x and y such that for each value of x may be 1 or more values of y.
Meanwhile function is a relationship between an independent variable x and an independent variable y such that for each 1 value of x there is 1 corresponding value of y.

~~8. i) The relations on set A is a function because each variable x have 1 variable y.~~
~~ii) It is a function because~~

8 i) It is a function because $f(2)=3$, $f(3)=4$, $f(4)=5$ and $f(5)=2$.
ii) It is a function because $f(2)=f(3)=f(5)=f(4)=4$.
iii) It is not a function because $(2,3)$ and $(2,4)$ in R but $3 \neq 4$ and domain of $\{2,5\}$ is not equal to A.
iv) It is not a function because $(2,3)$, $(2,2)$ in R but $3 \neq 2$ and $(4,4)$, $(4,5)$ in R but $4 \neq 5$.

9. $R = \{(x,y) \mid y = x+5,\ x \text{ is } Z^{+} < 6\}$

$y = x+5$, $x < 6$

$x = \{1,2,3,4,5\}$ $\quad$ $R = \{(1,6),(2,7),(3,8),(4,9),(5,10)\}$

~~$f(0) = 0+5$~~
~~$= 5$~~

$f(1) = 1+5$
$= 6$

$f(2) = 2+5$
$= 7$

$f(3) = 3+5$
$= 8$

$f(4) = 4+5$
$= 9$

$f(5) = 5+5$
$= 10$

$\therefore$ Domain of R is $\{1,2,3,4,5\}$
Range of R is $\{6,7,8,9,10\}$

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10. v) Let $x_1 = 0$, Let $x_2 = 1$, Let $x_3 = -1$,

$f(x) = 1 - 2x$ $f(x) = 1 - 2x$ $f(x) = 1 - 2x$

$f(0) = 1 - 2(0)$ $f(1) = 1 - 2(1)$ $f(-1) = 1 - 2(-1)$

$= 1$ $= -1$ $= 3$

$f(x_1) \neq f(x_2) \neq f(x_3)$ where $x_1 \neq x_2 \neq x_3$. Therefore, $f(x) = 1 - 2x$ is one-one.

$f(x) \in R$. Therefore, $f(x)$ is onto R.

$f(x) = 1 - 2x$ is bijective because $f(x)$ is one-one and onto.

vi) Let $x_1 = 1$, Let $x_2 = -1$,

$f(x) = 5x^2 - 1$ $f(x) = 5x^2 - 1$

$f(1) = 5(1)^2 - 1$ $f(-1) = 5(-1)^2 - 1$

$= 4$ $= 4$

$f(x_1) = f(x_2)$ where $x_1 \neq x_2$. Therefore, $f(x) = 5x^2 - 1$ is not one-one.

$f(x) \in R$. Therefore, $f(x)$ is onto R.

$f(x) = 5x^2 - 1$ is not bijective because $f(x)$ is not one-one.

vii) Let $x_1 = 1$, Let $x_2 = -1$,

$f(x) = x^4$ $f(x) = x^4$

$f(1) = 1^4$ $f(-1) = (-1)^4$

$= 1$ $= 1$

$f(x_1) = f(x_2)$ where $x_1 \neq x_2$. Therefore, $f(x) = x^4$ is not one-one.

$f(x) \in R$. Therefore, $f(x)$ is onto R.

$f(x) = x^4$ is not bijective because $f(x)$ is not one-one.

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10. viii) Let $x_1 = 1$, Let $x_2 = 0$, Let $x_3 = -1$,

$f(x) = \frac{x-2}{x-3}$ $f(x) = \frac{x-2}{x-3}$ $f(x) = \frac{x-2}{x-3}$

$f(1) = \frac{1-2}{1-3}$ $f(0) = \frac{0-2}{0-3}$ $f(-1) = \frac{-1-2}{-1-3}$

$= \frac{1}{2}$ $= \frac{2}{3}$ $= \frac{3}{4}$

$f(x_1) \neq f(x_2)$ where $x_1 \neq x_2$. Therefore, $f(x) = \frac{x-2}{x-3}$ is one-one.

For $f(x) = \frac{x-2}{x-3}$,

$x-3 \neq 0$

$x \neq 3$

Since $x \neq 3$, therefore $f(x)$ is not onto R.

$f(x) = \frac{x-2}{x-3}$ is not bijective because $f(x)$ is not one-one and not onto.

11. ix) $f(x) = 3x - 1$

$g(x) = x^2 - 1$

$fg(x) = f(x^2 - 1)$

$= 3(x^2 - 1) - 1$

$= 3x^2 - 3 - 1$

$= 3x^2 - 4$

Let $x = 0$, Let $x = 1$, Let $x = 2$, Let $x = 3$,

$fg(0) = 3(0)^2 - 4$ $fg(1) = 3(1)^2 - 4$ $fg(2) = 3(2)^2 - 4$ $fg(3) = 3(3)^2 - 4$

$= -4$ $= -1$ $= 8$ $= 23$

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11. x) $f(x) = x^2$

$g(x) = 5x - 6$

$fg(x) = f(5x-6)$

$= (5x-6)^2$

$= 25x^2 - 30x - 30x + 36$

$= 25x^2 - 60x + 36$

Let $x = 0$,
$fg(0) = 25(0)^2 - 60(0) + 36$
$= 36$

Let $x = 1$,
$fg(1) = 25(1)^2 - 60(1) + 36$
$= 1$

Let $x = 2$,
$fg(2) = 25(2)^2 - 60(2) + 36$
$= 16$

Let $x = 3$,
$fg(3) = 25(3)^2 - 60(3) + 36$
$= 81$

xi) $f(x) = x - 1$

$g(x) = x^3 + 1$

$fg(x) = f(x^3 + 1)$

$= x^3 + 1 - 1$

$= x^3$

Let $x = 0$,
$fg(0) = 0^3$
$= 0$

Let $x = 1$,
$fg(1) = 1^3$
$= 1$

Let $x = 2$,
$fg(2) = 2^3$
$= 8$

Let $x = 3$,
$fg(3) = 3^3$
$= 27$

12. xii) $a_n = 6a_{n-1} - 9a_{n-2}$; $a_0 = 1$, $a_1 = 6$

$a_2 = 6a_1 - 9a_0$

$= 6(6) - 9(1)$

$= 27$

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$a_3 = 6a_2 - 9a_1$

$a_3 = 6(27) - 9(6)$

$a_3 = 108$

$a_4 = 6a_3 - 9a_2$

$= 6(108) - 9(27)$

$= 405$

First few sequences of $a_n = 6a_{n-1} - 9a_{n-2}$ are $1, 6, 27, 108, 405, 1458, \ldots$

12. xiii) $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$; $a_0 = 2, a_1 = 5, a_2 = 15$

$a_3 = 6a_2 - 11a_1 + 6a_0$

$= 6(15) - 11(5) + 6(2)$

$= 47$

$a_4 = 6a_3 - 11a_2 + 6a_1$

$= 6(47) - 11(15) + 6(5)$

$= 147$

$a_5 = 6a_4 - 11a_3 + 6a_2$

$= 6(147) - 11(47) + 6(15)$

$= 455$

First few sequences of $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$ are $2, 5, 15, 47, 147, 455, \ldots$

xiv) $a_n = -3a_{n-1} - 3a_{n-2} + a_{n-3}$; $a_0 = 1, a_1 = -2, a_2 = -1$

$a_3 = -3a_2 - 3a_1 + a_0$

$= -3(-1) - 3(-2) + 1$

$= 10$

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$a_4 = -3a_3 - 3a_2 + a_1$

$= -3(10) - 3(-1) + (-2)$

$= -29$

$a_5 = -3a_4 - 3a_3 + a_2$

$= -3(-29) - 3(10) + (-1)$

$= 56$

The first few sequences of $a_n = -3a_{n-1} - 3a_{n-2} + a_{n-3}$ are $1, -2, -1, 10, -29, 56, \ldots$

13. i) $a_1 = k$

$a_2 = 5a_1 - 3$

$= 5k - 3$

$a_3 = 5a_2 - 3$

$= 5(5k-3) - 3$

$= 25k - 15 - 3$

$= 25k - 18$

$a_4 = 5a_3 - 3$

$= 5(25k - 18) - 3$

$= 125k - 90 - 3$

$= 125k - 93$

ii) $a_4 = 7$

$125k - 93 = 7$

$125k = 100$

$k = \frac{100}{125}$

$k = \frac{4}{5}$