Monte Carlo Integration

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In this project, we used the method of Monte Carlo integration to calculate the moment of inertia about x and z axis of a sphere with density which is not uniform. Also, we use the bin-average to calculate the average value of moment of inertia and its standard deviation.

1.Introduction and Theory

Monte Carlo integration is a technique for numerical integration using random numbers. It is a particular method of Monte Carlo methods that numerically computes a definite integral. While other algorithms usually evaluate the integrand at a regular grid, Monte Carlo algorithms randomly choose the points at which the integrand is evaluated. This method is particularly useful for higher dimensional integrals.

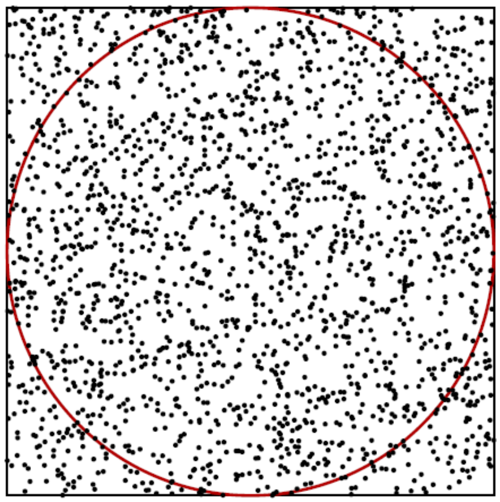

FIG. 1: The illustration of MC integration to get the area of a circle.

Monte Carlo integration is based on the simple fact that an integral can be expressed as an average of the integrand over the range, or volume, of integration, e.g., a one dimensional integral can be written as

$$A = \int_a^b f(x)\,\mathrm{d}x = (b-a) < f >$$

where $< f >$ is the average of the function in the range [a, b]. A statistical estimate of the average can be obtained by randomly generating N points $a < xi < b$ and calculating the arithmetic average

$$< f >= \frac{1}{N}\sum_{i=1}^{N} f(x_i)$$

This estimate approaches the true average $< f >$ as $N \longrightarrow \infty$, with a statistical error, which is proportional to $1/\sqrt{N}$. For example, like figure. 1 showed, in order to calculate the area of a circle,we can generate random numbers within a square. With the circle enclosed by a square, the fractional area inside the circle is estimated by generating points inside the square at random and counting the number of points that fall inside the circle. In one case, 2000 points were generated, and the fraction of points inside the circle is 0.791, which hence is the estimate of $\pi/4$ obtained in this calculation.

2.Problem and Programming

In this assignment, we will try to calculate the moment of inertial of the sphere(shown below) by using the method Monte Carlo integration.

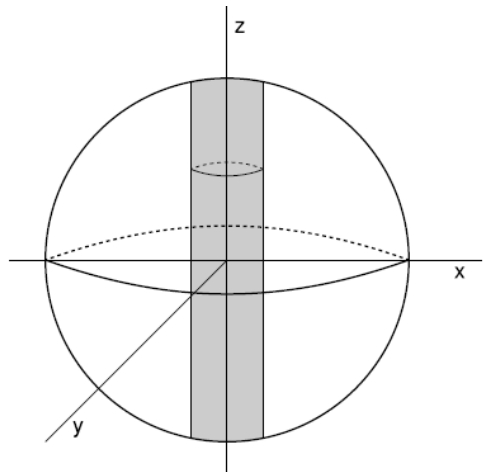

FIG. 2: The sphere with inuniform density.

Figure 2 showed a solid sphere of radius r1 with an inner solid cylinder of radius r2. The cylinder consists of a material with density $\rho 2$; the rest of the sphere consists of a material with density $\rho 1$.
To calculate the moment of inertia, we need to use the following integral:

$$A = \int_a^b \int_a^b \int_a^b f(x) * r^2(x,y,z)\, \mathrm{d}x \mathrm{d}y \mathrm{d}z.$$

3. Result

I used the MC integration method, and have found the minimum bin size which enable the relative standard deviation to be below 10^{-4}. By running the program, I got the following results(depend on different seed I found the result can be slightly different) : The bin number is 181 185. The moment of inertia around z axis is: $4.691(6) * 10^{-3}$ $kg * m^2$. The relative standard deviation of I(z) is : $9.0(6) * 10^{-5}$. The moment of inertia around x axis is: $4.947(7) * 10^{-3}$ $kg * m^2$. The relative standard deviation of I(x) is: $9.(8) * 10^{-5}$.