

INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI
DEPARTMENT OF MATHEMATICS

MA 322: SCIENTIFIC COMPUTING

Semester–II, Academic Year 2022-23

Lab – 1

L1.1. Some rational approximations to π are

$$\frac{22}{7}, \frac{333}{106}, \frac{355}{113}, \frac{104348}{33215}, \frac{1148183}{365478}, \frac{1252531}{398693}, \frac{2400714}{764171},$$
$$\frac{18057529}{5747890}, \frac{56573301}{18007841}, \frac{208236575}{66283474}, \frac{681280326}{216858263}.$$

Explore the absolute and relative errors involved in them. Use a symbolic manipulation program to compute these rational approximations for π .

L1.2. An interesting numerical experiment is to compute the dot product of the following two vectors:

$$x = [2.718281828, -3.141592653, 1.414213562, 0.5772156649, 0.3010299957]$$

$$y = [1486.2497, 878366.9879, -22.37492, 4773714.647, 0.000185049]$$

Compute the summation in four ways:

- (a) Forward order $\sum_{i=1}^n x_i y_i$
- (b) Backward order $\sum_{i=n}^1 x_i y_i$
- (c) Largest-to-smallest order (add positive numbers in order from largest to smallest, then add negative numbers in order from smallest to largest, and then add the two partial sums)
- (d) Smallest-to-largest (reverse the order of the addition in the previous method)

Compare the results with the correct value to seven decimal places, 0.1006571×10^{-8} .

L1.3. Write and execute a program to compute

$$f(x) = \sqrt{x^2 + 1} - 1$$

$$g(x) = x^2 / (\sqrt{x^2 + 1} + 1)$$

for a succession of values of x , such as $1/8, 1/8^2, 1/8^3, \dots$. Although $f = g$, the computer produces different results. Which results are reliable and which are not?

L1.4. Write and test a function for $f(x) = x^{-2}(1 - \cos x)$. Avoid loss of significance in subtraction for all arguments x and (of course) take care of the difficulty at $x = 0$.

L1.5. Using your computer, print the values of the functions

$$f(x) = x^8 - 8x^7 + 28x^6 - 56x^5 + 70x^4 - 56x^3 + 28x^2 - 8x + 1$$

$$g(x) = (((((((((x - 8)x + 28)x - 56)x + 70)x - 56)x + 28)x - 8)x + 1$$

$$h(x) = (x - 1)^8$$

at 101 equally spaced points covering the interval $[0.99, 1.01]$. Calculate each function in a straightforward way without rearranging or factoring. Observe that the three functions are identical. Account for the fact the printed values are not all positive as they should be. If a plotter is available, plot the functions near 1.0 using a magnified scale for the function values to see the variations involved.