INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI

DEPARTMENT OF MATHEMATICS

MA 322: SCIENTIFIC COMPUTING

Semester–II, Academic Year 2022-23

Lab – 6

L6_1. Write a subroutine to compute the integral

$$I(f) = \int_a^b w(x) f(x) \mathrm{d}x$$

using two-point (a) Gaussian-Laguerre quadrature formula, (b) Gaussian-Legendre formula, (c) Gaussian-Chebyshev formulas (both first and second kinds), and (d) Gaussian-Hermite formula. Here, $w(x)$ is the appropriate weight function and $a, b \in \mathbb{R}$. Compute $I(f)$ for $f(x) = e^x \sin x$ and display your results using six decimal places rounded arithmetic. Compute relative error for each of these quadrature formulas (whenever applicable) for $f(x) = e^x \sin x$.

(Write different subroutines for each quadrature formula. Use limit of integration that are standard for the respective quadrature formulas, e.g., for Gaussian-Legendre quadrature formula, $a = -1$ and $b = 1$.)

L6_2. $(n+1)$-point formula to approximate $f'(x_j)$ is given by

$$f'(x_j) = \sum_{k=0}^{n} f(x_k) L_k'(x_j) + \frac{f^{(n+1)}(\xi(x_j))}{(n+1)!} \prod_{k=0,\ k \neq j}^{n} (x_j - x_k).$$

The error in the finite difference approximation,

$$f' x_j = \sum_{k=0}^{n} f(x_k) L_k'(x_j),$$

is

$$E_n = \frac{f^{(n+1)(\xi(x_j))}}{(n+1)!} \prod_{k=0,\ k \neq j}^{n} (x_j - x_k).$$

Consider the following set of data Approximate $f'(2.0)$ using (a-b) three-point central

x_i	1.8	1.9	2.0	2.1	2.2
$f(x_i)$	10.889365	12.703199	14.778112	17.148957	19.855030

difference approximations, (c-d) three-point one-sided difference approximations, (e) five-point central difference approximations.

Observe that the data is drawn from the function $f(x) = xe^x$. Compute the absolute error of your approximation(s) and approximate the bound(s) on $|f^{(n+1)}(\xi(x_j))|$.