INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI
DEPARTMENT OF MATHEMATICS

MA 322: SCIENTIFIC COMPUTING
Semester–II, Academic Year 2022-23
Lab – 2

We are interested in finding the real roots of the following equations:

$$e^x - 3x^2 = 0, \tag{1}$$
$$x^3 = x^2 + x + 1, \tag{2}$$
$$e^x = \frac{1}{0.1 + x^2}, \tag{3}$$
$$x = 1 + 0.3\cos x. \tag{4}$$

L2_1. Write a program implementing the algorithm

(a) Bisection method,

(b) Newton's method,

(c) Secant method.

Use an error tolerance $\epsilon = 10^{-6}$. Compare the computational time in the above computations.

L2_2. Write a program implementing the fixed-point iteration to find real roots of equations (2) and (4) for appropriate $g(x)$ with an error tolerance $\epsilon = 10^{-6}$. Justify your answer.

L2_3. Write subprograms for carrying out the bisection method, Newton's method, and the secant method. They should apply to an arbitrary function F. In each case, the calling sequences should include a parameter $n_{\max}$ for the maximum number of steps the uses will allow. The user should be able to specify the accuracy desired.

L2_4. Test your subprograms on the function

(a)
$$f(x) = \tan^{-1} x - \frac{2x}{1 + x^2}.$$

Try to obtain the positive zero correct up to 8 decimal places.

(b)
$$f(x) = e^x - \tan x$$

to find the smallest positive zero (correct up to 8 decimal places), if exists.

L2_5. An iterative scheme is converges to α at a rate $p(\geq 1)$, provided

$$\lim_{n\to\infty} \frac{|\alpha - x_{n+1}|}{|\alpha - x_n|^p} = C, \tag{5}$$

for some constant $C \neq 0$. In practice, while using an iterative scheme to find roots of a nonlinear equation α is not known (in fact, if α is known, the iterative scheme is useless!). Therefore, we rather seek

$$|x_{n+1} - x_n| \approx C_n |x_n - x_{n-1}|^p, \qquad \forall n \geq n_0 \in \mathbb{N}$$

in our computations. We use this relation to determine the order of convergence of our iterative scheme as follows. Define $Y_n = \ln|x_{n+1} - x_n|$, $X_n = \ln|x_n - x_{n-1}|$. Fit a straight line passing through the points (X_n, Y_n) $(n \geq 1)$ – the slope of the fitted straight line corresponds to the order of convergence of the method p.

Find the positive roots of $x^3 + x^2 - 5x + 3 = 0$ using Newton's method using the tolerance $\epsilon = 10^{-7}$. Determine the rate of convergence. Compute the same root using the secant method and the same tolerance. Compare the order of convergence of the two methods. Finally, use the following iterates

$$x_{n+1} = x_n - 2\frac{f(x_n)}{f'(x_n)} \qquad n \geq 0$$

to find the positive root of the above equation starting with the same initial guess x_0 and for the same tolerance level. Do you observe any change in the order of convergence with this new iterates? What happens if the factor 2 in the second term is changed to an integer $m \geq 3$?

L2_6. **Newton's method for implicit functions.** Determine y (correct up to six decimal places, i.e., $\epsilon = 10^{-7}$) along the lines $x = 0$ through 20 where the function $G(x, y) = 3x^7 + 2y^5 - x^3 + y^3 - 3$ vanishes.