INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI

DEPARTMENT OF MATHEMATICS

MA 322: SCIENTIFIC COMPUTING

Semester–II, Academic Year 2022-23

Lab – 7

Taylor-Series Method: For the Taylor-series method, it is necessary to assume that various partial derivatives of f exist. Consider the initial value problem (IVP):

$$x' = f(x,t) \tag{1}$$

$$x(0) = x_0. \tag{2}$$

At the heart of the procedure is the Taylor series for x, which we write as

$$x(t+h) = x(t) + hx'(t) + \frac{h^2}{2!}x''(t) + \frac{h^3}{3!}x'''(t) + \frac{h^4}{4!}x^{(4)}(t) + \cdots \tag{3}$$

A Taylor-series method is called of order n, if we truncate terms of $\mathbf{O}(h^{n+1})$. For example, for a second order method,

$$\begin{aligned} x(t+h) &= x(t) + hx'(t) + \frac{h^2}{2!}x'' \\ &= x(t) + hf(x,t) + \frac{h^2}{2!}\left[f_t + f_x x'\right] \\ &= x(t) + hf(x,t) + \frac{h^2}{2!}\left[f_t + f_x f(x,t)\right]. \end{aligned} \tag{4}$$

L7_1. Write a computer program to solve the following initial value problem

$$x' = x + e^t + tx, \qquad x(1) = 2$$

on the interval [1, 3]. Use (a-b) Taylor-series methods of order 2 and 3, and (c-d) Euler methods. Test your method for different values of h. Determine the order of the numerical algorithms.

L7_2. Write a computer program to solve the following initial value problem

$$x' = \frac{t+x}{t-x}, \qquad x(1) = 0.$$

Prepare a table of the function $x(t)$ on the interval [0, 2] with steps of ± 0.01.

L7_3. The integral $\int \sqrt{1+x^3}\,\mathrm{d}x$ is one that integral that cannot be obtained by the method of elementary calculus. (It is an **elliptic integral**.) Prepare a table of the function

$$f(x) = \int_0^x \sqrt{1+t^3}\mathrm{d}t$$

on the interval $0 \le x \le 5$ by solving a suitable initial-value problem. Use the Taylor-series method of order 2 with $h = 1/64$ and forward Euler method with $h = 1/64$.

L7_4. Prepare a table of a **dilogarithm** function

$$f(x) = -\int_0^x \frac{\ln(1-t)}{t}\mathrm{d}t$$

on the interval $-2 \le x \le 0$ by solving a suitable initial-value problem.