GATE CSE 2006 | Question: 71

asked Sep 26, 2014 • retagged Jun 23, 2017 by **Arjun**

21,320 views

79

The 2^n vertices of a graph G corresponds to all subsets of a set of size n, for $n \geq 6$. Two vertices of G are adjacent if and only if the corresponding sets intersect in exactly two elements.
The number of vertices of degree zero in G is:

A. 1
B. n
C. $n+1$
D. 2^n

One modification for the above question is that they can ask the number of edges in the graph and it is equal to

(wihtout count self-loops) $n_{c_2} \ * \ (2^{n-1})_{c_2}$

Because we need to select two vertices and including these 2 vertices 2^{n-1}
Subsets are possible and we need to select any two vertices and there will be an edge between them. ChatGPT explanation

Add nc2 for the above eqn to self loops also

GO classes video solution

GATE CSE 1994 | Question: 24

asked Oct 5, 2014 • edited Dec 15, 2022 by **Gatecse**

7,355 views

33

An independent set in a graph is a subset of vertices such that no two vertices in the subset are connected by an edge. An incomplete scheme for a greedy algorithm to find a maximum independent set in a tree is given below:

```
V: Set of all vertices in the tree;
I := φ
while   V ≠ φ do
begin
    select a vertex u ∈ V such that
    _______;
    V := V - {u};
    if u is such that
    ________then I := I ∪ {u}
end;
Output(I);
```

A. Complete the algorithm by specifying the property of vertex u in each case.

B. What is the time complexity of the algorithm?

In the question we are asked to find the maximum independent set of a tree (a tree is a connected graph with no cycles). Finding the maximum independent set of a graph is an NP hard problem. But if the graph is restricted to a tree this problem not only becomes polynomial time solvable but can even be solved in linear time as shown here. The given algorithm in this question is using a greedy approach (not the optimal one). The greedy decision made here is to choose the vertex of minimum degree at any point. This greedy algorithm is guaranteed to work for trees and some other restricted class of graphs.

A. At each iteration we must select the vertex u with the least degree
B. u is added to I if there is no common edge between u and any vertex in I. For a single vertex this can take $O(|V|)$ time and hence for all the vertices this will take $O(|V|)^2$ time.

Complete algorithm is as follows:

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Complete algorithm is as follows:

```
V: Set of all vertices in the tree;
I := φ
while   V ≠ φ do
begin
    select a vertex u ∊ V such that
    degree(u) is minimum of all vertices in V
    V := V - {u};
    if u is such that
      no edge is common for u and any v ∊ I, then I := I ∪ {u}
end;
Output(I);
```

GATE CSE 2004 | Question: 81

asked Sep 18, 2014 • edited Jun 24, 2018 by **Shikha Mallick**

15,226 views

64

Let $G_1 = (V, E_1)$ and $G_2 = (V, E_2)$ be connected graphs on the same vertex set V with more than two vertices. If $G_1 \cap G_2 = (V, E_1 \cap E_2)$ is not a connected graph, then the graph $G_1 \cup G_2 = (V, E_1 \cup E_2)$

A. cannot have a cut vertex
B. must have a cycle
C. must have a cut-edge (bridge)
D. has chromatic number strictly greater than those of G_1 and G_2

We are given that G_1 and G_2 are connected. So, if we take any two vertices say v_i and v_j there must be path between them in both G_1 and G_2. Now, it is given that $G_1 \cap G_2$ is disconnected. That is, we have at least two vertices v_1 and v_2 such that there is no path between them in $G_1 \cap G_2$.

This means the path between v_1 and v_2 in G_1 and G_2 are **distinct**.

When we have two distinct paths between a pair of vertices in a graph, it forms a cycle.

answered Nov 24, 2015 • selected May 8, 2021 by **Gatecse**

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Srestha

Take example & Solve

Excellent Question ↓

GATE CSE 2014 Set 1 | Question: 51

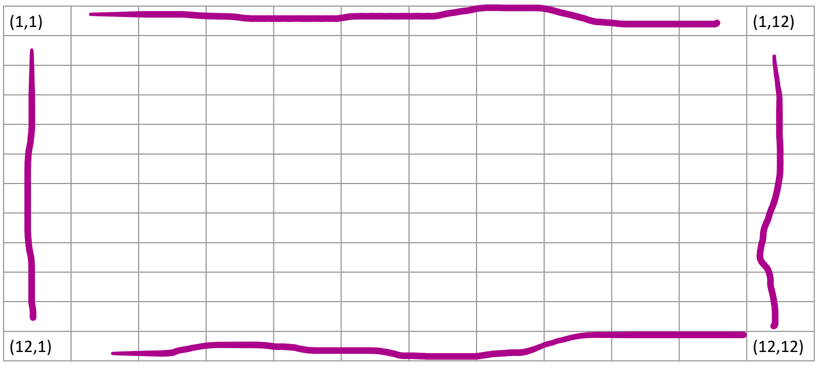
asked Sep 28, 2014 • retagged Jun 22, 2017 by **Silpa**

31,639 views

117

Consider an undirected graph G where self-loops are not allowed. The vertex set of G is $\{(i,j) \mid 1 \leq i \leq 12, 1 \leq j \leq 12\}$. There is an edge between (a,b) and (c,d) if $|a-c| \leq 1$ and $|b-d| \leq 1$. The number of edges in this graph is______.

Graph Theory #gatecse-2014-set1 #graph-theory #numerical-answers #normal #graph-connectivity



These corner vertices have degree 3 and there are 4 such vertices

Vertices in the line drawn have degree 5 each and there are 40 such vertices

And the remaining vertices have degree 8 and there are 144 - (40 +4) such vertices.

Total degree = 4*3 + 40*5 + 100 * 8
= 12 + 200 + 800
= 1012
Number of edges = 1012 /2
= 506

GATE CSE 2019 | Question: 38

asked Feb 7, 2019 • edited Dec 30, 2024 by **Deepak Poonia**

25,670 views

47

Let G be any connected, weighted, undirected graph.

I. G has a unique minimum spanning tree, if no two edges of G have the same weight.
II. G has a unique minimum spanning tree, if, for every cut of G, there is a unique minimum-weight edge crossing the cut.

Which of the following statements is/are TRUE?

A. I only
B. II only
C. Both I and II
D. Neither I nor II

34

1. If edge weights are distinct then there exist unique MST.

2. If for every cut of a graph there is a unique light edge crossing the cut then the graph has a unique minimum spanning tree but converse may not be true.

Proof by contradiction:
Lemma: If an edge e is contained in some minimum spanning tree, then it is a minimum cost edge crossing some cut of the graph.

Assume MST is not unique and there exist two MST's T_1 and T_2
Suppose $e_1 \in T_1$ but $e_1 \notin T_2$, if we remove e_1 from T_1, then we will have disconnected graph with two set of vertices V_1 and V_2. According to lemma e_1 is a minimum cost edge in the cut between V_1 and V_2.

Suppose $e_2 \in T_2$ but $e_2 \notin T_1$, if we remove e_2 from T_2, then we will have disconnected graph with two set of vertices V_1 and V_2. According to lemma e_2 is a minimum cost edge in the cut between V_1 and V_2.

Because the minimum cost edge is unique implies e_1 and e_2 is the same edge. $e_1 \in T_2$ and $e_2 \in T_1$. We have chosen e_1 at random, of all edges in T_1, also in T_2 and same for e_2. As a result, the MST is unique.

Why converse is not true always?

https://stanford.edu/~rezab/discrete/Midterm/midtermsoln.pdf

So both statements in the question are TRUE.

Answer is (C).

GATE CSE 2024 | Set 2 | Question: 7

asked Feb 16, 2024 • retagged Apr 27, 2024 by **Arjun**

13,508 views

14

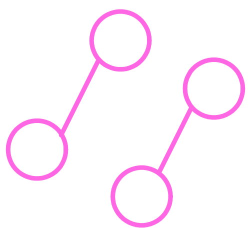
Let A be the adjacency matrix of a simple undirected graph G. Suppose A is its own inverse. Which one of the following statements is *always* TRUE?

A. G is a cycle
B. G is a perfect matching
C. G is a complete graph
D. There is no such graph G

The answer is based on $A^2 = I$

I Gives the number of two length walks between the same vertex

One such graph possible is given below :



This graph eliminates the options A,C and D. So the answer is B