

Implementation of Vector, Linear Transformation, and Quaternion Algebra in the Creation and Manipulation of Bézier Curves for 3D Computer Graphics

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Abstract—Untaian benang takdir mempersatukan kita, Teknik Informatika berjiwa kesatria. Takbir, Allahu Akbar!

Keywords—3D computer graphics, bézier curve, linear transformation, quaternion algebra, vector mathematics

I. INTRODUCTION

Computer graphics has been a broad and rapidly advancing field of research since the 1960s, contributing significantly to innovations across various fields of study [1], such as computer-aided design, virtual reality, and video game development. Computer graphics refers to the science and methods of creating and manipulating visual representations of computer objects [2]. It also includes the study of visual human-computer interaction, which involves the use of peripheral devices such as visual display units (VDUs) and keyboards to facilitate interaction. It is a multidisciplinary field of research involving engineering, physics, mathematics, computer science, visual art, and behavioral science [1]. In this paper, we will limit our discussions to 3D computer graphics exclusively. Hence, whenever we mention "computer graphics," we refer to "3D computer graphics".



Fig. 1. A 1963 demonstration of Ivan Sutherland's *Sketchpad*. Source: MIT Lincoln Laboratory

In 1963, Ivan Sutherland, an electrical engineering Ph.D. student at MIT Lincoln Laboratory, invented the *Sketchpad*. This innovative computer program marked the beginning of developments in computer graphics [3] [4]. *Sketchpad* significantly demonstrated the initial application of a graphical

user interface (GUI) and was an early form of computer-aided design (CAD) program, as it was specifically designed to create and interactively modify graphical objects represented as geometrical shapes. The program, written for an MIT Lincoln Laboratory TX-2 computer, involved the usage of a light pen as an input device enabling the creation of geometrical figures using line drawings instead of prompting through keyboard input [5], making it a real-time interactive GUI program.

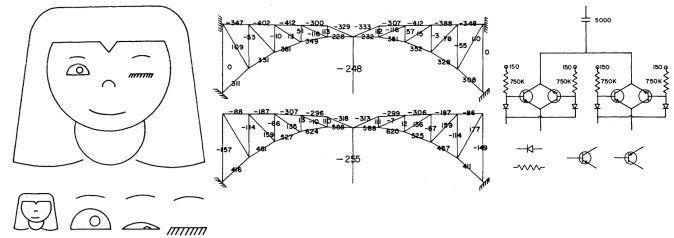


Fig. 2. Several examples of graphical illustration created in *Sketchpad* (from left): the winking lady "Nefertite" and her graphical component parts; truss diagrams of a cantilever and an arch bridge, with calculations of internal forces in the members; and an electrical circuit. Source: [5]

Sketchpad users could reuse and combine existing graphical objects to form a new one, letting the users structure its graphical objects hierarchically. Users could also create shapes using constraints. For instance, users could create a parallelogram by constraining the pair of sides to be parallel and having equal length. Sutherland's invention revolutionized human-computer interaction, object-oriented programming, and, especially, computer graphics modeling. Later on, Sutherland and David Evans, a computer science professor from the University of Utah, established Evans and Sutherland to manufacture hardware capable of running computer graphics programs. Due to his innovations, he was awarded the Turing Award in 1988 [4].

The next decades saw plentiful breakthroughs in the field. Better imagery could be generated due to the development of new methods and algorithms, including Z-buffer algorithm,

anti-aliasing, Phong shading, and ray tracing [6]. With a growing enthusiasm for research in computer graphics the Special Interest Group on Computer Graphics and Interactive Techniques (SIGGRAPH) was founded in 1974 as a platform for experts in computer graphics [7]. In the 1980s and 1990s, milestones in computer graphics included the release of 3D computer-animated films such as "Tron," "Toy Story," and "Jurassic Park"; the introduction of 3D modeling software such as Autodesk 3Ds Max and Maya and frameworks such as OpenGL and Direct3D; and the emergence of graphical processing units (GPUs) from companies like NVIDIA [6] [8]. These advancements laid the foundations for cutting-edge technologies in the field used today that can be seen everywhere, from entertainment, engineering, medical science, and many electronic appliances that we use daily.

One of the prominent concepts widely used in computer graphics is the Bézier curve. It was discovered in 1962 by the French engineer Pierre Bézier—hence its name. The mathematical groundwork to create such curve, de Casteljau's algorithm, had been discovered earlier in 1959 by Paul de Casteljau, a French mathematician. Bézier curves are defined by a parametric equation and a set of control points, with two points acting as the both ends of the curve while the rest acts as "weights". It is commonly available in computer graphics software, especially in 3D-modeling software like Blender and vector graphics software such as Adobe Illustrator.

II. THEORETICAL FOUNDATION

A. Vector

B. Linear Transformation

C. Quaternion

D. Spatial Rotation with Quaternion

E. Bézier Curve

III. ANALYSIS

IV. CONCLUSION

V. APPENDIX

The GitHub repository containing the source code for a simple demonstration, as well as the source of several images in this paper, can be accessed [here](#). The repository also includes detailed instructions on how to run the code.

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Teknik Informatika berjiwa kesatria ...*

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