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Homework 3

a) $f(n) = 9n$ and $g(n) = 5n^3$

$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{9n}{5n^3} = \lim_{n\to\infty} \frac{\frac{9}{n^2}}{\frac{5}{n}} = 0$, $\therefore f \in o(g)$

$f \notin O(g)$, $f \notin \Omega(g)$, $f \notin \Theta(g)$ and $f \notin \omega(g)$

$\lim_{n\to\infty} \frac{g(n)}{f(n)} = \lim_{n\to\infty} \frac{5n^3}{9n} = \lim_{n\to\infty} \frac{5n^2}{9} = \infty$, $\therefore g \in \omega(f)$

$g \notin o(f)$, $g \notin O(f)$, $g \notin \Omega(f)$, $g \notin \Theta(f)$

b) $f(n) = 9n^{0.8} + 2n^{0.3} + 14\log n$ and $g(n) = \sqrt{n}$

$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{9n^{0.8} + 2n^{0.3} + 14\log n}{\sqrt{n}}$

$= \lim_{n\to\infty} \left(\frac{9n^{0.8}}{n^{0.5}} + \frac{2n^{0.3}}{n^{0.5}} + \frac{14\log n}{n^{0.5}} \right)$

$= \lim_{n\to\infty} 9n^{0.3} + \lim_{n\to\infty} \frac{2}{n^{0.2}} + \lim_{n\to\infty} \frac{14\log n}{n^{0.5}}$

$= \infty + 0 + 0$

$= \infty$ $\therefore f \in \omega(g)$, $f \notin o(g)$, $f \notin O(g)$, $f \notin \Omega(g)$, $f \in \Theta(g)$

$\lim_{n\to\infty} \frac{g(n)}{f(n)} = \lim_{n\to\infty} \frac{\sqrt{n}}{9n^{0.8} + 2n^{0.3} + 14\log n}$

$= \lim_{n\to\infty} \frac{n^{0.5}}{9n^{0.8} + 2n^{0.3} + 14\log n}$

Here, we know that $(9n^{0.8} + 2n^{0.3} + 14\log n)$ approaches ∞ as n goes to ∞ faster than $n^{0.5}$.

$\therefore \lim_{n\to\infty} \frac{g(n)}{f(n)} = 0$

$\therefore$ ~~g~~ $g \in o(f)$, $g \notin O(f)$, $g \notin \Omega(f)$, $g \notin \Theta(f)$, $g \notin \omega(f)$

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c) $f(n) = \frac{n^2}{\log n}$ and $g(n) = n \log n$

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{n^{\cancel{2}}}{\log n} \times \frac{1}{\cancel{n}\log n} = \lim_{n\to\infty} \frac{n}{2\log n}$$

As $n \to \infty$ faster than $2\log n$, $\lim_{n\to\infty} \frac{f(n)}{g(n)} = \infty$

$\therefore f \in \omega(g), f \notin O(g), f \notin o(g), f \notin \Theta(g), f \notin \Omega(g)$

$$\lim_{n\to\infty} \frac{g(n)}{f(n)} = \lim_{n\to\infty} \frac{\cancel{n}\log n}{n^{\cancel{2}}} \times \log n = \lim_{n\to\infty} \frac{2\log n}{n}$$

As $n \to \infty$ faster than $2\log n$, $\lim_{n\to\infty} \frac{g(n)}{f(n)} = 0$

$\therefore g \in o(f), g \notin O(f), g \notin \Theta(f), g \notin \omega(f), g \notin \Omega(f)$

d) $f(n) = (\log(3n))^3$ and $g(n) = 9\log n$

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{(\log(3n))^3}{9\log n} = \lim_{n\to\infty} \frac{3(\log(3n))^3}{9\log n}$$

As $n \to \infty$, $(\cancel{3}/\cancel{x}\log 3n)^3 \to \infty$ faster than $9\log(3n)$,

$\therefore \lim_{n\to\infty} \frac{f(n)}{g(n)} = \infty$

$\therefore f \in \omega(g), f \notin o(g), f \notin O(g), f \notin \Omega(g), f \notin \Theta(g)$

$$\lim_{n\to\infty} \frac{g(n)}{f(n)} = \frac{9\log n}{(\log(3n))^3}$$

As $n \to \infty$, $(\log 3n)^3 \to \infty$ faster than $9\log(n)$,

$\therefore \lim_{n\to\infty} \frac{g(n)}{f(n)} = 0$

$\therefore g \in o(f), g \notin O(f), g \notin \Omega(f), g \notin \omega(f), g \notin \Theta(f)$