Homework 5
-Nayan Man Singh Pradhan

Problem 5.1

a.
All four method to compute Fibonacci numbers have been implemented in c++ file “5_1a.cpp”.

b.
The runtime is measured for all the methods using chrono standard library. To make the program stop after a certain amount of time, I have a while (1) loop that either stops when the max time has been reached or when all elements till n is executed in the loop. An example of the implementation is shown below (for naïve recursive method). I would recommend running the program with maxsize = 40 as the numbers start to wrap even when using long long as the data type. One can enter maxSize = 40 and time = 1 to see that the program stops after 1s.

```
auto indexTimeNaive = 0;
int nN=1;
while (1) {
    naive << nN << " ";
    auto startNaive = std::chrono::high_resolution_clock::now();
    naiveRecursive (nN);
    auto stopNaive = std::chrono::high_resolution_clock::now();
    auto durNaive = std::chrono::duration_cast
        <std::chrono::nanoseconds>(stopNaive - startNaive);
    naive << durNaive.count() << std::endl;
    indexTimeNaive += durNaive.count();
    if (indexTimeNaive >= maxTime) {
        std::cout << "Naive Recursive form stopped because max time reached!" << std::endl;
        break;
    }
    if (nN == userInput) {
        std::cout << "Naive Recursive form stopped because max n reached!" << std::endl;
        break;
    }
    nN++;
}
```

A table with the time taken for each method is made in a four different data files “naive.txt”, “bottom.txt”, “closed.txt”, “closed.txt”, and “matrix.txt” for naïve recursive method, bottom up method, closed form method, and matrix representation method respectively. The file should be created after the program “5_1b.cpp” is compiled and executed. I am using the “Power of Number” method while computing the power of terms in my matrix and closed form calculation in order to reach time complexity of $lg\ n$.

A table for the ‘n’ (in this case going from 1 to 40 because the data needs to fit in one page!) and the time taken to compute the Fibonacci series for the different methods. The time is calculated in nanoseconds! We notice that the time taken to compute the Fibonacci series gets considerably larger for the Naïve Recursive method (when comparting to the other data types).

n	Naïve Rec
1	139
2	168
3	152
4	196
5	320
6	401
7	480
8	533
9	716
10	987
11	1404
12	2182
13	3461
14	5572
15	8909
16	14340
17	22811
18	36842
19	59576
20	96197
21	156648
22	253129
23	407155
24	657429
25	1067920
26	1732901
27	2506440
28	4290888
29	5022036
30	7090633
31	10655819
32	15383734
33	23212773
34	37733338
35	59351110
36	95374154
37	152553353
38	259395961
39	416486242
40	673924852

n	Bottom up
1	265
2	881
3	697
4	1150
5	1120
6	1192
7	1122
8	1961
9	1976
10	1893
11	1910
12	1985
13	1924
14	2112
15	2022
16	3668
17	3559
18	3468
19	3523
20	3803
21	3533
22	3628
23	3493
24	3712
25	3499
26	3674
27	3556
28	3697
29	3520
30	3616
31	3589
32	7083
33	6790
34	6792
35	6723
36	7111
37	6733
38	6858
39	6744
40	7092

n	Matrix Rep
1	269
2	855
3	796
4	1317
5	1400
6	1354
7	1233
8	2391
9	2505
10	1938
11	2052
12	1762
13	1385
14	1407
15	1395
16	2640
17	2608
18	2674
19	2632
20	2741
21	2741
22	2698
23	2698
24	2694
25	2689
26	2750
27	2675
28	2710
29	2690
30	2677
31	2625
32	5099
33	5098
34	5085
35	5102
36	5082
37	5092
38	5092
39	5066
40	5091

n	Closed
1	212
2	721
3	602
4	1093
5	1101
6	1100
7	1127
8	1972
9	1995
10	2046
11	2158
12	2184
13	2165
14	2078
15	2082
16	3767
17	3729
18	3783
19	3694
20	3815
21	3841
22	3731
23	3791
24	3852
25	3851
26	3803
27	3799
28	3811
29	3799
30	3724
31	3746
32	7329
33	7311
34	7344
35	7255
36	7276
37	7299
38	7172
39	7284
40	7220

c.

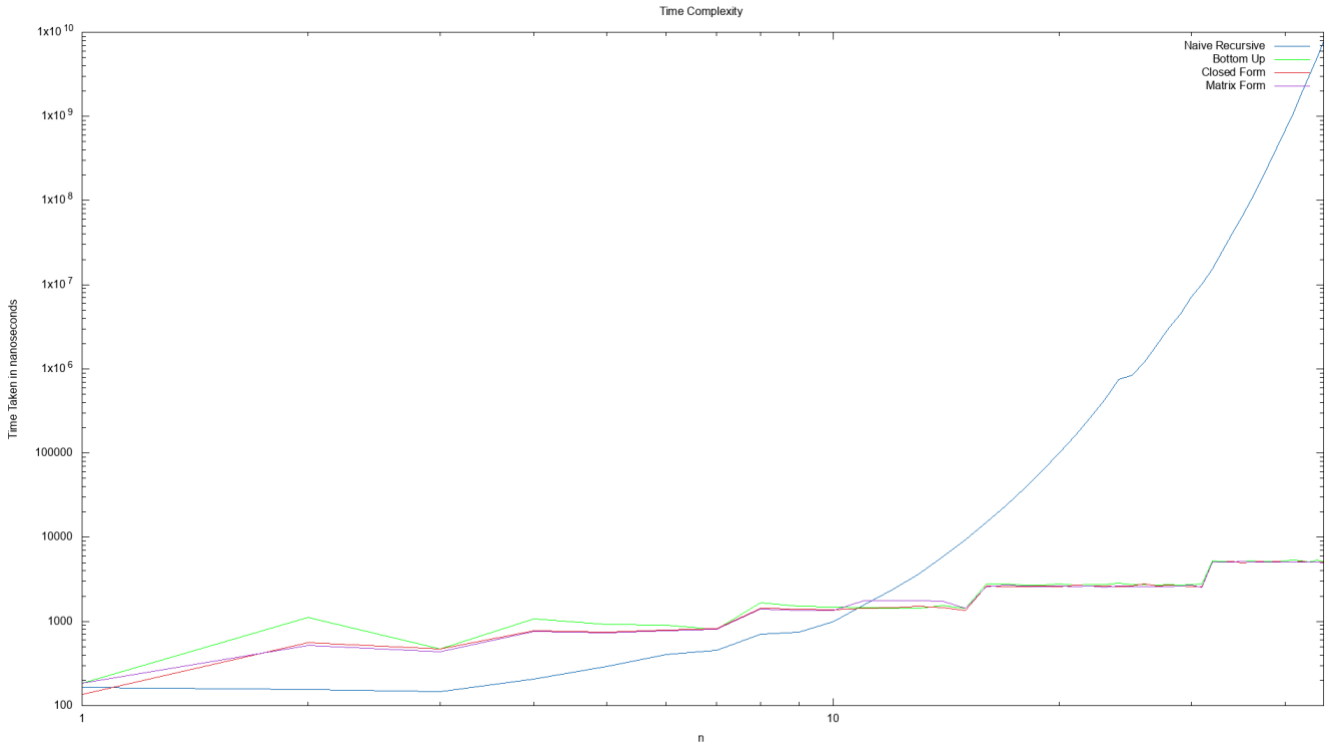
For the same value of n, all four methods might not return the same Fibonacci number! This is because, as the value of n gets larger and closer to infinity, the Closed Form approach contain floating-point errors. We use the equation involving the terms $pos = \frac{(1+\sqrt{5})}{2}$ and $neg = \frac{(1-\sqrt{5})}{2}$ as a long double values, then subtract neg from pos, compute the power of the term to n, then finally divide the term by the square root of 5! This value may round off certain terms based on the property and size of the data type "double", which may give possible errors.

```
long double pos = ((1 + sqrt(5))/2); //for formula in closed Form (global)
long double neg = ((1 - sqrt(5)))/2; //for formula in closed From (global)

 long long closedForm (int n) {
     if (n == 1 || n == 0) { //base cases
         return n;
     }
     else { //formula
         return ((powerClosed(pos, n) - powerClosed(neg, n))/sqrt(5));
     }
 }

 long double powerClosed (long double value, int n) {
     if (n == 0) { //base case
         return 1;
     }
     else if (n % 2 == 0) { //even case recursive
         return (powerClosed(value, n/2) * powerClosed(value, n/2));
     }
     else { //odd case recursive
         return (value * (powerClosed(value, (n-1)/2)) *
             (powerClosed(value, (n-1)/2)));
     }
 }
```

d.
The graph has been plotted using gnuplot. To get the graph, one must type “gnuplot plot.plt” into the terminal. I have attached a picture of the graph below (and also in my zip file for reference). The graph shows that Naïve Recursive takes exponential time (the most). The graph is made with logarithmic scales so that the values can be seen clearly.

Time complexity for the following methods:

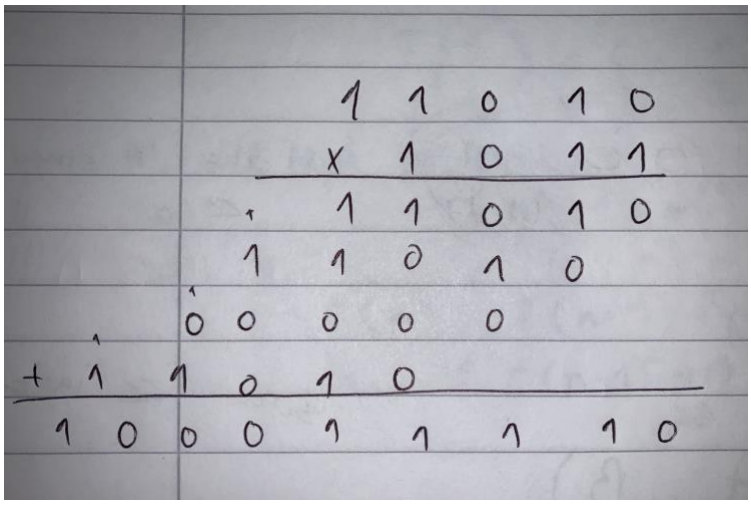
Naïve Recursive: $T(n) = \Omega(2^{\frac{n}{2}})$
Closed Form: $T(n) = \Omega(\lg n)$ when using the “power of number” recursion
Bottom Up Approach: $T(n) = \theta(\lg n)$
Matrix Representation: $T(n) = \theta(\lg n)$



Problem 5.2

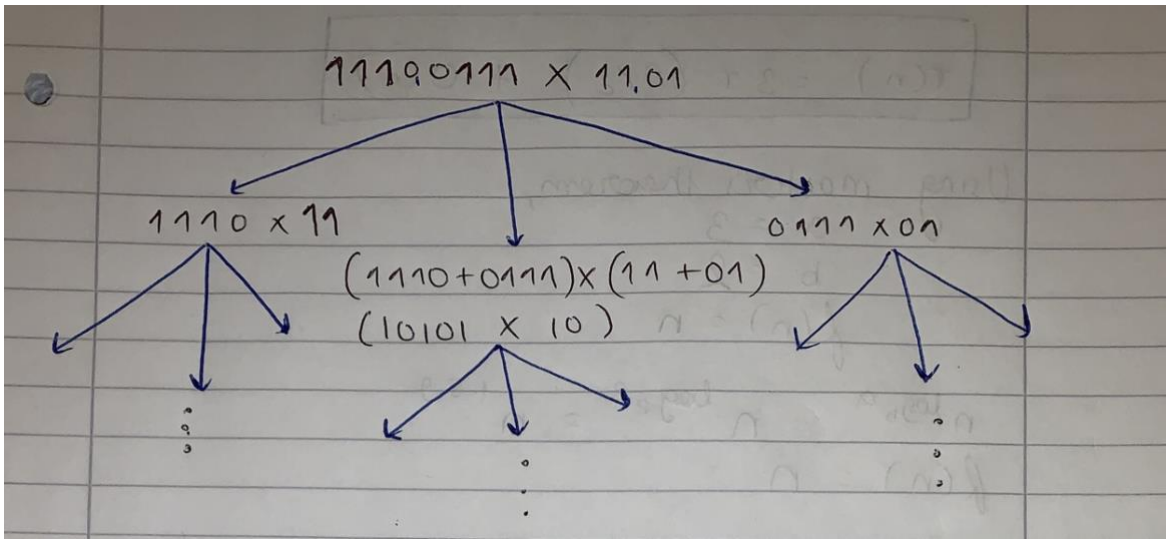
(I took help from https://people.eecs.berkeley.edu/~vazirani/algorithms/chap2.pdf)

a.




```
let firstNum = 1011
let secondNum = 11010
let sum = 0;
let tempSum = 0;
let totalSum = 0;

for i = 1 upto lengthof.firstNum
   for j = 1 upto lengthof.secondNum
      ans = firstNum[j] * secondNum[i]
      bitshift ans << j-1
      sum += ans;
   tempSum += ans << i-1
totalSum += tempSum;

//time complexity for one multiplicatin is n. There
are two nested loops running n times, therefore,
time complexity is bigO (n^2).
```

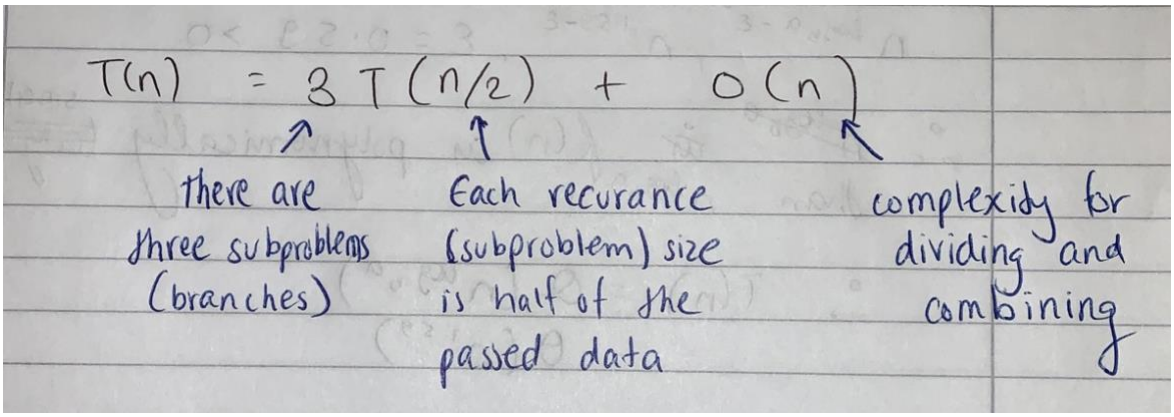
b.
//assume n to be a power of 2 = even




```
let firstNum = 11100111
let secondNum = 1101

divAndConq (firstElem, secondElem) {
   int len = maximumOf(lengthof.firstElem, lengthof.secondElem)
   if (len == 1) {
      return firstElem * secondElem
   }
   else {
      firstL = left [0 -> lengthof.firstElem/2]
      firstR = right [(lengthof.firstElem/2)+1 -> lengthof.firstElem]
      secondL = left [0 -> lengthof.secondElem/2]
      secondR = right [(lengthof.secondElem/2)+1 -> lengthof.secondElem]
      Mult1 = divAndConq (firstL, secondL)
      Mult2 = divAndConq (firstR, secondR)
      temp = divAndConq (firstL+firstR, secondL+secondR)
      return Mult1*2^n + (temp-Mult1-Mult2) * 2^n/2 + Mult2 //multiplied by 2^n to bit shift
   }
}
```

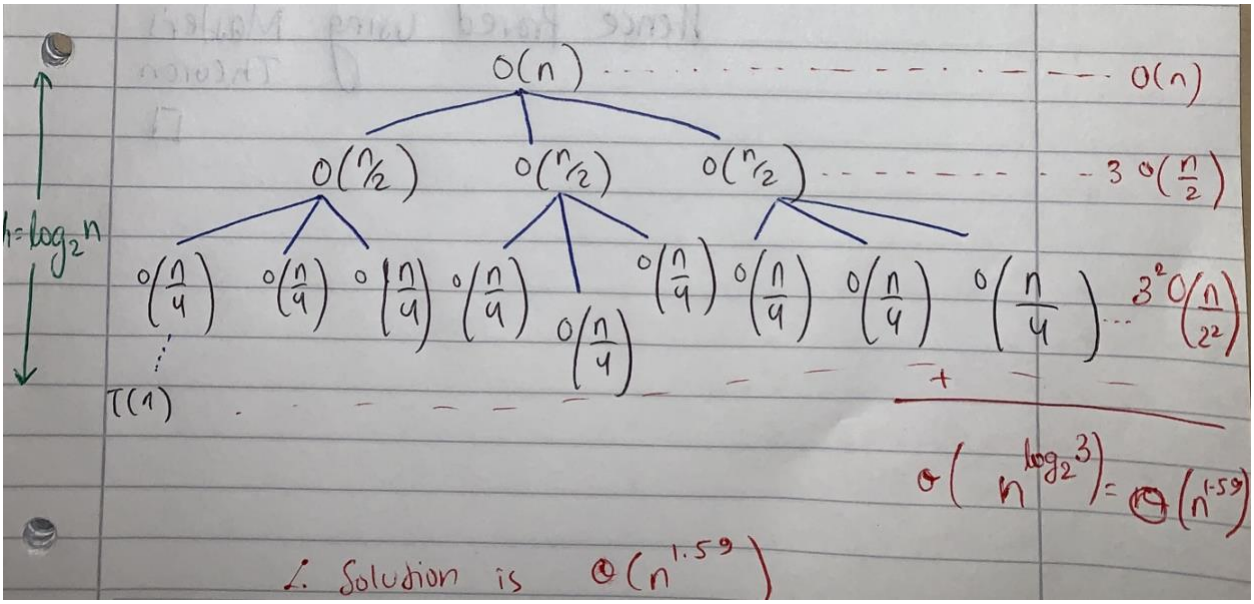
c.

The recurrence for the time complexity of the Divide and Conquer algorithm was created by looking at the tree I made in question 5.2b and the recurrence tree made in question 5.2c.

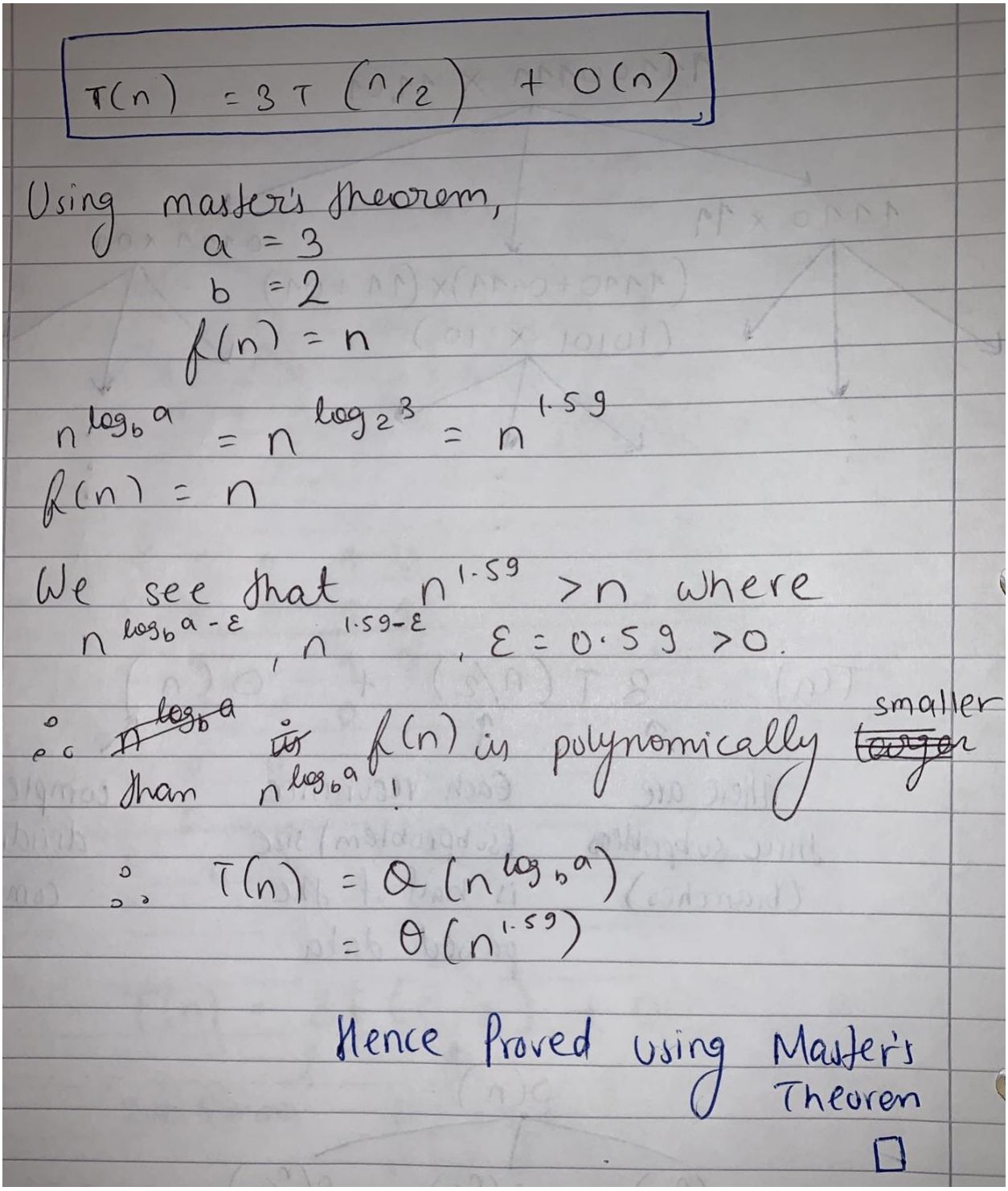


d.

Using recurrence tree:



e.



*I have included all the pictures in file "5_2pictures"