Assignment 8

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Submitted: 14th April 2020

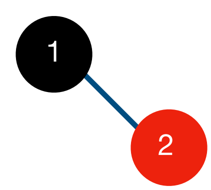
Problem 9.1 a, b
I have added photos named 9_1a_i, 9_1b_ii and 9_1b.

Problem 9.1c

```
RB-INSERT(T, z)
y = T.nil
x = T.root
while x ≠ T.nil
    y = x
    if z.key < x.key
        x = x.left
    else x = x.right
z.p = y
if y == T.nil
    T.root = z
elseif z.key < y.key
    y.left = z
else y.right = z
z.left = T.nil
z.right = T.nil
z.color = RED
RB-INSERT-FIXUP(T, z)
```

To Prove:
The red-black tree formed by inserting n nodes with the algorithm described in the lecture slides (left) contains at least one red node provided that n>1.

Base Case n = 2:
Let z = 1 and 2 for n = 1 and 2 respectively. When n = 1, z.parent = NULL and T.root = z. The second property of a red black tree says root needs to be black, therefore z.color = BLACK. When n = 2, z = 2 is inserted to the right of 1 and the z.color is RED by default. Because this does not violate any red black tree property, we have one red node.



Assume for n = k:
There is at least one red node.

Now for n = k+1:
We know that n = k contains at least one red node, therefore n = k+1 is just n = k + one extra term, which can be red or black. In either case, there is already a red node in the tree.

Therefore Proved

Problem 9.2

Implemented in cpp (makefile included).