Assignment 12

Done By: Nayan Man Singh Pradhan

Problem 12.1

A friend wants help in implementing an algorithm for finding the shortest path between two nodes u and v in a directed graph (possibly containing negative edge weights). He/she proposes the following:

- Add a large constant to each weight such that all weights become positive
- Run Dijkstra's algorithm for the shortest path from u to v

This method is not correct as it may not always work for all cases. A counterexample for when this method does not work:

Assume we have a start point "u" and an end point "v" with "n" vertices/nodes.

Path 1:

$$u \xrightarrow{-3} a \xrightarrow{8} v$$

Length of path 1: $-3 + 8 = 5$

Path 2:

$$u \xrightarrow{-3} a \xrightarrow{3} b \xrightarrow{4} v$$

Length of path 2: $-3 + 3 + 4 = 4$

We can see **that path 2 is the shorter path**. Now, we add the largest constant (in both cases -3) in both the paths!

Path 1:

$$u \xrightarrow{0} a \xrightarrow{11} v$$

Length of path 1: $0 + 11 = 11$

Path 2:

$$u \xrightarrow{0} a \xrightarrow{6} b \xrightarrow{7} v$$

Length of path 2: $0 + 6 + 7 = 13$

Now, in this case, when we add a positive large constant in both paths, **path 1 is the shorter path**. Therefore, we disprove the correctness of the given algorithm using a simple counter example!

Problem 12.2

Implemented in “OMP.cpp”. Execute make to run.

Problem 12.3

a.

The problem given to us can be represented as a graph problem.

i. Consider our board B.
ii. B has coordinates B[x][y] that represents the position of the player.
iii. Every coordinate B[x][y] (position) in the board is a node.
iv. All edges are 1 as the distance moved is 1 (in all directions form the current position)
v. The vertex $V = \{0, 1, 2, \ldots, n^2 - 1\}$
vi. Edges of vertex V are neighboring nodes, i.e. up, down, left, right.