hw3

November 16, 2020

1 Computer Vision

2 Jacobs University Bremen

3 Fall 2020

4 Homework 3

This notebook includes both coding and written questions. Please hand in this notebook file with all the outputs and your answers to the written questions.

This assignment covers Canny edge detector and Hough transform.

[1]:
```
# Done By: Nayan Man Singh Pradhan

# Setup
from __future__ import print_function
import numpy as np
import matplotlib.pyplot as plt
import matplotlib.image as mpimg
from time import time
from skimage import io

%matplotlib inline
plt.rcParams['figure.figsize'] = (15.0, 12.0) # set default size of plots
plt.rcParams['image.interpolation'] = 'nearest'
plt.rcParams['image.cmap'] = 'gray'

# for auto-reloading extenrnal modules
%load_ext autoreload
%autoreload 2
```

4.1 Part 1: Canny Edge Detector (75 points)

In this part, you are going to implement a Canny edge detector. The Canny edge detection algorithm can be broken down in to five steps: 1. Smoothing 2. Finding gradients 3. Non-maximum suppression 4. Double thresholding 5. Edge tracking by hysterisis

4.1.1 1.1 Smoothing (10 points)

Implementation (5 points) We first smooth the input image by convolving it with a Gaussian kernel. The equation for a Gaussian kernel of size $(2k+1) \times (2k+1)$ is given by:

$$h_{ij} = \frac{1}{2\pi\sigma^2} \exp\Big(-\frac{(i-k)^2 + (j-k)^2}{2\sigma^2}\Big), 0 \leq i, j < 2k+1$$

Implement **gaussian_kernel** in edge.py and run the code below.

[2]:
```
from edge import conv, gaussian_kernel

# Define 3x3 Gaussian kernel with std = 1
kernel = gaussian_kernel(3, 1)
kernel_test = np.array(
    [[ 0.05854983, 0.09653235, 0.05854983],
     [ 0.09653235, 0.15915494, 0.09653235],
     [ 0.05854983, 0.09653235, 0.05854983]]
)

# Test Gaussian kernel
if not np.allclose(kernel, kernel_test):
    print('Incorrect values! Please check your implementation.')
```

[3]:
```
# Test with different kernel_size and sigma
kernel_size = 5
sigma = 1.4

# Load image
img = io.imread('iguana.png', as_gray=True)

# Define 5x5 Gaussian kernel with std = sigma
kernel = gaussian_kernel(kernel_size, sigma)

# Convolve image with kernel to achieve smoothed effect
smoothed = conv(img, kernel)

plt.subplot(1,2,1)
plt.imshow(img)
plt.title('Original image')
plt.axis('off')

plt.subplot(1,2,2)
plt.imshow(smoothed)
plt.title('Smoothed image')
plt.axis('off')

plt.show()
```

Question (5 points) What is the effect of changing kernel_size and sigma?

Your Answer: In a Gaussian Kernel, the sigma determines how much the targeted pixel (center pixel in the kernel) is influenced/affected by the neighbouring pixels. If the value of sigma is high, the variance around the mean is wide, and therefore, the image is more blurred/smoothened as the central pixel is highly influenced by its neighbouring pixels. Likewise, if the value of sigma is low, the variance around the mean is thin, making the image less blurred/smoothened as the central pixel is only slightly influenced by its neighbouring pixels. The kernel_size, say n, is the dimension of the nxn kernel. The larger the kernel_size, the more smoother/blurrier the image can get. The value of sigma and kernel_size must be compatible for the most optimal result (ie, one cannot be too high and other cannot be too low). To get a high smoothening effect, both the kernel_size and sigma must be high.

4.1.2 1.2 Finding gradients (15 points)

The gradient of a 2D scalar function $I : \mathbb{R}^2 \rightarrow \mathbb{R}$ in Cartesian coordinate is defined by:

$$\nabla I(x,y) = [\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y}],$$

where

$$\frac{\partial I(x,y)}{\partial x} = \lim_{\Delta x \to 0} \frac{I(x+\Delta x, y) - I(x,y)}{\Delta x} \frac{\partial I(x,y)}{\partial y} = \lim_{\Delta y \to 0} \frac{I(x, y+\Delta y) - I(x,y)}{\Delta y}.$$

In case of images, we can approximate the partial derivatives by taking differences at one pixel intervals:

$$\frac{\partial I(x,y)}{\partial x} \approx \frac{I(x+1,y) - I(x-1,y)}{2} \frac{\partial I(x,y)}{\partial y} \approx \frac{I(x,y+1) - I(x,y-1)}{2}$$

Note that the partial derivatives can be computed by convolving the image I with some appropriate kernels D_x and D_y:

$$\frac{\partial I}{\partial x} \approx I * D_x = G_x \frac{\partial I}{\partial y} \approx I * D_y = G_y$$

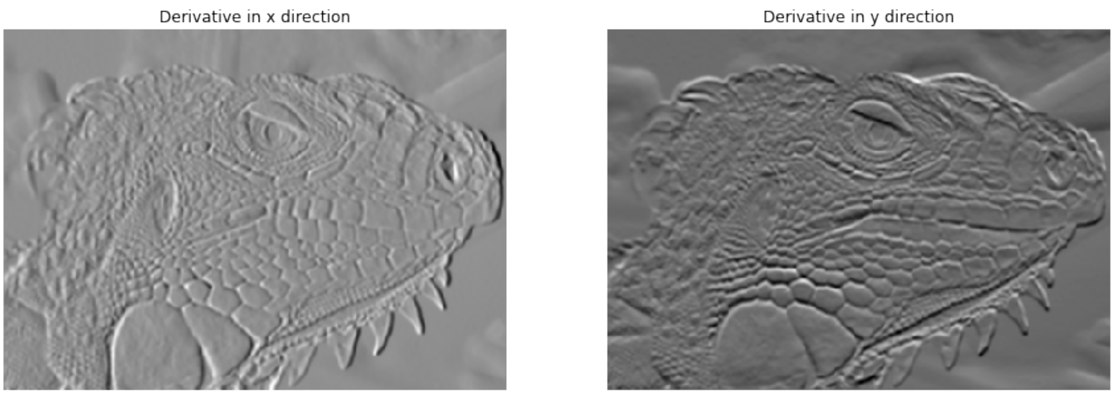
Implementation (5 points) Find the kernels D_x and D_y and implement **`partial_x`** and **`partial_y`** using `conv` defined in `edge.py`.

-Hint: Remeber that convolution flips the kernel.

[4]:
```
from edge import partial_x, partial_y

# Test input
I = np.array(
    [[0, 0, 0],
     [0, 1, 0],
     [0, 0, 0]]
)

# Expected outputs
I_x_test = np.array(
    [[ 0, 0, 0],
     [ 0.5, 0, -0.5],
     [ 0, 0, 0]]
)

I_y_test = np.array(
    [[ 0, 0.5, 0],
     [ 0, 0, 0],
     [ 0, -0.5, 0]]
)

# Compute partial derivatives
I_x = partial_x(I)
I_y = partial_y(I)

# Test correctness of partial_x and partial_y
if not np.all(I_x == I_x_test):
    print('partial_x incorrect')

if not np.all(I_y == I_y_test):
    print('partial_y incorrect')
```

[5]:
```
# Compute partial derivatives of smoothed image
Gx = partial_x(smoothed)
Gy = partial_y(smoothed)

plt.subplot(1,2,1)
plt.imshow(Gx)
```

```python
plt.title('Derivative in x direction')
plt.axis('off')

plt.subplot(1,2,2)
plt.imshow(Gy)
plt.title('Derivative in y direction')
plt.axis('off')

plt.show()
```



Question (5 points) What is the reason for performing smoothing prior to computing the gradients?

Your Answer: It is essential to perform image smoothing prior to computing the gradient. This is because, any sudden change in the image (for example salt-pepper noise) results in sudden change in intensity of neighbourhood pixels which causes abrupt ups and downs in the gradient of the image, which can be avoided if the image is smoothened before computing the gradient.

Implementation (5 points) Now, we can compute the magnitude and direction of gradient with the two partial derivatives:

$$G = \sqrt{G_x^2 + G_y^2} \Theta = arctan(\frac{G_y}{G_x})$$

Implement **gradient** in `edge.py` which takes in an image and outputs G and Θ.

[6]:
```python
from edge import gradient

G, theta = gradient(smoothed)

if not np.all(G >= 0):
    print('Magnitude of gradients should be non-negative.')
```

```
if not np.all((theta >= 0) * (theta < 360)):
    print('Direction of gradients should be in range 0 <= theta < 360')

plt.imshow(G)
plt.title('Gradient magnitude')
plt.axis('off')
plt.show()
```

Gradient magnitude

4.1.3 1.3 Non-maximum suppression (15 points)

You should be able to see that the edges extracted from the gradient of the smoothed image are quite thick and blurry. The purpose of this step is to convert the "blurred" edges into "sharp" edges. Basically, this is done by preserving all local maxima in the gradient image and discarding everything else. The algorithm is for each pixel (x,y) in the gradient image: 1. Round the gradient direction $\Theta[y, x]$ to the nearest 45 degrees, corresponding to the use of an 8-connected neighbourhood.

2. Compare the edge strength of the current pixel with the edge strength of the pixel in the positive and negative gradient directions. For example, if the gradient direction is south (theta=90), compare with the pixels to the north and south.

3. If the edge strength of the current pixel is the largest; preserve the value of the edge strength.

If not, suppress (i.e. remove) the value.

Implement **non_maximum_suppression** in `edge.py`.

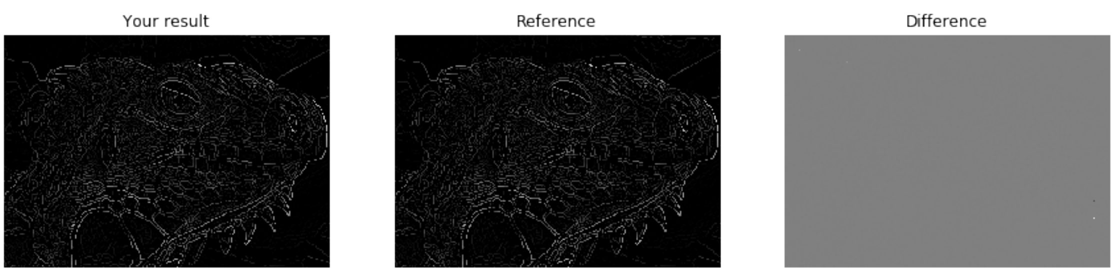
We provide the correct output and the difference between it and your result for debugging purposes. If you see white spots in the Difference image, you should check your implementation.

[7]:
```
from edge import non_maximum_suppression

# Test input
g = np.array(
    [[0.4, 0.5, 0.6],
     [0.3, 0.5, 0.7],
     [0.4, 0.5, 0.6]]
)
# Print out non-maximum suppressed output
# varying theta
for angle in range(0, 180, 45):
    print('Thetas:', angle)
    t = np.ones((3, 3)) * angle # Initialize theta
    print(non_maximum_suppression(g, t))
```

```
Thetas: 0
[[0.  0.  0.6]
 [0.  0.  0.7]
 [0.  0.  0.6]]
Thetas: 45
[[0.  0.  0.6]
 [0.  0.  0.7]
 [0.4 0.5 0.6]]
Thetas: 90
[[0.4 0.5 0. ]
 [0.  0.5 0.7]
 [0.4 0.5 0. ]]
Thetas: 135
[[0.4 0.5 0.6]
 [0.  0.  0.7]
 [0.  0.  0.6]]
```

[8]:
```
nms = non_maximum_suppression(G, theta)
plt.imshow(nms)
plt.title('Non-maximum suppressed')
plt.axis('off')
plt.show()

plt.subplot(1, 3, 1)
plt.imshow(nms)
plt.axis('off')
plt.title('Your result')
```

```python
plt.subplot(1, 3, 2)
reference = np.load('references/iguana_non_max_suppressed.npy')
plt.imshow(reference)
plt.axis('off')
plt.title('Reference')

plt.subplot(1, 3, 3)
plt.imshow(nms - reference)
plt.title('Difference')
plt.axis('off')
plt.show()
```



4.1.4 1.4 Double Thresholding (20 points)

The edge-pixels remaining after the non-maximum suppression step are (still) marked with their strength pixel-by-pixel. Many of these will probably be true edges in the image, but some may be caused by noise or color variations, for instance, due to rough surfaces. The simplest way to discern between these would be to use a threshold, so that only edges stronger that a certain value would be preserved. The Canny edge detection algorithm uses double thresholding. Edge pixels stronger than the high threshold are marked as strong; edge pixels weaker than the low threshold are suppressed and edge pixels between the two thresholds are marked as weak.

Implement **`double_thresholding`** in `edge.py`

[9]:
```
from edge import double_thresholding

low_threshold = 0.02
high_threshold = 0.03

strong_edges, weak_edges = double_thresholding(nms, high_threshold,␣
 ↪low_threshold)
assert(np.sum(strong_edges & weak_edges) == 0)

edges=strong_edges * 1.0 + weak_edges * 0.5

plt.subplot(1,2,1)
plt.imshow(strong_edges)
plt.title('Strong Edges')
plt.axis('off')

plt.subplot(1,2,2)
plt.imshow(edges)
plt.title('Strong+Weak Edges')
plt.axis('off')

plt.show()
```

[ ]:

```

```

4.1.5 1.5 Edge tracking (15 points)

Strong edges are interpreted as “certain edges”, and can immediately be included in the final edge image. Weak edges are included if and only if they are connected to strong edges. The logic is of course that noise and other small variations are unlikely to result in a strong edge (with proper adjustment of the threshold levels). Thus strong edges will (almost) only be due to true edges in the original image. The weak edges can either be due to true edges or noise/color variations. The latter type will probably be distributed independently of edges on the entire image, and thus only a small amount will be located adjacent to strong edges. Weak edges due to true edges are much more likely to be connected directly to strong edges.

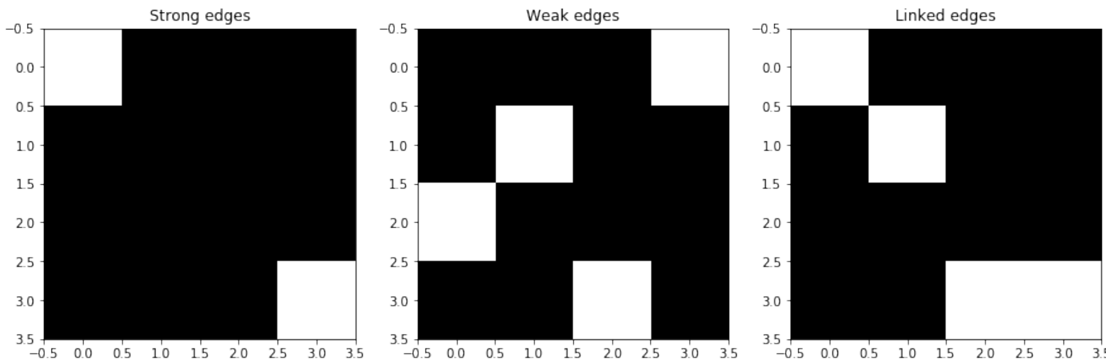
Implement **`link_edges`** in `edge.py`.

We provide the correct output and the difference between it and your result for debugging purposes. If you see white spots in the Difference image, you should check your implementation.

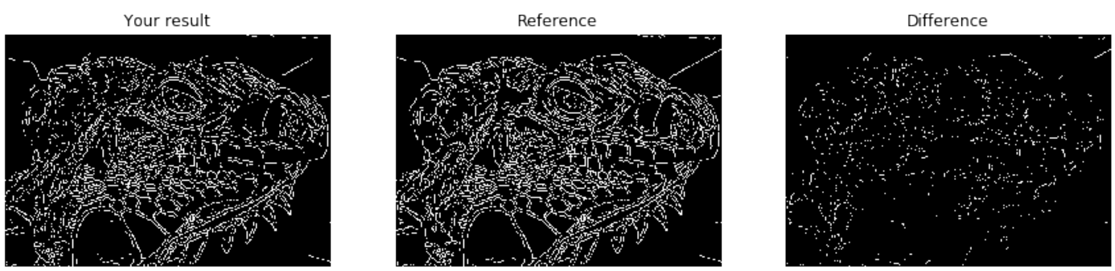
[10]:

```python
from edge import get_neighbors, link_edges

test_strong = np.array(
    [[1, 0, 0, 0],
     [0, 0, 0, 0],
     [0, 0, 0, 0],
     [0, 0, 0, 1]],
    dtype=np.bool
)

test_weak = np.array(
    [[0, 0, 0, 1],
     [0, 1, 0, 0],
     [1, 0, 0, 0],
     [0, 0, 1, 0]],
    dtype=np.bool
)

test_linked = link_edges(test_strong, test_weak)

plt.subplot(1, 3, 1)
plt.imshow(test_strong)
plt.title('Strong edges')

plt.subplot(1, 3, 2)
plt.imshow(test_weak)
plt.title('Weak edges')
```

```
plt.subplot(1, 3, 3)
plt.imshow(test_linked)
plt.title('Linked edges')
plt.show()
```



[11]:
```
edges = link_edges(strong_edges, weak_edges)

plt.imshow(edges)
plt.axis('off')
plt.show()

plt.subplot(1, 3, 1)
plt.imshow(edges)
plt.axis('off')
plt.title('Your result')

plt.subplot(1, 3, 2)
reference = np.load('references/iguana_edge_tracking.npy')
plt.imshow(reference)
plt.axis('off')
plt.title('Reference')

plt.subplot(1, 3, 3)
plt.imshow(edges ^ reference)
plt.title('Difference')
plt.axis('off')
plt.show()
```



4.1.6 1.6 Canny edge detector

Implement **`canny`** in `edge.py` using the functions you have implemented so far. Test edge detector with different parameters.

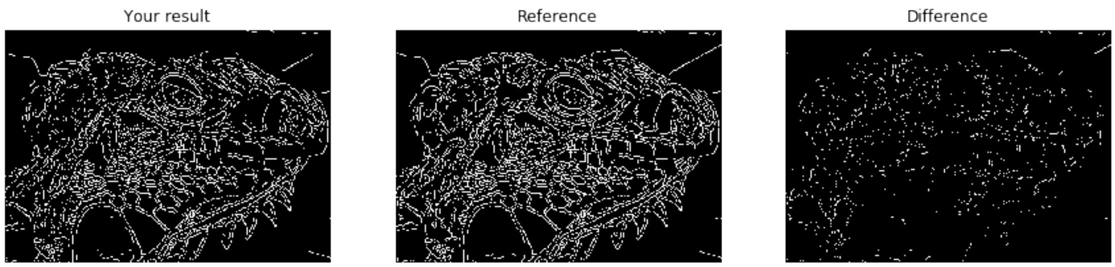
Here is an example of the output:

We provide the correct output and the difference between it and your result for debugging purposes. If you see white spots in the Difference image, you should check your implementation.

[12]:
```
from edge import canny

# Load image
img = io.imread('iguana.png', as_gray=True)

# Run Canny edge detector
edges = canny(img, kernel_size=5, sigma=1.4, high=0.03, low=0.02)
print (edges.shape)

plt.subplot(1, 3, 1)
plt.imshow(edges)
plt.axis('off')
plt.title('Your result')

plt.subplot(1, 3, 2)
reference = np.load('references/iguana_canny.npy')
plt.imshow(reference)
plt.axis('off')
plt.title('Reference')

plt.subplot(1, 3, 3)
```

```
plt.imshow(edges ^ reference)
plt.title('Difference')
plt.axis('off')
plt.show()
```

```
(310, 433)
```



4.2 Part2: Lane Detection (15 points)

In this section we will implement a simple lane detection application using Canny edge detector and Hough transform. Here are some example images of how your final lane detector will look like.

The algorithm can broken down into the following steps: 1. Detect edges using the Canny edge detector. 2. Extract the edges in the region of interest (a triangle covering the bottom corners and the center of the image). 3. Run Hough transform to detect lanes.

4.2.1 2.1 Edge detection

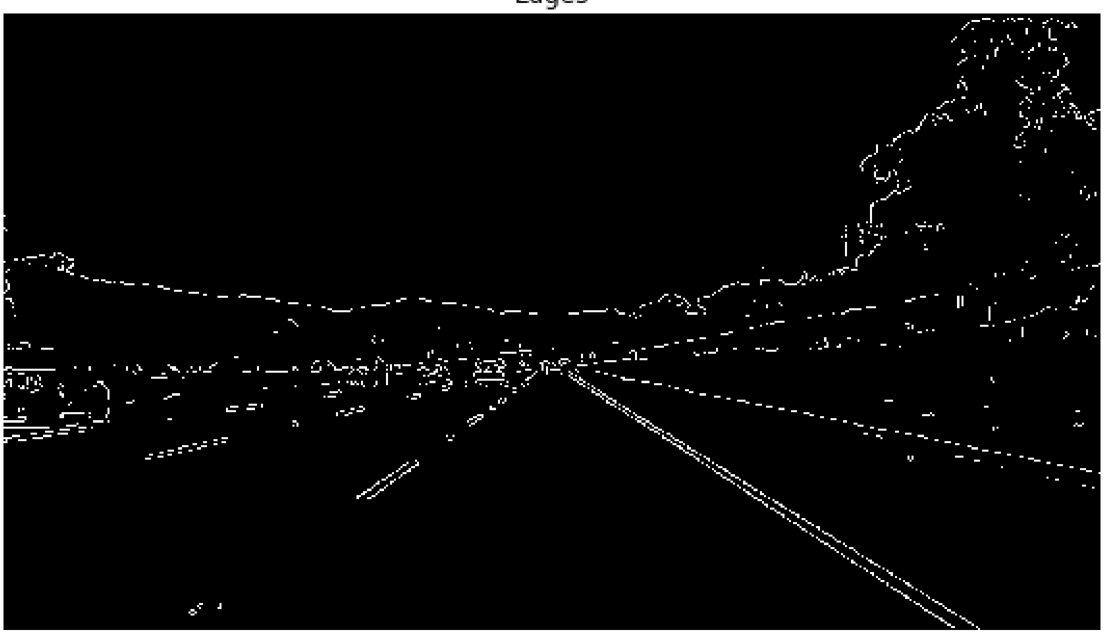
Lanes on the roads are usually thin and long lines with bright colors. Our edge detection algorithm by itself should be able to find the lanes pretty well. Run the code cell below to load the example image and detect edges from the image.

[13]:
```
from edge import canny

# Load image
img = io.imread('road.jpg', as_gray=True)

# Run Canny edge detector
edges = canny(img, kernel_size=5, sigma=1.4, high=0.03, low=0.02)

plt.subplot(211)
plt.imshow(img)
plt.axis('off')
plt.title('Input Image')

plt.subplot(212)
plt.imshow(edges)
plt.axis('off')
```

```
plt.title('Edges')
plt.show()
```

Input Image

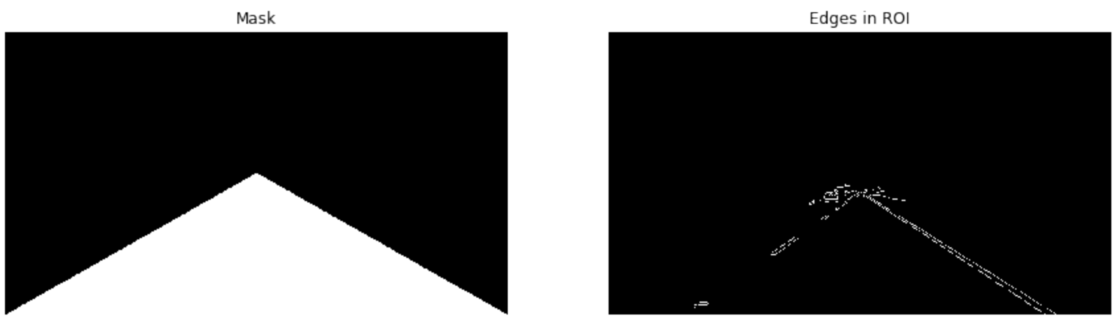
Edges



4.2.2 2.2 Extracting region of interest (ROI)

We can see that the Canny edge detector could find the edges of the lanes. However, we can also see that there are edges of other objects that we are not interested in. Given the position and orientation of the camera, we know that the lanes will be located in the lower half of the image. The code below defines a binary mask for the ROI and extract the edges within the region.

[14]:
```
H, W = img.shape

# Generate mask for ROI (Region of Interest)
mask = np.zeros((H, W))
for i in range(H):
    for j in range(W):
        if i > (H / W) * j and i > -(H / W) * j + H:
            mask[i, j] = 1

# Extract edges in ROI
roi = edges * mask

plt.subplot(1,2,1)
plt.imshow(mask)
plt.title('Mask')
plt.axis('off')

plt.subplot(1,2,2)
plt.imshow(roi)
plt.title('Edges in ROI')
plt.axis('off')
plt.show()
```



4.2.3 2.3 Fitting lines using Hough transform (15 points)

The output from the edge detector is still a collection of connected points. However, it would be more natural to represent a lane as a line parameterized as $y = ax+b$, with a slope a and y-intercept b. We will use Hough transform to find parameterized lines that represent the detected edges.

In general, a straight line $y = ax + b$ can be represented as a point (a, b) in the parameter space.

However, this cannot represent vertical lines as the slope parameter will be unbounded. Alternatively, we parameterize a line using $\theta \in [-\pi, \pi]$ and $\rho \in \mathbb{R}$ as follows:

$$\rho = x \cdot cos\theta + y \cdot sin\theta$$

Using this parameterization, we can map every point in xy-space to a sine-like line in $\theta\rho$-space (or Hough space). We then accumulate the parameterized points in the Hough space and choose points (in Hough space) with highest accumulated values. A point in Hough space then can be transformed back into a line in xy-space.

See notes on Hough transform.

Implement **hough_transform** in `edge.py`.

[15]:
```
from edge import hough_transform

# Perform Hough transform on the ROI
acc, rhos, thetas = hough_transform(roi)

# Coordinates for right lane
xs_right = []
ys_right = []

# Coordinates for left lane
xs_left = []
ys_left = []

for i in range(20):
    idx = np.argmax(acc)
    r_idx = idx // acc.shape[1]
    t_idx = idx % acc.shape[1]
    acc[r_idx, t_idx] = 0 # Zero out the max value in accumulator

    rho = rhos[r_idx]
    theta = thetas[t_idx]

    # Transform a point in Hough space to a line in xy-space.
    a = - (np.cos(theta)/np.sin(theta)) # slope of the line
    b = (rho/np.sin(theta)) # y-intersect of the line

    # Break if both right and left lanes are detected
    if xs_right and xs_left:
        break

    if a < 0: # Left lane
        if xs_left:
            continue
        xs = xs_left
```

```
        ys = ys_left
    else: # Right Lane
        if xs_right:
            continue
        xs = xs_right
        ys = ys_right

    for x in range(img.shape[1]):
        y = a * x + b
        if y > img.shape[0] * 0.6 and y < img.shape[0]:
            xs.append(x)
            ys.append(int(round(y)))

plt.imshow(img)
plt.plot(xs_left, ys_left, linewidth=5.0)
plt.plot(xs_right, ys_right, linewidth=5.0)
plt.axis('off')
plt.show()
```

[ ]:

```
```