ASSIGNMENT SHEET 12

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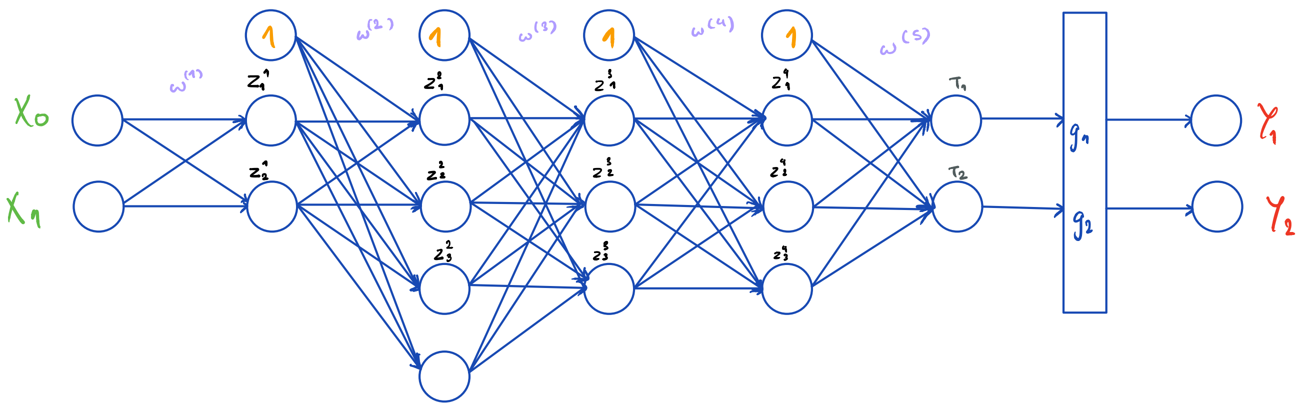
Excercise 1. (Visual representation of the MLP model)

Follow Example 11.3 from the lecture notes and draw a multilayer perceptron with input dimension $D = 2$, output dimension K =2, and $L = 4$ hidden layers. For the four hidden layers, it holds $M_1 = 2$, $M_2 = 4$, $M_3 = 3$ and $M_4 = 3$. Do not forget to also draw the intercept.

(4 Points)



Excercise 2. (Evaluating an MLP model)

You are given a multilayer perceptron with D = 2-dimensional input, K = 1-dimensional ouptut and two hidden layers with $M_1 = 3$ and $M_2 = 2$ that follows the simplified Definition 11.4 from the lecture notes. In the hidden layer, it further uses the sigmoid activation. The output activation is the identity function.

In this task, it is assumed that this artificial neural network model has been already trained, such that is has the weights given in matrices

$$\mathcal{W}^{(1)} = \begin{pmatrix} 0.2 & 0.1 \\ 0.3 & 0.0 \\ 0.1 & 0.4 \end{pmatrix}, \mathcal{W}^{(2)} = \begin{pmatrix} 0.1 & 0.2 & 0.5 \\ 0.2 & 0.1 & 0.4 \end{pmatrix}, \mathcal{W}^{(3)} = \begin{pmatrix} 0.1 & 0.1 \end{pmatrix}.$$

Evaluate the MLP given above for the input $\mathbf{x} = (1, 2)^\top$. Make sure to not only give the values at the output layer but also the intermediate results at the hidden layers.

(4 Points)

$$z^{(0)} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

for $\ell = 1$

i) $a^{(1)} = W^{(1)} z^{(1-1)} = \begin{pmatrix} 0.2 & 0.1 \\ 0.3 & 0.0 \\ 0.1 & 0.4 \end{pmatrix}_{3\times 2} \begin{pmatrix} 1 \\ 2 \end{pmatrix}_{2\times 1} = \begin{pmatrix} 0.4 \\ 0.3 \\ 0.9 \end{pmatrix}_{3\times 1}$

ii) $z^{(1)} = \sigma(a^{(1)}) = \begin{pmatrix} 0.4 \\ 0.3 \\ 0.9 \end{pmatrix}_{3\times 1}$

for $l=2$

i) $a^{(2)} = W^{(2)} z^{(1)} = \begin{pmatrix} 0.1 & 0.2 & 0.5 \\ 0.2 & 0.1 & 0.4 \end{pmatrix} \begin{pmatrix} 0.4 \\ 0.3 \\ 0.9 \end{pmatrix} = \begin{pmatrix} 0.55 \\ 0.47 \end{pmatrix}$

ii) $z^{(2)} = \sigma_2 (a^{(2)}) = \begin{pmatrix} 0.55 \\ 0.47 \end{pmatrix}$

for $l=3$

i) $a^{(3)} = w^{(3)} z^{(2)} = \begin{pmatrix} 0.1 & 0.1 \end{pmatrix} \begin{pmatrix} 0.55 \\ 0.47 \end{pmatrix} = \begin{pmatrix} 0.102 \end{pmatrix}$

ii) $z^{(3)} = \sigma_3 (a^{(3)}) = \begin{pmatrix} 0.102 \end{pmatrix}$

$f_w \begin{pmatrix} 1 \\ 2 \end{pmatrix} = 0.102 \,//$

Excercise 3. (Backward pass for classification)

In Theorem 11.1 from the lecture notes, we give the backward pass that is used to compute the gradients in the backpropagation algorithm. Here, we assume that even in the last layer, we have an activation that does not couple between the units in that layer. Hence, we explicitly exclude the case of the softmax activation in the output layer. Your task is do derive a modified version of Theorem 11.1, in which we explicitly use the softmax activation in the last layer. *Hint: You only have to modify the step for calculating the* $\delta_m^{(L+1)}$.

(4 Points)

Theorem 11.1 (Backward pass / propagagtion formula) Let the setting of Lemma 11.2 be given. Furthermore, we consider the L_2 loss. The $\delta_m^{(l)}$ are recursively given starting with

$$\delta_m^{(L+1)} = 2 \left(Z_m^{(L+1)} - y_{im} \right) \sigma'_{L+1} \left(A_m^{(L+1)} \right), \quad m = 1, \dots M_{L+1}$$

and continuing with $l = L+1, \dots, 2$ as

$$\boxed{\delta_v^{(l-1)} = \sigma'_{l-1} \left(A_v^{(l-1)} \right) \sum_{m=1}^{M_l} \delta_m^{(l)} \omega_{mv}^{(l)}, \quad v = 1, \dots, M_{l-1} \,,}$$

where σ' is the first derivative of the activation.