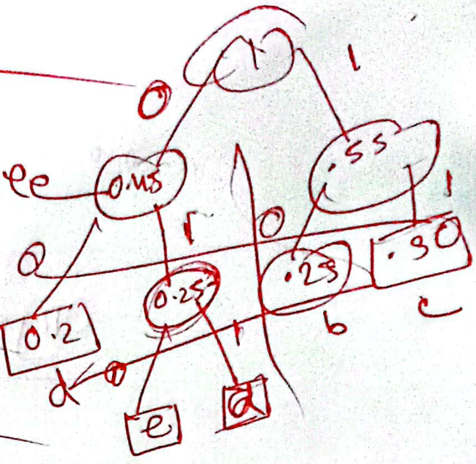
Math - 1

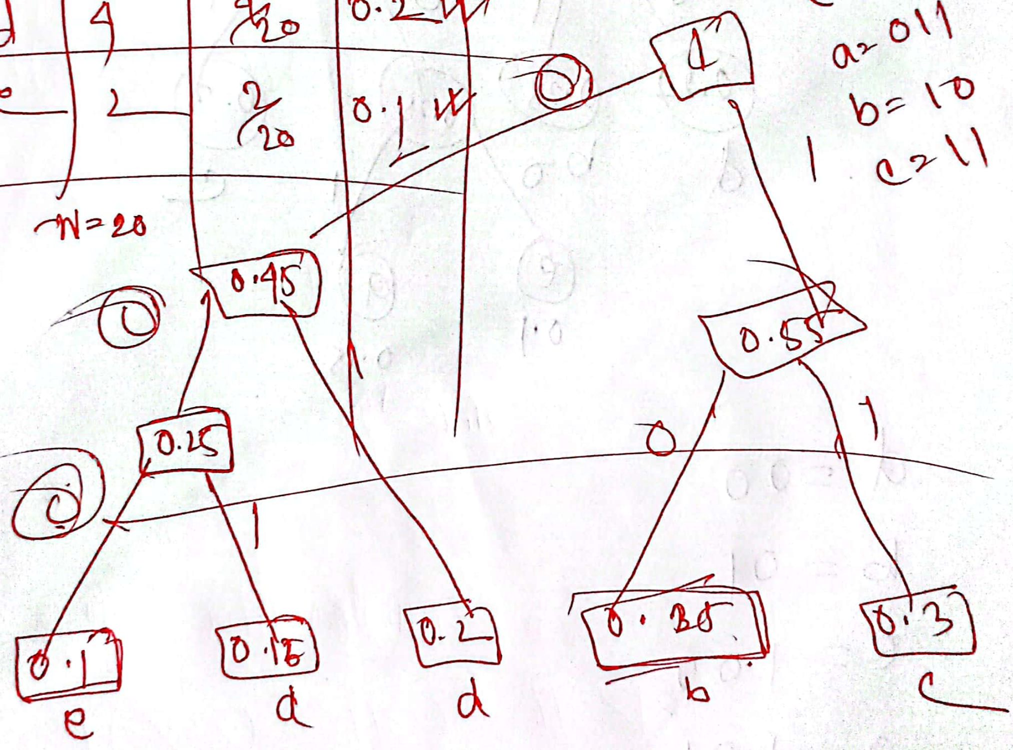
aaabbbbbccccccddddee



Symbol	Frequency	Probability p(i)	
a	3	3/20	0.15
b	5	5/20	0.25
c	6	6/20	0.3
d	4	4/20	0.2
e	2	2/20	0.1

N=20

d=00
e=010
a=011
b=10
c=11

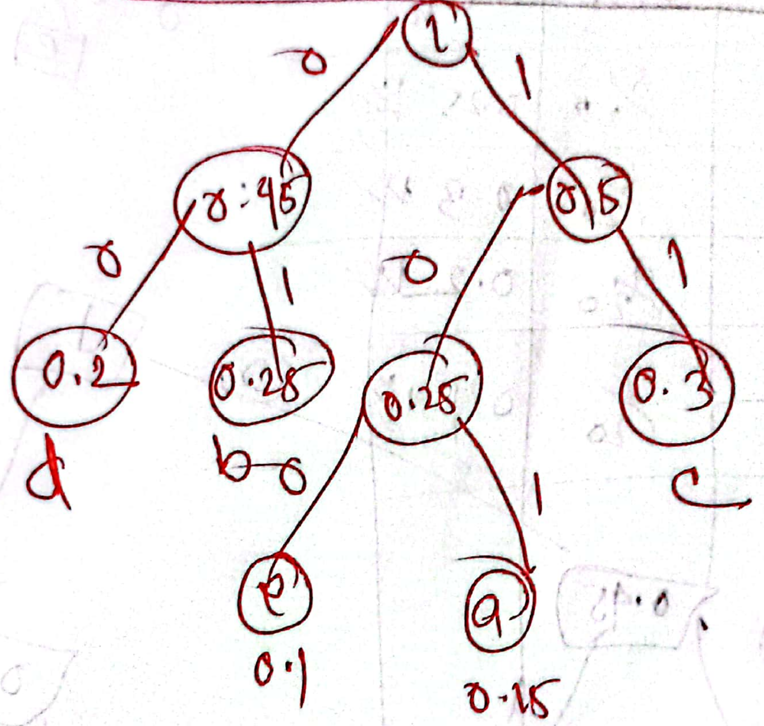


e=

d=

a = 0.10
b =

Original graph for Q1



d = 00

b = 01

e = 100

a = 101

c = 11

Math – 2

here, message, 10101

code rate $= \frac{1}{2}$

here,

$K = 4$

and, $m = 10101$

first bit generator, $0 = 1\ 1\ 1\ 1$

" " " $1 = 1\ 1\ 0\ 1$

here the message length $m = 5$

no of adder (or generators) $n = 2$

length of the code word,

$$L = n(l + K - 1)$$

$$= 2(5 + 4 - 1)$$

$$= 16$$

now, the message,

$$m = \overset{4}{1}\ \overset{3}{0}\ \overset{2}{1}\ \overset{1}{0}\ \overset{0}{1}$$

We know,

encoded output polynomial, $v(x)$

$$v_1(x) = g_1(x) \cdot m(x)$$

and, $v_2(x) = g_2(x) \cdot m(x)$

∴ here, $g_1(x)$ = generator polynomial [means bina polynomial constraint → num of

now, $m = 1\ 0\ 1\ 0\ 1$

$$m(x) = 1 \times x^4 + 0x^3 + 1x^2 + 0 \times x^1 + 1 \times x^0$$

$$= x^4 + x^2 + 1$$

now, $g_1 = (1\ 1\ 1\ 1)$

$$\therefore g_1(x) = 1 \times x^3 + 1 \times x^2 + 1 \times x^1 + 1 \times x^0$$

$$= x^3 + x^2 + x + 1$$

and, $g_2 = (1\ 1\ 0\ 1)$

$$\therefore g_2(x) = 1 \times x^3 + 1 \times x^2 + 0 \times x^1 + 1 \times x^0$$

$$= x^3 + x^2$$

Now, the output polynomial,

$$V_1(x) = g_1(x) \cdot m(x)$$

$$= \cancel{(x^4 + x^3 + x + 1)} \; (x^4 + x^2 + 1)$$

$x^3 + x^2 + x + 1$

$$= x^8 + x^7 + x^5 + x^4 + x^6 + x^5 + x^3 + x^2 + x^4 + x^3 + x + 1$$

$$= x^8 + x^7 + 2x^5 + 2x^4 + x^6 +$$

X-OR operation

$$= x^7 + x^6 + x^5 + x^4 + x^5 + x^4 + x^3 + x^2 + x^3 + x^2 + x + 1$$

thats why
$x^5 + x^5 = 0$
$1 + 1 = 0$
eliminate

$$= x^7 + x^6 + x + 1$$

and, $V_2(x) = g_2(x) \cdot m(x)$

$$= (x^3 + x^2 + 1)(x^4 + x^2 + 1)$$

$$= x^7 + x^6 + x^4 + x^5 + x^4 + x^2 + x^3 + x^2 + 1$$

$$= x^7 + x^6 + x^5 + x^3 + 1$$

now, $V_1(x) = x^7 + x^6 + x + 1$

$= x^7 + x^6 + \cdots\cdots + x + 1$

$= 1 + 1 + 0 + 0 + 0 + 0 + 1 + 1$

$V_1(x) =$ 1 1 0 0 0 0 1 1

and, $V_2(x) = x^7 + x^6 + x^5 + x^3 + 1$

$V_2(x) =$ 1 1 1 0 1 0 0 1

convolution code,

now, $C = V_1 \oplus V_2$

= 1111010000100010 11

Math-4

here, the message,

01101

here, bits = 5
data = 01101

now, find out number of parity bits →

Codition as follows,

$$2^P \geq P+k+1$$

$\Rightarrow$ $2^1 \geq 1+5+1$

not satisfy

now, $2^4 \geq 4+5+1$

$= 16 \geq 10$

$\therefore$ number of parity bit is 4.

Now find out the position of parity,

$2^0 = 1$, P_1

$2^1 = 2$, P_2

$2^2 = 4$, P_4

$2^3 = 8$, P_8

Now data bit and parity place in the message data,

9	8	7	6	5	4	3	2	1
D_9	P_8	D_7	D_6	D_5	P_4	D_3	P_2	P_1
0	0	1	1	0	0	1	1	0

$$P_1 = D_3 \oplus D_5 \oplus D_7 \oplus D_9$$
$$= 1 \oplus 0 \oplus 1 \oplus 0$$
$$= 1 \oplus 1$$
$$= 0$$

X-OR sum.

Clear

P_1 for first step
second check
at least all skip
iterate.

$P_2 = D_3 \oplus D_6 \oplus D_7$

$= 1 \oplus 1 \oplus 1$

$= 0 \oplus 1$

$= 1$

first 2 bit check and second 2 bit skip

$P_4 = D_5 \oplus D_6 \oplus D_7$

$= 0 \oplus 1 \oplus 1$

$= 0$

first 4 bit check and second 4 bit skip

$P_8 = D_9$

$= 0$

first 8 bit check and next 8 bits skip

now the encoded transmitted data,

code word = 0 0 1 1 0 0 1 1 0 →

Error check process

In user got

D9	P8	D7	P6	D5	P4	D3	P2	P1
0	0	1	1	0	1	1	1	0

where error

$P_4 = 1$

$P_1 = D_3 \oplus D_5 \oplus D_7 \oplus D_9$

$= 1 \oplus 0 \oplus 1 \oplus 0$

$= 0$

$P_2 = D_3 \oplus D_6 \oplus D_7$

$= 1 \oplus \oplus = 1 \oplus 1$

$= 0 \oplus 1$

$= 1$

$P_4 = D_5 \oplus D_6 \oplus D_7$

$= 0 \oplus 1 \oplus 1$

$= 0$

error

$P_8 = D_9 = 0$

$\therefore$ error in here, $\underbrace{0100}$ $\rightarrow$ 4

Now 4 number bits has an errors

	9	8	7	6	5	4	3	2	1
error Syndrome	0	0	0	0	0	1	0	0	0

⊕

	9	8	7	6	5	4	3	2	1
Received data	0	0	1	1	0	1	1	1	0

=

Correct Code =	0	0	1	1	0	0	1	1	0

Correction Code.

Limpel → 2 → Mash-3

0 | 1 | 1 1 | 0 0 | 1 0 | 1 0 0 | 1 0 0 1 |
1 2 3 4 5 6 7

→ Numerical Position fin → A | A B | A B B | B | A B A | A B A B | B B | A B B A | B B
1 2 3 4 5 6 7 8 9

1 2 3 4 5 6 7 8 9

Sequence	A	AB	ABB	B	ABA	ABAB	BB	ABBA BB
Numerical Representa	ϕA	1B	2B	ϕB	2A	5B	4B	3A 7
→ Code →	0000	0011	0101	0001	0100	1011	1001	0110 0111

A → 0
B → 1

0

⑤ A binary symmetric channel has the following noise matrix with probability →

$$P(Y/x) = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$$ Now find the channel capacity C.

Now

Given conditional probability matrix

$$P(Y/x) = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$$

we identify that the crossover probability p (the probability of error) is $p = \frac{1}{3}$. The probability of no we identify that the crossover probability p (the probability of error) is $p = \frac{1}{3}$. The probability of no error (receiving the correct bit) is $1-p = \frac{2}{3}$

Step-1 : calculate conditional ~~Enp~~ Entropy : $H(Y/x)$

we know-

$$H(Y/x) = (1-p) \cdot \log_2\left(\frac{1}{1-p}\right) + p \cdot \log_2\left(\frac{1}{p}\right)$$

$$= \frac{2}{3} \cdot \log_2\left(\frac{3}{2}\right) + \frac{1}{3} \cdot \log_2(3)$$

$$= 0.918 \text{ bits per symbol.}$$

llow,—

calculate channel capacity C —

The channel capacity C for a binary symmetric channel is ~~is~~ given by

$$C = 1 - H(Y/X)$$

$$= 1 - 0.918$$

$$= 0.082 \text{ bits per symbol}$$ Ans

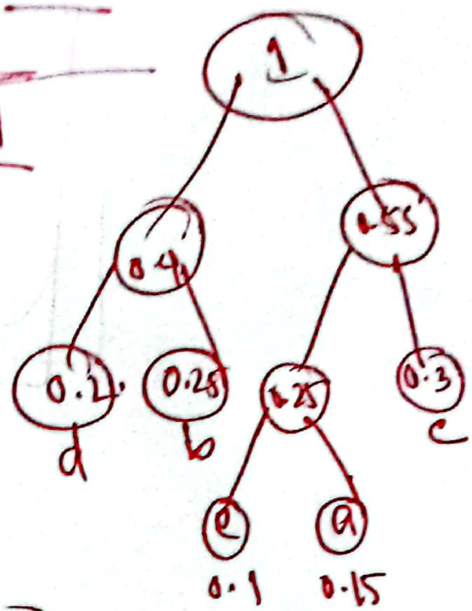
Math - 6

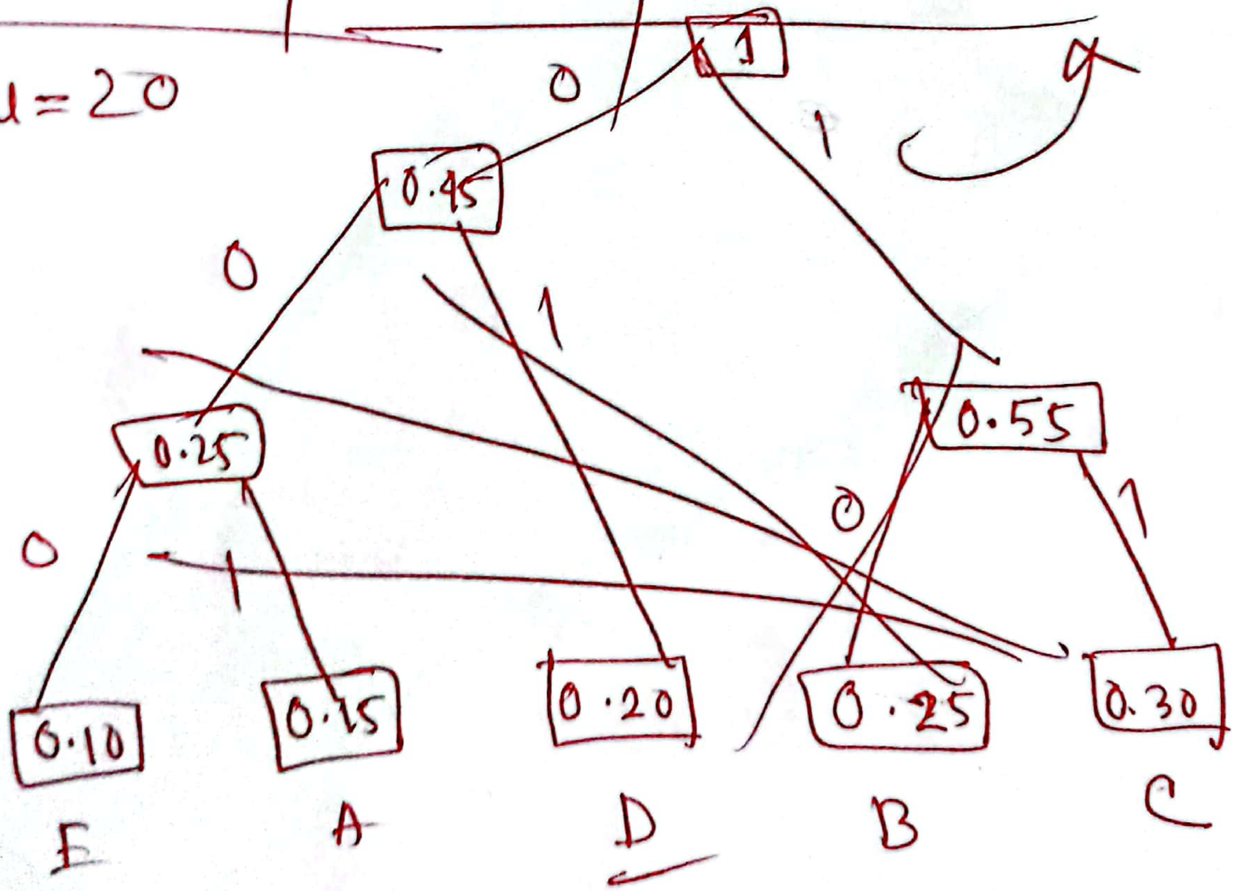
Message:

bccabbddabccbbaeddcc

Symbol	Frequency	Probability, p	Code
A	3	3/20 = 0.15	001
B	5	5/20 = 0.25	10
C	6	6/20 = 0.30	11
D	4	4/20 = 0.20	01
E	2	2/20 = 0.10	000

total = 20





Now, entropy of the message →

$$H(x) = - \sum_{i=1}^{n} \underline{p(x)} \; \underline{\log_2 p(x)}$$

$$\therefore H(x) = - \left\{ 0.15 \log_2(0.15) + 0.25 \log_2(0.25) + 0.30 \log_2(0.3) + 0.20 \log_2(0.20) + 0.10 \log_2(0.10) \right\}$$

$= 2.23$ bits → destination

Now, expected code length of huff man code

$a = 3$ ← bit
$b = 2$
$c = 2$
$d = 2$
$e = 3$

optional

So length expected,

$= (0.15 \times 3) + (2 \times 0.25) + (2 \times 0.30) + (2 \times 0.20) + (3 \times 0.10) = 0.25$ so its not

Math-8:

Math – 8

y \ x	1	2	3	4	
1	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{4}$
2	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{4}$
3	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{4}$
4	$\frac{1}{4}$	0	0	0	$\frac{1}{4}$
	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$	

→ Marginal distribution of $x : \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8}$

" " " $y : \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}$

$$\therefore H(X) = -\frac{1}{2}\log_2\frac{1}{2} - \frac{1}{4}\log_2\frac{1}{4} - \frac{1}{8}\log_2\frac{1}{8} - \frac{1}{8}\log_2\frac{1}{8}$$

$$= 0.15 + 0.15 \times 4 \qquad .11 \times 2$$

$$= 0.6$$

$$= \frac{1}{2}\log_2 2 + \frac{1}{4}\log_2 2^2 + \frac{1}{8}\log_2 2^3 + \frac{1}{8}\log_2 2^3$$

$$= \frac{1}{2} + \frac{1}{2} + \frac{3}{8} + \frac{3}{8}$$

$$= \frac{4+4+3+3}{8} = \frac{14}{8} = \frac{7}{4}$$

$$H(Y) = -\frac{1}{4}\log_2\frac{1}{4} - \frac{1}{4}\log_2\frac{1}{4} - \frac{1}{4}\log_2\frac{1}{4} - \frac{1}{4}\log_2\frac{1}{4}$$

$$= \frac{1}{4}\times 2 + \frac{1}{4}\times 2 + \frac{1}{4}\times 2 + \frac{1}{4}\times 2$$

$$= \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$$

$$= \frac{1+1+1+1}{2}$$

$$= \boxed{\frac{4}{2} = 2}$$

$H(Y) = 2$ bits

now, Condition entropy

$$H(X|Y) = \sum_{i=1}^{4} p(Y=i)\, H(X|Y=i)$$

$$= \frac{1}{4} H\left(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8}\right) + \frac{1}{4} H\left(\frac{1}{4}, \frac{1}{2}, \frac{1}{8}, \frac{1}{8}\right)$$

$$+ \frac{1}{4} H\left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right) + \frac{1}{4} H(1, 0, 0, 0)$$

একটা কাজ করি,

$$H\left(\frac{1}{4}, \frac{1}{2}, \frac{1}{8}, \frac{1}{8}\right)$$

$$= -\frac{1}{4}\log_2\frac{1}{4} - \frac{1}{2}\log_2\frac{1}{2} - \frac{1}{8}\log_2\frac{1}{8} - \frac{1}{8}\log_2\frac{1}{8}$$

$$= \frac{1}{4}\log_2 2^2 + \frac{1}{2}\log_2 2 + \frac{1}{8}\log_2 2^3 + \frac{1}{8}\log_2 2^3$$

$$= \frac{1}{4}\times 2 + \frac{1}{2}\times 1 + \frac{1}{8}\times 3 + \frac{1}{8}\times 3$$

$$= \frac{1}{2} + \frac{1}{2} + \frac{3}{8} + \frac{3}{8}$$

$$= \frac{\cancel{1+1+1+1+1+1}}{2}$$

$$= \cancel{2} \quad (1)\ \frac{4+4+3+3}{8}$$

$$= \cancel{2}\ \frac{14}{8}$$

$$= \frac{7}{4}.$$

Same as all →

$\therefore H(X|Y) = \frac{11}{8}$ bits.

Similiarly $\rightarrow$

$$H(Y|X) = \sum_{i=1}^{4} P(X=1)\, H(Y|X=1)$$

$$= \frac{1}{2} H\left(\frac{1}{4}, \frac{1}{8}, \frac{1}{8}, \frac{1}{2}\right) + \frac{1}{4} H\left(\frac{1}{4}, \frac{1}{2}, \frac{1}{4}, 0\right)$$

$$+ \frac{1}{8} H\left(\frac{1}{4}, \frac{1}{4}, \frac{1}{2}, 0\right) + \frac{1}{8} H\left(\frac{1}{4}, \frac{1}{4}, \frac{1}{2}, 0\right)$$

Same as,

$$H\left(\frac{1}{4}, \frac{1}{8}, \frac{1}{8}, \frac{1}{2}\right) = -\frac{1}{4}\log_2\frac{1}{4} - \frac{1}{8}\log_2\frac{1}{8} - \cdots\cdots$$

$\therefore H(Y|X) = \frac{13}{8}$ bits.

Now joint entropy,

$$\rightarrow H(X,Y) = H(X) + H(Y|X)$$

$$= \frac{7}{4} + \frac{13}{8}$$

$$= \frac{27}{8}$$

$$\rightarrow H(Y,X) = H(Y) + H(X|Y)$$

$$= 2 + \frac{11}{8}$$

$$= \frac{16+11}{8}$$

$$= \frac{27}{8} \text{ bits}$$

Now, mutual information,

$$I(X;Y) = H(X) - H(X|Y)$$

$$= \frac{7}{4} - \frac{11}{8}$$

$$= \frac{3}{8} \text{ bits}$$

or, $I(X;Y) = H(Y) - H(Y|X)$

$$= 2 - \frac{13}{8} = \frac{3}{8} \text{ bits.}$$