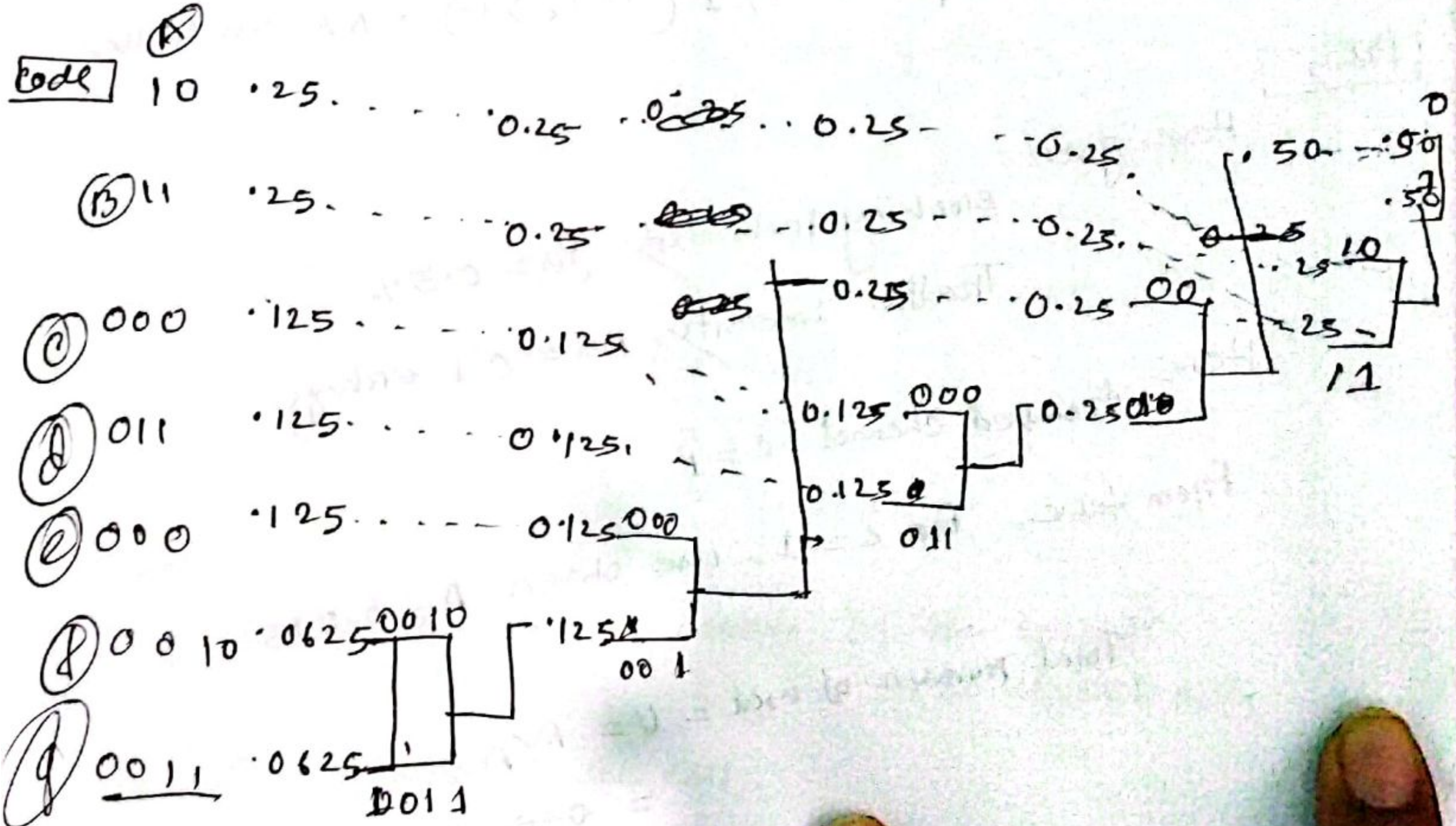
Information Theory

Experiment-2 Convolution Encoder

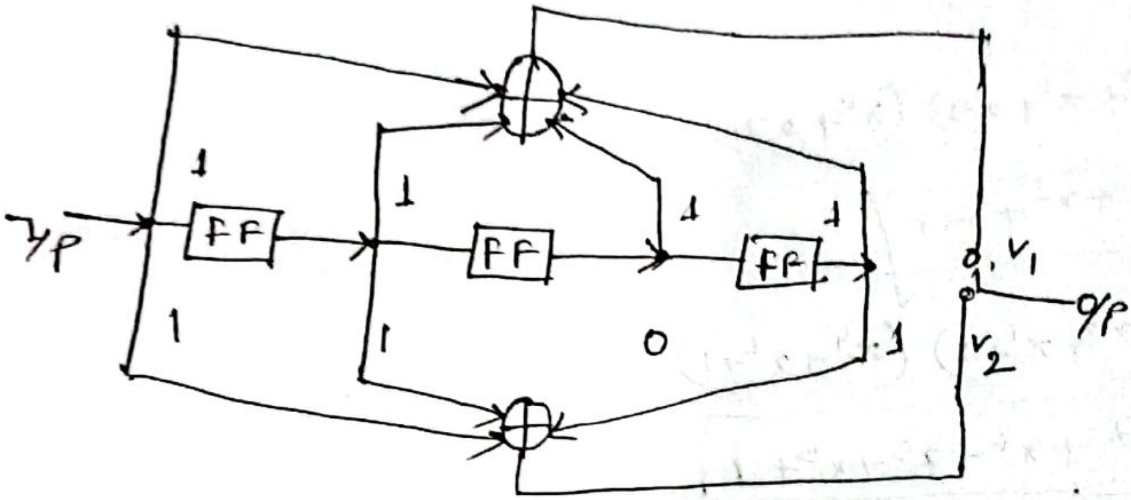
Ex→01 : Half man coding —

Symbol →	s_1	s_2	s_3	s_4	s_5	s_6	s_7
	0.25	0.25	0.125	0.125	0.123	0.0625	0.0625

Now —



Q2 The convolution encoder for rate = 1/2, constraint length k=4 determine the output codeword for the message 10101



$g_1=(1,1,1,1)$

$g_2=(1,1,0,1)$

code rate $= \frac{1}{2} \leftarrow \frac{k}{n}$

$K=4 \quad m = 10101$

message length $=5$

no. of adder $(n) = 2$

So, $v_1(x) = g_1(x)\, m(x)$

$v_2(x) = g_2(x) \cdot m(x)$

length of output codeword

$$L = n(l+k-1)$$
$$= 2(5+4-1)$$
$$= 16$$

$$m = \begin{array}{ccccc} 4 & 3 & 2 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{array}$$

So, $m(x) = 1\times x^4 + 0\times x^3 + 1\times x^2 + 0\times x^1 + 1\times x^0$

$$\boxed{\therefore m(x) = x^4+x^2+1}$$

∴ + ⇒ ⊕ (xor) ✓

Now

$g_1 = (\overset{3}{1}, \overset{2}{1}, \overset{1}{1}, \overset{0}{1}) \rightarrow g_1(x) = x^3 + x^2 + x + 1$

∴ $g_2 = (\overset{3}{1}, \overset{2}{1}, \overset{1}{0}, \overset{0}{1}) \rightarrow g_2(x) = x^3 + x^2 + 1$

$v(x) = (x^3 + x^2 + x + 1)(x^4 + x^2 + 1)$

$\boxed{v_1(x) = x^7 + x^6 + x + 1}$

$v_2(x) = (x^3 + x^2 + 1)(x^4 + x^2 + 1)$

$\boxed{\therefore v_2(x) = x^7 + x^6 + x^5 + x^3 + 1}$

∴ (Now)

$v_1(x) = x^7 + x^6 + x + 1$

$=$ [illegible] .

$\boxed{v_1 = 1\ 1\ 0\ 0\ 0\ 0\ 1\ 1}$

$v_2(x) = x^7 + x^6 + x^5 + x^3 + 1$

$\boxed{v_2 = 1\ 1\ 1\ 0\ 1\ 0\ 0\ 1}$

$\boxed{C = 11\ 11\ 01\ 0\ 0\ 01\ 00\ 10\ 1\ 1}$

2 A   A=0 2^1 1

$\boxed{1.\ 0}$.   A→0

B→1

③ Write a program to implement lempi-ziv. code ?

d| lossless compression

massage = A|A B|A B B|B|A B A|A B A B|B B|A B B A|B B| 7/p

1 2 3 4 5 6 7 8 9

Position:	1	2	3	4	5	6	7	8	9
sequence:	A	AB	ABB	B	AB	ABA	BB	ABBA	BB
Neumeric Reposition:	0A	1B	2B	0B	2A	5B	4B	3A	7
Digital code:	0000	0011	0101	0001	0100	1011	1001	0110	0111

④ Hamming code technique

Basic of Hamming code –

① It is given by RW Hamming

② It is used to detect and connect errors

③ In Hamming code, Codeword = Data bits + Parity bits

④ It is represented by (n, k) code

⑤ where, n = Total bits and k = message bit

⑥ Parity bits $p = n - k$

⑧ Parity bits condition in Hamming code

→ To identify parity bits, it should satisfy the given condition —

$\therefore 2^P \geq P + k + 1$ | P = redundency

message bits $k = 4$

$\Rightarrow 2^P \geq P + 4 + 1$

$\Rightarrow 2^P \geq P + 5$

→ If $P = 1$ → If $P = 2$ if $P = 3$

$2^1 \geq 6$ $2^2 \geq 7$ $2^3 \geq 8$

So, minimum value of $P = 3$ bits.

$\boxed{n = p + k}$

So Hamming code $(n, k) = (7, 4)$

⇒ Position of Parity bits and message bits in Hamming code

→ lets taken an example of (7-4) Hamming code

→ $P_1 = 2^0 = 1$ → $k = 4$

$P_2 = 2^1 = 2$

$P_4 = 2^2 = 4$

Bit position

7	6	5	4	3	2	1
D_7	D_6	D_5	P_4	D_3	P_2	P_1

⇒ calculation of Parity bits in Hamming code

⇒ lets take an example of (7-4) Hamming code

Bits position

7	6	5	4	3	2	1
D_7	D_6	D_5	P_4	D_3	P_2	P_1

$P_1 = D_3 \oplus D_5 \oplus D_7$ skip 1 then

$P_2 = D_3 \oplus D_6 \oplus D_7$ skip 2 then

$P_4 = D_5 \oplus D_6 \oplus D_7$ skip 3 then

even parity

odd parity use

reverse even parity

⇒ Generation of Hamming code

⇒ If 5 bits data 01101 is given represent given data in hammincode

⇒ k = 5 bits , data = 01101

⇒ condition of parity bits

$$2^P \geq P+k+1$$

$$\Rightarrow 2^P \geq P+6$$

⇒ $P = 1$ $\quad$ $P = 4$

$\Rightarrow 2^1 \geq 7$ $\quad$ $2^4 \geq 10$

✗ $\quad$ its

∴ $P = 4$ bit minimum bit are condition

∴ $P = 4$ bits

$P_1 = 2^0 = 1$

$P_2 = 2^1 = 2$

$P_4 = 2^2 = 4$

$P_8 = 2^3 = 8$

D= message bit

Bits position

9	8	7	6	5	4	3	2	1
D_9	P_8	D_7	P_6	D_5	P_4	D_3	P_2	P_1
0	0	1	1	0	0	1	1	0

⇒

⇒ $P_1 = P_3 \oplus D5 \oplus D_7 \oplus D_9$

$= 1 \oplus 0 \oplus 1 \oplus 0$

$= 0$

[P_1 = check 1 bits and skip one bits]

$P_2 = D_3 \oplus P_6 \oplus D_7$

$= 1 \oplus 1 \oplus 1$

$= 1$

[P_2 = chek 2 bits and skip 2 bits]

$P_4 = D_5 \oplus D_6 \oplus D_7$

$= 0 \oplus 1 \oplus 1$

$= 0$

[P_4 = check 4 bits and skip 4 bits]

$P_8 = D_9 = 0$

[P_8 = check 8 bits skip 8 bits]

⇒

∴ codeward = 0 0 1 1 0 0 1 1 0

⇒ Error Detection and Error correction by Hamming code

⇒ If received code as per Hamming code is 001101110. then find error in received codeword.

Bits position

9	8	7	6	5	4	3	2	1
0	0	1	1	0	1	1	1	0
D_9	P_8	D_7	P_6	D_5	P_4	D_3	P_2	P_1

→ $P_1 = 1 \oplus 0 + 1 \oplus 0 = 0$

$P_2 = 1 \oplus 1 \oplus 1 = 1$

$P_4 = 1 \oplus 1 \oplus 0 = 0$ ✗

$P_8 = ⓪$

∴ $P_8\ P_4\ P_2\ P_1$ = ✓ ✗ ✓ ✓ = $\overset{\text{means}}{\boxed{0\ 1\ 0\ 0}}$ = 4 it is Hamming error

Now, just =4 bits are 1 and other are 0

error syndrome

9	8	7	6	5	4	3	2	1
0	0	0	0	0	1	0	0	0

⊕

Received data

9	8	7	6	5	4	3	2	1
0	0	1	1	0	1	1	1	0

=

Correct data

9	8	7	6	5	4	3	2	1
0	0	1	1	0	0	1	1	0

⑤ A binary symmetric channel has the following noise matrix with probability →

$$P(Y/x) = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$$ Now find the channel capacity C.

Now.

Given conditional probability matrix

$$P(Y/x) = \begin{bmatrix} \frac{2}{3} , & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$$

we identify that the crossover probability p (the probability of error) is $p = \frac{1}{3}$. The probability of no error (receiving the correct bit) is $1-p = \frac{2}{3}$

Step-1 : calculate conditional ~~Enp~~ Entropy : H(Y/x)

we know-

$$H(Y/x) = (1-p) \cdot \log_2\left(\frac{1}{1-p}\right) + p \cdot \log_2\left(\frac{1}{p}\right)$$

$$= \frac{2}{3} \cdot \log_2\left(\frac{3}{2}\right) + \frac{1}{3} \cdot \log_2(3)$$

$$= 0.918 \text{ bits per symbol.}$$

Now,—

calculate channel capacity C —

The channel capacity C for a binary symmetric channel is ~~is~~ given by

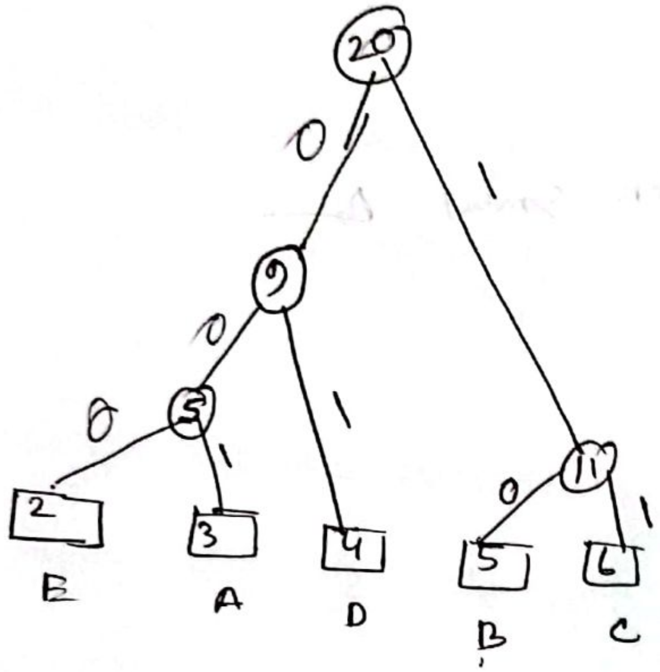
$$C = 1 - H(Y/x)$$

$$= 1 - 0.918$$

$$= 0.082 \text{ bits per symbol}$$

⑥ Write a program to check the optimality of Half man code.

message → BCCABBDDAECCBBAEDDCC



Char	Count	Code
A	3	001
B	5	10
C	6	11
D	4	01
E	2	000

① Entropy of the message

$$H(x) = -\sum P(x) \log_2 P(x)$$

where, $P(x)$ is the probability of each symbol in the message.

For, message

$a = 3$, $b = 5$ $c = 6$ $d = 4$, $e = 2$

Total $= 20$

The probability of symbol are —

$a = \frac{3}{20} = 0.15$

$b = \frac{5}{20} = 0.25$

. - . .

So,

$$H(x) = -\left[0.15\log_2(0.15) + 0.25\log_2(0.25) + 0.30\log_2(0.30) + 0.20\log_2(0.20) + 0.10\log_2(0.10)\right]$$

$$\approx 2.2282 \text{ bits}$$

So the entropy of the message is ~~2.2282~~ 2.228 bits.

No, expected code length of Halfman code.

Now,

$b = 2$

$c = 2$

$d = 2$

$a = 3$

$e = 3$

So, $L_{expected} = (0.15 \times 3) + (0.25 \times 2) + (0.30 \times 3) + (0.20 \times 2) + (0.10 \times 3)$

$= 2.25$ So not optimal