$$P = \begin{bmatrix} 0 & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{4} & 0 & \frac{1}{4} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \end{bmatrix}$$

Stationary distribution equations

$$\pi P = \pi$$

$$\begin{bmatrix} \pi_1, \pi_2, \pi_3, \pi_4 \end{bmatrix} \begin{bmatrix} 0 & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{4} & 0 & \frac{1}{4} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \end{bmatrix} = \begin{bmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \\ \pi_4 \end{bmatrix}^T$$

$= 0\pi_1 + \frac{1}{2}\pi_2 + \frac{1}{2}\pi_3 + \frac{1}{2}\pi_4 = \pi_1$

$\Rightarrow$

$\pi_2 = \frac{1}{2}\pi_1 + \frac{1}{2}\pi_3$

π_2

We know,

$\cancel{\pi_2 = \frac{1}{2}\pi_1 + \frac{1}{2}\pi_3}$

$\pi_1 + \pi_2 + \pi_3 + \pi_4 = 1$

In here,

$\pi_1 = \pi_3$ (Symmetric and same number of connection)

$\pi_2 = \pi_4$

$a = \frac{1}{2}b + \frac{1}{2}b + \frac{1}{3}a$

$\Rightarrow a = b + \frac{1}{3}a$

$\Rightarrow a - \frac{1}{3}a = b$

$\Rightarrow \frac{3a - a}{3}$

$\Rightarrow \boxed{\frac{2a}{3} = b}$

$\Rightarrow a = \frac{3}{2}$

$\Rightarrow$

Now, for,

$\pi_1 = \pi_3 = a$

$\pi_2 = \pi_4 = b$

$a + b + a + b = 1$

$\Rightarrow a + b = \frac{1}{2}$ ——— (1)

and,

$\pi_2 = \frac{1}{2}\pi_1 + \frac{1}{2}\pi_3$

in here,

$\pi_1 = \pi_3 = a$

and, $\pi_2 = \pi_4 = b$

$\therefore \; b = \frac{1}{2}a + \frac{1}{2}a$

$b = a$

now $b = a$ substitute, eqn (1)

$\therefore \; a + a = \frac{1}{2}$

$\Rightarrow 2a = \frac{1}{2}$

$\Rightarrow a = \frac{1}{4}$

substitute,

$b = \frac{1}{2} - \frac{1}{4}$

$= \frac{2-1}{4} = \frac{1}{4}$

$\pi_1 = \pi_3 = \frac{1}{4} = a$ and $\pi_2 = \pi_4 = \frac{1}{4} = b$

$- \frac{1}{4}\left(\frac{1}{4}\log_2\frac{1}{4} + \frac{1}{2}\log\frac{1}{2} + \frac{1}{4}\log\frac{1}{4}\right)$

$= \frac{1}{4}\left(\frac{1}{4}\times 2 + \frac{1}{2} + \frac{1}{4}\times 2\right)$

$= \frac{1}{4}\left(\frac{1}{2} + \frac{1}{2} + \frac{1}{2}\right)$

$= \frac{1}{4}\left(\frac{3}{2}\right) = 0.5$ bits encoder structure in convol × | Convolutional Encoder - an × | Information-Theory-a

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$$L = \sum_i p_i \cdot l_i$$

where p_i is the probability of symbol i and l_i is the length of the codeword for sym

- For Huffman coding, this average length L is minimized, meaning that there is no o code for the same set of symbol probabilities that can achieve a shorter average len
- This minimum average length approaches the entropy of the source $H(X) = -\sum p_i \log_2(p_i)$, but due to integer constraints on code lengths, the length L is typ slightly greater than $H(X)$:

$$H(X) \leq L < H(X) + 1$$

"

Summary of Optimality Conditions for Huffman Coding

To check if a Huffman code is optimal, ensure:

- It is prefix-free (no codeword is a prefix of another).
- It adheres to the greedy choice property (combines lowest-probability nodes iteratively)

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