25. Let (X,Y) have the following joint distribution:

Y \ X	1	2	3	4
1	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{32}$
2	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{32}$	$\frac{1}{32}$
3	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$
4	$\frac{1}{4}$	0	0	0

Calculate mutual information I(X;Y) and joint entropy H(X,Y).

Solution:

The marginal distribution of X is $(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8})$ and the marginal distribution of Y is $(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4})$.

H(X) = $\frac{7}{4}$ bits and H(Y) = 2 bits.

We obtain,

$$f(x_1) \geq f(x_o) + f'(x_o)(x_1 - \lambda x_1 - (1-\lambda)x_2)$$
$$\therefore f(x_1) \geq f(x_o) + f'(x_o)\{(1-\lambda)(x_1 - x_2)\} \ldots\ldots \text{(iii)}$$

$$H(X/Y) = \sum_{i=1}^{4} P(y=i)\ \ H(X/Y=i)$$
$$= \frac{1}{4} H(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8}) + \frac{1}{4} H(\frac{1}{4}, \frac{1}{2}, \frac{1}{8}, \frac{1}{8}) + \frac{1}{4} H(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}) + \frac{1}{4} H(1,0,0,0)$$
$$= \frac{1}{4} \times \frac{7}{4} + \frac{1}{4} \times \frac{7}{4} + \frac{1}{4} \times 2 + \frac{1}{4} \times 0$$
$$= \frac{11}{8} \text{ bits}$$

Similarly,

$$H(Y/X) = \sum_{i=1}^{4} P(x=i)\ \ H(Y/X=i)$$
$$= \frac{1}{2} H(\frac{1}{4}, \frac{1}{8}, \frac{1}{8}, \frac{1}{2}) + \frac{1}{4} H(\frac{1}{4}, \frac{1}{2}, \frac{1}{4}, 0) + \frac{1}{8} H(\frac{1}{4}, \frac{1}{4}, \frac{1}{2}, 0) + \frac{1}{8} H(\frac{1}{4}, \frac{1}{4}, \frac{1}{2}, 0)$$
$$= \frac{1}{2} \times \frac{14}{8} + \frac{1}{4} \times \frac{6}{4} + \frac{1}{8} \times \frac{6}{4} + \frac{1}{8} \times \frac{6}{4}$$
$$= \frac{13}{8} \text{ bits}$$

$$H(X,Y) = H(X) + H(Y/X) = \frac{7}{4} + \frac{13}{8} = \frac{27}{8} \text{ bits}$$

$$I(X;Y) = H(X) - H\left(\frac{X}{Y}\right) = \frac{7}{4} - \frac{11}{8} = \frac{3}{8} \text{ bits}$$

Or, $I(X;Y) = H(Y) - H\left(\frac{Y}{X}\right) = 2 - \frac{13}{8} = \frac{3}{8}$ bits