3. $7x, 21x^2$ and $14xy^2$

The Numerical Coefficients of given monomials are

7, 21, 14

Greatest Common factor of 7, 21, 14 is 7.

Common literals appearing in given monomials are, x, y.

Smallest Power of x in three monomials = 1

Smallest Power of y in three monomials = 0

Monomial of Common literals with smallest Powers = x

Hence, the greatest Common factor = $7x$.

4. $42x^2yz$ and $63x^3y^2z^3$.

The Numerical Coefficients of given monomials are 42, 63.

Greatest Common factor of 42, 63 is 21.

Common literals appearing in given monomials are x, y, z.

Smallest Power of x in two monomials = 2

Smallest Power of y in two monomials = 1

Smallest Power of z in two monomials = 1

Monomial of common literals with smallest Powers = x^2yz.

Hence, the greatest Common factor = $21x^2yz$.

5. $12ax^2$, $6a^2x^3$ and $2a^3x^5$.

The numerical coefficients of given monomials are 12, 6 and 2.

Greatest Common factor of 12, 6, 2 is 2.

Common literals appearing in given monomials are a, x.

Smallest power of a in three monomials = 1

Smallest power of x in three monomials = ~~x~~ 2

Monomials of common literals with smallest powers $= ax^2$.

Hence, the greatest common factor $= 2ax^2$.

6. $9x^2$, $15x^2y^3$, $6xy^2$ and $21x^2y^2$.

Numerical coefficients of given monomials are 9, 15, 6 and 21

Greatest common factor of 9, 15, 6, 21 is 3.

Common literals appearing in given monomials are x, y.

Smallest power of x in given monomials $= x1$

Smallest power of y in given monomials $= 0$

Monomials of common literals with smallest powers $= x$.

Hence, the greatest common factor $= 3x$.

7. $4a^2b^3$, $-12a^3b$, $18a^4b^3$.

Numerical coefficients of given monomials are

$4, -12$ and 18.

Greatest common factor of $4, -12, 18$ is 2.

Common literals appearing in given monomials are 'a' and 'b'.

Smallest power of a in three monomials $= 2$

Smallest power of b in three monomials $= 1$

Monomial of common literals with smallest powers

$= a^2b$

Hence, the greatest common factor $= 2a^2b$.

8. $6x^2y^2$, $9xy^3$, $3x^3y^2$.

Numerical coefficients of given monomilas are

$6, 9, 3$.

Greatest common factor of $6, 9, 3$ is 3

Common literals appearing in given monomials are x and y.

Smallest power of x in three monomials $= 1$

Smallest power of y in three monomials $= 2$

Monomial of common literals with smallest powers

$= x \cdot y^2$

Hence, the greatest common factor $= 3xy^2$

9. $a^2b^3,\ a^3b^2$

There are no numerical coefficients.

Common literals appearing in given monomials are a and b.

Smallest power of a in given monomials = 2

Smallest power of b in given monomials = 2

Monomial of common literals with smallest powers is the greatest common factor = a^2b^2.

10. $36a^2b^2c^4,\ 54a^5c^2,\ 90a^4b^2c^2$.

Numerical coefficients of given monomials are 36, 54, 90.

Greatest common factor of 36, 54, 90 is 18.

Common literals appearing in given monomials are a, b, c.

Smallest power of a in three monomials = 2

Smallest power of b in three monomials = 0

Smallest power of c in three monomials = 2

Monomial of common literales with smallest powers $= a^2c^2$

Hence, the greatest common factor = $18a^2c^2$

(11) $x^3, -yx^2$.

There are no numerical coefficients.

Common literals appearing in given monomials are x and y.

Smallest power of x in both monomials $= 2$.

Smallest power of y in both monomials $= 0$

$\therefore$ Greatest common factor $= x^2$.

(12) $15a^3, -45a^2, -150a$.

Numerical coefficients of given monomials are $15, -45, -150$

Greatest common factor of $15, -45, -150$ is 15

Common literal in given monomials is 'a'.

Smallest. Power of a in given monomials $= 1$.

Hence, the greatest common factor $= 15a$.

(13) $2x^3y^2, 10x^2y^3, 14xy$.

Numerical coefficients of given monomials are $2, 10, 14$.

Greatest common factor of $2, 10, 14$ is 2.

Common literals in given monomials are x and y.

Smallest power of x in three monomials $= 1$

Smallest power of y in three monomials $= 1$

Monomial of common literals with smallest powers $= xy$.

Hence, the greatest common factor $= 2xy$.

(14) $14x^3y^5, 10x^5y^3, 2x^2y^2$

Numerical coefficients of given monomials are 14, 10, 2.

Greatest common factor of 14, 10, 2 is 2.

Common literals appearing in given monomials are x and y.

Smallest power of x in three monomials is 2

Smallest power of y in three monomials is 2.

Monomial of common literals with smallest powers $= x^2y^2$.

Hence, the greatest common factor $= 2x^2y^2$.

Find the greatest common factor of terms in each of the following expressions:-

(15) $5a^4+10a^3-15a^2$

The greatest common factor of the terms $5a^4, 10a^3, -15a^2$ is $5a^2$. Also, $5a^4 = 5a^2(a^2)$

$10a^3 = 5a^2(2a)$

$-15a^2 = 5a^2(-3)$

$\therefore 5a^4+10a^3-15a^2 = 5a^2(a^2+2a-3)$.

$\therefore$ Greatest common factor $= 5a^2$.

(16) $2xyz+3x^2y+4y^2$

The greatest common factor of three terms namely $2xyz, 3x^2y, 4y^2$ is y

Also $2xyz = y(2xz)$, $3x^2y = y(3x^2)$, $4y^2 = y(4y)$

$2xyz+3x^2y+4y^2 = y(2xz+3x^2+4y)$

$\therefore$ Greatest common factor is y.

(17) $3a^2b^2+4b^2c^2+12a^2b^2c^2$

The greatest common factor of three terms namely $3a^2b^2, 4b^2c^2, 12a^2b^2c^2$ is b^2.

Also, $3a^2b^2 = b^2(3a^2)$, $4b^2c^2 = b^2(4c^2)$, $12a^2b^2c^2 = b^2(12a^2c^2)$

$3a^2b^2+4b^2c^2+12a^2b^2c^2 = b^2(3a^2+4c^2+12a^2c^2)$

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EXERCISE - 7.2.

(1) factorize the following

(1) $3x-9$.

Greatest Common factor of the two terms namely $3x$ and -9 of expression $3x-9$ is 3.

Also $3x = 3 \times x$ and $-9 = 3 \times (-3)$

$$3x-9 = 3(x-9)$$

and Greatest Common factor $= 3$.

(2) $5x - 15x^2$.

Greatest Common factor of two terms namely $5x$ and $-15x^2$ of expression $5x - 15x^2$ is $5x$.

Also $5x = 5x \times 1$ and $-15x^2 = 5x(-3x)$.

$$5x - 15x^2 = 5x(1-3x).$$

and Greatest Common factor $= 5x$.

(3) $20a^{12}b^2 - 15a^8b^4$

Greatest Common factor of two terms namely $20a^{12}b^2$ and $-15a^8b^4$ of expression $20a^{12}b^2 - 15a^8b^4$ is $5a^3b^2$.

Also $20a^{12}b^2 = 5a^3b^2(4a^9)$ and $-15a^8b^4 = 5a^3b^2(-3a^5b^2)$

$$\therefore 20a^{12}b^2 - 15a^8b^4 = 5a^3b^2(4 \cdot a^9 - 3a^5b^2)$$

(4) $72x^6y^7 - 96x^7y^6$.

The greatest common factor of the two terms namely $72x^6y^7$ and $96x^7y^6$ is $24x^6y^6$

Also $72x^6y^7 = 24x^6y^6(3y)$, $96x^7y^6 = 24x^6y^6(4x)$

$\therefore 72x^6y^7 - 96x^7y^6 = 24x^6y^6(3y - 4x)$.

(5) $20x^3 - 40x^2 + 80x$.

The greatest common factor of three terms namely $20x^3$, $40x^2$, $80x$ is $20x$.

Also $20x^3 = 20x(x^2)$, $40x^2 = 20x \times (2x)$, $80x = 20x(4)$

$\therefore 20x^3 - 40x^2 + 80x = 20x(x^2 - 2x + 4)$

(6) $2x^3y^2 - 4x^2y^3 + 8xy^4$.

The greatest common factor of three terms namely $2x^3y^2$, $4x^2y^3$, $8xy^4$ is $2xy^2$.

Also $2x^3y^2 = 2xy^2(x^2)$, $4x^2y^3 = 2xy^2(2xy)$, $8xy^4 = 2xy^2(4y^2)$

$\therefore 2x^3y^2 - 4x^2y^3 + 8xy^4 = 2xy^2(x^2 - 2xy - 4y^2)$.

(7) $10m^3n^2 + 15m^4n - 20m^2n^3$.

The greatest common factor of three terms namely $10m^3n^2$, $15m^4n$, $20m^2n^3$ is $5m^2n$.

Also, $10m^3n^2 = 5m^2n(2mn)$, $15m^4n = 5m^2n(3m^2)$, $20m^2n^3 = 5m^2n(4n^2)$

$\therefore 10m^3n^2 + 15m^4n - 20m^2n^3 = 5m^2n(2mn + 3m^2 - 4n^2)$

(8) $2a^4b^4 - 3a^3b^5 + 4a^2b^5$.

The greatest common factor of three terms namely $2a^4b^4$, $3a^3b^5$, $4a^2b^5$ is a^2b^4.

Also, $2a^4b^4 = a^2b^4(2a^2)$, $3a^3b^5 = a^2b^4(3ab)$, $4a^2b^5 = a^2b^4(4b)$

$\therefore 2a^4b^4 - 3a^3b^5 + 4a^2b^5 = a^2b^4(2a^2 - 3ab + 4b)$

(9) $28a^2 + 14a^2b^2 - 21a^4$

The greatest common factor of three terms namely $28a^2$, $14a^2b^2$, $21a^4$ is $7a^2$.

Also $28a^2 = 7a^2(4)$, $14a^2b^2 = 7a^2(2b^2)$, $21a^4 = 7a^2(3a^2)$

$\therefore 28a^2 + 14a^2b^2 - 21a^4 = 7a^2(4 + 2b^2 - 3a^2)$

(10) $a^4b - 3a^2b^2 - 6ab^3$.

The greatest common factor of three terms namely a^4b, $3a^2b^2$, $6ab^3$ is ab.

Also $a^4b = ab(a^3)$, $3a^2b^2 = ab(3ab)$, $6ab^3 = ab(6ab^2)$

$\therefore a^4b - 3a^2b^2 - 6ab^3 = ab(a^3 - 3ab - 6ab^2)$.

(11) $21lmn - 3lm^2n + 4lmn^2$

The greatest common factor of three terms namely $21lmn$, $3lm^2n$, $4lmn^2$ is lmn.

Also $21lmn = lmn(21)$, $3lm^2n = lmn(3m)$, $4lmn^2 = lmn(4n)$

$\therefore 21lmn - 3lm^2n + 4lmn^2 = lmn(21 - 3m + 4n)$

12. $x^4y^2 - x^2y^4 - x^4y^4$. 12

The greatest common factor of three terms namely x^4y^2, x^2y^4, x^4y^4 is x^2y^2.

Also $x^4y^2 = x^2y^2(x^2)$, $x^2y^4 = x^2y^2(y^2)$, $x^4y^4 = x^2y^2(x^2y^2)$

$\therefore x^4y^2 - x^2y^4 - x^4y^4 = x^2y^2(x^2 - y^2 - x^2y^2)$.

13. $9x^2y + 3axy$.

The greatest common factor of two terms namely $9x^2y$, $3axy$ is $3xy$

Also, $9x^2y = 3xy(3x)$, $3axy = 3xy(a)$

$9x^2y + 3axy = 3xy(3x+a)$

14. $16m - 4m^2$

The greatest common factor of two terms namely $16m$, $4m^2$ is $4m$.

Also, $16m = 4m(4)$, $4m^2 = 4m(m)$

$16m - 4m^2 = 4m(4 - 4m)$

15. $-4a^2 + 4ab - 4ca$.

The greatest common factor of two terms namely $-4a^2$, $4ab$, $-4ca$ is $-4a$

Also $-4a^2 = -4a(a)$, $4ab = -4a(-b)$, $-4ca = -4a(c)$

$\therefore -4a^2 + 4ab - 4ca = -4a(a - b + c)$

16. $x^2yz + xy^2z + xyz^2$

The greatest common factor is xyz, Also $x^2yz = xyz(x)$, $xy^2z = xyz(y)$, $xyz^2 = xyz(z)$

$\therefore x^2yz + xy^2z + xyz^2 = xyz(x+y+z)$

17. $ax^2y + bxy^2 + cxyz$.

Greatest common factor of given terms is xy.

Also $ax^2y = xy(ax)$, $bxy^2 = xy(by)$, $cxyz = xy(cz)$

$\therefore ax^2y + bxy^2 + cxyz = xy(ax + by + cz)$.

(16) $x^2yz + xy^2z + xyz^2$

The greatest common factor of given three terms is xyz.

Also $x^2yz = xyz(x)$, $xy^2z = xyz(y)$, $xyz^2 = xyz(z)$

$x^2yz + xy^2z + xyz^2 = xyz(x+y+z)$.

(17) $ax^2y + bxy^2 + cxyz$

The greatest common factor of given three terms is xy.

Also $ax^2y = xy(ax)$, $bxy^2 = xy(by)$, $cxyz = xy(cz)$

$\therefore ax^2y + bxy^2 + cxyz = xy(ax + by + cz)$.

EXERCISE - 7.3

(1) factorize each of the following algebraic expressions

(1) $6x(2x-y)+7y(2x-y)$

$= (6x+7y)(2x-y)$ [$\because$ Taking $(2x-y)$ common]

(2) $2r(y-x)+s(x-y)$

$= -2r(x-y)+s(x-y)$ [$\because$ Taking -1 common from $(y-x)$]

$= (x-y)(-2r+s)$ [$\because$ Taking $(x-y)$ common]

$= (x-y)(s-2r)$

(3) $7a(2x-3)+3b(2x-3)$

$= (7a+3b)(2x-3)$ [$\because$ Taking $(2x-3)$ common]

(4) $9a(6a-5b)-12a^2(6a-5b)$

$= (9a-12a^2)(6a-5b)$ [$\because$ Taking $(6a-5b)$ Common]

(5) $5(x-2y)^2+3(x-2y)$

$= (x-2y)[5(x-2y)+3]$ [$\because$ Taking $(x-2y)$ Common]

$= (x-2y)[5x-10y+3]$

(6) $16(2l-3m)^2-12(3m-2l)$

$= 16(2l-3m)^2+12(2l-3m)$ [$\because 3m-2l = -(2l-3m)$]

$= 4(2l-3m)[4(2l-3m)+3]$ [$\because$ Taking $4(2l-3m)$ common]

$= 4(2l-3m)[8l-12m+3]$

(7) $3a(x-2y) - b(x-2y)$

$= (3a-b)(x-2y)$ [∵ Taking $(x-2y)$ as common]

(8) $a^2(x+y) + b^2(x+y) + c^2(x+y)$

$= (a^2+b^2+c^2)(x+y)$ [∵ Taking $(x+y)$ common in each term].

(9). $(x-y)^2 + (x-y)$

$= (x-y)[x-y+1]$ [∵ Taking $(x-y)$ common]

(10) $6(a+2b) - 4(a+2b)^2$

$= [6 - 4(a+2b)](a+2b)$ [∵ Taking $(a+2b)$ as common]

$= (6 - 4a - 8b)(a+2b)$

(11) $a(x-y) + 2b(y-x) + c(x-y)^2$

$= a(x-y) - 2b(x-y) + c(x-y)^2$ [∵ $(y-x) = -(x-y)$]

$= (x-y)[a - 2b + c(x-y)]$

$= (x-y)[a - 2b + cx - cy]$

(12) $-4(x-2y)^2 + 8(x-2y)$

$= -(x-2y)[+4(x-2y) - 8]$ [∵ Taking $-(x-2y)$ as common]

$= -(x-2y)[4x - 8y - 8]$.

(13) $x^3(a-2b) + x^2(a-2b)$

$= x^2(a-2b)[x+1]$ [∵ Taking $x^2(a-2b)$ common]

(14) $(2x-3y)(a+b) + (3x-2y)(a+b)$

$= (a+b)[2x - 3y + 3x - 2y]$ [∵ By taking $(a+b)$ common]

$= (a+b)[5x - 5y]$

(15) $4(x+y)(3a-b) + 6(x+y)(2b-3a)$

$2(x+y)[2(3a-b) + 3(2b-3a)]$ [∵ By taking $2(x+y)$ common]

$2(x+y)[4b - 3a]$.

EXERCISE - 7.4.

factorize each of the following expressions.

① $qr - pr + qs - ps$

$= q(r+s) - p(r+s)$

$= (q-p)(r+s)$

② $p^2q - pr^2 - pq + r^2$

$= p(pq - r^2) - 1(pq - r^2)$

$= (p-1)(pq - r^2)$

③ $1 + x + xy + x^2y$.

$(1+xy) + x(1+xy)$

$= (1+x)(1+xy)$

④ $ax + ay - bx - by$.

$= a(x+y) - b(x+y)$

$= (a-b)(x+y)$

⑤ $xa^2 + xb^2 - ya^2 - yb^2$

$= x(a^2+b^2) - y(a^2+b^2)$

$= (x-y)(a^2+b^2)$

⑥ $x^2 + xy + xz + yz$

$x(x+z) + y(x+z)$

$= (x+y)(x+z)$

⑦ $2ax + bx + 2ay + by$.

$= 2a(x+y) + b(x+y)$

$= (2a+b)(x+y)$

(8) $ab - by - ay + y^2$

$= a(b-y) - y(b-y)$

$= (a-y)(b-y)$

(9) $axy + bcxy - az - bcz$

$= a(xy-z) + bc(xy-z)$

$= (a+bc)(xy-z)$

(10) $lm^2 - mn^2 - lm + n^2$

$= lm(m-1) - n^2(m-1)$

$= (lm-n^2)(m-1)$

(11) $x^3 - y^2 + x - x^2y^2$

$= -y^2(1+x^2) + x(x^2+1)$

$= (x-y^2)(1+x^2)$

(12) $6xy + 6 - 9y - 4x$

$= 2x(3y-2) - 3(3y-2)$

$= (2x-3)(3y-2)$

(13) $x^2 - 2ax - 2ab + bx$

$= x(x+b) - 2a(x+b)$

$= (x-2a)(x+b)$

(14) $x^3 - 2x^2y + 3xy^2 - 6y^3$

$= x(x^2+3y^2) - 2y(x^2+3y^2)$

$= (x-2y)(x^2+3y^2)$

(15) $abx^2 + (ay-b)x - y$

$= abx^2 + ayx - bx - y$

$= bx(ax-1) + y(ax-1)$

$= (bx+y)(ax-1)$

(16) $(ax+by)^2 + (bx-ay)^2$

$= a^2x^2 + b^2y^2 + 2ax\,by + b^2x^2 + a^2y^2 - 2\cdot bx\,a\cdot y.$

$= a^2(x^2+y^2) + b^2(x^2+y^2)$

$= (a^2+b^2)(x^2+y^2)$

(17) $16(a-b)^3 - 24(a-b)^2$

$= 8(a-b)^2[2(a-b)-3]$

$= 8(a-b)^2[2a-2b-3].$

(18) $ab(x^2+1) + x(a^2+b^2)$

$= a\cdot b\cdot x^2 + ab + x\cdot a^2 + x\cdot b^2.$

$= ax(bx+a) + b(bx+a)$

$= (ax+b)(bx+a)$

(19) $a^2x^2 + (ax^2+1)x + a.$

$= a^2x^2 + ax^3 + x + a$

$= x(ax^2+1) + a(ax^2+1)$

$= (x+a)(ax^2+1)$

(20) $a(a-2b-c)+2bc$

$= a^2-2ab-ac+2bc$.

$= a(a-c)-2b(a-c)$

$= (a-2b)(a-c)$

(21) $a(a+b+c)-bc$

$= a^2+ab+ac-bc$

$= a(a-c)+b(a-c)$

$= (a+b)(a-c)$

(22) $x^2-11xy-x+11y$.

$= x(x-1)-11y(x-1)$

$= (x-11y)(x-1)$

(23) $ab-a-b+1$

$= a(b-1)-1(b-1)$

$= (a-1)(b-1)$

(24) $x^2+y-xy-x$

$x(x-1)-y(x-1)$

$= (x-y)(x-1)$

EXERCISE - 7.5.

Factorize each of the following expressions.

(1) $16x^2 - 25y^2$

$= (4x)^2 - (5y)^2$

$= (4x+5y)(4x-5y)$.

(2) $3(9x^2 - 4y^2)$

$= 3((3x)^2 - (2y)^2)$

$= 3(3x+2y)(3x-2y)$

(3) $144a^2 - 289 \cdot b^2$

$(12a)^2 - (17b)^2$

$= (12a+17b)(12a-17b)$

(4) $12m^2 - 27$

$= 3(4m^2 - 9)$

$= 3((2m)^2 - 3^2)$

$= 3(2m+3)(2m-3)$

(5) $125x^2 - 45y^2$

$5(25x^2 - 9y^2)$

$5((5x)^2 - (3y)^2)$

$= 5(5x+3y)(5x-3y)$

(6) $144a^2 - 169b^2$

$= (12a)^2 - (13b)^2$

$= (12a + 13b)(12a - 13b)$

(7) $(2a-b)^2 - 16c^2$

$= (2a-b)^2 - (4c)^2$

$= (2a-b+4c)(2a-b-4c)$

(8) $(x+2y)^2 - 4(2x-y)^2$

$= (x+2y)^2 - [2(2x-y)]^2$

$= [(x+2y) + 2(2x-y)][x+2y-2(2x-y)]$

$= [x+4x+2y-2y][x-4x+2y+2y]$

$(5x)(4y-3x)$.

(9) $3a^5 - 48a^3$

$= 3a^3(a^2 - 16)$

$= 3a^3(a^2 - 4^2)$

$= 3a^3(a+4)(a-5)$

(10) $a^4 - 16b^4$

$= (a^2)^2 - (4b^2)^2$

$= (a^2+4b^2)(a^2-4b^2)$

(11) $x^8 - 1$

$= (x^4)^2 - 1^2$

$= (x^4+1)(x^4-1)$

(12). $64 - (a+1)^2$

$= 8^2 - (a+1)^2$

$= [8 + (a+1)][8 - (a+1)]$

$= (a+9)[7 - a]$.

(13) $36l^2 - (m+n)^2$.

$= (6l)^2 - (m+n)^2$

$= (6l+m+n)(6l-m-n)$.

(14) $25x^4y^4 - 1$

$(5x^2y^2)^2 - (1)^2$

$(5x^2y^2-1)(5x^2y^2+1)$.

(15) $a^4 - \frac{1}{b^4}$

$= (a^2)^2 - \left[\frac{1}{b^2}\right]^2$

$= \left[a^2 + \frac{1}{b^2}\right]\left[a^2 - \frac{1}{b^2}\right]$.

(16) $x^3 - 144x$

$= x\left[x^2 - (12)^2\right]$

$= x\left[x+12\right]\left[x-12\right].$

(17) $(x-4y)^2 - 625$

$= (x-4y)^2 - (25)^2$

$= \left[x-4y+25\right]\left[x-4y-25\right]$

(18) $9(a-b)^2 - 100(x-y)^2$

$= \left[3(a-b)\right]^2 - \left[10(x-y)\right]^2$

$= \left[3(a-b) + 10(x-y)\right]\left[3(a-b) - 10(x-y)\right].$

$= \left[3a-3b+10x-10y\right]\left[3a-3b-10x+10y\right].$

(19) $(3+2a)^2 - 25a^2$

$= (3+2a)^2 - (5a)^2$

$(3+2a+5a)(3+2a-5a)$

$= (7a+3)(3-3a)$

(20) $(x+y)^2 - (a-b)^2$

$\left[(x+y)+(a-b)\right]\left[(x+y)-(a-b)\right]$

$= (x+y+a-b)(x+y-a+b).$

(21) $\frac{1}{16}x^2y^2 - \frac{4}{49}y^2z^2$

$\left[\frac{1}{4}(xy)\right]^2 - \left[\frac{2}{7}(yz)\right]^2$

$\left[\frac{xy}{4} + \frac{2(yz)}{7}\right]\left[\frac{xy}{4} - \frac{2(yz)}{7}\right]$

$= y^2\left[\frac{x}{4} + \frac{2}{7}z\right]\left[\frac{x}{4} - \frac{2}{7}z\right]$

(22) $75a^3b^2 - 108a \cdot b^4$.

$= 3ab^2\left[25a^2 - 36b^2\right]$

$= 3ab^2\left[(5a)^2 - (6b)^2\right]$

$= 3ab^2\left[5a+6b\right]\left(5a-6b\right]$.

(23) $x^5 - 16x^3$.

$= x^3\left[x^2 - 16\right]$

$= x^3\left[x^2 - 4^2\right]$

$= x^3(x+4)(x-4)$

(24) $\frac{50}{x^2} - \frac{2x^2}{81}$

$= 2\left[\frac{25}{x^2} - \frac{x^2}{81}\right]$

$= 2\left[\left(\frac{5}{x}\right)^2 - \left(\frac{x}{9}\right)^2\right]$

$= 2\left[\frac{5}{x} + \frac{x}{9}\right]\left[\frac{5}{x} - \frac{x}{9}\right]$.

(25) $256x^5 - 81x.$

$= x(256x^4 - 81)$

$= x((16x^2)^2 - 9^2)$

$= x(16x+9)(16x-9)$

(26) $a^4 - (2b+c)^4$

$= (a^2)^2 - [(2b+c)^2]^2$

$= [a^2 + (2b+c)^2] \cdot [a^2 - (2b+c)^2].$

$= [a^2 + (2b+c)^2][a+2b+c][a-2b-c].$

(27) $(3x+4y)^4 - x^4.$

$[(3x+4y)^2]^2 - (x^2)^2$

$[(3x+4y)^2 + x^2][(3x+4y)^2 - x^2]$

$[(3x+4y)^2 + x^2][3x+4y+x][3x+4y-x].$

(28) $p^2q^2 - p^4q^4.$

$(pq)^2 - (p^2q^2)^2$

$(pq + p^2q^2)(pq - p^2q^2)$

$(pq)^2[1+pq][1-pq].$

29. $3x^2y - 243xy^3$

$3xy\left[x^2 - 81y^2\right]$

$= 3xy\left[x^2 - (9y)^2\right]$

$= (3xy)(x+9y)(x-9y)$

30. $a^4b^4 - 16c^4$

$= (a^2b^2)^2 - (4c^2)^2$

$= (a^2b^2 + 4c^2)(a^2b^2 - 4c^2)$

$= (a^2b^2 + 4c^2)(ab+2c)(ab-2c)$

31. $x^4 - 625$

$= (x^2)^2 - (25)^2$

$= (x^2+25)(x^2-25)$

$= (x^2+25)(x+5)(x-5)$

32. $x^4 - 1$

$= (x^2)^2 - (1)^2$

$= (x^2+1)(x^2-1)$

$= (x^2+1)(x+1)(x-1)$

(37) $(2x+1)^2 - 9x^4$.

$= (2x+1)^2 - (3x^2)^2$

$= (2x+1+3x^2)(2x+1-3x^2)$

$= (3x^2+2x+1)(-3x^2+2x+1)$.

(38) $x^4 - (2y-3z)^2$.

$= (x^2)^2 - (2y-3z)^2$.

$= (x^2+2y-3z)(x^2-2y+3z)$.

(39) $a^2 - b^2 + a - b$.

$(a+b)(a-b) + (a-b)$

$= (a-b)(a+b+1)$.

(40) $16a^4 - b^4$.

$= (4a^2)^2 - (b^2)^2$

$= (4a^2+b^2)(4a^2-b^2)$

$= (4a^2+b^2)(2a+b)(2a-b)$

(41) $a^4 - 16(b-c)^4$.

$= (a^2)^2 - [4(b-c)^2]^2$

$[a^2+4(b-c)^2][a^2-4(b-c)^2]$

$[a^2+4(b-c)^2][(a+2b-2c)(a-2b+2c)]$.

(42) $2a^5 - 32a$

$= 2a(a^4 - 16)$

$= 2a((a^2)^2 - 4^2)$

$= 2a(a^2+4)(a^2-4)$

$= 2a(a^2+4)(a+2)(a-2)$

(43) $a^4b^4 - 81c^4$.

$= (a^2b^2)^2 - (9c^2)^2$

$= (a^2b^2 + 9c^2)(a^2b^2 - 9c^2)$

$= (a^2b^2 + 9c^2)(ab+3c)(ab-3c)$

(44) $\cancel{x(y^9}$ $xy^9 - y \cdot x^9$.

$= xy(y^8 - x^8)$

$= xy((y^4)^2 - (x^4)^2)$

$= xy(y^4+x^4)(y^4-x^4)$

$= xy(y^4+x^4)(y^2+x^2)(y^2-x^2)$

$= xy(y^4+x^4)(y^2+x^2)(y+x)(y-x)$

(45) $x^3 - x$

$= x(x^2-1) = x(x+1)(x-1)$

(46) $18a^2x^2 - 32$

$= 2((3ax)^2 - 4^2)$

$= 2(3ax+4)(3ax-4)$.

EXERCISE - 7.6.

I. factorize each of the following expressions.

① $4x^2+12xy+9y^2$.

$= (2x)^2+(3y)^2+2(2x)(3y)$

$= (2x+3y)^2$

② $9a^2-24ab+16b^2$.

$= (3a)^2+(4b)^2-2(3a)(4b)$

$= (3a-4b)^2$.

③ $(pq)^2+(3r)^2-6pqr$.

$= (pq)^2+(3r)^2-2(pq)(3r)$

$= (pq-3r)^2$.

④ $36a^2+36a+9$.

$9(4a^2+4a+1)$

$9((2a)^2+2(2a)+1^2)$

$= 9(2a+1)^2$.

⑤ $a^2+2ab+b^2-16$

$= (a+b)^2-4^2$.

$= (a+b+4)(a+b-4)$.

⑥ $9z^2-x^2+4xy-4y^2$.

$(3z)^2-[x^2-2(x)(2y)+(2y)^2]$

$= (3z)^2-(x-2y)^2 = (3z+(x-2y))(3z-(x-2y))$.

(7) $9a^4 - 24a^2b^2 + 16b^4 - 256$.

$(3a^2)^2 - 2(4a^2)(3b^2) + (4b^2)^2 - (16)^2$.

$(3a^2 - 4b^2)^2 - (16)^2$.

$(3a^2 - 4b^2 + 16)(3a^2 - 4b^2 - 16)$

(8). $16 - a^6 + 4a^3b^3 - 4b^6$.

$= 4^2 - \left((a^3)^2 - 2(a^3)(2b^3) + (2b^3)^2\right)$

$= 4^2 - \left[a^3 - 2b^3\right]^2$.

$= \left[4 + (a^3 - 2b^3)\right]\left[4 - a^3 + 2b^3\right]$.

(9) $a^2 - 2ab + b^2 - c^2$.

$= (a+b)^2 - c^2$

$= (a+b+c)(a+b-c)$

(10) $x^2 + 2x + 1 - 9y^2$.

$= (x+1)^2 - (3y)^2$

$(x+3y+1)(x-3y+1)$.

(11) $a^2 + 4ab + 3b^2$.

$= a^2 + ab + 3ab + 3b^2$.

$= a(a+b) + 3b(a+b)$

$= (a+3b)(a+b)$.

(12) $96-4x-x^2$

$= -x^2-4x+96.$

$= -x^2-12x+8x+96.$

$= -x(x+12)+8(x+12)$

$= (x+12)(-x+8).$

(13) a^4+3a^2+4

$\cancel{(a^2)^2}+(a^2)^2+2\cdot 2a^2+4-a^2.$

$= (a^2+2)^2-a^2$

$= (a^2+2+a)(a^2+2-a)$

(14) $4x^4+1$

$(2x^2)^2+1+4x^2-4x^2$

$(2x^2+1)^2-4x^2$

$(2x^2+2x+1)(2x^2-2x+1)$

(15) $4x^4+y^4.$

$=(2x^2)^2+(y^2)^2+4x^2y^2-4x^2y^2.$

$=(2x^2+y^2)^2-4x^2y^2$

$= (2x^2+y^2+2xy)(2x^2+y^2-2xy)$

(16) $(x+2)^2-6(x+2)+9$

$=x^2+4+4x-6x-12+9$

$= x^2+1-2x$

$= (x-1)^2.$

(17) $25 - p^2 - q^2 - 2pq$

$= 25 - (p^2 + q^2 + 2pq)$

$= (5)^2 - (p+q)^2$

$= (5 + p + q)(5 - p - q)$

$= -(p + q - 5)(p + q + 5)$

(18) $x^2 + 9y^2 - 6xy - 25a^2$

$= (x - 3y)^2 - (5a)^2$

$= (x - 3y + 5a)(x - 3y - 5a)$

(19) $49 - a^2 + 8ab - 16b^2$

$= 49 - (a^2 - 8ab + 16b^2)$

$= 49 - (a - 4b)^2$

$= (7 + a - 4b)(7 - a + 4b)$

$-(a - 4b + 7)(a - 4b - 7)$

(20) $a^2 - 8ab + 16b^2 - 25c^2$

$(a - 4b)^2 - (5c)^2$

$(a - 4b + 5c)(a - 4b - 5c)$

(21) $x^2 - y^2 + 6y - 9$

$= x^2 + \cancel{6y} - (y^2 - 6y + 9)$

$= x^2 - (y - 3)^2$

$= (x + y - 3)(x - y + 3)$

22. $25x^2 - 10x + 1 - 36y^2$

$= (5x)^2 - 2(5x) + 1 - (6y)^2$

$= (5x - 1)^2 - (6y)^2$

$= (5x - 1 + 6y)(5x - 1 - 6y)$

(23) $a^2 - b^2 + 2bc - c^2$

$= a^2 - (b^2 - 2bc + c^2)$

$= a^2 - (b - c)^2$

$= (a + b - c)(a - b + c)$

(24) $a^2 + 2ab + b^2 - c^2$

$= (a + b)^2 - c^2$

$= (a + b + c)(a + b - c)$

(25) $49 - x^2 - y^2 + 2xy$

$= 49 - (x^2 + y^2 - 2xy)$

$= 7^2 - (x - y)^2$

$= [7 + (x - y)][7 - x + y]$

(26) $a^2 + 4b^2 - 4ab - 4c^2$

$a^2 - 2(a)(2b) + (2b)^2 - (2c)^2$

$(a - 2b)^2 - (2c)^2$

$(a - 2b + 2c)(a - 2b - 2c)$

(27) $x^2 - y^2 - 4xz + 4z^2$

$= x^2 - 2(x)(2z) + (2z)^2 - y^2$

$= (x - 2z)^2 - y^2$

$= (x - 2z + y)(x - 2z - y)$

EXERCISE- 7.7.

factorize each of the following algebraic expressions.

(1) $x^2+12x-45$.

In order to factorize the given expression, we have to find two numbers, P and Q such that.

$p+q=12, \quad pq=-45$.

Clearly $15-3=12, \quad 15(-3)=-45$.

$\therefore$ split $12x$ as $15x-3x$.

$\therefore x^2+12x-45 = x^2+15x-3x-45$
$= x(x+15)-3(x+15)$
$= (x-3)(x+15)$

(2) $40+3x-x^2$
$= -(x^2-3x-40)$

In order to find factors, we need P and Q, such that

$p+q=-3, \quad pq=-40$

Clearly $5-8=-3, \quad 5(-8)=-40$.

$\therefore$ split $-3x$ as $5x-8x$.

$x^2-3x-40 = x^2+5x-8x-40$
$= x(x+5)-8(x+5)$
$= (x-8)(x+5)$

$-(x^2-3x-40) = (x+5)(-x+8)$

(3) $a^2+3a-88$

In order to factorise it, we need to find P and Q, where $P+Q=3$, $PQ=-88$.

$\therefore$ split $3a$ as $11a-8a$.

$$\therefore a^2+3a-88 = a^2+11a-8a-88$$
$$= a(a+11)-8(a+11)$$
$$= (a-8)(a+11)$$

(4) $a^2-14a-51$

In order to factorize it, we need to find P and Q, where $P+Q=-14$, $PQ=-51$

Clearly $3-17=-14$, $3(-17)=-51$

$\therefore$ split $-14a$ as $3a-17a$.

$$\therefore a^2-14a-51 = a^2+3a-17a-51$$
$$= a(a+3)-17(a+3)$$
$$= (a-17)(a+3)$$

(5) $x^2+14x+45$.

In order to factorize it, we need to find P and Q, where $P+Q=14$, $PQ=45$.

Clearly $5+9=14$, $5(9)=45$.

$\therefore$ split $14x$ into $5x+9x$

$$x^2+14x+45 = x^2+5x+9x+45$$
$$= x(x+5)+9(x+5)$$
$$= (x+9)(x+5)$$

(6) $x^2-22x+120$

In order to factorise it, we need to find P and Q, where

$P+Q=-22, \quad P.Q=120.$

clearly $-12-10=-22, \quad (-12)(-10)=120.$

$\therefore$ split $(-22x)$ into $-12x-10x.$

$$\begin{aligned}\therefore x^2-22x+120 &= x^2-12x-10x+120\\ &= x(x-12)-10(x-12)\\ &= (x-10)(x-12)\end{aligned}$$

(7) $x^2-11x-42$

In order to factorise it, we need to find P and Q, where

$P+Q=-11, \quad PQ=-42.$

clearly $\cancel{6-7=4}$ $3-14=-11, \quad 3(-14)=-42.$

$\therefore$ split $(-11x)$ into $3x-14x$

$$\begin{aligned}x^2-11x-42 &= x^2+3x-14x-42\\ &= x(x+3)-14(x+3)\\ &= (x-14)(x+3)\end{aligned}$$

(8) $a^2+2a-3.$

In order to factorise it, we need to find P and Q, where $P+Q=2, \quad P.Q=-3$; Clearly $P=3, \; Q=-1$

$\therefore$ split $(2a)$ into $(3a-a)$

$$\begin{aligned}a^2+2a-3 &= a^2+3a-a-3\\ &= a(a+3)-(a+3)\\ &= (a-1)(a+3)\end{aligned}$$

(9) $a^2+14a+48$

In order to factorize the given polynomial, we need to find P and Q, where

$P+Q=14, \quad P.Q=48$

Clearly $8+6=14, \quad 8.6=48$

$\therefore$ split $(14a)$ into $8a+6a$.

$$\therefore a^2+14a+48 = a^2+8a+6a+48$$
$$= a(a+8)+6(a+8)$$
$$= (a+6)(a+8)$$

(10) $x^2-4x-21$

In order to factorize the given polynomial we need to find P and Q, where

$P+Q=-4, \quad P.Q=-21$

Clearly $3-7=-4, \quad (3)(-7)=-21$.

$\therefore$ split $(-4x)$ into $3x-7x$.

$$x^2-4x-21 = x^2+3x-7x-21$$
$$= x(x+3)-7(x+3)$$
$$= (x-7)(x+3)$$

(11) $y^2+5y-36$

In order to factorize the given polynomial, we need to find P and Q, where

$P+Q=5, \quad P.Q=-36$, clearly $9-4=5, \quad 9(-4)=-36$

$\therefore$ split $5y$ in to $9y-4y$

$$\therefore y^2+5y-36 = y^2+9y-4y-36 = y(y+9)-4(y+9)$$
$$= (y-4)(y+9).$$

(12) $(a^2-5a)^2-36$

It can be written as $(a^2-5a)^2-6^2$.

using $a^2-b^2=(a+b)(a-b)$, $(a^2-5a)^2-6^2=(a^2-5a+6)(a^2-5a-6)$

To factorize (a^2-5a+6), we need to find P and Q

where $P+Q=-5$, $P.Q=6$, Clearly $-2-3=-5$, $(-2)(-3)=6$.

$\therefore$ split $-5a$ into $-2a-3a$.

$$\therefore (a^2-5a+6) = a^2-2a-3a+6$$
$$= a(a-2)-3(a-2)$$
$$= (a-3)(a-2)$$

To factorize (a^2-5a-6), we need to find P and Q

where $P+Q=-5$, $P.Q=-6$, Clearly $1-6=-5$, $1(-6)=-6$.

$\therefore$ split $-5a = a-6a$.

$$\therefore (a^2-5a-6) = a^2-a-6a+6$$
$$= (a-6)(a-1)$$

$$\therefore (a^2-5a)^2-36 = (a^2-5a+6)(a^2-5a-6) = (a-1)(a-2)(a-3)(a-6).$$

(13) $(a+7)(a-10)+16$

$= a^2-3a-54$

To factorize it, we need to find P and Q, where

$P+Q=-3$, $P.Q=-54$, Clearly $6-9=-3$, $6(-9)=-54$.

$\therefore$ split $-3a$ into $6a-9a$.

$$\therefore a^2-3a-54 = a^2+6a-9a-54$$
$$=(a-9)(a+6)$$

$$\therefore (a+7)(a-10)+16 = (a-9)(a+6)$$

42.

(4) $7x-6-2x^2$, $= -2x^2+7x-6$.

Here Coefficient of $x^2=-2$, Coefficient of $x=7$, and Constant term $=-6$.

we shall now split up the Coefficient of x, i-e 7 into two parts whose sum is 7 and Product is $-2\times-6=12$, clearly $4+3=7$, $4\times3=12$.

So we write the middle term $7x$ as $4x+3x$

Thus we have

$$-2x^2+7x-6 = -2x^2+4x+3x-6$$
$$= -2x(x-2)+3(x-2)$$
$$= (x-2)(3-2x)$$

(5) $7x^2-19x-6$.

Here Coefficient of $x^2=7$, Coefficient of $x=-19$ and Constant term $=-6$.

we shall now split up the coefficient of x, i-e -19 into two parts, whose sum is -19 and product is $7\times-6=-42$, clearly $2-21=-19$ and $2(-21)=-42$. , So we write the middle term $-19x$ as $2x-21x$

Thus we have

$$7x^2-19x-6 = 7x^2+2x-21x-6$$
$$= x(7x+2)-3(7x+2)$$
$$= (x-3)(7x+2)$$

(6) $28 - 31x - 5x^2$

$= -5x^2 - 31x + 28$

Here Coefficient of $x^2 = -5$, Coefficient of $x = -31$ and Constant term $= 28$.

We shall now split up the Coefficient of x, i.e -31 into two parts whose sum is -31 and product is $-5(28) = -140$. Clearly, $4 - 35 = -31$ and $4(-35) = -140$. So we write the middle term $-31x$ as $4x - 35x$.

Thus we have

$$-5x^2 - 31x + 28 = -5x^2 + 4x - 35x + 28$$
$$= -x(5x - 4) - 7(5x - 4)$$
$$= -(x+7)(5x-4)$$

(7) $3 + 23y - 8y^2$

$= -8y^2 + 23y + 3$

Here Coefficient of $y^2 = -8$, Coefficient of $y = 23$, and Constant term $= 3$.

We shall now split up the Coefficient of y, ie 23 into two parts, whose sum is 23 and product is $-8(3) = -24$. Clearly $24 - 1 = 23$ and $24(-1) = -24$. So we write the middle term $23y$ as $24y - y$.

Thus, we have

$-8y^2+23y+3 = -8y^2+24y-y+3$

$= -8y(y-3)-1(y-3)$

$= -(8y+1)(y-3)$

(8) $11x^2-54x+63$

Here Coefficient of $x^2=11$, Coefficient of $x=-54$ and Constant team $=63$.

We shall now split up the Coefficient of x, ie -54 into two parts whose sum is -54 and product is 11×63, i.e 693, Clearly $-33x-21x=-54x$ and Product is $(-33)(-21)=693$

So, we write middle team $-54x$ as $-33x-21x$

Thus, we have

$11x^2-54x+63$

$= 11x^2-33x-21x+63$

$= 11x(x-3)-21(x-3)$

$= (11x-21)(x-3)$.

(9) $7x-6x^2+20$

$= -6x^2+7x+20$

Here Coefficient of $x^2=-6$, Coefficient of $x=7$, Constant team $=20$

we shall now split up the coefficient of x, ie 7 into two parts whose sum is 7 and product is $-6(20)=-120$, clearly $15-8=7$ and $15(-8)=-120$. clearly. so we write middle team $7x$ as $15x-8x$

Thus, we have

$$9-6x^2+7x+20 = -6x^2+15x-8x+20$$
$$= -3x(2x-5)-4(2x-5)$$
$$= -(3x+4)(2x-5)$$

(10) $3x^2+22x+35$.

Here Coefficient of $x^2=3$, Coefficient of $x=22$ and Constant team $=35$.

we shall now split up the Coefficient of x, ie 22 into two parts whose sum is 22 and product is $35(3)=105$. so we write middle team $22x$ as $15x+7x$

Thus we have

$$3x^2+22x+35 = 3x^2+15x+7x+35$$
$$= 3x(x+5)+7(x+5)$$
$$= (3x+7)(x+5)$$

(11) $12x^2 - 17xy + 6y^2$

Here Coefficient of $x^2 = 12$, Coefficient of $x = -17y$ and Constant term $= 6y^2$.

Now, we split coefficient of middle term i.e $-17y$ into two parts whose sum is $-17y$ and product $12 \times 6y^2 = 72y^2$.

clearly $-9y - 8y = -17y$ and $(-9y)(-8y) = 72y^2$.

So, we replace middle term, $-17xy = -9xy - 8xy$

Thus, we have

$$12x^2 - 17xy + 6y^2 = 12x^2 - 9xy - 8xy + 6y^2$$
$$= 3x(4x - 3y) - 2y(4x - 3y)$$
$$= (3x - 2y)(4x - 3y)$$

(12) $6x^2 - 5xy - 6y^2$

Here Coefficient of $x^2 = 6$, Coefficient of $x = -5y$ and Constant term $= -6y^2$.

Now, we split Coefficient of middle term i.e $-5y$ into two parts whose sum is $-5y$ and product is $6(-6y^2) = -36y^2$.

Clearly $4y - 9y = -5y$ and $(4y)(-9y) = -36y^2$.

So, we replace $-5xy = 4xy - 9xy$, Thus we have

$$6x^2 - 5xy - 6y^2 = 6x^2 + 4xy - 9xy - 6y^2$$
$$= (2x - 3y)(3x + 2y)$$

(13) $6x^2 - 13xy + 2y^2$.

Coefficient of $x^2 = 6$, Coefficient of $x = -13y$

Constant term $= 2y^2$.

Now, we split coefficient of middle term

i.e $-13y$ into two parts whose sum is $-13y$

and product $6 \times 2y^2 = 12y^2$.

clearly $-12y - y = -13y$, $(-12y)(-y) = 12y^2$.

So, we replace $-13xy = -12xy - xy$

Thus, we have

$$6x^2 - 13xy + 2y^2 = 6x^2 - 12xy - xy + 2y^2$$
$$= 6x(x-2y) - y(x-2y)$$
$$= (6x-y)(x-2y).$$

(14) $14x^2 + 11xy - 15y^2$

Coefficient of $x^2 = 14$, Coefficient of $x = 11y$

Constant term $= -15y^2$.

Now, we split coefficient of middle term

i.e $11y$ into two parts whose sum is $11y$

and product $= 14(-15)y^2 = -210y^2$.

clearly $21y - 10y = 11y$ and $(21y)(-10y) = -210y^2$.

So, we replace $11xy = 21y - 10y$.

Thus we have

$$14x^2+11xy-15y^2 = 14x^2+21xy-10xy-15y^2$$
$$= 2x(7x-5y)+3y(7x-5y)$$
$$= (2x+3y)(7x-5y).$$

(15) $6a^2+17ab-3b^2$

Coefficient of $a^2 = 6$, Coefficient of $a = 17b$

Constant term $= -3b^2$.

Now, we split coefficient of middle term i.e $17ab$ into two terms whose sum is $17b$ and product $= 6(-3b^2) = -18b^2$

Clearly $18b - b = 17b$ and $(18b)(-b) = -18b^2$.

So, we replace $17ab = 18ab - ab$

Thus, we have

$$6a^2+17ab-3b^2 = 6a^2+18ab-ab-3b^2$$
$$= 6a(a+3b)-b(a+3b)$$
$$= (6a-b)(a+3b)$$

(16) $36a^2+12abc-15b^2c^2$

Coefficient of $a^2 = 36$, Coefficient of $a = 12bc$,

Constant term $= -15b^2c^2$

Now, we split coefficient of middle term, i.e $12bc$ into two terms whose sum is $12bc$ and product $= 36(-15b^2c^2) = -540b^2c^2$

So, we replace $12abc = 30abc - 18abc$.

Thus, we have

$$36a^2 + 12abc - 15b^2c^2 = 36a^2 + 30abc - 18abc - 15b^2c^2$$

$$= (6a+5bc)(6a-3bc)$$

(17) $15x^2 - 16xyz - 15y^2z^2$

Coefficient of $x^2 = 15$, Coefficient of $x = -16yz$,

Coefficient Constant term $= -15y^2z^2$.

Now, we split coefficient of middle term i.e $-16yz$ into two parts whose sum is $-16yz$ and product is $15(-15y^2z^2) = -225y^2z^2$.

Clearly $-25yz + 9yz = -16yz$ and $(-25yz)(9yz) = -225y^2z^2$

So, we replace $-16xyz = -25yz + 9yz$

Thus, we have

$$15x^2 - 16xyz - 15y^2z^2 = 15x^2 - 25yz + 9yz - 15y^2z^2$$

$$= 5x(3x - 5yz) + 3yz(3x - 5yz)$$

$$= (5x + 3yz)(3x - 5yz)$$

(18) $(x-2y)^2-5(x-2y)+6$

$x^2+4y^2-4xy-5x+10y+6$

Coefficient of $(x-2y)^2=1$, Coefficient of $(x-2y)=-5$

Constant $=6$.

Now, we split Coefficient of middle term i.e -5 into two terms whose sum is -5 and product $=6(1)=6$

Clearly $-2-3=-5$, $-2(-3)=6$.

So we replace $-5(x-2y)=-2(x-2y)-3(x-2y)$

Thus we have,

$$(x-2y)^2-5(x-2y)+6=(x-2y)^2-2(x-2y)-3(x-2y)+6$$
$$=(x-2y-2)(x-2y-3)$$

(19) $(2a-b)^2+2(2a-b)-8$.

Coefficient of $(2a-b)^2=1$, Coefficient of $(2a-b)=2$

Constant $=-8$.

Now, we split coefficient of middle term i.e 2 into two terms whose sum is 2 and product $=-8(1)=-8$.

Clearly $4-2=2$ and $4(-2)=-8$.

So, we replace $2(2a-b)=4(2a-b)-2(2a-b)$

Thus we have,

$$(2a-b)^2+2(2a-b)-8=(2a-b)^2+4(2a-b)-2(2a-b)-8$$
$$=(2a-b)(2a-b+4)-2(2a-b+4)$$
$$=(2a-b-2)(2a-b+4)$$

EXERCISE - 7.9.

(1) p^2+6p+8

Here Coefficient of p^2 is unity. so we add and subtract square of half of coefficient of p.

$\therefore p^2+6p+8 = p^2+6p+3^2-3^2+8$ [Adding and subtracting $\left(\frac{6}{2}\right)^2 = 3^2$]

$= (p+3)^2 - 1^2$ [By completing the square]

$= [p+3-1][p+3+1]$

$= (p+2)(p+4)$

(2) $q^2-10q+21$

Coefficient of q^2 is 1, so we add and subtract square of half of Coefficient of q.

$q^2-10q+21 = q^2-10q+5^2-5^2+21$ [Adding and subtracting $\left(\frac{10}{2}\right)^2 = 5^2$]

$= (q-5)^2 - 2^2$ [By completing the square]

$= [q-5-2][q-5+2]$

$= (q-7)(q-3)$

③ $4y^2+12y+5$

we have $4y^2+12y+5 = 4\left[y^2+3y+\frac{5}{4}\right]$

[∴ making m^2 coefficient = 1].

$= 4\left[y^2+3y+\left(\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2+\frac{5}{4}\right]$

$= 4\left[\left(y+\frac{3}{2}\right)^2-1^2\right]$ (Completing the square)

$= 4\left[y+\frac{3}{2}+1\right]\left[y+\frac{3}{2}-1\right]$

$= (2y+5)(2y+1)$

④ $p^2+6p-16$.

Coefficient of $p^2=1$

∴ we have $p^2+6p+3^2-3^2-16$ [Adding and subtracting $\left(\frac{6}{2}\right)^2 = 3^2$]

$= (p+3)^2-5^2$ (Completing the square)

$= (p+3+5)(p+3-5)$

$= (p+8)(p-2)$

⑤ $x^2+12x+20$

Coefficient of $x^2=1$

∴ we have $x^2+12x+6^2-6^2+20$ [Adding and subtracting $\left(\frac{12}{2}\right)^2 = 6^2$]

$= (x+6)^2-4^2$ (Completing the square)

$= (x+6+4)(x+6-4)$

$= (x+10)(x+2)$.

$= 4\left[x-\frac{3}{2}+1\right]\left[x-\frac{3}{2}-1\right]$

$(2x-1)(2x-5)$.

(9) $y^2-7y+12$.

Coefficient of $y^2=1$

$\therefore$ we have $y^2-7y+12 = y^2-7\cdot y+\left(\frac{7}{2}\right)^2-\left(\frac{7}{2}\right)^2+12$

[$\because$ By adding and subtracting $\left(\frac{7}{2}\right)^2$]

$= \left(y-\frac{7}{2}\right)^2-\left(\frac{1}{2}\right)^2$ (Completing the square)

$= \left(y-\frac{7}{2}-\frac{1}{2}\right)\left(y-\frac{7}{2}+\frac{1}{2}\right)$

$= (y-4)(y-3)$

(10) $z^2-4z-12$.

Coefficient of $z^2=1$

$\therefore$ we have $z^2-4z-12 = z^2-4z+2^2-2^2-12$

[$\because$ By adding and subtracting $\left(\frac{4}{2}\right)^2=2^2$]

$(z-2)^2-4^2$ (Completing the square)

$= (z-2+4)(z-2-4)$

$= (z+2)(z-6)$

(6) $a^2 - 14a - 51$

Coefficient of $a^2 = 1$

$\therefore$ we have $a^2 - 14a - 51 = a^2 - 14a + 7^2 - 7^2 - 51$

$\left[\because \text{Adding and subtracting } \left(\frac{14}{2}\right)^2 = 7^2\right]$

$= (a-7)^2 - 10^2$ (Completing the square)

$= (a - 7 + 10)(a - 7 - 10)$

$= (a+3)(a-17)$

(7) $a^2 + 2a - 3.$

Coefficient of $a^2 = 1$

$\therefore$ we have $a^2 + 2a - 3 = a^2 + 2a + 1^2 - 1^2 - 3$

$\left[\because \text{Adding and subtracting } \left(\frac{2}{2}\right)^2 = 1^2\right]$

$= (a+1)^2 - 2^2$ (completing the square)

$= (a+1+2)(a+1-2)$

$(a+3)(a-1)$

(8) $4x^2 - 12x + 5$

we have $4x^2 - 12x + 5 = 4\left(x^2 - 3x + \frac{5}{4}\right)$

$= 4\left[x^2 - 3x + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + \frac{5}{4}\right]$

$\left(\because \text{adding and subtracting } \left(\frac{3}{2}\right)^2\right)$

$= 4\left[\left(x - \frac{3}{2}\right)^2 - 1^2\right]$ ($\because$ completing the square)