Ex 23.1

Q1

(i) Angle made with positive x axis is $\frac{-\pi}{4}$.

$\therefore m = \tan\theta = \tan\left(\frac{-\pi}{4}\right) = -1$

(ii) Angle made with positive x axis is $\frac{2\pi}{3}$

$\therefore m = \tan\theta = \tan\left(\frac{2\pi}{3}\right) = -\sqrt{3}$

(iii) Angle made with positive x axis is $\frac{3\pi}{4}$

$\therefore m = \tan\theta = \tan\left(\frac{3\pi}{4}\right) = \tan\left(\pi - \frac{\pi}{4}\right) = -1$

(iv) Angle made with positive x axis is $\frac{\pi}{3}$

$\therefore m = \tan\theta = \tan\left(\frac{\pi}{3}\right) = \sqrt{3}$

Q2

(i) $(-3,2)$ and $(1,4)$

slope of line $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-2}{1-(-3)} = \frac{2}{4} = \frac{1}{2}$

(ii) $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$

slope of line $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{2at_2 - 2at_1}{at_2^2 - at_1^2} = \frac{2}{t_2 + t_1}$

(iii) $(3, -5)$ and $(1,2)$

slope of line $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-(-5)}{1-3} = \frac{7}{-2} = \frac{-7}{2}$

Q3(i)

Slope of line joining $(5,6)$ and $(2,3)$

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-6}{2-5} = \frac{-3}{-3} = 1$$

Slope of line joining $(9,-2)$ and $(6,-5)$

$$m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5-(-2)}{6-9} = \frac{-5+2}{-3} = 1$$

Here $m_1 = m_2$

$\therefore$ The two lines are parallel.

Q3(ii)

Slope of line joining $(-1,1)$ and $(9,5)$

$$m_1 = \frac{5-1}{9-(-1)} = \frac{4}{10} = \frac{2}{5}$$

Slope of line joining $(3,-5)$ and $(8,-3)$

$$m_2 = \frac{-3-(-5)}{8-3} = \frac{-3+5}{5} = \frac{2}{5}$$

Here $m_1 = m_2$

$\therefore$ The two lines are parallel

Q3(iii)

Slope of line joining $(6,3)$ and $(1,1)$

$$m_1 = \frac{1-3}{1-6} = \frac{-2}{-5} = \frac{2}{5}$$

Slope of line joining $(-2,5)$ and $(2,-5)$

$$m_2 = \frac{-5-5}{2-(-2)} = \frac{-10}{4} = \frac{-5}{2}$$

Here $m_1 \times m_2 = \frac{2}{5} \times \frac{-5}{2} = -1$

$\therefore$ The lines are perpendicular to each other.

Q3(iv)

Slope of line joining $(3,15)$ and $(16,6)$

$$m_1 = \frac{6-15}{16-3} = \frac{-9}{13}$$

Slope of line joining $(-5,3)$ and $(8,2)$

$$m_2 = \frac{2-3}{8-(-5)} = \frac{-1}{13}$$

Here, neither $m_1 = m_2$ *nor* $m_1 \times m_2 = -1$

$\therefore$ The lines are neither parallel nor perpendicular.

Q4

(i) Line bisects first quadrant.

$\Rightarrow$ Angle between line and positive direction of x-axis $= \frac{90°}{2}$

$= 45°$

Slope of line $(m) = \tan\theta$

$m = \tan 45°$

$m = 1$

(ii) Line makes angle of $30°$ wiht the positve direction of y-axis.

= Angle between line and positive side of axis $= 90° + 30°$

$\theta° = 120°$

$m = \tan 120°$

$m = -\sqrt{3}$

Q5(i)

$A(4,8)$, $B(5,12)$ and $C(9,28)$

slope of $AB = \frac{12-8}{5-4} = \frac{4}{1} = 4$

slope of $BC = \frac{28-12}{9-5} = \frac{16}{4} = 4$

slope of $CA = \frac{8-28}{4-9} = \frac{-20}{-5} = 4$

Since all 3 line segments have the same slope, they are parallel.
Since they have a common point B, they are collinear.

Q5(ii)

$A(16,-18), B(3,-6)$ and $C(-10,6)$

slope of $AB = \dfrac{-6-(-18)}{3-16} = \dfrac{12}{-13}$

slope of $BC = \dfrac{6-(-6)}{-10-3} = \dfrac{12}{-13}$

slope of $CA = \dfrac{6-(-18)}{-10-16} = \dfrac{12}{-13}$

Since all 3 line segments have the same slope and share a common vertex B, they are collinear.

Q6

Slope of line joining $(-1,4)$ and $(0,6)$ is

$m_1 = \dfrac{6-4}{0-(-1)} = 2$

Slope of line joining $(3,y)$ and $(2,7)$ is

$m_2 = \dfrac{7-y}{2-3} = y-7$

Since the two lines are parallel $m_1 = m_2$

$\Rightarrow 2 = y-7$

$\Rightarrow y=9$

Q7

(i) If $slope = \tan\theta = 0 \Rightarrow \theta = 0$

When the slope of a line is zero then the line is parallel to x-axis.

(ii) If the slope is positive then $\tan\theta =$ positive $\Rightarrow \theta =$ acute

Thus the line makes an acute angle $\left(0<\theta<\dfrac{\pi}{2}\right)$ with the positive x-axis.

(iii) When the slope is negative then $\tan\theta =$ negative $\Rightarrow \theta$ is obtuse

Thus the line makes an obtuse angle angle $\left(\theta>\dfrac{\pi}{2}\right)$ with the positive x-axis.

Q8

Slope of line joining $(2,-3)$ and $(-5,1)$

$$m_1 = \frac{1-(-3)}{-5-2} = \frac{4}{-7}$$

Slope of line joining $(7,-1)$ and $(0,3)$

$$m_2 = \frac{3-(-1)}{0-7} = \frac{4}{-7}$$

Since $m_1 = m_2$, the two lines are parallel.

Q9

Slope of line joining $(2,-5)$ and $(-2,5)$ is

$$m_1 = \frac{5-(-5)}{-2-2} = \frac{-5}{2}$$

Slope of line joining $(6,3)$ and $(1,1)$

$$m_2 = \frac{1-3}{1-6} = \frac{2}{5}$$

$$m_1 \times m_2 = \frac{-5}{2} \times \frac{2}{5} = -1$$

$\therefore$ The two lines are perpendicular to each other

Q10

Slope of $AB = \dfrac{2-4}{1-0} = -2$

Slope of $BC = \dfrac{3-2}{3-1} = \dfrac{1}{2}$

slope of $AB \times$ slope of $BC = -2 \times \dfrac{1}{2} = -1$

$\therefore$ Angle between AB and $BC = \dfrac{\pi}{2}$

$\therefore$ ABC are the vertices of a right angled triangle.

Q11

Here $A(-4, -1), B(-2, -4), C(4, 0), D(2, 3)$

Slope of $AB = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4+1}{-2+4}$

$$M_{AB} = \frac{-3}{2}$$

Slope of $BC = \frac{0+4}{4+2}$

$$M_{BC} = \frac{2}{3}$$

Slope of $AD = \frac{3+1}{2+4}$

$$M_{AD} = \frac{2}{3}$$

Slope of $CD = \frac{3-0}{2-4}$

$$M_{CD} = \frac{-3}{2}$$

$\Rightarrow \quad M_{AB} = M_{CD}$ and $M_{BC} = M_{AD}$

$\Rightarrow \quad AB || CD$ and $BC || AD$

$$M_{AB} \times M_{BC} = \frac{-3}{2} \times \frac{2}{3}$$

$$M_{AB} \times M_{BC} = -1$$

$\Rightarrow \quad AB \perp BC$

$$M_{BC} \times M_{CD} = \frac{2}{3} \times \frac{-3}{2}$$

$$M_{BC} \times M_{CD} = -1$$

$\Rightarrow \quad BC \perp CD$

Thus,

$AB || CD$ and $BC || AD$

$AB \perp BC$, $BC \perp CD$, $CD \perp AD$

$\Rightarrow \quad ABCD$ is a rectangle

Q12

If 3 points lie on a line (ie they are collinear) lines joining these point have the same slope

$\therefore$ slope of AP = slope of PB = slope of BA

$$\Rightarrow \frac{b-0}{a-h} = \frac{k-b}{0-a} = \frac{k-0}{0-h} \text{..........(i)}$$

$$\Rightarrow \frac{k-b}{0-a} = \frac{k-0}{0-h}$$

$$\Rightarrow -kh + bh = -ka$$

$$\Rightarrow -1 + \frac{b}{k} = \frac{-a}{h} \qquad \text{(dividing by } kh\text{)}$$

$$\Rightarrow \frac{a}{h} + \frac{b}{k} = 1$$

Hence Proved

Q13

Let $m_1 = x$, $m_2 = 2x$

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\frac{1}{3} = \left|\frac{x - 2x}{1 + 2x^2}\right|$$

Case I:

$$\frac{1}{3} = \frac{x - 2x}{1 + 2x^2}$$

$$2x^2 + 1 = -3x$$

$$2x^2 + 3x + 1 = 0$$

$$2x^2 + 2x + x + 1 = 0$$

$$2x(x + 1) + 1(x + 1) = 0$$

$$(x + 1)(2x + 1) = 0$$

$$x = -1, -\frac{1}{2}$$

Case II:

$$\frac{1}{3} = \left(\frac{-x}{1 + 2x^2}\right)$$

$$\frac{1}{3} = \frac{x}{1 + 2x^2}$$

$$2x^2 + 1 = 3x$$

$$2x^2 - 3x + 1 = 0$$

$$2x^2 - 2x - x + 1 = 0$$

$$2x(x - 1) - 1(x - 1) = 0$$

$$(x - 1)(2x - 1) = 0$$

$$x = 1, \frac{1}{2}$$

Slope of other line is

$$1, \frac{1}{2} \quad \text{or} \quad -1, -\frac{1}{2}$$

Q14

Slope of $AB = \frac{97-92}{1995-1985} = \frac{5}{10} = \frac{1}{2}$

Population (p) in 2010 can be calculated using the slope of AC.

Slope of $AC = \frac{p-92}{2010-1985} = \frac{p-92}{25} = \frac{1}{2}$ = Slope of AB

$\Rightarrow \quad p - 92 = \frac{25}{2}$

$\Rightarrow \quad 2p - 184 = 25$

$\Rightarrow \quad 2p = 209$

$\Rightarrow \quad p = \frac{209}{2}$

$\therefore \quad p = 104.50$ crores

Q15

Let $A(-2,-1)$, $B(4,0)$, $C(3,3)$ and $D(-3,2)$ be a quadrilateral.

slop of $AB = \frac{0-(-1)}{4-(-2)} = \frac{1}{6}$

slop of $BC = \frac{3-0}{3-4} = -3$

slop of $CD = \frac{3-2}{3-(-3)} = \frac{1}{6}$

slop of $DA = \frac{2-(-1)}{-3-(-2)} = -3$

we observe that slope of opposite side of the quadrilateral $ABCD$ are equal.
Hence the quadrilateral $ABCD$ is a parallelogram.

Q16

Slope of the line segment joning the points $(3,-1)$ and $(4,-2)$ is

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-(-1)}{4-3} = \frac{-2+1}{4-3} = \frac{-1}{1} = -1$$

Slope of x axis is 0

$\Rightarrow m_2 = 0$

If θ is the angle between x-axis and the line segment then

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \left|\frac{-1-0}{1+(-1)(0)}\right|$$

$$= \frac{-1}{1} = -1$$

$\therefore \theta = 135^0$

Q17

The slope of the line joining $(-2,6)$ and $(4,8)$ is

$$m_1 = \frac{8-6}{4-(-2)} = \frac{2}{6} = \frac{1}{3}$$

The slope of the line joining $(8,12)$ *and* $(x,24)$ is

$$m_2 = \frac{24-12}{x-8} = \frac{12}{x-8}$$

Since the lines are perpendicular to each other

$m_1 \times m_2 = -1$

$$\frac{1}{3} \times \frac{12}{x-8} = -1$$

$\Rightarrow 4 = 8 - x$

$\Rightarrow x = 4$

Q18

The given points are $A(x,-1)$, $B(2,1)$ and $C(4,5)$

It is given that the points are collinear. So, the area of the triangle that they form must be zero.

Hence,

$x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)=0 \quad ---(1)$

Putting the value of $(x_1y_1),(x_2y_2),(x_3y_3)$ in (i)

$x(1-5)+(2)(5-(-1))+4(-1-1)=0$

$-4x+2(5+1)+4(-2)=0$

$-4x+12-8=0$

$-4x=-12+8$

$-4x=-4$

$x=1$

Q19

Slope of the line segment joning the points $(3,-1)$ and $(4,-2)$ is

$$m_1=\frac{y_2-y_1}{x_2-x_1}=\frac{-2-(-1)}{4-3}=\frac{-2+1}{4-3}=\frac{-1}{1}=-1$$

Slope of x axis is 0

$\Rightarrow m_2=0$

If θ is the angle between x-axis and the line segment then

$$\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|$$

$$=\left|\frac{-1-0}{1+(-1)(0)}\right|$$

$$=\frac{-1}{1}=-1$$

$\therefore\ \theta=135^0$

Q20

Let the vertices be $A(-2,-1), B(4,0), C(3,3), D(-3,2)$.

Using slope formula, $m = \dfrac{y_2 - y_1}{x_2 - x_1}$, we get:

Slope of AB $(m_1) = \dfrac{0-(-1)}{4-(-2)} = \dfrac{1}{6}$

Slope of CD $(m_2) = \dfrac{2-3}{-3-3} = \dfrac{-1}{-6} = \dfrac{1}{6}$

$\Rightarrow m_1 = m_2 \Rightarrow AB \parallel CD$

Also

Slope of AD $(m_3) = \dfrac{2-(-1)}{-3-(-2)} = \dfrac{3}{-1} = -3$

Slope of BC $(m_4) = \dfrac{3-0}{3-4} = \dfrac{3}{-1} = -3$

$\Rightarrow m_3 = m_4 \Rightarrow AD \parallel BC$

Hence, ABCD is a parrallelogram.

Q21

Let $ABCD$ be the given quadrilateral

E is mid point of AB

F is mid point of BC

G is mid point of CD

H is mid point of AD

Using mid point formula $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Coordinates of $E = \left(\frac{4+1}{2}, \frac{1+7}{2}\right) = \left(\frac{5}{2}, 4\right)$

Coordinates of $F = \left(\frac{1-6}{2}, \frac{7+0}{2}\right) = \left(\frac{-5}{2}, \frac{7}{2}\right)$

Coordinates of $G = \left(\frac{-6-1}{2}, \frac{0-9}{2}\right) = \left(\frac{-7}{2}, \frac{-9}{2}\right)$

Coordinates of $H = \left(\frac{-1+4}{2}, \frac{-9+1}{2}\right) = \left(\frac{3}{2}, -4\right)$

Now, $EFGH$ is parallelogram if diagonals EG and FH have the same mid-point.

Coordinates of mid-point of $EG = \left(\frac{\frac{5-7}{2}}{2}, \frac{4-\frac{9}{2}}{2}\right) = \left(\frac{-2}{4}, \frac{-1}{4}\right) = \left(\frac{-1}{2}, \frac{-1}{4}\right)$

Coordinates of mid-point of $FH = \left(\frac{\frac{-5+3}{2}}{2}, \frac{\frac{7-8}{2}}{2}\right) = \left(\frac{-2}{4}, \frac{-1}{4}\right)$

$\Rightarrow EFGH$ is parallelogram

Ex 23.2

Q1

Let the equation of the line be:

$y - y_1 = m(x - x_1)$

Now,

$m = 0$ $\quad [\because$ Parallel lines have equal slopes, the slope of x-axis is $0]$

$(x_1, y_1) = (3, -5)$

$\therefore y - y_1 = m(x - x_1)$

$y - (-5) = 0(x - 3)$

$y + 5 = 0$

Q2

The slope of x-axis is 0, any line perpendicular to it will have

slope $= \frac{-1}{0}$

Also the required line is passing through the point $(-2, 0)$

(because it is given it has x-intercept is -2)

The required equation of line is

$y - y_1 = m(x - x_1)$

where $m = \frac{-1}{0}, (x_1 y_1) \Rightarrow (-2, 0)$

$y - 0 = \frac{-1}{0}(x - (-2))$

$y - 0 = \frac{-1}{0}(x + 2)$

$-(x + 2) = 0$

$x + 2 = 0$

$x = -2$

Q3

The slope of x-axis is 0

Any line parallel to x-axis will also have the same slope.

therefore $m = 0$

Also line has y - intercept, ie. $(0, b)$

$\Rightarrow (0, -2) \Rightarrow (x_1 y_1)$

The required equation of the line is $y - y_1 = m(x - x_1)$

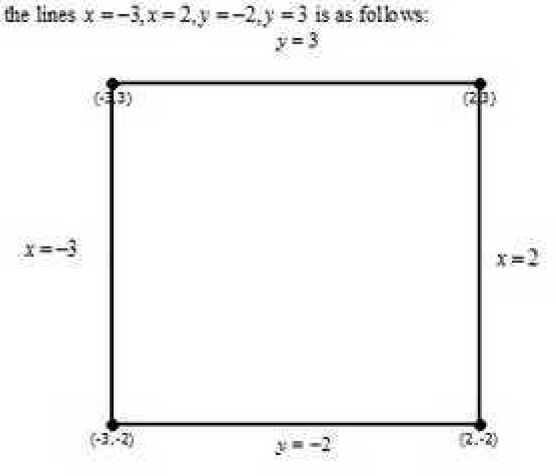
$y - (-2) = 0(x - 0)$

$y + 2 = 0$

$y = -2$

Q4

The figure with the lines $x=-3, x=2, y=-2, y=3$ is as follows:



From the figure, the co-ordinates of the vertices of the square are $(2,3),(-3,3),(-3,-2),(2,-2)$.

Q5

Slope of a line parallel to x-axis = 0

Since the line passes through $(4,3)$,

The required equation of the line parallel to x-axes is

$y - y_1 = m(x - x_1)$

$y - (3) = 0(x - 4)$

$y - 3 = 0$

$y = 3$

Slope of a line perpendicular to x-axis = $\frac{-1}{0}$

The required equation of the line perpendicular to x-axis is

$y - y_1 = m(x - x_1)$

$y - 3 = \frac{-1}{0}(x - 4)$

$x - 4 = 0$

$x = 4$

Q6

Let $x = \lambda$ be the line equidistant from

$x = -2$ and $x = 6$

so $\left|\frac{-2-\lambda}{\sqrt{1}}\right| = \left|\frac{\lambda - 6}{\sqrt{1}}\right|$

$-2 - \lambda = \lambda - 6$

$4 = 2\lambda$

$\therefore \lambda = 2$

$\therefore$ The line equidistant from $x = -2$ and $x = 6$ is $x = 2$

Q7

A line which is equidistant from two other lines, must have the same slope.

The slope of $y = 10$ and $y = -2$ is 0, ie line parallel to x-axis.

The required line is also parallel to $y = 10$ and $y = -2$

$\therefore \quad m = 0$

Also, the required line will pass from the mid-point of the line joining

(0, -2) and (0,10)

Coordinates of this point will be $\left(0, \frac{10-2}{2}\right) = \left(0, \frac{8}{2}\right) = (0, 4)$

$\therefore$ The equation of the require line is:

$y-4=0(x-x_1)$

$\Rightarrow y = 4$

Ex 23.3

Q1

The equation of the line having slope m and y-intercept (0, c) is given by:

$$y = mx + c$$

Now, $m = \tan\left(150^0\right) = \frac{-1}{\sqrt{3}}$

and

y-intercept is $(0, 2)$

The required equation of line is

$$y = mx + c$$

$$\Rightarrow \quad y = \frac{-x}{\sqrt{3}} + 2$$

$$\Rightarrow \quad \sqrt{3}y - 2\sqrt{3} + x = 0$$

$$\Rightarrow \quad x + \sqrt{3}y = 2\sqrt{3}$$

Q2

(i) With slope 2 and y intercept 3

$m = 2$, point is $(0, 3)$

The required equation of line is

$$y = mx + c$$
$$\Rightarrow \quad y = 2x + 3$$

(ii) slope $= \dfrac{-1}{3}$, y intercept $= (0, -4)$

$m = \dfrac{-1}{3}$, $c = -4$

The required equation of line is $y = mx + c$

$$\Rightarrow \quad y = \frac{-1}{3}x - 4$$
$$\Rightarrow \quad 3y + x = -12$$

(iii) $m = -2$, $c = -3$

The required equation of line is

$y - y_1 = m(x - x_1)$

Since the line cuts the x − *axis at* $(-3,0)$ *with slope* -2, *we have,*

$y - 0 = -2(x + 3)$

$\Rightarrow y = -2x - 6$

$\Rightarrow 2x + y + 6 = 0$

Q3

The given lines are $x = 0, y = 0$.

The equation of the bisectors of the angles between $x = 0$ and $y = 0$ are:

$$\frac{x}{\sqrt{(1)^2 + (0)^2}} = \pm \frac{y}{\sqrt{(0)^2 + (1)^2}}$$

$x = \pm y$

$x \pm y = 0$

Q4

$\theta = \tan^{-1} 3 \Rightarrow m = \tan\theta = 3$

Intercept in negative direction of $y - axis$ is $(0,-4)$

Hence, required equation of line is

$$y = mx + c$$

$$\Rightarrow \quad y = 3x - 4$$

Q5

Here, y intercept, c = -4

The required line is parallel to line joining $(2,-5)$ and $(1,2)$

Let m be the slope of the required line, then

m = slope of $(2,-5)$ and $(1,2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-5)}{1 - 2} = \frac{7}{-1} = -7$$

$\therefore$ the required equation of line is

$y = mx + c$

$y = -7x - 4$

$7x + y + 4 = 0$

Q6

The required equation of line is $y = mx + c$
Here, c = 3

Let m be slope of the required line.

Then,
m × slope of given line = -1

Slope of given line $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-2}{3-4} = \frac{3}{-1} = -3$

$\Rightarrow \quad m = \frac{1}{3}$

So, the required equation is:

$y = mx + c$

$y = \frac{1}{3}x + 3$

$x - 3y + 9 = 0$

Q7

The required equation of line is $y = mx + c$
Here, c = -3

Let m be slope of the required line.

Then,
m × slope of given line = -1

Slope of line joining $(4,3)$ and $(-1,1)$ $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-3}{-1-4} = \frac{-2}{-5} = \frac{2}{5}$

$\Rightarrow \quad m = -\frac{5}{2}$

So, the required equation is:

$y = mx + c$

$y = -\frac{5}{2}x - 3$

$y + 3 = \frac{-5x}{2}$

$2y + 5x + 6 = 0$

Q8

The required equation of line is

$y - y_1 = m(x - x_1)$

where $m = \tan 30^0 = \frac{1}{\sqrt{3}}$

point is $(x_1 y_1) = (0,2)$

$\Rightarrow y - 2 = \frac{1}{\sqrt{3}}(x - 0)$

$x - \sqrt{3}y + 2\sqrt{3} = 0$

Ex 23.4

Q1

Let the required equation of the line be

$y - y_1 = m(x - x_1)$

Now,
m = slope = -3
$(x_1 y_1) = (6, 2)$

$\therefore \quad y - y_1 = m(x - x_1)$
$\Rightarrow y - 2 = -3(x - 6)$
$\Rightarrow y - 2 = -3x + 18$
$\Rightarrow 3x + y = +20$
$\Rightarrow 3x + y - 20 = 0$

$\therefore$ The equation of the given line is $3x + y - 20 = 0$.

Q2

Let the required equation of the line be

$y - y_1 = m(x - x_1)$

Now,
The line is inclined at an angle of 45^0 with x-axis
$\therefore m = \tan 45^0 = 1$

$(x_1 y_1) = (-2, 3)$

$\therefore \quad y - y_1 = m(x - x_1)$
$\Rightarrow y - 3 = 1(x - (-2))$
$\Rightarrow y - 3 = x + 2$
$\Rightarrow x - y = -5$
$\therefore$ Equation of required line is $x - y + 5 = 0$

Q3

Therequiredequation of thelineis

$$y-y_1=m(x-x_1)$$

$(x_1,y_1)=(0,0)$ andslopeis m

Therefore, $y-y_1=m(x-x_1)$

$$y-0=m(x-0)$$

$$y=mx$$

Q4

Therequiredequation of thelineis

$$y-y_1=m(x-x_1)$$

Sincethelinemakes an angle 75° with $x-axis$

$m=\tan 75^\circ=3.73$

$(x_1,y_1)=(2,2\sqrt{3})$

Therefore, $y-y_1=m(x-x_1)$

$$y-2\sqrt{3}=(2+\sqrt{3})(x-2)$$

$$y-2\sqrt{3}=(2+\sqrt{3})x-7.46$$

$$(2+\sqrt{3})x-y-4=0$$

Q5

Let $\sin\theta=\frac{3}{4}$

Then,

$\Rightarrow m=\text{slope}=\tan\theta=\frac{3}{4}$

The equation of straight line with slope m and passing through (1, 2) is

$y-y_1=m(x-x_1)$

$y-2=\frac{3}{4}(x-1)$

$4y-8=3x-3$

$3x-4y=-5$

$3x-4y+5=0$

Q6

The required equation of the line is

$y - y_1 = m(x - x_1)$

Since the line makes an angle 60^0 with the positive direction of y axis, it makes 30^0 with the positive direction of x axis.

$\therefore m = \tan 30^0 = \frac{1}{\sqrt{3}}$ (angle with y-axis)

A point on the line is $(x_1 y_1) = (3, -2)$

Therefore, the equation of the line is:

$y - y_1 = m(x - x_1)$

$y - (-2) = \frac{1}{\sqrt{3}}(x - 3)$

$x - \sqrt{3}y - 3 - 2\sqrt{3} = 0$

Q7

Equation of the line passing through (x_1, y_1)
and making angle θ with the x-axis is,
$(y - y_1) = \tan\theta(x - x_1)$

For the first line: $(x_1, y_1) = (0, 2), \theta = \frac{\pi}{3}$
$(y - y_1) = \tan\theta(x - x_1)$
$(y - 2) = \left(\tan\frac{\pi}{3}\right)(x - 0)$
$y - 2 = \sqrt{3}x$
$\sqrt{3}x - y + 2 = 0$

For the second line: $(x_1, y_1) = (0, 2), \theta = \frac{2\pi}{3}$
$(y - y_1) = \tan\theta(x - x_1)$
$(y - 2) = \left(\tan\frac{2\pi}{3}\right)(x - 0)$
$y - 2 = -\sqrt{3}x$
$\sqrt{3}x + y - 2 = 0$

The line parallel to $\sqrt{3}x - y + 2 = 0$
and cutting y-axis at a distance of 2 units below the origin.
$y = \sqrt{3}x - 2$
$\sqrt{3}x + y - 2 = 0$

The line parallel to $\sqrt{3}x + y - 2 = 0$
and cutting y-axis at a distance of 2 units below the origin.
$y = -\sqrt{3}x - 2$
$\sqrt{3}x + y + 2 = 0$

Q8

If a line is equally inclined to axis, then

$\theta = 45^0$ or $\theta = 135^0 \Rightarrow m = \tan\theta = \pm 1$

Since, y intercept, $c = 5$

$\therefore$ We get the solution of the line as:

$y = mx + c$

$y = \pm 1x + 5$

$y - x = 5$ or $y + x = 5$

Q9

The line passes through the point $(2,0)$.

Also its inclination to $y-$*axis is* $135°$.

That is, the inclination of the given line with the $x-$*axis is* $180° - 135°$.

That is, the slope of the given line is $45°$

The equation of the line having slope 'm' and passing through the point (x_1, y_1) *is* $y - y_1 = m(x - x_1)$

Therefore, the required equation is

$y - 0 = \tan 45°(x - 2)$

$\Rightarrow y = 1 \times (x - 2)$

$\Rightarrow y = x - 2$

$\Rightarrow x - y - 2 = 0$

Q10

The coordinates of the point which divides the join of the points $(2,3)$ and $(-5,8)$ in the ratio 3:4 is given by (x, y) where,

$$x = \frac{lx_2 + mx_1}{l+m} = \frac{3(-5)+4(2)}{3+4} = \frac{-15+6}{7} = \frac{-9}{7}$$

$$y = \frac{ly_2 + my_1}{l+m} = \frac{3(8)+4(3)}{3+4} = \frac{24+12}{7} = \frac{36}{7}$$

Slope of the line joining the points $(2,3)$ and $(-5,8)$ $= \frac{8-3}{-5-2} = \frac{5}{-7} = \frac{-5}{7}$

$\therefore$ Slope of line perpendicular to line= m $= \frac{7}{5}$

The required equation is:

$$y - y_1 = m(x - x_1)$$

$$y - \frac{36}{7} = \frac{7}{5}\left(x - \left(\frac{-9}{7}\right)\right)$$

$$49x - 35y + 229 = 0$$

Q11

Let the perpendicular drawn from $P(4,1)$ on line joining $A(2,-1)$ and $B(6,5)$ divide in the ratio k:1 at the point R.

Using section formula, coordinates of R are:

$$x = \frac{6k+2}{k+1} \text{ and } y = \frac{5k-1}{k+1} \quad ---(1)$$

PR is perpendicular to AB

$\therefore$ (slope of PR) $\times$ (slope of AB) $= -1$

$\Rightarrow \left(\frac{y-1}{x-4}\right) \times \left(\frac{5-(-1)}{6-2}\right) = -1$

$\Rightarrow \frac{\frac{5k-1}{k+1}-1}{\frac{6k+2}{k+1}-4} \times \frac{6}{4} = -1$

$\Rightarrow \frac{5k-1-k-1}{6k+2-4k-4} = \frac{-4}{6}$

$\Rightarrow \frac{4k-2}{2k-2} = \frac{-2}{3}$

$\Rightarrow 3(2k-1) = -2(k-1)$

$\Rightarrow 6k-3 = -2k+2$

$\Rightarrow 8k = 5$

$\Rightarrow k = \frac{5}{8}$

ratio is 5 : 8

$\therefore$ R divides AB in the ratio 5 : 8

Q12

AD, BE and CF are the three altitudes of the triangle

We know,

Slope of AD $\times$ Slope of BC = -1; AD passes through A(2,-2)

Slope of BE $\times$ Slope of AC = -1; AD passes through B(1,1)

Slope of CF $\times$ Slope of AB = -1; AD passes through C(-1,0)

Slope of BC = $\frac{0-1}{-1-1} = \frac{-1}{-2} = \frac{1}{2}$ $\Rightarrow$ Slope of AD = -2

Slope of AC = $\frac{0-(-2)}{-1-2} = \frac{2}{-3} = \frac{-2}{3}$ $\Rightarrow$ Slope of BE = $\frac{3}{2}$

Slope of AB = $\frac{1+2}{1-2} = \frac{3}{-1} = -3$ $\Rightarrow$ Slope of CF = $\frac{1}{3}$

So, for AD, we have

$y - y_1 = m(x - x_1)$

$\Rightarrow y - (-2) = -2(x - 2)$

$\Rightarrow y + 2 = -2x + 4$

$\Rightarrow 2x + y - 2 = 0$

And, for BE, we have

$y - y_1 = m(x - x_1)$

$\Rightarrow y - 1 = \frac{3}{2}(x - 1)$

$\Rightarrow 2y - 3x + 1 = 0$

And, for CF, we have

$y - y_1 = m(x - x_1)$

$\Rightarrow y - 0 = \frac{1}{3}(x + 1)$

$\Rightarrow x - 3y + 1 = 0$

Q13

The right bisector PQ of AB bisects AB at C and is perpendicular to AB.

The co-ordinates of C are $= \left(\frac{3-1}{2}, \frac{4+2}{2}\right) = (1,3)$

And slope of $PQ = \frac{-1}{\text{slope of } AB} = \frac{-1}{2-4}(-1-3) = \frac{4}{-2} = -2$

The equation of PQ is

$(y-3) = -2(x-1)$

$y-3 = -2x+2$

$y+2x = 5$

Q14

The line passes through the point $(-3,5)$

So $(x_1, y_1) = (-3,5)$

The line is perpendicular to the line joining $(2,5)$ and $(-3,6)$.

$$\Rightarrow m = \frac{-1}{\text{slope of line joining } (2,5) \text{ and } (-3,6)} == \frac{-1}{\frac{y_2-y_1}{x_2-x_1}} = \frac{-1}{\frac{6-5}{-3-2}} = \frac{-1}{\frac{-1}{5}}$$

$\therefore m = 5$

Hence, equation of straight line is

$y - y_1 = m(x - x_1)$

$y - 5 = 5(x(-3))$

$y - 5 = 5x + 15$

$5x - y + 20 = 0$

Q15

The right bisector PQ of AB bisects AB at C and is also perpendicular to AB.

Slope of $AB = \frac{3-0}{2-1} = 3$

Now,

$(\text{slope of } AB) \times (\text{slope of } PQ) = -1$

$\therefore$ slope of $PQ = \frac{-1}{3}$

Co-ordinates of c are $= \left(\frac{1+2}{2}, \frac{3+0}{2}\right) = \left(\frac{3}{2}, \frac{3}{2}\right)$

$\therefore$ Equation of right bisector PQ is

$\left(y - \frac{3}{2}\right) = \frac{-1}{3}\left(x - \frac{3}{2}\right)$

$6y - 9 = -2x + 3$

$x + 3y = 6$

Q16

Equation of the line passing through (x_1, y_1)

and making angle θ with the x-axis is,

$(y - y_1) = \tan\theta(x - x_1)$

Here $(x_1, y_1) = (1, 2)$, angle with y-axis is $30°$

$\therefore$ angle with x-axis is $\theta = 90° - 30° = 60°$

$(y - y_1) = \tan\theta(x - x_1)$

$(y - 2) = (\tan 60°)(x - 1)$

$y - 2 = \sqrt{3}x - \sqrt{3}$

$\sqrt{3}x - y + 2 - \sqrt{3} = 0$

Ex 23.5

Q1(i)

Here,

$(x_1 y_1) = (0,0)$

$(x_2 y_2) = (2,-2)$

The equation of the given straight line is:

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow \quad y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\Rightarrow \quad y - 0 = \frac{-2 - 0}{2 - 0}(x - 0)$$

$$\Rightarrow \quad y = \frac{-2x}{2}$$

$$\Rightarrow \quad y = -x$$

$\therefore$ The equation of the line joining the points $(0,0)$ and $(2,-2)$ is $y = -x$

Q1(ii)

Let $A(a,b) = (x_1 y_1)$

$B(a + c\sin\alpha, b + c\cos\alpha) = (x_2 y_2)$

Then equation of line AB is

$$\Rightarrow \quad y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\Rightarrow \quad y - b = \frac{b + c\cos\alpha - b}{a + c\sin\alpha - a}(x - a)$$

$$\Rightarrow \quad y - b = \frac{c\cot\alpha}{c\sin\alpha}(x - a)$$

$$\Rightarrow \quad y - b = \cot\alpha(x - a)$$

$\therefore$ The equation of the line joining the points (a,b) and $(a + c\sin\alpha, b + c\cos\alpha)$ is $y - b = \cot\alpha(x - a)$

Q1(iii)

Let $A(a,-a)$ be $(x_1 y_1)$

$B(b,0)$ be $(x_2 y_2)$

Then equation of line AB is

$$\Rightarrow \quad y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\Rightarrow \quad y - (-a) = \frac{0 - (-a)}{b - 0}(x - 0)$$

$$\Rightarrow \quad y + a = \frac{a}{b}(x - 0)$$

$$\Rightarrow \quad ax - by = ab$$

$\therefore$ The equation of the line joining the points $(0,-a)$ and $(b,0)$ is $ax - by = ab$

Q1(iv)

Let $A(a,b)$ be $(x_1 y_1)$

$B(a+b, a-b)$ be $(x_2 y_2)$

Then equation of line AB is

$$\Rightarrow \quad y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\Rightarrow \quad y - b = \frac{a - b - b}{a + b - a}(x - a)$$

$$\Rightarrow \quad y - b = \frac{a - 2b}{b}(x - a)$$

$$\Rightarrow \quad by - b^2 = ax - a^2 - 2bx + 2ba$$

$$\Rightarrow \quad (a - 2b)x - by + b^2 - a^2 + 2ab = 0$$

$\therefore$ The equation of the line joining the points (a,b) and $(a+b, a-b)$ is $(a-2b)x - by + b^2 - a^2 + 2ab = 0$

Q1(v)

Let $A(x_1 y_1)$ be $\left(at_1, \frac{a}{t_1}\right)$

$B(x_2 y_2)$ be $\left(at_2, \frac{a}{t_2}\right)$

Then equation of line AB is

$$\Rightarrow \quad y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\Rightarrow \quad y - \frac{a}{t_1} = \frac{\frac{a}{t_2} - \frac{a}{t_1}}{at_2 - at_1}(x - at_1)$$

$$\Rightarrow \quad y - \frac{a}{t_1} = \frac{a(t_1 - t_2)}{at_1t_2(t_2 - t_1)}(x - at_1)$$

$$\Rightarrow \quad y - \frac{a}{t_1} = \frac{-1}{t_1t_2}(x - at_1)$$

$$\Rightarrow \quad t_1t_2y + x = a(t_1 + t_2)$$

$\therefore$ The equation of the line joining the points $\left(at_1, \frac{a}{t_1}\right)$ and $\left(at_2, \frac{a}{t_2}\right)$ is $t_1t_2y + x = a(t_1 + t_2)$

Q1(vi)

Let $A(x_1 y_1)$ be $(a\cos\alpha, a\sin\alpha)$

$B(x_2 y_2)$ be $(a\cos\beta, a\sin\beta)$

$$\Rightarrow \quad y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\Rightarrow \quad y - a\sin\alpha = \frac{a\sin\beta - a\sin\alpha}{a\cos\beta - a\cos\alpha}(x - a\cos\alpha)$$

$$\Rightarrow \quad y - a\sin\alpha = \frac{a\left(-2\sin\left(\frac{\beta-\alpha}{2}\right)\right)\cos\beta\left(\frac{\beta+\alpha}{2}\right)}{a\left(-2\sin\frac{\beta-\alpha}{2}\right)\sin\left(\frac{\beta+\alpha}{2}\right)}(x - a\cos\alpha)$$

$$\Rightarrow \quad y - a\sin\alpha = \frac{\cos\left(\frac{\alpha+\beta}{2}\right)}{\sin\left(\frac{\alpha+\beta}{2}\right)}(x - a\cos\alpha)$$

$$\Rightarrow \quad x\cos\left(\frac{\alpha+\beta}{2}\right) + y\sin\frac{\alpha+\beta}{2} = a\cos\frac{\alpha-\beta}{2}$$

$\therefore$ The equation of the line joining the points $(a\cos\alpha, a\sin\alpha)$ and $(a\cos\beta, a\sin\beta)$ is

$$x\cos\left(\frac{\alpha+\beta}{2}\right) + y\sin\frac{\alpha+\beta}{2} = a\cos\frac{\alpha-\beta}{2}$$

Q2(i)

Let $A(1,4)$, $B(2,-3)$, $C(-1,-2)$

Then equation of AB is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 4 = \frac{-3-4}{2-1}(x-1)$$

$$y - 4 = \frac{-7}{1}(x-1)$$

$$7x + y = 11$$

Equation of side BC is

$$y - y_2 = \frac{y_3 - y_2}{x_3 - x_2}(x - x_2)$$

$$y - (-3) = \frac{-2-(-3)}{-1-2}(x-2)$$

$$y + 3 = \frac{1}{-3}(x-2)$$

$$x + 3y + 7 = 0$$

Equation of side AC is

$$y - y_1 = \frac{y_3 - y_1}{x_3 - x_1}(x - x_1)$$

$$y - 4 = \frac{-2-4}{-1-1}(x-1)$$

$$y - 4 = 3(x-1)$$

$$y - 3x = 1$$

Q2(ii)

Let $A(0,1), B(2,0), C(-1,-2)$

then equation of side AB is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$
$$y - 1 = \frac{0 - 1}{2 - 0}(x - 0)$$
$$y - 1 = \frac{-1}{2}(x)$$
$$x + 2y = 2$$

Equation of side BC is

$$y - y_2 = \frac{y_3 - y_2}{x_3 - x_2}(x - x_1)$$
$$y - 0 = \frac{-2 - 0}{-1 - 2}(x - 2)$$
$$y = \frac{2}{3}(x - 2)$$
$$2x - 3y = 4$$

Equation of side AC is

$$y - y_1 = \frac{y_3 - y_1}{x_3 - x_1}(x - x_1)$$
$$y - 1 = \frac{-2 - 1}{-1 - 0}(x - 0)$$
$$y - 1 = 3(x - 0)$$

$$y - 3x = 1$$

Q3

Let $A(-1, 6)$ be $(x_1 y_1)$

$B(-3, -9)$ be $(x_2 y_2)$

$C(5, -8)$ be $(x_3 y_3)$

Median is a line segment which joins a vertex to the mid-point of the side opposite to it.
Let D, E and F be the mid points of sides AB, BC, and CA.

Then, using mid point formula $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ we can find the coordinates of D, E and F as:

$$D = \left(\frac{-3+5}{2}, \frac{-9-8}{2}\right) = \left(-1, \frac{-17}{2}\right)$$

$$E = \left(\frac{-1+5}{2}, \frac{6-8}{2}\right) = (2, -1)$$

$$F = \left(\frac{-1-3}{2}, \frac{6-9}{2}\right) = \left(-2, \frac{-3}{2}\right)$$

Equation of median AD is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 6 = \frac{\frac{-17}{2} - 6}{1 - (-1)}(x + 1) = \frac{-29}{4}(x + 1) \qquad \left[A(-1, 6), D\left(1, \frac{-17}{2}\right)\right]$$

$$29x + 4y + 5 = 0$$

Equation of median BE is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - (-9) = \frac{-1 - (-9)}{2 - (-3)}(x - (-3)) \qquad [B(-3, -9), E(2, -1)]$$

$$y + 9 = \frac{8}{5}(x + 3)$$

$$5y + 45 = 8x + 24$$

$$8x - 5y - 21 = 0$$

Equation of median CF is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - (-8) = \frac{\frac{-3}{2} - (-8)}{-2 - 5}(x - 5) \qquad \left[C(5, -8), F\left(-2, \frac{-3}{2}\right)\right]$$

$$y + 8 = \frac{-3 + 16}{2 \times (-7)}(x - 5)$$

$$y + 8 = \frac{-13}{14}(x - 5)$$

$$13x + 14y + 47 = 0$$

Q4

The rectangle ABCD will have diagonals AC and BD
AC passes through $A(a,b)$ and $C(a',b')$.
Thus equation of AC is:

$$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$$

$$\Rightarrow \frac{y-b}{b'-b}=\frac{x-a}{a'-a}$$

$$\Rightarrow (y-b)(a'-a)=(x-a)(b'-b)$$

$$\Rightarrow y(a'-a)-a'b+ab=x(b'-b)-ab'+ab$$

$$\Rightarrow y(a'-a)=x(b'-b)-ab'+a'b$$

$$\Rightarrow y(a'-a)-x(b'-b)=a'b-ab'$$

BD passes through $B(a',b)$ and $D(a,b')$.
Thus equation of BD is:

$$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$$

$$\Rightarrow \frac{y-b}{b'-b}=\frac{x-a'}{a-a'}$$

$$\Rightarrow (y-b)(a-a')=(x-a')(b'-b)$$

$$\Rightarrow -y(a'-a)-ab+a'b=x(b'-b)-a'b'+a'b$$

$$\Rightarrow a'b'-ab=x(b'-b)+y(a'-a)$$

$$\Rightarrow x(b'-b)+y(a'-a)=a'b'-ab$$

Q5

Equation of BC

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$y-1=\frac{0-1}{2-0}(x-0) \quad [\because B(0,1), C(2,0)]$$

$$2y-2=-x$$

$$x+2y=2$$

D is mid point of BC

So,

$$D=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{0+2}{2},\frac{1+0}{2}\right)=\left(1,\frac{1}{2}\right)$$

$\therefore$ Equation of the median AD :

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$y-(-2)=\frac{\frac{1}{2}-(-2)}{1-(-1)}\left(x-(-1)\right)=\frac{\frac{5}{2}}{2}(x+1) \qquad \left[\because A(-1,-2), D\left(1,\frac{1}{2}\right)\right]$$

$$4y+8=5x+5$$

$$5x-4y-3=0$$

Q6

The equation of the line passing through points $(-2,-2)$ and $(8,2)$ is

$$y+2=\frac{2+2}{8+2}(x+2)$$

$$2x-5y-6=0$$

Clearly, $(3,0)$ satisfies this equation which means that the line passing through $(-2,-2)$ and $(8,2)$ also passes through $(3,0)$.

Hence three points are collinear.

Q7

Let AB be the line segment
Let P be any point which divides the line segment in the ratio 2:3
then using section formula

$$x = \frac{lx_2 + mx_1}{l+m}, y = \frac{ly_2 + my_1}{l+m}$$

where $l : m :: 2 : 3$

$$\Rightarrow x = \frac{2(8)+3(3)}{2+3} = \frac{16+9}{5} = \frac{25}{5} = 5$$

$$y = \frac{2(9)+3(-1)}{2+3} = \frac{18-3}{5} = \frac{15}{5} = 3$$

Now P must lie on the line, where P is $(5,3)$

$$y - x + 2 = 0$$

$$\Rightarrow 3 - (5) + 2 = 0$$

$$-2 + 2 = 0$$

$$0 = 0$$

Hence, Proved

Q8

The line that bisects the distance between the points $A(a, b)$, $B(a'b')$ and between $C(-a, b)$, $D(a'-b')$ means a line passing through the mid-point of AB and CD

mid point of AB is $\left(\frac{a+a'}{2}, \frac{b+b'}{2}\right)$

mid point of CD is $\left(\frac{-a+a'}{2}, \frac{b-b'}{2}\right)$

Equation is $y - y_1 = m(x - x_1)$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - \left(\frac{b+b'}{2}\right) = \frac{\left(\frac{b-b'}{2}\right) - \left(\frac{b+b'}{2}\right)}{\left(\frac{-a+a'}{2} - \frac{a+a'}{2}\right)}\left(x - \left(\frac{a+a'}{2}\right)\right)$$

$$y - \left(\frac{b+b'}{2}\right) = \frac{\frac{b}{2} - \frac{b'}{2} - \frac{b}{2} - \frac{b'}{2}}{\frac{-a}{2} + \frac{a'}{2} - \frac{a}{2} - \frac{a'}{2}}\left(x - \left(\frac{a+a'}{2}\right)\right)$$

$$y - \left(\frac{b+b'}{2}\right) = \frac{+b'}{a}\left(x - \left(\frac{a+a'}{2}\right)\right)$$

$$2ay - 2b'x = ab - a'b'$$

Q9

In what ratio is the line joining the points $(2,3)$ and $(4,-5)$ divided by the line passing through the points $(6,8)$ and $(-3,-2)$.

Let the equation of line AB joining the points $(6,8)$ and $(-3,-2)$ be

$$y - y_1 = m(x - x_1)$$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 8 = \frac{-2-8}{-3-6}(x - 5)$$

$$y - 8 = \frac{10}{9}(x - 6)$$

$$9y - 10x = 12 \quad \text{---}(1)$$

Suppose the line joining $(2,3)$ and $(4,-5)$ is divided by the line $9y - 10x = 12$ in the ratio $k:1$ at the point (x, y), then

$$x = \frac{k(4) + 1(2)}{k+1}, y = \frac{k(-5) + 1(3)}{k+1}$$

Substituiting in equation (i), we get:

$$\frac{9(-5k + 3)}{k+1} - 10\left(\frac{4k + 2}{k+1}\right) = 12$$

$$\Rightarrow -45k + 27 - 40k - 20 = 12k + 12$$

$$\Rightarrow 97k = 5$$

$$\Rightarrow k = \frac{5}{97}$$

Q10

The quadrilateral $ABCD$ has diagonals AC and BD.
The required equation is

Since, $A(-2,6)$, $C(10,4)$, the equation for AC is:

$$y-6=\frac{4-6}{10-(-2)}(x-(-2)) \qquad \left[y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)\right]$$

$$y-6=\frac{-12}{6}(x+2)$$

$$y-6=\frac{-(x+2)}{6}$$

$$6y-36=-x-2$$

$$x+6y-34=0$$

Since, $B(1,2)$, $D(7,8)$, the equation for BD is:

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$y-2=\frac{8-2}{7-1}(x-1)$$

$$y-2=\frac{6}{6}(x-1)$$

$$y-2=x-1$$

$$x-y+1=0$$

Q11

$L_1 = 124.942,\ C_1 = 20$

$L_2 = 125.134,\ C_2 = 110$

Equation of line passing through

(L_1, C_1) and (L_2, C_2)

$$L - L_1 = \left(\frac{L_2 - L_1}{C_2 - C_1}\right)(C - C_1)$$

$$L - 124.942 = \left(\frac{125.134 - 124.942}{110 - 20}\right)(C - 20)$$

$$L - 124.942 = \frac{0.192}{90}(C - 20)$$

$$L - 124.942 = \frac{192}{90000}(C - 20)$$

$$L - 124.942 = \frac{4}{1875}(C - 20)$$

$$L = \frac{4}{1875}C + 124.942 - 4 \times \frac{20}{1875}$$

$$\Rightarrow L = \frac{4}{1875}C + 124.899$$

Q12

Assuming x be the price per litre and y be the quantity of the milk.

sold at this price.

So, the line representing the relationship passes through $(14,980)$ and $(16,1220)$.

So its equation is

$$y - 980 = \frac{1220 - 980}{16 - 14}(x - 14)$$

$$y - 980 = 120(x - 14)$$

$120x - y - 700 = 0$

When $x = 17, 120 \times 17 - y - 700 = 0$

$$y = 1340$$

Q13

Let AD be the bisector of $\angle A$

Then, $BD : DC = AB : AC$

Now,

$$|AB| = \sqrt{(4-0)^2 + (3-0)^2} = 5$$

$$|AC| = \sqrt{(4-2)^2 + (3-3)^2} = 2$$

$$\Rightarrow \frac{AB}{AC} = \frac{BD}{DC} = \frac{5}{2}$$

$\Rightarrow D$ divides BC in the ratio $5:2$

So, coordinates of D are $\left(\frac{5\times 2+0}{5+2}, \frac{5\times 3+0}{5+2}\right) = \left(\frac{10}{7}, \frac{15}{7}\right)$

$\therefore$ The equation of AD is

$$y-3 = \left(\frac{\frac{15}{7}-3}{\frac{10}{7}-4}\right)(x-4)$$

$$y-3 = \left(\frac{15-21}{10-28}\right)(x-4)$$

$$\Rightarrow y-3 = \frac{1}{3}(x-4)$$

$$\Rightarrow 3(y-3) = x-4$$

$$\Rightarrow x-3y+9-4=0$$

$$\Rightarrow x-3y+5=0$$

Q14

The required straight line passes through $(0,0)$ and trisects the part of the line $3x+y=12$ that lies between the axes of coordinates.

The line $3x+y=12$ has $A(4,0)$ and $B(0,12)$ as x and y intercepts.

Let P and Q be the points of trisection of AB.

Since P divides AB in the ratio 1:2, coordinates of P are:

$$P=\frac{1(0)+2(4)}{1+2},\frac{1(12)+2(0)}{1+2}=\left(\frac{8}{3},4\right)$$

Since Q divides BA in the ratio 1:2, coordinates of Q are:

$$Q=\frac{2(0)+1(4)}{1+2},\frac{1(0)+2(12)}{1+2}=\left(\frac{4}{3},8\right)$$

Equation of line through $(0,0)$ and $P\left(\frac{8}{3},4\right)$ is:

$$y-0=\frac{4-0}{\frac{8}{3}-0}(x-0)$$

$$y-0=\frac{12}{8}x$$

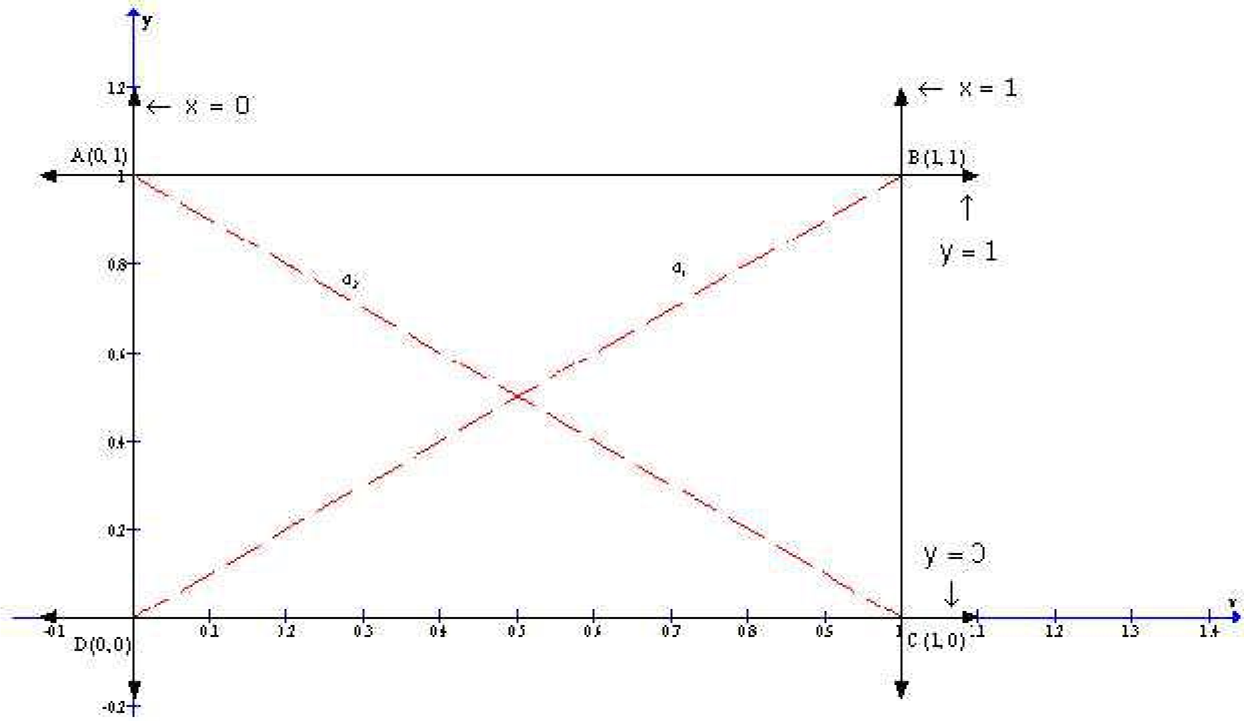
$$2y=3x$$

Equation of line through $(0,0)$ and $Q\left(\frac{4}{3},8\right)$ is:

$$y-0=\frac{8-0}{\frac{4}{3}-0}(x-0)=6x$$

$$y=6x$$

Q15



When we draw all the given equations of lines on the graph we get the points of intersection A(0, 1), B(1,1), C(1,0) and D(0,0).

Let d_1 be the diagonal fomed by joining the points B and D.
Let d_2 be the diagonal fomed by joining the points A and C.

Equation of the diagonal d_1 is given by,

$$(y - 1) = \frac{(0-1)}{(0-1)}(x-1)$$

$$(y-1) = 1(x-1)$$

$$y = x$$

Equation of the diagonal d_2 is given by,

$$(y - 1) = \frac{(0-1)}{(1-0)}(x-0)$$

$$(y-1) = -1(x)$$

$$y + x = 1$$

$\therefore$ The equations of the diagonals are $y = x$ and $y + x = 1$.

Ex 23.6

Q1

(i)
If $(a,0)$ and $(0,b)$ are the intercepts of a line then the intercept form of equation is

$\frac{x}{a}+\frac{y}{b}=1$

Here, $a=3, b=2$

$\therefore$ The required equation is

$\frac{x}{3}+\frac{y}{2}=1$

$\Rightarrow 2x+3y=6$

(ii) If $(a,0)$ and $(0,b)$ are the intercepts of a line then the intercept form of equation is

$\frac{x}{a}+\frac{y}{b}=1$

Here, $a=-5, b=6$

$\therefore$ The required equation is

$\frac{x}{-5}+\frac{y}{6}=1$

$\Rightarrow 6x-5y=-30$

Q2

The equation of straight line in the intercept form is

$\frac{x}{a}+\frac{y}{b}=1 \quad ---(1)$

If (1) passes through the point $(1,-2)$ and has equal intercepts $(a=b=k)$, we get,

$\frac{1}{k}+\frac{(-2)}{k}=1$

$\frac{1}{k}-\frac{2}{k}=1$

$1-2=k$

$k=-1$

$\Rightarrow a=b=-1$

Putting in (1)

$\frac{x}{-1}+\frac{y}{-1}=1$

$x+y=-1$

Q3

(i) Intercepts are equal and positive

$\Rightarrow a = b = k$

The equation of straight line is

$$\frac{x}{a} + \frac{y}{b} = 1 \quad ---(1)$$

Since this line passes through (5, 6) and a=b=k, we get:

$$\frac{5}{k} + \frac{6}{k} = 1$$
$$k = 1$$

3 $$\therefore \frac{x}{11} + \frac{y}{11} = 1$$

$\Rightarrow x + y = 11$

(ii) Intercepts are equal but opposite in sign

Let, $a = k, b = -k$

Putting in (1), we get,

$$\frac{5}{k} + \frac{6}{-k} = 1$$
$$\frac{5}{k} - \frac{6}{k} = 1$$
$$\Rightarrow \quad k = -1$$

thus from (1)

$x - y = -1$

Q4

The equation of the given line is,

$ax + by + 8 = 0$

$\Rightarrow -\dfrac{x}{\frac{8}{a}} - \dfrac{y}{\frac{8}{b}} = 1$

It cuts the axes at $A\left(\dfrac{-8}{a}, 0\right)$ and $B\left(0, \dfrac{-8}{b}\right)$.

The equation of the given line is,

$2x - 3y + 6 = 0$

$\Rightarrow \dfrac{-x}{3} + \dfrac{y}{2} = 1$

It cuts the axes at $C(-3, 0)$ and $D(0, 2)$.

The intercepts of both the lines are opposite in sign

$\Rightarrow \left(\dfrac{-8}{a}, 0\right) = -(-3, 0) \quad and \quad \left(0, \dfrac{-8}{b}\right) = -(0, 2)$

$\Rightarrow \dfrac{-8}{a} = 3 \quad and \quad \dfrac{-8}{b} = -2$

$\Rightarrow a = \dfrac{-8}{3} \quad and \quad b = 4$

Q5

Let the intercepts on the axes be $(a, 0)$ and $(0, a)$.

Then,

$a \times a = 25$

$a^2 = 25$

$a = 5$

(Ignoring negative sign because it is given that the intercepts are positive)

$\Rightarrow a = b = 5$ (given the intercepts are equal)

$\therefore$ Putting in equation of straight line

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{x}{5} + \frac{y}{5} = 1$$

$x + y = 5$

Q6

The equation of the given line is,

$$\frac{x}{a}+\frac{y}{b}=1$$

It cuts the axes at $A(a,0)$ and $B(0,b)$.

The portion of AB intercepted between the axis is 5:3.

$$\therefore h=\frac{3\times a+5\times 0}{8} \text{ and } k=\frac{3\times 0+5\times b}{8}$$

$$\Rightarrow p=\left(\frac{3a}{8},\frac{5b}{8}\right)$$

The line is passing through the point $(-4,3)$

$$\Rightarrow \frac{3a}{8}=-4 \qquad \frac{5b}{8}=3$$

$$\Rightarrow a=\frac{-32}{3} \qquad b=\frac{24}{5}$$

$\therefore$ The equation of the given line is,

$$\frac{x}{\frac{-32}{3}}+\frac{y}{\frac{24}{5}}=1$$

$$\frac{-3x}{32}+\frac{5y}{24}=1$$

$$9x-20y+96=0$$

Q7

The line intercepted by the axes are $(a,0)$ and $(0,b)$, if this line segment is bisected at point (α,β)

then $\frac{a+0}{2}=\alpha, \frac{0+b}{2}=\beta$ (Using mid point formula)

$a=2\alpha, b=2\beta$

The equation of straight line in the intercept form is

$$\frac{x}{a}+\frac{y}{b}=1$$

$$\frac{x}{2\alpha}+\frac{y}{2\beta}=1$$

Q8

Suppose $P = (3,4)$ divides the line joining the points $A(a,0)$ and $B(0,b)$ in the ration 2:3.

Then,

$$3 = \frac{2(0)+3(a)}{2+3} \Rightarrow 3 = \frac{3a}{5} \Rightarrow a = 5$$

$$4 = \frac{2(b)+3(0)}{2+3} \Rightarrow 4 = \frac{2b}{5} \Rightarrow b = 10$$

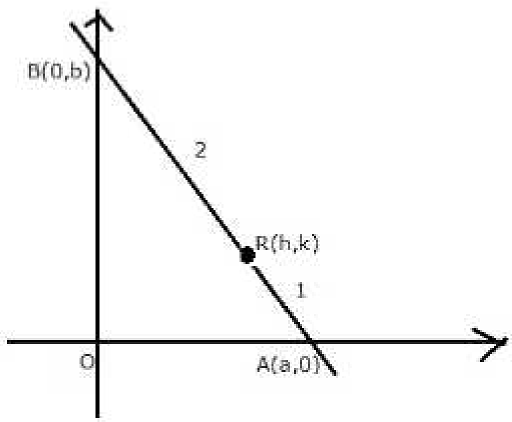
$\therefore A$ is $(5,0)$, B is $(0,10)$

Equation of line AB is

$$\frac{x}{5} + \frac{y}{10} = 1$$

$$2x + y = 10$$

Q9



Point (h,k) divides the line segment in the ratio 1:2

Thus, using section point formula, we have

$$h = \frac{2 \times a + 1 \times 0}{1+2}$$

and

$$k = \frac{2 \times 0 + 1 \times b}{1+2}$$

Therefore, we have,

$$h = \frac{2a}{3} \text{ and } k = \frac{b}{3}$$

$$\Rightarrow a = \frac{3h}{2} \text{ and } b = 3k$$

Thus, the corresponding points of A and B are $\left(\frac{3h}{2}, 0\right)$ *and* $(0,3k)$

Thus, the equation of the line joining the points A and B is

$$\frac{y-3k}{3k-0} = \frac{x-0}{0-\frac{3h}{2}}$$

$$\Rightarrow -\frac{3h}{2}(y-3k) = x \times 3k$$

$$\Rightarrow -3hy + 9hk = 6kx$$

$$\Rightarrow 2kx + hy = 3kh$$

Q10

Let equation of line be

$$\frac{x}{a}+\frac{y}{b}=1$$

then $a+b=7$ and $a \geq 0$ and $b \geq 0$

$$\therefore \frac{x}{a}+\frac{y}{7-a}=1 \quad ---(1)$$

The line passes through $(-3, 8)$

$$\Rightarrow \frac{-3}{a}+\frac{8}{7-a}=1$$

$$\Rightarrow -21+3a+8a=7a-a^2$$

$$\Rightarrow -21+11a=7a-a^2$$

$$\Rightarrow a^2+4A-21=0$$

$$\Rightarrow A=3 \text{ or } -7$$

$$a \neq -7 \ (as \ a \geq 0)$$

$\therefore$ $a=3$ and $b=4$

$\therefore$ Eequation of line is

$$\frac{x}{3}+\frac{y}{4}=1$$

or $4x+3y=12$

Q11

Let equation of line be $\frac{x}{a}+\frac{y}{b}=1$

It is given $(-4, 3)$ divides the line joining $A(a, 0)$ and $B(0, b)$ in ratio $5:3$

$$\therefore \left(\frac{3a}{8}, \frac{5b}{8}\right)=(-4, 3)$$

$$\Rightarrow \frac{3a}{8}=-4 \qquad \Rightarrow a=\frac{-32}{3}$$

And

$$\frac{5b}{8}=3 \qquad \Rightarrow b=\frac{24}{5}$$

$\therefore$ The equation of line is

$$\frac{3x}{-32}+\frac{5y}{24}=1$$

or $9x-20y+96=0$

Q12

Let equation of line be

$$\frac{x}{a}+\frac{y}{b}=1$$

then $a=b+5$

$$\therefore\ \frac{x}{b+5}+\frac{y}{b}=1$$

It passes through $(22,-6)$

$$\Rightarrow \frac{22}{b+5}-\frac{6}{b}=1$$

$$\Rightarrow 22b-6b-30=b^2+5b$$

$$\Rightarrow b^2-11b+30=0$$

$$\Rightarrow b=5 \quad \text{or } 6$$

$$\therefore\ a=10 \ or\ 11$$

$\therefore$ Equations of line are

$$\frac{x}{10}+\frac{y}{5}=1$$

or $x+2y-10=0$

and

$$\frac{x}{11}+\frac{y}{6}=1$$

$$6x+11y=66$$

Q13

The equation of straight line is

$y - y_1 = m(x - x_1)$

The line passes through (x, y) ie, $(1, -7)$ and meets the axes at A and B

$\Rightarrow A$ point is $(a, 0)$ and B *is* $(0, b)$

$$\frac{AP}{BP} = \frac{3}{4}$$

Using section formula $\frac{lx_2 + mx_1}{l + m}, \frac{ly_2 + my_1}{l + m}$

$l : m = 3 : 4, (a, 0) \Leftrightarrow (x_1 y_1), (0, b) \Leftrightarrow (x_2 y_2)$

$$\Rightarrow 1 = \frac{3(0) + 4(a)}{3+4}$$

$$\Rightarrow 1 = \frac{4a}{7}$$

$$\Rightarrow a = \frac{4}{7}$$

$$-7 = \frac{3(b) + 4(0)}{3 + 4}$$

$$\Rightarrow -7 = \frac{3b}{7}$$

$$\Rightarrow b = \frac{-49}{3}$$

then $A\left(\frac{7}{4}, 0\right), B\left(0, \frac{-49}{3}\right)$ putting in (1)

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 0 = \frac{\frac{-49}{3} - 0}{0 - \frac{7}{4}}\left(x - \frac{7}{4}\right)$$

$$y - 0 = \frac{49}{3} \times \frac{4}{7}\left(x - \frac{7}{4}\right)$$

$$y = \frac{28}{3}\left(x - \frac{7}{4}\right)$$

$3y - 28x + 49 = 0$

Q14

Let the equation of line be $\frac{x}{a} + \frac{y}{b} = 1$

and $a + b = 9$

$\therefore\ b = 9 - a$

$\therefore$ Equation is

$$\frac{x}{a} + \frac{y}{9 - a} = 1$$

and it passes through $(2, 2)$

$$\therefore\ \frac{2}{a} + \frac{2}{(a - a)} = 1$$

$18 - 2a + 2a = 9a - a^2$

$a^2 - 9a + 18 = 0$

$a = 6, 3$

$\therefore\ b = 3, 6$

The equation of line are

$\frac{x}{6} + \frac{y}{3} = 1$ or $\frac{x}{3} + \frac{y}{6} = 1$

$2x + y - 6 = 0$ or $x + 2y - 6 = 0$

Q15

$P(2,6)$ let A be the point on x-axis(x,y)

$\Rightarrow A\underset{(x_1,y_1)}{(a,0)}$

B be a point on y-axis

$\Rightarrow B\underset{(x_2,y_2)}{(0,b)}$

Using section formula $x = \frac{lx_2+mx_1}{l+m}, \frac{ly_2+my_1}{l+m}$

$l : m = 2 : 3$

$2 = \frac{2(0)+3(0)}{2+3}$

$\Rightarrow 10 = 3a$

$\Rightarrow a = \frac{10}{3}$

$6 = \frac{2(b)+3(0)}{2+3}$

$\Rightarrow 30 = 2b$

$\Rightarrow b = 15$

$\therefore$ Point A is $\left(\frac{10}{3},0\right),(0,15)$

equation of line AB is

$y\text{-}y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$

$y-0 = \frac{15-0}{0-\frac{10}{3}}\left(x-\frac{10}{3}\right)$

$y = \frac{-15\times 3}{10}\left(x-\frac{10}{3}\right)$

$2y = -9x+\frac{90}{3}$

$9x+2y = 30$

Q16

The equation of line is

$$\frac{x}{a}+\frac{y}{b}=1$$

Here,

$a-b=2$

or $a=2+b$

$$\therefore \frac{x}{b+2}+\frac{y}{b}=1 \quad ---(1)$$

It passes through $(3,2)$

$$\therefore \frac{3}{b+2}+\frac{2}{b}=1$$

$3b+2b+4=b^2+2b$

$\Rightarrow b^2-3b-4=0$

$\Rightarrow b=4$ or -1

$\Rightarrow a=6$ or $1.$

$\therefore$ Equations of lines are

$$\frac{x}{6}+\frac{y}{4}=1$$

$\Rightarrow 2x+3y=12$

or

$$\frac{x}{1}-\frac{y}{1}=1$$

$\therefore x-y=1$

Q17

The line $2x+3y=6$ cuts coordinate axis at A $(3,0)$ and B $(0,2)$.

The portion AB intercepted between the axis *is* trisected by points P and Q

$$\therefore \frac{AP}{PB} = \frac{1}{2} \text{ and } \frac{AQ}{QB} = \frac{2}{1}$$

$$\Rightarrow \text{ Coordinate of } P = \left(\frac{1 \times 0 + 3 \times 2}{3}, \frac{1 \times 2 + 0}{3}\right) = \left(\frac{1}{3}, \frac{2}{3}\right)$$

$$\Rightarrow \text{Coordinate of } Q\left(\frac{2 \times 0 + 3 \times 1}{3}, \frac{4+0}{3}\right) = \left(\frac{3}{3}, \frac{4}{3}\right)$$

$$\text{Equation of } OQ = y - 0 = \frac{\frac{4}{3} - 0}{\frac{3}{3} - 0}(x - 0)$$

$$3y = 4x$$

$$\text{Equation of } OP \Rightarrow y - 0 = \frac{\frac{2}{3} - 0}{\frac{1}{3} - 0}(x - 0)$$

$$x - 3y = 0$$

Q18

The equation of the given line is

$$3x - 5y = 15$$

$$\frac{x}{5} - \frac{y}{3} = 1$$

It cuts axis at $(5,0)$ and $(-3,0)$.

The position AB intercepted between the axis is $1:1$

$$\therefore P = \left(\frac{5}{2}, \frac{-3}{2}\right)$$

The equation of the line passing through point $(2,1)$

$$y-1 = \frac{1+\frac{3}{2}}{2-\frac{5}{2}}(x-2)$$

$$y-1 = -5(x-2)$$

$$5x+y = 11$$

Q19

The equation of the given line is,

$ax + by + c = 0$

$ax + by = -c$

$$\frac{x}{\frac{-c}{a}} + \frac{y}{\frac{-c}{b}} = 1$$

$$c = \left(\frac{\frac{-c}{a} + 0}{2}, \frac{0 - \frac{c}{b}}{2} \right)$$

$$c = \left(\frac{-c}{2a}, \frac{-c}{2b} \right)$$

The equation of the line is passing through the point $(0,0)$

and c = $\left(\frac{-c}{2a}, \frac{-c}{2b} \right)$,

$$\left(y + \frac{c}{2b} \right) = \left(\frac{\frac{-c}{2b}}{\frac{-c}{2a}} \right) \left(x + \frac{c}{2a} \right)$$

$$\Rightarrow \frac{-c}{2a} \left(y + \frac{c}{2b} \right) = \left(\frac{-c}{2b} \right) \left(x + \frac{c}{2a} \right)$$

$$\Rightarrow \frac{-y}{a} + \frac{x}{b} = 0$$

$$\Rightarrow ax - by = 0$$

Ex 23.7

Q1(i)

$P = 5, \alpha = 60^\circ$

$x\cos\alpha + y\sin\alpha = P$

$\Rightarrow \quad x\cos 60^\circ + y\sin 60^\circ = 5$

$\Rightarrow \quad x \times \frac{1}{2} + y \times \frac{\sqrt{3}}{2} = 5$

$\Rightarrow \quad x + \sqrt{3}y = 10$

Q1(ii)

$P = 4, \alpha = 150^\circ$

$x\cos\alpha + y\sin\alpha = P$

$\Rightarrow \quad x\cos 150^\circ + y\sin 150^\circ = 4$

$\Rightarrow \quad -x \times \frac{\sqrt{3}}{2} + y \times \frac{1}{2} = 4$

$\Rightarrow \quad -\sqrt{3}x + y = 8$

Q1(iii)

$P = 8, \alpha = 225^\circ$

$x\cos\alpha + y\sin\alpha = P$

$\Rightarrow \quad x\cos 225^\circ + y\sin 225^\circ = 8$

$\Rightarrow \quad -x \times \frac{1}{\sqrt{2}} - y \times \frac{1}{\sqrt{2}} = 8$

$\Rightarrow \quad x + y + 8\sqrt{2} = 0$

Q1(iv)

$P = 8, \alpha = 300°$

$x \cos\alpha + y \sin\alpha = P$

$\Rightarrow \quad x \cos 300° + y \sin 300° = 8$

$\Rightarrow \quad x \times \frac{1}{2} - y \times \frac{\sqrt{3}}{2} = 8$

$\Rightarrow \quad x - \sqrt{3}y = 16$

Q2

Given, Inclination of perpendicular line (L) passing through origin is 30°

$\Rightarrow$ Slope=Tan $30^{\circ} = \frac{1}{\sqrt{3}}$

Slope of perpedicular line (M) which is perpendicular to line L is $-\sqrt{3}$

So equation of line M is $y=-\sqrt{3}x+c$

Given perpendicular distance from origin to line M is 4

$4=\frac{c}{2} \Rightarrow c=8$

So equation of line M is $y=-\sqrt{3}x+8$

Q3

Here,

$p = 4$ and $\alpha = 15^\circ$

The equation of line is

$x\cos\alpha + y\sin\alpha = p \quad ---(1)$

$x\cos 15^\circ + y\sin 15^\circ = 4$

$\cos 15^\circ = \cos(45 - 30) = c\,os\,45\cos 30 + \sin 45\sin 30$

$(\because \cos(\theta - \phi) = \cos\theta\cos\phi + \sin\theta\sin\phi)$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$= \frac{1}{2\sqrt{2}}(\sqrt{3} + 1)$

$\sin 15^\circ = \sin(45 - 30) = \sin 45\cos 30 \qquad \cos 45\sin 30$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{1}{2\sqrt{2}}(\sqrt{3} - 1)$

Putting in (1)

$x \times \frac{1}{2\sqrt{2}}(\sqrt{3} + 1) + y \times \frac{1}{2\sqrt{2}}(\sqrt{3} - 1) = 4$

$x(\sqrt{3} + 1) + y(\sqrt{3} - 1) = 8\sqrt{2}$

Q4

Here $P = 3$

and $\alpha = \tan^{-1}\left(\frac{5}{12}\right)$

$\Rightarrow \cos\alpha = \frac{12}{13}, \sin\alpha = \frac{5}{13}$

Equation of straight line is:

$x\cos\alpha + y\sin\alpha = P$

$x\left(\frac{12}{13}\right) + y\left(\frac{5}{13}\right) = 3$

$12x + 5y = 39$

Q5

Here $P = 2, \ \sin\alpha = \frac{1}{3}$

$\Rightarrow \quad \cos\alpha = \frac{2\sqrt{2}}{3}$

The equation of straight line is

$x\cos\alpha + y\sin\alpha = P$

$x\left(\frac{2\sqrt{2}}{3}\right) + y\left(\frac{1}{3}\right) = 2$

$2\sqrt{2}x + y = 6$

Q6

Given:

$P = \pm 2$

$\tan\alpha = \frac{5}{12}$

The equation of line is

$x\cos\alpha + y\sin\alpha = \pm P$

$x\frac{12}{13} + y\frac{5}{13} = \pm 2$

$12x + 5y \pm 26 = 0$

Q7

Here,

P = perpendicular distance from origin = 7

Angle made with y-axis is 150°,

$\therefore$ Angle made with x-axis is 30°

$$\cos\alpha = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\sin\alpha = \sin 30^\circ = \frac{1}{2}$$

The equation of line is

$x\cos\alpha + y\sin\alpha = P$

$$x\left(\frac{\sqrt{3}}{2}\right) + y\left(\frac{1}{2}\right) = 7$$

$$\sqrt{3}x + y = 14$$

Q8

We have,

$$\sqrt{3}x + y + 2 = 0$$

$$-\sqrt{3}x - y = 2$$

$$\left(\frac{-\sqrt{3}}{2}\right)x + \left(\frac{-1}{2}\right)y = 1$$

This same as $x\cos\theta + y\sin\theta = p$

Therefore, $\cos\theta = \frac{-\sqrt{3}}{2}, \sin\theta = -\frac{1}{2}$ and $p = 1$

$\theta = 210^\circ$ and $p = 1$

$\theta = \frac{7\pi}{6}$ and $p = 1$

Q9

Perpendicular from origin makes an angle of $30°$ with y-axis, thus making $60°$ woth x-axis

Area of triangle is = $96\sqrt{3}$

$$\frac{1}{2} \times 2P \times \frac{2p}{\sqrt{3}} = 96\sqrt{3}$$

$$p^2 = \frac{96\sqrt{3} \times \sqrt{3}}{2} = 48 \times 3 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$$

$$p = 12$$

$$x\cos\alpha + y\sin\alpha = p$$

$$x\cos 60° + y\sin 60° = 12$$

$$x \times \frac{1}{2} + y\frac{\sqrt{3}}{2} = 12$$

$$x + \sqrt{3}y = 24$$

Q10

$$\alpha = 30°$$

$$\text{area of triangle} = \frac{50}{\sqrt{3}}$$

$$\text{area of triangle} = \frac{1}{2}r^2\sin\theta = \frac{50}{\sqrt{3}}$$

$$\sin 30 = \frac{1}{2}$$

$$\frac{1}{2} \times 2p \times \frac{2p}{\sqrt{3}} = \frac{50}{\sqrt{3}}$$

$$p^2 = \frac{50}{\sqrt{3}} \times \frac{\sqrt{3}}{2} = 25$$

$$p \pm 5$$

$$x\cos\alpha + y\sin\alpha = \pm 5$$

$$x\cos 30° + y\sin 30° = \pm 5$$

$$x\frac{\sqrt{3}}{2} + \frac{y}{2} = \pm 5$$

$$\sqrt{3}x + y = \pm 10$$

Ex 23.8

Q1

The equation of line through $(1,2)$ and making an angle of 60^0 with the x-axis is

$$\frac{x-1}{\cos 60^0} = \frac{y-2}{\sin 60^0} = r$$

$$\frac{x-1}{\frac{1}{2}} = \frac{y-2}{\frac{\sqrt{3}}{2}} = r$$

Where r is the distance of any point on the line from $A(1,2)$.

The coordinates of P on the line are

$$\left(1+\frac{1}{2}r, 2+\frac{\sqrt{3}}{2}r\right)$$

and

P lies on $x+y=6$

$$\therefore 1+\frac{r}{2}+2+\frac{\sqrt{3}r}{2} = 6$$

$$\text{or } r = \frac{6}{1+\sqrt{3}} = 3\left(\sqrt{3}-1\right)$$

Hence length $AP = 3\left(\sqrt{3}-1\right)$

Q2

The equation of line is

$$\frac{x-3}{\cos\frac{\pi}{6}} = \frac{y-4}{\sin\frac{\pi}{6}} = \pm r$$

$$\text{or } x = \pm\frac{\sqrt{3}}{2}r+3 \text{ and } y = \pm\frac{1}{2}r+4$$

$$Q\left(\pm\frac{\sqrt{3}r}{2}+3, \ \pm\frac{r}{2}+4\right) \text{ lie in } 12x+5y+10=0$$

$$\therefore 12\left(\pm\frac{\sqrt{3}r}{2}+3\right)+5\left(\pm\frac{r}{2}+4\right)+10=0$$

$$\pm\frac{12\sqrt{3}r}{2}+36\pm\frac{5r}{2}+20+10=0$$

$$r = \frac{\pm 132}{5+12\sqrt{3}}$$

Hence, length PQ is $\frac{132}{12\sqrt{3}-5}$

Q3

The equation of line is

$$\frac{x-2}{\cos\alpha} = \frac{y-1}{\sin\alpha} = r$$

$$\Rightarrow \frac{x-2}{\frac{1}{\sqrt{2}}} = \frac{y-1}{\frac{1}{\sqrt{2}}} = r$$

or $x = \frac{1}{\sqrt{2}}r + 2,\ y = \frac{1}{\sqrt{2}}r + 1$

$B\left(\frac{r}{\sqrt{2}} + 2,\ \frac{r}{\sqrt{2}} + 1\right)$ lie on $x + 2y + 1 = 0$

$$\therefore \frac{r}{\sqrt{2}} + 2 + \frac{2r}{\sqrt{2}} + 2 + 1 = 0$$

$$\frac{3r}{\sqrt{2}} = \pm 5$$

$$r = \frac{5\sqrt{2}}{3}$$

The length AB is $\frac{5\sqrt{2}}{3}$ units

Q4

The required line is parallel to $3x-4y+1=0$

$\therefore$ Slope of the line = slope of $3x - 4y + 1 = \frac{-3}{-4}$

$$\tan\alpha = \frac{3}{4}$$

$$\Rightarrow \sin\alpha = \frac{3}{5} \text{ and } \cos\alpha \, \frac{4}{5}$$

The equation of line is

$$\frac{x+4}{\cos\alpha} + \frac{y+1}{\sin\alpha} = r$$

$$\Rightarrow \frac{x-4}{\frac{4}{5}} + \frac{y+1}{\frac{3}{5}} = \pm 5$$

$$\Rightarrow x = 8 \text{ and } y = 2$$

or

$$x = 0 \text{ and } y = -4$$

$\therefore$ $(8, 2)$ and $(0, -4)$ are coordinates of two points on the line which are at a distance of 5 units from $(4, 1)$

Q5

The equation of line is

$$\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = \pm r$$

or

$$x = x_1 \pm r\cos\theta \text{ and } y = y_1 \pm r\sin\theta$$

$\therefore (x_1 \pm r\cos\theta, y_1 \pm r\sin\theta)$ lie on $ax + by + c = 0$

$$\Rightarrow a(x_1 \pm r\cos\theta) + b(y_1 \pm r\sin\theta) + c = 0$$

$$\Rightarrow \pm r(a\cos\theta + b\sin\theta) = -c - ax_1 - by_1$$

$$\Rightarrow r = \left|\frac{ax_1 + by_1 + c}{a\cos\theta + b\sin\theta}\right|$$

Q6

Equation of line is

$$\frac{x-2}{\cos 45^0} = \frac{y-3}{\sin 45^0} = r$$

$$x = \frac{r}{\sqrt{2}} + 2 \quad \text{and} \quad y = \frac{r}{\sqrt{2}} + 3$$

$$P\left(\frac{r}{\sqrt{2}} + 2, \frac{r}{\sqrt{2}} + 3\right) \text{ lie on } 2x - 3y + 9 = 0$$

$$\therefore\ 2\left(\frac{r + 2\sqrt{2}}{\sqrt{2}}\right) - 3\left(\frac{r + 3\sqrt{2}}{\sqrt{2}}\right) + 9 = 0$$

$$\Rightarrow 2r + 4\sqrt{2} - 3r - 9\sqrt{2} + 9\sqrt{2} = 0$$

$$\Rightarrow r = 4\sqrt{2}$$

$\therefore$ The point $(2,3)$ is at a distance of $4\sqrt{2}$ from $2x - 3y + 9 = 0$

Q7

Equation of the required line is

$$\frac{x-3}{\cos\alpha} = \frac{y-5}{\sin\alpha} = r \quad ---(1)$$

$$\tan\alpha = \frac{1}{2} \quad \Rightarrow \cos\alpha = \frac{2}{\sqrt{5}} \text{ and } \sin\alpha = \frac{1}{\sqrt{5}}$$

$\therefore$ equation is

$$\frac{x-3}{\frac{2}{\sqrt{5}}} = \frac{y-5}{\frac{1}{5}} = r$$

$$\text{or } x = \frac{2}{\sqrt{5}}r + 3, y = \frac{1}{\sqrt{5}}r + 5$$

$$P\left(\frac{2r}{\sqrt{5}} + 3, \frac{r}{\sqrt{5}} + 5\right) \text{ lie on } 2x + 3y = 14$$

$$\therefore\ \frac{4r}{\sqrt{5}} + 6 + \frac{3r}{\sqrt{5}} + 15 = 1 \pm 14$$

$$\frac{7r}{\sqrt{5}} = \pm 17$$

$$r = \pm\sqrt{5}$$

$$r = \sqrt{5} \quad \left(r \neq -\sqrt{5}\right)$$

$\therefore$ Distance of $(3,5)$ from $2x + 3y = 14$ is $\sqrt{5}$ units

Q8

Slope of the line = $\tan\alpha = \dfrac{3}{4}$

$\therefore\ \sin\alpha = \dfrac{3}{5}$ and $\cos\alpha = \dfrac{4}{5}$

$\therefore$ Equation of line is

$$\frac{x-2}{\cos\alpha} = \frac{y-5}{\sin\alpha} = r$$

$$\Rightarrow \frac{x-2}{\frac{4}{5}} = \frac{y-5}{\frac{3}{5}} = r$$

or $x = \dfrac{4r}{5} + 2$ and $y = \dfrac{3r}{5} + 5$

then $P\left(\dfrac{4r}{5} + 2, \dfrac{3r}{5} 5\right)$ lie on $3x + y + 4 = 0$

$$\therefore\ 3\left(\frac{4r}{5} + 2\right) + \left(\frac{3r}{5} + 5\right) + 4 = 0$$

$$\frac{15}{5} r = \pm 15$$

$$r = \pm \frac{15 \times 5}{15}$$

$= 5$ units

Q9

If m is the slope of the line $x - 2y = 1$, then

$m = \tan\theta = \frac{-1}{-2} = \frac{1}{2}$

$\therefore \sin\theta = \frac{1}{\sqrt{5}}$ and $\cos\theta = \frac{2}{\sqrt{5}}$

The equation of line is

$$\frac{x-3}{\cos\theta} = \frac{y-5}{\sin\theta} = \pm r$$

$$\Rightarrow x = \pm\frac{2}{\sqrt{5}}r + 3 \text{ and } y = \pm\frac{1}{\sqrt{5}}r + 5$$

$$P\left(\pm\frac{2}{\sqrt{5}}r + 3, \ \pm\frac{r}{\sqrt{5}} + 5\right) \text{ lie in } 2x + 3y = 14$$

$$2\left(\pm\frac{2}{\sqrt{5}}r + 3\right) + 3\left(\pm\frac{1}{\sqrt{5}}r + 5\right) = 14$$

$$4r + 6\sqrt{5} + 3r + 15\sqrt{5} = 14\sqrt{5}$$

$$7r = -7\sqrt{5}$$

$$r = \left|\sqrt{5}\right|$$

$$r = \sqrt{5}$$

Q10

Slope of required line

$= \textit{slope of } 3x - 4y + 8 = 0 = \frac{3}{4} = \tan\theta$

$\therefore$ Equation of required line

$$\frac{x-2}{\frac{4}{5}} = \frac{y-5}{\frac{3}{5}} = r$$

or

$$P\left(\frac{4}{5}r + 2, \ \frac{3r}{5} + 5\right)$$

and P lies in $3x + y + 4 = 0$

$$\therefore \ 3\left(\frac{4}{3}r + 2\right) + \left(\frac{3r}{5} + 5\right) + 4 = 0$$

$\Rightarrow 12r + 30 + 3r + 25 + 20 = 0$

$\Rightarrow 15r + 75 = 0$

$\Rightarrow r = 5$

Q11

The slope of the line $= 1$

$\tan\theta = 1$

or $\theta = \frac{\pi}{4}$

$\therefore$ Equation of line is

$$\frac{x+1}{\cos\frac{\pi}{4}} = \frac{y+3}{\sin\frac{\pi}{4}} = r$$

or

$x = \frac{r}{\sqrt{2}} - 1$ and $y = \frac{r}{\sqrt{2}} - 3$

$P\left(\frac{r}{\sqrt{2}} - 1, \ \frac{r}{\sqrt{2}} - 3\right)$ lie in $2x + y = 3$

$\therefore 2\left(\frac{r}{\sqrt{2}} - 1\right) + \left(\frac{r}{\sqrt{2}} - 3\right) = 3$

$\Rightarrow \frac{3r}{\sqrt{2}} = 8$

$r = \frac{8\sqrt{2}}{3}$

The distance of $2x + y = 3$ from $(-1, -3)$ is $\frac{8\sqrt{2}}{3}$ units

Q12

$5x-y-4=0$ ---1

$3x+4y-4=0$ ---2

P(1,5)

Let (a,b) lie on 2; (c,d) on 1

we get $3a+4b=4$;----3

$5c-d=4$ ------4

From midpoint formula, we have

$a+c=2$ -----5

$b+d=10$ -------6

Solving 3 and 5 we get $4b-3c=-2$ -----7

Solving 4 and 6 we get $5c+b=14$ ---- 8

Solving 7 and 8 we get $c=\frac{58}{23}$

Substitute c in 5 we get $a=\frac{-12}{23}$

Substitute above values similarly in other equations we get

$(a,b)=(\frac{-12}{23},\frac{32}{23})$

$(c,d)=(\frac{58}{23},\frac{198}{23})$

Slope of line connecting above points is $\frac{198-32}{58+12}=\frac{83}{35}$

Required equation of line is

$y-5=\frac{83}{35}(x-1)$

$35y-175=83x-83$

$83x-35y+92=0$

Q13

The equation of any line passing through $(-2,-7)$ is

$$\frac{x+2}{\cos\theta}=\frac{y+7}{\sin\theta}=r$$

B and C are at distance r and $(r+3)$

Thus, Coordinates of B and C are $(-2+r\cos\theta,\ -7+r\sin\theta)$ and $(-2+(r+3)\cos\theta, -7+(r+3)\sin\theta)$

B lies on $4x+3y=12$

$\Rightarrow 4(-2+r\cos\theta)+3(-7+r\sin\theta)=12 \quad ---(1)$

C lies on $4x+3y=3$

$\Rightarrow 4(-2+(r+3)\cos\theta)+3(-7+(r+3)\sin\theta)=3 \quad ---(1)$

Subtracting (i) from (2)

$12\cos\theta+9\sin\theta=-9$

$\Rightarrow 4\cos\theta=-3(1-\sin\theta)$

$\Rightarrow 16\cos^2\theta=9(1+\sin^2\theta-2\sin\theta)$

$\Rightarrow 16(1-\sin^2\theta)=9(1+\sin^2\theta-2\sin\theta)$

$\Rightarrow 16-16\sin^2\theta=9+9\sin^2\theta-18\sin\theta$

$\Rightarrow 25\sin^2\theta-18\sin\theta-7=0$

$\Rightarrow 25\sin^2\theta-25\sin\theta+7\sin\theta-7=0$

$\Rightarrow 25\sin\theta(\sin\theta-1)-7(\sin\theta-1)=0$

$\sin\theta=1,\ \ \sin\theta=\frac{7}{25}$

Now, $\sin\theta=1\Rightarrow\cos\theta=0$

$\therefore\ x+2=0 \quad ---(1)$

and if $\sin\theta=\frac{7}{25}$ then $\cos\theta=\frac{24}{5}$

$$\therefore\ \frac{x+2}{\frac{24}{25}}=\frac{y+7}{\frac{7}{25}}$$

$\Rightarrow 7x+24y+182=0 \quad ---(2)$

Ex 23.9

Q1

(i) Slope intercept form $(y = mx + c)$

$\sqrt{3}x + y + 2 = 0$

$\Rightarrow y = -\sqrt{3}x - 2$

$\Rightarrow m = -\sqrt{3},\ c = -\sqrt{2}$

y-intercept $= -2$, slope $= -\sqrt{3}$

(ii) Intercept form $\left(\frac{x}{a} + \frac{y}{b} = 1\right)$

$\sqrt{3}x + y + 2 = 0$

$\Rightarrow \sqrt{3}x + y = -2$

$\Rightarrow \frac{\sqrt{3}x}{-2} + \frac{y}{-2} = 1$

$\Rightarrow \frac{x}{\frac{-2}{\sqrt{3}}} + \frac{y}{-2} = 1$

$\Rightarrow x$ intercept $= \frac{-2}{\sqrt{3}}$, y intercept $= -2$

(iii) Normal form $(x\cos\alpha + y\sin\alpha = p)$

$\sqrt{3}x + y + 2 = 0$

$\Rightarrow -\sqrt{3}x - y = 2$

$\Rightarrow \left(\frac{-\sqrt{3}}{2}\right)x + \left(\frac{-1}{2}\right)y = 1$

$\Rightarrow \cos\alpha = \frac{-\sqrt{3}}{2} = \cos 210^0$ and $\sin\alpha = \frac{-1}{2} = \sin 210^0$

$\Rightarrow p = 1,\ \alpha = 210^0$

Q2(i)

$x + \sqrt{3}y - 4 = 0$

Divide the equation by 2, we get

$\frac{1}{2}x + \frac{\sqrt{3}}{2}y = 2$

$x\cos 60 + y\sin 60 = 2$

So, p=2 and ω=60

Q2(ii)

$$x + y + \sqrt{2} = 0$$
$$x + y = -\sqrt{2}$$

Dividing each term by $\sqrt{(1)^2 + (1)^2} = \sqrt{2}$

$$\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} = -1$$
$$\frac{-x}{\sqrt{2}} - \frac{y}{\sqrt{2}} = 1$$

Comparing with $x\cos\alpha + y\sin\alpha = p$

$$\cos\alpha = \frac{-1}{\sqrt{2}}, \quad \sin\alpha = \frac{-1}{\sqrt{2}}, \quad p = 1$$

Both are negative

α is in III quadrant

$$\Rightarrow \alpha = \pi\frac{\pi}{4} = \frac{5\pi}{4} = 225^0$$

Q2(iii)

$$x - y + 2\sqrt{2} = 0$$
$$-x + y = 2\sqrt{2}$$

Dividing each term by $\sqrt{(1)^2 + (1)^2} = \sqrt{2}$

$$\frac{-x}{\sqrt{2}} + \frac{y}{\sqrt{2}} = 2$$

Comparing with $x\cos\alpha + y\sin\alpha = p$

$$\cos\alpha = \frac{-1}{\sqrt{2}}, \quad \sin\alpha = \frac{-1}{\sqrt{2}}, \quad p = 2$$

α is in II quadrant

$$\Rightarrow \alpha = \frac{\pi}{4} + \frac{\pi}{2} = \frac{3\pi}{4} = 135^0, \quad p = 2$$

Q2(iv)

$x - 3 = 0$

$x = 3$

Comparing with $x\cos\alpha + y\sin\alpha = p$

$\cos\alpha = 1$

$= \cos 0$

$\Rightarrow \alpha = 0$

$p = 3$

Q2(v)

$y - 2 = 0$

$y = 2$

Comparing with $x\cos\alpha + y\sin\alpha = p$

$\sin\alpha = 1$

$\alpha = \frac{\pi}{2}, \quad p = 2$

Q3

$\frac{x}{a} + \frac{y}{b} = 1$

The slope intercept form is

$y = mx + c$

$\frac{x}{a} + \frac{y}{b} = 1$

$bx + ay = ab$

$ay = -bx + ab$

$y = \frac{-bx}{a} + b$

Thus y-intercept is b.

Slope $= \frac{-b}{a}$

Q4

The normal form is obtaned by dividing each term of the equation by $\sqrt{a^2+b^2}$,

a = coefficient of x

b = coefficient of y

$3x-4y+4=0 \quad ---(1)$

$3x-4y=-4$

$-3x+4y=4$

Dividing each term by $\sqrt{(-3)^2+(4)^2}=\sqrt{9+16}=\sqrt{25}=5$

$\frac{-3}{5}x+\frac{y}{5}y=\frac{4}{5}$

$\Rightarrow p=\frac{4}{5} \qquad$ for equation (1)

Also
$2x+4y-5=0 \quad ---(2)$

$2x+4y=5$

Dividing each term by $\sqrt{(2)^2+(4)^2}=\sqrt{4+16}=\sqrt{20}$

$\frac{2x}{\sqrt{20}}+\frac{4y}{\sqrt{20}}=\frac{5}{\sqrt{20}}$

$p=\frac{5}{\sqrt{20}}=\frac{5}{4.4} \qquad$ for equation (2)

Comparing P of (1) and(2)

We conclude that $3x-4y+4=0$ is nearest to origin

Q5

Reduce $4x + 3y + 10 = 0$ to perpendicular form

$4x + 3y = -10$

$-4x - 3y = 10$

Dividing each term by $\sqrt{(-4)^2 + (-3)^2} = \sqrt{16 + 9} = \sqrt{25} = 5$

$\frac{-4}{5}x - \frac{3}{5}y = \frac{10}{5} = 2$

$\Rightarrow p_1 = 2 \qquad ---(1)$

$5x - 12y + 26 = 0$

$5x - 12y = -26$

Dividing each term by $\sqrt{(-5)^2 + (12)^2} = \sqrt{25 + 144} = \sqrt{169} = 13$

$\frac{-5}{13}x + \frac{12}{13}y = \frac{26}{13} = 2$

$\Rightarrow p_2 = 2 \qquad ---(2)$

$7x + 24y = 50$

Dividing each term by $\sqrt{(7)^2 + (24)^2} = \sqrt{49 + 576} = \sqrt{625} = 25$

$\frac{7x}{25} + \frac{24}{25}y = \frac{50}{25} = 2$

$\Rightarrow p_3 = 2 \qquad ---(3)$

Hence, origin is equidistant from all three lines.

Q6

$$\sqrt{3}x + y + 2 = 0$$
$$\sqrt{3}x + y = 2$$
$$-\sqrt{3}x - y = 2 - - - - - (1)$$

So,

$$\cos\theta = -\sqrt{3}, \sin\theta = -1$$
$$\tan\theta = \frac{1}{\sqrt{3}}$$
$$\theta = \left(\pi + \frac{\pi}{6}\right)$$
$$= 180° + 30°$$

$$\theta = 210°$$
$$p = 2 \qquad \left[\text{From equation } (1)\right]$$

Q7

The intercept form of equation is

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$3x - 2y + 6 = 0$$
$$3x - 2y = -6$$

$$\frac{-3x}{-6} - \frac{2y}{-6} = 1$$

$$\frac{x}{\frac{-6}{3}} + \frac{y}{\frac{-6}{-2}} = 1$$

$$\frac{x}{-2} + \frac{y}{3} = 1$$

$$\Rightarrow x\text{-intercept} = a = -2$$
$$y\text{-intercept} = b = 3$$

Q8

Perpendicular distance from the origin to the line is 5, so

$$x\cos\alpha + y\sin\alpha = 5$$

$$y\sin\alpha = -x\cos\alpha + 5$$

$$y = -\frac{\cos\alpha}{\sin\alpha}x + 5$$

$$y = -\cot\alpha x + 5$$

Comparing with $y = mx + c$

$$m = -\cot\alpha$$

$$-1 = -\cot\alpha$$

$$\cot\alpha = 1$$

$$\alpha = \frac{\pi}{4}$$

So, equation of line is

$$x\cos\frac{\pi}{4} + y\sin\frac{\pi}{4} = 5$$

$$\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} = 5$$

$$x + y = 5\sqrt{2}$$

Ex 23.10

Q1(i)

$2x - y + 3 = 0 \Rightarrow y = 2x + 3$

Putting this value in the second equation, we get

$$x + y - 5 = 0$$
$$x + (2x + 3) - 5 = 0$$
$$x + 2x + 3 - 5 = 0$$
$$3x - 2 = 0$$
$$x = \frac{2}{3}$$

Putting this value in the first equation, we get

$$\Rightarrow y = 2x + 3 = \frac{2 \times 2}{3} + 3 = \frac{4}{3} + 3 = \frac{13}{3}$$

$\therefore$ Point of intersection is $\left(\frac{2}{3}, \frac{13}{3}\right)$

Q1(ii)

$bx + ay = ab \Rightarrow x = \frac{ab - ay}{b}$

Putting this value in the second equation, we get

$$ax + by = ab$$
$$a\left(\frac{ab - ay}{b}\right) + by = ab$$
$$a^2b - a^2y + b^2y = ab^2$$
$$y\left(b^2 - a^2\right) = ab(b - a)$$
$$y = \frac{ab(b - a)}{b^2 - a^2} = \frac{ab}{b + a}$$

Putting this value in the first equation, we get

$$\Rightarrow x = \frac{ab - \frac{a(ab)}{a + b}}{b} = \frac{ab}{a + b}$$

$\therefore$ Point is $\left(\frac{ab}{a + b}, \frac{ab}{a + b}\right)$

Q1(iii)

$y = m_1x + \frac{a}{m_1}$ and $y = m_2x + \frac{a}{m_2}$

Putting value of y from one equation to another

$$m_1x + \frac{a}{m_1} = m_2x + \frac{a}{m_2}$$

$$x\left(m_1 - m_2\right) = \frac{a}{m_2} - \frac{a}{m_1} = a\left(\frac{m_1 - m_2}{m_1m_2}\right)$$

$$\Rightarrow x = \frac{a}{m_1m_2}$$

$$\Rightarrow y = m_1x + \frac{a}{m_1}$$

$$= m_1\left(\frac{a}{m_1m_2}\right) + \frac{a}{m_1}$$

$$= \frac{a}{m_2} + \frac{a}{m_1}$$

$$= a\left(\frac{m_1 + m_2}{m_1m_2}\right)$$

Q2(i)

The point of intersection of two sides will give the vertex

$x + y - 4 = 0 \quad (1)$
$2x - y + 3 = 0 \quad (2)$
$x - 3y + 2 = 0 \quad (3)$

Solving (1) and (2)
$x + y = 4$
$y = 4 - x$
Putting y in (2)
$2x - (4 - x) + 3 = 0$
$2x - 4 + x + 3 = 0$
$3x - 1 = 0$
$x = \frac{1}{3}$
Putting x in (1)
$\frac{1}{3} + y - 4 = 0$
$y = 4 - \frac{1}{3} = \frac{11}{3}$
$\therefore$ One vertex is $\left(\frac{1}{3}, \frac{11}{3}\right)$

Solving (2) and (3), we get

$y = 2x + 3$ and putting in (3)

$x - 3y + 2 = 0$
$x - 3(2x + 3) + 2 = 0$
$x - 6x - 9 + 2 = 0$
$-5x = +7$
$x = \frac{-7}{5}$

Q2(ii)

$$y(t_1+t_2)=2x+2at_1t_2, \quad 1$$
$$y(t_2+t_3)=2x+2at_2t_3 \text{ and,} \quad 2$$
$$y(t_3+t_1)=2x+2at_1t_3 \quad 3$$

Solving 1 and 2 gives $(x_1, y_1)=(at_2^2, 2at_2)$

Solving 2 and 3 gives $(x_2, y_2)=(at_3^2, 2at_3)$

Solving 1 and 3 gives $(x_3, y_3)=(at_1^2, 2at_1)$

Above points are the vertices of the triangle

Q3(i)

$$y=m_1x+c_1 \quad 1$$
$$y=m_2x+c_2 \quad 2$$
$$x=0 \quad 3$$

Solving 1 and 2 gives $\left(\frac{c_2-c_1}{m_1-m_2}, \frac{m_1c_2-m_2c_1}{m_1-m_2}\right)$

Solving 2 and 3 gives $(0, c_2)$

Solving 1 and 3 gives $(0, c_1)$

Area of traingle formed by above vertices is

$$=\frac{1}{2}\left[\left(\frac{c_2-c_1}{m_1-m_2}\times c_1\right)-\left(\frac{c_2-c_1}{m_1-m_2}\times c_2\right)\right]$$

$$=\frac{(c_2-c_1)^2}{2(m_1-m_2)}$$

Q3(ii)

$y = 0,\ y = 2,\ x + 2y = 3$

$y = 0 \qquad ---(1)$

$y = 2 \qquad ---(2)$

$x + 2y = 3 \qquad ---(3)$

Solving (1) and (2)

$(2, 0) \qquad ---(A)$

Solving (2) and (3)

$2 + 2y = 3$

$\Rightarrow y = \frac{1}{2}$

$\Rightarrow y = 2$

$\therefore \left(2, \frac{1}{2}\right) \qquad ---(B)$

Solving (1) and (3)

$x + 0 = 3$

$\Rightarrow$ Point is $(3, 0) \qquad ---(C)$

Area of triangle is

$\frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$

and treating the points A, B, C as

$(x_1 - y_1), (x_2 - y_2)$ and $(x_3 - y_3)$

$= \frac{1}{2}\left[2\left(\frac{1}{2} - 0\right) + 2(0 - 0) + 3\left(0 - \frac{1}{2}\right)\right]$

$= \frac{1}{2}\left[1 - \frac{3}{2}\right]$

$= \frac{-1}{4}$

Q3(iii)

x+y-6=0, 1
x-3y-2=0, and 2
5x -3y+2= 0 3

Solving 1 and 2 gives us $(x_1,y_1)=(5,1)$
Solving 2 and 3 gives us $(x_2,y_2)=(-1,-1)$
Solving 3 and 1 gives us $(x_3,y_3)=(2,4)$

So Area of triangle when three vertices are given is

$$\frac{1}{2}(x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2))$$

$$=\frac{1}{2}[|-25-3+4|]$$

$$=12\,squnits$$

Q4

Solving the equations $3x + 2y + 6 = 0$ and $2x-5y+4=0$
we get $x = -2$ and $y = 0$.

Solving the equations $x-3y-6=0$ and $2x-5y+4=0$
we get $x = -42$ and $y = -16$.

Solving the equations $3x + 2y + 6 = 0$ and $x-3y-6=0$
we get $x = -6/11$ and $y = -24/11$.

So let the intersection points be A, B and C i.e. the triangle be ABC.
Coordinates of A, B and C will be
A(-2, 0); B(-42, -16) and C(-6/11, -24/11).

By mid-point formula the mid-point of AB will be (-22, -8)
Equation of line passing through this mid-point and
the opposite vertex C(-6/11, -24/11) will be the equation of
the median from C. The equation will be

$$\frac{y+8}{x+22} = \frac{-8+\frac{24}{11}}{-22+\frac{6}{11}}$$

$$\frac{y+8}{x+22} = \frac{-88+24}{-242+6} = \frac{16}{59}$$

$$16x - 59y + 352 - 472 = 0$$

$$16x - 59y - 120 = 0 \qquad \textit{Median through C}$$

Similar procedure has to be used for getting other medians as well
For getting median through B find midpoint of AC and
apply the two point form of line equation. Similarly for median through A

Final median equations are

$$41x - 112y - 70 = 0$$

$$25x - 53y + 50 = 0$$

$$16x - 59y - 120 = 0$$

Q5

Let the line be

$y = \sqrt{3}x + 1 \qquad --- (1)$

$y = 4 \qquad --- (2)$

$y = -\sqrt{3}x + 2 \qquad --- (3)$

Solve (1) and (2)

$4 = \sqrt{3}x + 1$

$x = \frac{4-1}{\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}$

$\therefore$ Point A is $(\sqrt{3}, 4)$

Solve (2) and (3)

$4 = -\sqrt{3}x + 2$

$\sqrt{3}x = -2$

$x = \frac{-2}{\sqrt{3}}$

$= \frac{-2\sqrt{3}}{3}$

$\therefore$ Point B is $\left(\frac{-2\sqrt{3}}{3}, 4\right)$

Solve (1) and (3)

$\sqrt{3}x + 1 = -\sqrt{3}x + 2$

$2\sqrt{3}x = 1$

$x = \frac{1}{2\sqrt{3}} = \frac{\sqrt{3}}{6}$

$y = \sqrt{3}\left(\frac{\sqrt{3}}{6}\right) + 1$

$= \frac{3}{2}$

Q6(i)

$2x + y - 1 = 0,\ 3x + 2y + 5 = 0$

Writing equation in the form $y = mx + c$

$y = -2x + 1 \quad , y = \frac{-3}{2}x - \frac{5}{2}$

$\Rightarrow m = -2 \quad , m' = \frac{-3}{2}$

$m \neq m',,\ m_1 m_2 \neq -1$

$\Rightarrow$ The lines are intersecting

Q6(ii)

$x - y = 0,\ 3x - 3y + 5 = 0$

$\Rightarrow y = mx + c \quad , 3x - 3y + 5 = 0$

$y = x \quad , y = x + \frac{5}{3}$

$\Rightarrow m = 1 \quad , m' = 1$

Slopes of both lines are equal

$\therefore$ Lines are parallel

Q6(iii)

$3x + 2y - 4 = 0,\ 6x + 4y - 8 = 0$

$y = \frac{-3}{2}x + \frac{4}{2} \quad , y = \frac{-6}{4}x + \frac{8}{4}$

$y = \frac{-3}{2}x + 2 \quad , y = \frac{-3}{2}x + 2$

$\Rightarrow$ Lines are coincident

Because $m_1 = m_2 = \frac{-3}{2}$

Intercept = 2 in both line

Q7

The point of intersection of the lines

$4x + y - 1 = 0$ and $7x - 3y - 35 = 0$
is $y = 1 - 4x$
$7x - 3(1 - 4x) - 35 = 0$
$7x - 3 + 12x - 35 = 0$
$19x = 38$
$x = 2$
$\Rightarrow y = 1 - 4x = 1 - 8 = -7$

$\therefore$ Let $P(2, -7)$ and $Q(3, 5)$
The equation of line PQ is
$y - y_1 = m(x - x_1)$
$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$
$y - (-7) = \frac{5 - (-7)}{3 - 2}(x - 2)$
$y + 7 = 12(x - 2)$
$y - 12x = -31$
$12x - y - 31 = 0$

Q8

Given lines are,
$4x - 7y = 3$
$2x - 3y = -1$
Solving these two, we get the
point of intersection,
$x = -8, y = -5$
Point of intersection of given lines is $(-8, -5)$ equation of line makeing equaol intercepts (a) on the coordinate axes is,
$\frac{x}{a} + \frac{y}{a} = 1$
$x + y = a$
$-8 - 5 = a$
$a = -13$
So,
Equation of required line is
$x + y = -13$

Q9

$y = m_1x$, $y = m_2x$ and $y = c$

Vertices of triangle formed by above lines are

$A(0,0);\ B(\frac{c}{m_1}, c);\ C(\frac{c}{m_2}, c)$

So Area of triangle when three vertices are given is

$\frac{1}{2}(x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2))$

$$= \frac{1}{2}\left[\left|\frac{c^2}{m_1} - \frac{c^2}{m_2}\right|\right] = \frac{c^2}{2}\left[\left|\frac{m_2 - m_1}{m_1 m_2}\right|\right]$$

Given m_1 and m_2 are roots of $x^2 + (\sqrt{3}+2)x + \sqrt{3} - 1 = 0$

Product of roots$= m_1 m_2 = \sqrt{3} - 1$

$|m_2 - m_1| = \sqrt{(m_2 + m_1)^2 - 4m_1m_2} = \sqrt{(\sqrt{3}+2)^2 - 4\sqrt{3} + 4}$

$|m_2 - m_1| = \sqrt{3 + 4 + 4\sqrt{3} - 4\sqrt{3} + 4} = \sqrt{11}$

$Area = \frac{c^2}{2}\left[\frac{\sqrt{11}}{\sqrt{3} - 1}\right]$

Rationalising denominator gives $\frac{c^2}{4}\left[\sqrt{33} + \sqrt{11}\right]$

Hence Proved

Q10

If point of intersection of lines $x + y = 3$ and $2x - 3y = 1$ is

$x = 3 - y$

$2(3 - y) - 3y = 1$

$6 - 2y - 3y = 1$

$-5y = -5$

$y = 1$

$\Rightarrow x = 3 - 1 = 2$

$\therefore$ Point is $(2,1)$

Any line parallel to $x - y - 6 = 0$

Will have the same slope $= 1$

$\therefore$ Equation of line parring through $(2,1)$ and having slope $= 1$

is $y - y_1 = m(x - x_1)$

$y - 1 = 1(x - 2)$

$y - 1 = x - 2$

$y - x = -2 + 1$

$y - x = -1$

$x - y = 1$

$\therefore$ $a = 1,\ b = -1$ $\left(\text{Comparring with } \frac{x}{a} + \frac{y}{b} = 1\right)$

Q11

x + y = 1,   AB   1

2x + 3y = 6 and   BC 2

4x - y + 4   AC   3

Solving 1 and 2 gives B (-3, 4)

Solving 1 and 3 gives A $(\frac{-3}{5}, \frac{8}{5})$

Altitude from A to BC is given by

$$y - \frac{8}{5} = \frac{3}{2}\left(x + \frac{3}{5}\right)$$

$$10y - 16 = 15x + 9$$

$$15x - 10y + 25 = 0$$

$$3x - 2y + 5 = 0 ------4$$

Similarly Altitude from B to AC is given by

$$y - 4 = \frac{-1}{4}(x + 3)$$

$$4y - 16 = -x - 3$$

$$x + 4y - 13 = 0 -----5$$

Solving 4 and 5 gives orthocentre

$O(\frac{3}{7}, \frac{22}{7})$

Q12

On solving the equation of AB, BC and CA we get

$B = (-1,-1)$

$A = (2,4)$

$C = (5,1)$

The slope of $BC = \frac{1}{3}$ then slope of $AE = -3$

slope of $AC = -1$ then slope of $BD = 1$

slope of $AB = \frac{5}{3}$ then slope of $CF = \frac{-3}{5}$

Where AD, BE, CF are altitudes of ΔABC

The equation of AD, BE and CF are

$BD = y + 1 = 1(x+1) \quad \Rightarrow x - y = 0$

$AE = y - 4 = -3(x-2) \quad \Rightarrow 3x + y = 10$

$CF = y - 1 = \frac{-3}{5}(x-5) \quad \Rightarrow 3x + 5y = 20$

Are the required equations, then equation through A is $3x + y = 10$.

Q13

$AD \perp BC, CF \perp AB, BE \perp AC$

Let G be the orthocentre of triangle

Let $G(h,k)$

Now, $AG \perp BC$

$\therefore$ (slope of AG) $\times$ (slope of BC) $= -1$

$$\left(\frac{k-3}{h+1}\right)\left(\frac{0+1}{0-2}\right) = -1$$

$k - 3 = 2(h+1)$

$k - 2h = 5 \qquad ---(1)$

And $BG \perp AC$

$\Rightarrow$ (slope of BG) $\times$ (slope of AC) $= -1$

$$\left(\frac{k+1}{h-2}\right)\left(\frac{0-3}{0+1}\right) = -1$$

$3(k+1) = h - 2$

$3k - h = -5 \qquad ---(2)$

from (1) and (2)

Orthocentre $(h,k) = (-4,-3)$

Q14

Let ABC be the triangle whose sides BC,CA and AB have the equations

$y - 15 = 0$, BC

$3x - 4y = 0$, AC

$5x + 12y = 0$ AB

Solving these equations pair wise we can obtain the coordinates of the vertices A,B,C as

A(0,0), B(-36,15), C(20,15) respectively

Centroid $\left(\frac{-36+20+0}{3}, \frac{15+15+0}{3}\right) = \left(\frac{-16}{3}, 10\right)$

For incentre, We have

$a = BC = \sqrt{56^2 + 0} = 56$

$b = CA = \sqrt{20^2 + 15^2} = 25$

$c = AB = \sqrt{36^2 + 16^2} = 39$

Coordinates of incentre are

$$\left(\frac{56\times 0+25\times -36+39\times 20}{36+25+39}, \frac{56\times 0+25\times 15+39\times 15}{36+25+39}\right)$$

$= (-1, 8)$

Q15

Let $ABCD$ be a quadrilateral with sides AB, BC, CD, DA as $\sqrt{3}x + y = 0$, $\sqrt{3}y + x = 0$, $-\sqrt{3}x + y = 1$ and $\sqrt{3}y + x = 1$ respectively.

The slope of $AB = -\sqrt{3}$ $\quad ---(1)$

The slope of $BC = \frac{-1}{\sqrt{3}}$ $\quad ---(2)$

The slope of $CD = -\sqrt{3}$ $\quad ---(3)$

The slope of $DA = \frac{-1}{\sqrt{3}}$ $\quad ---(4)$

From $(1),(2),(3)$ and (4) we observe the slope of opposite sides of quadrilateral are equal

$\therefore$ Opposite sides are parallel.

$\therefore$ $ABCD$ is a parallelogram.

We observe that distance between $(AD$ and $BC)$ and $(DC$ and $AB)$ is equal $= 1$ unit

$\therefore$ Sides $AD = AB = BC = DC$

$\therefore$ The given figure $ABCD$ is a rhombus

Hence, proved

Q16

$2x + y = 5$ and $x + 3y + 8 = 0$

Intersection point of above lines is $\left(\frac{23}{5}, \frac{-21}{5}\right)$

Required line is parallel to $3x + 4y = 7$ and passing through above point

So required line equation is

$$y + \frac{21}{5} = \frac{-3}{4}\left(x - \frac{23}{5}\right)$$

$$20y + 84 = -15x + 69$$

$$15x + 20y + 15 = 0$$

$$3x + 4y + 3 = 0$$

Q17

Solving equations $5x - 6y - 1 = 0$ and $3x + 2y + 5 = 0$, we get
$x = -1$ and $y = 1$

So, the given lines intersect at the point whose coordinates are $(-1,-1)$.
We know that, the equation of the required line is perpendicular to the line $3x - 5y + 11 = 0$.

Slope of the required line $= -\frac{5}{3}$

Equation of the required line is given by,

$(y + 1) = -\frac{5}{3}(x + 1)$

$3y + 5x + 8 = 0$

Ex 23.11

Q1(i)

If the lines are concurrent then point of intersection of any two lines satisfies the third line

$15x - 18y + 1 = 0 \qquad ---(1)$

$12x + 10y - 3 = 0 \qquad ---(2)$

$6x + 66y - 11 = 0 \qquad ---(3)$

Solving (1) and (2)

$$x = \frac{18y - 1}{15}$$

$$12\left(\frac{18y - 1}{15}\right) + 10y - 3 = 0$$

$$216y - 12 + 150y - 45 = 0$$

$$366y = 57$$

$$y = \frac{57}{366} = \frac{19}{122}$$

$$\Rightarrow x = \frac{18y - 1}{15}$$

$$= \frac{18 \times \frac{19}{122} - 1}{15}$$

$$= \frac{18 \times 19 - 122}{122 \times 15}$$

$$= \frac{342 - 122}{1730}$$

$$= \frac{220}{1730}$$

$$= \frac{22}{173}$$

Putting x and y in (3)

$$6\left(\frac{22}{173}\right) + 66\left(\frac{19}{122}\right) - 11 = 0$$

$$6 \times 22 \times 122 + 66 \times 19 \times 173 - 11 \times 173 \times 122 = 0$$

$$0 = 0$$

Q1(ii)

$3x - 5y - 11 = 0,\ 5x + 3y - 7 = 0,\ x + 2y = 0$

$3x - 5y - 11 \quad ---(1)$

$5x + 3y - 7 = 0 \quad ---(2)$

$x + 2y = 0 \quad ---(3)$

Solving (1) and (2)

$x = -2y$

$5(-2y) + 3y - 7 = 0$

$-10y + 3y - 7 = 0$

$-7y = y$

$y = -1$

$\Rightarrow x = 2$

substituting x and y in (1)

$3(2) - 5(-1) - 11 = 0$

$6 + 5 - 11 = 0$

$0 = 0$

Hence, the lines are concurrent

Q1(iii)

$\frac{x}{a} + \frac{y}{b} = 1,\ \frac{x}{b} + \frac{y}{a} = 1,\ y = x$

$bx + ay = ab,\ ax + by = ab$

Put $y = x$

$bx + ax = ab,\ ax + bx = ab$

Hence the lines are concurrent

Q2

The three lines are concurrent if they have the common point of intersection.

$2x-5y+3=0 \quad ---(1)$

$x-2y+1=0 \quad ---(2)$

Solving (1) and (2)

$2x = 5y - 3$

$x = \frac{5y-3}{2}$

$\frac{5y-3}{2} - 2y + 1 = 0$

$5y - 3 - 4y + 2 = 0$

$y = 0$

$\Rightarrow \quad x = \frac{5y-3}{2} = \frac{5-3}{2} = \frac{2}{2} = 1$

Substituting x and y is $5x - 9y + \lambda = 0$

$5(1) - 9(1) + \lambda = 0$

$5 - 9 + \lambda = 0$

$\lambda = 4$

Q3

The three lines are

$y = m_1x + c_1 \quad ---(1)$

$y = m_2x + c_2 \quad ---(2)$

$y = m_3x + c_3 \quad ---(3)$

Collinear or they meet at a point only when they have common point of intersection

Solving (1) and (2) for x and y

$m_1x + c_1 = m_2x + c_2$

$x(m_1 - m_2) = c_2 - c_1$

$x = \frac{c_2 - c_1}{m_1 - m_2}$

$\Rightarrow \quad y = m_1x + c_1$

$= m_1\left(\frac{c_2 - c_1}{m_1 - m_2}\right) + c_1$

$= m_1c_2 - m_1c_1 + m_1c_1 - m_2c_1$

Putting x and y in (3)

$m_1c_2 - m_1c_1 = m_3\frac{(c_2 - c_1)}{m_1 - m_2} + c_3$

$m_1^2c_2 - m_1m_2c_2 - m_1m_2c_1 + m_2^2c_1 = m_3c_2 - m_3c_1 + m_1c_3 - m_2c_3$

$\Rightarrow \quad m_1(c_2 - c_3) + m_2(c_3 - c_1) + m_3(c_1 - c_2) = 0$

Q4

If the lines are concurrent, then the lines have common point of intersection.
The given line are

$$p_1x + q_1y = 1 \qquad ---(1)$$
$$p_2x + q_2y = 1 \qquad ---(2)$$
$$p_3x + q_3y = 1 \qquad ---(3)$$

Solving (1) and (2)

$$x = \frac{1-q_1y}{p_1}$$
$$p_2\left(\frac{1-q_1y}{p_1}\right) + q_2y = 1$$
$$p_2 = p_2q_1y + p_1q_2y = p_1$$
$$y = \frac{p_1-p_2}{p_1q_2-p_2q_1} \Rightarrow x = \frac{1-q_1\left(\frac{p_1-p_2}{p_1q_2-p_2q_1}\right)}{p_1}$$

Putting $x,\ y$ in (3)

$$p_3\left[(p_1q_2-p_2q_1)-q_1p_1-q_1p_2\right]\left[p_1q_2-p_2q_1\right]+q_3p_1(p_1-p_2)=1$$
$$(p_1p_3q_2-p_2p_3q_1-p_1p_3q_1+p_2p_3q_1)(p_1q_2-p_2q_1)+q_3p_1^2-q_3p_1p_2=1$$
$$(p_1p_3q_2-p_1p_3q_1)(p_1q_2-p_2q_1)+q_3p_1^2-q_3p_1p_2=1$$
$$p_1^2p_3q_2^2-p_1p_2p_3q_1q_2-p_1^2p_3q_1q_2+p_1p_2p_3q_1^2+q_3p_1^2-q_3p_1p_2=1 \qquad ---(1)$$

Also if $(p_1q_1)(p_2q_2)(p_3q_3)$ are collinear

Then,

$$p_1(q_2-q_3)+p_2(q_3-q_1)+p_3(q_1-q_3)=0$$

From (1)

$$p_1\left[p_1p_3q_2^2-p_2p_3q_1q_2-p_1p_3q_1q_2+p_2p_3q_1^2+q_3p_1-q_3p_2\right]=1$$
$$p_1\left[p_3q_2(p_1q_2-p_2q_1)-p_3q_1(p_1q_2-p_2q_1)+q_3(p_1-p_2)\right]=1$$

Hence, the points are collinear

Q5

The three lines are concurrent if they have the common point of intersection

$$(b+c)x + ay + 1 = 0$$
$$(c+a)x + by + 1 = 0$$
$$(a+b)x + cy + 1 = 0$$

Solving (1) and (2)

$$y = \frac{-1-(b+c)x}{a}$$

Putting in (2)

$$(c+a)x + b\frac{(-1-(b+c)x)}{a} + 1 = 0$$
$$acx + a^2x + b - b^2x - bcx + a = 0$$
$$x\left(ac + a^2 - b^2 - bc\right) = b - a$$
$$x\left(ac - bc + a^2 - b^2\right) = b - a$$
$$x\left(c(a-b) + (a-b)(a+b)\right) = b - a$$
$$x(c+a+b) = -1 \qquad \left[\text{Cancelling}(a-b) \text{ both sides}\right]$$
$$x = \frac{-1}{a+b+c}$$
$$y = \frac{-1 + \frac{(b+c)(-1)}{a+b+c}}{a} = \frac{-a-b-c-b-c}{a(a+b+c)}$$

Putting the value of x, y in (3);

$$(a+b)\left(\frac{-1}{a+b+c}\right) + c\left(\frac{-a-2b-2c}{a(a+b+c)}\right) + 1 = 0$$
$$-a^2 - ba - ac - 2bc - 2c^2 + a^2 + ab + ac = 0$$
$$0 = 0$$

Hence, the lines are concurrent

Q6

If the three lines are concurrent then the point of intersection of (1) and (2) should verify the (3) line, where

$$ax + a^2y + 1 = 0 \qquad ---(1)$$

$$bx + b^2y + 1 = 0 \qquad ---(2)$$

$$cx + c^2y + 1 = 0 \qquad ---(3)$$

Solving (1) and (2)

$$x = \frac{-1 - a^2y}{a} \Rightarrow b\left(\frac{-1 - a^2y}{a}\right) + b^2y + 1 = 0$$

$$-b - a^2by + ab^2y + a = 0$$

$$y = \frac{b - a}{ab(b - a)} = \frac{1}{ab}$$

$$\Rightarrow \qquad x = \frac{1 - a^2 \times \frac{1}{ab}}{a} = \frac{1 - \frac{a}{b}}{a} = \frac{b - a}{ab}$$

Putting in (3)

$$c\left(\frac{b - a}{ab}\right) + c^2\left(\frac{1}{ab}\right) + 1 = 0$$

$$bc - ac + c^2 + ab = 0$$

$$bc + c^2 - ac + ab = 0$$

$$c(b + c) - a(c - b) = 0$$

$\Rightarrow$ Either $c = b \Rightarrow 2bc = 0 \Rightarrow 2c^2 = 0 \Rightarrow c = 0$

Q7

If a, b, c are in $A.P.$

$b - a = c - b$

$2b = a + c$ [Common difference]

To prove that the straight lines are concurrent then they have the common point of intersection.

$$ax + 2y + 1 = 0 \quad ---(1)$$
$$bx + 3y + 1 = 0 \quad ---(2)$$
$$cx + 4y + 1 = 0 \quad ---(3)$$

Solving (1) and (2)

$$x = \frac{-1 - 2y}{a}$$

Put in (2)

$$b\left(\frac{-1 - 2y}{a}\right) + 3y + 1 = 0$$

$$y = \frac{b - a}{3a - 2b} \Rightarrow x = \frac{-1 - \frac{2(b - a)}{3a - 2b}}{a} = \frac{-3a + 2b - 2b + 2a}{a(3a - 2b)}$$

$$x = \frac{-1}{3a - 2b}$$

Putting x, y in (3)

$$c\left(\frac{-1}{3a - 2b}\right) + 4\left(\frac{b - a}{3a - 2b}\right) + 1 = 0$$

$$-c + 4b - 4a + 3a - 2b = 0$$

$$-a + 2b - c = 0$$

$$-a + a + c - c = 0$$

$$0 = 0$$

Hence, Proved

Q8

Let coordinates of ΔABC be $A(0,0)$, $B(a,0)$, $C(0,b)$.

Then mid points of AB, BC and CA are $\rightarrow$.

$D\left(\frac{a}{2},0\right), E\left(\frac{a}{2},\frac{b}{2}\right)$ and $F\left(0,\frac{b}{2}\right)$

Then equation of CD, AE and BF are

$CD \Rightarrow y-b=\frac{a-b}{\frac{a}{2}-0}(x-0)$

$\Rightarrow \quad y-b=\frac{-2b}{a}(x)$

$\Rightarrow \quad ay-ab=-2bx$

$\Rightarrow \quad ay+2bx-ab=0 \qquad ---(1)$

$BF \Rightarrow y-0=\frac{\frac{b}{2}-0}{0-a}(x-a)$

$\Rightarrow \quad y=\frac{-b}{2a}(x-a)$

$\Rightarrow \quad -2ay-bx=ba \qquad ---(2)$

$AE \Rightarrow y-0=\frac{0-\frac{b}{2}}{0-\frac{a}{2}}(x-0)$

$\Rightarrow \quad ya=+bx \qquad ---(3)$

Adding (1), (2) and (3)

$ay+2bx-ab+2b^2-2ay-bx-ab+ay-bx=0$

then,

$\lambda_1 L_1+\lambda_2 L_2+\lambda_3 L_3=0.$ where $\lambda_1=\lambda_2=\lambda_3=1.$

Hence, lines are concurrent

Ex 23.12

Q1

Equation of line through $(2,3)$ is

$$y-y_1 = m(x - x_1) \qquad ---(1)$$

$(2, 3)$ is $(x_1 y_1)$

Since the line is parallel to $3x - 4y + 5 = 0$

$\Rightarrow$ The slope will be equal

Slope of $3x - 4y + 5 = 0$

$$4y = 3x + 5$$

$$y = \frac{3}{4}x + \frac{5}{4}$$

$$\Rightarrow \quad m = \frac{3}{4}$$

Substituting m and $(x_1 y_1)$ is (1)

$$y - 3 = \frac{3}{4}(x - 2)$$

$$4y - 12 = 3x - 6$$

$$3x - 4y = -12 + 6 = -6$$

$$3x - 4y + 6 = 0$$

Q2

Any equation passing through $(3,-2)$ and perpendicular to givven line is

$$y-y_1 = -\frac{1}{m}(x - x_1) \qquad ---(1)$$

Where $(x_1 - y_1)$ is $(3,-2)$ and m is slope of line.

$\frac{-1}{m}$ is taken as lines are perpendicular

Finding slope of line $x - 3y + 5 = 0$

$$3y = x + 5$$

$$y = \frac{x}{3} + \frac{5}{3}$$

$$\Rightarrow \quad m = \frac{1}{3}$$

Substituting the value of m and $(x_1 - y_1)$ in (1)

$$y - (-2) = -\frac{1}{\frac{1}{3}}(x - 3)$$

$$y + 2 = -3(x - 3) = -3x + 9$$

$$3x + y = 7$$

Q3

Any line which is perpendicular bisector means line is perpendicular to the given line and one end point is the mid point of that line.

The line joining $\underset{(x_1,y_1)}{(1,3)}$ and $\underset{(x_2,y_2)}{(3,1)}$.

Has the mid-point

$$x = \frac{x_1 + x_2}{2},\ y = \frac{y_1 + y_2}{2}$$

$$\Rightarrow \quad (x_1 y_1) = \left(\frac{1+3}{2}, \frac{3+1}{2}\right) = (2,2)$$

Also slope of line is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-3}{3-1} = \frac{-2}{2} = -1$$

So, the slope of required line is 1 (negative reciprocal of slope)

Thus, the equation of perpendicular bisector is

$$y - y_1 = \frac{-1}{m}(x - x_1)$$

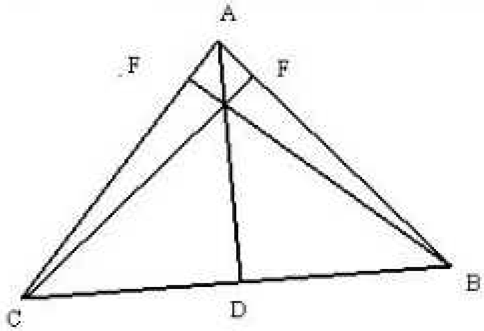
$$y - 2 = 1(x - 2)$$

$$y - 2 = x - 2$$

$$y = x$$

Q4

Let the perpendiculars of the triangle on the side AB, BC and AC be CF, AD and FB respectively.



Slope of the side AB $= \frac{4-2}{1+3} = \frac{2}{4} = \frac{1}{2}$

Corresponding slope of CF $= -\frac{1}{1/2} = -2$

[since $m_1 \times m_2 = -1$]

Equation of CF, $y - y_1 = m(x - x_1)$

$y+3 = -2(x+5)$ [Putting co-ordinates of C in place of x_1 and y_1]

$y+3 = -2x - 10$

$y = -2x - 13$

Slope of the side BC $= \frac{2+3}{-3+5} = \frac{5}{2}$

Corresponding slope of AD $= -\frac{1}{5/2} = -\frac{2}{5}$

Equation of AD,

$y - y_1 = m(x - x_1)$

$y - 4 = -\frac{2}{5}(x - 1)$

$5y - 20 = -2x + 2$

$5y = -2x - 22$

Slope of the side AC $= \frac{4+3}{1+5} = \frac{7}{6}$

Q5

Required equation of line is

$$y - y_1 = m'(x - x_1) \qquad ---(1)$$

Point is $(x_1 y_1) = (0, -4)$

It is perpendicular to line $\sqrt{3}x - y + 5 = 0$

$\Rightarrow$ Slope is $y = mx + c$

$$y = \sqrt{3}x + 5$$

$$m = \sqrt{3}$$

$$m' = \frac{-1}{m} = \frac{-1}{\sqrt{3}}$$

Putting m' and $(x_1 y_1)$ in (1)

$$y - (-4) = \frac{-1}{\sqrt{3}}(x - 0)$$

$$y + 4 = \frac{-x}{\sqrt{3}}$$

$$x + \sqrt{3}y + 4\sqrt{3} = 0$$

Q6

Here,

Let l be line mirror and B is image of A

Let m be slope of line l

So,

$$m(\text{slope of } AB) - -1$$

$$m\left(\frac{2-1}{5-2}\right) = -1$$

$$m\left(\frac{1}{3}\right) = -1$$

$$m = -3$$

M is mid point of AB

$$M = \left(\frac{2+5}{2}, \frac{2+1}{2}\right)$$

$$M = \left(\frac{7}{2}, \frac{3}{2}\right)$$

Equation line l is,

$$y - y_1 = m(x - x_1)$$

$$y - \frac{3}{2} = (-3)\left(x - \frac{7}{2}\right)$$

$$\frac{2y - 3}{2} = -3x + \frac{21}{2}$$

$$2y - 3 = -6x + 21$$

$$6x + 2y = 24$$

$$3x + y = 12$$

Q7

Any line is given by equation

$$y-y_1 = m(x - x_1) \qquad ---(1)$$

Where $(x_1 y_1)$ is (α, β)

And m is negative reciprocal of slope of line $lm + my + n = 0$.

i.e; $y = \frac{-lx}{m} - \frac{n}{m}$

$\Rightarrow$ Slope of line $= \frac{-l}{m}$

Putting the data in (i), we get

$$y - \beta = \frac{m}{l}(x - \alpha)$$

$$ly + mx = m\alpha + l\beta$$

$$m(x - \alpha) = l(y - \beta)$$

Q8

Let the equation of the required line be $y-y_1=m(x-x_1)$, where 'm' denotes the slope of the line and (x_1, y_1) be the point through which the line passes.

Since the x-intercept of the line is 1 on the positive direction of the x-axis therefore the line passes through (1,0)

Also, $2x-3y=5$

$$3y = 2x - 5$$

$$y = \frac{2x}{3} - \frac{5}{3}$$

Therefore, the slope of the given line is 2/3.

Slope of the required line = $\frac{-1}{2/3} = -\frac{3}{2}$

Therefore the equation of the required line is

$$y - y_1 = m(x - x_1)$$

$$y - 0 = \frac{2}{3}(x-1)$$

$$y = -\frac{3}{2}(x-1)$$

$$2y = -3x + 3$$

The equation of the required line is 3x+2y -3=0

Q9

Slope of line through the points $\underset{(x_1,\ y_1)}{(a,\ 2a)},\ \underset{(x_2,\ y_2)}{(-2,\ 3)}$

$$\Rightarrow \quad m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 2a}{-2 - a}$$

Also, slope of line $x - ay = 1$ in the form $y = mx + c$

$$4x + 3y + 5 = 0$$

$$y = \frac{-4}{3}x - \frac{5}{3}$$

$$\Rightarrow \quad m_2 = \frac{-4}{3}$$

If two lines are perpendicular then, $m_1 m_2 = -1$

$$\left(\frac{3 - 2a}{-2 - a}\right)\left(\frac{-4}{3}\right) = -1$$

$$-12 + 8a = 6 + 3a$$

$$5a = 18$$

$$a = \frac{18}{5}$$

Q10

Any line having y-intercept equal to $\frac{4}{3}$ passes through the point $\underset{(x_1, y_1)}{\left(0, \frac{4}{3}\right)}$

Slope of line $3x - 4y + 11 = 0$

$$y = \frac{3}{4}x + \frac{11}{4}$$

$$\Rightarrow \quad m = \frac{3}{4}$$

The required line is perpendicular to the given line, therefore its slope is $\frac{-4}{3}$

$\Rightarrow$ Equation of required line is

$$y - y_1 = m'(x - x_1)$$

$$y - \frac{4}{3} = \frac{-4}{3}(x - 0)$$

$$4x + 3y - 4 = 0$$

Q11

Any line which is right bisector to another line segment passes through the mid-point of end-points and is perpendicular to it.

$\Rightarrow$ mid point of (a,b) and (a_1,b_1) is

$$(x_1,y_1)=\left(\frac{a+a_1}{2},\ \frac{b+b_1}{2}\right)$$

Slope of line $(m)=\dfrac{b_1-b}{a_1-a}$

Slope of required line is $m'=\dfrac{a-a_1}{b-b_1}$

Equation of required line is

$$y-y_1=m'(x-x_1)$$

$$y-\left(\frac{b+b_1}{2}\right)=\frac{a-a_1}{b-b_1}\left(x-\frac{a+a_1}{2}\right)$$

$$2x(a_1-a)+2y(b_1-b)+a^2+b^2=a_1^2+b_1^2$$

Q12

Let the image of the point $P(2,1)$ in the line mirror AB be $Q(\alpha,\beta)$. Then, PQ is perpendicularly bisected at R.

The coordinates of R are

$$\left(\frac{\alpha+2}{2},\ \frac{\beta+1}{2}\right)$$

And lie on the line $x+y-5=0$

$$\left(\frac{\alpha+2}{2}\right)+\left(\frac{\beta+1}{2}\right)-5=0$$

$$\alpha+2+\beta+1-10=0$$

$$\alpha+\beta=7 \qquad ---(1)$$

Since PQ is $\perp$ to AB

$$(\text{Slope of } AB)\times(\text{Slope of } PQ)=-1$$

$$-1\times\left(\frac{\beta-1}{\alpha-2}\right)=-1$$

$$\beta-1=\alpha-2$$

$$\beta-\alpha=-1 \qquad ---(2)$$

Solving (1) and (2), we get

$$\alpha=5 \text{ and } \beta=2$$

$\therefore$ Image of $(1,2)$ in $x+y-5=0$ is $(4,3)$.

Q13

Let $Q(5,2)$ be the mirror image of $P(2,-1)$ with respect to the line mirror $AB \times (ax+by+c=0)$

Then,

$$(\text{Slope of } AB) \times (\text{Slope of } PQ) = -1$$

$$\frac{-a}{b} \times \left(\frac{2-1}{5-2}\right) = -1$$

$$\frac{-a}{b} \times \frac{1}{3} = -1$$

$$-a = -3b$$

$$a = 3b \qquad ---(1)$$

and

(R) mid point of PQ should line in AB, as PQ perpendicularlly biosects AB.

$\therefore$ Coordinates of R are $\left(\frac{5+2}{2}, \frac{2+1}{2}\right) = \left(\frac{7}{2}, \frac{3}{2}\right)$

$$\therefore \quad \frac{7}{2}a + \frac{3}{2}b + c = 0$$

$$7a + 3\left(\frac{a}{3}\right) + 2c = 0 \qquad \left[\because b = \frac{a}{3} \text{ from } (1)\right]$$

$$8a + 2c = 0$$

or, $\quad -4a = 6 \qquad ---(2)$

$\therefore$ equetion of line is $ax+by+c=0$

or, $\quad ax + \frac{a}{3}y - 4a = 0$

or, $\quad 3x + y - 12 = 0$

Q14

The slope of the given line is equal to the slope of line $3x - 4y + 6 = 0$ as the two lines are parallel to each other

$$\therefore \quad m_1 = m_2 = \frac{3}{4}$$

And the line passes through mid point of points $(2,3)$ and $(4,-1)$

i.e; $\quad \left(\frac{2+4}{2}, \frac{3-1}{2}\right) \qquad$ [Using mid point formula]

$\Rightarrow \quad (3,1)$

$\therefore$ using one point-slope equation of line

$$(y-1) = \frac{3}{4}(x-3)$$

$$4y - 4 = 3x - 9$$

$$3x - 4y = 5$$

Is the required line

Q15

In a paralleogram opposite sides are parallel and parallel sides have equal slope.

Slope of line $2x - 3y + 1 = 0$

$$m_1 = \frac{2}{3} \qquad ---(1)$$

Slope of line $x + y = 3$

$$m_2 = 1 \qquad ---(2)$$

Slope of line $2x - 3y - 2 = 0$

$$m_3 = \frac{2}{3} \qquad ---(3)$$

Slope of line $x + y = 4$

$$m_4 = -1 \qquad ---(4)$$

From $(1),(2),(3)$ and (4)

We observe that opposite sides of $ABCD$ have same slope and hence are parallel

Hence, proved, the given quadrilateral is a parallelogram

Q16

The required line is perpendicular to the given line $6x + 4y = 24$.

$\therefore$ $(\text{Slope of required line}) \times (\text{Slope of given line}) = -1$

$$m_1 = \frac{-1}{\left(\frac{-6}{4}\right)} = \frac{4}{6}$$

and

The required line passes through the point (x_1, y_1) where it meets the y-axis

$\therefore$ x coordinate at that point is zero. i.e; $x_1 = 0$

$$(y - y_1) = \frac{4}{6}(x - 0)$$

$$6y - 6y_1 = 4x$$

$$2x - 3y = -3y_1 \Rightarrow y_1 = 6$$

$$2x - 3y = -18$$

$$2x - 3y + 18 = 0$$

Q17

OP is perpendicular to the given line $y = mx + c$

$\therefore$ $(\text{Slope of } OP) \times (\text{Slope of line}) = -1$

$$\frac{2-0}{-1-0} \times m = -1$$

$$m = \frac{-1 \times -1}{2} = \frac{1}{2}$$

and $(-1, 2)$ lies on the line $y = \frac{1}{2} + c$

$\therefore$ $2 = \frac{1}{2}(-1) + c$

$$c = 2 + \frac{1}{2} = \frac{5}{2}$$

$\therefore$ $c = \frac{5}{2}$ and $m = \frac{1}{2}$

Q18

The slope of line joining $(3, 4)$ and $(-1, 2)$ is

$$\frac{2-4}{-1-3} = \frac{-2}{-4} = \frac{1}{2}$$

The required line is $\perp$ to the given line

$\therefore$ $(\text{Slope of required line}) \times \frac{1}{2} = -1$ $[\because m_1 \times m_2 = -1 \text{ for perpendicular lines}]$

$$m_1 = -2$$

And the line passes through the mid point of line joining $(3, 4)$ and $(-1, 2)$

i.e; $\left(\frac{3-1}{2}, \frac{4+2}{2}\right)$ or $(1, 3)$

$\therefore$ equation of the required line is

$$y - 3 = (-2)(x - 1)$$

or $y - 3 = -2x + 2$

or $2x + y - 5 = 0$

Q19

If two lines intersect at right angles, then product of their slope is -1.

Slope of $7x - 9y - 19 = 0$ is $m_1 = \frac{7}{9}$ $\quad ---(1)$

Slope of line joining $(h, 3)$ and $(4, 1) = \frac{1-3}{4-h}$

or, $\quad m_2 = \frac{2}{h-4}$ $\quad ---(2)$

$$m_1 \times m_2 = -1$$
$$\frac{7}{9} \times \frac{2}{h-4} = -1$$
$$14 = -9h + 36$$
$$9h = 36 - 14$$
$$h = \frac{22}{9}$$

Q20

Let the image of $P(3, 8)$ in $x + 3y = 7$ be $Q(\alpha, \beta)$.

Then,

PQ is perpendicularly bisected at R.

Then,

$$R = \left(\frac{\alpha+3}{2}, \ \frac{\beta+8}{2}\right)$$

and lie on $x + 3y = 7$

$$\frac{\alpha+3}{2} + \frac{3\beta+24}{2} = 7$$
$$\alpha + 3 + 3\beta + 24 = 14$$
$$\alpha + 3\beta = -13 \qquad ---(1)$$

And since PQ is perpendicular to

$$x + 3y = 7$$
$$(\text{Slope of line}) \times (\text{Slope of } PQ) = -1$$
$$\frac{-1}{3} \times \frac{\beta - 8}{\alpha - 3} = -1$$
$$\beta - 8 = 3\alpha - 9$$
$$\beta - 3\alpha = -1 \qquad ---(2)$$

Solving (1) and (2)

$$\beta = -4, \ \alpha = -1$$

$\therefore \quad Q$ is $(-1, -4)$

Q21

Let foot of perpendicular of $P\left(-1,3\right)$ on line $3x-4y=16$ be $Q\left(\alpha,\beta\right)$

Then,

$$\left(\text{Slope of line}\right)\times\left(\text{Slope of } PQ\right)=-1$$

$$\frac{3}{4}\times\frac{\beta-3}{\alpha+1}=-1$$

$$3\left(\beta-3\right)=-4\alpha-4$$

$$3\beta-9=-4\alpha-4$$

$$4\alpha+3\beta=5 \qquad ---(1)$$

α and β should lie on $3x-4y=16$

$\therefore \quad 3\alpha-4\beta=16 \qquad ---(2)$

From (1) and (2)

$$\alpha=\left(\frac{68}{25}\right) \qquad \beta=\left(\frac{-49}{25}\right)$$

$\therefore \quad Q$ is $\left(\frac{68}{25},\ \frac{-49}{25}\right)$

Q22

Let AB be the line, $A = (-1,2)$, $B = (5,4)$

Then, equation of line AB is

$$y - y_1 = m(x - x_1)$$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 2 = \frac{4-2}{5+1}(x+1)$$

$$y - 2 = \frac{2}{6}(x+1)$$

$$3y - x = 7 \qquad ---(1)$$

$$\text{Slope} = \frac{1}{3}.$$

Let P point $(1,0)$ be the given point

Let $Q(x_1, y_1)$ be the projection of P

$$\text{Slope of } PQ = -3 \qquad [PQ \perp AB,\ m_1m_2 = -1]$$

Eq of PQ,

$$y - 0 = -3(x-1)$$

$$y = -3x + 3 \qquad ---(2)$$

Solving (1) and (2)

$$3y - \left(\frac{y-3}{-3}\right) = 7$$

$$-9y - y + 3 = -21$$

$$-10y = -24$$

$$y = \frac{12}{5}$$

$$\Rightarrow \quad \frac{12}{5} = -3x + 3$$

$$-3x = +\frac{12}{5} - 3 = \frac{+12-15}{5} = \frac{-3}{5}$$

$$x = \frac{1}{5}$$

$$\therefore \quad N\left(\frac{1}{5}, \frac{12}{5}\right)$$

Q23

Any line perpendicular to line $\sqrt{3}x - y + 5 = 0$

Will have the slope $\frac{-1}{m}$

Where,

$$m \Rightarrow y = mx + c$$

$$y = \sqrt{3}x + 5$$

$$m = \sqrt{3}$$

Point is $(x_1 y_1) = (3, 3)$

$$y - y_1 = \frac{-1}{m}(x - x_1)$$

$$y - 3 = \frac{-1}{\sqrt{3}}(x - 3)$$

$$x + \sqrt{3}y + 6 = 0$$

Point can be $(-3, -3)$

Then, equation is

$$x + \sqrt{3}y - 6$$

$$\therefore \quad x + \sqrt{3}y \pm 6$$

Q24

The line $2x + 3y = 12$ meets the x-axis at A and y-axis at B

$\Rightarrow$ A is $2x = 12 = x = 6$

$\therefore$ A is $(6, 0)$

$\Rightarrow$ B is $3y = 12$

$y = 4$

$\therefore$ B is $(0, 4)$

Line through $(5, 5)$ perpendicular to $2x + 3y = 12$ will have slope $= \frac{3}{2}$

$$y - y_1 = m(x - x_1)$$

$$y - 5 = \frac{3}{2}(x - 5)$$

$2y - 3x = -5$ is eq of line which meets x-axis at C and the line at E

$\therefore$ C is $-3x = -5$

$$x = \frac{-5}{3}$$

$\therefore$ E is $\left(\frac{5}{3}, 0\right)$

$E \Rightarrow$ point of intersection of two lines

$$2x + 3y = 12$$

$$2y - 3x = -5$$

The area of $OBCE$ = are of AOB – area of ACE

$$\Rightarrow \frac{1}{2} \times AO \times OB - \frac{1}{2} \times AC \times CE$$

$$\Rightarrow \frac{24}{2} - \frac{1}{2} \times \sqrt{13} \times \frac{2}{3}\sqrt{13}$$

$$\Rightarrow \frac{24}{2} - \frac{1}{2} \times \frac{2}{3} \times 13$$

$$\Rightarrow 12 - \frac{13}{3}$$

$$\Rightarrow \frac{23}{3} \, sq\ units$$

Q25

The equation of line in intercept form is $\frac{x}{a}+\frac{y}{b}=1$

Intercept on y-axis = $2a$. (given)

$\therefore$ equation is

$$\frac{x}{2a}+\frac{y}{a}=1$$

$$ax+2ay=2a^2 \qquad ---(1)$$

Now, perpendicular distance of (1) from origin is given unity

$$\Rightarrow \quad \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}=1$$

$$a=a, b=2a, c=-2a^2, x_1=0, y_1=0$$

$$= \quad \frac{|a(0)+2a(0)-2a^2|}{\sqrt{(2a)^2+(a)^2}}=1$$

$$\Rightarrow \quad -2a^2=\sqrt{5}a$$

$$\Rightarrow \quad 4a^4=a^2 5$$

$$a^2=\frac{5}{4} \Rightarrow a=\pm\frac{\sqrt{5}}{4}$$

$\therefore$ the intercept form of straight line is

$$\frac{x}{2a}+\frac{y}{a}=1$$

$$\frac{x}{\pm\frac{2\sqrt{5}}{4}}+\frac{y}{\pm\frac{\sqrt{5}}{4}}=1$$

$$x+2y=\pm\sqrt{5}$$

$$x+2y\pm\sqrt{5}=0$$

Q26

Let (x_1, y_1) and (x_2, y_2) be the coordinates of B and C.

Perpendicular bisector of AB is $x - y + 5 = 0$

Its slope = 1

Coordinates of $F = \left(\frac{x_1 + 1}{2}, \frac{y_1 - 2}{2}\right)$

F lies on the $x - y + 5 = 0$

$$\Rightarrow \quad \frac{x_1 + 1}{2} - \frac{y_1 - 2}{2} + 5 = 0$$

$$\Rightarrow \quad x_1 + 1 - y_1 + 2 + 10 = 0$$

$$x_1 - y_1 + 13 = 0 \qquad ---(1)$$

AB is perpendicular to HF

$(\text{Slope of } AB)(\text{Slope of } HF) = -1$

$$\left(\frac{y_1 + 2}{x_1 - 1}\right)(1) = -1$$

$$x_1 + y_1 + 1 = 0 \qquad ---(2)$$

Solving equation (1) and (2),

$x_1 = -7$, $y_1 = 6$

Thus, B is $(-7, 6)$

Now, perpendicular bisector of AC is

$x + 2y = 0$

Slope of this is $= -\frac{1}{2}$

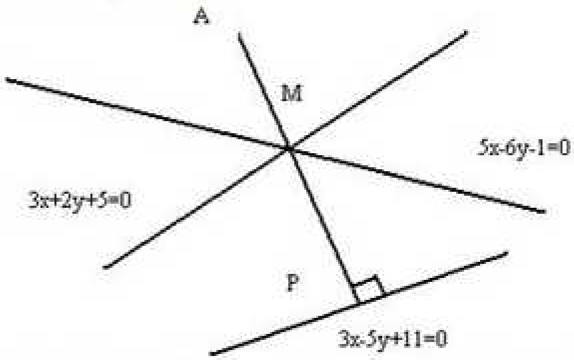
Mid-point of $ACE = \left(\frac{x_2 + 1}{2}, \frac{y_2 - 2}{2}\right)$

E lies on perpendicular bisector of AC

$$\Rightarrow \quad \left(\frac{x_2 + 1}{2}\right) + 2\left(\frac{y_2 - 2}{2}\right) = 0$$

Q27

Let M be the point o f intersection of the lines 5x-6y-1=0 and 3x+2y+5=0.



Solving the equations 5x-6y-1=0 and 3x+2y+5=0, we get the point of intersection as M (-1,-1).

$$3x-5y+11=0$$

Also, $\Rightarrow 5y=3x+11$

$$\Rightarrow y=\frac{3}{5}x+\frac{11}{5}$$

Therefore, slope = 3/5, Slope of AP = -5/3

Equation of AP, $y-y_1=m(x-x_1)$

$$y+1=-\frac{5}{3}(x+1)$$

$$3y+3=-5x-5$$

$$5x+3y+8=0$$

Therefore equation of the line AP, 5x+3y+8=0

Ex 23.13

Q1(i)

Writing the equation in the form

$$y = mx + c$$

$$3x + y + 12 = 0$$

$$y = -3x - 12$$

$\Rightarrow \quad m_1 = -3$

Also

$$x + 2y - 1 = 0$$

$$2y = 1 - x$$

$$y = \frac{1}{2} - \frac{x}{2}$$

$\Rightarrow \quad m_2 = \frac{-1}{2}$

Angle between the lines

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \left|\frac{-3 - \left(\frac{-1}{2}\right)}{1 + (-3)\left(\frac{-1}{2}\right)}\right|$$

$$= \left|\frac{-3 + \frac{1}{2}}{1 + \frac{3}{2}}\right| = \left|\frac{\frac{-6 + 12}{2}}{\frac{2 + 3}{2}}\right|$$

$$= \left|\frac{-5}{5}\right| = 1$$

$\Rightarrow \quad \text{angle} = \frac{\pi}{4}$

Q1(ii)

Finding slopes of the lines by converting the equation in the form

$$y = mx + c$$

$$3x - y + 5 = 0$$

$$\Rightarrow \quad y = 3x + 5$$

$$\Rightarrow \quad m_1 = 3$$

Also

$$x - 3y + 1 = 0$$

$$3y = x + 1$$

$$y = \frac{x}{3} + \frac{1}{3}$$

$$\Rightarrow \quad m_2 = \frac{1}{3}$$

Thus angle between the lines is

$$\tan\theta = \left|\frac{m_1 - m_2}{m_1 m_2}\right|$$

$$= \quad \left|\frac{3 - \frac{1}{3}}{1 + 3 \times \frac{1}{3}}\right| = \left|\frac{\frac{9-1}{3}}{1+1}\right|$$

$$= \quad \left|\frac{\frac{8}{3}}{2}\right| = \left|\frac{8}{6}\right| = \frac{4}{3}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{4}{3}\right)$$

Q1(iii)

To find angle between the lines, convert the equations in the form

$y = mx + c$

$3x + 4y - 7 = 0$

$\Rightarrow \quad 4y = -3x + 7$

$y = \frac{-3}{4}x + \frac{7}{4}$

$\Rightarrow \quad m_1 = \frac{-3}{4}$

Also, $4x - 3y + 5 = 0$

$\Rightarrow \quad 3y = 4x + 5$

$\Rightarrow \quad y = \frac{4}{3}x + \frac{5}{3}$

$\Rightarrow \quad m_1 = \frac{4}{3}$

The angle between the lines is given by $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$$= \left|\frac{\frac{-3}{4} - \frac{4}{3}}{1 + \frac{(-3)}{4}\left(\frac{4}{3}\right)}\right| = \left|\frac{\frac{-3}{4} - \frac{4}{3}}{1 - 1}\right|$$

$\Rightarrow \quad \theta = \frac{\pi}{2}$ or 90^0

Q1(iv)

To find angle convert the equation in the form $y = mx + c$

$$x - 4y = 3$$

$$\Rightarrow \quad 4y = x - 3$$

$$y = \frac{x}{4} - \frac{3}{4}$$

$$\Rightarrow \quad m_1 = \frac{1}{4}$$

Also, $6x - y = 11$

$$y = 6x - 11$$

$$\Rightarrow \quad m_2 = 6$$

Thus, angle between the lines is

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \left|\frac{\frac{1}{4} - 6}{1 + \frac{1}{4} \times 6}\right|$$

$$= \left|\frac{\frac{-23}{4}}{1 + \frac{3}{2}}\right| = \left|\frac{\frac{-23}{4}}{\frac{5}{2}}\right|$$

$$\theta = \tan^{-1}\left(\frac{23}{10}\right)$$

Q1(v)

Converting the equation in the form

$$y = mx + c$$

$$y = \frac{(mn + n^2)}{m^2 - mn}x + \frac{n^3}{(m^2 - mn)}$$

$$\Rightarrow \quad m_1 = \frac{mn + n^2}{m^2 - mn}$$

Also, $y = \dfrac{(mn - n^2)}{nm + m^2}x + \dfrac{m^3}{nm + m^2}$

$$\Rightarrow \quad m_2 = \frac{mn - n^2}{nm + m^2}$$

Thus, angle between 2 lines is $\tan\theta$

$$\Rightarrow \quad \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \left|\frac{\left(\dfrac{mn + n^2}{m^2 - mn}\right) - \left(\dfrac{mn - n^2}{nm + m^2}\right)}{1 + \left(\dfrac{mn + n^2}{m^2 - mn}\right)\left(\dfrac{mn - n^2}{nm + m^2}\right)}\right|$$

$$= \left|\frac{m^2n^2 + m^3n + n^3m + n^2m^2 - m^3n + m^2n^2 + n^2m^2 - mn^3}{m^3n + m^4 - m^2n^2 - m^3n + m^2n^2 - mn^3 + mn^3 - n^4}\right|$$

$$= \left|\frac{4m^2n^2}{m^4 - n^4}\right|$$

$$\Rightarrow \quad \theta = \tan^{-1}\left|\frac{4m^2n^2}{m^4 - n^4}\right|$$

Q2

Slope of line $2x - y + 3 = 0$

$$\text{is } \frac{-2}{-1} = \frac{(\text{coefficient of } x)}{(\text{coefficient of } y)} = 2$$

$$\therefore \quad m_1 = 2 \qquad \text{---(i)}$$

Slope of line $x + y + 2 = 0$

$$\text{is } \frac{-1}{1} = \frac{-(\text{coefficient of } x)}{(\text{coefficient of } y)}$$

$$\therefore \quad m_2 = -1 \qquad \text{---(ii)}$$

Acute angle between lines

$$\theta = \tan^{-1}\left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \tan^{-1}\left|\frac{2 - (-1)}{1 - (2)(-1)}\right|$$

$$= \tan^{-1}\left|\frac{3}{1-2}\right| = \tan^{-1}\left|\frac{3}{1}\right| = \tan^{-1}|3|$$

Q3

Let $ABCD$ be a quadrilateral

$$AB = \sqrt{(0-2)^2 + (2+1)^2}$$

Using distance formula

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{2^2 + 3^2} = \sqrt{4+9} = \sqrt{13}$$

$$BC = \sqrt{(2-0)^2 + (3-2)^2} = \sqrt{4+1} = \sqrt{5}$$

$$CD = \sqrt{(4-2)^2 + (0-3)^2} = \sqrt{4+9} = \sqrt{13}$$

$$DA = \sqrt{(4-2)^2 + (0+1)^2} = \sqrt{4+1} = \sqrt{5}$$

Since opposite sides $(AB \text{ and } CD)$ and $(BC \text{ and } DA)$ are equal

$\therefore$ The given quadrilateral is a parallelogram.

Q4

The equation between the points

$$\underset{(x_1,y_1)}{(2,0)} \text{ and } \underset{(x_2,y_2)}{(0,3)}$$

Slope of line $= \frac{y_2 - y_1}{x_2 - x_1}$

$$m_1 = \frac{3-0}{0-2} = \frac{-3}{2}$$

Also, slope of line $x + y = 1$

Converting in the form $y = mx + c$

$$y = 1 - x$$

$\Rightarrow \quad m_2 = -1$

Thus, $\tan\theta$ = angle between the lines

$$\Rightarrow \quad \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \left|\frac{\frac{-3}{2} - (-1)}{1 + \left(\frac{-3}{2}\right)(-1)}\right| = \left|\frac{\frac{-3}{2} + 1}{1 + \frac{3}{2}}\right|$$

$$= \left|\frac{\frac{-3+2}{2}}{\frac{2+3}{2}}\right| = \left|\frac{\frac{-1}{2}}{\frac{5}{2}}\right| = \frac{1}{5}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{1}{5}\right)$$

Q5

Let l_1, be the line joining AO and
Let l_2 be the line joining BO

Then, line l_1 is $y - 0 = \left(\frac{0 - x_1}{0 - y_1}\right)(x - 0)$

$$yy_1 - x_1x = 0$$

Then, $m_1 = \frac{x_1}{y_1}$

Then line l1 is $y - 0 = \left(\frac{0 - x_2}{0 - y_2}\right)(x - 0)$

Then, $m_2 = \frac{x_2}{y_2}$

Then, $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right| = \left|\frac{\frac{x_1}{y_1} - \frac{x_2}{y_2}}{1 + \frac{x_1}{y_1}\frac{x_2}{y_2}}\right|$

$$= \left|\frac{x_1y_2 - y_1x_2}{y_1y_2 + x_1x_2}\right|$$

From triangle,

$$AC = \sqrt{(AB)^2 + (BC)^2}$$

$$= \sqrt{\left(m_1^2 + m_2^2 - 2m_1m_2\right) + \left(1 + m_1m_2\right)^2}$$

$$= \sqrt{m_1^2 + m_2^2 - 2m_1m_2 + 1 + m_1^2m_2^2 + 2m_1m_2}$$

$$= \sqrt{m_1^2 + m_2^2 + 1 + m_1^2m_2^2}$$

$$\cos\theta = \frac{BC}{AC} = \frac{1 + m_1m_2}{\sqrt{m_1^2 + m_2^2 + m_1^2m_2^2 + 1}}$$

$$= \frac{1 + \frac{x_1}{y_1}\frac{x_2}{y_2}}{\sqrt{\frac{x_1^2}{y_1^2} + \frac{x_2^2}{y_2^2} + \frac{x_1^2x_2^2}{y_1^2y_2^2} + 1}}$$

$$= \frac{\frac{y_1y_2 + x_1x_2}{y_1y_2}}{\sqrt{\frac{x_1^2y_2^2 + x_2^2y_1^2 + x_1^2x_2^2 + y_1^2y_2^2}{y_1^2y_2^2}}}$$

$$= \frac{y_1y_2 + x_1x_2}{\sqrt{x_1^2\left(y_2^2 + x_2^2\right) + y_1^2\left(y_2^2 + x_2^2\right)}}$$

$$= \frac{y_1y_2 + x_1x_2}{\sqrt{x_1^2 + y_1^2}\sqrt{y_2^2 + x_2^2}}$$

Hence proved.

Q6

$$(a+b)x+(a-b)y=2ab \qquad \text{---(i)}$$
$$(a-b)x+(a+b)y=2ab \qquad \text{---(ii)}$$
$$x+y=0 \qquad \text{---(iii)}$$

Converting all the equation in the form

$$y=mx+c$$
$$y=\frac{-(a+b)x}{a-b}+\frac{2ab}{a+b}$$
$$\Rightarrow \quad m_1=\frac{-(a+b)}{a-b}$$

$$y=\frac{-(a-b)x}{a+b}+\frac{2ab}{a+b}$$
$$\Rightarrow \quad m_2=\frac{-(a-b)}{a+b}$$

$$y=-x$$
$$\Rightarrow \quad m_3=-1$$

Thus angle between (i) and (ii)

$$\tan\theta_1=\left|\frac{m_1-m_2}{1+m_1m_2}\right|$$
$$=\left|\frac{-\left(\frac{a+b}{a-b}\right)+\left(\frac{a-b}{a+b}\right)}{1+\left(\frac{a+b}{a-b}\times\frac{a-b}{a+b}\right)}\right|$$

$$=\frac{2ab}{b^2-a^2}$$
$$=\frac{2\frac{a}{b}}{1-\left(\frac{a}{b}\right)^2}$$
$$Tan\,\theta_1=Tan\left(2Tan^{-1}\left(\frac{a}{b}\right)\right)$$

Q7

$$x = a$$

$$\Rightarrow \quad m_1 = \frac{1}{0}$$

$$by + c = 0$$

$$y = \frac{-c}{b}$$

$$m_2 = 0$$

Comparing with $y = mx + c$

Then, putting in

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \left|\frac{\frac{1}{0} - 0}{1 + \frac{1}{0} \times 0}\right|$$

$$= \frac{1}{0} = \infty$$

$$\Rightarrow \quad \theta = 90°$$

Q8

Line$_1$ is $\frac{x}{3} + \frac{y}{4} = 1$

i.e $\quad 4x + 3y = 12$

Line$_2$ is $\frac{x}{1} + \frac{y}{8} = 1$

i.e $\quad 8x + y = 8$

Slope of line$_1$ and line$_2$ is $\frac{-4}{3}$ and $\frac{-8}{1}$ respectively.

Thus, $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$$= \left|\frac{\frac{-4}{3} - (-8)}{1 + \left(\frac{-4}{3}\right)(-8)}\right|$$

$$= \left|\frac{\frac{-4}{3} + 8}{1 + \frac{32}{3}}\right| = \left|\frac{-4 + 24}{3 + 32}\right|$$

$$= \left|\frac{20}{35}\right| = \frac{4}{7}$$

Thus, $\tan\theta = \frac{4}{7}$.

Q9

Slope of line through the points $\underset{(x_1,\ y_1)}{(a,\ 2a)}$, $\underset{(x_2,\ y_2)}{(-2,\ 3)}$

$$\Rightarrow \quad m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 2a}{-2 - a}$$

Also, slope of line $x - ay = 1$ in the form $y = mx + c$

$$4x + 3y + 5 = 0$$

$$y = \frac{-4}{3}x - \frac{5}{3}$$

$$\Rightarrow \quad m_2 = \frac{-4}{3}$$

If two lines are perpendicular then, $m_1 m_2 = -1$

$$\left(\frac{3 - 2a}{-2 - a}\right)\left(\frac{-4}{3}\right) = -1$$

$$-12 + 8a = 6 + 3a$$

$$5a = 18$$

$$a = \frac{18}{5}$$

Q10

$a^2x + ay + 1 = 0$

$x - ay = 1$

Converting these two equations in the form $y = mx + c$

$$y = -\frac{a^2}{a}x - \frac{1}{a} = -ax - \frac{1}{a}$$

$$\Rightarrow \quad m_1 = -a$$

Also, $y = \dfrac{x}{a} - \dfrac{1}{a}$

$$\Rightarrow \quad m_2 = \frac{1}{a}$$

Thus, $m_1 m_2 = -a \times \dfrac{1}{a} = -1$

The two lines are perpendicular as the product of slopes is -1.

Q11

Let the line $\frac{x}{a}+\frac{y}{b}=1$ be AB and the line $\frac{x}{a}-\frac{y}{b}=1$ be CD.

Equation of AB, $\frac{bx+ay}{ab}=1$

$\Rightarrow ay=-bx+ab$

$\Rightarrow y=-\frac{bx}{a}+b$

Therefore $m_1=-\frac{b}{a}$

Similarly, the equation of CD, $\frac{bx-ay}{ab}=1$

$\Rightarrow bx-ay=ab$

$\Rightarrow ay=\frac{bx}{a}-a$

Therefore, $m_2=\frac{b}{a}$

The tangent of angle between the lines AB and CD is

$$\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|=\left|\frac{-\frac{b}{a}-\frac{b}{a}}{1+\left(-\frac{b}{a}\right)\left(\frac{b}{a}\right)}\right|=\left|\frac{-\frac{2b}{a}}{\frac{a^2-b^2}{a^2}}\right|=\left|\frac{-2ab}{a^2-b^2}\right|=\frac{2ab}{a^2-b^2}$$

The tangent of the angle between the lines $=\frac{2ab}{a^2-b^2}$

Ex 23.14

Q1

Let ABC be the triangle of the equations whose sides AB, BC and CA are respectively
$x - 5y + 6 = 0$, $x - 3y + 2 = 0$ and $x - 2y - 3 = 0$

The coordinates of the vertices are $A(9,3)$, $B(4,2)$ and $C(13,5)$.

If the point $P(\alpha, \alpha^2)$ lies n side the ΔABC, then

(i) A and P must be on the same side of BC.

(ii) B and P must be on the same side of AC.

(iii) C and P must be on the same side of AB.

Now,

A and P are on the same side of BC if,

$$\left(9(1) + 3(-3) + 2\right)\left(\alpha^2 - 3\alpha + 2\right) > 0$$
$$\left(9 - 9 + 2\right)\left(\alpha^2 - 3\alpha + 2\right) > 0$$
$$\alpha^2 - 3\alpha + 2 > 0$$
$$(\alpha - 1)(\alpha - 2) > 0$$
$$\alpha \in (-\infty, 1) \ v\ (2, \infty) \qquad \text{---(i)}$$

B and P will lie on the same side of CA if,

$$\left(13(1) + 5(-5) + 6\right)\left(\alpha^2 - 5\alpha + 6\right) > 0$$
$$\Rightarrow \quad (-6)\left(\alpha^2 - 5\alpha + 6\right) > 0$$
$$\Rightarrow \quad \alpha^2 - 5\alpha + 6 < 0$$
$$\Rightarrow \quad (\alpha - 2)(\alpha - 3) < 0$$
$$\Rightarrow \quad \alpha \in (2, 3) \qquad \text{---(ii)}$$

C and P will lie on the same side of AB if,

$$\left(4(1) + 2(-2) - 3\right)\left(\alpha^2 - 2\alpha - 3\right) > 0$$
$$(-3)\left(\alpha^2 - 2\alpha - 3\right) > 0$$
$$\alpha^2 - 2\alpha - 3 < 0$$
$$(\alpha - 3)(\alpha + 1) < 0$$
$$\alpha \in (-1, 3) \qquad \text{---(iii)}$$

From i, ii, iii

$$\alpha \in [2, 3]$$

Q2

Let ABC be the triangle. The coordinates of the vertices of the triangle ABC are marked in the following figure.

Point $P(a,2)$ lie inside or on the triangle if.

(i) A and P lie on the same side of BC.

(ii) B and P lie on the same side of AC.

(iii) C and P lie on the same side of AB.

A and P will lie on the same side of BC if.

$$(7(3)-7(-3)-0)(3a-7(2)-0)>0$$
$$(21+21-8)(3a-14-8)>0$$
$$3a-22>0$$
$$a>\frac{22}{3} \qquad ---(i)$$

B and P will lie on the same side of AC if.

$$\left(4\left(\frac{18}{5}\right)-\left(\frac{2}{5}\right)-31\right)(4a-2-31)>0$$
$$4a-33>0$$
$$a>\frac{33}{4} \qquad ---(ii)$$

C and P will lie on the same side of BC if.

$$\left(\frac{209}{25}+\frac{61}{25}-4\right)(a+2-4)>0$$
$$a+2>0$$
$$a>-2 \qquad ---(iii)$$

From (, (ii), (iii)

$$? \in \left(\frac{22}{3},\ \frac{33}{4}\right)$$

Q3

Let ABC be the triangle, then coordinates of the vertices are marked in the following figure.

$P(-3,2)$ lie inside if.

(i) A and P, B and P, C and P lie on the same side of BC, AC and BA respectively.

If A and P lie on the same side of BC then,

$$(3(7)-7(-3)+8)(3(-3)-7(2)+8)>0$$
$$(21+21+8)(-9-14+8)>0$$

But, $(50)(-15)$ is not >0

$\therefore$ The point $(-3,2)$ is outside ABC.

Ex 23.15

Q1

Distance of a point (x_1, y_1) from $ax + by + c = 0$ is

$$= \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$$

Here, $a = 3,\ b = -5,\ c = 7,\ x_1 = 4,\ y_1 = 5$

$$\therefore \text{ Distance} = \left|\frac{3(4) - 5(5) + 7}{\sqrt{3^2 + 5^2}}\right|$$

$$= \left|\frac{12 - 25 + 7}{\sqrt{9 + 25}}\right| = \left|\frac{6}{\sqrt{34}}\right| \text{ units.}$$

Q2

Equation of line passing through $(\cos\theta, \sin\theta)$ and $(\cos\phi, \sin\phi)$ is

$$y - \sin\phi = \left(\frac{\sin\phi - \sin\theta}{\cos\phi - \cos\theta}\right)(x - \cos\phi)$$

$$y - \sin\phi = \left(\frac{2\cos\frac{\theta+\phi}{2}\sin\frac{\phi-\theta}{2}}{-2\sin\frac{\theta+\phi}{2}\sin\frac{\phi-\theta}{2}}\right)(x - \cos\phi)$$

$$y - \sin\phi = -\cot\left(\frac{\theta+\phi}{2}\right)(x - \cos\phi)$$

$$x\cot\left(\frac{\theta+\phi}{2}\right) + y - \sin\phi - \cos\phi\cot\left(\frac{\theta+\phi}{2}\right) = 0$$

Distance of this line from origin,

$$= \left|\frac{ax_1 + by_1 + c}{a^2 + b^2}\right|$$

$$= \left|\frac{0 + 0 - \sin\phi - \cos\phi\cot\left(\frac{\theta+\phi}{2}\right)}{\sqrt{\left(\cos\left(\frac{\theta+\phi}{2}\right)\right)^2 + 1}}\right|$$

$$= \frac{\sin\phi + \cos\phi\cot\left(\frac{\theta+\phi}{2}\right)}{\operatorname{cosec}\left(\frac{\theta+\phi}{2}\right)}$$

$$= \sin\phi\sin\left(\frac{\theta+\phi}{2}\right) + \cos\phi\operatorname{co}\left(\frac{\theta+\phi}{2}\right)$$

$$= \cos\left(\frac{\theta+\phi}{2} - \phi\right)$$

$$= \cos\left(\frac{\theta+\phi-2\phi}{2}\right)$$

$$D = \cos\left(\frac{\theta-\phi}{2}\right)$$

Q3

Line formed from joining $(a\cos\alpha, a\sin\alpha)$ and $(a\cos\beta, a\sin\beta)$

$$\Rightarrow \quad y - a\sin\beta = \frac{a\sin\beta - a\sin\alpha}{a\cos\beta - a\cos\alpha} \times x - a\cos\beta$$

$$\Rightarrow \quad y - a\sin\beta = \frac{2\sin\left(\frac{\beta-\alpha}{2}\right)\cos\left(\frac{\beta+\alpha}{2}\right)}{-2\sin\left(\frac{\beta-\alpha}{2}\right)\sin\left(\frac{\beta+\alpha}{2}\right)} \times (x - a\cos\beta)$$

$$\Rightarrow \quad y - a\sin\beta = -\cot\left(\frac{\beta+\alpha}{2}\right)(x - a\cos\beta)$$

$$\Rightarrow \quad y + \cot\left(\frac{\alpha+\beta}{2}\right)x - a\cos\beta\cot\left(\frac{\beta+\alpha}{2}\right) - a\sin\beta = 0$$

Then, the length of perpendicular

$$\Rightarrow \quad \left|\frac{0(y) + 0 - a\cos\beta\cot\left(\frac{\beta+\alpha}{2}\right) - a\sin\beta}{\sqrt{1+\cot^2\left(\frac{\alpha+\beta}{2}\right)}}\right|$$

$$\Rightarrow \quad \frac{a\cos\beta\cot\left(\frac{\alpha+\beta}{2}\right) + a\sin\beta}{\operatorname{cosec}\left(\frac{\alpha+\beta}{2}\right)}$$

$$\Rightarrow \quad a\cos\beta\cos\left(\frac{\alpha+\beta}{2}\right) + a\sin\beta\sin\left(\frac{\alpha+\beta}{2}\right)$$

$$\Rightarrow \quad a\cos\left(\frac{\alpha-\beta}{2}\right) \qquad [\text{using } \cos A\cos B + \sin A\sin B = \cos(A-B)]$$

Hence, proved.

Q4

Let (h,k) be the point on the line $2x + 11y - 5 = 0$

$\Rightarrow \quad 2h + 11k - 5 = 0 ------(1)$

Let p and q be length of perpendicular from (h,k) on lines $24x + 7y - 20 = 0$ and $4x - 3y - 2 = 0$ so,

$$p = q$$

$$\frac{24h + 7k - 20}{\sqrt{(24)^2 + (7)^2}} = \frac{4h - 3k - 2}{\sqrt{(4)^2 + (-3)^2}}$$

$$\frac{24h + 7k - 20}{\sqrt{576 + 49}} = \frac{4h - 3k - 2}{\sqrt{25}}$$

$$\frac{24h + 7k - 20}{25} = \frac{4h - 3k - 2}{5}$$

$$24h + 7k - 20 = 20h - 15k - 10$$

$$4h = -22k + 10$$

$$4\left(\frac{5 - 11k}{2}\right) = -22k + 10 \qquad [\text{Using equation (1)}]$$

$$10 - 22k = -22k - 10$$

$$LHS = RHS$$

So,

Distance $24x + 7y - 20$ and $4x - 3y - 2 = 0$ from any point on the line $2x + 11y - 5 = 0$ is equal.

Q5

The point of intersection of two lines can be calculated by solving the equations

Solving $2x + 3y = 21$ and $3x - 4y + 11 = 0$, we get the point of intersection as $P(3, -5)$

Distance of P from $8x - 6y + 5 = 0$ is

Here, $a = 8$, $b = -6$, $c = 5$, $x_1 = 3$, $y_1 = 5$

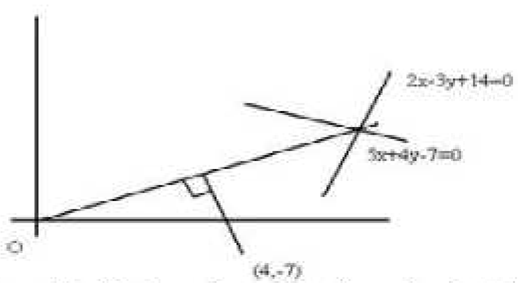
$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$$\Rightarrow \quad \frac{|8(3) - 6(-5) + 5|}{\sqrt{64 + 36}}$$

$$\Rightarrow \quad \frac{|24 + 30 + 5|}{\sqrt{100}} = \frac{|59|}{10}$$

$$\Rightarrow \quad \frac{59}{10}$$

Q6



The point of intersection of the lines $2x - 3y + 14 = 0$ and $5x + 4y - 7 = 0$ can be found out by solving these equations.

Solving these equations we get, $x = -\frac{35}{23}$ and $y = \frac{252}{69}$

Equation of line joining origin and the point $\left(-\frac{35}{23}, \frac{252}{69}\right)$

is $y = mx$, where $m = \frac{\frac{252}{69}}{-\frac{35}{23}} = -\frac{12}{5}$

Therefore the equation of required line is $y = -\frac{12x}{5}$

$12x + 5y = 0$

Perpendicular distance from (4,-7) to $12x + 5y = 0$ is

$$p = \left|\frac{12(4) + 5(-7)}{\sqrt{12^2 + (-5)^2}}\right| = \frac{13}{13} = 1$$

Q7

Any point on x-axis is $\underset{(x_1, y_1)}{(\pm a, 0)}$

Perpendicular distance from a line $bx + ay = ab$ is

$$\left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right| = a$$

Where,

$$a = b,\ b = a,\ c = -ab,\ x_1 = \pm a,\ y_1 = 0$$

$$= \left|\frac{b(x) + a(0) - ab}{\sqrt{a^2 + b^2}}\right| = a$$

$$a = 0 \quad \text{or}$$

$$\frac{b(x) + a(0) - ab}{\sqrt{a^2 + b^2}} = a$$

$$\frac{b}{a}x = \pm\sqrt{a^2 + b^2} + b$$

$$x = \frac{a}{b}\left(b \pm \sqrt{a^2 + b^2}\right)$$

$$x = 0$$

Q8

Perpendicular distance from $\left(\sqrt{a^2-b^2},0\right)$ to $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta-1=0$

$$\left|\frac{\frac{\sqrt{a^2-b^2}}{a}\cos\theta+\frac{0x}{b}\sin\theta-1}{\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}}\right|$$

$$=\frac{\frac{\sqrt{a^2-b^2}}{a}\cos\theta-1}{\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}} \qquad \text{---(i)}$$

Also, perpendicular distance from $\left(-\sqrt{a^2-b^2},0\right)$ to $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta-1=0$

$$\left|\frac{\frac{-\sqrt{a^2-b^2}}{a}\cos\theta+0-1}{\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}}\right| \qquad \text{---(ii)}$$

$(i)\times(ii)$

$$\frac{\left(\frac{a^2-b^2}{a^2}\right)\cos^2\theta-1}{\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}}=b^2$$

Q9

The perpendicular of $(1,2)$ on the straight line $x-\sqrt{3}y=-4$
Then, the equation is

$$y-y_1=m'(x-x_1)$$

$$x_1=1,\ y_1=2,\ m=\frac{1}{\sqrt{3}},\ m'=-\sqrt{3},$$

$$y-2=-\sqrt{3}(x-1)$$

$$y+\sqrt{3}x-\left(2+\sqrt{3}\right)=0 \qquad \text{——(i)}$$

The perpendicular distance from $(0,0)$ to (i) is

$$\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$$

$$a=\sqrt{3},\ b=1,\ c=-\left(2+\sqrt{3}\right)$$

$$x_1=0,\ y_1=0$$

$$=\frac{\left|\sqrt{3}(0)+1(0)+\left(-2-\sqrt{3}\right)\right|}{\sqrt{\left(\sqrt{3}\right)^2+(1)^2}}=\frac{2+\sqrt{3}}{2}$$

Q10

On solving $x + 2y = 5$ and $x - 3y = 7$ we get a point $A\left(\frac{29}{5}, \frac{-2}{5}\right)$

The line passing through $A\left(\frac{29}{5}, \frac{-2}{5}\right)$ and slope 5 is

$$y + \frac{2}{5} = 5\left(x - \frac{29}{5}\right)$$
$$5y + 2 = 25x - 145$$
$$25x - 5y - 147 = 0$$

The distance of $(1,2)$ from $25x - 5y - 147 = 0$ is

$$\Rightarrow \quad \left|\frac{25(1) - 5(2) - 147}{\sqrt{25^2 + 5^2}}\right| \qquad \text{[using distance formula]}$$

$$\Rightarrow \quad \left|\frac{-132}{\sqrt{650}}\right|$$

$$\Rightarrow \quad \left|\frac{132}{\sqrt{650}}\right|$$

Q11

Let the required point be $(0,a)$

Given, distance of $(0,a)$ from line $4x + 3y - 12 = 0$ is 4 units.

$$d = \left|\frac{ax_1 + by + c}{\sqrt{a^2 + b^2}}\right|$$
$$4 = \left|\frac{4(0) + 3(a) - 12}{\sqrt{4^2 + 3^2}}\right|$$
$$4 = \left|\frac{3a - 12}{5}\right|$$
$$\Rightarrow \quad 4 = \frac{-3a + 12}{5}$$
$$\Rightarrow \quad -3a = 20 - 12$$
$$a = -\frac{8}{3}$$

And $4 = \frac{3a - 12}{5}$

$$3a = 20 + 12$$
$$\Rightarrow \quad a = \frac{32}{3}$$

So, Required points are

$$\left(0, \frac{32}{3}\right), \left(0, \frac{-8}{5}\right)$$

Q12

The equation of BC is

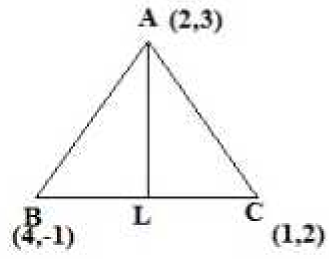
$$y+1=\frac{2+1}{1-4}(x-4)$$

$$y+1=-x+4$$

$$x+y-3=0$$

$$AL=\left|\frac{2+3-3}{\sqrt{1+1}}\right|$$

$$=\sqrt{2}$$



Clearly, slope of BC having equation $x+y-3=0$ is -1.

So, slope of AL is 1. As it passes through $A(2,3)$ so, its equation is

$y-3=1(x-2)$ or $x-y+1=0$

Q13

Let $P(h,k)$ be a moving point such that it is equidistant from the lines $3x-2y-5=0$ and $3x+2y-5=0$, then

$$\left|\frac{3h-2k-5}{\sqrt{9+4}}\right|=\left|\frac{3h+2k-5}{\sqrt{9+4}}\right|$$

$$|3h-2k-5|=|3h+2k-5|$$

$4k=0$ or $6h-10=0$

$k=0$ $\quad$ $3h=5$

Hence, the locus of (h,k) is $y=0$ or $3x=5$, which are straight lines.

Q14

It is given that the sum of the perpendicular distances of a variable point $P(x,y)$ from the lines $(x+y-5)=0$ and $3x-2y+7=0$ is always 10.

Therefore, $\dfrac{x+y-5}{\sqrt{2}}+\dfrac{3x-2y+7}{\sqrt{13}}=10$

$$\left(3\sqrt{2}+\sqrt{13}\right)x+\left(\sqrt{13}-2\sqrt{2}\right)y+\left(7\sqrt{2}-5\sqrt{13}-10\sqrt{26}\right)=0$$

Clearly, it is a straight line.

Q15

Length of perpendicular from $(1,1)$ to $ax - by + c = 0$

$$\Rightarrow \quad \left|\frac{a(1) - b(1) + c}{\sqrt{a^2 + b^2}}\right| = 1$$

$$a - b + c = \sqrt{a^2 + b^2}$$

$$(a - b + c)^2 = a^2 + b^2$$

$$a^2 + b^2 + c^2 + 2ac - 2bc - 2ab = a^2 + b^2$$

$$c^2 + 2ac - 2bc = 2ab$$

$$c + 2a - 2b = \frac{2ab}{c}$$

$$\frac{c}{2ab} + \frac{2a}{2ab} - \frac{2b}{2ab} = \frac{1}{c}$$

$$\frac{c}{2ab} = \frac{1}{c} + \frac{1}{a} - \frac{1}{b}$$

Hence, proved.

Ex 23.16

Q1

Determine between parallel lines

$ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is

$$\left|\frac{c_2 - c_1}{\sqrt{a^2 + b^2}}\right|$$

(i) $4x - 3y - 9 = 0$ and $4x - 3y - 24 = 0$

Distance between the two parallel lines is

$$\left|\frac{-24 - (-9)}{\sqrt{4^2 + 3^2}}\right| = \left|\frac{-24 + 9}{5}\right|$$

$= 3$ units.

(ii) Distance between $8x + 15y - 34 = 0$ and $8x + 15y + 31 = 0$

is $\left|\frac{-34 - 31}{\sqrt{8^2 + 15^2}}\right| = \frac{65}{17}$ units

(iii) Distance between $y = mx + c$ and $y = mx + d$

is $\left|\frac{c - d}{\sqrt{m^2 + 1}}\right|$

(iv) Distance between $4x + 3y - 11 = 0$ and $8x + 6y = 15$

is $\left|\frac{-11 - 15}{\sqrt{4^2 + 3^2}}\right| = \frac{7}{10}$ units.

Q2

The two sides of square are

$5x - 12y - 65 = 0$ and $5x - 12y + 26 = 0$

The distance between these two parallel sides $\left(\text{as both have slope } \frac{5}{12}\right)$ is

$$\left|\frac{-65 - 26}{\sqrt{5^2 + 12^2}}\right| = \left|\frac{-91}{13}\right| = 7 \text{ units.}$$

And all sides of square are equal.

$\therefore$ Area of the square is $7 \times 7 = 49$ sq units.

Q3

Let the required equation be $y = mx + c$ where m is slope of the line which is equal to slope of $x + 7y + 2 = 0\left(\text{i.e} \frac{-1}{7}\right)$ as the two lines are parallel.

$\therefore$ The required equation is $y = \frac{-1}{7}x + c$ which is a unit distance from $(1,1)$.

$$\therefore \quad \left|\frac{7(1) + (1) - 7c}{\sqrt{49+1}}\right| = 1$$

$$8 - 7c = \sqrt{50}$$

$$64 + 49c^2 - 112c = 50$$

$$49c^2 - 112C - 14 = 0$$

$$7c^2 - 16c - 2 = 0$$

$$C = \frac{6 \pm 5\sqrt{2}}{7} \qquad \left[\text{using } \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right]$$

$\therefore$ The required equation is.

$$y = \frac{-1}{7}x + \frac{6 \pm 5\sqrt{2}}{7}$$

or $\quad 7y + x + 6 \pm 5\sqrt{2} = 0$

Q4

Since the coefficient of x and y in the equations $2x + 3y - 19 = 0$, $2x + 3y - 6 = 0$ and $2x + 3y + 7 = 0$ are same, therefore all the lines are parallel.

Distance between parallel lines is $d = \left|\frac{c_2 - c_1}{\sqrt{a^2 + b^2}}\right|$, where $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ are the lines parallel to each other.

Distance between the lines $2x - 3y - 19 = 0$ and $2x + 3y - 6 = 0$ is

$$d_1 = \left|\frac{-19+6}{\sqrt{2^2 + 3^2}}\right| = \left|\frac{13}{\sqrt{13}}\right| = \sqrt{13}$$

Distance between the lines $2x - 3y + 7 = 0$ and $2x + 3y - 6 = 0$ is

$$d_2 = \left|\frac{7+6}{\sqrt{2^2 + 3^2}}\right| = \frac{13}{\sqrt{13}} = \sqrt{13}$$

Since the distances of both the lines $2x + 3y + 7 = 0$ and $2x + 3y - 19 = 0$ from the line $2x + 3y - 6 = 0$ are equal, therefore the lines are equidistant.

Q5

The equation of lines are

$$3x + 2y - \frac{7}{3} = 0 \qquad \text{---(i)}$$

$$3x + 2y + 6 = 0 \qquad \text{---(ii)}$$

Let equation of mid way be $3x + 2y + \lambda = 0$ ---(iii)

Then, distance between (i) and (iii) and (ii) and (iii) should be equal.

$$\left|\frac{\lambda + \frac{7}{3}}{\sqrt{9+4}}\right| = \left|\frac{\lambda - 6}{\sqrt{9+4}}\right|$$

$$\Rightarrow \quad \lambda + \frac{7}{3} = -\lambda + 6$$

$$\Rightarrow \quad \lambda = \frac{11}{6}$$

$\therefore$ The required line is $3x + 2y + \frac{11}{6} = 0$ or $18x + 12y + 11 = 0$.

Q6

Clearly, the slope of each of the given lines is same equal to $\frac{3}{4}$.
Hence, the line $3x + 4y + 2 = 0$ is parallel to each of the given lines.

Putting $y = 0$ in $3x + 4y + 2 = 0$, we get $x = \frac{2}{3}$.
So, the coordinates of a point on $3x + 4y + 2 = 0$ are $\left(\frac{2}{3}, 0\right)$.

The distance d_1 between the lines $3x + 4y + 2 = 0$ and $3x + 4y + 5 = 0$ is given by

$$d_1 = \left|\frac{3\left(-\frac{2}{3}\right) + 4(0) + 5}{\sqrt{3^2 + 4^2}}\right| = \frac{3}{5}$$

The distance d_2 between the lines $3x + 4y + 2 = 0$ and $3x + 4y - 5 = 0$ is given by

$$d_2 = \left|\frac{3\left(-\frac{2}{3}\right) + 4(0) - 5}{\sqrt{3^2 + 4^2}}\right| = \frac{7}{5}$$

$$\frac{d_1}{d_2} = \frac{\frac{3}{5}}{\frac{7}{5}} = \frac{3}{7}$$

So $3x + 4y + 2 = 0$ divides the distance between the lines $3x + 4y + 5 = 0$ and $3x + 4y - 5 = 0$ in the ratio 3:7.

Ex 23.17

Q1

Let $ABCD$ be a parallelogram the equation of whose sides AB, BC, CD and DA are a_1x_1, b_1y_1, $c_1 = 0$, $a_2x + b_2y + c_2 = 0$, $a_1x + b_1y + d_1 = 0$ and $a_2x - b_2y + d_2 = 0$.

Let p_1 and p_2 be the distance between the pairs of parallel side of $ABCD$

$$\sin\theta \frac{p_1}{AD} = \frac{p_2}{AB}$$

$$AD = \frac{p_1}{\sin\theta} \text{ and } AB = \frac{p_2}{\sin\theta}$$

Area of $ABCD = AB \times p_1 = \frac{p_1p_2}{\sin\theta}$

or $\Rightarrow AD \times p_2 = \frac{p_1p_2}{\sin\theta}$.

Now,

$$m_1 = \text{slope of } AB = \frac{a_1}{b_1}$$

$$m_2 = \text{slope of } AD = \frac{-a_2}{b_2}$$

Since θ is angle between AB and AC.

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1m_2}$$

$$= \frac{\frac{a_2}{b_2} + \frac{a_1}{b_1}}{1 - \frac{a_1a_2}{b_1b_2}}$$

$$\tan\theta = \frac{a_2b_1 - a_1b_2}{a_1a_2 + b_1b_2} \Rightarrow \sin\theta = \frac{a_2b_1 - a_1b_2}{\sqrt{\left(a_1^2 + b_1^2\right)\left(a_2^2 + b_2^2\right)}}$$

P_1 = Distance between AB and AD

$$= \left|\frac{c_1 - d_1}{\sqrt{a_1^2 + b_1^2}}\right|$$

P_2 = Distance between AD and BC.

$$= \left|\frac{c_2 - d_2}{\sqrt{a_2^2 + b_2^2}}\right|$$

$\therefore$ Area of parallelogram is

$$\frac{|c_1 - d_1||c_2 - d_2|}{|a_2b_1 - a_1b_2|}$$ Hence, proved.

(i) Rhombus is a parallelogram with all side equal.

$\therefore$ $P_1 = P_2$

$\therefore$ Modifing the formula of area of parallelogram devided above.

The area of rhombus

$$= \frac{P_1P_2}{\sin\theta}$$

$$= \frac{2p_1}{\sin\theta} = \frac{2p_2}{\sin\theta}$$

$$= 2\left|\frac{(c_1 - d_1)}{a_2b_1 - a_1b_2}\right| \text{ or } 2\left|\frac{(c_2 - d_2)}{a_2b_1 - b_2a_1}\right|$$

Q2

The area of a parallelogram is

$$= \frac{|c_1 - d_1||c_2 - d_2|}{|a_2 b_1 - b_2 a_1|}$$

$$= \frac{|-a + 2a||3a - a|}{|3(-3) - 4(-4)|}$$

$$= \frac{a \times 2a}{7}$$

$$= \frac{2}{7}a^2$$

Hence, proved.

Q3

Let $ABCD$ be a parallelogram as shown in the following figure.

We observe that the following parallelogram is a rhombus, as distance between opposite sides $(AB$ and $CD)$ and $(AD$ and $BC)$ is equal $= (n - n')$.

And in a Rhombus, diagnals are perpendicular to eah other.

$\therefore$ Angle between the two diagnals is 17/2.

Ex 23.18

Q1

Let the required equation be $ax + by = c$ but here it passes through origin $(0,0)$.

$\therefore \quad c = 0$

$\therefore$ Equaiton is $ax + by = 0$

Slope of the line $(m_1) = \frac{-a}{b}$ and $m_2 = \frac{-\sqrt{3}}{1}$

$\Rightarrow \quad$ Angle between $\sqrt{3}x + y = 11$ and $ax + by = 0$ is $45°$

$$\therefore \quad \tan 45° = \frac{m_1 \pm m_2}{1 \mp m_1 m_2}$$

$$1 = \frac{\frac{-a}{b} \pm \left(-\sqrt{3}\right)}{1 \mp \frac{a}{b} \times \sqrt{3}}$$

$$1 - \frac{\sqrt{3}a}{b} = \frac{-a}{b} - \sqrt{3} \text{ and } 1 + \frac{a}{b}\sqrt{3} = \frac{-a}{b} + \sqrt{3}$$

$$b - \sqrt{3}a = -a - \sqrt{3}b \text{ and } b + a\sqrt{3} = -a + b\sqrt{3}$$

$$a\left(1 - \sqrt{3}\right) = b\left(-\sqrt{3} - 1\right) \text{ and } a\left(\sqrt{3} + 1\right) = b\left(\sqrt{3} - 1\right)$$

$$\frac{a}{b} - \frac{1 - \sqrt{3}}{\sqrt{3} - 1} = \frac{\left(\sqrt{3} - 1\right)^2}{2} = 2 - \sqrt{3}$$

or

$$\frac{a}{b} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = -2 - \sqrt{3}$$

$\therefore$ Required lines are $\frac{y}{x} = \sqrt{3} \pm 2$ or $y = \left(\sqrt{3} \pm 2\right)x$

Q2

Let the required equation be $y = mx + c$

But, $c = 0$ as it passes through origin $(0, 0)$

$\therefore$ Equation of the lines is $y = mx$.

Slope of $x + y + \sqrt{3}y = \sqrt{3}x = a$

or $\left(\sqrt{3}+1\right)x + \left(1-\sqrt{3}\right)y = a$ is

$$\frac{\sqrt{3}+1}{\sqrt{3}-1} = \frac{4-2\sqrt{3}}{2} = 2-\sqrt{3}.$$

The angle between $x + y + \sqrt{3}y - \sqrt{3} = a$ and $y = mx$ is $75°$

$$\tan\left(75°\right) = \frac{m_1 \pm m_2}{1 \mp m_1 m_2}$$

$$\tan\left(30° + 45°\right) = \frac{m \pm \left(2-\sqrt{3}\right)}{1 - m\left(2-\sqrt{3}\right)}$$

$$\frac{\frac{1}{\sqrt{3}}+1}{1-\frac{1}{\sqrt{3}}\times 1} = \frac{m \pm 2 - \sqrt{3}}{1 - m\left(2-\sqrt{3}\right)}$$

$$2+\sqrt{3} = \frac{m+2-\sqrt{3}}{1+m\left(\sqrt{3}-2\right)} \text{ and } 2+\sqrt{3} = \frac{m+\sqrt{3}-2}{1+m\left(2-\sqrt{3}\right)}$$

$\therefore$ $\frac{1}{m} = 0$ or $m = -\sqrt{3}$

$\therefore$ $y = mx$ $y = -\sqrt{3}x$ and $x = 0$ are the required equatins.

Q3

Given equation is 6x + 5y - 8=0.

Slope of given line = m = $-\frac{6}{5}$

Equations of required line is,

$$y+1=\frac{-\frac{6}{5}-1}{1+\frac{6}{5}}(x-2)$$

$$y+1=\frac{-11}{11}(x-2)$$

$$y+1=-x+2$$

$$x+y-1=0$$

$$y+1=\frac{-\frac{6}{5}+1}{1-\frac{6}{5}}(x-2)$$

$$y+1=\frac{-1}{-1}(x-2)$$

$$y+1=x-2$$

$$x-y=3$$

Q4

The required equation is

$$y - k = m'(x - h)$$

And this line is inclined at $\tan^{-1} m$ to straight line $y = mx + c$.

Slope $= m = \tan\theta$

Passing through $\underset{(x_1,\, y_1)}{(h, k)}$

$\therefore$ Equation of line is

$$y - y_1 = m(x - x_1) \quad \text{---(i)}$$

Also, $\tan\theta = \left|\dfrac{m - m'}{1 + mm'}\right|$

Here, $m = m'$

$\therefore$ $\tan\theta = \dfrac{m - m}{1 + m^2}$ or $\left|\dfrac{-m - m}{1 - m^2}\right|$

$= 0$ or $\dfrac{+2m}{1 - m^2}$

Substituting in (i)

$$y - k = 0$$

$\Rightarrow$ $y = k$ or

$$y - k = \frac{+2m}{1 - m^2}(x - h)$$

$$(1 - m^2)(y - k) = +2m(x - h)$$

Q5

Here, $x_1 = 2,\ y_1 = 3,\ \alpha = 45°$

m = slope of line $3x + y - 5 = 0$

$$= \frac{-\text{coeff of x}}{\text{coeff of y}} = -3$$

The equations of the required line are

$$y - y_1 = \frac{-3 - \tan 45°}{1 + (-3)\tan 45°}(x - 2)$$

$$y - 3 = \frac{-3 - 1}{1 + (-3)(1)}(x - 2)$$

$$y - 3 = \frac{-4}{2}(x - 2) = 2x - 4$$

$$2x - y - 1 = 0$$

Also, $$y - 3 = \frac{-3 + \tan 45°}{1 - (-3)\tan 45}(x - 2)$$

$$y - 3 = \frac{-3 + 1}{1 + 3}(x - 2)$$

$$y - 3 = \frac{-2}{4}(x - 2) = \frac{-x}{2} + 1$$

$$x + 2y - 8 = 0$$

Q6

Let the isosceles right triangle be.

$AC = 3x + 4y = 4$

$c(2,2)$

Then, slope of $AC = \frac{-3}{4}$

$AB = BC$ $\quad$ [$\because$ It is an isoscales right triangle]

Then, angle between $(AB$ and $AC)$ and $(BC$ and $AC)$ is 45°.

$$\tan\frac{\pi}{4} = \frac{m_1 - \left(\frac{-3}{4}\right)}{1 + \left(\frac{-3}{4}\right)m_1} \qquad [\text{when } m_1 = \text{slope of } BC]$$

$$1 = \frac{m_1 + \frac{3}{4}}{1 - \frac{3}{4}m}$$

$4 - 3m_1 = 4m_1 + 3$

$7m_1 = 1 \qquad m_1 = \frac{1}{7}$

and, $\quad AB \perp BC$

$\therefore$ (slope of AB) $\times$ (slope of BC) $= -1$

$m_2 \times \frac{1}{7} = -1$

$m_2 = -7.$

The equation of BC is

$(y - 2) = \frac{1}{7}(x - 2)$

$7y - 14 = x - 2$

$x - 7y + 12 = 0$

and

The equation of AB is

$(y - 2) = -7(x - 2)$

$y - 2 = -x + 14$

$y + 7x - 16 = 0$

Q7

Let $C\left(2+\sqrt{3},5\right)$ be one vertex and $x = y$ be the opposite side of equilateral triangle ABC.

The other two sides makes an angle of 60° with other two sides.
slope of $x - y = 0$ is 1.

$$\therefore \quad y - 5 = \frac{1 \pm \tan 60^\circ}{1 \mp \tan 60^\circ}\left(x - 2 - \sqrt{3}\right)$$

$$y - 5 = \frac{1+\sqrt{3}}{1-\sqrt{3}}\left(x - 2 - \sqrt{3}\right) \text{ and } y - 5 = \frac{1-\sqrt{3}}{1+\sqrt{3}}\left(x - 2 - \sqrt{3}\right)$$

$$y - 5 = \left(\sqrt{3} - 2\right)\left(x - 2 - \sqrt{3}\right) \text{ and } y - 5 = \left(\sqrt{3} - 2\right)\left(x - 2 - \sqrt{3}\right)$$

$$y + \left(2+\sqrt{3}\right)x = 12 + 4\sqrt{3} \text{ and } y + \left(2-\sqrt{3}\right)x = 6$$

Hence proved the 2nd side of ΔABC is $y + \left(2-\sqrt{3}\right)x = 6$
and the 3rd side is $y + \left(2+\sqrt{3}\right)x = 12 + 4\sqrt{3}$.

Q8

Let $ABCD$ be a square whose diagnal BD is $4x + 7y = 12$
Then, slope of $BD = \frac{-4}{7}$
Let slope of $AB = m$

$$\text{Then,} \quad \tan 45^\circ = \frac{m + \frac{4}{7}}{1 - \frac{4}{7}m}$$

$$7 - 4m = 7m + 4$$

$$11m = 3$$

$$\therefore \quad m = \frac{3}{11}$$

$$\therefore \quad \text{Slope of } BC = \frac{-1}{\text{slope of } AB}$$

$$= \frac{-11}{3}$$

$\therefore$ Equationof of AB is

$$\left(y - 2\right) = \frac{3}{11}\left(x - 1\right)$$

$$11y - 22 = 3x - 3$$

$$3x - 11y + 19 = 0$$

and
Equation of BC is

$$\left(y - 2\right) = \frac{-11}{3}\left(x - 1\right)$$

$$11x + 3y - 17 = 0$$

Q9

AC and BC are inclided to $(AB)\, x + y = 0$ at an angle of $60°$

$\therefore \Delta ABC$ is equilateral triangle.

The slope of AB is -1 and let slope of AC be m_1

$$\tan 60° = \frac{m_1 + 1}{1 - m_1} \qquad \text{or} \qquad \sqrt{3}\left(1 - m_1\right) = m_1 + 1$$

$$\sqrt{3} - 1 = m_1 + \sqrt{3}m$$

$$\Rightarrow \qquad m_1 = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = 2 - \sqrt{3}$$

and, slope of BC is m_2

$$\tan 60° = \frac{m_2 - 1}{1 + m_1} = \sqrt{3}$$

$$\therefore \qquad m_2 = \sqrt{3} + 2$$

$\therefore$ Equations of AC and BC are

$$y - 2 = \left(2 - \sqrt{3}\right)\left(x - 1\right) \qquad \text{---(i)}$$

$$y - 2 = \left(2 + \sqrt{3}\right)\left(x - 1\right)$$

using (i) and $x + y = 0$

$$A \text{ is } \left(\frac{-1 - \sqrt{3}}{2}, \frac{1 + \sqrt{3}}{2}\right)$$

$$AC \text{ is } \sqrt{\left(\frac{2 + 1 + \sqrt{3}}{2}\right)^2 + \left(\frac{3 - \sqrt{3}}{2}\right)^2}$$

$$= \sqrt{\frac{9 + 3 + 6\sqrt{3} + 9 + 3 - 6\sqrt{3}}{4}}$$

$$AC = \sqrt{\frac{24}{6}}$$

$$= \sqrt{6}$$

The are of ΔABC

$$= \frac{\sqrt{3}}{4}\left(AC\right)^2$$

$$= \frac{\sqrt{3}}{4} \times \left(\sqrt{6}\right)^2$$

$$= \frac{3}{2}\sqrt{3} \text{ sq untis.}$$

Q10

Solving $7x - y + 3 = 0$ and $x + y - 3 = 0$ we get, $A(0, 3)$

The slope of $7x - y + 3 = 0\ (m_1)$ and $x + y - 3 = 0\ (m_2)$ are 7 and -1 respectively.

Any line through the point $(1, -10)$ is

$$y + 10 = m(x - 1) \qquad \text{---(i)}$$

Since it make equal angle say θ with the given lines, therefore

$$\tan\theta = \frac{m - 7}{1 + 7m} = -\frac{m - (-1)}{1 + m(-1)}$$

$$\Rightarrow \qquad m = -3 \text{ or } \frac{1}{3}$$

Putting in (i)

$$y + 10 = -3(x - 1)$$

$$y + 10 = -3x + 3$$

$$3x + y + 7 = 0$$

$$y + 10 = \frac{1}{3}(x - 1) \Rightarrow \frac{x}{3} - \frac{1}{3}$$

$$3y - x + 31 = 0$$

Q11

The distance from $(3, -5)$ to the line $2x + 3y - 7 = 0$ is

$$\left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$$

$$= \frac{|2(3) + 3(-5) - 7|}{\sqrt{(2)^2 + (3)^2}}$$

$$= \frac{|6 - 15 - 7|}{\sqrt{13}}$$

$$= \frac{16}{\sqrt{13}} \qquad \text{(i)}$$

Also distance of $(3, -5)$ from the second line $2x - 3y - 12 = 0$

$$\left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$$

$$= \frac{|2(3) + 3(-5) + 12|}{\sqrt{(2)^2 + (3)^2}}$$

$$= \frac{|6 - 15 + 12|}{\sqrt{13}}$$

$$= \frac{21}{\sqrt{13}} \qquad \text{(ii)}$$

Now $\quad c_1 - c_2 = 12 - 7 = 5$

Also difference between (i) and (ii) is 5

$\therefore$ $(3, -5)$ lies between the two lines equation of line through $(3, -5)$ cutting the lines at 45° is

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$\tan 45^\circ = \frac{m - \left(\frac{-2}{3}\right)}{1 - \frac{2}{3}m} = \pm 1$$

$$m + \frac{2}{3} = 1 - \frac{2}{3}m \qquad \text{or,} \qquad m + \frac{2}{3} = -1 + \frac{2}{3}m$$

$$m\left(1 + \frac{2}{3}\right) = 1 - \frac{2}{3} \qquad \text{or,} \qquad m\left(1 - \frac{2}{3}\right) = -1 - \frac{2}{3}$$

Q12

The slope of $AB = -1$

Let slope of AC be m

Then,

$$\tan 60° = \frac{m+1}{1-m}$$

$$m = 2 - \sqrt{3}$$

And similarly slope of $AB = 2 + \sqrt{3}$.

Equation of AC and AB are

$$(y+1) = \left(2-\sqrt{3}\right)(x-2)$$

or, $\left(2-\sqrt{3}\right)x - y - 5 + 2\sqrt{3} = 0$ ---(i)

and,

$$(y-1) = \left(2+\sqrt{3}\right)(x-2)$$

or, $\left(2+\sqrt{3}\right)x - y - 5 - 2\sqrt{3} = 0$ ---(ii)

On solving (i) with $x + y = 2$, we get

$$A\left(\frac{21-11\sqrt{3}}{6}, \ \frac{11\sqrt{3}-9}{6}\right)$$

$$AB = AC = BC$$

$$= \sqrt{\left(\frac{21-11\sqrt{3}-1}{6}\right)^2 + \left(\frac{11\sqrt{3}-9-1}{6}\right)^2}$$

$$= \sqrt{\frac{225+363-330\sqrt{3}+363+225-330\sqrt{3}}{36}}$$

$$= \sqrt{\frac{2}{3}}$$

Q13

Let $A(1,2)$, $C(5,8)$, $B(x_1,y_1)$, $D(x_2,y_2)$

Slope of $AC = \frac{8-2}{5-1} = \frac{6}{4} = \frac{3}{2}$

Let m be the slope of a line making on angle 45° with AC

$$\therefore \qquad \tan 45° = \left|\frac{m_1 - \frac{3}{2}}{1 + m \times \frac{3}{2}}\right|$$

$$1 = \frac{m - \frac{3}{2}}{1 + \frac{3m}{2}}$$

$1 + \frac{3m}{2} = m - \frac{3}{2}$ or, $1 + \frac{3m}{2} = -\left(m - \frac{3}{2}\right)$

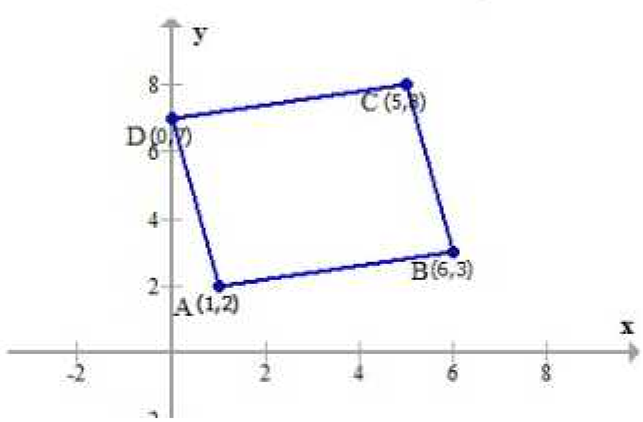
$\frac{3m}{2} - m = \frac{-3}{2} - 1$ or, $1 + \frac{3m}{2} = -m + \frac{3}{2}$

$\frac{1}{2}m = \frac{-5}{2}$ or, $\frac{3m}{2} + m = \frac{3}{2} - 1$

$m = -5$ or, $\frac{5m}{2} = \frac{1}{2}$

$m = \frac{1}{5}$

Consider the following figure:



Ex 23.19

Q1

Line through the intersection of $4x - 3y = 0$ and $2x - 5y + 3 = 0$ is

$$(4x - 3y) + \lambda(2x - 5y + 3) = 0 \qquad \text{---(i)}$$

or, $$x(4 + 2\lambda) - y(3 + 5\lambda) + 3\lambda = 0$$

And the required line is parallel to $4x + 5y + 6$

$\therefore$ slope of required = slope of $4x + 5y + 6 = \dfrac{-4}{3}$

$$\therefore \quad \frac{-(4 + 2\lambda)}{-(3 + 5\lambda)} = \frac{-4}{3}$$

$$\Rightarrow \quad 5(4 + 2\lambda) = -4(3 + 5\lambda)$$

$$\Rightarrow \quad 20 + 10\lambda = -12 - 20\lambda$$

$$\Rightarrow \quad 30\lambda = -32$$

$$\Rightarrow \quad \lambda = \frac{-16}{15}$$

Putting λ in equation (i)

$$(4x - 3y) - \frac{16}{15}(2x - 5y + 3) = 0$$

$$\Rightarrow \quad 60x - 45y - 32x + 80y - 48 = 0$$

$$\Rightarrow \quad 28x + 35y - 48 = 0$$

Is the required line

Q2

The equation of the required line is

$$(x + 2y + 3) + \lambda(3x - 4y + 7) = 0$$

or, $$x(1 - 3\lambda) + y(2 + 4\lambda) + 3 + 7\lambda = 0$$

$$m_1 = \text{slope of the line} = -\left(\frac{1 + 3\lambda}{2 + 4\lambda}\right)$$

The line is perpendicular to $x - y + 9 = 0$ whose slope $(m_2 = 1)$

$$\therefore \quad m_1 \times m_2 = -1$$

$$\Rightarrow \quad -\left(\frac{1 + 3\lambda}{2 + 4\lambda}\right) \times 1 = -1$$

$$\Rightarrow \quad 1 + 3\lambda = 2 + 4\lambda$$

$$\Rightarrow \quad \lambda = -1$$

$\therefore$ The required line is

$$x + 2y + 3 - (3x + 4y + 7) = 0$$

$$-2x - 2y - 4 = 0$$

or, $$x + y - 2 = 0$$

Q3

The required line is

$$2x - 7y + 11 + \lambda (x + 3y - 8) = 0$$

or, $$x(2 + \lambda) + y(-7 + 3\lambda) + 11 - 8\lambda = 0$$

(i) When the line is parallel to x-axis. It slope is 0

$$\therefore \quad -\frac{(2 + \lambda)}{3\lambda - 7} = 0$$

$$\lambda = -2$$

$\therefore$ Equation of line is

$$2x - 7y + 11 - 2(x + 3y - 8) = 0$$

$$-13y + 27 = 0$$

(ii) When the line is parallel to y-axis then,

$$\frac{-1}{\text{slope}} = 0$$

i.e $$\frac{3\lambda - 7}{2 + \lambda} = 0$$

$$\lambda = \frac{7}{3}$$

$\therefore$ Equation of line is

$$2x - 7y + 11 + \frac{7}{3}(x + 3y - 8) = 0$$

$$\Rightarrow \frac{6x - 21y + 33 + 7x + 21y - 56}{3} = 0$$

$$\Rightarrow 6x - 21y + 33 + 7x + 21y - 56 = 0$$

$$\Rightarrow 13x - 23 = 0$$

$$\Rightarrow 13x = 23$$

Q4

The required line is

$$(2x + 3y - 1) + \lambda(3x - 5y - 5) = 0$$

or, $$x(2 + 3\lambda) + y(3 - 5\lambda) - 1 - 5\lambda = 0$$

Since this lines is equally inclined to both the axes, it slope should be 1. or -1

$$\therefore \quad \frac{-2 - 3\lambda}{3 - 5\lambda} = 1 \qquad \text{or,} \qquad \frac{-2 - 3\lambda}{3 - 5\lambda} = -1$$

$$\Rightarrow \quad 3 - 5\lambda = -2 - 3\lambda \qquad \text{or,} \qquad \Rightarrow -2 - 3\lambda = -3 + 5\lambda$$

$$\Rightarrow \quad 5 = 2\lambda \qquad \text{or,} \qquad \Rightarrow 1 = 8\lambda$$

$$\Rightarrow \quad \lambda = \frac{5}{2} \qquad \text{or,} \qquad \Rightarrow \lambda = \frac{1}{8}$$

$\therefore$ The required line is

$$2x + 3y + 1 + \frac{5}{2}(3x - 5y - 5) = 0$$

$$4x + 6y + 2 + 15x - 25y - 25 = 0$$

$$19x - 19y - 23 = 0$$

or

$$(2x + 3y + 1) + \frac{1}{8}(3x - 5y - 5) = 0$$

$$16x + 24y + 8 + 3x - 5y - 5 = 0$$

$$19x + 19y + 3 = 0$$

$\therefore$ The two possible equation are

$$19x - 19y - 23 = 0 \qquad \text{or} \qquad 19x + 19y + 3 = 0$$

Q5

The required line is

$$(x + y - 4) + \lambda(2x - 3y - 1) = 0$$

or, $$x(1 + 2\lambda) + y(1 - 3\lambda) - 4 - \lambda = 0$$

And it is perpendicular to $\frac{x}{5} + \frac{y}{6} = 1$

$\therefore$ $$(\text{slope of required line}) \times \left(\text{slope of } \frac{x}{5} + \frac{y}{6} = 1\right) = -1$$

$\Rightarrow$ $$-\left(\frac{1 + 2\lambda}{1 - 3\lambda}\right) \times \frac{-6}{5} = -1$$

$\Rightarrow$ $$\frac{1 + 2\lambda}{1 - 3\lambda} = \frac{-5}{6}$$

$\Rightarrow$ $$6 + 12\lambda = -5 + 15\lambda$$

$\Rightarrow$ $$11 = 3\lambda \qquad \text{or } \lambda = \frac{11}{3}$$

$\therefore$ The required line is

$$(x + y - 4) + \frac{11}{3}(2x - 3y - 1) = 0$$

$$3x + 3y - 12 + 22x - 33y - 11 = 0$$

$$25x - 30y - 23 = 0$$

Q6

$$x(1 + \lambda) + y(2 - \lambda) + 5 = 0$$

$\Rightarrow$ $$x + x\lambda + 2y - \lambda y + 5 = 0$$

$\Rightarrow$ $$\lambda(x - y) + (x + 2y + 5) = 0$$

$\Rightarrow$ $$(x + 2y + 5) + \lambda(x - y) = 0$$

This is of the form $L_1 + \lambda L_2 = 0$

So it represents a line passing through the intersection of $x - y = 0$ and $x + 2y = -5$.

Solving the two equations, we get $\left(\frac{-5}{3}, \frac{-5}{3}\right)$ which is the fixed point through which the given family of lines passes for any value of λ.

Q7

$$(2+k)x+(1+k)y=5+7k$$

or, $$(2x+y-5)+k(x+y-7)=0$$

It is of the form $L_1+kL_2=0$ i.e., the equation of line passing through the intersection of 2 lines L_1 and L_2.

So, it represents a line passing through $2x+y-5=0$ and $x+y-7=0$.

Solving the two equation we get, $(-2,9)$. Which is the fixed point through which the given line pass. For any value of k.

Q8

$L_1+\lambda l_2=0$ is the equation of line passing through two lines. L_1 and L_2.

$\therefore$ $(2x+y-1)+\lambda(x+3y-2)=0$ is the required equation. ---(i)

or, $$x(2+\lambda)+y(1+3\lambda)-1-2\lambda=0$$

$$\frac{x}{\frac{1+2\lambda}{2+\lambda}}+\frac{4}{\frac{1+2\lambda}{1+3\lambda}}=1$$

Area of $\Delta=\frac{1}{2}\times OB\times OA$

$$\frac{8}{3}=\frac{1}{2}\times(y\ \text{intercept})\times(x\ \text{intercept})$$

$$\frac{8}{3}=\frac{1}{2}\times\left(\frac{1+2\lambda}{1+3\lambda}\right)\times\left(\frac{1+2\lambda}{2+\lambda}\right)$$

$$\frac{16}{3}=\frac{1+4\lambda^2+4\lambda}{2+3\lambda^2+7\lambda}$$

$$32+48\lambda^2+112\lambda=-3-12\lambda^2-12\lambda$$

$$60\lambda^2+124\lambda+35=0$$

$$\lambda=\frac{-124\pm\sqrt{(124)^2-4\times60\times35}}{2\times60}$$

$$=\frac{-124\pm\sqrt{15376-8400}}{120}$$

Approximately = 1

$\therefore$ Subtituting in (i) $\Rightarrow 3x+4y-3=0,\ 12x+y-3=0$

Q9

The required line is

$$(3x - y - 5) + \lambda(x + 3y - 1) = 0$$

or, $\quad (3 + \lambda)x + (-1 + 3\lambda)y - 5 - \lambda = 0$

or, $\quad \dfrac{x}{\left(\dfrac{5+\lambda}{3+\lambda}\right)} + \dfrac{y}{\dfrac{5+\lambda}{3\lambda - 1}} = 1$

And the line makes equal and positive intercepts with the line (given)

$\therefore \quad \dfrac{5+\lambda}{3+\lambda} = \dfrac{5+\lambda}{3\lambda - 1}$

$$3\lambda - 1 = 3 + \lambda$$
$$2\lambda = 4$$
$$\lambda = 2$$

$\therefore$ The required line is

$$3x - y - 5 + 2x + 6y - 2 = 0$$

or, $\quad 5x + 5y = 7$

Q10

The required line is

$$x - 3y + 1 + \lambda(2x + 5y - 9) = 0$$

or, $\quad (1 + 2\lambda)x + (-3 + 5\lambda)y + 1 - 9\lambda = 0$

Distance from origin of this line is

$$\left|\frac{(1+2\lambda)0 + (-3+5\lambda)0 + 1 - 9\lambda}{\sqrt{(1+2\lambda)^2 + (5\lambda - 3)^2}}\right| \qquad \left[\text{using } \frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}}\right]$$

$$\sqrt{5} = \left|\frac{1 - 9\lambda}{\sqrt{1 + 4\lambda^2 - 4\lambda + 25\lambda^2 + 9 - 30\lambda}}\right|$$

$\Rightarrow \quad \sqrt{5} = \left|\dfrac{1 - 9\lambda}{\sqrt{10 + 29\lambda^2 - 26\lambda}}\right|$

$\Rightarrow \quad 5(10 + 29\lambda^2 - 26\lambda) = (1 - 9\lambda)^2$

$\Rightarrow \quad 50 + 145\lambda^2 - 130\lambda = 1 - 81\lambda^2 - 18\lambda^2$

$\Rightarrow \quad 64\lambda^2 - 112\lambda + 49 = 0$

$\Rightarrow \quad (8\lambda - 7)^2 = 0 \quad$ or, $\quad \lambda = \dfrac{7}{8}$

$\therefore$ Required line is

$$x - 3y + 1 + \frac{7}{8}(2x + 5y - 9) = 0$$

$\Rightarrow \quad 8x - 24y + 8 + 14x - 35y - 63 = 0$

$\Rightarrow \quad 22x + 11y - 55 = 0$

$\Rightarrow \quad 2x + y - 5 = 0$

Q11

Solving two equations of lines $x - y + 1 = 0$ and $2x - 3y + 5 = 0$ we get,

intersection point (2,3).

Let equation of a line passing through (2,3) be y=mx+c

$\therefore$ 3= 2m+c

c=3-2m

Equation of the line is y= mx + 3 - 2m.......(1)

Perpendicular distance of above line from (3,2) $= \frac{7}{5}$

$$\left|\frac{3m - 2 + 3 - 2m}{\sqrt{m^2 + 1}}\right| = \frac{7}{5}$$

$$\left|\frac{m + 1}{\sqrt{m^2 + 1}}\right| = \frac{7}{5}$$

$$\frac{(m+1)^2}{m^2 + 1} = \frac{49}{25}$$

$$25(m^2 + 2m + 1) = 49m^2 + 49$$

$$25m^2 + 50m + 25 = 49m^2 + 49$$

$$24m^2 - 50m + 24 = 0$$

$$12m^2 - 25m + 12 = 0$$

$$m = \frac{4}{3}, m = \frac{3}{4}$$

Substituting m in (1), we get,

$$y = \frac{4}{3}x + 3 - \frac{2 \times 4}{3}$$

$$3y = 4x + 1$$

$$4x - 3y + 1 = 0$$

$$y = \frac{3}{4}x + 3 - \frac{2 \times 3}{4}$$

$$4y - 3y + 1 = 0$$

Equations of lines are $4x - 3y + 1 = 0$ and $4y - 3y + 1 = 0$