

Ex 8.1

Q1

$$\begin{aligned}
 \text{(i)} \quad & 2 \sin 3\theta \cos \theta \\
 &= \sin(3\theta + \theta) + \sin(3\theta - \theta) \quad \left[\because 2 \sin A \cos B = \sin(A+B) + \sin(A-B) \right] \\
 &= \sin 4\theta + \sin 2\theta
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad & 2 \cos 3\theta \sin \theta \\
 \therefore \quad & 2 \cos A \sin B = \sin(A+B) - \sin(A-B) \\
 \Rightarrow \quad & 2 \cos(3\theta) \sin(\theta) = \sin(3\theta + \theta) - \sin(3\theta - \theta) \\
 &= \sin 4\theta - \sin 2\theta
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad & 2 \sin 4\theta \sin 3\theta \\
 \therefore \quad & 2 \sin A \sin B = \cos(A-B) - \cos(A+B) \\
 \Rightarrow \quad & 2 \sin 4\theta \sin 3\theta = \cos(4\theta - 3\theta) - \cos(4\theta + 3\theta) \\
 &= \cos \theta - \cos 7\theta
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv)} \quad & 2 \cos 7\theta \cos 3\theta \\
 \therefore \quad & 2 \cos A \cos B = \cos(A+B) + \cos(A-B) \\
 \Rightarrow \quad & 2 \cos 7\theta \cos 3\theta = \cos(7\theta + 3\theta) + \cos(7\theta - 3\theta) \\
 &= \cos 10\theta + \cos 4\theta
 \end{aligned}$$

Q2

$$\begin{aligned}
 \text{(i)} \quad & 2 \sin \frac{5x}{12} \sin \frac{x}{12} \\
 \therefore \quad & 2 \sin A \sin B = \cos(A-B) - \cos(A+B) \\
 \Rightarrow \quad & 2 \sin \frac{5x}{12} \sin \frac{x}{12} = \cos\left(\frac{5x}{12} - \frac{x}{12}\right) - \cos\left(\frac{5x}{12} + \frac{x}{12}\right) \\
 &= \cos\left(\frac{4x}{12}\right) - \cos\left(\frac{6x}{12}\right) \\
 &= \cos\left(\frac{x}{3}\right) - \cos\left(\frac{x}{2}\right) \\
 &= \frac{1}{2} - 0 = \frac{1}{2} = \text{RHS}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad & 2 \cos \frac{5x}{12} \cos \frac{x}{12} = \frac{1}{2} \\
 \therefore \quad & 2 \cos A \cos B = \cos(A+B) + \cos(A-B) \\
 &= \cos\left(\frac{5x}{12} + \frac{x}{12}\right) + \cos\left(\frac{5x}{12} - \frac{x}{12}\right) \\
 &= \cos\left(\frac{x}{2}\right) + \cos\left(\frac{x}{3}\right) \\
 &= 1 + \frac{1}{2} = \frac{3}{2} = \text{LHS}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad & 2 \sin \frac{5x}{12} \cos \frac{x}{12} \\
 \therefore \quad & 2 \sin A \cos B = \sin(A+B) + \sin(A-B) \\
 &= \sin\left(\frac{5x}{12} + \frac{x}{12}\right) + \sin\left(\frac{5x}{12} - \frac{x}{12}\right) \\
 &= \sin \frac{x}{2} + \sin \frac{x}{3} \\
 &= 1 + \frac{\sqrt{3}}{2} = \frac{2+\sqrt{3}}{2} = \text{RHS (Taking LCM)}
 \end{aligned}$$

Q3(i)

$$\sin 50^\circ \cos 85^\circ = \frac{1 - \sqrt{2} \sin 35^\circ}{2\sqrt{2}}$$

$$\text{LHS} = \sin 50^\circ \cos 85^\circ = \frac{2 \sin 50^\circ \cos 85^\circ}{2}$$

$$\therefore 2 \sin A \cos B = \sin(A + B) + \sin(A - B)$$

$$\Rightarrow \frac{2 \sin 50^\circ \cos 85^\circ}{2} = \frac{1}{2} [\sin(50^\circ + 85^\circ) + \sin(50^\circ - 85^\circ)]$$

$$= \frac{1}{2} [\sin 135^\circ + \sin(-35^\circ)]$$

$$= \frac{1}{2} [\sin(90^\circ + 45^\circ) - \sin 35^\circ] \quad \left[\because \sin(-\theta) = -\sin \theta \right]$$

$$= \frac{1}{2} [\cos 45^\circ - \sin 35^\circ] \quad \left[\because \sin(90^\circ + \theta) = \cos \theta \right]$$

Now,

$$\cos 45^\circ = \frac{1}{\sqrt{2}}$$

$$= \frac{1}{2} \left[\frac{1}{\sqrt{2}} - \sin 35^\circ \right]$$

$$= \frac{1 - \sqrt{2} \sin 35^\circ}{2\sqrt{2}}$$

Q3(ii)

$$\begin{aligned}\text{LHS} &= \sin 25^\circ \cos 115^\circ \\ &= \frac{2 \sin 25^\circ \cos 115^\circ}{2}\end{aligned}$$

We know that

$$\begin{aligned}2 \sin A \cos B &= \sin(A+B) + \sin(A-B) \\ &= \frac{1}{2} [\sin(25^\circ + 115^\circ) + \sin(25^\circ - 115^\circ)] \\ &= \frac{1}{2} [\sin 140^\circ + \sin(-90^\circ)] \\ \sin(-\theta) &= -\sin \theta\end{aligned}$$

$$\text{And, } \sin(90^\circ + \theta) = \cos \theta$$

$$\begin{aligned}\Rightarrow & \frac{1}{2} [\sin(90^\circ + 50^\circ) - \sin 90^\circ] \\ &= \frac{1}{2} [\cos 50^\circ - 1]\end{aligned}$$

Also,

$$\begin{aligned}\cos \theta &= \sin(90^\circ - \theta) \\ \cos 50^\circ &= \sin(90^\circ - 50^\circ) = \sin 40^\circ \\ & \frac{1}{2} [\sin 40^\circ - 1]\end{aligned}$$

Q4

We have,

$$\begin{aligned}\text{LHS} &= 4 \cos \theta \cos \left(\frac{\pi}{3} + \theta \right) \cos \left(\frac{\pi}{3} - \theta \right) \\&= 2 \cos \theta \left[2 \cos \left(\frac{\pi}{3} + \theta \right) \cos \left(\frac{\pi}{3} - \theta \right) \right] \\&= 2 \cos \theta \left[2 \cos \left(\frac{\pi}{3} + \theta + \frac{\pi}{3} - \theta \right) + \cos \left(\frac{\pi}{3} + \theta - \frac{\pi}{3} + \theta \right) \right] \\&= 2 \cos \theta \left[\cos \frac{2\pi}{3} + \cos 2\theta \right] \\&= 2 \cos \theta \left[\cos \left(\frac{\pi}{2} + \frac{\pi}{6} \right) + \cos 2\theta \right] \\&= 2 \cos \theta \left[-\sin \frac{\pi}{6} + \cos 2\theta \right] \\&= 2 \cos \theta \left[-\frac{1}{2} + \cos 2\theta \right] \\&= -2 \cos \theta \times \frac{1}{2} + 2 \cos \theta \cos 2\theta \\&= -\cos \theta + [\cos (\theta + 2\theta) + \cos (2\theta - \theta)] \\&= -\cos \theta + \cos 3\theta + \cos \theta \\&= \cos 3\theta \\&= \text{RHS}\end{aligned}$$

\therefore LHS = RHS Hence proved.

Q5(i)

$$\cos 10^\circ \cos 30^\circ \cos 50^\circ \cos 70^\circ = \frac{3}{16}$$

$$\begin{aligned} \text{LHS} &= \cos 10^\circ \cos 30^\circ \cos 50^\circ \cos 70^\circ \\ &= \cos 30^\circ \cos 10^\circ \cos 50^\circ \cos 70^\circ \\ &= \frac{\sqrt{3}}{2} (\cos 10^\circ \cos 50^\circ \cos 70^\circ) \\ &= \frac{\sqrt{3}}{2} (\cos 10^\circ \cos 50^\circ) \cos 70^\circ \\ &= \frac{\sqrt{3}}{4} (2 \cos 10^\circ \cos 50^\circ) \cos 70^\circ \quad [\text{Multiplying and dividing by 2}] \end{aligned}$$

Also,

$$\begin{aligned} \Rightarrow 2 \cos A \cos B &= \cos(A+B) + \cos(A-B) \quad \text{---(i)} \\ &= \frac{\sqrt{3}}{4} \cos 70^\circ (\cos(50^\circ + 10^\circ) + \cos(10^\circ - 50^\circ)) \\ &= \frac{\sqrt{3}}{4} \cos 70^\circ (\cos 60^\circ + \cos(-40^\circ)) \end{aligned}$$

Now,

$$\begin{aligned} \cos(-\theta) &= \cos \theta \\ &= \frac{\sqrt{3}}{4} \cos 70^\circ \left(\frac{1}{2} + \cos 40^\circ \right) \quad \left[\because \cos 60^\circ = \frac{1}{2} \right] \\ &= \frac{\sqrt{3}}{8} \cos 70^\circ + \frac{\sqrt{3}}{4} \cos 70^\circ \cos 40^\circ \\ &= \frac{\sqrt{3}}{8} \cos 70^\circ + \frac{\sqrt{3}}{8} (2 \cos 70^\circ \cos 40^\circ) \\ &= \frac{\sqrt{3}}{8} [\cos 70^\circ + \cos(70^\circ + 40^\circ) + \cos(70^\circ - 40^\circ)] \quad [\text{from (i)}] \\ &= \frac{\sqrt{3}}{8} [\cos 70^\circ + \cos 110^\circ + \cos 30^\circ] \\ &= \frac{\sqrt{3}}{8} \left[\cos 70^\circ + \cos(180^\circ - 70^\circ) + \frac{\sqrt{3}}{2} \right] \\ &= \frac{\sqrt{3}}{8} \left[\cos 70^\circ - \cos 70^\circ + \frac{\sqrt{3}}{2} \right] \quad [\because \cos(180^\circ - \theta) = -\cos \theta] \\ &= \frac{\sqrt{3}}{8} \times \frac{\sqrt{3}}{2} = \frac{3}{16} \\ &= \text{RHS} \end{aligned}$$

Q5(ii)

$$\cos 40^\circ \cos 80^\circ \cos 160^\circ = -\frac{1}{8}$$

$$\begin{aligned} \text{LHS} &= \cos 40^\circ \cos 80^\circ \cos 160^\circ \\ &= \cos 80^\circ \cos 40^\circ \cos 160^\circ \end{aligned}$$

Multiplying and dividing by 2

$$\begin{aligned} &= \frac{1}{2} (\cos 80^\circ \times (2 \cos 40^\circ \cos 160^\circ)) \\ 2 \cos A \cos B &= \cos(A+B) + \cos(A-B) \\ &= \frac{1}{2} (\cos 80^\circ (\cos(40^\circ + 160^\circ) + \cos(40^\circ - 160^\circ))) \\ &= \frac{1}{2} (\cos 80^\circ (\cos 200^\circ + \cos(-120^\circ))) \\ &= \frac{1}{2} \cos 80^\circ (\cos(180^\circ + 20^\circ) + \cos(180^\circ - 60^\circ)) \\ &= \frac{1}{2} \cos 80^\circ (\cos 20^\circ + \cos 60^\circ) \\ &= \frac{1}{2} \cos 80^\circ \cos 20^\circ + \frac{1}{2} \cos 80^\circ + \cos 60^\circ \\ &= -\frac{1}{2} (2 \cos 80^\circ \cos 20^\circ) + \frac{1}{2} \cos 80^\circ + \cos 60^\circ \\ &= -\frac{1}{4} [2 \cos 80^\circ \cos 20^\circ + \cos 80^\circ] \\ &= -\frac{1}{4} [\cos(80^\circ + 20^\circ) + \cos(80^\circ - 20^\circ) + \cos 80^\circ] \\ &= -\frac{1}{4} [\cos 100^\circ + \cos 60^\circ + \cos 80^\circ] \\ &= -\frac{1}{4} [\cos(180^\circ - 80^\circ) + \cos 60^\circ + \cos 80^\circ] \\ &= -\frac{1}{4} [-\cos 80^\circ + \cos 60^\circ + \cos 80^\circ] \\ &= -\frac{1}{4} \cos 60^\circ \\ &= -\frac{1}{4} \times \frac{1}{2} \\ &= -\frac{1}{8} \quad \text{RHS} \end{aligned}$$

Q5(iii)

$$\sin 20^\circ \sin 40^\circ \sin 80^\circ$$

$$= \frac{1}{2} (2 \sin 20^\circ \sin 40^\circ) \sin 80^\circ$$

$$= \frac{1}{2} [\cos (40^\circ - 20^\circ) - \cos (40^\circ + 20^\circ)] \sin 80^\circ$$

$$[\because 2 \sin A \sin B = \cos (A - B) - \cos (A + B)]$$

$$= \frac{1}{2} [\cos 20^\circ - \cos 60^\circ] \sin 80^\circ$$

$$= \frac{1}{2} \left[\cos 20^\circ - \frac{1}{2} \right] \sin 80^\circ$$

$$= \frac{1}{2} [\cos 20^\circ \sin 80^\circ] - \frac{1}{4} \sin 80^\circ$$

$$= \frac{1}{4} [2 \cos 20^\circ \sin 80^\circ - \sin 80^\circ]$$

$$= \frac{1}{4} [\sin (80^\circ + 20^\circ) + \sin (80^\circ - 20^\circ) - \sin 80^\circ]$$

$$[\because 2 \sin A \cos B = \sin (A + B) + \sin (A - B)]$$

$$= \frac{1}{4} [\sin 100^\circ + \sin 60^\circ - \sin 80^\circ]$$

$$= \frac{1}{4} \left[\sin (180^\circ - 80^\circ) + \frac{\sqrt{3}}{2} - \sin 80^\circ \right]$$

$$= \frac{1}{4} \left[\sin 80^\circ + \frac{\sqrt{3}}{2} - \sin 80^\circ \right]$$

$$= \frac{\sqrt{3}}{8} = \text{RHS}$$

Q5(iv)

$$\cos 20^\circ \cos 40^\circ \cos 80^\circ$$

$$= \frac{1}{2} (2 \cos 20^\circ \cos 40^\circ) \cos 80^\circ$$

$$= \frac{1}{2} [\cos (40^\circ + 20^\circ) + \cos (40^\circ - 20^\circ)] \cos 80^\circ$$

$$[\because 2 \cos A \cos B = \cos (A + B) + \cos (A - B)]$$

$$= \frac{1}{2} [\cos 60^\circ + \cos 20^\circ] \cos 80^\circ$$

$$= \frac{1}{2} \left[\frac{1}{2} + \cos 20^\circ \right] \cos 80^\circ$$

$$= \frac{1}{2} [\cos 80^\circ + 2 \cos 20^\circ \cos 80^\circ]$$

$$= \frac{1}{4} [\cos 80^\circ + \cos (80^\circ + 20^\circ) + \cos (20^\circ - 80^\circ)]$$

$$= \frac{1}{4} [\cos 80^\circ + \cos 100^\circ + \cos 60^\circ]$$

$$= \frac{1}{4} [\cos 80^\circ + \cos (180^\circ - 80^\circ) + \cos 60^\circ]$$

$$= \frac{1}{4} [\cos 80^\circ - \cos 80^\circ + \cos 60^\circ]$$

$$= \frac{1}{4} \left[\frac{1}{2} \right] = \frac{1}{8} = \text{RHS}$$

Q5(v)

$$\tan 20^\circ \tan 40^\circ \tan 60^\circ \tan 80^\circ$$

$$= (\tan 20^\circ \tan 40^\circ \tan 80^\circ) \sqrt{3}$$

$$= \left(\frac{\sin 20^\circ \sin 40^\circ \sin 80^\circ}{\cos 20^\circ \cos 40^\circ \cos 80^\circ} \right) \sqrt{3}$$

$$= \frac{(2 \sin 20^\circ \sin 40^\circ) \sin 80^\circ \times \sqrt{3}}{(2 \cos 20^\circ \cos 40^\circ) \cos 80^\circ}$$

$$\left[\because \tan 60^\circ = \sqrt{3} \right]$$

Applying

$$\Rightarrow 2 \sin A \sin B = \cos(A - B) - \cos(A + B)$$

$$2 \cos A \cos B = \cos(A + B) + \cos(A - B)$$

$$= \frac{(\cos(40^\circ - 20^\circ) - \cos(40^\circ + 20^\circ)) \sin 80^\circ \times \sqrt{3}}{(\cos(20^\circ + 40^\circ) + \cos(40^\circ - 20^\circ)) \cos 80^\circ}$$

$$= \frac{(\cos 20^\circ - \cos 60^\circ) \sin 80^\circ \times \sqrt{3}}{(\cos 60^\circ + \cos 20^\circ) \cos 80^\circ}$$

$$= \frac{\left(\cos 20^\circ - \frac{1}{2} \right) \sin 80^\circ \times \sqrt{3}}{\left(\frac{1}{2} + \cos 20^\circ \right) \cos 80^\circ}$$

$$= \frac{(2 \sin 80^\circ \cos 20^\circ - \sin 80^\circ) \sqrt{3}}{\cos 80^\circ + 2 \cos 20^\circ \cos 80^\circ}$$

$$\Rightarrow 2 \sin A \cos B = \sin(A + B) + \sin(A - B)$$

$$= \frac{(\sin(80^\circ + 20^\circ) + \sin(80^\circ - 20^\circ) - \sin 80^\circ) \sqrt{3}}{\cos 80^\circ + (\cos(20^\circ + 80^\circ) + \cos(80^\circ - 20^\circ))}$$

$$= \frac{(\sin 100^\circ + \sin 60^\circ - \sin 80^\circ) \sqrt{3}}{\cos 80^\circ + \cos 100^\circ + \cos 60^\circ}$$

$$= \frac{\left(\sin(180^\circ - 80^\circ) + \frac{\sqrt{3}}{2} - \sin 80^\circ \right) \sqrt{3}}{\cos 80^\circ + \cos(180^\circ - 80^\circ) + \cos 60^\circ}$$

$$= \frac{\left(\sin 80^\circ + \frac{\sqrt{3}}{2} - \sin 80^\circ \right) \sqrt{3}}{\cos 80^\circ - \cos 80^\circ + \cos 60^\circ}$$

$$= \frac{\frac{3}{2}}{\frac{1}{2}} = 3 = \text{RHS}$$

Q5(vi)

$$\tan 20^\circ \tan 30^\circ \tan 40^\circ \tan 80^\circ$$

$$\begin{aligned} &= \frac{1}{\sqrt{3}} (\tan 20^\circ \tan 40^\circ \tan 80^\circ) \\ &= \frac{(\sin 20^\circ \sin 40^\circ \sin 80^\circ)}{(\cos 20^\circ \cos 40^\circ \cos 80^\circ) \sqrt{3}} \\ &= \frac{(2 \sin 20^\circ \sin 40^\circ) \sin 80^\circ}{\sqrt{3} (2 \cos 20^\circ \cos 40^\circ) \cos 80^\circ} \end{aligned}$$

$$\left[\because \tan 30^\circ = \frac{1}{\sqrt{3}} \right]$$

Applying

$$\Rightarrow 2 \sin A \sin B = \cos(A - B) - \cos(A + B)$$

$$\begin{aligned} 2 \cos A \cos B &= \cos(A + B) + \cos(A - B) \\ &= \frac{(\cos(40^\circ - 20^\circ) - \cos(20^\circ + 40^\circ)) \sin 80^\circ}{\cos(20^\circ + 40^\circ) + \cos(40^\circ - 20^\circ) \cos 80^\circ \sqrt{3}} \\ &= \frac{(\cos 20^\circ - \cos 60^\circ) \sin 80^\circ}{\sqrt{3} (\cos 60^\circ + \cos 20^\circ) \cos 80^\circ} \\ &= \frac{\left(\cos 20^\circ - \frac{1}{2}\right) \sin 80^\circ}{\sqrt{3} \left(\frac{1}{2} + \cos 20^\circ\right) \cos 80^\circ} \\ &= \frac{2 \sin 20^\circ \sin 80^\circ - \sin 80^\circ}{\sqrt{3} (\cos 80^\circ + 2 \cos 20^\circ \cos 80^\circ)} \end{aligned}$$

Now,

$$\Rightarrow 2 \sin A \cos B = \sin(A + B) + \sin(A - B)$$

$$\begin{aligned} &= \frac{\sin(80^\circ + 20^\circ) + \sin(80^\circ - 20^\circ) - \sin 80^\circ}{\sqrt{3} (\cos 80^\circ + \cos(20^\circ + 80^\circ) + \cos(80^\circ - 20^\circ))} \\ &= \frac{\sin 100^\circ + \sin 60^\circ - \sin 80^\circ}{\sqrt{3} (\cos 80^\circ + \cos 100^\circ + \cos 60^\circ)} \\ &= \frac{\sin 100^\circ + \sin 60^\circ - \sin(80^\circ - 100^\circ)}{\sqrt{3} (\cos 80^\circ + \cos(180^\circ - 80^\circ) + \sin 60^\circ)} \\ &= \frac{\sin 100^\circ + \frac{\sqrt{3}}{2} - \sin 100^\circ}{\sqrt{3} (\cos 80^\circ - \cos 80^\circ + \cos 60^\circ)} \end{aligned}$$

$$= \frac{\frac{\sqrt{3}}{2}}{\sqrt{3} \left(\frac{1}{2}\right)} = 1 = \text{RHS}$$

Q5(vii)

$$\sin 10^\circ \sin 50^\circ \sin 60^\circ \sin 70^\circ = \frac{\sqrt{3}}{16}$$

LHS

$$\begin{aligned} & \sin 10^\circ \sin 50^\circ \sin 70^\circ \frac{\sqrt{3}}{2} \\ &= \sin(90^\circ - 80^\circ) \sin(90^\circ - 40^\circ) \sin(90^\circ - 20^\circ) \frac{\sqrt{3}}{2} \\ &= \cos 80^\circ \cos 40^\circ \cos 20^\circ \frac{\sqrt{3}}{2} \\ &= \frac{\sqrt{3}}{2 \times 2} (2 \cos 40^\circ \cos 20^\circ) \cos 80^\circ \\ &= \frac{\sqrt{3}}{2 \times 2} [\cos(40^\circ + 20^\circ) + \cos(40^\circ - 20^\circ)] \cos 80^\circ \\ &= \frac{\sqrt{3}}{2 \times 2} [\cos 60^\circ + \cos 20^\circ] \cos 80^\circ \\ &= \frac{\sqrt{3}}{2 \times 2} \left[\frac{1}{2} + \cos 20^\circ \right] \cos 80^\circ \\ &= \frac{\sqrt{3}}{4} \left[\frac{1}{2} \cos 80^\circ + \cos 20^\circ \cos 80^\circ \right] \\ &= \frac{\sqrt{3}}{8} [\cos 80^\circ + 2 \cos 20^\circ \cos 80^\circ] \\ &= \frac{\sqrt{3}}{8} [\cos 80^\circ + \cos(80^\circ + 20^\circ) + \cos(80^\circ - 20^\circ)] \\ &= \frac{\sqrt{3}}{8} [\cos 80^\circ + \cos 100^\circ + \cos 60^\circ] \\ &= \frac{\sqrt{3}}{8} [\cos 80^\circ + \cos(180^\circ - 80^\circ) + \cos 60^\circ] \\ &= \frac{\sqrt{3}}{8} [\cos 60^\circ] = \frac{\sqrt{3}}{16} = \text{RHS} \end{aligned}$$

$$\left[\because \sin 60^\circ = \frac{\sqrt{3}}{2} \right]$$

$$\left[\because 2 \cos A \cos B = \cos(A+B) + \cos(A-B) \right]$$

Q5(viii)

$$\text{LHS} = \sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ$$

$$= \sin 20^\circ \sin 40^\circ \sin 80^\circ \times \frac{\sqrt{3}}{2}$$

$$\left[\because \sin 60^\circ = \frac{\sqrt{3}}{2} \right]$$

$$= \frac{\sqrt{3}}{2} \times \frac{1}{2} (2 \sin 20^\circ \sin 40^\circ) \sin 80^\circ$$

$$= \frac{\sqrt{3}}{4} [\cos (40^\circ - 20^\circ) - \cos (40^\circ + 20^\circ)] \sin 80^\circ$$

$$= \frac{\sqrt{3}}{4} [\cos 20^\circ - \cos 60^\circ] \sin 80^\circ$$

$$= \frac{\sqrt{3}}{4} \left[\cos 20^\circ \sin 80^\circ - \frac{1}{2} \sin 80^\circ \right]$$

$$= \frac{\sqrt{3}}{8} [2 \cos 20^\circ \sin 80^\circ - \sin 80^\circ]$$

$$= \frac{\sqrt{3}}{8} [\sin (80^\circ + 20^\circ) + \sin (80^\circ - 20^\circ) - \sin 80^\circ]$$

$$= \frac{\sqrt{3}}{8} [\sin 100^\circ + \sin 60^\circ - \sin 80^\circ]$$

$$= \frac{\sqrt{3}}{8} [\sin 80^\circ + \sin 60^\circ - \sin 80^\circ]$$

$$= \frac{\sqrt{3}}{8} \times \sin 60^\circ = \frac{\sqrt{3}}{8} \times \frac{\sqrt{3}}{2}$$

$$= \frac{3}{16}$$

Q6(i)

We have,

$$\begin{aligned}\text{LHS} &= \sin A \sin(B - C) + \sin B \sin(C - A) + \sin C \sin(A - B) \\&= \frac{1}{2} [2 \sin A \sin(B - C) + 2 \sin B \sin(C - A) + 2 \sin C \sin(A - B)] \\&= \frac{1}{2} [\cos(A - B + C) - \cos(A + B - C) + \cos(B - C + A) - \cos(B + C - A) \\&\quad + \cos(C - A + B) - \cos(C + A - B)] \\&= \frac{1}{2} [\cos(A - B + C) - \cos(A - B + C) - \cos(A + B - C) + \cos(A + B - C) \\&\quad - \cos(B + C - A) + \cos(B + C - A)] \\&= \frac{1}{2} \times 0 \\&= 0 \\&= \text{RHS}\end{aligned}$$

$\therefore \sin A \sin(B - C) + \sin B \sin(C - A) + \sin C \sin(A - B) = 0$ Hence proved.

Q6(ii)

We have,

$$\begin{aligned}\text{LHS} &= \sin(B - C) \cos(A - D) + \sin(C - A) \cos(B - D) + \sin(A - B) \cos(C - D) \\&= \frac{1}{2} [2 \sin(B - C) \cos(A - D) + 2 \sin(C - A) \cos(B - D) + 2 \sin(A - B) \cos(C - D)] \\&= \frac{1}{2} [\sin(B - C + A - D) + \sin(B - C - A + D) + \sin(C - A + B - D) + \sin(C - A - B + D) \\&\quad + \sin(A - B + C - D) + \sin(A - B - C + D)] \\&= \frac{1}{2} [\sin(A + B - C - D) + \sin(B + D - C - A) + \sin(B + C - A - D) + \sin(C + D - A - B) \\&\quad + \sin(A + C - B - D) + \sin(A + D - B - C)] \\&= \frac{1}{2} [\sin(A + B - C - D) + \sin(B + D - C - A) + \sin\{-(A + D - B - C)\} + \sin\{-(A + B - C - D)\} \\&\quad + \sin\{-(B + D - A - C)\} + \sin(A + D - B - C)] \\&= \frac{1}{2} [\sin(A + B - C - D) + \sin(B + D - C - A) - \sin(A + D - B - C) - \sin(A + B - C - D) \\&\quad - \sin(B + D - A - C) + \sin(A + D - B - C)] \\&= \frac{1}{2} \times 0 \quad [\because \sin(-\theta) = -\sin\theta] \\&= 0 \\&= \text{RHS}\end{aligned}$$

$\therefore \sin(B - C) \cos(A - D) + \sin(C - A) \cos(B - D) + \sin(A - B) \cos(C - D) = 0$ Hence proved.

Q7

We have,

$$\begin{aligned}
 \text{LHS} &= \tan \theta \tan (60^\circ - \theta) \tan (60^\circ + \theta) \\
 &= \frac{\sin \theta \sin (60^\circ - \theta) \sin (60^\circ + \theta)}{\cos \theta \cos (60^\circ - \theta) \cos (60^\circ + \theta)} \\
 &= \frac{2 \sin \theta \sin (60^\circ - \theta) \sin (60^\circ + \theta)}{2 \cos \theta \cos (60^\circ - \theta) \cos (60^\circ + \theta)} \\
 &= \frac{\sin \theta [2 \sin (60^\circ - \theta) \sin (60^\circ + \theta)]}{\cos \theta [2 \cos (60^\circ - \theta) \cos (60^\circ + \theta)]} \\
 &= \frac{\sin \theta [\cos \{(60^\circ - \theta) - (60^\circ + \theta)\} - \cos \{(60^\circ - \theta) + (60^\circ + \theta)\}]}{\cos \theta [\cos \{(60^\circ - \theta) + (60^\circ + \theta)\} + \cos \{(60^\circ - \theta) - (60^\circ + \theta)\}]} \\
 &= \frac{\sin \theta [\cos (-2\theta) - \cos 120^\circ]}{\cos \theta [\cos 120^\circ + \cos (-2\theta)]} \\
 &= \frac{\sin \theta [\cos 2\theta - \cos 120^\circ]}{\cos \theta [\cos 120^\circ + \cos 2\theta]} \quad [\because \cos(-\theta) = \cos \theta] \\
 &= \frac{\sin \theta [\cos 2\theta - \cos (90^\circ + 30^\circ)]}{\cos \theta [\cos (90^\circ + 30^\circ) + \cos 2\theta]} \\
 &= \frac{\sin \theta [\cos 2\theta + \sin 30^\circ]}{\cos \theta [-\sin 30^\circ + \cos 2\theta]} \quad [\because \cos \text{ is negative in IIInd quadrant}] \\
 &= \frac{\sin \theta \left[\cos 2\theta + \frac{1}{2} \right]}{\cos \theta \left[-\frac{1}{2} + \cos 2\theta \right]} \\
 &= \frac{\sin \theta \cos 2\theta + \frac{1}{2} \sin \theta}{-\frac{1}{2} \cos \theta + \cos \theta \cos 2\theta}
 \end{aligned}$$

Q8

Let $y = \cos \alpha \cos \beta$ then,

$$\begin{aligned}y &= \frac{1}{2}(2 \cos \alpha \cos \beta) \\&= \frac{1}{2}[\cos(\alpha + \beta) + \cos(\alpha - \beta)] \\&= \frac{1}{2}[\cos 90^\circ + \cos(\alpha - \beta)] && [\because \alpha + \beta = 90^\circ] \\&= \frac{1}{2}[0 + \cos(\alpha - \beta)] \\&= \frac{1}{2}\cos(\alpha - \beta)\end{aligned}$$

$$\Rightarrow y = \frac{1}{2}\cos(\alpha - \beta)$$

Now,

$$\begin{aligned}-1 &\leq \cos(\alpha - \beta) \leq 1 \\ \Rightarrow \quad \frac{-1}{2} &\leq \frac{1}{2}\cos(\alpha - \beta) \leq \frac{1}{2} \\ \Rightarrow \quad \frac{-1}{2} &\leq y \leq \frac{1}{2} \\ \Rightarrow \quad \frac{-1}{2} &\leq \cos \alpha \cos \beta \leq \frac{1}{2}\end{aligned}$$

Hence, the maximum values of $\cos \alpha \cos \beta$ is $\frac{1}{2}$.

Ex 8.2

Q1

$$(i) \sin 12\theta - \sin 4\theta$$

$$= 2 \sin \left(\frac{12\theta + 4\theta}{2} \right) \cos \left(\frac{12\theta - 4\theta}{2} \right) \\ = 2 \sin 8\theta \cos 4\theta$$

$$\left[\because \sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2} \right]$$

$$(ii) \sin 5\theta - \sin \theta$$

$$= 2 \cos \left(\frac{5\theta + \theta}{2} \right) \sin \left(\frac{5\theta - \theta}{2} \right) \\ = 2 \sin 2\theta \cos 3\theta$$

$$\left[\because \sin C - \sin D = 2 \sin \frac{C+D}{2} \sin \frac{C-D}{2} \right]$$

$$(iii) \cos 12\theta + \cos 8\theta$$

$$= 2 \cos 10\theta \cos 2\theta$$

$$\left[\because \cos C + \cos D = 2 \cos \frac{C+D}{2} \cos \frac{C-D}{2} \right]$$

$$(iv) \cos 12\theta - \cos 4\theta$$

$$= -2 \sin \left(\frac{12\theta + 4\theta}{2} \right) \sin \left(\frac{12\theta - 4\theta}{2} \right) \\ = -2 \sin 8\theta \sin 4\theta$$

$$\left[\because \cos C - \cos D = -2 \sin \frac{C+D}{2} \sin \frac{C-D}{2} \right]$$

$$(v) \sin 2\theta + \cos 4\theta$$

$$= \sin 2\theta - \sin (90^\circ - 4\theta) \\ = 2 \sin \frac{(2\theta + 90^\circ - 4\theta)}{2} \cos \frac{(2\theta - 90^\circ + 4\theta)}{2} \\ = 2 \sin \left(\frac{\pi}{4} + \theta \right) \cos \left(\frac{\pi}{4} - 3\theta \right)$$

Q2

$$\sin 38^\circ + \sin 22^\circ = \sin 82^\circ$$

$$\text{LHS} = \sin 38^\circ + \sin 22^\circ$$

$$\therefore \sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2}$$

$$\Rightarrow \sin 38^\circ + \sin 22^\circ = 2 \sin \frac{60^\circ}{2} \cos \frac{16^\circ}{2}$$

$$= 2 \sin 30^\circ \cos 8^\circ$$

$$= 2 \times \frac{1}{2} \cos 8^\circ$$

$$= \cos (90^\circ - 82^\circ)$$

$$= \sin 82^\circ = \text{RHS}$$

$$\left[\because \cos \theta = \sin (90^\circ - \theta) \right]$$

Q2(i)

$$\cos 100^\circ + \cos 20^\circ = \cos 40^\circ$$

$$\text{LHS} = \cos 100^\circ + \cos 20^\circ$$

$$\begin{aligned}\Rightarrow & 2 \cos \frac{(100^\circ + 20^\circ)}{2} \cos \frac{(100^\circ - 20^\circ)}{2} \\ &= 2 \cos 60^\circ \cos 40^\circ \\ &= 2 \times \frac{1}{2} \cos 40^\circ \\ &= \cos 40^\circ = \text{RHS}\end{aligned}$$

$$\left[\because \cos C + \cos D = 2 \cos \frac{C+D}{2} \cos \frac{C-D}{2} \right]$$

$$\left[\because \cos 60^\circ = \frac{1}{2} \right]$$

Q2(ii)

$$\sin 50^\circ - \sin 10^\circ = \cos 20^\circ$$

$$\text{LHS} = \sin 50^\circ + \sin 10^\circ$$

$$\begin{aligned}\sin 50^\circ + \sin 10^\circ &= 2 \sin \frac{60^\circ}{2} \cos 20^\circ \\ &= 2 \sin 30^\circ \cos 20^\circ \\ &= 2 \times \frac{1}{2} \cos 20^\circ \\ &= \cos 20^\circ = \text{RHS}\end{aligned}$$

$$\left[\because \sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2} \right]$$

$$\left[\because \sin 30^\circ = \frac{1}{2} \right]$$

Q2(iii)

$$\sin 37^\circ + \sin 37^\circ = \cos 7^\circ$$

$$\text{LHS} = \sin 37^\circ + \sin 37^\circ$$

$$\begin{aligned}&= 2 \sin \left(\frac{2 \times 37^\circ}{2} \right) \cos \left(\frac{2 \times 37^\circ - 37^\circ}{2} \right) \\ &= 2 \sin (30^\circ) \cos (-7^\circ) \\ &= 2 \times \frac{1}{2} \cos 7^\circ \\ &= \cos 7^\circ = \text{RHS}\end{aligned}$$

$$\left[\because \sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2} \right]$$

$$\left[\because \cos (-\theta) = \cos \theta, \sin 30^\circ = \frac{1}{2} \right]$$

Q2(iv)

$$\text{LHS} = \sin 105^\circ + \cos 105^\circ$$

$$= \sin 105^\circ + \cos (90^\circ + 15^\circ)$$

$$= \sin 105^\circ - \sin 15^\circ$$

$$\begin{aligned}&= 2 \sin \left(\frac{105^\circ - 15^\circ}{2} \right) \cos \left(\frac{105^\circ + 15^\circ}{2} \right) \\ &= 2 \sin 45^\circ \cos 60^\circ \\ &= 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{2} \\ &= \frac{1}{\sqrt{2}} \\ &= \cos 45^\circ\end{aligned}$$

Q2(v)

$$\sin 40^\circ + \sin 20^\circ = \cos 10^\circ$$

$$\text{LHS} = \sin 40^\circ + \sin 20^\circ$$

$$= 2 \sin \left(\frac{40^\circ + 20^\circ}{2} \right) \cos \left(\frac{40^\circ - 20^\circ}{2} \right)$$

$$= 2 \sin 30^\circ \cos 10^\circ$$

$$= 2 \times \frac{1}{2} \cos 10^\circ$$

$$= \cos 10^\circ$$

$$= \text{RHS}$$

$$\left[\because \sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2} \right]$$

$$\left[\because \sin 30^\circ = \frac{1}{2} \right]$$

Q3(i)

$$\cos 55^\circ + \cos 65^\circ + \cos 175^\circ = 0$$

$$\cos 175^\circ = -\cos 5^\circ$$

substitute above value in the equation we get

$$\cos 55^\circ + \cos 65^\circ = \cos 5^\circ$$

$$\text{applying rule } \cos A + \cos B = 2 \cos \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right)$$

$$\cos 55^\circ + \cos 65^\circ = 2 \cos \left(\frac{65^\circ + 55^\circ}{2} \right) \cos \left(\frac{65^\circ - 55^\circ}{2} \right) = 2 \cos 60^\circ \cos 5^\circ = 2 \times \frac{1}{2} \times \cos 5^\circ = \cos 5^\circ$$

Hence Proved

Q3(ii)

$$\sin 50^\circ - \sin 70^\circ + \sin 10^\circ = 0$$

$$(\sin 50^\circ - \sin 70^\circ) + \sin 10^\circ$$

$$\Rightarrow \left(2 \sin \left(\frac{50^\circ - 70^\circ}{2} \right) \cos \left(\frac{50^\circ + 70^\circ}{2} \right) \right) + \sin 10^\circ$$

$$= 2 \sin(-10^\circ) \cos 60^\circ + \sin 10^\circ$$

$$= -2 \sin 10^\circ \times \frac{1}{2} + \sin 10^\circ$$

$$= 0$$

$$= \text{RHS}$$

$$\left[\because \sin C - \sin D = 2 \sin \left(\frac{C-D}{2} \right) \cos \left(\frac{C+D}{2} \right) \right]$$

$$\left[\because \cos 60^\circ = \frac{1}{2} \right]$$

Q3(iii)

$$\begin{aligned}
& \cos 80^\circ + \cos 40^\circ - \cos 20^\circ = 0 \\
& (\cos 80^\circ + \cos 40^\circ) - \cos 20^\circ \\
& = 2 \cos\left(\frac{80^\circ + 40^\circ}{2}\right) \cos\left(\frac{80^\circ - 40^\circ}{2}\right) - \cos 20^\circ \quad \left[\because \cos C + \cos D = 2 \cos\left(\frac{C+D}{2}\right) \cos\left(\frac{C-D}{2}\right) \right] \\
& = 2 \cos 60^\circ \cos 20^\circ - \cos 20^\circ \\
& = 2 \times \frac{1}{2} \cos 20^\circ - \cos 20^\circ \\
& = \cos 20^\circ - \cos 20^\circ \\
& = 0 \\
& = \text{RHS}
\end{aligned}$$

Q3(iv)

$$\begin{aligned}
& \cos 20^\circ + \cos 100^\circ + \cos 140^\circ = 0 \\
\Rightarrow & (\cos 20^\circ + \cos 100^\circ) + \cos 140^\circ \\
& = 2 \cos\left(\frac{20^\circ + 100^\circ}{2}\right) \cos\left(\frac{20^\circ - 100^\circ}{2}\right) + \cos 140^\circ \quad \left[\because \cos C + \cos D = 2 \cos\left(\frac{C+D}{2}\right) \cos\left(\frac{C-D}{2}\right) \right] \\
& = 2 \cos 60^\circ \cos(-40^\circ) + \cos 140^\circ \\
& = 2 \times \frac{1}{2} \cos 40^\circ + \cos 140^\circ \quad \left[\because \cos 60^\circ = \frac{1}{2} \right] \\
& = \cos 40^\circ + \cos(180^\circ - 40^\circ) \\
& = \cos 40^\circ - \cos 40^\circ \\
& = 0 \\
& = \text{RHS}
\end{aligned}$$

Q3(v)

$$\begin{aligned}
& \sin \frac{5\pi}{18} - \cos \frac{4\pi}{9} = \sqrt{3} \sin \frac{\pi}{9} \\
\text{LHS} & = \sin \frac{5\pi}{18} - \cos \frac{4\pi}{9} \\
& = \sin 50^\circ - \cos 80^\circ \\
& = \sin 50^\circ - \sin 10^\circ \\
& = 2 \sin\left(\frac{50^\circ - 10^\circ}{2}\right) \cos\left(\frac{50^\circ + 10^\circ}{2}\right) \\
& = 2 \sin 20^\circ \cos 30^\circ \\
& = 2 \sin 20^\circ \times \frac{\sqrt{3}}{2} \\
& = \sqrt{3} \sin \frac{\pi}{9}
\end{aligned}$$

Q3(vi)

$$\cos \frac{\pi}{12} - \sin \frac{\pi}{12} = \frac{1}{\sqrt{2}}$$

Multiplying and dividing by $\sqrt{2}$ on LHS

$$= \sqrt{2} \left(\frac{1}{\sqrt{2}} \cos \frac{\pi}{12} - \frac{1}{\sqrt{2}} \sin \frac{\pi}{12} \right)$$

$$= \sqrt{2} \left(\sin \frac{\pi}{4} \cos \frac{\pi}{12} - \cos \frac{\pi}{4} \sin \frac{\pi}{12} \right)$$

$$= \sqrt{2} \left(\sin \left(\frac{\pi}{4} - \frac{\pi}{12} \right) \right)$$

$$= \sqrt{2} \left(\sin \frac{\pi}{6} \right)$$

$$= \sqrt{2} \times \frac{1}{2}$$

$$= \frac{1}{\sqrt{2}}$$

= RHS

$$\left[\because \frac{1}{\sqrt{2}} = \cos \frac{\pi}{4} = \sin \frac{\pi}{4} \right]$$

$$\left[\because \sin(A - B) = \sin A \cos B - \cos A \sin B \right]$$

Q3(vii)

$$\sin 80^\circ - \cos 70^\circ = \cos 50^\circ$$

$$\text{LHS} = \sin 80^\circ = \cos 50^\circ + \cos 70^\circ$$

Now,

$$\cos C + \cos D = 2 \cos \frac{C+D}{2} \cos \frac{C-D}{2}$$

$$\text{RHS} = \cos 50^\circ + \cos 70^\circ$$

$$= 2 \cos \left(\frac{50^\circ + 70^\circ}{2} \right) \cos \left(\frac{50^\circ - 70^\circ}{2} \right)$$

$$= 2 \cos 60^\circ \cos (-10^\circ)$$

$$= 2 \times \frac{1}{2} \cos 10^\circ$$

$$= \cos 10^\circ$$

$$= \sin 80^\circ$$

$$= \text{LHS}$$

$$[\cos(-\theta) = \cos \theta]$$

$$[\because \cos \theta = \sin(90 - \theta)]$$

Q3(viii)

$$\begin{aligned}\sin 51^\circ + \cos 81^\circ &= \cos 21^\circ \\ \sin 51^\circ &= \cos 21^\circ - \cos 81^\circ \\ \text{RHS} &= \cos 21^\circ - \cos 31^\circ \\ &= -2 \sin(51^\circ) \sin(-30^\circ) \\ &= +2 \sin 51^\circ \sin 30^\circ \\ &= 2 \sin 51^\circ \times \frac{1}{2} \\ &= \sin 51^\circ \\ &= \text{LHS}\end{aligned}$$

$$\left[\because \cos C - \cos D = -2 \sin \frac{C+D}{2} \sin \frac{C-D}{2} \right]$$

Q4

We have,

$$\begin{aligned}\text{LHS} &= \cos\left(\frac{3\pi}{4} + x\right) - \cos\left(\frac{3\pi}{4} - x\right) \\ &= -\left[\cos\left(\frac{3\pi}{4} - x\right) - \cos\left(\frac{3\pi}{4} + x\right)\right] \\ &= -\left[2 \sin \frac{3\pi}{4} \sin x\right] \\ &= -2 \sin \frac{3\pi}{4} \sin x \\ &= -2 \sin\left(\frac{\pi}{2} + \frac{\pi}{4}\right) \sin x \\ &= -2 \cos \frac{\pi}{4} \sin x \\ &= -2 \times \frac{1}{\sqrt{2}} \times \sin x \\ &= -\frac{\sqrt{2} \times \sqrt{2}}{\sqrt{2}} \sin x \\ &= -\sqrt{2} \sin x \\ &= \text{RHS}\end{aligned}$$

$$\left[\because \cos(A-B) - \cos(A+B) = 2 \sin A \sin B \right]$$

$$\therefore \cos\left(\frac{3\pi}{4} + x\right) - \cos\left(\frac{3\pi}{4} - x\right) = -\sqrt{2} \sin x \quad \text{Hence proved.}$$

Q4(i)

We have,

$$\begin{aligned}
 \text{LHS} &= \cos\left(\frac{\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} - x\right) \\
 &= 2 \cos \frac{\pi}{4} \cos x && \left[\because \cos(A+B) + \cos(A-B) = 2 \cos A \cos B \right] \\
 &= 2 \times \frac{1}{\sqrt{2}} \times \cos x \\
 &= \frac{\sqrt{2} \times \sqrt{2}}{\sqrt{2}} \cos x \\
 &= \sqrt{2} \cos x \\
 &= \text{RHS}
 \end{aligned}$$

$$\therefore \cos\left(\frac{\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} - x\right) = \sqrt{2} \cos x.$$

Q5(i)

We have,

$$\begin{aligned}
 \text{LHS} &= \sin 65^\circ + \cos 65^\circ \\
 &= \sin(45^\circ + 20^\circ) + \cos(90^\circ - 25^\circ) \\
 &= \sin(45^\circ + 20^\circ) + \sin 25^\circ \\
 &= \sin(45^\circ + 20^\circ) + \sin(45^\circ - 20^\circ) \\
 &= 2 \sin 45^\circ \cos 20^\circ \\
 &= 2 \times \frac{1}{\sqrt{2}} \cos 20^\circ \\
 &= \frac{\sqrt{2} \times \sqrt{2}}{\sqrt{2}} \times \cos 20^\circ \\
 &= \sqrt{2} \cos 20^\circ \\
 &= \text{RHS}
 \end{aligned}$$

$$\therefore \sin 65^\circ + \cos 65^\circ = \sqrt{2} \cos 20^\circ \quad \text{Hence proved.}$$

Q5(ii)

We have,

$$\begin{aligned}\text{LHS} &= \sin 47^\circ + \cos 77^\circ \\ &= \sin (90^\circ - 43^\circ) + \cos 77^\circ \\ &= \cos 43^\circ + \cos 77^\circ \\ &= \cos (60^\circ - 17^\circ) + \cos (60^\circ + 17^\circ) \\ &= 2 \cos 60^\circ \cos 17^\circ \\ &= 2 \times \frac{1}{2} \times \cos 17^\circ \\ &= \cos 17^\circ \\ &= \text{RHS}\end{aligned}$$

$\therefore \sin 47^\circ + \cos 77^\circ = \cos 17^\circ$ Hence proved.

Q6(i)

We have,

$$\begin{aligned}\text{LHS} &= \cos 3A + \cos 5A + \cos 7A + \cos 15A \\ &= [\cos 5A + \cos 3A] + [\cos 15A + \cos 7A] \\ &= \left[2 \cos \frac{(5A + 3A)}{2} \cos \frac{(5A - 3A)}{2} \right] + \left[2 \cos \frac{(15A + 7A)}{2} \cos \frac{(15A - 7A)}{2} \right] \\ &= 2 \cos 4A \cos A + 2 \cos 11A \cos 4A \\ &= 2 \cos 4A [\cos A + \cos 11A] \\ &= 2 \cos 4A [\cos 11A + \cos A] \\ &= 2 \cos 4A \left[2 \cos \frac{(11A + A)}{2} \cos \frac{(11A - A)}{2} \right] \\ &= 4 \cos A [\cos 6A \cos 5A] \\ &= 4 \cos 4A \cos 5A \cos 6A \\ &= \text{RHS}\end{aligned}$$

$\therefore \cos 3A + \cos 5A + \cos 7A + \cos 15A = 4 \cos 4A \cos 5A \cos 6A$ Hence proved.

Q6(ii)

We have,

$$\begin{aligned}
 \text{LHS} &= \cos A + \cos 3A + \cos 5A + \cos 7A \\
 &= (\cos 3A + \cos A) + (\cos 7A + \cos 5A) \\
 &= \left[2 \cos \left(\frac{3A + A}{2} \right) \cos \left(\frac{3A - A}{2} \right) \right] + \left[2 \cos \left(\frac{7A + 5A}{2} \right) \cos \left(\frac{7A - 5A}{2} \right) \right] \\
 &= 2 \cos 2A \cos A + 2 \cos 6A \cos A \\
 &= 2 \cos A [\cos 2A + \cos 6A] \\
 &= 2 \cos A [\cos 6A + \cos 2A] \\
 &= 2 \cos A \left[2 \cos \left(\frac{6A + 2A}{2} \right) \cos \left(\frac{6A - 2A}{2} \right) \right] \\
 &= 4 \cos A [\cos 4A \cos 2A] \\
 &= \text{RHS}
 \end{aligned}$$

$\therefore \cos A + \cos 3A + \cos 5A + \cos 7A = 4 \cos A \cos 2A \cos 4A$. Hence proved.

Q6(iii)

We have,

$$\begin{aligned}
 \text{LHS} &= \sin A + \sin 2A + \sin 4A + \sin 5A \\
 &= (\sin 2A + \sin A) + (\sin 5A + \sin 4A) \\
 &= \left[2 \sin \left(\frac{2A + A}{2} \right) \cos \left(\frac{2A - A}{2} \right) \right] + \left[2 \sin \left(\frac{5A + 4A}{2} \right) \cos \left(\frac{5A - 4A}{2} \right) \right] \\
 &= 2 \sin \frac{3A}{2} \cos \frac{A}{2} + 2 \sin \frac{9A}{2} \cos \frac{A}{2} \\
 &= 2 \cos \frac{A}{2} \left[\sin \frac{3A}{2} + \sin \frac{9A}{2} \right] \\
 &= 2 \cos \frac{A}{2} \left[\sin \frac{9A}{2} + \sin \frac{3A}{2} \right] \\
 &= 2 \cos \frac{A}{2} \left[2 \sin \left\{ \frac{1}{2} \left(\frac{9A}{2} + \frac{3A}{2} \right) \right\} \cos \left\{ \frac{1}{2} \left(\frac{9A}{2} - \frac{3A}{2} \right) \right\} \right] \\
 &= 4 \cos \frac{A}{2} \left[\sin \frac{12A}{4} \cos \frac{6A}{4} \right] \\
 &= 4 \cos \frac{A}{2} \sin 3A \cos \frac{3A}{2} \\
 &= 4 \cos \frac{A}{2} \cos \frac{3A}{2} \sin 3A \\
 &= \text{RHS}
 \end{aligned}$$

$\therefore \sin A + \sin 2A + \sin 4A + \sin 5A = 4 \cos \frac{A}{2} \cos \frac{3A}{2} \sin 3A$. Hence proved.

Q6(iv)

We have,

$$\begin{aligned}
 \text{L.H.S.} &= \sin 3A + \sin 2A - \sin A \\
 &= \sin 3A - \sin A + \sin 2A \\
 &= 2 \sin \left(\frac{3A - A}{2} \right) \cos \left(\frac{3A + A}{2} \right) + \sin 2A \\
 &= 2 \sin A \cos 2A + \sin 2A \\
 &= 2 \sin A \cos 2A + 2 \sin A \cos A \\
 &= 2 \sin A [\cos 2A + \cos A] \\
 &= 2 \sin A \cdot 2 \cos \left(\frac{2A + A}{2} \right) \cos \left(\frac{2A - A}{2} \right) \\
 &= 4 \sin A \cos \frac{3A}{2} \cos \frac{A}{2} \\
 &= 4 \sin A \cos \frac{A}{2} \cos \frac{3A}{2} \\
 &= \text{R.H.S.}
 \end{aligned}$$

$$\therefore \sin 3A - \sin A + \sin 2A = 4 \sin A \cos \frac{A}{2} \cos \frac{3A}{2} \quad \text{Hence proved.}$$

Q6(v)

We have,

$$\begin{aligned}
 \text{L.H.S.} &= \cos 20^\circ \cos 100^\circ + \cos 100^\circ \cos 140^\circ - \cos 140^\circ \cos 200^\circ \\
 &= \frac{1}{2} [2 \cos 100^\circ \cos 20^\circ + 2 \cos 140^\circ \cos 100^\circ - 2 \cos 200^\circ \cos 140^\circ] \\
 &= \frac{1}{2} [\cos (100^\circ + 20^\circ) + \cos (100^\circ - 20^\circ) - \cos (140^\circ + 100^\circ) + \cos (140^\circ - 100^\circ)] \\
 &\quad - \{ \cos (200^\circ - 140^\circ) + \cos (200^\circ + 140^\circ) \} \\
 &= \frac{1}{2} [\cos 120^\circ - \cos 80^\circ + \cos 240^\circ + \cos 40^\circ - \cos 340^\circ - \cos 60^\circ] \\
 &= \frac{1}{2} [\cos (90^\circ + 30^\circ) + \cos 0^\circ + \cos 40^\circ - \cos (100^\circ + 60^\circ) - \cos (360^\circ - 20^\circ) - \frac{1}{2}] \\
 &= \frac{1}{2} \left[-\sin 30^\circ + 2 \cos \left(\frac{30^\circ + 40^\circ}{2} \right) \cos \left(\frac{80^\circ - 40^\circ}{2} \right) - \cos 60^\circ - \cos 20^\circ - \frac{1}{2} \right] \\
 &= \frac{1}{2} \left[-\frac{1}{2} + 2 \cos 60^\circ \cos 20^\circ - \frac{1}{2} - \cos 20^\circ - \frac{1}{2} \right] \\
 &= \frac{1}{2} \left[-\frac{3}{2} + 2 \times \frac{1}{2} \times \cos 20^\circ - \cos 20^\circ \right] \\
 &= \frac{1}{2} \left[-\frac{3}{2} + \cos 20^\circ - \cos 20^\circ \right] \\
 &= \frac{1}{2} \left[-\frac{3}{2} + 0 \right] \\
 &= -\frac{3}{4} \\
 &= \text{R.H.S.}
 \end{aligned}$$

$$\therefore \cos 20^\circ \cos 100^\circ + \cos 100^\circ \cos 140^\circ - \cos 140^\circ \cos 200^\circ = -\frac{3}{4} \quad \text{Hence proved.}$$

Q6(vi)

We have,

$$\begin{aligned}
 \text{LHS} &= \sin \frac{\theta}{2} \sin \frac{7\theta}{2} + \sin \frac{3\theta}{2} \sin \frac{11\theta}{2} \\
 &= \frac{1}{2} \left[2 \sin \frac{7\theta}{2} \sin \frac{\theta}{2} + 2 \sin \frac{11\theta}{2} \sin \frac{3\theta}{2} \right] \\
 &= \frac{1}{2} \left[\cos \left(\frac{7\theta}{2} - \frac{\theta}{2} \right) - \cos \left(\frac{7\theta}{2} + \frac{\theta}{2} \right) + \cos \left(\frac{11\theta}{2} - \frac{3\theta}{2} \right) - \cos \left(\frac{11\theta}{2} + \frac{3\theta}{2} \right) \right] \\
 &= \frac{1}{2} \left[\cos \frac{6\theta}{2} - \cos \frac{8\theta}{2} + \cos \frac{8\theta}{2} - \cos \frac{14\theta}{2} \right] \\
 &= \frac{1}{2} [\cos 3\theta - \cos 4\theta + \cos 4\theta - \cos 7\theta] \\
 &= \frac{1}{2} [\cos 3\theta - \cos 7\theta] \\
 &= \frac{-1}{2} [\cos 7\theta - \cos 3\theta] \\
 &= \frac{1}{2} \left[2 \sin \left(\frac{7\theta + 3\theta}{2} \right) \sin \left(\frac{7\theta - 3\theta}{2} \right) \right] \\
 &= \sin \frac{10\theta}{2} \sin \frac{4\theta}{2} \\
 &= \sin 5\theta \sin 2\theta \\
 &= \sin 2\theta \sin 5\theta \\
 &= \text{RHS}
 \end{aligned}$$

$$\therefore \sin \frac{\theta}{2} \sin \frac{7\theta}{2} + \sin \frac{3\theta}{2} \sin \frac{11\theta}{2} = \sin 2\theta \sin 5\theta.$$

Hence proved.

Q7(i)

We have,

$$\begin{aligned}
 \text{LHS} &= \frac{\sin A + \sin 3A}{\cos A - \cos 3A} \\
 &= \frac{2 \sin \left(\frac{A+3A}{2} \right) \cos \left(\frac{A-3A}{2} \right)}{-2 \sin \left(\frac{A+3A}{2} \right) \sin \left(\frac{A-3A}{2} \right)} \\
 &= \frac{-\sin 2A \times \cos(-A)}{\sin 2A \sin(-A)} \\
 &= \frac{-\cos(-A)}{\sin(-A)} \\
 &= \frac{-\cos A}{-\sin A} \\
 &= \frac{\cos A}{\sin A} \\
 &= \cot A \\
 &= \text{RHS}
 \end{aligned}$$

$$[\because \cos(-\theta) = \cos \theta \text{ and } \sin(-\theta) = -\sin \theta]$$

$$\therefore \frac{\sin A + \sin 3A}{\cos A - \cos 3A} = \cot A. \quad \text{Hence proved.}$$

Q7(ii)

We have,

$$\begin{aligned}
 \text{LHS} &= \frac{\sin 9A - \sin 7A}{\cos 7A - \cos 9A} \\
 &= \frac{2 \sin \left(\frac{9A - 7A}{2} \right) \cos \left(\frac{9A + 7A}{2} \right)}{-2 \sin \left(\frac{7A + 9A}{2} \right) \sin \left(\frac{7A - 9A}{2} \right)} \\
 &= \frac{-\sin A \cos 8A}{\sin 8A \sin (-A)} \\
 &= \frac{-\sin A \cos 8A}{-\sin A \times \sin 8A} & [\because \sin(-\theta) = -\sin \theta] \\
 &= \frac{\cos 8A}{\sin 8A} \\
 &= \cot 8A \\
 &= \text{RHS}
 \end{aligned}$$

$$\therefore \frac{\sin 9A - \sin 7A}{\cos 7A - \cos 9A} = \cot 8A. \text{ Hence proved.}$$

Q7(iii)

We have,

$$\begin{aligned}
 \text{LHS} &= \frac{\sin A - \sin B}{\cos A + \cos B} \\
 &= \frac{2 \cos \left(\frac{A+B}{2} \right) \sin \left(\frac{A-B}{2} \right)}{2 \cos \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right)} \\
 &= \frac{\sin \left(\frac{A-B}{2} \right)}{\cos \left(\frac{A-B}{2} \right)} \\
 &= \tan \left(\frac{A-B}{2} \right) \\
 &= \text{RHS}
 \end{aligned}$$

$$\therefore \frac{\sin A - \sin B}{\cos A + \cos B} = \tan \left(\frac{A-B}{2} \right). \quad \text{Hence proved.}$$

Q7(iv)

We have,

$$\begin{aligned}
 \text{LHS} &= \frac{\sin A + \sin B}{\sin A - \sin B} \\
 &= \frac{2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)}{2 \sin\left(\frac{A-B}{2}\right) \cos\left(\frac{A+B}{2}\right)} \\
 &= \frac{\sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)}{\cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)} \\
 &= \tan\left(\frac{A+B}{2}\right) \cot\left(\frac{A-B}{2}\right) \\
 &= \text{RHS}
 \end{aligned}$$

$$\therefore \frac{\sin A + \sin B}{\sin A - \sin B} = \tan\left(\frac{A+B}{2}\right) \cot\left(\frac{A-B}{2}\right). \quad \text{Hence proved.}$$

Q7(v)

We have,

$$\begin{aligned}
 \text{LHS} &= \frac{\cos A + \cos B}{\cos B - \cos A} \\
 &= \frac{2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)}{2 \sin\left(\frac{B+A}{2}\right) \sin\left(\frac{B-A}{2}\right)} \\
 &= \frac{\cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)}{\sin\left(\frac{A+B}{2}\right) \sin\left(\frac{B-A}{2}\right)} \\
 &= \frac{-\cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)}{-\sin\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)} \\
 &= \cot\left(\frac{A+B}{2}\right) \cot\left(\frac{A-B}{2}\right) \\
 &= \text{RHS}
 \end{aligned}$$

$$[\because \sin(-\theta) = -\sin\theta]$$

$$\therefore \frac{\cos A + \cos B}{\cos B - \cos A} = \cot\left(\frac{A+B}{2}\right) \cot\left(\frac{A-B}{2}\right). \quad \text{Hence proved.}$$

Q8(i)

We have,

$$\begin{aligned}
 \text{LHS} &= \frac{\sin A + \sin 3A + \sin 5A}{\cos A + \cos 3A + \cos 5A} \\
 &= \frac{(\sin 5A + \sin A) + \sin 3A}{(\cos 5A + \cos A) + \cos 3A} \\
 &= \frac{2 \sin \left(\frac{5A + A}{2} \right) \cos \left(\frac{5A - A}{2} \right) + \sin 3A}{2 \cos \left(\frac{5A + A}{2} \right) \cos \left(\frac{5A - A}{2} \right) + \cos 3A} \\
 &= \frac{2 \sin 3A \cos 2A + \sin 3A}{2 \cos 3A \cos 2A + \cos 3A} \\
 &= \frac{\sin 3A (2 \cos 2A + 1)}{\cos 3A (2 \cos 2A + 1)} \\
 &= \frac{\sin 3A}{\cos 3A} \\
 &= \tan 3A \\
 &= \text{RHS}
 \end{aligned}$$

$$\therefore \frac{\sin A + \sin 3A + \sin 5A}{\cos A + \cos 3A + \cos 5A} = \tan 3A \quad \text{Hence proved.}$$

Q8(ii)

We have,

$$\begin{aligned}
 \text{LHS} &= \frac{\cos 3A + 2 \cos 5A + \cos 7A}{\cos A + 2 \cos 3A + \cos 5A} \\
 &= \frac{(\cos 7A + \cos 3A) + 2 \cos 5A}{(\cos 5A + \cos A) + 2 \cos 3A} \\
 &= \frac{2 \cos \left(\frac{7A + 3A}{2} \right) \cos \left(\frac{7A - 3A}{2} \right) + 2 \cos 5A}{2 \cos \left(\frac{5A + A}{2} \right) \cos \left(\frac{5A - A}{2} \right) + \cos 3A} \\
 &= \frac{2 \cos 5A \cos 2A + 2 \cos 5A}{2 \cos 3A \cos 2A + 2 \cos 3A} \\
 &= \frac{2 \cos 5A (\cos 2A + 1)}{2 \cos 3A (\cos 2A + 1)} \\
 &= \frac{\cos 5A}{\cos 3A} \\
 &= \text{RHS}
 \end{aligned}$$

$$\therefore \frac{\cos 3A + 2 \cos 5A + \cos 7A}{\cos A + 2 \cos 3A + \cos 5A} = \frac{\cos 5A}{\cos 3A} \quad \text{Hence proved.}$$

Q8(iii)

We have,

$$\begin{aligned}
 \text{LHS} &= \frac{\cos 4A + \cos 3A + \cos 2A}{\sin 4A + \sin 3A + \sin 2A} \\
 &= \frac{(\cos 4A + \cos 2A) + \cos 3A}{(\sin 4A + \sin 2A) + \sin 3A} \\
 &= \frac{2 \cos \left(\frac{4A + 2A}{2} \right) \cos \left(\frac{4A - 2A}{2} \right) + \cos 3A}{2 \sin \left(\frac{4A + 2A}{2} \right) \cos \left(\frac{4A - 2A}{2} \right) + \sin 3A} \\
 &= \frac{2 \cos 3A \cos A + \cos 3A}{2 \sin 3A \cos A + \sin 3A} \\
 &= \frac{\cos 3A (2 \cos A + 1)}{\sin 3A (2 \cos A + 1)} \\
 &= \frac{\cos 3A}{\sin 3A} \\
 &= \cot 3A \\
 &= \text{RHS}
 \end{aligned}$$

$$\therefore \frac{\cos 4A + \cos 3A + \cos 2A}{\sin 4A + \sin 3A + \sin 2A} = \cot 3A \text{ Hence proved.}$$

Q8(iv)

We have,

$$\begin{aligned}
 \text{LHS} &= \frac{\sin 3A + \sin 5A + \sin 7A + \sin 9A}{\cos 3A + \cos 5A + \cos 7A + \cos 9A} \\
 &= \frac{(\sin 9A + \sin 3A) + (\sin 7A + \sin 5A)}{(\cos 9A + \cos 3A) + (\cos 7A + \cos 5A)} \\
 &= \frac{2 \sin \left(\frac{9A + 3A}{2} \right) \cos \left(\frac{9A - 3A}{2} \right) + 2 \sin \left(\frac{7A + 5A}{2} \right) \cos \left(\frac{7A - 5A}{2} \right)}{2 \cos \left(\frac{9A + 3A}{2} \right) \cos \left(\frac{9A - 3A}{2} \right) + 2 \cos \left(\frac{7A + 5A}{2} \right) \cos \left(\frac{7A - 5A}{2} \right)} \\
 &= \frac{2 \sin 6A \cos 3A + 2 \sin 6A \cos A}{2 \cos 6A \cos 3A + 2 \cos 6A \cos A} \\
 &= \frac{2 \sin 6A (\cos 3A + \cos A)}{2 \cos 6A (\cos 3A + \cos A)} \\
 &= \frac{\sin 6A}{\cos 6A} \\
 &= \tan 6A \\
 &= \text{RHS}
 \end{aligned}$$

$$\therefore \frac{\sin 3A + \sin 5A + \sin 7A + \sin 9A}{\cos 3A + \cos 5A + \cos 7A + \cos 9A} = \tan 6A$$

Hence proved.

Q8(v)

We have,

$$\begin{aligned}
 \text{LHS} &= \frac{\sin 5A - \sin 7A - \sin 9A - \sin 11A}{\cos 4A + \cos 1A - \cos 1A - \cos 1A} \\
 &= \frac{-(\sin 7A - \sin 5A) + (\sin 9A - \sin 11A)}{-(\cos 1A - \cos 1A) - (\cos 1A - \cos 4A)} \\
 &= \frac{-\left[2 \sin \left(\frac{7A+5A}{2}\right) \cos \left(\frac{7A-5A}{2}\right)\right] + \left[2 \sin \left(\frac{9A+11A}{2}\right) \cos \left(\frac{9A-11A}{2}\right)\right]}{-2 \sin \left(\frac{7A+5A}{2}\right) \cos \left(\frac{7A-5A}{2}\right) - \left[-2 \sin \left(\frac{9A+11A}{2}\right) \cos \left(\frac{9A-11A}{2}\right)\right]} \\
 &= \frac{-2 \sin 6A \cos 6A - 2 \sin 10A \cos 10A}{-2 \sin 6A \cos 6A - 2 \sin 10A \cos 10A} \\
 &= \frac{2 \sin 6A [-\cos 6A + \cos 10A]}{2 \sin 6A [-\cos 6A + \cos 10A]} \\
 &= \frac{\cos 6A}{\sin 6A} \\
 &= \cot 6A \\
 &= \text{RHS}
 \end{aligned}$$

$$\therefore \frac{\sin 5A - \sin 7A + \sin 9A - \sin 11A}{\cos 1A + \cos 7A - \cos 5A - \cos 3A} = \cot 6A \quad \text{Hence proved.}$$

Q8(vi)

We have,

$$\begin{aligned}
 \text{LHS} &= \frac{\sin 5A \cos 2A - \sin 3A \cos 4A}{\sin A \sin 2A - \cos 2A \cos 3A} \\
 &= \frac{2(\sin 5A \cos 2A - \sin 3A \cos 4A)}{2(\sin A \sin 2A - \cos 2A \cos 3A)} \\
 &= \frac{2 \sin 5A \cos 2A - 2 \sin 3A \cos 4A}{2 \sin A \sin 2A - 2 \cos 2A \cos 3A} \\
 &= \frac{\sin (5A+2A) + \sin (5A-2A) - [\sin (6A+4A) - \sin (6A-4A)]}{\cos (2A-4A) - \cos (2A+A) - [\cos (3A-2A) - \cos (3A+2A)]} \\
 &= \frac{\sin 7A + \sin 3A - \sin 7A - \sin 5A}{\cos 4A - \cos 3A - \cos 5A - \cos A} \\
 &= \frac{\sin 3A - \sin 5A}{-\cos 3A - \cos 5A} \\
 &= \frac{-(\sin 5A - \sin 3A)}{-(\cos 5A - \cos 3A)} \\
 &= \frac{\sin 5A - \sin 3A}{\cos 5A - \cos 3A} \\
 &= \frac{2 \sin \left(\frac{5A+3A}{2}\right) \cos \left(\frac{5A-3A}{2}\right)}{2 \cos \left(\frac{5A+3A}{2}\right) \cos \left(\frac{5A-3A}{2}\right)} \\
 &= \frac{\sin 4A \cos 4A}{\cos 4A \cos 4A} \\
 &= \frac{\sin 4A}{\cos 4A} \\
 &= \tan 4A \\
 &= \text{RHS}
 \end{aligned}$$

$$\therefore \frac{\sin 5A \cos 2A - \sin 3A \cos 4A}{\sin A \sin 2A - \cos 2A \cos 3A} = \tan 4A \quad \text{Hence proved.}$$

Q8(vii)

We have,

$$\begin{aligned}
 \text{LHS} &= \frac{\sin 11A \sin A + \sin 7A \sin 3A}{\cos 11A \sin A + \cos 7A \sin 3A} \\
 &= \frac{2(\sin 11A \sin A + \sin 7A \sin 3A)}{2(\cos 11A \sin A + \cos 7A \sin 3A)} \\
 &= \frac{2 \sin 11A \sin A + 2 \sin 7A \sin 3A}{2 \cos 11A \sin A + 2 \cos 7A \sin 3A} \\
 &= \frac{\cos(11A - A) - \cos(11A + A) + \cos(7A - 3A) - \cos(7A + 3A)}{\sin(11A + A) - \sin(11A - A) + \sin(7A + 3A) - \sin(7A - 3A)} \\
 &= \frac{\cos 10A - \cos 12A + \cos 4A - \cos 10A}{\sin 12A - \sin 10A + \sin 10A - \sin 4A} \\
 &= \frac{-(\cos 12A - \cos 4A)}{\sin 12A - \sin 4A} \\
 &= \frac{-\left[-2 \sin\left(\frac{12A + 4A}{2}\right) \sin\left(\frac{12A - 4A}{2}\right)\right]}{2 \sin\left(\frac{12A - 4A}{2}\right) \cos\left(\frac{12A + 4A}{2}\right)} \\
 &= \frac{2 \sin 8A \sin 4A}{2 \sin 4A \cos 8A} \\
 &= \frac{\sin 8A}{\cos 8A} \\
 &= \tan 8A \\
 &= \text{RHS}
 \end{aligned}$$

$$\therefore \frac{\sin 11A \sin A + \sin 7A \sin 3A}{\cos 11A \sin A + \cos 7A \sin 3A} = \tan 8A$$

Hence proved.

Q8(viii)

$$\begin{aligned}
 \text{LHS} &= \frac{\sin 3A \cos 4A - \sin A \cos 2A}{\sin 4A \sin A + \cos 6A \cos A} \\
 &= \frac{2(\sin 3A \cos 4A - \sin A \cos 2A)}{2(\sin 4A \sin A + \cos 6A \cos A)} \\
 &= \frac{2\sin 3A \cos 4A - 2\sin A \cos 2A}{2\sin 4A \sin A + 2\cos 6A \cos A} \\
 &= \frac{\sin(4A + 3A) - \sin(4A - 3A) - [\sin(2A + A) - \sin(2A - A)]}{\cos(4A - A) - \cos(4A + A) + \cos(6A + A) + \cos(6A - A)} \\
 &= \frac{\sin(7A) - \sin(A) - \sin(3A) + \sin(A)}{\cos(3A) - \cos(5A) + \cos(7A) + \cos(5A)} \\
 &= \frac{\sin(7A) - \sin(3A)}{\cos(3A) - \cos(7A)} \\
 &= \frac{2\sin\left(\frac{7A-3A}{2}\right)\cos\left(\frac{7A+3A}{2}\right)}{2\cos\left(\frac{7A-3A}{2}\right)\cos\left(\frac{7A+3A}{2}\right)} \\
 &= \frac{\sin 2A}{\cos 2A} \\
 &= \tan 2A \\
 &= \text{RHS}
 \end{aligned}$$

Q8(ix)

We have

$$\begin{aligned}
 \text{LHS} &= \frac{\sin A \sin 2A + \sin 3A \sin 6A}{\sin A \cos 2A + \sin 3A \cos 6A} \\
 &= \frac{2[\sin A \sin 2A + \sin 3A \sin 6A]}{2[\sin A \cos 2A + \sin 3A \cos 6A]} \\
 &= \frac{2\sin 2A \sin A + 2\sin 6A \sin 3A}{2\cos 2A \sin A + 2\cos 6A \sin 3A} \\
 &= \frac{\cos(2A - A) - \cos(2A + A) - \cos(6A - 3A) - \cos(6A + 3A)}{\sin(2A + A) - \sin(2A - A) - \sin(6A + 3A) - \sin(6A - 3A)} \\
 &= \frac{\cos A - \cos 3A + \cos 3A - \cos 4A}{\sin 3A - \sin A + \sin 5A - \sin 9A} \\
 &= \frac{\cos A - \cos 4A}{\sin 5A - \sin 9A} \\
 &= \frac{\cos 9A - \cos A}{\sin 9A - \sin A} \\
 &= \frac{\left(-2\sin\left(\frac{9A-A}{2}\right) \times \sin\left(\frac{9A+A}{2}\right)\right)}{2\sin\left(\frac{9A-A}{2}\right) \times \cos\left(\frac{9A+A}{2}\right)} \\
 &= \frac{\sin 5A \sin A}{\sin 4A \cos 5A} \\
 &= \tan 5A \\
 &= \text{RHS}
 \end{aligned}$$

$$\frac{\sin A \sin 2A + \sin 3A \sin 6A}{\sin A \cos 2A + \sin 3A \cos 6A} = \tan 5A \quad \text{Hence proved}$$

Q8(x)

We have,

$$\begin{aligned}
\text{LHS} &= \frac{\sin A + 2 \sin 3A + \sin 5A}{\sin 3A + 2 \sin 5A + \sin 7A} \\
&= \frac{\sin 5A + \sin A + 2 \sin 3A}{\sin 7A + \sin 3A + 2 \sin 5A} \\
&= \frac{2 \sin \left(\frac{5A + A}{2} \right) \cos \left(\frac{5A - A}{2} \right) + 2 \sin 3A}{2 \sin \left(\frac{7A + 3A}{2} \right) \cos \left(\frac{7A - 3A}{2} \right) + 2 \sin 5A} \\
&= \frac{2 \sin 3A \cos 2A + 2 \sin 3A}{2 \sin 5A \cos 2A + 2 \sin 5A} \\
&= \frac{2 \sin 3A (\cos 2A + 1)}{2 \sin 5A (\cos 2A + 1)} \\
&= \frac{\sin 3A}{\sin 5A} \\
&= \text{RHS}
\end{aligned}$$

$$\therefore \frac{\sin A + 2 \sin 3A + \sin 5A}{\sin 3A + 2 \sin 5A + \sin 7A} = \frac{\sin 3A}{\sin 5A} \quad \text{Hence proved.}$$

Q8(xi)

We have,

$$\begin{aligned}
\text{LHS} &= \frac{\sin(\theta + \phi) - 2 \sin \theta + \sin(\theta - \phi)}{\cos(\theta + \phi) - 2 \cos \theta + \cos(\theta - \phi)} \\
&= \frac{\sin(\theta + \phi) + \sin(\theta - \phi) - 2 \sin \theta}{\cos(\theta + \phi) + \cos(\theta - \phi) - 2 \cos \theta} \\
&= \frac{2 \sin \left[\frac{(\theta + \phi) + (\theta - \phi)}{2} \right] \cos \left[\frac{(\theta + \phi) - (\theta - \phi)}{2} \right] - 2 \sin \theta}{2 \cos \left[\frac{(\theta + \phi) + (\theta - \phi)}{2} \right] \cos \left[\frac{(\theta + \phi) - (\theta - \phi)}{2} \right] - 2 \cos \theta} \\
&= \frac{2 \sin(\theta) \cos(\phi) - 2 \sin \theta}{2 \cos(\theta) \cos(\phi) - 2 \cos \theta} \\
&= \frac{2 \sin \theta (\cos \phi - 1)}{2 \cos \theta (\cos \phi - 1)} \\
&= \frac{\sin \theta}{\cos \theta} = \tan \theta \\
&= \text{RHS}
\end{aligned}$$

$$\therefore \frac{\sin(\theta + \phi) - 2 \sin \theta + \sin(\theta - \phi)}{\cos(\theta + \phi) - 2 \cos \theta + \cos(\theta - \phi)} = \tan \theta \quad \text{Hence proved.}$$

Q9(i)

We have,

$$\begin{aligned}
 \text{LHS} &= \sin \alpha + \sin \beta - \sin \gamma - \sin(\alpha + \beta - \gamma) \\
 &= (\sin \alpha + \sin \beta) + (\sin \gamma - \sin(\alpha + \beta - \gamma)) \\
 &= 2 \sin\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right) + 2 \sin\left(\frac{\gamma - (\alpha + \beta - \gamma)}{2}\right) \cos\left(\frac{\gamma + \alpha + \beta - \gamma}{2}\right) \\
 &= 2 \sin\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right) + 2 \sin\left(\frac{-\alpha - \beta}{2}\right) \cos\left(\frac{\alpha - \beta + 2\gamma}{2}\right) \\
 &= 2 \sin\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right) - 2 \sin\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha + \beta - 2\gamma}{2}\right) \\
 &= 2 \sin\left(\frac{\alpha + \beta}{2}\right) \left[\cos\left(\frac{\alpha - \beta}{2}\right) - \cos\left(\frac{\alpha + \beta - 2\gamma}{2}\right) \right] \\
 &= 2 \sin\left(\frac{\alpha + \beta}{2}\right) \left[\sin\left(\frac{\alpha - \beta - \alpha + \beta - 2\gamma}{2}\right) \right] \sin\left(\frac{\alpha - \beta - \alpha + \beta + 2\gamma}{2}\right) \\
 &= 2 \sin\left(\frac{\alpha + \beta}{2}\right) \left[\sin\left(\frac{-2\gamma}{2}\right) \sin\left(\frac{-2\gamma}{2}\right) \right] \\
 &= 4 \sin\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\alpha + \gamma}{2}\right) \sin\left(\frac{\beta + \gamma}{2}\right) \\
 &= 4 \sin\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\beta + \gamma}{2}\right) \sin\left(\frac{\alpha + \gamma}{2}\right) \\
 &= \text{RHS}
 \end{aligned}$$

$$\therefore \sin \alpha + \sin \beta - \sin \gamma - \sin(\alpha + \beta - \gamma) = 4 \sin\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\beta + \gamma}{2}\right) \sin\left(\frac{\alpha + \gamma}{2}\right) \quad \text{Hence proved.}$$

Q9(ii)

We have,

$$\begin{aligned}
 \text{LHS} &= \cos(A + B + C) + \cos(A - B + C) + \cos(A + B - C) + \cos(-A + B + C) \\
 &= [\cos(A + B - C) + \cos(A - B + C)] + [\cos(A + B - C) + \cos(-A + B + C)] \\
 &= 2 \cos\left\{\frac{A + B + C + A - B + C}{2}\right\} \cos\left\{\frac{A + B + C - A + B - C}{2}\right\} + 2 \left\{ \cos\left\{\frac{A + B - C - A + B + C}{2}\right\} \right. \\
 &\quad \left. \cos\left\{\frac{A + B - C + A - B - C}{2}\right\} \right\} \\
 &= 2 \cos\left\{\frac{2A + 2C}{2}\right\} \cos\left\{\frac{2B}{2}\right\} + 2 \cos\left\{\frac{2B}{2}\right\} \cos\left\{\frac{2A - 2C}{2}\right\} \\
 &= 2 \cos(A + C) \cos(B) + 2 \cos(B) \cos(A - C) \\
 &= 2 \cos(B) [\cos(A + C) + \cos(A - C)] \\
 &= 2 \cos(B) \left[2 \cos\left(\frac{A + C + A - C}{2}\right) \cos\left(\frac{A + C - A + C}{2}\right) \right] \\
 &= 2 \cos(B) [2 \cos A \cos C] \\
 &= 4 \cos A \cos B \cos C.
 \end{aligned}$$

Q10

We have,

$$\cos A + \cos B = \frac{1}{2}$$

and, $\sin A + \sin B = \frac{1}{4}$

Now,

$$\frac{\sin A + \sin B}{\cos A + \cos B} = \frac{\frac{1}{4}}{\frac{1}{2}}$$

[On dividing]

$$\Rightarrow \frac{2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)}{2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)} = \frac{1}{2}$$

$$\Rightarrow \frac{\sin\left(\frac{A+B}{2}\right)}{\cos\left(\frac{A+B}{2}\right)} = \frac{1}{2}$$

$$\Rightarrow \tan\left(\frac{A+B}{2}\right) = \frac{1}{2} \quad \text{Hence proved.}$$

Q11

We have,

$$\operatorname{cosec} A + \sec A = \operatorname{cosec} B + \sec B$$

$$\Rightarrow \sec A - \sec B = \operatorname{cosec} B - \operatorname{cosec} A$$

$$\Rightarrow \frac{1}{\cos A} - \frac{1}{\cos B} = \frac{1}{\sin B} - \frac{1}{\sin A}$$

$$\Rightarrow \frac{\cos B - \cos A}{\cos A \cos B} = \frac{\sin A - \sin B}{\sin A \sin B}$$

$$\Rightarrow \frac{\sin A \sin B}{\cos A \cos B} = \frac{\sin A - \sin B}{\cos B - \cos A}$$

$$\Rightarrow \tan A \tan B = \frac{2 \sin\left(\frac{A-B}{2}\right) \cos\left(\frac{A+B}{2}\right)}{-2 \sin\left(\frac{B-A}{2}\right) \sin\left(\frac{B+A}{2}\right)}$$

$$\Rightarrow \tan A \tan B = \frac{-\sin\left(\frac{A-B}{2}\right) \cos\left(\frac{A+B}{2}\right)}{-\sin\left(\frac{A-B}{2}\right) \sin\left(\frac{A+B}{2}\right)}$$

[$\because \sin(-\theta) = -\sin\theta$]

$$\Rightarrow \tan A \tan B = \cot\left(\frac{A+B}{2}\right) \quad \text{Hence proved.}$$

Q12

We have,

$$\sin 2A = \lambda \sin 2B$$

$$\Rightarrow \lambda = \frac{\sin 2A}{\sin 2B}$$

Now,

$$\begin{aligned} \frac{\lambda + 1}{\lambda - 1} &= \frac{\frac{\sin 2A}{\sin 2B} + 1}{\frac{\sin 2A}{\sin 2B} - 1} \\ &= \frac{\frac{\sin 2A + \sin 2B}{\sin 2B}}{\frac{\sin 2A - \sin 2B}{\sin 2B}} \\ &= \frac{\sin 2A + \sin 2B}{\sin 2A - \sin 2B} \\ &= \frac{2 \sin \left(\frac{2A + 2B}{2} \right) \cos \left(\frac{2A - 2B}{2} \right)}{2 \sin \left(\frac{2A - 2B}{2} \right) \cos \left(\frac{2A + 2B}{2} \right)} \\ &= \frac{\sin(A + B) \cos(A - B)}{\sin(A - B) \cos(A + B)} \\ &= \frac{\sin(A + B) \cos(A - B)}{\cos(A + B) \sin(A - B)} \\ &= \frac{\tan(A + B)}{\tan(A - B)} \end{aligned}$$

$$\therefore \frac{\lambda + 1}{\lambda - 1} = \frac{\tan(A + B)}{\tan(A - B)}$$

$$\Rightarrow \frac{\tan(A + B)}{\tan(A - B)} = \frac{\lambda + 1}{\lambda - 1} \quad \text{Hence proved.}$$

Q13(i)

We have,

$$\begin{aligned}
 \text{LHS} &= \frac{\cos(A+B+C) + \cos(-A+B+C) + \cos(A-B+C) + \cos(A+B-C)}{\sin(A+B+C) + \sin(-A+B+C) + \sin(A-B+C) - \sin(A+B-C)} \\
 &= \frac{2\cos\left\{\frac{A+B+C-A+B+C}{2}\right\} \cos\left\{\frac{A+B+C+A-B-C}{2}\right\} + 2\cos\left\{\frac{A-B+C+A+B-C}{2}\right\} \cos\left\{\frac{A-B+C-A-B+C}{2}\right\}}{2\sin\left\{\frac{A+B+C-A+B+C}{2}\right\} \cos\left\{\frac{A+B+C+A-B-C}{2}\right\} + 2\sin\left\{\frac{A-B+C-A-B+C}{2}\right\} \cos\left\{\frac{A-B+C+A+B-C}{2}\right\}} \\
 &= \frac{2\cos(B+C)\cos A + 2\cos A\cos(C-B)}{2\sin(B+C)\cos A + 2\sin(C-B)\cos A} \\
 &= \frac{2\cos A[\cos(B+C) + \cos(C-B)]}{2\cos A[\sin(B+C) + \sin(C-B)]} \\
 &= \frac{\cos(B+C) + \cos(C-B)}{\sin(B+C) + \sin(C-B)} \\
 &= \frac{2\cos\left\{\frac{B+C+C-B}{2}\right\} \cos\left\{\frac{B+C-C+B}{2}\right\}}{2\sin\left\{\frac{B+C+C-B}{2}\right\} \cos\left\{\frac{B+C-C+B}{2}\right\}} \\
 &= \frac{2\cos C \cos B}{2\sin C \cos B} \\
 &= \frac{\cos C}{\sin C} \\
 &= \cot C \\
 &= \text{RHS}
 \end{aligned}$$

$$\therefore \frac{\cos(A+B+C) + \cos(-A+B+C) + \cos(A-B+C) + \cos(A+B-C)}{\sin(A+B+C) + \sin(-A+B+C) + \sin(A-B+C) - \sin(A+B-C)} = \cot C.$$

Hence proved.

Q13(ii)

We have,

$$\begin{aligned}
 \text{LHS} &= \sin(B-C) \cos(A-D) + \sin(C-A) \cos(B-D) + \sin(A-B) \cos(C-D) \\
 &= \frac{1}{2} [2 \sin(B-C) \cos(A-D) + 2 \sin(C-A) \cos(B-D) + 2 \sin(A-B) \cos(C-D)] \\
 &= \frac{1}{2} \left[\sin(B-C+A-D) + \sin(B-C-A+D) + \sin(C-A+B-D) + \sin(C-A-B+D) \right. \\
 &\quad \left. + \sin(A-B+C-D) + \sin(A-B-C+D) \right] \\
 &= \frac{1}{2} \left[\sin(A+B-C-D) + \sin(B+C-A-D) + \sin(B+C-A-D) + \sin(C+D-A-B) \right. \\
 &\quad \left. + \sin(A-C-B-D) + \sin(A-D-B-C) \right] \\
 &= \frac{1}{2} \left[\sin(A+B-C-D) + \sin(A-C-B-D) + \sin(A-C-B-D) + \sin(A+B-C-D) \right. \\
 &\quad \left. + \sin(A-C-B-D) + \sin(A-D-B-C) \right] \\
 &= \frac{1}{2} [0] \\
 &= 0 \\
 &= \text{RHS}
 \end{aligned}$$

$$\sin(B-C) \cos(A-D) + \sin(C-A) \cos(B-D) + \sin(A-B) \cos(C-D) = 0 \quad \text{Hence proved.}$$

Q14

We have,

$$\begin{aligned}
 \frac{\cos(A-B)}{\cos(A+B)} &= \frac{\cos(C-D)}{\cos(C-D)} = 1 \\
 \Rightarrow \frac{\cos(A-B)}{\cos(A+B)} &= \frac{\cos(C-D)}{\cos(C-D)} \quad (i)
 \end{aligned}$$

Now,

$$\begin{aligned}
 \frac{\cos(A-B)}{\cos(A+B)} &= \frac{\cos(C-D)}{\cos(C-D)} \\
 \Rightarrow \frac{\cos(A-B)}{\cos(A+B)} &= 1 = \frac{-\cos(C+D)}{\cos(C-D)} \\
 \Rightarrow \frac{\cos(A-B) + \cos(A+B)}{\cos(A+B)} &= \frac{-\cos(C+D) + \cos(C-D)}{\cos(C-D)} \\
 \Rightarrow \frac{\cos(A+B) + \cos(A-B)}{\cos(A+B)} &= \frac{[\cos(C+D) - \cos(C-D)]}{\cos(C-D)} \quad \text{---(ii)}
 \end{aligned}$$

again,

$$\begin{aligned}
 \frac{\cos(A-B)}{\cos(A+B)} &= \frac{-\cos(C+D)}{\cos(C-D)} \quad [\text{By equation (i)}] \\
 \Rightarrow \frac{\cos(A-B)}{\cos(A+B)} &= 1 = \frac{-\cos(C+D)}{\cos(C-D)} \\
 \Rightarrow \frac{\cos(A-B) - \cos(A+B)}{\cos(A+B)} &= \frac{-\cos(C+D) - \cos(C-D)}{\cos(C-D)} \\
 \Rightarrow \frac{[\cos(A+B) - \cos(A-B)]}{\cos(A+B)} &= \frac{[\cos(C+D) - \cos(C-D)]}{\cos(C-D)} \\
 \Rightarrow \frac{\cos(A+B) - \cos(A-B)}{\cos(A+B)} &= \frac{\cos(C+D) + \cos(C-D)}{\cos(C-D)} \quad \text{---(iii)}
 \end{aligned}$$

Dividing equation (ii) by equation (iii), we get

$$\begin{aligned}
 \frac{\cos(A+B) + \cos(A-B)}{\cos(A+B) - \cos(A-B)} &= \frac{[\cos(C+D) - \cos(C-D)]}{\cos(C+D) + \cos(C-D)} \\
 \Rightarrow \frac{2 \cos \left\{ \frac{A+B+A-B}{2} \right\} \cos \left\{ \frac{A+B-A-B}{2} \right\}}{-2 \sin \left\{ \frac{A+B+A-B}{2} \right\} \sin \left\{ \frac{A+B-A-B}{2} \right\}} &= \frac{2 \sin \left\{ \frac{C+D+C-D}{2} \right\} \sin \left\{ \frac{C+D-C-D}{2} \right\}}{2 \cos \left\{ \frac{C+D+C-D}{2} \right\} \cos \left\{ \frac{C+D-C-D}{2} \right\}} \\
 \Rightarrow \frac{\cos A \cos B}{-\sin A \sin B} &= \frac{\sin C \sin D}{\cos C \cos D} \\
 \Rightarrow \frac{1}{-\tan A \tan B} &= \tan C \tan D \\
 \Rightarrow -1 &= \tan A \tan B \tan C \tan D
 \end{aligned}$$

$$\therefore \tan A \tan B \tan C \tan D = -1 \quad \text{Hence proved.}$$

Q15

We have,

$$\begin{aligned} \cos(\alpha + \beta) \sin(\gamma + \delta) &= \cos(\alpha - \beta) \sin(\gamma - \delta) \\ \Rightarrow \frac{\cos(\alpha + \beta)}{\cos(\alpha - \beta)} &= \frac{\sin(\gamma - \delta)}{\sin(\gamma + \delta)} \end{aligned} \quad \text{---(i)}$$

Now,

$$\begin{aligned} \frac{\cos(\alpha + \beta)}{\cos(\alpha - \beta)} &= \frac{\sin(\gamma - \delta)}{\sin(\gamma + \delta)} \\ \Rightarrow \frac{\cos(\alpha + \beta)}{\cos(\alpha - \beta)} + 1 &= \frac{\sin(\gamma - \delta)}{\sin(\gamma + \delta)} + 1 \\ \Rightarrow \frac{\cos(\alpha + \beta) + \cos(\alpha - \beta)}{\cos(\alpha - \beta)} &= \frac{\sin(\gamma - \delta) + \sin(\gamma + \delta)}{\sin(\gamma + \delta)} \end{aligned} \quad \text{---(ii)}$$

Again,

$$\begin{aligned} \frac{\cos(\alpha + \beta)}{\cos(\alpha - \beta)} &= \frac{\sin(\gamma - \delta)}{\sin(\gamma + \delta)} \quad [\text{By equation (i)}] \\ \Rightarrow \frac{\cos(\alpha + \beta)}{\cos(\alpha - \beta)} - 1 &= \frac{\sin(\gamma - \delta)}{\sin(\gamma + \delta)} - 1 \\ \Rightarrow \frac{\cos(\alpha + \beta) - \cos(\alpha - \beta)}{\cos(\alpha - \beta)} &= \frac{\sin(\gamma - \delta) - \sin(\gamma + \delta)}{\sin(\gamma + \delta)} \end{aligned} \quad \text{---(iii)}$$

Dividing equation (ii) by equation (iii), we get

$$\begin{aligned} \frac{\cos(\alpha + \beta) + \cos(\alpha - \beta)}{\cos(\alpha + \beta) - \cos(\alpha - \beta)} &= \frac{\sin(\gamma - \delta) + \sin(\gamma + \delta)}{\sin(\gamma - \delta) - \sin(\gamma + \delta)} \\ \Rightarrow \frac{\cos(\alpha + \beta) + \cos(\alpha - \beta)}{\cos(\alpha + \beta) - \cos(\alpha - \beta)} &= - \left[\frac{\sin(\gamma + \delta) + \sin(\gamma - \delta)}{\sin(\gamma + \delta) - \sin(\gamma - \delta)} \right] \\ \Rightarrow \frac{2 \cos \left\{ \frac{\alpha + \beta + \alpha - \beta}{2} \right\} \cos \left\{ \frac{\alpha + \beta - \alpha + \beta}{2} \right\}}{-2 \sin \left\{ \frac{\alpha + \beta + \alpha - \beta}{2} \right\} \sin \left\{ \frac{\alpha + \beta - \alpha + \beta}{2} \right\}} &= - \left[\frac{2 \sin \left\{ \frac{\gamma + \delta + \gamma - \delta}{2} \right\} \cos \left\{ \frac{\gamma + \delta - \gamma + \delta}{2} \right\}}{2 \sin \left\{ \frac{\gamma + \delta - \gamma + \delta}{2} \right\} \cos \left\{ \frac{\gamma + \delta + \gamma - \delta}{2} \right\}} \right] \\ \Rightarrow \frac{\cos \alpha \cos \beta}{\sin \alpha \sin \beta} &= \frac{\sin \gamma \cos \delta}{\sin \delta \cos \gamma} \\ \Rightarrow \cot \alpha \cot \beta &= \frac{\sin \gamma \cos \delta}{\cos \gamma \sin \delta} \\ \Rightarrow \cot \alpha \cot \beta &= \frac{\cot \delta}{\cot \gamma} \\ \Rightarrow \cot \alpha \cot \beta \cot \gamma &= \cot \delta \end{aligned}$$

$$\therefore \cot \alpha \cot \beta \cot \gamma = \cot \delta$$

Hence proved.

Q16

We have,

$$y \sin \phi = x \sin (2\theta + \phi)$$

$$\Rightarrow \frac{\sin \phi}{\sin (2\theta + \phi)} = \frac{x}{y} \quad \text{--- (i)}$$

Now,

$$\frac{\sin \phi}{\sin (2\theta + \phi)} = \frac{x}{y}$$

$$\Rightarrow \frac{\sin \phi}{\sin (2\theta + \phi)} + 1 = \frac{x}{y} + 1$$

$$\Rightarrow \frac{\sin \phi + \sin (2\theta + \phi)}{\sin (2\theta + \phi)} = \frac{x + y}{y} \quad \text{--- (ii)}$$

Again,

$$\frac{\sin \phi}{\sin (2\theta + \phi)} = \frac{x}{y} \quad \text{[By equation (i)]}$$

$$\Rightarrow \frac{\sin \phi}{\sin (2\theta + \phi)} - 1 = \frac{x}{y} - 1$$

$$\Rightarrow \frac{\sin \phi - \sin (2\theta + \phi)}{\sin (2\theta + \phi)} = \frac{x - y}{y} \quad \text{--- (iii)}$$

Dividing equation (ii) by equation (iii), we get

$$\frac{\sin \phi + \sin (2\theta + \phi)}{\sin \phi - \sin (2\theta + \phi)} = \frac{x + y}{x - y}$$

$$\Rightarrow \frac{2 \sin \left(\frac{\phi + 2\theta + \phi}{2} \right) \cos \left(\frac{\phi - 2\theta - \phi}{2} \right)}{2 \sin \left(\frac{\phi - 2\theta - \phi}{2} \right) \cos \left(\frac{\phi + 2\theta + \phi}{2} \right)} = \frac{x + y}{x - y}$$

$$\Rightarrow \frac{\sin (\theta + \phi) \cos (\theta - \phi)}{\sin (-\theta) \cos (\theta + \phi)} = \frac{x + y}{x - y}$$

$$\Rightarrow \frac{\sin (\theta + \phi) \cos (\theta)}{\cos (\theta + \phi) [-\sin (\theta)]} = \frac{x + y}{x - y}$$

$$\Rightarrow \frac{-\cot (\theta)}{\cot (\theta + \phi)} = \frac{x + y}{x - y}$$

$$\Rightarrow -(x - y) \cot \theta = (x + y) \cot (\theta + \phi)$$

$$\Rightarrow (y - x) \cot \theta = (x + y) \cot (\theta + \phi)$$

$$\Rightarrow (x + y) \cot (\theta + \phi) = (y - x) \cot \theta$$

Hence proved.

Q17

We have,

$$\begin{aligned} \cos(A+B) \sin(C-D) &= \cos(A-B) \sin(C+D) \\ \Rightarrow \frac{\cos(A+B)}{\cos(A-B)} &= \frac{\sin(C+D)}{\sin(C-D)} \end{aligned} \quad \text{---(i)}$$

Now,

$$\begin{aligned} \frac{\cos(A+B)}{\cos(A-B)} &= \frac{\sin(C+D)}{\sin(C-D)} \\ \Rightarrow \frac{\cos(A+B)}{\cos(A-B)} + 1 &= \frac{\sin(C+D)}{\sin(C-D)} + 1 \\ \Rightarrow \frac{\cos(A+B) + \cos(A-B)}{\cos(A-B)} &= \frac{\sin(C+D) + \sin(C-D)}{\sin(C-D)} \end{aligned} \quad \text{---(ii)}$$

Again,

$$\begin{aligned} \frac{\cos(A+B)}{\cos(A-B)} &= \frac{\sin(C+D)}{\sin(C-D)} \quad [\text{By equation (i)}] \\ \Rightarrow \frac{\cos(A+B)}{\cos(A-B)} - 1 &= \frac{\sin(C+D)}{\sin(C-D)} - 1 \\ \Rightarrow \frac{\cos(A+B) - \cos(A-B)}{\cos(A-B)} &= \frac{\sin(C+D) - \sin(C-D)}{\sin(C-D)} \end{aligned} \quad \text{---(iii)}$$

Dividing equation (ii) by equation (iii), we get

$$\begin{aligned} \frac{\cos(A+B) + \cos(A-B)}{\cos(A+B) - \cos(A-B)} &= \frac{\sin(C+D) + \sin(C-D)}{\sin(C+D) - \sin(C-D)} \\ \Rightarrow \frac{2 \cos \left\{ \frac{A+B+A-B}{2} \right\} \cos \left\{ \frac{A+B-A+B}{2} \right\}}{-2 \sin \left\{ \frac{A+B+A-B}{2} \right\} \sin \left\{ \frac{A+B-A+B}{2} \right\}} &= \frac{2 \sin \left\{ \frac{C+D+C-D}{2} \right\} \cos \left\{ \frac{C+D-C+D}{2} \right\}}{2 \sin \left\{ \frac{C+D-C+D}{2} \right\} \cos \left\{ \frac{C+D+C-D}{2} \right\}} \\ \Rightarrow \frac{\cos A \cos B}{-\sin A \sin B} &= \frac{\sin C \cos D}{\sin D \cos C} \\ \Rightarrow \frac{1}{-\tan A \tan B} &= \frac{\sin C \cos D}{\cos C \sin D} \\ \Rightarrow \frac{-1}{\tan A \tan B} &= \frac{\tan C}{\tan D} \\ \Rightarrow -\tan D &= \tan A \tan B \tan C \\ \Rightarrow \tan A \tan B \tan C &= -\tan D \\ \Rightarrow \tan A \tan B \tan C + \tan D &= 0 \end{aligned} \quad \text{Hence proved.}$$

Q18

$$\text{Given } x \cos \theta = y \cos \left(\theta + \frac{2\pi}{3} \right) = z \cos \left(\theta + \frac{4\pi}{3} \right) = k (\text{say})$$

$$x = \frac{k}{\cos \theta}$$

$$y = \frac{k}{\cos \left(\theta + \frac{2\pi}{3} \right)}$$

$$z = \frac{k}{\cos \left(\theta + \frac{4\pi}{3} \right)}$$

$$xy + yz + zx = k^2 \left[\frac{1}{\cos \theta \cos \left(\theta + \frac{2\pi}{3} \right)} + \frac{1}{\cos \left(\theta + \frac{2\pi}{3} \right) \cos \left(\theta + \frac{4\pi}{3} \right)} + \frac{1}{\cos \left(\theta + \frac{4\pi}{3} \right) \cos \theta} \right]$$

$$= k^2 \left[\frac{\cos \left(\theta + \frac{4\pi}{3} \right) + \cos \theta + \cos \left(\theta + \frac{2\pi}{3} \right)}{\cos \theta \cos \left(\theta + \frac{2\pi}{3} \right) \cos \left(\theta + \frac{4\pi}{3} \right)} \right]$$

$$= k^2 \left[\frac{\cos \theta \cos \frac{4\pi}{3} - \sin \theta \sin \frac{4\pi}{3} + \cos \theta + \cos \theta \cos \frac{2\pi}{3} - \sin \theta \sin \frac{2\pi}{3}}{\cos \theta \cos \left(\theta + \frac{2\pi}{3} \right) \cos \left(\theta + \frac{4\pi}{3} \right)} \right]$$

$$= k^2 \left[\frac{\cos \theta \left(\frac{-1}{2} \right) - \sin \theta \left(\frac{-\sqrt{3}}{2} \right) + \cos \theta + \cos \theta \left(\frac{-1}{2} \right) - \sin \theta \left(\frac{\sqrt{3}}{2} \right)}{\cos \theta \cos \left(\theta + \frac{2\pi}{3} \right) \cos \left(\theta + \frac{4\pi}{3} \right)} \right]$$

$$= k^2 \left[\frac{-\cos \theta + \sin \theta \left(\frac{\sqrt{3}}{2} \right) + \cos \theta + -\sin \theta \left(\frac{\sqrt{3}}{2} \right)}{\cos \theta \cos \left(\theta + \frac{2\pi}{3} \right) \cos \left(\theta + \frac{4\pi}{3} \right)} \right]$$

$$= 0$$

Hence Proved

Q19

Given that $m \sin \theta = n \sin(\theta + 2a)$,

We need to prove that $\tan(\theta + a) = \frac{m+n}{m-n} \tan a$

$$m \sin \theta = n \sin(\theta + 2a)$$

$$\Rightarrow \frac{\sin(\theta + 2a)}{\sin \theta} = \frac{m}{n}$$

Using Componendo – Dividendo, we have,

$$\Rightarrow \frac{\sin(\theta + 2a) + \sin \theta}{\sin(\theta + 2a) - \sin \theta} = \frac{m+n}{m-n} \dots (1)$$

We know that,

$$\sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2}$$

and

$$\sin C - \sin D = 2 \cos \frac{C+D}{2} \sin \frac{C-D}{2}$$

Applying the above formulae in equation (1), we have,

$$\frac{2 \sin \frac{\theta+2a+\theta}{2} \cos \frac{\theta+2a-\theta}{2}}{2 \cos \frac{\theta+2a+\theta}{2} \sin \frac{\theta+2a-\theta}{2}} = \frac{m+n}{m-n}$$

$$\Rightarrow \frac{2 \sin(\theta + a) \cos a}{2 \cos(\theta + a) \sin a} = \frac{m+n}{m-n}$$

$$\Rightarrow \frac{\tan(\theta + a)}{\tan a} = \frac{m+n}{m-n}$$

$$\Rightarrow \tan(\theta + a) = \frac{m+n}{m-n} \times \tan a$$

Hence proved.