EX - 31.1

Q1(i)

A statement or a proposition is an assertive (or a declarative) sentence which is either true or false but not both.

The sentence "Listen to me , Ravi!" is an exclamatory sentence. So, it is not a statement.

Q1(ii)

A statement or a proposition is an assertive (or a declarative) sentence which is either true or false but not both.

This sentence is always false, because there are sets which are not finite. Hence, it is a statement.

Q1(iii)

A statement or a proposition is an assertive (or a declarative) sentence which is either true or false but not both.

This sentence is always false, because there are non-empty sets whose intersection is empty. Hence, it is a statement.

Q1(iv)

A statement or a proposition is an assertive (or a declarative) sentence which is either true or false but not both.

Some cats are black and some not. So, the given sentence may or may not be true. Hence, it is not a statement

Q1(v)

A statement or a proposition is an assertive (or a declarative) sentence which is either true or false but not both.

The sentence "Are all circles round ?" is an interrogative sentence. So, it is not a statement.

Q1(vi)

A statement or a proposition is an assertive (or a declarative) sentence which is either true or false but not both.

This sentence is always false, because there are rhombuses that are not squares. Hence, it is a statement.

Q1(vii)

A statement or a proposition is an assertive (or a declarative) sentence which is either true or false but not both.

This sentence is always false, because there are rhombuses that are not squares. Hence, it is a statement.

Q1(viii)

A statement or a proposition is an assertive (or a declarative) sentence which is either true or false but not both.

If $x>0$,

$x^2 + 5|x| + 6 = 0$
$\Rightarrow x^2 + 5x + 6 = 0$
$\Rightarrow x = -3$ or $x = -2$
But, since $x > 0$, the equation has no roots.

If $x<0$,
$x^2 + 5|x| + 6 = 0$
$\Rightarrow x^2 - 5x + 6 = 0$
which has no real roots.
So, the sentence $x^2 + 5x + 6 = 0$ is always true.

Hence, it is a statement.

Q1(ix)

It is not a statement.

The sentence "This sentence is a statement." cannot be assigned a truth value of either true or false, because either assignment contradicts the sense of the sentence.

Q1(x)

A statement or a proposition is an assertive (or a declarative) sentence which is either true or false but not both.

The sentence "Is the earth round ?" is an interrogative sentence. So, it is not a statement.

Q1(xi)

A statement or a proposition is an assertive (or a declarative) sentence which is either true or false but not both.

The sentence "Go!" is an exclamatory sentence. So, it is not a statement.

Q1(xii)

A statement or a proposition is an assertive (or a declarative) sentence which is either true or false but not both.

"The real number x is less than 2" is not a statement, because its truth or falsity cannot be confirmed without knowing the value of x.

Q1(xiii)

A statement or a proposition is an assertive (or a declarative) sentence which is either true or false but not both.

"There are 35 days in a month" is a false declarative sentence. So, it is a false statement.

Q1(xiv)

A statement or a proposition is an assertive (or a declarative) sentence which is either true or false but not both.

"Mathematics is difficult" is true for those who may not like mathematics. But, for others, it may not be true. So, the given sentence may or may not be true. Hence, it is not a statement

Q1(xv)

A statement or a proposition is an assertive (or a declarative) sentence which is either true or false but not both.

This sentence is always true. Hence, it is a statement.

Q1(xvi)

A statement or a proposition is an assertive (or a declarative) sentence which is either true or false but not both.

This sentence is always false, because $\left(-1\times 8=-8\right)$. Hence, it is a statement.

Q2

A statement or a proposition is an assertive (or a declarative) sentence which is either true or false but not both.

Example $\left(1\right)$:
" Who lost this watch?" is an interrogative sentence. Hence, it is not a statement.

Example $\left(2\right)$:
The sentence: $x+2=9$ is an open sentence. Its truth value cannot be confirmed unless we are given the value of x. So, it is not a statement.

Example $\left(3\right)$:
The sentence "May god bless you!" is an exclamatory sentence. So, it is not a statement.

Ex 31.2

Q1

The negation of the statement:

Banglore is the capital of Karnataka.

is

Banglore is not the capital of Karnataka.

The negation of the statement:

It rained on July 4,2005.

is

It did not rain on July 4,2005.

The negation of the statement:

Ravish is honest.

is

Ravish is not honest.

The negation of the statement:

The earth is round.

is

The earth is not round.

The negation of the statement:

The sun is cold.

is

The sun is not cold.

Q2

The negation of the statements

All birds sing.

is

Not all birds sing.

The negation of the statements

Some even integers are prime.

is

No even integers is prime.

The negation of the statements

There is a complex number which is not a real number.

is

All complex numbers are real numbers.

The negation of the statements

I will not go to school.

is

I will go to school.

The negation of the statements

Both the diagonals of a rectangle have the same length.

is

There is at least one rectangle whose both diagonals do not have the same length.

The negation of the statements

All policemen are thieves.

is

No policemen is thief.

Q3

(i) The number x is not a rational number.

$\Rightarrow$ The number x is an irrational number.

$\therefore$ The statement "The number x is not an irrational number." is a negation of the first statement.

(ii) The number x is not a rational number.

$\Rightarrow$ The number x is an irrational number.

$\therefore$ The statement "The number x is an irrational number" is not a negation of the first statement.

Q4

(i)

The negation of the statement:

p : For every positive real number x, the number $(x-1)$ is also positive.

is

$\sim p$: There exists a positive real number x such that the number $(x-1)$ is not positive.

(ii)

The negation of the statement:

q : For every real number x, either $x > 1$ or $x < 1$.

is

$\sim q$: There exists a real number such that neither $x > 1$ or $x < 1$.

(iii)

The negation of the statement:

r : There are exists a number x such that $0 < x < 1$.

is

$\sim r$: For every real number x, either $x \leq$ or $x \geq 1$.

Q5

The negation of the statement

$a + b = b + a$ is true for every real number a and b.

is:

There exist real numbers a and b for which $a + b \neq b + a$.

So, the given statement is not the negation of the first statement.

Ex 31.3

Q1

(i)
The components of the compound statement:

The sky is blue and the grass is green.

are:

p : The sky is blue

q : The grass is green

(ii)
The components of the compound statement:

The earth is round or the sun is cold.

are:

p : The earth is round

q : The sun is cold.

(iii)
The components of the compound statement:

All rational numbers are real and all real numbers are complex.

are:

p : All rational numbers are real

q : All real numbers are complex

(iv)
The components of the compound statement:

25 is a multiple of 5 and 8.

are:

p : 25 is multiple of 5

q : 25 is multiple of 8.

Q2

(i)

In the statement

Students can take Hindi or Sanskrit as their third language.

an exclusive"OR" is used because

A student cannot take both Hindi and Sanskrit as the third language.

(ii)

In the statement

To entry a country, you need a passport or a voter registraion card.

an indusive "OR" is used because

Since a person can have both a passport and a voter registration card to enter a country.

(iii)

In the statement

A lady gives birth to a baby boy or a baby girl.

an exclusive"OR" is used because

A lady cannot give birth to a baby who is both a boy and a girl.

(iv)

In the statement

To apply for a driving licence, you should have a ration card or a passport.

an indusive "OR" is used because

A person can have both a ration card and passport to apply for a driving licence.

Q3

(i)

The component statements of the compound statement

To enter into a public library children need an identity card from the school or a letter from the school authorities.

are

p : To get into a public library children need an identity card.

q : To get into a public library children need a letter from the school authorities.

We know that if p and q are true then p or q must also be true.
Hence, the compound statement is true.

(ii)

The component statements of the compound statement

All rational numbers are real and all real numbers are not complex.

are

p : All rational numbers are real.

q : All real numbers are not complex.

We know that p is true and q is false.
∴ The compound statement "p and q" is false.

(iii)

The component statements of the compound statement

Square of an integer is positive or negative.

are

p : Square of an integer is positive.

q : Square of an integer is negative.

We know that if p and q are true then p or q must also be true.
Hence, the compound statement is true.

Q4

(i)

The component statements of the compound statement

Delhi is in India and $2+2=4$.

are

p : Delhi is in India.

q : $2+2=4$.

We know that if p and q are true then p or q must also be true.
Hence, the compound statement is true.

(ii)

The component statements of the compound statement

Delhi is in England and $2+2=4$.

are

p : Delhi is in England

q : $2+2=4$.

Here p is false and q is true. So, p and q must be false.
Hence, the compound statement is false.

Ex - 31.4

Q1

(i)

The negation of the statement

For every $x \in N, x + 3 < 10$

is

There exists $x \in N$ such that $x + 3 \geq 10$.

(ii)

The negation of the statement

There exists $x \in N, x + 3 = 10$

is

For every $x \in N, x + 3 \neq 10$

Q2

(i)

The negation of the statement

All the students completed their homework.

is

Some of the students did not complete their home work.

(ii)

The negation of the statement

There exists a number which is equal to its square.

is

For every real number $x, x^2 \neq x$.

Ex - 31.5

Q1

(i) If you access the website, then you pay a subcription fee.
(ii) If it rains, then there is traffic jam.
(iii) If you log on to the server, then you must have a passport.
(iv) If he is happy, then he is rich.
(v) If it is raining, then the game is cancelled.
(vi) If it rains, then it is cold.
(vii) If it rains, then it is cold.
(viii) If it is cold, then it never rains.

Q2

(i) Converse: If you feel thirsty, then it is hot outside.
Contrapositive: If you do not feel thirsty, thenit is not hot outside.
(ii) Converse: If I go to a beach,then it is a sunny day.
Contrapositive: If I do not go to a beach, then it is not a sunny day.
(iii) Converse: If an integer has no divisors other than 1 and itself, then it is prime.
Contrapositive: If an integer has some divisors other than 1 and itself, then it is not prime.
(iv) Converse: If you have winter clothes, then you live in Delhi.
Contrapositive: If you do not have winter clothes, then you do not live in Delhi.
(v) Converse: If the diagonals of a quadrilateral bisect each other, then is is a parallelogram.
Contrapositive: If the diagonals of a quadrilateral do not bisect each other, then the quadrilateral is not a parallelogram.

Q3

(i) You watch television if and only if your mind is free.
(ii) A quadrilateral is a rectangle if and only if it is equiangular.
(iii) You get an A grade if and only if you do all the homework regularly.
(iv) A tumbler is half empty if and only if it is half full.

Q4

(i) If Mohan is not poor, then he is not a poet.
(ii) If Max does not study, then he will not pass the test.
(iii) If she does not earn money, then she does not work.
(iv) If then they do not drive the car, then there is no snow.
(v) If it rains, then it is not cold.
(vi) If it did not snow, then Ravish will not ski.
(vii) If x is positive, then x is not less than zero.
(viii) If he does not win, then he does not have courage.
(ix) If he is not strong, then he is not a sailor.
(x) If he tires, then he will not win.
(xi) If x is even, then x^2 is even.

Q1

The statement is:

"100 is multiple of 4 and 5"

We know that 100 is a multiple of 4 as well as 5. So, p is true statement.
Hence, the statement is true i.e. the statement "p"
is a valid statement.

Q2

Let q and r be the statements given by
q : x and y are odd integers.
r : $x+y$ is an even integer.
Then, the given statement is

if q, then r.

Direct Method : Let q be true. Then,

q is true.

$\Rightarrow$ x and y are odd integers
$\Rightarrow$ $x=2m+1$, $y=2n+1$ for some integers m, n
$\Rightarrow$ $x+y=(2m+1)+(2n+1)$
$\Rightarrow$ $x+y=(2m+2n+2)$
$\Rightarrow$ $x+y=2(m+n+1)$
$\Rightarrow$ $x+y$ is an even integer
$\Rightarrow$ r is true.

Thus, q is true $\Rightarrow$ r is true.
Hence, "if q, then r" is a true statement.

Q3

Let q and r be the statements given

q : x is a real number such that $x^3 + x = 0$.

r : x is 0.

Then, p : if q, then r.

(i) *Direct Method* : Let q be true. Then,

q is true

$\Rightarrow$ x is a real number such that $x^3 + x = 0$

$\Rightarrow$ x is a real number such that $x\left(x^2 + 1\right) = 0$

$\Rightarrow$ $x = 0$

$\Rightarrow$ r is true.

Thus, q is true $\Rightarrow$ r is true.

Hence, p is true.

(ii) *Method of contrapositive* : Let r be not true. Then,

r is not true.

$\Rightarrow$ $x \neq 0, x \in R$

$\Rightarrow$ $x(x^2 + 1) \neq 0$, $x \in R$

$\Rightarrow$ q is not true

Thus, $-r \Rightarrow -q$.

Hence, $p : q \Rightarrow r$ is true.

(iii) *Method of contradiction* : If possible, let p be not true. Then,

p is not true

$\Rightarrow$ $-p$ is true

$\Rightarrow$ $-\left(p \Rightarrow r\right)$ is true

$\Rightarrow$ q and $-r$ is true

$\Rightarrow$ x is a real number such that $x^3 + x = 0$ and $x \neq 0$

$\Rightarrow$ $x = 0$ and $x \neq 0$

This a contradiction.

Hence, p is true.

Q4

Let q and r be the statements given by
q : If x is an integer and x^2 is odd
r : x is an odd integer.

Then, p : "If q, then r."
If possible, let r be false. Then,

r is false
$\Rightarrow$ x is not an odd integer
$\Rightarrow$ x is an even integer
$\Rightarrow$ $x = (2n)$ for some integer n
$\Rightarrow$ $x^2 = 4n^2$
$\Rightarrow$ x^2 is an even integer
$\Rightarrow$ q is false.

Thus, r is false $\Rightarrow$ q is false.
Hence, p : "if q, then r" is a true statement.

Q5

The given statement can be re-written as
"*The necessary and sufficient condition that the integer n is even is* n^2 *must be even*"

Let p and q be the statements given by

p : the integer n is even.

q : n^2 is even.

The given statement is

"*p if and only if q*"

In order to check its validity, we have to check the validity of the following statements.
(i) "*If p, then q*"
(ii) "*if q, then p*"

Checking the validity of "if p, then q":

The statement "if p, then q" is given by:

"*if the integer n is even, then* n^2 *is even*"

Let us assume that n is even. Then,

$n = 2m$, where m is an integer

$\Rightarrow \quad n^2 = (2m)^2$

$\Rightarrow \quad n^2 = 4m^2$

$\Rightarrow \quad n^2$ is an even integer

Thus, n is even $\Rightarrow$ n^2 is even

$\therefore$ "if p, then q" is true.

Checking the validity of "if q, then p":

"*if n is an integer and* n^2 *is even, then n is* even"

Q6

Consider a triangle ABC with all angles equal. Then each angle of the triangle is equal to 60°.
Hence, ABC is not an obtuse angle triangle.
Therefore the following statement is false.
p : "*if all the angles of a triangle are equal, then the triangle is an obtuse angled triangle*".

Q7

(i) False. Because, no radius of a circle is its chord.
(ii) False. Because, a chord does not have to pass through the centre.
(iii) True. Because a circle is an ellipse that has equal axes.
(iv) True. Because, for any two integers, if $x - y$ is positive then $-(x - y)$ is negative.
(v) False. Because square roots of prime numbers are irrational numbers.

Q8

The argument used to check the validity of the given statement is not correct because it does not produce a contradiction.