

Ex 9.1

Continuity Ex 9.1 Q1

We have to check the continuity of function at $x = 0$.

$$\text{L.H.L} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0 - h) = \lim_{h \rightarrow 0} \frac{-h}{|-h|} = \lim_{h \rightarrow 0} \frac{-h}{h} = -1$$

$$\text{R.H.L} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0 + h) = \lim_{h \rightarrow 0} \frac{h}{|h|} = 1$$

Thus, $\text{LHL} \neq \text{RHL}$

So, the given function is discontinuous and the discontinuity is of first kind.

Continuity Ex 9.1 Q2

We have, to check the continuity at $x = 3$.

$$\text{L.H.L} = \lim_{x \rightarrow 3^-} f(x) = \lim_{h \rightarrow 0} f(3 - h) = \lim_{h \rightarrow 0} \frac{(3 - h)^2 - (3 - h) - 6}{(3 - h) - 3} = \lim_{h \rightarrow 0} \frac{h^2 - 5h}{-h} = \lim_{h \rightarrow 0} -h + 5 = 5$$

$$\text{R.H.L} = \lim_{x \rightarrow 3^+} f(x) = \lim_{h \rightarrow 0} f(3 + h) = \lim_{h \rightarrow 0} \frac{(3 + h)^2 - (3 + h) - 6}{(3 + h) - 3} = \lim_{h \rightarrow 0} \frac{h^2 + 5h}{h} = \lim_{h \rightarrow 0} h + 5 = 5$$

$$f(3) = 5$$

Thus, we have, $\text{LHL} = \text{RHL} = f(3) = 5$

So, The function is continuous at $x = 3$

Continuity Ex 9.1 Q3

We have, to check the continuity of the function at $x = 3$,

$$\text{LHL} = \lim_{x \rightarrow 3^-} f(x) = \lim_{h \rightarrow 0} f(3-h) = \lim_{h \rightarrow 0} \frac{(3-h)^2 - 9}{(3-h) - 3} = \lim_{h \rightarrow 0} \frac{h^2 - 6h}{-h} = \lim_{h \rightarrow 0} -h + 6 = 6$$

$$\text{RHL} = \lim_{x \rightarrow 3^+} f(x) = \lim_{h \rightarrow 0} f(3+h) = \lim_{h \rightarrow 0} \frac{(3+h)^2 - 9}{(3+h) - 3} = \lim_{h \rightarrow 0} \frac{h^2 + 6h}{h} = \lim_{h \rightarrow 0} h + 6 = 6$$

$$f(3) = 6$$

Thus, we have, $\text{LHL} = \text{RHL} = f(3) = 6$

So, the given function is continuous at $x = 3$.

Continuity Ex 9.1 Q4

We want, to check the continuity of the function at $x = 1$.

$$\text{LHL} = \lim_{x \rightarrow 1^-} f(x) = \lim_{h \rightarrow 0} f(1-h) = \lim_{h \rightarrow 0} \frac{(1-h)^2 - 1}{(1-h) - 1} = \lim_{h \rightarrow 0} \frac{h^2 - 2h}{-h} = \lim_{h \rightarrow 0} -h + 2 = 2$$

$$\text{RHL} = \lim_{x \rightarrow 1^+} f(x) = \lim_{h \rightarrow 0} f(1+h) = \lim_{h \rightarrow 0} \frac{(1+h)^2 - 1}{(1+h) - 1} = \lim_{h \rightarrow 0} \frac{h^2 + 2h}{h} = \lim_{h \rightarrow 0} h + 2 = 2$$

$$f(1) = 2$$

we find that $\text{LHL} = \text{RHL} = f(1) = 2$

Hence, $f(x)$ is continuous at $x = 1$.

Continuity Ex 9.1 Q5

We have, to check the continuity of the function at $x = 0$.

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} \frac{\sin 3(-h)}{-h} = \lim_{h \rightarrow 0} \frac{\sin 3h}{-h} = 3$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0+h) = \lim_{h \rightarrow 0} \frac{\sin 3h}{h} = 3$$

$$f(0) = 1$$

$$\text{LHL} = \text{RHL} \neq f(0)$$

\Rightarrow Function is discontinuous at $x = 0$. It is removable discontinuity.

Continuity Ex 9.1 Q6

We have, to check the continuity of the function at $x = 0$.

$$\text{L.H.L} = \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0} f(x) = \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} e^{\frac{1}{-h}} = e^{-\infty} = 0$$

$$\text{R.H.L} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0+h) = \lim_{h \rightarrow 0} e^{\frac{1}{h}} = e^{\infty} = \infty$$

So, $\text{LHL} \neq \text{RHL}$

Hence, the function is discontinuous at $x = 0$. This is discontinuity of 1st kind.

Continuity Ex 9.1 Q7

We want, to check the continuity of the given function at $x = 0$.

$$\begin{aligned}\text{LHL} &= \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} \frac{1 - \cos(-h)}{(-h)^2} \\ &= \lim_{h \rightarrow 0} \frac{1 - \cosh}{h^2} \quad [\because \cos(-\theta) = \cos \theta] \\ &= \lim_{h \rightarrow 0} \frac{2 \sin^2 \frac{h}{2}}{h^2} \quad \left[\because 1 - \cos \theta = 2 \sin^2 \frac{\theta}{2} \right] \\ &= \lim_{h \rightarrow 0} 2 \left(\frac{\sin \frac{h}{2}}{h} \right)^2 = 2 \times \frac{1}{4} = \frac{1}{2} \\ \text{RHL} &= \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0+h) = \lim_{h \rightarrow 0} \frac{1 - \cosh}{h^2} = \lim_{h \rightarrow 0} \frac{2 \sin^2 \frac{h}{2}}{h^2} = \lim_{h \rightarrow 0} 2 \left(\frac{\sin \frac{h}{2}}{h} \right)^2 = 2 \times \frac{1}{4} = \frac{1}{2}\end{aligned}$$

$$f(0) = 1$$

$$\text{LHL} = \text{RHL} \neq f(0)$$

Hence, the function is discontinuous at $x = 0$

This is removable discontinuity.

Continuity Ex 9.1 Q8

We want, to check the continuity of the function at $x = 0$.

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} \frac{-h - |-h|}{2} = \lim_{h \rightarrow 0} \frac{-h - h}{2} = 0$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0+h) = \lim_{h \rightarrow 0} \frac{h - (|h|)}{2} = 0$$

$$f(0) = 2$$

$$\text{Thus, LHL} = \text{RHL} \neq f(0)$$

Hence, The function is discontinuous at $x = 0$

This is removable discontinuity.

Continuity Ex 9.1 Q9

We want, to check the continuity of the function at $x = a$.

$$\text{LHL} = \lim_{x \rightarrow a^-} f(x) = \lim_{h \rightarrow 0} f(a-h) = \lim_{h \rightarrow 0} \frac{|a-h-a|}{a-h-a} = \lim_{h \rightarrow 0} \frac{h}{-h} = -1$$

$$\text{RHL} = \lim_{x \rightarrow a^+} f(x) = \lim_{h \rightarrow 0} f(a+h) = \lim_{h \rightarrow 0} \frac{|a+h-a|}{a+h-a} = \lim_{h \rightarrow 0} \frac{h}{h} = 1$$

$$\text{Thus, LHL} \neq \text{RHL}$$

Hence, function is discontinuous at $x = a$. And the discontinuity is of first kind.

Continuity Ex 9.1 Q10(i)

We want, to check the continuity at $x = 0$.

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} |-h| \cos \left(\frac{1}{-h} \right) = \lim_{h \rightarrow 0} h \cos \left(\frac{1}{h} \right) = 0$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0+h) = \lim_{h \rightarrow 0} |h| \cos \left(\frac{1}{h} \right) = 0$$

$$f(0) = 0$$

$$\text{Thus, LHL} = \text{RHL} = f(0) = 0$$

Hence, function is continuous at $x = 0$.

Continuity Ex 9.1 Q10(ii)

We want, to check the continuity at $x = 0$.

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0 - h) = \lim_{h \rightarrow 0} (-h)^2 \sin\left(\frac{1}{-h}\right) = 0$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0 + h) = \lim_{h \rightarrow 0} h^2 \sin\left(\frac{1}{h}\right) = 0$$

$$f(0) = 0$$

Thus, $\text{LHL} = \text{RHL} = f(0) = 0$

Hence, the function is continuous at $x = 0$.

Continuity Ex 9.1 Q10(iii)

We want, to check the continuity of the function at $x = a$.

$$\text{LHL} = \lim_{x \rightarrow a^-} f(x) = \lim_{h \rightarrow 0} f(a - h) = \lim_{h \rightarrow 0} (a - h - a) \sin\left(\frac{1}{a - h - a}\right) = \lim_{h \rightarrow 0} -h \sin\left(\frac{-1}{h}\right) = 0$$

$$\text{RHL} = \lim_{x \rightarrow a^+} f(x) = \lim_{h \rightarrow 0} f(a + h) = \lim_{h \rightarrow 0} (a + h - a) \sin\left(\frac{1}{a + h - a}\right) = \lim_{h \rightarrow 0} h \sin\left(\frac{1}{h}\right) = 0$$

$$f(a) = 0$$

Thus, $\text{LHL} \neq \text{RHL} = f(a) = 0$

Hence, the function is continuous at $x = a$.

Continuity Ex 9.1 Q10(iv)

We want, to check the continuity of the function at $x = 0$.

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0 - h) = \lim_{h \rightarrow 0} \frac{e^{-h} - 1}{\log(1 + 2(-h))} = \lim_{h \rightarrow 0} \frac{e^{-h} - 1}{\log(1 - 2h)} = DNE$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0 + h) = \lim_{h \rightarrow 0} \frac{e^h - 1}{\log(1 + 2h)} = DNE$$

Thus, Both LHL and RHL do not exist

\therefore Function is discontinuous and the discontinuity is of IInd kind.

Continuity Ex 9.1 Q10(v)

We want, to check the continuity at $x = 1$

$$\begin{aligned} \text{LHL} = \lim_{x \rightarrow 1^-} f(x) &= \lim_{h \rightarrow 0} f(1-h) = \lim_{h \rightarrow 0} \frac{1 - (1-h)^n}{1 - (1-h)} = \lim_{h \rightarrow 0} \frac{1 - \left[1 - nh + \frac{n(n-1)}{2!}h^2 + \dots\right]}{h} \\ &= \lim_{h \rightarrow 0} n - \frac{n(n-1)}{2!}h + \dots \\ &= n \end{aligned}$$

$$\begin{aligned} \text{RHL} = \lim_{x \rightarrow 1^+} f(x) &= \lim_{h \rightarrow 0} f(1+h) = \lim_{h \rightarrow 0} \frac{1 - (1+h)^n}{1 - (1+h)} = \lim_{h \rightarrow 0} \frac{1 - \left[1 + nh + \frac{n(n-1)}{2!}h^2 + \dots\right]}{-h} \\ &= \lim_{h \rightarrow 0} n + \frac{n(n-1)}{2!}h + \dots \\ &= n \end{aligned}$$

$$f(1) = n - 1$$

Thus, $\text{LHL} = \text{RHL} \neq f(1)$

Hence, function is discontinuous at $x = 1$

This is removable discontinuity.

Continuity Ex 9.1 Q10(vi)

We want, to check the continuity at $x = 1$.

$$\text{LHL} = \lim_{x \rightarrow 1^-} f(x) = \lim_{h \rightarrow 0} f(1-h) = \lim_{h \rightarrow 0} \frac{|(1-h)^2 - 1|}{(1-h) - 1} = \lim_{h \rightarrow 0} \frac{|h^2 - 2h|}{-h} = \lim_{h \rightarrow 0} -(h-2) = 2$$

$$\text{RHL} = \lim_{x \rightarrow 1^+} f(x) = \lim_{h \rightarrow 0} f(1+h) = \lim_{h \rightarrow 0} \frac{|(1+h)^2 - 1|}{1+h-1} = \lim_{h \rightarrow 0} \frac{h^2 + 2h}{h} = 2$$

$$f(1) = 2$$

$$\therefore \text{LHL} = \text{RHL} = f(1) = 2$$

Hence, function is continuous.

Continuity Ex 9.1 Q10(vii)

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} \frac{2(-h) + (-h)^2}{-h} = \lim_{h \rightarrow 0} \frac{2h + h^2}{-h} = -2$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0+h) = \lim_{h \rightarrow 0} \frac{2 \times |h| + h^2}{h} = 2$$

Thus, $\text{LHL} \neq \text{RHL}$

Function is not continuous at $x = 0$

This is discontinuity of I^{st} kind.

Continuity Ex 9.1 Q11

We want to check the continuity at $x = 1$.

$$\text{LHL} = \lim_{x \rightarrow 1^-} f(x) = \lim_{h \rightarrow 0} f(1-h) = \lim_{h \rightarrow 0} 1 + (1-h)^2 = \lim_{h \rightarrow 0} 1 + 1 - 2h + h^2 = 2$$

$$\text{RHL} = \lim_{x \rightarrow 1^+} f(x) = \lim_{h \rightarrow 0} f(1+h) = \lim_{h \rightarrow 0} 2 - (1+h) = 1$$

$\text{LHL} \neq \text{RHL}$

Hence, the function is discontinuous at $x = 1$

This is discontinuity of I^{st} kind.

Continuity Ex 9.1 Q12

We want to check the continuity at $x = 0$.

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} \frac{\sin(3 \times (-h))}{\tan(2 \times (-h))} = \lim_{h \rightarrow 0} \frac{-\sin 3h}{-\tan 2h} = \lim_{h \rightarrow 0} \frac{\frac{\sin 3h}{3h} \times 3h}{\frac{\tan 2h}{2h} \times 2h} = \frac{3}{2}$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0+h) = \lim_{h \rightarrow 0} \frac{\log(1+3h)}{e^{2h}-1} = \lim_{h \rightarrow 0} \frac{\frac{\log(1+3h)}{3h} \times 3h}{\frac{e^{2h}-1}{2h} \times 2h} = \frac{3}{2}$$

$$f(0) = \frac{3}{2}$$

$$\text{Thus, LHL} = \text{RHL} = f(0) = \frac{3}{2}$$

Hence, the function is continuous at $x = 0$

Continuity Ex 9.1 Q13

We want to check the continuity of the function at $x = 0$.

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} 2(-h) - |-h| = \lim_{h \rightarrow 0} -2h - h = 0$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0+h) = \lim_{h \rightarrow 0} 2h - |h| = 0$$

$$f(0) = 0$$

$$\text{Thus, LHL} = \text{RHL} = f(0) = 0$$

Hence, the function is continuous at $x = 0$

Continuity Ex 9.1 Q14

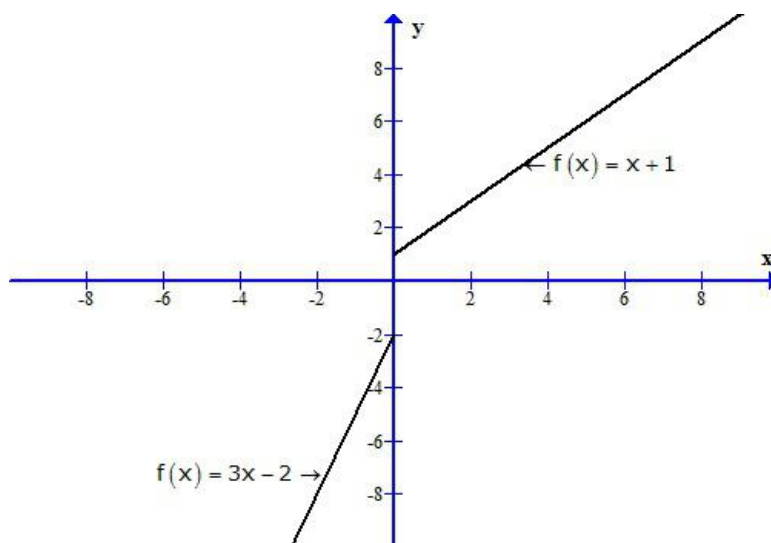
We want to check the continuity of the function at $x = 0$.

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} 3(-h) - 2 = \lim_{h \rightarrow 0} -3h - 2 = -2$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0+h) = \lim_{h \rightarrow 0} h + 1 = 1 \neq 0$$

$$\text{LHL} \neq \text{RHL}$$

So, the function is discontinuous



Continuity Ex 9.1 Q15

We want to discuss the continuity of the function at $x = 0$.

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} (-h) = 0$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0+h) = \lim_{h \rightarrow 0} h = 0$$

$$f(0) = 1$$

Thus, $\text{LHL} = \text{RHL} \neq f(0)$

Hence, the function is discontinuous at $x = 0$. And this is removable discontinuity.

Continuity Ex 9.1 Q16

We want to discuss the continuity of the function at $x = \frac{1}{2}$.

$$\text{LHL} = \lim_{x \rightarrow \frac{1}{2}^-} f(x) = \lim_{h \rightarrow 0} f\left(\frac{1}{2} - h\right) = \lim_{h \rightarrow 0} \frac{1}{2} - h = \frac{1}{2}$$

$$\text{RHL} = \lim_{x \rightarrow \left(\frac{1}{2}\right)^+} f(x) = \lim_{h \rightarrow 0} f\left(\frac{1}{2} + h\right) = \lim_{h \rightarrow 0} 1 - \left(\frac{1}{2} + h\right) = \frac{1}{2}$$

$$f\left(\frac{1}{2}\right) = \frac{1}{2}$$

$$\text{Thus, } \text{LHL} = \text{RHL} = f\left(\frac{1}{2}\right) = \frac{1}{2}$$

Hence, the function is continuous at $x = \frac{1}{2}$.

Continuity Ex 9.1 Q17

We want to check the continuity of the function at $x = 0$.

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} 2(-h) - 1 = -1$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0+h) = \lim_{h \rightarrow 0} 2h + 1 = 1$$

Thus, $\text{LHL} \neq \text{RHL}$

Hence, the function is discontinuous at $x = 0$. This is discontinuity of 1st kind.

Continuity Ex 9.1 Q18

We have given that the function is continuous at $x = 1$

$$\text{LHL} = \text{RHL} = f(1) \dots (1)$$

$$\text{Now, } \text{LHL} = \lim_{x \rightarrow 1^-} f(x) = \lim_{h \rightarrow 0} f(1-h) = \lim_{h \rightarrow 0} \frac{(1-h)^2 - 1}{(1-h) - 1} = \lim_{h \rightarrow 0} \frac{h^2 - 2h}{-h} = 2$$

$$f(1) = k$$

$$\text{From (1), } \text{LHL} = f(1)$$

$$\therefore 2 = k$$

Continuity Ex 9.1 Q19

We have that the function is continuous at $x = 1$

$$\therefore \text{LHL} = \text{RHL} = f(1) \dots (1)$$

Now,

$$\text{LHL} = \lim_{x \rightarrow 1^-} f(x) = \lim_{h \rightarrow 0} f(1-h) = \lim_{h \rightarrow 0} \frac{(1-h)^2 - 3(1-h) + 2}{(1-h) - 1} = \lim_{h \rightarrow 0} \frac{h^2 + h}{-h} = \lim_{h \rightarrow 0} -h - 1 = -1$$

$$f(1) = k$$

From (1), we get,

$$k = -1$$

Continuity Ex 9.1 Q20

We know that a function is continuous at 0 if

$$\text{LHL} = \text{RHL} = f(0) \quad \dots (1)$$

Now,

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} \frac{\sin 5(-h)}{3(-h)} = \lim_{h \rightarrow 0} \frac{-\sin 5h}{-3h} = \lim_{h \rightarrow 0} \frac{\sin 5h}{5h} \times \frac{5h}{3h} = \frac{5}{3}$$

$$f(0) = k$$

Thus, from (1),

$$k = \frac{5}{3}$$

Continuity Ex 9.1 Q21

$$\text{The given function is } f(x) = \begin{cases} kx^2, & \text{if } x \leq 2 \\ 3, & \text{if } x > 2 \end{cases}$$

The given function f is continuous at $x = 2$, if f is defined at $x = 2$ and if the value of f at $x = 2$ equals the limit of f at $x = 2$

It is evident that f is defined at $x = 2$ and $f(2) = k(2)^2 = 4k$

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^+} f(x) = f(2)$$

$$\Rightarrow \lim_{x \rightarrow 2^-} (kx^2) = \lim_{x \rightarrow 2^+} (3) = 4k$$

$$\Rightarrow k \times 2^2 = 3 = 4k$$

$$\Rightarrow 4k = 3 = 4k$$

$$\Rightarrow 4k = 3$$

$$\Rightarrow k = \frac{3}{4}$$

Therefore, the required value of k is $\frac{3}{4}$.

Continuity Ex 9.1 Q22

We have given that the function is continuous at $x = 0$

$$\text{So, LHL} = \text{RHL} = f(0) \dots (1)$$

Now,

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} \frac{\sin 2(-h)}{5(-h)} = \lim_{h \rightarrow 0} \frac{-\sin 2h}{-5h} = \lim_{h \rightarrow 0} \frac{\sin 2h}{2h} \times \frac{2h}{5h} = \frac{2}{5}$$

$$f(0) = k$$

$$\text{Using (1), } k = \frac{2}{5}$$

Continuity Ex 9.1 Q23

We have given that the function is continuous at $x = 2$

$$\text{LHL} = \text{RHL} = f(2) \dots (1)$$

Now,

$$\text{LHL} = \lim_{x \rightarrow 2^-} f(x) = \lim_{h \rightarrow 0} f(2-h) = \lim_{h \rightarrow 0} a(2-h) + 5 = 2a + 5$$

$$f(2) = 2a + 5$$

$$\text{RHL} = \lim_{x \rightarrow 2^+} f(x) = \lim_{h \rightarrow 0} f(2+h) = \lim_{h \rightarrow 0} 2+h-1 = 1$$

\therefore Using (1),

$$2a + 5 = 1 \Rightarrow a = -2$$

Continuity Ex 9.1 Q24

We have, at $x = 0$

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} \frac{-h}{|-h| + 2(-h)^2} = \lim_{h \rightarrow 0} \frac{-h}{h + 2h^2} = \lim_{h \rightarrow 0} \frac{-1}{1 + 2h} = -1$$

$$f(0) = k$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0+h) = \lim_{h \rightarrow 0} \frac{h}{|h| + 2h^2} = \lim_{h \rightarrow 0} \frac{1}{1 + 2h} = 1$$

Since, $\text{LHL} \neq \text{RHL}$, function will remain discontinuous at $x = 0$, regardless the choice of k .

Continuity Ex 9.1 Q25

Since $f(x)$ is continuous at $x = \frac{\pi}{2}$, $\text{L.H.Limit} = \text{R.H.Limit}$.

$$\Rightarrow \lim_{x \rightarrow \frac{\pi}{2}^-} f(x) = \lim_{x \rightarrow \frac{\pi}{2}^+} f(x) = \lim_{x \rightarrow \frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right)$$

$$\Rightarrow \lim_{x \rightarrow \frac{\pi}{2}^-} \frac{k \cos x}{\pi - 2x} = 3$$

$$\Rightarrow k \lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2} - x\right)}{2\left(\frac{\pi}{2} - x\right)} = 3$$

$$\Rightarrow \frac{k}{2} \lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2} - x\right)}{\left(\frac{\pi}{2} - x\right)} = 3$$

$$\Rightarrow \frac{k}{2} = 3$$

$$\Rightarrow k = 6$$

Continuity Ex 9.1 Q26

We have given that the function is continuous at $x = 0$

$$\text{LHL} = \text{RHL} = f(0) \dots (1)$$

$$f(0) = c$$

$$\begin{aligned} \text{LHL} = \lim_{x \rightarrow 0^-} f(x) &= \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} \frac{\sin(a+1)(-h) + \sin(-h)}{-h} = \lim_{h \rightarrow 0} \frac{-\sin(a+1)h - \sinh}{-h} \\ &= \lim_{h \rightarrow 0} \frac{\sin(a+1)h}{h} + \lim_{h \rightarrow 0} \frac{\sinh}{h} \\ &= a+1 + 1 = a+2 \end{aligned}$$

$$\begin{aligned} \text{RHL} = \lim_{x \rightarrow 0^+} f(x) &= \lim_{h \rightarrow 0} f(0+h) = \lim_{h \rightarrow 0} \frac{\sqrt{h+bh^2} - \sqrt{h}}{bh^2} \\ &= \lim_{h \rightarrow 0} \frac{\sqrt{h+bh^2} - \sqrt{h}}{bh^2} \times \frac{\sqrt{h+bh^2} + \sqrt{h}}{\sqrt{h+bh^2} + \sqrt{h}} \\ &= \lim_{h \rightarrow 0} \frac{h+bh^2-h}{bh^2(\sqrt{h+bh^2} + \sqrt{h})} = \lim_{h \rightarrow 0} \frac{bh^2}{bh^2(\sqrt{1+bh} + 1)} = \frac{1}{2} \end{aligned}$$

\therefore from (1),

$$a+2 = \frac{1}{2} \Rightarrow a = -\frac{3}{2}$$

$$c = \frac{1}{2} \text{ and}$$

$$b \in \mathbb{R} - \{0\}$$

$$\text{Hence, } a = -\frac{3}{2}, \quad b \in \mathbb{R} - \{0\}, \quad c = \frac{1}{2}$$

Continuity Ex 9.1 Q27

We have given that the function is continuous at $x = 0$

$$\therefore \text{LHL} = \text{RHL} = f(0) \dots (1)$$

$$f(0) = \frac{1}{2}$$

$$\begin{aligned} \text{LHL} = \lim_{x \rightarrow 0^-} f(x) &= \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} \frac{1 - \cos k(-h)}{-h \sin(-h)} = \lim_{h \rightarrow 0} \frac{1 - \cos kh}{+h \sinh} \\ &= \lim_{h \rightarrow 0} \frac{2 \sin^2 \frac{kh}{2}}{h \cdot 2 \sin \frac{h}{2} \cdot \cos \frac{h}{2}} \\ &= \lim_{h \rightarrow 0} \left(\frac{\sin \frac{kh}{2}}{\frac{kh}{2}} \right)^2 \times \frac{\frac{k^2 h^2}{4}}{\sin \frac{h}{2} \times \frac{h}{2}} \cdot \frac{1}{h} \\ &= \lim_{h \rightarrow 0} \left(\frac{\sin \frac{kh}{2}}{\frac{kh}{2}} \right)^2 \cdot \frac{\frac{k^2}{4}}{\frac{\sin \frac{h}{2}}{\frac{h}{2}} \cdot \frac{1}{2}} \\ &= \frac{k^2}{2} \end{aligned}$$

\therefore Using (1) we get,

$$\frac{k^2}{2} = \frac{1}{2} \Rightarrow k = \pm 1$$

Continuity Ex 9.1 Q28

We have given that the function is continuous at $x = 4$

$$\therefore \text{LHL} = \text{RHL} = f(4) \dots (1)$$

$$f(4) = a + b \dots (A)$$

$$\text{LHL} = \lim_{x \rightarrow 4^-} f(x) = \lim_{h \rightarrow 0} f(4-h) = \lim_{h \rightarrow 0} \frac{(4-h)-4}{|(4-h)-4|} + a = \lim_{h \rightarrow 0} \frac{-h}{h} + a = a - 1 \dots (B)$$

$$\text{RHL} = \lim_{x \rightarrow 4^+} f(x) = \lim_{h \rightarrow 0} f(4+h) = \lim_{h \rightarrow 0} \frac{(4+h)-4}{|(4+h)-4|} + b = \lim_{h \rightarrow 0} \frac{h}{h} + b = b + 1 \dots (C)$$

$$\therefore \text{from (1)}$$

$$a - 1 = b + 1 \Rightarrow a - b = 2 \dots (D)$$

from (A) and (B)

$$a + b = a - 1 \Rightarrow b = -1$$

from (A) and (C)

$$a + b = b + 1 \Rightarrow a = 1$$

Thus, $a = 1$ and $b = -1$

Continuity Ex 9.1 Q29

We have given that the function is continuous at $x = 0$

$$\therefore \text{LHL} = \text{RHL} = f(0) \dots (1)$$

$$f(0) = k$$

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} \frac{\sin 2(0-h)}{-h} = \lim_{h \rightarrow 0} \frac{-\sin 2h}{-h} = 2$$

$$\therefore \text{using (1), we get } k = 2$$

Continuity Ex 9.1 Q30

We know that a function is continuous at $x = 0$ if,

$$\text{LHL} = \text{RHL} = f(0) \dots (1)$$

$$\begin{aligned} \text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0-h) &= \lim_{h \rightarrow 0} \frac{\log\left(1-\frac{h}{a}\right) - \log\left(1+\frac{h}{b}\right)}{(-h)} = \lim_{h \rightarrow 0} \frac{\log\left(1+\left(\frac{-h}{a}\right)\right)}{\left(\frac{-h}{a}\right) \times a} + \frac{\log\left(1+\frac{h}{b}\right)}{h} \\ &= \frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab} \end{aligned}$$

from (1),

$$f(0) = \frac{a+b}{ab}$$

Continuity Ex 9.1 Q31

We are given that the function is continuous at $x = 2$

$$\therefore \text{LHL} = \text{RHL} = f(2) \quad \dots (1)$$

Now,

$$f(2) = k \quad \dots (A)$$

$$\begin{aligned} \text{LHL} &= \lim_{x \rightarrow 2^-} f(x) = \lim_{h \rightarrow 0} f(2-h) = \lim_{h \rightarrow 0} \frac{2^{(2-h)+2} - 16}{4^{(2-h)} - 16} = \lim_{h \rightarrow 0} \frac{2^{4-h} - 16}{4^{2-h} - 16} \\ &= \lim_{h \rightarrow 0} \frac{2^4 \cdot 2^{-h} - 16}{4^2 \cdot 4^{-h} - 16} \\ &= \lim_{h \rightarrow 0} \frac{16 \cdot 2^{-h} - 16}{16 \cdot 4^{-h} - 16} \\ &= \lim_{h \rightarrow 0} \frac{16(2^{-h} - 1)}{16(4^{-h} - 1)} \\ &= \lim_{h \rightarrow 0} \frac{2^{-h} - 1}{(2^{-h})^2 - 1^2} \quad \left[\because 2^{-2h} = (2^{-h})^2 = 4^{-h} \right] \\ &= \lim_{h \rightarrow 0} \frac{2^{-h} - 1}{(2^{-h} - 1)(2^{-h} + 1)} = \frac{1}{2} \quad \dots (B) \end{aligned}$$

\therefore Using (1) from (A) & (B)

$$k = \frac{1}{2}$$

Continuity Ex 9.1 Q33

We know that a function is said to be continuous at $x = \pi$ if

$$\text{LHL} = \text{RHL} = \text{value of the function at } x = \pi \dots (1)$$

$$\begin{aligned} \text{LHL} &= \lim_{x \rightarrow \pi^-} f(x) = \lim_{h \rightarrow 0} f(\pi - h) = \lim_{h \rightarrow 0} \frac{1 - \cos 7(\pi - h - \pi)}{5((\pi - h) - \pi)^2} = \lim_{h \rightarrow 0} \frac{1 - \cos 7h}{5h^2} \\ &= \lim_{h \rightarrow 0} \frac{2 \sin^2 \frac{7}{2}h}{5h^2} \\ &= \lim_{h \rightarrow 0} \frac{2}{5} \left(\frac{\sin \frac{7}{2}h}{\frac{7}{2}h} \right)^2 \times \left(\frac{7}{2} \right)^2 \\ &= \frac{2}{5} \times \frac{49}{4} = \frac{49}{10} \dots (B) \end{aligned}$$

Thus, using (1) we get,

$$f(\pi) = \frac{49}{10}$$

Continuity Ex 9.1 Q34

It is given that the function is continuous at $x = 0$

$$\therefore \text{LHL} = \text{RHL} = f(0) \dots (1)$$

$$\begin{aligned} \text{LHL} &= \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} \frac{2(-h) + 3 \sin(-h)}{3(-h) + 2 \sin(-h)} = \lim_{h \rightarrow 0} \frac{-2h - 3 \sin h}{-3h - 2 \sin h} \\ &= \lim_{h \rightarrow 0} \frac{2h + 3 \sin h}{3h + 2 \sin h} \\ &= \lim_{h \rightarrow 0} \frac{2 + 3 \frac{\sin h}{h}}{3 + 2 \frac{\sin h}{h}} = \frac{2+3}{3+2} = 1 \quad \left[\because \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1 \right] \end{aligned}$$

Using (1) we get,

$$f(0) = 1$$

Continuity Ex 9.1 Q35

It is given that the function is continuous at $x = 0$.

$$\text{LHL} = \text{RHL} = f(0) \dots (1)$$

$$f(0) = k \dots (A)$$

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} \frac{1 - \cos 4(-h)}{8(-h)^2} = \lim_{h \rightarrow 0} \frac{1 - \cos 4h}{8h^2} = \lim_{h \rightarrow 0} \frac{2\sin^2 2h}{8h^2} = \lim_{h \rightarrow 0} \left(\frac{\sin 2h}{2h} \right)^2 = 1$$

Thus, using (1) we get,

$$k = 1$$

Continuity Ex 9.1 Q36

The given function will be continuous at $x = 0$ if

$$\text{LHL} = \text{RHL} = f(0) \dots (1)$$

$$f(0) = 8 \dots (A)$$

$$\begin{aligned} \text{LHL} = \lim_{x \rightarrow 0^-} f(x) &= \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} \frac{1 - \cos 2k(-h)}{(-h)^2} = \lim_{h \rightarrow 0} \frac{1 - \cos 2kh}{h^2} = \lim_{h \rightarrow 0} \frac{2\sin^2 kh}{h^2} \\ &= \lim_{h \rightarrow 0} 2 \left(\frac{\sin kh}{kh} \right)^2 \cdot k^2 \\ &= 2k^2 \end{aligned}$$

Thus, using (1) we get,

$$2k^2 = 8 \Rightarrow k^2 = 4 \Rightarrow k = \pm 2$$

Hence, $k = \pm 2$

$$\text{Let } x - 1 = y$$

$$\Rightarrow x = y + 1$$

Thus,

$$\begin{aligned} \lim_{x \rightarrow 1} (x - 1) \tan \frac{\pi x}{2} &= \lim_{y \rightarrow 0} y \tan \frac{\pi(y + 1)}{2} \\ &= \lim_{y \rightarrow 0} y \tan \left(\frac{\pi y}{2} + \frac{\pi}{2} \right) \\ &= - \lim_{y \rightarrow 0} y \cot \frac{\pi y}{2} \\ &= - \lim_{y \rightarrow 0} y \frac{\cos \frac{\pi y}{2}}{\sin \frac{\pi y}{2}} \\ &= - \lim_{y \rightarrow 0} y \frac{\cos \frac{\pi y}{2}}{\frac{\left(\sin \frac{\pi y}{2} \right) \frac{\pi}{2}}{\frac{\pi}{2}}} \\ &= - \lim_{y \rightarrow 0} \frac{\cos \frac{\pi y}{2}}{\frac{\left(\sin \frac{\pi y}{2} \right) \frac{\pi}{2}}{\frac{\pi y}{2}}} \\ &= - \lim_{y \rightarrow 0} \frac{2}{\pi} \frac{\cos \frac{\pi y}{2}}{\left(\sin \frac{\pi y}{2} \right)} \\ &= - \frac{2}{\pi} \lim_{y \rightarrow 0} \cos \frac{\pi y}{2} \\ &= - \frac{2}{\pi} \end{aligned}$$

Since the function is continuous, L.H.Limit = R.H.Limit

$$\text{Thus, } k = - \frac{2}{\pi}$$

Since the function is continuous at every point, therefore

$$LHL = RHL = f(0)$$

Now

$$\begin{aligned} f(0) &= \cos 0 \\ &= 1 \end{aligned}$$

Again

$$\begin{aligned} LHL &= \lim_{x \rightarrow 0} k(x^2 - 2x) \\ &= \lim_{h \rightarrow 0^+} k(h^2 - 2h) \\ &= 0 \end{aligned}$$

Therefore there is no value of k

Since the function is continuous at every point, therefore

$$LHL = RHL = f(\pi)$$

Now

$$f(\pi) = k\pi + 1$$

Again

$$\begin{aligned} RHL &= \lim_{x \rightarrow \pi^-} \cos x \\ &= \lim_{h \rightarrow 0^+} \cos(\pi - h) \\ &= - \lim_{h \rightarrow 0^+} \cos h \\ &= -1 \end{aligned}$$

Therefore we can write

$$k\pi + 1 = -1$$

$$k = - \frac{2}{\pi}$$

We are given that function is continuous at $x = 5$.

$$\therefore \text{LHL} = \text{RHL} = f(5) \dots (1)$$

$$f(5) = 5k + 1$$

$$\text{LHL} = \lim_{x \rightarrow 5^+} f(x) = \lim_{h \rightarrow 0} f(5+h) = \lim_{h \rightarrow 0} 3(5+h) - 5 = 10$$

Thus, using (1), we get,

$$5k + 1 = 10$$

$$5k = 9$$

$$k = \frac{9}{5}$$

We know that the function will be continuous at $x = 5$, if

$$\text{LHL} = \text{RHL} = f(5) \dots (1)$$

$$f(5) = k$$

$$\text{LHL} = \lim_{x \rightarrow 5^-} f(x) = \lim_{h \rightarrow 0} f(5-h) = \lim_{h \rightarrow 0} \frac{(5-h)^2 - 25}{(5-h) - 5} = \lim_{h \rightarrow 0} \frac{h^2 - 10h}{-h} = \lim_{h \rightarrow 0} -h + 10 = 10$$

Thus, using (1), we get,

$$k = 10$$

We know that a function will be continuous at $x = 1$, if

$$\text{LHL} = \text{RHL} = f(1) \dots (1)$$

$$f(1) = k \cdot 1^2 = k$$

$$\text{LHL} = \lim_{x \rightarrow 1^-} f(x) = \lim_{h \rightarrow 0} f(1-h) = \lim_{h \rightarrow 0} 4 = 4$$

Thus, using (1), we get,

$$k = 4$$

We know that a function will be continuous at $x = 0$, if

$$\text{LHL} = \text{RHL} = f(0) \dots (1)$$

$$f(0) = k(0+2) = 2k$$

$$\text{LHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0+h) = \lim_{h \rightarrow 0} 3(h) + 1 = 1$$

Thus, using (1), we get,

$$2k = 1$$

$$k = \frac{1}{2}$$

Continuity Ex 9.1 Q37

It is given that the function is continuous at $x = 3$ and at $x = 5$

$$\therefore \text{LHL} = \text{RHL} = f(3) \dots (1) \text{ and}$$

$$\text{LHL} = \text{RHL} = f(5) \dots (2)$$

Now,

$$f(3) = 1$$

$$\text{RHL} = \lim_{x \rightarrow 3^+} f(x) = \lim_{h \rightarrow 0} f(3+h) = \lim_{h \rightarrow 0} a(3+h) + b = 3a + b$$

Thus, using (1), we get,

$$3a + b = 1 \dots (3)$$

$$f(5) = 7$$

$$\text{LHL} = \lim_{x \rightarrow 5^-} f(x) = \lim_{h \rightarrow 0} f(5-h) = \lim_{h \rightarrow 0} a(5-h) + b = 5a + b$$

Thus, using (2), we get

$$5a + b = 7 \dots (4)$$

Now, solving (3) and (4) we get,

$$a = 3 \text{ and } b = -8$$

Continuity Ex 9.1 Q38

We want to discuss the continuity of the function at $x = 1$

We need to prove that

$$\text{LHL} = \text{RHL} = f(1)$$

$$f(1) = \frac{1^2}{2} = \frac{1}{2}$$

$$\text{LHL} = \lim_{x \rightarrow 1^-} f(x) = \lim_{h \rightarrow 0} f(1-h) = \lim_{h \rightarrow 0} \frac{(1-h)^2}{2} = \frac{1}{2}$$

$$\text{RHL} = \lim_{x \rightarrow 1^+} f(x) = \lim_{h \rightarrow 0} f(1+h) = \lim_{h \rightarrow 0} 2(1+h)^2 - 3(1+h) + \frac{3}{2} = 2 - 3 + \frac{3}{2} = \frac{1}{2}$$

$$\text{Thus, LHL} = \text{RHL} = f(1) = \frac{1}{2}$$

Hence, function is continuous at $x = 1$

Continuity Ex 9.1 Q39

We want to discuss the continuity at $x = 0$ and $x = 1$

Now,

$$f(0) = 1$$

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} |-h| + |-h-1| = 1.$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0+h) = \lim_{h \rightarrow 0} |h| + |h-1| = 1$$

$\therefore \text{LHL} = \text{RHL} = f(0) = 1$, function is continuous at $x = 0$.

For $x = 1$.

$$f(1) = 1$$

$$\text{LHL} = \lim_{x \rightarrow 1^-} f(x) = \lim_{h \rightarrow 0} f(1-h) = \lim_{h \rightarrow 0} |1-h| + |1-h-1| = 1$$

$$\text{RHL} = \lim_{x \rightarrow 1^+} f(x) = \lim_{h \rightarrow 0} f(1+h) = \lim_{h \rightarrow 0} |1+h| + |1+h-1| = 1$$

$\therefore \text{LHL} = \text{RHL} = f(1) = 1$ function is continuous at $x = 1$.

For $x = -1$

$$f(-1) = |-1 - 1| + |-1 + 1| = 2$$

$$\text{LHL} = \lim_{x \rightarrow 1^-} f(x) = \lim_{h \rightarrow 0} f(-1 - h) = \lim_{h \rightarrow 0} |-1 - h - 1| + |-1 - h + 1| = 2$$

$$\text{RHL} = \lim_{x \rightarrow 1^+} f(x) = \lim_{h \rightarrow 0} f(-1 + h) = \lim_{h \rightarrow 0} |-1 + h - 1| + |-1 + h + 1| = 2$$

Thus, $\text{LHL} = \text{RHL} = f(-1) = 2$

Hence, function is continuous at $x = -1$

For $x = 1$

$$f(1) = |1 - 1| + |1 + 1| = 2$$

$$\text{LHL} = \lim_{x \rightarrow 1^-} f(x) = \lim_{h \rightarrow 0} f(1 - h) = \lim_{h \rightarrow 0} |1 - h - 1| + |1 - h + 1| = 2$$

$$\text{RHL} = \lim_{x \rightarrow 1^+} f(x) = \lim_{h \rightarrow 0} f(1 + h) = \lim_{h \rightarrow 0} |1 + h - 1| + |1 + h + 1| = 2$$

Thus, $\text{LHL} = \text{RHL} = f(1) = 2$

Hence, function is continuous at $x = 1$

Continuity Ex 9.1 Q40

Since $f(x)$ is continuous at $x=0$, $L.H.Limit = R.H.Limit$.

Thus, we have

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x)$$

$$\Rightarrow \lim_{x \rightarrow 0^-} a \sin \frac{\pi}{2}(x+1) = \lim_{x \rightarrow 0^+} \frac{\tan x - \sin x}{x^3}$$

$$\Rightarrow a \times 1 = \lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3}$$

$$\Rightarrow a = \lim_{x \rightarrow 0} \frac{\frac{\sin x}{\cos x} - \sin x}{x^3}$$

$$\Rightarrow a = \lim_{x \rightarrow 0} \frac{\frac{\sin x}{x} \left(\frac{1}{\cos x} - 1 \right)}{x^2}$$

$$\Rightarrow a = \lim_{x \rightarrow 0} \frac{\frac{\sin x}{x} \left(\frac{1 - \cos x}{\cos x} \right)}{x^2}$$

$$\Rightarrow a = \lim_{x \rightarrow 0} \frac{\sin x}{x} \times \lim_{x \rightarrow 0} \frac{1}{\cos x} \times \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$$

$$\Rightarrow a = 1 \times 1 \times \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$$

$$\Rightarrow a = \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} \times \frac{1 + \cos x}{1 + \cos x}$$

$$\Rightarrow a = \lim_{x \rightarrow 0} \frac{1 - \cos^2 x}{x^2(1 + \cos x)}$$

$$\Rightarrow a = \lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2(1 + \cos x)}$$

$$\Rightarrow a = \lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2} \times \lim_{x \rightarrow 0} \frac{1}{1 + \cos x}$$

$$\Rightarrow a = 1 \times \lim_{x \rightarrow 0} \frac{1}{1 + \cos x}$$

$$\Rightarrow a = 1 \times \frac{1}{1 + 1}$$

$$\Rightarrow a = \frac{1}{2}$$

Continuity Ex 9.1 Q41

It is given that function is continuous at $x = 0$. then,

$$LHL = RHL = f(0) \dots (1)$$

Now,

$$f(0) = 2 \cdot 0 + k = k$$

$$LHL = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} 2(-h)^2 + k = k$$

$$RHL = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0+h) = \lim_{h \rightarrow 0} 2(h^2) + k = k$$

Thus, the function will be continuous for any $k \in R$.

Continuity Ex 9.1 Q42

The given function f is $f(x) = \begin{cases} \lambda(x^2 - 2x), & \text{if } x \leq 0 \\ 4x + 1, & \text{if } x > 0 \end{cases}$

If f is continuous at $x = 0$, then

$$\begin{aligned} \lim_{x \rightarrow 0^-} f(x) &= \lim_{x \rightarrow 0^-} f(x) = f(0) \\ \Rightarrow \lim_{x \rightarrow 0^-} \lambda(x^2 - 2x) &= \lim_{x \rightarrow 0^-} (4x + 1) = \lambda(0^2 - 2 \times 0) \\ \Rightarrow \lambda(0^2 - 2 \times 0) &= 4 \times 0 + 1 = 0 \\ \Rightarrow 0 &= 1 = 0, \text{ which is not possible} \end{aligned}$$

Therefore, there is no value of λ for which f is continuous at $x = 0$

At $x = 1$,

$$f(1) = 4x + 1 = 4 \times 1 + 1 = 5$$

$$\begin{aligned} \lim_{x \rightarrow 1} (4x + 1) &= 4 \times 1 + 1 = 5 \\ \therefore \lim_{x \rightarrow 1} f(x) &= f(1) \end{aligned}$$

Therefore, for any values of λ , f is continuous at $x = 1$

Continuity Ex 9.1 Q43

The function will be continuous at $x = 2$

if $\text{LHL} = \text{RHL} = f(2) \dots (1)$

Now,

$$f(2) = k$$

$$\text{LHL} = \lim_{x \rightarrow 2^-} f(x) = \lim_{h \rightarrow 0} f(2 - h) = \lim_{h \rightarrow 0} 2(2 - h) + 1 = 5.$$

Thus, using (1) we get,

$$k = 5$$

Continuity Ex 9.1 Q44

It is given that the function is continuous at $x = \frac{\pi}{2}$

$$\therefore \text{LHL} = \text{RHL} = f\left(\frac{\pi}{2}\right) \dots\dots (1)$$

Now,

$$f\left(\frac{\pi}{2}\right) = a$$

$$\begin{aligned} \text{LHL} &= \lim_{x \rightarrow \frac{\pi}{2}} f(x) = \lim_{h \rightarrow 0} f\left(\frac{\pi}{2} - h\right) = \lim_{h \rightarrow 0} \frac{1 - \sin^3\left(\frac{\pi}{2} - h\right)}{3\cos^2\left(\frac{\pi}{2} - h\right)} = \lim_{h \rightarrow 0} \frac{1 - \cos^3 h}{3\sin^2 h} \\ &= \lim_{h \rightarrow 0} \frac{(1 - \cosh)(1 + \cos^2 h + \cosh)}{3\sin^2 h} \\ &= \lim_{h \rightarrow 0} \frac{2\sin^2 \frac{h}{2} (1 + \cos^2 h + \cosh)}{3\sin^2 h} \\ &= \lim_{h \rightarrow 0} \frac{2\left(\frac{\sin \frac{h}{2}}{\frac{h}{2}}\right)^2 \times \frac{h^2}{4} (1 + \cos^2 h + \cosh)}{3\left(\frac{\sin h}{h}\right)^2 \cdot h^2} \\ &= \lim_{h \rightarrow 0} \frac{2 \cdot \frac{1}{4} (1 + \cos^2 h + \cosh)}{3} = \frac{1}{2} \end{aligned}$$

$$\begin{aligned} \text{RHL} &= \lim_{x \rightarrow \frac{\pi}{2}} f(x) = \lim_{h \rightarrow 0} f\left(\frac{\pi}{2} + h\right) = \lim_{h \rightarrow 0} \frac{b\left(1 - \sin\left(\frac{\pi}{2} + h\right)\right)}{\left(\pi - 2\left(\frac{\pi}{2} + h\right)\right)^2} = \lim_{h \rightarrow 0} \frac{b(1 - \cosh)}{(\pi - \pi - 2h)^2} \\ &= \lim_{h \rightarrow 0} \frac{b \cdot 2\sin^2 \frac{h}{2}}{(2h)^2} \\ &= \lim_{h \rightarrow 0} \frac{b}{2} \left(\frac{\sin \frac{h}{2}}{\frac{h}{2}}\right)^2 \times \frac{1}{4} \\ &= \lim_{h \rightarrow 0} \frac{b}{8} = \frac{b}{8} \end{aligned}$$

Thus, using (1) we get,

$$a = \frac{1}{2}$$

And

$$\frac{b}{8} = \frac{1}{2} \Rightarrow b = 4$$

$$\text{Thus, } a = \frac{1}{2} \text{ and } b = 4$$

Continuity Ex 9.1 Q45

It is given that the function is continuous at $x = 0$, then

$$\text{LHL} = \text{RHL} = f(0) \dots\dots (1)$$

Now,

$$f(0) = k$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0 + h) = \lim_{h \rightarrow 0} \frac{h}{|h|} = 1 \dots\dots (B)$$

Thus, using (1) we get,

$$k = 1$$

Continuity Ex 9.1 Q46

Since the function is continuous at $x = 3$, therefore

$$LHL = RHL = f(3)$$

Now

$$\begin{aligned} RHL &= \lim_{x \rightarrow 3^+} f(x) \\ &= \lim_{h \rightarrow 0} f(3+h) \\ &= \lim_{h \rightarrow 0} b(3+h) + 3 \\ &= \lim_{h \rightarrow 0} 3b + 3h + 3 \\ &= 3b + 3 \end{aligned}$$

Again

$$\begin{aligned} f(3) &= a(3) + 1 \\ &= 3a + 1 \end{aligned}$$

Thus we can write

$$\begin{aligned} f(3) &= RHL \\ 3a + 1 &= 3b + 3 \\ 3a - 3b &= 2 \end{aligned}$$

Ex 9.2

Chapter 9 Continuity Ex 9.2 Q1

When $x < 0$, we have, $f(x) = \frac{\sin x}{x}$

We know that $\sin x$ and the identity function x both are everywhere continuous.

So, the quotient function $\frac{\sin x}{x} = f(x)$ is continuous for $x < 0$

When $x > 0$, we have $f(x) = x + 1$, which being a polynomial, is continuous for $x > 0$

Let us now consider $x = 0$

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0 - h) = \lim_{h \rightarrow 0} \frac{\sin(-h)}{-h} = \lim_{h \rightarrow 0} \frac{-\sin h}{-h} = 1$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0 + h) = \lim_{h \rightarrow 0} \frac{\sin h}{h} = 1$$

$$f(0) = 0 + 1 = 1$$

$$\text{Thus, } \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x) = f(0) = 1$$

$\therefore f(x)$ is continuous at $x = 0$

Hence, $f(x)$ is continuous everywhere.

Chapter 9 Continuity Ex 9.2 Q2

When $x \neq 0$,

$$f(x) = \frac{x}{|x|} = \begin{cases} \frac{-x}{x} = -1 & ; x < 0 \\ \frac{x}{x} = 1 & ; x > 0 \end{cases}$$

So, $f(x)$ is a constant function when $x \neq 0$
hence, is continuous for all $x < 0$ and $x > 0$

Now,

Consider the point $x = 0$.

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} \frac{-h}{|-h|} = -1$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0+h) = \lim_{h \rightarrow 0} \frac{h}{|h|} = 1$$

So, $\text{LHL} \neq \text{RHL}$

Hence, function is discontinuous at $x = 0$

Chapter 9 Continuity Ex 9.2 Q3(i)

When $x \neq 1$

$f(x) = x^3 - x^2 + 2x - 2$ is a polynomial, so is continuous for $x < 1$ and $x > 1$

Now, consider the point $x = 1$

$$\text{LHL} = \lim_{x \rightarrow 1^-} f(x) = \lim_{h \rightarrow 0} f(1-h) = \lim_{h \rightarrow 0} (1-h)^3 - (1-h)^2 + 2(1-h) - 2 = 1 - 1 + 2 - 2 = 0$$

$$\text{RHL} = \lim_{x \rightarrow 1^+} f(x) = \lim_{h \rightarrow 0} f(1+h) = \lim_{h \rightarrow 0} (1+h)^3 - (1+h)^2 + 2(1+h) - 2 = 1 - 1 + 2 - 2 = 0$$

$$f(1) = 4$$

$$\text{LHL} = \text{RHL} \neq f(1)$$

Thus, function is not discontinuous at $x = 1$

Chapter 9 Continuity Ex 9.2 Q3(ii)

When $x \neq 2$, we have,

$$f(x) = \frac{x^4 - 16}{x - 2} = \frac{(x^2 + 4)(x^2 - 4)}{x - 2} = \frac{(x^2 + 4)(x + 2)(x - 2)}{x - 2} = f(x) = (x^2 + 4)(x + 2)$$

which is a polynomial, so the function is continuous when $x < 2$ or $x > 2$

Now, consider the point $x = 2$

$$\text{LHL} = \lim_{x \rightarrow 2^-} f(x) = \lim_{h \rightarrow 0} f(2-h) = \lim_{h \rightarrow 0} \frac{(2-h)^4 - 16}{(2-h) - 2}$$

$$= \lim_{h \rightarrow 0} \frac{2^4 - 4.8h + 6.4h^2 - 4.2h^3 + h^4 - 16}{-h}$$

$$= \lim_{h \rightarrow 0} \frac{16 - 32h + 24h^2 - 8h^3 + h^4 - 16}{-h}$$

$$= \lim_{h \rightarrow 0} 32 - 24h + 8h^2 - h^3 = 32$$

$$\text{RHL} = \lim_{x \rightarrow 2^+} f(x) = \lim_{h \rightarrow 0} f(2+h) = \lim_{h \rightarrow 0} \frac{(2+h)^4 - 16}{(2+h) - 2} = \lim_{h \rightarrow 0} \frac{16 + 32h + 24h^2 + 8h^3 + h^4 - 16}{h}$$

$$= \lim_{h \rightarrow 0} 32 + 24h + 8h^2 + h^3$$

$$= 32$$

$$\text{Also, } f(2) = 16$$

$$\text{Thus, } \text{LHL} = \text{RHL} \neq f(2)$$

Hence, the function is discontinuous at $x = 2$

Chapter 9 Continuity Ex 9.2 Q3(iii)

When $x < 0$, we have, $f(x) = \frac{\sin x}{x}$

We know that $\sin x$ and the identity function continuous for $x < 0$, so the quotient function

$f(x) = \frac{\sin x}{x}$ is continuous for $x < 0$.

When $x > 0$ $f(x) = 2x + 3$, which is a polynomial of degree 1 so $f(x) = 2x + 3$ is continuous for $x > 0$.

Now, consider the point $x = 0$

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0 - h) = \lim_{h \rightarrow 0} \frac{\sin(-h)}{-h} = \lim_{h \rightarrow 0} \frac{-\sin h}{-h} = 1$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0 + h) = \lim_{h \rightarrow 0} \frac{\sin h}{h} = 1$$

$$f(0) = 2 \times 0 + 3 = 3$$

Thus, L.H.L = R.H.L $\neq f(0)$

Hence, $f(x)$ is discontinuous at $x = 0$

Chapter 9 Continuity Ex 9.2 Q3(iv)

When $x \neq 0$ $f(x) = \frac{\sin 3x}{x}$

We know that $\sin 3x$ and the identity function x are continuous for $x < 0$ and $x > 0$.

So, the quotient function $f(x) = \frac{\sin 3x}{x}$ is continuous for $x < 0$ and $x > 0$.

Now, consider the point $x = 0$

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0 - h) = \lim_{h \rightarrow 0} \frac{\sin 3(-h)}{-h} = \lim_{h \rightarrow 0} \frac{-\sin 3h}{-h} = 3$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0 + h) = \lim_{h \rightarrow 0} \frac{\sin 3h}{h} = 3$$

$$f(0) = 4$$

Thus, LHL = RHL $\neq f(0)$

Hence, $f(x)$ is discontinuous at $x = 0$

Chapter 9 Continuity Ex 9.2 Q3(v)

When $x \neq 0$, we have, $f(x) = \frac{\sin x}{x} + \cos x$

We know that

$\sin x$ and $\cos x$ is continuous for $x < 0$ and $x > 0$.

The identity function x is also continuous for $x < 0$ and $x > 0$.

\therefore The quotient function $f(x) = \frac{\sin x}{x}$ is continuous for $x < 0$ and $x > 0$.

And, the sum $\frac{\sin x}{x} + \cos x$ is also continuous for each $x < 0$ and $x > 0$.

Now, consider the point $x = 0$

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0 - h) = \lim_{h \rightarrow 0} \frac{\sin(-h)}{-h} + \cos(-h) = \lim_{h \rightarrow 0} \frac{-\sin h}{-h} + \cos h = 1 + 1 = 2$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0 + h) = \lim_{h \rightarrow 0} \frac{\sin h}{h} + \cos h = 1 + 1 = 2$$

$$f(0) = 5$$

Thus, LHL = RHL $\neq f(0)$

Hence, $f(x)$ is discontinuous at $x = 0$

Chapter 9 Continuity Ex 9.2 Q3(vi)

When $x \neq 0$, we have, $f(x) = \frac{x^4 + x^3 + 2x^2}{\tan^{-1} x}$

We know that a polynomial is continuous for $x < 0$ and $x > 0$. Also the inverse trigonometric function is continuous in its domain.

Here, $x^4 + x^3 + 2x^2$ is polynomial, so is continuous for $x < 0$ and $x > 0$ and $\tan^{-1} x$ is also continuous for $x < 0$ and $x > 0$

So, the quotient function $f(x) = \frac{x^4 + x^3 + 2x^2}{\tan^{-1} x}$ is continuous for each $x < 0$ and $x > 0$.

Now, consider the point $x = 0$

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0 - h) = \lim_{h \rightarrow 0} \frac{(-h)^4 + (-h)^3 + 2(-h)^2}{\tan^{-1}(-h)} = \lim_{h \rightarrow 0} \frac{h^4 - h^3 + 2h^2}{\tan^{-1} h} = 0$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0 + h) = \lim_{h \rightarrow 0} \frac{h^4 + h^3 + 2h^2}{\tan^{-1} h} = 0$$

$$f(0) = 10$$

Thus, $\text{LHL} = \text{RHL} \neq f(0)$

Hence, the function is not continuous at $x = 0$

Chapter 9 Continuity Ex 9.2 Q3(vii)

When $x \neq 0$, we have,

$$f(x) = \frac{e^x - 1}{\log_e(1 + 2x)}$$

We know that e^x and the constant function is continuous for $x < 0$ and $x > 0$

$\Rightarrow e^x - 1$ is continuous for $x < 0$ and $x > 0$

Again, logarithmic function is continuous for $x < 0$ and $x > 0$

$\Rightarrow \log_e(1 + 2x)$ is continuous for $x > 0$ and $x < 0$

So, the quotient function $f(x) = \frac{e^x - 1}{\log_e(1 + 2x)}$ is continuous for each $x < 0$ and $x > 0$.

Now, consider the point $x = 0$

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0 - h) = \lim_{h \rightarrow 0} \frac{e^{-h} - 1}{\log_e(1 - 2h)} = \lim_{h \rightarrow 0} \frac{\frac{e^{-h} - 1}{-h}}{\frac{-2h}{\log_e(1 - 2h)} \times -2} = \frac{1}{2}$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0 + h) = \lim_{h \rightarrow 0} \frac{e^h - 1}{\log_e(1 + 2h)} = \lim_{h \rightarrow 0} \frac{\frac{e^h - 1}{h}}{\frac{2h}{\log_e(1 + 2h)} \times 2} = \frac{1}{2}$$

$$f(0) = 7$$

Thus, $\text{LHL} = \text{RHL} \neq f(0)$

Hence, $f(x)$ is not continuous at $x = 0$

Chapter 9 Continuity Ex 9.2 Q3(viii)

We know that

(i) The absolute value function $g(x) = |x|$ is continuous on \mathbb{R}

(ii) Polynomial function are every where continuous.

So, the only possible point of discontinuity of $f(x)$ can be $x = 1$

Now

$$f(1) = |1 - 3| = |-2| = 2$$

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} |x - 3| = 2$$

$$\begin{aligned}\lim_{x \rightarrow 1^-} f(x) &= \lim_{x \rightarrow 1^-} \left(\frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4} \right) \\ &= \frac{1}{4} - \frac{3}{2} + \frac{13}{4} = \frac{8}{4} = 2\end{aligned}$$

Since

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x) = f(1) = 2$$

$\therefore f(x)$ is continuous at x

Hence $f(x)$ has no point of discontinuity.

Chapter 9 Continuity Ex 9.2 Q3(ix)

When $x < -3$,

$$f(x) = |x| + 3$$

We know that $|x|$ is continuous for $x < -3$

$\therefore |x| + 3$ is continuous for $x < -3$

When $x > 3$

$f(x) = 6x + 2$ which is a polynomial of degree 1, so $f(x) = 6x + 2$ is continuous for $x > 3$

When $-3 < x < 3$

$f(x) = -2x$ which is again a polynomial so, it is continuous for $-3 < x < 3$

Now, consider the point $x = -3$

$$\text{LHL} = \lim_{x \rightarrow -3^-} f(x) = \lim_{h \rightarrow 0} f(-3 - h) = \lim_{h \rightarrow 0} |-3 - h| + 3 = \lim_{h \rightarrow 0} |3 + h| + 3 = 6$$

$$\text{RHL} = \lim_{x \rightarrow -3^+} f(x) = \lim_{h \rightarrow 0} f(-3 + h) = \lim_{h \rightarrow 0} -2(-3 + h) = 6$$

$$f(-3) = |-3| + 3 = 6$$

Thus, $\text{LHL} = \text{RHL} = f(-3) = 6$

So, the function is continuous at $x = -3$

Now, consider the point $x = 3$

$$\text{LHL} = \lim_{x \rightarrow 3^-} f(x) = \lim_{h \rightarrow 0} f(3 - h) = \lim_{h \rightarrow 0} -2(3 - h) = -6$$

Chapter 9 Continuity Ex 9.2 Q3(xi)

The given function is $f(x) = \begin{cases} 2x, & \text{if } x < 0 \\ 0, & \text{if } 0 \leq x \leq 1 \\ 4x, & \text{if } x > 1 \end{cases}$

The given function is defined at all points of the real line.

Let c be a point on the real line.

Case I:

If $c < 0$, then $f(c) = 2c$

$$\lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} (2x) = 2c$$

$$\therefore \lim_{x \rightarrow c} f(x) = f(c)$$

Therefore, f is continuous at all points x , such that $x < 0$

Chapter 9 Continuity Ex 9.2 Q3(xii)

The given function f is $f(x) = \begin{cases} \sin x - \cos x, & \text{if } x \neq 0 \\ -1 & \text{if } x = 0 \end{cases}$

It is evident that f is defined at all points of the real line.

Let c be a real number.

Case I:

If $c \neq 0$, then $f(c) = \sin c - \cos c$

$$\lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} (\sin x - \cos x) = \sin c - \cos c$$

$$\therefore \lim_{x \rightarrow c} f(x) = f(c)$$

Therefore, f is continuous at all points x , such that $x \neq 0$

Chapter 9 Continuity Ex 9.2 Q3(xiii)

The given function f is $f(x) = \begin{cases} -2, & \text{if } x \leq -1 \\ 2x, & \text{if } -1 < x \leq 1 \\ 2, & \text{if } x > 1 \end{cases}$

The given function is defined at all points of the real line.

Let c be a point on the real line.

Case I:

If $c < -1$, then $f(c) = -2$ and $\lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} (-2) = -2$

$$\therefore \lim_{x \rightarrow c} f(x) = f(c)$$

Therefore, f is continuous at all points x , such that $x < -1$

Case II:

$$\text{If } c = -1, \text{ then } f(c) = f(-1) = -2$$

The left hand limit of f at $x = -1$ is,

$$\lim_{x \rightarrow -1^-} f(x) = \lim_{x \rightarrow -1^-} (-2) = -2$$

The right hand limit of f at $x = -1$ is,

$$\lim_{x \rightarrow -1^+} f(x) = \lim_{x \rightarrow -1^+} (2x) = 2 \times (-1) = -2$$

$$\therefore \lim_{x \rightarrow -1} f(x) = f(-1)$$

Therefore, f is continuous at $x = -1$

Case III:

$$\text{If } -1 < c < 1, \text{ then } f(c) = 2c$$

$$\lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} (2x) = 2c$$

$$\therefore \lim_{x \rightarrow c} f(x) = f(c)$$

Therefore, f is continuous at all points of the interval $(-1, 1)$.

Case IV:

$$\text{If } c = 1, \text{ then } f(c) = f(1) = 2 \times 1 = 2$$

The left hand limit of f at $x = 1$ is,

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} (2x) = 2 \times 1 = 2$$

The right hand limit of f at $x = 1$ is,

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} 2 = 2$$

$$\therefore \lim_{x \rightarrow 1} f(x) = f(c)$$

Therefore, f is continuous at $x = 2$

Case V:

$$\text{If } c > 1, \text{ then } f(c) = 2 \text{ and } \lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} (2) = 2$$

$$\lim_{x \rightarrow c} f(x) = f(c)$$

Therefore, f is continuous at all points x , such that $x > 1$

Thus, from the above observations, it can be concluded that f is continuous at all points of the real line

Chapter 9 Continuity Ex 9.2 Q4(i)

We have given that the function is continuous at $x = 0$

$$\therefore \text{LHL} = \text{RHL} = f(0) \quad \dots (1)$$

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0 - h) = \lim_{h \rightarrow 0} \frac{\sin(-2h)}{5(-h)} = \lim_{h \rightarrow 0} \frac{-\sin 2h}{-5h} = \lim_{h \rightarrow 0} \frac{\sin 2h}{2h} \times \frac{2h}{5h} = \frac{2}{5}$$

$$f(0) = 3k$$

So, using (1) we get,

$$\frac{2}{5} = 3k$$

$$k = \frac{2}{15}$$

Chapter 9 Continuity Ex 9.2 Q4(ii)

It is given that the function is continuous

$$\therefore \text{LHL} = \text{RHL} = f(2) \dots (1)$$

$$\text{LHL} = \lim_{x \rightarrow 2^-} f(x) = \lim_{h \rightarrow 0} f(2-h) = \lim_{h \rightarrow 0} k(2-h) + 5 = 2k + 5$$

$$\text{RHL} = \lim_{x \rightarrow 2^+} f(x) = \lim_{h \rightarrow 0} f(2+h) = \lim_{h \rightarrow 0} (2+h) - 1 = 1$$

Thus, using (1), we get,

$$2k + 5 = 1$$

$$k = -2$$

Chapter 9 Continuity Ex 9.2 Q4(iii)

It is given that the function is continuous

$$\text{LHL} = \text{RHL} = f(0) \dots (1)$$

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} k((-h)^2 + 3(-h)) = \lim_{h \rightarrow 0} k(h^2 - 3h) = 0$$

$$f(0) = \cos 2 \times 0 = \cos 0^\circ = 1$$

$$\text{LHL} \neq f(0)$$

Hence, no value of k can make f continuous

Chapter 9 Continuity Ex 9.2 Q4(iv)

First check the continuity of the function at $x = 3$

$$f(3) = 2 \dots (A)$$

$$\text{RHL} = \lim_{x \rightarrow 3^+} f(x) = \lim_{h \rightarrow 0} f(3+h) = \lim_{h \rightarrow 0} a(3+h) + b = 3a + b \dots (B)$$

$$\therefore f(x) \text{ will be continuous at } x = 3 \text{ if } 3a + b = 2 \dots (1)$$

Now, check the continuity at $x = 5$

$$f(5) = 9 \dots (C)$$

$$\text{LHL} = \lim_{x \rightarrow 5^-} f(x) = \lim_{h \rightarrow 0} f(5-h) = \lim_{h \rightarrow 0} a(5-h) + b = 5a + b$$

$$f(x) \text{ will be continuous at } x = 5 \text{ if } 5a + b = 9 \dots (2)$$

Solving (1) & (2), we get

$$a = \frac{7}{2} \text{ and } b = \frac{-17}{2}$$

Chapter 9 Continuity Ex 9.2 Q4(v)

It is given that the function is continuous

$$\text{At } x = -1$$

$$f(-1) = 4$$

$$\text{RHL} = \lim_{x \rightarrow -1^+} f(x) = \lim_{h \rightarrow 0} f(-1+h) = \lim_{h \rightarrow 0} a(-1+h)^2 + b = a + b$$

Since, $f(x)$ is continuous at $x = -1$

$$\therefore a + b = 4 \dots (A)$$

Now, at $x = 0$,

$$f(0) = \cos 0^\circ = 1$$

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} a(-h)^2 + b = b$$

Since, $f(x)$ is continuous at $x = 0$

$$\therefore f(0) = \text{LHL}$$

$$\Rightarrow b = 1$$

$$\therefore \text{from (A)}$$

$$a = 3$$

$$\text{Thus, } a = 3, b = 1$$

Chapter 9 Continuity Ex 9.2 Q4(vi)

It is given that the function is continuous.

At $x = 0$

$$\begin{aligned} \text{LHL} &= \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} \frac{\sqrt{1-ph} - \sqrt{1+ph}}{-h} = \lim_{h \rightarrow 0} \frac{(\sqrt{1-ph} - \sqrt{1+ph})}{-h} \times \frac{(\sqrt{1-ph} + \sqrt{1+ph})}{(\sqrt{1-ph} + \sqrt{1+ph})} \\ &= \lim_{h \rightarrow 0} \frac{(1-ph) - (1+ph)}{-h(\sqrt{1-ph} + \sqrt{1+ph})} = \frac{2p}{2} = p \end{aligned}$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0+h) = \lim_{h \rightarrow 0} \frac{2h+1}{h-2} = \frac{-1}{2}$$

Since, $f(x)$ is continuous so,

$$p = \frac{-1}{2}$$

Chapter 9 Continuity Ex 9.2 Q4(vii)

$$\text{The given function } f \text{ is } f(x) = \begin{cases} 5, & \text{if } x \leq 2 \\ ax+b, & \text{if } 2 < x < 10 \\ 21, & \text{if } x \geq 10 \end{cases}$$

It is evident that the given function f is defined at all points of the real line.

If f is a continuous function, then f is continuous at all real numbers.

In particular, f is continuous at $x = 2$ and $x = 10$

Since f is continuous at $x = 2$, we obtain

$$\begin{aligned} \lim_{x \rightarrow 2^-} f(x) &= \lim_{x \rightarrow 2^+} f(x) = f(2) \\ \Rightarrow \lim_{x \rightarrow 2^-} (5) &= \lim_{x \rightarrow 2^+} (ax+b) = 5 \\ \Rightarrow 5 &= 2a+b = 5 \\ \Rightarrow 2a+b &= 5 \quad \dots(1) \end{aligned}$$

Since f is continuous at $x = 10$, we obtain

$$\begin{aligned} \lim_{x \rightarrow 10^-} f(x) &= \lim_{x \rightarrow 10^+} f(x) = f(10) \\ \Rightarrow \lim_{x \rightarrow 10^-} (ax+b) &= \lim_{x \rightarrow 10^+} (21) = 21 \\ \Rightarrow 10a+b &= 21 = 21 \\ \Rightarrow 10a+b &= 21 \quad \dots(2) \end{aligned}$$

On subtracting equation (1) from equation (2), we obtain

$$8a = 16$$

$$a = 2$$

By putting $a = 2$ in equation (1), we obtain

$$2 \times 2 + b = 5$$

$$4 + b = 5$$

$$b = 1$$

Therefore, the values of a and b for which f is a continuous function are 2 and 1 respectively.

Chapter 9 Continuity Ex 9.2 Q4(viii)

Since the function is continuous at $x = \frac{\pi}{2}$ therefore

$$\begin{aligned} \text{LHL of } f(x) \text{ at } x = \frac{\pi}{2} & \text{ is} \\ &= \lim_{x \rightarrow \frac{\pi}{2}} f(x) \\ &= \lim_{h \rightarrow 0} f\left(h - \frac{\pi}{2}\right) \\ &= \lim_{h \rightarrow 0} \frac{k \cos\left(h - \frac{\pi}{2}\right)}{\pi - 2\left(h - \frac{\pi}{2}\right)} \\ &= \lim_{h \rightarrow 0} \frac{k \sinh}{2\pi - 2h} \\ &= \frac{k}{2} \lim_{h \rightarrow 0} \frac{\sin(\pi - h)}{(\pi - h)} \\ &= \frac{k}{2} \end{aligned}$$

Again

$$f\left(\frac{\pi}{2}\right) = 3$$

Hence

$$\begin{aligned} \text{LHL} &= f\left(\frac{\pi}{2}\right) \\ \frac{k}{2} &= 3 \\ k &= 6 \end{aligned}$$

Chapter 9 Continuity Ex 9.2 Q5

We have given that $f(x)$ is continuous on $[0, \infty]$

$\therefore f(x)$ is continuous at $x = 1$ and $x = \sqrt{2}$

\therefore At $x = 1$, LHL = RHL = $f(1)$ (A)

$$f(1) = a \quad \dots (1)$$

$$\text{LHL} = \lim_{x \rightarrow 1^-} f(x) = \lim_{h \rightarrow 0} f(1 - h) = \lim_{h \rightarrow 0} \frac{(1 - h)^2}{a} = \frac{1}{a}$$

Using (A) we get,

$$a = \frac{1}{a} \Rightarrow a^2 = 1 \Rightarrow a = \pm 1$$

$$\text{At } x = \sqrt{2} \text{ LHL} = \text{RHL} = f(\sqrt{2}) \quad \dots (B)$$

$$f(\sqrt{2}) = \frac{2b^2 - 4b}{(\sqrt{2})^2} = \frac{2b^2 - 4b}{2} = b^2 - 2b \quad \dots (2)$$

$$\text{LHL} = \lim_{x \rightarrow \sqrt{2}^-} f(x) = \lim_{h \rightarrow 0} f(\sqrt{2} - h) = \lim_{h \rightarrow 0} a = a$$

So, using (B), we get,

$$b^2 - 2b = a$$

$$\text{For } a = 1, \quad b^2 - 2b - 1 = 0$$

$$\Rightarrow b = \frac{2 \pm \sqrt{4 + 4}}{2} = 1 \pm \sqrt{2}$$

$$\text{For } a = -1, \quad b^2 - 2b + 1 = 0$$

$$\Rightarrow (b - 1)^2 = 0 \Rightarrow b = 1$$

$$\text{Thus, } a = -1, \quad b = 1 \text{ or } a = 1, \quad b = 1 \pm \sqrt{2}$$

Chapter 9 Continuity Ex 9.2 Q6

Since, $f(x)$ is continuous on $[0, \pi]$

$f(x)$ is continuous at $x = \frac{\pi}{4}$ and $x = \frac{\pi}{2}$

At $x = \frac{\pi}{4}$,

$$\text{LHL} = \text{RHL} = f\left(\frac{\pi}{4}\right) \dots \dots (A)$$

$$\text{Now, } f\left(\frac{\pi}{4}\right) = 2 \cdot \frac{\pi}{4} \cdot \cot\left(\frac{\pi}{4}\right) + b = \frac{\pi}{2} \cdot 1 + b = \frac{\pi}{2} + b \dots \dots (1)$$

$$\text{LHL} = \lim_{x \rightarrow \frac{\pi}{4}^-} f(x) = \lim_{h \rightarrow 0} f\left(\frac{\pi}{4} - h\right) = \lim_{h \rightarrow 0} \left(\frac{\pi}{4} - h\right) + a\sqrt{2} \sin\left(\frac{\pi}{4} - h\right) = \frac{\pi}{4} + a\sqrt{2} \cdot \frac{1}{\sqrt{2}} = \frac{\pi}{4} + a$$

Thus, using (A)

$$\frac{1}{2} + b = \frac{\pi}{4} + a$$

$$a - b = \frac{\pi}{4} \dots \dots (B)$$

At $x = \frac{\pi}{2}$

$$\text{LHL} = \text{RHL} = f\left(\frac{\pi}{2}\right) \dots \dots (C)$$

$$\text{Now, } f\left(\frac{\pi}{2}\right) = a \cos 2 \cdot \frac{\pi}{2} - b \sin \frac{\pi}{2} = -a - b \dots \dots (2)$$

$$\begin{aligned} \text{LHL} &= \lim_{x \rightarrow \frac{\pi}{2}^-} f(x) = \lim_{h \rightarrow 0} f\left(\frac{\pi}{2} - h\right) \\ &= \lim_{h \rightarrow 0} 2\left(\frac{\pi}{2} - h\right) \cot\left(\frac{\pi}{2} - h\right) + b = \pi \times 0 + b = b \end{aligned}$$

using (C), we get,

$$-a - b = b \Rightarrow 2b = -a \Rightarrow b = \frac{-a}{2}$$

$$\text{from (B), } a + \frac{a}{2} = \frac{\pi}{4}$$

$$\Rightarrow \frac{3}{2}a = \frac{\pi}{4}$$

$$\Rightarrow a = \frac{\pi}{6}$$

$$\text{and } b = \frac{-a}{2} = \frac{-\pi}{12}$$

$$\text{Thus, } a = \frac{\pi}{6}, \quad b = \frac{-\pi}{12}$$

Chapter 9 Continuity Ex 9.2 Q7

It is given that the $f(x)$ is continuous on $[0, 8]$

$f(x)$ is continuous at $x = 2$ and $x = 4$.

Now, At $x = 2$

$$\text{LHL} = \text{RHL} = f(2) \dots (A)$$

$$f(2) = 3 \times 2 + 2 = 8 \dots (1)$$

$$\text{LHL} = \lim_{x \rightarrow 2^-} f(x) = \lim_{h \rightarrow 0} f(2-h) = \lim_{h \rightarrow 0} (2-h)^2 + a(2-h) + b = 4 + 2a + b$$

from (A)

$$4 + 2a + b = 8$$

$$2a + b = 4 \dots (B)$$

Now, At $x = 4$

$$\text{LHL} = \text{RHL} = f(4) \dots (C)$$

$$f(4) = 3 \times 4 + 2 = 14 \dots (2)$$

$$\text{RHL} = \lim_{x \rightarrow 4^+} f(x) = \lim_{h \rightarrow 0} f(4+h) = \lim_{h \rightarrow 0} 2a(4+h) + 5b = 8a + 5b$$

From (C), we get,

$$8a + 5b = 14 \dots (D)$$

Solving (B) and (D), we get,

$$a = 3 \text{ and } b = -2$$

Chapter 9 Continuity Ex 9.2 Q8

The function will be continuous on $\left[0, \frac{\pi}{2}\right]$ if it is continuous at every point in $\left[0, \frac{\pi}{2}\right]$

Let us consider the point $x = \frac{\pi}{4}$,

We must have,

$$\text{LHL} = \text{RHL} = f\left(\frac{\pi}{4}\right) \dots (A)$$

$$\text{LHL} = \lim_{x \rightarrow \frac{\pi}{4}^-} f(x) = \lim_{h \rightarrow 0} f\left(\frac{\pi}{4} - h\right) = \lim_{h \rightarrow 0} \frac{\tan\left(\frac{\pi}{4} - \frac{\pi}{4} + h\right)}{\cot + 2\left(\frac{\pi}{4} - h\right)} = \lim_{h \rightarrow 0} \frac{\tan h}{\tan 2h} \quad \left[\because \cot\left(\frac{\pi}{2} - \theta\right) = \tan \theta \right]$$

$$= \lim_{h \rightarrow 0} \frac{\frac{\tan h}{h}}{\frac{\tan 2h}{h}} = \frac{1}{2}$$

Thus, using (A) we get,

$$f\left(\frac{\pi}{4}\right) = \frac{1}{2}$$

Hence, $f(x)$ will be continuous on $\left[0, \frac{\pi}{2}\right]$ if $f\left(\frac{\pi}{4}\right) = \frac{1}{2}$.

Chapter 9 Continuity Ex 9.2 Q9

When $x < 2$, we have

$f(x) = 2x - 1$, which is a polynomial of degree 1.

So, $f(x)$ is continuous for $x < 2$.

When $x > 2$, we have

$f(x) = \frac{3x}{2}$, which is again a polynomial of degree 1.

So, $f(x)$ is continuous for $x > 2$.

Now, consider the point $x = 2$

$$f(2) = \frac{3 \times 2}{2} = 3$$

$$\text{LHL} = \lim_{x \rightarrow 2^-} f(x) = \lim_{h \rightarrow 0} f(2 - h) = \lim_{h \rightarrow 0} 2(2 - h) - 1 = 3$$

$$\text{RHL} = \lim_{x \rightarrow 2^+} f(x) = \lim_{h \rightarrow 0} f(2 + h) = \lim_{h \rightarrow 0} \frac{3(2 + h)}{2} = 3$$

$$\text{LHL} = \text{RHL} = f(2) = 3$$

Thus, $f(x)$ is continuous at $x = 2$

Hence, $f(x)$ is continuous every where.

Chapter 9 Continuity Ex 9.2 Q10

Let $f(x) = \sin|x|$

This function f is defined for every real number and f can be written as the composition of two functions as,

$f = g \circ h$, where $g(x) = |x|$ and $h(x) = \sin x$

$$[\therefore (g \circ h)(x) = g(h(x)) = g(\sin x) = |\sin x| = f(x)]$$

It has to be proved first that $g(x) = |x|$ and $h(x) = \sin x$ are continuous functions.

$g(x) = |x|$ can be written as

$$g(x) = \begin{cases} -x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$$

Clearly, g is defined for all real numbers.

Let c be a real number.

Case I:

If $c < 0$, then $g(c) = -c$ and $\lim_{x \rightarrow c} g(x) = \lim_{x \rightarrow c} (-x) = -c$

$$\therefore \lim_{x \rightarrow c} g(x) = g(c)$$

Therefore, g is continuous at all points x , such that $x < 0$

Case II:

If $c > 0$, then $g(c) = c$ and $\lim_{x \rightarrow c} g(x) = \lim_{x \rightarrow c} x = c$

$$\therefore \lim_{x \rightarrow c} g(x) = g(c)$$

Therefore, g is continuous at all points x , such that $x > 0$

Case III:

If $c = 0$, then $g(c) = g(0) = 0$

$$\lim_{x \rightarrow 0^-} g(x) = \lim_{x \rightarrow 0^-} (-x) = 0$$

$$\lim_{x \rightarrow 0^+} g(x) = \lim_{x \rightarrow 0^+} (x) = 0$$

$$\therefore \lim_{x \rightarrow 0} g(x) = \lim_{x \rightarrow 0} (x) = g(0)$$

Therefore, g is continuous at $x = 0$

From the above three observations, it can be concluded that g is continuous at all points.

$$h(x) = \sin x$$

It is evident that $h(x) = \sin x$ is defined for every real number.

Let c be a real number. Put $x = c + k$

If $x \rightarrow c$, then $k \rightarrow 0$

$$h(c) = \sin c$$

$$h(c) = \sin c$$

$$\lim_{x \rightarrow c} h(x) = \lim_{x \rightarrow c} \sin x$$

$$= \lim_{k \rightarrow 0} \sin(c + k)$$

$$= \lim_{k \rightarrow 0} [\sin c \cos k + \cos c \sin k]$$

$$= \lim_{k \rightarrow 0} (\sin c \cos k) + \lim_{k \rightarrow 0} (\cos c \sin k)$$

$$= \sin c \cos 0 + \cos c \sin 0$$

$$= \sin c + 0$$

$$= \sin c$$

$$\therefore \lim_{x \rightarrow c} h(x) = g(c)$$

Therefore, h is a continuous function.

It is known that for real valued functions g and h , such that $(g \circ h)$ is defined at c , if g is continuous at c and if h is continuous at $g(c)$, then $(g \circ h)$ is continuous at c .

Therefore, $f(x) = (g \circ h)(x) = g(h(x)) = g(\sin x) = |\sin x|$ is a continuous function.

Chapter 9 Continuity Ex 9.2 Q11

When $x < 0$, we have,

$$f(x) = \frac{\sin x}{x}$$

We know that the $\sin x$ and the identity function x are continuous for $x < 0$.

So, the quotient function $f(x) = \frac{\sin x}{x}$ is continuous for $x < 0$.

When $x > 0$, we have,

$f(x) = x + 1$, which is a polynomial of degree 1. So, $f(x)$ is continuous for $x > 0$

Now, consider the point $x = 0$.

$$f(0) = 0 + 1 = 1.$$

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0 - h) = \lim_{h \rightarrow 0} \frac{\sin(-h)}{-h} = \lim_{h \rightarrow 0} \frac{-\sin h}{-h} = 1$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0 + h) = \lim_{h \rightarrow 0} h + 1 = 1$$

Thus, $\text{LHL} = \text{RHL} = f(0) = 1$

So, $f(x)$ is continuous at $x = 0$.

Hence, $f(x)$ is continuous everywhere

Chapter 9 Continuity Ex 9.2 Q12

The given function is $g(x) = x - [x]$

It is evident that g is defined at all integral points.

Let n be an integer.

Then,

$$g(n) = n - [n] = n - n = 0$$

The left hand limit of f at $x = n$ is,

$$\lim_{x \rightarrow n^-} g(x) = \lim_{x \rightarrow n^-} (x - [x]) = \lim_{x \rightarrow n^-} (x) - \lim_{x \rightarrow n^-} [x] = n - (n - 1) = 1$$

The right hand limit of f at $x = n$ is,

$$\lim_{x \rightarrow n^+} g(x) = \lim_{x \rightarrow n^+} (x - [x]) = \lim_{x \rightarrow n^+} (x) - \lim_{x \rightarrow n^+} [x] = n - n = 0$$

It is observed that the left and right hand limits of f at $x = n$ do not coincide.

Therefore, f is not continuous at $x = n$

Hence, g is discontinuous at all integral points

Chapter 9 Continuity Ex 9.2 Q13

It is known that if g and h are two continuous functions, then

$g + h$, $g - h$, and $g \cdot h$ are also continuous.

It has to be proved first that $g(x) = \sin x$ and $h(x) = \cos x$ are continuous functions.

Let $g(x) = \sin x$

It is evident that $g(x) = \sin x$ is defined for every real number.

Let c be a real number. Put $x = c + h$

If $x \rightarrow c$, then $h \rightarrow 0$

$$g(c) = \sin c$$

$$\begin{aligned}\lim_{x \rightarrow c} g(x) &= \lim_{x \rightarrow c} \sin x \\ &= \lim_{h \rightarrow 0} \sin(c + h) \\ &= \lim_{h \rightarrow 0} [\sin c \cos h + \cos c \sin h] \\ &= \lim_{h \rightarrow 0} (\sin c \cos h) + \lim_{h \rightarrow 0} (\cos c \sin h) \\ &= \sin c \cos 0 + \cos c \sin 0 \\ &= \sin c + 0 \\ &= \sin c\end{aligned}$$

$$\therefore \lim_{x \rightarrow c} g(x) = g(c)$$

Therefore, g is a continuous function.

Let $h(x) = \cos x$

It is evident that $h(x) = \cos x$ is defined for every real number.

Let c be a real number. Put $x = c + h$

If $x \rightarrow c$, then $h \rightarrow 0$

$$h(c) = \cos c$$

$$\begin{aligned}\lim_{x \rightarrow c} h(x) &= \lim_{x \rightarrow c} \cos x \\ &= \lim_{h \rightarrow 0} \cos(c + h) \\ &= \lim_{h \rightarrow 0} [\cos c \cos h - \sin c \sin h] \\ &= \lim_{h \rightarrow 0} \cos c \cos h - \lim_{h \rightarrow 0} \sin c \sin h \\ &= \cos c \cos 0 - \sin c \sin 0 \\ &= \cos c \times 1 - \sin c \times 0 \\ &= \cos c\end{aligned}$$

$$\therefore \lim_{x \rightarrow c} h(x) = h(c)$$

Therefore, h is a continuous function.

Therefore, it can be concluded that

(a) $f(x) = g(x) + h(x) = \sin x + \cos x$ is a continuous function

(b) $f(x) = g(x) - h(x) = \sin x - \cos x$ is a continuous function

(c) $f(x) = g(x) \times h(x) = \sin x \times \cos x$ is a continuous function

Chapter 9 Continuity Ex 9.2 Q14

The given function is $f(x) = \cos(x^2)$

This function f is defined for every real number and f can be written as the composition of two functions as,

$f = g \circ h$, where $g(x) = \cos x$ and $h(x) = x^2$

$$\left[\because (goh)(x) = g(h(x)) = g(x^2) = \cos(x^2) = f(x) \right]$$

It has to be first proved that $g(x) = \cos x$ and $h(x) = x^2$ are continuous functions.

It is evident that g is defined for every real number.

Let c be a real number.

Then, $g(c) = \cos c$

Put $x = c + h$

If $x \rightarrow c$, then $h \rightarrow 0$

$$\begin{aligned} \lim_{x \rightarrow c} g(x) &= \lim_{x \rightarrow c} \cos x \\ &= \lim_{h \rightarrow 0} \cos(c + h) \\ &= \lim_{h \rightarrow 0} [\cos c \cos h - \sin c \sin h] \\ &= \lim_{h \rightarrow 0} \cos c \cos h - \lim_{h \rightarrow 0} \sin c \sin h \\ &= \cos c \cos 0 - \sin c \sin 0 \\ &= \cos c \times 1 - \sin c \times 0 \\ &= \cos c \end{aligned}$$

$$\therefore \lim_{x \rightarrow c} g(x) = g(c)$$

Therefore, $g(x) = \cos x$ is continuous function.

$$h(x) = x^2$$

Clearly, h is defined for every real number.

Let k be a real number, then $h(k) = k^2$

$$\begin{aligned} \lim_{x \rightarrow k} h(x) &= \lim_{x \rightarrow k} x^2 = k^2 \\ \therefore \lim_{x \rightarrow k} h(x) &= h(k) \end{aligned}$$

Therefore, h is a continuous function.

It is known that for real valued functions g and h , such that $(g \circ h)$ is defined at c , if g is continuous at c and if f is continuous at $g(c)$, then $(f \circ g)$ is continuous at c .

Therefore, $f(x) = (goh)(x) = \cos(x^2)$ is a continuous function.

Chapter 9 Continuity Ex 9.2 Q15

The given function is $f(x) = |\cos x|$

This function f is defined for every real number and f can be written as the composition of two functions as,

$f = g \circ h$, where $g(x) = |x|$ and $h(x) = \cos x$

$$\left[\because (goh)(x) = g(h(x)) = g(\cos x) = |\cos x| = f(x) \right]$$

It has to be first proved that $g(x) = |x|$ and $h(x) = \cos x$ are continuous functions.

$g(x) = |x|$ can be written as

$$g(x) = \begin{cases} -x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$$

Clearly, g is defined for all real numbers.

Let c be a real number.

Case I:

$$\begin{aligned}\text{If } c < 0, \text{ then } g(c) = -c \text{ and } \lim_{x \rightarrow c} g(x) &= \lim_{x \rightarrow c} (-x) = -c \\ \therefore \lim_{x \rightarrow c} g(x) &= g(c)\end{aligned}$$

Therefore, g is continuous at all points x , such that $x < 0$

Case II:

$$\begin{aligned}\text{If } c > 0, \text{ then } g(c) = c \text{ and } \lim_{x \rightarrow c} g(x) &= \lim_{x \rightarrow c} x = c \\ \therefore \lim_{x \rightarrow c} g(x) &= g(c)\end{aligned}$$

Therefore, g is continuous at all points x , such that $x > 0$

Case III:

$$\text{If } c = 0, \text{ then } g(c) = g(0) = 0$$

$$\begin{aligned}\lim_{x \rightarrow 0^-} g(x) &= \lim_{x \rightarrow 0^-} (-x) = 0 \\ \lim_{x \rightarrow 0^+} g(x) &= \lim_{x \rightarrow 0^+} (x) = 0 \\ \therefore \lim_{x \rightarrow 0} g(x) &= \lim_{x \rightarrow 0} (x) = g(0)\end{aligned}$$

Therefore, g is continuous at $x = 0$

From the above three observations, it can be concluded that g is continuous at all points.

$$h(x) = \cos x$$

It is evident that $h(x) = \cos x$ is defined for every real number.

Let c be a real number. Put $x = c + h$

If $x \rightarrow c$, then $h \rightarrow 0$

$$h(c) = \cos c$$

$$\begin{aligned}\lim_{x \rightarrow c} h(x) &= \lim_{x \rightarrow c} \cos x \\ &= \lim_{h \rightarrow 0} \cos(c + h) \\ &= \lim_{h \rightarrow 0} [\cos c \cos h - \sin c \sin h] \\ &= \lim_{h \rightarrow 0} \cos c \cos h - \lim_{h \rightarrow 0} \sin c \sin h \\ &= \cos c \cos 0 - \sin c \sin 0 \\ &= \cos c \times 1 - \sin c \times 0 \\ &= \cos c \\ \therefore \lim_{x \rightarrow c} h(x) &= h(c)\end{aligned}$$

Therefore, $h(x) = \cos x$ is a continuous function.

It is known that for real valued functions g and h , such that $(g \circ h)$ is defined at c , if g is continuous at c and if h is continuous at $g(c)$, then $(g \circ h)$ is continuous at c .

Therefore, $f(x) = (g \circ h)(x) = g(h(x)) = g(\cos x) = |\cos x|$ is a continuous function

The given function is $f(x) = |x| - |x+1|$

The two functions, g and h , are defined as

$$g(x) = |x| \text{ and } h(x) = |x+1|$$

Then, $f = g - h$

The continuity of g and h is examined first.

$g(x) = |x|$ can be written as

$$g(x) = \begin{cases} -x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$$

Clearly, g is defined for all real numbers.

Let c be a real number.

Case I:

If $c < 0$, then $g(c) = -c$ and $\lim_{x \rightarrow c} g(x) = \lim_{x \rightarrow c} (-x) = -c$

$$\therefore \lim_{x \rightarrow c} g(x) = g(c)$$

Therefore, g is continuous at all points x , such that $x < 0$

Case II:

If $c > 0$, then $g(c) = c$ and $\lim_{x \rightarrow c} g(x) = \lim_{x \rightarrow c} x = c$

$$\therefore \lim_{x \rightarrow c} g(x) = g(c)$$

Therefore, g is continuous at all points x , such that $x > 0$

Case III:

If $c = 0$, then $g(c) = g(0) = 0$

$$\lim_{x \rightarrow 0^-} g(x) = \lim_{x \rightarrow 0^-} (-x) = 0$$

$$\lim_{x \rightarrow 0^+} g(x) = \lim_{x \rightarrow 0^+} (x) = 0$$

$$\therefore \lim_{x \rightarrow 0^-} g(x) = \lim_{x \rightarrow 0^+} g(x) = g(0)$$

Therefore, g is continuous at $x = 0$

From the above three observations, it can be concluded that g is continuous at all points.

$h(x) = |x+1|$ can be written as

$$h(x) = \begin{cases} -(x+1), & \text{if } x < -1 \\ x+1, & \text{if } x \geq -1 \end{cases}$$

Clearly, h is defined for every real number.

Let c be a real number.

Case I:

$$\text{If } c < -1, \text{ then } h(c) = -(c+1) \text{ and } \lim_{x \rightarrow c} h(x) = \lim_{x \rightarrow c} [-(x+1)] = -(c+1) \\ \therefore \lim_{x \rightarrow c} h(x) = h(c)$$

Therefore, h is continuous at all points x , such that $x < -1$

Case II:

$$\text{If } c > -1, \text{ then } h(c) = c+1 \text{ and } \lim_{x \rightarrow c} h(x) = \lim_{x \rightarrow c} (x+1) = c+1 \\ \therefore \lim_{x \rightarrow c} h(x) = h(c)$$

Therefore, h is continuous at all points x , such that $x > -1$

Case III:

$$\text{If } c = -1, \text{ then } h(c) = h(-1) = -1+1 = 0$$

$$\lim_{x \rightarrow -1^-} h(x) = \lim_{x \rightarrow -1^-} [-(x+1)] = -(-1+1) = 0$$

$$\lim_{x \rightarrow -1^+} h(x) = \lim_{x \rightarrow -1^+} (x+1) = (-1+1) = 0$$

$$\therefore \lim_{x \rightarrow -1^-} h(x) = \lim_{h \rightarrow -1^+} h(x) = h(-1)$$

Therefore, h is continuous at $x = -1$

From the above three observations, it can be concluded that h is continuous at all points of the real line.

g and h are continuous functions. Therefore, $f = g - h$ is also a continuous function.

Therefore, f has no point of discontinuity.

Chapter 9 Continuity Ex 9.2 Q17

$$\text{The given function } f \text{ is } f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

It is evident that f is defined at all points of the real line.

Let c be a real number.

Case I:

$$\text{If } c \neq 0, \text{ then } f(c) = c^2 \sin \frac{1}{c}$$

$$\lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} \left(x^2 \sin \frac{1}{x} \right) = \left(\lim_{x \rightarrow c} x^2 \right) \left(\lim_{x \rightarrow c} \sin \frac{1}{x} \right) = c^2 \sin \frac{1}{c}$$

$$\therefore \lim_{x \rightarrow c} f(x) = f(c)$$

Therefore, f is continuous at all points $x \neq 0$

Case II:

If $c = 0$, then $f(0) = 0$

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \left(x^2 \sin \frac{1}{x} \right) = \lim_{x \rightarrow 0} \left(x^2 \sin \frac{1}{x} \right)$$

It is known that, $-1 \leq \sin \frac{1}{x} \leq 1$, $x \neq 0$

$$\Rightarrow -x^2 \leq \sin \frac{1}{x} \leq x^2$$

$$\Rightarrow \lim_{x \rightarrow 0} (-x^2) \leq \lim_{x \rightarrow 0} \left(x^2 \sin \frac{1}{x} \right) \leq \lim_{x \rightarrow 0} x^2$$

$$\Rightarrow 0 \leq \lim_{x \rightarrow 0} \left(x^2 \sin \frac{1}{x} \right) \leq 0$$

$$\Rightarrow \lim_{x \rightarrow 0} \left(x^2 \sin \frac{1}{x} \right) = 0$$

$$\therefore \lim_{x \rightarrow 0^-} f(x) = 0$$

$$\text{Similarly, } \lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \left(x^2 \sin \frac{1}{x} \right) = \lim_{x \rightarrow 0} \left(x^2 \sin \frac{1}{x} \right) = 0$$

$$\therefore \lim_{x \rightarrow 0} f(x) = f(0) = \lim_{x \rightarrow 0^+} f(x)$$

Therefore, f is continuous at $x = 0$

From the above observations, it can be concluded that f is continuous at every point of the real line.

Thus, f is a continuous function.

Chapter 9 Continuity Ex 9.2 Q18

$$f(x) = \frac{1}{x+2}$$

$$\lim_{x \rightarrow -2^-} f(x) = \lim_{h \rightarrow 0} \frac{1}{-2-h+2} = \lim_{h \rightarrow 0} -\frac{1}{h} \rightarrow -\infty$$

$$\lim_{x \rightarrow -2^+} f(x) = \lim_{h \rightarrow 0} \frac{1}{-2+h+2} = \lim_{h \rightarrow 0} \frac{1}{h} \rightarrow \infty$$

$\therefore f(x)$ is discontinuous at $x = -2$

$$\text{Let } g(x) = f(f(x)) = \frac{x+2}{2x+5}$$

$$\lim_{x \rightarrow -\frac{5}{2}^-} g(x) = \lim_{h \rightarrow 0} \frac{-\frac{5}{2}-h+2}{-5-h+5} = \lim_{h \rightarrow 0} -\frac{-\frac{5}{2}-h+2}{h} \rightarrow -\infty$$

$$\lim_{x \rightarrow -\frac{5}{2}^+} f(x) = \lim_{h \rightarrow 0} \frac{-\frac{5}{2}+h+2}{-5+h+5} = \lim_{h \rightarrow 0} \frac{-\frac{5}{2}-h+2}{h} \rightarrow \infty$$

$\therefore g(x)$ is discontinuous at $x = -\frac{5}{2}$

$\therefore f(f(x))$ is discontinuous at $x = -\frac{5}{2}$

$\therefore f(x)$ is discontinuous at $x = -2$ and $-\frac{5}{2}$.

Chapter 9 Continuity Ex 9.2 Q19

$$f(t) = \frac{1}{t^2 + t - 2}, \text{ where } t = \frac{1}{x-1}$$

Clearly $t = \frac{1}{x-1}$ is discontinuous at $x = 1$.

For $x \neq 1$, we have

$$f(t) = \frac{1}{t^2 + t - 2} = \frac{1}{(t+2)(t-1)}$$

This is discontinuous at $t = -2$ and $t = 1$

$$\text{For } t = -2, t = \frac{1}{x-1} \Rightarrow x = \frac{1}{2}$$

$$\text{For } t = 1, t = \frac{1}{x-1} \Rightarrow x = 2$$

Hence f is discontinuous at $x = \frac{1}{2}$, $x = 1$ and $x = 2$.