Ex 10.1

Chapter 10 Differentiability Ex 10.1 Q1

$$f(x) = |x - 3|$$
$$= \begin{cases} -(x-3), & \text{if } x < 3 \\ x - 3, & \text{if } x \ge 3 \end{cases}$$
$$f(3) = 3 - 3 = 0$$

$$\text{LHL} = \lim_{x \to 3^-} f(x) = \lim_{h \to 0} f(3-h) = \lim_{h \to 0} 3 - (3-h) = \lim_{h \to 0} 0$$

$$\text{RHL} = \lim_{x \to 3^+} f(x) = \lim_{h \to 0} f(3+h) = \lim_{h \to 0} 3 + h - 3 = 0$$

LHL = $f(3)$ = RHL

$\therefore$ $f(x)$ is continuous at $x = 3$

$$(\text{LHD at } x = 3) = \lim_{x \to 3^-} \frac{f(x) - f(3)}{x - 3} = \lim_{h \to 0} \frac{f(3-h) - f(3)}{3 - h - 3} = \lim_{h \to 0} \frac{3 - (3-h) - 0}{-h} = \lim_{h \to 0} \frac{h}{-h} = -1$$

$$(\text{RHD at } x = 3) = \lim_{x \to 3^+} \frac{f(x) - f(3)}{x - 3} = \lim_{h \to 0} \frac{f(3+h) - f(3)}{3 + h - 3} = \lim_{h \to 0} \frac{3 + h - 3 - 0}{h} = \lim_{h \to 0} \frac{h}{h} = 1$$

$(\text{LHD at } x = 3) \neq (\text{RHD at } x = 3)$

$\therefore$ $f(x)$ is continuous but not differentiable at $x = 3$.

Chapter 10 Differentiability Ex 10.1 Q2

$$f(x) = x^{\frac{1}{3}}$$

$$\begin{aligned}
(\text{LHD at } x = 0) &= \lim_{x \to 0^-} \frac{f(x) - f(0)}{x - 0} \\
&= \lim_{h \to 0} \frac{f(0 - h) - f(0)}{0 - h - 0} \\
&= \lim_{h \to 0} \frac{(-h)^{\frac{1}{3}} - 0}{-h} \\
&= \lim_{h \to 0} \frac{(-h)^{\frac{1}{3}}}{-h} \\
&= \lim_{h \to 0} \frac{(-1)^{\frac{1}{3}} h^{\frac{1}{3}}}{(-1)h} \\
&= \lim_{h \to 0} (-1)^{\frac{-2}{3}} h^{\frac{-2}{3}} \\
&= \text{Not defined}
\end{aligned}$$

$$\begin{aligned}
(\text{RHD at } x = 0) &= \lim_{x \to 0^+} \frac{f(x) - f(0)}{x - 0} \\
&= \lim_{h \to 0} \frac{f(0 + h) - f(0)}{0 + h - 0} \\
&= \lim_{h \to 0} \frac{h^{\frac{1}{3}} - 0}{h} \\
&= \lim_{h \to 0} h^{\frac{-2}{3}} \\
&= \text{Not defined}
\end{aligned}$$

Since,

LHD and RHD does not exists at $x = 0$

$\therefore \quad f(x)$ is not differentiable at $x = 0$

Chapter 10 Differentiability Ex 10.1 Q3

$$f(x) = \begin{cases} 12x - 13, & \text{if } x \le 3 \\ 2x^2 + 5, & \text{if } x > 3 \end{cases}$$

$$\begin{aligned}
(\text{LHD at } x = 3) &= \lim_{x \to 3^-} \frac{f(x) - f(3)}{x - 3} \\
&= \lim_{h \to 0} \frac{f(3 - h) - f(3)}{3 - h - 3} \\
&= \lim_{h \to 0} \frac{[12(3 - h) - 13] - [12(3) - 13]}{-h} \\
&= \lim_{h \to 0} \frac{36 - 12h - 13 - 36 + 13}{-h} \\
&= \lim_{h \to 0} \frac{-12h}{-h} \\
&= 12
\end{aligned}$$

$$\begin{aligned}
(\text{RHD at } x = 3) &= \lim_{x \to 3^+} \frac{f(x) - f(3)}{x - 3} \\
&= \lim_{h \to 0} \frac{f(3 + h) - f(3)}{x - 3} \\
&= \lim_{h \to 0} \frac{\left[2\left(3 + h^2\right) + 5\right] - [12(3) - 13]}{3 + h - 3} \\
&= \lim_{h \to 0} \frac{18 + 12h + 2h^2 + 5 - 36 + 13}{h} \\
&= \lim_{h \to 0} \frac{2h^2 + 12h}{h} \\
&= \lim_{h \to 0} \frac{h(2h + 12)}{h} \\
&= 12
\end{aligned}$$

Now,

$(\text{LHD at } x = 3) = (\text{RHD at } x = 3)$

$\therefore \quad f(x)$ is differentiable at $x = 3$

$\quad f'(x) = 12$

Chapter 10 Differentiability Ex 10.1 Q4

$$f(x) = \begin{cases} 3x - 2 \ , \ 0 < x \le 1 \\ 2x^2 - x \qquad , \ 1 < x \le 2 \\ 5x - 4 \ , \ x > 2 \end{cases}$$

$$f(2) = 2(2)^2 - 2$$
$$= 8 - 2 = 6$$

$$\text{LHL} = \lim_{x \to 2^-} f(x)$$
$$= \lim_{h \to 0} f(2 - h)$$
$$= \lim_{h \to 0} \left[2(2 - h)^2 - (2 - h)\right]$$
$$= 8 - 2$$
$$= 6$$

$$\text{RHL} = \lim_{x \to 2^+} f(x)$$
$$= \lim_{h \to 0} f(2 + h)$$
$$= \lim_{h \to 0} 5(2 + h) - 4$$
$$= 6$$

$$\text{LHL} = f(2) = \text{RHL}$$

$f(x)$ is continuous at $x = 2$

$$(\text{LHD at } x = 2) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x - 2}$$
$$= \lim_{h \to 0} \frac{f(2 - h) - f(2)}{2 - h - 2}$$
$$= \lim_{h \to 0} \frac{\left[2(2 - h)^2 - (2 - h)\right] - [8 - 2]}{-h}$$
$$= \lim_{h \to 0} \frac{8 - 8h + 2h^2 - h - 6}{-h}$$
$$= \lim_{h \to 0} \frac{2h^2 - 6h}{-h}$$
$$= \lim_{h \to 0} \frac{h(2h - 6)}{-h}$$
$$= \lim_{h \to 0} (6 - 2h)$$
$$= 6$$

$$(\text{RHD at } x = 2) = \lim_{x \to 2^+} \frac{f(x) - f(2)}{x - 2}$$
$$= \lim_{h \to 0} \frac{f(2 + h) - f(2)}{2 + h - 2}$$
$$= \lim_{h \to 0} \frac{[5(2 + h) - 4] - [8 - 2]}{h}$$
$$= \lim_{h \to 0} \frac{10 + 5h - 4 - 6}{h}$$
$$= 5$$

Chapter 10 Differentiability Ex 10.1 Q5

$f(x) = |x|+|x-1|$ in theinterval $(-1, 2)$.

$$f(x)=\begin{cases} x+x+1 & -1<x<0 \\ 1 & 0\le x\le 1 \\ -x-x+1 & 1<x<2 \end{cases}$$

$$f(x)=\begin{cases} 2x+1 & -1<x<0 \\ 1 & 0\le x\le 1 \\ -2x+1 & 1<x<2 \end{cases}$$

We know that a polynomial and a constant function is continuous and differentiable everywhere.
So, $f(x)$ is continuous and differentiable for $x \in (-1, 0)$, $x \in (0, 1)$ and $(1, 2)$.

We need to check continuity and differentiability at $x = 0$ and $x = 1$.

Continuity at $x = 0$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} 2x+1 = 1$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} 1 = 1$$

$$f(0) = 1$$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0)$$
$\therefore$ $f(x)$ is continuous at $x = 0$.

Continuity at $x = 1$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} 1 = 1$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} 1 = 1$$

$$f(1) = 1$$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(1)$$
$\therefore$ $f(x)$ is continuous at $x = 1$.

Differentiability at $x = 0$

$$(\text{LHD at } x = 0) = \lim_{x\to 0^-} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0^-} \frac{2x+1-1}{x-0} = \lim_{x\to 0^-} \frac{2x}{x} = 2$$
$$(\text{RHD at } x = 0) = \lim_{x\to 0^+} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0^+} \frac{1-1}{x} = \lim_{x\to 0^+} \frac{0}{x} = 0$$

$\therefore$ $(\text{LHD at } x = 0) \ne (\text{RHD at } x = 0)$
So, $f(x)$ is differentiable at $x = 0$.

Differentiability at $x = 1$

$$(\text{LHD at } x = 1) = \lim_{x\to 1^-} \frac{f(x)-f(1)}{x-1} = \lim_{x\to 1^-} \frac{1-1}{x-1} = 0$$
$$(\text{RHD at } x = 1) = \lim_{x\to 1^+} \frac{f(x)-f(1)}{x-1} = \lim_{x\to 1^+} \frac{-2x+1-1}{x-1} \to \infty$$

$\therefore$ $(\text{LHD at } x = 1) \ne (\text{RHD at } x = 1)$
So, $f(x)$ is not differentiable at $x = 1$.

So, $f(x)$ is continuous on $(-1, 2)$ but not differentiable at $x = 0, 1$.

Chapter 10 Differentiability Ex 10.1 Q6

$$f(x)=\begin{cases} x, & x \le 1 \\ 2-x, & 1 \le x \le 2 \\ -2+3x-x^2, & x > 2 \end{cases}$$

Differentiability at x = 1

$$(\text{LHD at } x = 1) = \lim_{x\to 1^-} \frac{f(x)-f(1)}{x-1} = \lim_{x\to 1^-} \frac{x-1}{x-1} = 1$$

$$(\text{RHD at } x = 1) = \lim_{x\to 1^+} \frac{f(x)-f(1)}{x-1} = \lim_{x\to 1^+} \frac{2-x-1}{x-1} = \lim_{x\to 1^+} \frac{1-x}{x-1} = -1$$

$\therefore$ (LHD at x = 1) $\ne$ (RHD at x = 1)

So, f(x) is not differentiable at x = 1.

Differentiability at x = 2

$$(\text{LHD at } x = 2) = \lim_{x\to 2^-} \frac{f(x)-f(2)}{x-2} = \lim_{x\to 2^-} \frac{2-x-0}{x-2} = \lim_{x\to 2^-} \frac{2-x}{x-2} = -1$$

$$(\text{RHD at } x = 2) = \lim_{x\to 2^+} \frac{f(x)-f(2)}{x-2} = \lim_{x\to 2^+} \frac{-2+3x-x^2-0}{x-2} = \lim_{x\to 2^+} \frac{(1-x)(x-2)}{x-2} = -1$$

$\therefore$ (LHD at x = 2) = (RHD at x = 2)

So, f(x) is differentiable at x = 2.

Chapter 10 Differentiability Ex 10.1 Q7(i)

$$f(x)=\begin{cases} x^m \sin\left(\frac{1}{x}\right), & x \ne 0 \\ 0, & x = 0 \end{cases}$$

$$\begin{aligned}
(\text{LHD at } x = 0) &= \lim_{x\to 0^-} \frac{f(x)-f(0)}{x-0} \\
&= \lim_{h\to 0} \frac{f(0-h)-f(0)}{(0-h)-0} \\
&= \lim_{h\to 0} \frac{(0-h)^m \sin\left(\frac{1}{-h}\right)-0}{-h} \\
&= \lim_{h\to 0} \frac{(-h)^m \sin\left(-\frac{1}{h}\right)}{-h} \\
&= \lim_{h\to 0} (-h)^{m-1} \sin\left(-\frac{1}{h}\right) \\
&= \lim_{h\to 0} -(-h)^{m-1} \sin\left(\frac{1}{h}\right) \\
&= 0 \times k \qquad [\text{When } -1 \le k \le 1] \\
&= 0
\end{aligned}$$

$$\begin{aligned}
(\text{RHD at } x = 0) &= \lim_{x\to 0^+} \frac{f(x)-f(0)}{x-0} \\
&= \lim_{h\to 0} \frac{f(0+h)-f(0)}{(0+h)-0} \\
&= \lim_{h\to 0} \frac{h^m \sin\left(\frac{1}{h}\right)-0}{h} \\
&= \lim_{h\to 0} \left(h^{m-1}\right) \sin\left(\frac{1}{h}\right) \\
&= 0 \times k' \qquad [\text{Since } -1 \le k' \le 1] \\
&= 0
\end{aligned}$$

(LHD at $x = 0$) = (RHD at $x = 0$)

$\therefore$ $f(x)$ is differentiable at $x = 0$

Chapter 10 Differentiability Ex 10.1 Q7(ii)

LHL $= \lim_{x\to 0^-} f(x)$

$= \lim_{h\to 0} f(0-h)$

$= \lim_{h\to 0} (-h)^m \sin\left(-\frac{1}{h}\right)$

$= -\lim_{h\to 0} (-h)^m \sin\left(\frac{1}{h}\right)$

$= 0\times k$ [When $-1 \le k \le 1$]

$= 0$

RHL $= \lim_{x\to 0^+} f(x)$

$= \lim_{h\to 0} f(0+h)$

$= \lim_{h\to 0} (0+h)^m \sin\frac{1}{(0+h)}$

$= \lim_{h\to 0} h^m \sin\left(\frac{1}{h}\right)$

$= 0\times k'$ [Where $-1 \le k' \le 1$]

$= 0$

LHL $= f(0) =$ RHL

$\therefore$ $f(x)$ is continuons at $x = 0$

For differentiablility at $x = 0$

(LHD at $x = 0$) $= \lim_{x\to 0^{-1}} \frac{f(x)-f(0)}{x-0}$

$= \lim_{h\to 0} \frac{(0-h)-f(0)}{(0-h)-0}$

$= \lim_{h\to 0} \frac{(-h)^m \sin\left(-\frac{1}{h}\right)}{-h}$

$= \lim_{h\to 0} -(-h)^{m-1} \sin\left(\frac{1}{h}\right)$

= Not definded [Since $0 < m < 1$]

(RHD at $x = 0$) $= \lim_{x\to 0^+} \frac{f(x)-f(0)}{x-0}$

$= \lim_{h\to 0} \frac{f(0+h)-f(0)}{0+h-0}$

$= \lim_{h\to 0} \frac{h^m \sin\left(\frac{1}{h}\right)}{h}$

Chapter 10 Differentiability Ex 10.1 Q7(iii)

$$\text{LHL} = \lim_{x \to 0^-} f(x)$$
$$= \lim_{h \to 0} f(0-h)$$
$$= \lim_{h \to 0} (-h)^m \sin\left(-\frac{1}{h}\right)$$
$$= \text{Not defined as } m \le 0$$
$$\text{RHL} = \lim_{x \to 0^+} f(x)$$
$$= \lim_{h \to 0} f(0+h)$$
$$= \lim_{h \to 0} h^m \sin\left(\frac{1}{h}\right)$$
$$= \text{Not defined, as } m \le 0$$

Since RHL and LHL are not difined, so $f(x)$ is not continuous

Let $x = 0$ for $m \le 0$.

Now,

$$(\text{LHD at } x = 0) = \lim_{x \to 0^{-1}} \frac{f(x) - f(0)}{x - 0}$$
$$= \lim_{h \to 0} \frac{f(0-h) - 0}{0 - h - 0}$$
$$= \lim_{h \to 0} \frac{(-h)^m \sin\left(-\frac{1}{h}\right)}{-h}$$
$$= \lim_{h \to 0} - (-h)^{m-1} \sin\left(\frac{1}{h}\right)$$
$$= \text{Not defined, as } m \le 0$$
$$\text{RHD} = \lim_{x \to 0^+} \frac{f(x) - f(0)}{x - 0}$$
$$= \lim_{h \to 0} \frac{f(0+h) - f(0)}{0 + h - 0}$$
$$= \lim_{h \to 0} \frac{h^m \sin\left(\frac{1}{h}\right)}{h}$$
$$= \lim_{h \to 0} \left(h^{m-1}\right) \sin\left(\frac{1}{h}\right)$$
$$= \text{Not defined, as } m \le 0$$

Thus,

$f(x)$ is neither continuous not differentiable at $x = 0$ for $m \le 0$.

Chapter 10 Differentiability Ex 10.1 Q8

$$f(x) = \begin{cases} x^2 + 3x + a, & \text{if } x \le 1 \\ bx + 2 \quad , & \text{if } x > 1 \end{cases}$$

$$(\text{LHD at } x = 1) = \lim_{x \to 1^-} \frac{f(x) - f(1)}{x - 1}$$

$$= \lim_{h \to 0} \frac{f(1-h) - f(1)}{1 - h - 1}$$

$$= \lim_{h \to 0} \frac{\left[(1-h)^2 + 3(1-h) + a\right] - [1 + 3 + a]}{-h}$$

$$= \lim_{h \to 0} \frac{h^2 - 5h}{-h}$$

$$= -5$$

$$(\text{RHD at } x = 1) = \lim_{x \to 1^+} \frac{f(x) - f(1)}{x - 1}$$

$$= \lim_{h \to 0} \frac{f(1+h) - f(1)}{1 + h - 1}$$

$$= \lim_{h \to 0} \frac{\left[b(1+h) + 2\right] - (b + 2)}{h}$$

$$= \lim_{h \to 0} \frac{b + bh + 2 - b - 2}{h}$$

$$= b$$

Since $f(x)$ is differentiable, so

$$(\text{LHD at } x = 1) = (\text{RHD at } x = 1)$$

$$5 = b$$

$$f(1) = 1 + 3 + a$$

$$= 4 + a$$

LHL $= \lim_{x \to 1^{-1}} f(x)$

$$= \lim_{h \to 0} f(1-h)$$

$$= \lim_{h \to 0} (1-h)^2 + 3(1-h) + a$$

$$= 4 + a$$

RHL $= \lim_{x \to 1^+} f(x)$

$$= \lim_{h \to 0} f(1+h)$$

$$= \lim_{h \to 0} b(1+b) + 2$$

$$= b + 2$$

Chapter 10 Differentiability Ex 10.1 Q9

$$f(x)=\begin{cases}|2x-3|\,[x], & x\geq 1\\ \sin\left(\frac{\pi x}{2}\right) & , \quad x<1\end{cases}$$

$$f(x)=\begin{cases}(2x-3)[x]\ , & x\geq\frac{3}{2}\\ -(2x-3) & , \quad 1\leq x\leq\frac{3}{2}\\ \sin\left(\frac{\pi x}{2}\right) & , \quad x<1\end{cases}$$

For continuity at $x = 1$

$f(1) = -(2.1 - 3) = 1$

LHL $= \lim_{x\to 1^{-1}} f(x)$

$= \lim_{h\to 0} f(1-h)$

$= \lim_{h\to 0} \sin\left(\frac{\pi(1-h)}{2}\right)$

$= \sin\frac{\pi}{2}$

$= 1$

RHL $= \lim_{x\to 0^{+}} f(x)$

$= \lim_{h\to 0} f(1+h)$

$= \lim_{h\to 0} -\left(2(1+h)-3\right)$

$= -1(-1)$

$= 1$

$$\text{LHL} = f(1) = \text{RHL}$$

So, $f(x)$ is continuous at $x = 1$

For differentiablility at $x = 1$

$$(\text{LHD at } x = 1) = \lim_{x \to 0^-} \frac{f(x) - f(1)}{x - 1}$$

$$= \lim_{h \to 0} \frac{f(1-h) - 1}{1 - h - 1}$$

$$= \lim_{h \to 0} \frac{\sin\left(\frac{\pi(1-h)}{2}\right) - 1}{-h}$$

$$= \lim_{h \to 0} \frac{\sin\left(\frac{\pi}{2} - \frac{\pi}{2}h\right) - 1}{-h}$$

$$= \lim_{h \to 0} \frac{\cos\left(\frac{\pi}{2}h\right) - 1}{\frac{-h}{2}}$$

$$= \lim_{h \to 0} \frac{2\sin^2\left(\frac{\pi}{4}h\right)}{h} \times \frac{\left(\frac{\pi}{4}h\right)^2}{\left(\frac{\pi}{4}h\right)^2}$$

$$= 0$$

$$(\text{RHD at } x = 1) = \lim_{x \to 1^+} \frac{f(x) - f(1)}{x - 1}$$

$$= \lim_{h \to 0} \frac{f(1+h) - f(1)}{1 + h - 1}$$

$$= \lim_{h \to 0} \frac{-\left[2(1+h) - 3\right] - 1}{h}$$

$$= \lim_{h \to 0} \frac{-2 - 2h + 3 - 1}{h}$$

$$= \lim_{h \to 0} \frac{-2h}{h}$$

$$= -2$$

$$(\text{LHD at } x = 1) \neq (\text{RHD at } x = 1)$$

$\therefore$ $f(x)$ is continuous but differentiable at $x = 1$.

Chapter 10 Differentiability Ex 10.1 Q10

Her,e
$$f(x) = \begin{cases} ax^2 - b & , \text{ if } |x| < 1 \\ \frac{1}{|x|} & , \text{ if } |x| \geq 1 \end{cases}$$

$$= \begin{cases} -\frac{1}{x} & , \text{ if } x \leq -1 \\ ax^2 - b & , \text{ if } -1 < x < 1 \\ \frac{1}{x} & , \text{ if } x \geq 1 \end{cases}$$

$$\text{LHL} = \lim_{x \to 1^-} f(x)$$

$$= \lim_{h \to 0} f(1-h)$$

$$= \lim_{h \to 0} a(1-h)^2 - b$$

$$= a - b$$

$$\text{RHL} = \lim_{x \to 1^+} f(x)$$

$$= \lim_{h \to 0} f(1+h)$$

$$= \lim_{h \to 0} \frac{1}{1+h}$$

$$= 1$$

Since, $f(x)$ is continuous, so

$$\text{LHS} = \text{RHS}$$

$$a - b = 1 \qquad \text{---(i)}$$

$$\left(\text{LHD at } x = 1\right) = \lim_{x \to 1^-} \frac{f(x) - f(1)}{x - 1}$$
$$= \lim_{h \to 0} \frac{f(1-h) - 1}{1 - h - 1}$$
$$= \lim_{h \to 0} \frac{a(1-h)^2 - b - 1}{-h}$$
$$= \lim_{h \to 0} \frac{a(1-h)^2 - (a-1) - 1}{-h}$$
Using equation (i),
$$= \lim_{h \to 0} \frac{a + ah^2 - 2ah - a + 1 - 1}{-h}$$
$$= \lim_{h \to 0} \frac{ah^2 - 2ah}{-h}$$
$$= \lim_{h \to 0} (2a - ah)$$
$$= 2a$$

$$\left(\text{RHD at } x = 1\right) = \lim_{x \to 1^+} \frac{f(x) - f(1)}{x - 1}$$
$$= \lim_{h \to 0} \frac{f(1+h) - f(1)}{1 + h - 1}$$
$$= \lim_{h \to 0} \frac{\frac{1}{1+h} - 1}{h}$$
$$= \lim_{h \to 0} \frac{1 - 1 - h}{(1+h)h}$$
$$= \lim_{h \to 0} \frac{-1}{1+h}$$
$$= -1$$
Since $f(x)$ is differentiable at $x = 1$,
$$\left(\text{LHD at } x = 1\right) = \left(\text{RHD at } x = 1\right)$$
$$2a = -1$$
$$a = \frac{-1}{2}$$

Put $a = \frac{-1}{2}$ in equation (i),
$$a - b = 1$$
$$\left(\frac{-1}{2}\right) - b = 1$$
$$b = \frac{-1}{2} - 1$$
$$b = \frac{-3}{2}$$
$$a = \frac{-1}{2}$$

Ex 10.2

Differentiability Ex 10.2 Q1

Here, $f(x) = x^2$ is a polynomial function so, it is differentiable at $x = 2$.

$$f'(2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$$
$$= \lim_{h \to 0} \frac{(2+h)^2 - (2)^2}{h}$$
$$= \lim_{h \to 0} \frac{4 + h^2 + 4h - 4}{h}$$
$$= \lim_{h \to 0} \frac{4h + h^2}{h}$$
$$= \lim_{h \to 0} (4 + h)$$
$$= 4$$
$$\therefore \quad f'(2) = 4$$

Chapter 10 Differentiability Ex 10.2 Q2

$f(x) = x^2 - 4x + 7$ is a polynomial function, So it is differentiable everywhere.

$$f'(5) = \lim_{h \to 0} \frac{f(5+h) - f(5)}{h}$$
$$= \lim_{h \to 0} \frac{\left\{(5+h)^2 - 4(5+h) + 7\right\} - [25 - 20 + 7]}{h}$$
$$= \lim_{h \to 0} \frac{h^2 + 25 + 10h - 20 - 4h + 7 - 12}{h}$$
$$= \lim_{h \to 0} \frac{h^2 + 6h}{h}$$
$$= \lim_{h \to 0} (h + 6)$$
$$= 6$$

$$f'\left(\frac{7}{2}\right) = \lim_{h \to 0} \frac{f\left(\frac{7}{2} + h\right) - f\left(\frac{7}{2}\right)}{h}$$
$$= \lim_{h \to 0} \frac{\left[\left(\frac{7}{2} + h\right)^2 - 4\left(\frac{7}{2} + h\right) + 7\right] - \left[\left(\frac{7}{2}\right)^2 - 4\left(\frac{7}{2}\right) + 7\right]}{h}$$
$$= \lim_{h \to 0} \frac{\left[\frac{49}{2} + h^2 + 7h - 14 - 4h + 7\right] - \left[\frac{49}{4} - 14 + 7\right]}{h}$$
$$= \lim_{h \to 0} \frac{\frac{49}{4} + h^2 + 7h - 14 - 4h + 7 - \frac{49}{4} + 14 - 7}{h}$$
$$= \lim_{h \to 0} \frac{h^2 + 3h}{h}$$
$$= \lim_{h \to 0} (h + 3)$$
$$= 3$$

Now,

$$f'(5) = 6$$
$$= 2(3)$$
$$f'(5) = 2f'\left(\frac{7}{2}\right)$$

Chapter 10 Differentiability Ex 10.2 Q3

We know that, $f(x) = 2x^3 - 9x^2 + 12x + 9$ is a polynomial function. So, it is differentiable every where. For $x = 1$

$$f'(1) = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$$
$$= \lim_{h\to 0} \frac{\left[2(1+h)^3 - 9(1+h)^2 + 12(1+h) + 9\right] - [2 - 9 + 12 + 9]}{h}$$
$$= \lim_{h\to 0} \frac{\left[2\left(1 + h^3 + 3h^2 + 3h\right) - 9\left(1 + h^2 + 2h\right) + 12 + 12h + 9\right] - [14]}{h}$$
$$= \lim_{h\to 0} \frac{\left[2 + 2h^3 + 6h^2 + 6h - 9 - 9h^2 - 18h + 12 + 12h + 9 - 14\right]}{h}$$
$$= \lim_{h\to 0} \frac{2h^3 - 3h^2}{h}$$
$$= \lim_{h\to 0} \frac{h^2(2h-3)}{h}$$
$$= \lim_{h\to 0} h(2h-3)$$
$$f'(1) = 0 \qquad \text{---(i)}$$

For $x = 2$

$$f'(2) = \lim_{h\to 0} \frac{\left[2(2+h)^3 - 9(2+h)^2 + 12(12+h) + 9\right] - [16 - 36 + 24 + 9]}{h}$$
$$= \lim_{h\to 0} \frac{\left[2\left(8 + h^3 + 12h + 6h^2\right) - 9\left(4 + h^2 + 4h\right) + 24 + 12h + 9\right] - [13]}{h}$$
$$= \lim_{h\to 0} \frac{\left[16 + 2h^3 + 24h + 12h^2 - 36 - 9h^2 - 36h + 33 + 12h - 13\right]}{h}$$
$$= \lim_{h\to 0} \frac{2h^3 + 3h^2}{h}$$
$$= \lim_{h\to 0} \frac{h^2(2h+3)}{h}$$
$$= \lim_{h\to 0} h(2h+3)$$
$$f'(2) = 0 \qquad \text{---(ii)}$$

From equation (i) and (ii),

$$f'(1) = f'(2)$$

Chapter 10 Differentiability Ex 10.2 Q4

$\Phi(x) = \lambda x^2 + 7x - 4$ and $\Phi'(5) = 97$

$$\Phi'(5) = \lim_{h\to 0} \frac{\left[\lambda(5+h)^2 + 7(5+h) - 4\right] - [25\lambda + 35 - 4]}{h}$$
$$97 = \lim_{h\to 0} \frac{\lambda\left(25 + h^2 + 10h\right) + 35 + 7h - 4 - 25\lambda - 35 + 4}{h}$$
$$= \lim_{h\to 0} \frac{25\lambda + \lambda h^2 + 10\lambda h - 25\lambda + 7h}{h}$$
$$= \lim_{h\to 0} \frac{\lambda h^2 + h(10\lambda + 7)}{h}$$
$$= \lim_{h\to 0} \frac{f(\lambda h + 10\lambda + 7)}{h}$$
$$97 = 10\lambda + 7$$
$$10\lambda = 97 + 7$$
$$\lambda = \frac{90}{10}$$
$$\lambda = 9$$

Chapter 10 Differentiability Ex 10.2 Q5

$f(x) = x^3 + 7x^2 + 8x - 9$ is a polynomial function. So, it is differentiable every where.

$$f'(4) = \lim_{h \to 0} \frac{f(4+h) - h(4)}{h}$$

$$= \lim_{h \to 0} \frac{\left[(4+h)^3 + 7(4+h)^2 + 8(4+h) - 9\right] - [64 + 112 + 32 - 9]}{h}$$

$$= \lim_{h \to 0} \frac{\left[64 + h^3 + 48h + 12h^2 + 112 + 7h^2 + 56h + 32 + 8h - 9\right] - [210 - 9]}{h}$$

$$= \lim_{h \to 0} \frac{h^3 + 19h^2 + 112h + 210 - 9 - 210 + 9}{h}$$

$$= \lim_{h \to 0} \frac{h^3 + 19h^2 + 112h}{h}$$

$$= \lim_{h \to 0} \frac{h\left(h^2 + 19h + 112\right)}{h}$$

$$f'(4) = 112$$

Chapter 10 Differentiability Ex 10.2 Q6

$$f(x) = mx + c$$

$$f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{f(h) - h(0)}{h}$$

$$= \lim_{h \to 0} \frac{(mh + c) - (m \times 0 + c)}{h}$$

$$= \lim_{h \to 0} \frac{mh + c - c}{h}$$

$$= \lim_{h \to 0} \frac{mh}{h}$$

$$= m$$

$$f'(0) = m$$

Chapter 10 Differentiability Ex 10.2 Q7

$$f(x) = \begin{cases} 2x + 3, \text{ if} -3 \le x < -2 \\ x + 1, \text{ if} -2 \le x < 0 \\ x + 2, \text{ if } 0 \le x \le 1 \end{cases}$$

We know that polynomial funtiona are continuous and differentiable everywhere.
So f(x) is differentiable on $x \in [-3, 2)$, $x \in (-2, 0)$ and $x \in (0, 1]$.
We need to check the differentiability at x = -2 and x = 0

Differentiability at x = -2

$$(\text{LHD at } x = -2) = \lim_{x \to -2^-} \frac{f(x) - f(-2)}{x - (-2)} = \lim_{x \to -2^-} \frac{2x + 3 + 1}{x + 2} = \lim_{x \to -2^-} \frac{2(x+2)}{x + 2} = 2$$

$$(\text{RHD at } x = -2) = \lim_{x \to -2^+} \frac{f(x) - f(-2)}{x - (-2)} = \lim_{x \to -2^+} \frac{x + 1 + 1}{x + 2} = \lim_{x \to -2^+} \frac{x + 2}{x + 2} = 1$$

$\therefore$ (LHD at x = -2) $\neq$ (RHD at x = -2)
So, f(x) is not differentiable at x = -2.

Differentiability at x = 0

$$(\text{LHD at } x = 0) = \lim_{x \to 0^-} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0^-} \frac{x + 1 - 2}{x} = \lim_{x \to 0^-} \frac{x - 1}{x} \to \infty$$

$$(\text{RHD at } x = 0) = \lim_{x \to 0^+} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0^+} \frac{x + 2 - 2}{x} = \lim_{x \to 0^+} \frac{x}{x} = 1$$

$\therefore$ (LHD at x = 0) $\neq$ (RHD at x = 0)
So, f(x) is not differentiable at x = 0.

Chapter 10 Differentiability Ex 10.2 Q8

We know that, modulus function

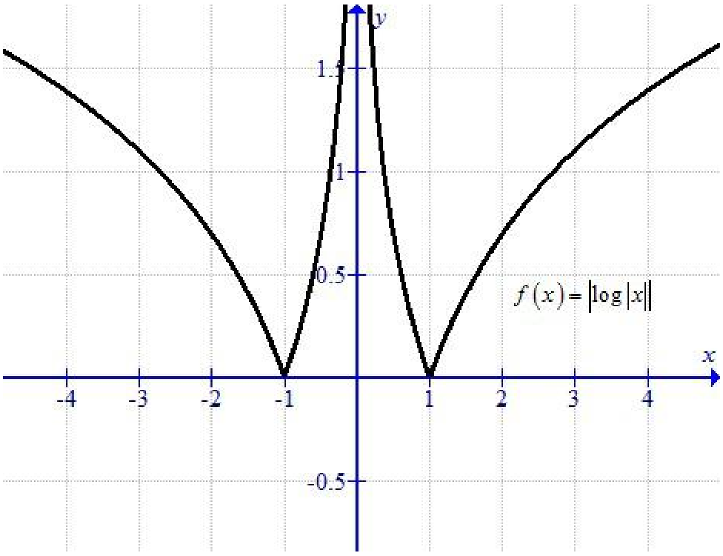
$f(x) = |x|$ is continuous but bot differentiable at $x = 0$,

So,

$f(x) = |x| + |x-1| + |x-2| + |x-3| + |x-4|$ is continuous but not differentiable

$x = 0, 1, 2, 3, 4.$

Chapter 10 Differentiability Ex 10.2 Q9



$f(x) = |\log|x||$

Since, it is an absolute function. So, it is continuous function.

The graph of the function is as below:-

Chapter 10 Differentiability Ex 10.2 Q10

$$f(x) = e^{|x|}$$

$$f(x) = \begin{cases} e^{-x} & , x < 0 \\ e^{x} & , x \geq 0 \end{cases}$$

For continuity at $x = 0$

RHL $= \lim_{x \to 0^+} f(x)$

$= \lim_{h \to 0} f(0+h)$

$= \lim_{h \to 0} e^{(0+h)}$

$= \lim_{h \to 0} e^{h}$

$= e^{0}$

RHL $= 1$

LHL $= \lim_{x \to 0^-} f(x)$

$= \lim_{h \to 0} f(0-h)$

$= \lim_{h \to 0} e^{-(0-h)}$

$= \lim_{h \to 0} e^{h}$

LHL $= 1$

$f(0) = e^{0}$

$= 1$

Now,

LHL $= f(0) =$ RHL

So, $f(x)$ is continuous at $x = 0$

For differentiablility at $x = 0$

LHD $= \lim_{x \to 0^-} \frac{f(x) - f(0)}{x - 0}$

$= \lim_{h \to 0} \frac{f(0-h) - e^{0}}{(0-h) - 0}$

$= \lim_{h \to 0} \frac{e^{-(0-h)} - 1}{-h}$

$= \lim_{h \to 0} \frac{e^{h} - 1}{-h}$

$= 1$ $\left[\text{Since } \lim_{x \to 0} \frac{e^{x} - 1}{x} = 1\right]$

RHD $= \lim_{hx \to 0^+} \frac{f(x) - f(0)}{x - 0}$

$= \lim_{h \to 0} \frac{f(0+h) - f(0)}{(0+h) - 0}$

$= \lim_{h \to 0} \frac{e^{x} - e^{0}}{h}$

$= \lim_{h \to 0} \frac{e^{h} - 1}{h}$

$= 1$ $\left[\text{Since } \lim_{x \to 0} \frac{e^{x} - 1}{x} = 1\right]$

Clearly,

LHD $\neq$ RHD

So,

$f(x)$ is not differentiable at $x = 0$.

Differentiability Ex 10.2 Q11

$$f(x)=\begin{cases}(x-c)\cos\frac{1}{(x-c)} & ,x\neq c\\ 0 & ,x=c\end{cases}$$

$$(\text{LHL at } x=c)=\lim_{x\to c^-} f(x)$$
$$=\lim_{h\to 0} f(-h)$$
$$=\lim_{h\to 0}(c-h-c)\cos\left(\frac{1}{c-h-c}\right)$$
$$=\lim_{h\to 0}-h\cos\left(-\frac{1}{h}\right)$$
$$=\lim_{h\to 0}-h\cos\left(\frac{1}{h}\right)$$
$$=0$$

$$(\text{RHL at } x=c)=\lim_{x\to c^+} f(x)$$
$$=\lim_{h\to 0} f(c+h)$$
$$=\lim_{h\to 0}(c+h-c)\cos\left(\frac{1}{c+h-c}\right)$$
$$=\lim_{h\to 0} h\cos\left(\frac{1}{h}\right)$$
$$=0$$

$$f(e)=0$$

Since, LHL $= f(x) =$ RHL at $x=c$

$\Rightarrow$ $f(x)$ is continuous at $x=c$

$$(\text{LHD at } x=c)=\lim_{h\to 0}\frac{f(c-h)-f(c)}{-h}$$
$$=\lim_{h\to 0}\frac{(c-h-c)\cos\left(\frac{1}{c-h-c}\right)-0}{-h}$$
$$=\lim_{h\to 0}\cos\left(-\frac{1}{h}\right)$$
$$=\lim_{h\to 0}\cos\left(\frac{1}{h}\right)$$
$$=k$$

$$(\text{RHD at } x=c)=\lim_{h\to 0}\frac{f(c+h)-f(c)}{h}$$
$$=\lim_{h\to 0}\frac{(c+h-c)\cos\left(\frac{1}{c+h-c}\right)-0}{h}$$
$$=\lim_{h\to 0}\frac{h\cos\left(\frac{1}{h}\right)}{h}$$
$$=\lim_{h\to 0}\cos\left(\frac{1}{h}\right)$$
$$=k$$

$$(\text{LHD at } x=c)=(\text{RHD at } x=c)$$

So,

$f(x)$ is differentiable and continuous at $x=c$.

Differentiability Ex 10.2 Q12

$$f(x) = |\sin x| = \begin{cases} -\sin x & , \ x < n\pi \\ \sin x & , \ x \geq n\pi \end{cases}$$

For $x = n\pi$ (n even)

$$(\text{LHD at } x = n\pi) = \lim_{x \to n\pi^{-1}} \frac{f(x) - f(n\pi)}{x - n\pi}$$
$$= \lim_{h \to 0} \frac{-\sin(n\pi - h) - \sin n\pi}{n\pi - h - n\pi}$$
$$= \lim_{h \to 0} \frac{\sin h - 0}{-h}$$
$$= -1$$

$$(\text{RHD at } x = n\pi) = \lim_{h \to 0} \frac{\sin(n\pi + h) - \sin n\pi}{h}$$
$$= \lim_{h \to 0} \frac{\sin h}{h}$$
$$= 1$$

$(\text{LHD at } x = n\pi) \neq (\text{RHD at } x = n\pi)$

For $x = n\pi$ (n is odd)

$$(\text{LHD at } x = n\pi) = \lim_{h \to 0} \frac{-\sin(n\pi - h) - \sin n\pi}{-h}$$
$$= \lim_{h \to 0} \frac{-\sin h}{-h}$$
$$= 1$$

$$(\text{RHD at } x = n\pi) = \lim_{h \to 0} \frac{\sin(n\pi + h) - \sin n\pi}{h}$$
$$= \lim_{h \to 0} \frac{-\sin h - 0}{h}$$
$$= -1$$

$(\text{LHD at } x = n\pi) \neq (\text{RHD at } x = n\pi)$

Thus,

$f(x) = |\sin x|$ is not differentiable at $x = n\pi$

$f(x) = \cos|x|$

Since, $\cos(-x) = \cos x$

$\Rightarrow \quad f(x) = \cos x$

$\Rightarrow \quad f(x) = \cos|x|$ is differnetiable everywhere.