Ex 29.1

Q1

$$\lim_{x \to 0} \frac{x}{|x|}$$

We know that $|x| = \begin{cases} x, \text{if } x \geq 0 \\ -x, \text{if } x < 0 \end{cases}$

$$\therefore \quad \lim_{x \to 0^+} \frac{x}{|x|} = \lim_{x \to 0^+} \frac{x}{x} = \lim_{x \to 0^+} 1 = 1$$

Also, $\lim_{x \to 0^-} \frac{x}{|x|} = \lim_{x \to 0^-} \frac{x}{-x} = \lim_{x \to 0^-} -1 = -1$

$\Rightarrow$ LHL of $f(x) \neq$ RHL of $f(x)$

$\Rightarrow \quad \lim_{x \to 0} \frac{x}{|x|}$ does not exist

Q2

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (2x + 3)$$
$$= 2(2) + 3$$
$$= 7$$

$\therefore \quad \lim_{x \to 2^-} f(x) = 7$

Also,

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (x + k)$$
$$= (2 + k)$$

Since, $\lim_{x \to 2} f(x)$ exists (given)

$\therefore \quad \lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$

$\Rightarrow \quad 7 = 2 + k$
$\Rightarrow \quad k = 5$

Q3

Let $f(x) = \dfrac{1}{x}$, this function is defined for every value of x except at $x = 0$

As $x \to 0^+, \dfrac{1}{x} \to \infty$.

As $x \to 0^-, \dfrac{1}{x} \to -\infty$

$\therefore \quad \lim\limits_{x \to 0} \dfrac{1}{x}$ does not exist.

Q4

$\lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0^-} \dfrac{3x}{-x + 2x} = \lim\limits_{x \to 0^-} \dfrac{3x}{x} = 3 \qquad \left[\because \text{ as } x \to 0^-, |x| = -x \right]$

$\lim\limits_{x \to 0^+} f(x) = \lim\limits_{x \to 0^+} \dfrac{3x}{x + 2x} = 1 \qquad \left[\because \text{ as } x \to 0^+, |x| = x \right]$

thus, $\lim\limits_{x \to 0^-} f(x) \neq \lim\limits_{x \to 0^+} f(x)$

$\therefore \quad \lim\limits_{x \to 0} f(x)$ does not exist.

Q5

$\lim\limits_{x \to 0^+} f(x) = \lim\limits_{x \to 0^+} x + 1$

$\qquad = \lim\limits_{h \to 0} (0 + h) + 1 = 1$

Also, $\lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0^-} x - 1$

$\qquad \lim\limits_{h \to 0} (0 - h) - 1 = -1$

$\Rightarrow \quad \lim\limits_{x \to 0^+} f(x) \neq \lim\limits_{x \to 0^-} f(x)$

Hence, *limit does not exist.*

Q6

$$LHL = \lim_{x \to 0^-} f(x)$$
$$= \lim_{h \to 0} f(0-h)$$
$$= \lim_{h \to 0} -h - 4$$
$$= 0 - 4$$
$$= -4$$

$$RHL = \lim_{x \to 0^+} f(x)$$
$$= \lim_{h \to 0} f(0+h)$$
$$= \lim_{h \to 0} h + 5$$
$$= 0 + 5$$
$$= 5$$

$$\therefore \lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x)$$

Hence $\lim_{x \to 0} f(x)$ does not exist.

Q7

$$\lim_{x \to 3^+} f(x) = 4$$

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} (x+1) = \lim_{h \to 0} (3-h+1) = 3+1 = 4$$

Since, $\lim_{x \to 3^+} f(x) = 4 = \lim_{x \to 3^-} f(x)$

$\therefore \quad \lim_{x \to 3} f(x)$ is 4

Q8

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} 3(x + 1) = 3$$

and

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} 2(x) + 3 = 3$$

$\therefore$ $\lim_{x \to 0} f(x) = 3$

$$\lim_{x \to 1} f(x) = \lim_{x \to 1} 3(x + 1) = 3 + 3 = 6$$

$\therefore$ $\lim_{x \to 1} f(x) = 6$

Q9

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} x^2 - 1 = \lim_{h \to 0} (-1 - h)^2 - 1 = 1 - 1 = 0$$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} -x^2 - 1 = \lim_{h \to 0} -(1 + h)^2 - 1 = -2$$

Since, $\lim_{x \to 1^-} f(x) \ne \lim_{x \to 1^+} f(x)$

$\therefore$ $\lim_{x \to 1} f(x)$ does not exist.

Q10

LHL = $\lim\limits_{x \to 0^-} f(x)$

$\Rightarrow \quad \lim\limits_{x \to 0^-} \dfrac{|x|}{x}$

$\Rightarrow \quad \lim\limits_{h \to 0} \dfrac{|0-h|}{0-h}$

$\Rightarrow \quad \lim\limits_{h \to 0} \dfrac{+h}{-h} = -1 \qquad\qquad ---(i)$

and,

RHL = $\lim\limits_{x \to 0^+} f(x)$

$\Rightarrow \quad \lim\limits_{x \to 0^+} \dfrac{|x|}{x}$

$\Rightarrow \quad \lim\limits_{h \to 0} \dfrac{|0+h|}{0+h} = \lim\limits_{h \to 0} \dfrac{h}{h} = 1 \qquad\qquad ---(ii)$

So, LHL $\ne$ RHL

$\therefore \quad \lim\limits_{x \to 0} f(x)$ does not exist.

Q11

$\lim\limits_{x \to a_1} f(x)$

$\Rightarrow \quad \lim\limits_{x \to a_1} (x - a_1)(x - a_2)\ldots(x - a_n)$ $\qquad\qquad$ [Putting limit $x \to a_1$]

$\Rightarrow \quad (a_1 - a_1)(a_1 - a_2)\ldots(a_1 - a_n)$

$\Rightarrow \quad 0$

And, $\quad \lim\limits_{x \to a} f(x)$

$\Rightarrow \quad \lim\limits_{x \to a} (x - a_1)(x - a_2)\ldots(x - a_n)$ $\qquad\qquad$ [Putting limit $x \to a$]

$\Rightarrow \quad (a - a_1)(a - a_2)\ldots(a - a_n).$

Q12

$\lim\limits_{x \to 1^+} \dfrac{1}{(x-1)} = \lim\limits_{h \to 0} \dfrac{1}{(1+h-1)} = \lim\limits_{h \to 0} \dfrac{1}{h} = \infty$

Q13(i)

$$\lim_{x \to 2^+} \frac{x-3}{x^2-4}$$

$$= \lim_{h \to 0} \frac{(2+h)-3}{(2+h)^2-2^2}$$

$$= \lim_{h \to 0} \frac{(2-3+h)}{(2+h-2)(2+h+2)}$$

$$= \lim_{h \to 0} \frac{(h-1)}{(h)(4+h)}$$

$$= \lim_{h \to 0} \frac{1-\frac{1}{h}}{4+h}$$

$$= \frac{1-\frac{1}{0}}{4} = -\infty$$

$$\left[\because \lim_{x \to 2^+} f(x) = \lim_{h \to 0} f(2+h) \right]$$

Q13(ii)

$$\lim_{x \to 2^-} \frac{x-3}{\left(x^2-4\right)} = \lim_{h \to 0} \frac{(2-h)-3}{(2-h)^2-4}$$

$$= \lim_{h \to 0} \frac{(2-h-3)}{(2-h+2)(2-h-2)}$$

$$= \lim_{h \to 0} \frac{-1-h}{(4-h)(-h)}$$

$$= \lim_{h \to 0} \frac{\frac{1}{h}+1}{(4-h)}$$

$$= \frac{\frac{1}{0}+1}{4} = \infty$$

$$\left[\because \lim_{x \to 2^-} f(x) = \lim_{h \to 0} f(2-h) \right]$$

Q13(iii)

$$\lim_{x \to 0^+} \frac{1}{3x}$$

$$= \lim_{h \to 0} \frac{1}{3(0+h)}$$

$$= \lim_{h \to 0} \frac{1}{0+3h}$$

$$= \frac{1}{0} = \infty$$

Q13(iv)

$$\lim_{x \to -8^+} \frac{2x}{x+8}$$

$$= \lim_{h \to 0} \frac{2(-8+h)}{(-8+h)+8}$$

$$= \lim_{h \to 0} \frac{-16+2h}{h}$$

$$= \lim_{h \to 0} \frac{-16}{h}+2$$

$$\Rightarrow \quad \frac{-16}{0}+2 = \infty$$

Q13(v)

$$\lim_{x \to 0^+} \frac{2}{x^{\frac{1}{5}}}$$

$$= \lim_{h \to 0} \frac{2}{(0+h)^{\frac{1}{5}}}$$

$$\Rightarrow \frac{2}{0} = \infty$$

Q13(vi)

$$\lim_{x \to \frac{\pi}{2}} \tan x$$

$$= \lim_{h \to 0} \tan\left(\frac{\pi}{2}-h\right)$$

$$= \tan\left(\frac{\pi}{2}-0\right)$$

$$\Rightarrow \quad \tan\frac{\pi}{2} = \infty$$

Q13(vii)

$$\lim_{x \to -\frac{\pi}{2}^{+}} \sec x$$

$$= \lim_{h \to 0} \sec\left(-\frac{\pi}{2} + h\right)$$

$$= \sec\left(-\frac{\pi}{2} + 0\right)$$

$$= \sec\left(-\frac{\pi}{2}\right)$$

$$= \frac{1}{\cos\left(-\frac{\pi}{2}\right)}$$

$$= \frac{-1}{\left(\cos\frac{\pi}{2}\right)}$$

$$= \frac{-1}{0} = -\infty$$

Q13(viii)

$$\lim_{x \to 0^{-}} \frac{x^2 - 3x + 2}{x^3 - 2x^2}$$

$$= \lim_{x \to 0^{-}} \frac{x^2 - x - 2x + 2}{x^2(x - 2)}$$

$$= \lim_{x \to 0^{-}} \frac{x(x - 1) - 2(x - 1)}{x^2(x - 2)}$$

$$= \lim_{x \to 0^{-}} \frac{(x - 1)(x - 2)}{x^2(x - 2)}$$

$$= \lim_{x \to 0^{-}} \frac{(x - 1)}{x^2}$$

$$= \lim_{h \to 0} \frac{(0 - h - 1)}{(0 - h)^2}$$

$$= \frac{-h}{h^2} = \frac{-1}{h} = \frac{-1}{0} = -\infty$$

Q13(ix)

$$\lim_{x \to -2^+} \frac{x^2 - 1}{2x + 4}$$

$$= \lim_{x \to 2^+} \frac{(x - 1)(x + 1)}{2(x + 2)}$$

$$= \lim_{h \to 0} \frac{(-2 + h - 1)(-2 + h + 1)}{2(-2 + h + 2)}$$

$$= \lim_{h \to 0} \frac{(-3 + h)(h - 1)}{2h}$$

$$\Rightarrow \quad \frac{-3 \times -1}{2 \times 0} = \frac{1}{0} = \infty$$

Q13(x)

$$\lim_{x \to 0^-} 2 - \cot x$$

$$= \lim_{h \to 0} 2 - \cot(0 - h)$$

$$= \lim_{h \to 0} 2 - (-1) \coth$$

$$= \lim_{h \to 0} 2 + \coth$$

$$= \lim_{h \to 0} 2 + \frac{1}{\tanh}$$

$$\Rightarrow \quad 2 + \frac{1}{0} \Leftarrow \infty$$

Q13(xi)

$$\lim_{x \to 0^-} 1 + \operatorname{cosec} x$$

$$= \lim_{x \to 0^-} 1 + \operatorname{cosec}(0 - h)$$

$$= \lim_{h \to 0} 1 - \operatorname{cosech}$$

$$= \lim_{h \to 0} 1 - \frac{1}{\sinh}$$

$$\Rightarrow \quad 1 - \frac{1}{0} = -\infty$$

Q14

$$\lim_{x \to 0} e^{\frac{-1}{x}}$$

$$\lim_{x \to 0^+} e^{\frac{-1}{x}} = \lim_{x \to 0^+} \frac{1}{e^{\frac{1}{x}}} = \lim_{h \to 0} \frac{1}{e^{\frac{1}{0+h}}} = \frac{1}{e^{\frac{1}{0}}} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$$

$$And, \lim_{x \to 0^-} e^{\frac{-1}{x}} = \lim_{x \to 0^-} \frac{1}{e^{\frac{1}{x}}} = \lim_{h \to 0} \frac{1}{e^{\frac{1}{0-h}}} = \lim_{h \to 0} \frac{1}{e^{-\frac{1}{h}}} = \frac{1}{e^{-\frac{1}{0}}} = \frac{1}{e^{-\infty}} = e^{\infty} = \infty$$

$$\Rightarrow \quad \lim_{x \to 0^+} e^{\frac{-1}{x}} \neq \lim_{x \to 0^-} e^{\frac{-1}{x}}$$

$$\therefore \quad \lim_{x \to 0} e^{\frac{-1}{x}} \text{ does not exist .}$$

Q15

(i) $\lim_{x \to 2}[x]$

$\lim_{x \to 2^-}[x] = 1$

$\lim_{x \to 2^+}[x] = 2$

Thus, $\lim_{x \to 2}[x]$ does not exist.

(ii) $\lim_{x \to \frac{5}{2}}[x]$

$\lim_{x \to \frac{5}{2}^+}[x] = 2$

$\lim_{x \to \frac{5}{2}^-}[x] = 2$

$\Rightarrow \quad \lim_{x \to \frac{5}{2}}[x] = 2$

(iii) $\lim_{x \to 1}[x]$

$\lim_{x \to 1^-}[x] = 0$

$\lim_{x \to 1^+}[x] = 1$

$\Rightarrow \quad \lim_{x \to 1^-}[x] \neq \lim_{x \to 1^+}[x]$

Thus, $\lim_{x \to 1}[x]$ does not exist

Q16

$\lim_{x \to a^+}[x]$

$\Rightarrow \quad \lim_{h \to 0}[a+h] = [a]$

$\Rightarrow \quad \lim_{h \to 0}[x] = [a] \forall a \in R$

Also, $\lim_{x \to 1^-}[x]$

$= \lim_{h \to 0}[1-h]$

$= 0$

$\Rightarrow \quad \lim_{x \to 1^-}[x] = 0$

Q17

$$\lim_{x \to 2^-} \frac{x}{[x]} \qquad = \lim_{x \to 2^-} \frac{x}{1} = \frac{2}{1} = 2 \qquad\qquad \left[\because \lim_{x \to k^-}[x] = k - 1 \right]$$

$$\text{Also, } \lim_{x \to 2^+} \frac{x}{[x]} = \lim_{x \to 2^+} \frac{x}{3} = \frac{2}{3} \qquad\qquad \left[\lim_{x \to k^+}[x] = k + 1 \right]$$

$$\Rightarrow \qquad \lim_{x \to 2^-} \frac{x}{[x]} \neq \lim_{x \to 2^+} \frac{x}{[x]}$$

Q18

$$\lim_{x \to 3^+} \frac{x}{[x]} = \lim_{x \to 3^+} \frac{x}{3} = \frac{3}{3} = 1$$

$$\lim_{x \to 3^-} \frac{x}{[x]} = \lim_{x \to 3^-} \frac{x}{2} = \frac{3}{2} = 1.5$$

$$\text{Therefore, } \lim_{x \to 3^+} \frac{x}{[x]} \neq \lim_{x \to 3^-} \frac{x}{[x]}$$

Q19

$$\lim_{x \to \frac{5}{2}}[x]$$

$$\lim_{x \to \frac{5}{2}}[x] = \left[\frac{5}{2}\right],$$

$$= [2.5] = 2 \qquad\qquad [\text{By definition of greatest integer function}]$$

$$\Rightarrow \qquad \lim_{x \to \frac{5}{2}}[x] = 2$$

Q20

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} \left(x - [x] \right)$$

$$= \lim_{x \to 2^-} x - \lim_{x \to 2^-} [x]$$

$$= 2 - 1 = 1 \qquad\qquad \left[\because \lim_{x \to k^-} [x] = k - 1 \right]$$

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (3x - 5)$$

$$= 3(2) - 5 \qquad\qquad [\because x > 2]$$

$$= 6 - 5$$

$$= 1$$

Thus, $\lim_{x \to 2^-} f(x) = 1 = \lim_{x \to 2^+} f(x)$

$\Rightarrow \qquad \lim_{x \to 2} f(x) = 1$

Q21

$$\lim_{x \to 0^-} \sin\frac{1}{x} = \lim_{h \to 0} \sin\frac{1}{0-h} = -\lim_{h \to 0} \sin\frac{1}{h}$$

$$= -\left(\text{An oscillating number which oscillates between} -1 \text{ and } 1 \right).$$

So, $\lim_{x \to 0^-} \sin\frac{1}{x}$ does not exist.

Similarly, $\lim_{x \to 0^+} \sin\frac{1}{x}$ does not exist.

$\lim_{x \to 0} \sin\frac{1}{x}$ does not exist.

Q22

Let $f(x) = \begin{cases} \dfrac{k\cos x}{\pi - 2x}, & \text{where } x \neq \dfrac{\pi}{2} \\ 3, & \text{where } x = \dfrac{\pi}{2} \end{cases}$ and if $\lim\limits_{x \to \pi/2} f(x) = f\left(\dfrac{\pi}{2}\right)$, find the value of k.

$\text{LHL} = \lim\limits_{x \to \frac{\pi}{2}^-} f(x)$

$= \lim\limits_{h \to \frac{\pi}{2}} f\left(\dfrac{\pi}{2} - h\right)$

$= \lim\limits_{h \to \frac{\pi}{2}} \dfrac{k\cos\left(\dfrac{\pi}{2} - h\right)}{\pi - 2\left(\dfrac{\pi}{2} - h\right)}$

$= \lim\limits_{h \to \frac{\pi}{2}} \dfrac{k\cos\left(\dfrac{\pi}{2} - h\right)}{2h}$

$= \dfrac{k\cos\left(\dfrac{\pi}{2} - \dfrac{\pi}{2}\right)}{\pi}$

$= \dfrac{k}{\pi}$

Ex 29.2

Q1

$$\lim_{x \to 1} \frac{x^2 + 1}{x + 1} = \frac{(1)^2 + 1}{1 + 1} = \frac{1 + 1}{2} = \frac{2}{2} = 1$$

Q2

$$\lim_{x \to 0} \frac{2x^2 + 3x + 4}{x^2 + 3x + 2} = \lim_{x \to 0} \frac{2x^2 + 3x + 4}{(x + 2)(x + 1)} = \frac{2(0) + 3(0) + 4}{(0 + 2)(0 + 1)} = \frac{4}{2} = 2$$

Q3

$$\lim_{x \to 3} \frac{\sqrt{2x + 3}}{x + 3} = \frac{\sqrt{9}}{6} = \frac{1}{2}$$

Q4

$$\lim_{x \to 1} \frac{\sqrt{x + 8}}{\sqrt{x}} = \frac{\sqrt{(1 + 8)}}{\sqrt{1}} = \sqrt{9} = 3$$

Q5

$$\lim_{x \to a} \frac{\sqrt{x} + \sqrt{a}}{x + a} = \frac{\sqrt{a} + \sqrt{a}}{a + a} = \frac{2\sqrt{a}}{2a} = \frac{1}{\sqrt{a}}$$

Q6

$$\lim_{x \to 1} \frac{1 + (x - 1)^2}{1 + x^2} = \frac{1 + 0^2}{1 + 1} = \frac{1}{2}$$

Q7

$$\lim_{x \to 0} \frac{x^{\frac{2}{3}} - 9}{x - 27} = \frac{-9}{-27} = \frac{1}{3}$$

Q8

$$\lim_{x \to 0} 9 = 9$$

Q9

$$\lim_{x \to 2} (3 - x) = (3 - 2) = 1$$

Q10

$$\lim_{x \to -1} \left(4x^2 + 2\right) = \left(4(-1)^2 + 2\right) = 6$$

Q11

$$\lim_{x \to -1} \frac{x^3 - 3x + 1}{x - 1} = \frac{(-1)^3 - 3(-1) + 1}{(-1 - 1)} = \frac{-1 + 3 + 1}{-2} = \frac{3}{-2} = \frac{-3}{2}$$

Q12

$$\lim_{x \to 0} \frac{3x + 1}{x + 3} = \frac{3(0) + 1}{(0 + 3)} = \frac{1}{3}$$

Q13

$$\lim_{x \to 3} \frac{x^2 - 9}{x + 2} = \frac{3^2 - 9}{3 + 2} = 0$$

Q14

$$\lim_{x \to 0} \frac{ax + b}{(x + d)} = \frac{a \times 0 + b}{(0 + d)} = \frac{b}{d}$$

Ex 29.3

Q1

$$\lim_{x \to -5} \frac{2x^2 + 9x - 5}{x + 5} = \lim_{x \to -5} \frac{(x+5)(2x-1)}{(x+5)} = \lim_{x \to -5} (2x - 1) = 2(-5) - 1 = -10 - 1 = -11$$

Q2

$$\lim_{x \to 3} \frac{x^2 - 4x + 3}{x^2 - 2x - 3}$$

$$= \lim_{x \to 3} \frac{x^2 - 3x - x + 3}{x^2 + x - 3x - 3}$$

$$= \lim_{x \to 3} \frac{x(x-1) - 3(x-1)}{x(x+1) - 3(x+1)}$$

$$= \lim_{x \to 3} \frac{(x-1)(x-3)}{(x+1)(x-3)}$$

$$= \lim_{x \to 3} \frac{x-1}{x+1}$$

$$= \frac{3-1}{3+1}$$

$$= \frac{2}{4} = \frac{1}{2}$$

Q3

$$\lim_{x \to 3} \frac{x^4 - 81}{x^2 - 9} = \lim_{x \to 3} \frac{(x^2 - 9)(x^2 + 9)}{(x^2 - 9)} = \lim_{x \to 3} x^2 + 9 = (3)^2 + 9 = 9 + 9 = 18$$

Q4

$$\lim_{x \to 2} \frac{x^3 - 8}{x^2 - 4} = \lim_{x \to 2} \frac{(x-2)(x^2 + 4 + 2x)}{(x-2)(x+2)} = \frac{(2)^2 + 4 + 2(2)}{2+2} = \frac{4+4+4}{4} = \frac{12}{4} = 3$$

Q5

$$\lim_{x \to -\frac{1}{2}} \frac{8x^3 + 1}{2x + 1} = \lim_{x \to -\frac{1}{2}} \frac{8\left(x^3 + \frac{1}{8}\right)}{2\left(x + \frac{1}{2}\right)}$$

$$= \frac{8}{2} \lim_{x \to -\frac{1}{2}} \frac{\left(x^3 + \left(\frac{1}{2}\right)^3\right)}{x + \frac{1}{2}}$$

$$= 4 \lim_{x \to -\frac{1}{2}} \frac{\left(x + \frac{1}{2}\right)\left(x^2 + \frac{1}{4} - \frac{1}{2}x\right)}{\left(x + \frac{1}{2}\right)}$$

$$= 4\left(\left(\frac{-1}{2}\right)^2 + \frac{1}{4} - \frac{1}{2}\left(\frac{-1}{2}\right)\right)$$

$$= 4\left(\frac{1}{4} + \frac{1}{4} + \frac{1}{4}\right)$$

$$= 3$$

Q6

$$\lim_{x \to 4} \frac{x^2 - 7x + 12}{x^2 - 3x - 4}$$

$$= \lim_{x \to 4} \frac{x^2 - 3x - 4x + 12}{x^2 + x - 4x - 4}$$

$$= \lim_{x \to 4} \frac{x(x - 3) - 4(x - 3)}{x(x + 1) - 1(x + 1)}$$

$$= \lim_{x \to 4} \frac{(x - 3)(x - 4)}{(x - 4)(x + 1)}$$

$$= \lim_{x \to 4} \frac{x - 3}{x + 1}$$

$$= \frac{4 - 3}{4 + 1}$$

$$= \frac{1}{5}$$

Q7

$$\lim_{x \to 2} \frac{x^4 - 16}{x - 2}$$

$$= \lim_{x \to 2} \frac{\left(x^2 - 4\right)\left(x^2 + 4\right)}{(x - 2)}$$

$$= \lim_{x \to 2} \frac{(x - 2)(x + 2)\left(x^2 + 4\right)}{(x - 2)}$$

$$= \lim_{x \to 2} (x + 2)\left(x^2 + 4\right)$$

$$= (2 + 2)(4 + 4)$$

$$= 4(8)$$

$$= 32$$

Q8

$$\lim_{x \to 5} \frac{x^2 - 9x + 20}{x^2 - 6x + 5}$$

$$= \lim_{x \to 5} \frac{x^2 - 4x - 5x + 20}{x^2 - x - 5x + 5}$$

$$= \lim_{x \to 5} \frac{x(x - 4) - 5(x - 4)}{x(x - 1) - 5(x - 1)}$$

$$= \lim_{x \to 5} \frac{(x - 5)(x - 4)}{(x - 5)(x - 1)}$$

$$= \lim_{x \to 5} \frac{x - 4}{x - 1}$$

$$= \frac{5 - 4}{5 - 1}$$

$$= \frac{1}{4}$$

Q9

$$\lim_{x \to -1} \frac{x^3 + 1}{x + 1}$$

$$= \lim_{x \to -1} \frac{(x + 1)(x^2 - x + 1)}{(x + 1)} \qquad \left[a^3 + b^3 = (a + b)(a^2 + b^2 - ab) \right]$$

$$= \lim_{x \to -1} (x^2 - x + 1)$$

$$= (-1)^2 - (-1) + 1$$

$$= 1 + 1 + 1$$

$$= 3$$

Q10

$$\lim_{x \to 5} \frac{x^3 - 125}{x^2 - 7x + 10} = \lim_{x \to 5} \frac{(x - 5)(x^2 + 25 + 5x)}{(x - 2)(x - 5)} = \frac{(5)^2 + 25 + 5(5)}{(5 - 2)} = \frac{25 + 25 + 25}{3} = \frac{75}{3} = 25$$

Q11

$$\lim_{x \to \sqrt{2}} \frac{x^2 - 2}{x^2 + \sqrt{2}x - 4}$$

$$= \lim_{x \to \sqrt{2}} \frac{(x - \sqrt{2})(x + \sqrt{2})}{x^2 + 2\sqrt{2}x - \sqrt{2}x - 4}$$

$$= \lim_{x \to \sqrt{2}} \frac{(x - \sqrt{2})(x + \sqrt{2})}{x(x + 2\sqrt{2}) - \sqrt{2}(x + 2\sqrt{2})}$$

$$= \lim_{x \to \sqrt{2}} \frac{(x - \sqrt{2})(x + \sqrt{2})}{(x + 2\sqrt{2})(x - \sqrt{2})}$$

$$= \frac{\sqrt{2} + \sqrt{2}}{\sqrt{2} + 2\sqrt{2}} = \frac{2\sqrt{2}}{3\sqrt{2}}$$

$$= \frac{2}{3}$$

Q12

$$\lim_{x \to \sqrt{3}} \frac{x^2 - 3}{x^2 + 3\sqrt{3}x - 12}$$

$$= \lim_{x \to \sqrt{3}} \frac{\left(x - \sqrt{3}\right)\left(x + \sqrt{3}\right)}{x^2 + 4\sqrt{3}x - \sqrt{3}x - 12}$$

$$= \lim_{x \to \sqrt{3}} \frac{\left(x - \sqrt{3}\right)\left(x + \sqrt{3}\right)}{x\left(x + 4\sqrt{3}\right) - \sqrt{3}\left(x + 4\sqrt{3}\right)}$$

$$= \lim_{x \to \sqrt{3}} \frac{\left(x - \sqrt{3}\right)\left(x + \sqrt{3}\right)}{\left(x - \sqrt{3}\right)\left(x + 4\sqrt{3}\right)}$$

$$= \frac{\sqrt{3} + \sqrt{3}}{\sqrt{3} + 4\sqrt{3}} = \frac{2\sqrt{3}}{5\sqrt{3}}$$

$$= \frac{2}{5}$$

Q13

$$\lim_{x \to \sqrt{3}} \frac{x^4 - 9}{x^2 + 4\sqrt{3}x - 15} = \lim_{x \to \sqrt{3}} \frac{\left(x - \sqrt{3}\right)\left(x + \sqrt{3}\right)\left(x^2 + 3\right)}{\left(x - \sqrt{3}\right)\left(x + 5\sqrt{3}\right)} = \lim_{x \to \sqrt{3}} \frac{\left(x + \sqrt{3}\right)\left(x^2 + 3\right)}{\left(x + 5\sqrt{3}\right)}$$

$$= \frac{\left(\sqrt{3} + \sqrt{3}\right)(3 + 3)}{\left(\sqrt{3} + 5\sqrt{3}\right)} = \frac{\left(2\sqrt{3}\right)(6)}{6\sqrt{3}} = 2$$

Q14

$$\lim_{x \to 2}\left(\frac{x}{x-2} - \frac{4}{x^2 - 2x}\right)$$

$$= \lim_{x \to 2}\left(\frac{x}{x-2} - \frac{4}{x(x-2)}\right)$$

$$= \lim_{x \to 2}\left(\frac{x(x) - 4}{x(x-2)}\right)$$

$$= \lim_{x \to 2}\left(\frac{x^2 - 4}{x(x-2)}\right)$$

$$= \lim_{x \to 2}\frac{(x-2)(x+2)}{x(x-2)}$$

$$= \lim_{x \to 2}\frac{(x+2)}{x}$$

$$= \frac{2+2}{2}$$

$$= \frac{4}{2}$$

$$= 2$$

Q15

$$\lim_{x \to 1}\left(\frac{1}{x^2 + x - 2} - \frac{x}{x^3 - 1}\right)$$

$$= \lim_{x \to 1}\left(\frac{1}{x^2 + x - 2} - \frac{x}{(x-1)(x^2 + x + 1)}\right)$$

$$= \lim_{x \to 1}\left(\frac{x^3 - 1 - x^3 - x^2 + 2x}{(x^3 - 1)(x^2 + x - 2)}\right)$$

$$= \lim_{x \to 1}\left(-\frac{(x^2 - 2x + 1)}{(x^3 - 1)(x^2 + x - 2)}\right)$$

$$= \lim_{x \to 1}\left(-\frac{(x-1)(x-1)}{(x-1)(x^2 + 1 + x)(x^2 + x - 2)}\right)$$

$$= -\lim_{x \to 1}\left(\frac{x-1}{(x^2 + 1 + x)(x+2)(x-1)}\right)$$

$$= -\frac{1}{(1 + 1 + 1)(1 + 2)}$$

$$= -\frac{1}{9}$$

Q16

$$\lim_{x \to 3} \left(\frac{1}{x-3} - \frac{2}{x^2 - 4x + 3} \right)$$

$$= \lim_{x \to 3} \left(\frac{1}{x-3} - \frac{2}{(x-3)(x-1)} \right)$$

$$= \lim_{x \to 3} \left(\frac{x-1-2}{(x-1)(x-3)} \right)$$

$$= \lim_{x \to 3} \left(\frac{x-3}{(x-1)(x-3)} \right)$$

$$= \lim_{x \to 3} \frac{1}{x-1}$$

$$= \frac{1}{3-1}$$

$$= \frac{1}{2}$$

Q17

$$\lim_{x \to 2} \left(\frac{1}{x-2} - \frac{2}{x^2 - 2x} \right)$$

$$= \lim_{x \to 2} \left(\frac{1}{x-2} - \frac{2}{x(x-2)} \right)$$

$$= \lim_{x \to 2} \left(\frac{x-2}{(x-2)(x)} \right)$$

$$= \lim_{x \to 2} \left(\frac{(x-2)}{(x-2)(x)} \right)$$

$$= \lim_{x \to 2} \frac{1}{x}$$

$$= \frac{1}{2}$$

Q18

$$\lim_{x \to \frac{1}{4}} \frac{4x-1}{2\sqrt{x}-1} = \lim_{x \to \frac{1}{4}} \frac{4\left(x - \frac{1}{4}\right)}{2\left(\sqrt{x} - \frac{1}{2}\right)} = 4 \lim_{x \to \frac{1}{4}} \frac{\left(\sqrt{x} - \frac{1}{2}\right)\left(\sqrt{x} + \frac{1}{2}\right)}{2\left(\sqrt{x} - \frac{1}{2}\right)} = 4 \lim_{x \to \frac{1}{4}} \frac{\left(\sqrt{x} + \frac{1}{2}\right)}{2} = \frac{4\left(\frac{1}{2} + \frac{1}{2}\right)}{2} = \frac{4(1)}{2} = 2$$

Q19

$$\lim_{x \to 4} \frac{x^2 - 16}{\sqrt{x} - 2}$$

$$= \lim_{x \to 4} \frac{(x - 4)(x + 4)}{\left(\sqrt{x} - 2\right)}$$

$$= \lim_{x \to 4} \frac{\left(\sqrt{x} - 2\right)\left(\sqrt{x} + 2\right)(x + 4)}{\left(\sqrt{x} - 2\right)}$$

$$= \lim_{x \to 4} \left(\sqrt{x} + 2\right)(x + 4)$$

$$= (2 + 2)(4 + 4)$$

$$= 4(8)$$

$$= 32$$

Q20

$$\lim_{x \to 0} \frac{(a + x)^2 - a^2}{x}$$

$$= \lim_{x \to 0} \frac{(a + x - a)(a + x + a)}{x}$$

$$= \lim_{x \to 0} \frac{(x)(2a + x)}{x}$$

$$= \lim_{x \to 0} (2a + x)$$

$$= 2a$$

Q21

$$\lim_{x \to 2} \left(\frac{1}{x - 2} - \frac{4}{x^3 - 2x^2}\right)$$

$$= \lim_{x \to 2} \left(\frac{1}{x - 2} - \frac{4}{x^2(x - 2)}\right)$$

$$= \lim_{x \to 2} \left(\frac{x^2 - 4}{x^2(x - 2)}\right)$$

$$= \lim_{x \to 2} \frac{(x - 2)(x + 2)}{x^2(x - 2)}$$

$$= \lim_{x \to 2} \frac{(x + 2)}{x^2}$$

$$= \frac{(2 + 2)}{2^2} = \frac{4}{4}$$

$$= 1$$

Q22

$$\lim_{x \to 3}\left(\frac{1}{x-3} - \frac{3}{x^2 - 3x}\right)$$

$$= \lim_{x \to 3}\left(\frac{1}{x-3} - \frac{3}{x(x-3)}\right)$$

$$= \lim_{x \to 3}\left(\frac{x-3}{x(x-3)}\right)$$

$$= \lim_{x \to 3}\left(\frac{1}{x}\right)$$

$$= \frac{1}{3}$$

Q23

$$\lim_{x \to 1}\left(\frac{1}{x-1} - \frac{2}{x^2 - 1}\right)$$

$$= \lim_{x \to 1}\left(\frac{1}{x-1} - \frac{2}{(x-1)(x+1)}\right)$$

$$= \lim_{x \to 1}\left(\frac{x+1-2}{(x-1)(x+1)}\right)$$

$$= \lim_{x \to 1}\left(\frac{x-1}{(x-1)(x+1)}\right)$$

$$= \lim_{x \to 1}\left(\frac{1}{x+1}\right)$$

$$= \frac{1}{1+1} = \frac{1}{2}$$

Q24

$$\lim_{x \to 3}\left(x^2 - 9\right)\left[\frac{1}{x+3} + \frac{1}{x-3}\right]$$

$$= \lim_{x \to 3}\left(x^2 - 9\right)\left[\frac{x-3+x+3}{(x+3)(x-3)}\right]$$

$$= \lim_{x \to 3}\left(x^2 - 9\right)\left[\frac{2x}{(x+3)(x-3)}\right]$$

$$= \lim_{x \to 3}\frac{(x-3)(x+3)(2x)}{(x+3)(x-3)}$$

$$= \lim_{x \to 3}2(x) = 2(3) = 6$$

Q25

$$\lim_{x \to 1} \frac{x^4 - 3x^3 + 2}{x^3 - 5x^2 + 3x + 1}$$

Dividing $x^4 - 3x^3 + 2$ by $x^3 - 5x^2 + 3x + 1$

$$
\begin{array}{r}
x + 2 \\
x^3 - 5x^3 + 3x + 1 \overline{\smash{)}\; x^4 - 3x^3 + 2} \\
\underline{\pm x^4 \mp 5x^3 \pm 3x^2 \pm x} \\
2x^3 - 3x^2 - x + 2 \\
\underline{\pm 2x^3 \mp 10x^2 \pm 6x \pm 2} \\
7x^2 - 7x
\end{array}
$$

$$\Rightarrow \lim_{x \to 1} \frac{x^4 - 3x^3 + 2}{x^3 - 5x^2 + 3x + 1} = \lim_{x \to 1} x + 2 + \lim_{x \to 1} \frac{7x^2 - 7x}{x^3 - 5x^3 + 3x + 1}$$

$$= \lim_{x \to 1} x + 2 + \lim_{x \to 1} \frac{7x(x - 1)}{x^3 - 5x^3 + 3x + 1}$$

Dividing $x^3 - 5x^2 + 3x + 1$ by $x - 1$

$$
\begin{array}{r}
x^2 - 4x - 1 \\
x - 1 \overline{\smash{)}\; x^3 - 5x^2 + 3x + 1} \\
\underline{\pm x^3 \mp x^2} \\
-4x^2 + 3x + 1 \\
\underline{-4x^2 + 4x} \\
^{+}^{-} \\
-x + 1 \\
\underline{-x + 1} \\
\times
\end{array}
$$

Q26

$$\lim_{x \to 2} \frac{x^3 + 3x^2 - 9x - 2}{x^3 - x - 6}$$

Divide $x^3 + 3x^2 - 9x - 2$ by $x^3 - x - 6$

$$x^3 - x - 6 \overline{)\begin{array}{r} 1 \\ x^3 + 3x^2 - 9x - 2 \end{array}}$$
$$\underline{+x^3 \quad -x \quad -6}$$
$$3x^2 - 8x + 4$$

$$\Rightarrow \quad \lim_{x \to 2} \frac{x^3 + 3x^2 - 9x - 2}{x^3 - x - 6} = \lim_{x \to 2} 1 + \lim_{x \to 2} \frac{3x^2 - 8x + 4}{x^3 - x - 6}$$

$$= 1 + \lim_{x \to 2} \frac{3x^2 - 2x - 6x + 4}{x^3 - x - 6}$$

$$= 1 + \lim_{x \to 2} \frac{3x^2 - 2x - 6x + 4}{x^3 - x - 6}$$

$$\Rightarrow \quad \lim_{x \to 2} \frac{x^3 + 3x^2 - 9x - 2}{x^3 - x - 6} = 1 + \lim_{x \to 2} \frac{(3x - 2)(x - 2)}{x^3 - x - 6}$$

Divide $x^3 - x - 6$ by $x - 2$

$$x - 2 \overline{)\begin{array}{l} x^2 + 2x + 3 \\ x^3 - x - 6 \end{array}}$$
$$\underline{+x^3 - 2x^2}$$
$$2x^2 - x - 6$$
$$\underline{2x^2 + 4x}$$
$$3x - 6$$
$$\underline{3x - 6}$$
$$\times$$

$$\Rightarrow \quad \lim_{x \to 2} \frac{x^3 + 3x^2 - 9x - 2}{x^3 - x - 6} = 1 + \lim_{x \to 2} \frac{(3x - 2)(x - 2)}{(x - 2)(x^2 + 2x + 3)}$$

$$= 1 + \lim_{x \to 2} \frac{(3x - 2)}{(x^2 + 2x + 3)}$$

$$= 1 + \frac{3 \times 2 - 2}{2^2 + 2 \times 2 + 3}$$

$$= 1 + \frac{4}{11}$$

$$= \frac{15}{11}$$

Q27

$$\lim_{x \to 1} \frac{1 - x^{\frac{-1}{3}}}{1 - x^{\frac{-2}{3}}}$$

$$= \lim_{x \to 1} \frac{1 - \dfrac{1}{x^{\frac{1}{3}}}}{1 - \dfrac{1}{x^{\frac{2}{3}}}}$$

$$= \lim_{x \to 1} \frac{\left(x^{\frac{1}{3}} - 1\right)}{\left(x^{\frac{1}{3}} - 1\right)\left(x^{\frac{1}{3}} + 1\right)} \times x^{\frac{1}{3}}$$

$$= \lim_{x \to 1} \frac{x^{\frac{1}{3}}}{x^{\frac{1}{3}} + 1}$$

$$= \frac{1}{1 + 1}$$

$$= \frac{1}{2}$$

Q28

$$\lim_{x \to 3} \frac{x^2 - x - 6}{x^3 - 3x^2 + x - 3}$$

Now
$$x^2 - x - 6$$
$$= x^2 - 3x + 2x - 6$$
$$= x(x - 3) + 2(x - 3)$$
$$= (x + 2)(x - 3) \qquad\qquad ---(i)$$

Dividing $x^3 - 3x^2 + x - 3$ by $(x - 3)$, we get

$$
\begin{array}{r}
x^2 + 1 \\
x - 3 \overline{) x^3 - 3x^2 + x - 3} \\
\underline{\pm x^3 \mp 3x^2} \\
x - 3 \\
\underline{x - 3} \\
\times
\end{array}
$$

Thus $(x - 3)$ is a factor of $x^3 - 3x^2 + x - 3$ $\qquad\qquad ---(ii)$

Substituting (i) and (ii) in the given expression

$$= \lim_{x \to 3} \frac{(x + 2)(x - 3)}{(x^2 + 1)(x - 3)}$$

$$= \frac{x + 2}{x^2 + 1} = \frac{3 + 2}{9 + 1} = \frac{5}{10}$$

$$= \frac{1}{2}$$

Q29

$$\lim_{x \to -2} \frac{x^3 + x^2 + 4x + 12}{x^3 - 3x + 2} = \lim_{x \to -2} \frac{(x + 2)(x^2 - x + 6)}{(x + 2)(x^2 - 2x + 1)}$$

$$\lim_{x \to -2} \frac{x^2 - x + 6}{x^2 - 2x + 1}$$

$$= \frac{(-2)^2 - (-2) + 6}{(-2)^2 - 2(-2) + 1}$$

$$= \frac{4 + 2 + 6}{4 + 4 + 1}$$

$$= \frac{12}{9}$$

$$= \frac{4}{3}$$

Q30

$$\lim_{x \to 1} \frac{x^3 + 3x^2 - 6x + 2}{x^3 + 3x^2 - 3x - 1} = \lim_{x \to 1} \frac{(x - 1)(x^2 + 4x - 2)}{(x - 1)(x^2 + 4x + 1)}$$

$$= \frac{(1)^2 + 4(1) - 2}{(1)^2 + 4(1) + 1} = \frac{1 + 4 - 2}{1 + 4 + 1}$$

$$= \frac{3}{6}$$

$$= \frac{1}{2}$$

Q31

$$\lim_{x \to 2}\left[\frac{1}{x-2} - \frac{2(2x-3)}{x^3 - 3x^2 + 2x}\right]$$

$$= \lim_{x \to 2}\left[\frac{1}{x-2} - \frac{2(2x-3)}{x(x^2 - 2x - x + 2)}\right]$$

$$= \lim_{x \to 2}\left[\frac{1}{x-2} - \frac{2(2x-3)}{x(x-2)(x-1)}\right]$$

$$= \lim_{x \to 2}\left[\frac{x(x-1) - 2(2x-3)}{x(x-1)(x-2)}\right]$$

$$= \lim_{x \to 2}\left[\frac{x^2 - x - 4x + 6}{x(x-1)(x-2)}\right]$$

$$= \lim_{x \to 2}\left[\frac{x^2 - 2x - 3x + 6}{x(x-1)(x-2)}\right]$$

$$= \lim_{x \to 2}\left[\frac{x(x-2) - 3(x-2)}{x(x-1)(x-2)}\right]$$

$$= \lim_{x \to 2}\left[\frac{(x-2)(x-3)}{x(x-1)(x-2)}\right]$$

$$= \lim_{x \to 2}\left[\frac{1}{x-2} - \frac{2(2x-3)}{x(x-2)(x-1)}\right]$$

$$= \frac{-1}{2}$$

Q32

$$\lim_{x \to 1} \frac{\sqrt{x^2 - 1} + \sqrt{x - 1}}{\sqrt{x^2 - 1}}$$

$$= \lim_{x \to 1} \frac{\left(\sqrt{x^2 - 1} + \sqrt{x - 1}\right)}{\sqrt{x^2 - 1}} \times \frac{\left(\sqrt{x^2 - 1} - \sqrt{x - 1}\right)}{\left(\sqrt{x^2 - 1} - \sqrt{x - 1}\right)} \times \frac{\sqrt{x^2 - 1}}{\sqrt{x^2 - 1}}$$

$$= \lim_{x \to 1} \frac{\left[\left(x^2 - 1\right) - (x - 1)\right] \times \sqrt{x^2 - 1}}{\left(x^2 - 1\right)\left(\sqrt{x^2 - 1} - \sqrt{x - 1}\right)}$$

$$= \lim_{x \to 1} \frac{\left(x^2 - x\right)\sqrt{x^2 - 1}}{\left(x^2 - 1\right)\left(\sqrt{x^2 - 1} - \sqrt{x - 1}\right)}$$

$$= \lim_{x \to 1} \frac{x(x - 1)\sqrt{x^2 - 1}}{(x - 1)(x + 1)\left(\sqrt{x^2 - 1} - \sqrt{x - 1}\right)}$$

$$\lim_{x \to 1} \frac{x\left(\sqrt{x - 1}\right)\left(\sqrt{x + 1}\right)}{(x + 1)\left(\sqrt{x - 1}\right)\left(\sqrt{x + 1} - 1\right)}$$

$$= \frac{\sqrt{2}}{2\left(\sqrt{2} - 1\right)}$$

$$= \frac{\sqrt{2}}{2 \times \left(\sqrt{2} - 1\right)} \times \frac{\sqrt{2} + 1}{\sqrt{2} + 1}$$

$$= \frac{\sqrt{2} + 1}{\sqrt{2}}$$

Q33

$$\lim_{x \to 1} \left\{ \frac{x-2}{x^2-x} - \frac{1}{x^3-3x^2+2x} \right\}$$

$$= \lim_{x \to 1} \left\{ \frac{x-2}{x(x-1)} - \frac{1}{x(x^2-3x+2)} \right\}$$

$$= \lim_{x \to 1} \left\{ \frac{x-2}{x(x-1)} - \frac{1}{x(x^2-3x+2)} \right\}$$

$$= \lim_{x \to 1} \left\{ \frac{x-2}{x(x-1)} - \frac{1}{x(x^2-1x-2x+2)} \right\}$$

$$= \lim_{x \to 1} \left\{ \frac{x-2}{x(x-1)} - \frac{1}{x(x-1)(x-2)} \right\}$$

$$= \lim_{x \to 1} \left\{ \frac{(x-2)^2-1}{x(x-1)(x-2)} \right\}$$

$$= \lim_{x \to 1} \left\{ \frac{x^2+4-4x-1}{x(x-1)(x-2)} \right\}$$

$$= \lim_{x \to 1} \left\{ \frac{x^2-4x+3}{x(x-1)(x-2)} \right\}$$

$$= \lim_{x \to 1} \left[\frac{x^2-x-3x+3}{x(x-1)(x-2)} \right]$$

$$= \lim_{x \to 1} \left[\frac{x(x-1)-3(x-1)}{x(x-1)(x-2)} \right]$$

$$= \lim_{x \to 1} \left[\frac{(x-3)(x-1)}{x(x-1)(x-2)} \right]$$

$$= \frac{(1-3)}{1(1-2)}$$

$$= \frac{-2}{-1}$$

$$= 2$$

Q34

$$\lim_{x \to 1} \frac{x^7 - 2x^5 + 1}{x^3 - 3x^2 + 2} = \lim_{x \to 1} \frac{(x-1)(x^6 + x^5 - x^4 - x^3 - x^2 - x - 1)}{(x-1)(x^2 - 2x - 2)}$$

$$= \lim_{x \to 1} \frac{(x^6 + x^5 - x^4 - x^3 - x^2 - x - 1)}{(x^2 - 2x - 2)}$$

$$= \frac{(1 + 1 - 1 - 1 - 1 - 1 - 1)}{(1 - 2 - 2)}$$

$$= \frac{-3}{-3}$$

$$= 1$$

Ex 29.4

Q1

$$\lim_{x \to 0} \frac{\sqrt{1+x+x^2}-1}{x}$$

$$= \lim_{x \to 0} \frac{\left(\sqrt{1+x+x^2}-1\right)}{x} \frac{\left(\sqrt{1+x+x^2}+1\right)}{\left(\sqrt{1+x+x^2}+1\right)}$$

$$= \lim_{x \to 0} \frac{\left(\left(1+x+x^2\right)-1\right)}{x\left(\sqrt{1+x+x^2}+1\right)}$$

$$= \lim_{x \to 0} \frac{x(1+x)}{x\left(\sqrt{1+x+x^2}+1\right)}$$

$$= \lim_{x \to 0} \frac{1+x}{\sqrt{1+x+x^2}+1}$$

$$= \frac{1+0}{\sqrt{1+0+0}+1}$$

$$= \frac{1}{1+1}$$

$$= \frac{1}{2}$$

Q2

$$\lim_{x \to 0} \frac{2x}{\sqrt{a+x}-\sqrt{a-x}}$$

$$= \lim_{x \to 0} \frac{2x}{\left(\sqrt{a+x}-\sqrt{a-x}\right)} \times \frac{\sqrt{a+x}+\sqrt{a-x}}{\left(\sqrt{a+x}+\sqrt{a-x}\right)}$$

$$= \lim_{x \to 0} \frac{2x\left(\sqrt{a+x}+\sqrt{a-x}\right)}{\left((a+x)-(a-x)\right)}$$

$$= \lim_{x \to 0} \frac{2x\left(\sqrt{a+x}+\sqrt{a-x}\right)}{2x}$$

$$= \lim_{x \to 0} \left(\sqrt{a+x}+\sqrt{a-x}\right)$$

$$= \sqrt{a}+\sqrt{a}$$

$$= 2\sqrt{a}$$

Q3

$$\lim_{x \to 0} \frac{\sqrt{a^2 + x^2} - a}{x^2}$$

$$= \lim_{x \to 0} \frac{\left(\sqrt{a^2 + x^2} - a\right)}{x^2} \times \frac{\left(\sqrt{a^2 + x^2} + a\right)}{\left(\sqrt{a^2 + x^2} + a\right)}$$

$$= \lim_{x \to 0} \frac{\left(a^2 + x^2 - a^2\right)}{x^2 \sqrt{a^2 + x^2} + a}$$

$$= \lim_{x \to 0} \frac{x^2}{x^2 \left(\sqrt{a^2 + x^2} + a\right)}$$

$$= \lim_{x \to 0} \frac{1}{\sqrt{a^2 + x^2} + a}$$

$$= \frac{1}{a + a}$$

$$= \frac{1}{2a}$$

Q4

$$\lim_{x \to 0} \frac{\sqrt{1 + x} - \sqrt{1 - x}}{2x}$$

$$= \lim_{x \to 0} \frac{\left(\sqrt{1 + x} - \sqrt{1 - x}\right)}{2x} \times \frac{\left(\sqrt{1 + x} + \sqrt{1 - x}\right)}{\left(\sqrt{1 + x} + \sqrt{1 - x}\right)}$$

$$= \lim_{x \to 0} \frac{(1 + x) - (1 - x)}{2x \left(\sqrt{1 + x} + \sqrt{1 - x}\right)}$$

$$= \lim_{x \to 0} \frac{2x}{2x \left(\sqrt{1 + x} + \sqrt{1 - x}\right)}$$

$$= \lim_{x \to 0} \frac{1}{\left(\sqrt{1 + x} + \sqrt{1 - x}\right)}$$

$$= \frac{1}{\sqrt{1} + \sqrt{1}}$$

$$= \frac{1}{2}$$

Q5

$$\lim_{x \to 2} \frac{\sqrt{3-x} - 1}{2-x}$$

$$= \lim_{x \to 2} \frac{\left(\sqrt{3-x} - 1\right)}{2-x} \times \frac{\left(\sqrt{3-x} + 1\right)}{\left(\sqrt{3-x} + 1\right)}$$

$$= \lim_{x \to 2} \frac{(3-x) - 1}{(2-x)\left(\sqrt{3-x} + 1\right)}$$

$$= \lim_{x \to 2} \frac{(2-x)}{(2-x)\left(\sqrt{3-x} + 1\right)}$$

$$= \lim_{x \to 2} \frac{1}{\left(\sqrt{3-x} + 1\right)}$$

$$= \frac{1}{\sqrt{3-2} + 1} = \frac{1}{1+1}$$

$$= \frac{1}{2}$$

Q6

$$\lim_{x \to 3} \frac{x-3}{\sqrt{x-2} - \sqrt{4-x}}$$

$$= \lim_{x \to 3} \frac{x-3}{\left(\sqrt{x-2} - \sqrt{4-x}\right)} \times \frac{\left(\sqrt{x-2} + \sqrt{4-x}\right)}{\left(\sqrt{x-2} + \sqrt{4-x}\right)}$$

$$= \lim_{x \to 3} \frac{(x-3)\left(\sqrt{x-2} + \sqrt{4-x}\right)}{(x-2) - (4-x)}$$

$$= \lim_{x \to 3} \frac{(x-3)\left(\sqrt{x-2} + \sqrt{4-x}\right)}{x-2-4+x}$$

$$= \lim_{x \to 3} \frac{(x-3)\left(\sqrt{x-2} + \sqrt{4-x}\right)}{2(x-3)}$$

$$= \frac{1}{2} \lim_{x \to 3} \left(\sqrt{x-2} + \sqrt{4-x}\right)$$

$$= \frac{1}{2}\left(\sqrt{3-2} + \sqrt{4-3}\right)$$

$$= \frac{1}{2}\left(\sqrt{1} + \sqrt{1}\right)$$

$$= \frac{1}{2}(1+1) = \frac{2}{2}$$

$$= 1$$

Q7

$$\lim_{x \to 0} \frac{x}{\left(\sqrt{1+x} - \sqrt{1-x}\right)}$$

$$= \lim_{x \to 0} \frac{x}{\left(\sqrt{1+x} - \sqrt{1-x}\right)} \times \frac{\left(\sqrt{1+x} + \sqrt{1-x}\right)}{\left(\sqrt{1+x} + \sqrt{1-x}\right)}$$

$$= \lim_{x \to 0} \frac{x\left(\sqrt{1+x} + \sqrt{1-x}\right)}{\left(\sqrt{1+x}\right)^2 - \left(\sqrt{1-x}\right)^2}$$

$$= \lim_{x \to 0} \frac{x\left(\sqrt{1+x} + \sqrt{1-x}\right)}{1+x-1+x}$$

$$= \lim_{x \to 0} \frac{x\left(\sqrt{1+x} + \sqrt{1-x}\right)}{2x}$$

$$= \frac{1}{2} \lim_{x \to 0} \left(\frac{\sqrt{1+x} + \sqrt{1-x}}{x}\right) x$$

$$= \frac{1}{2} \lim_{x \to 0} \left(\sqrt{1+x} + \sqrt{1-x}\right)$$

$$= \frac{1}{2}\left(\sqrt{1} + \sqrt{1}\right)$$

$$= \frac{2}{2} = 1$$

Q8

$$\lim_{x \to 1} \frac{\left(\sqrt{5x-4} - \sqrt{x}\right)}{x-1}$$

$$= \lim_{x \to 1} \frac{\left(\sqrt{5x-4} - \sqrt{x}\right)}{x-1} \times \frac{\left(\sqrt{5x-4} + \sqrt{x}\right)}{\left(\sqrt{5x-4} + \sqrt{x}\right)}$$

$$= \lim_{x \to 1} \frac{\left((5x-4) - x\right)}{(x-1)\left(\sqrt{5x-4} + \sqrt{x}\right)}$$

$$= 4 \lim_{x \to 1} \frac{(x-1)}{(x-1)\left(\sqrt{5x-4} + \sqrt{x}\right)}$$

$$= 4 \lim_{x \to 1} \frac{1}{\sqrt{5x-4} + \sqrt{x}}$$

$$= 4 \times \frac{1}{\sqrt{5-4} + \sqrt{1}}$$

$$= 4 \times \frac{1}{\sqrt{1} + \sqrt{1}}$$

$$= \frac{4}{2} = 2$$

Q9

$$\lim_{x \to 1} \frac{(x-1)}{\left(\sqrt{x^2+3}-2\right)}$$

$$= \lim_{x \to 1} \frac{(x-1) \times \left(\sqrt{x^2+3}+2\right)}{\left(\sqrt{x^2+3}-2\right)\left(\sqrt{x^2+3}+2\right)}$$

$$= \lim_{x \to 1} \frac{(x-1)\left(\sqrt{x^2+3}+2\right)}{\left(x^2+3-4\right)}$$

$$= \lim_{x \to 1} \frac{(x-1)\left(\sqrt{x^2+3}+2\right)}{\left(x^2-1\right)}$$

$$= \lim_{x \to 1} \frac{\sqrt{x^2+3}+2}{x+1}$$

Putting the value $x = 1$

$$\Rightarrow \quad \frac{\sqrt{1+3}+2}{1+1}$$

$$= \frac{2+2}{2}$$

$$= \frac{4}{2} = 2$$

Q10

$$\lim_{x \to 3} \frac{\left(\sqrt{x+3} - \sqrt{6}\right)}{\left(x^2 - 9\right)}$$

$$= \lim_{x \to 3} \frac{\left(\sqrt{x+3} - \sqrt{6}\right)}{(x-3)(x+3)} \frac{\left(\sqrt{x+3} - \sqrt{6}\right)}{\left(\sqrt{x+3} + \sqrt{6}\right)}$$

$$= \lim_{x \to 3} \frac{\left((x+3) - 6\right)}{(x-3)(x+3)\left(\sqrt{x+3} + \sqrt{6}\right)}$$

$$= \lim_{x \to 3} \frac{(x-3)}{(x-3)(x+3)\left(\sqrt{x+3} + \sqrt{6}\right)}$$

$$= \lim_{x \to 3} \frac{1}{(x+3)\left(\sqrt{x+3} + \sqrt{6}\right)}$$

$$= \frac{1}{(3+3)\sqrt{3+3} + \sqrt{6}}$$

$$= \frac{1}{6\left(\sqrt{6} + \sqrt{6}\right)} = \frac{1}{6 \times 2\sqrt{6}}$$

$$= \frac{1}{12\sqrt{6}}$$

Q11

$$\lim_{x \to 1} \frac{\left(\sqrt{5x-4} - \sqrt{x}\right)}{\left(x^2 - 1\right)}$$

$$= \lim_{x \to 1} \frac{\left(\sqrt{5x-4} - \sqrt{x}\right)}{(x-1)(x+1)} \times \frac{\left(\sqrt{5x-4} + \sqrt{x}\right)}{\left(\sqrt{5x-4} + \sqrt{x}\right)}$$

$$= \lim_{x \to 1} \frac{\left((5x-4) - x\right)}{(x-1)(x+1)\left(\sqrt{5x-4} + \sqrt{x}\right)}$$

$$= \lim_{x \to 1} \frac{4(x-1)}{(x-1)(x+1)\left(\sqrt{5x-4} + \sqrt{x}\right)}$$

$$= \lim_{x \to 1} \frac{4}{(x+1)\left(\sqrt{5x-4} + \sqrt{x}\right)}$$

$$= \frac{4}{(1+1)\left(\sqrt{5-4} + \sqrt{1}\right)}$$

$$= \frac{4}{2(1+1)}$$

$$= \frac{4}{4} = 1$$

Q12

$$\lim_{x \to 0} \frac{\left(\sqrt{1+x} - 1\right)}{x}$$

$$= \lim_{x \to 0} \frac{\left(\sqrt{1+x} - 1\right)}{x} \times \frac{\left(\sqrt{1+x} + 1\right)}{\left(\sqrt{1+x} + 1\right)}$$

$$= \lim_{x \to 0} \frac{(1+x-1)}{x\left(\sqrt{1+x} + 1\right)}$$

$$= \lim_{x \to 0} \frac{x}{x\left(\sqrt{1+x} + 1\right)}$$

$$= \lim_{x \to 0} \frac{1}{\left(\sqrt{1+x} + 1\right)}$$

$$= \frac{1}{\sqrt{1} + 1} = \frac{1}{2}$$

Q13

$$\lim_{x \to 2} \frac{\sqrt{x^2+1} - \sqrt{5}}{(x-2)}$$

$$= \lim_{x \to 2} \frac{\sqrt{x^2+1} - \sqrt{5}}{(x-2)} \times \frac{\left(\sqrt{x^2+1} + \sqrt{5}\right)}{\left(\sqrt{x^2+1} + \sqrt{5}\right)}$$

$$= \lim_{x \to 2} \frac{\left(x^2+1-5\right)}{(x-2)\left(\sqrt{x^2+1} + \sqrt{5}\right)}$$

$$= \lim_{x \to 2} \frac{(x+2)(x-2)}{(x-2)\left(\sqrt{x^2+1} + \sqrt{5}\right)}$$

$$= \lim_{x \to 2} \frac{(x+2)}{\left(\sqrt{x^2+1} + \sqrt{5}\right)}$$

$$= \frac{(2+2)}{\sqrt{4+1} + \sqrt{5}}$$

$$= \frac{4}{2\sqrt{5}} = \frac{2}{\sqrt{5}}$$

Q14

$$\lim_{x \to 2} \frac{x-2}{\sqrt{x} - \sqrt{2}}$$

$$= \lim_{x \to 2} \frac{(x-2)\left(\sqrt{x} + \sqrt{2}\right)}{\left(\sqrt{x} - \sqrt{2}\right)\left(\sqrt{x} + \sqrt{2}\right)}$$

$$= \lim_{x \to 2} \frac{(x-2)\left(\sqrt{x} + \sqrt{2}\right)}{(x-2)}$$

$$= \sqrt{2} + \sqrt{2}$$

$$= 2\sqrt{2}$$

Q15

$$\lim_{x \to 7} \frac{4 - \sqrt{9 + x}}{1 - \sqrt{8 - x}}$$

$$= \lim_{x \to 7} \frac{\left(4 - \sqrt{9 + x}\right)}{1 - \sqrt{8 - x}} \times \frac{\left(4 + \sqrt{9 + x}\right)}{\left(4 + \sqrt{9 + x}\right)} \times \frac{\left(1 + \sqrt{8 - x}\right)}{\left(\sqrt{1 + \sqrt{8 - x}}\right)}$$

$$= \lim_{x \to 7} \frac{\left((4)^2 - \left(\sqrt{9 + x}\right)^2\right)}{\left((1)^2 - \left(\sqrt{8 - x}\right)^2\right)} \times \frac{1 + \sqrt{8 - x}}{4 + \sqrt{9 + x}}$$

$$= \lim_{x \to 7} \frac{(16 - 9 - x) \times \left(1 + \sqrt{8 - x}\right)}{(1 - 8 + x) \times \left(4 + \sqrt{9 + x}\right)}$$

$$= \lim_{x \to 7} \frac{7 - x}{(-1)(7 - x)} \frac{\left(1 + \sqrt{8 - x}\right)}{\left(4 + \sqrt{9 + x}\right)}$$

$$= \frac{1}{(-1)} \times \frac{(1 + 1)}{(4 + 4)} = \frac{-1}{4}$$

Q16

$$\lim_{x \to 0} \frac{\sqrt{a + x} - \sqrt{a}}{x\sqrt{a^2 + ax}}$$

$$= \lim_{x \to 0} \frac{\left(\sqrt{a + x} - \sqrt{a}\right)}{x\sqrt{a^2 + ax}} \times \frac{\left(\sqrt{a + x} + \sqrt{a}\right)}{\left(\sqrt{a + x} + \sqrt{a}\right)}$$

$$= \lim_{x \to 0} \frac{(a + x) - a}{x\sqrt{a^2 + ax}\left(\sqrt{a + x} + \sqrt{a}\right)}$$

$$= \lim_{x \to 0} \frac{x}{x\sqrt{a^2 + ax}\left(\sqrt{a + x} + \sqrt{a}\right)}$$

$$= \lim_{x \to 0} \frac{1}{\left(\sqrt{a^2 + ax}\right)\left(\sqrt{a + x} + \sqrt{a}\right)}$$

$$= \frac{1}{a\left(2\sqrt{a}\right)}$$

$$= \frac{1}{2a\sqrt{a}}$$

Q17

$$\lim_{x \to 5} \frac{x - 5}{\sqrt{6x - 5} - \sqrt{4x + 5}}$$

$$= \lim_{x \to 5} \frac{x - 5}{\left(\sqrt{6x - 5} - \sqrt{4x + 5}\right)} \times \frac{\left(\sqrt{6x - 5} + \sqrt{4x + 5}\right)}{\left(\sqrt{6x - 5} + \sqrt{4x + 5}\right)}$$

$$= \lim_{x \to 5} \frac{(x - 5)\left(\sqrt{6x - 5} + \sqrt{4x + 5}\right)}{(6x - 5) - (4x + 5)}$$

$$= \lim_{x \to 5} \frac{(x - 5)\left(\sqrt{6x - 5} + \sqrt{4x + 5}\right)}{2x - 10}$$

$$= \lim_{x \to 5} \frac{(x - 5)\left(\sqrt{6x - 5} + \sqrt{4x + 5}\right)}{2(x - 5)}$$

$$= \frac{\sqrt{6(5) - 5} + \sqrt{4(5) + 5}}{2}$$

$$= \frac{\sqrt{25} + \sqrt{25}}{2}$$

$$= \frac{5 + 5}{2} = 5$$

Q18

$$\lim_{x \to 1} \frac{\sqrt{5x - 4} - \sqrt{x}}{x^3 - 1}$$

$$= \lim_{x \to 1} \frac{\left(\sqrt{5x - 4} - \sqrt{x}\right)}{(x - 1)\left(x^2 + 1 + 1\right)} \times \frac{\left(\sqrt{5x - 4} + \sqrt{x}\right)}{\left(\sqrt{5x - 4} + \sqrt{x}\right)}$$

$$= \lim_{x \to 1} \frac{(5x - 4 - x)}{(x - 1)\left(x^2 + 1 + x\right)\left(\sqrt{5x - 4} + \sqrt{x}\right)}$$

$$= \lim_{x \to 1} \frac{4(x - 1)}{(x - 1)\left(x^2 + x + 1\right)\left(\sqrt{5x - 4} + \sqrt{x}\right)}$$

$$= \frac{4}{(1 + 1 + 1)\left(\sqrt{5 - 4} + \sqrt{1}\right)}$$

$$= \frac{4}{(3)(1 + 1)}$$

$$= \frac{4}{3 \times 2} = \frac{2}{3}$$

Q19

$$\lim_{x \to 2} \frac{\sqrt{1 + 4x} - \sqrt{5 + 2x}}{x - 2}$$

$$= \lim_{x \to 2} \frac{\left(\sqrt{1 + 4x} - \sqrt{5 + 2x}\right)}{(x - 2)} \times \frac{\left(\sqrt{1 + 4x} + \sqrt{5 + 2x}\right)}{\left(\sqrt{1 + 4x} + \sqrt{5 + 2x}\right)}$$

$$= \lim_{x \to 2} \frac{(1 + 4x) - (5 + 2x)}{(x - 2)\left(\sqrt{1 + 4x} + \sqrt{5 + 2x}\right)}$$

$$= \lim_{x \to 2} \frac{-4 + 2x}{(x - 2)\left(\sqrt{1 + 4x} + \sqrt{5 + 2x}\right)}$$

$$= \lim_{x \to 2} \frac{2(x - 2)}{(x - 2)\left(\sqrt{1 + 4x} + \sqrt{5 + 2x}\right)}$$

$$= \frac{2}{\sqrt{1 + 8} + \sqrt{5 + 4}} = \frac{2}{\sqrt{9} + \sqrt{9}}$$

$$= \frac{2}{6} = \frac{1}{3}$$

Q20

$$\lim_{x \to 1} \frac{\sqrt{3+x} - \sqrt{5-x}}{x^2 - 1}$$

$$= \lim_{x \to 1} \frac{\left(\sqrt{3+x} - \sqrt{5-x}\right)}{\left(x^2 - 1\right)} \times \frac{\left(\sqrt{3+x} + \sqrt{5-x}\right)}{\left(\sqrt{3+x} + \sqrt{5-x}\right)}$$

$$= \lim_{x \to 1} \frac{(3+x) - (5-x)}{(x-1)(x+1)\left(\sqrt{3+x} + \sqrt{5-x}\right)}$$

$$= \lim_{x \to 1} \frac{-2 + 2x}{(x-1)(x+1)\left(\sqrt{3+x} + \sqrt{5-x}\right)}$$

$$= \lim_{x \to 1} \frac{2(x-1)}{(x-1)(x+1)\left(\sqrt{3+x} + \sqrt{5-x}\right)}$$

$$= \lim_{x \to 1} \frac{2}{(x+1)\left(\sqrt{3+x} + \sqrt{5-x}\right)}$$

$$= \frac{2}{(1+1)\left(\sqrt{3+1} + \sqrt{5-1}\right)} = \frac{2}{(2)(2+2)}$$

$$= \frac{1}{4}$$

Q21

$$\lim_{x \to 0} \frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{x}$$

$$= \lim_{x \to 0} \frac{\left(\sqrt{1+x^2} - \sqrt{1-x^2}\right)}{x} \times \frac{\left(\sqrt{1+x^2} + \sqrt{1-x^2}\right)}{\left(\sqrt{1+x^2} + \sqrt{1-x^2}\right)}$$

$$= \lim_{x \to 0} \frac{\left(1+x^2\right) - \left(1-x^2\right)}{x\left(\sqrt{1+x^2} + \sqrt{1-x^2}\right)}$$

$$= \lim_{x \to 0} \frac{2x^2}{x\left(\sqrt{1+x^2} + \sqrt{1-x^2}\right)}$$

$$= \frac{2 \times 0}{\left(\sqrt{1} + \sqrt{1}\right)}$$

$$= \frac{2}{2} \times 0$$

$$= 0$$

Q22

$$\lim_{x \to 0} \frac{\sqrt{1+x+x^2} - \sqrt{x+1}}{2x^2}$$

$$= \lim_{x \to 0} \frac{\left(\sqrt{1+x+x^2} - \sqrt{x+1}\right) \times \left(\sqrt{1+x+x^2} + \sqrt{x+1}\right)}{2x^2\left(\sqrt{1+x+x^2} + \sqrt{x+1}\right)}$$

$$= \lim_{x \to 0} \frac{\left(1+x+x^2\right) - (x+1)}{2x^2\left(\sqrt{1+x+x^2} + \sqrt{x+1}\right)}$$

$$= \lim_{x \to 0} \frac{x^2}{2x^2\left(\sqrt{1+x+x^2} + \sqrt{x+1}\right)}$$

$$= \frac{1}{2\left(\sqrt{1} + \sqrt{1}\right)}$$

$$= \frac{1}{2 \times 2}$$

$$= \frac{1}{4}$$

Q23

$$\lim_{x \to 4} \frac{2 - \sqrt{x}}{4 - x}$$

$$= \lim_{x \to 4} \frac{\left(2 - \sqrt{x}\right)\left(2 + \sqrt{x}\right)}{\left(4 - x\right)\left(2 + \sqrt{x}\right)}$$

$$= \lim_{x \to 4} \frac{(2)^2 - \left(\sqrt{x}\right)^2}{\left(4 - x\right)\left(2 + \sqrt{x}\right)}$$

$$= \lim_{x \to 4} \frac{\left(4 - x\right)}{\left(4 - x\right)\left(2 + \sqrt{x}\right)}$$

$$= \frac{1}{2 + \sqrt{4}}$$

$$= \frac{1}{2 + 2}$$

$$= \frac{1}{4}$$

Q24

$$\lim_{x \to a} \frac{x - a}{\sqrt{x} - \sqrt{a}}$$

$$= \lim_{x \to a} \frac{\left(x - a\right)\left(\sqrt{x} + \sqrt{a}\right)}{\left(\sqrt{x} - \sqrt{a}\right)\left(\sqrt{x} + \sqrt{a}\right)}$$

$$= \lim_{x \to a} \frac{\left(x - a\right)\left(\sqrt{x} + \sqrt{a}\right)}{\left(x - a\right)}$$

$$= \lim_{x \to a} \left(\sqrt{x} + \sqrt{a}\right)$$

$$= 2\sqrt{a}$$

Q25

$$\lim_{x \to 0} \frac{\sqrt{1 + 3x} - \sqrt{1 - 3x}}{x}$$

$$= \lim_{x \to 0} \frac{\left(\sqrt{1 + 3x} - \sqrt{1 - 3x}\right)}{x} \times \frac{\left(\sqrt{1 + 3x} + \sqrt{1 - 3x}\right)}{\left(\sqrt{1 + 3x} + \sqrt{1 - 3x}\right)}$$

$$= \lim_{x \to 0} \frac{\left(1 + 3x\right) - \left(1 - 3x\right)}{x\left(\sqrt{1 + 3x} + \sqrt{1 - 3x}\right)}$$

$$= \lim_{x \to 0} \frac{6x}{x\left(\sqrt{1 + 3x} + \sqrt{1 - 3x}\right)}$$

$$= \lim_{x \to 0} \frac{6}{\left(\sqrt{1 + 3x} + \sqrt{1 - 3x}\right)}$$

$$= \frac{6}{\sqrt{1} + \sqrt{1}}$$

$$= \frac{6}{2}$$

$$= 3$$

Q26

$$\lim_{x \to 0} \frac{\sqrt{2-x} - \sqrt{2+x}}{x}$$

$$= \lim_{x \to 0} \frac{\left(\sqrt{2-x} + \sqrt{2+x}\right) \times \left(\sqrt{2-x} + \sqrt{2+x}\right)}{x \times \left(\sqrt{2-x} + \sqrt{2+x}\right)}$$

$$= \lim_{x \to 0} \frac{(2-x) - (2+x)}{x \left(\sqrt{2-x} + \sqrt{2+x}\right)}$$

$$= \lim_{x \to 0} \frac{-2x}{x \left(\sqrt{2-x} + \sqrt{2+x}\right)}$$

$$= \frac{-2}{\sqrt{2} + \sqrt{2}}$$

$$= \frac{-2}{2\sqrt{2}}$$

$$= \frac{-1}{\sqrt{2}}$$

Q27

$$\lim_{x \to 1} \frac{\sqrt{3+x} - \sqrt{5-x}}{x^2 - 1}$$

$$= \lim_{x \to 1} \frac{\left(\sqrt{3+x} - \sqrt{5-x}\right)}{(x-1)(x+1)} \times \frac{\left(\sqrt{3+x} + \sqrt{5-x}\right)}{\left(\sqrt{3+x} + \sqrt{5-x}\right)}$$

$$= \lim_{x \to 1} \frac{\left((3+x) - (5-x)\right)}{(x-1)(x+1)\left(\sqrt{3+x} + \sqrt{5-x}\right)}$$

$$= \lim_{x \to 1} \frac{-2+2x}{(x-1)(x+1)\left(\sqrt{3+x} + \sqrt{5-x}\right)}$$

$$= \lim_{x \to 1} \frac{-2(x-1)}{(x-1)(x+1)\left(\sqrt{3+x} + \sqrt{5-x}\right)}$$

$$= \lim_{x \to 1} \frac{-2}{(x+1)\left(\sqrt{3+x} + \sqrt{5-x}\right)}$$

$$= \frac{2}{(1+1)\left(\sqrt{3+1} + \sqrt{5-1}\right)}$$

$$= \frac{2}{(2)(2+2)}$$

$$= \frac{1}{4}$$

Q28

$$\lim_{x \to 1} \frac{(2x-3)(\sqrt{x}-1)}{3x^2+3x-6}$$

$$= \lim_{x \to 1} \frac{(2x-3)(\sqrt{x}-1)}{3[x^2+x-2]}$$

$$= \lim_{x \to 1} \frac{(2x-3)(\sqrt{x}-1)}{3[x^2+2x-x-2]}$$

$$= \lim_{x \to 1} \frac{(2x-3)(\sqrt{x}-1)}{3(x+2)(x-1)}$$

$$= \lim_{x \to 1} \frac{(2x-3)(\sqrt{x}-1)}{3(x+2)(\sqrt{x}-1)(\sqrt{x}+1)}$$

$$= \lim_{x \to 1} \frac{(2x-3)}{3(x+2)(\sqrt{x}+1)}$$

$$= \frac{(2-3)}{3(1+2)(1+1)}$$

$$= \frac{-1}{3 \times 3 \times 2}$$

$$= -\frac{1}{18}$$

Q29

$$\lim_{x \to 0} \frac{\sqrt{1+x^2}-\sqrt{1+x}}{\sqrt{1+x^3}-\sqrt{1+x}}$$

$$= \lim_{x \to 0} \frac{\left(\sqrt{1+x^2}-\sqrt{1+x}\right)}{\left(\sqrt{1+x^3}-\sqrt{1+x}\right)} \times \frac{\left(\sqrt{1+x^2}+\sqrt{1+x}\right)}{\left(\sqrt{1+x^2}+\sqrt{1+x}\right)}$$

$$= \lim_{x \to 0} \frac{(1+x^2)-(1+x) \times \left(\sqrt{1+x^3}+\sqrt{1+x}\right)}{\left(\sqrt{1+x^3}-\sqrt{1+x}\right)\left(\sqrt{1+x^2}+\sqrt{1+x}\right)\left(\sqrt{1+x^3}+\sqrt{1+x}\right)}$$

$$= \lim_{x \to 0} \frac{(x^2-x)\left(\sqrt{1+x^3}+\sqrt{1+x}\right)}{\left(\sqrt{1+x^2}+\sqrt{1+x}\right)(1+x^3-1-x)}$$

$$= \lim_{x \to 0} \frac{x(x-1)\left(\sqrt{1+x^3}+\sqrt{1+x}\right)}{\left(\sqrt{1+x^2}+\sqrt{1+x}\right) \times (x^2-1)}$$

$$= \lim_{x \to 0} \frac{x(x-1)\left(\sqrt{1+x^3}+\sqrt{1+x}\right)}{\left(\sqrt{1+x^2}+\sqrt{1+x}\right)(x)(x-1)(x+1)}$$

$$= \lim_{x \to 0} \frac{\left(\sqrt{1+x^3}+\sqrt{1+x}\right)}{\left(\left(\sqrt{1+x^2}+\sqrt{1+x}\right)(x+1)\right)}$$

$$= \frac{2}{2} = 1$$

Q30

$$\lim_{x \to 1} \frac{x^2 - \sqrt{x}}{\sqrt{x} - 1}$$

$$= \lim_{x \to 1} \frac{\left(x^2 - \sqrt{x}\right)\left(x^2 + \sqrt{x}\right)}{\left(\sqrt{x} - 1\right)\left(x^2 + \sqrt{x}\right)}$$

$$= \lim_{x \to 1} \frac{x^4 - x}{\left(\sqrt{x} - 1\right)\left(x^2 + \sqrt{x}\right)}$$

$$= \lim_{x \to 1} \frac{x\left(x^3 - 1\right)}{\left(\sqrt{x} - 1\right)\left(x^2 + \sqrt{x}\right)}$$

$$= \lim_{x \to 1} \frac{x\left(x - 1\right)\left(x^2 + 1 + x\right)}{\left(\sqrt{x} - 1\right)\left(x^2 + \sqrt{x}\right)}$$

$$= \lim_{x \to 1} \frac{x\left(\sqrt{x} - 1\right)\left(\sqrt{x} + 1\right)\left(x^2 + 1 + x\right)}{\left(\sqrt{x} - 1\right)\left(x^2 + \sqrt{x}\right)}$$

$$= \lim_{x \to 1} \frac{x\left(\sqrt{x} + 1\right)\left(x^2 + 1 + x\right)}{\left(x^2 + \sqrt{x}\right)}$$

$$= \frac{1(1+1)(1+1+1)}{1+1}$$

$$= \frac{6}{2}$$

$$= 3$$

Q31

$$\lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

$$= \lim_{h \to 0} \frac{\left(\sqrt{x+h} - \sqrt{x}\right)}{h} \times \frac{\left(\sqrt{x+h} + \sqrt{x}\right)}{\left(\sqrt{x+h} + \sqrt{x}\right)}$$

$$= \lim_{h \to 0} \frac{(x+h-x)}{h\left(\sqrt{x+h} + \sqrt{x}\right)}$$

$$= \frac{1}{\sqrt{x} + \sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}}$$

Q32

$$\lim_{x\to\sqrt{10}} \frac{\sqrt{7+2x}-\left(\sqrt{5}+\sqrt{2}\right)}{x^2-10}$$

$$= \lim_{x\to\sqrt{10}} \frac{\sqrt{7+2x}-\left(\sqrt{\left(\sqrt{5}+\sqrt{2}\right)^2}\right)}{x^2-10}$$

$$= \lim_{x\to\sqrt{10}} \frac{\sqrt{7+2x}-\left(\sqrt{7+2\sqrt{10}}\right)}{x^2-10}$$

$$= \lim_{x\to\sqrt{10}} \frac{\sqrt{7+2x}-\left(\sqrt{7+2\sqrt{10}}\right)}{x^2-10} \times \frac{\sqrt{7+2x}+\left(\sqrt{7+2\sqrt{10}}\right)}{\sqrt{7+2x}+\left(\sqrt{7+2\sqrt{10}}\right)}$$

$$= \lim_{x\to\sqrt{10}} \frac{7+2x-7-2\sqrt{10}}{(x^2-10)\left(\sqrt{7+2x}+\left(\sqrt{7+2\sqrt{10}}\right)\right)}$$

$$= \lim_{x\to\sqrt{10}} \frac{2\left(x-\sqrt{10}\right)}{(x^2-10)\left(\sqrt{7+2x}+\left(\sqrt{7+2\sqrt{10}}\right)\right)}$$

$$= \lim_{x\to\sqrt{10}} \frac{2}{\left(x+\sqrt{10}\right)\left(\sqrt{7+2x}+\left(\sqrt{7+2\sqrt{10}}\right)\right)}$$

$$= \frac{2}{\left(\sqrt{10}+\sqrt{10}\right)\left(\sqrt{7+2\sqrt{10}}+\left(\sqrt{7+2\sqrt{10}}\right)\right)}$$

$$= \frac{2}{\left(2\sqrt{10}\right)\left(2\sqrt{7+2\sqrt{10}}\right)}$$

$$= \frac{1}{\left(2\sqrt{10}\right)\left(\sqrt{7+2\sqrt{10}}\right)}$$

$$= \frac{1}{\left(2\sqrt{10}\right)\left(\sqrt{5}+\sqrt{2}\right)}$$

$$= \frac{\left(\sqrt{5}-\sqrt{2}\right)}{\left(6\sqrt{10}\right)}$$

Q33

$$\lim_{x \to \sqrt{6}} \frac{\sqrt{5+2x} - \left(\sqrt{3} + \sqrt{2}\right)}{x^2 - 6}$$

$$= \lim_{x \to \sqrt{6}} \frac{\sqrt{5+2x} - \left(\sqrt{\left(\sqrt{3} + \sqrt{2}\right)^2}\right)}{x^2 - 6}$$

$$= \lim_{x \to \sqrt{6}} \frac{\sqrt{5+2x} - \left(\sqrt{5 + 2\sqrt{6}}\right)}{x^2 - 6}$$

$$= \lim_{x \to \sqrt{6}} \frac{\sqrt{5+2x} - \left(\sqrt{5 + 2\sqrt{6}}\right)}{x^2 - 6} \times \frac{\sqrt{5+2x} + \left(\sqrt{5 + 2\sqrt{6}}\right)}{\sqrt{5+2x} + \left(\sqrt{5 + 2\sqrt{6}}\right)}$$

$$= \lim_{x \to \sqrt{6}} \frac{5 + 2x - 5 - 2\sqrt{6}}{\left(x^2 - 6\right)\left(\sqrt{5+2x} + \left(\sqrt{5 + 2\sqrt{6}}\right)\right)}$$

$$= \lim_{x \to \sqrt{6}} \frac{2\left(x - \sqrt{6}\right)}{\left(x^2 - 6\right)\left(\sqrt{5+2x} + \left(\sqrt{5 + 2\sqrt{6}}\right)\right)}$$

$$= \lim_{x \to \sqrt{6}} \frac{2}{\left(x + \sqrt{6}\right)\left(\sqrt{5+2x} + \left(\sqrt{5 + 2\sqrt{6}}\right)\right)}$$

$$= \frac{2}{\left(\sqrt{6} + \sqrt{6}\right)\left(\sqrt{5 + 2\sqrt{6}} + \left(\sqrt{5 + 2\sqrt{6}}\right)\right)}$$

$$= \frac{2}{\left(2\sqrt{6}\right)\left(2\sqrt{5 + 2\sqrt{6}}\right)}$$

$$= \frac{1}{\left(2\sqrt{6}\right)\left(\sqrt{5 + 2\sqrt{6}}\right)}$$

$$= \frac{1}{\left(2\sqrt{6}\right)\left(\sqrt{3} + \sqrt{2}\right)}$$

$$= \frac{\left(\sqrt{3} - \sqrt{2}\right)}{\left(2\sqrt{6}\right)}$$

Q34

$$\lim_{x \to \sqrt{2}} \frac{\sqrt{3+2x} - (\sqrt{2}+1)}{x^2 - 2}$$

$$= \lim_{x \to \sqrt{2}} \frac{\sqrt{3+2x} - \left(\sqrt{(\sqrt{2}+1)^2}\right)}{x^2 - 2}$$

$$= \lim_{x \to \sqrt{2}} \frac{\sqrt{3+2x} - \left(\sqrt{3+2\sqrt{2}}\right)}{x^2 - 2}$$

$$= \lim_{x \to \sqrt{2}} \frac{\sqrt{3+2x} - \left(\sqrt{3+2\sqrt{2}}\right)}{x^2 - 2} \times \frac{\sqrt{3+2x} + \left(\sqrt{3+2\sqrt{2}}\right)}{\sqrt{3+2x} + \left(\sqrt{3+2\sqrt{2}}\right)}$$

$$= \lim_{x \to \sqrt{2}} \frac{3+2x - 3 - 2\sqrt{2}}{(x^2 - 2)\left(\sqrt{3+2x} + \left(\sqrt{3+2\sqrt{2}}\right)\right)}$$

$$= \lim_{x \to \sqrt{2}} \frac{2(x - \sqrt{2})}{(x^2 - 2)\left(\sqrt{3+2x} + \left(\sqrt{3+2\sqrt{2}}\right)\right)}$$

$$= \lim_{x \to \sqrt{2}} \frac{2}{(x + \sqrt{2})\left(\sqrt{3+2x} + \left(\sqrt{3+2\sqrt{2}}\right)\right)}$$

$$= \frac{2}{(\sqrt{2} + \sqrt{2})\left(\sqrt{3+2\sqrt{2}} + \left(\sqrt{3+2\sqrt{2}}\right)\right)}$$

$$= \frac{2}{(2\sqrt{2})\left(2\sqrt{3+2\sqrt{2}}\right)}$$

$$= \frac{1}{(2\sqrt{2})\left(\sqrt{3+2\sqrt{2}}\right)}$$

$$= \frac{1}{(2\sqrt{2})(\sqrt{2}+1)}$$

$$= \frac{(\sqrt{2}-1)}{(2\sqrt{2})}$$

Ex 29.5

Q1

$$\lim_{x \to a} \frac{(x+2)^{\frac{5}{2}} - (a+2)^{\frac{5}{2}}}{x-a}$$

$$= \lim_{x \to a} \frac{(x+2)^{\frac{5}{2}} - (a+2)^{\frac{5}{2}}}{(x+2) - (a+2)}$$

$$= \lim_{y \to b} \frac{y^{\frac{5}{2}} - b^{\frac{5}{2}}}{y-b}, \text{ where } x+2 = y \text{ and } a+2 = b$$

$$= \frac{5}{2} b^{\frac{5}{2}-1} \qquad \left[\text{Using formula } \lim_{x \to a} \frac{x^n - a^n}{x-a} = na^{n-1} \right]$$

$$= \frac{5}{2}(a+2)^{\frac{5}{2}-1}$$

$$= \frac{5}{2}(a+2)^{\frac{3}{2}}$$

Q2

$$\lim_{x \to a} \frac{(x+2)^{\frac{3}{2}} - (a+2)^{\frac{3}{2}}}{x-a}$$

$$= \lim_{x \to a} \frac{(x+2)^{\frac{3}{2}} - (a+2)^{\frac{3}{2}}}{(x+2) - (a+2)}$$

Let $x+2 = y$, $a+2 = b$

$$\Rightarrow \lim_{(x+2) \to (a+2)} \frac{(y)^{\frac{3}{2}} - (b)^{\frac{3}{2}}}{(y) - (b)} \qquad \left[\text{Using formula } \lim_{x \to a} \frac{x^n - a^n}{x-a} = na^{n-1} \right]$$

$$= \frac{3}{2}(b)^{\frac{3}{2}-1}$$

$$= \frac{3}{2}(a+2)^{\frac{3}{2}-1}$$

$$= \frac{3}{2}(a+2)^{\frac{1}{2}}$$

Q3

$$\lim_{x \to 0} \frac{(1+x)^6 - 1}{(1+x)^2 - 1}$$

$$= \lim_{x \to 0} \frac{\dfrac{(1+x)^6 - 1^6}{1+x-1}}{\dfrac{(1+x)^2 - 1^2}{1+x-1}}$$

$\Rightarrow$ Let $1 + x = y$, as $x \to 0$, $y \to 1$

$$= \frac{\lim\limits_{y \to 1} \dfrac{y^6 - 1^6}{y-1}}{\lim\limits_{y \to 1} \dfrac{y^2 - 1}{y-1}}$$

$$= \frac{6(1)^{6-1}}{2(1)^{2-1}} \qquad \left[\text{Using formula } \lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1} \right]$$

$$= \frac{6}{2}$$

$$= 3$$

Q4

$$\lim_{x \to a} \frac{x^{\frac{2}{7}} - a^{\frac{2}{7}}}{x - a}$$

Applying formula $\lim\limits_{x \to a} \dfrac{x^n - a^n}{x - a} = na^{n-1}$, here, $n = \dfrac{2}{7}$

$$\Rightarrow \lim_{x \to a} \frac{x^{\frac{2}{7}} - a^{\frac{2}{7}}}{x - a} = \frac{2}{7}(a)^{\frac{2}{7}-1}$$

$$= \frac{2}{7} a^{\frac{-5}{7}}$$

Q5

$$\lim_{x \to a} \frac{x^{\frac{5}{7}} - a^{\frac{5}{7}}}{x^{\frac{2}{7}} - a^{\frac{2}{7}}}$$

$$= \lim_{x \to a} \frac{\dfrac{x^{\frac{5}{7}} - a^{\frac{5}{7}}}{x - a}}{\dfrac{x^{\frac{2}{7}} - a^{\frac{2}{7}}}{x - a}} \qquad \text{[Dividing numerator and denominator by } x - a]$$

$$= \frac{\displaystyle\lim_{x \to a} \frac{x^{\frac{5}{7}} - a^{\frac{5}{7}}}{x - a}}{\displaystyle\lim_{x \to a} \frac{x^{\frac{2}{7}} - a^{\frac{2}{7}}}{x - a}}$$

Applying formula $\displaystyle\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$

Here, $n = \dfrac{5}{7}$ is numerator and applying $\displaystyle\lim_{x \to a} \frac{x^m - a^m}{x - a} = ma^{m-1}$ in denominator, where $m = \dfrac{2}{7}$

$$\Rightarrow \frac{\displaystyle\lim_{x \to a} \frac{x^{\frac{5}{7}} - a^{\frac{5}{7}}}{x - a}}{\displaystyle\lim_{x \to a} \frac{x^{\frac{2}{7}} - a^{\frac{2}{7}}}{x - a}} = \frac{\dfrac{5}{7}(a)^{\frac{5}{7}-1}}{\dfrac{2}{7}(a)^{\frac{2}{7}-1}}$$

$$= \frac{\dfrac{5}{7} a^{\frac{-2}{7}}}{\dfrac{2}{7} a^{\frac{-5}{7}}}$$

$$= \frac{5}{2} a^{\frac{-2}{7}+\frac{5}{7}}$$

$$= \frac{5}{2} a^{\frac{3}{7}}$$

Q6

$$\lim_{x \to -\frac{1}{2}} \frac{8x^3 + 1}{2x + 1}$$

$$= \frac{8}{2} \lim_{x \to -\frac{1}{2}} \frac{x^3 + \left(\frac{1}{2}\right)^3}{x + \frac{1}{2}}$$

$$= 4 \lim_{x \to -\frac{1}{2}} \frac{x^3 - \left(-\frac{1}{2}\right)^3}{x - \left(-\frac{1}{2}\right)}$$

Applying formula $\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$

Here, $n = 3$, $a = \frac{-1}{2}$

$$\Rightarrow \quad 4 \lim_{x \to -\frac{1}{2}} \frac{x^3 - \left(-\frac{1}{2}\right)^3}{x - \left(-\frac{1}{2}\right)} = 4 \times 3 \left(-\frac{1}{2}\right)^{3-1}$$

$$= 4 \times 3 \times \frac{1}{4}$$

$$= 3$$

Q7

$$\lim_{x \to 27} \frac{\left(x^{\frac{1}{3}} + 3\right)\left(x^{\frac{1}{3}} - 3\right)}{x - 27}$$

$$= \lim_{x \to 27} \frac{\left(x^{\frac{2}{3}} - 9\right)}{x - 27}$$

$$= \lim_{x \to 27} \frac{x^{\frac{2}{3}} - 27^{\frac{2}{3}}}{x - 27}$$

Applying formula $\lim_{x \to 27} \frac{x^n - a^n}{x - a} = na^{n-1}$

$$= \frac{2}{3}(27)^{\frac{2}{3} - 1}$$

$$= \frac{2}{3}(27)^{\frac{-1}{3}}$$

$$= \frac{2}{3} \times \frac{1}{(27)^{\frac{1}{3}}}$$

$$= \frac{2}{3} \times \frac{1}{3}$$

$$= \frac{2}{9}$$

Q8

$$\lim_{x \to 4} \frac{x^3 - 64}{x^2 - 16}$$

$$= \lim_{x \to 4} \frac{x^3 - 4^3}{x^2 - 4^2}$$

$$= \lim_{x \to 4} \frac{\dfrac{x^3 - 4^3}{x - 4}}{\dfrac{x^2 - 4^2}{x - 4}} \qquad \text{[Dividing numerator and denominator by } x - 4\text{]}$$

$$= \frac{\lim_{x \to 4} \dfrac{x^3 - 4^3}{x - 4}}{\lim_{x \to 4} \dfrac{x^2 - 4}{x - 4}}$$

Applying $\lim_{x \to a} \dfrac{x^n - a^n}{x - a} = na^{n-1}$ in numerator and $\lim_{x \to a} \dfrac{x^m - a^m}{x - a} = ma^{m-1}$ in denominator

$$\Rightarrow \quad n = 3, \ m = 2$$

$$\Rightarrow \quad \frac{\lim_{x \to 4} \dfrac{x^3 - 4^3}{x - 4}}{\lim_{x \to 4} \dfrac{x^2 - 4}{x - 4}} = \frac{3(4)^{3-1}}{2(4)^{2-1}} = \frac{3(4)^2}{2(4)}$$

$$= 6$$

Q9

$$\lim_{x \to 1} \frac{x^{15} - 1}{x^{10} - 1}$$

$$= \lim_{x \to 1} \frac{\dfrac{x^{15} - 1^{15}}{x - 1}}{\dfrac{x^{10} - 1^{10}}{x - 1}}$$

[Dividing numerator and denomintoar by $(x - 1)$]

$$= \frac{\displaystyle\lim_{x \to 1} \frac{x^{15} - 1^{15}}{x - 1}}{\displaystyle\lim_{x \to 1} \frac{x^{10} - 1^{10}}{x - 1}}$$

Applying formula $\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$ in numerator and $\lim_{x \to a} \frac{x^m - a^m}{x - a} = ma^{m-1}$ in denominator

Here, $n = 15, m = 10$

$$\Rightarrow \quad \frac{\displaystyle\lim_{x \to 1} \frac{x^{15} - 1^{15}}{x - 1}}{\displaystyle\lim_{x \to 1} \frac{x^{10} - 1^{10}}{x - 1}} = \frac{15(1)^{15-1}}{10(1)^{10-1}}$$

$$= \frac{15}{10}$$

$$= \frac{3}{2}$$

Q10

$$\lim_{x \to -1} \frac{x^3 + 1}{x + 1}$$

$$= \lim_{x \to -1} \frac{x^3 - (-1)^3}{x - (-1)}$$

Applying formula $\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$

Here, $n = 3, a = -1$

$$\Rightarrow \quad \lim_{x \to -1} \frac{x^3 - (-1)^3}{x - (-1)} = na^{n-1}$$

$$= 3(-1)^{3-1}$$

$$= 3(-1)^2$$

$$= 3$$

Q11

$$\lim_{x \to a} \frac{x^{\frac{2}{3}} - a^{\frac{2}{3}}}{x^{\frac{3}{4}} - a^{\frac{3}{4}}}$$

$$= \lim_{x \to a} \frac{\dfrac{x^{\frac{2}{3}} - a^{\frac{2}{3}}}{x - a}}{\dfrac{x^{\frac{3}{4}} - a^{\frac{3}{4}}}{x - a}}$$

[Dividing numerator and denominator by $x - a$]

$$= \frac{\lim_{x \to a} \dfrac{x^{\frac{2}{3}} - a^{\frac{2}{3}}}{x - a}}{\lim_{x \to a} \dfrac{x^{\frac{3}{4}} - a^{\frac{3}{4}}}{x - a}}$$

Applying the formula $\lim_{x \to a} \dfrac{x^n - a^n}{x - a} = na^{n-1}$ in numerator and $\lim_{x \to a} \dfrac{x^m - a^m}{x - a} = ma^{m-1}$ in denominator respectively

Here, $n = \dfrac{2}{3}$, $m = \dfrac{3}{4}$

$$\Rightarrow \quad \frac{\lim_{x \to a} \dfrac{x^{\frac{2}{3}} - a^{\frac{2}{3}}}{x - a}}{\lim_{x \to a} \dfrac{x^{\frac{3}{4}} - a^{\frac{3}{4}}}{x - a}} = \frac{\dfrac{2}{3}(a)^{\frac{2}{3}-1}}{\dfrac{3}{4}(a)^{\frac{3}{4}-1}}$$

$$= \frac{8}{9} a^{\frac{-1}{3}+\frac{1}{4}}$$

$$= \frac{8}{9} a^{\frac{-1}{12}}$$

Q12

$$\lim_{x \to 3} \frac{x^n - 3^n}{x - 3} = 108$$

LHS $= \lim_{x \to 3} \frac{x^n - 3^n}{x - 3}$

Applying the formula $\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$

Here, $n = n$, $a = 3$

$\Rightarrow \quad \lim_{x \to 3} \frac{x^n - 3^n}{x - 3} = n(3)^{n-1}$

It is given that $n(3)^{n-1} = 108$

$\Rightarrow \quad n(3)^{n-1} = 2 \times 2 \times 3 \times 3 \times 3$

$\qquad\qquad = (2)^2 \times (3)^3$

$\qquad\qquad = 4(3)^{4-1}$

$\Rightarrow \qquad n = 4$

Q13

If $\lim_{x \to a} \frac{x^9 - a^9}{x - a} = 9$ \qquad\qquad ---(i)

LHS $= \lim_{x \to a} \frac{x^9 - a^9}{x - a}$

$\qquad = 9(a)^{9-1}$

$\qquad = 9a^8$

It is given that $9a^8 = 9$ \qquad\qquad [From (i)]

$\Rightarrow \qquad a^8 = \frac{9}{9} = 1$

$\Rightarrow \qquad a^4 = 1$

$\qquad\qquad a^2 = 1$

$\qquad\qquad a = \pm 1$

$\Rightarrow \qquad a = 1$ and $a = -1$

Q14

If $\lim\limits_{x \to a} \dfrac{x^5 - a^5}{x - a} = 405$ ---(i)

$\text{LHS} = \lim\limits_{x \to a} \dfrac{x^5 - a^5}{x - a}$

$\quad\quad = 5(a)^{5-1}$

$\quad\quad = 5a^4$

It is given that $5a^4 = 405$

$\Rightarrow \quad 5a^4 = 405$

$\quad\quad a^4 = \dfrac{405}{5} = 81$

$\quad\quad a^4 = (3)^4, \; a^2 = 9$

$\quad\quad a = \pm 3$

$\Rightarrow \quad a = 3 \text{ and } a = -3$

Q15

If $\lim\limits_{x \to a} \dfrac{x^9 - a^9}{x - a} = \lim\limits_{x \to 5}(4 + x)$ ---(i)

$\text{LHS} = \lim\limits_{x \to a} \dfrac{x^9 - a^9}{x - a}$

$\quad\quad = 9(a)^{9-1} = 9a^8$ ---(ii)

$\text{RHS} = \lim\limits_{x \to 5}(4 + x)$

$\quad\quad = 4 + 5 = 9$ ---(iii)

Substituting (ii) and (iii) in (i),

$\quad\quad 9a^8 = 9$

$\quad\quad a^8 = 1$

$\Rightarrow \quad a^2 = 1$

$\Rightarrow \quad a = 1, \; a = -1$

Q16

If $\lim\limits_{x \to a} \dfrac{x^3 - a^3}{x - a} = \lim\limits_{x \to 1} \dfrac{x^4 - 1}{x - 1}$ --- (i)

$\text{LHS} = \text{If } \lim\limits_{x \to a} \dfrac{x^3 - a^3}{x - a}$

$= 3(a)^{3-1}$

$= 3a^2$ --- (ii)

$\text{RHS} = \lim\limits_{x \to a} \dfrac{x^4 - 1}{x - 1}$

$= \lim\limits_{x \to a} \dfrac{x^4 - 1}{x - 1}$

$= 4(1)^{4-1}$

$= 4$ --- (iii)

Substituting (ii) and (iii) in (i),

$3a^2 = 4$

$a^2 = \dfrac{4}{3}$

$a = \pm \dfrac{2}{\sqrt{3}}$

Ex 29.6

Q1

$$\lim_{x \to \infty} \frac{(3x-1)(4x-2)}{(x+8)(x-1)} \qquad \left[\text{Expression is } \frac{\infty}{\infty}\right]$$

$$= \lim_{x \to \infty} \frac{(12x^2 - 10x + 2)}{(x^2 + 9x - 8)}$$

$$= \lim_{x \to \infty} \left(\frac{12 - \dfrac{10}{x} + \dfrac{2}{x^2}}{1 + \dfrac{9}{x} - \dfrac{8}{x^2}} \right)$$

$$= \frac{12 - 0 + 0}{1 + 0 - 1}$$

$$= 12$$

Q2

$$\lim_{x \to \infty} \frac{3x^3 - 4x^2 + 6x - 1}{2x^3 + x^2 - 5x + 7}$$

$$= \lim_{x \to \infty} \frac{3 - \dfrac{4}{x} + \dfrac{6}{x^2} - \dfrac{1}{x^3}}{2 + \dfrac{1}{x} - \dfrac{5}{x^2} + \dfrac{7}{x^3}}$$

$$= \frac{3 - 0 + 0 - 0}{2 + 0 - 0 + 0}$$

$$= \frac{3}{2}$$

Q3

$$\lim_{x \to \infty} \frac{5x^3 - 6}{\sqrt{9 + 4x^6}}$$

$$= \lim_{x \to \infty} \frac{5 - \dfrac{6}{x^3}}{\sqrt{\dfrac{9}{x^6} + \dfrac{4x^6}{x^6}}}$$

$$= \lim_{x \to \infty} \frac{\left(5 - \dfrac{6}{x^3}\right)}{\sqrt{\dfrac{9}{x^6} + 4}}$$

$$= \frac{5}{\sqrt{4}} = \frac{5}{2}$$

Q4

$$\lim_{x \to \infty} \sqrt{x^2 + cx} - x$$

$$= \lim_{x \to \infty} \left[\left(\sqrt{x^2 + cx} - x \right) \frac{\left(\sqrt{x^2 + cx} + x \right)}{\sqrt{x^2 + cx} + x} \right]$$

$$= \lim_{x \to \infty} \frac{\left(x^2 + cx - x^2 \right)}{\sqrt{x^2 + cx} + x}$$

$$= \lim_{x \to \infty} \frac{cx}{\sqrt{x^2 + cx} + x} \qquad \left[\frac{\infty}{\infty} \text{ form} \right]$$

$$= \lim_{x \to \infty} \frac{c}{\sqrt{1 + \dfrac{c}{x}} + 1}$$

$$= \frac{c}{1 + 1} = \frac{c}{2}$$

Q5

$$\lim_{x \to \infty} \sqrt{x + 1} - \sqrt{x}$$

$$= \lim_{x \to \infty} \frac{\left(\sqrt{x + 1} - \sqrt{x} \right) \left(\sqrt{x + 1} + \sqrt{x} \right)}{\left(\sqrt{x + 1} + \sqrt{x} \right)}$$

$$= \lim_{x \to \infty} \frac{(x + 1 - x)}{\sqrt{x + 1} + \sqrt{x}}$$

$$= \lim_{x \to \infty} \left(\frac{1}{\sqrt{x + 1} + \sqrt{x}} \right)$$

$$= \frac{1}{\infty}$$

$$= 0$$

Q6

$$\lim_{x \to \infty} \sqrt{x^2 + 7x} - x$$

$$= \lim_{x \to \infty} \left(\frac{\left(\sqrt{x^2 + 7x} - x\right)\left(\sqrt{x^2 + 7x} + x\right)}{\sqrt{x^2 + 7x} + x} \right)$$

$$= \lim_{x \to \infty} \left(\frac{\left(x^2 + 7x\right) - x^2}{\sqrt{x^2 + 7x} + x} \right)$$

$$= \lim_{x \to \infty} \frac{7x}{\sqrt{x^2 + 7x} + x}$$

$$= \lim_{x \to \infty} \frac{7}{\sqrt{\frac{x^2}{x^2} + \frac{7x}{x^2}} + 1}$$

$$= \lim_{x \to \infty} \frac{7}{\sqrt{1 + \frac{7}{x}} + 1}$$

$$= \frac{7}{2}$$

Q7

$$\lim_{x \to \infty} \frac{x}{\sqrt{4x^2 + 1} - 1}$$

$$= \lim_{x \to \infty} \frac{1}{\sqrt{4 + \frac{1}{x^2}} - \frac{1}{x}}$$

$$= \frac{1}{\sqrt{4} - 0}$$

$$= \frac{1}{2}$$

Q8

$$\lim_{n\to\infty} \frac{n^2}{1+2+3+\ldots+n}$$

$$= \lim_{n\to\infty} \frac{n^2}{\frac{1}{2}n(n+1)}$$

$$\left[\because 1+2+3+\ldots+n = \frac{n(n+1)}{2}\right]$$

$$= \lim_{n\to\infty} \frac{2n^2}{n^2+n}$$

$$= 2\lim_{n\to\infty} \frac{n^2}{n^2+n}$$

$$= 2\lim_{n\to\infty} \frac{n^2}{n^2\left(1+\frac{1}{n}\right)}$$

$$= 2\times\frac{1}{1+0}$$

$$= 2$$

Q9

$$\lim_{n\to\infty} \frac{\frac{3}{x}+\frac{4}{x^2}}{\frac{5}{x}+\frac{6}{x^2}}$$

$$\left[\frac{0}{0} \text{ form}\right]$$

$$= \lim_{x\to\infty} \frac{\frac{1}{x}\left(3+\frac{4}{x}\right)}{\frac{1}{x}\left(5+\frac{6}{x}\right)}$$

$$= \lim_{x\to\infty} \frac{(3+0)}{(5+0)} = \frac{3}{5}$$

Q10

$$\lim_{x \to \infty} \frac{\sqrt{x^2+a^2}-\sqrt{x^2+b^2}}{\sqrt{x^2+c^2}-\sqrt{x^2+d^2}} \qquad \left[\frac{\infty}{\infty} \text{ form}\right]$$

$$\lim_{x \to \infty} \frac{\left(\sqrt{x^2+a^2}-\sqrt{x^2+b^2}\right)}{\left(\sqrt{x^2+c^2}-\sqrt{x^2+d^2}\right)} \times \frac{\left(\sqrt{x^2+a^2}+\sqrt{x^2+b^2}\right)}{\left(\sqrt{x^2+a^2}+\sqrt{x^2+b^2}\right)}$$

$$= \lim_{x \to \infty} \frac{\left(\left(x^2+a^2\right)-\left(x^2-b^2\right)\right)}{\left(\sqrt{x^2+c^2}-\sqrt{x^2+d^2}\right)\left(\sqrt{x^2+a^2}+\sqrt{x^2+b^2}\right)}$$

$$= \lim_{x \to \infty} \frac{\left(a^2-b^2\right)}{\left(\sqrt{x^2+c^2}-\sqrt{x^2+d^2}\right)\left(\sqrt{x^2+a^2}+\sqrt{x^2+b^2}\right)}$$

$$= \lim_{x \to \infty} \frac{\left(a^2-b^2\right)\left(\sqrt{x^2+c^2}+\sqrt{x^2+d^2}\right)}{\left(\sqrt{x^2+c^2}-\sqrt{x^2+d^2}\right)\left(\sqrt{x^2+c^2}+\sqrt{x^2+d^2}\right)\left(\sqrt{x^2+a^2}+\sqrt{x^2+b^2}\right)}$$

$$= \lim_{x \to \infty} \frac{\left(a^2-b^2\right)\left(\sqrt{x^2+c^2}+\sqrt{x^2+d^2}\right)}{\left(x^2+c^2-x^2-d^2\right)\left(\sqrt{x^2+a^2}+\sqrt{x^2+b^2}\right)}$$

$$= \lim_{x \to \infty} \frac{\left(a^2-b^2\right)\left(\sqrt{x^2+c^2}+\sqrt{x^2+d^2}\right)}{\left(c^2-d^2\right)\left(\sqrt{x^2+a^2}+\sqrt{x^2+b^2}\right)}$$

$$= \lim_{x \to \infty} \frac{\left(a^2-b^2\right)\left(x\sqrt{1+\frac{c^2}{x^2}}+x\sqrt{1+\frac{d^2}{x^2}}\right)}{\left(c^2-d^2\right)\left(x\sqrt{1+\frac{a^2}{x^2}}+x\sqrt{1+\frac{b^2}{x^2}}\right)}$$

$$= \lim_{x \to \infty} \frac{\left(a^2-b^2\right)\left(\sqrt{1+\frac{c^2}{x^2}}+\sqrt{1+\frac{d^2}{x^2}}\right)}{\left(c^2-d^2\right)\left(\sqrt{1+\frac{a^2}{x^2}}+\sqrt{1+\frac{b^2}{x^2}}\right)}$$

$$= \frac{\left(a^2-b^2\right)\left(\sqrt{1+0}+\sqrt{1+0}\right)}{\left(c^2-d^2\right)\left(\sqrt{1+0}+\sqrt{1+0}\right)}$$

$$= \frac{\left(a^2-b^2\right)(1+1)}{\left(c^2-d^2\right)(1+1)}$$

$$= \frac{\left(a^2-b^2\right)(2)}{\left(c^2-d^2\right)(2)} = \frac{a^2-b^2}{c^2-d^2}$$

Q11

$$\lim_{n \to \infty} \frac{(n+2)!+(n+1)!}{(n+2)!-(n+1)!}$$

We know that $(n+2)! = (n+2)(n+1)!$

$$\Rightarrow \quad \lim_{n \to \infty} \frac{(n+2)(n+1)!+(n+1)!}{(n+2)(n+1)!-(n+1)!}$$

$$= \lim_{n \to \infty} \frac{(n+1)![(n+2)+1]}{(n+1)![(n+2)-1]}$$

$$= \lim_{n \to \infty} \frac{n+3}{n+1} \qquad \left[\frac{\infty}{\infty} \text{ form}\right]$$

$$= \lim_{n \to \infty} \frac{1+\frac{3}{n}}{1+\frac{1}{n}}$$

$$= \frac{1+0}{1+0} = 1$$

$$= 1$$

Q12

$$\lim_{x \to \infty} x\left[\sqrt{x^2+1}-\sqrt{x^2-1}\right]$$

$$= \lim_{x \to \infty} x\left[\sqrt{x^2+1}-\sqrt{x^2-1}\right] \times \frac{\left(\sqrt{x^2+1}+\sqrt{x^2-1}\right)}{\left(\sqrt{x^2+1}+\sqrt{x^2+1}\right)}$$

$$= \lim_{x \to \infty} x \frac{x\left(x^2+1-x^2+1\right)}{\sqrt{x^2+1}+\sqrt{x^2-1}}$$

$$= \lim_{x \to \infty} \frac{x(2)}{x\left(\sqrt{1+\frac{1}{x^2}}+\sqrt{1-\frac{1}{x^2}}\right)}$$

$$= \lim_{x \to \infty} \frac{2}{\sqrt{1+\frac{1}{x^2}}+\sqrt{1-\frac{1}{x^2}}}$$

$$= \frac{2}{2} = 1$$

Q13

$$\lim_{x \to \infty} \left[\sqrt{x+1} - \sqrt{x} \right] \sqrt{x+2}$$

$$= \lim_{x \to \infty} \left[\sqrt{x+1} - \sqrt{x} \right] \frac{\left[\sqrt{x+1} + \sqrt{x} \right]}{\left[\sqrt{x+1} + \sqrt{x} \right]} \times \frac{\sqrt{x+2} \times \sqrt{x+2}}{\sqrt{x+2}}$$

$$= \lim_{x \to \infty} \frac{(x+1-x)}{\sqrt{x+1} + \sqrt{x}} \times \frac{(x+2)}{\sqrt{x+2}}$$

$$= \lim_{x \to \infty} \frac{1(x+2)}{\left(\sqrt{x+1} + \sqrt{x} \right)\left(\sqrt{x+2} \right)}$$

$$= \lim_{x \to \infty} \frac{x\left(1 + \dfrac{2}{x} \right)}{\sqrt{x}\left(\sqrt{1 + \dfrac{1}{x}} + \sqrt{1} \right)\left(\sqrt{1 + \dfrac{2}{x}} \right)\sqrt{x}}$$

$$= \lim_{x \to \infty} \frac{\left(1 + \dfrac{2}{x} \right)}{\left(\sqrt{1 + \dfrac{1}{x}} + \sqrt{1} \right)\left(\sqrt{1 + \dfrac{2}{x}} \right)}$$

$$= \lim_{x \to \infty} \frac{(1+0)}{(1+1) \times 1} = \frac{1}{2}$$

Q14

$$\lim_{n \to \infty} \frac{1^2 + 2^2 + 3^2 + \ldots + n^2}{n^3}$$

$$= \lim_{n \to \infty} \frac{\frac{1}{6} n(n+1)(2n+1)}{n^3}$$

$$\left| 1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{1}{6} n(n+1)(2n+1) \right|$$

$$= \frac{1}{6} \lim_{n \to \infty} \frac{(n^2 + n)(2n+1)}{n^3}$$

$$= \frac{1}{6} \lim_{n \to \infty} \frac{(2n^3 + n^2 + 2n^2 + n)}{n^3}$$

$$= \frac{1}{6} \lim_{n \to \infty} \frac{(2n^3 + 3n^2 + n)}{n^3}$$

$$\left[\frac{\infty}{\infty} \text{ form} \right]$$

$$= \frac{1}{6} \lim_{n \to \infty} \frac{\left(2 + \dfrac{3}{n} + \dfrac{1}{n^2} \right)}{1}$$

$$= \frac{1}{6} \frac{(2+0+0)}{1} = \frac{1}{3}$$

Q15

$$\lim_{n \to \infty} \left(\frac{1}{n^2} + \frac{2}{n^2} + \frac{3}{n^2} + \ldots + \frac{n-1}{n^2} \right)$$

$$\lim_{n \to \infty} \left(\frac{1 + 2 + 3 + \ldots + (n-1)}{n^2} \right)$$

$$= \lim_{n \to \infty} \frac{(n-1)(n)}{2 \times n^2}$$

$$\left[1 + 2 + 3 + \ldots + (n-1) - \frac{(n-1)(n)}{2} \right]$$

$$= \lim_{n \to \infty} \frac{n^2 - n}{2n^2}$$

$$\left[\frac{\infty}{\infty} \text{ form} \right]$$

$$= \lim_{n \to \infty} \frac{1 - \frac{1}{n}}{2}$$

$$= \frac{1 - 0}{2} = \frac{1}{2}$$

$$= \frac{1}{2}$$

Q16

$$\lim_{n \to \infty} \frac{1^3 + 2^3 + 3^3 - \ldots + n^3}{n^4}$$

$$= \lim_{n \to \infty} \frac{\left[\frac{1}{2} n (n+1) \right]^2}{n^4}$$

$$\left[1^3 + 2^3 + 3^3 + \ldots + n^3 = \left(\frac{1}{2} n (n+1) \right)^2 \right]$$

$$= \lim_{n \to \infty} \frac{\frac{1}{4} n^2 (n+1)^2}{n^4}$$

$$= \lim_{n \to \infty} \frac{1}{4} \frac{\left(n^2 \left(n^2 + 1 + 2n \right) \right)}{n^4}$$

$$\left[\text{Multiplying the term} \right] \left[\frac{\infty}{\infty} \text{ form} \right]$$

$$= \frac{1}{4} \lim_{n \to \infty} \frac{\left(1 + \frac{1}{n^2} + \frac{2}{n} \right)}{1}$$

$$= \frac{1}{4}$$

Q17

$$\lim_{n \to \infty} \frac{1^3 + 2^3 + \dots + n^3}{(n-1)^4}$$

$$= \lim_{n \to \infty} \frac{\left[\frac{1}{2}(n)(n+1)\right]^2}{(n-1)^4}$$

$$\left[1^3 + 2^3 + 3^3 + \dots + n^3 = \left[\frac{1}{2}n(n+1)\right]^2\right]$$

$$= \lim_{n \to \infty} \left(\frac{\frac{1}{4}n^2(n^2 + 1 + 2n)}{(n-1)^4}\right)$$

$$= \frac{1}{4} \lim_{n \to \infty} \left(\frac{n^4 + n^2 + 2n^3}{(n-1)^2(n-1)^2}\right)$$

$$= \frac{1}{4} \lim_{n \to \infty} \left(\frac{n^4 + n^2 + 2n^3}{n^4 + n^2 - 2n^3 + n^2 + 1 - 2n - 2n^3 - 2n + 4n^2}\right) \qquad \left[\frac{\infty}{\infty} \text{ form}\right]$$

$$= \frac{1}{4} \lim_{n \to \infty} \frac{\left(1 + \frac{1}{n^2} + \frac{2}{n}\right)}{\left(1 + \frac{1}{n^2} - \frac{2}{n} + \frac{1}{n^2} + \frac{1}{n^4} - \frac{2}{n^3} - \frac{2}{n} - \frac{2}{n^3} + \frac{4}{n^2}\right)}$$

$$= \frac{1}{4}\left(\frac{1}{1}\right)$$

$$= \frac{1}{4}$$

Q18

$$\lim_{n \to \infty} \sqrt{x} \left(\sqrt{x+1} - \sqrt{x} \right)$$

$$= \lim_{n \to \infty} \left(\sqrt{x^2 + x} - x \right)$$

$$= \lim_{n \to \infty} \left(\left(\sqrt{x^2 + x} - x \right) \times \frac{\left(\sqrt{x^2 + x} + x \right)}{\sqrt{x^2 + x} + x} \right)$$

$$= \lim_{n \to \infty} \left(\frac{\left(x^2 + x \right) - x^2}{\sqrt{x^2 + x} + x} \right)$$

$$= \lim_{n \to \infty} \left(\frac{x}{\sqrt{x^2 + x} + x} \right) \qquad \left[\frac{\infty}{\infty} \ \text{form} \right]$$

$$= \lim_{n \to \infty} \left(\frac{1}{\sqrt{\frac{x^2}{x^2} + \frac{x}{x^2}} + 1} \right)$$

$$= \lim_{n \to \infty} \left(\frac{1}{\sqrt{1 + \frac{1}{x}} + 1} \right)$$

$$= \frac{1}{1 + 1}$$

$$= \frac{1}{2}$$

Q19

$$\lim_{n \to \infty} \left(\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \ldots + \frac{1}{3^n} \right) \qquad \text{---(i)}$$

This is G.P of common ratio $\frac{1}{3}$

$\therefore$ Sum of n terms of G.P with $a = \frac{1}{3}$, $r = \frac{-1}{3}$

$$S_n = a \left(\frac{1 - r^n}{1 - r} \right)$$

$$S_n = \frac{1}{3} \left(\frac{1 - \left(\frac{1}{3} \right)^n}{1 - \frac{1}{3}} \right)$$

$$= \frac{1}{3} \left(\frac{1 - \frac{1}{3^n}}{\frac{2}{3}} \right)$$

$$= \frac{1}{3} \times \frac{3}{2} \left(1 - \frac{1}{3^n} \right)$$

$$S_n = \frac{1}{2} \left(1 - \frac{1}{3^n} \right)$$

Substituting value of S_n in (i), we get

$$\lim_{n \to \infty} S_n = \lim_{n \to \infty} \frac{1}{2} \left(1 - \frac{1}{3^n} \right)$$

$$= \frac{1}{2} \lim_{n \to \infty} \left(1 - \frac{1}{3^n} \right)$$

$$= \frac{1}{2} (1 - 0)$$

$$= \frac{1}{2}$$

Q20

$$\lim_{x \to \infty} \frac{x^4 + 7x^3 + 46x + a}{x^4 + 6} \qquad \left[\frac{\infty}{\infty} \text{ form}\right]$$

$$= \lim_{x \to \infty} \frac{1 + \dfrac{7}{x} + \dfrac{46}{x^3} + \dfrac{a}{x^4}}{1 + \dfrac{6}{x^4}}$$

$$= 1$$

Q21

$f(x) = \dfrac{ax^2 + b}{x^2 + 1}$

Also $\lim\limits_{x \to 0} f(x) = 1$ ---(i)   [given]

$\Rightarrow \quad \lim\limits_{x \to 0} \dfrac{ax^2 + b}{x^2 + 1} = 1$

$\Rightarrow \quad \dfrac{\lim\limits_{x \to 0} ax^2 + b}{\lim\limits_{x \to 0} x^2 + 1} = 1$

$\Rightarrow \quad b = 1$ $\left[\text{from (i)} \right]$

Also, it is given that $\lim\limits_{x \to \infty} f(x) = 1$

$\therefore \quad \lim\limits_{x \to \infty} \dfrac{ax^2 + b}{x^2 + 1} = 1$ ---(ii)

$\Rightarrow \quad \lim\limits_{x \to \infty} \dfrac{ax^2 + 1}{x^2 + 1} = 1$

$\Rightarrow \quad \lim\limits_{x \to \infty} \dfrac{a + \dfrac{1}{x^2}}{1 + \dfrac{1}{x^2}} = 1$ $\left[\dfrac{\infty}{\infty} \text{ form} \right]$

$\Rightarrow \quad a = 1$

Thus, $f(x) = \dfrac{ax^2 + b}{x^2 + 1} = \dfrac{x^2 + 1}{x^2 + 1} = 1$ $\left[\text{from (ii)} \right]$

$\qquad f(-2) = 1$
$\qquad f(2) = 1$
$\qquad f(-2) = 1 = f(2)$

Hence, proved.

Q22

RHS $= \lim_{x \to \infty} \left(\sqrt{x^2 + 1} - x \right)$

$= \lim_{x \to \infty} \frac{\left(\sqrt{x^2 + 1} - x \right)\left(\sqrt{x^2 + 1} + x \right)}{\left(\sqrt{x^2 + 1} + x \right)}$

$= \lim_{x \to \infty} \frac{x^2 + 1 - x^2}{x\sqrt{1 + \frac{1}{x^2}} + x}$

$= \lim_{x \to \infty} \frac{1}{x\sqrt{1 + \frac{1}{x^2}} + x}$

$= \lim_{x \to \infty} \frac{1}{x} \left(\frac{1}{\sqrt{1 + \frac{1}{x^2}} + 1} \right)$

$= 0$

Also,

LHS $= \lim_{x \to \infty} \left(\sqrt{x^2 + x + 1} - x \right)$

$= \lim_{x \to \infty} \frac{\left(\sqrt{x^2 + x + 1} - x \right) \left(\sqrt{x^2 + x + 1} + x \right)}{\left(\sqrt{x^2 + x + 1} + x \right)}$

$= \lim_{x \to \infty} \frac{\left(x^2 + x + 1 - x^2 \right)}{\sqrt{x^2 + x + 1} + x}$

$= \lim_{x \to \infty} \frac{x \left(1 + \frac{1}{x} \right)}{x \left(\sqrt{1 + \frac{1}{x} + \frac{1}{x^2}} + 1 \right)}$

$= \lim_{x \to \infty} \frac{1 + \frac{1}{x}}{\sqrt{1 + \frac{1}{x} + \frac{1}{x^2}} + 1}$

$= \frac{1}{1 + 1} = \frac{1}{2}$

LHS $\neq$ RHS

$\therefore \quad \lim_{x \to \infty} \left(\sqrt{x^2 + x + 1} - x \right)$ is not equal to $\lim_{x \to \infty} \left(\sqrt{x^2 + 1} - x \right)$.

Q23

$$\lim_{x \to -\infty} \left(\sqrt{4x^2 - 7x} + 2x \right)$$

Substitute $y = -x$

$$= \lim_{y \to \infty} \left(\sqrt{4y^2 + 7y} - 2y \right)$$

$$= \lim_{y \to \infty} \frac{\left(\sqrt{4y^2 + 7y} - 2y \right)\left(\sqrt{4y^2 + 7y} + 2y \right)}{\sqrt{4y^2 + 7y} + 2y}$$

$$= \lim_{y \to \infty} \frac{\left(4y^2 + 7y - 4y^2 \right)}{\sqrt{4y^2 + 7y} + 2y}$$

$$= \lim_{y \to \infty} \frac{(7y)}{\sqrt{4y^2 + 7y} + 2y}$$

$$= \lim_{y \to \infty} \frac{7}{\sqrt{4 + \dfrac{7}{y}} + 2}$$

$$= \frac{7}{2 + 2} = \frac{7}{4}$$

Q24

$$\lim_{x \to -\infty} \left(\sqrt{x^2 - 8x} + x \right)$$

$$= \lim_{y \to \infty} \left(\sqrt{y^2 + 8y} - y \right), \text{ where } y = -x \text{ on rationalising}$$

$$= \lim_{y \to \infty} \frac{\left\{ \sqrt{y^2 + 8y} - y \right\}\left\{ \sqrt{y^2 + 8y} + y \right\}}{\left\{ \sqrt{y^2 + 8y} + y \right\}}$$

$$= \lim_{y \to \infty} \frac{y^2 + 8y - y^2}{\sqrt{y^2 + 8y} + y}$$

$$= \lim_{y \to \infty} \frac{8y}{y\sqrt{1 + \dfrac{8}{y}} + y} \qquad \left[\frac{\infty}{\infty} \text{ form} \right]$$

$$= \lim_{y \to \infty} \frac{8}{\sqrt{1 + \dfrac{8}{y}} + 1}$$

$$= \frac{8}{1 + 1} = \frac{8}{2}$$

$$= 4$$

Q25

$$\lim_{n \to \infty} \frac{1^4 + 2^4 + 3^4 + \dots + n^4}{n^5} - \lim_{n \to \infty} \frac{1^3 + 2^3 + \dots + n^3}{n^5}$$

$$= \lim_{n \to \infty} \frac{\dfrac{n(1+n)(1+2n)(-1 + 3n + 3n^2)}{30}}{n^5} - \lim_{n \to \infty} \frac{\left(\dfrac{n(n+1)}{2} \right)^2}{n^5}$$

$$= \lim_{n \to \infty} \frac{\left(\dfrac{1}{n} + 1 \right)\left(\dfrac{1}{n} + 2 \right)\left(-\dfrac{1}{n^2} + \dfrac{3}{n} + 3 \right)}{30} - \lim_{n \to \infty} \frac{1}{n^5}\left(\dfrac{n^2(n^2 + 2n + 1)}{4} \right)$$

$$= \lim_{n \to \infty} \frac{\left(\dfrac{1}{n} + 1 \right)\left(\dfrac{1}{n} + 2 \right)\left(-\dfrac{1}{n^2} + \dfrac{3}{n} + 3 \right)}{30} - \lim_{n \to \infty} \frac{\left(\dfrac{1}{n} + \dfrac{2}{n^2} + \dfrac{1}{n^3} \right)}{4}$$

$$= \frac{1 \times 2 \times 3}{30} - 0$$

$$= \frac{1}{5}$$

Q26

$$\lim_{n\to\infty}\frac{1.2+2.3+3.4+\ldots+n(n+1)}{n^3} = \lim_{n\to\infty}\frac{\frac{n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}}{n^3}$$

$$= \lim_{n\to\infty}\frac{n(n+1)\left[\frac{(2n+1)+3}{6}\right]}{n^3}$$

$$= \lim_{n\to\infty}\frac{\frac{n(n+1)(2n+4)}{6}}{n^3}$$

$$= \lim_{n\to\infty}\frac{\left(1+\frac{1}{n}\right)\left(2+\frac{4}{n}\right)}{6}$$

$$= \frac{1\times 2}{6}$$

$$= \frac{1}{3}$$

Ex 29.7

Q1

$$\lim_{x \to 0} \frac{\sin 3x}{5x}$$

$$= \frac{1}{5} \lim_{x \to 0} \frac{\sin 3x}{x}$$

$$= \frac{1}{5} \lim_{x \to 0} \frac{\sin 3x}{3x} \times 3$$

$$= \frac{3}{5} \lim_{3x \to 0} \frac{\sin 3x}{3x}$$

$$= \frac{3}{5} \times 1 \qquad\qquad \left[\because \lim_{x \to 0} \frac{\sin x}{x} = 1 \right]$$

$$= \frac{3}{5}$$

Q2

$$\lim_{x \to 0} \frac{\sin x°}{x}$$

$$= \lim_{x \to 0} \frac{\sin \frac{x \times \pi}{180}}{x} \qquad\qquad \left[\because 1° = \frac{\pi}{180} \text{ radians} \right]$$

$$= \lim_{x \to 0} \frac{\sin \frac{\pi x}{180}}{x \times \frac{\pi}{180}} \times \frac{\pi}{180}$$

$$= \frac{\pi}{180} \lim_{x \to 0} \frac{\sin \frac{\pi x}{180}}{\frac{\pi x}{180}}$$

$$= \frac{\pi}{180} \lim_{x \to 0} \frac{\sin \frac{\pi x}{180}}{\frac{\pi x}{180}}$$

$$= \frac{\pi}{180} \times 1 = \frac{\pi}{180} \qquad\qquad \left[\because \lim_{x \to 0} \frac{\sin x}{x} = 1 \right]$$

$$= \frac{\pi}{180}$$

Q3

$$\lim_{x \to 0} \frac{x^2}{\sin x^2}$$

$$= \lim_{x \to 0} \frac{1}{\frac{\sin x^2}{x^2}}$$

$$= \frac{1}{\lim_{x \to 0} \frac{\sin x^2}{x^2}}$$

$$= \frac{1}{1} \qquad\qquad \left[\because \lim_{x \to 0} \frac{\sin x}{x} = 1 \right]$$

$$= 1$$

Q4

$$\lim_{x \to 0} \frac{\sin x \cos x}{3x}$$

$$= \frac{1}{3} \lim_{x \to 0} \frac{\sin x \cos x}{x}$$

$$= \frac{1}{3} \lim_{x \to 0} \frac{\sin x}{x} \times \lim_{x \to 0} \cos x$$

$$= \frac{1}{3} \times 1 \times 1 \qquad\qquad \left[\because \lim_{x \to 0} \frac{\sin x}{x} = 1, \ \lim_{x \to 0} \cos x = \cos 0° = 1 \right]$$

$$= \frac{1}{3}$$

Q5

$$\lim_{x \to 0} \frac{3 \sin x - 4 \sin^3 x}{x}$$

$$= \lim_{x \to 0} \frac{\sin 3x}{x} \qquad\qquad \left[\because \sin 3x = 3 \sin x - 4 \sin^3 x \right]$$

$$= \lim_{x \to 0} \frac{\sin 3x}{3x} \times 3$$

$$= 3 \times \lim_{x \to 0} \frac{\sin 3x}{3x}$$

$$= 3 \times \lim_{3x \to 0} \frac{\sin 3x}{3x} \qquad\qquad \left[\because x \to 0, \ 3x \to 0 \right]$$

$$= 3 \times 1 \qquad\qquad \left[\because \lim_{x \to 0} \frac{\sin x}{x} = 1 \right]$$

$$= 3$$

Q6

$$\lim_{x \to 0} \frac{\tan 8x}{\sin 2x}$$

$$= \frac{\lim\limits_{x \to 0} \tan 8x}{\lim\limits_{x \to 0} \sin 2x}$$

$$= \frac{\lim\limits_{x \to 0} \dfrac{\tan 8x}{8x} \times 8x}{\lim\limits_{x \to 0} \dfrac{\sin 2x}{2x} \times 2x}$$

$$= \frac{\lim\limits_{8x \to 0} \dfrac{\tan 8x}{8x} \times \dfrac{8x}{2x}}{\lim\limits_{2x \to 0} \dfrac{\sin 2x}{2x}} \qquad\qquad \begin{bmatrix} \because x \to 0 \\ 8x \to 0 \\ 2x \to 0 \end{bmatrix}$$

$$= \frac{1 \times \dfrac{8}{2}}{1} \qquad\qquad \left[\because \lim_{x \to 0} \frac{\sin x}{x} = 1 \text{ and } \lim_{x \to 0} \frac{\tan x}{x} = 1 \right]$$

$$= \frac{8}{2}$$

$$= 4$$

Q7

$$\lim_{x \to 0} \frac{\tan mx}{\tan nx}$$

$$= \frac{\lim\limits_{x \to 0} \tan mx}{\lim\limits_{x \to 0} \tan nx}$$

$$= \frac{\lim\limits_{mx \to 0} \dfrac{\tan mx}{mx} \times mx}{\lim\limits_{nx \to 0} \dfrac{\tan nx}{nx} \times nx} \qquad\qquad \left[\because \text{ If } x \to 0 \text{ then } mx \to 0 \text{ also } nx \to 0 \right]$$

$$= \frac{1 \times m}{1 \times n} \qquad\qquad \left[\because \lim_{x \to 0} \frac{\tan x}{x} = 1 \right]$$

$$= \frac{m}{n}$$

Q8

$$\lim_{x \to 0} \frac{\sin 5x}{\tan 3x}$$

$$= \frac{\lim\limits_{x \to 0} \sin 5x}{\lim\limits_{3x \to 0} \tan 3x}$$

$$= \frac{\lim\limits_{5x \to 0} \dfrac{\sin 5x}{5x} \times 5}{\lim\limits_{3x \to 0} \dfrac{\tan 3x}{3x} \times 3} \qquad \left[\because \text{If } x \to 0 \text{ then } 3x \to 0, 5x \to 0 \right]$$

$$= \frac{5}{3} \times 1 \qquad \left[\because \lim_{x \to 0} \frac{\sin x}{x} = 1 \text{ also } \lim_{x \to 0} \frac{\tan x}{x} = 1 \right]$$

$$= \frac{5}{3}$$

Q9

$$\lim_{x \to 0} \frac{\sin x^\circ}{x^\circ}$$

$$= \lim_{x \to 0} \frac{\sin \dfrac{x \times \pi}{180}}{x \times \dfrac{\pi}{180}} \qquad \left[\because 1^\circ = \frac{\pi}{180} \text{ radians} \right]$$

$$= \lim_{\frac{\pi x}{180} \to 0} \frac{\sin \dfrac{\pi x}{180}}{\dfrac{\pi x}{180}} \qquad \left[\because \text{If } x \to 0 \text{ then } \frac{\pi x}{180} \to 0 \right]$$

$$= 1 \qquad \left[\because \lim_{x \to 0} \frac{\sin x}{x} = 1 \right]$$

Q10

$$\lim_{x \to 0} \frac{7x \cos x - 3 \sin x}{4x + \tan x}$$

$$= \lim_{x \to 0} \frac{7 \cos x - \dfrac{3 \sin x}{x}}{4 + \dfrac{\tan x}{x}}$$

$$= \frac{\lim\limits_{x \to 0} 7 \cos x - \lim\limits_{x \to 0} \dfrac{3 \sin x}{x}}{\lim\limits_{x \to 0} 4 + \lim\limits_{x \to 0} \dfrac{\tan x}{x}}$$

$$= \frac{7 \times \lim\limits_{x \to 0} \cos x - 3 \lim\limits_{x \to 0} \dfrac{\sin x}{x}}{4 + \lim\limits_{x \to 0} \dfrac{\tan x}{x}}$$

$$= \frac{7 \times 1 - 3 \times 1}{4 + 1}$$

$$\left[\because \lim_{x \to 0} \frac{\sin x}{x} = 1 \atop \lim_{x \to 0} \frac{\tan x}{x} = 1 \right]$$

$$= \frac{4}{5}$$

Q11

$$\lim_{x \to 0} \frac{\cos ax - \cos bx}{\cos cx - \cos dx} = \lim_{x \to 0} \frac{\left(-2 \sin\left(\frac{a+b}{2}\right) x \sin\left(\frac{a-b}{2}\right) x\right)}{-2 \sin\left(\frac{c+d}{2}\right) x \sin\left(\frac{c-d}{2}\right) x}$$

$$= \frac{\lim\limits_{x \to 0} \sin\left(\frac{a+b}{2}\right) x \lim\limits_{x \to 0} \sin\left(\frac{a+b}{2}\right) x}{\lim\limits_{x \to 0} \sin\left(\frac{c+d}{2}\right) x \lim\limits_{x \to 0} \sin\left(\frac{c-d}{2}\right) x}$$

$$= \frac{\left(\lim\limits_{x \to 0} \frac{\sin\left(\frac{a+b}{2}\right)x}{\left(\frac{a+b}{2}\right)x} \times \left(\frac{a+b}{2}\right)x\right)\left(\lim\limits_{x \to 0} \frac{\sin\left(\frac{a-b}{2}\right)x}{\left(\frac{a-b}{2}\right)x} \times \left(\frac{a-b}{2}\right)x\right)}{\left(\lim\limits_{x \to 0} \frac{\sin\left(\frac{c+d}{2}\right)x}{\left(\frac{c+d}{2}\right)x} \times \left(\frac{c+d}{2}\right)x\right)\left(\lim\limits_{x \to 0} \frac{\sin\left(\frac{c-d}{2}\right)x}{\left(\frac{c-d}{2}\right)x} \times \left(\frac{c-d}{2}\right)x\right)}$$

$$= \frac{(a+b)(a-b)}{(c+d)(c-d)} \qquad\qquad \left[\because \lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1\right]$$

$$= \frac{a^2 - b^2}{c^2 - d^2}.$$

Q12

$$\lim_{x \to 0} \frac{\tan^2 3x}{x^2}$$

$$= \left(\lim_{x \to 0} \frac{\tan 3x}{x}\right)^2$$

$$= \left(\lim_{x \to 0} \frac{\tan 3x}{3x}\right)^2 \times 9$$

$$= 1 \times 9 \qquad\qquad \left[\because \lim_{x \to 0} \frac{\tan x}{x} = 1\right]$$

$$= 9$$

Q13

$$\lim_{x \to 0} \frac{1 - \cos mx}{x^2}$$

$$= \lim_{x \to 0} \frac{2 \sin^2 \frac{mx}{2}}{x^2}$$

$$= 2 \times \left(\lim_{x \to 0} \frac{\sin \frac{mx}{2}}{x} \right)^2$$

$$= 2 \times \left(\lim_{x \to 0} \frac{\sin \frac{mx}{2}}{\frac{mx}{2}} \right)^2 \times \left(\frac{m}{2} \right)^2$$

$$= 2 \times \frac{m^2}{4} = \frac{m^2}{2} \qquad \left[\because \lim_{x \to 0} \frac{\sin x}{x} = 1 \right]$$

$$= \frac{m^2}{2}$$

Q14

$$\lim_{x \to 0} \frac{3 \sin 2x + 2x}{3x + 2 \tan 3x}$$

$$= \lim_{x \to 0} \frac{3 \frac{\sin 2x}{x} + 2}{3 + 2 \tan \frac{3x}{x}}$$

$$= \frac{\lim_{x \to 0} \frac{3 \sin 2x}{x} + \lim_{x \to 0} 2}{\lim_{x \to 0} 3 + \lim_{x \to 0} \frac{2 \tan 3x}{x}}$$

$$= \frac{\left(3 \lim_{2x \to 0} \frac{\sin 2x}{2x} \times 2 \right) + 2}{3 + \left(2 \times \lim_{3x \to 0} \frac{\tan 3x}{3x} \times 3 \right)}$$

$$= \frac{(3 \times 2) + 2}{3 + (2 \times 3)} \qquad \left[\because \lim_{x \to a} \frac{\sin x}{x} = 1 \text{ also } \lim_{x \to 0} \frac{\tan x}{x} = 1 \right]$$

$$= \frac{8}{9}$$

Q15

$$\lim_{x \to 0} \frac{\cos 3x - \cos 7x}{x^2}$$

$$= \lim_{x \to 0} \left(\frac{-2 \sin\left(\frac{3x + 7x}{2}\right) \sin\left(\frac{3x - 7x}{2}\right)}{x^2} \right)$$

$$= \lim_{x \to 0} \left(\frac{-2 \sin 5x \sin\left(\frac{-4x}{2}\right)}{x^2} \right)$$

$$= \left(\lim_{x \to 0} \frac{-2 \sin 5x}{x} \right) \times \left(\lim_{x \to 0} \frac{\sin(-2x)}{x} \right)$$

$$= \left(-2 \lim_{x \to 0} \frac{\sin 5x}{5x} \times 5 \right) \times \left(-1 \lim_{x \to 0} \frac{\sin 2x}{2x} \times 2 \right)$$

$$= (-2 \times 5)(-1 \times 2)$$

$$= 20$$

Q16

$$\frac{\lim\limits_{\theta \to 0} \sin 3\theta}{\lim\limits_{\theta \to 0} \tan 2\theta}$$

$$= \frac{\lim\limits_{3\theta \to 0} \frac{\sin 3\theta}{3\theta} \times 3\theta}{\lim\limits_{2\theta \to 0} \frac{\tan 2\theta}{2\theta} \times 2\theta}$$

$$= \frac{\left(\lim\limits_{3\theta \to 0} \frac{\sin 3\theta}{3\theta} \right)}{\left(\lim\limits_{2\theta \to 0} \frac{\sin 2\theta}{2\theta} \right)} \times \frac{3\theta}{2\theta}$$

$$= \frac{1}{1} \times \frac{3}{2} \qquad \left[\because \lim_{x \to 0} \frac{\sin x}{x} = 1 \text{ and } \lim_{x \to 0} \frac{\tan x}{x} = 1 \right]$$

$$= \frac{3}{2}$$

Q17

$$\lim_{x \to 0} \frac{\sin x^2 \left(1 - \cos x^2\right)}{x^6}$$

$$= \lim_{x \to 0} \frac{\sin x^2 \times 2 \sin^2 \frac{x^2}{2}}{x^6}$$

$$= \lim_{x \to 0} \frac{\sin x^2}{x^2} \times \lim_{x \to 0} \frac{2 \sin^2 \frac{x^2}{2}}{x^4}$$

$$= \left(\lim_{x \to 0} \frac{\sin x}{x}\right)^2 \times 2 \times \left(\lim_{x \to 0} \frac{\sin \frac{x^2}{2}}{\frac{x^2}{2}}\right)^2 \times \frac{1}{4}$$

$$= (1)^2 \times 2 \times 1 \times \frac{1}{4} \qquad\qquad \left[\because \lim_{x \to 0} \frac{\sin x}{x} = 1\right]$$

$$= \frac{1}{2}$$

Q18

$$\lim_{x \to 0} \frac{\sin^2 4x^2}{x^4}$$

$$= \lim_{x \to 0} \frac{\left(\sin 4x^2\right)^2}{x^4}$$

$$= \lim_{x \to 0} \frac{\left(\sin 4x^2\right)^2}{\left(x^2\right)^2}$$

$$= \left(\lim_{x \to 0} \frac{\sin 4x^2}{x^2}\right)^2$$

$$= \left(\lim_{4x^2 \to 0} \frac{\sin 4x^2}{4x^2}\right) \times 16$$

$$= 1 \times 16 = 16 \qquad\qquad \left[\because \lim_{x \to 0} \frac{\sin x}{x} = 1\right]$$

$$= 16$$

Q19

$$\lim_{x \to 0} \frac{x \cos x + 2 \sin x}{x^2 + \tan x}$$

Dividing each term by x

$$\lim_{x \to 0} \frac{\cos x + \dfrac{2 \sin x}{x}}{x + \dfrac{\tan x}{x}}$$

$$= \frac{\lim\limits_{x \to 0} \cos x + \lim\limits_{x \to 0} \dfrac{2 \sin x}{x}}{\lim\limits_{x \to 0} x + \lim\limits_{x \to 0} \dfrac{\tan x}{x}}$$

$$= \frac{\cos 0 + 2 \lim\limits_{x \to 0} \dfrac{\sin x}{x}}{0 + \lim\limits_{x \to 0} \dfrac{\tan x}{x}}$$

$$= \frac{1 + 2}{0 + 1} = 3 \qquad \left[\because \lim_{x \to 0} \frac{\sin x}{x} = 1, \ \lim_{x \to 0} \frac{\tan x}{x} = 1 \right]$$

$$= 3$$

Q20

$$\lim_{x \to 0} \frac{2x - \sin x}{\tan x + x}$$

Dividing each term by x

$$= \lim_{x \to 0} \frac{2 - \dfrac{\sin x}{x}}{\dfrac{\tan x}{x} + 1}$$

$$= \frac{\lim\limits_{x \to 0} 2 - \lim\limits_{x \to 0} \dfrac{\sin x}{x}}{\lim\limits_{x \to 0} \dfrac{\tan x}{x} + \lim\limits_{x \to 0} 1}$$

$$= \frac{2 - 1}{1 + 1} \qquad \left[\because \lim_{x \to 0} \frac{\sin x}{x} = 1, \lim_{x \to 0} \frac{\tan x}{x} = 1 \right]$$

$$= \frac{1}{2}$$

Q21

$$\lim_{x \to 0} \frac{5x \cos x + 3 \sin x}{3x^2 + \tan x}$$

$$\lim_{x \to 0} \frac{5 \cos x + 3 \dfrac{\sin x}{x}}{3x + \dfrac{\tan x}{x}}$$

$$= \frac{\lim\limits_{x \to 0} 5 \cos x + \lim\limits_{x \to 0} \dfrac{3 \sin x}{x}}{\lim\limits_{x \to 0} 3x + \lim\limits_{x \to 0} \dfrac{\tan x}{x}}$$

$$= \frac{5 \lim\limits_{x \to 0} \cos x + 3 \lim\limits_{x \to 0} \dfrac{\sin x}{x}}{3 \lim\limits_{x \to 0} x + \lim\limits_{x \to 0} \dfrac{\tan x}{x}}$$

$$= \frac{5 \times \cos 0 + 3 \times 1}{3 \times 0 + 1}$$

$$= \frac{5 + 3}{1}$$

$$= 8$$

$$\left[\because \lim_{x \to 0} \frac{\sin x}{x} = 1 \text{ and } \lim_{x \to 0} \frac{\tan x}{x} = 1 \right]$$

Q22

$$\lim_{x \to 0} \frac{\sin 3x - \sin x}{\sin x}$$

$$= \lim_{x \to 0} \left(\frac{2 \cos \left(\dfrac{3x + x}{2} \right) \sin \left(\dfrac{3x - x}{2} \right)}{\sin x} \right)$$

$$= \lim_{x \to 0} \left(\frac{2 \cos 2x \sin x}{\sin x} \right)$$

$$= \lim_{x \to 0} \left(2 \cos x \right)$$

$$= 2 \lim_{x \to 0} \cos 2x$$

$$= 2 \times \cos 0$$

$$= 2 \times 1 = 2$$

$$= 2$$

Q23

$$\lim_{x \to 0} \frac{\sin 5x - \sin 3x}{\sin x}$$

$$= \lim_{x \to 0} \frac{2 \cos\left(\frac{5x + 3x}{2}\right) \sin\left(\frac{5x - 3x}{2}\right)}{\sin x}$$

$$= \lim_{x \to 0} \frac{2 \cos 4x \sin x}{\sin x}$$

$$= 2 \lim_{x \to 0} \cos 4x$$

$$= 2 \times \cos 0$$

$$= 2$$

Q24

$$\lim_{x \to 0} \frac{\cos 3x - \cos 5x}{x^2}$$

$$= \lim_{x \to 0} \frac{\left(-2 \sin\left(\frac{3x + 5x}{2}\right) \sin\left(\frac{3x - 5x}{2}\right)\right)}{x^2}$$

$$= \lim_{x \to 0} \left(\frac{-2 \sin 4x \sin(-x)}{x^2}\right)$$

$$= \lim_{x \to 0} \frac{2 \sin 4x \sin x}{x^2}$$

$$= \lim_{x \to 0} \frac{2 \sin 4x}{x} \times \lim_{x \to 0} \frac{\sin x}{x}$$

$$= \left(2 \lim_{x \to 0} \frac{\sin 4x}{4x} \times 4\right) \times \left(\lim_{x \to 0} \frac{\sin x}{x}\right) \qquad \left[\because \lim_{x \to 0} \frac{\sin x}{x} = 1\right]$$

$$= 8$$

Q25

$$\lim_{x \to 0} \frac{\dfrac{\tan 3x}{x} - \dfrac{\tan 2x}{x}}{\dfrac{3x}{x} - \dfrac{\sin^2 x}{x}}$$

$$= \frac{\left(\lim\limits_{x \to 0} \dfrac{\tan 3x}{3x} \times 3\right) - \left(\lim\limits_{x \to 0} \dfrac{\tan 2x}{2x} \times 2\right)}{\left(\lim\limits_{x \to 0} \dfrac{3x}{2}\right) - \lim\limits_{x \to 0} \dfrac{(\sin x)^2}{x}}$$

$$= \frac{3 - 2}{3 - \left(\dfrac{\sin x}{x}\right)^2 \times x} \qquad \left[\because \lim_{x \to 0} \frac{\tan x}{x} = 1\right]$$

$$= \frac{3 - 2}{3 - 0} = \frac{1}{3} \qquad \left[\because \lim_{x \to 0} \frac{\sin x}{x} = 1\right]$$

$$= \frac{1}{3}$$

Q26

$$\lim_{x \to 0} \frac{\sin(2 + x) - \sin(2 - x)}{x}$$

$$= \lim_{x \to 0} 2\cos\frac{\left(\dfrac{2 + x + 2 - x}{2}\right) \times \sin\left(\dfrac{2 + x - 2 + x}{2}\right)}{x}$$

$$= 2 \lim_{x \to 0} \frac{\cos(2) \times \sin x}{x}$$

$$= 2 \lim_{x \to 0} \cos 2 \times \lim_{x \to 0} \frac{\sin x}{x} \qquad \left[\because \lim_{x \to 0} \frac{\sin x}{x} = 1\right]$$

$$= 2\cos 2 \times 1$$

$$= 2\cos 2$$

Q27

$$\lim_{h \to 0} \frac{(a+h)^2 \sin(a+h) - a^2 \sin a}{h}$$

$$= \lim_{h \to 0} \frac{\left(a^2 + h^2 + 2ah\right) \sin(a+h) - a^2 \sin a}{h}$$

$$= \lim_{h \to 0} \frac{a^2 \sin(a+h) + h^2 \sin(a+h) + 2ah \sin(a+h) - a^2 \sin a}{h}$$

$$= \lim_{h \to 0} \left[\frac{a^2 \left(\sin(a+h) - \sin a\right)}{h} + \frac{h^2 \sin(a+h)}{h(a+h)} \times (a+h) + \frac{2ah}{h} \left(\sin(a+h)\right) \right]$$

$$= \left[a^2 \lim_{h \to 0} \frac{2 \cos\left(\frac{a+h+a}{2}\right) \sin\left(\frac{a+h-a}{2}\right)}{h} \right] + [0] + 2a \lim_{h \to 0} \sin(a+h)$$

$$= \left(2a^2 \lim_{h \to 0} \cos\left(a + \frac{h}{2}\right) \frac{\sin\frac{h}{2}}{\frac{h}{2}} \times \frac{1}{2} \right) + (2a \times \sin a)$$

$$= \left(2a^2 \cos a \times \frac{1}{2} \right) + (2a \sin a)$$

$$= a^2 \cos a + 2a \sin a$$

Q28

$$\lim_{x \to 0} \frac{\tan x - \sin x}{\sin 3x - 3\sin x}$$

$$\lim_{x \to 0} \frac{\dfrac{\sin x}{\cos x} - \sin x}{3\sin x - 4\sin^3 x - 3\sin x}$$

$$= \lim_{x \to 0} \frac{\sin x \left(\dfrac{1}{\cos x} - 1\right)}{-4\sin^3 x}$$

$$= \frac{-1}{4} \lim_{x \to 0} \frac{\dfrac{1}{\cos x} - 1}{\sin^2 x}$$

$$= \frac{-1}{4} \lim_{x \to 0} \frac{\dfrac{1 - \cos x}{\cos x}}{1 - \cos^2 x}$$

$$= \frac{-1}{4} \lim_{x \to 0} \frac{1 - \cos x}{(\cos x)(1 - \cos x)(1 + \cos x)}$$

$$= \frac{-1}{4} \lim_{x \to 0} \frac{1}{(\cos x)(1 + \cos x)}$$

$$= \frac{-1}{4} \times \frac{1}{1(1+1)}$$

$$= \frac{-1}{4} \times \frac{1}{2}$$

$$= -\frac{1}{8}$$

Q29

$$\lim_{x \to 0} \frac{\sec 5x - \sec 3x}{\sec 3x - \sec x} = \lim_{x \to 0} \left(\frac{\dfrac{\cos 3x - \cos 5x}{\cos 3x \cos 5x}}{\dfrac{\cos x - \cos 3x}{\cos x \cos 3x}} \right)$$

$$= \lim_{x \to 0} \left(\frac{\cos 3x - \cos 5x}{\cos x - \cos 3x} \times \frac{\cos x \cos 3x}{\cos 3x \cos 5x} \right)$$

$$= \lim_{x \to 0} \left(\frac{-2 \sin 4x \sin(-x)}{-2 \sin(2x) \sin(-x)} \times \frac{\cos x}{\cos 5x} \right)$$

$$= \lim_{x \to 0} \left(\frac{\sin 4x}{\sin 2x} \times \frac{\cos x}{\cos 5x} \right)$$

$$= \frac{\lim_{x \to 0} \sin 4x \times \lim_{x \to 0} \cos x}{\lim_{x \to 0} \sin 2x \times \lim_{x \to 0} \cos 5x}$$

$$= \frac{\left(\lim_{4x \to 0} \dfrac{\sin 4x}{4x} \times 4x \right) \times \left(\lim_{x \to 0} \cos x \right)}{\left(\lim_{2x \to 0} \dfrac{\sin 2x}{2x} \times 2x \right) \times \left(\lim_{x \to 0} \cos 5x \right)}$$

$$= \frac{(1 \times 4x) \times 1}{1 \times 2x \times 1} \qquad\qquad \left[\because \lim_{x \to 0} \frac{\sin x}{x} = 1 \text{ and } \lim_{x \to 0} \cos x = \cos 0 = 1 \right]$$

$$= \frac{4x}{2x}$$

$$= 2$$

Q30

$$\lim_{x \to 0} \frac{1 - \cos 2x}{\cos 2x - \cos 8x}$$

$$= \lim_{x \to 0} \frac{2 \sin^2 x}{-2 \sin\left(\frac{2x + 8x}{2}\right) \sin\left(\frac{2x - 8x}{2}\right)}$$

$$= \lim_{x \to 0} \frac{\sin^2 x}{\sin 5x \times \sin(-3x)}$$

$$= \frac{\lim_{x \to 0} \sin^2 x}{-\left(\lim_{x \to 0} \sin 5x\right)\left(-\lim_{x \to 0} \sin 3x\right)}$$

$$= \frac{\left(\lim_{x \to 0} \frac{\sin x}{x}\right)^2 \times x^2}{\left(\lim_{x \to 0} \frac{\sin 5x}{5x}\right) \times 5x \left(\lim_{x \to 0} \frac{\sin 3x}{3x}\right) \times 3x}$$

$$= \frac{1 \times x^2}{1 \times 5x \times 1 \times 3x} \qquad\qquad \left[\because \lim_{x \to 0} \frac{\sin x}{x} = 1\right]$$

$$= \frac{x^2}{15x^2}$$

$$= \frac{1}{15}$$

Q31

$$\lim_{x \to 0} \frac{1 - \cos 2x + \tan^2 x}{x \sin x}$$

$$= \lim_{x \to 0} \frac{2 \sin^2 x + \tan^2 x}{x \sin x}$$

$$= \frac{2 \lim_{x \to 0} \sin^2 x + \lim_{x \to 0} \tan^2 x}{\lim_{x \to 0} x \sin x}$$

$$= \frac{\left(2\left(\lim_{x \to 0} \frac{\sin x}{x}\right)^2 \times x^2\right) + \left(\lim_{x \to 0} \frac{\tan x}{x}\right)^2 \times x^2}{\left(\lim_{x \to 0} \frac{\sin x}{x}\right) \times x^2}$$

$$= \frac{(2 \times 1 \times x^2) + (1 \times x^2)}{(1 \times x^2)} \qquad\qquad \left[\because \lim_{x \to 0} \frac{\sin x}{x} = 1 \text{ and } \lim_{x \to 0} \frac{\tan x}{x} = 1\right]$$

$$= \frac{3x^2}{x^2}$$

$$= 3$$

Q32

$$\lim_{x \to 0} \frac{\sin(a+x) + \sin(a-x) - 2\sin a}{x \sin x}$$

$$= \lim_{x \to 0} \frac{2\sin\left(\frac{a+x+a-x}{2}\right)\cos\left(\frac{a+x-a+x}{2}\right) - 2\sin a}{x \sin x}$$

$$= \lim_{x \to 0} \frac{2\sin a(\cos x - 1)}{x \sin x}$$

$$= \lim_{x \to 0} \frac{-2\sin a(1 - \cos x)}{x \sin x}$$

$$= -2\sin a \lim_{x \to 0} \frac{2\sin^2 \frac{x}{2}}{\left(2\sin\frac{x}{2}\cos\frac{x}{2}\right)x}$$

$$= -2\sin a \lim_{x \to 0} \frac{2\sin\frac{x}{2}}{\left(\cos\frac{x}{2}\right)x}$$

$$= -2\sin a \lim_{x \to 0} \frac{\tan\frac{x}{2}}{\frac{x}{2}} \times \frac{1}{2}$$

$$= -2\sin a \times 1 \times \frac{1}{2} \qquad\qquad \left[\because \lim_{x \to 0} \frac{\tan x}{x} = 1\right]$$

$$= -\sin a$$

$$= -\sin a$$

Q33

$$\lim_{x \to 0} \frac{x^2 - \tan 2x}{\tan x}$$

$$= \lim_{x \to 0} \frac{\left(\frac{x^2}{2x} - \frac{\tan 2x}{2x}\right) \times 2x}{\frac{\tan x}{x} \times x}$$

$$= \lim_{x \to 0} \frac{2\left(\frac{x}{2} - \frac{\tan 2x}{2x}\right)}{\frac{\tan x}{x}}$$

$$= 2\left(\frac{0-1}{1}\right)$$

$$= -2$$

Q34

$$\lim_{x \to 0} \frac{\sqrt{2} + \sqrt{1 + \cos x}}{\sin^2 x} = \lim_{x \to 0} \frac{\sqrt{2} + \sqrt{1 + \cos x}}{\sin^2 x} \times \frac{\sqrt{2} - \sqrt{1 + \cos x}}{\sqrt{2} - \sqrt{1 + \cos x}}$$

$$= \lim_{x \to 0} \frac{2 - 1 - \cos x}{\sin^2 x}$$

$$= \lim_{x \to 0} \frac{1 - \cos x}{\sin^2 x}$$

$$= \lim_{x \to 0} \frac{1 - \cos x}{1 - \cos^2 x}$$

$$= \lim_{x \to 0} \frac{1}{1 + \cos x}$$

$$= \frac{1}{1 + 1}$$

$$= \frac{1}{2}$$

Q35

$$\lim_{x \to 0} \frac{x \tan x}{1 - \cos x}$$

$$= \lim_{x \to 0} \frac{x \sin x}{\cos x (1 - \cos x)}$$

$$= \lim_{x \to 0} \frac{x \sin x}{\cos x \left(2 \sin^2 \frac{x}{2} \right)}$$

$$= \lim_{x \to 0} \frac{x \left(2 \sin \frac{x}{2} \cos \frac{x}{2} \right)}{\cos x \left(2 \sin^2 \frac{x}{2} \right)}$$

$$= \lim_{x \to 0} \frac{x \cos \frac{x}{2}}{\cos x \frac{\sin x}{2}}$$

$$= \lim_{x \to 0} \frac{1}{\cos x \times \frac{\tan x}{\frac{2}{x}}}$$

$$= \lim_{x \to 0} \frac{1}{\cos x} \times \frac{1}{\frac{\tan x}{\frac{2}{x}}}$$

$$\lim_{x \to 0} \frac{\frac{2}{x}}{\frac{x}{2}} \times \frac{1}{2}$$

$$= 1 \times 1 \times 2$$

$$= 2$$

Q36

$$\lim_{x \to 0} \frac{x^2 + 1 - \cos x}{x \sin x}$$

$$= \lim_{x \to 0} \frac{x^2 + 2\sin^2 \frac{x}{2}}{x \sin x}$$

$$= \lim_{x \to 0} \frac{x^2 \left[1 + 2\left(\frac{\sin \frac{x}{2}}{x}\right)^2\right]}{x \sin x}$$

$$= \lim_{x \to 0} \frac{1 + 2\left(\frac{\sin \frac{x}{2}}{\frac{x}{2}}\right)^2 \times \frac{1}{4}}{\frac{\sin x}{x}}$$

$$= \lim_{x \to 0} \frac{1 + \lim_{x \to 0} 2\left(\frac{\sin \frac{x}{2}}{\frac{x}{2}}\right)^2 \times \frac{1}{4}}{\lim_{x \to 0} \frac{\sin x}{x}}$$

$$= \frac{1 + 2 \times 1 \times \frac{1}{4}}{1} = \frac{1 + \frac{1}{2}}{1}$$

$$= \frac{3}{2}$$

Q37

$$\lim_{x\to 0} \frac{\sin 2x \left(\cos 3x - \cos x\right)}{x^3}$$

$$= \lim_{x\to 0} \frac{\sin 2x \left(-2 \sin\left(\frac{3x+x}{2}\right) \sin\left(\frac{3x-x}{2}\right)\right)}{x^3}$$

$$= \lim_{x\to 0} \frac{\sin 2x \left(-2 \sin 2x \sin x\right)}{x^3}$$

$$= \frac{-2 \lim\limits_{x\to 0} \sin 2x \times \lim\limits_{x\to 0} \sin 2x \times \lim\limits_{x\to 0} \sin x}{x^3}$$

$$= -2\left(\lim_{x\to 0} \frac{\sin 2x}{2x} \times 2\right) \times \left(2 \lim_{x\to 0} \frac{\sin 2x}{2x}\right) \times \left(\lim_{x\to 0} \frac{\sin x}{x}\right)$$

$$= -2\left(1 \times 2\right) \times (2) \times (1)$$

$$= -8$$

Q38

$$\lim_{x\to 0} \frac{2 \sin x° - \sin 2x°}{x^3}$$

$$= \lim_{x\to 0} \frac{2 \sin \frac{\pi x}{180} - \sin \frac{2\pi x}{180}}{x^3}$$

$$= \lim_{x\to 0} \frac{2 \sin \frac{\pi x}{180} - 2 \sin \frac{\pi x}{180} \cos \frac{\pi x}{180}}{x^3}$$

$$= \lim_{x\to 0} \frac{2 \sin \frac{\pi x}{180} \left(2 \sin^2 \frac{\pi x}{360}\right)}{x^3}$$

$$= 4\left(\lim_{x\to 0} \frac{\sin \frac{\pi x}{180}}{x}\right) \times \left(\lim_{x\to 0} \frac{\sin \frac{\pi x}{360}}{x}\right) \times \left(\lim_{x\to 0} \frac{\sin \frac{\pi x}{360}}{x}\right)$$

$$= 4\left(\lim_{x\to 0} \frac{\sin \frac{\pi x}{180}}{\frac{\pi x}{180}} \times \frac{\pi}{180}\right) \times \left(\lim_{x\to 0} \frac{\sin \frac{\pi x}{360}}{\frac{\pi x}{360}} \times \frac{\pi}{360}\right) \times \left(\lim_{x\to 0} \frac{\sin \frac{\pi x}{360}}{\frac{\pi x}{360}} \times \frac{\pi}{360}\right)$$

$$= 4 \times \frac{\pi}{180} \times \frac{\pi}{360} \times \frac{\pi}{360}$$

$$= \left(\frac{\pi}{180}\right)^3$$

Q39

$$\lim_{x \to 0} \frac{x^3 \cot x}{1 - \cos x}$$

$$= \lim_{x \to 0} \frac{x^3}{\tan x \, (1 - \cos x)}$$

$$= \lim_{x \to 0} \frac{x^3}{\tan x \cdot 2 \sin^2 \frac{x}{2}}$$

$$= \lim_{x \to 0} \frac{1}{\dfrac{\tan x}{x} \times \dfrac{2 \sin^2 \frac{x}{2}}{x^2}}$$

$$= \frac{1}{\left(\lim\limits_{x \to 0} \dfrac{\tan x}{x}\right) \times 2 \left(\lim\limits_{x \to 0} \dfrac{\sin \frac{x}{2}}{\frac{x}{2}}\right)^2 \times \dfrac{1}{4}}$$

$$= \frac{1}{1 \times 2 \times 1 \times \dfrac{1}{4}} \qquad \left[\because \lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1, \ \lim_{\theta \to 0} \frac{\tan \theta}{\theta} = 1 \right]$$

$$= 2$$

Q40

$$\lim_{x \to 0} \frac{x \tan x}{1 - \cos 2x}$$

$$= \lim_{x \to 0} \frac{x \tan x}{2 \sin^2 x}$$

$$= \lim_{x \to 0} \frac{\dfrac{\tan x}{x}}{\dfrac{2 \sin^2 x}{x^2}}$$

$$= \frac{\lim\limits_{x \to 0} \dfrac{\tan x}{x}}{2 \lim\limits_{x \to 0} \left(\dfrac{\sin x}{x}\right)^2} \qquad \left[\because \lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1, \ \lim_{\theta \to 0} \frac{\tan \theta}{\theta} = 1 \right]$$

$$= \frac{1}{2 \times 1}$$

$$= \frac{1}{2}$$

Q41

$$\lim_{x \to 0} \frac{\sin(3+x) - \sin(3-x)}{x}$$

$$= \lim_{x \to 0} \frac{2\cos\left(\dfrac{3+x+3-x}{2}\right)\sin\left(\dfrac{3+x-3+x}{2}\right)}{x}$$

$$= 2\lim_{x \to 0} \frac{\cos 3 . \sin x}{x}$$

$$= 2\cos 3 \lim_{x \to 0} \frac{\sin x}{x}$$

$$= 2\cos 3 \times 1$$

$$= 2\cos 3$$

Q42

$$\lim_{x \to 0} kx \cos ecx = \lim_{x \to 0} x \cos eckx$$

$$\text{LHS} = \lim_{x \to 0} kx \cos ecx = k \lim_{x \to 0} x \cos ecx$$

$$= k \lim_{x \to 0} \frac{x}{\sin x}$$

$$= k \lim_{x \to 0} \frac{1}{\frac{\sin x}{x}}$$

$$= k \times 1 \qquad\qquad \left[\because \lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1 \right]$$

$$\text{Also, RHS} = \lim_{x \to 0} x \cos eckx$$

$$= \lim_{x \to 0} \frac{x}{\sin kx}$$

$$= \lim_{x \to 0} \frac{x}{\frac{\sin kx}{kx} \times kx}$$

$$= \frac{1}{k} \qquad\qquad \left[\because \lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1 \right]$$

As, LHS = RHS

$$\Rightarrow \quad k = \frac{1}{k}$$

$$k^2 = 1$$

$$k = \pm 1$$

Q43

$$\lim_{x \to 0} \frac{3 \sin^2 x - 2 \sin x^2}{3x^2}$$

$$= \lim_{x \to 0} \frac{3 \sin^2 x}{3x^2} - \lim_{x \to 0} \frac{2 \sin x^2}{3x^2}$$

$$= \lim_{x \to 0} \left(\frac{\sin x}{x} \right)^2 - \frac{2}{3} \lim_{x \to 0} \frac{\sin x^2}{x^2}$$

$$= 1 - \frac{2}{3} \times 1 \qquad\qquad \left[\because \lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1 \right]$$

$$= 1 - \frac{2}{3}$$

$$= \frac{3 - 2}{3}$$

$$= \frac{1}{3}$$

Q44

$$\lim_{x \to 0} \frac{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}}{x}$$

$$= \lim_{x \to 0} \frac{\left(\sqrt{1 + \sin x} - \sqrt{1 - \sin x} \right) \left(\sqrt{1 + \sin x} + \sqrt{1 - \sin x} \right)}{x \left(\sqrt{1 + \sin x} + \sqrt{1 - \sin x} \right)}$$

$$= \lim_{x \to 0} \frac{\left((1 + \sin x) - (1 - \sin x) \right)}{x \sqrt{1 + \sin x} + \sqrt{1 - \sin x}}$$

$$= \lim_{x \to 0} \frac{2 \sin x}{x \left(\sqrt{1 + \sin x} + \sqrt{1 - \sin x} \right)}$$

$$= 2 \lim_{x \to 0} \frac{\sin x}{x} \times \frac{1}{\lim_{x \to 0} \left(\sqrt{1 + \sin x} + \sqrt{1 - \sin x} \right)}$$

$$= 2 \times 1 \times \frac{1}{2}$$

$$= 1$$

Q45

$$\lim_{x \to 0} \frac{1 - \cos 4x}{x^2}$$

$$= \lim_{x \to 0} \frac{2 \sin^2 2x}{x^2}$$

$$= 2 \lim_{x \to 0} \left(\frac{\sin 2x}{x} \right)^2$$

$$= 2 \lim_{x \to 0} \left(\frac{\sin 2x}{2x} \right)^2 \times (2)^2$$

$$= 2 \times 1 \times 4$$

$$= 8$$

Q46

$$\lim_{x \to 0} \frac{x \cos x + \sin x}{x^2 + \tan x}$$

$$= \lim_{x \to 0} \frac{\cos x + \dfrac{\sin x}{x}}{x + \dfrac{\tan x}{x}}$$

$$= \frac{\lim\limits_{x \to 0} \cos x + \lim\limits_{x \to 0} \dfrac{\sin x}{x}}{\lim\limits_{x \to 0} x + \lim\limits_{x \to 0} \dfrac{\tan x}{x}}$$

$$= \frac{1 + 1}{0 + 1} = \frac{2}{1}$$

$$= 2$$

Q47

$$\lim_{x \to 0} \frac{1 - \cos 2x}{3 \tan^2 x}$$

$$= \lim_{x \to 0} \frac{2 \sin^2 x}{3 \tan^2 x}$$

$$= \frac{2}{3} \lim_{x \to 0} \frac{\sin^2 x}{\left(\dfrac{\sin^2 x}{\cos^2 x} \right)}$$

$$= \frac{2}{3} \lim_{x \to 0} \cos^2 x$$

$$= \frac{2}{3}$$

Q48

$$\lim_{\theta \to 0} \frac{1 - \cos 4\theta}{1 - \cos 6\theta}$$

$$= \lim_{\theta \to 0} \frac{2 \sin^2 2\theta}{2 \sin^2 3\theta}$$

$$= \lim_{\theta \to 0} \frac{(\sin 2\theta)^2}{(\sin 3\theta)^2}$$

$$= \frac{\lim_{\theta \to 0} \left(\dfrac{\sin 2\theta}{2\theta} \right)^2 \times 4\theta^2}{\lim_{\theta \to 0} \left(\dfrac{\sin 3\theta}{3\theta} \right)^2 \times 9\theta^2}$$

$$= \frac{1 \times 4\theta^2}{1 \times 9\theta^2}$$

$$= \frac{4}{9}$$

Q49

$$\lim_{x \to 0} \frac{ax + x \cos x}{b \sin x}$$

$$= \lim_{x \to 0} \frac{a + \cos x}{\dfrac{b \sin x}{x}}$$

$$= \frac{\lim\limits_{x \to 0} a + \lim\limits_{x \to 0} \cos x}{\lim\limits_{x \to 0} \dfrac{b \sin x}{x}}$$

$$= \frac{a+1}{b} \qquad\qquad \left[\because \lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1 \right]$$

$$= \frac{a+1}{b}$$

Q50

$$\lim_{\theta \to 0} \frac{\sin 4\theta}{\tan 3\theta}$$

$$= \frac{\lim\limits_{\theta \to 0} \sin 4\theta}{\lim\limits_{\theta \to 0} \tan 3\theta}$$

$$= \frac{\left(\lim\limits_{\theta \to 0} \dfrac{\sin 4\theta}{4\theta} \right) \times 4\theta}{\left(\lim\limits_{\theta \to 0} \dfrac{\tan 3\theta}{3\theta} \right) \times 3\theta}$$

$$= \frac{1 \times 4\theta}{1 \times 3\theta}$$

$$= \frac{4}{3}$$

Q51

$$\lim_{x \to 0} \frac{2\sin x - \sin 2x}{x^3} = \lim_{x \to 0} \frac{2\sin x - 2\sin x \cos x}{x^3}$$

$$= \lim_{x \to 0} \frac{2\sin x (1 - \cos x)}{x^3}$$

$$= \lim_{x \to 0} \frac{2\sin x (1 - \cos x)}{x^3} \times \frac{1 + \cos x}{1 + \cos x}$$

$$= \lim_{x \to 0} \frac{2\sin x (1 - \cos^2 x)}{x^3 (1 + \cos x)}$$

$$= \lim_{x \to 0} \frac{2\sin x (\sin^2 x)}{x^3 (1 + \cos x)}$$

$$= \lim_{x \to 0} \frac{2\sin^3 x}{x^3 (1 + \cos x)}$$

$$= 2\lim_{x \to 0} \left(\frac{\sin x}{x}\right)^3 \times \lim_{x \to 0} \frac{1}{(1 + \cos x)}$$

$$= 2 \times 1 \times \frac{1}{(1 + 1)}$$

$$= 1$$

Q52

$$\lim_{x \to 0} \frac{1 - \cos 5x}{1 - \cos 6x}$$

$$= \frac{\lim_{x \to 0} 2\sin^2 \frac{5x}{2}}{\lim_{x \to 0} 2\sin^2 3x}$$

$$= \frac{2\left(\lim_{x \to 0} \frac{\sin \frac{5x}{2}}{\frac{5x}{2}}\right)^2 \times \frac{25}{4} x^2}{2\left(\lim_{x \to 0} \frac{\sin 3x}{3x}\right)^2 \times 9x^2}$$

$$= \frac{2 \times 1 \times \frac{25}{4} x^2}{2 \times 1 \times 9x^2}$$

$$= \frac{25}{4 \times 9}$$

$$= \frac{25}{36}$$

Q53

$$\lim_{x \to 0} \frac{\operatorname{cosec} x - \cot x}{x}$$

$$= \lim_{x \to 0} \left(\frac{1}{\sin x} - \frac{\cos x}{\sin x} \right) \times \frac{1}{x}$$

$$= \lim_{x \to 0} \left(\frac{1}{\sin x} \left(\frac{1 - \cos x}{x} \right) \right)$$

$$= \lim_{x \to 0} \left(\frac{1}{\sin x} \left(\frac{2 \sin^2 \frac{x}{2}}{x} \right) \right)$$

$$= 2 \lim_{x \to 0} \left(\frac{1}{\frac{\sin x}{x}} \times x \times \left(\frac{\sin \frac{x}{2}}{\frac{x}{2}} \right)^2 \times \frac{x}{4} \right)$$

$$= 2 \left(\lim_{x \to 0} \frac{1}{\frac{\sin x}{x}} \right) \times \frac{1}{x} \times \left(\lim_{x \to 0} \frac{\sin \frac{x}{2}}{\frac{x}{2}} \right)^2 \times \frac{x}{4}$$

$$= 2 \times \frac{1}{x} \times \frac{x}{4}$$

$$= \frac{1}{2}$$

Q54

$$\lim_{x \to 0} \frac{\sin 3x + 7x}{4x + \sin 2x}$$

$$= \lim_{x \to 0} \frac{\left(\frac{\sin 3x}{x} + \frac{7x}{x} \right)}{\left(\frac{4x}{x} + \frac{\sin 2x}{x} \right)}$$

$$= \frac{\left(\lim_{x \to 0} \frac{\sin 3x}{3x} \times 3 \right) + 7}{4 + \left(\lim_{x \to 0} \frac{\sin 2x}{2x} \right) \times 2}$$

$$= \frac{3 + 7}{4 + 2}$$

$$= \frac{10}{6}$$

$$= \frac{5}{3}$$

Q55

$$\lim_{x \to 0} \frac{5x + 4\sin 3x}{4\sin 2x + 7x}$$

$$= \lim_{x \to 0} \frac{5 + \dfrac{4\sin 3x}{x}}{\dfrac{4\sin 2x}{x} + 7}$$

$$= \frac{\lim_{x \to 0} 5 + 4 \lim_{x \to 0} \dfrac{\sin 3x}{3x} \times 3}{4 \lim_{x \to 0} \dfrac{\sin 2x}{2x} \times 2 + 7}$$

$$= \frac{5 + 4 \times 1 \times 3}{4 \times 2 + 7}$$

$$= \frac{5 + 12}{8 + 7}$$

$$= \frac{17}{15}$$

Q56

$$\lim_{x \to 0} \frac{3\sin x - \sin 3x}{x^3}$$

$$= \lim_{x \to 0} \frac{3\sin x - \left(3\sin x - 4\sin^3 x\right)}{x^3}$$

$$= \lim_{x \to 0} \frac{4\sin^3 x}{x^3}$$

$$= 4 \left(\lim_{x \to 0} \frac{\sin x}{x} \right)^3$$

$$= 4 \times 1$$

$$= 4$$

Q57

$$\lim_{x \to 0} \frac{\tan 2x - \sin 2x}{x^3}$$

$$= \lim_{x \to 0} \frac{\frac{\sin 2x}{\cos 2x} - \sin 2x}{x^3}$$

$$= \lim_{x \to 0} \frac{\sin 2x \left(\frac{1}{\cos 2x} - 1\right)}{x^3}$$

$$= \lim_{x \to 0} \frac{\sin 2x \left(1 - \cos 2x\right)}{x^3 \cos 2x}$$

$$= \lim_{x \to 0} \frac{\sin 2x \left(2 \sin^2 x\right)}{x^3 \cos 2x}$$

$$= \frac{\left(\lim_{x \to 0} \frac{\sin 2x}{x}\right)\left(\lim_{x \to 0} \frac{2 \sin^2 x}{x^2}\right)}{\left(\lim_{x \to 0} \cos 2x\right)}$$

$$= \frac{\left(\lim_{x \to 0} \frac{\sin 2x}{2x} \times 2\right)\left(\lim_{x \to 0} 2 \left(\frac{\sin x}{x}\right)^2\right)}{\lim_{x \to 0} \cos 2x}$$

$$= \frac{\left(2 \times 1\right)\left(2 \times 1\right)}{1}$$

$$= 4$$

Q58

$$\lim_{x \to 0} \frac{\sin ax + bx}{ax + \sin bx}$$

$$= \lim_{x \to 0} \frac{\frac{\sin ax}{x} + b}{a + \frac{\sin bx}{x}}$$

$$= \frac{\lim_{x \to 0} \frac{\sin ax}{ax} \times a + b}{a + \lim_{x \to 0} \frac{\sin bx}{bx} \times b}$$

$$= \frac{a + b}{a + b}$$

$$= 1$$

Q59

$$\lim_{x \to 0} (\operatorname{cosec} x - \cot x)$$

$$= \lim_{x \to 0} \left(\frac{1}{\sin x} - \frac{\cos x}{\sin x} \right)$$

$$= \lim_{x \to 0} \left(\frac{1 - \cos x}{\sin x} \right)$$

$$= \lim_{x \to 0} \frac{2 \sin^2 \frac{x}{2}}{2 \sin \frac{x}{2} \cos \frac{x}{2}}$$

$$= \lim_{x \to 0} \frac{\tan x}{2}$$

$$= \left(\lim_{x \to 0} \frac{\frac{\tan x}{2}}{\frac{x}{2}} \right) \times \frac{x}{2}$$

$$= \lim_{x \to 0} 1 \times \frac{x}{2}$$

$$= 0$$

Q60

$$\lim_{x \to 0} \frac{\{\sin(\alpha + \beta)x + \sin(\alpha - \beta)x + \sin 2\alpha x\}}{\cos^2 \beta x - \cos^2 \alpha x}$$

$$= \lim_{x \to 0} \frac{\left\{ 2 \sin \frac{(\alpha + \beta + \alpha - \beta)}{2} x \times \cos \frac{(\alpha + \beta - \alpha + \beta)}{2} x + 2 \sin \alpha \cos \alpha x \right\}}{(\cos \beta x - \cos \alpha x)(\cos \beta x + \cos \alpha x)}$$

$$= \lim_{x \to 0} \frac{\{2 \sin \alpha x \cos \beta x + 2 \sin \alpha x \cos \alpha x\}}{(\cos \beta x - \cos \alpha x)(\cos \beta x + \cos \alpha x)}$$

$$= \lim_{x \to 0} \frac{2 \sin \alpha x (\cos \beta x + \cos \alpha x)}{(\cos \beta x - \cos \alpha x)(\cos \beta x + \cos \alpha x)}$$

$$= \lim_{x \to 0} \frac{2 \sin \alpha x}{(\cos \beta x - \cos \alpha x)}$$

$$= \lim_{x \to 0} \frac{2 \sin \alpha x}{\left(1 - 2 \sin^2 \left(\frac{\beta x}{2} \right) - 1 + 2 \sin^2 \left(\frac{\alpha x}{2} \right) \right)}$$

$$= \lim_{x \to 0} \frac{2 \sin \alpha x}{2 \sin^2 \left(\frac{\alpha x}{2} \right) - 2 \sin^2 \left(\frac{\beta x}{2} \right)}$$

$$= \frac{2\alpha}{\alpha^2 - \beta^2}$$

Q61

$$\lim_{x\to 0}\frac{\cos ax - \cos bx}{\cos cx - 1} = \lim_{x\to 0}\frac{1 - 2\sin^2\left(\frac{ax}{2}\right) - 1 + 2\sin^2\left(\frac{bx}{2}\right)}{1 - 2\sin^2\left(\frac{cx}{2}\right) - 1}$$

$$= \lim_{x\to 0}\frac{-2\sin^2\left(\frac{ax}{2}\right) + 2\sin^2\left(\frac{bx}{2}\right)}{-2\sin^2\left(\frac{cx}{2}\right)}$$

$$= \lim_{x\to 0}\frac{-\sin^2\left(\frac{ax}{2ax}\right)4a^2x^2 + \sin^2\left(\frac{bx}{2}\right)4b^2x^2}{-\sin^2\left(\frac{cx}{2}\right)4c^2x^2}$$

$$= \frac{-a^2 + b^2}{-c^2}$$

$$= \frac{a^2 - b^2}{c^2}$$

Q62

$$\lim_{h\to 0}\frac{(a+h)^2\sin(a+h) - a^2\sin a}{h} = \lim_{h\to 0}\frac{(a+h)^2(\sin a\cos h + \cos a\sin h) - a^2\sin a}{h}$$

$$= \lim_{h\to 0}\frac{(a+h)^2(\sin a\cos h) - a^2\sin a + (a+h)^2\cos a\sin h}{h}$$

$$= \lim_{h\to 0}\frac{(a^2 + 2ah + h^2)(\sin a\cos h) - a^2\sin a - (a+h)^2\cos a\sin h}{h}$$

$$= \lim_{h\to 0}\frac{a^2\sin a(\cos h - 1) + 2ah\sin a\cos h + h^2\sin a\cos h + (a+h)^2\cos a\sin h}{h}$$

$$= \lim_{h\to 0}\frac{a^2\sin a(\cos h - 1)}{h} + \lim_{h\to 0}\frac{2ah\sin a\cos h}{h} - \lim_{h\to 0}\frac{h^2\sin a\cos h}{h} + \lim_{h\to 0}\frac{(a+h)^2\cos a\sin h}{h}$$

$$= \lim_{h\to 0}\frac{-a^2\sin a\sin^2\left(\frac{h}{2}\right)}{\frac{h}{2}} + 2a\sin a + 0 + a^2\cos a$$

$$= 0 + 2a\sin a + a^2\cos a$$

$$= 2a\sin a + a^2\cos a$$

Q63

$$\lim_{x \to 0} kx \cosec x = \lim_{x \to 0} x \cosec kx$$

$$\lim_{x \to 0} kx \frac{1}{\sin x} = \lim_{x \to 0} x \frac{1}{\sin kx}$$

$$k\lim_{x \to 0}\left(\frac{x}{\sin x}\right) = \frac{1}{k}\lim_{x \to 0}\left(\frac{kx}{\sin kx}\right)$$

$$k = \frac{1}{k}$$

$$k^2 = 1$$

$$k = \pm 1$$

Ex 29.8

Q1

$$\lim_{x \to \pi/2}\left(\frac{\pi}{2} - x\right)\tan x$$

Let $y = \frac{\pi}{2} - x$

as $x \to \pi/2, \quad y \to 0$

$$\lim_{x \to \pi/2}\left(\frac{\pi}{2} - x\right)\tan x$$

$$= \lim_{y \to 0} y \tan\left(\frac{\pi}{2} - y\right)$$

$$= \lim_{y \to 0} y \frac{\sin\left(\frac{\pi}{2} - y\right)}{\cos\left(\frac{\pi}{2} - y\right)}$$

$$= \lim_{y \to 0} y \frac{\cos y}{\sin y}$$

$$= \lim_{y \to 0} \cos y = \lim_{y \to 0} \frac{y}{\sin y}$$

$$= 1$$

Q2

$$\lim_{x \to \frac{\pi}{2}} \frac{\sin 2x}{\cos x}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{2 \sin x \cos x}{\cos x}$$

$$= 2 \lim_{x \to \frac{\pi}{2}} \sin x$$

$$= 2 \times \sin \frac{\pi}{2}$$

$$= 2 \times 1$$

$$= 2$$

Q3

$$\lim_{x \to \frac{\pi}{2}} \frac{\cos^2 x}{1 - \sin x}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{1 - \sin^2 x}{1 - \sin x}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{(1 - \sin x)(1 + \sin x)}{(1 - \sin x)}$$

$$= \lim_{x \to \frac{\pi}{2}} (1 + \sin x)$$

$$= 1 + \sin \frac{\pi}{2}$$

$$= 1 + 1$$

$$= 2$$

Q4

$$\lim_{x \to \frac{\pi}{2}} \frac{1 - \sin x}{\cos^2 x}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{1 - \sin x}{1 - \sin^2 x}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{(1 - \sin x)}{(1 - \sin x)(1 + \sin x)}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{1}{1 + \sin x}$$

$$= \frac{1}{1 + \sin \frac{\pi}{2}}$$

$$= \frac{1}{1 + 1}$$

$$= \frac{1}{2}$$

Q5

$$\lim_{x \to a} \frac{\cos x - \cos a}{x - a}$$

$$= \lim_{x \to a} \frac{\left(-2 \sin\left(\frac{x+a}{2}\right) \sin\left(\frac{x-a}{2}\right)\right)}{x-a}$$

$$= -2 \lim_{x \to a} \sin\left(\frac{x+a}{2}\right) \lim_{x \to a} \frac{\sin\left(\frac{x-a}{2}\right)}{x-a}$$

$$= -2 \times \sin\left(\frac{a+a}{2}\right) \times \left(\lim_{x-a \to 0} \frac{\sin\left(\frac{x-a}{2}\right)}{\frac{x-a}{2}}\right) \times \frac{1}{2}$$

$$= -2 \sin a \times 1 \times \frac{1}{2} \qquad\qquad \left[\because \lim_{x \to a} \frac{\sin x}{x} = 1\right]$$

$$= -\sin a$$

Q6

$$\lim_{x \to \frac{\pi}{4}} \frac{1 - \tan x}{x - \frac{\pi}{4}}$$

If $x \to \frac{\pi}{4}$, then $x - \frac{\pi}{4} \to 0$

Let $x - \frac{\pi}{4} = y \Rightarrow y \to 0$

$$= \lim_{y \to 0} \frac{1 - \tan\left(y + \frac{\pi}{4}\right)}{y}$$

$$= \lim_{y \to 0} \frac{1 - \left(\frac{\tan y + \tan\frac{\pi}{4}}{1 - \tan y \tan\frac{\pi}{4}}\right)}{y}$$

$$= \lim_{y \to 0} \frac{(1 - \tan y - \tan y - 1)}{y(1 - \tan y)} \qquad\qquad \left[\because \tan\frac{\pi}{4} = 1\right]$$

$$= \lim_{y \to 0} \frac{(-2 \tan y)}{y(1 - \tan y)}$$

$$= -2 \lim_{y \to 0} \frac{\tan y}{y} \times \frac{1}{\lim_{y \to 0}(1 - \tan y)}$$

$$= -2 \times 1 \frac{1}{(1 - 0)} \qquad\qquad \left[\because \lim_{\theta \to 0} = \frac{\tan\theta}{\theta} = 1\right]$$

$$= -2$$

Q7

$$\lim_{x \to \frac{\pi}{2}} \frac{1 - \sin x}{\left(\frac{\pi}{2} - x\right)^2}$$

If $x \to \frac{\pi}{2}, \frac{\pi}{2} - x \to 0$

Let $\frac{\pi}{2} - x = y$ they $y \to 0$

$$= \lim_{y \to 0} \frac{1 - \sin\left(\frac{\pi}{2} - y\right)}{y^2}$$

$$= \lim_{y \to 0} \frac{1 - \cos y}{y^2}$$

$$= \lim_{y \to 0} \frac{2 \sin^2 \frac{y}{2}}{y^2}$$

$$= 2 \left(\lim_{y \to 0} \frac{\sin \frac{y}{2}}{\frac{y}{2}}\right)^2 \times \frac{1}{4} \qquad \left[\because \lim_{x \to 0} \frac{\sin x}{x} = 1\right]$$

$$= 2 \times 1 \times \frac{1}{4}$$

$$= \frac{1}{2}$$

Q8

$$\lim_{x \to \frac{\pi}{3}} \frac{\sqrt{3} - \tan x}{\pi - 3x}$$

If $x \to \frac{\pi}{3}, \frac{\pi}{3} - x \to 0, \pi - 3x \to 0$

Let $\frac{\pi}{3} - x = y$ then $y \to 0$

$$= \lim_{y \to 0} \frac{\sqrt{3} - \tan\left(\frac{\pi}{3} - y\right)}{3\left(\frac{\pi}{3} - x\right)}$$

$$= \lim_{y \to 0} \frac{\left(\sqrt{3} - \dfrac{\tan\frac{\pi}{3} - \tan y}{1 + \tan\frac{\pi}{3}.\tan y}\right)}{3y}$$

$$= \lim_{y \to 0} \frac{\left(\sqrt{3} - \dfrac{\sqrt{3} - \tan y}{1 + \sqrt{3}\,\tan y}\right)}{3y}$$

$$= \lim_{y \to 0} \frac{\left(\sqrt{3} + 3\tan y - \sqrt{3} + \tan y\right)}{3\left(1 + \sqrt{3}\,\tan y\right)y}$$

$$= \lim_{y \to 0} \frac{4\tan y}{3\left(1 + \sqrt{3}\,\tan y\right)y}$$

$$= \frac{4}{3} \times \lim_{y \to 0} \frac{\tan y}{y} \times \frac{1}{\lim_{y \to 0}\left(1 + \sqrt{3}\,\dfrac{\tan y}{y} \times y\right)}$$

$$= \frac{4 \times 1}{3} \times \frac{1}{1 + 0}$$

$$= \frac{4}{3}$$

Q9

$$\lim_{x \to a} \frac{a \sin x - x \sin a}{ax^2 - xa^2} = \lim_{x \to a} \frac{(a \sin x - x \sin a)}{ax(x-a)}$$

Let $\quad t = x - a$

Then, as $x \to a$, $t \to 0$

$$\therefore \lim_{x \to a} \frac{(a \sin x - x \sin a)}{ax(x-a)} = \lim_{t \to 0} \frac{(a \sin(t+a) - (t+a) \sin a)}{a(t+a)t}$$

$$= \lim_{t \to 0} \frac{a \sin t \cos a + a \sin a \cos t - t \sin a - a \sin a}{a(t+a)t}$$

$$= \lim_{t \to 0} \frac{a \sin t \cos a + a \sin a (\cos t - 1) - t \sin a}{a(t+a)t}$$

$$= \lim_{t \to 0} \frac{a \sin t \cos a + a \sin a (2 \sin^2(t/2)) - t \sin a}{a(t+a)t}$$

$$= \lim_{t \to 0} \frac{a \sin t \cos a}{a(t+a)t} + \lim_{t \to 0} \frac{a \sin a (2 \sin^2(t/2))}{a(t+a)t} - \lim_{t \to 0} \frac{t \sin a}{a(t+a)t}$$

$$= \frac{a \cos a}{a^2} + 0 - \frac{\sin a}{a^2}$$

$$= \frac{a \cos a - \sin a}{a^2}$$

Q10

$$\lim_{x \to \frac{\pi}{2}} \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x} \frac{\sqrt{2} + \sqrt{1+\sin x}}{\sqrt{2} + \sqrt{1+\sin x}}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{2 - 1 - \sin x}{\cos^2 x \left(\sqrt{2} + \sqrt{1+\sin x} \right)}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{1 - \sin x}{\left(1 - \sin^2 x \right) \left(\sqrt{2} + \sqrt{1+\sin x} \right)}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{1}{\left(1 + \sin x \right) \left(\sqrt{2} + \sqrt{1+\sin x} \right)}$$

$$= \frac{1}{(1+1) \left(\sqrt{2} + \sqrt{2} \right)}$$

$$= \frac{1}{\left(4\sqrt{2} \right)}$$

Q11

$$\lim_{x \to \frac{\pi}{2}} \frac{\sqrt{2 - \sin x} - 1}{\left(\frac{\pi}{2} - x\right)^2}$$

$\Rightarrow \quad x \to \frac{\pi}{2}$, then $\frac{\pi}{2} - x \to 0$, let $\frac{\pi}{2} - x = y$

$$\lim_{x - \frac{\pi}{2} \to 0} = \lim_{y \to 0} \frac{\sqrt{2 - \sin\left(\frac{\pi}{2} - y\right)} - 1}{y^2}$$

$$= \lim_{y \to 0} \frac{\sqrt{2 - \cos y} - 1}{y^2}$$

$$= \lim_{y \to 0} \frac{\left(\sqrt{2 - \cos y} - 1\right)}{y^2} \times \frac{\left(\sqrt{2 - \cos y} + 1\right)}{\left(\sqrt{2 - \cos y} + 1\right)}$$

$$= \lim_{y \to 0} \frac{(2 - \cos y - 1)}{y^2\left(\sqrt{2 - \cos y} + 1\right)}$$

$$= \lim_{y \to 0} \frac{(1 - \cos y)}{\left(\sqrt{2 - \cos y} + 1\right) y^2}$$

$$= \lim_{y \to 0} \frac{2 \sin^2 \frac{y}{2}}{y^2\left(\sqrt{2 - \cos y} + 1\right)}$$

$$= 2 \times \left(\lim_{y \to 0} \frac{\sin \frac{y}{2}}{\frac{y}{2}}\right)^2 \times \frac{1}{4} \times \frac{1}{\lim_{y \to 0} \sqrt{2 - \cos y} + 1}$$

$$= 2 \times 1 \times \frac{1}{4} \times \frac{1}{1 + 1} = \frac{1}{4}$$

Q12

$$\lim_{x \to \frac{\pi}{4}} \frac{\sqrt{2} - \cos x - \sin x}{\left(\frac{\pi}{4} - x\right)^2}$$

As $x \to \frac{\pi}{4}, \frac{\pi}{4} - x \to 0$, let $\frac{\pi}{4} - x = y$

$$= \lim_{y \to 0} \frac{\sqrt{2} - \cos\left(\frac{\pi}{4} - y\right) - \sin\left(\frac{\pi}{4} - y\right)}{y^2}$$

$$= \lim_{y \to 0} \frac{\sqrt{2} - \left[\cos\frac{\pi}{4}\cos y + \sin\frac{\pi}{4}\sin y + \sin\frac{\pi}{4}\cos y - \cos\frac{\pi}{4}\sin y\right]}{y^2}$$

$$= \lim_{y \to 0} \frac{\sqrt{2} - \left(\frac{\cos y}{\sqrt{2}} + \frac{\sin y}{\sqrt{2}} + \frac{\cos y}{\sqrt{2}} - \frac{\sin y}{\sqrt{2}}\right)}{y^2}$$

$$= \lim_{y \to 0} \frac{\sqrt{2} - \frac{2\cos y}{\sqrt{2}}}{y^2} = \lim_{y \to 0} \frac{\sqrt{2} - \sqrt{2}\cos y}{y^2}$$

$$= \sqrt{2} \lim_{y \to 0} \frac{(1 - \cos y)}{y^2}$$

$$= \sqrt{2} \lim_{y \to 0} \frac{2\sin^2\frac{y}{2}}{\frac{y^2}{4}} \times \frac{1}{4}$$

$$= \sqrt{2} \times 2 \times \frac{1}{4}\left(\lim_{y \to 0} \frac{\sin\frac{y}{2}}{\frac{y}{2}}\right)^2$$

$$= \sqrt{2} \times 2 \times \frac{1}{4} \times 1 \qquad\qquad \left[\because \lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1\right]$$

$$= \frac{1}{\sqrt{2}}$$

Q13

$$\lim_{x \to \frac{\pi}{8}} \frac{\cot 4x - \cos 4x}{8^3\left(\frac{\pi}{8} - x\right)^3}$$

When $x \to \frac{\pi}{8}, \frac{\pi}{8} - x \to 0$, let $\frac{\pi}{8} - x = y$

$$= \lim_{y \to 0} \frac{\cot 4\left(\frac{\pi}{8} - y\right) - \cos 4\left(\frac{\pi}{8} - y\right)}{(8)^3 y^3}$$

$$= \lim_{y \to 0} \frac{\cot\left(\frac{\pi}{2} - 4y\right) - \cos\left(\frac{\pi}{2} - 4y\right)}{(8)^3 y^3}$$

$$= \lim_{y \to 0} \frac{\tan 4y - \sin 4y}{(8)^3 y^3}$$

$$= \lim_{y \to 0} \frac{\frac{\sin 4y}{\cos 4y} - \sin 4y}{8^3 y^3}$$

$$= \lim_{y \to 0} \frac{\sin 4y - \sin 4y \cos 4y}{\cos 4y \times y^3 \times 8^3}$$

$$= \lim_{y \to 0} \frac{\sin 4y (1 - \cos 4y)}{\cos 4y \times y^3 \times 8^3}$$

$$= \lim_{y \to 0} \frac{\sin 4y \left(2\sin^2 2y\right)}{\cos 4y \times y^3 \times 8^3}$$

$$= \frac{2}{8^3} \lim_{y \to 0} \frac{\sin 4y}{y} \times \frac{\sin^2 2y}{y^2} \times \frac{1}{\cos 4y}$$

$$= \frac{2}{8^3} \left(\lim_{y \to 0} \frac{\sin 4y}{4y} \times 4\right) \times \left(\lim_{y \to 0} \frac{\sin 2y}{2y}\right)^2 \times 4 \times \frac{1}{\lim_{y \to 0} \cos 4y}$$

$$= \frac{2}{8^3} (1 \times 4) \times (1) \times 4 \times \frac{1}{1}$$

$$= \frac{2 \times 4 \times 4}{8 \times 8 \times 8}$$

$$= \frac{1}{16}$$

$$\left[\because \lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1, \lim_{\theta \to 0} \cos \theta = 1\right]$$

Q14

$$\lim_{x \to a} \frac{\cos x - \cos a}{\sqrt{x} - \sqrt{a}}$$

$$= \lim_{x \to a} \frac{\left(-2\sin\left(\frac{x+a}{2}\right)\sin\left(\frac{x-a}{2}\right)\right) \times \left(\sqrt{x} + \sqrt{a}\right)}{\left(\sqrt{x} - \sqrt{a}\right)\left(\sqrt{x} + \sqrt{a}\right)}$$

$$= -2\lim_{x \to a} \frac{\sin\left(\frac{x+a}{2}\right)\sin\left(\frac{x-a}{2}\right) \times \left(\sqrt{x} + \sqrt{a}\right)}{(x - a)}$$

$$= -2\lim_{x \to a} \sin\left(\frac{x+a}{2}\right) \times \lim_{x \to a} \frac{\sin\left(\frac{x-a}{2}\right) \times \frac{1}{2}}{\left(\frac{x-a}{2}\right)} \lim_{x \to a}\left(\sqrt{x} + \sqrt{a}\right)$$

$$= -2 \times \sin(a) \times 1 \times \frac{1}{2} \times 2\sqrt{a}$$

$$= -2\sqrt{a}\sin a$$

$$\left[\because \lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1\right]$$

Q15

$$\lim_{x \to \pi} \frac{\sqrt{5 + \cos x} - 2}{(\pi - x)^2}$$

$\Rightarrow \quad x \to \pi, \text{ then } \pi - x \to 0, \text{ let } \pi - x = y$

$$= \lim_{y \to 0} \frac{\sqrt{5 + \cos(\pi - y)} - 2}{y^2}$$

$$= \lim_{y \to 0} \frac{\sqrt{5 - \cos y} - 2}{y^2}$$

$$= \lim_{y \to 0} \frac{\left(\sqrt{5 - \cos y} - 2\right)}{y^2} \times \frac{\left(\sqrt{5 - \cos y} + 2\right)}{\left(\sqrt{5 - \cos y} + 2\right)}$$

$$= \lim_{y \to 0} \frac{(5 - \cos y - 4)}{y^2 \left(\sqrt{5 - \cos y} + 2\right)}$$

$$= \lim_{y \to 0} \frac{2\sin^2 \frac{y}{2}}{y^2 \left(\sqrt{5 - \cos y} + 2\right)}$$

$$= 2 \times \left(\lim_{y \to 0} \frac{\sin \frac{y}{2}}{\frac{y}{2}}\right)^2 \times \frac{1}{4} \frac{1}{\lim_{y \to 0} \left(\sqrt{5 - \cos y} + 2\right)}$$

$$= 2 \times \frac{1}{4} \times \frac{1}{\sqrt{4} + 2} = 2 \times \frac{1}{4} \times \frac{1}{4}$$

$$= \frac{1}{8}$$

Q16

$$\lim_{x \to a} \frac{\cos \sqrt{x} - \cos \sqrt{a}}{x - a}$$

$$= \lim_{x \to a} \frac{-2 \sin\left(\frac{\sqrt{x} + \sqrt{a}}{2}\right) \sin\left(\frac{\sqrt{x} - \sqrt{a}}{2}\right)}{\left(\sqrt{x} - \sqrt{a}\right)\left(\sqrt{x} + \sqrt{a}\right)}$$

$$= -2 \lim_{x \to a} \frac{\sin\left(\frac{\sqrt{x} + \sqrt{a}}{2}\right) \times \lim_{x \to a} \sin\left(\frac{\sqrt{x} - \sqrt{a}}{2}\right)}{\lim_{x \to a} \left(\sqrt{x} + \sqrt{a}\right) \times \left(\frac{\sqrt{x} - \sqrt{a}}{2}\right)} \times \frac{1}{2}$$

$$= -2 \sin \sqrt{a} \times 1 \times \frac{1}{2\sqrt{a}} \times \frac{1}{2}$$

$$= -\frac{1}{2\sqrt{a}} \sin \sqrt{a}$$

Q17

$$\lim_{x \to a} \frac{\sin \sqrt{x} - \sin \sqrt{a}}{x - a}$$

$$= \lim_{x \to a} \frac{\sin \sqrt{x} - \sin \sqrt{a}}{\left(\sqrt{x} - \sqrt{a}\right)\left(\sqrt{x} + \sqrt{a}\right)}$$

$$= \lim_{x \to a} \frac{2 \sin\left(\frac{\sqrt{x} - \sqrt{a}}{2}\right) \cos\left(\frac{\sqrt{x} + \sqrt{a}}{2}\right)}{\left(\sqrt{x} - \sqrt{a}\right)\left(\sqrt{x} + \sqrt{a}\right)}$$

$$= 2 \left(\lim_{x \to a} \frac{\sin \frac{\sqrt{x} - \sqrt{a}}{2}}{\left(\frac{\sqrt{x} - \sqrt{a}}{2}\right)} \right) \times \frac{1}{2} \frac{\lim_{x \to a} \cos\left(\frac{\sqrt{x} + \sqrt{a}}{2}\right)}{\lim_{x \to a}\left(\sqrt{x} + \sqrt{a}\right)}$$

$$= 2 \times 1 \times \frac{1}{2} \times \cos \sqrt{a} \times \frac{1}{2\sqrt{a}}$$

$$= \frac{\cos \sqrt{a}}{2\sqrt{a}}$$

Q18

$$\lim_{x \to 1} \frac{1 - x^2}{\sin 2\pi x}$$

$\Rightarrow \quad x \to 1, \text{ then } x - 1 \to 0, \text{ let } x - 1 = y$

$$= \lim_{(x-1) \to 0} \frac{(1 - x)(1 + x)}{\sin 2\pi x}$$

$$= \lim_{y \to 0} \frac{-y(1 + y + 1)}{\sin 2\pi (y + 1)}$$

$$= - \lim_{y \to 0} \frac{y(y + 2)}{\sin(2\pi y + 2\pi)}$$

$$= - \lim_{y \to 0} \frac{y(y + 2)}{\sin 2\pi y}$$

$$= - \lim_{y \to 0} (y + 2) \times \frac{y}{\left(\lim_{y \to 0} \sin \frac{2\pi y}{y \times 2\pi}\right) \times 2\pi y}$$

$$= -2 \times \frac{1}{1 \times 2\pi}$$

$$= -\frac{1}{\pi}$$

Q19

$$\lim_{x \to \frac{\pi}{4}} \left(\frac{\sin 2x - \sin 2\left(\frac{\pi}{4}\right)}{x - \frac{\pi}{4}} \right)$$

$$[\because \text{ given } f(x) = \sin 2x]$$

$$= \lim_{x \to \frac{\pi}{4}} \left(\frac{\sin 2x - \sin \frac{\pi}{2}}{x - \frac{\pi}{4}} \right)$$

$$\Rightarrow \quad x \to \frac{\pi}{4} \Rightarrow x - \frac{\pi}{4} \to 0, \text{ let } x - \frac{\pi}{4} = y$$

$$= \lim_{y \to 0} \left(\frac{\sin 2\left(y + \frac{\pi}{4}\right) - 1}{y} \right)$$

$$= \lim_{y \to 0} \frac{\sin\left(\frac{\pi}{2} + 2y\right) - 1}{y}$$

$$= \lim_{y \to 0} \frac{\cos 2y - 1}{y}$$

$$= -\lim_{y \to 0} \frac{1 - \cos 2y}{y}$$

$$= -\lim_{y \to 0} \frac{2 \sin^2 y}{y}$$

$$= -2 \left(\lim_{y \to 0} \frac{\sin y}{y}\right)^2 \times y$$

$$\left[\because \lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1\right]$$

$$= -2 \times 0$$

$$= 0$$

Q20

$$\lim_{x \to 1} \frac{1 + \cos \pi x}{(1 - x)^2}$$

$$\Rightarrow \quad x \to 1, x - 1 \to 0, \text{ let } x - 1 = y$$

$$= \lim_{y \to 0} \frac{1 + \cos \pi (y + 1)}{(-y)^2}$$

$$= \lim_{y \to 0} \frac{1 + \cos(\pi + \pi y)}{y^2}$$

$$= \lim_{y \to 0} \frac{1 - \cos(\pi y)}{y^2}$$

$$= \lim_{y \to 0} \frac{2 \sin^2 \frac{\pi y}{2}}{y^2}$$

$$= 2 \left(\lim_{y \to 0} \frac{\sin \frac{\pi y}{2}}{\frac{\pi y}{2}}\right)^2 \times \frac{\pi^2}{4}$$

$$= 2 \times 1 \times \frac{\pi^2}{4}$$

$$= \frac{\pi^2}{2}$$

Q21

$$\lim_{x \to 1} \frac{1 - x^2}{\sin \pi x}$$

$\Rightarrow \quad x \to 1 \Rightarrow x - 1 \to 0,\ \text{let } x - 1 = y \Rightarrow y \to 0$

$\Rightarrow \quad \lim_{x \to 1} \frac{1 - x^2}{\sin \pi x} = \lim_{x-1 \to 0} \frac{(1 - x)(1 + x)}{\sin \pi x}$

$= \lim_{y \to 0} \frac{(-y)(1 + y + 1)}{\sin \pi (y + 1)}$

$= -\lim_{y \to 0} \frac{y(y + 2)}{\sin(\pi y + \pi)}$

$= -\lim_{y \to 0} \frac{y(y + 2)}{-\sin \pi y}$

$= \lim_{y \to 0} \frac{y(y + 2)}{\dfrac{\sin \pi y}{2}}$

$= \dfrac{\lim_{y \to 0} y(y + 2)}{\lim_{y \to 0} \dfrac{\sin \pi y}{\pi y} \times \pi y}$

$= \dfrac{2}{\pi}$

Q22

$$\lim_{x \to \frac{\pi}{4}} \frac{1 - \sin 2x}{1 + \cos 4x}$$

$$x \to \frac{\pi}{4}, x - \frac{\pi}{4} \to 0, \text{ let } x - \frac{\pi}{4} = y$$

$$\Rightarrow \lim_{x \to \frac{\pi}{4}} \frac{1 - \sin 2x}{1 + \cos 4x} = \lim_{x - \frac{\pi}{4} \to 0} \frac{\left(1 - \sin 2\left(y + \frac{\pi}{4}\right)\right)}{1 + \cos 4\left(y + \frac{\pi}{4}\right)}$$

$$= \lim_{x - \frac{\pi}{4} \to 0} \left[\frac{1 - \sin\left(\frac{\pi}{2} + 2y\right)}{1 + \cos(\pi + 4y)}\right]$$

$$= \lim_{x - \frac{\pi}{4} \to 0} \frac{1 - \cos 2y}{1 - \cos 4y}$$

$$= \lim_{y \to 0} \frac{2 \sin^2 y}{2 \sin^2 2y}$$

$$= \frac{\lim_{y \to 0} \sin^2 y}{\lim_{y \to 0} \sin^2 2y}$$

$$= \frac{\left(\lim_{y \to 0} \frac{\sin y}{y}\right)^2 \times y^2}{\left(\lim_{y \to 0} \frac{\sin 2y}{2y}\right)^2 \times 4y^2}$$

$$= \frac{1 \times y^2}{1 \times 4y^2} \qquad \left[\because \lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1\right]$$

$$= \frac{1}{4}$$

Q23

$$\lim_{x \to \frac{\pi}{6}} \frac{\cot^2 x - 3}{\text{cosec} x - 2}$$

$$= \lim_{x \to \frac{\pi}{6}} \frac{\left(\text{cosec}^2 x - 1\right) - 3}{\text{cosec} x - 2}$$

$$= \lim_{x \to \frac{\pi}{6}} \frac{\left(\text{cosec}^2 x - 4\right)}{\text{cosec} x - 2}$$

$$= \lim_{x \to \frac{\pi}{6}} \frac{(\text{cosec} x - 2)(\text{cosec} x + 2)}{(\text{cosec} x - 2)}$$

$$= \lim_{x \to \frac{\pi}{6}} \text{cosec} x + 2$$

$$= \text{cosec} \frac{\pi}{6} + 2$$

$$= 2 + 2$$

$$= 4$$

Q24

$$\lim_{n \to \infty} n \sin\left(\frac{\pi}{4n}\right) \cos\left(\frac{\pi}{4n}\right)$$

$$= \lim_{n \to \infty} 2 \left(n \sin\frac{\pi}{4n} \cos\frac{\pi}{4n}\right) \times \frac{1}{2}$$

$$= \lim_{n \to \infty} n \times \sin\frac{\pi}{2n} \times \frac{1}{2}$$

$n \to \infty$, then $\frac{1}{n} \to 0$, let $\frac{1}{n} = y$

$$= \frac{1}{2} \lim_{\frac{1}{n} \to 0} \frac{1}{y} \sin\left(\frac{\pi}{2}\right)\left(\frac{1}{n}\right)$$

$$= \frac{1}{2} \lim_{y \to 0} \frac{\sin\left(\frac{\pi}{2}\right)y}{y}$$

$$= \frac{1}{2} \left[\lim_{y \to 0} \frac{\sin\left(\frac{\pi y}{2}\right)}{\frac{\pi y}{2}}\right] \times \frac{\pi}{2}$$

$$= \frac{1}{2} \times 1 \times \frac{\pi}{2} \qquad\qquad \left[\because \lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1\right]$$

$$= \frac{\pi}{4}$$

Q25

$$\lim_{n \to \infty} 2^{n-1} \sin\left(\frac{a}{2^n}\right)$$

$$= \lim_{n \to \infty} \frac{2^n}{2} \sin\left(\frac{a}{2^n}\right)$$

$$= \lim_{n \to \infty} \frac{2^n}{2} \sin\frac{a}{2^n}$$

$n \to \infty, \frac{1}{n} = 0, let\, h = \frac{1}{n}$

$$= \lim_{h \to 0} \frac{2^{\frac{1}{h}}}{2} \sin\frac{a}{2^{\frac{1}{h}}}$$

$$= \lim_{h \to 0} \frac{2^{\frac{1}{h}}}{2} \frac{\sin\frac{a}{2^{\frac{1}{h}}}}{\frac{a}{2^{\frac{1}{h}}}} \times \frac{a}{2^{\frac{1}{h}}} \qquad\qquad \left[\because \lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1\right]$$

$$= \frac{a}{2}$$

Q26

$$= \frac{\lim\limits_{n \to \infty} \sin\left(\dfrac{a}{2^n}\right)}{\lim\limits_{n \to \infty} \sin\left(\dfrac{b}{2^n}\right)}$$

$$n \to \infty, \frac{1}{n} = h \to 0$$

$$= \frac{\lim\limits_{h \to 0} \sin\left(\dfrac{a}{2^{\frac{1}{h}}}\right)}{\lim\limits_{h \to 0} \sin\left(\dfrac{b}{2^{\frac{1}{h}}}\right)}$$

$$= \frac{\left\{ \lim\limits_{h \to 0} \dfrac{\sin\dfrac{a}{2^{\frac{1}{h}}}}{\dfrac{a}{2^{\frac{1}{h}}}} \times \dfrac{a}{2^{\frac{1}{h}}} \right\}}{\left\{ \lim\limits_{h \to 0} \dfrac{\sin\dfrac{b}{2^{\frac{1}{h}}}}{\dfrac{b}{2^{\frac{1}{h}}}} \times \dfrac{b}{2^{\frac{1}{h}}} \right\}}$$

$$= \frac{1 \times \dfrac{a}{2^{\frac{1}{h}}}}{1 \times \dfrac{b}{2^{\frac{1}{h}}}} = \frac{a}{b}$$

Q27

$$\lim_{x \to -1} \frac{x^2 - x - 2}{\left(x^2 + x\right) + \sin(x + 1)}$$

$$= \lim_{x \to -1} \frac{(x - 2)(x + 1)}{x(x + 1) + \sin(x + 1)}$$

$$= \lim_{x \to -1} \frac{1}{\dfrac{x(x + 1)}{(x - 2)(x + 1)} + \dfrac{\sin(x + 1)}{(x - 2)(x + 1)}}$$

$$= \lim_{x \to -1} \frac{1}{\dfrac{x}{x - 2} + \dfrac{\sin(x + 1)}{(x - 2)(x + 1)}}$$

$$= \lim_{x \to -1} \frac{1}{(x - 2)\left(x + \dfrac{\sin(x + 1)}{x + 1}\right)}$$

$$= \lim_{x \to -1} \frac{1}{x - 2} \times \frac{1}{\lim\limits_{x \to -1}(x) + \lim\limits_{x + 1 \to 0} \sin\dfrac{x + 1}{x + 1}}$$

$$= \left(\frac{1}{-1 - 2}\right) \times \frac{1}{(-1) + 1} \qquad \left[\because \lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1\right]$$

$$= \frac{1}{0} \qquad\qquad\qquad\qquad \left[\because \frac{1}{0} = \infty\right]$$

$$= \infty$$

Q28

$$\lim_{x \to 2} \frac{x^2 - x - 2}{x^2 - 2x + \sin(x-2)}$$

$$= \lim_{x \to 2} \frac{(x-2)(x+1)}{x^2 - 2x + \sin(x-2)}$$

$$= \lim_{x \to 2} \frac{1}{\dfrac{x}{x+1} + \dfrac{\sin(x-2)}{(x-2)(x+1)}}$$

$$= \lim_{x \to 2} (x+1) \left(\frac{1}{x + \dfrac{\sin(x-2)}{x-2}} \right)$$

$$= \lim_{x \to 2} (x+1) \times \frac{1}{\displaystyle\lim_{x \to 2}(x) + \lim_{x \to 2} \sin \frac{(x-2)}{x-2}}$$

$$= (2+1) \times \frac{1}{(2) + \displaystyle\lim_{x-2 \to 0} \frac{\sin(x-2)}{(x-2)}} \qquad \left[\because \lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1 \right]$$

$$= 3 \times \frac{1}{2+1}$$

$$= 1$$

Q29

$$\lim_{x \to 1} (1-x) \tan \frac{\pi x}{2}$$

When $x \to 1, x - 1 \to 0$, let $x - 1 = y$, then $y \to 0$

$$= \lim_{(x-1) \to 0} - (x-1) \tan \frac{\pi x}{2}$$

$$= - \lim_{y \to 0} y \tan \frac{\pi}{2} (y+1)$$

$$= - \lim_{y \to 0} y \times \tan \left(\frac{\pi}{2} + \frac{\pi}{2} y \right)$$

$$= \lim_{y \to 0} y \times \cot \frac{\pi}{2} y$$

$$= \lim_{y \to 0} \frac{y}{\tan \dfrac{\pi y}{2}}$$

$$= \lim_{y \to 0} \frac{\dfrac{\pi y}{2} \times \dfrac{2}{\pi}}{\tan \dfrac{\pi y}{2}}$$

$$= \frac{2}{\pi}$$

Q30

$$\lim_{x \to \frac{\pi}{4}} \frac{1 - \tan x}{1 - \sqrt{2}\sin x}$$

$x \to \frac{\pi}{4}$, then $x - \frac{\pi}{4} \to 0$, let $x - \frac{\pi}{4} = y$

$\Rightarrow \quad \lim_{x \to \frac{\pi}{4}} \frac{1 - \tan x}{1 - \sqrt{2}\sin x} = \lim_{x - \frac{\pi}{4} \to 0} \frac{1 - \tan x}{1 - \sqrt{2}\sin x}$

$$= \lim_{y \to 0} \frac{1 - \tan\left(y + \frac{\pi}{4}\right)}{1 - \sqrt{2}\sin\left(y + \frac{\pi}{4}\right)}$$

$$= \lim_{y \to 0} \frac{1 - \left(\dfrac{\tan\frac{\pi}{4} + \tan y}{1 + \tan\frac{\pi}{4}\tan y}\right)}{1 - \sqrt{2}\left(\sin y \cos\frac{\pi}{4} + \cos y \sin\frac{\pi}{4}\right)}$$

$$= \lim_{y \to 0} \frac{\left(1 - \left(\dfrac{1 + \tan y}{1 - \tan y}\right)\right)}{1 - \sqrt{2}\left(\dfrac{\sin y}{\sqrt{2}} + \dfrac{\cos y}{\sqrt{2}}\right)}$$

$$= \lim_{y \to 0} \frac{(1 - \tan y - 1 - \tan y)}{(1 - \tan y)(1 - \sin y + \cos y)}$$

$$= \lim_{y \to 0} \left(\frac{-2\tan y}{(1 - \tan y)(1 - \sin y - \cos y)}\right)$$

$$= -2 \lim_{y \to 0} \frac{\tan y \times 1}{\lim_{y \to 0}(1 - \tan y) \times \lim_{y \to 0}(1 - \sin y - \cos y)}$$

$$= \frac{-2\left(\lim_{y \to 0}\dfrac{\tan y}{y}\right) \times y}{\left(\lim_{y \to 0}(1) - \lim_{y \to 0}\tan y\right) \times \left(1 - \lim_{y \to 0}\dfrac{\sin y}{y} \times y - \cos 0\right)}$$

$$= \frac{-2}{(1 - y)(1 - y - 1)} = \frac{-2y}{(1 - y)(-y)} = \frac{2}{1 - y}$$

$$= \lim_{y \to 0} \frac{2}{1 - y} = 2$$

$= 2$

Q31

$$\lim_{x \to \pi} \frac{\left(\sqrt{2 + \cos x} - 1\right)}{(\pi - x)^2}$$

$\Rightarrow \quad x \to \pi$ then $x - \pi \to 0$ or let $x - \pi = y$

$\Rightarrow \quad \lim_{x \to \pi} \frac{\left(\sqrt{2 + \cos x} - 1\right)}{(\pi - x)^2} = \lim_{x - \pi \to 0} \frac{\sqrt{2 + \cos(x)} - 1}{(-1)^2 (x - \pi)^2}$

$= \lim_{y \to 0} \frac{\sqrt{2 + \cos(\pi + y)} - 1}{y^2}$

$= \lim_{y \to 0} \frac{\sqrt{2 - \cos y} - 1}{y^2}$

$= \lim_{y \to 0} \frac{\left(\sqrt{2 - \cos y} - 1\right)\left(\sqrt{2 - \cos y} + 1\right)}{y^2 \left(\sqrt{2 - \cos y} + 1\right)}$

$= \lim_{y \to 0} \frac{(2 - \cos y - 1)}{\left(\sqrt{2 - \cos y} + 1\right) y^2}$

$= \lim_{y \to 0} \frac{(1 - \cos y)}{\left(\sqrt{2 - \cos y} + 1\right) y^2}$

$= \lim_{y \to 0} \frac{2 \sin^2 \frac{y}{2}}{y^2 \left(\sqrt{2 - \cos y} + 1\right)}$

$= 2 \lim_{y \to 0} \left(\frac{\sin \frac{y}{2}}{\frac{y}{2}}\right)^2 \times \frac{1}{4} \times \frac{1}{\lim_{y \to 0} \sqrt{2 - \cos 0} + 1}$

$= 2 \times \frac{1}{4} \times \frac{1}{\sqrt{2 - 1} + 1} \qquad \left[\because \lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1\right]$

$= \frac{1}{4}$

Q32

$$\lim_{x \to \frac{\pi}{4}} \frac{\sqrt{\cos x} - \sqrt{\sin x}}{x - \frac{\pi}{4}}$$

$$= \lim_{x - \frac{\pi}{4} \to 0} \frac{\left(\sqrt{\cos x} - \sqrt{\sin x}\right)}{\left(x - \frac{\pi}{4}\right)} \times \frac{\left(\sqrt{\cos x} + \sqrt{\sin x}\right)}{\left(\sqrt{\cos x} + \sqrt{\sin x}\right)}$$

$$= \lim_{x - \frac{\pi}{4} \to 0} \frac{(\cos x - \sin x)}{\left(x - \frac{\pi}{4}\right)\left(\sqrt{\cos x} + \sqrt{\sin x}\right)}$$

$$As \; x \to \frac{\pi}{4} \Rightarrow x - \frac{\pi}{4} \to 0 \Rightarrow \; let \; x - \frac{\pi}{4} = y$$

$$= \lim_{y \to 0} \frac{\left(\cos\left(\frac{\pi}{4} + y\right) - \sin\left(\frac{\pi}{4} + y\right)\right)}{y\left(\sqrt{\cos\left(\frac{\pi}{4} + y\right)} + \sqrt{\sin\left(\frac{\pi}{4} + y\right)}\right)}$$

$$= \lim_{y \to 0} \frac{\left(\cos\frac{\pi}{4}\cos y - \sin\frac{\pi}{4}\sin y\right) - \left(\sin\frac{\pi}{4}\cos y + \cos\frac{\pi}{4}\sin y\right)}{y\left(\sqrt{\cos\left(\frac{\pi}{4} + y\right)} + \sqrt{\sin\left(\frac{\pi}{4} + y\right)}\right)}$$

$$= \lim_{y \to 0} \frac{\left(\frac{\cos y}{\sqrt{2}} - \frac{\sin y}{\sqrt{2}} - \frac{\cos y}{\sqrt{2}} - \frac{\sin y}{\sqrt{2}}\right)}{y\left(\sqrt{\cos\left(\frac{\pi}{4} + y\right)} + \sqrt{\sin\left(\frac{\pi}{4} + y\right)}\right)}$$

$$= \lim_{y \to 0} \frac{\left(-2\frac{\sin y}{\sqrt{2}}\right)}{y\left(\sqrt{\cos\left(y + \frac{\pi}{4}\right)} + \sqrt{\sin\left(y + \frac{\pi}{4}\right)}\right)}$$

$$= -\sqrt{2}\left(\lim_{y \to 0} \frac{\sin y}{y}\right) \times \frac{1}{\lim_{y \to 0}\sqrt{\cos\left(y + \frac{\pi}{4}\right)} + \lim_{y \to 0}\sqrt{\sin\left(y + \frac{\pi}{4}\right)}}$$

$$= -\sqrt{2} \times 1 \times \frac{1}{\sqrt{\cos\frac{\pi}{4}} + \sqrt{\sin\frac{\pi}{4}}}$$

$$= -\sqrt{2} \times \frac{1}{\left(\frac{1}{\sqrt{2}}\right)^{\frac{1}{2}} + \left(\frac{1}{\sqrt{2}}\right)^{\frac{1}{2}}}$$

$$= \frac{-\sqrt{2}}{\left(\frac{1}{\sqrt{2}}\right)^{\frac{1}{2}}(1 + 1)^{\frac{1}{2}}}$$

$$= \frac{-\sqrt{2}}{\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)^{\frac{1}{2}}}$$

$$= -\frac{1}{2^{\frac{1}{4}}}$$

$$\left[\because \cos\frac{\pi}{4} = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}\right]$$

Q33

$$\lim_{x \to 1} \frac{1 - \frac{1}{x}}{\sin \pi (x - 1)}$$

As $x \to 1$, then $x - 1 \to 0$ let $x - 1 = y$

$$= \lim_{x-1 \to 0} \frac{(x - 1)}{x \sin \pi (x - 1)}$$

$$= \lim_{y \to 0} \frac{y}{(y + 1) \sin(\pi y)}$$

$$= \lim_{y \to 0} \frac{y}{(y + 1)(\sin \pi y)}$$

$$= \lim_{y \to 0} \frac{1}{\dfrac{(y + 1) \sin \pi y}{y}}$$

$$= \frac{1}{\left(\lim_{y \to 0} (y + 1) \right) \times \left(\lim_{y \to 0} \dfrac{\sin \pi y}{y \times \pi} \times \pi \right)}$$

$$= \frac{1}{(1)(1 \times \pi)} \qquad \left[\because \lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1 \right]$$

$$= \frac{1}{\pi}$$

Q34

$$\lim_{x \to \frac{\pi}{4}} \frac{\sqrt{2} - \cos x - \sin x}{(4x - \pi)^2}$$

$x \to \frac{\pi}{4}$ then $x - \frac{\pi}{4} \to 0$, also $4x - \pi \to 0$ let $x - \frac{\pi}{4} \to y$

$$\lim_{x - \frac{\pi}{4} \to 0} \frac{\sqrt{2} - \cos x - \sin x}{(4)^2 \left(x - \frac{\pi}{4}\right)^2}$$

$$= \lim_{y \to 0} \frac{\sqrt{2} - \cos\left(y + \frac{\pi}{4}\right) - \sin\left(y + \frac{\pi}{4}\right)}{16 \times y^2}$$

$$= \frac{1}{16} \lim_{y \to 0} \frac{\sqrt{2} - \left(\cos y \cos \frac{\pi}{4} - \sin y \sin \frac{\pi}{4}\right) - \left(\sin y \cos \frac{\pi}{4} + \cos \sin \frac{\pi}{4}\right)}{y^2}$$

$$= \frac{1}{16} \lim_{y \to 0} \frac{\sqrt{2} - \left(\cos y \times \frac{1}{\sqrt{2}} - \sin y \times \frac{1}{\sqrt{2}}\right) - \left(\frac{\sin y}{\sqrt{2}} + \frac{\cos y}{\sqrt{2}}\right)}{y^2}$$

$$= \frac{1}{16} \lim_{y \to 0} \frac{\sqrt{2} - \frac{1}{\sqrt{2}}(\cos y - \sin y) - \frac{1}{\sqrt{2}}(\sin y + \cos y)}{y^2}$$

$$= \frac{1}{16} \lim_{y \to 0} \frac{\sqrt{2} - \frac{1}{\sqrt{2}}[(\cos y - \sin y) - (\sin y + \cos y)]}{y^2}$$

$$= \frac{1}{16} \lim_{y \to 0} \frac{\left(\sqrt{2} - \frac{1}{\sqrt{2}}\cos y + \frac{1}{\sqrt{2}}\sin y - \frac{1}{\sqrt{2}}\sin y - \frac{1}{\sqrt{2}}\cos y\right)}{y^2}$$

$$= \frac{1}{16} \lim_{y \to 0} \frac{\sqrt{2} + \frac{2}{\sqrt{2}}\cos y}{y^2}$$

$$= \frac{1}{16} \lim_{y \to 0} \frac{\sqrt{2}(1 - \cos y)}{y^2}$$

$$= \frac{\sqrt{2}}{16} \lim_{y \to 0} \frac{2\sin^2 \frac{y}{2}}{y^2}$$

$$= \frac{\sqrt{2}}{8} \lim_{y \to 0} \left(\frac{\sin \frac{y}{2}}{\frac{y}{2}}\right)^2 \times \frac{1}{4}$$

$$= \frac{1}{16\sqrt{2}}$$

Q35

$$\lim_{x \to \frac{\pi}{2}} \frac{\left(\frac{\pi}{2} - x\right) \sin x - 2\cos x}{\left(\frac{\pi}{2} - x\right) + \cot x}$$

If $x \to \frac{\pi}{2}$, then $\frac{\pi}{2} - x \to 0$

Let $\frac{\pi}{2} - x = y$

$$= \lim_{y \to 0} \frac{\left(y \sin\left(\frac{\pi}{2} - y\right) - 2\cos\left(\frac{\pi}{2} - y\right)\right)}{y + \cot\left(\frac{\pi}{2} - y\right)}$$

$$= \lim_{y \to 0} \left(\frac{y \cos y - 2\sin y}{1 + \tan y}\right)$$

$$= \lim_{y \to 0} \left(\frac{\cos y - 2\dfrac{\sin y}{y}}{1 + \dfrac{\tan y}{y}}\right)$$

$$= \frac{\lim_{y \to 0} \cos y - 2 \lim_{y \to 0} \dfrac{\sin y}{y}}{1 + \lim_{y \to 0} \dfrac{\tan y}{y}}$$

$$= \frac{1 - 2}{1 + 1} = \frac{-1}{2}$$

$$= -\frac{1}{2}$$

$\left[\because \lim_{y \to 0} \dfrac{\sin y}{y} = 1, \lim_{y \to 0} \dfrac{\tan y}{y} = 1\right]$

Q36

$$\lim_{x \to \frac{\pi}{4}} \frac{\cos x - \sin x}{\left(\frac{\pi}{4} - x\right)(\cos x + \sin x)}$$

$\Rightarrow \quad x \to \frac{\pi}{4}$ then $x - \frac{\pi}{4} \to 0$, let $x - \frac{\pi}{4} = y$

$$= \lim_{y \to 0} \frac{\cos\left(\frac{\pi}{4} + y\right) - \sin\left(\frac{\pi}{4} + y\right)}{-y\left(\cos\left(\frac{\pi}{4} + y\right) + \sin\left(\frac{\pi}{4} + y\right)\right)}$$

$$= \lim_{y \to 0} \frac{\left[\left(\cos\frac{\pi}{4}\cos y - \sin\frac{\pi}{4}\sin y\right) - \left(\sin\frac{\pi}{4}\cos y + \cos\frac{\pi}{4}\sin y\right)\right]}{-y\left(\cos\left(\frac{\pi}{4} + y\right) + \sin\left(\frac{\pi}{4} + y\right)\right)}$$

$$= \lim_{y \to 0} \frac{\left[\frac{\cos y}{\sqrt{2}} - \frac{\sin y}{\sqrt{2}} - \frac{\cos y}{\sqrt{2}} - \frac{\sin y}{\sqrt{2}}\right]}{-y\left(\cos\left(\frac{\pi}{4} + y\right) + \sin\left(\frac{\pi}{4} + y\right)\right)}$$

$$= \lim_{y \to 0} \frac{\frac{-2\sin y}{\sqrt{2}}}{-y\left(\cos\left(\frac{\pi}{4} + y\right) + \sin\left(\frac{\pi}{4} + y\right)\right)}$$

$$= \sqrt{2}\lim_{y \to 0}\left(\frac{\sin y}{y}\right) \times \frac{1}{\lim_{y \to 0}\left(\cos\left(\frac{\pi}{4} + y\right) + \sin\left(\frac{\pi}{4} + y\right)\right)}$$

$$= \sqrt{2} \times 1 \times \frac{1}{\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}} = \sqrt{2} \times \frac{1}{\frac{2}{\sqrt{2}}}$$

$$= \frac{\sqrt{2} \times \sqrt{2}}{2} = 1$$

Q37

$$\lim_{x \to \pi} \frac{1 - \sin\frac{x}{2}}{\cos\frac{x}{2}\left(\cos\frac{x}{4} \quad \sin\frac{x}{4}\right)} = \lim_{h \to 0} \frac{1 - \sin\left(\frac{\pi + h}{2}\right)}{\cos\left(\frac{\pi + h}{2}\right)\left(\cos\left(\frac{\pi + h}{4}\right) - \sin\left(\frac{\pi + h}{4}\right)\right)}$$

$$- \lim_{h \to 0} \frac{1 - \cos\left(\frac{h}{2}\right)}{-\sin\left(\frac{h}{2}\right)\left(\frac{1}{\sqrt{2}}\cos\left(\frac{h}{4}\right) - \frac{1}{\sqrt{2}}\sin\left(\frac{h}{4}\right) - \frac{1}{\sqrt{2}}\sin\left(\frac{h}{4}\right) - \frac{1}{\sqrt{2}}\cos\left(\frac{h}{4}\right)\right)}$$

$$= \lim_{h \to 0} \frac{1 - \cos\left(\frac{h}{2}\right)}{\sqrt{2}\sin\left(\frac{h}{2}\right)\sin\left(\frac{h}{4}\right)}$$

$$= \lim_{h \to 0} \frac{2\sin^2\left(\frac{h}{4}\right)}{\sqrt{2}\sin\left(\frac{h}{2}\right)\sin\left(\frac{h}{4}\right)}$$

$$= \sqrt{2}\lim_{h \to 0} \frac{\sin\left(\frac{h}{4}\right)}{\sin\left(\frac{h}{2}\right)}$$

Ex 29.9

Q1

$$= \lim_{x \to \pi} \frac{1 + \cos x}{\tan^2 x}$$

As $x \to \pi$, $x - \pi \to 0$, let $x - \pi = y$

$$= \lim_{y \to 0} \frac{1 + \cos(\pi + y)}{\tan^2(\pi + y)}$$

$$= \lim_{y \to 0} \frac{1 - \cos y}{\tan^2 y}$$

$$= \lim_{y \to 0} \frac{2 \sin^2 \frac{y}{2}}{\tan^2 y}$$

$$= \frac{\lim\limits_{y \to 0} 2 \sin^2 \frac{y}{2}}{\lim\limits_{y \to 0} \tan^2 y}$$

$$= \frac{2 \left(\lim\limits_{y \to 0} \dfrac{\sin \frac{y}{2}}{\frac{y}{2}} \right)^2 \times \dfrac{y^2}{4}}{\left(\lim\limits_{y \to 0} \dfrac{\tan y}{y} \right) \times y^2}$$

$$= \frac{2 \times 1 \times \dfrac{y^2}{4}}{1 \times y^2}$$

$$\left[\because \lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1 \atop \lim_{\theta \to 0} \frac{\tan \theta}{\theta} = 1 \right]$$

$$= 2 \times 1 \times \frac{1}{4}$$

$$= \frac{1}{2}$$

Q2

$$\lim_{x \to \frac{\pi}{4}} \frac{\csc^2 x - 2}{\cot x - 1}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{\cot^2 x + 1 - 2}{\cot x - 1}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{\cot^2 x - 1}{\cot x - 1}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{(\cot x - 1)(\cot x + 1)}{(\cot x - 1)}$$

$$= \lim_{x \to \frac{\pi}{4}} (\cot x + 1)$$

$$= \cot \frac{\pi}{4} + 1$$

$$= 1 + 1$$

$$= 2$$

Q3

$$\lim_{x \to \frac{\pi}{6}} \frac{\csc^2 x - 1 - 3}{\csc x - 2}$$

$$= \lim_{x \to \frac{\pi}{6}} \frac{\csc^2 x - 4}{\csc x - 2}$$

$$= \lim_{x \to \frac{\pi}{6}} \frac{(\csc x - 2)(\csc x + 2)}{(\csc x - 2)}$$

$$= \lim_{x \to \frac{\pi}{6}} (\csc x + 2)$$

$$= \csc \frac{\pi}{6} + 2$$

$$= 2 + 2$$

$$= 4$$

Q4

$$\lim_{x \to \frac{\pi}{4}} \frac{2 - cosec^2 x}{1 - \cot x}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{2 - \left(1 + \cot^2 x\right)}{1 - \cot x}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{2 - 1 - \cot^2 x}{1 - \cot x}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{1 - \cot^2 x}{1 - \cot x}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{\left(1 - \cot x\right)\left(1 + \cot x\right)}{\left(1 - \cot x\right)}$$

$$= \lim_{x \to \frac{\pi}{4}} \left(1 + \cot x\right)$$

$$= 1 + \cot \frac{\pi}{4}$$

$$= 1 + 1$$

$$= 2$$

Q5

$$\lim_{x \to \pi} \frac{\sqrt{2 + \cos x} - 1}{(\pi - x)^2}$$

$$= \lim_{x \to \pi} \frac{\sqrt{2 + \cos x} - 1}{(\pi - x)^2} \times \frac{\sqrt{2 + \cos x} + 1}{\sqrt{2 + \cos x} + 1}$$

$$= \lim_{x \to \pi} \frac{(2 + \cos x) - 1}{(\pi - x)^2 \left(\sqrt{2 + \cos x} + 1\right)}$$

$$= \lim_{x \to \pi} \frac{1 + \cos x}{(\pi - x)^2 \left(\sqrt{2 + \cos x} + 1\right)}$$

Let $\pi - x = y, x \to \pi, y \to 0$

$$\Rightarrow \quad \lim_{x \to \pi} \frac{1 + \cos x}{(\pi - x)^2 \left(\sqrt{2 + \cos x} + 1\right)} = \lim_{y \to 0} \frac{1 + \cos(\pi - y)}{y^2 \left(\sqrt{2 + \cos(\pi - y)} + 1\right)}$$

$$= \lim_{y \to 0} \frac{1 - \cos y}{y^2 \sqrt{2 - \cos y} + 1}$$

$$= \lim_{y \to 0} \frac{\dfrac{2 \sin^2 y}{2}}{y^2 \sqrt{2 - \cos y} + 1}$$

$$= 2 \lim_{y \to 0} \left(\frac{\sin \dfrac{y}{2}}{\dfrac{y}{2}}\right)^2 \times \frac{1}{4} \frac{1}{\sqrt{2 - \cos y} + 1}$$

$$= 2 \times \left(\lim_{y \to 0} \frac{\sin y}{2}\right)^2 \times \frac{1}{4} \frac{1}{\lim_{y \to 0} \sqrt{2 - \cos y} + 1}$$

$$= 2 \times 1 \times \frac{1}{4} \times \frac{1}{\sqrt{2 - \cos 0} + 1}$$

$$= 2 \times 1 \times \frac{1}{4} \times \frac{1}{\sqrt{2 - 1} + 1}$$

$$= 2 \times 1 \times \frac{1}{4} \times \frac{1}{1 + 1}$$

$$= 2 \times 1 \times \frac{1}{4} \times \frac{1}{2}$$

$$= \frac{1}{4}$$

Q6

$$\lim_{x \to \frac{3\pi}{2}} \frac{1 + cosec^3 x}{cot^2 x}$$

$$= \lim_{x \to \frac{3\pi}{2}} \frac{(1 + cosec x)(1 + cosec^2 x - cosec x)}{(cosec^2 x - 1)}$$

$$= \lim_{x \to \frac{3\pi}{2}} \frac{(cosec x + 1)(1 + cosec^2 x - cosec x)}{(cosec x - 1)(cosec x + 1)}$$

$$= \lim_{x \to \frac{3\pi}{2}} \frac{(1 + cosec^2 x - cosec x)}{cosec x - 1}$$

$$= \frac{1 + cosec^2 \frac{3\pi}{2} - cosec \frac{3\pi}{2}}{cosec \frac{3\pi}{2} - 1}$$

$$= \frac{1 + (-1)^2 - (-1)}{(-1) - 1} \qquad \left[\because cosec \frac{3\pi}{2} = -1 \right]$$

$$= \frac{1 + 1 + 1}{-2}$$

$$= \frac{-3}{2}$$

Ex 29.10

Q1

$$\lim_{x \to 0} \frac{5^x - 1}{\sqrt{4+x} - 2}$$

$$= \lim_{x \to 0} \frac{(5^x - 1)(\sqrt{4+x} + 2)}{(\sqrt{4+x} - 2)(\sqrt{4+x} + 2)}$$

$$= \lim_{x \to 0} \frac{(5^x - 1)(\sqrt{4+x} + 2)}{x}$$

$$= 4 \log 5$$

Q2

$$\lim_{x \to 0} \frac{\log(1+x)}{3^x - 1}$$

$$= \lim_{x \to 0} \frac{\log(1+x)}{x} \times \frac{1}{\lim\limits_{x \to 0} \frac{3^x - 1}{x}}$$

$$= \frac{1}{\log 3}$$

Q3

$$\lim_{x \to 0} \frac{a^x + a^{-x} - 2}{x^2}$$

$$= \lim_{x \to 0} \frac{a^{2x} - 2a^x + 1}{a^x \cdot x^2}$$

$$= \lim_{x \to 0} \left(\frac{a^x - 1}{x}\right)^2 \times \frac{1}{a^x}$$

$$= (\log_e a)^2 \times \frac{1}{a^0}$$

$$= (\log_e a)^2$$

Q4

$$\lim_{x \to 0} \frac{a^{mx} - 1}{b^{nx} - 1}, n \neq 0$$

$$= \lim_{x \to 0} \frac{a^{mx} - 1}{mx} \times \frac{1}{\lim\limits_{x \to 0} \frac{b^{nx} - 1}{nx}} \times \frac{m}{n}$$

$$= \frac{m \log a}{n \log b}, n \neq 0$$

Q5

$$\lim_{x \to 0} \frac{a^x + b^x - 2}{x}$$

$$= \lim_{x \to 0} \frac{a^x - 1}{x} + \lim_{x \to 0} \frac{b^x - 1}{x}$$

$$= \log a + \log b$$

$$= \log (ab)$$

Q6

$$\lim_{x \to 0} \frac{9^x - 2.6^x + 4^x}{x^2}$$

$$= \lim_{x \to 0} \frac{(3^x)^2 - 2.3^x 2^x + (2^x)^2}{x^2}$$

$$= \lim_{x \to 0} \left(\frac{3^x - 2^x}{x} \right)^2$$

$$= \left(\lim_{x \to 0} \frac{3^x - 1}{x} - \lim_{x \to 0} \left(\frac{2^x - 1}{x} \right) \right)^2$$

$$= \left(\log \frac{3}{2} \right)^2$$

Q7

$$\lim_{x \to 0} \frac{8^x - 4^x - 2^x + 1}{x^2}$$

$$= \lim_{x \to 0} \frac{(2^x - 1)^2 (2^x + 1)}{x^2}$$

$$= \lim_{x \to 0} \left(\frac{(2^x - 1)}{x} \right)^2 \lim_{x \to 0} (2^x + 1)$$

$$= 2 \,(\log 2)^2$$

Q8

$$\lim_{z \to 0} \frac{a^{mx} - b^{nx}}{x}$$

$$= \lim_{x \to 0} m \frac{a^{mx} - 1}{mx} - \lim_{x \to 0} n \frac{b^{nx} - 1}{nx}$$

$$= m \log a - n \log b$$

$$= \log \left(\frac{a^m}{b^n} \right)$$

Q9

$$\lim_{x \to 0} \frac{a^x + b^x + c^x - 3}{x}$$

$$= \lim_{x \to 0} \frac{a^x - 1}{x} + \lim_{x \to 0} \frac{b^x - 1}{x} + \lim_{x \to 0} \frac{c^x - 1}{x}$$

$$= \log a + \log b + \log c$$

$$= \log (abc)$$

Q10

Let $x - 2 = h$

$$\lim_{h \to 0} \frac{h}{\log_a (h + 1)}$$

$$= \lim_{h \to 0} \frac{\log a}{\frac{\log(h + 1)}{h}}$$

$$= \log a$$

Q11

$$\lim_{x \to 0} \frac{5^x + 3^x + 2^x - 3}{x}$$

$$= \lim_{x \to 0} \frac{5^x - 1}{x} + \lim_{x \to 0} \frac{3^x - 1}{x} + \lim_{x \to 0} \frac{2^x - 1}{x}$$

$$= \log 5 + \log 3 + \log 2$$

$$= \log 30$$

Q12

$$Let \; \frac{1}{x} = h$$

$$\lim_{h \to 0} \frac{(a^h - 1)}{h}$$

$$= \log a$$

Q13

$$\lim_{x \to 0} \frac{a^{mx} - b^{nx}}{\sin kx}$$

$$= \lim_{x \to 0} \frac{a^{mx} - b^{nx}}{kx \; \frac{\sin kx}{kx}}$$

$$= \frac{1}{k} \lim_{x \to 0} \frac{\frac{(a^{mx} - b^{nx})}{x}}{\frac{\sin kx}{kx}}$$

$$= \frac{1}{k} \log \frac{a^m}{b^n}$$

Q14

$$\lim_{x \to 0} \frac{a^x + b^x - c^x - d^x}{x}$$

$$= \lim_{x \to 0} \frac{a^x - 1}{x} + \lim_{x \to 0} \frac{a^x - 1}{x} - \lim_{x \to 0} \frac{c^x - 1}{x} - \lim_{x \to 0} \frac{d^x - 1}{x}$$

$$= \log a + \log b - \log c - \log d$$

$$= \log \left(\frac{ab}{cd} \right)$$

Q15

$$\lim_{x \to 0} \frac{e^x - 1 + \sin x}{x}$$

$$= \lim_{x \to 0} \frac{e^x - 1}{x} + \lim_{x \to 0} \frac{\sin x}{x}$$

$$= \log e + 1$$

$$= 2$$

Q16

$$\lim_{x \to 0} \frac{\sin 2x}{e^x - 1}$$

$$= \lim_{x \to 0} \frac{\sin 2x}{2x} \times 2 \lim_{x \to 0} \frac{x}{e^x - 1}$$

$$= 1 \times 2 \times \log e$$

$$= 2$$

Q17

$$\lim_{x \to 0} \frac{e^{\sin x} - 1}{x}$$

$$= \lim_{x \to 0} \frac{e^{\sin x} - 1}{\sin x} \times \lim_{x \to 0} \frac{\sin x}{x}$$

$$= \log e \times 1$$

$$= 1$$

Q18

$$\lim_{x \to 0} \frac{e^{2x} - e^x}{\sin 2x}$$

$$= \lim_{x \to 0} \frac{e^{2x} - 1}{\sin 2x} - \lim_{x \to 0} \frac{e^x - 1}{\sin 2x}$$

$$= \left(\lim_{x \to 0} \frac{e^{2x} - 1}{2x} \times \lim_{x \to 0} \frac{2x}{\sin 2x} \right) - \frac{1}{2} \left(\lim_{x \to 0} \frac{e^x - 1}{x} \times \lim_{x \to 0} \frac{2x}{\sin 2x} \right)$$

$$= 1 - \frac{1}{2}$$

$$= \frac{1}{2}$$

Q19

$$\lim_{x \to a} \frac{\log x - \log a}{x - a}$$

$$= \lim_{x \to a} \frac{\log \dfrac{x}{a}}{a \left(\dfrac{x}{a} - 1 \right)}$$

$$let \ h = \frac{x}{a} - 1$$

$$= \frac{1}{a} \lim_{x \to a} \frac{\log(h + 1)}{h}$$

$$= \frac{1}{a}$$

Q20

$$\lim_{x \to 0} \frac{\log(a + x) - \log(a - x)}{x}$$

$$= \lim_{x \to 0} \frac{\log \left(\dfrac{a + x}{a - x} \right)}{x}$$

$$= \lim_{x \to 0} \frac{\log \left(1 + \dfrac{2x}{a - x} \right)}{\dfrac{2x}{a - x}} \times \lim_{x \to 0} \frac{2}{a - x}$$

$$= \frac{2}{a}$$

Q21

$$\lim_{x \to 0} \frac{\log(2 + x) + \log(0.5)}{x}$$

$$= \lim_{x \to 0} \frac{\log\left(1 + \dfrac{x}{2}\right)}{2\left(\dfrac{x}{2}\right)}$$

$$= \frac{1}{2}$$

Q22

$$\lim_{x \to 0} \frac{\log(a + x) - \log(a)}{x}$$

$$= \lim_{x \to 0} \frac{\log\left(1 + \dfrac{x}{a}\right)}{a\left(\dfrac{x}{a}\right)}$$

$$= \frac{1}{a}$$

Q23

$$\lim_{x \to 0} \frac{\log(3 + x) - \log(3 - x)}{x}$$

$$= \lim_{x \to 0} \frac{\log\left(\dfrac{3 + x}{3 - x}\right)}{x}$$

$$= \lim_{x \to 0} \frac{\log\left(1 + \dfrac{2x}{3 - x}\right)}{x}$$

$$= \lim_{x \to 0} \frac{\log\left(1 + \dfrac{2x}{3 - x}\right)}{\dfrac{2x}{3 - x}} \times \lim_{x \to 0} \frac{2}{3 - x}$$

$$= \frac{2}{3}$$

Q24

$$\lim_{x \to 0} \frac{8^x - 2^x}{x}$$

$$= \lim_{x \to 0} \frac{8^x - 1}{x} - \lim_{x \to 0} \frac{2^x - 1}{x}$$

$$= \log 8 - \log 2$$

$$= \log 4$$

Q25

$$\lim_{x \to 0} \frac{x(2^x - 1)}{1 - \cos x}$$

$$= \lim_{x \to 0} \frac{x(2^x - 1)}{2\sin^2\left(\frac{x}{2}\right)}$$

$$= \lim_{x \to 0} \frac{(2^x - 1)}{x} \times \lim_{x \to 0} \frac{x^2}{\left(\frac{\sin\left(\frac{x}{2}\right)}{\frac{x}{2}}\right)^2 \times \frac{x^2}{2}}$$

$$= 2\log 2$$

$$= \log 4$$

Q26

$$\lim_{x \to 0} \frac{\sqrt{1 + x} - 1}{\log(1 + x)}$$

$$= \lim_{x \to 0} \frac{(\sqrt{1 + x} - 1)(\sqrt{1 + x} + 1)}{\log(1 + x)(\sqrt{1 + x} + 1)}$$

$$= \lim_{x \to 0} \frac{x}{\log(1 + x)(\sqrt{1 + x} + 1)}$$

$$= \lim_{x \to 0} \frac{1}{\frac{\log(1 + x)}{x}} \times \lim_{x \to 0} \frac{1}{(\sqrt{1 + x} + 1)}$$

$$= 1 \times \frac{1}{2}$$

$$= \frac{1}{2}$$

Q27

$$\lim_{x \to 0} \frac{\log|1 + x^3|}{\sin^3 x}$$

$$= \lim_{x \to 0} \frac{\log|1 + x^3|}{\sin^3 x} \times \frac{1}{\lim\limits_{x \to 0}\left(\frac{\sin x}{x}\right)^3}$$

$$= 1 \times 1$$

$$= 1$$

Q28

$$\lim_{x \to \frac{\pi}{2}} \frac{a^{\cot x} - a^{\cos x}}{\cot x - \cos x}$$

$$= \lim_{x \to \frac{\pi}{2}} a^{\cos x}\left[\frac{a^{\cot x - \cos x} - 1}{\cot x - \cos x}\right]$$

$$= 1 \times \log a$$

$$= \log a$$

Q29

$$\lim_{x \to 0} \frac{e^x - 1}{\sqrt{1 - \cos x}}$$

$$= \lim_{x \to 0} \frac{(e^x - 1)(\sqrt{1 + \cos x})}{(\sqrt{1 - \cos x})(\sqrt{1 + \cos x})}$$

$$= \lim_{x \to 0} \frac{(e^x - 1)(\sqrt{1 + \cos x})}{\sin x}$$

Both numerator and denominator are both zeros for $x = 0$
hence limit can not exist

Q30

$$\lim_{x \to 0} \frac{e^{5+h} - e^5}{h}$$

$$= e^5 \lim_{x \to 0} \frac{e^h - 1}{h}$$

$$= e^5 \times 1$$

$$= e^5$$

Q31

$$\lim_{x \to 0} \frac{e^{x+2} - e^2}{x}$$

$$= e^2 \lim_{x \to 0} \frac{e^x - 1}{x}$$

$$= e^2$$

Q32

$$Let \ x - \frac{\pi}{2} = h$$

$$\lim_{h \to 0} \frac{e^{-\sin h} - 1}{-\sin h}$$

$$= \lim_{\sin h \to 0} \frac{e^{-\sin h} - 1}{-\sin h}$$

$$= 1$$

Q33

$$\lim_{x \to 0} \frac{e^{3+x} - \sin x - e^3}{x}$$

$$= e^3 \lim_{x \to 0} \frac{e^x - 1}{x} - \lim_{x \to 0} \frac{\sin x}{x}$$

$$= e^3 \log e - 1$$

$$= e^3 - 1$$

Q34

$$\lim_{x \to 0} \frac{e^x - x - 1}{x}$$

$$= \lim_{x \to 0} \frac{e^x - 1}{x} - 1$$

$$= 1 - 1$$

$$= 0$$

Q35

$$\lim_{x \to 0} \frac{e^{3x} - e^{2x}}{x}$$

$$= 3 \lim_{x \to 0} \frac{e^{3x} - 1}{3x} - \lim_{x \to 0} \frac{e^{2x} - 1}{2x}$$

$$= 3 - 2$$

$$= 1$$

Q36

$$\lim_{x \to 0} \frac{e^{\tan x} - 1}{\tan x}$$

$$= \lim_{\tan x \to 0} \frac{e^{\tan x} - 1}{\tan x}$$

$$= 1$$

Q37

$$\lim_{x \to 0} \frac{e^{bx} - e^{ax}}{x}$$

$$= b \lim_{x \to 0} \frac{e^{bx} - 1}{bx} - a \lim_{x \to 0} \frac{e^{ax} - 1}{ax}$$

$$= b - a$$

Q38

$$\lim_{x \to 0} \frac{e^{\tan x} - 1}{x}$$

$$= \lim_{x \to 0} \frac{e^{\tan x} - 1}{\tan x} \times \lim_{x \to 0} \frac{\tan x}{x}$$

$$= \log e \times 1$$

$$= 1$$

Q39

$$\lim_{x \to 0} \frac{e^x - e^{\sin x}}{x - \sin x}$$

$$= \lim_{x \to 0} e^{\sin x} \left[\frac{e^{x - \sin x} - 1}{x - \sin x} \right]$$

$$= 1 \times \log e$$

$$= 1$$

Q40

$$\lim_{x \to 0} \frac{3^{2+x} - 9}{x} = \lim_{x \to 0} \frac{3^2 \cdot 3^x - 9}{x}$$

$$= 9 \lim_{x \to 0} \frac{3^x - 1}{x}$$

$$= 9 \log_e 3$$

Q41

$$\lim_{x \to 0} \frac{a^x - a^{-x}}{x} = \lim_{x \to 0} \frac{a^{2x} - 1}{x a^x}$$

$$= \lim_{x \to 0} \frac{2(a^{2x} - 1)}{2x} \lim_{x \to 0} \frac{1}{a^x}$$

$$= 2 \log_e 2$$

Q42

$$\lim_{x \to 0} \frac{x(e^x - 1)}{1 - \cos x}$$

$$= \lim_{x \to 0} \frac{x(e^x - 1)}{2\sin^2\left(\dfrac{x}{2}\right)}$$

$$= \lim_{x \to 0} \frac{(e^x - 1)}{2x} \times \lim_{x \to 0} \frac{4}{\left(\dfrac{\sin\left(\dfrac{x}{2}\right)}{\dfrac{x}{2}}\right)^2}$$

$$= \frac{1}{2} \times 4$$

$$= 2$$

Q43

$$\lim_{x \to \frac{\pi}{2}} \frac{2^{-\cos x} - 1}{x\left(x - \dfrac{\pi}{2}\right)} = \lim_{x \to \frac{\pi}{2}} \frac{2^{\sin\left(x - \frac{\pi}{2}\right)} - 1}{\left(x - \dfrac{\pi}{2}\right)} \times \frac{1}{x}$$

$$= \frac{2}{\pi} \log_e 2$$

Ex 29.11

Q1

$$\lim_{n \to \infty} \left(1 + \frac{x}{n}\right)^n = e^{\lim_{n \to \infty} \left(\frac{x}{n}\right) n}$$

$$= e^x$$

Q2

$$\lim_{x \to 0^+} \left\{1 + \tan^2 \sqrt{x}\right\}^{1/2x}$$

$$= e^{\lim_{x \to 0^+} \left\{\frac{\tan^2 \sqrt{x}}{2x}\right\}}$$

$$= e^{\lim_{x \to 0^+} \left\{\frac{\tan^2 \sqrt{x}}{2x}\right\}}$$

$$= e^{\lim_{x \to 0^+} \left\{\frac{\sin^2 \sqrt{x}}{2x \cos^2 \sqrt{x}}\right\}}$$

$$= e^{\frac{1}{2} \lim_{x \to 0^+} \left\{\left(\frac{\sin \sqrt{x}}{\sqrt{x}}\right)^2\right\} \lim_{x \to 0^+} \left\{\frac{1}{\cos^2 \sqrt{x}}\right\}}$$

$$= e^{\frac{1}{2}}$$

$$= \sqrt{e}$$

Q3

$$\lim_{x \to 0} (\cos x)^{1/\sin x} = \lim_{x \to 0} (1 + \cos x - 1)^{1/\sin x}$$

$$= \lim_{x \to 0} (1 - (1 - \cos x))^{1/\sin x}$$

$$= \lim_{x \to 0} \left(1 - 2\sin^2\left(\frac{x}{2}\right)\right)^{1/\sin x}$$

$$= e^{\lim_{x \to 0}\left(-2\sin^2\left(\frac{x}{2}\right)\right) \times (1/\sin x)}$$

$$= e^{\lim_{x \to 0}\left(\frac{-2\sin^2\left(\frac{x}{2}\right)}{\sin x}\right)}$$

$$= e^{\lim_{x \to 0}\left(\frac{-2\sin^2\left(\frac{x}{2}\right)}{2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)}\right)}$$

$$= e^{\lim_{x \to 0} -\tan x}$$

$$= e^0$$

$$= 1$$

Q4

$$\lim_{x \to 0} (\cos x + \sin x)^{1/x}$$

$$= \lim_{x \to 0} (1 + (\cos x + \sin x - 1))^{1/x}$$

$$= e^{\lim_{x \to 0}\frac{(\cos x + \sin x - 1)}{x}}$$

$$= e^{\lim_{x \to 0}\frac{(\sin x - (1 - \cos x))}{x}}$$

$$= e^{\lim_{x \to 0}\frac{(\sin x - 2\sin^2(x/2))}{x}}$$

$$= e^{\lim_{x \to 0}\frac{\sin x}{x} - \lim_{x \to 0}\frac{2\sin(x/2)\sin(x/2)}{2\left(\frac{x}{2}\right)}}$$

$$= e^{\lim_{x \to 0}\frac{\sin x}{x} - \lim_{x \to 0}\frac{\sin(x/2)\sin(x/2)}{\left(\frac{x}{2}\right)}}$$

$$= e^{1-0}$$

$$= e$$

Q5

$$\lim_{x \to 0} (\cos x + a \sin bx)^{1/x}$$

$$= \lim_{x \to 0} (1 + (\cos x + a \sin bx - 1))^{1/x}$$

$$= e^{\lim\limits_{x \to 0} \frac{(\cos x + a \sin bx - 1)}{x}}$$

$$= e^{\lim\limits_{x \to 0} \frac{(a \sin bx - (1 - \cos x))}{x}}$$

$$= e^{\lim\limits_{x \to 0} \frac{(a \sin bx - 2\sin^2(x/2))}{x}}$$

$$= e^{\lim\limits_{x \to 0} \frac{ab \sin bx}{bx} - \lim\limits_{x \to 0} \frac{2\sin(x/2)\sin(x/2)}{2\left(\frac{x}{2}\right)}}$$

$$= e^{\lim\limits_{x \to 0} \frac{ab \sin bx}{bx} - \lim\limits_{x \to 0} \frac{\sin(x/2)\sin(x/2)}{\left(\frac{x}{2}\right)}}$$

$$= e^{ab - 0}$$

$$= e^{ab}$$

Q6

$$\lim_{x \to \infty} \left\{ \frac{x^2 + 2x + 3}{2x^2 + x + 5} \right\}^{\frac{3x-2}{3x+2}}$$

$$= e^{\lim\limits_{x \to \infty} \left\{ \left(\frac{3x-2}{3x+2}\right) \ln\left(\frac{x^2+2x+3}{2x^2+x+5}\right) \right\}}$$

$$= e^{\lim\limits_{x \to \infty} \left\{ \left(\frac{3 - \frac{2}{x}}{3 + \frac{2}{x}}\right) \left(\ln\left(\frac{x^2+2x+3}{2x^2+x+5}\right) \right) \right\}}$$

$$= e^{\lim\limits_{x \to \infty} \left\{ \left(\frac{3 - \frac{2}{x}}{3 + \frac{2}{x}}\right) \left(\ln\left(\frac{1 + \frac{2}{x} + \frac{3}{x^2}}{2 + \frac{1}{x} + \frac{5}{x^2}}\right) \right) \right\}}$$

$$= e^{1 \ln\left(\frac{1}{2}\right)}$$

$$= \frac{1}{2}$$

Q7

$$\lim_{x \to 1} \left\{ \frac{x^3 + 2x^2 + x + 1}{x^2 + 2x + 3} \right\}^{\frac{1-\cos(x-1)}{(x-1)^2}}$$

$$= e^{\lim_{x \to 1} \left\{ \frac{1-\cos(x-1)}{(x-1)^2} \ln\left(\frac{x^3 + 2x^2 + x + 1}{x^2 + 2x + 3} \right) \right\}}$$

$$= e^{\lim_{x \to 1} \left\{ \frac{2\sin^2(x-1)}{4\left(\frac{x-1}{2}\right)^2} \ln\left(\frac{x^3 + 2x^2 + x + 1}{x^2 + 2x + 3} \right) \right\}}$$

$$= e^{\frac{2}{4} \ln\left(\frac{5}{6}\right)}$$

$$= e^{\ln\left(\frac{5}{6}\right)^{\frac{1}{2}}}$$

$$= \left(\frac{5}{6}\right)^{\frac{1}{2}} = \sqrt{\frac{5}{6}}$$

Q8

$$\lim_{x \to 0} \left\{ \frac{e^x + e^{-x} - 2}{x^2} \right\}^{1/x^2}$$

$$= e^{\lim_{x \to 0} \frac{\left\{ \frac{e^x + e^{-x} - 2}{x^2} \right\} - 1}{x^2}}$$

Applying L'Hospital's Rule

$$= e^{\lim_{x \to 0} \left\{ \frac{2 + e^x(-2+x) + x}{2(-1+e^x)x^2} \right\}}$$

Applying L'Hospital's Rule again

$$= e^{\frac{1}{2} \lim_{x \to 0} \left\{ \frac{1 + e^x(-1+x)}{x(-2 + e^x(2+x))} \right\}}$$

Applying L'Hospital's Rule again

$$= e^{\frac{1}{2} \lim_{x \to 0} \left\{ \frac{e^x x}{-2 + e^x(2 + 4x + x^2)} \right\}}$$

Applying L'Hospital's Rule again

$$= e^{\frac{1}{2} \lim_{x \to 0} \left\{ \frac{1 + x}{6 + 6x + x^2} \right\}}$$

$$= e^{\frac{1}{2} \left\{ \frac{\lim_{x \to 0}(1+x)}{\lim_{x \to 0}(6 + 6x + x^2)} \right\}}$$

$$= e^{\frac{1}{12}}$$

$$= \sqrt[12]{e}$$

Q9

$$\lim_{x \to a} \left\{ \frac{\sin x}{\sin a} \right\}^{\frac{1}{x-a}}$$

$$= \lim_{x \to a} \left\{ 1 + \left(\frac{\sin x}{\sin a} - 1 \right) \right\}^{\frac{1}{x-a}}$$

$$= e^{\lim\limits_{x \to a} \left\{ \frac{\left(\frac{\sin x}{\sin a} - 1 \right)}{x-a} \right\}}$$

$$= e^{\lim\limits_{x \to a} \left\{ \frac{\left(\frac{\sin x - \sin a}{\sin a} \right)}{x-a} \right\}}$$

$$= e^{\lim\limits_{x \to a} \left\{ \left(\frac{\sin x - \sin a}{\sin a (x-a)} \right) \right\}}$$

$$= e^{\lim\limits_{x \to a} \left\{ \left(\frac{2\cos\left(\frac{x+a}{2}\right) \sin\left(\frac{x-a}{2}\right)}{\sin a (x-a)} \right) \right\}}$$

$$= e^{\lim\limits_{x \to a} \left\{ \left(\frac{2\cos\left(\frac{x+a}{2}\right)}{\sin a} \right) \right\} \lim\limits_{x \to a} \left\{ \left(\frac{\sin\left(\frac{x-a}{2}\right)}{2\left(\frac{x-a}{2}\right)} \right) \right\}}$$

$$= e^{\frac{2\cos a}{2\sin a}}$$

$$= e^{\cot a}$$

Q10

$$\lim_{x \to \infty} \left\{ \frac{3x^2 + 1}{4x^2 - 1} \right\}^{\frac{x^3}{1+x}}$$

$$= \lim_{x \to \infty} \left\{ 1 + \frac{-x^2 + 2}{4x^2 - 1} \right\}^{\frac{x^3}{1+x}}$$

$$= e^{\lim_{x \to \infty} \left\{ \left(\frac{-x^2+2}{4x^2-1} \right)\left(\frac{x^3}{1+x} \right) \right\}}$$

$$= e^{\lim_{x \to \infty} \left\{ \left(\frac{-x^5+2x^3}{4x^2-1+4x^3-x} \right) \right\}}$$

$$= e^{-\infty}$$

$$= 0$$