Ex 28.1

Q1(i)

All are positive, so octant is XOYZ

Q1(ii)

X is negative and rest are positive, so octant is $X'OYZ$

Q1(iii)

Y is negative and rest are positive, so octant is $XOY'Z$

Q1(iv)

Z is negative and rest are positive, so octant is $XOYZ'$

Q1(v)

> X and Y are negative and Z is positive, so octant is $X'OY'Z$

Q1(vi)

All are negative, so octant is $X'OY'Z'$

Q1(vii)

X and Z are negative, so octant is $X'OYZ'$

Q2(i)

YZ plane is x-axis, so sign of x will be changed. So answer is (2, 3, 4)

Q2(ii)

XZ plane is y-axis, so sign of y will be changed. So answer is (-5, -4, -3)

Q2(iii)

> XY-plane is z-axis, so sign of Z will change. So answer is (5, 2, 7)

Q2(iv)

XZ plane is y-axis, so sign of Y will change, So answer is (-5, 0, 3)

Q2(v)

XY plane is Z-axis, so sign of Z will change So answer is (-4, 0, 0)

Q3

Vertices of cube are

(1, 0, -1) (1, 0, 4) (1, -5, -1)

(1, -5, 4) (-4, 0, -1) (-4, -5, -4)

(-4, -5, -1) (4, 0, 4) (1, 0, 4)

Q4

3-(-2)=5, |0-5|=5, |-1-4|=5

5, 5, 5 are lengths of edges

Q5

5-3=2, 0-(-2)=2, 5-2=3

2, 2, 3 are lengths of edges

Q6

(-4, 3, 5)

x-axis: $\sqrt{9+25} = \sqrt{34}$

y-axis: $\sqrt{16+25} = \sqrt{41}$

z-axis= $\sqrt{9+16} = 5$

Q7

(-3, -2, -5) (-3, -2, 5) (3, -2, -5) (-3, 2, -5) (3, 2, 5)

(3, 2, -5) (-3, 2, 5)

Ex 28.2

Q1

(i) Distance between points P and Q

$$PQ = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

$$= \sqrt{(1-2)^2 + (-1-1)^2 + (0-2)^2}$$

$$= \sqrt{(-1)^2 + (-2)^2 + (-2)^2}$$

$$= \sqrt{1+4+4}$$

$$PQ = 3 \text{ units}$$

(ii) Distance between points A and B

$$AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 + -z_2)^2}$$

$$= \sqrt{(3+1)^2 + (2+1)^2 + (-1+1)^2}$$

$$= \sqrt{(4)^2 + (3)^2 + (0)^2}$$

$$= \sqrt{16+9+0}$$

$$= \sqrt{25}$$

$$AB = 5 \text{ units}$$

Q2

Distance between points P and Q

$$PQ = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

$$= \sqrt{(-2-2)^2 + (3-1)^2 + (1-2)^2}$$

$$= \sqrt{(-4)^2 + (2)^2 + (-1)^2}$$

$$= \sqrt{16+4+1}$$

$$PQ = \sqrt{21} \text{ units}$$

Q3(i)

$A(4,-3,-1)$, $B(5,-7,6)$ and $C(3,1,-8)$

$$AB=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$$

$$=\sqrt{(4-5)^2+(-3+7)^2+(-1-6)^2}$$

$$=\sqrt{(-1)^2+(4)^2+(-7)^2}$$

$$=\sqrt{1+16+49}$$

$$=\sqrt{66}\text{ units}$$

$$BC=\sqrt{(5-3)^2+(-7-1)^2+(6+8)^2}$$

$$=\sqrt{(2)^2+(-8)^2+(14)^2}$$

$$=\sqrt{4+64+196}$$

$$=\sqrt{264}$$

$$=2\sqrt{66}\text{ units}$$

$$AC=\sqrt{(4-3)^2+(-3-1)^2+(-1+8)^2}$$

$$=\sqrt{(1)^2+(-4)^2+(7)^2}$$

$$=\sqrt{1+16+49}$$

$$=\sqrt{66}\text{ units}$$

Since $AC+AB=BC$
so, A, B, C are collinear.

Q3(ii)

$P(0,7,-7),\ Q(1,4,-5),\ R(-1,10,-9)$

$$PQ = \sqrt{(0-1)^2 + (7-4)^2 + (-7+5)^2}$$

$$= \sqrt{(1)^2 + (3)^2 + (-2)^2}$$

$$= \sqrt{1+9+4}$$

$$= \sqrt{14} \text{ units}$$

$$QR = \sqrt{(1+1)^2 + (4-10)^2 + (-5+9)^2}$$

$$= \sqrt{(2)^2 + (-6)^2 + (4)^2}$$

$$= \sqrt{4+36+16}$$

$$= 2\sqrt{14} \text{ units}$$

$$PR = \sqrt{(0+1)^2 + (7-10)^2 + (-7+9)^2}$$

$$= \sqrt{1^2 + (-3)^2 + (2)^2}$$

$$= \sqrt{1+9+4}$$

$$= \sqrt{14} \text{ units}$$

Since $PQ + PR = QR$
so, P, Q, R are collinear

Q3(iii)

$A(3,-5,1)$, $B(-1,0,8)$, and $C(7,-10,-6)$

$$AB = \sqrt{(3+1)^2 + (-5-0)^2 + (1-8)^2}$$

$$= \sqrt{(4)^2 + (-5)^2 + (-7)^2}$$

$$= \sqrt{16+25+49}$$

$$= \sqrt{90}$$

$$= 3\sqrt{10} \text{ units}$$

$$BC = \sqrt{(-1-7)^2 + (0+10)^2 + (8+6)^2}$$

$$= \sqrt{(-8)^2 + (10)^2 + (14)^2}$$

$$= \sqrt{64+100+196}$$

$$= \sqrt{360}$$

$$= 6\sqrt{10} \text{ units}$$

$$CA = \sqrt{(3-7)^2 + (-5+10)^2 + (1+6)^2}$$

$$= \sqrt{(-4)^2 + (5)^2 + (7)^2}$$

$$= \sqrt{16+25+49}$$

$$= \sqrt{90}$$

$$= 3\sqrt{10} \text{ units}$$

Since $AB + AC = BC$
so, $A, B,$ and C are collinear

Q4(i)

Let the point on xy – plane be $P(x, y, 0)$.

Now P is equidistance from $A(1, -1, 0)$, $B(2, 1, 2)$ and $C(3, 2, -1)$.

So, $AP = BP = CP$

Now,

$(AP)^2 = (x-1)^2 + (y+1)^2 + (0-0)^2$

$(BP)^2 = (x-2)^2 + (y-1)^2 + (0-2)^2$

$(CP)^2 = (x-3)^2 + (y-2)^2 + (0+1)^2$

$(AP)^2 = (BP)^2 \Rightarrow (x-1)^2 + (y+1)^2 = (x-2)^2 + (y-1)^2 + 4$

$\Rightarrow x^2 + 1 - 2 + y^2 + 1 + 2y + z^2 = x^2 + 4 - 4x + y^2 + 1 - 2y + 4$

$\Rightarrow 2x + 4y = 7 \quad \dots(1)$

$(BP)^2 = (CP)^2 \Rightarrow (x-2)^2 + (y-1)^2 + 4 = (x-3)^2 + (y-2)^2 + 1$

$\Rightarrow x^2 + 4 - 4x + y^2 + 1 - 2y + z^2 + 4 = x^2 + 9 - 6x + y^2 + 4 - 4y + 1$

$\Rightarrow 2x + 2y = 5 \quad \dots(2)$

$(AP)^2 = (CP)^2 \Rightarrow (x-1)^2 + (y+1)^2 = (x-3)^2 + (y-2)^2 + 1$

$\Rightarrow x^2 + 1 - 2x + y^2 + 1 + 2y = x^2 + 9 - 6x + y^2 + 4 - 4y + 1$

$\Rightarrow 4x + 6y = 12 \quad \dots(3)$

Solving equation (1) and (2) we get

$y = 1, \quad x = 3/2$

Put x and y in equation (3)

$4(3/2) + 6(1) = 12$

$12 = 12$

So, the required point is $(3/2,\ 1, 0)$

Q4(ii)

Let $Q(0, y, z)$ be the required point.

So

$(AQ)^2 = (BQ)^2 \Rightarrow (0-1)^2 + (y+1)^2 + (z-0)^2 = (0-2) + (y-1)^2 + (z-2)^2$

$\Rightarrow \quad 1 + y^2 + 1 + 2y + z^2 = 4 + y^2 + 1 - 2y + z^2 + 4 - 42$

$\Rightarrow \quad 4y + 4z = 7 \ldots (1)$

$(BQ)^2 = (CQ)^2 \Rightarrow (0-z)^2 + (y-1)^2 + (z-2)^2 = (0-3)^2 + (y-2)^2 + (z+1)^2$

$\Rightarrow \quad 4 + y^2 + 1 - 2y + z^2 + 4 - 4z = 9 + y^2 + 4 - 4y + z^2 + 1 + 2z$

$\Rightarrow \quad 2y - 6z = 5 \ \ldots (2)$

$(AQ)^2 = (CQ)^2 \Rightarrow (0-1)^2 + (y+1)^2 + (z-0)^2 = (0-3)^2 + (y-2)^2 + (z+1)^2$

$\Rightarrow \quad 1 + y^2 + 2y + 1 + z^2 = 9 + y^2 - 4y + 4 + z^2 + 1 + 2z$

$\Rightarrow \quad 6y - 2z = 12 \ \ldots (3)$

Solving equation (1) and (2), we get

$$z = \frac{-3}{16} \text{ and } y = \frac{31}{16}$$

Put the value of y and z is equation (3)

$6y - 2z = 12 = 12$

$$6\left(\frac{31}{16}\right) - 2\left(\frac{-3}{16}\right) = 12$$

$$\frac{186}{16} + \frac{6}{16} = 12$$

$$\frac{192}{16} = 12$$

$12 = 12$

LHS = RHS.

so,

$$y = \frac{31}{16}, z = \frac{13}{16}$$

Required point $= \left(0, \frac{31}{16}, \frac{-3}{16}\right)$

Q4(iii)

Let $R(x, 0, z)$ be the required point.

So

$(AR)^2 = (BR)^2 \Rightarrow (1-x)^2 + (-1-0)^2 + (0-z)^2 = (2-x) + (1-0)^2 + (2-z)^2$

$\Rightarrow \quad 1 + x^2 - 2x + 1 + z^2 = 4 + x^2 - 4x + 1 + 4 + z^2 - 4z$

$\Rightarrow \quad 2x + 4z = 7 \ldots(1)$

$(BR)^2 = (CR)^2 \Rightarrow (z-z)^2 + (1-0)^2 + (2-z)^2 = (3-x)^2 + (2-0)^2 + (-1-z)^2$

$\Rightarrow \quad 4 + x^2 - 4x + 4 + z^2 - 4z = 9 + x^2 - 6x + 4 + 1 + z^2 + 2z$

$\Rightarrow \quad 2x - 6z = 5 \ \ldots(2)$

$(AR)^2 = (CR)^2 \Rightarrow (1-x)^2 + (1-0)^2 + (0-z)^2 = (3-x)^2 + (2-0)^2 + (-1-z)^2$

$\Rightarrow \quad 1 + x^2 - 2x + 1 + z^2 = 9 + 6x + 4 + 1 + z^2 + 2z$

$\Rightarrow \quad 4x - 2z = 12 \ \ldots(3)$

Solving equation (1) and (2), we get

$$z = \frac{1}{5}, \ x = \frac{31}{10}$$

Put the value of x and z is equation (3)

$4x - 2z = 12$

$$4\left(\frac{31}{10}\right) - 2\left(\frac{1}{5}\right) = 12$$

$$\frac{124}{10} - \frac{2}{10} = 12$$

$$\frac{120}{10} = 12$$

$12 = 12$

LHS = RHS.

so,

$$x = \frac{31}{10}, z = \frac{1}{5}$$

Required point $= \left(\frac{31}{10}, 0, \frac{1}{5}\right)$

Q5

Let $P(0,0,z)$ be the point equidistant from $Q(1,5,7)$ and $R(5,1,-4)$.

So,

$(PQ)^2 = (PR)^2 \Rightarrow (0-1)^2+(0-5)^2+(z-7)^2 = (0-5)^2+(0-1)^2+(z+4)^2$

$\Rightarrow 1+25+(z-7)^2 = 25+1+(z+4)^2$

$\Rightarrow 26+z^2+49-14z = 26+z^2+8z+16$

$\Rightarrow \quad -14z-8z = 16-49$

$\Rightarrow \quad -22z = -33$

$\Rightarrow \quad z = \frac{-33}{-22}$

$\Rightarrow \quad z = \frac{3}{2}$

Required point $= (0,0,\ 3/2)$

Q6

Let $P(0,y,0)$ be a point on y-axis which is equidistant from $Q(3,1,2)$ and $R(5,5,2)$.

So,

$(PR)^2 = (PQ)^2 \Rightarrow (0-5)^2+(y-5)^2+(0-2)^2 = (0-3)^2+(y-1)^2+(0-2)^2$

$\Rightarrow 25+y^2+25-10y+4 = 9+y^2+1-2y+4$

$\Rightarrow \quad -10y+2y = 14-54$

$\Rightarrow \quad -14z-8z = 16-49$

$\Rightarrow \quad -8y = -40$

$\Rightarrow \quad y = 5$

So, the required point is $(0,5,0)$

Q7

Let $P(0,0,z)$ be at a distance of $\sqrt{21}$ from $Q(1,2,3)$.

So

$PQ = \sqrt{(0-1)^2+(0-2)^2+(z-3)^2}$

$\sqrt{21} = \sqrt{(1)^2+(2)^2+(z-3)^2}$

$21-5 = (z-3)^2$

$16 = (z-3)^2$

$z-3 = \pm 4$

$z = 7$ and $z = -1$

So, the required points are $(0,0,7)$ and $(0,0,-1)$

Q8

Let the triangle formed be $\triangle ABC$

$$(AB) = \sqrt{(1-2)^2 + (2-3)^2 + (3-1)^2}$$
$$= \sqrt{(-1)^2 + (-1)^2 + (2)^2}$$
$$= \sqrt{6} \text{ units}$$

$$BC = \sqrt{(2-3)^2 + (3-1)^2 + (1-2)^2}$$
$$= \sqrt{(-1)^2 + (2)^2 + (-1)^2}$$
$$= \sqrt{6} \text{ units}$$

$$AC = \sqrt{(1-3)^2 + (2-1)^2 + (3-2)^2}$$
$$= \sqrt{(-2)^2 + (1)^2 + (1)^2}$$
$$= \sqrt{6} \text{ units}$$

since, $AB = BC = CA$

So, $\triangle ABC$ is an equilateral $\triangle$

Q9

Let $A = (0,7,10)$, $B=(-1,6,6)$ and $C = (-4,9,6)$

$AB = \sqrt{(0+1)^2 + (7-6)^2 + (10-6)^2}$
$= \sqrt{(1)^2 + (1)^2 + (4)^2}$
$= \sqrt{18}$
$= 3\sqrt{2}$ units

$BC = \sqrt{(-1+4)^2 + (6-9)^2 + (6-6)^2}$
$= \sqrt{(3)^2 + (3)^2 + 0}$
$= \sqrt{18}$
$= 3\sqrt{2}$ units

$AC = \sqrt{(0+4)^2 + (7-9)^2 + (10-6)^2}$
$= \sqrt{(4)^2 + (-2)^2 + (4)^2}$
$= \sqrt{36}$
$= 6$ units

$(AB)^2 + (BC)^2$
$= \left(3\sqrt{2}\right)^2 + \left(3\sqrt{2}\right)^2$
$= 18 + 18$
$= 36$
$= (AC)^2$

Also $l(AB) = l(BC)$

Hence $(0,7,10)$, $(-1,6,6)$ and $(-4,9,6)$ are the vertices of an isosceles right-angled triangle.

Q10

Here points are $A(3,3,3)$, $B(0,6,3)$, $C(1,7,7)$ and $D(4,4,7)$.

$$AB = \sqrt{(3-0)^2+(3-6)^2+(3-3)^2}$$
$$= \sqrt{9+9}$$
$$= 3\sqrt{2} \text{ units}$$

$$BC = \sqrt{(0-1)^2+(6-7)^2+(3-7)^2}$$
$$= \sqrt{1+1+16}$$
$$= 3\sqrt{2} \text{ units}$$

$$AC = \sqrt{(3-1)^2+(3-7)^2+(3-7)^2}$$
$$= \sqrt{4+16+16}$$
$$= 6 \text{ units}$$

$$BD = \sqrt{(0-4)^2+(6-4)^2+(3-7)^2}$$
$$= \sqrt{16+4+16}$$
$$= 6 \text{ units}$$

$$CD = \sqrt{(1-4)^2+(7-4)^2+(7-7)^2}$$
$$= \sqrt{9+9}$$
$$= 3\sqrt{2} \text{ units}$$

$$AD = \sqrt{(3-4)^2+(3-4)^2+(3-7)^2}$$
$$= \sqrt{1+1+16}$$
$$= 3\sqrt{2} \text{ units}$$

Since,

$AB = BC = CD = DA$

And $AC = BD$

So,

A, B, C, D are vertices of a square.

Q11

Here,

$$AB = \sqrt{(1+5)^2 + (3-5)^2 + (0-2)^2}$$
$$= \sqrt{36+4+4}$$
$$= \sqrt{44}$$
$$= 2\sqrt{11} \text{ units}$$

$$BC = \sqrt{(-5+9)^2 + (5+1)^2 + (2-2)^2}$$
$$= \sqrt{16+36}$$
$$= \sqrt{52}$$
$$= 2\sqrt{13} \text{ units}$$

$$CD = \sqrt{(-9+3)^2 + (-1+3)^2 + (2-0)^2}$$
$$= \sqrt{36+4+4}$$
$$= 2\sqrt{11} \text{ units}$$

$$DA = \sqrt{(-3-4)^2 + (-3-3)^2 + 0}$$
$$= \sqrt{16+36}$$
$$= \sqrt{52}$$
$$= 2\sqrt{13} \text{ units}$$

$$AC = \sqrt{(1+9)^2 + (3+1)^2 + (0-2)^2}$$
$$= \sqrt{150+16+4}$$
$$= \sqrt{120}$$
$$= 4\sqrt{5} \text{ units}$$

$$BD = \sqrt{(-3+5)^2 + (-3-5)^2 + (0-2)^2}$$
$$= \sqrt{4+64+4}$$
$$= \sqrt{72}$$
$$= 6\sqrt{2} \text{ units}$$

Since,

$AB = CD$ and $BC = DA$

$\Rightarrow$ $ABCD$ is a parallelgram = BD

but, $AC \neq BD$

$\Rightarrow$ $ABCD$ is not a rectangle.

Q12

Here,

$$AB = \sqrt{(1+1)^2 + (3-6)^2 + (4-10)^2}$$
$$= \sqrt{4+9+36}$$
$$= 7\text{units}$$

$$BC = \sqrt{(-1+7)^2 + (6-4)^2 + (0-7)^2}$$
$$= \sqrt{36+4+9}$$
$$= 7\text{units}$$

$$CD = \sqrt{(-7+5)^2 + (4-1)^2 + (7-1)^2}$$
$$= \sqrt{4+9+36}$$
$$= 7\text{units}$$

$$DA = \sqrt{(-5-1)^2 + (1-3)^2 + (1-4)^2}$$
$$= \sqrt{36+4+9}$$
$$= \sqrt{52}$$
$$= 7\text{units}$$

Since, $AB = BC = CD = DA$

So, $ABCD$ is a rhombus.

Q13

Here,

$$AB = \sqrt{(0-1)^2 + (1-0)^2 + (1-1)^2}$$
$$= \sqrt{1+1}$$
$$= \sqrt{2} \text{ units}$$

$$BC = \sqrt{(1-1)^2 + (0-1)^2 + (1-0)^2}$$
$$= \sqrt{1+1}$$
$$= \sqrt{2} \text{ units}$$

$$CA = \sqrt{(1-0)^2 + (1-1)^2 + (0-1)^2}$$
$$= \sqrt{1+0+1}$$
$$= \sqrt{2} \text{ units}$$

$$DA = \sqrt{(0-0)^2 + (0-1)^2 + (0-1)^2}$$
$$= \sqrt{1+1}$$
$$= \sqrt{2} \text{ units}$$

$$OB = \sqrt{(0-1)^2 + (0-0)^2 + (0-1)^2}$$
$$= \sqrt{1+1}$$
$$= \sqrt{2} \text{ units}$$

$$OC = \sqrt{(0-1)^2 + (0-1)^2 + (0-0)^2}$$
$$= \sqrt{1+1}$$
$$= \sqrt{2} \text{ units}$$

Since, $OA = OB = OC = AB = BC = CA$

So, O, A, B, C represent a regular tetrahedron

Q14

Here,

$$OA = \sqrt{(1-3)^2 + (3-2)^2 + (4-2)^2}$$
$$= \sqrt{4+1+4}$$
$$= 3 \text{units}$$

$$OB = \sqrt{(1+1)^2 + (3-1)^2 + (4-3)^2}$$
$$= \sqrt{4+4+1}$$
$$= 3 \text{units}$$

$$OC = \sqrt{(1-0)^2 + (3-5)^2 + (4-6)^2}$$
$$= \sqrt{1+4+4}$$
$$= 3 \text{units}$$

$$OD = \sqrt{(1-2)^2 + (3-1)^2 + (4-2)^2}$$
$$= \sqrt{1+4+4}$$
$$= 3 \text{units}$$

Since, $OA = OC = OD = OB$, points A, B, C, D lie on a sphere with centre O.

Radius = 3 units

Q15

Let the required point be $P(x_1 y_1 z)$

Here, $O(0,0,0)$, $A(2,0,0)$, $B(0,3,0)$, $C(0,0,8)$

Since, $(OP)^2 = (PA)^2$

$$(x-0)^2 + (y-0)^2 + (z-0)^2 = (x-2)^2 + (y-0)^2 + (z-0)^2$$
$$x^2 + y^2 + z^2 = x^2 - 4x + 4 + y^2 + z^2$$
$$4x = 4$$
$$x = 1$$

$$(OP)^2 = (PB)^2$$
$$(x-0)^2 + (y-0)^2 + (z-0)^2 = (x-0)^2 + (y-3)^2 + (z-0)^2$$
$$x^2 + y^2 + z^2 = x^2 + y^2 - 6y + 9 + z^2$$
$$6y = 9$$
$$y = \frac{3}{2}$$

$$(OP)^2 = (PC)^2$$
$$(x-0)^2 + (y-0)^2 + (z-0)^2 = (x-0)^2 + (y-0)^2 + (z-8)^2$$
$$x^2 + y^2 + z^2 = x^2 + y^2 + z^2 - 16z + 64$$
$$16z = 64$$
$$z = 4$$

The required point $= \left(1, \frac{3}{2}, 4\right)$

Q16

Let P be $(x_1 y_1 z)$, here, $A(-2,2,3)$ and $B(13,-3,13)$
and $3PA = 2PB$

$$\Rightarrow \quad 3\sqrt{(x+2)^2 (y-2)^2 + (z-3)^2} = 2\sqrt{(x-13)^2 + (y+3)^2 + (z-13)^2}$$

squaring both the sides,

$$\Rightarrow \quad 9\left[x^2 + 4x + 4 + y^2 + 4 - 4y + z^2 + 9 - 6z\right]$$
$$= 4\left[x^2 + 169 - 26x + y^2 + 9 + 6y + z^2 + 169 - 26z\right]$$
$$\Rightarrow \quad 9x^2 - 4x^2 + 36x + 104x + 36 - 676 + 9y^2 - 4y^2$$
$$+36 - 36 - 36y - 24y + 9z^2 - 4z^2 + 81 - 676 - 54z + 6yz = 0$$
$$\Rightarrow \quad 5x^2 + 5y^2 + 5z^2 + 140x - 60y + 50z - 1235 = 0$$
$$\Rightarrow \quad 5\left(x^2 + y^2 + z^2\right) + 140x - 60y + 50z - 1235 = 0$$

Q17

Let $P(x_1 y_1 z)$, here, $A(3,4,5)$, $B(-1,3,-7)$
$PA^2 + PB^2 = 2k^2$

$$\Rightarrow \quad (x-3)^2 + (y-4)^2 + (z-5)^2 + (x+1)^2 + (y-3)^2 + (2+7)^2 = 2k^2$$
$$\Rightarrow \quad x^2 + 9 - 6x + y^2 + 16 - 8y + z^2 + 25 - 10z + x^2 + 1 + 2x$$
$$+y^2 + 9 - 6y + z^2 + 49 + 14z = 2k^2$$
$$\Rightarrow \quad 2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z + 109 = 2k^2$$
$$\Rightarrow \quad 2\left(x^2 + y^2 + z^2\right) - 4x - 14y + 4z + 109 - 2k^2 = 0$$

Q18

Here, $A(a,b,c)$, $B(b,c,a)$, $C(c,a,b)$

$$AB = \sqrt{(a-b)^2+(b-c)^2+(c-a)^2}$$
$$= \sqrt{a^2+b^2-2ab+b^2+c^2-2bc+c^2+a^2-2ac}$$

$$AB = \sqrt{2a^2+2b^2+2c^2-2ab-2bc-2ac}$$
$$BC = \sqrt{(b-c)^2+(c-a)^2+(a-b)^2}$$
$$= \sqrt{b^2+c^2-2bc+c^2+a^2-2ca+a^2+b^2-2ab}$$
$$BC = \sqrt{2a^2+2b^2+2c^2-2ab-2bc-2ca}$$

$$CA = \sqrt{(a-c)^2+(b-a)^2+(c-b)^2}$$
$$= \sqrt{a^2+c^2-2ac+b^2+a^2-2ab+b^2+c^2-2bc}$$

$$CA = \sqrt{2a^2+2b^2+2c^2-2ab-2bc-2ca}$$

Since, $AB = BC = CA$, so
$\triangle$ABC is an isosceles A

Q19

Here, $A(3, 6, 9)$, $B(10.20.30)$, $C(25, 41, 5)$

$$\begin{aligned}
(AB)^2 &= (3-10)^2 + (6-20)^2 + (9-30)^2 \\
&= (-7)^2 + (-14)^2 + (-21)^2 \\
&= 49 + 196 + 441 \\
&= 586 \\
(BC)^2 &= (10-25)^2 + (20+41)^2 + (30-5)^2 \\
&= (-15)^2 + (61)^2 + (25)^2 \\
&= 225 + 3721 + 625 \\
&= 4571 \\
(CA)^2 &= (3-25)^2 + (6+41)^2 + (9-5)^2 \\
&= (-22)^2 + (47)^2 + (4)^2 \\
&= 484 + 2209 + 16 \\
&= 2709
\end{aligned}$$

Since, $AB^2 + BC^2 \neq AC^2$

$AB^2 + AC^2 \neq BC^2$

$BC^2 + AC^2 \neq AB^2$

So, $\triangle$ABC is not a right triangle.

Q20(i)

Here, $A(0,7,-10)$, $B(1,6,-6)$, $C(4,9,-6)$

$$AB = \sqrt{(0-1)^2+(7-6)^2+(-10+6)^2}$$
$$= \sqrt{1+1+16}$$
$$= 3\sqrt{2} \text{ units}$$

$$BC = \sqrt{(1-4)^2+(6-9)^2+(-6+6)^2}$$
$$= \sqrt{9+9}$$
$$= 3\sqrt{2} \text{ units}$$

$$AC = \sqrt{(0,-4)^2+(7-9)^2+(-10+6)^2}$$
$$= \sqrt{16+4+16}$$
$$= 6 \text{ units}$$

Since, $AB = BC$

So, $\triangle$ABC is an isosceles $\triangle$

Q20(ii)

Here, $A(0,7,10)$, $B(-1,6,6)$, $C(-4,9,6)$

$$AB = \sqrt{(0+1)^2+(7-6)^2+(10-6)^2}$$
$$= \sqrt{1+1+16}$$
$$= 3\sqrt{2} \text{ units}$$

$$BC = \sqrt{(-1+4)^2+(6-9)^2+(6-6)^2}$$
$$= \sqrt{9+9+0}$$
$$= 3\sqrt{2} \text{ units}$$

$$CA = \sqrt{(-4-0)^2+(9-7)^2+(6+10)^2}$$
$$= \sqrt{16+4+16}$$
$$= \sqrt{36} \text{ units}$$

Since, $(AB)^2+(BC)^2=(AC)^2$

So, $\triangle$ABC is a right triangle.

Q20(iii)

Here, $A(-1,2,1)$, $B(1,-2,5)$, $C(4,-7,8)$, $D(2,-3,4)$

$$AB = \sqrt{(-1-1)^2 + (2+2)^2 + (1-5)^2}$$
$$= \sqrt{4+16+16}$$
$$= 6 \text{ units}$$

$$BC = \sqrt{(1-4)^2 + (-2+7)^2 + (5-8)^2}$$
$$= \sqrt{9+25+9}$$
$$= \sqrt{43} \text{ units}$$

$$CD = \sqrt{(4-2)^2 + (-7+3)^2 + (8-4)^2}$$
$$= \sqrt{4+16+16}$$
$$= 6 \text{ units}$$

$$DA = \sqrt{(2+1)^2 + (-3-2)^2 + (4-1)^2}$$
$$= \sqrt{9+25+9}$$
$$= \sqrt{43} \text{ units}$$

Since, $AB = CD$ and $BC = DA$

So, $\triangle ABC$ is a parallelogram

Q20(iv)

Let A(5, -1, 1), B(7, -4, 7), C(1, -6, 10) and D(-1, -3, 4) be the given points.

$$AB = \sqrt{(7-5)^2 + (-4+1)^2 + (7-1)^2} = \sqrt{4+9+36} = 7$$

$$BC = \sqrt{(1-7)^2 + (-6+4)^2 + (10-7)^2} = \sqrt{36+4+9} = 7$$

$$CD = \sqrt{(-1-1)^2 + (-3+6)^2 + (4-10)^2} = \sqrt{4+9+36} = 7$$

$$AD = \sqrt{(-1-5)^2 + (-3+1)^2 + (4-1)^2} = \sqrt{36+4+9} = 7$$

So AB = BC = CD = AD

Hence ABCD is a rhombus.

Q21

Let the point $P(x_1 y_1 z)$ which is equidistance from $A(1, 2, 3)$ and $B(3, 2, -1)$, so

$$AP = BP$$
$$(AP)^2 = (BP)^2$$

$$(x-1)^2 + (y-2)^2 + (z-3)^2 = (x-3)^2 + (y-2)^2 + (z+1)^2$$
$$x^2 + 1 - 2x + y^2 + 4 - 4y + z^2 + 9 - 6z = x^2 + 9 - 6x + y^2 + 4 - 4y + z^2 + 1 + 2z$$
$$4x - 8z = 14 - 14$$
$$4x - 8z = 0$$
$$x - 2z = 0$$

Q22

Let locus of $P(x_1 y_1 z)$ is the required locus, so

$$PA + P = 10$$

$$\Rightarrow \quad \sqrt{(x-4)^2 + (y-0)^2 + (z-0)^2} + \sqrt{(x+4)^2 + (y-0)^2 + (z-0)^2} = 10$$
$$\Rightarrow \quad \sqrt{x^2 + 16 - 8x + y^2 + z^2} = 10 - \sqrt{x^2 + 8x + 16 + y^2 + z^2}$$

$$\Rightarrow \quad x^2 + y^2 + z^2 - 8x + 16 = (10)^2 + \left(x^2 + y^2 + z^2 + 8x + 16\right)$$
$$-20\sqrt{x^2 + y^2 + z^2 + 8x + 16}$$
$$\Rightarrow \quad -8x + 16 - 100 - 8x - 16 = -20\sqrt{x^2 + y^2 + z^2 + 8x + 16}$$
$$\Rightarrow \quad -16x - 100 = -20\sqrt{x^2 + y^2 + z^2 + 8x + 16}$$
$$\Rightarrow \quad -4(4x + 25) = -20\sqrt{x^2 + yy^2 + z^2 + 8x + 16}$$
$$\Rightarrow \quad (4x + 25) = 5\sqrt{x^2 + y^2 + z^2 + 8x + 16}$$

squiring both the sides,

$$(4x+25)^2 = 25\left(x^2 + y^2 + z^2 + 8x + 16\right)$$
$$\Rightarrow \quad 16x^2 + 625 + 200x = 25\left(x^2 + y^2 + z^2 + 8x + 16\right)$$
$$\Rightarrow \quad 16x^2 + 625 + 200x = 25x^2 + 25y^2 + 25z^2 + 200x + 400$$
$$\Rightarrow \quad 9x^2 + 25y^2 + 25z^2 - 225 = 0$$

Q23

$$AB = \sqrt{(1+1)^2 + (2+2)^2 + (3+1)^2}$$
$$= \sqrt{4+16+16}$$
$$= 6 \text{ units}$$

$$BC = \sqrt{(-1-2)^2 + (-2-3)^2 + (-1-2)^2}$$
$$= \sqrt{9+25+9}$$
$$= \sqrt{43} \text{ units}$$

$$CD = \sqrt{(2-4)^2 + (3-7)^2 + (2-6)^2}$$
$$= \sqrt{4+16+16}$$
$$= 6 \text{ units}$$

$$DA = \sqrt{(4-1)^2 + (7-2)^2 + (6-3)^2}$$
$$= \sqrt{9+25+9}$$
$$= \sqrt{43} \text{ units}$$

Q24

Let the point be P (x, y, z)

Given

A=(3, 4, -5)

B=(-2, 1, 4)

PA=PB $\Rightarrow PA^2=PB^2$

$$PA^2 = (x-3)^2 + (y-4)^2 + (z+5)^2$$

$$PB^2 = (x+2)^2 + (y-1)^2 + (z-4)^2$$

$PA^2=PB^2 \Rightarrow$

$$(x-3)^2 + (y-4)^2 + (z+5)^2 = (x+2)^2 + (y-1)^2 + (z-4)^2$$

All square terms will be cancelled on both sides, we get

$$-6x+9-8y+16+10z+25 = 4x+4-2y+1-8z+16$$

$10x+6y-18z-29=0$ is the required equation

Ex 28.3

Q1

We know that angle bisector divides opposite side in ratio of other two sides

$\Rightarrow$ D divides BC in ratio of AB : AC

A(5, 4, 6), B(1, -1, 3) and C(4, 3, 2)

$AB=\sqrt{16+25+9}=\sqrt{50}=5\sqrt{2}$

$AC=\sqrt{1+1+16}=\sqrt{18}=3\sqrt{2}$

AB:AC=5:3=m:n

$$D(x,y,z)=\left(\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n},\frac{mz_2+nz_1}{m+n}\right)$$

Substitute values for m:n=5:3,

$(x_1, y_1, z_1)=(1, -1, 3)$

$(x_2, y_2, z_2)=(4, 3, 2)$

$$D=\left(\frac{23}{8},\frac{3}{2},\frac{19}{8}\right)$$

Q2

z-coordinate 8

A(2, -3, 4) and B(8, 0, 10)

DR's of AB=(6, 3, 6)

DR's of BC= (x-8, y-0, 8-10)

Given A, B, C lie on same line

So values of DR's should be proportional

$$\frac{x-8}{6}=\frac{y}{3}=\frac{8-10}{6}$$

$So\ x=6, y=-1$

point is (6, -1, 8)

Q3

If points are collinear then all points lie on same line

and DR's should be proportional

A(2, 3, 4), B(-1, 2, -3) and C(-4, 1, -10)

DR's of AB=(3, 1, 7)

DR's of BC=(3, 1, 7)

So A, B , C are collinear

Length of AC=$\sqrt{36+4+196} = \sqrt{236}$

Length of AB=$\sqrt{9+1+49} = \sqrt{59}$

Ratio is AC:AB=2:1

So C divides AB in ratio 2:1 externally

Q4

yz plane means x=0

Given (2, 4, 5) and (3, 5, 4)

assume ratio to be m:n

lets equate x-term

$0=\frac{3m+2n}{m+n}$

$3m=-2n$

$m:n=-2:3$

which means yz plane divides the line in 2:3 ratio externally

Q5

(2, -1, 3) and (-1, 2, 1)

$x+y+z=5$

Assume plane divides line in ratio λ:1

so point P which is diving line in $\lambda:1$ ratio is

$P=(\frac{-\lambda+2}{\lambda+1}, \frac{2\lambda-1}{\lambda+1}, \frac{\lambda+3}{\lambda+1})$

P lies on plane x+y+z=5

$-\lambda+2+2\lambda-1+\lambda+3=5\lambda+5$

$3\lambda=-1 \Rightarrow \lambda=-1:3$

So plane diving line in 1:3 ratio externally

Q6

A(3, 2, -4), B(9, 8, -10) and C(5, 4, -6)

AC=$\sqrt{4+4+4} = 2\sqrt{3}$

AB=$\sqrt{36+36+36} = 6\sqrt{3}$

$BC = \sqrt{16+16+16} = 4\sqrt{3}$

$AC : BC = 1 : 2$

Q7

Given midpoints D(-2, 3, 5), E(4, -1, 7) and F(6, 5, 3)

Assume D is midpoint of AB, E is midpoint of BC

F is midpoint of CA

$A(x_1, y_1, z_1)$ $B(x_2, y_2, z_2)$ $C(x_3, y_3, z_3)$

From midpoint formula, we get following equations

$x_1+x_2=-4$; $x_2+x_3=8$; $x_3+x_1=12$

$y_1+y_2=6$; $y_2+y_3=-2$; $y_3+y_1=10$

$z_1+z_2=10$; $z_2+z_3=14$; $z_3+z_1=6$

Solving above set of equations we get

A=(0, 9, 1)

B=(-4, -3, 9)

C=(12, 1, 5)

Q8

A(1, 2, 3), B(0, 4, 1), C(-1, -1, -3)

Angle bisctor at A divides BC in ratio of AB:AC

AB=$\sqrt{1+4+4} = 3$

$AC = \sqrt{4+9+36} = 7$

Assume D divides BC

m:n=3:7

so D=$\left(\frac{-3}{10}, \frac{25}{10}, \frac{-2}{10}\right)$

Q9

(12, -4, 8) and (27, -9, 18)

Assume point P is dividing line in λ:1 ratio, we get

$$P=\left(\frac{27\lambda+12}{\lambda+1},\frac{-9\lambda-4}{\lambda+1},\frac{18\lambda+8}{\lambda+1}\right)$$

P lies on Sphere, so substitute in Sphere equation

$$x^2+y^2+z^2=504$$

$$9(9\lambda+4)^2+(9\lambda+4)^2+4(9\lambda+4)^2=504(\lambda+1)^2$$

$$729\lambda^2+81\lambda^2+324\lambda^2+648\lambda+72\lambda+288\lambda+144+16+64=504\lambda^2+1008\lambda+504$$

$$(1134-504)\lambda^2+(1008-1008)\lambda+224-504=0$$

$$630\lambda^2=280$$

$$\lambda^2=\frac{4}{9}$$

$$\lambda=2:3$$

Q10

Assume ratio is $\lambda:1$

Plane is $ax+by+cz+d=0$

points (x_1,y_1,z_1) and (x_2,y_2,z_2)

Assume point of intersection of line and plane is D

$$D=\left(\frac{\lambda x_2+x_1}{\lambda+1},\frac{\lambda y_2+y_1}{\lambda+1},\frac{\lambda z_2+z_1}{\lambda+1}\right)$$

As D lies on plane, substitute D in plane equation, we get

$$\lambda\left(ax_2+by_2+cz_2+d\right)+ax_1+by_1+cz_1+d=0$$

$$\Rightarrow \lambda=-\frac{ax_1+by_1+cz_1+d}{ax_2+by_2+cz_2+d}$$

Q11

(1, 2, -3), (3, 0, 1) and (-1, 1, -4)

Centroid of Traingle is given by

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$$

We know that

$x_1+x_2=2$

$x_2+x_3=6$

$x_1+x_3=-2$

Adding all gives $\Rightarrow 2\left(x_1+x_2+x_3\right)=6$

so $x_1+x_2+x_3=3$

similarly, $y_1+y_2+y_3=3; z_1+z_2+z_3=-6$

$Centroid=(1,1,-2)$

Q12

Given Centroid (1, 1, 1)

A(3, -5, 7) and B(-1, 7, -6)

Equating terms, we get

$$1=\frac{3-1+x_3}{3}$$

$$1=\frac{-5+7+y_3}{3}$$

$$1=\frac{7-6+z_3}{3}$$

$(x_3, y_3, z_3)=(1,1,2)$

Q13

Trisection points are those which divide line in ratio 1:2 or 2:1

P(4, 2, -6) and Q(10, -16, 6)

Consider 1:2 case, we get

$$\left(\frac{10+8}{3}, \frac{-16+4}{3}, \frac{6-12}{3}\right)=(6,-4,-2)$$

Consider 2:1 case, we get

$$\left(\frac{20+4}{3}, \frac{-32+2}{3}, \frac{12-6}{3}\right)=(8,-10,2)$$

$(6,-4,-2)$ and $(8,-10,2)$ are trisection points

Q14

A(2, -3, 4), B(-1, 2, 1) and C(0, 1/3, 2)

DR's of AB are (3, -5, 3)

DR's of BC are $(-1, \frac{5}{3}, -1)$

DR's of AC are $(2, \frac{-10}{3}, 2)$

Its clear that all DR's are proportional

Q15

P(3, 2, -4), Q(5, 4, -6) and R(9, 8, -10)

$PQ=\sqrt{4+4+4} = 2\sqrt{3}$

$QR = \sqrt{16+16+16} = 4\sqrt{3}$

$PQ : QR = 1 : 2$

Q16

(4, 8, 10) and (6, 10, -8) is divided by the yz-plane.

Equation of yz-plane is x=0

assume ratio is m:n

Equating x-term, we get

$0=\frac{6m+4n}{m+n}$

$m : n = -2 : 3$

So YZ plane divides the line segment in ratio 2:3 externally