Ex 25.1

Q1(i)

Let $P(x, y)$ be any point on the parabola whose focus is $S(3,0)$ and the directrix $3x + 4y = 1$. Draw PM perpendicular from $P(x, y)$ on the directrix $3x + 4y = 1$.

Then by definition

$$SP = PM$$

$$\Rightarrow \quad SP^2 = PM^2$$

$$\Rightarrow \quad (x-3)^2 + (y-0)^2 = \left(\frac{3x + 4y - 1}{\sqrt{(3)^2 + (4)^2}}\right)^2$$

$$\Rightarrow \quad x^2 + 9 - 6x + y^2 = \left(\frac{3x + 4y - 1}{\sqrt{9+16}}\right)^2$$

$$\Rightarrow \quad x^2 - 6x + 9 + y^2 = \frac{(3x + 4y - 1)^2}{\left(\sqrt{25}\right)^2}$$

$$\Rightarrow \quad x^2 - 6x + y^2 + 9 = \frac{(3x + 4y - 1)^2}{25}$$

$$\Rightarrow \quad 25\left(x^2 - 6x + y^2 + 9\right) = (3x + 4y - 1)^2$$

$$\Rightarrow \quad 25x^2 - 150x + 25y^2 + 225 = (3x)^2 + (4y)^2 + (-1)^2 + 2 \times 3x \times 4y + 2 \times 4y \times (-1) + 2 \times (-1) \times 3x$$

$$\Rightarrow \quad 25x^2 - 150x + 25y^2 + 225 = 9x^2 + 16y^2 + 1 + 24xy - 8y - 6x$$

$$\Rightarrow \quad 25x^2 - 9x^2 + 25y^2 - 16y^2 - 150x + 6x + 8y - 24xy + 225 - 1 = 0$$

$$\Rightarrow \quad 16x^2 + 9y^2 - 144x + 8y - 24xy + 224 = 0$$

$$\Rightarrow \quad 16x^2 + 9y^2 - 24xy - 144x + 8y + 224 = 0$$

This is the equation of the required parabola.

Q1(ii)

Let $P(x, y)$ be any point on the parabola whose focus is $S(1, 1)$ and the directrix $x + y + 1 = 0$. Draw PM perpendicular from $P(x, y)$ on the directrix $x + y + 1 = 0$. Then by definition

$$SP = PM$$

$$\Rightarrow \quad SP^2 = PM^2$$

$$\Rightarrow \quad (x-1)^2 + (y-1)^2 = \left(\frac{x+y+1}{\sqrt{1^2+1^2}}\right)^2$$

$$\Rightarrow \quad x^2 + 1 - 2x + y^2 + 1 - 2y = \left(\frac{x+y+1}{\sqrt{2}}\right)^2$$

$$\Rightarrow \quad x^2 + y^2 - 2x - 2y + 2 = \frac{(x+y+1)^2}{2}$$

$$\Rightarrow \quad 2(x^2 + y^2 - 2x - 2y + 2) = x^2 + y^2 + 1 + 2xy + 2y + 2x$$

$$\Rightarrow \quad 2x^2 + 2y^2 - 4x - 4y + 4 = x^2 + y^2 + 1 + 2xy + 2y + 2x$$

$$\Rightarrow \quad 2x^2 - x^2 + 2y^2 - y^2 - 2xy - 4x - 2x - 4y - 2y + 4 - 1 = 0$$

$$\Rightarrow \quad x^2 + y^2 - 2xy - 6x - 6y + 3 = 0$$

This is the equation of the required parabola.

Q1(iii)

Let $P(x, y)$ be any point on the parabola whose focus is $S(0, 0)$ and the directrix $2x - y - 1 = 0$. Draw PM perpendicular from $P(x, y)$ on the directrix $2x - y - 1 = 0$.

Then by definition

$$SP = PM$$

$$\Rightarrow \quad SP^2 = PM^2$$

$$\Rightarrow \quad (x-0)^2 + (y-0)^2 = \left(\frac{2x - y - 1}{\sqrt{(2)^2 + (-1)^2}}\right)^2$$

$$\Rightarrow \quad x^2 + y^2 = \frac{(2x - y - 1)^2}{(\sqrt{5})^2}$$

$$\Rightarrow \quad 5(x^2 + y^2) = (2x - y - 1)^2$$

$$\Rightarrow \quad 5x^2 + 5y^2 = (2x)^2 + (-y)^2 + (-1)^2 + 2 \times 2x \times (-y) + 2 \times (-y) \times (-1) + 2 \times (-1) \times 2x$$

$$\Rightarrow \quad 5x^2 + 5y^2 = 4x^2 + y^2 + 1 - 4xy + 2y - 4x$$

$$\Rightarrow \quad 5x^2 - 4x^2 + 5y^2 - y^2 + 4xy + 4x - 2y - 1 = 0$$

$$\Rightarrow \quad x^2 + 4y^2 + 4xy + 4x - 2y - 1 = 0$$

This is the equation of the required parabola.

Q1(iv)

Let $P(x, y)$ be any point on the parabola whose focus is $S(2, 3)$ and the directrix $x - 4y + 3 = 0$. Draw PM perpendicular from $P(x, y)$ on the directrix $x - 4y + 3 = 0$.
Then by definition

$$SP = PM$$

$$\Rightarrow \quad SP^2 = PM^2$$

$$\Rightarrow \quad (x-2)^2 + (y-3)^2 = \left(\frac{x - 4y + 3}{\sqrt{1^2 + (-4)^2}}\right)^2$$

$$\Rightarrow \quad x^2 + 4 - 4x + y^2 + 9 - 6y = \frac{(x - 4y + 3)^2}{(\sqrt{17})^2}$$

$$\Rightarrow \quad x^2 + y^2 - 4x - 6y + 4 + 9 = \frac{(x - 4y + 3)^2}{17}$$

$$\Rightarrow \quad 17(x^2 + y^2 - 4x - 6y + 13) = (x - 4y + 3)^2$$

$$\Rightarrow \quad 17x^2 + 17y^2 - 68x - 102y + 221 = x^2 + (-4y)^2 + 3^2 + 2 \times x \times (-4y) + 2 \times (-4y) \times 3 + 2 \times 3 \times x$$

$$\Rightarrow \quad 17x^2 + 17y^2 - 68x - 102y + 221 = x^2 + 16y^2 + 9 - 8xy - 24y + 6x$$

$$\Rightarrow \quad 17x^2 - x^2 + 17y^2 - 16y^2 + 8xy - 68x - 6x - 102y + 24y + 221 - 9 = 0$$

$$\Rightarrow \quad 16x^2 + y^2 + 8xy - 74x - 78y + 212 = 0$$

This is the equation of the required parabola.

Q2

Let $P(x, y)$ be any point on the parabola whose focus is $S(2, 3)$ and the directrix $x - 4y + 3 = 0$. Draw PM perpendicular from $P(x, y)$ on the directrix $x - 4y + 3 = 0$.

Then by definition

$$SP = PM$$

$$\Rightarrow \quad SP^2 = PM^2$$

$$\Rightarrow \quad (x-2)^2 + (y-3)^2 = \left(\frac{x - 4y + 3}{\sqrt{1^2 + (-4)^2}}\right)^2$$

$$\Rightarrow \quad x^2 + 4 - 4x + y^2 + 9 - 6y = \frac{(x - 4y + 3)^2}{(\sqrt{17})^2}$$

$$\Rightarrow \quad x^2 + y^2 - 4x - 6y + 4 + 9 = \frac{(x - 4y + 3)^2}{17}$$

$$\Rightarrow \quad 17(x^2 + y^2 - 4x - 6y + 13) = (x - 4y + 3)^2$$

$$\Rightarrow \quad 17x^2 + 17y^2 - 68x - 102y + 221 = x^2 + (-4y)^2 + 3^2 + 2 \times x \times (-4y) + 2 \times (-4y) \times 3 + 2 \times 3 \times x$$

$$\Rightarrow \quad 17x^2 + 17y^2 - 68x - 102y + 221 = x^2 + 16y^2 + 9 - 8xy - 24y + 6x$$

$$\Rightarrow \quad 17x^2 - x^2 + 17y^2 - 16y^2 + 8xy - 68x - 6x - 102y + 24y + 221 - 9 = 0$$

$$\Rightarrow \quad 16x^2 + y^2 + 8xy - 74x - 78y + 212 = 0$$

This is the equation of the required parabola.

Latus Rectum = Length of perpendicular from focus $(2,3)$ *on directrix* $x - 4y + 3 = 0$

$$= 2\left|\frac{2 - 12 + 3}{\sqrt{1 + 16}}\right|$$

$$= 2\left|\frac{-7}{\sqrt{17}}\right|$$

$$= \frac{14}{\sqrt{17}}$$

Q3(i)

Given focus (-6, -6)

Vertex(-2,2)

Slope of line connecting vertex and focus is $\frac{2+6}{-2+6}=2$

Slope of directrix will be $-\frac{1}{2}$, because both lines are perpendicular

Vertex is the midpoint of focus and point on directrix which passes through axis

$-2=\frac{-6+x}{2};\ 2=\frac{-6+y}{2}$

$(x,y)=(2,10)$

Equation of directrix is given by

$y-10=\frac{-1}{2}(x-2)$

$2y-20=-x+2$

$x+2y=22$

Equation of Parabola is $(x+6)^2+(y+6)^2=\frac{(x+2y-22)^2}{5}$

$5\left[x^2+y^2+36+36+12x+12y\right]=\left[x^2+4y^2+484+4xy-88y-44x\right]$

$4x^2+y^2-124-4xy+104x+148y=0$

$(2x-y)^2+4(26x+37y-31)=0$

Q3(ii)

In a parabola, vertex is the mid-point of the focus and the point of the intersection of the axis and directrix. so, let (x_1,y_1) be the coordinate of the point of intersection of the axis and directrix. Then $(0,0)$ is the mid-point of the line segment joining $(0,-3)$ and (x_1,y_1).

$\therefore\quad \frac{x_1-0}{2}=0$ and $\frac{y_1-3}{2}=0$

$\Rightarrow\quad x_1=0$ and $y_1=3$

Thus, the directrix meets the axis at $(0,3)$

$\therefore$ The equation of the directrix is $y=3$

Clearly, the required parabola is of the form $x^2=-4ay$, where $a=3$

$\therefore$ equation of parabola is $x^2=-4\times3\times y$

$\Rightarrow\quad x^2=-12y$

Q3(iii)

In a parabola, vertex is the mid-point of the focus and the point of intersection of the axis and directrix. So, let (x_1, y_1) be the coordinate of the point of intersection of the axis and directrix. Then $(-1,-3)$ is the mid-point of the line segment joining $(0,-3)$ and (x_1, y_1).

$$\therefore \quad \frac{x_1+0}{2} = -1 \quad \text{and} \quad \frac{y_1-3}{2} = -3$$
$$\Rightarrow \quad x_1 = -2 \quad \text{and} \quad y_1 = -3$$

Thus, the directrix meets the axis at $(-2,-3)$.
Let A be the vertex and S be the focus of the required parabola.
Then,

$$m_1 = \text{slope of } AS = \frac{-3-(-3)}{0-(-1)} = 0$$

$$\therefore \quad \text{slope of the directrix} = \frac{-1}{0} = \infty \qquad [\because \text{ Directrix is perpendicular to the axis}]$$

Thus, the directrix passes through $(-2,-3)$ and has slope ∞, so its equation is

$$y-(-3) = \infty(x-(-2))$$

$$\frac{y+3}{\infty} = x+2$$
$$\Rightarrow \quad x+2 = 0$$

Let $P(x, y)$ be a point on the parabola.
Then, PS = Distance of P from the directrix.

$$\sqrt{(x-0)^2+(y+3)^2} = \left|\frac{x+2}{\sqrt{1^2}}\right|$$

$$\Rightarrow \quad x^2+(y+3)^2 = (x+2)^2$$

$$\Rightarrow \quad x^2+y^2+9+6y = x^2+4+4x$$

$$\Rightarrow \quad y^2-4x+6y+9-4 = 0$$

$$\Rightarrow \quad y^2-4x+6y+5 = 0$$

Q3(iv)

In a parabola, vertex is the mid-point of the focus and the point of intersection of the axis and directrix. so, let (x_1, y_1) be the coordinates of the point of intersection of the axis and directrix. Then $(a', 0)$ is the mid-point of the line segment joining $(a, 0)$ and (x_1, y_1).

$$\therefore \quad \frac{x_1 + a}{2} = a' \quad \text{and} \quad \frac{y_1 + 0}{2} = 0$$

$$\Rightarrow \quad x_1 = 2a' - a \quad \text{and} \quad y_1 = 0$$

Thus, the directrix meets the axis at $(2a' - a, 0)$.

So the equation of directrix is $x = 2a' - a$

Let $P(x, y)$ be any point on the parabola. Then

$$SP = PM$$

$$\Rightarrow \quad SP^2 = PM^2$$

$$\Rightarrow \quad (x - a)^2 + (y - 0)^2 = \left[\frac{x - 2a' + a}{\sqrt{1^2}}\right]^2$$

$$\Rightarrow \quad x^2 + a^2 - 2ax + y^2 = (x - 2a' + a)^2$$

$$\Rightarrow \quad x^2 + a^2 - 2ax + y^2 = x^2 + (-2a')^2 + a^2 + 2 \times x \times (-2a') + 2 \times (-2a') \times a + 2 \times (a) \times (x)$$

$$\Rightarrow \quad x^2 + a^2 - 2ax + y^2 = x^2 + 4(a')^2 + a^2 - 4xa' - 4a'a + 2ax$$

$$\Rightarrow \quad y^2 = x^2 - x^2 + a^2 - a^2 + 2ax + 4(a')^2 - 4xa' - 4a'a + 2ax$$

$$\Rightarrow \quad y^2 = 4ax - 4xa' + 4(a')^2 - 4a'a$$

$$\Rightarrow \quad y^2 = 4ax - 4a'a - 4xa' + 4(a')^2$$

$$= 4a(x - a') - 4a'(x - a')$$

$$= (4a - 4a')(x - a')$$

$$= 4(a - a')(x - a')$$

$$\therefore \quad y^2 = 4(a - a')(x - a')$$

$$\Rightarrow \quad y^2 = -4(a' - a)(x - a')$$

Hence, required equation of parabola is $y^2 = -4(a' - a)(x - a')$

Q3(v)

x + y = 1 and x - y = 3

Intersecting point of above lines is

(x,y)=(2,-1)--------vertex

Focus (0,0)

Vertex is the midpoint of focus and point on directrix which passes through

$2=\frac{0+x}{2};\ -1=\frac{0+y}{2}$

$(x,y)=(4,-2)$

Slope of line passing through focus and vertex is $\frac{-1}{2}$

Slope of directrix is 2, as both are perpendicular lines

y+2=2(x-4)

2x-y=10------ directrix

$SP^2 = PM^2$

$5\left(x^2+y^2\right)=\left(2x-y-10\right)^2$

$x^2+4y^2-100+4xy-20y+40x=0$

$\left(x+2y\right)^2+20(2x-y-5)=0$

Q4(i)

The given parabola $y^2 = 8x$ is of the form $y^2 = 4ax$, where $4a = 8$

$\Rightarrow \quad a = \frac{8}{4} = 2.$

Vertex: The coordinates of the vertex are $(0,0)$.

Focus: The coordinates of the focus are $(2,0)$.

Axes: The equation of the axis is $y = 0$.

Directrix: The equation of the directrix is $x = -2$

Latus-rectum: The length of the latus-rectum = $4a = 4\times 2 = 8$.

Q4(ii)

In the given parabola, $a = \frac{1}{16}$

$Focus(0, -\frac{1}{16})$

$vertex(0,0)$

$Directrix,\ y = \frac{1}{16}$

$aixs,\ x = 0$

$LR = \frac{1}{4}$

Q4(iii)

Axis: Equation of the axis of the parabola w.r.t new axes is $Y = 0$

$\therefore \quad y = 0 + 2$

$\Rightarrow \quad y = 2$

$\therefore$ equation of axis w.r.t old axes is $y = 2$

Directrix: Equation of the directrix of the parabola w.r.t new axes is $X = \frac{-3}{4}$

$\therefore \quad x = \frac{-3}{4} - 1$

$\Rightarrow \quad x = \frac{-7}{4}$

$\therefore$ Equation of the directrix of the parabola w.r.t old axes is $x = \frac{-7}{4}$

Latus-rectum: The length of the latus-rectum $= 4a$

$$= 4 \times \frac{3}{4}$$

$$= 3.$$

Q4(iv)

The given equation is

$$y^2 - 4y + 4x = 0$$

$\Rightarrow \quad y^2 - 4y = -4x$

$\Rightarrow \quad y^2 - 2 \times y \times 2 + (2)^2 = -4x + (2)^2$

$\Rightarrow \quad (y-2)^2 = -4x + 4$

$\Rightarrow \quad (y-2)^2 = -4(x-1) \qquad \ldots(i)$

Shifting the origin to the point $(1,2)$ without rotating the axes and denoting the new coordinates with respect to these axes by X and Y, we have

$x = X + 1, \quad y = Y + 2 \qquad \ldots(ii)$

Using these relations equation (i), reduces to

$Y^2 = -4X \qquad \ldots(iii)$

This is of the form $Y^2 = -4aX$.

on comparing, we get, $a = 1$

Now,

Vertex: The coordinates of the vertex w.r.t to new axes are $(X = 0, \quad y = 0)$.

$\therefore \quad x = 0 + 1, \qquad y = 0 + 2 \qquad$ [Using equation ii]

$\Rightarrow \quad x = 1, \qquad y = 2$

$\therefore$ Coordinates of the vertex w.r.t old axes are, $(1,2)$

Focus: The coordinates of the focus with respect to new axes are $(X = 1, \quad Y = 0)$.

Putting $X = -1$ and $Y = 0$ in equation (ii), we get

$x = -1 + 1, \quad y = 0 + 2$

$\Rightarrow \quad x = 0, \qquad y = 2$

$\therefore$ Coordinates of the focus w.r.t old axes are $(0,2)$.

Axis: Equation of the axis of the parabola w.r.t new axes is $Y = 0$

$\therefore \quad y = 0 + 2 \qquad$ [Using equation ii]

$\Rightarrow \quad y = 2$

$\therefore$ equation of axis w.r.t old axes is $y = 2$

Directrix: Equation of the directrix of the parabola w.r.t new axes is $X = 1$

Q4(v)

The given equation is

$y^2 + 4x + 4y - 3 = 0.$

$\Rightarrow \quad y^2 + 4y = -4x + 3$

$\Rightarrow \quad y^2 + 2 \times y \times 2 + 2^2 = -4x + 3 + 2^2$

$\Rightarrow \quad (y + 2)^2 = -4x + 3 + 4$

$\Rightarrow \quad (y + 2)^2 = -4x + 7$

$\Rightarrow \quad (y + 2)^2 = -4\left(x - \frac{7}{4}\right) \qquad \ldots(i)$

Shifting the origin to the point $\left(\frac{7}{4}, -2\right)$ without rotating the axes and denoting the new coordinates with respect to these axes by X and Y, we have

$x = X + \frac{7}{4}, \quad y = Y - 2 \qquad \ldots(ii)$

Using these relations equation (i), reduces to $Y^2 = -4X \qquad \ldots(iii)$

This is of the form $Y^2 = -4aX$ on comparing, we get $a = 1$

Now,

Vertex: The coordinates of the vertex w.r.t new axes are $(X = 0, \quad Y = 0)$

$\therefore \quad x = 0 + \frac{7}{4}, \quad y = 0 - 2 \qquad$ [Using (ii)]

$\Rightarrow \quad x = \frac{7}{4}, \quad y = -2$

$\therefore$ Coordinates of the vertex w.r.t old axes are $\left(\frac{7}{4}, -2\right)$.

Focus: The coordinates of the focus w.r.t new axes are $(X = -1, \quad = 0)$

$\therefore \; x = -1 + \frac{7}{4}$ and $y = 0 - 2 \qquad$ [Using (ii)]

$\Rightarrow \quad x = \frac{3}{4},$ and $y = -2$

$\therefore$ Coordinates oof the focus w.r.t old axes are $\left(\frac{3}{4}, -2\right)$.

Axis: Equation of the axis of the parabola w.r.t new axes is

$Y = 0$

$\therefore \quad y = 0 - 2 \qquad$ [Using equation (ii)]

$\Rightarrow \quad y = -2$

$\therefore$ equation of the w.r.t old axes is $y + 2 = 0$.

Q4(vi)

The given equation is

$$y^2 = 8x + 8y$$

$\Rightarrow \quad y^2 - 8y = 8x$

$\Rightarrow \quad y^2 - 2 \times 4 \times y + 16 = 8x + 16$

$\Rightarrow \quad (y-4)^2 = 8(x+2) \qquad \ldots(i)$

Shifting the origin to the point $(-2, 4)$ without rotating the axes and denoting the new coordinates w.r.t these axes by X and Y, we have

$$x = X - 2, \ y = Y + 4 \qquad \ldots(ii)$$

Using these relations equation (i), reduces to

$$Y^2 = 8X \qquad \ldots(iii)$$

This is of the form $Y^2 = 4aX$, on comparing, we get

$$4a = 8$$

$\Rightarrow \quad a = 2$

Now,

Vertex: The coordinates of the vertex w.r.t new axes are $(X = 0, \ Y = 0)$

$\therefore \quad x = 0 - 2, \ y = 0 + 4$

$\Rightarrow \quad x = -2, \ y = 4$

$\therefore$ Coordinates of the vertex w.r.t old axes are $(-2, 4)$

Focus: The coordinates of the focus w.r.t new axes are $(X = 2, \ Y = 0)$

$\therefore \quad x = 2 - 2$ and $y = 0 + 4 \qquad$ [Using equation (ii)]

$\Rightarrow \quad x = 0,$ and $y = 4$

$\therefore$ Coordinates of the focus w.r.t old axes are $(0, 4)$.

Axis: Equation of the axis of the parabola w.r.t new axes is $Y = 0$

$\therefore \quad y = 0 + 4 \qquad$ [Using equation (ii)]

$\Rightarrow \quad y = 4$

$\therefore$ equation of axis w.r.t old axes is $y = 4$

Directrix: Equation of the directrix of the parabola w.r.t new axes is

$$X = -2$$

Q4(vii)

The given system of equation is

$$4(y-1)^2 = -7(x-3)$$

$\Rightarrow \quad (y-1)^2 = \frac{-7}{4}(x-3) \qquad \ldots()$

Shifting the origin to the point $(3,1)$ without rotating the axes and denoting the new coordinates w.r.t these axes by X and Y,

we have,

$x = X+3, \quad y = Y+1 \qquad \ldots(\text{ii})$

Using these relation (i), redus to

$Y^2 = \frac{-7}{4}X \qquad \ldots(\text{iii})$

This is of the form $Y^2 = -4aX$, on comparing, we get

$$4a = \frac{7}{4}$$

$\Rightarrow \quad a = \frac{7}{16}$

Now,

Vertex: The coordinates of the vertex w.r.t new axes are $(X = 0,\ Y = 0)$

$\therefore \quad x = 0+3, \quad y = 0+1 \qquad$ [Using equation (iii)]

$\Rightarrow \quad x = 3, \quad y = 1$

$\therefore$ Coordinates of the vertex w.r.t old axes are $(3,1)$.

Focus: The coordinates of the focus w.r.t new axes are $\left(X = -\frac{7}{16},\ Y = 0\right)$

$\therefore \quad x = \frac{-7}{16}+3, \quad y = 0+1$

$\Rightarrow \quad x = \frac{41}{16}, \quad y = 1$

$\therefore$ Coordinates of the focus w.r.t old axes are $\left(\frac{41}{16}, 1\right)$.

Axis: Equation of the axis of the parabola w.r.t new axes is

Q4(viii)

The given system of equation is

$$y^2 = 5x - 4y - 9$$

$$\Rightarrow \quad y^2 + 4y = 5x - 9$$

$$\Rightarrow \quad y^2 + 4y + 4 = 5x - 9 + 4$$

$$\Rightarrow \quad (y+2)^2 = 5x - 5$$

$$\Rightarrow \quad (y+2)^2 = 5(x-1) \qquad \ldots(\mathrm{i})$$

Shifting the origin to the point $(1, -2)$ without rotating the axes and denoting the new coordinates w.r.t these axes by X and Y,
we have,

$$x = X + 1, \quad y = Y - 2 \qquad \ldots(\mathrm{ii})$$

using these relations, equation (i) reduces to

$$Y^2 = 5X \qquad \ldots(\mathrm{iii})$$

This is of the from $Y^2 = 4aX$ on comparing we get

$$4a = 5$$

$$\Rightarrow \quad a = \frac{5}{4}$$

Now,

Vertex: The coordinates of the vertex w.r.t new axes are $(X = 0, \ Y = 0)$

$$\therefore \quad x = 0 + 1, \ y = 0 - 2 \qquad [\text{Using equation } (\mathrm{ii})]$$

$$\Rightarrow \quad x = 1, \quad y = -2$$

$\therefore$ Coordinates of the vertex w.r.t old axes are $(1, -2)$.

Focus: The coordinates of the focus w.r.t new are $\left(X = \frac{5}{4}, \quad Y = 0\right)$

$$\therefore \quad x = \frac{5}{4} + 1, \quad y = 0 - 2$$

$$\Rightarrow \quad x = \frac{9}{4}, \quad y = -2$$

Q4(ix)

The given of equation is

$$x^2 + y = 6x - 14$$

$\Rightarrow \quad x^2 - 6x = -y - 14$

$\Rightarrow \quad x^2 - 2 \times x \times 3 + 9 = -y - 14 + 9$

$\Rightarrow \quad (x-3)^2 = -y - 5$

$\Rightarrow \quad (x-3)^2 = -1(y+5) \qquad \ldots(\text{i})$

Shifting the origin to the point $(3, -5)$ without rotating the axes and denotinh the new coordinates w.r.t these axes by X and Y,

we have,

$$x = X + 3, \quad y = Y - 5 \qquad \ldots(\text{ii})$$

Using these relations, equation (i) reduces to

$$X^2 = -y \qquad \ldots(\text{iii})$$

This is of the form $X^2 = -4aY$, on comparing, we get

$$4a = 1$$

$\Rightarrow \quad a = \frac{1}{4}$

Now,

Vertex: The coordinates of the vertex w.r.t new are $(X = 0, \ Y = 0)$

$\therefore \quad x = 0 + 3, \quad y = 0 - 5$

$\Rightarrow \quad x = 3, \ y = -5$

$\therefore$ Coordinates of the vertex w.r.t old axes are $(3, -5)$.

Focus: The coordinates of the focus w.r.t new axes are $\left(X = 0, \ Y = \frac{-1}{4}\right)$

$\therefore \quad x = 0 + 3, \quad y = \frac{-1}{4} - 5$

Q5

Let PQ be the double ordinate of length $8p$ of the parabola $y^2 = 4px$.
Then, $PR = QR = 4p$.

let $AR = x_1$. Then, the coordinates of P and Q are $(x_1, 4p)$ and, $(x_1, -4p)$ respectively.

Since P lies on $y^2 = 4px$

$\therefore \quad (4p)^2 = 4px_1$

$\Rightarrow \quad x_1 = 4p.$

So, coordinates of P and Q are $(4p, 4p)$ and $(4p, -4p)$ respectively.

$\therefore$ The extremities of a double ordinate are $(4p, 4p)$ and $(4p, -4p)$.

Also, the coordinates of the vertex A are $(0,0)$.

$$\therefore \quad m_1 = \text{slope of } AP = \frac{4p - 0}{4p - 0} = 1$$

$$\text{and,} \quad m_2 = \text{slope of } AQ = \frac{-4p - 0}{4p - 0} = -1$$

Clearly, $m_1 m_2 = -1$.
Hence, $AP \perp AQ$

$\therefore$ The lines from the vertex to its extremities are at right angles.

Q6

The given equation of the parabola is

$$x^2 = 12y$$

This is of the form $x^2 = 4ay$, on comparing, we get

$$4a = 12$$

$$\Rightarrow \quad a = \frac{12}{4} = 3$$

$\therefore$ Coordinates of the focus S is $(0,3)$.

P and Q lies on the parabola.

$$\therefore \quad x^2 = 12 \times 3$$

$$\Rightarrow \quad x^2 = 36$$

$$\Rightarrow \quad x = \pm 6$$

$\therefore$ $P(-6,3)$ and $Q(6,3)$.

Now, $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$= \sqrt{(6+6)^2 + (3-3)^2}$$

$$= \sqrt{(12)^2}$$

$$= 12$$

and, $OS = 3$.

$\therefore$ Area of $\triangle POQ = \frac{1}{2} \times PQ \times OS$

$$= \frac{1}{2} \times 12 \times 3$$

$$= 6 \times 3 = 18 \text{ sq. units.}$$

Q7

The axis of the parabola is a line $\perp$ to the directrix and passing through focus. The equation of a line $\perp$ to $3x - 4y - 2 = 0$ is

$$y = \frac{-4}{3} + \lambda \qquad \left[\begin{array}{l} \because m_1 m_2 = -1 \\ \Rightarrow m_2 = \dfrac{-1}{m_1} \text{ and } y = m_2 x + \lambda \end{array}\right]$$

$$\Rightarrow \quad 3y + 4x = 3\lambda$$

This will pass through focus $(3, 3)$ if,

$$3 \times 3 + 4 \times 3 = 3\lambda$$

$$\Rightarrow \quad 9 + 12 = 3\lambda$$

$$\Rightarrow \quad 21 = 3\lambda$$

$$\Rightarrow \quad \lambda = \frac{21}{3} = 7$$

so, the equation of axis is $3y + 4x = 3 \times 7 = 21$

$$\Rightarrow \quad 3y + 4x = 21 \qquad \ldots(i)$$

And the equation of directrix is

$$3x - 4y = 2 \qquad \ldots(ii)$$

Mutiplying equation (i) by 4 and equation (ii) by 3, we get

$$16x + 12y = 84 \qquad \ldots(iii)$$

$$9x - 12y = 6 \qquad \ldots(iv)$$

Adding equation (iii) and (iv), we get

$$16x + 9x = 84 + 6$$

$$\Rightarrow \quad 25x = 90$$

$$\Rightarrow \quad x = \frac{90}{25} = \frac{18}{5}$$

Putting $x = \frac{18}{5}$ in equation (i), we get

Q8

Let the ordinates of the required point is y.

$\therefore$ abscissa = $3y$

$\therefore$ The coordinates of the points are $(3y, y)$.

These points lies on the parabola $x^2 = 9y$.

$\therefore \quad (3y)^2 = 9y$

$\Rightarrow \quad 9y^2 = 9y$

$\Rightarrow \quad 9y^2 - 9y = 0$

$\Rightarrow \quad 9y(y-1) = 0$

$\Rightarrow \quad y - 1 = 0 \qquad [\because y \neq 0]$

$\Rightarrow \quad y = 1$

$\therefore$ abscissa $= 3 \times y = 3$

Hence, the required point is $(3,1)$.

Q9

Let the equation of parabola be

$y^2 = 4ax \qquad \ldots(i) \qquad [\because$ axis along x-axis$]$

If passes through $(2,3)$.

$\therefore \quad (3)^2 = 4 \times a \times 2$

$\Rightarrow \quad 9 = 8a$

$\Rightarrow \quad a = \frac{9}{8}$

Putting the value of a in equation (i), we get

$y^2 = 4 \times \frac{9}{8} \times x$

$\Rightarrow \quad y^2 = \frac{9}{2} \times x$

$\Rightarrow \quad 2y^2 = 9x$

Hence, the required equation of parabola is $2y^2 = 9x$.

Q10

Let (x_1, y_1) be the coordinates of the point intersection of the axis and the directrix.

$\therefore \quad (x_1, y_1) = (0, 2) \qquad [\because\ y = 2]$

we know that, the vertex is the mid-point of the line segment joining $(0, 2)$ and focus (x_2, y_2)

$\therefore \quad \frac{x_2 + 0}{2} = 0 \quad \text{and} \quad \frac{y_2 + 2}{2} = 0 \qquad [\because \text{vertex at the origin}]$

$\therefore \quad x_2 = 0, \quad \text{and } y_2 = -2$

$\therefore$ The coordinates of focus is $(0, -2)$.

By the definition of parabola

$$PS = PM$$

$$\Rightarrow \quad PS^2 = PM^2$$

$$\Rightarrow \quad (x - 0)^2 + (y + 2)^2 = \left[\frac{y - 2}{\sqrt{1}}\right]^2$$

$$\Rightarrow \quad x^2 + y^2 + 4 + 4y = (y - 2)^2$$

$$\Rightarrow \quad x^2 + y^2 + 4 + 4y = y^2 + 4 - 4y$$

$$\Rightarrow \quad x^2 = -4y - 4y$$

$$\Rightarrow \quad x^2 = -8y$$

Hence, The required equation of parabola is $x^2 = -8y$.

Q11

In a parabola, vertex is the mid point of the focus and the point of intersection of the axis and directrix. So let (x, y) be the coordinates of the point of intersection of the axis and directrix.

Then $(3,2)$ is the mid point of the line segment joining $(5,2)$ and (x_1, y_1)

$$\frac{x_1+5}{2}=3 \quad \text{and} \quad \frac{y_1+2}{2}=2$$
$$x_1+5=6 \quad \text{and} \quad y_1+2=4$$
$$x_1=1 \quad \text{and} \quad y_1=2$$

The directrix meets the axis at $(1,2)$

Let A be the vertex and S be the focus of the required parabola

Then

$$m_1 = \text{slope of } AS = \frac{2-2}{5-3}$$
$$=0$$

Let m_2 be the slope of the directrix

Then

$$m_2=\infty \qquad [\because \text{Directrix is perpendicular to the axis}]$$

Thus the directrix passes through $(1,2)$ and the slope ∞, so its equation is

$$y-2=\infty(x-1)$$
$$\frac{y-2}{\infty}=x-1$$
$$x-1=0$$

Let $P(x, y)$ be a point on the parabola

Then PS=distance of P from the directrix

$$\sqrt{(x-5)^2+(y-2)^2}=\left|\frac{x-1}{\sqrt{1^2}}\right|$$
$$(x-5)^2+(y-2)^2=(x-1)^2$$
$$x^2+25-10x+y^2+4-4y=x^2+1-2x$$
$$y^2-4y-8x+28=0$$

Hence the required equation of the parabola is $y^2-4y-8x+28=0$

Q12

Let CAB be the bridge and LOX be the road way. Let A be the centre of the bridge. we find that the coordinates of A $are\left(0,6\right)$.

Clearly, the bridgeis in the shope of a parabola having its vertex at A $\left(0,6\right)$.
Let its equation be $x^2 = 4a\left(y-6\right)$ $\quad\quad\quad\ldots\left(\text{i}\right)$

It posses through $B\left(50,30\right)$. Therefore, $\left(50\right)^2 = 4a\left(30-6\right)$

$$\Rightarrow \quad 2500 = 4a \times 24$$

$$\Rightarrow \quad \frac{2500}{4\times24} = a$$

$$\Rightarrow \quad a = \frac{625}{24}$$

Putting the value of a in $\left(\text{i}\right)$, we get

$$x^2 = 4\times\frac{625}{24}\left(y-6\right)$$

$$\Rightarrow \quad x^2 = \frac{625}{6}\left(y-6\right) \quad\quad\quad \ldots\left(\text{ii}\right)$$

Let l metres be the length of the vertical supporting cable 18 metres from the centre. Then, P $\left(18,l\right)$ lies on $\left(\text{ii}\right)$. Therefore

$$\left(18\right)^2 = \frac{625}{6}\left(l-6\right)$$

$$\Rightarrow \quad 324\times6 = 625\left(l-6\right)$$

Q13

When x=24, then y=$\pm$12

So two points are A(24, 12) and B(24, -12)

Equation of lines joining vertex and A is

$$y=\frac{1}{2}x$$

Equation of lines joining vertex and B is

$$y=-\frac{1}{2}x$$

Q14

In given parabola

a=2

Given focal distance=a+x=4, so x=2

So points are (2, 4) and (2, -4)

Q15

$$y = x\tan\theta$$
$$y^2 = 4ax$$

Intersection point of both the curves are $\left(\frac{4a}{\tan^2\theta}, \frac{4a}{\tan\theta}\right)$

So Distance from origin to the above point is

$$\sqrt{\left(\frac{4a}{\tan^2\theta}\right)^2+\left(\frac{4a}{\tan\theta}\right)^2} = \frac{4a}{\tan^2\theta}\sqrt{1+\tan^2\theta} = 4a\cot\theta\operatorname{cosec}\theta$$

Q16

The vertex and focus of the parabola are $A(0, 4)$ and $F(0,2)$ respectively.
$AF = 2$
As point A and F lie on y-axis, so y-axis is the axis of the parabola.
If the diretrix meets the axis of parabola at point Z, then $AZ = AF = 2$.
$\therefore OZ = OF + FA + AZ = 2 + 2 + 2 = 6$
So equation of diretrix is $y = 6$

Let $P(x, y)$ be any point in the plane of focus and diretrix,
and MP be the perpendicular distance from P to the diretrix,
then P lies on parabola iff $FP = MP$

$$\Leftrightarrow \sqrt{(x-0)^2 + (y-2)^2} = \frac{|y-6|}{1}$$
$$\Leftrightarrow x^2 + (y-2)^2 = (y-6)^2$$
$$\Leftrightarrow x^2 + y^2 - 4y + 4 = y^2 - 12y + 36$$
$$\Leftrightarrow x^2 + 8y = 32$$

$x^2 + 8y = 32$ is the required equation of the parabola.

Q17

The line $y = mx + 1$ is tangent to the parabola $y^2 = 4x$.

$$\therefore (mx+1)^2 = 4x$$
$$m^2x^2 + 2mx + 1 = 4x$$
$$m^2x^2 + (2m-4)x + 1 = 0$$

As we know tangent touches the parabola, so the roots of the
above quadratic will be equal.

$$\Rightarrow D = b^2 - 4ac = 0$$
$$\Rightarrow (2m-4)^2 - 4(m^2)(1) = 0$$
$$\Rightarrow 4m^2 + 16 - 16m - 4m^2 = 0$$
$$\Rightarrow m = 1$$