

Ex 21.1

Q1

Let T_n be the n th term of this series then,

$$\begin{aligned} T_n &= [1 + (n-1)2]^3 \\ &= (2n-1)^3 \\ &= (2n)^3 - 3(2n)^2 \cdot 1 + 3 \cdot 1^2 \cdot 2n - 1^3 \quad \left[\because (a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3 \right] \\ &= 8n^3 - 12n^2 + 6n - 1 \end{aligned}$$

$\therefore 1^3 + 3^3 + 5^3 + \dots$ to n terms

$$\begin{aligned} &= \sum_{k=1}^n T_k \\ &= \sum_{k=1}^n (8k^3 - 12k^2 + 6k - 1) \\ &= 8 \sum_{k=1}^n k^3 - 12 \sum_{k=1}^n k^2 + 6 \sum_{k=1}^n k - \sum_{k=1}^n 1 \\ &= 8 \left[\frac{n(n+1)}{2} \right]^2 - 12 \left[\frac{n(n+1)(2n+1)}{6} \right] + 6 \left[\frac{n(n+1)}{2} \right] - n \\ &= 8 \frac{n^2(n+1)^2}{4} - 12[n(n+1)(2n+1)] + 3[n(n+1)] - n \\ &= 2n^2(n+1)^2 - 2n(n+1)(2n+1) + 3(n+1) - n \\ &= (n+1)[2n^2(n+1) - 2n(2n+1) + 3n] - n \\ &= (n+1)[2n^3 + 2n^2 - 4n^2 - 2n + 3n] - n \\ &= (n+1)[2n^3 - 2n^2 + n] - n \\ &= 2n^4 - 2n^3 + n^2 + 2n^3 - 2n^2 + n - n \\ &= 2n^4 + n^2 - 2n^2 \\ &= 2n^4 - n^2 \\ &= n^2(2n^2 - 1) \end{aligned}$$

$\therefore 1^3 + 3^3 + 5^3 + 7^3 + \dots$ to n terms $= n^2(2n^2 - 1)$.

Q2

Let T_n be the n th term of this series. Then,

$$\begin{aligned}T_n &= (2n)^3 \\&= 8n^3\end{aligned}$$

Let S_n be the sum to n terms of the given series; Then,

$$S_n = \sum_{k=1}^n 8k^3$$

$$= 8 \sum_{k=1}^n k^3$$

$$= 8 \left[\frac{n(n+1)}{2} \right]^2$$

$$= 8 \times \frac{n^2(n+1)^2}{4}$$

$$= 2n^2(n+1)^2$$

$$\text{Hence, } S_n = 2n^2(n+1)^2$$

Q3

Let T_n be the n th term of the given series. Then,

$$\begin{aligned}T_n &= (\text{nth term of } 1, 2, 3, \dots) \times (\text{nth term of } 2, 3, 4, \dots) \times (\text{nth term of } 5, 6, 7, \dots) \\&= [1 + (n-1) \times 1] \times [2 + (n-1) \times 1] \times [5 + (n-1) \times 1] \\&= [1 + n - 1] \times [2 + n - 1] \times [5 + n - 1] \\&= n \times (n+1) (n+4) \\&= (n^2 + n) (n+4) \\&= n^3 + 4n^2 + n^2 + 4n \\&= n^3 + 5n^2 + 4n \\&= T_n = n^3 + 5n^2 + 4n\end{aligned}$$

Let S_n denote the sum to n terms of the give series. Then,

$$\begin{aligned}S_n &= \sum_{n=1}^n T_n = \sum_{n=1}^n (n^3 + 5n^2 + 4n) \\&= \sum_{n=1}^n n^3 + \sum_{n=1}^n 5n^2 + \sum_{n=1}^n 4n \\&= \sum_{n=1}^n n^3 + 5 \sum_{n=1}^n n^2 + 4 \sum_{n=1}^n n \\&= \left[\frac{n(n+1)}{2} \right]^2 + 5 \left[\frac{n(n+1)(2n+1)}{6} \right] + 4 \left[\frac{n(n+1)}{2} \right] \\&= \frac{n^2(n+1)^2}{4} + \frac{5n(n+1)(2n+1)}{6} + 2n(n+1) \\&= \frac{3n^2(n+1)^2 + 10n(n+1)(2n+1) + 24n(n+1)}{12} \\&= \frac{n(n+1)}{12} [3n(n+1) + 10(2n+1) + 24] \\&= \frac{n(n+1)}{12} [3n^2 + 3n + 20n + 10 + 24] \\&= \frac{n(n+1)}{12} [3n^2 + 23n + 34] \\&= \frac{n(n+1)(3n^2 + 23n + 34)}{12}\end{aligned}$$

$$\text{Hence, } S_n = \frac{n(n+1)(3n^2 + 23n + 34)}{12}$$

Q4

Let T_n be the n th term of the given series. Then,

$$\begin{aligned}T_n &= (n\text{th term of } 1, 2, 3, \dots) \times (n\text{th term of } 2, 3, 4, \dots) \times (n\text{th term of } 4, 7, 10, \dots) \\&= [1 + (n-1) \times 1] \cdot [2 + (n-1) \times 1] \cdot [4 + (n-1) \times 3] \\&= [1 + n - 1] \cdot [2 + n - 1] \cdot [4 + 3n - 3] \\&= n(n+1)(3n+1) \\&= (n^2 + n)(3n+1) \\&= 3n^3 + n^2 + 3n^2 + n \\&= 3n^3 + 4n^2 + n \\ \therefore &= T_n = 3n^3 + 4n^2 + n\end{aligned}$$

Let S_n denote the sum to n terms of the given series. Then,

$$\begin{aligned}S_n &= \sum_{n=1}^n T_n = \sum_{n=1}^n (3n^3 + 4n^2 + n) \\&= \sum_{n=1}^n 3n^3 + \sum_{n=1}^n 4n^2 + \sum_{n=1}^n n \\&= 3 \sum_{n=1}^n n^3 + 4 \sum_{n=1}^n n^2 + \sum_{n=1}^n n \\&= 3 \left[\frac{n(n+1)}{2} \right]^2 + 4 \left[\frac{n(n+1)(2n+1)}{6} \right] + \left[\frac{n(n+1)}{2} \right] \\&= \frac{3}{4} [n(n+1)]^2 + \frac{2n(n+1)(2n+1)}{3} + \frac{n(n+1)}{2} \\&= \frac{9[n(n+1)]^2 + 8n(n+1)(2n+1) + 6n(n+1)}{12} \\&= \frac{n(n+1)}{12} [9n(n+1) + 8(2n+1) + 6] \\&= \frac{n}{12} (n+1) [9n^2 + 9n + 16n + 8 + 6] \\&= \frac{n}{12} (n+1) [9n^2 + 25n + 14]\end{aligned}$$

$$\text{Hence, } S_n = \frac{n}{12} (n+1) (9n^2 + 25n + 14)$$

Q5

Let T_n be the n th term of the given series, Then,

$$\begin{aligned}T_n &= 1 + 2 + 3 + \dots + n \\&= \frac{n}{2} [2 \times 1 + (n - 1) \times 1] \\&= \frac{n}{2} [2 + n - 1] \\&= \frac{n}{2} (n + 1) \\&= \frac{n^2}{2} + \frac{n}{2}\end{aligned}$$

Let S_n denote the sum to n terms of the given series. Then,

$$\begin{aligned}S_n &= \sum_{k=1}^n T_k = \sum_{k=1}^n \left[\frac{k^2}{2} + \frac{k}{2} \right] \\&= \sum_{k=1}^n \frac{k^2}{2} + \sum_{k=1}^n \frac{k}{2}\end{aligned}$$

$$\begin{aligned}\Rightarrow S_n &= \frac{1}{2} \sum_{k=1}^n k^2 + \frac{1}{2} \sum_{k=1}^n k \\&= \frac{1}{2} \left[\frac{n(n+1)(2n+1)}{6} \right] + \frac{1}{2} \left[\frac{n(n+1)}{2} \right] \\&= \frac{n(n+1)(2n+1)}{12} + \frac{n(n+1)}{4} \\&= \frac{n(n+1)(2n+1) + 3n(n+1)}{12} \\&= \frac{n(n+1)}{12} [2n+1+3] \\&= \frac{n(n+1)}{12} [2n+4] \\&= \frac{n(n+1)}{12} \times 2(n+2) \\&= \frac{n}{6} (n+1)(n+2)\end{aligned}$$

$$\text{Hence, } S_n = \frac{n}{6} (n+1)(n+2)$$

Q6

Let T_n be the n th term of the given series. Then,

$$T_n = (n\text{th term of } 1, 2, 3, \dots) \times (n\text{th term of } 2, 3, 4, \dots)$$

$$= [1 + (n+1) \times 1] \cdot [2 + (n+1) \times 1]$$

$$= [1 + n - 1] \cdot [2 + n - 1]$$

$$= n(n+1)$$

$$= n^2 + n$$

Let S_n denote the sum to n terms of the given series. Then,

$$S_n = \sum_{n=1}^n T_n = \sum_{n=1}^n (n^2 + n) = \sum_{n=1}^n n^2 + \sum_{n=1}^n n$$

$$= \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$$

$$= \frac{n(n+1)(2n+1) + 3n(n+1)}{6}$$

$$= \frac{n(n+1)[2n+1+3]}{6}$$

$$= \frac{n(n+1)[2n+4]}{6}$$

$$= \frac{n(n+1) \times 2(n+2)}{6}$$

$$= \frac{n}{3} (n+1)(n+2)$$

$$\text{Hence, } S_n = \frac{n}{3} (n+1)(n+2)$$

Q7

Let T_n the n th term of the given series. Then,

$$\begin{aligned}T_n &= (n\text{th term of } 3, 5, 7, \dots) \times (n\text{th term of } 1^2, 2^2, 3^2, \dots) \\&= [3 + (n-1)2] \cdot [n^2] \\&= [3 + 2n - 2] \cdot [n^2] \\&= [2n + 1][n^2] \\&= 2n^3 + n^2\end{aligned}$$

$$\therefore T_n = 2n^3 + n^2$$

Let S_n denote the sum of n terms of the given series. Then,

$$\begin{aligned}S_n &= \sum_{n=1}^n T_n = \sum_{n=1}^n (2n^3 + n^2) \\&= \sum_{n=1}^n 2n^3 + \sum_{n=1}^n n^2 = 2 \sum_{n=1}^n n^3 + \sum_{n=1}^n n^2 \\&= 2 \left[\frac{n(n+1)}{2} \right]^2 + \left[\frac{n(n+1)(2n+1)}{6} \right] \\&= \frac{2}{4} [n(n+1)]^2 + \frac{[n(n+1)(2n+1)]}{6} \\&= \frac{[n(n+1)]^2}{2} + \frac{n(n+1)(2n+1)}{6} \\&= \frac{3[n(n+1)]^2 + n(n+1)(2n+1)}{6} \\&= \frac{n(n+1)}{6} [3n(n+1) + (2n+1)] \\&= \frac{n(n+1)}{6} [3n^2 + 3n + 2n + 1] \\&= \frac{n}{6} (n+1) (3n^2 + 5n + 1)\end{aligned}$$

$$\text{Hence, } S_n = \frac{n}{6} (n+1) (3n^2 + 5n + 1)$$

Q8(i)

We have,

$$T_n = 2n^3 + 3n^2 - 1$$

Let S_n denote the sum of n terms of the series whose n th term is T_n . Then,

$$S_n = \sum_{k=1}^n T_k = \sum_{k=1}^n (2k^3 + 3k^2 - 1) = \sum_{k=1}^n 2k^3 + \sum_{k=1}^n 3k^2 - \sum_{k=1}^n 1$$

$$= 2 \sum_{k=1}^n k^3 + 3 \sum_{k=1}^n k^2 - \sum_{k=1}^n 1$$

$$= 2 \left[\frac{n(n+1)}{2} \right]^2 + 3 \left[\frac{n(n+1)(2n+1)}{6} \right] - n$$

$$= \frac{2}{4} [n(n+1)]^2 + \frac{n(n+1)(2n+1) - n}{2}$$

$$= \frac{[n(n+1)]^2 + n(n+1)(2n+1) - n}{2}$$

$$= \frac{[n(n+1)]^2 + (n+1)(2n+1) - 2n}{2}$$

$$= \frac{n}{2} [n(n+1)^2 + (n+1)(2n+1) - 2]$$

$$= \frac{n}{2} [n^3 + n + 2n^2 + 2n^2 + 3n - 1]$$

$$= \frac{n}{2} [n^3 + 4n^2 + 4n - 1]$$

Hence, $S_n = \frac{n}{2} (n^3 + 4n^2 + 4n - 1)$

Q8(ii)

We have,

$$T_n = n^3 - 3^n$$

Let S_n denote the sum of n terms of the series whose n th term is T_n . Then,

$$S_n = \sum_{k=1}^n T_k = \sum_{k=1}^n (k^3 - 3^k) = \sum_{k=1}^n k^3 - \sum_{k=1}^n 3^k$$

$$\Rightarrow S_n = \sum_{k=1}^n k^3 - \sum_{k=1}^n 3^k$$

$$= \left[\frac{n(n+1)}{2} \right]^2 - (3^1 + 3^2 + \dots + 3^n)$$

$$= \frac{n^2(n+1)^2}{4} - 3 \left(\frac{3^n - 1}{3 - 1} \right)$$

$$= \frac{n^2(n+1)^2}{4} - \frac{3}{2}(3^n - 1)$$

$$\text{Hence, } S_n = \left[\frac{n(n+1)}{2} \right]^2 - \frac{3}{2}(3^n - 1)$$

Q8(iii)

We have,

$$T_n = n(n+1)(n+4) = (n^2+n)(n+4) = n^3 + 5n^2 + 4n$$

Let S_n denote the sum of n terms of the series n th term is T_n . Then,

$$\begin{aligned} S_n &= \sum_{k=1}^n T_k = \sum_{k=1}^n (k^3 + 5k^2 + 4k) \\ &= \sum_{k=1}^n k^3 + \sum_{k=1}^n 5k^2 + \sum_{k=1}^n 4k \\ &= \sum_{k=1}^n k^3 + 5 \sum_{k=1}^n k^2 + 4 \sum_{k=1}^n k \\ \Rightarrow S_n &= \sum_{k=1}^n k^3 + 5 \sum_{k=1}^n k^2 + 4 \sum_{k=1}^n k \\ &= \left[\frac{n(n+1)}{2} \right]^2 + 5 \left[\frac{n(n+1)(2n+1)}{6} \right] + \frac{4n(n+1)}{2} \\ &= \frac{1}{4} [n(n+1)]^2 + \frac{5n(n+1)(2n+1)}{6} + 2n(n+1) \\ &= \frac{3[n(n+1)]^2 + 10n(n+1)(2n+1) + 24n(n+1)}{12} \\ &= \frac{n(n+1)}{12} [3n(n+1) + 10(2n+1) + 24] \\ &= \frac{n(n+1)}{12} [3n^2 + 3n + 20n + 10 + 24] \\ &= \frac{n(n+1)}{12} [3n^2 + 23n + 34] \end{aligned}$$

$$\text{Hence, } S_n = \frac{n}{12} (n+1) (3n^2 + 23n + 34)$$

Q8(iv)

We have,

$$\begin{aligned}T_n &= (2n-1)^2 \\&= (2n)^2 + 1 - 2 \times 2n \times 1 \\&= 4n^2 + 1 - 4n \\&= 4n^2 - 4n + 1\end{aligned}$$

$$\therefore T_n = 4n^2 - 4n + 1$$

Let S_n denote the sum of n terms of the series whose n th term is T_n . Then,

$$S_n = \sum_{k=1}^n T_k = \sum_{k=1}^n 4k + \sum_{k=1}^n 1$$

$$S_n = \sum_{k=1}^n k^2 - 4 \sum_{k=1}^n k + \sum_{k=1}^n 1$$

$$\begin{aligned}\Rightarrow S_n &= 4 \sum_{k=1}^n k^2 - 4 \sum_{k=1}^n k + \sum_{k=1}^n 1 \\&= 4 \left[\frac{n(n+1)(2n+1)}{6} \right] - 4 \frac{n(n+1)}{2} + n \\&= \frac{2}{3} n(n+1)(2n+1) - 2n(n+1) + n \\&= \frac{2}{3} n(n+1)(2n+1) - 2n^2 - 2n + n \\&= \frac{2}{3} n(n+1)(2n+1) - 2n^2 - n \\&= \frac{2n}{3} (n+1)(2n+1) - n(2n+1) \\&= \frac{2n(n+1)(2n+1) - 3n(2n+1)}{3} \\&= \frac{n}{3} (2n+1) [2(n+1) - 3] \\&= \frac{n}{3} (2n+1) (2n+2-3) \\&= \frac{n}{3} (2n+1) (2n-1)\end{aligned}$$

$$\text{Hence, } S_n = \frac{n}{3} (2n+1) (2n-1)$$

Q9

Here the n th term of the series is:

$$T_n = 2n(2n+2)$$

Thus the 20th term will be:

$$T_{20} = 2 \times 20(2 \times 20 + 2) = 1680$$

The infinite series can be written as:

$$2 \times 4 + 4 \times 6 + 6 \times 8 + \dots = \sum_{n=1}^{\infty} 2n(2n+2)$$

Therefore the sum up to 20th term will be:

$$\begin{aligned} \sum_{n=1}^{20} 2n(2n+2) &= \sum_{n=1}^{20} 4n^2 + \sum_{n=1}^{20} 4n \\ &= 4 \sum_{n=1}^{20} n^2 + 4 \sum_{n=1}^{20} n \\ &= 4 \cdot \frac{20(20+1)(2 \cdot 20+1)}{6} + 4 \cdot \frac{20(20+1)}{2} \\ &= 12320 \end{aligned}$$

Ex 21.2

Q1

we have,

$$3 + 5 + 9 + 15 + 23 + \dots + T_{n-1} + T_n$$

The difference between the successive terms are $5 - 3 = 2$, $9 - 5 = 4$, $15 - 9 = 6$...dearly, these difference are in A.P.

Let, S_n denote the sum to n terms of the given series.

Then,

$$S_n = 3 + 5 + 9 + 15 + 23 + \dots + T_{n-1} + T_n \dots (i)$$

$$\text{Also, } S_{n-1} = 3 + 5 + 9 + 15 + \dots + T_{n-1} \dots (ii)$$

Subtracting (ii) from (i), we get

$$0 = 3 + [2 + 4 + 6 + \dots + (T_n - T_{n-1})] - T_n$$

$$T_n = 3 + \frac{(n-1)}{2} [2 \times 2 + (n-1-1) \times 2]$$

$$T_n = 3 + \frac{(n-1)}{2} \times 2 [2 + n - 2]$$

$$= 3 + (n-1)(n)$$

$$= 3 + n^2 - n$$

$$= n^2 - n + 3$$

$$\therefore S_n = \sum_{k=1}^n T_k = \sum_{k=1}^n (k^2 - k + 3)$$

$$= \sum_{k=1}^n k^2 - \sum_{k=1}^n k + \sum_{k=1}^n 3$$

$$= \sum_{k=1}^n k^2 - \sum_{k=1}^n k + \sum_{k=1}^n 3$$

Q2

We have,

$$2 + 5 + 10 + 17 + 26 + \dots$$

The sequence of the differences between the successive terms of this series is 3, 5, 7, 9,

Clearly, it is an A.P. with common difference 2.

Let T_n be the n th term and S_n denote the sum of n terms of the given series.

$$\text{Then, } S_n = 2 + 5 + 10 + 17 \dots T_{n-1} + T_n \dots (i)$$

$$\text{Also, } S_n = 2 + 5 + 10 + 17 \dots + T_{n-1} + T_n \dots (ii)$$

Subtracting (ii) from (i), we get.

$$0 = 2 + [3 + 5 + 7 \dots (T_n - T_{n-1})] - T_n$$

$$\Rightarrow T_n = 2 + (3 + 5 + 7 + \dots T_n - T_{n-1})$$

$$= 2 + \frac{(n-1)}{2} [2 \times 3 + (n-1-1) \times 2]$$

$$= 2 + \frac{(n-1)}{2} \times 2 [3 + n - 2]$$

$$= 2 + (n-1)(n+1)$$

$$= 2 + n^2 + n - n - 1$$

$$= 2 + n^2 - 1$$

$$= n^2 + 1$$

Q3

We have,

$$1+3+7+13+21+\dots$$

The sequence of the differences between the successive terms of this series is 2, 4, 6, 8.... clearly, it is an A.P. with common difference 2.

Let T_n be the n th term and S_n denote the sum of n terms of the given series.

$$\text{Then, } S_n = 1+3+7+13+21+\dots T_{n-1}+T_n \dots \dots (i)$$

$$\text{Also, } S_n = 1+3+7+13 \dots T_{n-1}+T_n \dots (ii)$$

Subtracting (ii) from (i), we get

$$0 = 1 + [2+4+6+8 \dots (T_n - T_{n-1})] - T_n$$

$$\Rightarrow T_n = 1 + [2+4+6+8 \dots (T_n - T_{n-1})]$$

$$\Rightarrow T_n = 1 + \frac{(n-1)}{2} [2 \times 2 + (n-1-1) \times 2]$$

$$= 1 + \frac{(n-1)}{2} \times 2 [2 + (n-2)]$$

$$= 1 + (n-1)(n)$$

$$= 1 + n^2 - n$$

$$= n^2 - n + 1$$

$$\Rightarrow S_n = \sum_{k=1}^n T_k = \sum_{k=1}^n (k^2 - k + 1) = \sum_{k=1}^n k^2 - \sum_{k=1}^n k + \sum_{k=1}^n 1$$

Q4

We have,

$$3 + 7 + 14 + 24 + 37 + \dots$$

The sequence of the differences between the successive terms of this series is 4, 7, 10, 13, clearly, it is an A.P. with common difference 3.

Let T_n be the n th term and S_n denote the sum of n terms of the given series.

$$\text{Then, } S_n = 3 + 7 + 14 + 24 + 37 + \dots + T_{n-1} + T_n \dots \dots (i)$$

$$\text{Also, } S_n = 3 + 7 + 14 + 24 + \dots + T_{n-1} + T_n \dots (ii)$$

Subtracting (ii) from (i), we get

$$0 = 3 + [4 + 7 + 10 \dots + (T_n - T_{n-1})] - T_n$$

$$\Rightarrow T_n = 3 + [4 + 7 + 10 \dots + (T_n - T_{n-1})]$$

$$\Rightarrow T_n - 3 + \frac{(n-1)}{2} [2 \times 4 + (n-1-1) \times 3]$$

$$= 3 + \frac{(n-1)}{2} [8 + (n-2) 3]$$

$$= 3 + \frac{(n-1)}{2} [8 + 3n - 6]$$

$$= 3 + \frac{(n-1)}{2} [2 + 3n]$$

$$= \frac{6 + (n-1)(2+3n)}{2}$$

$$= \frac{6 + 2n + 3n^2 - 2 - 3n}{2}$$

$$= \frac{6 + 3n^2 - n - 2}{2}$$

$$= \frac{3n^2 - n + 4}{2}$$

Q5

We have,

$$1 + 3 + 6 + 10 + 15 + \dots$$

The sequence of the differences between the successive terms of this series is 2, 3, 4, 5 + clearly, it is an A.P. with common difference 1.

Let T_n be the n th term and S_n denote the sum of n terms of the given series.

$$\text{Then, } S_n = 1 + 3 + 6 + 10 + 15 + \dots + T_{n-1} + T_n \dots \dots (i)$$

$$\text{Also, } S_n = 1 + 3 + 6 + 10 + \dots + T_{n-1} + T_n \dots (ii)$$

Subtracting (ii) from (i), we get

$$0 = 1 + [2 + 3 + 4 + 5 \dots (T_n - T_{n-1})] - T_n$$

$$\Rightarrow T_n = 1 + [2 + 3 + 4 + 5 \dots (T_n - T_{n-1})]$$

$$\Rightarrow T_n = 1 + \frac{(n-1)}{2} [2 \times 2 + (n-1-1) \times 1]$$

$$= 1 + \frac{(n-1)}{2} [4 + n - 2]$$

$$= 1 + \frac{(n-1)}{2} (n+2)$$

$$= 1 + \frac{n^2 + 2n - n - 2}{2}$$

$$= 1 + \frac{n^2 + n - 2}{2}$$

$$= \frac{2 + n^2 + n - 2}{2}$$

$$= \frac{n^2 + n}{2}$$

$$\therefore = \sum_{k=1}^n T_k = \sum_{k=1}^n \left(\frac{k^2 + k}{2} \right) = \frac{1}{2} \sum_{k=1}^n k^2 + \frac{1}{2} \sum_{k=1}^n k$$

Q6

We have,

$$1 + 4 + 13 + 40 + 121 + \dots$$

The sequence of the differences between the successive terms of this series is 3, 9, 27, 81, ... clearly, it is a G.P. with common difference 3.

Let T_n be the n th term and S_n denote the sum of n terms of the given series.

$$\text{Then, } S_n = 1 + 4 + 13 + 40 + \dots + T_{n-1} + T_n \dots \dots (i)$$

$$\text{Also, } S_n = 1 + 4 + 13 \dots + T_{n-1} + T_n \dots (ii)$$

Subtracting (ii) from (i), we get

$$0 = 1 + [3 + 9 + 27 + 81 \dots (T_n - T_{n-1})] - T_n$$

$$\Rightarrow T_n = 1 + [3 + 9 + 27 + 81 \dots (T_n - T_{n-1})]$$

$$\Rightarrow T_n = 1 + \frac{3(3^{n-1} - 1)}{(3 - 1)}$$

$$\Rightarrow T_n = 1 + \frac{3}{2}(3^{n-1} - 1)$$

$$= 1 + \frac{3}{2} - 3^{n-1} - \frac{3}{2}$$

$$= 1 - \frac{3}{2} + \frac{3^n}{2}$$

$$= -\frac{1}{2} + \frac{3^n}{2}$$

$$= \frac{3^n}{2} - \frac{1}{2}$$

$$S_n = \sum_{k=1}^n T_k = \sum_{k=1}^n \left(\frac{3^k}{2} - \frac{1}{2} \right)$$

Q7

We have,

$$4 + 6 + 9 + 13 + 18 + \dots$$

The sequence of the differences between the successive terms of this series is 2, 3, 4, 5, ..., clearly, it is an A.P. with common difference 1.

Let T_n be the n th term and S_n denote the sum of n terms of the given series.

$$\text{Then, } S_n = 4 + 6 + 9 + 13 + \dots + T_{n-1} + T_n \dots \dots (i)$$

$$\text{Also, } S_n = 4 + 6 + 9 + 13 + \dots + T_{n-1} + T_n \dots \dots (ii)$$

Subtracting (ii) from (i), we get

$$0 = 4 + [2 + 3 + 4 + 5 + \dots + (T_n - T_{n-1})] - T_n$$

$$\Rightarrow T_n = 4 + [2 + 3 + 4 + 5 + \dots + (T_n - T_{n-1})]$$

$$\Rightarrow T_n = 4 + \frac{(n-1)}{2} [2 \times 2 + (n-1-1) \times 1]$$

$$= 4 + \frac{(n-1)}{2} [4 + n - 2]$$

$$= 4 + \frac{(n-1)}{2} (n+2)$$

$$= \frac{8 + n^2 + 2n - n - 2}{2}$$

$$= \frac{n^2 + n + 6}{2}$$

$$\therefore S_n = \sum_{k=1}^n T_k = \sum_{k=1}^n \left(\frac{k^2 + k + 6}{2} \right)$$

$$= \frac{1}{2} \sum_{k=1}^n k^2 + \frac{1}{2} \sum_{k=1}^n k + \sum_{k=1}^n 3$$

$$\Rightarrow S_n = \frac{1}{2} \left[\frac{n(n+1)(2n+1)}{6} \right] + \frac{n(n+1)}{2 \times 2} + 3n$$

Q8

We have,

$$2 + 4 + 7 + 11 + 16 + \dots$$

The sequence of the differences between the successive terms of this series is 2, 3, 4, 5, ... clearly, it is an A.P. with common difference 1.

Let T_n be the n th term and S_n denote the sum of n terms of the given series.

$$\text{Then, } S_n = 2 + 4 + 7 + 11 + 16 + \dots + T_{n-1} + T_n \dots \text{ (i)}$$

$$\text{Also, } S_n = 2 + 4 + 7 + 11 + \dots + T_{n-1} + T_n \dots \text{ (ii)}$$

Subtracting (ii) from (i), we get

$$0 = 2 + [2 + 3 + 4 + 5 + \dots + (T_n - T_{n-1})] - T_n$$

$$\Rightarrow T_n = 2 + \frac{(n-1)}{2} [2 \times 2 + (n-1-1) \times 1]$$

$$= 2 + \frac{(n-1)}{2} [4 + n - 2]$$

$$= 2 + \frac{(n-1)}{2} (n+2)$$

$$= \frac{4 + n^2 + 2n - n - 2}{2}$$

$$= \frac{n^2 + n + 2}{2}$$

$$= \frac{n^2}{2} + \frac{n}{2} + 1$$

$$\Rightarrow S_n = \sum_{k=1}^n T_k = \sum_{k=1}^n \left(\frac{k^2}{2} + \frac{k}{2} + 1 \right)$$

Q9

We have,

$$\frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \dots$$

Let T_r be the r th term of the given series. Then,

$$T_r = \frac{1}{(3r-2)(3r+1)}, r = 1, 2, \dots, n$$

$$\Rightarrow T_r = \frac{1}{3} \left[\frac{1}{3r-2} - \frac{1}{3r+1} \right]$$

$$\therefore \text{required sum} = \frac{1}{3} \sum_{r=1}^n T_r$$

$$= \frac{1}{3} \sum_{r=1}^n \left[\frac{1}{3r-2} - \frac{1}{3r+1} \right]$$

$$= \frac{1}{3} \left[\left(1 - \frac{1}{4} \right) + \left(\frac{1}{4} - \frac{1}{7} \right) + \left(\frac{1}{7} - \frac{1}{10} \right) + \dots + \left(\frac{1}{3n-2} - \frac{1}{3n+1} \right) \right]$$

$$= \frac{1}{3} \left[1 - \frac{1}{3n+1} \right]$$

$$= \frac{1}{3} \left[\frac{3n+1-1}{3n+1} \right]$$

$$= \frac{1}{3} \times \frac{3n}{3n+1}$$

$$= \frac{n}{3n+1}$$

$$\text{Hence, required sum} = \frac{n}{3n+1}$$

Q10

We have,

$$\frac{1}{1.6} + \frac{1}{6.11} + \frac{1}{11.14} + \frac{1}{14.19} + \dots + \frac{1}{(5n-4)(5n+1)}$$

Let T_r be the r th term of the given series. Then,

$$T_r = \frac{1}{(5r-4)(5r+1)}, r = 1, 2, \dots, n$$

$$\Rightarrow T_r = \frac{1}{5} \left[\frac{1}{5r-4} - \frac{1}{5r+1} \right]$$

$$\therefore \text{required sum} = \frac{1}{5} \sum_{r=1}^n T_r$$

$$= \frac{1}{5} \sum_{r=1}^n \left[\frac{1}{5r-4} - \frac{1}{5r+1} \right]$$

$$= \frac{1}{5} \left[\left(1 - \frac{1}{6} \right) + \left(\frac{1}{6} - \frac{1}{11} \right) + \left(\frac{1}{11} - \frac{1}{14} \right) + \dots + \left(\frac{1}{5n-4} - \frac{1}{5n+1} \right) \right]$$

$$= \frac{1}{5} \left[1 - \frac{1}{5n+1} \right]$$

$$= \frac{1}{5} \left[\frac{5n+1-1}{5n+1} \right]$$

$$= \frac{1}{5} \times \frac{5n}{5n+1}$$

$$= \frac{n}{5n+1}$$

$$\text{Hence, required sum} = \frac{n}{5n+1}$$