Ex 16.1

Tangents and Normals Ex 16.1 Q1(i)

We know that the slope of the tangent to the curve $y = f(x)$ is

$$\frac{dy}{dx} = f'(x) \qquad \text{---(A)}$$

And the slope of the normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} \qquad \text{---(B)}$$

Now,

$$y = \sqrt{x^3}$$

$$\therefore \quad \frac{dy}{dx} = \frac{3x^2}{2\sqrt{x^3}}$$

$\therefore$ Slope of tangent at $x = 4$ is

$$\left(\frac{dy}{dx}\right)_{x=4} = \frac{3.16}{2\sqrt{64}} = \frac{48}{16} = 3$$

Slope of normal at $x = 4$ is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} = \frac{-1}{3}$$

Tangents and Normals Ex 16.1 Q1(ii)

We know that the slope of the tangent to the curve $y = f(x)$ is

$$\frac{dy}{dx} = f'(x) \qquad \text{---(A)}$$

And the slope of the normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} \qquad \text{---(B)}$$

$y = \sqrt{x}$

$\therefore \quad \frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

$\therefore$ Slope oftangent at $x = 9$.

$\therefore \quad \left(\frac{dy}{dx}\right)_{x=9} = \frac{1}{2\sqrt{9}} = \frac{1}{6}$

Also, the slope of normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} = -6$$

Tangents and Normals Ex 16.1 Q1(iii)

We know that the slope of the tangent to the curve $y = f(x)$ is

$$\frac{dy}{dx} = f'(x) \qquad \text{---(A)}$$

And the slope of the normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} \qquad \text{---(B)}$$

$y = x^3 - x$

$\therefore \quad \frac{dy}{dx} = 3x^2 - 1$

$\therefore$ Slope of tangent at $x = 2$ is

$$\left(\frac{dy}{dx}\right)_{x=2} = 3.2^2 - 1 = 11$$

Slope of normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} = \frac{-1}{11}$$

Tangents and Normals Ex 16.1 Q1(iv)

We know that the slope of the tangent to the curve $y = f(x)$ is

$$\frac{dy}{dx} = f'(x) \qquad \text{---(A)}$$

And the slope of the normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} \qquad \text{---(B)}$$

$y = 2x^2 + 3\sin x$

$\therefore \quad \frac{dy}{dx} = 4x + 3\cos x$

So, slope of tangent of $x = 0$ is

$$\left(\frac{dy}{dx}\right)_{x=0} = 4.0 + 3\cos 0° = 3$$

And slope of normanl is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} = \frac{-1}{3}$$

Tangents and Normals Ex 16.1 Q1(v)

We know that the slope of the tangent to the curve $y = f(x)$ is

$$\frac{dy}{dx} = f'(x) \qquad ---(A)$$

And the slope of the normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} \qquad ---(B)$$

$x = a(\theta - \sin\theta)$, $y = a(1 + \cos\theta)$

Now,

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-a\sin\theta}{a(1-\cos\theta)}$$

$\therefore$ Slope of tangent of $\theta = -\frac{\pi}{2}$

$$\left(\frac{dy}{dx}\right)_{\theta=-\frac{\pi}{2}} = \frac{-a\sin\left(-\frac{\pi}{2}\right)}{a\left(1-\cos\left(-\frac{\pi}{2}\right)\right)}$$

$$= \frac{a}{a(1-0)} = 1$$

Also, the slope of normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} = -1$$

Tangents and Normals Ex 16.1 Q1(vi)

We know that the slope of the tangent to the curve $y = f(x)$ is

$$\frac{dy}{dx} = f'(x) \qquad ---(A)$$

And the slope of the normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} \qquad ---(B)$$

$x = a\cos^3\theta$, $y = a\sin^3\theta$

$\therefore$ $\frac{dx}{d\theta} = 3a\cos^2\theta \times (-\sin\theta) = -3a\sin\theta \times \cos^2\theta$

and $\frac{dy}{dx} = 3a\sin^2\theta \times \cos\theta$

Now,

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{3a\sin^2\theta \times \cos\theta}{-3a\sin\theta \times \cos^2\theta}$$

$$= -\tan\theta$$

$\therefore$ Slope of tangent at $\theta = \frac{\pi}{4}$ is

$$\left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{4}} = 1$$

Also, the slope of normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} = -1$$

Tangents and Normals Ex 16.1 Q1(vii)

We know that the slope of the tangent to the curve $y = f(x)$ is

$$\frac{dy}{dx} = f'(x) \qquad ---(A)$$

And the slope of the normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} \qquad ---(B)$$

$x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$

$\therefore \quad \frac{dx}{d\theta} = a(1 - \cos\theta)$, $\frac{dy}{d\theta} = a(0 + \sin\theta) = a\sin\theta$

Now,

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a\sin\theta}{a(1 - \cos\theta)}$$

Now, the slope of tangent at $\theta = \frac{\pi}{2}$ is

$$\left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{2}} = \frac{a\sin\frac{\pi}{2}}{a\left(1 - \cos\frac{\pi}{2}\right)} = \frac{a}{a} = 1$$

And, the slope of normal is

$$\frac{-1}{\frac{dy}{dx}} = -1$$

Tangents and Normals Ex 16.1 Q1(viii)

We know that the slope of the tangent to the curve $y = f(x)$ is

$$\frac{dy}{dx} = f'(x) \qquad ---(A)$$

And the slope of the normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} \qquad ---(B)$$

$y = (\sin 2x + \cot x + 2)^2$

$\therefore \quad \frac{dy}{dx} = 2(\sin 2x + \cot x + 2)\left(2\cos 2x - \operatorname{cosec}^2 x\right)$

$\therefore \quad$ Slope of tangent of $x = \frac{\pi}{2}$ is

$$\left(\frac{dy}{dx}\right)_{x=\frac{\pi}{2}} = 2\left(\sin\pi + \cos\frac{\pi}{2} + 2\right)\left(2\cos\pi - \operatorname{cosec}^2\frac{\pi}{2}\right)$$

$$= 2(0 + 0 + 2)(-2 - 1)$$

$$= -12$$

$\therefore \quad$ Slope of normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{1}{12}$$

Tangents and Normals Ex 16.1 Q1(ix)

We know that the slope of the tangent to the curve $y = f(x)$ is

$$\frac{dy}{dx} = f'(x) \qquad \text{---(A)}$$

And the slope of the normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} \qquad \text{---(B)}$$

$x^2 + 3y + y^2 = 5$

Differentiating with respect to x, we get

$$2x + 3\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx}(3 + 2y) = -2x$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-2x}{3 + 2y}$$

So, the slope of tangent at $(1,1)$ is

$$\frac{dy}{dx} = \frac{-2.1}{3 + 2.1} = \frac{-2}{5}$$

The slope of normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{5}{2}$$

Tangents and Normals Ex 16.1 Q1(x)

We know that the slope of the tangent to the curve $y = f(x)$ is

$$\frac{dy}{dx} = f'(x) \qquad \text{---(A)}$$

And the slope of the normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} \qquad \text{---(B)}$$

$xy = 6$

Differentiating with respect to x, we get

$$y + x\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-y}{x}$$

$\therefore$ Slope of tangent at $(1,6)$ is

$$\frac{dy}{dx} = -6 \text{ and}$$

Slope of normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{1}{6}$$

Tangents and Normals Ex 16.1 Q2

Differentiating with respect to x, we get

$$y + x\frac{dy}{dx} + a + b\frac{dy}{dx} = 0$$

$\Rightarrow$ $\frac{dy}{dx}(x+b) = -(a+y)$

$\Rightarrow$ $\frac{dy}{dx} = \frac{-(a+y)}{x+b}$

$\therefore$ Slope of tangent $= \left(\frac{dy}{dx}\right)_{x=1,\,y=1} = \frac{-(a+1)}{b+1} = 2$ [given]

$\Rightarrow$ $-(a+1) = 2b+2$

$\Rightarrow$ $2b + a = -3$ ---(i)

Also, $(1,1)$ lies on the curve, so x = 1, y = 1 satisfies the equation

$$xy + ax + by = 2$$

$\Rightarrow$ $1 + a + b = 2$

$\Rightarrow$ $a + b = 1$ ---(ii)

Solving (i) and (ii), we get

$$a = 5,\ b = -4$$

Tangents and Normals Ex 16.1 Q3

We have,

$y = x^3 + ax + b$ ---(i)

$x - y + 5 = 0$ ---(ii)

Now,

Point $(1,-6)$ lies on (i), so,

$-6 = 1 + a + b$

$\Rightarrow$ $a + b = -7$ ---(iii)

Also,

Slope of tangent to (i) is

$$\frac{dy}{dx} = 3x^2 + a$$

$\Rightarrow$ $\left(\frac{dy}{dx}\right)_{(1,-6)} = 3 + a$

And slope of tangent to (ii) is

$$\frac{dy}{dx} = 1$$

According to the question slope of (i) and (ii) are parallel

$\therefore$ $3 + a = 1$

$\Rightarrow$ $a = -2$

From (iii)

$$b = -5$$

Tangents and Normals Ex 16.1 Q4

We have,

$$y = x^3 - 3x \qquad \text{---(i)}$$

$\therefore$ Slope of (i) is

$$\frac{dy}{dx} = 3x^2 - 3 \qquad \text{---(ii)}$$

Also,

The slope of the chord obtained by joining the points $(1,-2)$ and $(2,2)$ is

$$\frac{2-(-2)}{2-1} \qquad \left[\text{Slope } \frac{y_2 - y_1}{x_2 - x_1}\right]$$

$$= 4$$

According to the question slope of tangent to (i) and the chord are parallel

$\therefore$ $3x^2 - 3 = 4$

$\Rightarrow$ $3x^2 = 7$

$\Rightarrow$ $x = \pm\sqrt{\frac{7}{3}}$

From (i)

$$y = \pm\sqrt{\frac{7}{3}} \mp 3\sqrt{\frac{7}{3}}$$

$$= \mp\frac{2}{3}\sqrt{\frac{7}{3}}$$

Thus, the required point is

$$\pm\sqrt{\frac{7}{3}}, \ \mp\frac{2}{3}\sqrt{\frac{7}{3}}$$

Tangents and Normals Ex 16.1 Q5

The given equations are

$$y = x^3 - 2x^2 - 2x \qquad \text{---(i)}$$

$$y = 2x - 3 \qquad \text{---(ii)}$$

Slope to the tangents of (i) and (ii) are

$$\frac{dy}{dx} = 3x^2 - 4x - 2 \qquad \text{---(iii)}$$

and $$\frac{dy}{dx} = 2 \qquad \text{---(iv)}$$

According to the question slope to (i) and (ii) are parallel, so

$3x^2 - 4x - 2 = 2$

$\Rightarrow$ $3x^2 - 4x - 4 = 0$

$\Rightarrow$ $3x^2 - 6x + 2x - 4 = 0$

$\Rightarrow$ $3x(x-2) + 2(x-2) = 0$

$\Rightarrow$ $(3x+2)(x-2) = 0$

$\Rightarrow$ $x = \frac{-2}{3}$ or 2

From (i)

$$y = \frac{4}{27} \text{ or } -4$$

Thus, the points are

$$\left(\frac{-2}{3}, \ \frac{4}{27}\right) \text{ and } (2,-4)$$

Tangents and Normals Ex 16.1 Q6

We have,

$$y^2 = 2x^3 \qquad ---(i)$$

Differentiating (i) with respect to x, we get

$$2y\frac{dy}{dx} = 6x^2$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{3x^2}{y} \qquad ---(ii)$$

According to the question

$$\frac{3x^2}{y} = 3$$

$$\Rightarrow \quad x^2 = y \qquad ---(iii)$$

From (i) and (ii)

$$\left(x^2\right)^2 = 2x^3$$

$$\Rightarrow \quad x^4 - 2x^3 = 0$$

$$\Rightarrow \quad x^3\left(x - 2\right) = 0$$

$$\Rightarrow \quad x = 0 \text{ or } 2$$

If $x = 0$, then

$\frac{dy}{dx} = \frac{3x^2}{y} \Rightarrow \frac{dy}{dx} = 0$

Which is not possible.

$\therefore$ x = 2.

Putting x = 2 in the equation of the curve $y^2 = 2x^3$, we get y = 4.

Hence the required point is $(2, 4)$

Tangents and Normals Ex 16.1 Q7

We know that the slope to any curve is $\frac{dy}{dx} = \tan\theta$ where θ is the angle with possitive direction of x-axis.

Now,

The given curve is

$$xy + 4 = 0 \qquad ---(i)$$

Differentiating with respect to x, we get

$$y + x\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-y}{x} \qquad ---(ii)$$

Also,

$$\frac{dy}{dx} = \tan 45° = 1 \qquad ---(iii)$$

$\therefore$ From (ii) and (iii)

$$\frac{-y}{x} = 1$$

$$\Rightarrow \quad x = -y \qquad ---(iv)$$

From (i) and (iv), we get

$$-y^2 + 4 = 0$$

$$\Rightarrow \quad y = \pm 2$$

$$\therefore \quad x = \mp 2$$

Thus, the points are

$(2, -2)$ and $(-2, 2)$

Tangents and Normals Ex 16.1 Q8

The given equation of the curve is

$$y = x^2 \qquad \text{---(i)}$$

$\therefore$ Slope of tangent to (i) is

$$\frac{dy}{dx} = 2x \qquad \text{---(ii)}$$

According to the question

$$\frac{dy}{dx} = x \qquad \text{---(iii)} \qquad [\text{Slope} = x\text{-coordinate}]$$

From (ii) and (iii)

$$2x = x$$

$$\Rightarrow \quad x = 0 \ \& \ y = 0$$

Thus, the required point is $(0,0)$

Tangents and Normals Ex 16.1 Q9

The given equation of the curve is

$$x^2 + y^2 - 2x - 4y + 1 = 0 \qquad \text{---(i)}$$

Differentiating with respect is x, we get

$$2x + 2y\frac{dy}{dx} - 2 - 4\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx}(2y - 4) = 2 - 2x$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{2(1-x)}{2(y-2)} \qquad \text{---(ii)}$$

According to the question the tangent is parallel to x-axis, so $\theta = 0°$

$$\therefore \quad \text{Slope} = \tan\theta = \tan 0° = 0 \qquad \text{---(iii)}$$

From (ii) and (iii), we get

$$\frac{1-x}{y-2} = 0$$

$$\Rightarrow \quad 1 - x = 0$$

$$\Rightarrow \quad x = 1$$

$\therefore$ from (i)

$$y = 0, 4$$

Thus, the points are $(1,0)$ and $(1,4)$

Tangents and Normals Ex 16.1 Q10

The given equation of curve is

$$y = x^2 \qquad \text{---(i)}$$

$$\therefore \quad \text{Slope} = \frac{dy}{dx} = 2x \qquad \text{---(ii)}$$

As per question

$$\text{slope} = \tan 45° = 1 \qquad \text{---(iii)}$$

From (ii) and (iii), we have

$$2x = 1$$

$$\Rightarrow \quad x = \frac{1}{2}$$

$\therefore$ From (i)

$$y = \frac{1}{4}$$

Thus, the required point is

$$\left(\frac{1}{2}, \ \frac{1}{4}\right)$$

Tangents and Normals Ex 16.1 Q11

The given equation of the curve is

$$y = 3x^2 - 9x + 8 \qquad \text{---(i)}$$

$$\text{Slope} = \frac{dy}{dx} = 6x - 9 \qquad \text{---(ii)}$$

As per question

The tangent is equally inclined to the axes

$$\therefore \quad \theta = \frac{\pi}{4} \text{ or } \frac{-\pi}{4}$$

$$\therefore \quad \text{Slope} = \tan\theta$$

$$= \tan\frac{\pi}{4} \text{ or } \tan\left(\frac{-\pi}{4}\right)$$

$$= 1 \text{ or } -1 \qquad \text{---(iii)}$$

From (ii) and (iii), we have,

$$6x - 9 = 1 \quad \text{or} \quad 6x - 9 = -1$$

$$\Rightarrow \quad x = \frac{5}{3} \quad \text{or} \quad x = \frac{4}{3}$$

So, from (i)

$$y = \frac{4}{3} \quad \text{or} \quad y = \frac{4}{3}$$

Thus, the points are

$$\left(\frac{5}{3}, \frac{4}{3}\right) \text{ or } \left(\frac{4}{3}, \frac{4}{3}\right)$$

Tangents and Normals Ex 16.1 Q12

The given equation are

$$y = 2x^2 - x + 1 \qquad \text{---(i)}$$

$$y = 3x + 4 \qquad \text{---(ii)}$$

Slope to (i) is

$$\frac{dy}{dx} = 4x - 1 \qquad \text{---(iii)}$$

Slope to (ii) is

$$\frac{dy}{dx} = 3 \qquad \text{---(iv)}$$

According to the question

$$4x - 1 = 3$$

$$\Rightarrow \quad x = 1$$

Thus from (i)

$$y = 2$$

Hence, the point is $(1,2)$.

Tangents and Normals Ex 16.1 Q13

The given equation of curve is

$$y = 3x^2 + 4 \qquad \text{---(i)}$$

$$\text{Slope} = m_1 = \frac{dy}{dx} = 6x \qquad \text{---(ii)}$$

Now,

The given slope $m_2 = \frac{-1}{6}$

We have,

tangent to (i) is perpendicular to the tangent whose slope is $\frac{-1}{6}$

$\therefore \quad m_1 \times m_2 = -1$

$\Rightarrow \quad 6x \times \frac{-1}{6} = -1$

$\Rightarrow \quad x = 1$

From (i)

$y = 7$

Thus, the required point is $(1, 7)$.

Tangents and Normals Ex 16.1 Q14

The given equation of curve and the line is

$$x^2 + y^2 = 13 \qquad \text{---(i)}$$

and $\quad 2x + 3y = 7 \qquad \text{---(ii)}$

Slope = m_1 for (i)

$$m_1 = \frac{dy}{dx} = \frac{-x}{y} \qquad \text{---(iii)}$$

Slope = m_2 for (ii)

$$m_2 = \frac{dy}{dx} = \frac{-2}{3} \qquad \text{---(iv)}$$

According to the question

$m_1 = m_2$

$\Rightarrow \quad \frac{-x}{y} = \frac{-2}{3}$

$\Rightarrow \quad x = \frac{2}{3}y$

From (i)

$\frac{4}{9}y^2 + y^2 = 13$

$\Rightarrow \quad \frac{13y^2}{9} = 13$

$\Rightarrow \quad y = \pm 3$

$\therefore \quad x = \pm 2$

Thus, the points are $(2, 3)$ and $(-2, -3)$.

Tangents and Normals Ex 16.1 Q15

The given equation of the curve is

$$2a^2y = x^3 - 3ax^2 \quad ---(i)$$

Differentiating with respect to x, we get

$$2a^2\frac{dy}{dx} = 3x^2 - 6ax$$

$$\therefore \text{ Slope } m_1 = \frac{dy}{dx} = \frac{1}{2a^2}\left[3x^2 - 6ax\right] \quad ---(ii)$$

Also,

$$\text{Slope } m_2 = \frac{dy}{dx} = \tan\theta$$

$$= \tan 0° = 0 \quad \left[\because \text{Slope is parallel to } x\text{-axis}\right]$$

$$\therefore \quad m_1 = m_2$$

$$\Rightarrow \quad \frac{1}{2a^2}\left[3x^2 - 6ax\right] = 0$$

$$\Rightarrow \quad 3x\left[x - 2a\right] = 0$$

$$\Rightarrow \quad x = 0 \text{ or } 2a$$

$\therefore$ From (i)

$$y = 0 \text{ or } -2a$$

Thus, the required points are $(0,0)$ or $(2a,-2a)$.

Tangents and Normals Ex 16.1 Q16

The given equations of curve and the line are

$$y = x^2 - 4x + 5 \quad ---(i)$$

$$2y + x = 7 \quad ---(ii)$$

Slope of the tangent to (i) is

$$m_1 = \frac{dy}{dx} = 2x - 4 \quad ---(iii)$$

Slope of the line (ii) is

$$m_2 = \frac{dy}{dx} = \frac{-1}{2} \quad ---(iv)$$

We have given that slope of (i) and (ii) are perpendicular to each other.

$$\therefore \quad m_1 \times m_2 = -1$$

$$\Rightarrow \quad (2x - 4)\left(\frac{-1}{2}\right) = -1$$

$$\Rightarrow \quad -2x + 4 = -2$$

$$\Rightarrow \quad x = 3$$

From (i)

$$y = 2$$

Thus, the required point is $(3,2)$.

Tangents and Normals Ex 16.1 Q17

Differentiating $\frac{x^2}{4}+\frac{y^2}{25}=1$ with respect to x, we get

$$\frac{x}{2}+\frac{2y}{25}\cdot\frac{dy}{dx}=0$$

or $$\frac{dy}{dx}=\frac{-25}{4}\cdot\frac{x}{y}$$

(i) Now, the tangent is parallel to the x – axis if the slope of the tangent is zero.

$$\therefore \quad \frac{-25}{4}\cdot\frac{x}{y}=0$$

This is possible if $x = 0$.

Then $\frac{x^2}{4}+\frac{y^2}{25}=1$ for $x = 0$ gives $y^2 = 25$

$$\therefore \quad y=\pm 5$$

Thus, the points at which the tangents are parallel to the x – axis are $(0,5)$ and $(0,-5)$.

(ii) Now, the tangent is parallel to the y – axis if the slope of the normal is zero.

$$\therefore \quad \frac{4y}{25x}=0$$

This is possible if $y = 0$.

Then $\frac{x^2}{4}+\frac{y^2}{25}=1$ for $y = 0$ gives $x^2 = 4$

$$\therefore \quad x=\pm 2$$

Thus, the points at which the tangents are parallel to the y – axis are $(2,0)$ and $(-2,0)$.

Tangents and Normals Ex 16.1 Q18

The equation of the given curve is $x^2+y^2-2x-3=0$.

On differentiating with respect to x, we have:

$$2x+2y\frac{dy}{dx}-2=0$$

$$\Rightarrow y\frac{dy}{dx}=1-x$$

$$\Rightarrow \frac{dy}{dx}=\frac{1-x}{y}$$

Now, the tangents are parallel to the x-axis if the slope of the tangent is 0.

$$\therefore \frac{1-x}{y}=0 \Rightarrow 1-x=0 \Rightarrow x=1$$

But, $x^2+y^2-2x-3=0$ for $x=1$.

$$\Rightarrow y^2=4 \Rightarrow y=\pm 2$$

Hence, the points at which the tangents are parallel to the x-axis are $(1, 2)$ and $(1, -2)$

(b)

Now, the tangents are parallel to the x-axis if the slope of the tangents is 0

$$\frac{y}{1-x}=0$$

$$y=0$$

But,

$$x^2+y^2-2x-3=0 \text{ for } y=0$$

$$x^2-2x-3=0$$

$$x=-1,3$$

Hence, the points at which the tangents are parallel to the y-axis are,

$(-1,0),(3,0)$

Tangents and Normals Ex 16.1 Q19

The equation of the given curve is $\frac{x^2}{9}+\frac{y^2}{16}=1$.

On differentiating both sides with respect to x, we have:

$$\frac{2x}{9}+\frac{2y}{16}\cdot\frac{dy}{dx}=0$$
$$\Rightarrow \frac{dy}{dx}=\frac{-16x}{9y}$$

(i) The tangent is parallel to the x-axis if the slope of the tangent is i.e., $0\,\frac{-16x}{9y}=0$, which is possible if $x = 0$.

Then, $\frac{x^2}{9}+\frac{y^2}{16}=1$ for $x = 0$

$$\Rightarrow y^2=16 \Rightarrow y=\pm 4$$

Hence, the points at which the tangents are parallel to the x-axis are

(0, 4) and (0, − 4).

(ii) The tangent is parallel to the y-axis if the slope of the normal is 0, which gives $\frac{-1}{\left(\frac{-16x}{9y}\right)}=\frac{9y}{16x}=0 \Rightarrow y=0$.

Then, $\frac{x^2}{9}+\frac{y^2}{16}=1$ for $y = 0$.

$$\Rightarrow x=\pm 3$$

Hence, the points at which the tangents are parallel to the y-axis are

(3, 0) and (− 3, 0).

Tangents and Normals Ex 16.1 Q20

The equation of the given curve is $y = 7x^3 + 11$.

$$\therefore \frac{dy}{dx}=21x^2$$

The slope of the tangent to a curve at (x_0, y_0) is $\left.\frac{dy}{dx}\right]_{(x_0,y_0)}$.

Therefore, the slope of the tangent at the point where $x = 2$ is given by,

$$\left.\frac{dy}{dx}\right]_{x=-2}=21(2)^2=84$$

It is observed that the slopes of the tangents at the points where $x = 2$ and $x = -2$ are equal.

Hence, the two tangents are parallel.

Tangents and Normals Ex 16.1 Q21

The given equation of curve is

$$y = x^3 \qquad \text{---(i)}$$

Slope of the tangent to (i) is

$$m_1 = \frac{dy}{dx} = 3x^2 \qquad \text{---(ii)}$$

Also,

given that slope of the tangent is parallel to x-coordinate of the point.

$$\therefore \quad m_2 = \frac{dy}{dx} = x \qquad \text{---(iii)}$$

From (ii) and (iii)

$$m_1 = m_2$$

$$\Rightarrow \quad 3x^2 = x$$

$$\Rightarrow \quad 3x^2 - x = 0$$

$$\Rightarrow \quad x(3x - 1) = 0$$

$$\Rightarrow \quad x = 0 \quad \text{or} \quad \frac{1}{3}$$

$\therefore$ From (i)

$$y = 0 \quad \text{or} \quad \frac{1}{27}$$

Thus, the required point is $(0, 0)$ or $\left(\frac{1}{3}, \frac{1}{27}\right)$.

Ex 16.2

Tangents and Normals Ex 16.2 Q1

The given equation of the curve is

$$\sqrt{x} + \sqrt{y} = a \qquad ---(i)$$

Differentiating with respect to x, we get

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \times \frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}}$$

$$\therefore \quad m = \left(\frac{dy}{dx}\right)_{\left(\frac{a^2}{4}, \frac{a^2}{4}\right)} = -\frac{\frac{a}{2}}{\frac{a}{2}} = -1$$

Thus,

the equation of tangent is

$$y - y_1 = m\left(x - x_1\right)$$

$$\Rightarrow \quad y - \frac{a^2}{4} = (-1)\left(x - \frac{a^2}{4}\right)$$

$$\Rightarrow \quad x + y = \frac{a^2}{4} + \frac{a^2}{4}$$

$$\Rightarrow \quad x + y = \frac{a^2}{2}$$

Tangents and Normals Ex 16.2 Q2

The equation of the curve is

$$y = 2x^3 - x^2 + 3 \qquad \text{---(i)}$$

Slope $= m = \frac{dy}{dx} = 6x^2 - 2x$

$\therefore \quad m = \left(\frac{dy}{dx}\right)_{(1,4)} = 4$

Now,

The equation of normal is (i) is

$$y - y_1 = \frac{-1}{m}(x - x_1)$$

$\Rightarrow \quad (y - 4) = \frac{-1}{4}(x - 1)$

$\Rightarrow \quad x + 4y = 16 + 1$

$\Rightarrow \quad x + 4y = 17$

Tangents and Normals Ex 16.2 Q3(i)

(i) The equation of the curve is $y = x^4 - 6x^3 + 13x^2 - 10x + 5$.

On differentiating with respect to x, we get:

$$\frac{dy}{dx} = 4x^3 - 18x^2 + 26x - 10$$

$$\left.\frac{dy}{dx}\right]_{(0,\ 5)} = -10$$

Thus, the slope of the tangent at (0, 5) is -10. The equation of the tangent is given as:

$y - 5 = -10(x - 0)$

$\Rightarrow y - 5 = -10x$

$\Rightarrow 10x + y = 5$

The slope of the normal at (0, 5) is $\frac{-1}{\text{Slope of the tangent at }(0,\ 5)} = \frac{1}{10}$.

Therefore, the equation of the normal at (0, 5) is given as:

$$y - 5 = \frac{1}{10}(x - 0)$$

$\Rightarrow 10y - 50 = x$

$\Rightarrow x - 10y + 50 = 0$

Tangents and Normals Ex 16.2 Q3(ii)

(ii) The equation of the curve is $y = x^4 - 6x^3 + 13x^2 - 10x + 5$.

On differentiating with respect to x, we get:

$$\frac{dy}{dx} = 4x^3 - 18x^2 + 26x - 10$$
$$\left.\frac{dy}{dx}\right]_{(1,\ 3)} = 4 - 18 + 26 - 10 = 2$$

Thus, the slope of the tangent at (1, 3) is 2. The equation of the tangent is given as:

$$y - 3 = 2(x - 1)$$
$$\Rightarrow y - 3 = 2x - 2$$
$$\Rightarrow y = 2x + 1$$

The slope of the normal at (1, 3) is $\dfrac{-1}{\text{Slope of the tangent at } (1,\ 3)} = \dfrac{-1}{2}$.

Therefore, the equation of the normal at (1, 3) is given as:

$$y - 3 = -\frac{1}{2}(x - 1)$$
$$\Rightarrow 2y - 6 = -x + 1$$
$$\Rightarrow x + 2y - 7 = 0$$

Tangents and Normals Ex 16.2 Q3(iii)

The equation of the curve is $y = x^2$.

On differentiating with respect to x, we get:

$$\frac{dy}{dx} = 2x$$
$$\left.\frac{dy}{dx}\right]_{(0,\ 0)} = 0$$

Thus, the slope of the tangent at (0, 0) is 0 and the equation of the tangent is given as:

$$y - 0 = 0\ (x - 0)$$

$$\Rightarrow y = 0$$

The slope of the normal at (0, 0) is $\dfrac{-1}{\text{Slope of the tangent at } (0,\ 0)} = -\dfrac{1}{0}$, which is not defined.

Therefore, the equation of the normal at $(x_0, y_0) = (0, 0)$ is given by

$$x = x_0 = 0.$$

Tangents and Normals Ex 16.2 Q3(iv)

We know that the equation of tangent and the normal to any curve is given by

$$y - y_1 = m(x - x_1) \qquad \text{(A)} \quad \text{Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad \text{(B)} \quad \text{Normal}$$

Where m is the slope

We have,

$$y = 2x^2 - 3x - 1 \qquad P = (1, -2)$$

Slope $m = \frac{dy}{dx} = 4x - 3$

$$m = \left(\frac{dy}{dx}\right)_P = 1$$

$\therefore$ equation of tangent from (A)

$$(y + 2) = 1(x - 1)$$

$\Rightarrow \quad x - y = 3$

And equation of normal from (B)

$$(y + 2) = -1(x - 1)$$

$\Rightarrow \quad x + y + 1 = 0$

Tangents and Normals Ex 16.2 Q3(v)

We know that the equation of tangent and the normal to any curve is given by

$$y - y_1 = m(x - x_1) \qquad \text{(A)} \quad \text{Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad \text{(B)} \quad \text{Normal}$$

Where m is the slope

We have,

$$y^2 = \frac{x^3}{4 - x} \qquad P - (2, -2)$$

Differentiating with respect to x, we get

$$2y\frac{dy}{dx} = \frac{3x^2(4 - x) + x^3}{(4 - x)^2}$$

$\Rightarrow \quad \frac{dy}{dx} = \frac{3x^2(4 - x) + x^3}{2y(4 - x)^2}$

$\therefore$ Slope $m = \left(\frac{dy}{dx}\right)_P = \frac{3 \times 4(4 - 2) + 8}{-2 \times 2(4 - 2)^2}$

$$= \frac{32}{-16} = -2$$

From (A)

Equation of tangent is

$$(y + 2) = -2(x - 2)$$

$\Rightarrow \quad 2x + y = 2$

From (B)

Equation of Normal is

$$(y + 2) = \frac{1}{2}(x - 2)$$

$\Rightarrow \quad x - 2y = 6$

Tangents and Normals Ex 16.2 Q3(vi)

We know that the equation of tangent and the normal to any curve is given by

$$y - y_1 = m(x - x_1) \qquad \text{(A)} \quad \text{Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad \text{(B)} \quad \text{Normal}$$

Where m is the slope

We have,

$$y = x^2 + 4x + 1 \qquad \text{and} \qquad P = (x = 3)$$

$$\text{Slope} = \frac{dy}{dx} = 2x + 4$$

$$\therefore \qquad m = \left(\frac{dy}{dx}\right)_P = 10$$

From (A)

Equation of tangent is

$$(y - 22) = 10(x - 3)$$

$$\Rightarrow \qquad 10x - y = 8$$

From (B)

Equation of normal is

$$(y - 22) = \frac{-1}{10}(x - 3)$$

$$\Rightarrow \qquad x + 10y = 223$$

Tangents and Normals Ex 16.2 Q3(vii)

We know that the equation of tangent and the normal to any curve is given by

$$y - y_1 = m(x - x_1) \qquad (A) \qquad \text{Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad (B) \qquad \text{Normal}$$

Where m is the slope

We have,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{and} \quad P = (a\cos\theta, b\sin\theta)$$

Differentiating with respect to x, we get

$$\frac{2x}{a^2} + \frac{2y}{b^2}\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{xb^2}{ya^2}$$

$$\therefore \quad \text{Slope } m = \left(\frac{dy}{dx}\right)_P = \frac{-a\cos\theta b^2}{b\sin\theta a^2}$$

$$= \frac{-b}{a}\cot\theta$$

From (A)

Equation of tangent is,

$$(y - b\sin\theta) = \frac{-b}{a}\cot\theta(x - a\cos\theta)$$

$$\Rightarrow \quad \frac{b}{a}x\cot\theta + y = b\sin\theta + b\cot\theta \times \cos\theta$$

$$\Rightarrow \quad \frac{x}{a}\cot\theta + \frac{y}{b} = \frac{\sin^2\theta + \cos^2\theta}{\sin\theta}$$

$$\Rightarrow \quad \frac{x}{a}\cot\theta + \frac{y}{b} = \frac{1}{\sin\theta}$$

$$\Rightarrow \quad \frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$$

From (B)

Equation of normal is

$$(y - b\sin\theta) = \frac{a}{b}\frac{\sin\theta}{\cos\theta}(x - a\cos\theta)$$

$$\Rightarrow \quad \frac{a}{b}x\tan\theta - y = \frac{a^2}{b}\sin\theta - b\sin\theta$$

$$\Rightarrow \quad \frac{a}{b}x\tan\theta - y = \frac{a^2 - b^2}{b}\sin\theta$$

$$\Rightarrow \quad \frac{a}{b}x\sec\theta - y\,\mathrm{cosec}\theta = \frac{a^2 - b^2}{b}$$

$$\Rightarrow \quad ax\sec\theta - by\,\mathrm{cosec}\theta = a^2 - b^2$$

Tangents and Normals Ex 16.2 Q3(viii)

We know that the equation of tangent and the normal to any curve is given by

$$y - y_1 = m(x - x_1) \qquad \text{(A)} \quad \text{Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad \text{(B)} \quad \text{Normal}$$

Where m is the slope

We have,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad P = (a\sec\theta, b\tan\theta)$$

Differentiating with respect to x, we get

$$\frac{2x}{a^2} - \frac{2y}{b^2}\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{xb^2}{ya^2}$$

$$\therefore \quad \text{Slope } m = \left(\frac{dy}{dx}\right)_P = \frac{a\sec\theta b^2}{b\tan\theta a^2}$$

$$= \frac{b}{a\sin\theta}$$

From (A)

Equation of tangent is,

$$(y - b\tan\theta) = \frac{b}{a\sin\theta}(x - a\sec\theta)$$

$$\Rightarrow \quad \frac{b}{a}\frac{x}{\sin\theta} - y = \frac{b\sec\theta}{\sin\theta} - b\tan\theta$$

$$\Rightarrow \quad \frac{bx}{a\sin\theta} - y = \frac{b\sec\theta}{\sin\theta}\left(1 - \sin^2\theta\right)$$

$$\Rightarrow \quad \frac{x}{a} - \frac{y}{b}\sin\theta = \cos\theta$$

$$\Rightarrow \quad \frac{x}{a}\sec\theta - \frac{y}{b}\tan\theta = 1$$

From (B)

Equation of normal is

$$y - b\tan\theta = \frac{-a\sin\theta}{b}(x - a\sec\theta)$$

$$\Rightarrow \quad ax\sin\theta + by = b^2\tan\theta + a^2\tan\theta$$

$$\Rightarrow \quad ax\cos\theta + by\cot\theta = a^2 + b^2$$

Tangents and Normals Ex 16.2 Q3(ix)

We know that the equation of tangent and the normal to any curve is given by

$$y - y_1 = m(x - x_1) \qquad (A) \quad \text{Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad (B) \quad \text{Normal}$$

Where m is the slope

We have,

$$y^2 = 4ax \qquad P\left(\frac{a}{m^2}, \frac{2a}{m}\right)$$

Differentiating with respect to x, we get

$$2y\frac{dy}{dx} = 4a$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{2a}{y}$$

$$\therefore \quad \text{Slope } m = \left(\frac{dy}{dx}\right)_P = m$$

From (A)

Equation of tangent is

$$\left(y - \frac{2a}{m}\right) = m\left(x - \frac{a}{m^2}\right)$$

$$\Rightarrow \quad m^2x - my = 2a - a$$

$$\Rightarrow \quad m^2x - my = a$$

From (B)

Equation of normal is

$$\left(y - \frac{2a}{m}\right) = \frac{-1}{m}\left(x - \frac{a}{m^2}\right)$$

$$\Rightarrow \quad (my - 2a) = \frac{-m^2x + a}{m^2}$$

$$\Rightarrow \quad m^2x + m^3y = 2am^2 + a$$

$$\Rightarrow \quad m^2x + m^3y - 2am^2 - a = 0$$

Tangents and Normals Ex 16.2 Q3(x)

We know that the equation of tangent and the normal to any curve is given by

$$y - y_1 = m(x - x_1) \qquad \text{(A)} \quad \text{Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad \text{(B)} \quad \text{Normal}$$

Where m is the slope

We have,

$$c^2\left(x^2 + y^2\right) = x^2y^2 \qquad P = \left(\frac{c}{\cos\theta}, \frac{c}{\sin\theta}\right)$$

Differentiating with respect to x, we get

$$c^2\left(2x + 2y\frac{dy}{dx}\right) = 2xy^2 + 2x^2y\frac{dy}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx}\left(2yc^2 - 2x^2y\right) = 2xy^2 - 2xc^2$$

$$\therefore \quad \frac{dy}{dx} = \frac{x\left(y^2 - c^2\right)}{y\left(c^2 - x^2\right)}$$

$$\therefore \quad \text{Slope } m = \left(\frac{dy}{dx}\right)_P = \frac{\frac{c}{\cos\theta}\left(\frac{c^2}{\sin^2\theta} - c^2\right)}{\frac{c}{\sin\theta}\left(c^2 - \frac{c^2}{\cos^2\theta}\right)}$$

$$= \frac{c^2\tan\theta\left(1 - \sin^2\theta\right)}{c^2\tan^2\theta\left(\cos^2\theta - 1\right)}$$

$$= \frac{1}{-\tan\theta} \times \frac{\cos^2\theta}{\sin^2\theta}$$

$$= \frac{-\cos^3\theta}{\sin^3\theta}$$

From (A)

Equation of tangent is

$$\left(y - \frac{c}{\sin\theta}\right) = \frac{-\cos^3\theta}{\sin^3\theta}\left(x - \frac{c}{\cos\theta}\right)$$

$$\Rightarrow \quad x\cos^3\theta + y\sin^3\theta = c\sin^2\theta + c\cos^2\theta$$

$$\Rightarrow \quad x\cos^3\theta + y\sin^3\theta = c$$

From (B)

Equation of normal is

$$\left(y - \frac{c}{\sin\theta}\right) = \frac{\sin^3\theta}{\cos^3\theta}\left(x - \frac{c}{\cos\theta}\right)$$

$$\Rightarrow \quad x\sin^3\theta - y\cos^3\theta = \frac{c\sin^3\theta}{\cos\theta} - \frac{c\cos^3\theta}{\sin\theta}$$

$$\Rightarrow \quad x\sin^3\theta - y\cos^3\theta = \frac{c\left(\sin^4\theta - \cos^4\theta\right)}{\cos\theta \times \sin\theta}$$

$$= \frac{c\left(\sin^2\theta - \cos^2\theta\right)\left(\sin^2\theta + \cos^2\theta\right)}{\frac{1}{2}\sin 2\theta}$$

$$= \frac{-2c\cos 2\theta}{\sin 2\theta} = -2c\cot 2\theta$$

$$\therefore \quad x\sin^3\theta - y\cos^3\theta + 2c\cot 2\theta = 0$$

Tangents and Normals Ex 16.2 Q3(xi)

We know that the equation of tangent and the normal to any curve is given by

$$y - y_1 = m(x - x_1) \qquad (A) \quad \text{Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad (B) \quad \text{Normal}$$

Where m is the slope

We have,

$$xy = c^2 \qquad P = \left(ct, \frac{c}{t}\right)$$

Differentiating with respect to x, we get

$$y + x\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{y}{x}$$

$$\therefore \quad \text{Slope } m = \left(\frac{dy}{dx}\right)_P = \frac{\frac{-c}{t}}{ct} = \frac{-1}{t^2}$$

From (A)

Equation of tangent is

$$\left(y - \frac{c}{t}\right) = \frac{-1}{t^2}(x - ct)$$

$$\Rightarrow \quad x + t^2 y = tc + ct$$

$$\Rightarrow \quad x + t^2 y = 2ct$$

From (B)

Equation of normal is

$$\left(y - \frac{c}{t}\right) = t^2(x - ct)$$

$$\Rightarrow \quad xt^3 - ty = ct^3 \times t - c$$

$$\Rightarrow \quad xt^3 - ty = ct^4 - c$$

Tangents and Normals Ex 16.2 Q3(xii)

We know that the equation of tangent and the normal to any curve is given by

$$y - y_1 = m(x - x_1) \qquad \text{(A)} \quad \text{Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad \text{(B)} \quad \text{Normal}$$

Where m is the slope

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad \text{---(i)} \qquad P = (x_1, y_1)$$

Differentiating with resect to x, we get

$$\frac{2x}{a^2} + \frac{2y}{b^2}\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{xb^2}{ya^2}$$

$$\therefore \quad \text{Slope } m = \left(\frac{dy}{dx}\right)_P = -\frac{x_1 b^2}{y_1 a^2}$$

From (A)

Equation of tangent is

$$(y - y_1) = -\frac{x_1 b^2}{y_1 a^2}(x - x_1)$$

$$\Rightarrow \quad xx_1 b^2 + yy_1 a^2 = x_1^2 b^2 + y_1^2 a^2$$

Divide by $a^2 b^2$ both side

$$\Rightarrow \quad \frac{xx_1}{a^2} + \frac{yy_1}{b^2} = \frac{x_1^2}{a^2} + \frac{y_1^2}{b^2}$$

$$= 1 \qquad [\because (x_1, y_1) \text{ lies on (i)}]$$

$$\therefore \quad \frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$$

From (B)

Equation of normal is

$$(y - y_1) = \frac{y_1 a^2}{x_1 b^2}(x - x_1)$$

$$xy_1 a^2 - yx_1 b^2 = x_1 y_1 a^2 - y_1 x_1 b^2$$

Dividing by $x_1 y_1$ both side

$$\frac{xa^2}{x_1} - \frac{yb^2}{y_1} = a^2 - b^2$$

Tangents and Normals Ex 16.2 Q3(xiii)

Differentiating $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ with respect to x, we have:

$$\frac{2x}{a^2} - \frac{2y}{b^2}\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{2y}{b^2}\frac{dy}{dx} = \frac{2x}{a^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{b^2 x}{a^2 y}$$

Therefore, the slope of the tangent at (x_0, y_0) is $\left.\frac{dy}{dx}\right]_{(x_0, y_0)} = \frac{b^2 x_0}{a^2 y_0}$.

Then, the equation of the tangent at (x_0, y_0) is given by,

$$y - y_0 = \frac{b^2 x_0}{a^2 y_0}(x - x_0)$$

$$\Rightarrow a^2 yy_0 - a^2 y_0^2 = b^2 xx_0 - b^2 x_0^2$$

$$\Rightarrow b^2 xx_0 - a^2 yy_0 - b^2 x_0^2 + a^2 y_0^2 = 0$$

Tangents and Normals Ex 16.2 Q3(xiv)

Differentiating $x^{\frac{2}{3}}+y^{\frac{2}{3}}=2$ with respect to x, we get

$$\frac{2}{3}x^{\frac{-1}{3}}+\frac{2}{3}y^{\frac{-1}{3}}\frac{dy}{dx} = 0$$

$$\Rightarrow \qquad \frac{dy}{dx}=-\left(\frac{y}{x}\right)^{\frac{1}{3}}$$

Therefore, the slope of the tangent at $(1,1)$ is $\left.\frac{dy}{dx}\right]_{(1,1)} = -1$

So, the equation of the tangent at $(1,1)$ is

$$y-1=-1(x-1)$$

$$\Rightarrow \qquad y+x-2=0$$

Also, the slope of the normal at $(1,1)$ is given by $\frac{-1}{\text{slope of tangent at }(1,1)}=1$

$\therefore$ the equation of the normal at $(1,1)$ is

$$y-1=1(x-1)$$

$$\Rightarrow \qquad y-x=0$$

Tangents and Normals Ex 16.2 Q3(xv)

We know that the equation of tangent and the normal to any curve is given by

$$y-y_1=m(x-x_1) \qquad (A) \quad \text{Tangent}$$

$$y-y_1=\frac{-1}{m}(x-x_1) \qquad (B) \quad \text{Normal}$$

Where m is the slope

We have,

$$x^2=4y \qquad P=(2,1)$$

$$\therefore \qquad 2x=\frac{4dy}{dx}$$

$$\Rightarrow \qquad \frac{dy}{dx}=\frac{x}{2}$$

$$\therefore \qquad \text{Slope } m=\left(\frac{dy}{dx}\right)_P = 1$$

From (A)

Equation of tangent is

$$y-1=1(x-2)$$

$$\Rightarrow \qquad x-y=1$$

From (B)

Equation of normal is

$$(y-1)=-1(x-2)$$

$$\Rightarrow \qquad x+y=3$$

Tangents and Normals Ex 16.2 Q3(vi)

The equation of the given curve is $y^2 = 4x$.

Differentiating with respect to x, we have:

$$2y\frac{dy}{dx} = 4$$
$$\Rightarrow \frac{dy}{dx} = \frac{4}{2y} = \frac{2}{y}$$
$$\therefore \left.\frac{dy}{dx}\right]_{(1,2)} = \frac{2}{2} = 1$$

Now, the slope at point (1, 2) is $\frac{-1}{\left.\frac{dy}{dx}\right]_{(1,2)}} = \frac{-1}{1} = -1$.

$\therefore$Equation of the tangent at (1, 2) is $y - 2 = -1(x - 1)$.

$\Rightarrow y - 2 = -x + 1$

$\Rightarrow x + y - 3 = 0$

Equation of the normal is ,

$$y - 2 = -(-1)(x - 1)$$
$$y - 2 = x - 1$$
$$x - y + 1 = 0$$

Tangents and Normals Ex 16.2 Q3(xix)

Let $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be the equation of the curve.

Rewriting the above equation as,

$$\frac{y^2}{b^2} = \frac{x^2}{a^2} - 1$$
$$\Rightarrow y^2 = \frac{b^2}{a^2}x^2 - b^2$$

Differentiating the above function w.r.t. x, we get,

$$2y\frac{dy}{dx} = \frac{b^2}{a^2}2x$$
$$\Rightarrow \frac{dy}{dx} = \frac{b^2}{a^2}\frac{x}{y}$$

$$\Rightarrow \left[\frac{dy}{dx}\right]_{(\sqrt{2}a, b)} = \frac{b^2}{a^2}\frac{\sqrt{2}a}{b} = \frac{\sqrt{2}b}{a}$$

Slope of the tangent $m = \frac{\sqrt{2}b}{a}$

Equation of the tangent is

$$(y - y_1) = m(x - x_1)$$
$$\Rightarrow (y - b) = \frac{\sqrt{2}b}{a}(x - \sqrt{2}a)$$
$$\Rightarrow a(y - b) = \sqrt{2}b(x - \sqrt{2}a)$$
$$\Rightarrow \sqrt{2}bx - ay + ab - 2ab = 0$$
$$\Rightarrow \sqrt{2}bx - ay - ab = 0$$

Slope of the normal is $-\frac{1}{\frac{\sqrt{2}b}{a}} = -\frac{a}{b\sqrt{2}}$

Equation of the normal is

$$(y - y_1) = m(x - x_1)$$
$$\Rightarrow (y - b) = \frac{-a}{\sqrt{2}b}(x - \sqrt{2}a)$$
$$\Rightarrow \sqrt{2}b(y - b) = -a(x - \sqrt{2}a)$$
$$\Rightarrow ax + \sqrt{2}by - \sqrt{2}b^2 - \sqrt{2}a^2 = 0$$
$$\Rightarrow ax + \sqrt{2}by - \sqrt{2}(a^2 + b^2) = 0$$

Tangents and Normals Ex 16.2 Q4

The given equations are,

$$x=\theta+\sin\theta \quad , \quad y=1+\cos\theta$$

$$\frac{dx}{d\theta}=1+\cos\theta \quad , \quad \frac{dy}{d\theta}=-\sin\theta$$

$$\therefore \quad \frac{dx}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=\frac{-\sin\theta}{1+\cos\theta}$$

Slope,

$$m=\left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{4}}=\frac{-\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}}$$

$$=-1+\frac{1}{\sqrt{2}}$$

Thus, equation of tangent is,

$$y-y_1=m(x-x_1)$$

$$y-\left(1+\frac{1}{\sqrt{2}}\right)=\left(-1+\frac{1}{\sqrt{2}}\right)\left(x-\left(\frac{\pi}{4}+\frac{1}{\sqrt{2}}\right)\right)$$

$$y-1-\frac{1}{\sqrt{2}}=\left(-1+\frac{1}{\sqrt{2}}\right)\left(x-\frac{\pi}{4}-\frac{1}{\sqrt{2}}\right)$$

Tangents and Normals Ex 16.2 Q5(i)

We know that the equation of tangent and normal to any curve at the point (x_1, y_1) is

$$y-y_1=m(x-x_1) \qquad \text{---(A) Tangent}$$

$$y-y_1=\frac{-1}{m}(x-x_1) \qquad \text{---(B) Normal}$$

Where m is slope.

$x=\theta+\sin\theta,\quad y=1+\cos\theta,\quad \theta=\frac{\pi}{2}$

$$\therefore \quad P=\left[\left(\frac{\pi}{2}+1\right),1\right]$$

and $\quad \frac{dx}{d\theta}=1+\cos\theta, \frac{dy}{d\theta}=-\sin\theta$

$$\therefore \quad \text{Slope } m=\left(\frac{dy}{dx}\right)_P=\left(\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}\right)=\frac{-1}{+1}=-1$$

Equation of tangent from (A)

$$(y-1)=-1\left(x-\left(\frac{\pi}{2}+1\right)\right)$$

$$\Rightarrow \quad x+y=\frac{\pi}{2}+1+1$$

$$\Rightarrow \quad 2(x+y)=\pi+4$$

From (B)

Equation of normal is

$$(y-1)=1\left(x-\left(\frac{\pi}{2}+1\right)\right)$$

$$\Rightarrow \quad 2(x-y)=\pi$$

Tangents and Normals Ex 16.2 Q5(ii)

We know that the equation of tangent and normal to any curve at the point (x_1, y_1) is

$$y - y_1 = m(x - x_1) \qquad \text{---(A) Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad \text{---(B) Normal}$$

Where m is slope.

$$x = \frac{2at^2}{1+t^2}, \quad y = \frac{2at^3}{1+t^2}, \quad t = \frac{1}{2}$$

$$\therefore \quad P = \left(x = \frac{a}{2+\frac{1}{2}} = \frac{2a}{5}, y = \frac{a}{4+1} = \frac{a}{5}\right)$$

Now,

$$\frac{dx}{dt} = \frac{4a + \left(1+t^2\right) - 2at^2\,(2t)}{\left(1+t^2\right)^2}$$

$$= \frac{4at}{\left(1+t^2\right)^2}$$

$$\frac{dy}{dt} = \frac{6at^2\left(1+t^2\right) - \left(2at^3\right)(2t)}{\left(1+t^2\right)^2}$$

$$= \frac{6at^2 - 2at^4}{\left(1+t^2\right)^2}$$

$$\therefore \quad \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{6at^2 + 2at^4}{4at}$$

$$\therefore \quad \text{Slope } m = \left(\frac{dy}{dx}\right)_P = \frac{\frac{3a}{2} + \frac{a}{8}}{2a} = \frac{13}{16}$$

From (A)

Equation of tangent is,

$$\left(y - \frac{a}{5}\right) = \frac{13}{16}\left(x - \frac{2a}{5}\right)$$

$$16y - \frac{16a}{5} = 13x - \frac{26a}{5}$$

$$13x - 16y - 2a = 0$$

Equation of normal is,

$$\left(y - \frac{a}{5}\right) = -\frac{16}{13}\left(x - \frac{2a}{5}\right)$$

$$13y - \frac{13a}{5} = -16x + \frac{32a}{5}$$

$$16x + 13y - 9a = 0$$

Tangents and Normals Ex 16.2 Q5(iii)

We know that the equation of tangent and normal to any curve at the point (x_1, y_1) is

$$y - y_1 = m(x - x_1) \qquad \text{---(A) Tangent}$$
$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad \text{---(B) Normal}$$

Where m is slope.

$x = at^2, \qquad y = 2at, \qquad t = 1$

$\therefore \qquad P = (a, 2a)$

and

$$\frac{dx}{dt} = 2at, \qquad \frac{dy}{dt} = 2a$$

$$\therefore \qquad \text{Slope } m = \left(\frac{dy}{dx}\right)_P = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2a}{2a} = 1$$

From (A)

Equation of tangent is

$$(y - 2a) = 1(x - a)$$
$$\Rightarrow \qquad x - y + a = 0$$

From (B)

Equation of normaol is

$$(y - 2a) = -1(x - a)$$
$$\Rightarrow \qquad x + y = 3a$$

Tangents and Normals Ex 16.2 Q5(iv)

We know that the equation of tangent and normal to any curve at the point (x_1, y_1) is

$$y - y_1 = m(x - x_1) \qquad \text{---(A) Tangent}$$
$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad \text{---(B) Normal}$$

Where m is slope.

$x = a\sec t, \qquad y = b\tan t, \qquad t = t$

$$\therefore \qquad \frac{dx}{dt} = a\sec t \times \tan t$$

and

$$\frac{dy}{dt} = b\sec^2 t$$

$$\therefore \qquad \text{Slope } m = \frac{dy}{dx} = \frac{b\sec^2 t}{a\sec t \times \tan t}$$
$$= \frac{b}{a}\operatorname{cosec} t$$

From (A)

Equatin of tangent

$$(y - b\tan t) = \frac{b}{a}\operatorname{cosec} t\,(x - a\sec t)$$
$$\Rightarrow \qquad bx\operatorname{cosec} t - ay = ab\operatorname{cosec} t \times \sec t - ab\tan t$$
$$= \frac{ab\left[1 - \sin^2 t\right]}{\sin t \times \cos t}$$
$$= \frac{ab\cos t}{\sin t}$$
$$\Rightarrow \qquad bx\sec t - ay\tan t = ab$$

From (B)

Equation of normal is

$$(y - b\tan t) = \frac{-a\sin t}{b}(x - a\sec t)$$
$$\Rightarrow \qquad ax\sin t + by = a^2\tan t + b^2\tan t$$

$$\Rightarrow \qquad ax\cos t + by\cot t = a^2 + b^2$$

Tangents and Normals Ex 16.2 Q5(v)

We know that the equation of tangent and normal to any curve at the point (x_1, y_1) is

$$y - y_1 = m(x - x_1) \qquad \text{---(A) Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad \text{---(B) Normal}$$

Where m is slope.

$$x = a(\theta + \sin\theta), y = a(1 - \cos\theta)$$

$$\frac{dx}{d\theta} = a(1 + \cos\theta), \frac{dy}{d\theta} = a\sin\theta$$

$$\therefore \quad \text{Slope } m = \frac{dy}{dx} = \frac{\sin\theta}{1 + \cos\theta} = \frac{\frac{2\sin\theta}{2} \times \frac{\cos\theta}{2}}{\frac{2\cos^2\theta}{2}}$$

$$= \frac{\tan\theta}{2}$$

Now,

From (A)

Equation of tangent

$$y - a(1 - \cos\theta) = \frac{\tan\theta}{2}\left(x - a(\theta + \sin\theta)\right)$$

$$\Rightarrow \quad \frac{x\tan\theta}{2} - y = a(\theta + \sin\theta)\frac{\tan\theta}{2} - a(1 - \cos\theta)$$

From (B)

Equation of normal is

$$y - a(1 - \cos\theta) = \frac{-\cot\theta}{2}\left(x - a(\theta + \sin\theta)\right)$$

$$\Rightarrow \quad (y - 2a)\frac{\tan\theta}{2} + x - a\theta = 0$$

Tangents and Normals Ex 16.2 Q5(vi)

$$x = 3\cos\theta - \cos^3\theta, \; y = 3\sin\theta - \sin^3\theta$$

$$\Rightarrow \frac{dx}{d\theta} = -3\sin\theta + 3\cos^2\theta\sin\theta \text{ and } \frac{dy}{d\theta} = 3\cos\theta - 3\sin^2\theta\cos\theta$$

$$\Rightarrow \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{3\cos\theta - 3\sin^2\theta\cos\theta}{-3\sin\theta + 3\cos^2\theta\sin\theta} = \frac{\cos\theta\left(1 - \sin^2\theta\right)}{-\sin\theta\left(1 - \cos^2\theta\right)} = \frac{\cos^3\theta}{-\sin^3\theta} = -\tan^3\theta$$

So equation of the tangent at θ is

$$y - 3\sin\theta + \sin^3\theta = -\tan^3\theta\left(x - 3\cos\theta + \cos^3\theta\right)$$

$$\Rightarrow 4\left(y\cos^3\theta - x\sin^3\theta\right) = 3\sin4\theta$$

So equation of normal at θ is

$$y - 3\sin\theta + \sin^3\theta = \frac{1}{\tan^3\theta}\left(x - 3\cos\theta + \cos^3\theta\right)$$

$$\Rightarrow y\cos^3\theta - x\cos^3\theta = 3\sin^4\theta - \sin^6\theta - 3\cos^4\theta + \cos^6\theta$$

$$\Rightarrow y\sin^3\theta - x\cos^3\theta = 3\sin^4\theta - \sin^6\theta - 3\cos^4\theta + \cos^6\theta$$

Tangents and Normals Ex 16.2 Q6

The given equation of curve is

$$x^2 + 2y^2 - 4x - 6y + 8 = 0 \qquad \text{---(i) at } x = 2$$

Differentiating with respect to x, we get

$$2x + 4y\frac{dy}{dx} - 4 - 6\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx}[4y - 6] = 4 - 2x$$

$$\therefore \quad \frac{dy}{dx} = \frac{2 - x}{2y - 3}$$

Now,

From (i) at $x = 2$

$$4 + 2y^2 - 8 - 6y + 8 = 0$$

$$\Rightarrow \quad 2y^2 - 6y + 4 = 0$$

$$\Rightarrow \quad y^2 - 3y + 2 = 0$$

$$\Rightarrow \quad (y - 2)(y - 1) = 0$$

$$\Rightarrow \quad y = 2, 1$$

Thus,

$$\text{Slope } m_1 = \left(\frac{dy}{dx}\right)_{(2,2)} = 0$$

$$m_2 = \left(\frac{dy}{dx}\right)_{(2,1)} = 0$$

Thus, the equation of normal is

$$(y - y_1) = \frac{-1}{0}(x - 2)$$

$$\Rightarrow \quad x = 2$$

Tangents and Normals Ex 16.2 Q7

The equation of the given curve is $ay^2 = x^3$.

On differentiating with respect to x, we have:

$$2ay\frac{dy}{dx} = 3x^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{3x^2}{2ay}$$

The slope of a tangent to the curve at (x_0, y_0) is $\left.\frac{dy}{dx}\right]_{(x_0, y_0)}$.

$\Rightarrow$ The slope of the tangent to the given curve at (am^2, am^3) is

$$\left.\frac{dy}{dx}\right]_{(am^2, am^3)} = \frac{3(am^2)^2}{2a(am^3)} = \frac{3a^2m^4}{2a^2m^3} = \frac{3m}{2}.$$

$\therefore$ Slope of normal at (am^2, am^3)

$$= \frac{-1}{\text{slope of the tangent at } (am^2, am^3)} = \frac{-2}{3m}$$

Hence, the equation of the normal at (am^2, am^3) is given by,

$$y - am^3 = \frac{-2}{3m}(x - am^2)$$

$$\Rightarrow 3my - 3am^4 = -2x + 2am^2$$

$$\Rightarrow 2x + 3my - am^2(2 + 3m^2) = 0$$

Tangents and Normals Ex 16.2 Q8

The given equations are

$$y^2 = ax^3 + b \qquad \text{---(i)}$$

$$y = 4x - 5 \qquad \text{---(ii)} \qquad P = (2, 3)$$

Differentiating (i) with respect to x, we get

$$2y\frac{dy}{dx} = 3ax^2$$

$$\therefore \quad \frac{dy}{dx} = \frac{3ax^2}{2y}$$

$$\therefore \quad m_1 = \left(\frac{dy}{dx}\right)_P = \frac{12a}{6} = 2a$$

$$m_2 = \text{slope of (ii)} = 4$$

According to the question

$$m_1 = m_2 \Rightarrow 2a = 4 \Rightarrow a = 2$$

From (i)

$$y^2 = 2 \times 2^3 + b$$

$$\Rightarrow \quad 9 = 16 + b$$

$$\Rightarrow \quad b = -7$$

Thus,

$$a = 2, b = -7$$

Tangents and Normals Ex 16.2 Q9

The given equatioins are,

$$y = x^2 + 4x - 16 \qquad \text{---(i)}$$

$$3x - y + 1 = 0 \qquad \text{---(ii)}$$

Slope m_1 of (i)

$$m_1 = \frac{dy}{dx} = 2x + 4$$

Slope m_2 of (ii)

$$m_2 = 3$$

As per question

$$m_1 = m_2$$

$$\Rightarrow \quad 2x + 4 = 3$$

$$\Rightarrow \quad x = \frac{-1}{2}$$

From (i)

$$y = \frac{1}{4} - 2 - 16 = -\frac{71}{4}$$

$$\therefore \quad P = \left(\frac{-1}{2}, \frac{-71}{4}\right)$$

Thus, the equation of tangent

$$\left(y + \frac{71}{4}\right) = 3\left(x + \frac{1}{2}\right)$$

$$\Rightarrow \quad 3x - y = \frac{71}{4} - \frac{3}{2}$$

$$\Rightarrow \quad 3x - y = \frac{65}{4}$$

$$\Rightarrow \quad 12x - 4y - 65 = 0$$

Tangents and Normals Ex 16.2 Q10

The given equation is

$$y = x^3 + 2x + 6 \qquad ---(i)$$
$$x + 14y + 4 = 0 \qquad ---(ii)$$

Slope m_1 of (i)

$$m_1 = \frac{dy}{dx} = 3x^2 + 2$$

Slope m_2 of (ii)

$$m_2 = \frac{-1}{14}$$

$\therefore$ Slope of normal to (i) is

$$\frac{-1}{m_1} = \frac{-1}{3x^2 + 2}$$

According to the question

$$\frac{-1}{3x^2 + 2} = \frac{-1}{14}$$

$\Rightarrow \quad 3x^2 2 = 14$

$\Rightarrow \quad x^2 = 4$

$\Rightarrow \quad x = \pm 2$

From (i)

$$y = 8 + 4 + 6 \quad \text{or} \quad -8 - 4 + 6$$
$$= 18 \quad \text{or} \quad -6$$

so, $\quad P = (2, 18)$ and $Q = (-2, -6)$

Thus, the equation of normal is

$$(y - 18) = \frac{-1}{14}(x - 2) \quad \Rightarrow \quad x + 14y + 86 = 0$$

or $\quad (y + 6) = \frac{-1}{14}(x + 2) \quad \Rightarrow \quad x + 14y - 254 = 0$

Tangents and Normals Ex 16.2 Q11

The given equations are,

$$y = 4x^3 - 3x + 5 \qquad ---(i)$$
$$9y + x + 3 = 0 \qquad ---(ii)$$

Slope m_1 of (i)

$$m_1 = \frac{dy}{dx} = 12x^2 - 3$$

Slope m_2 of (ii)

$$m_2 = \frac{-1}{9}$$

According to the question

$$m_1 \times m_2 = -1$$

$\Rightarrow \quad \left(12x^2 - 3\right)\left(-\frac{1}{9}\right) = -1$

$\Rightarrow \quad 4x^2 - 1 = 3$

$\Rightarrow \quad x^2 = 1$

$\Rightarrow \quad x = \pm 1$

From (i)

$$y = 4 - 3 + 5 \quad \text{or} \quad -4 + 3 + 5$$
$$= 6 \quad \text{or} \quad 4$$

$\therefore \quad P = (1, 6)$ or $Q = (-, 1, 4)$

Thus, the equation of tangent is

$$(y - 6) = 9(x - 1) \quad \Rightarrow \quad 9x - y - 3 = 0$$
$$(y - 4) = 9(x + 1) \quad \Rightarrow \quad 9x - y + 13 = 0$$

Tangents and Normals Ex 16.2 Q12

The given equations are,

$y = x \log_e x$ --- (i)

$2x - 2y + 3 = 0$ --- (ii)

Slope m_1 of (i)

$$m_1 = \frac{dy}{dx} = \log_e x + 1$$

slope m_2 of (ii)

$$m_2 = 1$$

Tangents and Normals Ex 16.2 Q13

The equation of the given curve is $y = x^2 - 2x + 7$.

On differentiating with respect to x, we get:

$$\frac{dy}{dx} = 2x - 2$$

(a) The equation of the line is $2x - y + 9 = 0$.

$2x - y + 9 = 0 \Rightarrow y = 2x + 9$

This is of the form $y = mx + c$.

$\therefore$Slope of the line $= 2$

If a tangent is parallel to the line $2x - y + 9 = 0$, then the slope of the tangent is equal to the slope of the line.

Therefore, we have:

$2 = 2x - 2$

$\Rightarrow 2x = 4$

$\Rightarrow x = 2$

Now, $x = 2$

$\Rightarrow y = 4 - 4 + 7 = 7$

Thus, the equation of the tangent passing through (2, 7) is given by,

$$y - 7 = 2(x - 2)$$
$$\Rightarrow y - 2x - 3 = 0$$

Hence, the equation of the tangent line to the given curve (which is parallel to line $2x - y + 9 = 0$) is $y - 2x - 3 = 0$.

(b) The equation of the line is $5y - 15x = 13$.

$$5y - 15x = 13 \Rightarrow y = 3x + \frac{13}{5}$$

This is of the form $y = mx + c$.

$\therefore$Slope of the line = 3

If a tangent is perpendicular to the line $5y - 15x = 13$, then the slope of the tangent is $\frac{-1}{\text{slope of the line}} = \frac{-1}{3}$.

$$\Rightarrow 2x - 2 = \frac{-1}{3}$$

$$\Rightarrow 2x = \frac{-1}{3} + 2$$
$$\Rightarrow 2x = \frac{5}{3}$$
$$\Rightarrow x = \frac{5}{6}$$
$$\text{Now, } x = \frac{5}{6}$$
$$\Rightarrow y = \frac{25}{36} - \frac{10}{6} + 7 = \frac{25 - 60 + 252}{36} = \frac{217}{36}$$

Thus, the equation of the tangent passing through $\left(\frac{5}{6}, \frac{217}{36}\right)$ is given by,

$$y - \frac{217}{36} = -\frac{1}{3}\left(x - \frac{5}{6}\right)$$
$$\Rightarrow \frac{36y - 217}{36} = \frac{-1}{18}(6x - 5)$$
$$\Rightarrow 36y - 217 = -2(6x - 5)$$
$$\Rightarrow 36y - 217 = -12x + 10$$
$$\Rightarrow 36y + 12x - 227 = 0$$

Hence, the equation of the tangent line to the given curve (which is perpendicular to line $5y - 15x = 13$) is $36y + 12x - 227 = 0$.

Tangents and Normals Ex 16.2 Q14

The equation of the given curve is $y = \frac{1}{x-3}$, $x \neq 3$.

The slope of the tangent to the given curve at any point (x, y) is given by,

$$\frac{dy}{dx} = \frac{-1}{(x-3)^2}$$

If the slope of the tangent is 2, then we have:

$$\frac{-1}{(x-3)^2} = 2$$

$$\Rightarrow 2(x-3)^2 = -1$$
$$\Rightarrow (x-3)^2 = \frac{-1}{2}$$

This is not possible since the L.H.S. is positive while the R.H.S. is negative.

Hence, there is no tangent to the given curve having slope 2.

Tangents and Normals Ex 16.2 Q15

The slope of the tangent to the given curve at any point (x, y) is given by,

$$\frac{dy}{dx} = \frac{-(2x-2)}{(x^2-2x+3)^2} = \frac{-2(x-1)}{(x^2-2x+3)^2}$$

If the slope of the tangent is 0, then we have:

$$\frac{-2(x-1)}{(x^2-2x+3)^2} = 0$$

$$\Rightarrow -2(x-1) = 0$$
$$\Rightarrow x = 1$$

When $x = 1$, $y = \frac{1}{1-2+3} = \frac{1}{2}$.

$\therefore$The equation of the tangent through $\left(1, \frac{1}{2}\right)$ is given by,

$$y - \frac{1}{2} = 0(x-1)$$
$$\Rightarrow y - \frac{1}{2} = 0$$
$$\Rightarrow y = \frac{1}{2}$$

Hence, the equation of the required line is $y = \frac{1}{2}$.

Tangents and Normals Ex 16.2 Q16

The equation of the given curve is $y=\sqrt{3x-2}$.

The slope of the tangent to the given curve at any point (x, y) is given by,

$$\frac{dy}{dx}=\frac{3}{2\sqrt{3x-2}}$$

The equation of the given line is $4x - 2y + 5 = 0$.

$4x-2y+5=0 \Rightarrow y=2x+\frac{5}{2}$ (which is of the form $y=mx+c$)

$\therefore$Slope of the line = 2

Now, the tangent to the given curve is parallel to the line $4x - 2y - 5 = 0$ if the slope of the tangent is equal to the slope of the line.

$$\frac{3}{2\sqrt{3x-2}}=2$$
$$\Rightarrow \sqrt{3x-2}=\frac{3}{4}$$
$$\Rightarrow 3x-2=\frac{9}{16}$$
$$\Rightarrow 3x=\frac{9}{16}+2=\frac{41}{16}$$
$$\Rightarrow x=\frac{41}{48}$$

When $x=\frac{41}{48}$, $y=\sqrt{3\left(\frac{41}{48}\right)-2}=\sqrt{\frac{41}{16}-2}=\sqrt{\frac{41-32}{16}}=\sqrt{\frac{9}{16}}=\frac{3}{4}$.

$\therefore$Equation of the tangent passing through the point $\left(\frac{41}{48}, \frac{3}{4}\right)$ is given by,

$$y-\frac{3}{4}=2\left(x-\frac{41}{48}\right)$$

$$\Rightarrow \frac{4y-3}{4}=2\left(\frac{48x-41}{48}\right)$$
$$\Rightarrow 4y-3=\frac{48x-41}{6}$$
$$\Rightarrow 24y-18=48x-41$$
$$\Rightarrow 48x-24y=23$$

Hence, the equation of the required tangent is $48x-24y=23$

Tangents and Normals Ex 16.2 Q17

The given equations are,

$$x^2 + 3y - 3 = 0 \qquad \text{---(i)}$$
$$y = 4x - 5 \qquad \text{---(ii)}$$

Slope m_1 of (i)

$$m_1 = \frac{dy}{dx} = -\frac{2x}{3}$$

Slope m_2 of (ii)

$$m_2 = 4$$

According to the question

$$m_1 = m_2$$
$$\Rightarrow \quad -\frac{2x}{3} = 4$$
$$\Rightarrow \quad x = -6$$

From (i)

$$36 + 3y - 3 = 0$$
$$\Rightarrow \quad 3y = -33$$
$$\therefore \quad y = -11$$

So, $P = (-6, -11)$

Thus, the equation of tangent is

$$(y + 11) = 4(x + 6)$$
$$\Rightarrow \quad 4x - y + 13 = 0$$

Tangents and Normals Ex 16.2 Q18

The equations are

$$\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2 \qquad \text{---(i)}$$
$$\frac{x}{a} + \frac{y}{b} = 2 \qquad \text{---(ii)}$$

$$P = (a, b)$$

We need to prove (ii) is the tangent to (i)

Differentiating (i) with respect to x, we get

$$n\left(\frac{x}{a}\right)^n \times \frac{1}{a} + n\left(\frac{y}{b}\right)^{n-1} \times \frac{1}{b} \times \frac{dy}{dx} = 0$$
$$\Rightarrow \quad \frac{x^{n-1}}{a^n} + \frac{y^{n-1}}{b^n} \times \frac{dy}{dx} = 0$$
$$\Rightarrow \quad \frac{dy}{dx} = -\left(\frac{x}{y}\right)^{n-1} \times \left(\frac{b}{a}\right)^n$$
$$\therefore \quad \text{Slope } m = \left(\frac{dy}{dx}\right)_P = -\left(\frac{a}{b}\right)^{n-1} \times \left(\frac{b}{a}\right)^n$$
$$= -\frac{b}{a}$$

Thus, the equation of tangent is

$$(y - b) = -\frac{b}{a}(x - a)$$
$$\Rightarrow \quad bx + ay = ab + ab$$
$$\Rightarrow \quad bx + ay = 2ab$$
$$\Rightarrow \quad \frac{x}{a} + \frac{y}{b} = 2$$

Tangents and Normals Ex 16.2 Q19

We have,

$$x = \sin 3t, \qquad y = \cos 2t, \qquad t = \frac{\pi}{4}$$

$$\therefore \qquad P = \left(x = \frac{1}{\sqrt{2}}, y = 0\right)$$

Now,

$$\frac{dx}{dt} = 3\cos 3t, \quad \frac{dy}{dt} = -2\sin 2t$$

$$\therefore \quad \text{Slope } m = \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-2\sin 2t}{3\cos 3t}$$

$$= \frac{-2}{-3 \times \frac{1}{\sqrt{2}}}$$

$$= \frac{+2\sqrt{2}}{3}$$

Thus, equation of tangent is

$$(y - 0) = \frac{+2\sqrt{2}}{3}\left(x - \frac{1}{\sqrt{2}}\right)$$

$$2\sqrt{2}x - 3y = 2$$

Ex 16.3

Tangents and Normals Ex 16.3 Q1(i)

We know that angle of intersection of two curves is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \qquad \text{---(A)}$$

Where m_1 and m_2 are slopes of curves.

The given equations are

$$y^2 = x \qquad \text{---(i)}$$

$$x^2 = y \qquad \text{---(ii)}$$

$$m_1 = \frac{dy}{dx} = \frac{1}{2y}$$

$$m_2 = \frac{dy}{dx} = 2x$$

Solving (i) and (ii)

$$x^4 - x = 0 \quad \Rightarrow \quad x\left(x^3 - 1\right) = 0$$

and $y = 0, 1$

$\therefore \quad m_1 = \frac{1}{2}, \infty \quad$ and $\quad m_2 = 0$ or 2

$$\therefore \quad \tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| = \frac{3}{4}$$

$$\therefore \quad \theta = \tan^{-1}\left(\frac{3}{4}\right)$$

and $\quad \tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| = \infty$

$$\theta = \frac{\pi}{2}$$

Tangents and Normals Ex 16.3 Q1(ii)

We know that angle of intersection of two curves is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \qquad \text{---(A)}$$

Where m_1 and m_2 are slopes of curves.

$$y = x^2 \qquad \text{---(i)}$$

$$x^2 + y^2 = 20 \qquad \text{---(ii)}$$

Solving (i) and (ii)

$$y + y^2 = 20$$

$$\Rightarrow \quad y^2 + y - 20 = 0$$

$$\Rightarrow \quad (y + 5)(y - 4) = 0$$

$$\Rightarrow \quad y = -5, 4$$

$$\therefore \quad x = \sqrt{-5}, \pm 2$$

$\therefore$ Points are $P = (2, 4)$, $Q = (-2, 4)$

Now,

Slope m_1 for (i)

$$m_1 = 2x = 4$$

Slope m_2 for (ii)

$$m_2 = \frac{dy}{dx} = \frac{-x}{y} = \frac{-1}{2}$$

Now,

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| = \left|\frac{\frac{-1}{2} - 4}{1 - \frac{1}{2} \times 4}\right|$$

$$= \frac{9}{2}$$

$$\therefore \quad \theta = \tan^{-1}\frac{9}{2}$$

Tangents and Normals Ex 16.3 Q1(iii)

We know that angle of intersection of two curves is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \qquad \text{---(A)}$$

Where m_1 and m_2 are slopes of curves.

$$2y^2 = x^3 \qquad \text{---(i)}$$
$$y^2 = 32x \qquad \text{---(ii)}$$

Solving (i) and (ii)

$$x^3 = 64x$$
$$\Rightarrow \quad x\left(x^2 - 64\right) = 0$$
$$\Rightarrow \quad x(x+8)(x-8) = 0$$
$$\Rightarrow \quad x = 0, -8, 8$$

$$\therefore \quad y = 0, -, 16$$

$$\therefore \quad P = (0,0),\ Q = (8,16)$$

Now,

$$m_1 = \frac{dy}{dx} = \frac{3x^2}{4y} = 0 \text{ or } 3$$
$$m_2 = \frac{dy}{dx} = \frac{32}{2y} = \infty \text{ or } 1$$

From (A)

$$\tan\theta = \left|\frac{\infty - 0}{10}\right| = \infty \Rightarrow \theta = \frac{\pi}{2}$$

and $\tan\theta = \left|\frac{3-1}{13} = \frac{2}{4} = \frac{1}{2}\right|$

$$\therefore \quad \theta = \tan^{-1}\left(\frac{1}{2}\right)$$

Thus,

$$\theta = \frac{\pi}{2} \text{ and } \tan^{-1}\left(\frac{1}{2}\right)$$

Tangents and Normals Ex 16.3 Q1(iv)

We have,

$$x^2+y^2-4x-1=0 \qquad \text{---(i)}$$

and $$x^2+y^2-2y-9=0 \qquad \text{---(ii)}$$

Equation (i) can be written as

$$(x-2)^2+y^2-5=0 \qquad \text{---(iii)}$$

Subtracting (ii) from (i), we get

$$-4x+2y+8=0$$

$$\Rightarrow \quad y=2x-4$$

Substituting in (iii), we get

$$(x-2)^2+(2x-4)^2-5=0$$

$$\Rightarrow \quad (x-2)^2+4(x-2)^2-5=0$$

$$\Rightarrow \quad (x-2)^2=1$$

$$\Rightarrow \quad x-2=1,\ x-2=-1$$

$$\Rightarrow \quad x=3 \text{ or } x=1$$

$$\therefore \quad y=2(3)-4=2 \text{ or } y=-2$$

$\therefore$ The points of intersection of the two curves are $(3,2)$ and $(-1,-2)$

Differentiation (i) and (ii), w.r.t x we get

$$2x+2y\frac{dy}{dx}-4=0 \qquad \text{---(iv)}$$

and $$2x+2y\frac{dy}{dx}-2\frac{dy}{dx}=0 \qquad \text{---(v)}$$

$\therefore$ At $(3,2)$, from equation (iv) we have,

$$\left(\frac{dy}{dx}\right)_{C_1}=\frac{4-2(3)}{2(2)}=\frac{-1}{2}$$

$$\left(\frac{dy}{dx}\right)_{C_2}=\frac{-2(3)}{(2\times 2-3)}=\frac{-6}{2}=-3$$

$\therefore$ If φ is the angle between the curves

Then,

$$\tan\varphi=\frac{\left(\frac{dy}{dx}\right)_{C_1}-\left(\frac{dy}{dx}\right)_{C_2}}{1+\left(\frac{dy}{dx}\right)_{C_1}\left(\frac{dy}{dx}\right)_{C_2}}$$

Tangents and Normals Ex 16.3 Q1(v)

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad \text{---(i)}$$

$$x^2+y^2=ab \qquad \text{---(ii)}$$

From (ii), we get

$$y^2=ab-x^2$$

$\therefore$ From (i), we get

$$\frac{x^2}{a^2}+\frac{ab-x^2}{b^2}=1$$

$\Rightarrow$ $b^2x^2+a^3b-a^2x^2=a^2b^2$

$\Rightarrow$ $\left(b^2-a^2\right)x^2=a^2b^2-a^3b$

$$\Rightarrow \quad x^2=\frac{a^2b^2-a^3b}{b^2-a^2}$$

$$=\frac{a^2b\left(b-a\right)}{\left(b-a\right)\left(b+a\right)}$$

$$=\frac{a^2b}{b+a}$$

$$\therefore \quad x=\pm\sqrt{\frac{a^2b}{a+b}}$$

$$\therefore \quad y^2=ab-x^2=ab-\frac{a^2b}{a+b}$$

$$=\frac{a^2b+ab^2-a^2b}{a+b}=\frac{ab^2}{a+b}$$

Differentiating (i) and (ii) w.r.t x we get

$$\frac{2x}{a^2}+\frac{2y}{b^2}\times\left(\frac{dy}{dx}\right)_{C_1}=0$$

and $2x+2y\left(\frac{dy}{dx}\right)_{C_2}=0$

$$\therefore \quad \left(\frac{dy}{dx}\right)_{C_1}=\frac{-x}{a^2}\times\frac{b^2}{y}=\frac{-b^2x}{a^2y}$$

$$\left(\frac{dy}{dx}\right)_{C_2}=\frac{-x}{y}$$

At $\left(\pm\sqrt{\frac{a^2b}{a+b}},\pm\sqrt{\frac{ab^2}{a+b}}\right)$ we get

$$\left(\frac{dy}{dx}\right)_{C_1}=\frac{-b^2}{a^2}\sqrt{\frac{a}{b}}=\frac{-b^2\sqrt{a}}{a^2\sqrt{b}}$$

$$\left(\frac{dy}{dx}\right)_{C_2}=-\sqrt{\frac{a}{b}}$$

Tangents and Normals Ex 16.3 Q1(vi)

We know that angle of intersection of two curves is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \qquad \text{---(A)}$$

Where m_1 and m_2 are slopes of curves.

$$x^2 + 4y^2 = 8 \qquad \text{---(i)}$$
$$x^2 - 2y^2 = 2 \qquad \text{---(ii)}$$

Solving (i) and (ii)

$$6y^2 = 6 \Rightarrow \quad y = \pm 1$$
$$\therefore \quad x^2 = 2 + 2 \Rightarrow \quad x = \pm 2$$

$\therefore$ Point of intersection are

$$P = (2, 1) \text{ and } (-2, -1)$$

Now,

Slope m_1 for (i)

$$8y\frac{dy}{dx} = -2x \Rightarrow \frac{dy}{dx} = -\frac{x}{4y}$$
$$\therefore \quad m_1 = \frac{1}{2}$$

Slope m_2 for (ii)

$$4y\frac{dy}{dx} = 2x \Rightarrow \frac{dy}{dx} = \frac{x}{2y}$$
$$\therefore \quad m_2 = 1$$

From (A)

$$\tan\theta = \left|\frac{1 - \frac{1}{2}}{1 + 1 \times \frac{1}{2}}\right| = \frac{1}{3}$$

$$\therefore \quad \theta = \tan^{-1}\left(\frac{1}{3}\right)$$

Tangents and Normals Ex 16.3 Q1(vii)

We know that angle of intersection of two curves is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \qquad \text{---(A)}$$

Where m_1 and m_2 are slopes of curves.

$$x^2 = 27y \qquad \text{---(i)}$$
$$y^2 = 8x \qquad \text{---(ii)}$$

Solving (i) and (ii) are

$$\frac{y^4}{64} = 27y$$
$$\Rightarrow \quad y\left(y^3 - 27 \times 64\right) = 0$$
$$\Rightarrow \quad y = 0 \text{ or } 12$$

$$\therefore \quad x = 0 \text{ or } 18$$

$\therefore$ Points or intersection is $(0,0)$ and $(18,12)$

Now,

Slope of (i)

$$m_1 = \frac{2x}{27} = \frac{12}{9} = \frac{4}{3}$$

Slope of (ii)

$$m_2 = \frac{8}{2y} = \frac{8}{24} = \frac{1}{3}$$

From (A)

$$\tan\theta = \left|\frac{\frac{4}{3} - \frac{1}{3}}{1 + \frac{4}{3} \times \frac{1}{3}}\right| = \frac{9}{13}$$

$$\therefore \quad \theta = \tan^{-1}\left(\frac{9}{13}\right)$$

Tangents and Normals Ex 16.3 Q1(viii)

We know that angle of intersection of two curves is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \qquad ---(A)$$

Where m_1 and m_2 are slopes of curves.

$$x^2 + y^2 = 2x \qquad ---(i)$$
$$y^2 = x \qquad ---(ii)$$

Solving (i) and (ii)

$$x^2 + x = 2x$$
$\Rightarrow$ $x^2 - x = 0$
$\Rightarrow$ $x(x-1) = 0$
$\Rightarrow$ $x = 0, 1$

$\therefore$ $y = 0$ or 1

$\therefore$ The points of intersection is $P = (0,0)$, $Q = (1,1)$

$\therefore$ Slope of (i)

$$2y\frac{dy}{dx} = 2 - 2x$$
$\therefore$ $\frac{dy}{dx} = \frac{2-2x}{2y} = \frac{1-x}{y}$
$\therefore$ $m_1 = 0$

Slope of (ii)

$$m_2 = \frac{1}{2y} = \frac{1}{2}$$

From (A)

$$\tan\theta = \left|\frac{\frac{1}{2} - 0}{1 + \frac{1}{2}\times 0}\right| = \frac{1}{2}$$

$\therefore$ $\theta = \tan^{-1}\left(\frac{1}{2}\right)$

Tangents and Normals Ex 16.3 Q1(ix)

$y = 4 - x^2$......(i)
$y = x^2$.......(ii)

Substituting eq (ii) in (i) we get,

$x^2 = 4 - x^2$

$\Rightarrow x^2 = 2$

$\Rightarrow x = \pm\sqrt{2}$

From(i) when $x = \sqrt{2}$,we get $y = 2$ and when $x = -\sqrt{2}$,we get $y = 2$

Thus the two curves intersect at $\left(\sqrt{2},\ 2\right)$ and $\left(-\sqrt{2}, 2\right)$.

Differnentiating (i) wrt x, we get

$$\frac{dy}{dx} = 0 - 2x = -2x$$

Differnentiating (ii) wrt x, we get

$$\frac{dy}{dx} = 2x$$

Angle of intersection at $\left(\sqrt{2},\ 2\right)$

$$m_1 = \left(\frac{dy}{dx}\right)_{\left(\sqrt{2},\ 2\right)} = -2\sqrt{2}$$

Angle of intersection at $\left(-\sqrt{2},\ 2\right)$

$$m_2 = \left(\frac{dy}{dx}\right)_{\left(-\sqrt{2},\ 2\right)} = 2\sqrt{2}$$

Let θ be the angle of intersection of the two curves.

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| = \left|\frac{2\sqrt{2} + 2\sqrt{2}}{1 + \left(2\sqrt{2}\right)\left(-2\sqrt{2}\right)}\right| = \left|\frac{4\sqrt{2}}{-7}\right| = \frac{4\sqrt{2}}{7}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{4\sqrt{2}}{7}\right)$$

Tangents and Normals Ex 16.3 Q2(i)

We know that two curves intersects orthogonally if

$$m_1 \times m_2 = -1 \qquad \text{---(A)}$$

Where m_1 and m_2 are the slopes of two curves

$$y = x^3 \qquad \text{---(i)}$$
$$6y = 7 - x^2 \qquad \text{---(ii)}$$

Slope of (i)

$$\frac{dy}{dx} = 3x^2 = m_1$$

Slope of (ii)

$$\frac{dy}{dx} = -\frac{2}{6}x = m_2$$

Point of intersection of (i) and (ii) is

$$6x^3 = 7 - x^2$$
$$\Rightarrow \quad 6x^3 + x^2 - 7 = 0$$
$$\Rightarrow \quad x = 1$$

$$\therefore \quad y = 1$$
$$\therefore \quad P = (1,1)$$
$$\therefore \quad m_1 = 3 \text{ and } m_2 = -\frac{1}{3}$$

Now,

$$m_1 \times m_2 = 3 \times -\frac{1}{3} = -1$$

$\therefore$ (i) and (ii) cuts orthogonally.

Tangents and Normals Ex 16.3 Q2(ii)

We know that two curves intersects orthogonally if

$$m_1 \times m_2 = -1 \qquad ---(A)$$

Where m_1 and m_2 are the slopes of two curves

$$x^3 - 3xy^2 = -2 \qquad ---(i)$$

$$3x^2y - y^3 = 2 \qquad ---(ii)$$

Point of intersection of (i) and (ii)

(i) + (ii)

$\Rightarrow \quad x^3 - 3xy^2 + 3x^2y - y^3 = 0$

$\Rightarrow \quad (x - y)^3 = 0$

$\Rightarrow \quad x = y$

$\therefore$ from (i)

$x^3 - 3x^2 = -2$

$\Rightarrow \quad -2x^3 = -2$

$\Rightarrow \quad x = 1$

$\therefore$ $P = (1,1)$ is the point of intersection

Now,

Slope of (i)

$$3x^2 - 3y^2 - 6xy\frac{dy}{dx} = 0$$

$$\therefore \quad m_1 = \frac{dy}{dx} = \frac{3(x^2 - y^2)}{6xy}$$

Slope of (ii)

$$6xy + 3x^2\frac{dy}{dx} - 3y^2\frac{dy}{dx} = 0$$

$$\Rightarrow \quad m_2 = \frac{dy}{dx} = \frac{-6xy}{3(x^2 - y^2)}$$

$$\therefore \quad m_1 \times m_2 = \frac{(x^2 - y^2)}{2xy} \times \frac{-2xy}{(x^2 - y^2)} = -1$$

Tangents and Normals Ex 16.3 Q2(iii)

We know that two curves intersects orthogonally if

$$m_1 \times m_2 = -1 \qquad \text{---(A)}$$

Where m_1 and m_2 are the slopes of two curves

$$x^2 + 4y^2 = 8 \qquad \text{---(i)}$$
$$x^2 - 2y^2 = 4 \qquad \text{---(ii)}$$

Point of intersection of (i) and (ii) is (i) − (ii), we get

$$6y^2 = 4$$
$$\Rightarrow \quad y = \sqrt{\frac{2}{3}}$$

$$\therefore \quad x^2 = 4 + \frac{8}{6}$$
$$x^2 = \frac{32}{6}$$
$$\Rightarrow \quad x = \frac{4}{\sqrt{3}}$$

Now,
Slope of (i)

$$2x + 8y\frac{dy}{dx} = 0$$
$$\Rightarrow \quad \frac{dy}{dx} = -\frac{x}{4y}$$
$$\Rightarrow \quad m_1 = -\frac{1}{4} \times \frac{4}{\sqrt{2}} = -\frac{1}{\sqrt{2}} \qquad \left[\because \frac{x}{y} = \frac{4}{\sqrt{2}}\right]$$

Slope of (ii)

$$2x - 4y\frac{dy}{dx} = 0$$
$$\Rightarrow \quad \frac{dy}{dx} = \frac{x}{2y}$$
$$\Rightarrow \quad m_2 = \frac{1}{2} \times \frac{4}{\sqrt{2}} = \sqrt{2}$$
$$\therefore \quad m_1 \times m_2 = -\frac{1}{\sqrt{2}} \times \sqrt{2} = -1$$

$\therefore$ (i) and (ii) cuts orthogonally.

Tangents and Normals Ex 16.3 Q3(i)

We have,

$$x^2 = 4y \qquad \text{---(i)}$$
$$4y + x^2 = 8 \qquad \text{---(ii)} \qquad P = (2,1)$$

Slope of (i)

$$2x = 4\frac{dy}{dx}$$
$$\therefore \quad m_1 = \left(\frac{dy}{dx}\right)_P = \left(\frac{x}{2}\right)_P = 1$$

Slope of (ii)

$$4\frac{dy}{dx} + 2x = 0$$
$$\therefore \quad m_2 = \left(\frac{dy}{dx}\right)_P = \left(-\frac{x}{2}\right)_P = -1$$
$$\therefore \quad m_1 \times m_2 = 1 \times -1 = -1$$

Hence the result.

Tangents and Normals Ex 16.3 Q3(ii)

We have,

$$x^2 = y \qquad \text{---(i)}$$

$$x^3 + 6y = 7 \qquad \text{---(ii)} \qquad P = (1,1)$$

Slope of (i)

$$2x = \frac{dy}{dx}$$

$$\therefore \quad m_1 = \left(\frac{dy}{dx}\right)_P = 2$$

Slope of(ii)

$$3x^2 + 6\frac{dy}{dx} = 0$$

$$\therefore \quad m_2 = \left(\frac{dy}{dx}\right)_P = \left(-\frac{x^2}{2}\right)_P = \frac{-1}{2}$$

$$\therefore \quad m_1 \times m_2 = 2 \times \frac{-1}{2} = -1$$

Tangents and Normals Ex 16.3 Q3(iii)

We have,

$$y^2 = 8x \qquad \text{---(i)}$$

$$2x^2 + y^2 = 10 \qquad \text{---(ii)} \qquad P\left(1, 2\sqrt{2}\right)$$

Slope of (i)

$$2y\frac{dy}{dx} = 8$$

$$\therefore \quad m_1 = \left(\frac{dy}{dx}\right)_P = \left(\frac{4}{y}\right)_P = \sqrt{2}$$

Slope of (ii)

$$4x + 2y\frac{dy}{dx} = 0$$

$$\therefore \quad m_2 = \left(\frac{dy}{dx}\right)_P = \left(-\frac{2x}{y}\right)_P = \frac{-1}{\sqrt{2}}$$

$$\therefore \quad m_1 \times m_2 = \sqrt{2} \times \frac{-1}{\sqrt{2}} = -1$$

Tangents and Normals Ex 16.3 Q4

We have,

$$4x = y^2 \qquad \text{---(i)}$$
$$4xy = k \qquad \text{---(ii)}$$

Slope of (i)

$$4 = 2y\frac{dy}{dx}$$
$$\Rightarrow \quad m_1 = \frac{dy}{dx} = \frac{2}{y}$$

Slope of (ii)

$$y + x\frac{dy}{dx} = 0$$
$$\Rightarrow \quad m_2 = \frac{dy}{dx} = \frac{-y}{x}$$

Solving (i) and (ii)

$$\frac{k}{y} = y^2$$
$$\Rightarrow \quad y^3 = k$$
$$k = \frac{k^{\frac{2}{3}}}{4}$$

$\therefore$ (i) and (ii) cuts orthogonolly

$$\therefore \quad m_1 \times m_2 = -1$$
$$\Rightarrow \quad \frac{2}{y} \times \frac{-y}{x} = -1$$
$$\Rightarrow \quad \frac{2}{x} = 1$$
$$\Rightarrow \quad x = 2$$
$$\Rightarrow \quad \frac{k^{\frac{2}{3}}}{4} = 2$$
$$\Rightarrow \quad k^{\frac{2}{3}} = 8$$
$$\therefore \quad k^2 = 512$$

Tangents and Normals Ex 16.3 Q5

We have,

$$2x = y^2 \qquad \text{---(i)}$$
$$2xy = k \qquad \text{---(ii)}$$

Slope of (i)

$$2 = 2y\frac{dy}{dx}$$
$$\Rightarrow \quad m_1 = \frac{dy}{dx} = \frac{1}{y}$$

Slope of (ii)

$$y + x\left(\frac{dy}{dx}\right) = 0$$
$$\therefore \quad m_2 = \frac{dy}{dx} = \frac{-y}{x}$$

Now,
Solving (i) and (ii)

$$\frac{k}{y} = y^2$$
$$\Rightarrow \quad y^3 = k$$
$$\therefore \quad x = \frac{y^2}{2} = \frac{k^{\frac{2}{3}}}{2}$$

$\therefore$ (i) and (ii) cuts orthogonolly

$$\therefore \quad m_1 \times m_2 = -1$$
$$\frac{1}{y} \times \frac{-y}{x} = -1$$
$$\Rightarrow \quad \frac{1}{x} = 1$$
$$\Rightarrow \quad x = 1$$
$$\Rightarrow \quad \frac{k^{\frac{2}{3}}}{2} = 1$$
$$\Rightarrow \quad k^{\frac{2}{3}} = 2$$

Closing both side, we get

$$k^2 = 8$$

Tangents and Normals Ex 16.3 Q6

$xy = 4$

$\Rightarrow x = \frac{4}{y} \ldots\ldots(i)$

$x^2 + y^2 = 8 \ldots\ldots(ii)$

Substituting eq (i) in (ii) we get,

$x^2 + y^2 = 8$

$\Rightarrow \left(\frac{4}{y}\right)^2 + y^2 = 8$

$\Rightarrow 16 + y^4 = 8y^2$

$\Rightarrow y^4 - 8y^2 + 16 = 0$

$\Rightarrow \left(y^2 - 4\right)^2 = 0$

$\Rightarrow y^2 = 4$

$\Rightarrow y = \pm 2$

From(i) when $y = 2$,we get $x = 2$ and when $y = -2$,we get $x = -2$

Thus the two curves intersect at $(2, 2)$ and $(-2, 2)$.

Differnentiating (i) wrt x, we get

$y + x\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{y}{x}$

Differnentiating (i) wrt x, we get

$y + x\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{y}{x}$

Differnentiating (ii) wrt x, we get

$2x + 2y\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{x}{y}$

At $(2, 2)$

$\left(\frac{dy}{dx}\right)_{C_1} = -1$

$\left(\frac{dy}{dx}\right)_{C_2} = -1$

Clearly $\left(\frac{dy}{dx}\right)_{C_1} = \left(\frac{dy}{dx}\right)_{C_2}$ at $(2, 2)$

So given two curves touch each other at $(2, 2)$.

Simillarly, it can be seen that two curves touch each other at $(-2, -2)$.

Tangents and Normals Ex 16.3 Q7

$y^2 = 4x$......(i)

$x^2 + y^2 - 6x + 1 = 0$.......(ii)

Differnentiating (i) wrt x, we get

$$2y\frac{dy}{dx} = 4$$

$$\Rightarrow \frac{dy}{dx} = \frac{2}{y}$$

Differnentiating (ii) wrt x, we get

$$2x + 2y\frac{dy}{dx} - 6 + 0 = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{3-x}{y}$$

At (1, 2)

$$\left(\frac{dy}{dx}\right)_{c_1} = \frac{2}{2} = 1$$

$$\left(\frac{dy}{dx}\right)_{c_2} = \frac{3-1}{2} = \frac{2}{2} = 1$$

Clearly $\left(\frac{dy}{dx}\right)_{c_1} = \left(\frac{dy}{dx}\right)_{c_2}$ at (1, 2)

So given two curves touch each other at (1, 2).

Tangents and Normals Ex 16.3 Q8

We have,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad \text{---(i)}$$

$$xy = c^2 \qquad \text{---(ii)}$$

Slope of (i)

$$\frac{2x}{a^2} - \frac{2y}{b^2}\frac{dy}{dx} = 0$$

$$\therefore \quad m_1 = \frac{dy}{dx} = \frac{x}{y} \times \frac{b^2}{a^2}$$

Slope of (ii)

$$y + x\frac{dy}{dx} = 0$$

$$\therefore \quad m_2 = \frac{dy}{dx} = \frac{-y}{x}$$

$\therefore$ (i) and (ii) cuts orthogonally

$$\therefore \quad m_1 \times m_2 = -1$$

$$\Rightarrow \quad \frac{x}{y} \times \frac{-y}{x} \times \frac{a^2}{b^2} = -1$$

$$\Rightarrow \quad a^2 = b^2$$

We have,

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad \text{---(i)}$$

$$\frac{x^2}{A^2}-\frac{y^2}{B^2}=1 \qquad \text{---(ii)}$$

Slope of (i)

$$\frac{2x}{a^2}+\frac{2y}{b^2}\times\frac{dy}{dx}=0$$

$$\therefore \quad m_1=\frac{dy}{dx}=-\frac{x}{y}\frac{b^2}{a^2}$$

Slope of (ii)

$$\frac{2x}{A^2}-\frac{2y}{B^2}\times\frac{dy}{dx}=0$$

$$\therefore \quad m_2=\frac{dy}{dx}=\frac{x}{y}\times\frac{B^2}{A^2}$$

$\therefore$ (i) and (ii) cuts orthogonally

$$\therefore \quad m_1\times m_2=-1$$

$$\therefore \quad \frac{-x}{y}\frac{b^2}{a^2}\times\frac{x}{y}\times\frac{B^2}{A^2}=-1$$

$$\Rightarrow \quad \frac{x^2}{y^2}\times\frac{b^2B^2}{a^2A^2}=1$$

$$\Rightarrow \quad \frac{x^2}{y^2}=\frac{a^2A^2}{b^2B^2} \qquad \text{---(iii)}$$

Now,

(i) − (ii) gives

$$x^2\left[\frac{1}{a^2}-\frac{1}{A^2}\right]+y^2\left[\frac{1}{b^2}+\frac{1}{B^2}\right]=0$$

$$\Rightarrow \quad \frac{x^2}{y^2}=\frac{B^2+b^2}{b^2B^2}\times\frac{a^2A^2}{a^2-A^2}$$

Put in (iii), we get

$$\frac{\left(B^2+b^2\right)}{b^2B^2}\times\frac{a^2A^2}{\left(a^2-A^2\right)}=\frac{a^2A^2}{b^2B^2}$$

$$\Rightarrow \quad B^2+b^2=a^2-A^2$$

$$\Rightarrow \quad a^2-b^2=A^2+B^2$$

Tangents and Normals Ex 16.3 Q9

We have,

$$\frac{x^2}{a^2+\lambda_1}+\frac{y^2}{b^2+\lambda_1}=1 \qquad \text{---(i)}$$

$$\frac{x^2}{a^2+\lambda_2}+\frac{y^2}{b^2+\lambda_2}=1 \qquad \text{---(ii)}$$

slope of (i)

$$\frac{2x}{a^2+\lambda_1}+\frac{2y}{b^2+\lambda_1}\times\frac{dy}{dx}=0$$

$$\Rightarrow \quad m_1=\frac{dy}{dx}=\frac{-x}{y}\times\frac{b^2+\lambda_1}{a^2+\lambda_1}$$

Slope of (ii)

$$\frac{2x}{a^2+\lambda_2}+\frac{2y}{b^2+\lambda_2}\times\frac{dy}{dx}=0$$

$$\Rightarrow \quad m_2=\frac{dy}{dx}=\frac{-x}{y}\times\frac{b^2+\lambda_2}{a^2+\lambda_2}$$

Now,

Subtracting (ii) from (i), we get

$$x^2\left[\frac{1}{a^2+\lambda_1}-\frac{1}{a^2+\lambda_2}\right]+y^2\left[\frac{1}{b^2+\lambda_1}-\frac{1}{b^2+\lambda_2}\right]=0$$

$$\Rightarrow \quad \frac{x^2}{y^2}=\frac{\lambda_2-\lambda_1}{\left(b^2+\lambda_1\right)\left(b^2+\lambda_2\right)}\times\frac{1}{\dfrac{\lambda_1-\lambda_2}{\left(a^2+\lambda_1\right)\left(a^2+\lambda_2\right)}}$$

Now,

$$m_1\times m_2=\frac{x^2}{y^2}\times\frac{\left(b^2+\lambda_1\right)\left(b^2+\lambda_2\right)}{\left(a^2+\lambda_1\right)\left(a^2+\lambda_2\right)}$$

$$=\frac{\left(\lambda_2-\lambda_1\right)}{\left(b^2+\lambda_1\right)\left(b^2+\lambda_2\right)}\times-\frac{\left(a^2+\lambda_1\right)\left(a^2+\lambda_2\right)}{\lambda_2-\lambda_1}\times\frac{\left(b^2+\lambda_1\right)\left(b^2+\lambda_2\right)}{\left(a^2+\lambda_1\right)\left(a^2+\lambda_2\right)}$$

$$=-1$$

$\therefore$ (i) and (ii) cuts orthogonolly

Tangents and Normals Ex 16.3 Q10

Suppose the straight line $x\cos\alpha+y\sin\alpha=p$ touches the curve at $Q(x_1,y_1)$.

But equation of tangent to $\frac{x^2}{a^2}+\frac{y}{b^2}=1$ at $Q(x_1,y_1)$ is

$$\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$$

Thus equation $\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$ and $x\cos\alpha+y\sin\alpha=p$ represent the same line.

$$\therefore \frac{x_1/a^2}{\cos\alpha}+\frac{y_1/b^2}{\sin\alpha}=\frac{1}{p}$$

$$\Rightarrow x_1=\frac{a^2\cos\alpha}{p},\quad y_1=\frac{b^2\sin\alpha}{p}\ \ldots\ldots\ldots\ldots\text{(i)}$$

The point $Q(x_1,y_1)$ lies on the curve $\frac{x^2}{a^2}+\frac{y}{b^2}=1$

$$\therefore \frac{a^4\cos^2\alpha}{p^2a^2}+\frac{b^4\sin^2\alpha}{p^2b^2}=1$$

$$\Rightarrow a^2\cos^2\alpha-b^2\sin^2\alpha=p^2$$