Ex 27.1

Q1

Let $S(-1,1)$ be the focus and $P(x,y)$ be a point on the hyperbola Draw PM perpendicular from P on the directrix. Then, by definition.

$$SP = ePM$$
$$\Rightarrow \quad SP^2 = e^2PM^2$$
$$\Rightarrow \quad (x+1)^2 + (y-1)^2 = (3)^2 \left[\frac{x-y+3}{\sqrt{1^2+(-1)^2}}\right]^2 \qquad [\because e = 3]$$
$$\Rightarrow \quad x^2+1+2x+y^2+1-2y = \frac{9[x-y+3]^2}{2}$$
$$\Rightarrow \quad 2\left[x^2+y^2+2x-2y+2\right] = 9[x-y+3]^2$$
$$\Rightarrow \quad 2x^2+2y^2+4x-4y+4 = 9\left[x^2(-y)^2+3^2+2\times x\times(-y)+2\times(-y)\times 3+2\times 3\times x\right]$$
$$\Rightarrow \quad 2x^2+2y^2+4x-4y-4 = 9\left[x^2+y^2+9-2xy-6y+6x\right]$$
$$\Rightarrow \quad 2x^2+2y^2+4x-4y+4 = 9x^2+9y^2+81-18xy-54y+4y+81-4=0$$
$$\Rightarrow \quad 7x^2+7y^2-18xy+50x-50y+77=0$$

This is the required equation of the hyperbota

Q2(i)

Let $S(0,3)$ be the focus and $P(x,y)$ be a point on the hyperbola.
Draw PM perpendicular from P on the directrix. Then, by definition

$$sP = ePM$$
$$\Rightarrow \quad sP^2 = e^2PM^2$$
$$\Rightarrow \quad (x-0)^2+(y-3)^2 = 2^2\left[\frac{x+y-1}{\sqrt{1^2+1^2}}\right]^2 \qquad [\because e = 2]$$
$$\Rightarrow \quad x^2+y^2+9-6y = \frac{4[x+y-1]^2}{2}$$
$$\Rightarrow \quad x^2+y^2-6y+9 = 2(x+y-1)^2$$
$$\Rightarrow \quad x^2+y^2-6y+9 = 2\left[x^2+y^2+(-1)^2+2xy+2\times y\times(-1)+2\times(-1)\times x\right]$$
$$\Rightarrow \quad x^2+y^2-6y+9 = 2\left[x^2+y^2+1+2xy-2y-2x\right]$$
$$\Rightarrow \quad x^2+y^2-6y+9 = 2x^2+2y^2+2+4xy-4y-4x$$
$$\Rightarrow \quad 2x^2-x^2+2y^2-y^2+4xy-4x-4y+6y+2-9=0$$
$$\Rightarrow \quad x^2+y^2+4xy-4x+2y-7=0$$

This is the required equation of the hyperbola.

Q2(ii)

Let $S(1,1)$ be the focus and $P(x,y)$ be a point on the hyperbola.
Draw PM perpendicular from P on the directrix. Then, by definition

$$
\begin{aligned}
& sP = ePM \\
\Rightarrow \quad & sP^2 = e^2PM^2 \\
\Rightarrow \quad & (x-1)^2 + (y-1)^2 = 2^2\left[\frac{3x+4y+8}{\sqrt{3^2+4^2}}\right]^2 \qquad [\because e = 2] \\
\Rightarrow \quad & x^2+1-2x+y^2+1-2y = 4\left[\frac{3x+4y+8}{\sqrt{25}}\right] \\
\Rightarrow \quad & x^2+y^2-2x-2y+2 = \frac{4(3x+4y+8)^2}{25} \\
\Rightarrow \quad & 25x^2+25y^2-50x-50y+50 = 4(3x+4y+8)^2 \\
\Rightarrow \quad & 25x^2+25y^2-50x-50y+50 = 4\left[9x^2+16y^2+6y+24xy+64y+48x\right] \\
\Rightarrow \quad & 25x^2+25y^2-50x-50y+50 = 36x^2+64y^2+256+96xy+256y+192x \\
\Rightarrow \quad & 36x^2-25x^2+64y^2-25y^2+96xy+192x+50x+256y+50y+256-50 = 0 \\
\Rightarrow \quad & 11x^2+39y^2+96xy+242x+306y+206 = 0
\end{aligned}
$$

This is the required equation of the hyperbola.

Q2(iii)

Let $S(1,1)$ be the focus and $P(x,y)$ be a point on the hyperbola.
Draw PM perpendicular from P on the directrix. Then, by definition

$$
\begin{aligned}
& sP = ePM \\
\Rightarrow \quad & sP^2 = e^2PM^2 \\
\Rightarrow \quad & (x-1)^2 + (y-1)^2 = \left(\sqrt{3}\right)^2\left[\frac{2x+y-1}{\sqrt{2^2+1^2}}\right]^2 \qquad [\because e = 2] \\
\Rightarrow \quad & x^2+1-2x+y^2+1-2y = \frac{3[2x+y-1]^2}{5} \\
\Rightarrow \quad & 5\left[x^2+y^2-2x-2y+2\right] = 3(2x+y-1)^2 \\
\Rightarrow \quad & 5x^2+5y^2-10x-10y+10 = 3\left[(2x)^2+y^2+(-1)^2+2\times 2x\times y+2\times y\times(-1)+2\times(-1)\times 2x\right] \\
\Rightarrow \quad & 5x^2+5y^2-10x-10y+10 = 3\left[4x^2+y^2+1+4xy-2y-4x\right] \\
\Rightarrow \quad & 5x^2+5y^2-10x-10y+10 = 12x^2+3y^2+3+12xy-6y-12x \\
\Rightarrow \quad & 12x^2-5x^2+3y^2-5y^2+12xy-12x+10x-6y+10y+3-10 = 0 \\
\Rightarrow \quad & 7x^2-2y^2+12xy-2x+4y-7 = 0
\end{aligned}
$$

This is the required equation of the hyperbola.

Q2(iv)

Let $S(2,-1)$ be the focus and $P(x,y)$ be a point on the hyperbola.
Draw PM perpendicular from P on the directrix. Then, by definition

$$sP = ePM$$
$$\Rightarrow \quad sP^2 = e^2PM^2$$
$$\Rightarrow \quad (x-2)^2 + (y+1)^2 = 2^2\left[\frac{2x+3y-1}{\sqrt{2^2+3^2}}\right]^2 \qquad [\because e = 2]$$
$$\Rightarrow \quad x^2 + 4 - 4x + y^2 + 1 + 2y = \frac{4[2x+3y-1]^2}{13}$$
$$\Rightarrow \quad 13\left[x^2+y^2-4x+2y+5\right] = 4(2x+3y-1)^2$$
$$\Rightarrow \quad 13x^2 + 13y^2 - 52x + 26y + 65 = 4[2x+3y-1]^2$$
$$\Rightarrow \quad 13x^2 + 13y^2 - 52x + 26y + 65 = 4\left[(2x)^2 + (3y)^2 + (-1)^2 + 2\times 2x \times 3y + 2\times 3y \times (-1) + 2\times(-1)\times 2x\right]$$
$$\Rightarrow \quad 13x^2 + 13y^2 - 52x + 26y + 65 = 4\left[4x^2 + 9y^2 + 1 + 12xy - 6y - 4x\right]$$
$$\Rightarrow \quad 13x^2 + 13y^2 - 52x + 26y + 65 = 16x^2 + 36y^2 + 4 + 48xy - 24y - 16x$$
$$\Rightarrow \quad 16x^2 - 13x^2 + 36y^2 - 13y^2 + 48xy - 16x + 52x - 24y - 26y + 4 - 65 = 0$$
$$\Rightarrow \quad 3x^2 + 23y^2 + 48xy + 36x - 50y - 61 = 0$$

This is the required equation of the hyperbola.

Q2(v)

Let $S(a,0)$ be the focus and $P(x,y)$ be a point on the hyperbola.
Draw PM perpendicular from P on the directrix. Then, by definition

$$sP = ePM$$
$$\Rightarrow \quad sP^2 = e^2PM^2$$
$$\Rightarrow \quad (x-a)^2 + (y-0)^2 = \left(\frac{4}{3}\right)^2\left[\frac{2x-y+a}{\sqrt{2^2+(-1)^2}}\right]^2 \qquad \left[\because e = \frac{4}{3}\right]$$
$$\Rightarrow \quad x^2 + a^2 - 2ax + y^2 = \frac{16}{9}\times\frac{[2x-y+a]^2}{5}$$
$$\Rightarrow \quad 45\left[x^2+y^2-2ax+a^2\right] = 16[2x-y+a]^2$$
$$\Rightarrow \quad 45x^2 + 45y^2 - 90ax + 45a^2 = 16\left[(2x)^2 + (-y)^2 + a^2 + 2\times 2x(-y) + 2\times(-y)\times a + 2\times a\times 2x\right]$$
$$\Rightarrow \quad 45x^2 + 45y^2 - 90ax + 45a^2 = 16\left[4x^2 + y^2 + a^2 - 4xy - 2ay + 4ax\right]$$
$$\Rightarrow \quad 45x^2 + 45y^2 - 90ax + 45a^2 = 64x^2 + 16y^2 + 16a^2 - 64xy - 32ay + 64ax$$
$$\Rightarrow \quad 64x^2 - 45x^2 + 16y^2 - 45y^2 - 64xy + 64ax + 90ax - 32ay + 16a^2 - 45a^2 = 0$$
$$\Rightarrow \quad 19x^2 - 29y^2 - 64xy + 154ax - 32ay - 29a^2 = 0$$

This is the required equation of the hyperbola.

Q2(vi)

Let $S(2,2)$ be the focus and $P(x,y)$ be a point on the hyperbola.
Draw PM perpendicular from P on the directrix. Then, by definition

$$sP = ePM$$

$$\Rightarrow \quad sP^2 = e^2PM^2$$

$$\Rightarrow \quad (x-2)^2 + (y-2)^2 = 2^2\left[\frac{x+y-9}{\sqrt{1^2+1^2}}\right]^2 \qquad \left[\because e = \frac{4}{3}\right]$$

$$\Rightarrow \quad x^2 + 4 - 4x + y^2 + 4 - 4y = \frac{4[x+y-9]^2}{2}$$

$$\Rightarrow \quad x^2 + y^2 - 4x - 4y + 8 = 2[x+y-9]^2$$

$$\Rightarrow \quad x^2 + y^2 - 4x - 4y + 8 = 2\left[x^2 + y^2 + (-9)^2 + 2 \times x \times y + 2 \times y \times (-9) + 2 \times (-9) \times x\right]$$

$$\Rightarrow \quad x^2 + y^2 - 4x - 4y + 8 = 2\left[x^2 + y^2 + 81 + 2xy - 18y + 18x\right]$$

$$\Rightarrow \quad x^2 + y^2 - 4x - 4y + 8 = \left[2x^2 + 2y^2 + 162 + 4xy - 36y - 36x\right]$$

$$\Rightarrow \quad 2x^2 - x^2 + 2y^2 - y^2 + 4xy - 36x + 4x - 36y + 4y + 162 - 8 = 0$$

$$\Rightarrow \quad x^2 + y^2 + 4xy - 32x - 32y + 154 = 0$$

This is the required equation of the hyperbola.

Q3(i)

We have,

$$9x^2 - 16y^2 = 144$$

$$\Rightarrow \quad \frac{9x^2}{144} - \frac{16y^2}{144} = 1$$

$$\Rightarrow \quad \frac{x^2}{16} - \frac{y^2}{9} = 1$$

This is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, where $a^2 = 16$ and $b^2 = 9$

Eccentricity: The eccentricity e is given by

$$\begin{aligned} e &= \sqrt{1 + \frac{b^2}{a^2}} \\ &= \sqrt{1 + \frac{9}{16}} \\ &= \sqrt{\frac{25}{16}} \\ &= \frac{5}{4} \end{aligned}$$

Foci: The coordinates of the foci are $(\pm ae, 0)$ i.e., $(\pm 5, 0)$

Equations of the directrices: The equations of the directrices are

$$x = \pm\frac{a}{e} \text{ i.e., } x = \pm\frac{16}{5}$$

$$\therefore \quad 5x = \pm 16$$

$$\Rightarrow \quad 5x \mp 16 = 0$$

Length of latus-rectum: The length of the latus-rectum

$$= \frac{2b^2}{a} = \frac{2 \times 9}{4} = \frac{9}{2}$$

Q3(ii)

We have,

$$16x^2 - 9y^2 = -144$$

$$\Rightarrow \quad \frac{16x^2}{144} - \frac{9y^2}{144} = -1$$

$$\Rightarrow \quad \frac{x^2}{9} - \frac{y^2}{16} = -1$$

This is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$, where $a^2 = 9$ and $b^2 = 16$

$\therefore \quad a = 3$ and $b = 4$

Eccentricity: The eccentricity e is given by

$$e = \sqrt{1 + \frac{a^2}{b^2}}$$

$$= \sqrt{1 + \frac{9}{16}}$$

$$= \sqrt{\frac{25}{16}}$$

$$= \frac{5}{4}$$

Foci: The coordinates of the foci are $(0, \pm be)$.

$$\therefore \quad (0, \pm be) = \left(0, \pm 4 \times \frac{5}{4}\right)$$

$$= (0, \pm 5)$$

$\therefore \quad$ the coordinates of the foci are $(0, \pm 5)$

Equations of the directrices: The equations of the directrices are

$$y = \frac{\pm b}{e}$$

$$\Rightarrow \quad y = \pm \frac{4}{\frac{5}{4}} = \pm \frac{16}{5}$$

$$\Rightarrow \quad 5y \mp 16 = 0$$

Latus-rectum : The length of the latus-rectum

$$= \frac{2a^2}{b}$$

$$= \frac{2 \times 9}{4} = \frac{9}{2}$$

Q3(iii)

We have,

$$4x^2 - 3y^2 = 36$$

$$\Rightarrow \quad \frac{4x^2}{36} - \frac{3y^2}{36} = 1$$

$$\Rightarrow \quad \frac{x^2}{9} - \frac{y^2}{12} = 1$$

This is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$, where $a^2 = 9$ and $b^2 = 12$

$\therefore \quad a = 3$ and $b = \sqrt{12} = 2\sqrt{3}$

Eccentricity: The eccentricity e is given by

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$

$$= \sqrt{1 + \frac{12}{9}}$$

$$= \sqrt{1 + \frac{4}{3}}$$

$$= \sqrt{\frac{7}{3}}$$

Q3(iv)

We have,

$$3x^2 - y^2 = 4$$

$$\Rightarrow \quad \frac{3x^2}{4} - \frac{y^2}{4} = 1$$

$$\Rightarrow \quad \frac{x^2}{\frac{4}{3}} - \frac{y^2}{4} = 1$$

$$\Rightarrow \quad \frac{x^2}{\left(\frac{2}{\sqrt{3}}\right)^2} - \frac{y^2}{2^2} = 1$$

This is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, where $a = \frac{2}{\sqrt{3}}$ and $b = 2$

Eccentricity: The eccentricity e is given by

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$

$$= \sqrt{1 + \frac{4}{\frac{4}{3}}}$$

$$= \sqrt{1+3}$$

$$= \sqrt{4}$$

$$= 2$$

Q3(v)

Foci: The coordinates of the foci are $(\pm ae, 0)$

$$\therefore \qquad \pm ae = \pm \frac{2}{\sqrt{3}} \times 2 = \pm \frac{4}{\sqrt{3}}$$

The coordinates of the foci are $\left(\pm \frac{4}{\sqrt{3}}, 0\right)$

Equations of the directirices: The equations of the directrices are

$$\begin{aligned} x &= \pm \frac{a}{e} \\ &= \pm \frac{2}{\frac{\sqrt{3}}{2}} \\ &= \pm \frac{1}{\sqrt{3}} \end{aligned}$$

$$\Rightarrow \qquad \sqrt{3}x \mp 1 = 0$$

Latus-rectum: The length of the latus-rectum $= \frac{2b^2}{a}$.

$$\begin{aligned} \therefore \qquad \frac{2b^2}{a} &= 2 \times \frac{4}{\frac{2}{\sqrt{3}}} \\ &= 4\sqrt{3} \end{aligned}$$

Q4

We have,

$$25x^2 - 36y^2 = 225$$

$$\Rightarrow \quad \frac{25x^2}{225} - \frac{36^2}{225} = 1$$

$$\Rightarrow \quad \frac{x^2}{9} - \frac{4y^2}{25} = 1$$

$$\Rightarrow \quad \frac{x^2}{9} - \frac{y^2}{\frac{25}{4}} = 1$$

$$\Rightarrow \quad \frac{x^2}{(3)^2} - \frac{y^2}{\left(\frac{5}{2}\right)^2} = 1$$

This is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, where $a = 3$ and $b = \frac{5}{2}$

Length of the transverse axis: The length of the transverse axis

$= 2a$

$= 2 \times 3 = 6$

Q5(i)

We have,

$$16x^2 - 9y^2 + 32x + 36y - 164 = 0$$

$$\Rightarrow \quad 16x^2 + 32x - 9y^2 + 36y - 14 = 0$$

$$\Rightarrow \quad 16\left(x^2 + 2x\right) - 9\left(y^2 + 4y\right) - 164 = 0$$

$$\Rightarrow \quad 16\left[x^2 + 2x + 1 - 1\right] - 9\left[y^2 - 4y + 4 - 4\right] - 164 = 0$$

$$\Rightarrow \quad 16\left[(x+1)^2 - 1\right] - 9\left[(y-2)^2 - 4\right] - 164 = 0$$

$$\Rightarrow \quad 16(x+1)^2 - 16 - 9(y-2)^2 + 36 - 164 = 0$$

$$\Rightarrow \quad 16(x+1)^2 - 9(y-2)^2 + 20 - 164 = 0$$

$$\Rightarrow \quad 16(x+1)^2 - 9(y-2)^2 - 144 = 0$$

$$\Rightarrow \quad 16(x+1)^2 - 9(y-2)^2 = 144$$

$$\Rightarrow \quad \frac{16(x+1)^2}{144} - \frac{9(y-2)^2}{144} = 1$$

$$\Rightarrow \quad \frac{(x+1)^2}{9} - \frac{(y-2)}{16} = 1 \qquad \text{---(i)}$$

Shifting the origin at $(-1,2)$ without rotating the coordinate axes and denoting the new coordinates with respect to the new axes by X and y,

We have,

$$x = X - 1 \text{ and } y = Y + 2 \qquad \text{---(ii)}$$

Q5(ii)

We have,

$$x^2 - y^2 + 4x = 0$$

$$\Rightarrow \quad x^2 + 4x - y^2 = 0$$

$$\Rightarrow \quad x^2 + 4x + 4 - 4 - y^2 = 0$$

$$\Rightarrow \quad (x+2)^2 - y^2 = 4$$

$$\Rightarrow \quad \frac{(x+2)^2}{4} - \frac{y^2}{4} = 1 \qquad \text{---(i)}$$

Shifting the origin at $(-2, 0)$ without rotating the axes and denoting the new coordinates w.r.t these axes by X and y,

We have,

$$x = X - 2 \text{ and } y = Y \qquad \text{---(ii)}$$

Using these relations, equation (i) reduces to

$$\frac{X^2}{4} - \frac{Y^2}{4} = 1 \qquad \text{---(ii)}$$

This is of the form $\frac{X^2}{a^2} - \frac{Y^2}{b^2} = 1$, where $a^2 = 4$ and $b^2 = 4$. so,

We have,

Centre: The coordinates of the centre w.r.t the new axes are $(X = 0, Y = 0)$

Putting $X = 0$ and $Y = 0$ in equation (ii), we get

$$x = -2 \text{ and } y = 0.$$

So, the coordinates of the centre w.r.t the old axes are $(-2, 0)$.

Q5(iii)

We have,

$$x^2 - 3y^2 - 2x = 8$$

$\Rightarrow \quad x^2 - 2x - 3y^2 = 8$

$\Rightarrow \quad x^2 - 2x + 1 - 1 - 3y^2 = 8$

$\Rightarrow \quad (x-1)^2 - 1 - 3y^2 = 8$

$\Rightarrow \quad (x-1)^2 - 3y^2 = 9$

$\Rightarrow \quad \dfrac{(x-1)^2}{9} - \dfrac{3y^2}{9} = 1$

$\Rightarrow \quad \dfrac{(x-1)^2}{9} - \dfrac{y^2}{3} = 1 \qquad$ ---(i)

Shifting the origin at $(1,0)$ without rotating the axes and denoting the new coordinates w.r.t these axes by X and y, We have,

$$x = X + 1 \text{ and } y = Y \qquad \text{---(ii)}$$

Using these relations, equation (i) reduces to

$$\frac{X^2}{9} - \frac{Y^2}{3} = 1 \qquad \text{---(ii)}$$

This is of the form $\dfrac{X^2}{a^2} - \dfrac{Y^2}{b^2} = 1$, where $a^2 = 9$ and $b^2 = 3$. so,

We have,

Centre: The coordinates of the centre w.r.t the new axes are $(X = 0, Y = 0)$

Putting $X = 0$ and $Y = 0$ in equation (ii), we get

$x = 1$ and $y = 0$.

So, the coordinates of the centre w.r.t the old axes are $(1,0)$.

Q6(i)

Let the equation of the hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad \text{---(i)}$$

Then,

Distance between the foci = 16

$\Rightarrow \quad 2ae = 16 \qquad [\because \text{Distance between foci} = 2ae]$

$\Rightarrow \quad ae = 8$

$\Rightarrow \quad a \times \sqrt{2} = 8 \qquad [\because e = \sqrt{2}]$

$\Rightarrow \quad a = \dfrac{8}{\sqrt{2}}$

$\Rightarrow \quad a^2 = \dfrac{64}{2} = 32$

Now,

$$b^2 = a^2\left(e^2 - 1\right)$$
$$= 32\left(\left(\sqrt{2}\right)^2 - 1\right)$$
$$= 32 \times (2 - 1)$$
$$= 32$$

Putting $a^2 = 32$ and $b^2 = 32$ in equation (i), we get

$$\frac{x^2}{32} - \frac{y^2}{32} = 1$$

$\Rightarrow \quad x^2 - y^2 = 32$

Hence, the equation of the required hyperbola is $x^2 - y^2 = 32$.

Q6(ii)

Let the equation of the hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad \text{---(i)}$$

Then,

The length of the conjugate axis = $2b$

$\therefore \quad 2b = 5 \qquad [\because \text{Conjugate axis} = 5]$

$\Rightarrow \quad b = \frac{5}{2}$

$\Rightarrow \quad b^2 = \frac{25}{4}$

And, the distance between foci = $2ae$

$\therefore \quad 2ae = 13 \qquad [\because \text{ The distance between foci is } 13]$

$\Rightarrow \quad a^2e^2 = \frac{169}{4}$

Q6(iii)

Let the equation of the hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad \text{---(i)}$$

Then,

The length of the conjugate axis = $2b$

$\therefore \quad 2b = 7 \qquad \left[\because \text{Conjugate axis is} = 5\right]$

$\Rightarrow \quad b = \frac{7}{2}$

$\Rightarrow \quad b^2 = \frac{49}{4} \qquad \text{---(ii)}$

The required hyperbola passes through the point $(3, -2)$.

$\therefore \quad \frac{(3)^2}{a^2} - \frac{(-2)^2}{b^2} = 1$

$\Rightarrow \quad \frac{a}{a^2} - \frac{y}{\frac{49}{4}} = 1$

$\Rightarrow \quad \frac{9}{a^2} - \frac{16}{49} = 1$

$\Rightarrow \quad \frac{9}{a^2} = 1 + \frac{16}{49}$

$\Rightarrow \quad \frac{9}{a^2} = \frac{65}{49}$

$\Rightarrow \quad a^2 = \frac{49 \times 9}{65}$

$\Rightarrow \quad a^2 = \frac{441}{65}$

Putting $a^2 = \frac{441}{65}$ and $b^2 = \frac{49}{4}$ in equation (i), we get

$$\frac{x^2}{\frac{441}{65}} - \frac{y^2}{\frac{49}{4}} = 1$$

$\Rightarrow \quad \frac{65x^2}{441} - \frac{4y^2}{49} = 1$

$\Rightarrow \quad \frac{65x^2 - 36y^2}{441} = 1$

$\Rightarrow \quad 65x^2 - 36y^2 = 441$

Hence, the equation of the required hyperbola is $65x^2 - 36y^2 = 441$.

Q7(i)

The centre of the hyperbola is the mid-point of the line joining the two foci.

So, the coordinates of the centre are $\left(\frac{6-4}{2}, \frac{4+4}{2}\right)$ i.e., $(1, 4)$.

Let $2a$ and $2b$ be the length of transverse and conjugate axes and let e be the eccentricity.
Then, the equation of the hyperbola is

$$\frac{(x-1)^2}{a^2} - \frac{(y-4)^2}{b^2} = 1 \qquad \text{---(i)}$$

Now, distance between two foci = $2ae$

$\Rightarrow \quad \sqrt{(6+4)^2 + (4-4)^2} = 2ae \qquad [\because \text{Foci} = (6, 4) \text{ and } (-4, 4)]$

$\Rightarrow \quad \sqrt{(10)^2} = 2ae$

$\Rightarrow \quad 10 = 2ae$

$\Rightarrow \quad 2ae = 10$

$\Rightarrow \quad 2a \times 2 = 10 \qquad [\because e = 2]$

$\Rightarrow \quad a = \frac{10}{4}$

$\Rightarrow \quad a = \frac{5}{2}$

$\Rightarrow \quad a^2 = \frac{25}{4}$

Q7(ii)

The centre of the hyperbola is the mid-point of the line line joining the two vertices.
So, the coordinates of the centre are $\left(\frac{16-8}{2}, \frac{-1-1}{2}\right)$ i.e., $(4,-1)$.

Let $2a$ and $2b$ be the length of transverse and conjugate axes and let e be the eccentricity.
Then, the equation of the hyperbola is

$$\frac{(x-4)^2}{a^2} - \frac{(y+1)^2}{b^2} = 1 \qquad \text{---(i)}$$

Now,

The distance between two vertices = $2a$

$$\therefore \quad \sqrt{(16+8)^2 + (-1+1)^2} = 2ae \qquad \left[\because \text{ vertices} = (-8,-1) \text{ and } (16,-1)\right]$$

$$\Rightarrow \quad 24 = 2a$$

$$\Rightarrow \quad a = 12$$

$$\Rightarrow \quad a^2 = 144$$

and, the distance between the focus and vertex is = $ae - a$

Q7(iii)

The centre of the hyperbola is the mid-point of the line line joining the two foci.
So, the coordinates of the centre are $\left(\frac{4+8}{2}, \frac{2+2}{2}\right)$ i.e., $(6, 2)$.

Let $2a$ and $2b$ be the length of transverse and conjugate axes and let e be the eccentricity.
Then, the equation of the hyperbola is

$$\frac{(x-6)^2}{a^2} - \frac{(y-2)^2}{b^2} = 1 \qquad \text{---(i)}$$

Now, distance between two foci = $2ae$

$$\Rightarrow \quad \sqrt{(8-4)^2 + (2-2)^2} = 2ae \qquad [\because \text{Foci} = (4,2) \text{ and } (8,2)]$$
$$\Rightarrow \quad \sqrt{(4)^2} = 2ae$$
$$\Rightarrow \quad 2ae = 4$$
$$\Rightarrow \quad 2 \times a \times 2 = 4 \qquad [\because e = 2]$$
$$\Rightarrow \quad a = \frac{4}{4} = 1$$
$$\Rightarrow \quad a^2 = 1$$

Now,

$$b^2 = a^2\left(e^2 - 1\right)$$
$$\Rightarrow \quad b^2 = 1\left(2^2 - 1\right) \qquad [\because e = 2]$$
$$\Rightarrow \quad b^2 = 4 - 1$$
$$\Rightarrow \quad b^2 = 3$$

Q7(iv)

Since, the vertices are on y-axis, so let the equation of the required hyperbola is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1 \qquad \text{---(i)}$$

The coordinates of its vertices and foci are $(0, \pm b)$ and $(0, \pm be)$ respectively.

$\therefore \quad b = 7 \qquad [\because \text{vertices} = (0, \pm 7)]$

$\Rightarrow \quad b^2 = 49$

and,

$be = \frac{28}{3} \qquad \left[\because \text{Foci} = \left(0, \pm\frac{28}{3}\right)\right]$

$\Rightarrow \quad 7 \times e = \frac{28}{3}$

$\Rightarrow \quad e = \frac{4}{3}$

$\Rightarrow \quad e^2 = \frac{16}{9}$

Now,

$a^2 = b^2(e^2 - 1)$

$\Rightarrow \quad a^2 = 49\left(\frac{16}{9} - 1\right)$

$\Rightarrow \quad a^2 = 49 \times \frac{7}{9}$

$\Rightarrow \quad a^2 = \frac{343}{9}$

Putting $a^2 = \frac{343}{9}$ and $b^2 = 49$ in equation (i), we get

$$\frac{x^2}{\frac{343}{9}} - \frac{y^2}{49} = -1$$

This is the equation of the required hyperbola.

Q8

Let $2a$ and $2b$ be the transverse and conjugate axes and e be the eccentricity. Then,

The length of conjugate axis $= \frac{3}{4}$[length of transverse axis]

$$\Rightarrow \quad 2b = \frac{3}{4} \times (2a)$$

$$\Rightarrow \quad \frac{b}{a} = \frac{3}{4}$$

$$\Rightarrow \quad \frac{b^2}{a^2} = \frac{9}{16}$$

Now,

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$

$$= \sqrt{1 + \frac{9}{16}}$$

$$= \sqrt{\frac{25}{16}}$$

$$= \frac{5}{4}$$

Hence, $e = \frac{5}{4}$

Q9(i)

Let (x_2, y_2) be the coordinates of the second vertex.

We know that, the ventre of the hyperbola is the mid-point of the line-joining the two vertices.

$\therefore \quad \frac{x_1 + 4}{2} = 3$ and $\frac{y_1 + 2}{2} = 2$ $\quad$ [$\because$ Centre $= (3,2)$ and vertiex $= (4,2)$]

$\Rightarrow \quad x_1 = 2$ and $y_2 = 2$

$\therefore$ The coordinates of the second vertex is $(2,2)$

Let $2a$ and $2b$ be the length of transverse and conjugate axes and let e be eccentricity. Then, the equation of hyperbola is

$$\frac{(x-3)^2}{a^2} - \frac{(y-2)^2}{b^2} = 1 \qquad \text{---(i)}$$

Now, distance between the two vertices $= 2a$

$\Rightarrow \quad \sqrt{(4-2)^2 + (2-2)^2} = 2a$ $\quad$ [$\because$ Vertices $= (4,2)$ and $(2,2)$]

$\Rightarrow \quad \sqrt{2^2} = 2a$

$\Rightarrow \quad 2a = 2$

$\Rightarrow \quad a = 1 \qquad$ ---(ii)

Q9(ii)

Let (x_1, y_1) be the coordinates of the second focus of the required hyperbola.

We know that, the ventre of the hyperbola is the mid-point of the line-joining the two foci.

$\therefore \quad \dfrac{x_1 + 4}{2} = 6$ and $\dfrac{y_1 + 2}{2} = 2 \qquad \left[\because \text{Centre} = (6,2) \text{ and focus} = (4,2)\right]$

$\Rightarrow \quad x_1 = 8$ and $y_2 = 2$

$\therefore \quad$ The coordinates of the second focus is $(8,2)$

Let $2a$ and $2b$ be the length of transverse and conjugate axes and let e be the eccentricity. Then, the equation of hyperbola is

$$\frac{(x-6)^2}{a^2} - \frac{(y-2)^2}{b^2} = 1 \qquad \text{---(i)}$$

Now, distance between the two vertices = $2ae$

$\Rightarrow \quad \sqrt{(8-4)^2 + (2-2)^2} = 2ae \qquad \left[\because \text{foci} = (4,2) \text{ and } (8,2)\right]$

$\Rightarrow \quad \sqrt{2^2} = 2a$

$\Rightarrow \quad 2a = 2$

$\Rightarrow \quad a = 1 \qquad \text{---(ii)}$

Nwo, the distance between the vertex and focus is = $ae - a$

$\Rightarrow \quad \sqrt{(5-4)^2 + (2-2)^2} = ae - a \qquad \left[\because \text{Focus} = (5,2) \text{ and vertex} = (4,2)\right]$

$\Rightarrow \quad \sqrt{(4)^2} = 2ae$

$\Rightarrow \quad 2ae = 4$

$\Rightarrow \quad 2 \times a \times 2 = 4 \qquad [\because e = 2]$

$\Rightarrow \quad a = 1$

$\Rightarrow \quad a^2 = 1$

Q10

For a hyperbole if the length of semi transverse and semi conjugate axes are equal.
Then $a = b$

Equation of the given hyperbole is

$x^2 - y^2 = a^2$ ......(1)

Then $e = \sqrt{2}$, $C = (0, 0)$, $S = \left(\sqrt{2}a, 0\right)$, $S' = \left(-\sqrt{2}a, 0\right)$

Let coordinates of any point P on hyperbole be (α, β). Since P lies on (1)

? $\alpha^2 - \beta^2 = a^2$ ......(2)

Now $SP^2 = \left(\sqrt{2}a - \alpha\right)^2 + \beta^2 = 2a^2 + \alpha^2 + \beta^2 - 2\sqrt{2}a\alpha$

and $S'P^2 = -\left(-\sqrt{2}a - \alpha\right)^2 + \beta^2 = 2a^2 + \alpha^2 + \beta^2 + 2\sqrt{2}a\alpha$

Now $SP^2 . S'P^2 = (2a^2 + \alpha^2 + \beta^2)^2 - 8a^2\alpha^2$

$= 4a^4 + 4a^2(\alpha^2 + \beta^2) + (\alpha^2 + \beta^2)^2 - 8a^2\alpha^2$

$= 4a^2(a^2 - 2\alpha^2) + 4a^2(\alpha^2 + \beta^2) + (\alpha^2 + \beta^2)^2$

$= 4a^2(\alpha^2 - \beta^2 - 2\alpha^2) + 4a^2(\alpha^2 + \beta^2) + (\alpha^2 + \beta^2)^2$

$= (\alpha^2 + \beta^2)^2 = CP^4$

$SP . S'P = CP^2$

Q11(i)

Let the equation of hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad \text{---(i)}$$

The coordinates of its vertices and foci are $(\pm a, 0)$ and $(\pm ae, 0)$ respectively.

$\therefore \quad a = 2 \qquad \left[\because \text{vertices} = (\pm 2, 0)\right]$

$\Rightarrow \quad a^2 = 4$

and,

$ae = 3 \qquad \left[\because \text{Foci} = (\pm 3, 0)\right]$

$\Rightarrow \quad 2 \times e = 3 \qquad \left[\because a = 2\right]$

$\Rightarrow \quad e = \frac{3}{2}$

Now,

$b^2 = a^2\left(e^2 - 1\right)$

$\Rightarrow \quad b^2 = 2^2\left[\left(\frac{3}{2}\right)^2 - 1\right]$

$\Rightarrow \quad b^2 = 4\left[\frac{9}{4} - 1\right]$

$\Rightarrow \quad b^2 = 4\left[\frac{9-4}{4}\right]$

$= 4 \times \frac{5}{4}$

$= 5$

Putting $a^2 = 4$ and $b^2 = 5$ in equation (1), we get

$$\frac{x^2}{4} - \frac{y^2}{5} = 1$$

Hence, the equation of the required hyperbola is $\frac{x^2}{4} - \frac{y^2}{5} = 1$.

Q11(ii)

Since, the vertices line on y-axis, so let the equation of the required hyperbola be

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1 \qquad \text{---(i)}$$

The coordinates of its vertices and foci are $(0,\pm b)$ and $(0,\pm be)$ respectively.

$$\therefore \quad b=5 \qquad [\because \text{vertices} = (0,\pm 5)]$$

$$\Rightarrow \quad b^2=25$$

$$\text{and,} \quad be=8 \qquad [\because \text{Foci} = (0,\pm 8)]$$

$$\Rightarrow \quad 5\times e=8 \qquad [\because b=5]$$

$$\Rightarrow \quad e=\frac{8}{5}$$

$$\Rightarrow \quad e^2=\frac{64}{25}$$

Now,

$$a^2=b^2\left(e^2-1\right)$$

$$\Rightarrow \quad a^2=25\left(\frac{64}{25}-1\right) \qquad \left[\because b^2=25 \text{ and } e^2=\frac{64}{25}\right]$$

$$\Rightarrow \quad a^2=25\times\frac{39}{25}$$

$$\Rightarrow \quad a^2=39$$

Putting $a^2=39$ and $b^2=25$ in equatoin (i), we get

$$\frac{x^2}{39}-\frac{y^2}{25}=-1$$

Hence, the equaton of the required hyperbola is

$$\frac{x^2}{39}-\frac{y^2}{25}=-1.$$

Q11(iii)

Since, the vertices line on y-axis, so let the equation of the required hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1 \qquad \text{---(i)}$$

The coordinates of its vertices and foci are $(0, \pm b)$ and $(0, \pm be)$ respectively.

$\therefore \quad b = 3 \qquad \left[\because \text{vertices} = (0, \pm 3)\right]$

$\Rightarrow \quad b^2 = 9$

and, $\quad be = 5 \qquad \left[\because \text{Foci} = (0, \pm 5)\right]$

$\Rightarrow \quad e \times 3 = 5$

$\Rightarrow \quad e = \frac{5}{3}$

$\Rightarrow \quad e^2 = \frac{25}{9}$

Now,

$$a^2 = b^2\left(e^2 - 1\right)$$

$$\Rightarrow \quad a^2 = 9\left(\frac{25}{9} - 1\right)$$

$$= 9 \times \left(\frac{25 - 9}{9}\right)$$

$$= 9 \times \frac{16}{9}$$

$$= 16$$

Putting $a^2 = 16$ and $b^2 = 9$ in equatoin (i), we get

$$\frac{x^2}{16} - \frac{y^2}{9} = -1$$

Hence, the equaton of the required hyperbola is

$$\frac{x^2}{16} - \frac{y^2}{9} = -1.$$

Q11(iv)

Since, the vertices line on x-axis, so let the equation of the required hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad \text{---(i)}$$

The length of transverse axis = 8

$\therefore \quad 2a = 8 \qquad [\because \text{transverse axis is } 2a]$

$\Rightarrow \quad a = 4$

$\Rightarrow \quad a^2 = 16$

This coordinates of foci of the required hyperbola is $(\pm ae, 0)$

$\therefore \quad ae = 5 \qquad [\because \text{foci} = (\pm 5, 0)]$

$\Rightarrow \quad 4 \times e = 5 \qquad [\because a = 4]$

$\Rightarrow \quad e = \frac{5}{4}$

$\rightarrow \quad e^2 = \frac{25}{16}$

Now,

$$\begin{aligned} b^2 &= a^2\left(e^2 - 1\right) \\ &= 16\left(\frac{25}{16} - 1\right) \\ &= 16 \times \frac{9}{16} \\ &= 9 \end{aligned}$$

Putting $a^2 = 16$ and $b^2 = 9$ in equation (i), we get

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

Hence, the equation of the required hyperbola is

$$\frac{x^2}{16} - \frac{y^2}{9} = 1.$$

Q11(v)

Since, the vertices line on x-axis, so let the equation of the required hyperbola be

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1 \qquad \text{---(i)}$$

The length of conjugater axis of the required hyperbola is 24.

$\therefore \quad 2a = 24 \qquad$ [$\because$ conjugate axis is $2a$]

$\Rightarrow \quad a = \frac{24}{2} = 12$

$\Rightarrow \quad a^2 = 144$

This coordinates of foci of the required hyperbola is $(0, \pm be)$

$\therefore \quad be = 13$

$b^2e^2 = 169$

Now,

$a^2 = b^2(e^2-1)$

$\Rightarrow \quad 144 = b^2e^2 - b^2$

$\Rightarrow \quad 144 = 169 - b^2$

$\Rightarrow \quad b^2 = 169 - 144 = 25$

Putting $a^2 = 144$ and $b^2 = 25$ in equation (i), we get

$$\frac{x^2}{144}-\frac{y^2}{25}=-1$$

Hence, the equation of the required hyperbola is

$$\frac{x^2}{144}-\frac{y^2}{25}=-1.$$

Q11(vi)

Since, the vertices line on x-axis, so let the equation of the required hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad ---(i)$$

The length of conjugater axis of the required hyperbola is 8.

$$\therefore \quad \frac{2b^2}{a} = 8$$

$$\Rightarrow \quad b^2 = \frac{8}{2} \times a$$

$$\Rightarrow \quad b^2 = 4a \qquad ---(ii)$$

Now,
This coordinates of foci of the required hyperbola is $(\pm ae, 0)$

$$\therefore \quad ae = 3\sqrt{5} \qquad \left[\because \text{Foci} = \left(\pm 3\sqrt{5}, 0\right)\right]$$

$$\Rightarrow \quad e = \frac{3\sqrt{5}}{a}$$

$$\Rightarrow \quad e^2 = \frac{45}{a^2} \qquad ---(iii)$$

Q11(vii)

Since, the vertices line on x-axis, so let the equation of the required hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad ---(i)$$

The length of the latus-rectum of the required hyperbola is 12

$$\therefore \quad \frac{2b^2}{a} = 12$$

$$\Rightarrow \quad b^2 = 6a \qquad ---(ii)$$

Now,
The coordinates of foci of the required hyperbola is $(\pm ae, 0)$

$$\therefore \quad ae = 4$$

$$\Rightarrow \quad e = \frac{4}{a}$$

$$\Rightarrow \quad e^2 = \frac{16}{a^2} \qquad ---(iii)$$

Q11(viii)

Since, the vertices line on x-axis, so let the equation of the required hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad \text{---(i)}$$

The length of the vertices of the required hyperbola are $(\pm a, 0)$.

$$\therefore \quad a = 7 \qquad [\because \text{vertices} = (\pm 7, 0)]$$

$$\Rightarrow \quad a^2 = 49 \qquad \text{---(ii)}$$

Now,

$$b^2 = a^2(e^2 - 1)$$

$$\Rightarrow \quad b^2 = 49\left[\left(\frac{4}{3}\right)^2 - 1\right] \qquad \left[\because e = \frac{4}{3}\right]$$

$$\Rightarrow \quad b^2 = 49\left[\frac{16}{9} - 1\right]$$

$$\Rightarrow \quad b^2 = 49\left[\frac{7}{9}\right]$$

$$\Rightarrow \quad b^2 = \frac{343}{9}$$

Putting $a^2 = 49$ and $b^2 = \frac{343}{9}$ in equation (i), we get

$$\frac{x^2}{49} - \frac{y^2}{\frac{343}{9}} = 1$$

$$\Rightarrow \quad \frac{x^2}{49} - \frac{9y^2}{343} = 1$$

Hence, the equation of the required hyperbola is

$$\frac{x^2}{49} - \frac{9y^2}{343} = 1.$$

Q11(ix)

Since, the vertices line on y-axis, so let the equation of the required hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1 \qquad \text{---(i)}$$

It passes through $(2,3)$

$$\therefore \quad \frac{(2)^2}{a^2} - \frac{(3)^2}{b^2} = -1$$

$$\Rightarrow \quad \frac{4}{a^2} - \frac{9}{b^2} = -1$$

$$\Rightarrow \quad \frac{4}{a^2} - \frac{9}{a^2\left(e^2-1\right)} = -1 \qquad \left[\because b^2 = a^2\left(e^2-1\right)\right]$$

$$\Rightarrow \quad \frac{4}{a^2} - \frac{9}{a^2e^2 - a^2} = -1 \qquad \text{---(ii)}$$

The coordinates of foci of the required hyperbola are $(0, \pm ae)$.

$$\therefore \quad ae = \sqrt{10}$$

$$\Rightarrow \quad a^2e^2 = 10 \qquad \text{---(iii)}$$

Q11(x)

Since, the vertices lie on x- axis, so let the equation of the required hyperbola be

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1 \qquad \text{--------- (i)}$$

The length of the latus-rectum of the required hyperbola is 36.

$$\frac{2a^2}{b}=36$$

$$a^2=18b \qquad \text{----(ii)}$$

Now,

The coordinates of foci of the required hyperbola is $(0,\pm be)$.

$$be=12$$

$$e=\frac{12}{b}$$

$$e^2=\frac{144}{b^2}$$

Now,

$$a^2=b^2\left(e^2-1\right)$$

$$18b=b^2\left(\frac{144}{b^2}-1\right)$$

$$18b=144-b^2$$

$$b^2+18b-144=0$$

$$(b-6)(b+24)=0$$

$$b_{1,2}=6,-24$$

Consider the positive value of $b=6$.

On putting $b^2=36$, $a^2=18(6)=108$ in equation (i), we get

$$\frac{x^2}{108}-\frac{y^2}{36}=-1$$

$$\frac{x^2-3y^2}{108}=-1$$

$$x^2-3y^2=-108$$

$$3y^2-x^2=108$$

Therefore, the equation of the hyperbola is $3y^2-x^2=108$.

Q12

Eccentricity $= e = \sqrt{2}$

Distance between foci is

$2ae = 16$

$2a\sqrt{2} = 16$

$$a = \frac{16}{2\sqrt{2}} = 4\sqrt{2}$$

$$e = \frac{\sqrt{a^2 + b^2}}{a}$$

$$\sqrt{2} = \frac{\sqrt{32 + b^2}}{4\sqrt{2}}$$

$$8 = \sqrt{32 + b^2}$$

$$64 = 32 + b^2$$

$$b^2 = 32$$

Equation of hyperbola is $\frac{x^2}{32} - \frac{y^2}{32} = 1$

Rewriting we get, $x^2 - y^2 = 32$

Q13

Let P (x,y) be a point of the set.

Distance of P(x,y) from (4,0) = $\sqrt{(x-4)^2+y^2}$

Distance of P(x,y) from (-4,0)= $\sqrt{(x+4)^2+y^2}$

Difference between distance =2

$\sqrt{(x-4)^2+y^2}$ - $\sqrt{(x+4)^2+y^2}$ =2

$\sqrt{(x-4)^2+y^2}$ =2+ $\sqrt{(x+4)^2+y^2}$

Squaring both sides, we get,

$(x-4)^2+y^2 = 4 + 4\sqrt{(x+4)^2+y^2} + (x+4)^2+y^2$

$(x-4)^2+y^2-(x+4)^2-y^2 = 4 + 4\sqrt{(x+4)^2+y^2}$

$(x-4-x-4)(x-4+x+4) = 4 + 4\sqrt{(x+4)^2+y^2}$

-16x -4 =4 $\sqrt{(x+4)^2+y^2}$

-4x -1 = $\sqrt{(x+4)^2+y^2}$

Squaring both sides, we get,

$16x^2+8x+1= x^2+8x+16+y^2$

$15x^2-y^2=15$

This is a hyperbola.