

Ex 12.1

Higher Order Derivatives Ex 12.1 Q1(i)

We have $f(x) = x^3 + \tan x$

$$\Rightarrow f'(x) = 3x^2 + \sec^2 x$$

$$\Rightarrow f''(x) = 6x + 2 \sec x \times \sec x \tan x$$

$$\Rightarrow f''(x) = 6x + 2 \sec^2 x \tan x.$$

Higher Order Derivatives Ex 12.1 Q1(ii)

Let $y = \sin(\log x)$

Then,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} [\sin(\log x)] = \cos(\log x) \cdot \frac{d}{dx} (\log x) = \frac{\cos(\log x)}{x} \\ \therefore \frac{d^2 y}{dx^2} &= \frac{d}{dx} \left[\frac{\cos(\log x)}{x} \right] \\ &= \frac{x \cdot \frac{d}{dx} [\cos(\log x)] - \cos(\log x) \cdot \frac{d}{dx} (x)}{x^2} \\ &= \frac{x \cdot \left[-\sin(\log x) \cdot \frac{d}{dx} (\log x) \right] - \cos(\log x) \cdot 1}{x^2} \\ &= \frac{-x \sin(\log x) \cdot \frac{1}{x} - \cos(\log x)}{x^2} \\ &= \frac{-[\sin(\log x) + \cos(\log x)]}{x^2}\end{aligned}$$

Higher Order Derivatives Ex 12.1 Q1(iii)

Let $y = \log(\sin x)$

Differentiating with respect to x , we get,

$$\frac{dy}{dx} = \frac{\cos x}{\sin x}$$

Again differentiating with respect to x , we get,

$$\frac{d^2y}{dx^2} = \frac{-\sin x \times \sin x - \cos x \times \cos x}{\sin^2 x}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-1}{\sin^2 x}$$

$$\Rightarrow \frac{d^2y}{dx^2} = -\operatorname{cosec}^2 x$$

Higher Order Derivatives Ex 12.1 Q1(iv)

Let $y = e^x \sin 5x$

Then,

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx}(e^x \sin 5x) = \sin 5x \cdot \frac{d}{dx}(e^x) + e^x \frac{d}{dx}(\sin 5x) \\ &= \sin 5x \cdot e^x + e^x \cdot \cos 5x \cdot \frac{d}{dx}(5x) = e^x \sin 5x + e^x \cos 5x \cdot 5 \\ &= e^x (\sin 5x + 5 \cos 5x) \\ \therefore \frac{d^2y}{dx^2} &= \frac{d}{dx}[e^x (\sin 5x + 5 \cos 5x)] \\ &= (\sin 5x + 5 \cos 5x) \cdot \frac{d}{dx}(e^x) + e^x \cdot \frac{d}{dx}(\sin 5x + 5 \cos 5x) \\ &= (\sin 5x + 5 \cos 5x)e^x + e^x \left[\cos 5x \cdot \frac{d}{dx}(5x) + 5(-\sin 5x) \cdot \frac{d}{dx}(5x) \right] \\ &= e^x (\sin 5x + 5 \cos 5x) + e^x (5 \cos 5x - 25 \sin 5x) \\ &= e^x (10 \cos 5x - 24 \sin 5x) = 2e^x (5 \cos 5x - 12 \sin 5x) \end{aligned}$$

Higher Order Derivatives Ex 12.1 Q1(v)

Let $y = e^{6x} \cos 3x$

Then,

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx}(e^{6x} \cdot \cos 3x) = \cos 3x \cdot \frac{d}{dx}(e^{6x}) + e^{6x} \cdot \frac{d}{dx}(\cos 3x) \\ &= \cos 3x \cdot e^{6x} \cdot \frac{d}{dx}(6x) + e^{6x} \cdot (-\sin 3x) \cdot \frac{d}{dx}(3x) \\ &= 6e^{6x} \cos 3x - 3e^{6x} \sin 3x \quad \dots (1) \\ \therefore \frac{d^2y}{dx^2} &= \frac{d}{dx}(6e^{6x} \cos 3x - 3e^{6x} \sin 3x) = 6 \cdot \frac{d}{dx}(e^{6x} \cos 3x) - 3 \cdot \frac{d}{dx}(e^{6x} \sin 3x) \\ &= 6 \cdot [6e^{6x} \cos 3x - 3e^{6x} \sin 3x] - 3 \cdot \left[\sin 3x \cdot \frac{d}{dx}(e^{6x}) + e^{6x} \cdot \frac{d}{dx}(\sin 3x) \right] \quad [\text{Using (1)}] \\ &= 36e^{6x} \cos 3x - 18e^{6x} \sin 3x - 3 \left[\sin 3x \cdot e^{6x} \cdot 6 + e^{6x} \cdot \cos 3x \cdot 3 \right] \\ &= 36e^{6x} \cos 3x - 18e^{6x} \sin 3x - 18e^{6x} \sin 3x - 9e^{6x} \cos 3x \\ &= 27e^{6x} \cos 3x - 36e^{6x} \sin 3x \\ &= 9e^{6x} (3 \cos 3x - 4 \sin 3x) \end{aligned}$$

Higher Order Derivatives Ex 12.1 Q1(vi)

Let $y = x^3 \log x$

Then,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} [x^3 \log x] = \log x \cdot \frac{d}{dx} (x^3) + x^3 \cdot \frac{d}{dx} (\log x) \\ &= \log x \cdot 3x^2 + x^3 \cdot \frac{1}{x} = \log x \cdot 3x^2 + x^2 \\ &= x^2 (1 + 3 \log x) \\ \therefore \frac{d^2 y}{dx^2} &= \frac{d}{dx} [x^2 (1 + 3 \log x)] \\ &= (1 + 3 \log x) \cdot \frac{d}{dx} (x^2) + x^2 \cdot \frac{d}{dx} (1 + 3 \log x) \\ &= (1 + 3 \log x) \cdot 2x + x^2 \cdot \frac{3}{x} \\ &= 2x + 6x \log x + 3x \\ &= 5x + 6x \log x \\ &= x(5 + 6 \log x)\end{aligned}$$

Higher Order Derivatives Ex 12.1 Q1(vii)

Let $y = \tan^{-1} x$

Then,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} (\tan^{-1} x) = \frac{1}{1+x^2} \\ \therefore \frac{d^2 y}{dx^2} &= \frac{d}{dx} \left(\frac{1}{1+x^2} \right) = \frac{d}{dx} (1+x^2)^{-1} = (-1) \cdot (1+x^2)^{-2} \cdot \frac{d}{dx} (1+x^2) \\ &= \frac{-1}{(1+x^2)^2} \times 2x = \frac{-2x}{(1+x^2)^2}\end{aligned}$$

Higher Order Derivatives Ex 12.1 Q1(viii)

Let $y = x \cdot \cos x$

Then,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} (x \cdot \cos x) = \cos x \cdot \frac{d}{dx} (x) + x \cdot \frac{d}{dx} (\cos x) = \cos x \cdot 1 + x(-\sin x) = \cos x - x \sin x \\ \therefore \frac{d^2 y}{dx^2} &= \frac{d}{dx} [\cos x - x \sin x] = \frac{d}{dx} (\cos x) - \frac{d}{dx} (x \sin x) \\ &= -\sin x - \left[\sin x \cdot \frac{d}{dx} (x) + x \cdot \frac{d}{dx} (\sin x) \right] \\ &= -\sin x - (\sin x + x \cos x) \\ &= -(x \cos x + 2 \sin x)\end{aligned}$$

Higher Order Derivatives Ex 12.1 Q1(ix)

Let $y = \log(\log x)$

Then,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} [\log(\log x)] = \frac{1}{\log x} \cdot \frac{d}{dx} (\log x) = \frac{1}{x \log x} = (x \log x)^{-1} \\ \therefore \frac{d^2 y}{dx^2} &= \frac{d}{dx} [(x \log x)^{-1}] = (-1) \cdot (x \log x)^{-2} \cdot \frac{d}{dx} (x \log x) \\ &= \frac{-1}{(x \log x)^2} \cdot \left[\log x \cdot \frac{d}{dx} (x) + x \cdot \frac{d}{dx} (\log x) \right] \\ &= \frac{-1}{(x \log x)^2} \cdot \left[\log x \cdot 1 + x \cdot \frac{1}{x} \right] = \frac{-(1 + \log x)}{(x \log x)^2}\end{aligned}$$

Higher Order Derivatives Ex 12.1 Q2

$$y = e^{-x} \cos x$$

differentiating both sides w.r.t. x

$$\Rightarrow \frac{dy}{dx} = e^{-x} (-\sin x) + (\cos x) (-e^{-x})$$

$$\Rightarrow \frac{dy}{dx} = -e^{-x} \sin x - e^{-x} \cos x = -e^{-x} (\sin x + \cos x)$$

again differentiating both sides w.r.t. x

$$\Rightarrow \frac{d^2y}{dx^2} = -e^{-x} (\cos x - \sin x) + e^{-x} (\sin x + \cos x)$$

$$\Rightarrow \frac{d^2y}{dx^2} = 2e^{-x} \sin x$$

Higher Order Derivatives Ex 12.1 Q3

$$y = x + \tan x$$

differentiating both sides w.r.t. x

$$\Rightarrow \frac{dy}{dx} = 1 + \sec^2 x$$

differentiating w.r.t. x

$$\Rightarrow \frac{d^2y}{dx^2} = 0 + 2 \sec^2 x \tan x$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{2 \sin x}{\cos^3 x}$$

$$\Rightarrow \cos^2 x \frac{d^2y}{dx^2} = 2 \tan x + 2x - 2x$$

$$\Rightarrow \cos^2 x \frac{d^2y}{dx^2} = 2(x + \tan x) - 2x$$

$$\Rightarrow \cos^2 x \frac{d^2y}{dx^2} = 2y - 2x$$

$$\Rightarrow \cos^2 x \frac{d^2y}{dx^2} - 2y + 2x = 0$$

Higher Order Derivatives Ex 12.1 Q4

$$y = x^3 \log x$$

differentiating w.r.t. x

$$\Rightarrow \frac{dy}{dx} = 3x^2 \log x + \frac{x^3}{x}$$

$$\Rightarrow \frac{dy}{dx} = 3x^2 \log x + x^2$$

differentiating w.r.t. x

$$\Rightarrow \frac{d^2y}{dx^2} = (\log x) (3 \times 2x) + \frac{3x^2}{x} + 2x$$

$$\Rightarrow \frac{d^2y}{dx^2} = 6x \log x + 5x$$

differentiating w.r.t. x

$$\Rightarrow \frac{d^3y}{dx^3} = \frac{6x}{x} + 6 \log x + 5$$

$$\Rightarrow \frac{d^3y}{dx^3} = 6 \log x + 11$$

differentiating w.r.t. x

$$\Rightarrow \frac{d^4y}{dx^4} = \frac{6}{x} + 0$$

$$\Rightarrow \frac{d^4y}{dx^4} = \frac{6}{x}$$

Higher Order Derivatives Ex 12.1 Q5

$$y = \log(\sin x)$$

differentiating w.r.t. x

$$\Rightarrow \frac{dy}{dx} = \frac{d(\log(\sin x))}{d(\sin x)} \times \frac{d(\sin x)}{dx} \text{ (chain rule)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\sin x} \times \cos x = \cot x$$

differentiating w.r.t. x

$$\Rightarrow \frac{d^2y}{dx^2} = -\operatorname{cosec}^2 x$$

differentiating w.r.t. x

$$\Rightarrow \frac{d^3y}{dx^3} = (-2 \operatorname{cosec} x) \times (-\cot x \operatorname{cosec} x)$$

$$\Rightarrow \frac{d^3y}{dx^3} = \frac{2 \operatorname{cosec}^2 x \cos x}{\sin x}$$

$$\Rightarrow \frac{d^3y}{dx^3} = 2 \operatorname{cosec}^3 x \cos x$$

Higher Order Derivatives Ex 12.1 Q6

$$y = 2 \sin x + 3 \cos x$$

differentiating w.r.t. x

$$\Rightarrow \frac{dy}{dx} = 2 \cos x + 3(-\sin x) = 2 \cos x - 3 \sin x$$

differentiating w.r.t. x

$$\Rightarrow \frac{d^2y}{dx^2} = 2(-\sin x) - 3 \cos x = -(2 \sin x + 3 \cos x) = -y$$

$$\Rightarrow \frac{d^2y}{dx^2} + y = 0$$

Higher Order Derivatives Ex 12.1 Q7

$$y = \frac{\log x}{x}$$

differentiating w.r.t. x

$$\Rightarrow \frac{dy}{dx} = \frac{x \left(\frac{1}{x} \right) - (\log x)(1)}{x^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1 - \log x}{x^2}$$

differentiating w.r.t. x

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{x^2 \left(-\frac{1}{x^2} \right) - (1 - \log x)(2x)}{x^4}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-x - 2x + 2x \log x}{x^4} = \frac{x(2 \log x - 3)}{x^4}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{2 \log x - 3}{x^3}$$

Higher Order Derivatives Ex 12.1 Q8

$$x = a \sec \theta \quad y = b \tan \theta$$

differentiating both w.r.t. θ

$$\Rightarrow \frac{dx}{d\theta} = a \sec \theta \tan \theta \quad \dots\dots\dots (1)$$

$$\Rightarrow \frac{dy}{d\theta} = b \sec^2 \theta \quad \dots\dots (2)$$

Dividing (2) by (1)

$$\Rightarrow \frac{dy}{dx} = \frac{b \sec^2 \theta}{a \sec \theta \tan \theta} \quad \dots\dots (3)$$

Differentiating (3) w.r.t. θ

$$\Rightarrow \frac{d\left(\frac{dy}{dx}\right)}{d\theta} = \frac{b}{a} \left[\frac{\tan \theta (\sec \theta \tan \theta) - \sec \theta (\sec^2 \theta)}{\tan^2 \theta} \right]$$

$$\Rightarrow \frac{d\left(\frac{dy}{dx}\right)}{d\theta} = \frac{b}{a} \left[\frac{\sec \theta (\tan^2 \theta) - \sec^2 \theta}{\tan^2 \theta} \right] \quad \dots\dots (4)$$

Dividing (4) by (1)

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{b \sec \theta (\tan^2 \theta - \sec^2 \theta)}{a \times a \sec \theta \tan \theta \times \tan^2 \theta}$$

Multiplying & dividing RHS by b^3

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-b^4}{a^2 \times b^3 \tan^3 \theta}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-b^4}{a^2 y^3}$$

Higher Order Derivatives Ex 12.1 Q9

It is given that, $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$

$$\begin{aligned} \therefore \frac{dx}{dt} &= a \cdot \frac{d}{dt}(\cos t + t \sin t) \\ &= a \left[-\sin t + \sin t \cdot \frac{d}{dt}(t) + t \cdot \frac{d}{dt}(\sin t) \right] \\ &= a [-\sin t + \sin t + t \cos t] = at \cos t \end{aligned}$$

$$\begin{aligned} \frac{dy}{dt} &= a \cdot \frac{d}{dt}(\sin t - t \cos t) \\ &= a \left[\cos t - \left\{ \cos t \cdot \frac{d}{dt}(t) + t \cdot \frac{d}{dt}(\cos t) \right\} \right] \\ &= a [\cos t - \{\cos t - t \sin t\}] = at \sin t \end{aligned}$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{at \sin t}{at \cos t} = \tan t$$

$$\begin{aligned} \text{Then, } \frac{d^2y}{dx^2} &= \frac{d}{dx} \left(\frac{dy}{dx} \right) = \frac{d}{dx} (\tan t) = \sec^2 t \cdot \frac{dt}{dx} \\ &= \sec^2 t \cdot \frac{1}{at \cos t} \quad \left[\frac{dx}{dt} = at \cos t \Rightarrow \frac{dt}{dx} = \frac{1}{at \cos t} \right] \\ &= \frac{\sec^3 t}{at}, 0 < t < \frac{\pi}{2} \end{aligned}$$

Higher Order Derivatives Ex 12.1 Q10

$$y = e^x \cos x$$

differentiating w.r.t. x

$$\Rightarrow \frac{dy}{dx} = e^x (-\sin x) + e^x \cos x = e^x (\cos x - \sin x)$$

differentiating w.r.t. x

$$\Rightarrow \frac{d^2y}{dx^2} = e^x (-\cos x - \sin x) + e^x (\cos x - \sin x)$$

$$\Rightarrow \frac{d^2y}{dx^2} = -2e^x \sin x$$

$$\Rightarrow \frac{d^2y}{dx^2} = 2e^x \cos \left(x + \frac{\pi}{2} \right)$$

Higher Order Derivatives Ex 12.1 Q11

$$x = a \cos \theta$$

differentiating w.r.t. θ

$$\Rightarrow \frac{dy}{d\theta} = -a \sin \theta \dots\dots (1)$$

$$\Rightarrow \frac{dy}{d\theta} = b \cos \theta \dots\dots (2)$$

Dividing (2) by (1)

$$\Rightarrow \frac{dy}{dx} = \frac{-b \cos \theta}{a \sin \theta} \dots\dots (3)$$

differentiating (3) w.r.t. θ

$$\Rightarrow \frac{d\left(\frac{dy}{dx}\right)}{d\theta} = \frac{-b}{a} \left\{ \frac{\sin \theta (-\sin \theta) - \cos \theta (\cos \theta)}{\sin^2 \theta} \right\} = \frac{b}{a} \frac{(\sin^2 \theta + \cos^2 \theta)}{\sin^2 \theta} = \frac{b}{a \sin^2 \theta} \dots\dots (4)$$

Dividing (4) by (1)

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-b}{a^2 \sin^3 \theta} \times \frac{b^3}{b^3}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-b^4}{a^2 y^3}$$

Higher Order Derivatives Ex 12.1 Q12

$$x = a(1 - \cos^3 \theta); \quad y = a \sin^3 \theta$$

differentiating both w.r.t. θ

$$\Rightarrow \frac{dx}{d\theta} = a(0 - 3\cos^2 \theta (-\sin \theta)); \quad \frac{dy}{d\theta} = a(3\sin^2 \theta \times \cos \theta) \dots\dots (2)$$

$$\Rightarrow \frac{dy}{d\theta} = 3a \sin \theta \cos^2 \theta; \quad \frac{dy}{d\theta} = 3a \sin^2 \theta \cos \theta$$

$$\Rightarrow \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx} = \frac{3a \sin^2 \theta \cos \theta}{3a \sin \theta \cos^2 \theta} = \frac{\sin \theta}{\cos \theta} = \tan \theta$$

Differentiating w.r.t. θ

$$\Rightarrow \frac{d\left(\frac{dy}{dx}\right)}{d\theta} = \sec^2 \theta \dots\dots (3)$$

Dividing (3) by (1)

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{\sec^2 \theta}{3a \sin \theta \cos^2 \theta}$$

Putting $\theta = \pi/6$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{\frac{2}{\sqrt{3}} \times \frac{2}{\sqrt{3}}}{3a \times \frac{1}{2} \times \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}} = \frac{2^5}{3a \times (\sqrt{3})^4} = \frac{32}{27a}$$

Higher Order Derivatives Ex 12.1 Q13

$$x = a(\theta + \sin \theta); \quad y = a(1 + \cos \theta)$$

differentiating both w.r.t. θ

$$\Rightarrow \frac{dx}{d\theta} = a(1 + \cos \theta); \quad (1)$$

$$\Rightarrow \frac{dy}{d\theta} = a(0 - \sin \theta) \quad (2)$$

Dividing (2) by (1)

$$\Rightarrow \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx} = \frac{-a \sin \theta}{a(1 + \cos \theta)}$$

Differentiating w.r.t. θ

$$\begin{aligned} \Rightarrow \frac{d\left(\frac{dy}{dx}\right)}{d\theta} &= - \left\{ \frac{(1 + \cos \theta)(\cos \theta) - (\sin \theta)(0 - \sin \theta)}{(1 + \cos \theta)^2} \right\} = - \left\{ \frac{\cos \theta + \cos^2 \theta + \sin^2 \theta}{(1 + \cos \theta)^2} \right\} \\ &= - \left\{ \frac{\cos \theta + 1}{(\cos \theta + 1)^2} \right\} \\ &= \frac{-1}{1 + \cos \theta} \quad \dots\dots(3) \end{aligned}$$

dividing (3) by (1)

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-1 \times a}{a(1 + \cos \theta)^2 \times a} = \frac{-a}{y^2}$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q14

$$x = a(\theta - \sin\theta); y = a(1 + \cos\theta)$$

Differentiating the above functions with respect to θ , we get,

$$\frac{dx}{d\theta} = a(1 - \cos\theta) \quad \dots(1)$$

$$\frac{dy}{d\theta} = a(-\sin\theta) \quad \dots(2)$$

Dividing equation (2) by (1), we have,

$$\frac{dy}{dx} = \frac{a(-\sin\theta)}{a(1 - \cos\theta)} = \frac{-\sin\theta}{1 - \cos\theta}$$

Differentiating with respect to θ , we have,

$$\frac{d\left(\frac{dy}{dx}\right)}{d\theta} = \frac{(1 - \cos\theta)(-\cos\theta) + \sin\theta(\sin\theta)}{(1 - \cos\theta)^2}$$

$$= \frac{-\cos\theta + \cos^2\theta + \sin^2\theta}{(1 - \cos\theta)^2}$$

$$= \frac{1 - \cos\theta}{(1 - \cos\theta)^2}$$

$$\frac{d\left(\frac{dy}{dx}\right)}{d\theta} = \frac{1}{1 - \cos\theta} \quad \dots(3)$$

Dividing equation (3) by (1), we have,

$$\frac{d^2y}{dx^2} = \frac{1}{1 - \cos\theta} \times \frac{1}{a(1 - \cos\theta)}$$

$$= \frac{1}{a(1 - \cos\theta)^2}$$

$$= \frac{1}{a\left(2\sin^2\frac{\theta}{2}\right)^2}$$

$$= \frac{1}{4a\sin^4\left(\frac{\theta}{2}\right)}$$

$$= \frac{1}{4a} \operatorname{cosec}^4\left(\frac{\theta}{2}\right)$$

Higher Order Derivatives Ex 12.1 Q15

$$x = a(1 - \cos \theta); \quad y = a(\theta + \sin \theta)$$

Differentiating both w.r.t. θ

$$\Rightarrow \quad \frac{dx}{d\theta} = a(0 + \sin \theta); \quad \frac{dy}{d\theta} = a(1 + \cos \theta)$$

Dividing (2) by (1)

$$\Rightarrow \quad \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx} = \frac{a(1 + \cos \theta)}{a \sin \theta}$$

Differentiating w.r.t. θ

$$\begin{aligned} \Rightarrow \quad \frac{d\left(\frac{dy}{dx}\right)}{d\theta} &= \frac{\sin \theta (0 - \sin \theta) - (1 + \cos \theta) \cos \theta}{\sin^2 \theta} = - \frac{\sin^2 \theta - \cos \theta - \cos^2 \theta}{\sin^2 \theta} \\ &= - \frac{(1 + \cos \theta)}{\sin^2 \theta} \quad \dots\dots (3) \end{aligned}$$

dividing (3) by (1)

$$\Rightarrow \quad \frac{d^2y}{dx^2} = - \frac{(1 + \cos \theta)}{\sin^2 \theta \times a \sin \theta}$$

Putting $\theta = \pi/2$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = - \frac{1}{a}$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q17

$$x = \cos \theta; y = \sin^3 \theta$$

Differentiating both w.r.t. θ

$$\Rightarrow \frac{dx}{d\theta} = -\sin \theta; \quad (1)$$

$$\frac{dy}{d\theta} = 3 \sin^2 \theta \cos \theta \quad (2)$$

Dividing (2) by (1)

$$\Rightarrow \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx} = -\frac{3 \sin^2 \theta \cos \theta}{\sin \theta} = -3 \sin \theta \cos \theta$$

Differentiating w.r.t. θ

$$\Rightarrow \frac{d\left(\frac{dy}{dx}\right)}{d\theta} = -3\{\sin \theta(-\sin \theta) + \cos \theta(\cos \theta)\} = -3\{\cos^2 \theta - \sin^2 \theta\} \dots \dots \dots (3)$$

Dividing (3) by (1)

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{+3\{\cos^2 \theta - \sin^2 \theta\}}{\sin \theta} \times \frac{\sin^2 \theta}{\sin^2 \theta}$$

$$\Rightarrow \sin^3 \theta \frac{d^2y}{dx^2} = 3 \sin^2 \theta \{\cos^2 \theta - \sin^2 \theta\}$$

$$\Rightarrow y \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 3 \sin^2 \theta \{\cos^2 \theta - \sin^2 \theta\} + \left(\frac{dy}{dx}\right)^2$$

$$\Rightarrow y \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 3 \sin^2 \theta \cos^2 \theta - 3 \sin^4 \theta + 9 \sin^2 \theta \cos^2 \theta$$

adding and subtracting $3 \sin^2 \theta \cos^2 \theta$ on RHS

$$\Rightarrow y \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 12 \sin^2 \theta \cos^2 \theta - 3 \sin^4 \theta + 3 \sin^2 \theta \cos^2 \theta - 3 \sin^2 \theta \cos^2 \theta$$

$$\Rightarrow y \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 15 \sin^2 \theta \cos^2 \theta - 3 \sin^2 \theta \{\sin^2 \theta + \cos^2 \theta\}$$

$$= 15 \sin^2 \theta \cos^2 \theta - 3 \sin^2 \theta$$

$$\Rightarrow y \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 3 \sin^2 \theta \{5 \cos^2 \theta - 1\}$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q18

$$y = \sin(\sin x)$$

differentiating w.r.t. x

$$\Rightarrow \frac{dy}{dx} = \frac{d(\sin(\sin x))}{d(\sin x)} \times \frac{d(\sin x)}{dx} = \cos(\sin x) \times \cos x$$

differentiating w.r.t. x

$$\Rightarrow \frac{d^2y}{dx^2} = \{\cos(\sin x)\}(-\sin x) + \{\cos x\}\{-\sin(\sin x)\}(\cos x)$$

$$\Rightarrow \frac{d^2y}{dx^2} = -\sin x \cos(\sin x) \times \frac{\cos x}{\cos x} - y \cos^2 x$$

$$\Rightarrow \frac{d^2y}{dx^2} = -\tan x \frac{dy}{dx} - y \cos^2 x$$

$$\Rightarrow \frac{d^2y}{dx^2} + \tan x \frac{dy}{dx} + y \cos^2 x = 0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q19

$$x = \sin t; \quad y = \sin pt$$

differentiating both w.r.t. t

$$\Rightarrow \frac{dy}{dt} = \cos t \dots\dots (1); \quad \frac{dy}{dt} = p \cos pt \dots\dots (2)$$

dividing (2) by (1)

$$\Rightarrow \frac{dy}{dx} = p \frac{\cos pt}{\cos t}$$

differentiating w.r.t. x

$$\begin{aligned} \Rightarrow \frac{d\left(\frac{dy}{dx}\right)}{dt} &= p \left\{ \frac{p \cos t (-\sin pt) - (\cos pt) (-\sin t)}{\cos^2 t} \right\} \\ &= p \left\{ \frac{\sin t \cos pt - p \cos t \sin pt (-\sin t)}{\cos^2 t} \right\} \dots\dots (3) \end{aligned}$$

\Rightarrow dividing (3) by (1)

$$\Rightarrow \frac{d^2y}{dx^2} = p \left\{ \frac{\sin t \cos pt - p \cos t \sin pt}{\cos^3 t} \right\} = \left\{ \frac{\tan t \cos t - p \sin pt}{\cos^2 t} \right\}$$

$$\therefore \sin^2 t + \cos^2 t = 1$$

$$\Rightarrow 1 - \sin^2 t = \cos^2 t$$

$$\Rightarrow 1 - x^2 = \cos^2 t$$

$$\Rightarrow \frac{d^2y}{dx^2} = p \left\{ \frac{\tan t \cos pt - p \sin pt}{1 - x^2} \right\}$$

$$\Rightarrow (1 - x^2) \frac{d^2y}{dx^2} = p \frac{\sin t \cos pt}{\cos t} - p^2 \sin pt = \frac{dy}{dx} - p^2 y$$

$$\Rightarrow (1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + p^2 y = 0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q20

$$y = e^{\tan^{-1} x}$$

differentiating w.r.t. x

$$\Rightarrow \frac{dy}{dx} = e^{\tan^{-1} x} \left(\frac{1}{1+x^2} \right)$$

differentiating w.r.t. x

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{(1+x^2) \left(e^{\tan^{-1} x} \right) \times \frac{1}{1+x^2} - e^{\tan^{-1} x} (2x)}{(1+x^2)^2}$$

$$\Rightarrow (1+x^2) \frac{d^2y}{dx^2} = \frac{e^{\tan^{-1} x} - 2xe^{\tan^{-1} x}}{1+x^2}$$

$$\Rightarrow (1+x^2) \frac{d^2y}{dx^2} = \frac{e^{\tan^{-1} x}}{1+x^2} (1-2x) = \frac{dy}{dx} (1-2x)$$

$$\Rightarrow (1+x^2) \frac{d^2y}{dx^2} + (2x-1) \frac{dy}{dx} = 0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q21

$$y = e^{\tan^{-1} x}$$

differentiating w.r.t. x

$$\Rightarrow \frac{dy}{dx} = e^{\tan^{-1} x} \left(\frac{1}{1+x^2} \right)$$

differentiating w.r.t. x

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{(1+x^2) \left(e^{\tan^{-1} x} \right) \times \frac{1}{1+x^2} - e^{\tan^{-1} x} (2x)}{(1+x^2)^2}$$

$$\Rightarrow (1+x^2) \frac{d^2y}{dx^2} = \frac{e^{\tan^{-1} x} - 2xe^{\tan^{-1} x}}{1+x^2}$$

$$\Rightarrow (1+x^2) \frac{d^2y}{dx^2} = \frac{e^{\tan^{-1} x}}{1+x^2} (1-2x) = \frac{dy}{dx} (1-2x)$$

$$\Rightarrow (1+x^2) \frac{d^2y}{dx^2} + (2x-1) \frac{dy}{dx} = 0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q22

It is given that, $y = 3 \cos(\log x) + 4 \sin(\log x)$

Then,

$$\begin{aligned}
 y_1 &= 3 \cdot \frac{d}{dx} [\cos(\log x)] + 4 \cdot \frac{d}{dx} [\sin(\log x)] \\
 &= 3 \cdot \left[-\sin(\log x) \cdot \frac{d}{dx} (\log x) \right] + 4 \cdot \left[\cos(\log x) \cdot \frac{d}{dx} (\log x) \right] \\
 \therefore y_1 &= \frac{-3 \sin(\log x)}{x} + \frac{4 \cos(\log x)}{x} = \frac{4 \cos(\log x) - 3 \sin(\log x)}{x} \\
 \therefore y_2 &= \frac{d}{dx} \left(\frac{4 \cos(\log x) - 3 \sin(\log x)}{x} \right) \\
 &= \frac{x \{ 4 \cos(\log x) - 3 \sin(\log x) \}' - \{ 4 \cos(\log x) - 3 \sin(\log x) \} (x)'}{x^2} \\
 &= \frac{x \left[4 \{ \cos(\log x) \}' - 3 \{ \sin(\log x) \}' \right] - \{ 4 \cos(\log x) - 3 \sin(\log x) \} \cdot 1}{x^2} \\
 &= \frac{x \left[-4 \sin(\log x) \cdot (\log x)' - 3 \cos(\log x) \cdot (\log x)' \right] - 4 \cos(\log x) + 3 \sin(\log x)}{x^2} \\
 &= \frac{x \left[-4 \sin(\log x) \cdot \frac{1}{x} - 3 \cos(\log x) \cdot \frac{1}{x} \right] - 4 \cos(\log x) + 3 \sin(\log x)}{x^2} \\
 &= \frac{-4 \sin(\log x) - 3 \cos(\log x) - 4 \cos(\log x) + 3 \sin(\log x)}{x^2} \\
 &= \frac{-\sin(\log x) - 7 \cos(\log x)}{x^2} \\
 \therefore x^2 y_2 + x y_1 + y &= x^2 \left(\frac{-\sin(\log x) - 7 \cos(\log x)}{x^2} \right) + x \left(\frac{4 \cos(\log x) - 3 \sin(\log x)}{x} \right) + 3 \cos(\log x) + 4 \sin(\log x) \\
 &= -\sin(\log x) - 7 \cos(\log x) + 4 \cos(\log x) - 3 \sin(\log x) + 3 \cos(\log x) + 4 \sin(\log x) \\
 &= 0
 \end{aligned}$$

Hence, proved.

Higher Order Derivatives Ex 12.1 Q23

$$y = e^{2x} (ax + b)$$

differentiating w.r.t. x

$$\Rightarrow \frac{dy}{dx} = e^{2x} (a) + 2(ax + b)(e^{2x})$$

$$\Rightarrow \frac{dy}{dx} = ae^{2x} + 2y$$

differentiating w.r.t. x

$$\Rightarrow \frac{d^2y}{dx^2} = 2ae^{2x} + 2 \frac{dy}{dx}$$

$$\Rightarrow \frac{d^2y}{dx^2} = 2 \frac{dy}{dx} + 2ae^{2x} + 4y - 4y = 2 \frac{dy}{dx} + 2 \frac{dy}{dx} - 4y$$

$$\Rightarrow \frac{d^2y}{dx^2} - 4 \frac{dy}{dx} + 4y = 0$$

$$\Rightarrow y_2 - 4y_1 + 4y = 0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q24

$$x = \sin\left(\frac{1}{a} \log y\right)$$

$$\Rightarrow \sin^{-1} x = \frac{1}{a} \log y$$

differentiating w.r.t. x

$$\Rightarrow \frac{1}{\sqrt{1-x^2}} = \frac{1}{ay} \frac{dy}{dx}$$

$$\Rightarrow y_1 = \frac{dy}{dx} = \frac{ay}{\sqrt{1-x^2}}$$

differentiating w.r.t. x

$$\Rightarrow y_2 = \frac{d^2y}{dx^2} = a \left[\frac{\sqrt{1-x^2} \frac{dy}{dx} + \frac{y \times 2x}{2\sqrt{1-x^2}}}{1-x^2} \right]$$

$$\Rightarrow (1-x^2)y_2 = a\sqrt{1-x^2} \frac{dy}{dx} + \frac{ayx}{\sqrt{1-x^2}}$$

$$\Rightarrow (1-x^2)y_2 = x \frac{dy}{dx} + a\sqrt{1-x^2} \times \frac{ay}{\sqrt{1-x^2}}$$

$$\Rightarrow (1-x^2)y_2 - xy_1 - a^2y = 0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q25

$$\log y = \tan^{-1} x$$

differentiating w.r.t. x

$$\Rightarrow \frac{1}{y} \frac{dy}{dx} = \frac{1}{1+x^2}$$

$$\Rightarrow (1+x^2) \frac{dy}{dx} = y$$

differentiating w.r.t. x

$$\Rightarrow (1+x^2) \frac{d^2y}{dx^2} + 2x \frac{dy}{dx} = \frac{dy}{dx}$$

$$\Rightarrow (1+x^2) \frac{d^2y}{dx^2} + (2x-1) \frac{dy}{dx} = 0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q26

$$y = \tan^{-1} x$$

differentiating w.r.t. x

$$\Rightarrow \frac{dy}{dx} = \frac{1}{1+x^2}$$

$$\Rightarrow (1+x^2) \frac{dy}{dx} = 1$$

differentiating w.r.t. x

$$\Rightarrow (1+x^2) \frac{d^2y}{dx^2} + 2x \frac{dy}{dx} = 0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q27

$$y = \left[\log \left(x + \sqrt{1+x^2} \right) \right]^2$$

differentiating w.r.t. x

$$\Rightarrow \frac{dy}{dx} = 2 \log \left(x + \sqrt{1+x^2} \right) \times \frac{1}{x + \sqrt{1+x^2}} \times \left(1 + \frac{1 \times 2x}{2\sqrt{1+x^2}} \right)$$

$$\Rightarrow y_1 = \frac{2 \log \left(x + \sqrt{1+x^2} \right)}{x + \sqrt{1+x^2}} \times \frac{\sqrt{1+x^2} + x}{\sqrt{1+x^2}} = \frac{2 \log \left(x + \sqrt{1+x^2} \right)}{\sqrt{1+x^2}}$$

squaring both sides

$$\Rightarrow (y_1)^2 = \frac{4}{1+x^2} \left[\log \left(x + \sqrt{1+x^2} \right) \right]^2 = \frac{4y}{1+x^2}$$

$$\Rightarrow (1+x^2)(y_1)^2 = 4y$$

differentiating w.r.t. x

$$\Rightarrow (1+x^2)2y_1y_2 + 2x(y_1)^2 = 4y_1$$

$$\Rightarrow (1+x^2)y_2 + xy_1 = 2$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q28

The given relationship is $y = (\tan^{-1} x)^2$

Then,

$$y_1 = 2 \tan^{-1} x \frac{d}{dx} (\tan^{-1} x)$$

$$\Rightarrow y_1 = 2 \tan^{-1} x \cdot \frac{1}{1+x^2}$$

$$\Rightarrow (1+x^2)y_1 = 2 \tan^{-1} x$$

Again differentiating with respect to x on both the sides, we obtain

$$(1+x^2)y_2 + 2xy_1 = 2 \left(\frac{1}{1+x^2} \right)$$

$$\Rightarrow (1+x^2)^2 y_2 + 2x(1+x^2)y_1 = 2$$

Hence, proved.

Higher Order Derivatives Ex 12.1 Q29

$$y = \cot x$$

differentiating w.r.t. x

$$\Rightarrow \frac{dy}{dx} = -\operatorname{cosec}^2 x$$

differentiating w.r.t. x

$$\Rightarrow \frac{d^2y}{dx^2} = -[2 \cos \operatorname{cosec} \{-\operatorname{cosec} \cot x\}] = 2 \cos \operatorname{cosec}^2 x \cot x = -2 \frac{dy}{dx} \cdot y$$

$$\Rightarrow \frac{d^2y}{dx^2} + 2y \frac{dy}{dx} = 0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q30

$$y = \log \left(\frac{x^2}{e^2} \right)$$

differentiating w.r.t. x

$$\Rightarrow \frac{dy}{dx} = \frac{1}{x^2/e^2} \times \frac{1}{e^2} \times 2x = \frac{2}{x}$$

differentiating w.r.t. x

$$\Rightarrow \frac{d^2y}{dx^2} = 2 \left(\frac{-1}{x^2} \right) = \frac{-2}{x^2}$$

Higher Order Derivatives Ex 12.1 Q31

$$y = ae^{2x} + be^{-x}$$

differentiating w.r.t. x

$$\Rightarrow \frac{dy}{dx} = 2ae^{2x} + be^{-x}(-1) = 2ae^{2x} - be^{-x}$$

differentiating w.r.t. x

$$\Rightarrow \frac{d^2y}{dx^2} = 2ae^{2x}(2) - be^{-x}(-1) = 4ae^{2x} + be^{-x}$$

Adding and subtracting be^{-x} on RHS

$$\Rightarrow \frac{d^2y}{dx^2} = 4ae^{2x} + 2be^{-x} - be^{-x} = 2(ae^{2x} + be^{-x}) + 2ae^{2x} - be^{-x} = 2y + \frac{dy}{dx}$$

$$\Rightarrow \frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$$

Higher Order Derivatives Ex 12.1 Q32

$$y = e^x (\sin x + \cos x)$$

differentiating w.r.t. x

$$\Rightarrow \frac{dy}{dx} = e^x (\cos x - \sin x) + (\sin x + \cos x)e^x$$

$$\Rightarrow \frac{dy}{dx} = y + e^x (\cos x - \sin x)$$

differentiating w.r.t. x

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{dy}{dx} + e^x (-\sin x - \cos x) + (\cos x - \sin x)e^x$$

$$= \frac{dy}{dx} - y + (\cos x - \sin x)e^x$$

Adding and subtracting y on RHS

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{dy}{dx} - y + (\cos x - \sin x)e^x + y - y = 2 \frac{dy}{dx} - 2y$$

$$\Rightarrow \frac{d^2y}{dx^2} - 2 \frac{dy}{dx} + 2y = 0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q33

It is given that, $y = \cos^{-1} x$

Then,

$$\frac{dy}{dx} = \frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}} = -(1-x^2)^{-\frac{1}{2}}$$

$$\begin{aligned} \frac{d^2y}{dx^2} &= \frac{d}{dx} \left[-(1-x^2)^{-\frac{1}{2}} \right] \\ &= -\left(-\frac{1}{2}\right) \cdot (1-x^2)^{-\frac{3}{2}} \cdot \frac{d}{dx}(1-x^2) \\ &= \frac{1}{2\sqrt{(1-x^2)^3}} \times (-2x) \end{aligned}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-x}{\sqrt{(1-x^2)^3}} \quad \dots (i)$$

$$y = \cos^{-1} x \Rightarrow x = \cos y$$

Putting $x = \cos y$ in equation (i), we obtain

$$\begin{aligned}\frac{d^2 y}{dx^2} &= \frac{-\cos y}{\sqrt{(1 - \cos^2 y)^3}} \\ \Rightarrow \frac{d^2 y}{dx^2} &= \frac{-\cos y}{\sqrt{(\sin^2 y)^3}} \\ &= \frac{-\cos y}{\sin^3 y} \\ &= \frac{-\cos y}{\sin y} \times \frac{1}{\sin^2 y} \\ \Rightarrow \frac{d^2 y}{dx^2} &= -\cot y \cdot \operatorname{cosec}^2 y\end{aligned}$$

Higher Order Derivatives Ex 12.1 Q34

It is given that, $y = e^{a \cos^{-1} x}$

Taking logarithm on both the sides, we obtain

$$\log y = a \cos^{-1} x \log e$$

$$\log y = a \cos^{-1} x$$

Differentiating both sides with respect to x , we obtain

$$\begin{aligned}\frac{1}{y} \frac{dy}{dx} &= a \times \frac{-1}{\sqrt{1-x^2}} \\ \Rightarrow \frac{dy}{dx} &= \frac{-ay}{\sqrt{1-x^2}}\end{aligned}$$

By squaring both the sides, we obtain

$$\begin{aligned}\left(\frac{dy}{dx}\right)^2 &= \frac{a^2 y^2}{1-x^2} \\ \Rightarrow (1-x^2) \left(\frac{dy}{dx}\right)^2 &= a^2 y^2 \\ (1-x^2) \left(\frac{dy}{dx}\right)^2 &= a^2 y^2\end{aligned}$$

Again differentiating both sides with respect to x , we obtain

$$\begin{aligned}\left(\frac{dy}{dx}\right)^2 \frac{d}{dx}(1-x^2) + (1-x^2) \times \frac{d}{dx} \left[\left(\frac{dy}{dx}\right)^2 \right] &= a^2 \frac{d}{dx}(y^2) \\ \Rightarrow \left(\frac{dy}{dx}\right)^2 (-2x) + (1-x^2) \times 2 \frac{dy}{dx} \cdot \frac{d^2 y}{dx^2} &= a^2 \cdot 2y \cdot \frac{dy}{dx} \\ \Rightarrow \left(\frac{dy}{dx}\right)^2 (-2x) + (1-x^2) \times 2 \frac{dy}{dx} \cdot \frac{d^2 y}{dx^2} &= a^2 \cdot 2y \cdot \frac{dy}{dx} \\ \Rightarrow -x \frac{dy}{dx} + (1-x^2) \frac{d^2 y}{dx^2} &= a^2 \cdot y \quad \left[\frac{dy}{dx} \neq 0 \right] \\ \Rightarrow (1-x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - a^2 y &= 0\end{aligned}$$

Hence, proved.

Higher Order Derivatives Ex 12.1 Q35

It is given that, $y = 500e^{7x} + 600e^{-7x}$

Then,

$$\begin{aligned}\frac{dy}{dx} &= 500 \cdot \frac{d}{dx}(e^{7x}) + 600 \cdot \frac{d}{dx}(e^{-7x}) \\ &= 500 \cdot e^{7x} \cdot \frac{d}{dx}(7x) + 600 \cdot e^{-7x} \cdot \frac{d}{dx}(-7x) \\ &= 3500e^{7x} - 4200e^{-7x} \\ \therefore \frac{d^2y}{dx^2} &= 3500 \cdot \frac{d}{dx}(e^{7x}) - 4200 \cdot \frac{d}{dx}(e^{-7x}) \\ &= 3500 \cdot e^{7x} \cdot \frac{d}{dx}(7x) - 4200 \cdot e^{-7x} \cdot \frac{d}{dx}(-7x) \\ &= 7 \times 3500 \cdot e^{7x} + 7 \times 4200 \cdot e^{-7x} \\ &= 49 \times 500e^{7x} + 49 \times 600e^{-7x} \\ &= 49(500e^{7x} + 600e^{-7x}) \\ &= 49y\end{aligned}$$

Hence, proved

Higher Order Derivatives Ex 12.1 Q36

$$y = 2 \cos t - \cos 2t; \quad y = 2 \sin t - \sin 2t$$

differentiating w.r.t. t

$$\Rightarrow \frac{dy}{dt} = 2(-\sin t) - 2(-\sin 2t); \quad \frac{dy}{dt} = 2 \cos t - 2 \cos 2t$$

dividing (2) by (1)

$$\Rightarrow \frac{dy}{dx} = \frac{2(\cos t - \cos 2t)}{2(\sin 2t - \sin t)}$$

differentiating w.r.t. t

$$\Rightarrow \frac{d\left(\frac{dy}{dx}\right)}{dt} = \frac{(\sin 2t - \sin t)(-\sin t + 2 \sin 2t) - (\cos t - \cos 2t)(2 \cos 2t - \cos t)}{(\sin 2t - \sin t)^2} \dots\dots\dots (3)$$

dividing (3) by (1)

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{(\sin 2t - \sin t)(2 \sin 2t - \sin t) - (\cos t - \cos 2t)(2 \cos 2t - \cos t)}{2(\sin 2t - \sin t)^3}$$

Putting $t = \pi/2$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{(0-1)(0-1) - (0-(-1))(2(-1)-0)}{2(0-1)^3} = \frac{1+2}{-2} = \frac{-3}{2}$$

Higher Order Derivatives Ex 12.1 Q37

$$x = 4z^2 + 5 \qquad y = 6z^2 + 7z + 3$$

differentiating both w.r.t. z

$$\Rightarrow \frac{dx}{dz} = 8z + 0 \qquad \frac{dy}{dz} = 12z + 7$$

$$\Rightarrow \frac{dx}{dz} = \frac{12z + 7}{8z} = \frac{12z}{8z} + \frac{7}{8z}$$

differentiating w.r.t. z

$$\Rightarrow \frac{d\left(\frac{dy}{dx}\right)}{dz} = 0 + \frac{7}{8} \left(\frac{-1}{z^2}\right) \dots\dots (3)$$

dividing (3) by (1)

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-7}{8z^2 \times 8z} = \frac{-7}{64z^3}$$

Higher Order Derivatives Ex 12.1 Q38

$$y = \log(1 + \cos x)$$

differentiating w.r.t. x

$$\Rightarrow \frac{dy}{dx} = \frac{1}{1 + \cos x} \times -\sin x = \frac{-\sin x}{1 + \cos x}$$

differentiating w.r.t. x

$$\Rightarrow \frac{d^2y}{dx^2} = - \left[\frac{(1 + \cos x) \cos x - \sin x (-\sin x)}{(1 + \cos x)^2} \right]$$

$$\Rightarrow \frac{d^2y}{dx^2} = - \left[\frac{\cos x + \cos^2 x + \sin^2 x}{(1 + \cos x)^2} \right] = - \left[\frac{1 + \cos x}{(1 + \cos x)^2} \right] = \frac{-1}{1 + \cos x}$$

differentiating w.r.t. x

$$\Rightarrow \frac{d^3y}{dx^3} = - \left(\frac{+1}{(1 + \cos x)^2} \times +\sin x \right) = - \left(\frac{-\sin x}{1 + \cos x} \right) \times \left(\frac{-1}{1 + \cos x} \right) = - \frac{dy}{dx} \cdot \frac{d^2y}{dx^2}$$

$$\Rightarrow \frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} \cdot \frac{dy}{dx} = 0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q39

$$y = \sin(\log x)$$

$$\Rightarrow \frac{dy}{dx} = \cos(\log x) \times \frac{1}{x}$$

$$\Rightarrow x \frac{dy}{dx} = \cos(\log x)$$

$$\Rightarrow x \frac{d^2y}{dx^2} + \frac{dy}{dx} = -\sin(\log x) \times \frac{1}{x}$$

$$\Rightarrow x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} = -y$$

$$\Rightarrow x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q40

Given $y = 3e^{2x} + 2e^{3x}$

Then, $\frac{dy}{dx} = 6e^{2x} + 6e^{3x} = 6(e^{2x} + e^{3x})$

$$\therefore \frac{d^2y}{dx^2} = 12e^{2x} + 18e^{3x} = 6(2e^{2x} + 3e^{3x})$$

Hence,

$$\frac{d^2y}{dx^2} - 5 \frac{dy}{dx} + 6y = 6(2e^{2x} + 3e^{3x}) - 30(e^{2x} + e^{3x}) + 6(3e^{2x} + 2e^{3x})$$

$$= 0$$

Higher Order Derivatives Ex 12.1 Q41

$$y = (\cot^{-1} x)^2$$

differentiating w.r.t. x

$$\Rightarrow \frac{dy}{dx} = y_1 = 2 \cot^{-1} x \times \frac{-1}{1+x^2}$$

$$= \frac{-2 \cot^{-1} x}{1+x^2} \text{ (chain rule)}$$

$$\Rightarrow (1+x^2) \frac{dy}{dx} = -2 \cot^{-1} x$$

differentiating w.r.t. x

$$\Rightarrow (1+x^2) y_2 + 2x y_1 = +2 \left(\frac{-1}{1+x^2} \right)$$

(multiplication rule on LHS)

$$\Rightarrow (1+x^2)^2 y_2 + 2x(1+x^2) y_1 = 2$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q42

We know that, $\frac{d}{dx}(\operatorname{cosec}^{-1}x) = \frac{-1}{|x|\sqrt{x^2-1}}$

Let $y = \operatorname{cosec}^{-1}x$

$$\frac{dy}{dx} = \frac{-1}{|x|\sqrt{x^2-1}}$$

Since $x > 1$, $|x| = x$

Thus,

$$\frac{dy}{dx} = \frac{-1}{x\sqrt{x^2-1}} \dots(1)$$

Differentiating the above function with respect to x , we have,

$$\frac{d^2y}{dx^2} = \frac{x \frac{2x}{2\sqrt{x^2-1}} + \sqrt{x^2-1}}{x^2(x^2-1)}$$

$$= \frac{\frac{x^2}{\sqrt{x^2-1}} + \sqrt{x^2-1}}{x^2(x^2-1)}$$

$$= \frac{x^2 + x^2 - 1}{x^2(x^2-1)^{\frac{3}{2}}}$$

$$= \frac{2x^2-1}{x^2(x^2-1)^{\frac{3}{2}}}$$

$$\text{Thus, } x(x^2-1) \frac{d^2y}{dx^2} = \frac{2x^2-1}{x\sqrt{x^2-1}} \dots(2)$$

Similarly, from (1), we have

$$(2x^2-1) \frac{dy}{dx} = \frac{-2x^2+1}{x\sqrt{x^2-1}} \dots(3)$$

Thus, from (2) and (3), we have,

$$x(x^2-1) \frac{d^2y}{dx^2} + (2x^2-1) \frac{dy}{dx} = \frac{2x^2-1}{x\sqrt{x^2-1}} + \left(\frac{-2x^2+1}{x\sqrt{x^2-1}} \right) = 0$$

Hence proved.

Higher Order Derivatives Ex 12.1 Q43

Given that, $x = \cos t + \log \tan \frac{t}{2}$, $y = \sin t$

Differentiating with respect to t , we have,

$$\frac{dx}{dt} = -\sin t + \frac{1}{\tan \frac{t}{2}} \times \sec^2 \frac{t}{2} \times \frac{1}{2}$$

$$= -\sin t + \frac{1}{\frac{\sin \frac{t}{2}}{\cos \frac{t}{2}}} \times \frac{1}{\cos^2 \frac{t}{2}} \times \frac{1}{2}$$

$$= -\sin t + \frac{1}{\frac{\sin \frac{t}{2}}{\cos \frac{t}{2}}} \times \frac{1}{\cos^2 \frac{t}{2}} \times \frac{1}{2}$$

$$= -\sin t + \frac{1}{2 \sin \frac{t}{2} \cos \frac{t}{2}}$$

$$= -\sin t + \frac{1}{\sin t}$$

$$= \frac{1 - \sin^2 t}{\sin t}$$

$$= \frac{\cos^2 t}{\sin t}$$

$$= \cos t \times \cot t$$

Now find the value of $\frac{dy}{dx}$:

$$\frac{dy}{dt} = \cos t$$

$$\text{Thus, } \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \cos t \times \frac{1}{\cos t \times \cot t}$$

$$\Rightarrow \frac{dy}{dx} = \tan t$$

$$\text{Since } \frac{dy}{dt} = \cos t, \text{ we have } \frac{d^2 y}{dt^2} = -\sin t$$

$$\text{At } t = \frac{\pi}{4}, \left(\frac{d^2 y}{dt^2} \right)_{t=\frac{\pi}{4}} = -\sin \left(\frac{\pi}{4} \right) = \frac{-1}{\sqrt{2}}$$

$$\frac{d^2 y}{dx^2} = \frac{\frac{d}{dt} \left(\frac{dy}{dx} \right)}{\frac{dx}{dt}}$$

$$= \frac{\frac{d}{dt} (\tan t)}{\cos t \times \cot t}$$

$$= \frac{\sec^2 t}{\cos t \times \cot t}$$

$$= \frac{\sec^2 t}{\cos t \times \frac{\cos t}{\sin t}}$$

$$= \frac{\sec^2 t}{\cos^2 t} \times \sin t$$

$$= \sec^4 t \times \sin t$$

$$\text{Thus, } \left(\frac{d^2 y}{dx^2} \right)_{t=\frac{\pi}{4}} = \sec^4 \left(\frac{\pi}{4} \right) \times \sin \frac{\pi}{4} = 2$$

$$x = a \sin t \text{ and } y = a \left(\cos t + \log \tan \frac{t}{2} \right)$$

$$\frac{dx}{dt} = a \cos t$$

$$\frac{d^2x}{dt^2} = -a \sin t$$

$$\frac{dy}{dt} = -a \sin t + a \frac{1}{\tan \frac{t}{2}} \times \sec^2 \frac{t}{2} \times \frac{1}{2}$$

$$= -a \sin t + a \frac{1}{2 \sin \frac{t}{2} \cos \frac{t}{2}}$$

$$= -a \sin t + a \operatorname{cosec} t$$

$$\frac{d^2y}{dt^2} = -a \cos t - a \operatorname{cosec} t \cot t$$

$$\frac{d^2y}{dx^2} = \frac{\frac{dx}{dt} \frac{d^2y}{dt^2} - \frac{dy}{dt} \frac{d^2x}{dt^2}}{\left(\frac{dx}{dt} \right)^3}$$

$$= \frac{a \cos t (-a \cos t - a \operatorname{cosec} t \cot t) - (-a \sin t + a \operatorname{cosec} t)(-a \sin t)}{(a \cos t)^3}$$

$$= \frac{-a^2 \cos^2 t - a^2 \cot^2 t - a^2 \sin^2 t + a^2}{a^3 \cos^3 t}$$

$$= \frac{-a^2 \cos^2 t - a^2 \sin^2 t - a^2 \cot^2 t + a^2}{a^3 \cos^3 t}$$

$$= \frac{-a^2 (\cos^2 t + \sin^2 t) - a^2 \cot^2 t + a^2}{a^3 \cos^3 t}$$

$$= -\frac{1}{a \sin^2 t \cos t}$$

Higher Order Derivatives Ex 12.1 Q45

$$x = a (\cos t + t \sin t)$$

$$\frac{dx}{dt} = -a \sin t + a \cos t + a \sin t$$

$$= a t \cos t$$

$$\frac{d^2x}{dt^2} = -a t \sin t + a \cos t$$

$$y = a (\sin t - t \cos t)$$

$$\frac{dy}{dt} = a \cos t - a \cos t + a t \sin t$$

$$= a t \sin t$$

$$\frac{d^2y}{dt^2} = a t \cos t + a \sin t$$

$$\frac{d^2y}{dx^2} = \frac{\frac{dx}{dt} \frac{d^2y}{dt^2} - \frac{dy}{dt} \frac{d^2x}{dt^2}}{\left(\frac{dx}{dt} \right)^3}$$

$$= \frac{a t \cos t (a t \cos t + a \sin t) - a t \sin t (-a t \sin t + a \cos t)}{(a t \cos t)^3}$$

$$= \frac{a^2 t^2 \cos^2 t + a^2 t \cos t \sin t + a^2 t^2 \sin^2 t - a^2 t \sin t \cos t}{(a t \cos t)^3}$$

$$= \frac{a^2 t^2}{a^3 t^3 \cos^3 t} = \frac{1}{a t \cos^3 t}$$

$$\left. \frac{d^2y}{dx^2} \right|_{t=\frac{\pi}{4}} = \frac{1}{a \times \frac{\pi}{4} \cos^3 \frac{\pi}{4}} = \frac{8\sqrt{2}}{\pi a}$$

Higher Order Derivatives Ex 12.1 Q46

$$x = a \left(\cos t + \log \tan \frac{t}{2} \right) \text{ and } y = a \sin t$$

$$\frac{dx}{dt} = -a \sin t + a \frac{1}{\tan \frac{t}{2}} \times \sec^2 \frac{t}{2} \times \frac{1}{2}$$

$$= -a \sin t + a \frac{1}{2 \sin \frac{t}{2} \cos \frac{t}{2}}$$

$$= -a \sin t + a \operatorname{cosec} t$$

$$\frac{d^2x}{dt^2} = -a \cos t - a \operatorname{cosec} t \cot t$$

$$\frac{dy}{dt} = a \cos t$$

$$\frac{d^2y}{dt^2} = -a \sin t$$

$$\frac{d^2y}{dx^2} = \frac{\frac{dx}{dt} \frac{d^2y}{dt^2} - \frac{dy}{dt} \frac{d^2x}{dt^2}}{\left(\frac{dx}{dt} \right)^3}$$

$$= \frac{(-a \sin t + a \operatorname{cosec} t)(-a \sin t) - (a \cos t)(-a \cos t - a \operatorname{cosec} t \cot t)}{(-a \sin t + a \operatorname{cosec} t)^3}$$

$$= \frac{a^2 \sin^2 t + a^2 \cos^2 t - a^2 + a^2 \cot^2 t}{\left(-a \sin t + \frac{a}{\sin t} \right)^3}$$

$$= \frac{a^2 \cot^2 t}{a^3 \cos^6 t} \times \sin^3 t = \frac{1}{a} \times \frac{\sin t}{\cos^4 t}$$

$$\left. \frac{d^2y}{dx^2} \right|_{t=\frac{\pi}{3}} = \frac{1}{a} \times \frac{\sin \frac{\pi}{3}}{\cos^4 \frac{\pi}{3}} = \frac{8\sqrt{3}}{a}$$

Higher Order Derivatives Ex 12.1 Q47

$$x = a (\cos 2t + 2t \sin 2t)$$

$$\frac{dx}{dt} = -2a \sin 2t + 2a \sin 2t + 4at \cos 2t = 4at \cos 2t$$

$$y = a (\sin 2t - 2t \cos 2t)$$

$$\frac{dy}{dt} = 2a \cos 2t - 2a \cos 2t + 4at \sin 2t = 4at \sin 2t$$

$$\frac{dy}{dx} = \tan 2t$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx} (\tan 2t)$$

$$\frac{d^2y}{dx^2} = \sec^2 2t \frac{d}{dx} (2t)$$

$$\frac{d^2y}{dx^2} = 2 \sec^2 2t \frac{d}{dx} (t)$$

$$\frac{d^2y}{dx^2} = 2 \sec^2 2t \times \frac{1}{4at \cos 2t}$$

$$\frac{d^2y}{dx^2} = \frac{1}{2a} \sec^3 2t$$

Higher Order Derivatives Ex 12.1 Q48

$$x = a \sin t - b \cos t; \quad y = a \cos t + b \sin t$$

Differentiating both w.r.t. t

$$\Rightarrow \quad \frac{dx}{dt} = a \cos t + b \sin t; \quad \frac{dy}{dt} = -a \sin t + b \cos t$$

$$\Rightarrow \quad \frac{dx}{dt} = y \dots\dots\dots(1) \quad ; \quad \frac{dy}{dt} = -x \dots\dots\dots(2)$$

Dividing (2) by (1)

$$\Rightarrow \quad \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = -\frac{x}{y}$$

Differentiating w.r.t. t

$$\Rightarrow \quad \frac{d\left(\frac{dy}{dx}\right)}{dt} = -\left\{\frac{y \frac{dx}{dt} - x \frac{dy}{dt}}{y^2}\right\}$$

Putting values from (1) and (2)

$$\Rightarrow \quad \frac{d\left(\frac{dy}{dx}\right)}{dt} = -\left\{\frac{y^2 + x^2}{y^2}\right\} \dots\dots\dots(3)$$

Dividing (3) by (1)

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -\left\{\frac{y^2 + x^2}{y^2 \times y}\right\} = -\left\{\frac{x^2 + y^2}{y^3}\right\}$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q49

$$y = A \sin 3x + B \cos 3x$$

differentiating w.r.t. x

$$\Rightarrow \frac{dy}{dx} = 3A \cos 3x + 3B (-\sin 3x)$$

again differentiating w.r.t. x

$$\Rightarrow \frac{d^2y}{dx^2} = 3A (-\sin 3x) \times 3 - 3B (\cos 3x) \times 3$$

$$\Rightarrow \frac{d^2y}{dx^2} = -9(A \sin 3x + B \cos 3x) = -9y$$

Now adding $\frac{d^2y}{dx^2} + \frac{4dy}{dx} + 3y$

$$\begin{aligned} \Rightarrow \frac{d^2y}{dx^2} + \frac{4dy}{dx} + 3y &= -9y + 4(3A \cos 3x - 3B \sin 3x) + 3y \\ &= 12(A \cos 3x - B \sin 3x) - 6(A \sin 3x + B \cos 3x) \end{aligned}$$

$$\Rightarrow \frac{d^2y}{dx^2} + \frac{4dy}{dx} + 3y = (12A - 6B) \cos 3x - (12B + 6A) \sin 3x$$

But given,

$$\frac{d^2y}{dx^2} + \frac{4dy}{dx} + 3y = 10 \cos 3x$$

Thus, $12A - 6B = 10$ (1)

and $-(12B + 6A) = 0$ (2)

solving (2)

$$12B + 6A = 0 \Rightarrow 6A = -12B \Rightarrow A = -2B$$

Putting value of A in (1)

$$\Rightarrow 12(-2B) - 6B = 10$$

$$\Rightarrow -24B - 6B = 10$$

$$\Rightarrow -30B = 10$$

$$\Rightarrow B = \frac{-1}{3}$$

$$\Rightarrow \therefore A = -2 \times \frac{-1}{3} = \frac{2}{3}$$

and $A = \frac{2}{3}; \quad B = \frac{-1}{3}$

Higher Order Derivatives Ex 12.1 Q50

$$y = Ae^{-kt} \cos(pt + c)$$

differentiating w.r.t. t

$$\Rightarrow \frac{dy}{dt} = A \left\{ e^{-kt} (-\sin(pt + c) \times p) + (\cos(pt + c))(-ke^{-kt}) \right\}$$

$$\Rightarrow -Ape^{-kt} \sin(pt + c) - kAe^{-kt} \cos(pt + c)$$

$$\Rightarrow \frac{dy}{dt} = -Ape^{-kt} \sin(pt + c) - ky$$

differentiating w.r.t. t

$$\begin{aligned} \Rightarrow \frac{d^2y}{dt^2} &= -Ap \left\{ e^{-kt} (\cos(pt + c) \times p) + (\sin(pt + c))(-ke^{-kt}) \right\} - ky^1 \\ &= -p^2y + Apke^{-kt} \sin(pt + c) - ky^1 \end{aligned}$$

Adding & subtracting ky^1 on RHS

$$\Rightarrow \frac{d^2y}{dt^2} = +Apke^{-kt} \sin(pt + c) - p^2y - 2ky^1 + ky^1$$

$$\frac{d^2y}{dt^2} = Apke^{-kt} \sin(pt + c) - p^2y - 2ky^1 - kApe^{-kt} \sin(pt + c) - k^2y$$

$$\Rightarrow \frac{d^2y}{dt^2} = -(p^2 + k^2)y - 2k \frac{dy}{dx}$$

$$\Rightarrow \frac{d^2y}{dt^2} + 2k \frac{dy}{dt} + n^2y = 0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q51

$$y = x^n \{a \cos(\log x) + b \sin(\log x)\}$$

$$y = ax^n \cos(\log x) + bx^n \sin(\log x)$$

$$\frac{dy}{dx} = anx^{n-1} \cos(\log x) - ax^{n-1} \sin(\log x) + bnx^{n-1} \sin(\log x) + bx^{n-1} \cos(\log x)$$

$$\frac{dy}{dx} = x^{n-1} \cos(\log x)(na + b) + x^{n-1} \sin(\log x)(bn - a)$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx} (x^{n-1} \cos(\log x)(na + b) + x^{n-1} \sin(\log x)(bn - a))$$

$$\frac{d^2y}{dx^2} = (na + b) [(n-1)x^{n-2} \cos(\log x) - x^{n-2} \sin(\log x)] + (bn - a) [(n-1)x^{n-2} \sin(\log x) + x^{n-2} \cos(\log x)]$$

$$\frac{d^2y}{dx^2} = (na + b)x^{n-2} [(n-1) \cos(\log x) - \sin(\log x)] + (bn - a)x^{n-2} [(n-1) \sin(\log x) + \cos(\log x)]$$

$$x^2 \frac{d^2y}{dx^2} + (1-2n) \frac{dy}{dx} + (1+n^2)y$$

$$= (na + b)x^n [(n-1) \cos(\log x) - \sin(\log x)] + (bn - a)x^n [(n-1) \sin(\log x) + \cos(\log x)]$$

$$+ (1-2n)x^{n-1} \cos(\log x)(na + b) + (1-2n)x^{n-1} \sin(\log x)(bn - a)$$

$$+ a(1+n^2)x^n \cos(\log x) + b(1+n^2)x^n \sin(\log x)$$

$$= 0$$

Higher Order Derivatives Ex 12.1 Q52

$$y = a\left(x + \sqrt{x^2 + 1}\right)^n + b\left(x - \sqrt{x^2 + 1}\right)^{-n},$$

$$\frac{dy}{dx} = na\left(x + \sqrt{x^2 + 1}\right)^{n-1}\left[1 + x(x^2 + 1)^{-\frac{1}{2}}\right] - nb\left(x - \sqrt{x^2 + 1}\right)^{-n-1}\left[1 - x(x^2 + 1)^{-\frac{1}{2}}\right]$$

$$\frac{dy}{dx} = \frac{na}{\sqrt{x^2 + 1}}\left(x + \sqrt{x^2 + 1}\right)^n + \frac{nb}{\sqrt{x^2 + 1}}\left(x - \sqrt{x^2 + 1}\right)^{-n}$$

$$\frac{dy}{dx} = \frac{n}{\sqrt{x^2 + 1}}\left[a\left(x + \sqrt{x^2 + 1}\right)^n + b\left(x - \sqrt{x^2 + 1}\right)^{-n}\right]$$

$$x \frac{dy}{dx} = \frac{nx}{\sqrt{x^2 + 1}} y$$

$$\frac{d^2y}{dx^2} = \frac{nx}{\sqrt{x^2 + 1}} \frac{dy}{dx} + y \left[\frac{\sqrt{x^2 + 1} - x^2(x^2 + 1)^{-\frac{1}{2}}}{x^2 + 1} \right]$$

$$\frac{d^2y}{dx^2} = \frac{n^2x^2}{x^2 + 1} + y \left[\frac{1}{(x^2 + 1)\sqrt{x^2 + 1}} \right]$$

$$\frac{d^2y}{dx^2} = \frac{n^2x^2(\sqrt{x^2 + 1}) + y}{(x^2 + 1)\sqrt{x^2 + 1}}$$

$$(x^2 - 1) \frac{d^2y}{dx^2} = \frac{n^2x^4(\sqrt{x^2 + 1}) + x^2y}{(x^2 + 1)\sqrt{x^2 + 1}} - \frac{n^2x^2(\sqrt{x^2 + 1}) + y}{(x^2 + 1)\sqrt{x^2 + 1}}$$

Now

$$\begin{aligned} & (x^2 - 1) \frac{d^2y}{dx^2} + x \frac{dy}{dx} - ny \\ &= \frac{n^2x^4(\sqrt{x^2 + 1}) + x^2y}{(x^2 + 1)\sqrt{x^2 + 1}} - \frac{n^2x^2(\sqrt{x^2 + 1}) + y}{(x^2 + 1)\sqrt{x^2 + 1}} + \frac{nx}{\sqrt{x^2 + 1}} y - ny \\ &= 0 \end{aligned}$$