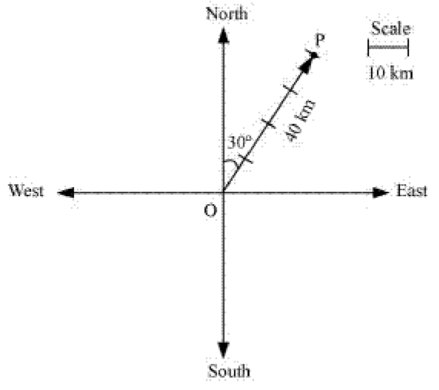
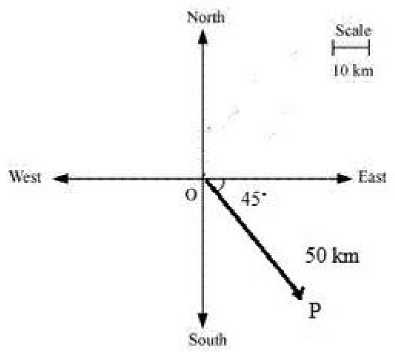
Ex 23.1

Algebra of Vectors Ex 23.1 Q1(i)



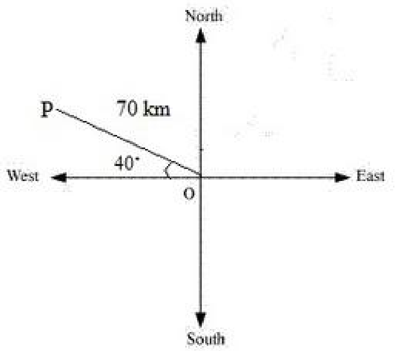
Here, vector $\overrightarrow{OP}$ represents the displacement of 40 km, 30° East of North.

Algebra of Vectors Ex 23.1 Q1(ii)



Here, vector $\overrightarrow{OP}$ represents the displacement of 50 km, south-east

Algebra of Vectors Ex 23.1 Q1(iii)



Here, vector $\overrightarrow{OP}$ represents the displacement of 70 km, 40° north of west

Algebra of Vectors Ex 23.1 Q2

(i) 15 kg is a scalar quantity because it involves only mass

(ii) 20 kg weight is a vector quantity as it involves both magnitude and direction.

(iii) 45° is a scalar quantity as it involves only magnitude.

(iv) 10 meters south-east is a vector quantity as it involve direction.

(v) 50 m/s² is a scalar quantity as it involves magnitude of acceleration.

Algebra of Vectors Ex 23.1 Q3

(i) Time period is a scalar quantity as it involves only magnitude.

(ii) Distance is a scalar quantity as it involves only magnitude.

(iii) Displacement is vector quantity as it involves both magnitude and direction.

(iv) Force is a vector quantity as it involves both magnitude and direction.

(v) Work done is a scalar quantity as it involves only magnitude.

(vi) Velocity is a vector quantity as it involves both magnitude as well as direction.

(vii) Acceleration is a vector quantity because it involves both magnitude as well as direction.

Algebra of Vectors Ex 23.1 Q4

(i)
Collinear vectors are
$\vec{x}, \vec{z}$ and $\vec{b}$
$\vec{y}, \vec{c}$
$\vec{a}, \vec{d}$

(ii)
Equal vectors are
$\vec{y}$ and $\vec{c}$
$\vec{x}$ and $\vec{b}$
$\vec{a}$ and $\vec{d}$

(iii)
Coinitial vector are
$\vec{a}$, $\vec{y}$ and $\vec{z}$

(iv)
Collinear but not equal
$\vec{b}$ and $\vec{z}$
$\vec{x}$ and $\vec{z}$

Algebra of Vectors Ex 23.1 Q5

(i) a and b are collinear, it is true.

(ii) Two collinear vectors are may not be equal in magnitude, so it is false.

(iii) Zero vector may not be unique, so it is false.

(iv) Two vectors having same magnitude are may not be collinear so it is false.

(v) Two collinear vectors having the same magnitude are may not be equal, so it is false.

Ex 23.2

Algebra of Vectors Ex 23.2 Q1

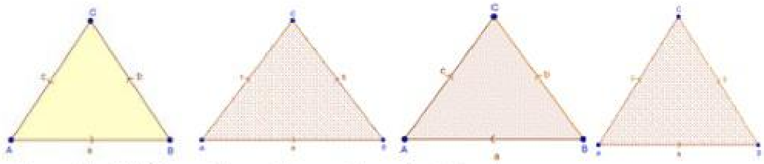
Given that, P, Q, R are collinear.

It also given that, $\overrightarrow{PQ} = \vec{a}$ and $\overrightarrow{QR} = \vec{b}$

$$\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR}$$
$$= \vec{a} + \vec{b}$$

$$\overrightarrow{PR} = \vec{a} + \vec{b}$$

Algebra of Vectors Ex 23.2 Q2



Given that, $\vec{a}, \vec{b}$, and $\vec{c}$ are three sides of a triangle.

$\vec{a} + \vec{b} + \vec{c}$

$= \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA}$

$= \overrightarrow{AC} + \overrightarrow{CA}$ [Since $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$]

$= \overrightarrow{AC} - \overrightarrow{AC}$ [Since $\overrightarrow{CA} = \overrightarrow{AC}$]

$= \vec{0}$

So, $\vec{a} + \vec{b} + \vec{c} = \vec{0}$

Triangle law says that, if vectors are represented in magnitude and direction by the two sides of triangle taken is same order, then their sum is represented by the third side taken in reverse order.

Thus,
$\vec{a} + \vec{b} = \vec{c}$
or
$\vec{a} + \vec{c} = \vec{b}$
$\vec{b} + \vec{c} = \vec{a}$

Algebra of Vectors Ex 23.2 Q3

Here, it is given that $\vec{a}$ and $\vec{b}$ are two non-collinear vectors having the same initial point.

Let $\vec{a} = \overrightarrow{AB}$ and $\vec{b} = \overrightarrow{AD}$, So we can draw a parallelogram ABCD as above.

By the properties of parallelogram
$\overrightarrow{BC} = \vec{b}$ and $\overrightarrow{DC} = \vec{a}$

In ΔABC,
Using triangle law,
$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$
$$\vec{a} + \vec{b} = \overrightarrow{AC} \quad --(i)$$

In ΔABD,
Using triangle law,
$$\overrightarrow{AD} + \overrightarrow{DB} = \overrightarrow{AB}$$
$$\vec{b} + \overrightarrow{DB} = \vec{a}$$
$$\overrightarrow{DB} = \vec{a} - \vec{b} \quad --(ii)$$

From equation (i) and (ii), we get that

$\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are diagonals of a parallelogram whose adjacent sides are $\vec{a}$ and $\vec{b}$

Algebra of Vectors Ex 23.2 Q4

Given that m is a scalar and $\vec{a}$ is a vector such that

$m\vec{a} = \vec{0}$

$m\left(a_1\hat{i} + b_1\hat{j} + c_1\hat{k}\right) = 0 \times \hat{i} + 0 \times \hat{j} + 0 \times \hat{k}$ $\left[\text{since let } \vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}\right]$

$ma_1\hat{i} + mb_1\hat{j} + mc_1\hat{k} = 0 \times \hat{i} + 0 \times \hat{j} + 0 \times \hat{k}$

Comparing the coefficients of $\hat{i}, \hat{j}, \hat{k}$ of LHS and RHS,

$ma_1 = 0 \Rightarrow m = 0$ or $a_1 = 0$ (i)

$mb_1 = 0 \Rightarrow m = 0$ or $b_1 = 0$ (ii)

$mc_1 = 0 \Rightarrow m = 0$ or $c_1 = 0$ (iii)

From (i), (ii) and (iii)

$m = 0$ or $a_1 = b_1 = c_1 = 0$

$\Rightarrow m = 0$ or $\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{j} = 0$

$\Rightarrow m = 0$ or $\vec{a} = 0$

Algebra of Vectors Ex 23.2 Q5

(i)

Let $\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$

$\vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$

Given that, $a = -b$

$a_1\hat{i} + b_1\hat{j} + c_1\hat{k} = -a_2\hat{i} - b_2\hat{j} - c_2\hat{k}$

Comparing the coefficients of i, j, k in LHS and RHS,

$a_1 = -a_2$ (1)

$b_1 = -b_2$ (2)

$c_1 = -c_2$ (3)

$\left|\vec{a}\right| = \sqrt{a_1^2 + b_1^2 + c_1^2}$

Using (1), (2)and (3),

$\left|\vec{a}\right| = \sqrt{(-a_2)^2 + (-b_2)^2 + (-c_2)^2}$

$\left|\vec{a}\right| = \sqrt{a_2^2 + b_2^2 + c_2^2}$

$\therefore \left|\vec{a}\right| = \left|\vec{b}\right|$

(ii)

Given a and b are two vectors such that $\left|\vec{a}\right| = \left|\vec{b}\right|$

It means magnitude of vector $\vec{a}$ is equal to the magnitude of vector $\vec{b}$, but we cannot conclude anything about the direction of the vector.

So,it is false that

$\left|\vec{a}\right| = \left|\vec{b}\right| \Rightarrow \vec{a} = \pm\vec{b}$

(iii)

Given for any vector $\vec{a}$ and $\vec{b}$

$\left|\vec{a}\right| = \left|\vec{b}\right|$

It means magnitude of the vector $\vec{a}$ and $\vec{b}$ are equal but we cannot say any thing about the direction of the vector $\vec{a}$ and $\vec{b}$. And we know that $\vec{a} = \vec{b}$ means magnitude and same direction. So, it is false.

Algebra of Vectors Ex 23.2 Q6

Here it is given that $ABCD$ is a quadrilateral.

In ΔADC, using triangle law, we get

$\overrightarrow{CD} + \overrightarrow{DA} = \overrightarrow{CA}$ --(i)

In ΔABC, using triangle law, we get

$\overrightarrow{BC} + \overrightarrow{CA} = \overrightarrow{BA}$ --(ii)

Put value of $\overrightarrow{CA}$ in equation (ii),

$\overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DA} = \overrightarrow{BA}$

Adding $\overrightarrow{BA}$ on both the sides,

$\overrightarrow{BA} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DA} = \overrightarrow{BA} + \overrightarrow{BA}$

$\therefore\ \overrightarrow{BA} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DA} = 2\overrightarrow{BA}$

Algebra of Vectors Ex 23.2 Q7

(i)

Given that $ABCDE$ is a pentagon.

$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EA}$

$= \left(\overrightarrow{AB} + \overrightarrow{BC}\right) + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EA}$

$= \overrightarrow{AC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EA}$ $\left[\text{Using triangle law in } \Delta ABC,\ \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}\right]$

$= \left(\overrightarrow{AC} + \overrightarrow{CD}\right) + \overrightarrow{DE} + \overrightarrow{EA}$

$= \left(\overrightarrow{AD}\right) + \overrightarrow{DE} + \overrightarrow{EA}$ $\left[\text{Using triangle law in } \Delta ACD,\ \overrightarrow{AC} + \overrightarrow{CD} = \overrightarrow{AD}\right]$

$= \overrightarrow{AD} + \overrightarrow{DA}$

$= \overrightarrow{AD} - (-\overrightarrow{AD})$

$= \vec{0}$

$\therefore\ \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EA} = \vec{0}$

(ii)

It is given that ABCDE is a pentagon, So

$\overrightarrow{AB} + \overrightarrow{AE} + \overrightarrow{BC} + \overrightarrow{DC} + \overrightarrow{ED} + \overrightarrow{AC}$

$= \left(\overrightarrow{AB} + \overrightarrow{BC}\right) + \overrightarrow{AE} + \overrightarrow{DC} + \overrightarrow{ED} + \overrightarrow{AC}$

$= \overrightarrow{AC} + \overrightarrow{DC} + \left(\overrightarrow{AE} + \overrightarrow{ED}\right) + \overrightarrow{AC}$ $\left[\text{Using triangle law in } \Delta ABC,\ \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}\right]$

$= \overrightarrow{AC} + \overrightarrow{DC} + \left(\overrightarrow{AD}\right) + \overrightarrow{AC}$

$= \overrightarrow{AC} + \overrightarrow{DC} - \overrightarrow{DA} + \overrightarrow{AC}$

$= \overrightarrow{AC} + \overrightarrow{DC} + \overrightarrow{AD} + \overrightarrow{AC}$

$= \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{DC} + \overrightarrow{AC}$

$= \overrightarrow{AC} + \overrightarrow{AC} + \overrightarrow{AC}$

$= 3\overrightarrow{AC}$

So,

$\overrightarrow{AB} + \overrightarrow{AE} + \overrightarrow{BC} + \overrightarrow{DC} + \overrightarrow{ED} + \overrightarrow{AC} = 3\overrightarrow{AC}$

Algebra of Vectors Ex 23.2 Q8

Let O be the centre of a regular octagon, we know that the centre of a regular octagon bisects all the diagonals passing through it.

Thus,

$\overrightarrow{OA} = -\overrightarrow{OE}$ (i)

$\overrightarrow{OB} = -\overrightarrow{OF}$ (ii)

$\overrightarrow{OC} = -\overrightarrow{OG}$ (iii)

$\overrightarrow{OD} = -\overrightarrow{OH}$ (iv)

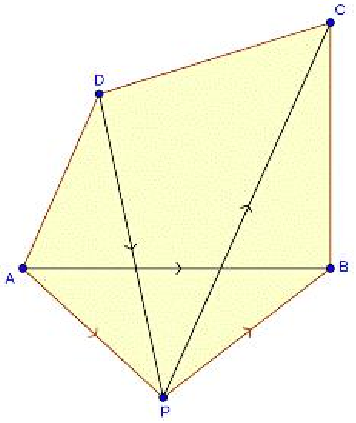
Adding equation (i), (ii), and (iv),

$\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD} = -\overrightarrow{OE} - \overrightarrow{OF} - \overrightarrow{OG} - \overrightarrow{OH}$

$\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD} + = -\left(\overrightarrow{OE} + \overrightarrow{OF} + \overrightarrow{OG} + \overrightarrow{OH}\right)$

$\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD} + \overrightarrow{OE} + \overrightarrow{OF} + \overrightarrow{OG} + \overrightarrow{OH} = \vec{0}$

Algebra of Vectors Ex 23.2 Q9



Given,

$$\overrightarrow{AP}+\overrightarrow{PB}+\overrightarrow{PD}=\overrightarrow{PC}$$
$$\overrightarrow{AP}+\overrightarrow{PB}=\overrightarrow{PC}-\overrightarrow{PD}$$
$$\overrightarrow{AP}+\overrightarrow{PB}=\overrightarrow{PC}+\overrightarrow{DP}\quad\left[\text{Since }\overrightarrow{DP}=-\overrightarrow{PD}\right]$$
$$\overrightarrow{AP}+\overrightarrow{PB}=\overrightarrow{DP}+\overrightarrow{PC}$$
$$\overrightarrow{AB}=\overrightarrow{DC}\quad\left[\begin{array}{l}\text{Using triangle law in }\Delta APB,\ \overrightarrow{AP}+\overrightarrow{PB}=\overrightarrow{AB}\\ \text{Using triangle law in }\Delta DPC,\overrightarrow{DP}+\overrightarrow{PC}=\overrightarrow{DC}\end{array}\right]$$

Therefore, AB is parallel to DC and equal is magnitude.
Hence, $ABCD$ is a parallelogram.

Algebra of Vectors Ex 23.2 Q10

We need to show that

$$\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}=6\overrightarrow{AO}$$

We know that centre O of the hexagon bisects the diagonal $\overrightarrow{AD}$

$$\therefore\quad \overrightarrow{AO}=\frac{1}{2}\overrightarrow{AD};\ \overrightarrow{BO}=-\overrightarrow{EO};\ \overrightarrow{CO}=-\overrightarrow{FO}$$

Now

$$\overrightarrow{AB}+\overrightarrow{BO}=\overrightarrow{AO}$$
$$\overrightarrow{AC}+\overrightarrow{CO}=\overrightarrow{AO}$$
$$\overrightarrow{AD}+\overrightarrow{DO}=\overrightarrow{AO}$$
$$\overrightarrow{AE}+\overrightarrow{EO}=\overrightarrow{AO}$$
$$\overrightarrow{AF}+\overrightarrow{FO}=\overrightarrow{AO}$$

Adding these equations we get

$$\left(\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}\right)+\left(\overrightarrow{BO}+\overrightarrow{CO}+\overrightarrow{DO}+\overrightarrow{EO}+\overrightarrow{FO}\right)$$
$$=5\overrightarrow{AO}$$
$$\Rightarrow\quad\left(\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}\right)+\overrightarrow{DO}=5\overrightarrow{AO}$$

But $\overrightarrow{DO}=-\overrightarrow{AO}$

$$\therefore\quad \overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}=6\overrightarrow{AO}.$$

Ex 23.3

Algebra of Vectors Ex 23.3 Q1

Point R divides the line joining the two points P and Q in the ratio 1:2 internally.

Position vector of point R $=\dfrac{1\left(\vec{a}-2\vec{b}\right)+2\left(2\vec{a}+\vec{b}\right)}{1+2}=\dfrac{5\vec{a}}{3}$

Point R divides the line joining the two points P and Q in the ratio 1:2 externally.

Position vector of point R $=\dfrac{1\left(\vec{a}-2\vec{b}\right)-2\left(2\vec{a}+\vec{b}\right)}{1-2}=\dfrac{-3\vec{a}-4\vec{b}}{-1}=3\vec{a}+4\vec{b}$

Algebra of Vectors Ex 23.3 Q2

Here it is given that $\vec{a},\vec{b},\vec{c}$ and $\vec{d}$ be the position vectors of the four distinct points A,B,C,D such that
$\vec{b}-\vec{a}=\vec{c}-\vec{d}$

Given that,
$\vec{b}-\vec{a}=\vec{c}-\vec{d}$
$\overrightarrow{AB}=\overrightarrow{DC}$

So, AB is parallel and equal to DC (in magnitude).

Hence,
$ABCD$ *is* a parallelogram.

Algebra of Vectors Ex 23.3 Q3

Here, it is given that $\vec{a},\vec{b}$ are position vector of A and B.

Let C be a point in AB produced such that $AC=3AB$.

It is clear that point C divides the line AB in ratio $3:2$ externally.
So position vector point C is given by

$$\vec{c}=\frac{m\vec{b}-n\vec{a}}{m-n}$$
$$=\frac{3\vec{b}-2\vec{a}}{3-2}$$
$$\vec{c}=3\vec{b}-2\vec{a}$$

Again, let D be a point in BA produced such that $BD=2BA$.

Let $\vec{d}$ be the position vector of D. It is clear that point D divides the line AB in $1:2$ externally. So position vector of D is given by

$$\vec{d}=\frac{m\vec{a}-n\vec{b}}{m-n}$$
$$=\frac{2\vec{a}-\vec{b}}{2-1}$$
$$\vec{d}=2\vec{a}-\vec{b}$$

$$\vec{c}=3\vec{b}-2\vec{a}$$
$$\vec{d}=2\vec{a}-\vec{b}$$

Algebra of Vectors Ex 23.3 Q4

We have given that

$3\vec{a} - 2\vec{b} + 5\vec{c} - 6\vec{d} = \vec{0}$

$3\vec{a} + 5\vec{c} = 2\vec{b} + 6\vec{d}$ (i)

Sum of the coefficients on both the sides of the equation (i) is 8, so divide equation (i) by 8 on both the sides,

$$\frac{3\vec{a} + 5\vec{c}}{8} = \frac{2\vec{b} + 6\vec{d}}{8}$$

$$\frac{3\vec{a} + 5\vec{c}}{3+5} = \frac{2\vec{b} + 6\vec{d}}{2+6}$$

It shows that position vector of a point p dividing AC in the ratio 3 : 5, is same as that of a point dividing BD in the ratio of 2 : 6.

Hence, point P is common to AC and BD. Therefore, P is the point of intersection of AC and BD.

So, A, B, C and D are coplanar.

Position vector of point P is given by

$$\frac{3\vec{a} + 5\vec{c}}{8} \quad \text{or} \quad \frac{2\vec{b} + 6\vec{d}}{8}$$

Algebra of Vectors Ex 23.3 Q5

We have given that

$5\vec{p} - 2\vec{q} + 6\vec{r} - 9\vec{s} = \vec{0}$

Where $\vec{p}, \vec{q}, \vec{r}$ and $\vec{s}$ are the position vectors of point P, Q, R and S.

$5\vec{p} + 6\vec{r} = 2\vec{q} + 9\vec{s}$ (i)

Sum of the coefficients on both the sides of the equation (i) is 11. So divide equation (i) by 11 on both the sides.

$$\frac{5\vec{p} + 6\vec{r}}{11} = \frac{2\vec{q} + 9\vec{s}}{11}$$

$$\frac{5\vec{p} + 6\vec{r}}{5+6} = \frac{2\vec{q} + 9\vec{s}}{2+9}$$

It shows that position vector of a point A dividing PR in the ratio of 6 : 5 and QS in the ratio of 9 : 2. Thus, A is the common point to PR and QS and it is also point of intersection of PQ and QS.

So,

P, Q, R and S are coplanar

Position vector of point A is given by

$$\frac{5p + 6q}{11} \quad \text{or} \quad \frac{2\vec{q} + 9\vec{s}}{11}$$

Algebra of Vectors Ex 23.3 Q6

Let ABC be a triangle.

Let the position vectors of A, B and C with respect to some origin, O be $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively.

Let D be the point on BC where the bisector of $\angle A$ meets.

Let $\vec{d}$ position vector of D which divides BC internally in the ratio β and γ, where $\beta = |\overrightarrow{AC}|$ and $\gamma = |\overrightarrow{AB}|$

Thus, $\beta = |\vec{c} - \vec{a}|$ and $\gamma = |\vec{b} - \vec{a}|$

Thus, by section formula, the position vector of D is given by

$$\overrightarrow{OD} = \frac{\beta\vec{b} + \gamma\vec{c}}{\beta + \gamma}$$

Let $\alpha = |\vec{b} - \vec{c}|$

Incentre is the concurrent point of angle bisectors.

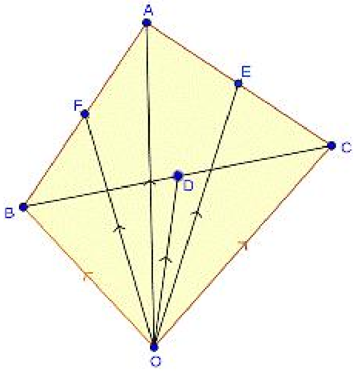
Thus, Incentre divides the line AD in the ratio $\alpha : \beta + \gamma$

Thus, the position vector of incentre is

$$\text{equal to } \frac{\alpha\vec{a} + \frac{\beta\vec{b} + \gamma\vec{c}}{(\beta + \gamma)} \times (\beta + \gamma)}{\alpha + \beta + \gamma} = \frac{\alpha\vec{a} + \beta\vec{b} + \gamma\vec{c}}{\alpha + \beta + \gamma}$$

Ex 23.4

Algebra of Vectors Ex 23.4 Q1



Here, in ΔABC, D, E, F are the mid points of the sides of BC, CA and AB respectively. And O is any point in space.

Let $\vec{a}, \vec{b}, \vec{c}, \vec{d}, \vec{e}, \vec{f}$ be the position vector of point A, B, C, D, E, F with respect to O.

So, $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$, $\overrightarrow{OC} = \vec{c}$
$\overrightarrow{OD} = \vec{d}$, $\overrightarrow{OE} = \vec{e}$, $\overrightarrow{OF} = \vec{f}$

$$\vec{d} = \frac{\vec{b} + \vec{c}}{2}$$

$$\vec{e} = \frac{\vec{a} + \vec{c}}{2} \qquad \text{[Using mid point formula]}$$

$$\vec{f} = \frac{\vec{a} + \vec{b}}{2}$$

$$\begin{aligned}
\overrightarrow{OD} + \overrightarrow{OE} + \overrightarrow{OF} &= \vec{d} + \vec{e} + \vec{f} \\
&= \frac{\vec{b} + \vec{c}}{2} + \frac{\vec{a} + \vec{c}}{2} + \frac{\vec{a} + \vec{b}}{2} \\
&= \frac{\vec{b} + \vec{c} + \vec{a} + \vec{c} + \vec{a} + \vec{b}}{2} \\
&= \frac{2\left(\vec{a} + \vec{b} + \vec{c}\right)}{2} \\
&= \vec{a} + \vec{b} + \vec{c} \\
&= \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC}
\end{aligned}$$

So,
$\overrightarrow{OD} + \overrightarrow{OE} + \overrightarrow{OF} = \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC}$

Algebra of Vectors Ex 23.4 Q2

Here, we have to show that the sum of the three vectors ditermined by medians of a triangle directed from the vertices is zero.

Let ABC is triangle such that position vector of A, B and C are $\vec{a}, \vec{b}$ and $\vec{c}$ respectively.

As AD, BE, CF are medians, D, E and F are mid points.

Position vector of $D = \dfrac{\vec{b}+\vec{c}}{2}$ [Using mid point formula]

Position vector of $E = \dfrac{\vec{c}+\vec{a}}{2}$ [Using mid point formula]

Position vector of $F = \dfrac{\vec{a}+\vec{b}}{2}$ [Using mid point formula]

Now,

$$\begin{aligned}
&\overrightarrow{AD}+\overrightarrow{BE}+\overrightarrow{CF}\\
&= \left(\frac{\vec{b}+\vec{c}}{2}-\vec{a}\right)+\left(\frac{\vec{c}+\vec{a}}{2}-\vec{b}\right)+\left(\frac{\vec{a}+\vec{b}}{2}-\vec{c}\right)\\
&= \frac{\vec{b}+\vec{c}-2\vec{a}}{2}+\frac{\vec{c}+\vec{a}-2\vec{b}}{2}+\frac{\vec{a}+\vec{b}-2\vec{c}}{2}\\
&= \frac{\vec{b}+\vec{c}-2\vec{a}+\vec{c}+\vec{a}-2\vec{b}+\vec{a}+\vec{b}-2\vec{c}}{2}\\
&= \frac{2\vec{b}+2\vec{c}+2\vec{a}-2\vec{b}-2\vec{a}-2\vec{c}}{2}\\
&= \frac{\vec{0}}{2}\\
&= \vec{0}
\end{aligned}$$

$$\therefore\ \overrightarrow{AD}+\overrightarrow{BE}+\overrightarrow{CF}=\vec{0}$$

Algebra of Vectors Ex 23.4 Q3

Here, it is given that $ABCD$ is a parallelogram, P is the point of intersection of diagonals and O be the point of reference.

Using triangle law in ΔAOP,

$$\overrightarrow{OP}+\overrightarrow{PA}=\overrightarrow{OA} \qquad \text{(i)}$$

Using triangle law in ΔOBP,

$$\overrightarrow{OP}+\overrightarrow{PB}=\overrightarrow{OB} \qquad \text{(ii)}$$

Using triangle law in ΔOPC,

$$\overrightarrow{OP}+\overrightarrow{PC}=\overrightarrow{OC} \qquad \text{(iii)}$$

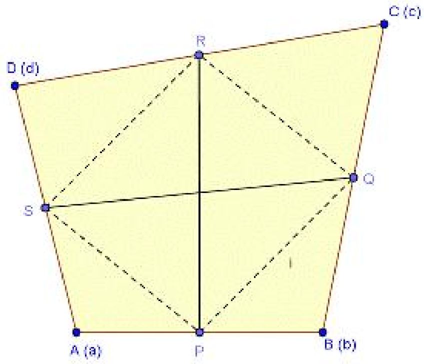
Using triangle law in ΔOPD,

$$\overrightarrow{OP}+\overrightarrow{PD}=\overrightarrow{OD} \qquad \text{(iv)}$$

Adding equation (i), (ii), (iii), and (iv),

$$\begin{aligned}
\overrightarrow{OP}+\overrightarrow{PA}+\overrightarrow{OP}+\overrightarrow{PB}+\overrightarrow{OP}+\overrightarrow{PC}+\overrightarrow{OP}+\overrightarrow{PD} &= \overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}+\overrightarrow{OD}\\
4\overrightarrow{OP}+\overrightarrow{PA}+\overrightarrow{PB}+\overrightarrow{PC}+\overrightarrow{PD} &= \overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}+\overrightarrow{OD}\\
4\overrightarrow{OP}+\overrightarrow{PA}+\overrightarrow{PB}-\overrightarrow{PA}-\overrightarrow{PB} &= \overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}+\overrightarrow{OD} && \left[\begin{array}{l}\text{Since } \overrightarrow{PC}=-\overrightarrow{PA} \text{ and } \overrightarrow{PD}=-\overrightarrow{PB}\\ \text{as } P \text{ is mid point of } AC, BD\end{array}\right]\\
4\overrightarrow{OP} &= \overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}+\overrightarrow{OD}
\end{aligned}$$

Algebra of Vectors Ex 23.4 Q4



Let $ABCD$ be a quadrilateral and P, Q, R, S be the mid points of sides AB, BC, CD and DA respectively.

Let position vector of A, B, C and D be $\vec{a}, \vec{b}, \vec{c}$, and $\vec{d}$.

So position vector of P, Q, R and S are $\left(\frac{\vec{a}+\vec{b}}{2}\right)$, $\left(\frac{\vec{b}+\vec{c}}{2}\right)$, $\left(\frac{\vec{c}+\vec{d}}{2}\right)$ and $\left(\frac{\vec{d}+\vec{a}}{2}\right)$ respectively.

Position vector of $\overrightarrow{PQ}$

= Position vector of Q − Position vector of P

$= \left(\frac{\vec{b}+\vec{c}}{2}\right) - \left(\frac{\vec{a}+\vec{b}}{2}\right)$

$= \frac{\vec{b}+\vec{c}-\vec{a}-\vec{b}}{2}$

$= \frac{\vec{c}-\vec{a}}{2}$ (i)

Position vector of $\overrightarrow{SR}$

= Position vector of R − Position vector of S

$= \left(\frac{\vec{c}+\vec{d}}{2}\right) - \left(\frac{\vec{a}+\vec{d}}{2}\right)$

$= \frac{\vec{c}+\vec{d}-\vec{a}-\vec{d}}{2}$

$= \frac{\vec{c}-\vec{a}}{2}$ (ii)

Using (i) and (ii) ,

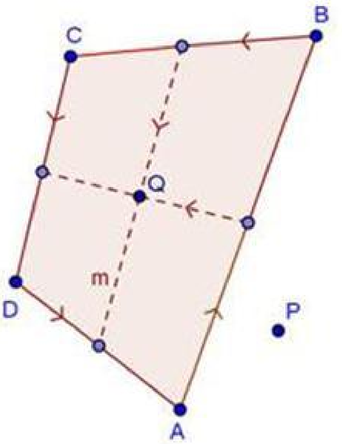
$\overrightarrow{PQ} = \overrightarrow{SR}$

So, $PQRS$ is a parallelogram.

Therefore, PR bisects QS [as diagonals of parallelogram]

Line segment joining the mid point of opposite sides of a quadrilateral bisects each other.

Algebra of Vectors Ex 23.4 Q5



Let $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ be the position vectors of the points $A, B, C,$ and D respectively.

Then, position vector of

$$\text{mid point of } AB = \frac{\vec{a}+\vec{b}}{2}$$

$$\text{mid point of } BC = \frac{\vec{b}+\vec{c}}{2}$$

$$\text{mid point of } CD = \frac{\vec{c}+\vec{d}}{2}$$

$$\text{mid point of } DA = \frac{\vec{a}+\vec{d}}{2}$$

Q is the mid point of the line joining the mid points of AB and CD

$$\therefore \quad p.r. \text{ or } Q = \frac{\frac{\vec{a}+\vec{a}}{2}+\frac{\vec{c}+\vec{d}}{2}}{2}$$

$$= \frac{\vec{a}+\vec{b}+\vec{c}+\vec{d}}{4}$$

Let $\vec{p}$ be the position vector of P.

Then,

$$\overrightarrow{PA}+\overrightarrow{PB}+\overrightarrow{PC}+\overrightarrow{PD}$$

$$= \vec{a}-\vec{p}+\vec{b}-\vec{p}+\vec{c}-\vec{p}+\vec{d}-\vec{p}$$

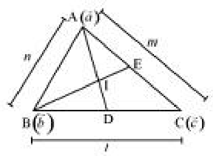
$$= \left(\vec{a}+\vec{b}+\vec{c}+\vec{d}\right)-4\vec{p}$$

$$= 4\left(\frac{\vec{a}+\vec{b}+\vec{c}+\vec{d}}{4}-\vec{p}\right)$$

$$= 4\overrightarrow{PQ}$$

Algebra of Vectors Ex 23.4 Q6

Let $A\left(\vec{a}\right), B\left(\vec{b}\right)$ and $C\left(\vec{c}\right)$ be the position vectors of the vertices of the triangle ΔABC and the length of the sides BC, CA and AB be l, m and n respectively.



The internal bisector of a triangle divides the opposite side in the ratio of the sides containing the angles.

Since AD is the internal bisector of the $\angle ABC$,

$$\frac{BD}{DC}=\frac{AB}{AC}=\frac{n}{m} \qquad (1)$$

Therefore position vector of $D = \dfrac{n\vec{c}+m\vec{b}}{m+n}$

Let the internal bisector intersect at I.

$$\frac{ID}{AI}=\frac{BD}{AB} \qquad (2)$$

$$\frac{BD}{DC}=\frac{n}{m}$$

Therefore,

$$\frac{CD}{BD}=\frac{m}{n}$$

$$\frac{CD+BD}{BD}=\frac{m+n}{n}$$

$$\frac{BC}{BD}=\frac{m+n}{n}$$

$$BD=\frac{\ln}{m+n} \qquad (3)$$

From (2) and (3), we get

$$\frac{ID}{AI}=\frac{\ln}{m+n}$$

Therefore,

$$\text{Position vector of } I = \frac{\left(\frac{nc+mb}{m+n}\right)(m+n)+la}{l+m+n}=\frac{la+mb+nc}{l+m+n}$$

Similarly, we can prove that I lie on the internal bisectors of angles B and C.

Hence the three bisectors are concurrent.

Ex 23.5

Algebra of Vectors Ex 23.5 Q1

Here $\vec{a} = -4\hat{i} - 3\hat{j}$

$$|\vec{a}| = \sqrt{(-4)^2 + (-3)^2}$$
$$= \sqrt{16+9}$$
$$= \sqrt{25}$$
$$= 5$$

$\therefore |\vec{a}| = 5$

Algebra of Vectors Ex 23.5 Q2

Here $\vec{a} = 12\hat{i} + n\hat{j}$

$$|\vec{a}| = \sqrt{(12)^2 + (n)^2}$$
$$13 = \sqrt{144 + n^2} \qquad \left[\text{Since } |\vec{a}| = 13\right]$$

Squaring both sides,

$$(13)^2 = \left(\sqrt{144+n^2}\right)^2$$
$$169 = 144 + n^2$$
$$n^2 = 169 - 144$$
$$n^2 = 25$$
$$n = \pm\sqrt{25}$$
$$n = \pm 5$$

Algebra of Vectors Ex 23.5 Q3

Here, $\vec{a} = \sqrt{3}\hat{i} + \hat{j}$

Let $\vec{b}$ is any vector parallel to $\vec{a}$

So, $\vec{b} = \lambda\vec{a}$ (where λ is any scalar)

$$= \lambda\left(\sqrt{3}\hat{i} + \hat{j}\right)$$
$$\vec{b} = \lambda\sqrt{3}\hat{i} + \lambda\hat{j}$$
$$|\vec{b}| = \sqrt{\left(\lambda\sqrt{3}\right)^2 + (\lambda)^2}$$
$$= \sqrt{3\lambda^2 + \lambda^2}$$
$$= \sqrt{4\lambda^2}$$
$$|\vec{b}| = 2\lambda$$
$$4 = 2\lambda$$
$$\lambda = \frac{4}{2}$$
$$\lambda = 2$$

$$\therefore \vec{b} = \lambda\sqrt{3}\hat{i} + \lambda\hat{j}$$
$$\vec{b} = 2\sqrt{3}\hat{i} + 2\hat{j}$$

Algebra of Vectors Ex 23.5 Q4

(i) Here, $A = (4, -1)$

$B = (1, 3)$

Position vector of $A = 4\hat{i} - \hat{j}$

Position vector of $B = \hat{i} + 3\hat{j}$

$\overrightarrow{AB}$ = Position vector of B – Position vector of A

$= (\hat{i} + 3\hat{j}) - (4\hat{i} - \hat{j})$

$= \hat{i} + 3\hat{j} - 4\hat{i} + \hat{j}$

$\overrightarrow{AB} = -3\hat{i} + 4\hat{j}$

$|\overrightarrow{AB}| = \sqrt{(-3)^2 + (4)^2}$

$= \sqrt{9 + 16}$

$= \sqrt{25}$

$|\overrightarrow{AB}| = 5$

$\overrightarrow{AB} = -3\hat{i} + 4\hat{j}$

(ii) Here, $A = (-6, 3)$

$B = (-2, -5)$

Position vector of $A = -6\hat{i} + 3\hat{j}$

Position vector of $B = -2\hat{i} - 5\hat{j}$

$\overrightarrow{AB}$ = Position vector of B – Position vector of A

$= (-2\hat{i} - 5\hat{j}) - (-6\hat{i} + 3\hat{j})$

$= -2\hat{i} - 5\hat{j} + 6\hat{i} - 3\hat{j}$

$\overrightarrow{AB} = 4\hat{i} - 8\hat{j}$

$|\overrightarrow{AB}| = \sqrt{(4)^2 + (-8)^2}$

$= \sqrt{16 + 64}$

$= \sqrt{80}$

$= \sqrt{16 \times 5}$

$= 4\sqrt{5}$

$|\overrightarrow{AB}| = 4\sqrt{5}$

$\overrightarrow{AB} = 4\hat{i} - 8\hat{j}$

Algebra of Vectors Ex 23.5 Q5

Here, $A = (-1, 3)$

$B = (-2, 1)$

Position vector of $A = -\hat{i} + 3\hat{j}$

Position vector of $B = -2\hat{i} + 1\hat{j}$

$\overrightarrow{AB}$ = Position vector of B – Position vector of A

$= (-2\hat{i} + \hat{j}) - (-\hat{i} + 3\hat{j})$

$= -2\hat{i} + \hat{j} + \hat{i} - 3\hat{j}$

$= -\hat{i} - 2\hat{j}$

So,

Coordinate of the position vector equivalent to $\overrightarrow{AB} = (-1, -2)$

Algebra of Vectors Ex 23.5 Q6

Here, $A = (-2,-1)$

$B = (3,0)$

$C = (1,-2)$

Let $D = (x,y)$

$\overrightarrow{AB}$ = Position vector of B – Position vector of A

$= \left(3\hat{i} - 0\times\hat{j}\right) - \left(-2\hat{i} - \hat{j}\right)$

$= 3\hat{i} - 0\times\hat{j} + 2\hat{i} + \hat{j}$

$\overrightarrow{AB} = 5\hat{i} + \hat{j}$

$\overrightarrow{DC}$ = Position vector of C – Position vector of D

$= \left(\hat{i} - 2\hat{j}\right) - \left(x\hat{i} + y\hat{j}\right)$

$= \hat{i} - 2\hat{j} - x\hat{i} - y\hat{j}$

$\overrightarrow{DC} = (1-x)\hat{i} + (-2-y)\hat{j}$

Since $ABCD$ is a parallelogram, which have equal and parallel opposite sides.

So, $\overrightarrow{AB} = \overrightarrow{DC}$

$5\hat{i} + \hat{j} = (1-x)\hat{i} + (-2-y)\hat{j}$

Comparing components of LHS and RHS

$5 = 1 - x$

$x = 1 - 5$

$x = -4$

$1 = -2 - y$

$y = -2 - 1$

$y = -3$

So, coordinate of D is $(-4,-3)$

Algebra of Vectors Ex 23.5 Q7

Here, $A(3,4), B(5,-6), C(4,-1)$

$\vec{a} = 3\hat{i} + 4\hat{j}$

$\vec{b} = 5\hat{i} - 6\hat{j}$

$\vec{c} = 4\hat{i} - \hat{j}$

Now,

$\vec{a} + 2\vec{b} - 3\vec{c} = \left(3\hat{i} + 4\hat{j}\right) + 2\left(5\hat{i} - 6\hat{j}\right) - 3\left(4\hat{i} - \hat{j}\right)$

$= 3\hat{i} + 4\hat{j} + 10\hat{i} - 12\hat{j} - 12\hat{i} + 3\hat{j}$

$= \hat{i} - 5\hat{j}$

$\therefore\ \vec{a} + 2\vec{b} - 3\vec{c} = \hat{i} - 5\hat{j}$

Algebra of Vectors Ex 23.5 Q9

$\left|\overrightarrow{AB}\right| = 5$ units

$\left|\overrightarrow{BC}\right| = \sqrt{(8)^2}$

$\left|\overrightarrow{BC}\right| = 8$ units

$\left|\overrightarrow{AC}\right| = \sqrt{(-3)^2 + (8)^2}$

$= \sqrt{9 + 16}$

$= \sqrt{25}$

$\left|\overrightarrow{AC}\right| = 5$ units

Here, $\left|\overrightarrow{AB}\right| = \left|\overrightarrow{AC}\right|$

$5 = 5$

So, there are two sides AB,and BC of ΔABC have same length.

ΔABC is an isosceles triangle.

Algebra of Vectors Ex 23.5 Q10

Let $\vec{a} = \hat{i} + \sqrt{3}\hat{j}$

Suppose $\vec{b}$ is any vector parallel to $\vec{a}$

$\vec{b} = \lambda\vec{a}$ where λ is a scalar

$$= \lambda\left(\hat{i} + \sqrt{3}\hat{j}\right)$$

$$\vec{b} = \lambda\hat{i} + \sqrt{3}\lambda\hat{j}$$

$$\left|\vec{b}\right| = \sqrt{(\lambda)^2 + \left(\sqrt{3}\lambda\right)^2}$$

$$= \sqrt{\lambda^2 + 3\lambda^2}$$

$$= \sqrt{4\lambda^2}$$

$$= 2\lambda$$

Unit vector of $\vec{b} = \frac{\vec{b}}{\left|\vec{b}\right|}$

$$\hat{b} = \frac{\lambda\hat{i} + \sqrt{3}\lambda\hat{j}}{2\lambda}$$

$$\hat{b} = \frac{\left(\hat{i} + \sqrt{3}\ \hat{j}\right)}{2}$$

$$\hat{b} = \frac{1}{2}\left(\hat{i} + \sqrt{3}\ \hat{j}\right)$$

Algebra of Vectors Ex 23.5 Q11

(i) Here, $P = (3, 2)$

Position vector of $P = 3\hat{i} + 2\hat{j}$

Component of P along x-axis = $3\hat{i}$

Component of P along y-axis = $2\hat{j}$

(ii) Here, $Q = (-5, 1)$

Position vector of $Q = -5\hat{i} + \hat{j}$

Component of Q along x-axis = $-5\hat{i}$

Component of Q along y-axis = $\hat{j}$

(ii) Here, $R = (-11, -9)$

Position vector of $R = -11\hat{i} - 9\hat{j}$

Component of R along x-axis = $-11\hat{i}$

Component of R along y-axis = $-9\hat{j}$

(iv) Here, $S = (4, -3)$

Position vector of $S = 4\hat{i} - 3\hat{j}$

Component of S along x-axis = $4\hat{i}$

Component of S along y-axis = $-3\hat{j}$

Ex 23.6

Algebra of Vectors Ex 23.6 Q1

Magnitude of a vector $x\hat{i} + y\hat{j} + z\hat{k}$ is given by $\sqrt{(x)^2 + y^2 + z^2}$.

So,

$$\left|\vec{a}\right| = \sqrt{(2)^2 + (3)^2 + (-6)^2}$$
$$= \sqrt{4 + 9 + 36}$$
$$= \sqrt{49}$$
$$= 7$$

$$\therefore \left|\vec{a}\right| = 7$$

Algebra of Vectors Ex 23.6 Q2

Unit vector of $\vec{a} = \dfrac{\vec{a}}{\left|\vec{a}\right|}$

$$\text{Unit vector of } 3\hat{i} + 4\hat{j} - 12\hat{k} = \frac{3\hat{i} + 4\hat{j} - 12\hat{k}}{\sqrt{(3)^2 + (4)^2 + (-12)^2}}$$
$$= \frac{3\hat{i} + 4\hat{j} - 12\hat{k}}{\sqrt{9 + 16 + 144}}$$
$$= \frac{3\hat{i} + 4\hat{j} - 12\hat{k}}{\sqrt{169}}$$

$$\text{Unit vector of } \left(3\hat{i} + 4\hat{j} - 12\hat{k}\right) = \frac{1}{13}\left(3\hat{i} + 4\hat{j} - 12\hat{k}\right)$$

Algebra of Vectors Ex 23.6 Q3

Let $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$

$\vec{b} = 2\hat{i} + \hat{j} - 2\hat{k}$

$\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$

Let d be the resultant of $\vec{a}, \vec{b},$ and $\vec{c}$,

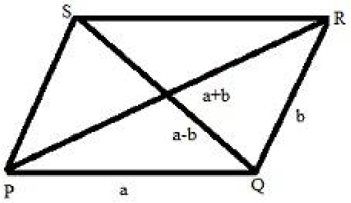
$$\vec{d} = \vec{a} + \vec{b} + \vec{c}$$
$$= \left(\hat{i} - \hat{j} + 3\hat{k}\right) + \left(2\hat{i} + \hat{j} - 2\hat{k}\right) + \left(\hat{i} + 2\hat{j} - 2\hat{k}\right)$$
$$\vec{d} = 4\hat{i} + 2\hat{j} - \hat{k}$$

$$\text{Unit vector } \vec{d} = \frac{\vec{d}}{\left|\vec{d}\right|}$$
$$= \frac{4\hat{i} + 2\hat{j} - \hat{k}}{\sqrt{(4)^2 + (2)^2 + (-1)^2}}$$
$$\vec{d} = \frac{4\hat{i} + 2\hat{j} - \hat{k}}{\sqrt{16 + 4 + 1}}$$

Algebra of Vectors Ex 23.6 Q4

Let $PQRS$ be a parallelogram such that $PQ = \vec{a} = \hat{i} + \hat{j} - \hat{k}$ and

$QR = \vec{b} = -2\hat{i} + \hat{j} + 2\hat{k}$



In $\triangle PQR$

$$\overrightarrow{PQ} + \overrightarrow{QR} = \overrightarrow{PR}$$

$$\overrightarrow{PR} = \vec{a} + \vec{b} = \hat{i} + \hat{j} - \hat{k} + \left(-2\hat{i} + \hat{j} + 2\hat{k}\right)$$

$$\overrightarrow{PR} = -\hat{i} + 2\hat{j} + \hat{k}$$

In $\triangle PSQ$

$$\overrightarrow{PS} + \overrightarrow{SQ} = \overrightarrow{PQ}$$

$$\overrightarrow{SQ} = \vec{a} - \vec{b} = \hat{i} + \hat{j} - \hat{k} - \left(-2\hat{i} + \hat{j} + 2\hat{k}\right)$$

$$\overrightarrow{SQ} = 3\hat{i} + 0\hat{j} - 3\hat{k}$$

The unit vector along $\overrightarrow{PR} = \dfrac{\overrightarrow{PR}}{\left|\overrightarrow{PR}\right|} = \dfrac{-\hat{i} + 2\hat{j} + \hat{k}}{\sqrt{1+4+1}} = \dfrac{1}{\sqrt{6}}\left(-\hat{i} + 2\hat{j} + \hat{k}\right)$

The unit vector along $\overrightarrow{SQ} = \dfrac{\overrightarrow{SQ}}{\left|\overrightarrow{SQ}\right|} = \dfrac{3\hat{i} + 0\hat{j} - 3\hat{k}}{\sqrt{9+0+9}} = \dfrac{1}{\sqrt{2}}\left(\hat{i} - \hat{k}\right)$

Algebra of Vectors Ex 23.6 Q5

$$\begin{aligned} 3\vec{a} - 2\vec{b} + 4\vec{c} &= 3\left(3\hat{i} - \hat{j} - 4\hat{k}\right) - 2\left(-2\hat{i} + 4\hat{j} - 3\hat{k}\right) + 4\left(\hat{i} + 2\hat{j} - \hat{k}\right) \\ &= 9\hat{i} - 3\hat{j} - 12\hat{k} + 4\hat{i} - 8\hat{j} + 6\hat{k} + 4\hat{i} + 8\hat{j} - 4\hat{k} \\ &= 17\hat{i} - 3\hat{j} - 10\hat{k} \end{aligned}$$

$$\begin{aligned} \left|3\vec{a} - 2\vec{b} + 4\vec{c}\right| &= \sqrt{(17)^2 + (-3)^2 + (10)^2} \\ &= \sqrt{289 + 9 + 100} \\ &= \sqrt{398} \end{aligned}$$

$$\left|3\vec{a} - 2\vec{b} + 4\vec{c}\right| = \sqrt{398}$$

Algebra of Vectors Ex 23.6 Q6

Here, $\overrightarrow{PQ} = 3\hat{i} + 2\hat{j} - \hat{k}$

Position vector of $P = \hat{i} - \hat{j} + 2\hat{k}$

$\overrightarrow{PQ}$ = Position vector of Q – Position vector of P

$3\hat{i} + 2\hat{j} - \hat{k}$ = Position vector of $Q - \left(\hat{i} - \hat{j} + 2\hat{k}\right)$

Position vector of $Q = \left(3\hat{i} + 2\hat{j} - \hat{k}\right) + \left(\hat{i} - \hat{j} + 2\hat{k}\right)$

$= 4\hat{i} + \hat{j} + \hat{k}$

Coordinates of $Q = (4, 1, 1)$

Algebra of Vectors Ex 23.6 Q7

Let $\vec{A} = \hat{i} - \hat{j}$

$\vec{B} = 4\hat{i} + 3\hat{j} + \hat{k}$

$\vec{C} = 2\hat{i} - 4\hat{j} + 5\hat{k}$

$$\begin{aligned}\overrightarrow{AB} &= \vec{B} - \vec{A}\\ &= \left(4\hat{i} + 3\hat{j} + \hat{k}\right) - \left(\hat{i} - \hat{j}\right)\\ &= 4\hat{i} + 3\hat{j} + \hat{k} - \hat{i} + \hat{j}\\ &= 3\hat{i} + 4\hat{j} + \hat{k}\end{aligned}$$

$$\left|\overrightarrow{AB}\right| = \sqrt{(3)^2 + (4)^2 + (1)^2} = \sqrt{9 + 16 + 1} = \sqrt{26}$$

$$\begin{aligned}\overrightarrow{BC} &= \vec{C} - \vec{B}\\ &= \left(2\hat{i} - 4\hat{j} + 5\hat{k}\right) - \left(4\hat{i} + 3\hat{j} + \hat{k}\right)\\ &= 2\hat{i} - 4\hat{j} + 5\hat{k} - 4\hat{i} - 3\hat{j} - \hat{k}\\ &= -2\hat{i} - 7\hat{j} + 4\hat{k}\end{aligned}$$

$$\left|\overrightarrow{BC}\right| = \sqrt{(2)^2 + (-7)^2 + (4)^2} = \sqrt{4 + 49 + 16} = \sqrt{69}$$

$$\begin{aligned}\overrightarrow{CA} &= \vec{A} - \vec{C}\\ &= \hat{i} - \hat{j} - \left(2\hat{i} - 4\hat{j} + 5\hat{k}\right)\\ &= \hat{i} - \hat{j} - 2\hat{i} + 4\hat{j} - 5\hat{k}\\ &= -\hat{i} + 3\hat{j} - 5\hat{k}\end{aligned}$$

$$\left|\overrightarrow{CA}\right| = \sqrt{(-1)^2 + (3)^2 + (-5)^2} = \sqrt{1 + 9 + 25} = \sqrt{35}$$

Here, $\left|AB\right|^2 + \left|\overrightarrow{CA}\right|^2 = \left|\overrightarrow{BC}\right|^2$

$$26 + 35 = 69$$
$$61 \neq 69$$
$$\text{LHS} \neq \text{RHS}$$

Since sum of square of two sides is not equal to the square of third sides. So, ΔABC is not a right triangle

Algebra of Vectors Ex 23.6 Q8

Here,

Let vertex $\vec{A} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

vertex $\vec{B} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

vertex $\vec{C} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$

Side $\overrightarrow{AB} = \vec{B} - \vec{A}$

$$= \left(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\right) - \left(a_1\hat{i} + a_2\hat{j} + a_3\hat{k}\right)$$

$$\overrightarrow{AB} = (b_1 - a_1)\hat{i} + (b_2 - a_2)\hat{j} + (b_3 - a_3)\hat{k}$$

$$\begin{aligned}\overrightarrow{BC} &= \vec{C} - \vec{B}\\ &= \left(c_1\hat{i} + c_2\hat{j} + c_3\hat{k}\right) - \left(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\right)\\ &= c_1\hat{i} + c_2\hat{j} + c_3\hat{k} - b_1\hat{i} - b_2\hat{j} - b_3\hat{k}\end{aligned}$$

$$\overrightarrow{BC} = (c_1 - b_1)\hat{i} + (c_2 - b_2)\hat{j} + (c_3 - b_3)\hat{k}$$

$$\begin{aligned}\overrightarrow{AC} &= \left(c_1\hat{i} + c_2\hat{j} + c_3\hat{k}\right) - \left(a_1\hat{i} + a_2\hat{j} + a_3\hat{k}\right)\\ &= c_1\hat{i} + c_2\hat{j} + c_3\hat{k} - a_1\hat{i} - a_2\hat{j} - a_3\hat{k}\end{aligned}$$

$$\overrightarrow{AC} = (c_1 - a_1)\hat{i} + (c_2 - a_2)\hat{j} + (c_3 - a_3)\hat{k}$$

$$\left|\overrightarrow{AB}\right| = \sqrt{(b_1 - a_1)^2 + (b_2 - a_2)^2 + (b_3 - a_3)^2}$$

$$\left|\overrightarrow{BC}\right| = \sqrt{(c_1 - b_1)^2 + (c_2 - b_2)^2 + (c_3 - b_3)^2}$$

$$\left|\overrightarrow{AC}\right| = \sqrt{(c_1 - a_1)^2 + (c_2 - a_2)^2 + (c_3 - a_3)^2}$$

Algebra of Vectors Ex 23.6 Q9

Here, given vertex $A = (1,-1,2)$

$$\vec{A} = \hat{i} - \hat{j} + 2\hat{k}$$

vertex $B = (2,1,3)$

$$\vec{B} = 2\hat{i} + \hat{j} + 3\hat{k}$$

vertex $C = (-1,2,-1)$

$$\vec{C} = -\hat{i} + 2\hat{j} - \hat{k}$$

$$\text{Centroid } \vec{O} = \frac{\vec{A}+\vec{B}+\vec{C}}{3}$$

$$= \frac{\left(\hat{i} - \hat{j} + 2\hat{k}\right) + \left(2\hat{i} + \hat{j} + 3\hat{k}\right) + \left(-\hat{i} + 2\hat{j} - \hat{k}\right)}{3}$$

$$= \frac{2\hat{i} + 2\hat{j} + 4\hat{k}}{3}$$

$$\text{Centroid } \vec{O} = \frac{2\hat{i}}{3} + \frac{2\hat{j}}{3} + \frac{4\hat{k}}{3}$$

Algebra of Vectors Ex 23.6 Q10

The position vector of point R dividing the line segment joining two points

P and Q in the ratio $m: n$ is given by:

i. Internally:

$$\frac{m\vec{b}+n\vec{a}}{m+n}$$

ii. Externally:

$$\frac{m\vec{b}-n\vec{a}}{m-n}$$

Position vectors of P and Q are given as:

$$\overrightarrow{OP} = \hat{i} + 2\hat{j} - \hat{k} \text{ and } \overrightarrow{OQ} = -\hat{i} + \hat{j} + \hat{k}$$

(i) The position vector of point R which divides the line joining two points P and Q internally in the ratio 2: 1 is given by,

$$\overrightarrow{OR} = \frac{2\left(-\hat{i}+\hat{j}+\hat{k}\right)+1\left(\hat{i}+2\hat{j}-\hat{k}\right)}{2+1} = \frac{\left(-2\hat{i}+2\hat{j}+2\hat{k}\right)+\left(\hat{i}+2\hat{j}-\hat{k}\right)}{3}$$

$$= \frac{-\hat{i}+4\hat{j}+\hat{k}}{3} = -\frac{1}{3}\hat{i} + \frac{4}{3}\hat{j} + \frac{1}{3}\hat{k}$$

Algebra of Vectors Ex 23.6 Q11

Here, $P\left(2\hat{i} - 3\hat{j} + 4\hat{k}\right)$ and

$Q\left(4\hat{i} + \hat{j} - 2\hat{k}\right)$

We know that,
If A and B are two points with position vector $\vec{a}$ and $\vec{b}$ then the position vector of mid point C is given by

$$\frac{\vec{a}+\vec{b}}{2}$$

Let R is the mid point of PQ.

$$\text{Position vector of } R = \frac{\vec{P}+\vec{Q}}{2}$$

$$\vec{R} = \frac{2\hat{i} - 3\hat{j} + 4\hat{k} + 4\hat{i} + \hat{j} - 2\hat{k}}{2}$$

$$= \frac{6\hat{i} - 2\hat{j} + 2\hat{k}}{2}$$

$$= \frac{2\left(3\hat{i} - \hat{j} + \hat{k}\right)}{2}$$

Position vector of mid point $= 3\hat{i} - \hat{j} + \hat{k}$

Algebra of Vectors Ex 23.6 Q12

Here, point $P = (1, 2, 3)$

$$\vec{P} = \hat{i} + 2\hat{j} + 3\hat{k}$$

Point $Q = (4, 5, 6)$

$$\vec{Q} = 4\hat{i} + 5\hat{j} + 6\hat{k}$$

$$\begin{aligned}\overrightarrow{PQ} &= \text{Position vector of } Q - \text{Position vector of } P\\ &= \left(4\hat{i} + 5\hat{j} + 6\hat{k}\right) - \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)\\ &= 4\hat{i} + 5\hat{j} + 6\hat{k} - \hat{i} - 2\hat{j} - 3\hat{k}\\ &= 3\hat{i} + 3\hat{j} + 3\hat{k}\\ &= 3\left(i + \hat{j} + k\right)\end{aligned}$$

$$\begin{aligned}\left|\overrightarrow{PQ}\right| &= 3\sqrt{(1)^2 + (1)^2 + (1)^2}\\ &= 3\sqrt{1+1+1}\\ \left|\overrightarrow{PQ}\right| &= 3\sqrt{3}\end{aligned}$$

$$\text{Unit vector in the direction of } \overrightarrow{PQ} = \frac{\overrightarrow{PQ}}{\left|\overrightarrow{PQ}\right|} = \frac{3\left(\hat{i} + \hat{j} + \hat{k}\right)}{3\sqrt{3}}$$

$$\text{Unit vector is the direction of } \overrightarrow{PQ} = \frac{1}{\sqrt{3}}\left(\hat{i} + \hat{j} + \hat{k}\right)$$

Algebra of Vectors Ex 23.6 Q13

The position vectors of A, B and C are $2\hat{i} - \hat{j} + \hat{k}, \hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ respectively.

Therefore,

$\overrightarrow{AB} = -\hat{i} - 2\hat{j} - 6\hat{k}, \overrightarrow{BC} = 2\hat{i} - \hat{j} + \hat{k}$ and $\overrightarrow{CA} = -\hat{i} + 3\hat{j} + 5\hat{k}$

Clearly, $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = 0$.

So, $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{CA}$ form a triangle.

Now

$$\left|\overrightarrow{AB}\right| = \sqrt{1+4+36} = \sqrt{41}$$

$$\left|\overrightarrow{BC}\right| = \sqrt{4+1+1} = \sqrt{6}$$

$$\left|\overrightarrow{CA}\right| = \sqrt{1+9+25} = \sqrt{35}$$

Clearly,

$$\left|\overrightarrow{AB}\right|^2 = \left|\overrightarrow{BC}\right|^2 + \left|\overrightarrow{CA}\right|^2$$

$$AB^2 = BC^2 + CA^2$$

Hence ΔABC is a right triangle right angle at C.

Algebra of Vectors Ex 23.6 Q14

Find the position vector of the mid point of the vector joining the points P (2, 3, 4) and Q (4, 1, − 2).

Solution 16:

The position vector of mid-point R of the vector joining points P (2, 3, 4) and Q (4, 1, − 2) is given by,

$$\overline{OR} = \frac{\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right) + \left(4\hat{i} + \hat{j} - 2\hat{k}\right)}{2} = \frac{(2+4)\hat{i} + (3+1)\hat{j} + (4-2)\hat{k}}{2}$$

$$= \frac{6\hat{i} + 4\hat{j} + 2\hat{k}}{2} = 3\hat{i} + 2\hat{j} + \hat{k}$$

Algebra of Vectors Ex 23.6 Q15

$x\left(\hat{i}+\hat{j}+\hat{k}\right)$ is a unit vector if $\left|x\left(\hat{i}+\hat{j}+\hat{k}\right)\right|=1$.

Now,

$$\left|x\left(\hat{i}+\hat{j}+\hat{k}\right)\right|=1$$
$$\Rightarrow \sqrt{x^2+x^2+x^2}=1$$
$$\Rightarrow \sqrt{3x^2}=1$$
$$\Rightarrow \sqrt{3}\,x=1$$
$$\Rightarrow x=\pm\frac{1}{\sqrt{3}}$$

Hence, the required value of x is $\pm\frac{1}{\sqrt{3}}$.

Algebra of Vectors Ex 23.6 Q16

We have,

$\vec{a}=\hat{i}+\hat{j}+\hat{k},\ \vec{b}=2\hat{i}-\hat{j}+3\hat{k}$ and $\vec{c}=\hat{i}-2\hat{j}+\hat{k}$

$$\begin{aligned}2\vec{a}-\vec{b}+3\vec{c}&=2\left(\hat{i}+\hat{j}+\hat{k}\right)-\left(2\hat{i}-\hat{j}+3\hat{k}\right)+3\left(\hat{i}-2\hat{j}+\hat{k}\right)\\&=2\hat{i}+2\hat{j}+2\hat{k}-2\hat{i}+\hat{j}-3\hat{k}+3\hat{i}-6\hat{j}+3\hat{k}\\&=3\hat{i}-3\hat{j}+2\hat{k}\end{aligned}$$
$$\left|2\vec{a}-\vec{b}+3\vec{c}\right|=\sqrt{3^2+(-3)^2+2^2}=\sqrt{9+9+4}=\sqrt{22}$$

Hence, the unit vector along $2\vec{a}-\vec{b}+3\vec{c}$ is

$$\frac{2\vec{a}-\vec{b}+3\vec{c}}{\left|2\vec{a}-\vec{b}+3\vec{c}\right|}=\frac{3\hat{i}-3\hat{j}+2\hat{k}}{\sqrt{22}}=\frac{3}{\sqrt{22}}\hat{i}-\frac{3}{\sqrt{22}}\hat{j}+\frac{2}{\sqrt{22}}\hat{k}.$$

Algebra of Vectors Ex 23.6 Q17

Here, $\vec{a} = \hat{i} + \hat{j} + \hat{k}$

$\vec{b} = 4\hat{i} - 2\hat{j} + 3\hat{k}$

$\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$

$2\vec{a} - \vec{b} + 3\vec{c}$

$= 2\left(\hat{i} + \hat{j} + \hat{k}\right) - \left(4\hat{i} - 2\hat{j} + 3\hat{k}\right) + 3\left(\hat{i} - 2\hat{j} + \hat{k}\right)$

$= 2\hat{i} + 2\hat{j} + 2\hat{k} - 4\hat{i} + 2\hat{j} - 3\hat{k} + 3\hat{i} - 6\hat{j} + 3\hat{k}$

$= \hat{i} - 2\hat{j} + 2\hat{k}$

Let $\vec{d}$ is a vector parallel to $2\vec{a} - \vec{b} + 3\vec{c}$

So, $\vec{d} = \lambda\left(2\vec{a} - \vec{b} + 3\vec{c}\right)$

Where λ is any scalar

$= \lambda\left(\hat{i} - 2\hat{j} + 2\hat{k}\right)$

$\vec{d} = \lambda\hat{i} - \lambda 2\hat{j} + \lambda 2\hat{k}$ (i)

Given that $\left|\vec{d}\right| = 6$

$\sqrt{(\lambda)^2 + (-2\lambda)^2 + (2\lambda)^2} = 6$

$\sqrt{\lambda^2 + 4\lambda^2 + 4\lambda^2} = 6$

$\sqrt{9\lambda^2} = 6$

$3\lambda = 6$

$\lambda = \frac{6}{3}$

$\lambda = 2$

Put the value of λ in equation (i)

$\vec{d} = 2\hat{i} - 2(2)\hat{j} + 2(2)\hat{k}$

$= 2\hat{i} - 4\hat{j} + 4\hat{k}$

A vector of magnitude 6 which is parallel to $2\vec{a} - \vec{b} + 3\vec{c}$ is given by

$2\hat{i} - 4\hat{j} + 4\hat{k}$

Algebra of Vectors Ex 23.6 Q18

Given that

$\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$

and

$\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$

Thus, Find a vector of magnitude of 5 *units parallel to the resultant of the vectors*

$\vec{a} + \vec{b} = 2\hat{i} + 3\hat{j} - \hat{k} + \hat{i} - 2\hat{j} + \hat{k}$

$\Rightarrow \vec{a} + \vec{b} = 3\hat{i} + \hat{j}$

$\Rightarrow \left|\vec{a} + \vec{b}\right| = \sqrt{9 + 1} = \sqrt{10}$

Thus, the unit vector along the resultant vector $\vec{a} + \vec{b}$ *is*

$$\frac{3\hat{i} + \hat{j}}{\sqrt{10}}$$

The vector of magnitude of 5 *units parallel to the resultant*

$$vector = \frac{3\hat{i} + \hat{j}}{\sqrt{10}} \times 5 = \sqrt{\frac{5}{2}}\left(3\hat{i} + \hat{j}\right)$$

Algebra of Vectors Ex 23.6 Q19

Let D be the point at which median drawn from A touches side BC.

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be the position vectors of the vertices A, B and C.

Position vector of D $= \dfrac{\vec{b}+\vec{c}}{2}$.........[Since D is midpoint of B and C]

$$\overrightarrow{AD} = \frac{\vec{b}+\vec{c}}{2} - \vec{a} = \frac{\vec{b}+\vec{c}-2\vec{a}}{2} = \frac{\vec{b}-\vec{a}+\vec{c}-\vec{a}}{2} = \frac{\overrightarrow{AB}+\overrightarrow{AC}}{2} = \frac{\hat{j}+\hat{i}+3\hat{i}-\hat{j}+4\hat{k}}{2}$$

$$\overrightarrow{AD} = 2\,\hat{i}+2\hat{k}$$

$$\left|\overrightarrow{AD}\right| = \sqrt{4+4} = 4\sqrt{2} \text{ units}$$

Note : Answer given in the book is incorrect.

Ex 23.7

Algebra of Vectors Ex 23.7 Q1

Here, position vector of A = Position vector of $A = \vec{a} - 2\vec{b} + 3\vec{c}$

position vector of B = Position vector of $B = 2\vec{a} + 3\vec{b} - 4\vec{c}$

position vector of C = Position vector of $C = -7\vec{b} + 10\vec{c}$

$\overrightarrow{AB}$ = position vector of B – position vector of A

$$= \left(2\vec{a} + 3\vec{b} - 4\vec{c}\right) - \left(\vec{a} - 2\vec{b} + 3\vec{c}\right)$$

$$= 2\vec{a} + 3\vec{b} - 4\vec{c} - \vec{a} + 2\vec{b} - 3\vec{c}$$

$$\overrightarrow{AB} = \vec{a} + 5\vec{b} - 7\vec{c}$$

$\overrightarrow{BC}$ = position vector of C – position vector of B

$$= \left(-7\vec{b} + 10\vec{c}\right) - \left(2\vec{a} + 3\vec{b} - 4\vec{c}\right)$$

$$= -7\vec{b} + 10\vec{c} - 2\vec{a} - 3\vec{b} + 4\vec{c}$$

$$\overrightarrow{BC} = -2\vec{a} - 10\vec{b} + 14\vec{c}$$

From $\overrightarrow{AB}$ and $\overrightarrow{BC}$, we get

$$\overrightarrow{BC} = -2\left(\overrightarrow{AB}\right)$$

So, $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are parallel but $\vec{B}$ is a common vector. Hence, A, B, C are collinear.

Algebra of Vectors Ex 23.7 Q2(i)

Let the points be A, B, C

Position vector of $A = \vec{a}$

Position vector of $B = \vec{b}$

Position vector of $C = 3\vec{a} - 2\vec{b}$

$\overrightarrow{AB}$ = Position vector of B – Position vector of A

$$= \vec{b} - \vec{a}$$

$\overrightarrow{BC}$ = Position vector of C – Position vector of B

$$= 3\vec{a} - 2\vec{b} - \vec{b}$$

$$= 3\vec{a} - 3\vec{b}$$

Using $\overrightarrow{AB}$ and $\overrightarrow{BC}$

Let $\overrightarrow{BC} = \lambda\left(\overrightarrow{AB}\right)$ [where λ is and scalar]

$$3\vec{a} - 3\vec{b} = \lambda\left(\vec{b} - \vec{a}\right)$$

$$3\vec{a} - 3\vec{b} = \lambda\vec{b} - \lambda\vec{a}$$

$$3\vec{a} - 3\vec{b} = \lambda\vec{a} + \lambda\vec{b}$$

Comparing the coefficients of LHS and RHS, we get

$$-\lambda = 3$$

$$\lambda = 3$$

$$\lambda = -3$$

Since the value of λ are different.

Therefore,

A, B, C are not collinear.

Algebra of Vectors Ex 23.7 Q2(ii)

Let the points be A, B, C

Position vector of $A = \vec{a} + \vec{b} + \vec{c}$
Position vector of $B = 4\vec{a} + 3\vec{b}$
Position vector of $C = 10\vec{a} + 7\vec{b} - 2\vec{c}$

$\overrightarrow{AB}$ = Position vector of B – Position vector of A

$$= \left(4\vec{a} + 3\vec{b}\right) - \left(\vec{a} + \vec{b} + \vec{c}\right)$$
$$= 4\vec{a} + 3\vec{b} - \vec{a} - \vec{b} - \vec{c}$$
$$\overrightarrow{AB} = 3\vec{a} + 2\vec{b} - \vec{c}$$

$\overrightarrow{BC}$ = Position vector of C – Position vector of B

$$= \left(10\vec{a} + 7\vec{b} - 2\vec{c}\right) - \left(4\vec{a} + 3\vec{b}\right)$$
$$= 10\vec{a} + 7\vec{b} - 2\vec{c} - 4\vec{a} - 3\vec{b}$$
$$\overrightarrow{BC} = 6\vec{a} + 4\vec{b} - 2\vec{c}$$

Using $\overrightarrow{AB}$ and $\overrightarrow{BC}$

$$\overrightarrow{BC} = 2\left(\overrightarrow{AB}\right)$$

So, $\overrightarrow{AB}$ is parallel to $\overrightarrow{BC}$ but $\vec{B}$ is a common vector. Hence,
A,B,C are collinear.

Algebra of Vectors Ex 23.7 Q3

Let the points be A, B, C

Position vector of $A = \hat{i} + 2\hat{j} + 3\hat{k}$
Position vector of $B = 3\hat{i} + 4\hat{j} + 7\hat{k}$
Position vector of $C = -3\hat{i} - 2\hat{j} - 5\hat{k}$

$\overrightarrow{AB}$ = Position vector of B – Position vector of A

$$= \left(3\hat{i} + 4\hat{j} + 7\hat{k}\right) - \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$$
$$= 3\hat{i} + 4\hat{j} + 7\hat{k} - \hat{i} - 2\hat{j} - 3\hat{k}$$
$$\overrightarrow{AB} = 2\hat{i} + 2\hat{j} + 4\hat{k}$$

$\overrightarrow{BC}$ = Position vector of C – Position vector of B

$$= \left(-3\hat{i} - 2\hat{j} - 5\hat{k}\right) - \left(3\hat{i} + 4\hat{j} + 7\hat{k}\right)$$
$$= -3\hat{i} - 2\hat{j} - 5\hat{k} - 3\hat{i} - 4\hat{j} - 7\hat{k}$$
$$\overrightarrow{BC} = -6\hat{i} - 6\hat{j} - 12\hat{k}$$

Using $\overrightarrow{AB}$ and $\overrightarrow{BC}$ we get

$$\overrightarrow{BC} = -3\left(\overrightarrow{AB}\right)$$

So, $\overrightarrow{AB}$ is parallel to $\overrightarrow{BC}$ but $\vec{B}$ is a common vector. Hence,
A, B, C are collinear.

Algebra of Vectors Ex 23.7 Q4

Let the points be A, B, C

Position vector of $A = 10\hat{i} + 3\hat{j}$

Position vector of $B = 12\hat{i} - 5\hat{j}$

Position vector of $C = a\hat{i} + 11\hat{j}$

Given that, A,B,C are collinear

$\Rightarrow \overrightarrow{AB}$ and $\overrightarrow{BC}$ are collinear

$\Rightarrow \overrightarrow{AB} = \lambda\left(\overrightarrow{BC}\right)$ (Where λ is same scalar)

$\Rightarrow$ Position vector of B – Position vector of $A = \lambda$ – (Position vector of C – Position vector of B)

$$\Rightarrow \quad \left(12\hat{i} - 5\hat{j}\right) - \left(10\hat{i} + 3\hat{j}\right) = \lambda\left[\left(a\hat{i} + 11\hat{j}\right) - \left(12\hat{i} - 5\hat{j}\right)\right]$$

$$\Rightarrow \quad 12\hat{i} - 5\hat{j} - 10\hat{i} - 3\hat{j} = \lambda\left(a\hat{i} + 11\hat{j} - 12\hat{i} + 5\hat{j}\right)$$

$$\Rightarrow \quad 2\hat{i} - 8\hat{j} = \left(\lambda a - 12\lambda\right)\hat{i} = \left(11\lambda + 5\lambda\right)\hat{j}$$

Comparing the coefficients of LHS and RHS, we get

$\lambda a - 12\lambda = 2$ (i)

$-8 = 11\lambda + 5\lambda$ (ii)

$-8 = 16\lambda$

$\lambda = \frac{-8}{16}$

$\lambda = -\frac{1}{2}$

Put the value of λ in equation (i),

$$\lambda a - 12\lambda = 2$$

$$\left(-\frac{1}{2}\right)a - 12\left(-\frac{1}{2}\right) = 2$$

$$-\frac{1}{2}a + \frac{12}{2} = 2$$

$$-\frac{1}{2}a + 6 = 2$$

$$-\frac{1}{2}a = 2 - 6$$

$$-\frac{1}{2}a = -4$$

$$a = (-4) \times (-2)$$

$$a = 8$$

Algebra of Vectors Ex 23.7 Q5

Let A, B, C be the points then

Position vector of $A = \vec{a} + \vec{b}$
Position vector of $B = \vec{a} - \vec{b}$
Position vector of $C = \vec{a} + \lambda\vec{b}$

$$\overrightarrow{AB} = \text{Position vector of } B - \text{Position vector of } A$$
$$= \left(\vec{a} - \vec{b}\right) - \left(\vec{a} + \vec{b}\right)$$
$$= \vec{a} - \vec{b} - \vec{a} - \vec{b}$$
$$\overrightarrow{AB} = -2\vec{b}$$

$$\overrightarrow{BC} = \text{Position vector of } C - \text{Position vector of } B$$
$$= \left(\vec{a} + \lambda\vec{b}\right) - \left(\vec{a} - \vec{b}\right)$$
$$= \vec{a} + \lambda\vec{b} - \vec{a} + \vec{b}$$
$$= \lambda\vec{b} + \vec{b}$$
$$\overrightarrow{BC} = \left(\lambda + 1\right)\vec{b}$$

Using $\overrightarrow{AB}$ and $\overrightarrow{BC}$, we get
$$\overrightarrow{AB} = \left[\frac{(\lambda + 1)}{-2}\right]\left(\overrightarrow{BC}\right)$$

Let $\left(\frac{\lambda + 1}{-2}\right) = \mu$

Since λ is a real number. So,
μ is also a real no.

So, $\overrightarrow{AB}$ is parallel to $\overrightarrow{BC}$, but $\vec{B}$ is a common vector. Hence,
A, B, C are collinear.

Algebra of Vectors Ex 23.7 Q6

Here, $\overrightarrow{OA} + \overrightarrow{OB} = \overrightarrow{OB} + \overrightarrow{OC}$
$$\overrightarrow{OA} - \overrightarrow{BO} = \overrightarrow{BO} - \overrightarrow{CO}$$
$$\overrightarrow{AB} = \overrightarrow{BC}$$

So, $\overrightarrow{AB}$ is parallel to $\overrightarrow{BC}$ but $\vec{B}$ is a common vector. Hence,
A, B, C are collinear.

Algebra of Vectors Ex 23.7 Q7

Let the given points be A and B

Position vector of $A = 2\hat{i} - 3\hat{j} + 4\hat{k}$
Position vector of $B = -4\hat{i} + 6\hat{j} - 8\hat{k}$

Let O be the initial point having postion vector
$$0 \times \hat{i} + 0 \times \hat{j} + 0 \times \hat{k}$$

$$\overrightarrow{OA} = \text{Position vector of } A - \text{Position vector of } O$$
$$= \left(2\hat{i} - 3\hat{j} + 4\hat{k}\right) - \left(0 \times \hat{i} + 0 \times \hat{j} + 0 \times \hat{k}\right)$$
$$= 2\hat{i} - 3\hat{j} + 4\hat{k}$$

$$\overrightarrow{OB} = \text{Position vector of } B - \text{Position vector of } O$$
$$= \left(-4\hat{i} + 6\hat{j} - 8\hat{k}\right) - \left(0 \times \hat{i} + 0 \times \hat{j} + 0 \times \hat{k}\right)$$
$$\overrightarrow{OB} = -4\hat{i} + 6\hat{j} - 8\hat{k}$$

Using OA and OB, we get
$$\overrightarrow{OB} = -2\left(\overrightarrow{OA}\right)$$

Therefore, $\overrightarrow{OA}$ is parallel to $\overrightarrow{OB}$ but O is the common point to them. Hence,
A and B are collinear.

Algebra of Vectors Ex 23.7 Q8

Here, $A = (m, -1)$

$B = (2, 1)$

$C = (4, 5)$

$\overrightarrow{AB}$ = Position vector of B – Position vector of A

$= (2\hat{i} + \hat{j}) - (m\hat{i} - \hat{j})$

$= 2\hat{i} + \hat{j} - m\hat{i} + \hat{j}$

$= (2 - m)\hat{i} + 2\hat{j}$

$\overrightarrow{BC}$ = Position vector of C – Position vector of B

$= (4\hat{i} + 5\hat{j}) - (2\hat{i} + \hat{j})$

$= 4\hat{i} + 5\hat{j} - 2\hat{i} - \hat{j}$

$\overrightarrow{BC} = 2\hat{i} + 4\hat{j}$

A, B, C are collinear. So, $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are collinear.

So, $\overrightarrow{AB} = \lambda(\overrightarrow{BC})$

$(2 - m)\hat{i} + 2\hat{j} = \lambda(2\hat{i} + 4\hat{j})$, for λ scalar

$(2 - m)\hat{i} + 2\hat{j} = 2\lambda i + 4\lambda\hat{j}$

Comparing the coefficient of LHS and RHS.

$2 - m = 2\lambda$

$\frac{2 - m}{2} = \lambda \qquad \text{(i)}$

$2 = 4\lambda$

$\frac{2}{4} = \lambda$

$\frac{1}{2} = \lambda \qquad \text{(ii)}$

Using (i) and (ii)

$\frac{2 - m}{2} = \frac{1}{2}$

$4 - 2m = 2$

$-2m = 2$

$-2m = 2 - 4$

$-2m = -2$

$m = \frac{-2}{-2}$

$m = 1$

$\therefore m = 1$

Algebra of Vectors Ex 23.7 Q9

Here, let $A = (3, 4)$

$B = (-5, 16)$

$C = (5, 1)$

$\overrightarrow{AB}$ = Position vector of B − Position vector of A

$= \left(-5\hat{i} + 16\hat{j}\right) - \left(3\hat{i} + 4\hat{j}\right)$

$= -5\hat{i} + 16\hat{j} - 3\hat{i} - 4\hat{j}$

$\overrightarrow{AB} = -8\hat{i} + 12\hat{j}$

$\overrightarrow{BC}$ = Position vector of C − Position vector of B

$= \left(5\hat{i} + \hat{j}\right) - \left(-5\hat{i} + 16\hat{j}\right)$

$= 5\hat{i} + \hat{j} + 5\hat{i} - 16\hat{j}$

$\overrightarrow{BC} = 10\hat{i} - 15\hat{j}$

So, $4\left(\overrightarrow{AB}\right) = -5\left(\overrightarrow{BC}\right)$

$\overrightarrow{AB}$ is parallel to $\overrightarrow{BC}$ but B is a common point.

Hence, A, B, C are collinear.

Algebra of Vectors Ex 23.7 Q10

Here, it is given that vectors

$a = 2\hat{i} - 3\hat{j}$ and $b = -6\hat{i} + m\hat{j}$ are collinear.

So, $a = \lambda b$, for a scalar λ

$2\hat{i} - 3\hat{j} = \lambda\left(-6\hat{i} + m\hat{j}\right)$

$2\hat{i} - 3\hat{j} = -6\lambda\hat{i} + \lambda m\hat{j}$

Comparing the coefficients of LHS and RHS,

$2 = -6\lambda$

$\lambda = \frac{2}{-6}$

$\lambda = \frac{-1}{3}$ (i)

$-3 = \lambda m$

$\lambda = \frac{-3}{m}$ (ii)

From (i) and (ii),

$\frac{-1}{3} = \frac{-3}{m}$

$m = 3 \times 3$

$= 9$

$\therefore m = 9$

Algebra of Vectors Ex 23.7 Q11

The given points are A (1, −2, −8), B (5, 0, −2), and C (11, 3, 7).

$$\therefore \overrightarrow{AB} = (5-1)\hat{i} + (0+2)\hat{j} + (-2+8)\hat{k} = 4\hat{i} + 2\hat{j} + 6\hat{k}$$
$$\overrightarrow{BC} = (11-5)\hat{i} + (3-0)\hat{j} + (7+2)\hat{k} = 6\hat{i} + 3\hat{j} + 9\hat{k}$$
$$\overrightarrow{AC} = (11-1)\hat{i} + (3+2)\hat{j} + (7+8)\hat{k} = 10\hat{i} + 5\hat{j} + 15\hat{k}$$
$$\left|\overrightarrow{AB}\right| = \sqrt{4^2 + 2^2 + 6^2} = \sqrt{16+4+36} = \sqrt{56} = 2\sqrt{14}$$
$$\left|\overrightarrow{BC}\right| = \sqrt{6^2 + 3^2 + 9^2} = \sqrt{36+9+81} = \sqrt{126} = 3\sqrt{14}$$
$$\left|\overrightarrow{AC}\right| = \sqrt{10^2 + 5^2 + 15^2} = \sqrt{100+25+225} = \sqrt{350} = 5\sqrt{14}$$
$$\therefore \left|\overrightarrow{AC}\right| = \left|\overrightarrow{AB}\right| + \left|\overrightarrow{BC}\right|$$

Thus, the given points A, B, and C are collinear.

Now, let point B divide AC in the ratio $\lambda : 1$. Then, we have:

$$\overrightarrow{OB} = \frac{\lambda\overrightarrow{OC} + \overrightarrow{OA}}{(\lambda+1)}$$
$$\Rightarrow 5\hat{i} - 2\hat{k} = \frac{\lambda\left(11\hat{i} + 3\hat{j} + 7\hat{k}\right) + \left(\hat{i} - 2\hat{j} - 8\hat{k}\right)}{\lambda+1}$$
$$\Rightarrow (\lambda+1)\left(5\hat{i} - 2\hat{k}\right) = 11\lambda\hat{i} + 3\lambda\hat{j} + 7\lambda\hat{k} + \hat{i} - 2\hat{j} - 8\hat{k}$$
$$\Rightarrow 5(\lambda+1)\hat{i} - 2(\lambda+1)\hat{k} = (11\lambda+1)\hat{i} + (3\lambda-2)\hat{j} + (7\lambda-8)\hat{k}$$

On equating the corresponding components, we get:

$$5(\lambda+1) = 11\lambda + 1$$
$$\Rightarrow 5\lambda + 5 = 11\lambda + 1$$
$$\Rightarrow 6\lambda = 4$$
$$\Rightarrow \lambda = \frac{4}{6} = \frac{2}{3}$$

Hence, point B divides AC in the ratio $2:3$.

Algebra of Vectors Ex 23.7 Q12

We have

$\overrightarrow{AP}$ = *Position vector of P − Position vector of A*

$\Rightarrow \overrightarrow{AP} = \hat{i} + 2\hat{j} + 3\hat{k} - \left(-2\hat{i} + 3\hat{j} + 5\hat{k}\right) = 3\hat{i} - \hat{j} - 2\hat{k}$

$\overrightarrow{PB}$ = *Position vector of B − Position vector of P*

$\Rightarrow \overrightarrow{PB} = 7\hat{i} - \hat{k} - \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) = 6\hat{i} - 2\hat{j} - 4\hat{k}$

Clearly, $\overrightarrow{PB} = 2\overrightarrow{AP}$

so vectors $\overrightarrow{AP}$ *and* $\overrightarrow{PB}$ *are collinear.*

But P is a point common to $\overrightarrow{AP}$ *and* $\overrightarrow{PB}$.

Hence P, A, B are collinear points.

Similarly, $\overrightarrow{CP} = \hat{i} + 2\hat{j} + 3\hat{k} - \left(-3\hat{i} - 2\hat{j} - 5\hat{k}\right) = 4\hat{i} + 4\hat{j} + 8\hat{k}$

and $\overrightarrow{PD} = 3\hat{i} + 4\hat{j} + 7\hat{k} - \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) = 2\hat{i} + 2\hat{j} + 4\hat{k}$

So vectors $\overrightarrow{CP}$ *and* $\overrightarrow{PD}$ *are collinear.*

But P is a common point to $\overrightarrow{CP}$ *and* $\overrightarrow{CD}$.

Hence, C, P, D are collinear points.

Thus, A, B, C, D and P are points such that A, P, B and C, P, D are two sets of collinear points. Hence AB and CD intersect at the point P

Algebra of Vectors Ex 23.7 Q13

Points (λ, - 10, 3), (1 -1, 3) and (3, 5, 3) are collinear.

$\therefore$ (λ, - 10, 3) = x(1 -1, 3) + y(3, 5, 3) for some scalars x and y.
$\Rightarrow \lambda = x + 3y$, $-10 = -x + 5y$ and $3 = 3x + 3y$

Solving $-10 = -x + 5y$ and $3 = 3x + 3y$ for x and y we get,
$x = \frac{5}{2}$ and $y = -\frac{3}{2}$

Now,
$\lambda = x + 3y$
$\Rightarrow \lambda = \frac{5}{2} + 3\left(-\frac{3}{2}\right) = -2$

Ex 23.8

Algebra of Vectors Ex 23.8 Q1

(i) Let P, Q, R be the points whose position vectors are $2\hat{i}+\hat{j}-\hat{k}$, $3\hat{i}-2\hat{j}+\hat{k}$ and $\hat{i}+4\hat{j}-3\hat{k}$ respectively.

$$\begin{aligned}
\overrightarrow{PQ} &= \text{Position vector of } Q - \text{Position vector of } P\\
&= \left(3\hat{i}-2\hat{j}+\hat{k}\right)-\left(2\hat{i}+\hat{j}-\hat{k}\right)\\
&= 3\hat{i}-2\hat{j}+\hat{k}-2\hat{i}-\hat{j}+\hat{k}\\
\overrightarrow{PQ} &= \hat{i}-3\hat{j}+2\hat{k}
\end{aligned}$$

$$\begin{aligned}
\overrightarrow{QR} &= \text{Position vector of } R - \text{Position vector of } Q\\
&= \left(\hat{i}+4\hat{j}-3\hat{k}\right)-\left(3\hat{i}-2\hat{j}+\hat{k}\right)\\
&= \hat{i}+4\hat{j}-3\hat{k}-3\hat{i}+2\hat{j}-\hat{k}\\
&= -2\hat{i}+6\hat{j}-4\hat{k}\\
\overrightarrow{QR} &= -2\overrightarrow{PQ}
\end{aligned}$$

Therefore, $\overrightarrow{QR}$ is parallel to $\overrightarrow{PQ}$ but there is a common point Q. So, P, Q, R are collinear.

(ii) Let P, Q, R be the points represented be the vectors are $3\hat{i}-2\hat{j}+4\hat{k}$, $\hat{i}+\hat{j}+\hat{k}$ and $-\hat{i}+4\hat{j}-2\hat{k}$ respectively.

$$\begin{aligned}
\overrightarrow{PQ} &= \text{Position vector of } Q - \text{Position vector of } P\\
&= \left(-\hat{i}+4\hat{j}-2\hat{k}\right)-\left(3\hat{i}-2\hat{j}+4\hat{k}\right)\\
&= \hat{i}+\hat{j}+\hat{k}-3\hat{i}+2\hat{j}-4\hat{k}\\
&= -2\hat{i}-3\hat{j}-3\hat{k}
\end{aligned}$$

$$\begin{aligned}
\overrightarrow{QR} &= \text{Position vector of } R - \text{Position vector of } Q\\
&= \left(-\hat{i}+4\hat{j}-2\hat{k}\right)-\left(\hat{i}+\hat{j}+\hat{k}\right)\\
&= -\hat{i}+4\hat{j}-2\hat{k}-\hat{i}-\hat{j}-\hat{k}\\
&= -2\hat{i}+3\hat{j}-3\hat{k}\\
\overrightarrow{PQ} &= \overrightarrow{QR}
\end{aligned}$$

So, $\overrightarrow{PQ}$ is parallel to $\overrightarrow{QR}$ but Q is the common point Q. So, P, Q, R are collinear.

Algebra of Vectors Ex 23.8 Q2(i)

Here,
$$\begin{aligned}
\vec{A} &= 6\hat{i}-7\hat{j}-\hat{k}\\
\vec{B} &= 2\hat{i}-3\hat{j}+\hat{k}\\
\vec{C} &= 4\hat{i}-5\hat{j}-0\times\hat{k}
\end{aligned}$$

$$\begin{aligned}
\overrightarrow{AB} &= \vec{B}-\vec{A}\\
&= \left(2\hat{i}-3\hat{j}+\hat{k}\right)-\left(6\hat{i}-7\hat{j}-\hat{k}\right)\\
&= 2\hat{i}-3\hat{j}+\hat{k}-6\hat{i}+7\hat{j}+\hat{k}\\
\overrightarrow{AB} &= -4\hat{i}+4\hat{j}+2\hat{k}
\end{aligned}$$

$$\begin{aligned}
\overrightarrow{BC} &= \vec{C}-\vec{B}\\
&= \left(4\hat{i}-5\hat{j}-0\times\hat{k}\right)-\left(2\hat{i}-3\hat{j}+\hat{k}\right)\\
&= 4\hat{i}-5\hat{j}-0\times\hat{k}-2\hat{i}+3\hat{j}-\hat{k}\\
\overrightarrow{BC} &= 2\hat{i}-2\hat{j}-\hat{k}
\end{aligned}$$

$$\overrightarrow{AB} = -2\left(\overrightarrow{BC}\right)$$

So, $\overrightarrow{AB}$ is parallel to $\overrightarrow{BC}$ but B is the common point. So, A, B, C are collinear.

Algebra of Vectors Ex 23.8 Q2(ii)

Here, $\vec{A} = 2\hat{i} - \hat{j} + 3\hat{k}$

$\vec{B} = 4\hat{i} + 3\hat{j} + \hat{k}$

$\vec{C} = 3\hat{i} + \hat{j} + 2\hat{k}$

$$\overrightarrow{AB} = \vec{B} - \vec{A}$$
$$= \left(4\hat{i} + 3\hat{j} + \hat{k}\right) - \left(2\hat{i} - \hat{j} + 3\hat{k}\right)$$
$$= 4\hat{i} + 3\hat{j} + \hat{k} - 2\hat{i} + \hat{j} - 3\hat{k}$$
$$\overrightarrow{AB} = 2\hat{i} + 4\hat{j} - 2\hat{k}$$

$$\overrightarrow{BC} = \vec{C} - \vec{B}$$
$$= \left(3\hat{i} + \hat{j} + 2\hat{k}\right) - \left(4\hat{i} + 3\hat{j} + \hat{k}\right)$$
$$= 3\hat{i} + \hat{j} + 2\hat{k} - 4\hat{i} - 3\hat{j} - \hat{k}$$
$$\overrightarrow{BC} = -\hat{i} - 2\hat{j} + \hat{k}$$

So, $\overrightarrow{AB} = -2\left(\overrightarrow{BC}\right)$

$\overrightarrow{AB}$ is parallel to $\overrightarrow{BC}$ but $\vec{B}$ is a common vector.

Therefore, A, B, C are collinear.

Algebra of Vectors Ex 23.8 Q2(iii)

Here, $\vec{A} = \hat{i} + 2\hat{j} + 7\hat{k}$

$\vec{B} = 2\hat{i} + 6\hat{j} + 3\hat{k}$

$\vec{C} = 3\hat{i} + 10\hat{j} - \hat{k}$

$$\overrightarrow{AB} = \vec{B} - \vec{A}$$
$$= \left(2\hat{i} + 6\hat{j} + 3\hat{k}\right) - \left(\hat{i} + 2\hat{j} + 7\hat{k}\right)$$
$$= 2\hat{i} + 6\hat{j} + 3\hat{k} - \hat{i} - 2\hat{j} - 7\hat{k}$$
$$\overrightarrow{AB} = \hat{i} + 4\hat{j} - 4\hat{k}$$

$$\overrightarrow{BC} = \vec{C} - \vec{B}$$
$$= \left(3\hat{i} + 10\hat{j} - \hat{k}\right) - \left(2\hat{i} + 6\hat{j} + 3\hat{k}\right)$$
$$= 3\hat{i} + \hat{j} + 2\hat{k} - 2\hat{i} - 6\hat{j} - 3\hat{k}$$
$$\overrightarrow{BC} = \hat{i} + 4\hat{j} - 4\hat{k}$$

$\overrightarrow{AB} = \overrightarrow{BC}$

So, $\overrightarrow{AB}$ is parallel to $\overrightarrow{BC}$ but $\vec{B}$ is a common vector.

So, A, B, C are collinear.

Algebra of Vectors Ex 23.8 Q2(iv)

Here, $\vec{A} = -3\hat{i} - 2\hat{j} - 5\hat{k}$

$\vec{B} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{C} = 3\hat{i} + 4\hat{j} + 7\hat{k}$

$$\overrightarrow{AB} = \vec{B} - \vec{A}$$
$$= \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) - \left(-3\hat{i} - 2\hat{j} - 5\hat{k}\right)$$
$$= \hat{i} + 2\hat{j} + 3\hat{k} + 3\hat{i} + 2\hat{j} + 5\hat{k}$$
$$\overrightarrow{AB} = 4\hat{i} + 4\hat{j} + 8\hat{k}$$

$$\overrightarrow{BC} = \vec{C} - \vec{B}$$
$$= \left(3\hat{i} + 4\hat{j} + 7\hat{k}\right) - \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$$
$$= 3\hat{i} + 4\hat{j} + 7\hat{k} - \hat{i} - 2\hat{j} - 3\hat{k}$$
$$\overrightarrow{BC} = 2\hat{i} + 2\hat{j} + 4\hat{k}$$

So, $\overrightarrow{AB} = 2\overrightarrow{BC}$

Hence, $\overrightarrow{AB}$ is parallel to $\overrightarrow{BC}$ but $\vec{B}$ is a common vector.

Therefore, A, B, C are collinear.

Algebra of Vectors Ex 23.8 Q2(v)

Here, $\vec{A} = 2\hat{i} - \hat{j} + 3\hat{k}$

$\vec{B} = 3\hat{i} - 5\hat{j} + \hat{k}$

$\vec{C} = -\hat{i} + 11\hat{j} + 9\hat{k}$

$$\overrightarrow{AB} = \vec{B} - \vec{A}$$
$$= \left(3\hat{i} - 5\hat{j} + \hat{k}\right) - \left(2\hat{i} - \hat{j} + 3\hat{k}\right)$$
$$= 3\hat{i} - 5\hat{j} + \hat{k} - 2\hat{i} + \hat{j} - 3\hat{k}$$
$$\overrightarrow{AB} = \hat{i} - 4\hat{j} - 2\hat{k}$$

$$\overrightarrow{BC} = \vec{C} - \vec{B}$$
$$= \left(-\hat{i} + 11\hat{j} + 9\hat{k}\right) - \left(3\hat{i} - 5\hat{j} + \hat{k}\right)$$
$$= -\hat{i} + 11\hat{j} + 9\hat{k} - 3\hat{i} + 5\hat{j} - \hat{k}$$
$$= -4\hat{i} + 16\hat{j} + 8\hat{k}$$

So, $\overrightarrow{AB} = -4\left(\overrightarrow{BC}\right)$

$\overrightarrow{AB}$ is parallel to vector $\overrightarrow{BC}$ but $\vec{B}$ is a common vector.

So, A, B, C are collinear

Algebra of Vectors Ex 23.8 Q3(i)

We know that,

Three vectors are coplanar if one of the vector can be expressed as the linear combination of other two.

Let,

$$5\vec{a} + 6\vec{b} + 7\vec{c} = x\left(7\vec{a} - 8\vec{b} + 9\vec{c}\right) + y\left(3\vec{a} + 20\vec{b} + 5\vec{c}\right)$$
$$5\vec{a} + 6\vec{b} + 7\vec{c} = 7\vec{a}x - 8\vec{b}x + 9\vec{c}x + 3\vec{a}y + 20\vec{b}y + 5\vec{c}y$$
$$5\vec{a} + 6\vec{b} + 7\vec{c} = \left(7x + 3y\right)\vec{a} + \left(-8x + 20y\right)\vec{b} + \left(9x + 5y\right)\vec{c}$$

Comparing the LHS and RHS,

$7x + 3y = 5 \qquad (i)$

$-8x + 20y = 6 \qquad (ii)$

$9x + 5y = 7 \qquad (iii)$

For solving (i) and (ii),

Subtract $-8 \times (i)$ from $7 \times (ii)$,

$$-56x + 140y = 42$$
$$\underset{(+)}{-}\,56x \underset{(+)}{-} 24y = \underset{(+)}{-}\,40$$
$$164y = 82$$

$$y = \frac{82}{164}$$

$$y = \frac{1}{2}$$

Put $y = \frac{1}{2}$ in equation (i),

$$7x + 3y = 5$$
$$7x + 3\left(\frac{1}{2}\right) = 5$$
$$7x + \frac{3}{2} = 5$$
$$7x = \frac{5}{1} - \frac{3}{2}$$
$$7x = \frac{10-3}{2}$$
$$7x = \frac{7}{2}$$
$$x = \frac{7}{14}$$
$$x = \frac{1}{2}$$

Now, put $x = \frac{1}{2}$ and $y = \frac{1}{2}$ in equation (iii),

$$9x + 5y = 7$$
$$9\left(\frac{1}{2}\right) + 5\left(\frac{1}{2}\right) = 7$$
$$\frac{9}{2} + \frac{5}{2} = 7$$
$$\frac{14}{2} = 7$$
$$7 = 7$$
$$\text{LHS} = \text{RHS}$$

$\therefore$ The value of x, y satisfy equation (iii).

So,
$5\vec{a} + 6\vec{b} + 7\vec{c},\ 7\vec{a} - 8\vec{b} + 9\vec{c},\ 3\vec{a} + 20\vec{b} + 5\vec{c}$ are coplanar.

Algebra of Vectors Ex 23.8 Q3(ii)

We know that,
Three vectors are coplanar if one of them can be expressed as the linear combination of other two.

Let
$$\vec{a} - 2\vec{b} + 3\vec{c} = x\left(-3\vec{b} + 5\vec{c}\right) + y\left(-2\vec{a} + 3\vec{b} - 4\vec{c}\right)$$
$$\vec{a} - 2\vec{b} + 3\vec{c} = -3\vec{b}x + 5\vec{c}x + 2\vec{a}y + 3\vec{b}y - 4\vec{c}y$$
$$\vec{a} - 2\vec{b} + 3\vec{c} = (-2y)\vec{a} + (-3x + 3y)\vec{b} + (5x - 4y)\vec{c}$$

Comparing the LHS and RHS,
$$-2y = 1 \quad \text{(i)}$$
$$-3x + 3y = -2 \quad \text{(ii)}$$
$$5x - 4y = 3 \quad \text{(iii)}$$

From solving (i) and $y = -\frac{1}{2}$

Put value of y in equation (ii),
$$-3x + 3y = -2$$
$$-3x + 3\left(-\frac{1}{2}\right) = -2$$
$$-3x - \frac{3}{2} = -2$$
$$-3x = \frac{-2}{1} + \frac{3}{2}$$
$$-3x = \frac{-4+3}{2}$$
$$-3x = \frac{-1}{2}$$
$$x = \frac{-1}{-6}$$
$$x = \frac{1}{6}$$

Put value of x and y in equation (iii)
$$5x - 4y = 3$$
$$5\left(\frac{1}{6}\right) - 4\left(-\frac{1}{2}\right) = 3$$
$$\frac{5}{6} + \frac{4}{2} = 3$$
$$\frac{5+12}{6} = 3$$
$$\frac{17}{6} = 3$$
$$\text{LHS} \neq \text{RHS}$$

So, value of x and y do not satisfy the equation (iii).

So,
vectors $\vec{a} - 2\vec{b} + 3\vec{c},\ -3\vec{b} + 5\vec{c},$ and $-2\vec{a} + 3\vec{b} - 4\vec{c}$ are not coplanar.

Algebra of Vectors Ex 23.8 Q4

Here,

Position vector of $P = 6\hat{i} - 7\hat{j}$

Position vector of $Q = 16\hat{i} - 19\hat{j} - 4\hat{k}$

Position vector of $R = 3\hat{j} - 6\hat{k}$

Position vector of $S = 2\hat{i} - 5\hat{j} + 10\hat{k}$

$\overrightarrow{PQ}$ = Position vector of Q – Position vector of P

$$= \left(16\hat{i} - 19\hat{j} - 4\hat{k}\right) - \left(6\hat{i} - 7\hat{j}\right)$$
$$= 16\hat{i} - 19\hat{j} - 4\hat{k} - 6\hat{i} + 7\hat{j}$$
$$= 10\hat{i} - 12\hat{j} - 4\hat{k}$$

$\overrightarrow{PR}$ = Position vector of R – Position vector of P

$$= \left(3\hat{j} - 6\hat{k}\right) - \left(6\hat{i} - 7\hat{j}\right)$$
$$= 3\hat{j} - 6\hat{k} - 6\hat{i} + 7\hat{j}$$
$$= -6\hat{i} + 10\hat{j} - 6\hat{k}$$

$\overrightarrow{PS}$ = Position vector of S – Position vector of P

$$= \left(2\hat{i} - 5\hat{j} + 10\hat{k}\right) - \left(6\hat{i} - 7\hat{j}\right)$$
$$= 2\hat{i} - 5\hat{j} + 10\hat{k} - 6\hat{i} + 7\hat{j}$$
$$= -4\hat{i} + 2\hat{j} + 10\hat{k}$$

Let $\overrightarrow{PQ} = x\overrightarrow{PR} + y\overrightarrow{PS}$

$$10\hat{i} - 12\hat{j} - 4\hat{k} = x\left(-6\hat{i} + 10\hat{j} - 6\hat{k}\right) + \left(-4\hat{i} + 2\hat{j} + 10\hat{k}\right)$$
$$= -6x\hat{i} + x10\hat{j} - 6x\hat{k} - 4y\hat{i} + 2y\hat{j} + 10y\hat{k}$$
$$10\hat{i} - 12\hat{j} - 4\hat{k} = \left(-6x - 4y\right)\hat{i} + \left(10x + 2y\right)\hat{j} + \left(-6x + 10y\right)\hat{k}$$

Comparing the coefficients of $\hat{i}, \hat{j}$ and $\hat{k}$ of LHS and RHS,

$$-6x - 4y = 10$$
$$3x + 2y = -5 \qquad (i)$$
$$10x + 2y = -12 \qquad (ii)$$
$$-6x + 10y = -4 \qquad (iii)$$

Substracting (i) from (ii),

$$\begin{array}{r} 10x + 2y = -12 \\ 3x + 2y = -5 \\ (-) \quad (-) \qquad (+) \\ \hline 7x \qquad = -7 \end{array}$$

$$x = \frac{-7}{7}$$
$$x = -1$$

Put $x = -1$ in equation (i)

$$3x + 2y = -5$$
$$3(-1) + 2y = -5$$
$$-3 + 2y = -5$$
$$2y = -5 + 3$$
$$2y = -2$$
$$y = \frac{-2}{2}$$
$$y = -1$$

Put $x = -1$ and $y = -1$ in equation (iii),

$$-6x + 10y = -4$$
$$-6(-1) + 10(-1) = -4$$
$$6 - 10 = -4$$
$$-4 = -4$$
$$\text{LHS} = \text{RHS}$$

Therefore,

P, Q, R, S are coplanar.

Algebra of Vectors Ex 23.8 Q5(i)

We know that, three vectors are coplanar if one of the vector can be expressed as linear combination of other two.

Let,

$$2\hat{i} - \hat{j} + \hat{k} = x\left(\hat{i} - 3\hat{j} - 5\hat{k}\right) + y\left(3\hat{i} - 4\hat{j} - 4\hat{k}\right)$$

$$2\hat{i} - \hat{j} + \hat{k} = x\hat{i} - 3x\hat{j} - 5x\hat{k} + 3y\hat{i} - 4y\hat{j} - 4y\hat{k}$$

$$2\hat{i} - \hat{j} + \hat{k} = (x + 3y)\hat{i} + (-3x - 4y)\hat{j} + (-5x - 4y)\hat{k}$$

Comparing the coefficients of LHS and RHS,

$$x + 3y = 2 \qquad (i)$$

$$-3x - 4y = -1 \qquad (ii)$$

$$-5x - 4y = 1 \qquad (iii)$$

For solving equation (i) and (ii),

Add $3 \times (i)$ with equation (ii),

$$\begin{array}{r} 3x + 9y = 6 \\ \underline{-3x - 4y = -1} \\ 5y \;\; = 5 \end{array}$$

$$y = \frac{5}{5}$$

$$y = 1$$

Put y in equation (i),

$$x + 3y = 2$$

$$x + 3(1) = 2$$

$$x + 3 = 2$$

$$x = 2 - 3$$

$$x = -1$$

Put the value of x and y in equation (iii),

$$-5x - 4y = 1$$

$$-5(-1) - 4(1) = 1$$

$$5 - 4 = 1$$

$$1 = 1$$

LHS = RHS

So, the value of x and y satisfy equation (iii).

Hence, vectors are coplanar.

Algebra of Vectors Ex 23.8 Q5(ii)

We know that,

Three vectors are coplanar if one of the vector can be expressed as the linear combination of other two vectors.

Let,

$\hat{i} + \hat{j} + \hat{k} = x\left(2\hat{i} + 3\hat{j} - \hat{k}\right) + y\left(-\hat{i} - 2\hat{j} + 2\hat{k}\right)$

$\hat{i} + \hat{j} + \hat{k} = 2x\hat{i} + 3x\hat{j} - x\hat{k} + -y\hat{i} - 2y\hat{j} + 2y\hat{k}$

$\hat{i} + \hat{j} + \hat{k} = \left(2x - y\right)\hat{i} + \left(3x - 2y\right)\hat{j} + \left(-x + 2y\right)\hat{k}$

Comparing the coefficients of LHS and RHS,

$2x - y = 1 \qquad (i)$

$3x - 2y = 1 \qquad (ii)$

$-x + 2y = 1 \qquad (iii)$

For solving (i) and (ii),

Subtracting $2 \times (i)$ from (ii),

$$\begin{array}{rcl} 3x - 2y &=& 1 \\ 4x - 2y &=& 2 \\ (-)\quad (+) & & (-) \\ \hline -x &=& -1 \end{array}$$

$x = 1$

Put the value of x in equation (i),

$2x - y = 1$

$2(1) - y = 1$

$2 - y = 1$

$-y = 1 - 2$

$-y = -1$

$y = 1$

Put the value of x and y in equation (iii),

$-x + 2y = 1$

$-(1) + 2(1) = 1$

$-1 + 2 = 1$

$1 = 1$

$\text{LHS} = \text{RHS}$

The value of x and y satisfy equation (iii).

Hence, vectors are coplanar.

Algebra of Vectors Ex 23.8 Q6(i)

We know that,

Three vectors are coplanar if one of them vector can be expressed as the linear combination of the other two.

Let,

$$\left(3\hat{i}+\hat{j}-\hat{k}\right)=x\left(2\hat{i}-\hat{j}+7\hat{k}\right)+y\left(7\hat{i}-\hat{j}+23\hat{k}\right)$$

$$=2x\hat{i}-x\hat{j}+7x\hat{k}+7y\hat{i}-y\hat{j}+23y\hat{k}$$

$$\left(3\hat{i}+\hat{j}-\hat{k}\right)=\left(2x+7y\right)\hat{i}+\left(-x-y\right)\hat{j}+\left(7x+23y\right)\hat{k}$$

Equating the coefficients of LHS and RHS,

$$2x+7y=3 \qquad (i)$$

$$-x-y=1 \qquad (ii)$$

$$7x+23y=-1 \qquad (iii)$$

For solving (i) and (ii),

Add (i) and $2\times(ii)$,

$$\begin{array}{r} 2x+7y=3 \\ -2x-2y=2 \\ \hline 5y=5 \end{array}$$

$$y=\frac{5}{5}$$

$$y=1$$

Put the value of y in equation (i),

$$2x+7y=3$$

$$2x+7(1)=3$$

$$2x+7=3$$

$$2x=3-7$$

$$2x=-4$$

$$x=\frac{-4}{2}$$

$$x=-2$$

Put the value of x and y in equation (iii),

$$7x+23y=-1$$

$$7(2)+23(1)=-1$$

$$14+23=-1$$

$$37=-1$$

$$\text{LHS}\neq\text{RHS}$$

The value of x and y do not satisfy the equation (iii).

Hence, vectors are non-coplanar.

Algebra of Vectors Ex 23.8 Q6(ii)

We know that,

Three vectors are coplanar if any one of the vector can be expressed as the linear combination of other two vectors.

Let,

$$\hat{i} + 2\hat{j} + 3\hat{k} = x\left(2\hat{i} + \hat{j} + 3\hat{k}\right) + y\left(\hat{i} + \hat{j} + \hat{k}\right) = 2x\hat{i} + x\hat{j} + 3x\hat{k} + y\hat{i} + y\hat{j} + y\hat{k}$$

$$\therefore \quad \hat{i} + 2\hat{j} + 3\hat{k} = (2x + y)\hat{i} + (x + 2y)\hat{j} + (3x + y)\hat{k}$$

Comparing the coefficients of LHS and RHS,

$2x + y = 1 \qquad \text{(i)}$

$x + 2y = 2 \qquad \text{(ii)}$

$3x + y = 3 \qquad \text{(iii)}$

Subtracting $2 \times \text{(ii)}$ from equation (i),

$$\begin{array}{r} 2x + 4y = 4 \\ 2x + y = 1 \\ (-)\quad(-)\quad(-) \\ \hline 3y = 3 \end{array}$$

$y = \frac{3}{3}$

$y = 1$

Put the value of y in equation (i),

$2x + y = 1$

$2x + 1 = 1$

$2x = 1 - 1$

$2x = 0$

$x = \frac{0}{2}$

$x = 0$

Put the value of x and y in equation (iii),

$3x + y = 3$

$3(0) + 1 = 3$

$0 + 1 = 3$

$1 = 3$

$\text{LHS} \neq \text{RHS}$

The value of x and y do not satisfy the equation (iii).

Hence, vectors are non-coplanar.

Algebra of Vectors Ex 23.8 Q7(i)

We know that,
Three vectors are coplanar if any one of them can be expressed as the linear combination of other two vectors.

Let,

$$\left(2\vec{a}-\vec{b}+3\vec{c}\right)=x\left(\vec{a}+\vec{b}-2\vec{c}\right)+y\left(\vec{a}+\vec{b}-3\vec{c}\right)$$
$$=\vec{a}x+\vec{b}x-2\vec{c}x+\vec{a}y+\vec{b}y-3\vec{c}y$$
$$\left(2\vec{a}-\vec{b}+3\vec{c}\right)=\left(x+y\right)\vec{a}+\left(x+y\right)\vec{b}+\left(-2x-3y\right)\vec{c}$$

Comparing the coefficients of LHS and RHS,

$x+y=2$ (i)

$x+y=-1$ (ii)

$-2x-3y=3$ (iii)

For solving the equation (i) and (ii),
Subtracting (ii) from (i),

$$\begin{array}{r} x+y=2 \\ \underset{(-)}{x}\underset{(-)}{+y}=\underset{(+)}{-1} \\ \hline 0=3 \end{array}$$

There is no value of x and y that can satisfy the equation (iii).
Hence, vectors are non-coplanar.

Algebra of Vectors Ex 23.8 Q7(ii)

We know that,
Three vectors are coplanar if any one of them can be expressed as the linear combination of other two vectors.

Let

$$\vec{a}+2\vec{b}+3\vec{c}=x\left(2\vec{a}+\vec{b}+3\vec{c}\right)+y\left(\vec{a}+\vec{b}+\vec{c}\right)$$
$$=2\vec{a}x+\vec{b}x+3\vec{c}x+\vec{a}y+\vec{b}y+\vec{c}y$$
$$\vec{a}+2\vec{b}+3\vec{c}=\left(2x+y\right)\vec{a}+\left(x+y\right)\vec{b}+\left(3x+y\right)\vec{c}$$

Comparing the coefficients of LHS and RHS,

$2x+y=1$ (i)

$x+y=2$ (ii)

$3x+y=3$ (iii)

For solving the equation (i) and (ii),
Subtracting equation (i) from equation (ii),

$$\begin{array}{r} x+y=2 \\ \underset{(-)}{2x}\underset{(-)}{+y}=\underset{(-)}{1} \\ \hline -x=1 \end{array}$$

$x = -1$

Put the value of x in equation (i)

$x + y = 2$

$-1 + y = 2$

$y = 2 + 1$

$y = 3$

Put the x and y in equation (iii),

$3x + y = 3$

$3(-1) + 3 = 3$

$-3 + 3 = 3$

$0 = 3$

LHS $\neq$ RHS

The value of x and y do not satisfy the equation (iii).

Hence, vectors are non-coplanar.

Algebra of Vectors Ex 23.8 Q8

We know that,

Three vectors are coplanar if any one of them can be expressed as the linear combination of other two vectors.

Let

$\vec{a} = x\vec{b} + y\vec{c}$

$= x\left(2\hat{i} + \hat{j} + 3\hat{k}\right) + y\left(\hat{i} + \hat{j} + \hat{k}\right)$

$= 2\hat{i}x + \hat{j}x + 3\hat{k}x + \hat{i}y + \hat{j}y + \hat{k}y$

$\hat{i} + 2\hat{j} + 3\hat{k} = (2x + y)\hat{i} + (x + y)\hat{j} + (3x + y)\hat{k}$

Comparing the coefficient of LHS and RHS,

$2x + y = 1 \quad \text{(i)}$

$x + y = 2 \quad \text{(ii)}$

$3x + y = 3 \quad \text{(iii)}$

For solving (i) and (ii),

Subtracting (i) from (ii),

$$\begin{array}{rcl} x + y &=& 2 \\ 2x + y &=& 1 \\ (-)\ \ (-) && (-) \\ \hline -x &=& 1 \\ x &=& -1 \end{array}$$

Put the value of x in equation (i),

$x + y = 2$

$-1 + y = 2$

$y = 2 + 1$

$y = 3$

Put the values of x and y in equation (iii)

$3x + y = 3$

$3(-1) + 3 = 3$

$-3 + 3 = 3$

$0 = 3$

LHS $\neq$ RHS

The values of x and y do not satisfy equation (iii).

Hence,

$\vec{a}, \vec{b}, \vec{c}$ are non coplanar.

Let,

$\vec{d} = x\vec{b} + y\hat{j} + z\hat{k}$

$= x\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + y\left(2\hat{i} + \hat{j} + 3\hat{k}\right) + z\left(\hat{i} + \hat{j} + \hat{k}\right)$

$= x\hat{i} + 2x\hat{j} + 3x\hat{k} + 2y\hat{i} + \hat{j}y + 3y\hat{k} + z\hat{i} + z\hat{j} + z\hat{k}$

$2\hat{i} - \hat{j} - 3\hat{k} = \left(x + 2y + z\right)\hat{i} + \left(2x + y + z\right)\hat{j} + \left(3x + 3y + z\right)\hat{k}$

Comparing the coefficient of LHS and RHS,

$x + 2y + z = 2 \qquad (i)$

$2x + y + z = -1 \qquad (ii)$

$3x + 3y + z = -3 \qquad (iii)$

Subtracting equation (i) from (ii),

$$\begin{array}{l} 2x + y + z = -1 \\ \underset{(-)}{x} \underset{(-)}{+ 2y} \underset{(-)}{+ z} = \underset{(-)}{2} \\ \hline x - y \quad = -3 \end{array} \qquad (iv)$$

Subtracting equation (ii) from (iii),

$$\begin{array}{l} 3x + 3y + z = -3 \\ \underset{(-)}{2x} \underset{(-)}{+ y} \underset{(-)}{+ z} = \underset{(+)}{-1} \\ \hline x + 2y \quad = -2 \end{array} \qquad (v)$$

Subtracting (iv) from (v),

$$\begin{array}{l} x + 2y = -2 \\ \underset{(-)}{x} \underset{(+)}{- y} = \underset{(+)}{-3} \\ \hline 3y = 1 \\ y = \frac{1}{3} \end{array}$$

Put y in equation (v),

$x + 2y = -2$

$x + 2\left(\frac{1}{3}\right) = -2$

$2 + \frac{2}{3} = -2$

$x = \frac{-2}{1} - \frac{2}{3}$

$= \frac{-6 - 2}{3}$

$x = \frac{-8}{3}$

Put value of x and y in equation (i),

$x + 2y + z = 2$

$\frac{-8}{3} + 2\left(\frac{1}{3}\right) + z = 2$

$\frac{-8}{3} + \frac{2}{3} + z = 2$

$z = \frac{2}{1} + \frac{8}{3} - \frac{2}{3}$

$z = \frac{6 + 8 - 2}{3}$

$z = \frac{14 - 2}{3}$

$z = \frac{12}{3}$

$z = 4$

So,

$\vec{d} = x\vec{a} + y\vec{b} + z\vec{c}$

$\vec{d} = \left(\frac{-8}{3}\right)\vec{a} + \left(\frac{1}{3}\right)\vec{b} + \left(4\right)\vec{c}$

Algebra of Vectors Ex 23.8 Q9

Necessary Condition: Let $\vec{a}, \vec{b}, \vec{c}$ are three coplanar vectors. Then one of them can be expressed as the linear combination of other two vectors.

Let, $\vec{c} = x\vec{a} + y\vec{b}$

$x\vec{a} + y\vec{b} - \vec{c} = 0$

Put $x = l$, $y = m$, $(-1) = n$

$l\vec{a} + m\vec{b} + n\vec{c} = 0$

Thus, if $\vec{a}, \vec{b}, \vec{c}$ are coplanar vectors,
then there exist scalars l, m, n

$l\vec{a} + m\vec{b} + n\vec{c} = 0$

Such that l,m,n are not all zero simultaneously.

Sufficient Condition: Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors such that there exist scalars l, m, n not all zero simultaneously satisfying $l\vec{a} + m\vec{b} + n\vec{c} = 0$

$l\vec{a} + m\vec{b} + n\vec{c} = 0$

$n\vec{c} = -l\vec{a} - m\vec{b}$

Dividing by n, both the sides

$$\frac{n\vec{c}}{n} = \frac{-l\vec{a}}{n} - \frac{m\vec{b}}{n}$$

$$\vec{c} = \left(-\frac{l}{n}\right)\vec{a} + \left(-\frac{m}{n}\right)\vec{b}$$

$\vec{c}$ is a linear combination of $\vec{a}$ and $\vec{b}$

Hence, $\vec{a}, \vec{b}, \vec{c}$ are coplanar vectors.

Algebra of Vectors Ex 23.8 Q10

Given that, A, B, C and D are four points with position vector $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ respectively.

Let A, B, C, D are coplanar.

If so, there exists x, y, z, u not all zero such that

$x\vec{a} + y\vec{b} + z\vec{c} + u\vec{d} = 0$

$x + y + z + u = 0$

Let, $x = 3,\ y = -2,\ z = 1,\ u = -2$

$3\vec{a} - 2\vec{b} + \vec{c} - 2\vec{d} = 0$

and, $x + y + z + u = 3 + (-2) + 1 + (-2)$

$= 4 - 4$

$= 0$

Thus, A, B, C, D are coplanar.

if $3\vec{a} - 2\vec{b} + \vec{c} - 2\vec{d} = 0$

Let $3\vec{a} - 2\vec{b} + \vec{c} - 2\vec{d} = 0$

$3\vec{a} + \vec{c} = 2\vec{b} + 2\vec{d}$

Divide by sum of the coefficients that is by 4 on both sides,

$$\frac{3\vec{a} + \vec{c}}{4} = \frac{2\vec{b} + 2\vec{d}}{4}$$

$$\frac{3\vec{a} + \vec{c}}{3 + 1} = \frac{2\vec{b} + 2\vec{d}}{2 + 2}$$

It shows that P is the point which divides AC in ratio $1:3$ internally as well as BD in ratio $2:2$ internally.

Thus, P is the point of intersection of AC and BD.

Hence,

A, B, C, D are coplanar.

We can say that,

A, B, C, D are coplanar if and only if Let $3\vec{a} - 2\vec{b} + \vec{c} - 2\vec{d} = \vec{0}$

Ex 23.9

Algebra of Vectors Ex 23.9 Q1

We know that, If l, m, n are the direction cosine of a vector and α, β, γ can the direction angle, then

$$l = \cos\alpha,\ m = \cos\beta\ n = \cos\gamma$$

and, $l^2 + m^2 + n^2 = 1 \qquad \text{(i)}$

$\therefore \quad l = \cos 45°,\ m = \cos 60°,\ n = \cos 120°$

$$l = \frac{1}{\sqrt{2}},\ m = \frac{1}{2}, n = -\frac{1}{2}$$

Put l, m, n in equation (i)

$$l^2 + m^2 + n^2 = 1$$
$$\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{-1}{2}\right)^2 = 1$$
$$\frac{1}{2} + \frac{1}{4} + \frac{1}{4} = 1$$
$$\frac{2+1+1}{4} = 1$$
$$\frac{4}{4} = 1$$
$$1 = 1$$

LHS = RHS

Therefore,a vector can have direction angle 45°, 60°,120°.

Algebra of Vectors Ex 23.9 Q2

Here, $l = 1,\ m = 1,\ n = 1$

Put it in

$$l^2 + m^2 + n^2 = 1$$
$$(1)^2 + (1)^2 + (1)^2 = 1$$
$$1 + 1 + 1 = 1$$
$$3 = 1$$

LHS $\neq$ RHS

Therefore,
1,1,1 can not be direction cosines of a straight line.

Algebra of Vectors Ex 23.9 Q3

Here, $\alpha = \frac{\pi}{4},\ \beta = \frac{\pi}{4},\ \gamma = ?$

$$l = \cos\alpha = \cos\frac{\pi}{4}$$
$$l = \frac{1}{\sqrt{2}}$$

$$m = \cos\beta = \cos\frac{\pi}{4}$$
$$m = \frac{1}{\sqrt{2}}$$

$$n = \cos\gamma$$

Put value of $l, m,$ and n in

$$l^2 + m^2 + n^2 = 1$$
$$\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \cos^2\gamma = 1$$
$$\frac{1}{2} + \frac{1}{2} + \cos^2\gamma = 1$$
$$1 + \cos^2\gamma = 1$$
$$\cos^2\gamma = 1 - 1$$
$$\cos^2\gamma = 0$$
$$\cos\gamma = 0$$
$$\gamma = \cos^{-1}(0)$$
$$\gamma = \frac{\pi}{2}$$

The angle made by the vector with the z-axis $= \frac{\pi}{2}$

Algebra of Vectors Ex 23.9 Q4

Here, $\alpha = \beta = \gamma$

$\Rightarrow \quad \cos\alpha = \cos\beta = \cos\gamma$

$\Rightarrow \quad l = m = n = x$ (say)

We know that,

$$l^2 + m^2 + n^2 = 1$$
$$x^2 + x^2 + x^2 = 1$$
$$3x^2 = 1$$
$$x^2 = \frac{1}{3}$$
$$x = \pm\frac{1}{\sqrt{3}}$$

$$l = \pm\frac{1}{\sqrt{3}},\ m = \pm\frac{1}{\sqrt{3}},\ n = \pm\frac{1}{\sqrt{3}}$$

Hence, direction cosiner of $\vec{r}$ are,

$$\pm\frac{1}{\sqrt{3}},\ \pm\frac{1}{\sqrt{3}},\ \pm\frac{1}{\sqrt{3}}$$

Vector $\vec{r} = |\vec{r}|\left(l\hat{i} + m\hat{j} + n\hat{k}\right)$

$$= 6\left(\pm\frac{1}{\sqrt{3}}\hat{i} + \pm\frac{1}{\sqrt{3}}\hat{j} + \pm\frac{1}{\sqrt{3}}\hat{k}\right)$$

$$= \frac{\pm 6 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}\left(\hat{i} + \hat{j} + \hat{k}\right) \qquad \text{[Rationalizing the denominator]}$$

$$= \frac{\pm 6\sqrt{3}}{3}\left(\hat{i} + \hat{j} + \hat{k}\right)$$

$$\vec{r} = \pm 2\sqrt{3}\left(\hat{i} + \hat{j} + \hat{k}\right)$$

Algebra of Vectors Ex 23.9 Q5

Here, $\alpha = 45°,\ \beta = 60°,\ \gamma = \theta$ (say)

$$l = \cos\alpha$$
$$= \cos 45°$$
$$l = \frac{1}{\sqrt{2}}$$

$$m = \cos\beta$$
$$= \cos 60°$$
$$m = \frac{1}{2}$$

$$n = \cos\theta$$

Put l, m, and n in

$$l^2 + m^2 + n^2 = 1$$
$$\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 + \cos^2\theta = 1$$
$$\frac{2+1}{4} + \cos^2\theta = 1$$
$$\frac{3}{4} + \cos^2\theta = 1$$
$$\cos^2\theta = \frac{1}{1} - \frac{3}{4}$$
$$= \frac{4-3}{4}$$
$$\cos^2\theta = \frac{1}{2}$$

So, $\quad l = \frac{1}{\sqrt{2}},\ m = \frac{1}{2},\ n = \pm\frac{1}{2}$

The required,

vector $\vec{r} = |\vec{r}|\left(l\hat{i} + m\hat{j} + n\hat{k}\right)$

$$= 8\left(\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{2}\hat{j} \pm \frac{1}{2}\hat{k}\right)$$

$$= 8\,\frac{\sqrt{2}\hat{i} + \hat{j} \pm \hat{k}}{2}$$

$$\vec{r} = 4\left(\sqrt{2}\hat{i} + \hat{j} \pm \hat{k}\right)$$

Algebra of Vectors Ex 23.9 Q6

(i)

Here, the direction ratios of the vector

$2\hat{i} + 2\hat{j} - \hat{k} = 2,\ 2,\ -1$

The direction cosines of the vector

$= \frac{2}{|\vec{r}|}, \frac{2}{|\vec{r}|}, \frac{-1}{|\vec{r}|}$

$= \frac{2}{\sqrt{(2)^2 + (2)^2 + (-1)^2}}, \frac{2}{\sqrt{(2)^2 + (2)^2 + (-1)^2}}, \frac{-1}{\sqrt{(2)^2 + (2)^2 + (-1)^2}}$

$= \frac{2}{\sqrt{4+4+1}}, \frac{2}{\sqrt{4+4+1}}, \frac{-1}{\sqrt{4+4+1}}$

$= \frac{2}{\sqrt{9}}, \frac{2}{\sqrt{9}}, \frac{-1}{\sqrt{9}}$

$= \frac{2}{3}, \frac{2}{3}, \frac{-1}{3}$

(ii)

Here, let $\vec{r} = 6\hat{i} - 2\hat{j} - 3\hat{k}$

and, $|\vec{r}| = \sqrt{(6)^2 + (-2)^2 + (-3)^2}$

$= \sqrt{36 + 4 + 9}$

$= \sqrt{49}$

$|\vec{r}| = 7$

The direction cosines of $\vec{r}$ are given by

$= \frac{6}{|\vec{r}|}, \frac{-2}{|\vec{r}|}, \frac{-3}{|\vec{r}|}$

$= \frac{6}{7}, \frac{-2}{7}, \frac{-3}{7}$

Algebra of Vectors Ex 23.9 Q7(i)

Let, $\vec{r} = \hat{i} - \hat{j} + \hat{k}$

The direction ratios of the vector $\vec{r} = 1, -1, 1$

And, $|\vec{r}| = \sqrt{(1)^2 + (-1)^2 + (1)^2}$

$= \sqrt{1+1+1}$

$= \sqrt{3}$

The direction cosines of the vector $\vec{r}$

$= \frac{1}{|\vec{r}|}, \frac{-1}{|\vec{r}|}, \frac{1}{|\vec{r}|}$

$= \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

So, $l = \cos\alpha = \frac{1}{\sqrt{3}}$

$\alpha = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$m = \cos\beta = \frac{-1}{\sqrt{3}}$

$\beta = \cos^{-1}\left(\frac{-1}{\sqrt{3}}\right)$

$n = \cos\gamma = \frac{1}{\sqrt{3}}$

$\gamma = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Thus, angles made by $\vec{r}$ with the coordinate axes are given by

$\cos^{-1}\left(\frac{1}{\sqrt{3}}\right), \cos^{-1}\left(-\frac{1}{\sqrt{3}}\right), \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Algebra of Vectors Ex 23.9 Q7(ii)

Let, $\vec{r} = \hat{j} - \hat{k}$

$\vec{r} = 0 \times \hat{i} + \hat{j} - \hat{k}$

The direction ratios of $\vec{r} = 0, 1, -1$

and, $\left|\vec{r}\right| = \sqrt{(0)^2 + (1)^2 + (-1)^2}$

$= \sqrt{0 + 1 + 1}$

$\left|\vec{r}\right| = \sqrt{2}$

The direction cosines of the $\vec{r}$ are given by

$= \frac{0}{\left|\vec{r}\right|}, \frac{1}{\left|\vec{r}\right|}, \frac{-1}{\left|\vec{r}\right|}$

$= \frac{0}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}$

So, $l = \cos\alpha = 0$

$\alpha = \cos^{-1}(0)$

$\alpha = \frac{\pi}{2}$

$m = \cos\beta = \frac{1}{\sqrt{2}}$

$\beta = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$\beta = \frac{\pi}{4}$

$n = \cos\gamma = -\frac{1}{\sqrt{2}}$

$\gamma = \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

$\lambda = \pi - \frac{\pi}{7}$

$\gamma = \frac{3\pi}{4}$

So, angles made by the vector $\vec{r}$ with coordinate axes are given by

$\frac{\pi}{2}, \frac{\pi}{4}, \frac{3\pi}{4}$

Algebra of Vectors Ex 23.9 Q7(iii)

Let, $4\hat{i} + 8\hat{j} + \hat{k} = \vec{r}$

The direction ratios of $\vec{r} = 4, 8, 1$

And, $\left|\vec{r}\right| = \sqrt{(4)^2 + (8)^2 + (1)^2}$

$= \sqrt{16 + 64 + 1}$

$= \sqrt{81}$

$\left|\vec{r}\right| = 9$

The direction cosines of the $\vec{r}$ are given by

$= \frac{4}{\left|\vec{r}\right|}, \frac{8}{\left|\vec{r}\right|}, \frac{1}{\left|\vec{r}\right|}$

$= \frac{4}{9}, \frac{8}{9}, \frac{1}{9}$

Now, $l = \cos\alpha = \frac{4}{9}$

$\alpha = \cos^{-1}\left(\frac{4}{9}\right)$

$m = \cos\beta = \frac{8}{9}$

$\beta = \cos^{-1}\left(\frac{8}{9}\right)$

$n = \cos\gamma = \frac{1}{9}$

$\gamma = \cos^{-1}\left(\frac{1}{9}\right)$

The angles made by the vector $\vec{r}$ with the coordinate axes are given by

$\cos^{-1}\left(\frac{4}{9}\right), \cos^{-1}\left(\frac{8}{9}\right), \cos^{-1}\left(\frac{1}{9}\right)$

Algebra of Vectors Ex 23.9 Q8

Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$.

Then,

$|\vec{a}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$

Therefore, the direction cosines of $\vec{a}$ are $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$.

Now, let α, β, and γ be the angles formed by $\vec{a}$ with the positive directions of x, y, and z axes.

Then, we have $\cos\alpha = \frac{1}{\sqrt{3}}, \cos\beta = \frac{1}{\sqrt{3}}, \cos\gamma = \frac{1}{\sqrt{3}}$.

Hence, the given vector is equally inclined to axes OX, OY, and OZ.

Algebra of Vectors Ex 23.9 Q9

Let a vector be equally inclined to axes OX, OY, and OZ at angle α.

Then, the direction cosines of the vector are $\cos\alpha$, $\cos\alpha$, and $\cos\alpha$.

Now,

$\cos^2\alpha + \cos^2\alpha + \cos^2\alpha = 1$

$\Rightarrow 3\cos^2\alpha = 1$

$\Rightarrow \cos\alpha = \frac{1}{\sqrt{3}}$

Hence, the direction cosines of the vector which are equally inclined to the axes are $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$.

Algebra of Vectors Ex 23.9 Q10

Let unit vector $\vec{a}$ have (a_1, a_2, a_3) components.

$\Rightarrow \vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

Since $\vec{a}$ is a unit vector, $|\vec{a}| = 1$.

Also, it is given that $\vec{a}$ makes angles $\frac{\pi}{3}$ with $\hat{i}$, $\frac{\pi}{4}$ with $\hat{j}$, and an acute angle θ with $\hat{k}$.

Then, we have:

$\cos\frac{\pi}{3} = \frac{a_1}{|\vec{a}|}$

$\Rightarrow \frac{1}{2} = a_1 \qquad [|\vec{a}| = 1]$

$\cos\frac{\pi}{4} = \frac{a_2}{|\vec{a}|}$

$\Rightarrow \frac{1}{\sqrt{2}} = a_2 \qquad [|\vec{a}| = 1]$

Also, $\cos\theta = \frac{a_3}{|\vec{a}|}$.

$\Rightarrow a_3 = \cos\theta$

Now,

$|a| = 1$

$\Rightarrow \sqrt{a_1^2 + a_2^2 + a_3^2} = 1$

$$\Rightarrow \left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \cos^2\theta = 1$$

$$\Rightarrow \frac{1}{4} + \frac{1}{2} + \cos^2\theta = 1$$

$$\Rightarrow \frac{3}{4} + \cos^2\theta = 1$$

$$\Rightarrow \cos^2\theta = 1 - \frac{3}{4} = \frac{1}{4}$$

$$\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

$$\therefore a_3 = \cos\frac{\pi}{3} = \frac{1}{2}$$

Hence, $\theta = \frac{\pi}{3}$ and the components of $\vec{a}$ are $\left(\frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{1}{2}\right)$.

The Plane Ex 23.9 Q11

Let l, m, n be the direction cosines of the vector $\vec{r}$.

$l = \cos\alpha$, $m = \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$ and $n = \cos\left(\frac{\pi}{2}\right) = 0$

$$l^2 + m^2 + n^2 = 1$$

$$l^2 + \frac{1}{2} + 0 = 1$$

$$l = \pm\frac{1}{\sqrt{2}}$$

$$\vec{r} = |\vec{r}|\left(l\hat{i} + m\hat{j} + n\hat{k}\right)$$

$$\vec{r} = 3\sqrt{2}\left(\pm\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j} + 0\right)$$

$$\vec{r} = \pm 3\hat{i} + 3\hat{j}$$

The Plane Ex 23.9 Q12

Let l, m, n be the direction cosines of the vector $\vec{r}$.

Vector $\vec{r}$ is inclined at equal angles to the three axes.

$l = \cos\alpha$, $m = \cos\alpha$ and $n = \cos\alpha$

$\Rightarrow l = m = n$

$$l^2 + m^2 + n^2 = 1$$

$$\Rightarrow l = m = n = \pm\frac{1}{\sqrt{3}}$$

$$\vec{r} = |\vec{r}|\left(l\hat{i} + m\hat{j} + n\hat{k}\right)$$

$$\vec{r} = 2\sqrt{3}\left(\pm\frac{1}{\sqrt{3}}\hat{i} \pm \frac{1}{\sqrt{3}}\hat{j} \pm \frac{1}{\sqrt{3}}\hat{k}\right)$$

$$\vec{r} = \pm 2\hat{i} \pm 2\hat{j} \pm 2\hat{k}$$