Statistics for Economics

CLASS - XI

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Essential Practicals

Solutions

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Chapter – 1

Concept of Economics and Significance of Statistics in Economics

Essential Practical:

Q.1. Complete the following observation:

a. Statistics means

b. Statistics and economics are

c. The term population refers to

d. Descriptive Statistics means those methods

Solution:

i. Statistics means **quantitative information.**

It deals with quantitative information only and does not take qualitative data into consideration. Qualitative variables such as beauty, honesty and kindness cannot be studied.

ii. Statistics and economics are **complementary to each other**.

Statistics plays a significant role in Economics. It

- Expresses economic problems quantitatively.
- Facilitates inter-sectoral and inter-temporal comparison.
- Helps in studying the cause and effect relationship between different economic variables which leads to construction of economic theories.

iii. The term population refers to the **aggregate of all items relating to statistical study.** Thus, if statistical study comprises 200 items, then population is 200.

iv. Descriptive statistics means those methods **which are used for collection, presentation and analysis of data**. Under this, following estimations are done:

- Measurement of central tendencies such as mean, median, mode, quartiles, deciles and percentiles
- Measurement of dispersion such as range, quartile deviation, mean deviation and standard deviation
- Measurement of correlation through a scatter diagram and correlation coefficient.

Q.2. Prepare a list of statistical information that will facilitate comparison of academic performance of your school with the other schools in your neighbourhood.

Solution:

List of statistical information which will be required to facilitate comparison of academic performance of my school with the other schools in my neighbourhood:

i) Data of the total number of schools and the total number of students in each school

ii) Average marks obtained by students in each class in each school

iii) Average marks of all students in each school

iv) Coefficient of variation of marks

Academic performance of a school is better when average marks are highest and coefficient of variation is lowest.

Q.3. Write two such pairs of statistical variables that show cause and effect relationship with each other.

Solution:

The following are two pairs of statistical variables that show 'cause and effect relationship' with each other:

i) ***Unemployment rate and inflation rate***: Unemployment and inflation are negatively related to each other. In other words, with a decrease in unemployment rate, the inflation rate rises and vice-versa. For instance, with the decrease in the unemployment, the income of people increases. This results in an increase in the demand for goods and services, which in turn leads to an increase in the general price level (inflation).

ii) ***Wage rate and price level****:* Wage rate and price level are positively related to each other. In other words, an increase in the general wage rates, results in a rise in the general price level. With an increase in the wage rate of workers, the cost of production increases. To maintain their level of profit margins, the producers pass on this increase in the cost of production to consumers in the form of higher prices.

Chapter – 2
Collection of Data

Essential Practical:

Q.1. Frame five two-way questions and five 'multiple choice' questions relating to a questionnaire that you intend to design for collecting primary data on the Level and Composition of expenditure of the people in your locality.
Example: Two-way question: Does your monthly expenditure ever exceed your monthly income during a year?

Yes	No

Multiple choice questions: Which of the following is your major item of expenditure during the month?

School Fee	Tuition Fee	Food
Clothing	House Rent	Others

Solution:

QUESTIONNAIRE
on
Level and Composition of Expenditure of People
Two-way Questions

Name: _____
Age: _____
Address: _____

1. Does your monthly expenditure exceed Rs 20,000?
   Yes ☐ No ☐
2. Do you save anything from your monthly income?
   Yes ☐ No ☐
3. How much do you spend on food and clothing?
   Less than 50% ☐ More than 50% ☐
4. Does your basket of goods contain luxury items?
   Yes ☐ No ☐
5. Does your monthly expenditure include educational expenses?
   Yes ☐ No ☐

QUESTIONNAIRE
on
Level and Composition of Expenditure of People
Multiple Choice Questions

Name: _____
Age: _____
Address: _____

1. **What percentage of your income do you spend on purchasing goods and services?**
   a. **10-30%**
   b. **30-50%**
   c. **50-70%**
   d. **70-100%**

2. **What percentage do you save from your monthly income?**
   a. **0-10%**
   b. **10-20%**
   c. **20-30%**
   d. **30% and above**

3. **How many times in a month do you go for an outing?**
   a. **Never**
   b. **Once**
   c. **Twice**
   d. **Thrice and above**

4. **Which of the items account for maximum monthly expenditure?**
   a. **Food**
   b. **Clothing**
   c. **Rent**
   d. **School fees**
   e. **Others (specify) _____**

5. **How much do you spend on luxury items?**
   a. **0-10%**
   b. **10-20%**
   c. **20-30%**
   d. **30% and above**

Q.2. Complete the following observations:

i) **Secondary data is the data_____.**
ii) **Sources of secondary data are_____.**
iii) **Primary data are always collected directly from the individuals____.**
iv) **Pilot survey is conducted to assess_____.**

Solution:

i) Secondary data is the data ***collected by other persons***. That is, secondary data is collected by an individual or a group of individuals or any agency in the past for their own purpose but later on used by some other party for some other or same purpose. If the data is collected on a special request then it will be treated as primary data rather than secondary data.

ii) Sources of secondary data are ***government publications, semi-government publications and private publications.*** Data collected from such sources are not original as they have not been collected directly by the investigator rather they had been collected by someone else in the past. The secondary sources of data can be in published or unpublished form.

iii) Primary data are always collected directly from the individuals ***who constitute the universe of study***. Primary data means collecting data directly from the actual source of information.

iv) Pilot survey is conducted to assess ***the quality of questionnaire***. Pilot survey are conducted before the final statistical investigation is done. It is a form of pre-testing of the questionnaire on a small sample of the universe that helps in judging the quality of the questionnaire and accordingly, getting to know if any modifications are required in it before the final survey takes place.

Chapter – 3
Census and Sample Methods of Collection of Data

Essential Practical:

Q.1. There are 10 students in your school who excel in the game of cricket. All are equally brilliant, but you have to select only 3 out of 10 for representing your school in the inter-zonal cricket tournament. How would you do it? Give details with reason.

Solution:

Lottery method of random sampling should be adopted to select 3 students out of 10 students. This is because all are equally brilliant, and thus, all of them should get an equal chance of being selected in the sample. Thus,

Selection process:

1. Write the name of each student on paper slips.
2. Put the slips in a box and shuffle them.
3. Select 3 slips one by one.
4. Students with their names written on the slips will be selected for the inter-zonal tournament.

Q.2. There are 40 students in your school who are cricket players. You are to form a team of 11 players. How would you do it? Would you resort to random sampling technique? Give reasons in support of your answer.

Solution:

We can use the random sampling technique if and only if all the students of the school are equally brilliant in the game of cricket, otherwise, if no such information is available non-random sampling technique can be used. In this regard, any of the non-random sampling, namely, judgement, quota or convenience sampling technique can be used. For instance, in accordance to the judgement sampling, based on your own opinion and judgement, any 11 students that in your opinion are the best, can be selected out of the total 40 students.

Q.3. You want to conduct a survey on the popularity of noodles among the students of different schools in your town. How would you design and choose your sample of study? Give reasons in support of your answer.

Solution:

To conduct a survey on the popularity of noodles among students of different schools in my town, the sample of study would be designed as follows:

1. Quota sampling method will be adopted.
2. Population will be divided into different groups according to different characteristics of population. Characteristics of students will include age-group, school in which the student studies etc.
3. A sample will be taken from each group.

Q.4. In a village comprising 300 small and big farmers, you are to select a sample of 10% farming households. The idea is to study the cropping pattern in the village. How would you take your decisions on selecting the sample?

Solution:

Stratified or mixed sampling will be used to study the cropping pattern in the village. According to this method, the population will be divided into different strata comprising different characteristics. Thus, in the given case, two groups will be formed of small farmers and big farmers. A sample will be selected from each stratum. Hence, the required sample will be received by combining the samples from both strata.

Q.5. How would you use the random sampling method when you are to select a sample of 3 out of 10 students in your class?

Solution:

Using the random sampling method (lottery method) to select 3 students out of 10 in the class:

i) Make ten paper slips of equal size and write the name of each student on each slip.
ii) Put all the slips in a box and mix them.
iii) Draw three slips at random without replacement.
iv) Students whose names are written on the slips are selected.

Chapter – 4
Organisation of Data

Essential Practical:

Q.1.

In an examination, 25 students secured the following marks:

23	28	30	32	35	35	36	40	41	43	44	45	45
48	49	52	53	54	56	56	58	61	62	65	68	

i. Arrange these data in the form of a frequency distribution using the following class as intervals: 20-29, 30-39, 40-49, 50-59, and 60-69.

ii. Arrange the data with cumulative frequencies.

Solution:

i. Datain the form of a frequency disribution :

Marks	Tally Bars	No. of students
20 - 29	II	2
30 - 39	𝍸	5
40 - 49	𝍸 III	8
50 - 59	𝍸 I	6
60 - 69	IIII	4
		$\sum f = 25$

ii. Data with cumulative frequencies:

Marks	No. of Students	Marks	No. of Students
Less than 29	0 +2 =2	More than 20	25
Less than 39	2 + 5 =7	More than 30	25 - 2 =23
Less than 49	7 + 8 =15	More than 40	23 - 5 =18
Less than 59	15 + 6 =21	More than 50	18 - 8 =10
Less than 69	21 + 4 =25	More than 60	10 - 6 =4

Q.2.

The following data is of the age of 25 students of Class XI.

Arrange these data in the form of a frequency distribution.

15	16	16	17	18	18	17	15	15	16	16	17	15
16	15	16	16	18	15	17	17	18	10	16	15	

Solution:

Data in the form of frequency distribution:

Age	Tally Bars	Frequency
10	I	1
15	𝍸 II	7
16	𝍸 III	8
17	𝍸	5
18	IIII	4
		$\sum f = 25$

Q.3.

Students of Class XI obtained following marks in economics. Classify the data in the form of individual series, discrete series, continuous series and cumulative frequency series.

15	16	16	17	18	18	17	15	15	16	16	17	15
16	15	16	16	18	15	17	17	18	10	16	15	

Solution:

Individual series:

15	15	15	15	16	16	16	17	18	18	18	18	19	20	20
20	21	22	22	22	22	23	24	24	24	25	25	25	25	25

Discrete series :

Marks	Tally Bars	No.of students
15	IIII	4
16	III	3
17	I	1
18	IIII	4
19	I	1
20	III	3
21	I	1
22	IIII	4
23	I	1
24	III	3
25	𝍸	5
		$\Sigma f = 30$

Continuous series :

Marks	Tally Bars	No. of Students
12 - 15	IIII	4
16 - 19	𝍸 IIII	9
20 - 23	𝍸 IIII	9
24 - 27	𝍸 III	8
		$\Sigma f = 30$

Cumulative frequency series:

Marks	No. of Students	Marks	No. of Students
Less than 15	0	More than 12	30
Less than 19	9 + 4 =13	More than 16	30 - 4 =26
Less than 23	13 + 9 =22	More than 20	26 - 9 =17
Less than 27	22 + 8 = 30	More than 24	17 - 9 =8

Q.4.

Arrange the following data in the form of an exclusive frequency distribution, using 5-10 as the initial class interval:

12	36	40	30	28	20	19	10	10	19	27	15	26	10
19	7	45	33	26	37	5	20	11	17	37	30	20	

Solution: Exclusive frequency distribution:

Class Interval	Tally Bars	Frequency
5 - 10	II	2
10 - 15	𝍸	5
15 - 20	𝍸	5
20 - 25	III	3
25 - 30	IIII	4
30 - 35	III	3
35 - 40	III	3
40 - 45	I	1
45 - 50	I	1
		$\Sigma f = 27$

Q.5.

Weight of 20 students is given in kilograms. Using class interval of 5, make a frequency distribution.

30	45	26	25	42	33	15	35	45	45
45	39	42	40	18	35	41	20	36	48

Solution:

Frequency distribution:

Weight (kg)	Tally Bars	No. of Students
15 - 20	II	2
20 - 25	I	1
25 - 30	II	2
30 - 35	II	2
35 - 40	IIII	4
40 - 45	IIII	4
45 - 50	I 𝍸 I	5
Total		$\sum f = 20$

Q.6.

Convert the following data in a simple frequency distribution:

5 students obtained less than 3 marks
12 students obtained less than 6 marks
25 students obtained less than 9 marks
33 students obtained less than 12 marks

Solution:

Given data can be written as:

Marks	Cumulative Frequency (c.f)
Less than 3	5
Less than 6	12
Less than 9	25
Less than 12	33

Simple frequency distribution:

Marks	Frequency (f)
0 - 3	5
3 - 6	7 (= 12 - 5)
6 - 9	13(= 25 - 12)
9 - 12	8(= 33 - 25)
	$\sum f = 33$

Q.7.

In the following statement, take the number of letters in a word a items and numbers of times a word (of the same size) repeats itself as frequencies. Prepare a discrete series.

"Success in the examination confers no absolute right to appointment unless government is satisfied after such an enquiry as may be considered necessary that the candidate is suitable in all respects for appointment."

Solution:

Discrete series:

Size of item	Tally Bars	Frequency
2	𝍸 IIII	9
3	𝍸	5
4	II	2
5	II	2
6	I	1
7	III	3
8	III	3
9	III	3
10	II	2
11	III	3
Total		$\sum f = 33$

Q.8.

An economics survey revealed that 30 families in a town incur following expenditure in a day (rupees)

11	12	14	16	16	17	18	18	20	20	20	21	21	22	22
23	23	24	25	25	26	27	28	28	31	32	32	33	36	38

i. Convert these data in the form of a frequency distribution, using the following class intervals 10-14, 15-19, 20-24,25-29,30-34 and 35-39.

ii. How many families spend more than 29 rupees a day?

Solution:

i. **Frequency distribution:**

Expenditure	Tally Bars	No. of Families
10-14	III	3
15-19	𝍸	5
20-24	𝍸 𝍸	10
25-29	𝍸 I	6
30-34	IIII	4
35-39	II	2
		$\sum f = 30$

ii. Families spending more than Rs 29 per day = 4 + 2 = 6

Percentage of families spending more than Rs 29

$$= \frac{\text{Families spending more than Rs 29}}{\text{Total families}} \times 100$$

$$= \frac{6}{30} \times 100 = 20$$

Hence, 20% of the families spend more than Rs 29 per day.

Q.9.

From the following data related to the weight of college students in kg, prepare a frequency distribution with a class interval of 10 on exclusive and inclusive basis:

40	70	63	53	85
92	72	65	53	79
49	42	43	47	50
52	50	48	65	42
69	60	54	82	55

Solution:

Frequency distribution on exclusive basis:

Weight (in kg)	No. of students (f)
40 - 50	7
50 - 60	7
60 - 70	5
70 - 80	3
80 - 90	2
90 - 100	1
	∑f = 25

Frequency distribution on inclusive basis:

Weight (in kg)	No. of students (f)
40 - 50	9
51 - 61	6
62 - 72	6
73 - 83	2
84 - 94	2
95 - 105	0
	∑f = 25

Q.10.

Construct the sample frequency distribution from the following data:

Mid-value	5	15	25	35	45	55
Frequency	2	8	15	12	7	6

Solution:

Lower limits and upper limits of class intervals are calculated using the following formula.

$$\text{Lower limit}(l_1) = m - \frac{1}{2}i$$

$$\text{Upper limit}(l_2) = m + \frac{1}{2}i$$

where m is mid value and i is the difference between mid-values.

Mid-value	Class-interval	Frequency (f)
5	0 - 10	2
15	10 - 20	8
25	20 - 30	15
35	30 - 40	12
45	40 - 50	7
55	50 - 60	6
		∑f = 50

Q.11.

Classify the following data by taking class interval such that their mid-values are 17,22,27,32 and so on:

30	42	30	54	40	48	14	17	51	42	25	41
30	27	42	36	28	28	37	54	44	31	36	40
36	22	30	31	19	48	16	42	32	21	22	40
33	41	21	16	17	36	37	41	46	47	52	53

Solution:

Lower limits and upper limits of class intervals are calculated using the following formula.

$$\text{Lower limit}\,(l_1) = m - \frac{1}{2}i$$

$$\text{Upper limit}\,(l_2) = m + \frac{1}{2}i$$

Where m is mid value and i is the difference between mid-values.

Mid-value	Class-interval	Frequency (f)
12	9.5 - 14.5	1
17	14.5 - 19.5	5
22	19.5- 24.5	4
27	24.5 - 29.5	4
32	29.5 - 34.5	8
37	34.5 - 39.5	6
42	39.5 - 44.5	11
47	44.5 - 49.5	4
52	49.5 - 54.5	5
		∑f = 48

Chapter – 5

Presentation of Data- Textual and Tabular Presentation

Essential Practical:

Q.1. In 2009-10, the contribution of primary, secondary and tertiary sector to India's GDP was17.6%, 28.2% and 54.2% respectively. In 2010-11 these shares were 17.7%, 27.0% and 55.3% respectively. This information is based on the India 2013 (EPW Research Foundation). Present this information in the form of a table.

Solution:

Contribution of primary, Secondary and Tertiary Sector to India's GDP

(in percentage)

Sector	2009-10	2010-11
Primary Sector	17.6	17.7
Secondary Sector	28.2	27
Tertiary Sector	54.2	55.3
Total	100	100

Source: India 2013, EPW Research Foundation

Q.2. Prepare a sketch of such a table that exhibits the following types of information on the numbers of students of your college:

i) Faculty-wise: Art, Commerce and Science'
ii) Class-wise: XI; B.A (I); B.A (II); and B.A (III).
iii) Sex-wise: Boys and Girls.

Solution:

Distribution of Students (according to faculty, class and sex)									
	ARTS			**Commerce**			**Science**		
Class	**Boys**	**Girls**	**Total**	**Boys**	**Girls**	**Total**	**Boys**	**Girls**	**Total**
XI									
B.A (I)									
B.A (II)									
B.A (III)									
Total									

Q.3. Following information related to the exports from India to USSR and UK in the years 2011, 2012, 2013 and 2014. Present it in the form of a table.

Year	**2011**	**2012**	**2013**	**2014**
Exports to USSR (Rs crore)	209	416	1,305	1,655
Exports to UK (Rs crore)	170	421	550	670

Solution:

Exports from India to USSR and UK		
	(2011-2014)	
Year	**Export to USSR (Rs crore)**	**Export to UK (Rs Crore)**
2011	209	170
2012	416	421
2013	1305	550
2014	1655	670

Q.4. Point out the mistakes in the following table. Rearrange it correctly.

Number of students	Subjects			
	Economics	English	Hindi	History
Boys				
Girls				

Solution:

Mistakes in the given table:

1. Title and head note are not provided.
2. Sub-entries and the captions are not written correctly.
3. Total of the rows and the columns is missing.

Number of Students (according to subject and sex)			
Number of Students			
Subjects	**Boys**	**Girls**	**Total**
Economics			
English			
Hindi			
History			
Total			

Q.5. Following information related to the marks secured by 50 students in Economics. Present the information in the form of a table.

Marks	0-10	10-20	20-30	30-40
Students	15	12	18	5

Solution:

Marks Secured by Students in Economics

Marks	Number of Students
0 - 10	15
10 - 20	12
20 - 30	18
30 -40	5

Q.6. Following information related to the marks secured by 50 boys and girls in their paper in Economics. Present the information in the form of a two-way table.

Marks	0-10	10-20	20-30	30-40
Boys	10	7	6	1
Girls	5	5	12	4

Solution:

Marks Secured by Students in Economics

Marks	Numbers of Students		Total
	Boys	**Girls**	
0 - 10	10	5	15
10 - 20	7	5	12
20 - 30	6	12	18
30 - 40	1	4	5
Total	24	26	50

Q.7. Draw a blank to show the distribution of population according to sex, literacy, and income.

Solution:

Distribution of Population (according to sex, literacy and income)						
Income (in'000)	**Literate**			**Illiterate**		
(0 - 20)	**Male**	**Female**	**Total**	**Male**	**Female**	**Total**
(20 - 40)						
(40 - 60)						
(60 - 80)						
(80 - 100)						
Above 100						
Total						

Q.8. Present the following information in a suitable tabular form:

i. **In 2007, out of total 2,000 workers in a factory, 1,550 were members of a trade union. The number of woman workers employees was 250, out of which 200 did not belong to any trade union.**

ii. **In 2013, the number of union workers was 1,725 of which 1,600 were men. The number of non-union workers was 380, among which 155 were women.**

Solution:

Distribution of Workers in a Factory (according to member of trade union and sex)									
	Trade Union			**Non-Trade Union**			**Total**		
Year	**Male**	**Female**	**Total**	**Male**	**Female**	**Total**	**Male**	**Female**	**Total**
2007	1500	50	1550	250	200	450	1750	250	2000
2013	1600	125	1725	225	155	380	1825	280	2105
Total	3100	175	3275	475	355	830	3575	530	4105

Chapter – 6

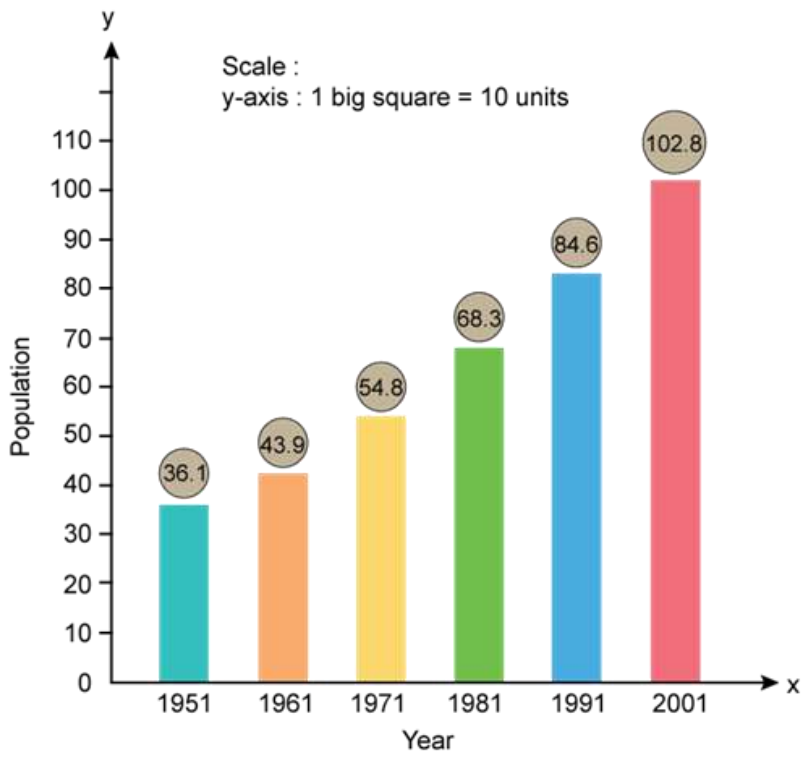
Diagrammatic Presentation of Data: Bar Diagrams

Essential Practical:

Q.1. Make a suitable diagram of the following data on population in India.

Year	1951	1961	1971	1981	1991	2001	2011
Population (crore)	36.1	43.9	54.8	68.3	84.6	102.8	121.0

Solution: Population of India (Rs crore)

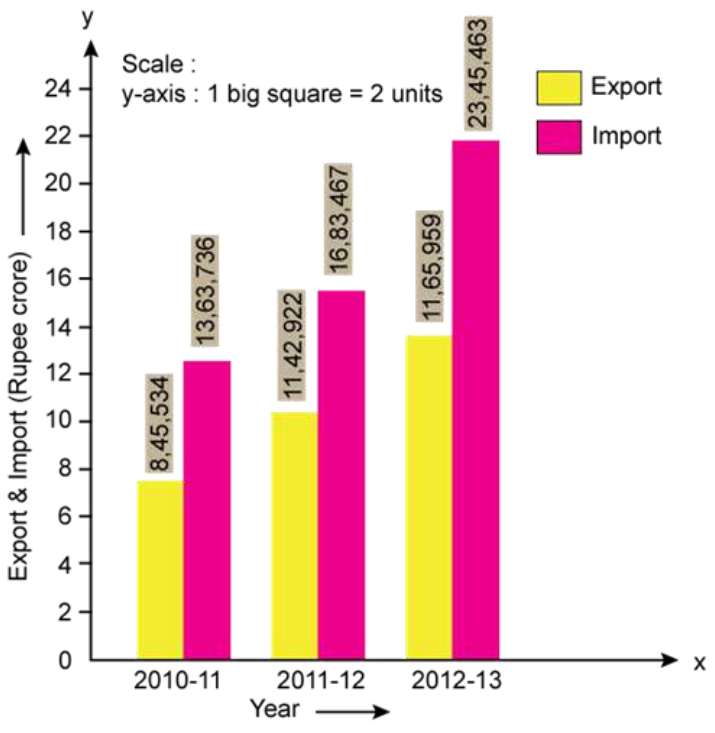


Q.2. Give a diagrammatic presentation of the following data on India's Exports and Imports:

Year	Export (Rs crore)	Import (Rs crore)
2010-11	8,45,534	13,63,736
2011-12	11,42,922	16,83,467
2012-13	14,65,959	23,45,463

Solution:

India's Exports and Imports (Rs crore)

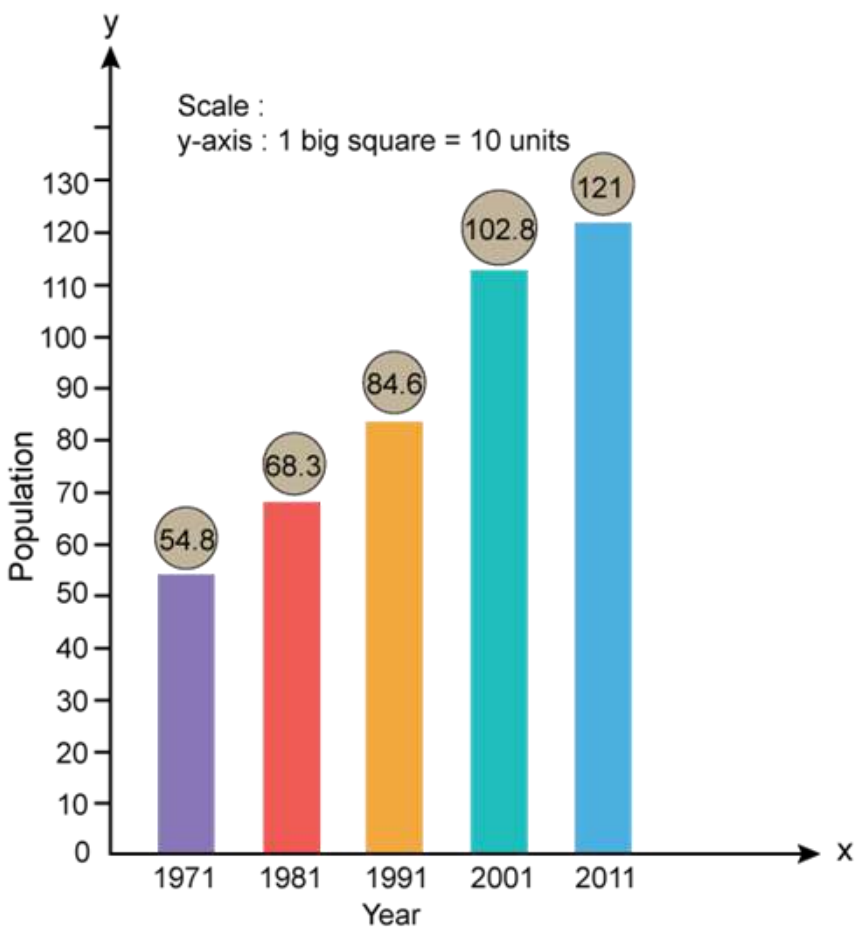


Q.3. Make a bar diagram of the following data on India's population:

Year	1971	1981	1991	2001	2011
Population (crore)	54.8	68.3	84.6	102.8	121.0

Solution:

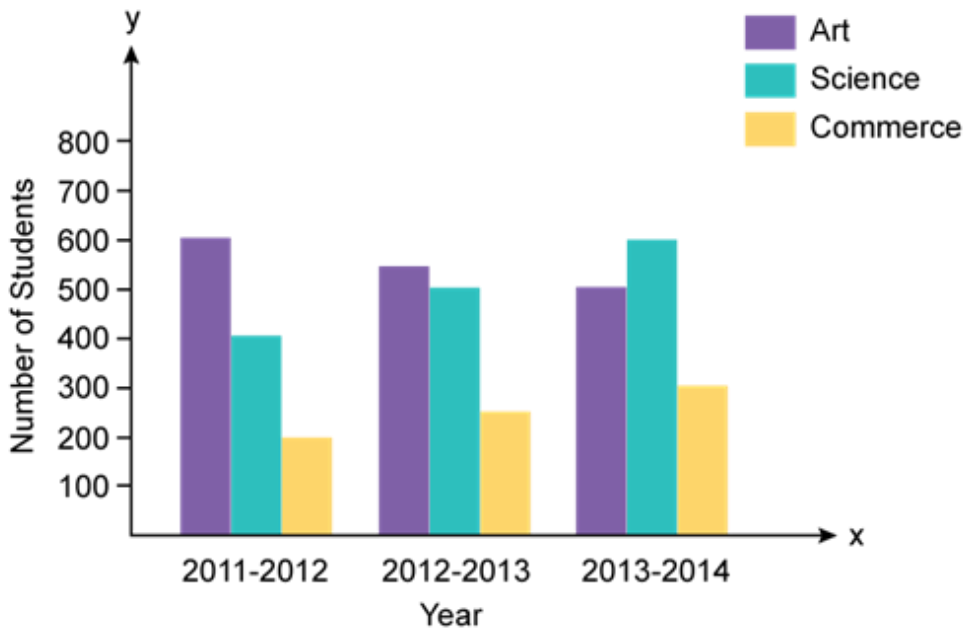
India's Population (Rs crore)



Q.4. Make a multiple bar diagram of the following data:

Faculty	Number of Students		
	2011-12	2012-13	2013-14
Art	600	550	500
Science	400	500	600
commerce	200	250	300

Solution: Number of Students in different Streams



Q.5. Following table shows estimates of cost of production of certain commodities. Present the data in the form of a sub-divided bar diagram:

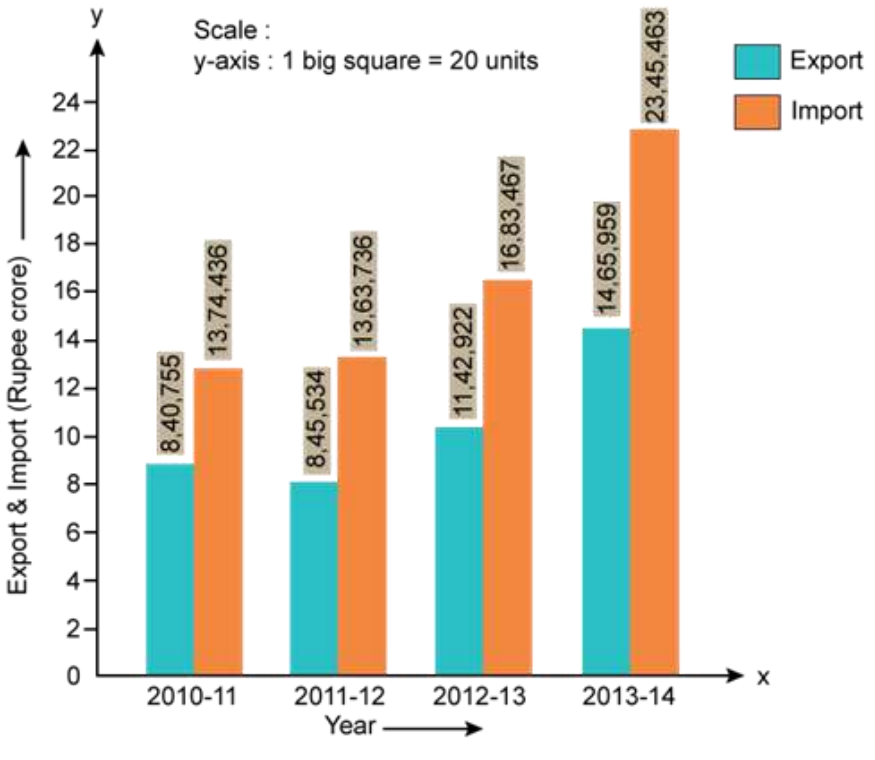
Estimate of Cost	Goods			
	A	B	C	D
Raw material	50	40	45	50
Wages	40	40	40	40
Fixed cost	10	12	15	15
Office expenses	10	8	10	5
Total	110	100	110	110

Solution: Cost of Production of Commodities

Q.6. Present the following data in the form of a multiple bar diagram:

Year	Export (Rs crore)	Import (Rs crore)
2010-11	8,40,755	13,74,436
2011-12	8,45,534	13,63,736
2012-13	11,42,922	16,83,467
2013-14	14,65,959	23,45,463

Solution: Multiple Bar Diagram



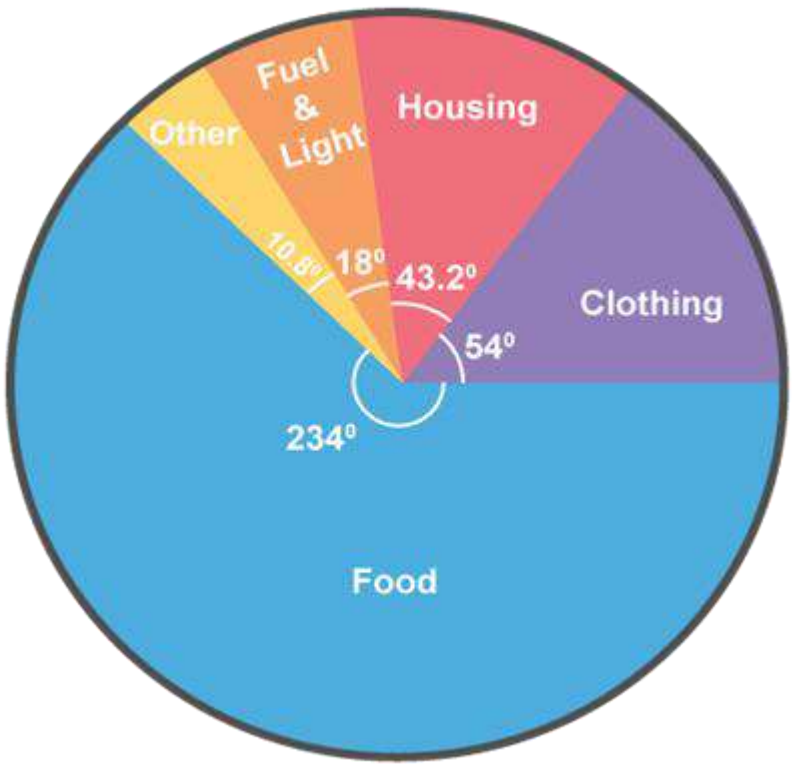
Q.7. What do you mean by a circular diagram? Present the data on the expenditure of labour-family in the form of a circular diagram:

Items of Expenditure	Food	Clothing	Housing	Fuel and Light	Others
Percentage of Income Spent	65	15	12	5	3

Solution: Circular diagram also known as pie diagram shows the percentage values of a series which is presented in a circle divided into various segments.

Item	Percentage Share	Share in terms of degree
Food	65	Degree share of Food $= \frac{65}{100} \times 360 = 234°$
Clothing	15	Degree share of Clothing $= \frac{15}{100} \times 360 = 54°$
Housing	12	Degree share of Housing $= \frac{12}{100} \times 360 = 43.2°$
Fuel & Light	5	Degree share of Fuel and Light $= \frac{5}{100} \times 360 = 18°$
Others	3	Degree share of Others $= \frac{3}{100} \times 360 = 10.8°$

Pie Diagram



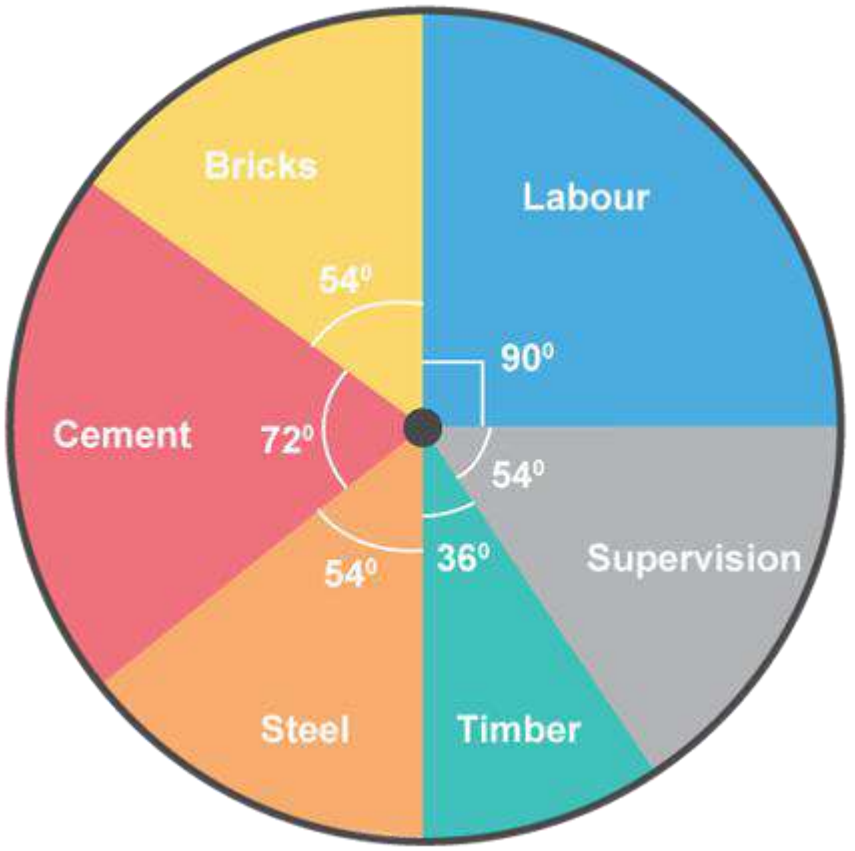
Q.8. Following data relate to the construction of a house in Delhi. Present the information in the form of a pie diagram:

Items	labour	Bricks	Cement	steel	timber	Supervision
Percentage Expenditure	25	15	20	15	10	15

Solution:

Item	Percentage Share	Share in terms of degree
Labour	25	Degree share of Labour $= \frac{25}{100} \times 360 = 90°$
Bricks	15	Degree share of Bricks $= \frac{15}{100} \times 360 = 54°$
Cement	20	Degree share of Cement $= \frac{20}{100} \times 360 = 72°$
Steel	15	Degree share of Steel $= \frac{15}{100} \times 360 = 54°$
Timber	10	Degree share of Timber $= \frac{10}{100} \times 360 = 36°$
Supervision	15	Degree share of Supervision $= \frac{15}{100} \times 360 = 54°$

Pie diagram



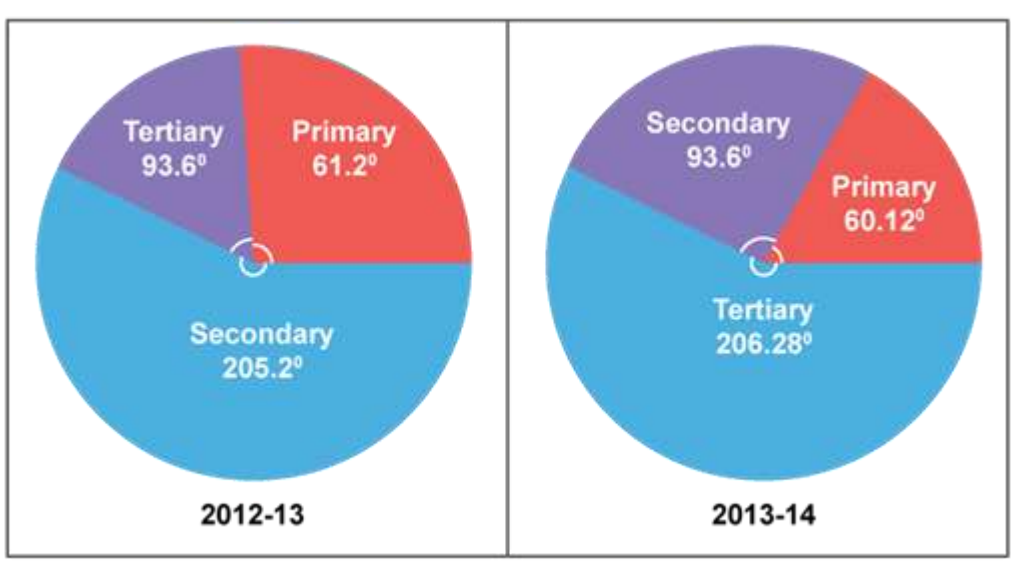
Q.9. For the years 2012-13 and 2013-14, value of gross domestic product at factor cost by the industry of origin is given in the following table. Present the information in the form of Pie Diagram showing difference in the percentage contribution of different sectors between the said years.

Sector	Year	
	2012-13	2013-14
Primary	17	16.7
Secondary	57	26
Tertiary	26	57.3
Total	100	100

[Hint: Make separate Pie diagrams for the year 2012-13 and 2013-14.]

Solution:

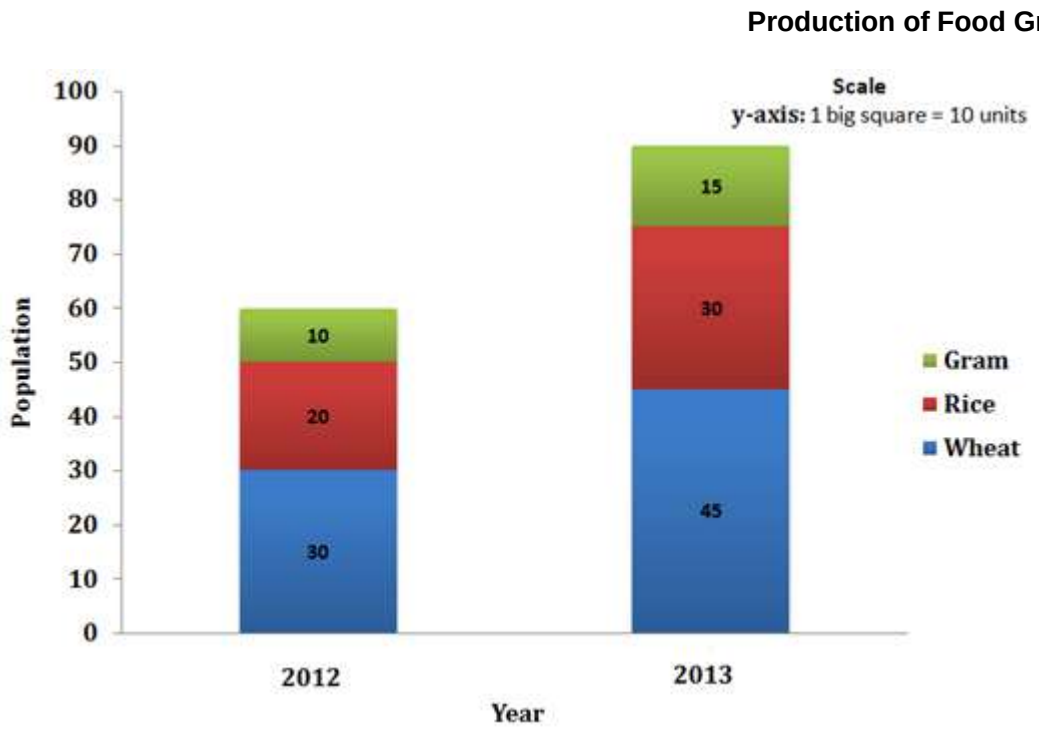
2012-2013			2013-2014		
Sector	**% contribution**	**Degree Share**	**Sector**	**% contribution**	**Degree Share**
Primary	17	17 ×36/100=61.2	Primary	16.7	16.7 ×36/100=60.12^0
Secondary	57	57 × 36/100=205	Secondary	26	26 × 36/100=93.6^0
Tertiary	26	26 ×36/100=93.6	Tertiary	57.3	57.3 ×36/100=206.28^0



Q.10. Present the following data on the production of food grains in the form of a sub-divided bar diagram:

Year	Wheat	Rice	Gram	Total
2012	30	20	10	60
2013	45	30	15	90

Solution:

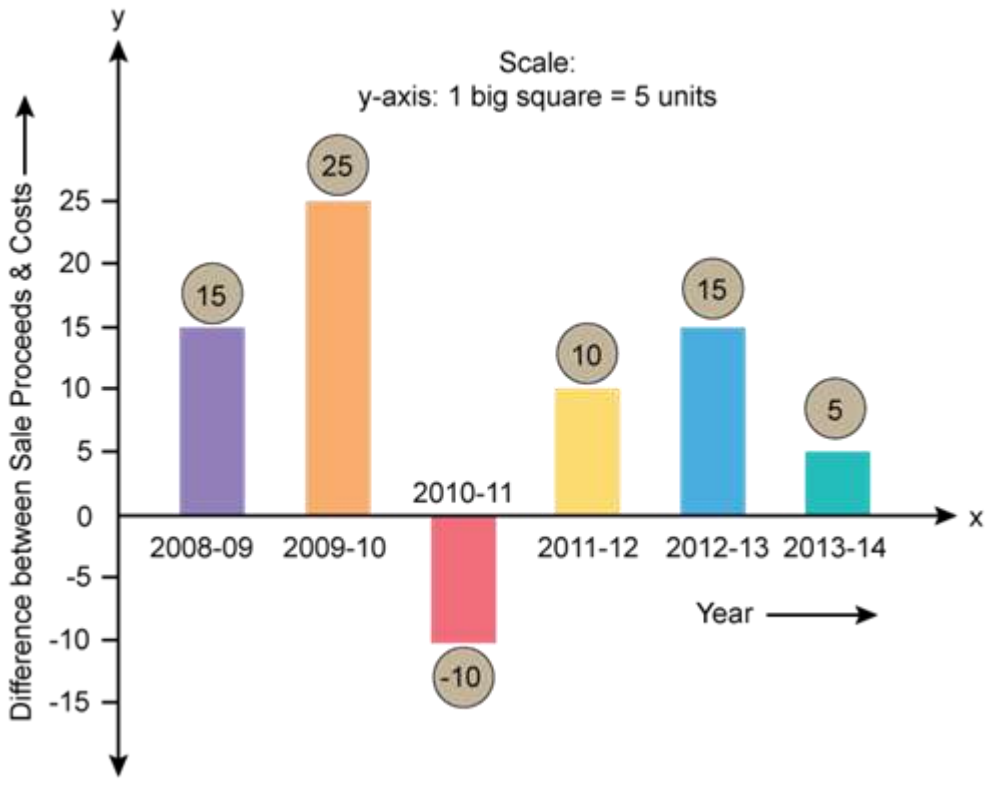


Q.11. Present the following data by a deviation bar diagram, showing the difference between sale proceeds and cost of firm.

Year	Sale Proceeds (Rs in lakh)	Costs (Rs in lakh)
2008 - 09	115	100
2009 - 10	140	115
2010 - 11	145	155
2011 - 12	150	140
2012 - 13	160	145
2013 - 14	170	165

Solution:

Year	Sale Proceeds	Costs	Sale Proceeds -Cost
2008 - 09	115	100	15
2009 - 10	140	115	25
2010 - 11	145	155	-10
2011 - 12	150	140	10
2012 - 13	160	145	15
2013 - 14	170	165	5


y
Scale:
y-axis: 1 big square = 5 units
Difference between Sale Proceeds & Costs
25
20
15
10
5
0
-5
-10
-15
15
25
2010-11
10
15
5
-10
x
2008-09
2009-10
2011-12
2012-13
2013-14
Year

Chapter – 7

Frequency Diagrams: Histogram, Polygon and Ogive

Essential Practical:

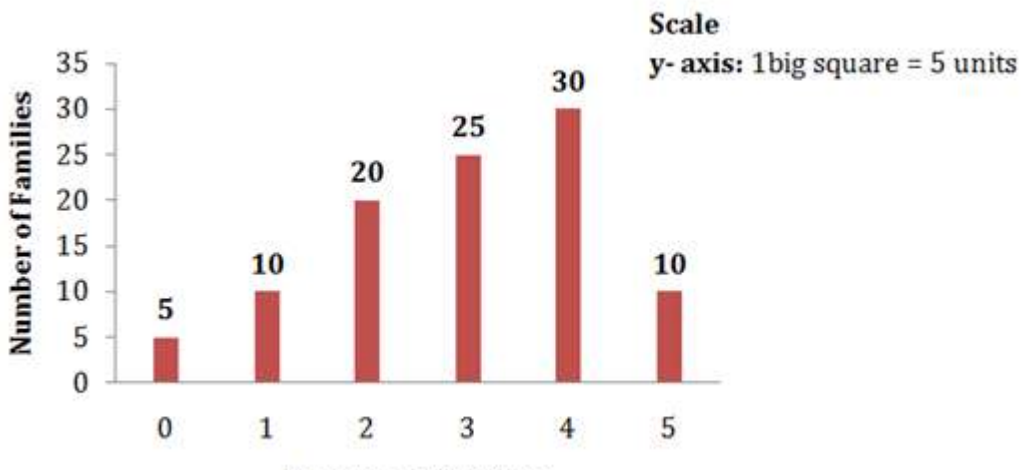
Q.1. Represent the following frequency distribution graphically:

Number of Children	Number of families
0	5
1	10
2	20
3	25
4	30
5	10
	Total = 100

Solution:

The given distribution can be represented with the help of a bar diagram as follows:

Bar Diagram

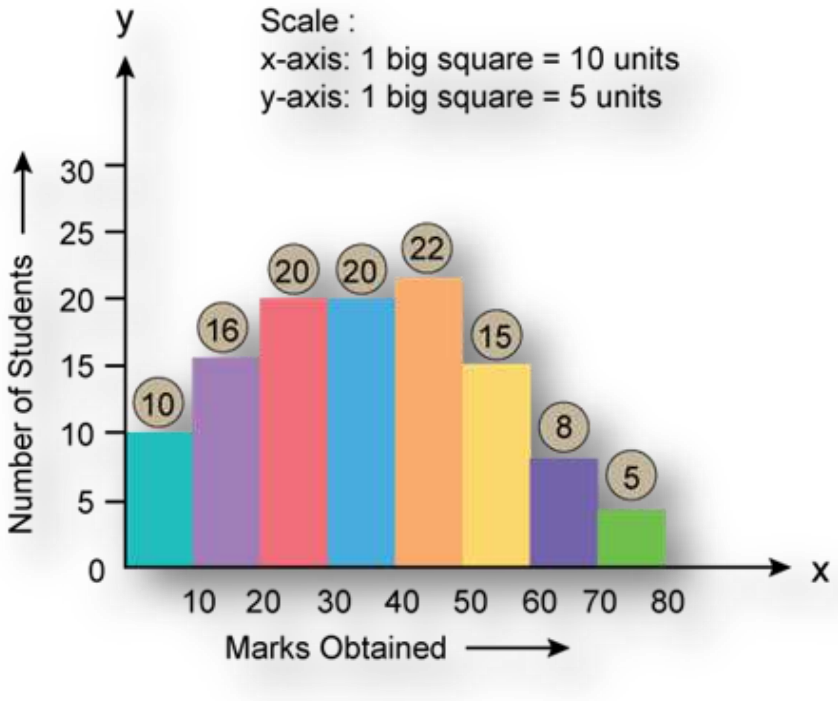


Q.2. Construct histogram, frequency polygon and frequency curve from the following data:

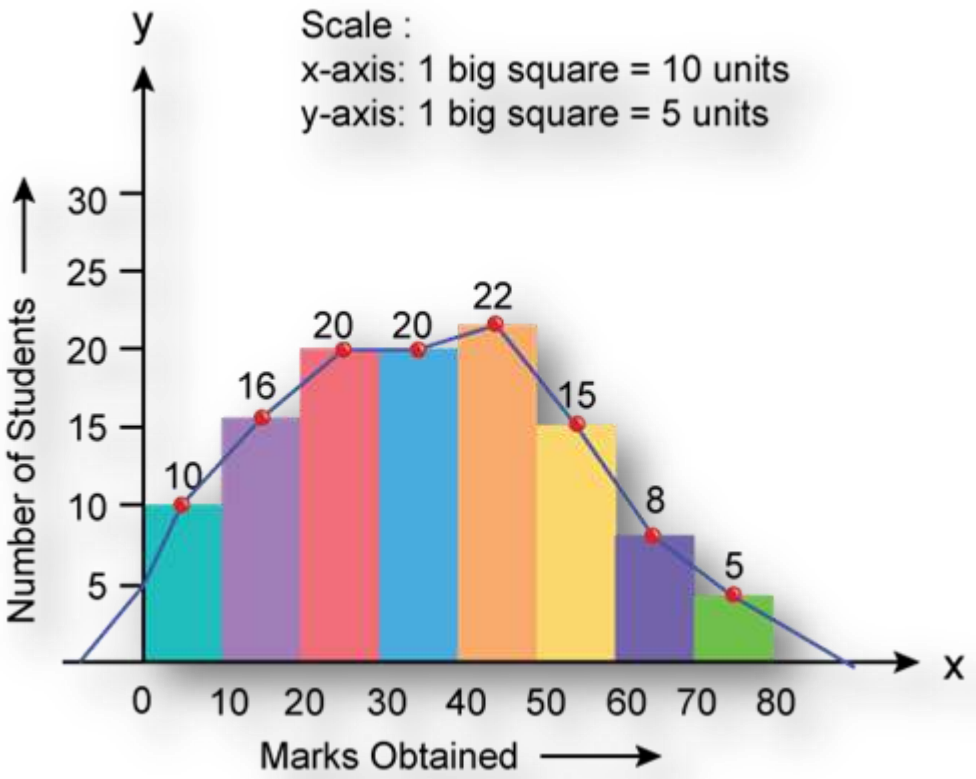
Marks Obtained	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Number of Students	10	16	20	20	22	15	8	5

Solution:

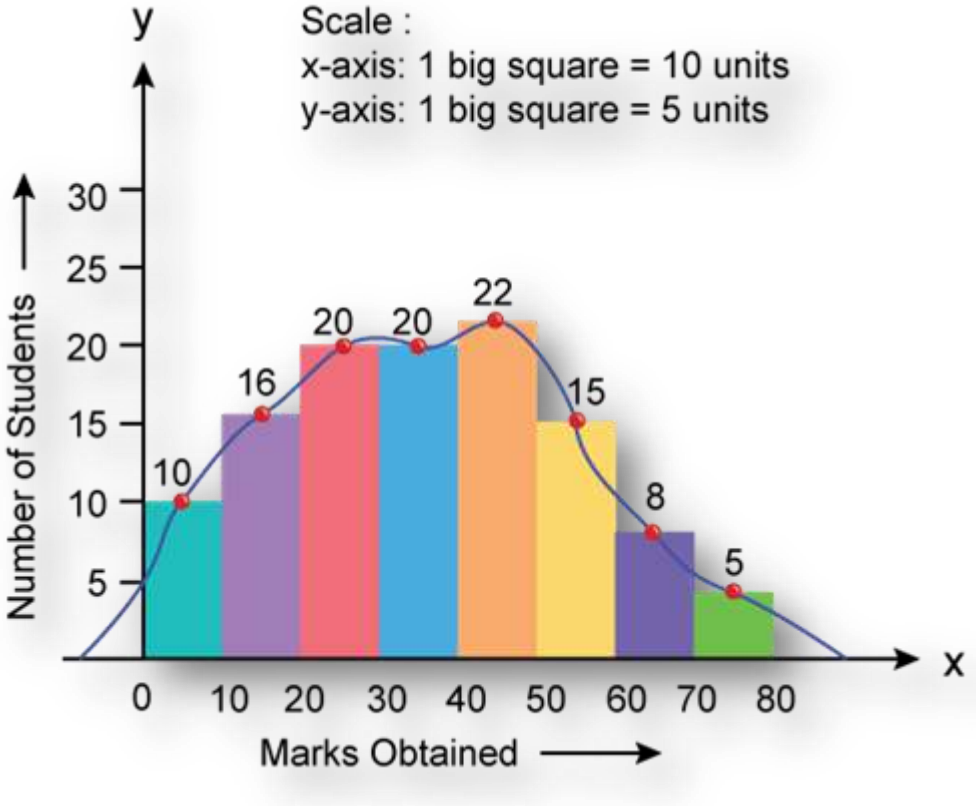
i. **Histogram:** A histogram is two dimensional diagrams.



ii. **Frequency Polygon:** A frequency polygon is drawn by joining the mid-points of all tops of a histogram. Here, the points are joined by using a foot rule.



iii. **Frequency Curve:** Similar to frequency polygon, a frequency curve is drawn by joining the mid-points of all tops of a histogram. But, the points are joined using a free hand.

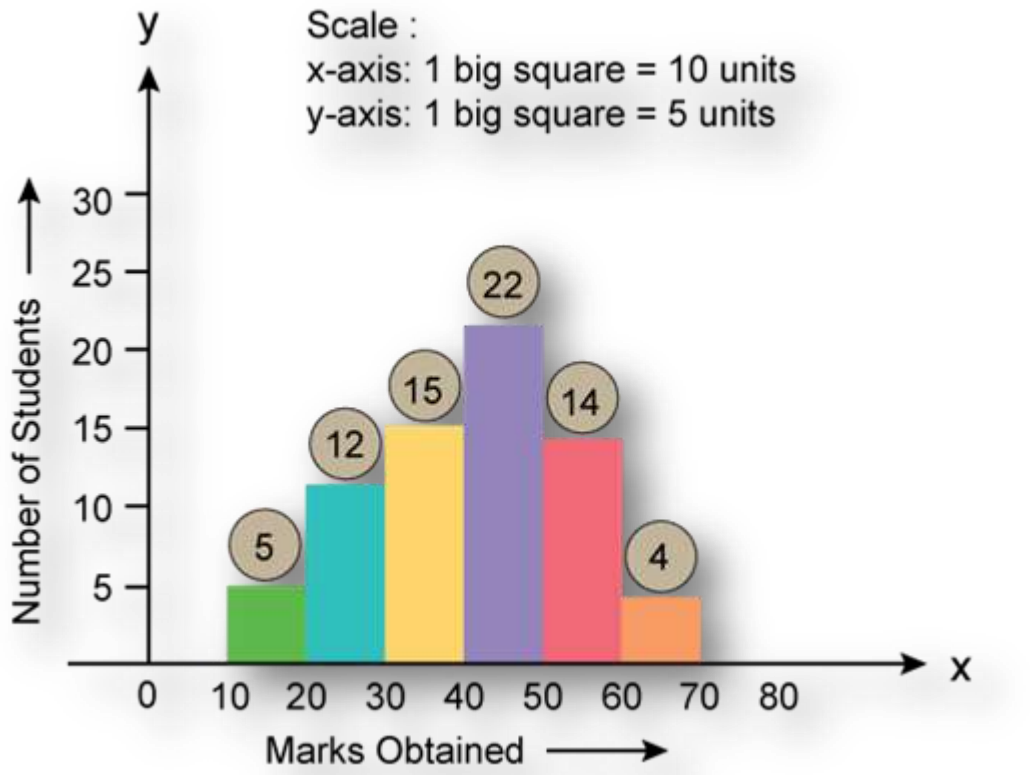


Q.3. Make a frequency polygon and histogram using the given data:

Marks Obtained	10-20	20-30	30-40	40-50	50-60	60-70
Number of Students	5	12	15	22	14	4

Solution:

Histogram:



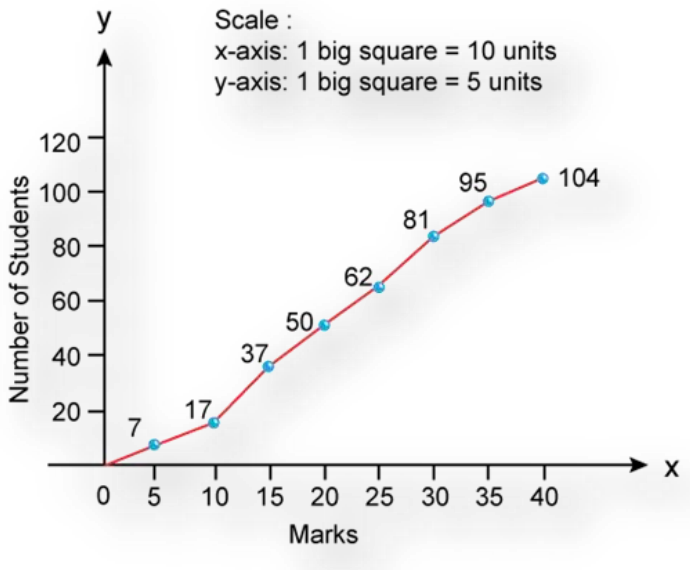
Q.4. Draw 'less than 'and more than' ogive curves from the following data:

Marks	0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40
Number of Students	7	10	20	13	12	19	14	9

Solution: i. Less than ogive curves: In this method, frequencies are cumulated and presented in a graph corresponding to upper limits of the classes in a frequency distribution. Firstly, all the data are converted into less than cumulative frequency distribution as follows-

Marks	**Cumulative Frequency**
Less than 5	7
Less than 10	7 + 10 = 17
Less than 15	17 + 20 = 37
Less than 20	37 + 13 = 50
Less than 25	50 + 12 = 62
Less than 30	62 + 19 = 81
Less than 35	81 + 14 = 95
Less than 40	95 + 9 = 104

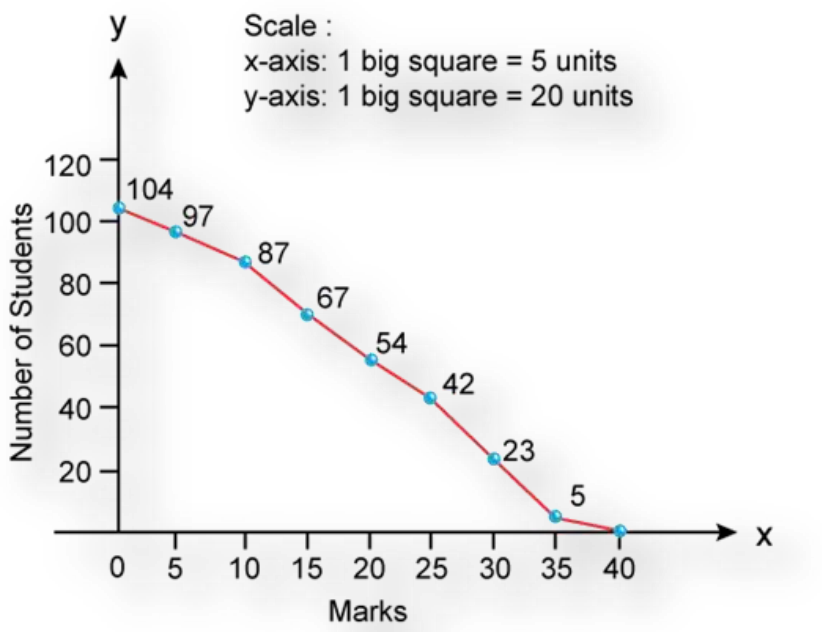
This curve is drawn by plotting cumulative frequencies against the upper limit of the class intervals. And these points are joined to obtain the less than ogive curve.



ii. **More than ogive curves:** In this method, frequencies are cumulated and presented in a graph corresponding to lower limits of the classes in a frequency distribution. Firstly, all the data are converted into more than cumulative frequency distribution as follows-

Marks	Cumulative Frequency
More than 0	104
More than 5	104 - 7 = 97
More than 10	97 - 10 = 87
More than 15	87 - 20 = 67
More than 20	67 - 13 = 54
More than 25	54 - 12 = 42
More than 30	42 - 19 = 23
More than 35	23 - 14 = 9
More than 40	9 - 9 =0

This curve is drawn by plotting cumulative frequencies against the lower limit of the class intervals. And these points are joined to obtain more than ogive curve.



Q.5. What is meant by ogive or cumulative frequency curve? From the following distribution construct the 'less than' ogive:

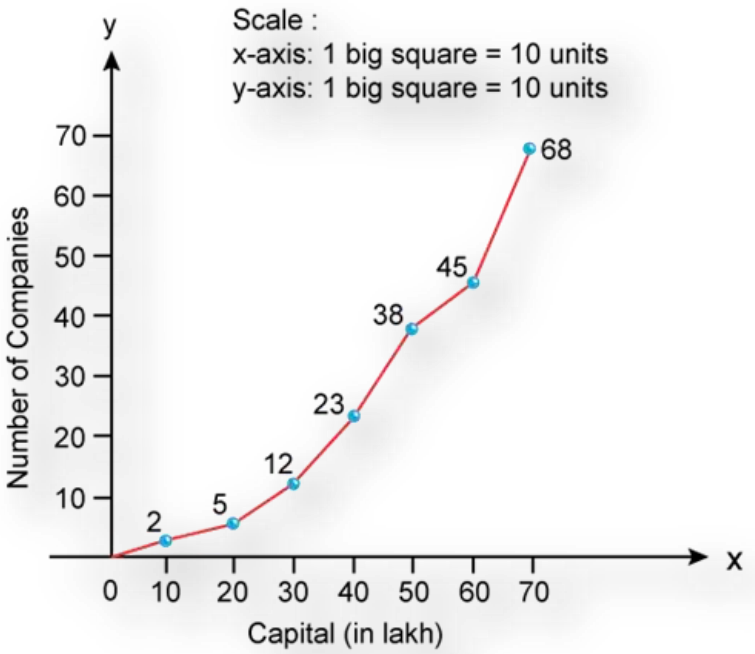
Capital (in lakh)	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Number of Companies	2	3	7	11	15	7	23

Solution: Ogive curve is a smooth curve presented by plotting the frequency data on a graph. This curve represents the frequencies corresponding to lower limits or upper limits in the distribution of data.

Less than ogive curves: In this method, frequencies are cumulated and presented in a graph corresponding to upper limits of the classes in a frequency distribution. Firstly, all the data are converted into less than cumulative frequency distribution as follows-

Capital (in Lakh)	Cumulative Frequency
Less than 10	2
Less than20	2 + 3 = 5
Less than 30	5 + 7 = 12
Less than 40	12 + 11 = 23
Less than 50	23 + 15 = 38
Less than 60	38 + 7 = 45
Less than 70	45 + 23 = 68

This curve is drawn by plotting cumulative frequencies against the upper limit of the class intervals. And these points are joined to obtain the less than ogive curve.

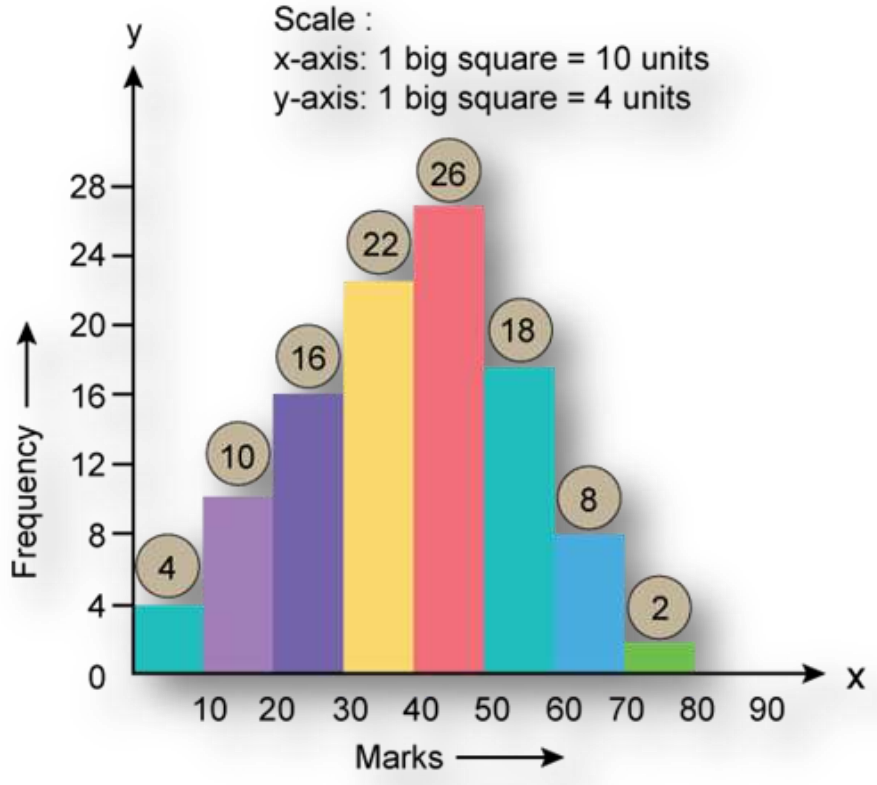


Q.6. Present the data given in the table below in a histogram:

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	4	10	16	22	26	18	8	2

Solution:

Histogram

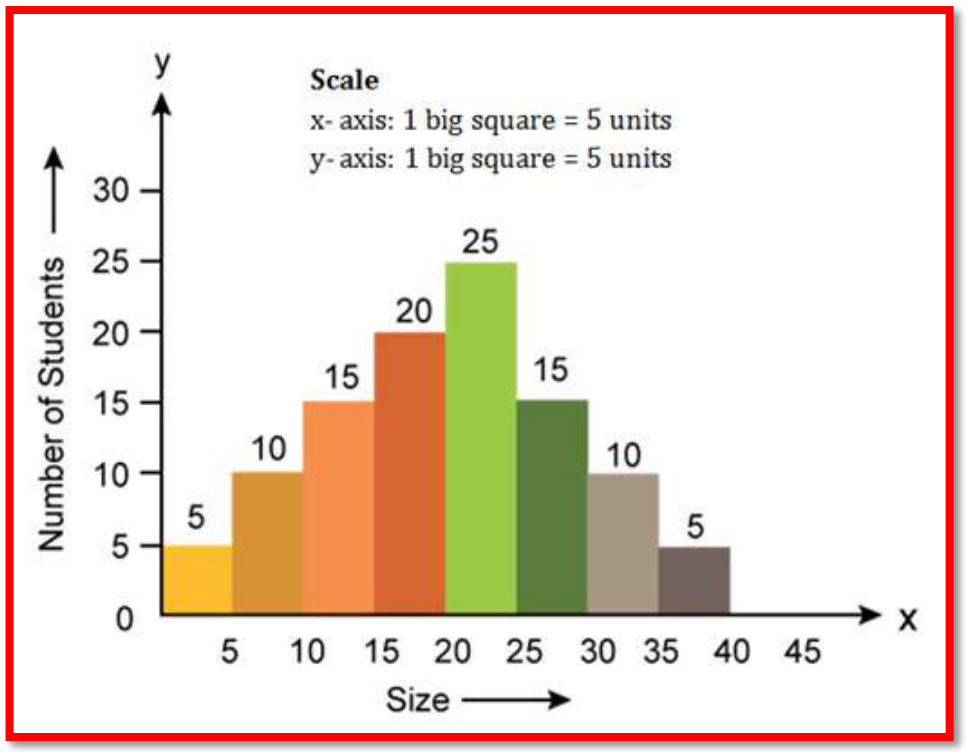


Q.7. Draw a histogram from the following data relating to the monthly pocket allowance of the students of Class XI of a school:

Size	0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40
Number of student	5	10	15	20	25	15	10	5

Solution:

Histogram



Q.8. We are given the following marks secured by 25 students in an examination.
23,28,30,32,35,36,40,41,43,44,44,45,48,49,52,53,54,56,56,58,61,62,65,68.

i. **Arrange this data in the form of a frequency distribution taking the following class intervals.**
20-29,30-39,40-49,50-59 and 60-69

ii. **Draw the frequency polygon and ogive for the above data.**

Solution:

i. The frequency distribution of the given data is as follows:

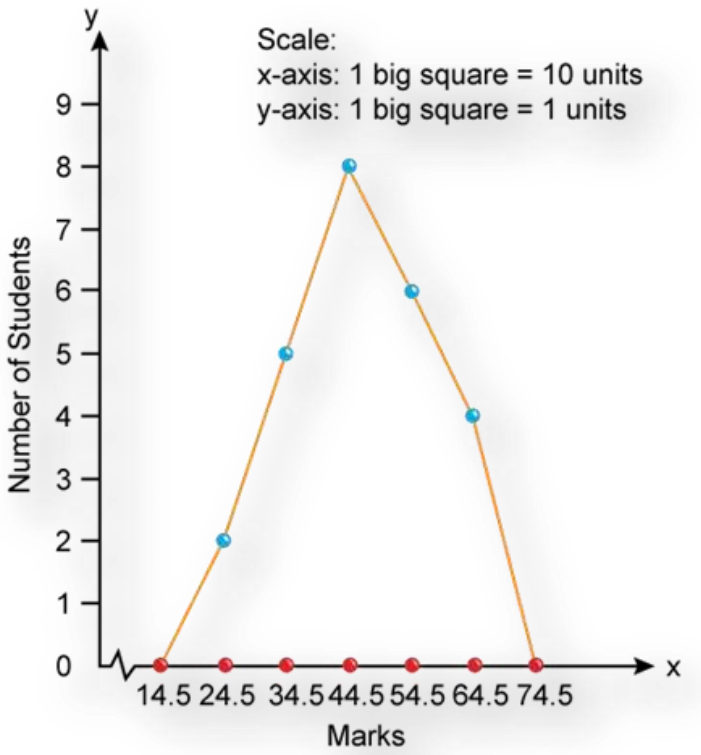
Class Interval (Marks)	Frequency (No. of Student)
20 - 29	2
30 - 39	5
40 - 49	8
50 - 59	6
60 - 69	4
	∑f = 25

ii.

a. **Frequency polygon:** Presenting the frequencies in the form of rectangle and joining the mid-points of the tops of the consecutive rectangles is known as frequency polygon. It is an alternative to histogram which is derived from histogram itself. However, frequency polygon can be drawn even without presenting the histogram.

Firstly, the mid points of the respective class intervals are calculated and presented graphically against their respective frequencies. Here, the points are joined by using a foot rule to obtain the frequency polygon curve.

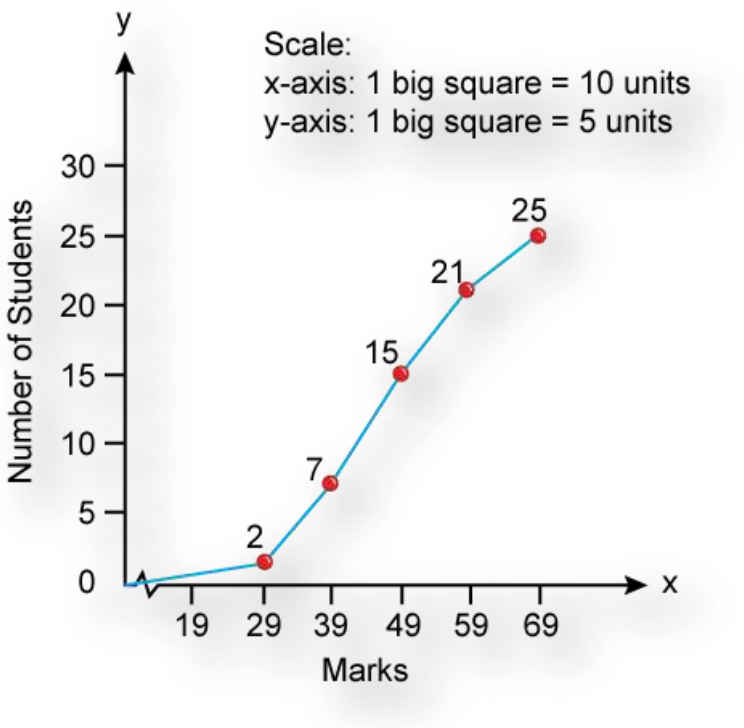
Marks	Mid Value	No. of Students
20 – 29	$\frac{20+20}{2} = 24.5$	2
30 – 39	$\frac{30+39}{2} = 34.5$	5
40 – 49	$\frac{40+49}{2} = 44.5$	8
50 – 59	$\frac{50+59}{2} = 54.5$	6
60 – 69	$\frac{60+69}{2} = 64.5$	4
Total		25



b. **Less than ogive curves:** In this method, frequencies are cumulated and presented in a graph corresponding to upper limits of the classes in a frequency distribution. Firstly, all the data are converted into less than cumulative frequency distribution as follows-

Less than Ogive	Cumulative Frequency
Less than 29	2
Less than 39	2 + 5 = 7
Less than 49	7 + 8 = 15
Less than 59	15+6=21
Less than 69	21+4=25

This curve is drawn by plotting cumulative frequencies against the upper limit of the class intervals. And these points are joined to obtain the less than ogive curve.



Q.9. Present the following data in the form of a histogram:

Mid-point	115	125	135	145	155	165	175	185	195
Size	6	55	48	72	116	60	38	22	3

[Hint: First, convert the data into a continuous frequency table; e.g., 115 will be 110-120]

Solution:

Only mid-points are given to draw a ***histogram***. So these mid-points are converted into class intervals.

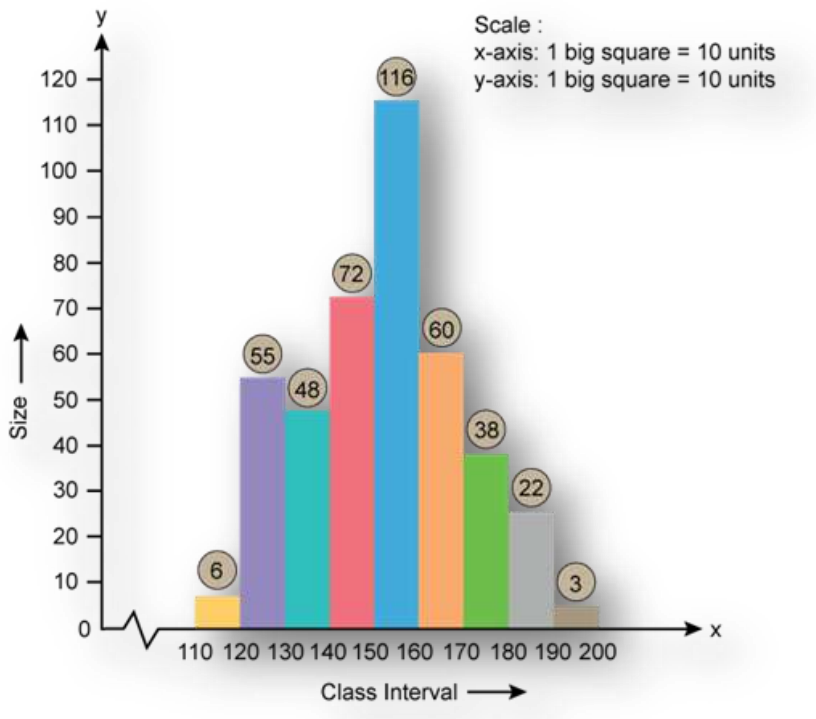
Procedure

Step 1: Formula to derive the class intervals corresponding to each mid-point is

$$\text{Value of Adjustment} = \frac{125 - 115}{2} = 5$$

Step 2: Add and subtract 5 to each mid-point to get the following class intervals.

Mid point	Class Interval	Size
115	110 - 120	6
125	120 - 130	55
135	130 - 140	48
145	140 - 150	72
155	150 - 160	116
165	160 - 170	60
175	170 - 180	38
185	180 - 190	22
195	190 - 200	3



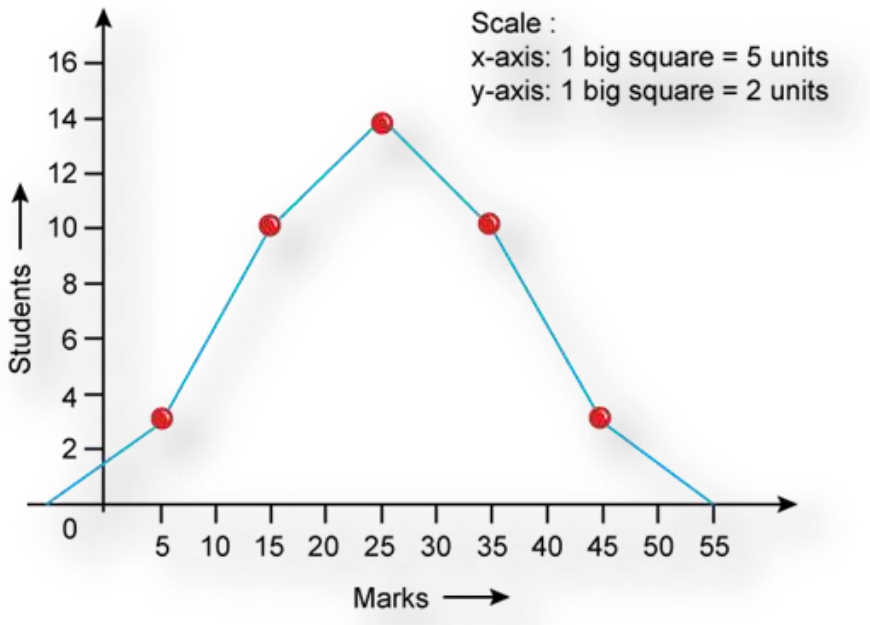
Q.10. The frequency distribution of marks obtained by students in a class test is given below. Draw frequency polygon and ogive.

Marks	0-10	10-20	20-30	30-40	40-50
Number of Student	3	10	14	10	3

Solution:

i. **Frequency polygon:** Frequency polygon can be drawn even without presenting the histogram. Firstly, the mid points of the respective class intervals are calculated and presented graphically against their respective frequencies. Here, the points are joined by using a foot rule to obtain the frequency polygon curve.

Marks	Mid Point	Students
0 – 10	$\frac{0-10}{2} = 5$	3
10 – 20	$\frac{10-20}{2} = 15$	10
20 – 30	$\frac{20-30}{2} = 25$	14
30 – 40	$\frac{30-40}{2} = 35$	10
40 – 50	$\frac{40-50}{2} = 45$	3

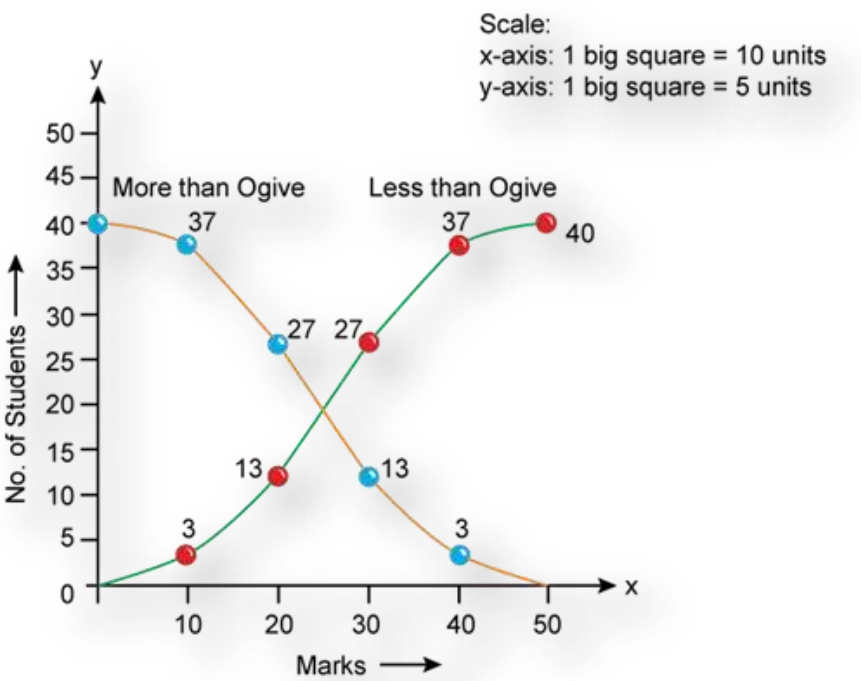


ii. **Ogive curve:** It is a smooth curve presented by plotting cumulative frequency data on a graph. This can be drawn by understanding the frequencies corresponding to lower limits and upper limits in the distribution of data.
Firstly, all the data are converted into more than and less than cumulative frequency distribution as follows-

Marks	**Cumulative Frequency**
Less than 10	3
Less than 20	3 + 10 = 13
Less than 30	13 + 14 = 27
Less than 40	27 + 10 = 37
Less than 50	37 + 3 =40

Marks	**Cumulative Frequency**
More than 0	40
More than 10	40 - 3 = 37
More than 20	37 + 10 = 27
More than 30	27 + 14 = 13
More than 40	13 + 10 = 3

This less than curve is drawn by plotting cumulative frequencies against the upper limit of the class intervals. And these points are joined to obtain the less than ogive curve. And more than curve is drawn by plotting cumulative frequencies against the lower limit of the class intervals. And these points are joined to obtain more than ogive curve.



Q.11.

i. Construct a histogram and frequency polygon of the following distribution:

Marks	0-10	10-20	20-30	30-40	40-50
Number of Student	8	18	35	25	14

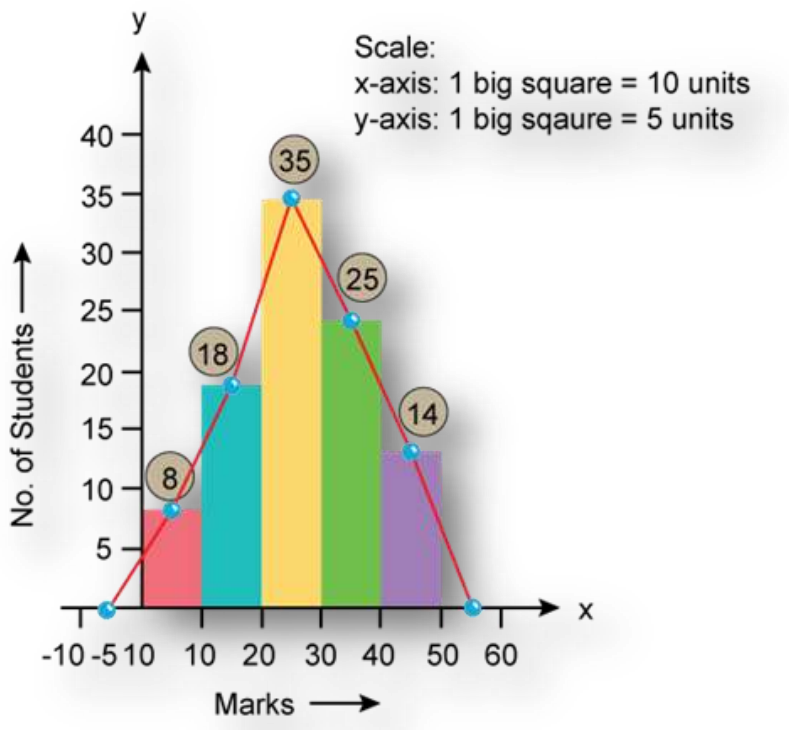
ii. Show that the area under frequency polygon is equal to the area under histogram.

[Hints: (a) Make of intervals of 10 taking first class interval as 0-10.

(b) Area of histogram and polygon are equal, because the area which is left out of the histogram is included in the polygon.]

Solution:

i. **Histogram and Frequency Polygon**



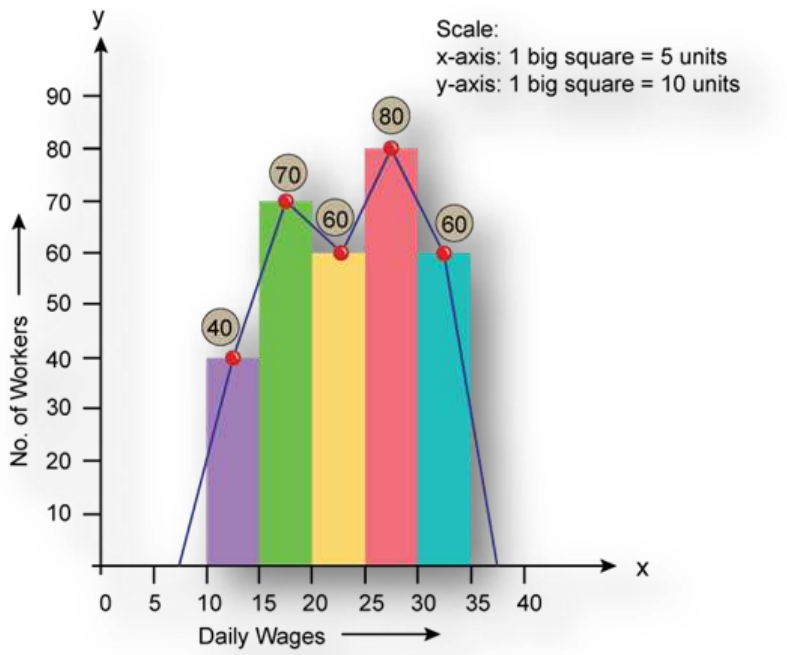
ii. The first class interval is extended to the left side by half the size of class interval which denotes the initial point of the frequency polygon. And the last class interval is extended to the right side by half the size of the class interval which denotes the final point of the frequency polygon. This implies that the area not included at the time of joining the mid-points is now included in the frequency polygon. Thus, the area under frequency polygon is equal to the area under histogram.

Q.12. Draw a frequency polygon from the following data by using (i) histogram, and (ii) without using histogram:

Daily Wages (in Rs)	10-15	15-20	20-25	25-30	30-35
Number of Workers	40	70	60	80	60

Solution:

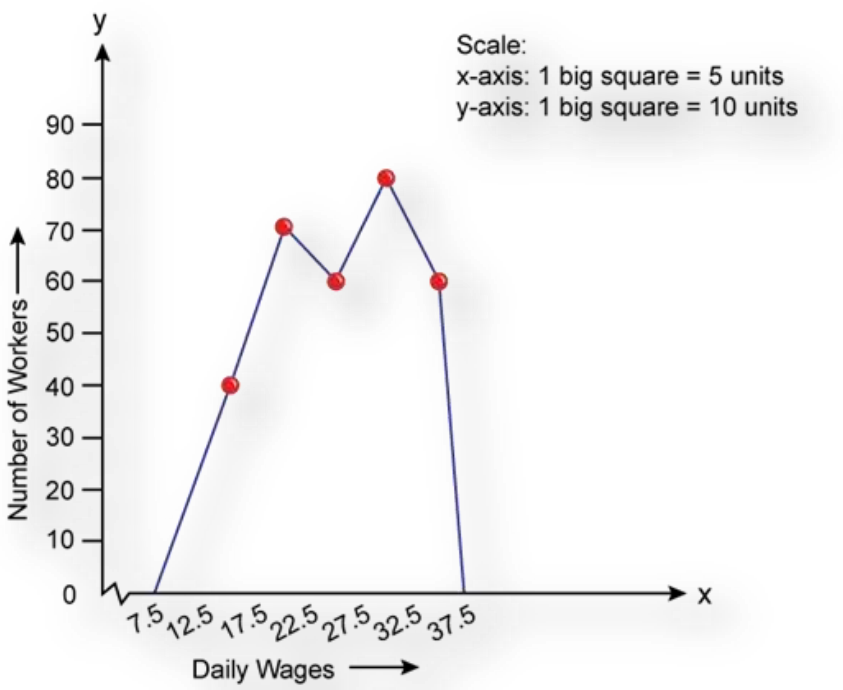
i. **Frequency polygon using histogram**



By plotting the mid-points of the class intervals with their respective frequencies, the mid points are joined to draw a frequency polygon.

Daily Wages	Mid – Point	No. of Workers
10 – 15	$\frac{10+15}{2} = 12.5$	40
15 – 20	$\frac{10+15}{2} = 17.5$	70
20 – 25	$\frac{10+15}{2} = 22.5$	60
25 – 30	$\frac{10+15}{2} = 27.5$	80
30 – 35	$\frac{10+15}{2} = 32.5$	60

ii. **Frequency polygon without using histogram**



Q.13.

Draw 'less than' as well as 'more than' ogive for the following data:

Weight (in kg)	30-34	35-39	40-44	45-49	50-54	55-59	60-64
Frequency	3	5	12	18	14	6	2

[Hint: Convert into Exclusive Series]

Solution:

If the class intervals are equal but the series are inclusive, then inclusive series are converted into an exclusive series.

Step1: Apply the formula to convert into exclusive series

$$\text{Value of Adjustment} = \frac{\text{Value of lower limit of one class} - \text{Value of upper limit of the preceeding class}}{2}$$

$$= \frac{35-34}{2} = 0.5$$

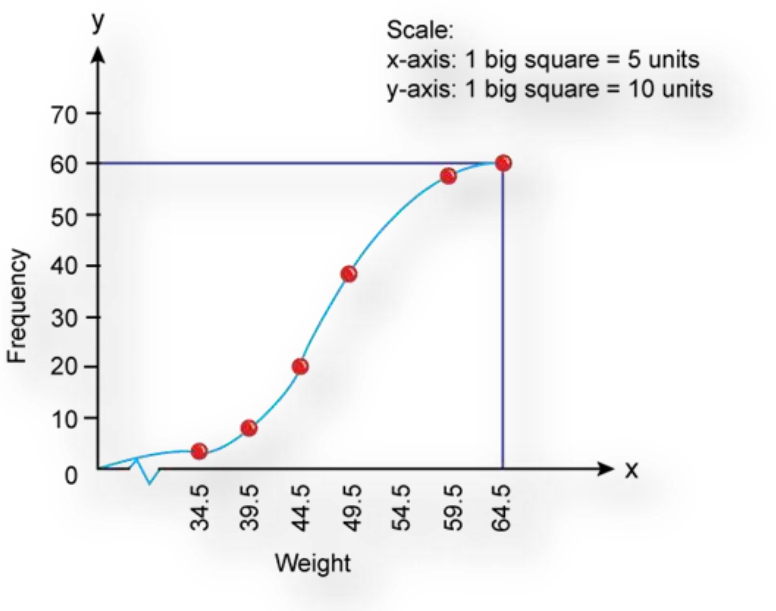
Step 2: Add and subtract 0.5 to each class intervals.

Weight	Frequency
29.5 - 34.5	3
34.5 - 39.5	5
39.5 - 44.5	12
44.5 - 49.5	18
49.5 - 54.5	14
54.5 - 59.5	6
59.5 - 64.5	2

i. Less than ogive curves: In this method, frequencies are cumulated and presented in a graph corresponding to upper limits of the classes in a frequency distribution. Firstly, all the data are converted into less than cumulative frequency distribution as follows-

Weight	Cumulative Frequency
Less than 34.5	3
Less than 39.5	3 + 5 = 8
Less than 44.5	8 + 12 = 20
Less than 49.5	20 + 18 = 38
Less than 54.5	38 + 14 = 52
Less than 59.5	52 + 6 = 58
Less than 64.5	58 + 2 = 60

This curve is drawn by plotting cumulative frequencies against the upper limit of the class intervals. And these points are joined to obtain the less than ogive curve.

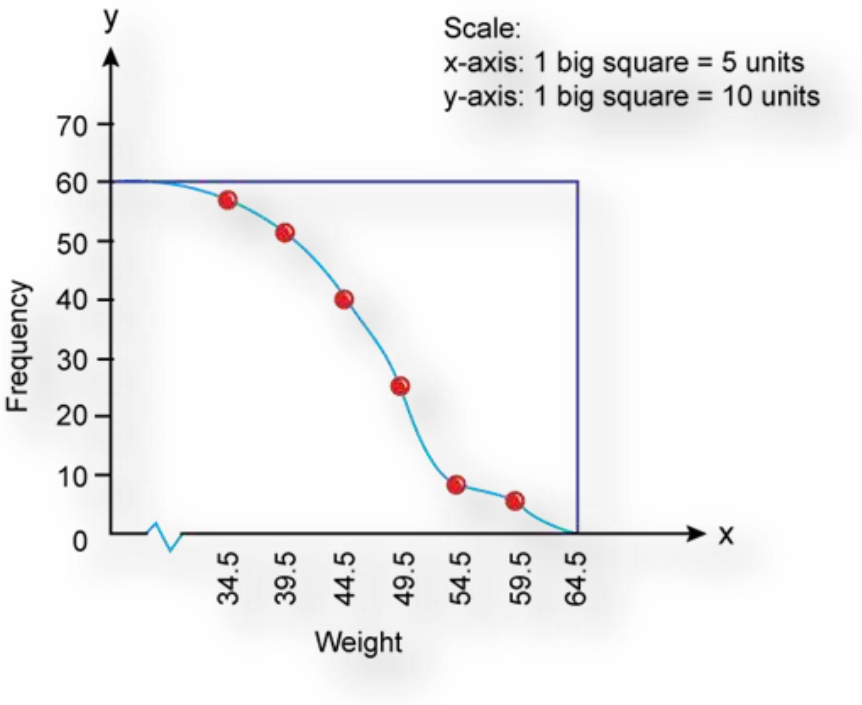


ii. More than ogive curves: In this method, frequencies are cumulated and presented in a graph corresponding to lower limits of the classes in a frequency distribution. Firstly, all the data are converted into more than cumulative frequency distribution as follows-

Weight	Cumulative Frequency
More than 0	60

More than 34.5	60 - 3 = 57
More than 39.5	57 - 5 = 52
More than 44.5	52 - 12 = 40
More than 49.5	40 - 18 = 22
More than 54.5	22 - 14 = 8
More than 59.5	8 - 6 =2
More than 64.5	2 - 2 = 0

This curve is drawn by plotting cumulative frequencies against the lower limit of the class intervals. And these points are joined to obtain more than ogive curve.



Chapter – 8

Arithmetic Line-Graphs or Time Series Graphs

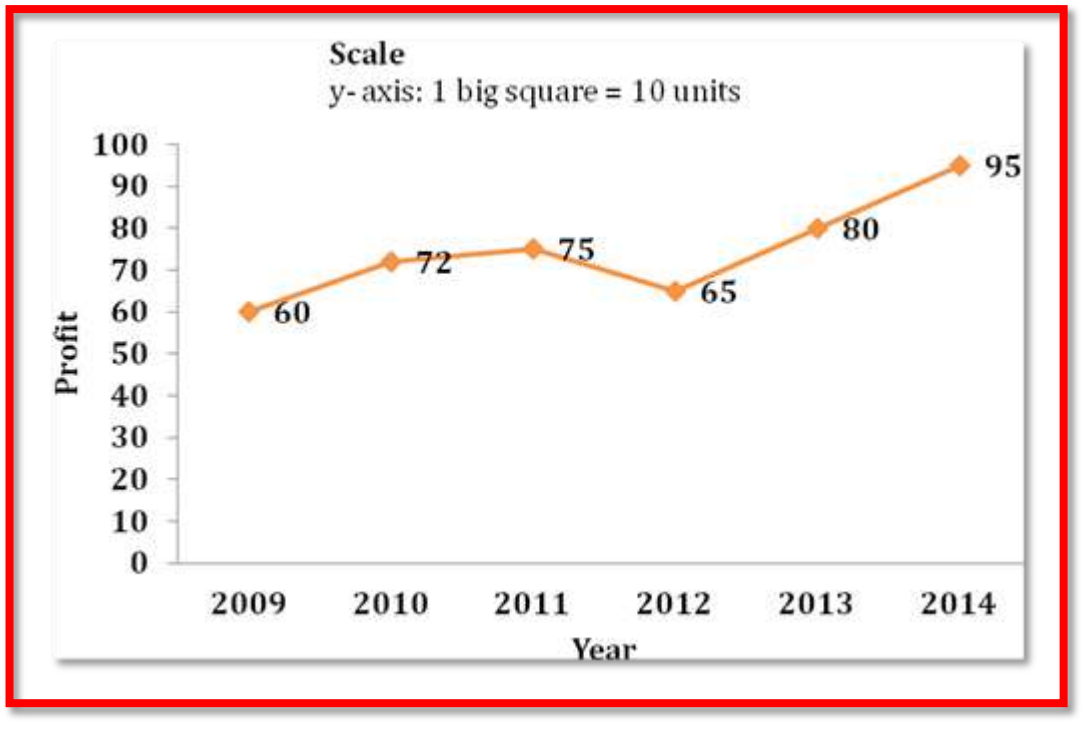
Essential Practical:

Q.1. Plot the annual profit of a firm on a time series graph

Year	2009	2010	2011	2012	2013	2014
Profit ('000 Rs)	60	72	75	65	80	95

Solution:

Annual Profit of a Firm

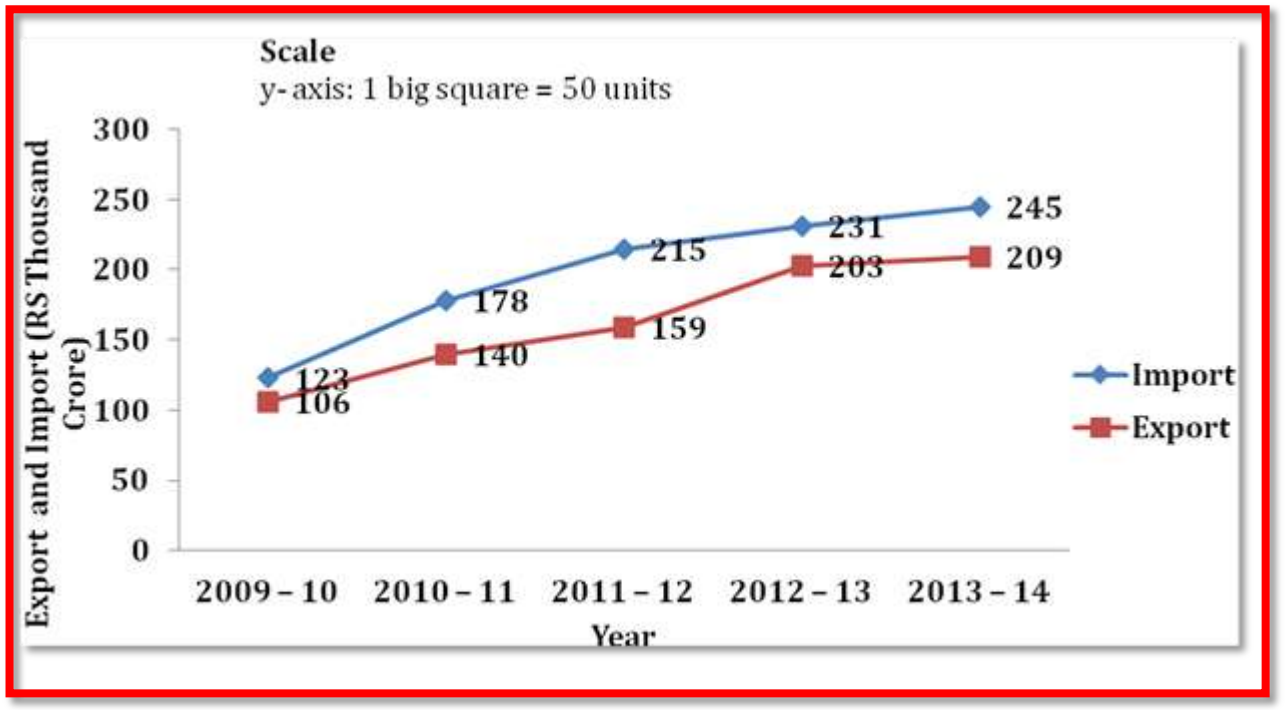


Q.2. Plot the following hypothetical figures on a time series graph:

Year	Import (Rs thousand crore)	Exports (Rs thousand crore)
2009 - 10	123	106
2010 - 11	178	140
2011 - 12	215	159
2012 - 13	231	203
2013 - 14	245	209

Solution:

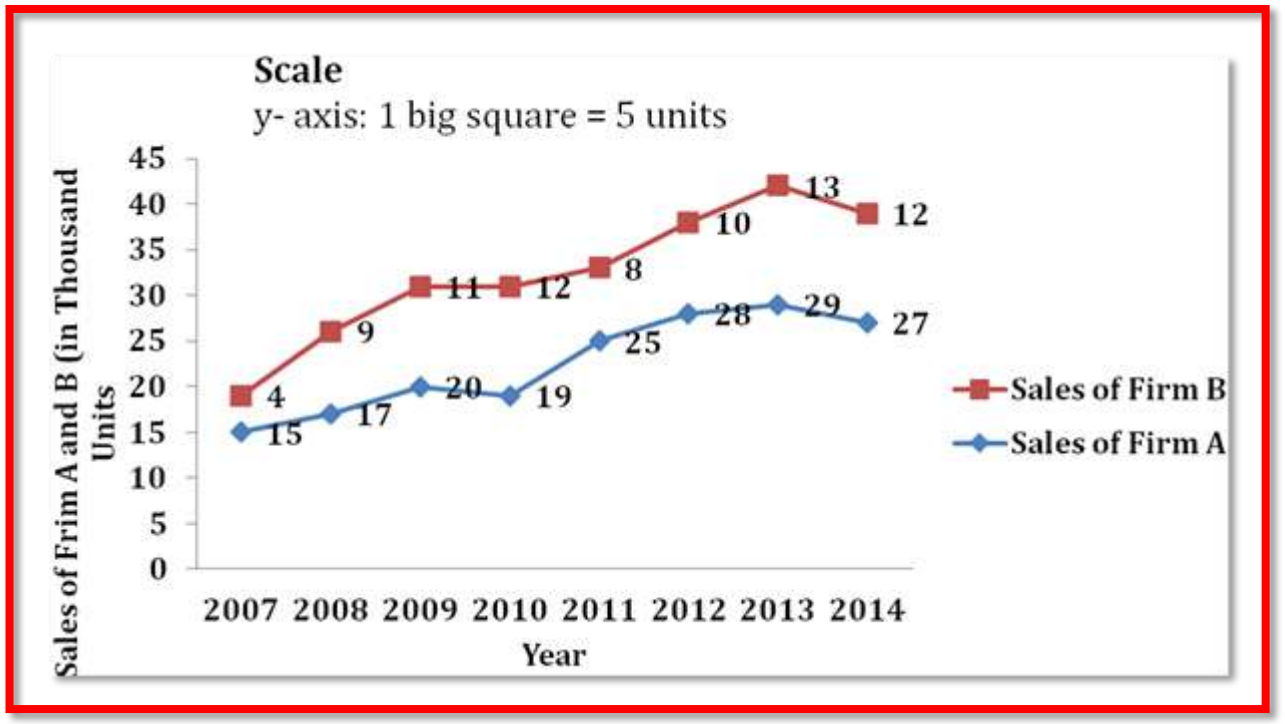
Export and Import



Q.3. The following are the figure of sales of two firms A and B for the years 2007-2014. Present the data graphically.

Year	Sales of Firm A (in thousand units)	Sales of Firm B (in thousand units)
2007	15	4
2008	17	9
2009	20	11
2010	19	12
2011	25	8
2012	28	10
2013	29	13
2014	27	12

Solution: **Sales of Firm A and Firm B**

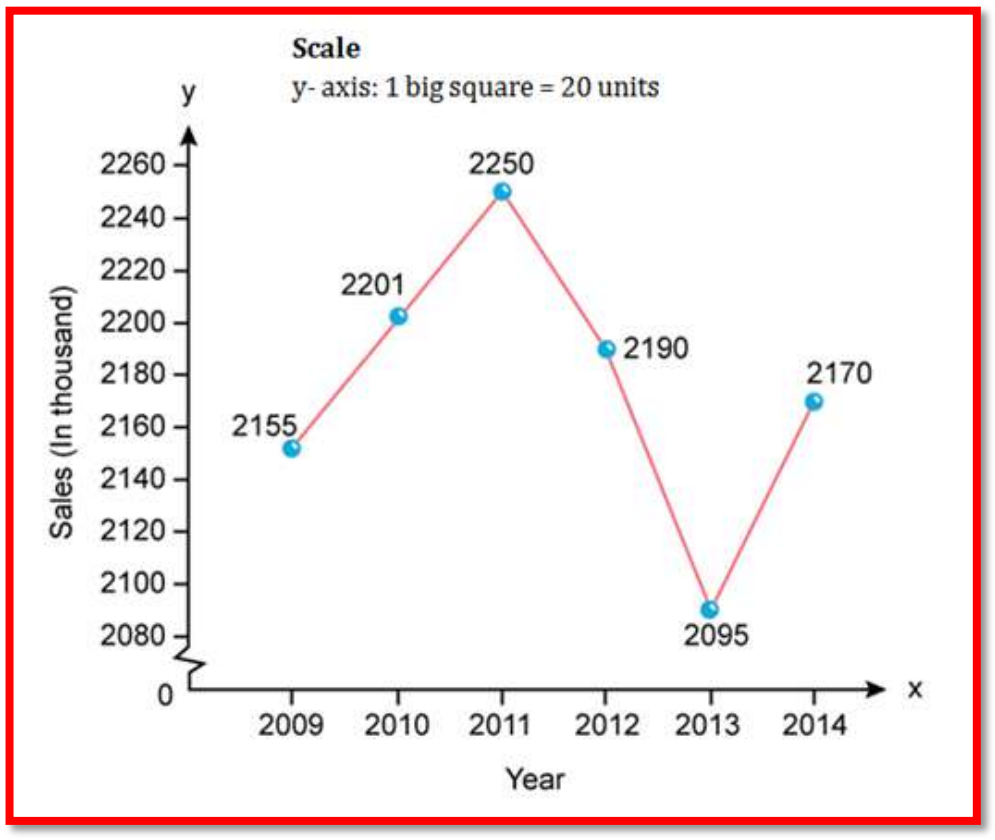


Q.4. The following are the sales figures of TVs of Firm A, during 2009-2014:

Year	2009	2010	2011	2012	2013	2014
Sale (in thousand units)	2,155	2,201	2,250	2,190	2,095	2,170

Plot the above data on time series graph.

Solution: **Sale of TV of Firm A (Year 2009-14)**



Chapter – 9

Measures of Central Tendency- Arithmetic Mean

Essential Practical:

Q.1. Eight workers earn the following income:

30,36,34,40,42,46,54,62

Find out arithmetic mean.

Solution:

$$\text{Arithmetic Mean } (\bar{X}) = \frac{\Sigma X}{N} = \frac{344}{8} = 43$$

$$\therefore \boxed{\bar{X} = 43}$$

Q.2. Pocket allowance of 5 students respectively are:

125,75,150,175,200

Find out arithmetic mean.

Solution:

$$\text{Arithmetic Mean } (\bar{X}) = \frac{\Sigma X}{N} = \frac{725}{5} = 145$$

$$\therefore \boxed{\bar{X} = 145}$$

Q.3. Following is the height of 10 students:

Students	A	B	C	D	E	F	G	H	I	J
Height (cm)	155	153	168	160	162	166	164	180	157	165

Calculate arithmetic mean using Direct and Short-cut Methods.

Solution:

Students	Height (x)	d = X - A
A	155	-7
B	153	-9
C	168	6
D	160	-2
E	**162 = A**	0
F	166	4
G	164	2
H	180	18
I	157	-5
J	165	3
	∑X = 1630	∑d = 10

Direct Method:

$$\bar{X} = \frac{\Sigma X}{N} = \frac{1630}{10}$$

$$\therefore \boxed{\bar{X} = 163}$$

Shortcut method:

$$\bar{X} = A + \frac{\Sigma d}{N}$$

$$\bar{X} = 162 + \frac{10}{10} = 162 + 1$$

$$\therefore \boxed{\bar{X} = 163}$$

Q.4. Weight of 15 persons is as follows:

Weight (kg)	20	28	34	39	42	50	53	54	59	64	72	74	74	78	79

Find out mean weight, using Direct Method as well as short-cut Method.

Solution:

Weight (X)	d = X - A

20	-30
28	-22
34	-16
39	-11
42	-8
50 = A	0
53	3
54	4
59	9
64	14
72	22
74	24
74	24
78	28
79	29
∑X = 820	∑d = 70

Direct Method:

$$\overline{X} = \frac{\Sigma X}{N} = \frac{820}{15}$$

$$\therefore \boxed{\overline{X} = 54.67}$$

Shortcut Method:

$$\overline{X} = A + \frac{\Sigma d}{N}$$

$$\overline{X} = 50 + \frac{70}{15}$$

$$\therefore \boxed{\overline{X} = 54.67}$$

Q.5. Calculate average of the following discrete series. Use Short-cut Method by taking 25 as assumed average.

Size	30	29	28	27	26	25	24	23	22	21
Frequency (f)	2	4	5	3	2	7	1	4	5	7

Solution:

Size (X)	**Frequency**	**Deviation d = X-A**	**fd**
30	2	5	10
29	4	4	16
28	5	3	15
27	3	2	6
26	2	1	2
25 = A	7	0	0
24	1	-1	-1
23	4	-2	-8
22	5	-3	-15
21	7	-4	-28
	∑f = 40		∑fd = -3

Shortcut Method :

$$\overline{X} = A + \frac{\Sigma fd}{\Sigma f}$$

$$\overline{X} = 25 + \left(\frac{-3}{40}\right)$$

$$\therefore \boxed{\overline{X} = 24.92}$$

Q.6. Marks secured by 42 students in Economics are:

Marks	15	20	22	23	27	35	18

Number of students	8	4	7	3	8	7	5

Find average marks.

Solution:

Marks (X)	No. of Students (f)	fx
15	8	120
20	4	80
22	7	154
23	3	69
27	8	216
35	7	245
18	5	90
	∑f = 42	∑fX = 974

$$\bar{X} = \frac{\sum fx}{\sum f} = \frac{974}{42}$$

$$\therefore \boxed{\bar{X} = 23.2}$$

Q.7. Average age of the people of a country is shown in the following table:

Age (Years)	10-20	20-30	30-40	40-50	50-60
People ('000)	30	32	15	12	9

Find out mean age by Direct Method.

Solution:

Age (X)	Mid-values (m)	People (f)	fm
10 - 20	15	30	450
20 - 30	25	32	800
30 - 40	35	15	525
40 - 50	45	12	540
50 - 60	55	9	495
		∑f = 98	∑fm = 2810

$$\bar{X} = \frac{\sum fm}{\sum f} = \frac{2810}{98}$$

$$\therefore \boxed{\bar{X} = 28.7}$$

Q.8. Calculate the arithmetic mean of the following frequency distribution by Direct Method:

Class Interval	10-20	20-40	40-70	70-120	120-200
Frequency	4	10	26	8	2

Solution:

Class Interval	Mid-values (m)	Frequency (f)	fm
10 - 20	15	4	60
20 - 40	30	10	300
40 - 70	55	26	1430
70 - 120	95	8	760
120 - 200	160	2	320
		∑f = 50	∑fm = 2870

$$\bar{X} = \frac{\sum fm}{\sum f} = \frac{2870}{50}$$

$$\therefore \boxed{\bar{X} = 57.4}$$

Q.9. Calculate arithmetic mean from the following data by short-cut Method:

Class Interval	20-25	25-35	30-35	35-40	40-45	45-50	50-55
Frequency	10	12	8	20	11	4	5

Solution:

Class Interval	Mid- Value (m)	Frequency (f)	d = m-A	fd
20 - 25	22.5	10	-15	-150
25 - 30	27.5	12	-10	-120
30 - 35	32.5	8	-5	-40
35 - 40	**37.5 = A**	20	0	0
40 - 45	42.5	11	5	55
45 - 50	47.5	4	10	40
50 - 55	52.5	5	15	75
		∑f = 70		∑fd = -140

$$\bar{X} = A + \frac{\sum fd}{\sum f}$$

$$\bar{X} = 37.5 + \left(\frac{-140}{70}\right)$$

$$\therefore \boxed{\bar{X} = 35.5}$$

Q.10. Find out arithmetic mean from the following distribution by short-cut Method:

Items	10-8	8-6	6-4	4-2	2-0
Frequency	10	8	6	4	2

Solution:

Items (x)	Mid-Values (m)	Frequency (f)	d = m-A	fd
0 - 2	1	2	-4	-8
2 - 4	3	4	-2	-8
4 - 6	**5 =A**	6	0	0
6 - 8	7	8	2	16
8 - 10	9	10	4	40
		∑f = 30		∑fd = 40

$$\bar{X} = A + \frac{\sum fd}{\sum f}$$

$$\bar{X} = 5 + \frac{40}{30}$$

$$\therefore \boxed{\bar{X} = 6.33}$$

Q.11. Sachin made the following runs in different matches:

Runs	5-15	15-25	25-35	35-45	45-55
Frequency	10	12	17	19	22

Calculated the average mean of the runs by Step-deviation Method.

Solution:

Run	Mid-Values	Frequency	Deviation	Step deviation	*fd'*

(*X*)	(*m*)	(*f*)	*m* – A (*d*)	d'=m-Ai	
5 – 15	10	10	−20	−2	−20
15 – 25	20	12	−10	−1	−12
25 – 35	**30 = *A***	17	0	0	0
35 – 45	40	19	10	1	19
45 – 55	50	22	20	2	44
		$\sum f = 80$			$\sum fd' = 31$

$$\bar{X} = A + \frac{\sum fd'}{\sum fd} \times c$$

$$\bar{X} = 30 + \frac{31}{80} \times 10$$

$$\therefore \boxed{\bar{X} = 33.875}$$

Thus, average mean of the runs is 33.875

Q.12. Calculated arithmetic mean of the following frequency distribution:

Class	Less than 10	10-20	20-30	30-40	40-50	50-60	More than 60
Frequency	5	12	18	22	6	4	3

Solution:

Let first and the last class intervals be (0- 10) and (60 - 70) respectively.

Class Interval	Mid-Values (m)	Frequency (f)	fm
0 - 10	5	5	25
10 - 20	15	12	180
20 - 30	25	18	450
30 - 40	35	22	770
40 - 50	45	6	270
50 - 60	55	4	220
60 - 70	65	3	195
		$\sum f = 70$	$\sum fm = 2110$

$$\bar{X} = \frac{\sum fm}{\sum f}$$

$$\bar{X} = \frac{2110}{70}$$

$$\therefore \boxed{\bar{X} = 30.14}$$

Q.13. Mean marks obtained by a student in his five subject are 15. In English he secures 8 marks, in Economics 12, in Mathematics 18, and in commerce 9. Find out the marks he secured in statistics.

Solution:

Mean marks $(\bar{X}) = 15$

Marks in English $(x_1) = 8$

Marks in Economics $(x_2) = 12$

Marks in Mathematics $(x_3) = 18$

Marks in Commerce $(x_4) = 9$

Marks in Statistics $(x_5) = ?$

$$\bar{X} = \frac{\Sigma x}{N}$$

$$15 = \frac{8+12+18+9+x_5}{5}$$

$$75 = x_5 + 47$$

$$\therefore \boxed{x_5 = 28}$$

Thus, marks scored in statistics is 28.

Q.14. Mean value of the weekly income of 40 families is 265. But in the calculation, income of one family was read as 150 instead of 115. Find the "Corrected" mean.

Solution:

Given:

$\bar{X} = 265$

N=40

Incorrect item = 150

Correct item = 115

$$\bar{X} = \frac{\Sigma X}{N}$$

Wrong $\Sigma X = 265 \times 40 = 10,600$

Correct ΣX = 10600 + Correct item - Incorrect item

Correct $\Sigma X = 10600 + 115 - 150$

Correct $\Sigma X = 10565$

$$\text{Correct } \bar{X} = \frac{\text{Correct } \Sigma X}{N}$$

$$\text{Correct } \bar{X} = \frac{10565}{40}$$

$$\therefore \boxed{\text{Correct } \bar{X} = 264.125}$$

Q.15. Average pocket allowance of 6 students is Rs 45. Of these, pocket allowance of 5 students is 20, 30, 22, 24 and 32 respectively. What is the pocket allowance of the sixth student?

Solution:

Given :

$\bar{X} = 45$

N =6

Let the pocket allowance of sixth student be x_6.

$$\bar{X} = \frac{\Sigma X}{N}$$

$$45 = \frac{20+30+22+24+32+x_6}{6}$$

$$270 = 128 + x_6$$

$$\therefore \boxed{x_6 = 142}$$

Q.16. The following table shows wages of the workers. Calculated the average wage of the workers.

Wages (Rs)	10-19	20-29	30-39	40-49	50-59
Number of workers	8	9	12	11	6

Solution:

Wages (X)	Mid Value (m)	Frequency (f)	Deviation $d = m - A$	d'=di	fd'
10 – 19	14.5	8	−20	−2	−16
20 – 29	24.5	9	−10	−1	−9
30 – 39	**34.5 =*A***	12	0	0	0
40 – 49	44.5	11	10	1	11
50 – 59	54.5	6	20	2	12
		$\sum f = 46$			$\sum fd = -2$

$$\bar{X} = A + \frac{\sum fd'}{\sum f} \times c$$

$$\bar{X} = 34.5 + \frac{-2}{46} \times 10$$

$$\therefore \boxed{\bar{X} = 34.07}$$

Q.17. Ten player of the Australian team made an average of 63 runs and ten players of the Indian team made an average of 77 runs. Calculated the average run made by both the teams.

Solution:

Let :

$\bar{X}_1 = 63$

$\bar{X}_2 = 77$

$N_1 = 10$

$N_2 = 10$

$$\bar{X}_{12} = \frac{N_1\bar{X}_1 + N_2\bar{X}_2}{N_1 + N_2}$$

$$\bar{X}_{12} = \frac{10 \times 63 + 10 \times 77}{10 + 10}$$

$$\therefore \boxed{\bar{X}_{12} = 70}$$

Q.18. Average income of 50 families is Rs 3,000. Of these average income of 12 families is Rs 1,800. Find out the average income of the remaining families.

Solution:

Given :

Average income 50 families $= \bar{X}_{12} = 3000$

$\bar{X}_1 = 1800$

$\bar{X}_2 = ?$

$N_1 = 12$

$N_2 = 38$

$$\bar{X}_{12} = \frac{N_1\bar{X}_1 + N_2\bar{X}_2}{N_1 + N_2}$$

$$3000 = \frac{12 \times 1800 + 38 \times \bar{X}_2}{50}$$

$$38\bar{X}_2 = 150000 - 21600$$

$$\therefore \boxed{\bar{X}_2 = 3378.95}$$

Q.19. In the following frequency distribution, if the arithmetic mean is 45.6, find out missing frequency.

Wages (Rs)	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Number of workers	5	6	7	X	4	3	9

Solution:

Wages (X)	Mid Value (m)	No. of worker (f)	fm
10 – 20	15	5	75
20 – 30	25	6	150
30 – 40	35	7	245
40 – 50	45	X	45
50 – 60	55	4	220
60 – 70	65	3	195
70 – 80	75	9	675
		$\sum f = 34 + X$	$\sum fm = 1560 + 45X$

$$\bar{X} = \frac{\sum fm}{\sum f}$$

$$45.6 = \frac{1560 + 45x}{34 + x}$$

$$1550.4 + 45.6\,x = 1560 + 45\,x$$

$$0.6\,x = 9.6$$

$$\therefore \boxed{x = 16}$$

Thus, missing frequency is 16.

Q.20. Calculate the weighted mean of the following data:

Item	96	102	104	124	148	164
Weight	5	6	3	7	12	9

Solution:

Items (X)	Weight (W)	WX
96	5	480
102	6	612
104	3	312
124	7	868
148	12	1776
164	9	1476
	$\sum W = 42$	$\sum WX = 5524$

$$\bar{X}_w = \frac{\sum WX}{\sum X}$$

$$\bar{X}_w = \frac{5524}{42}$$

$$\therefore \boxed{\bar{X}_w = 131.52}$$

Q.21. A student obtained 60 marks in English, 75 in Hindi, 63 in Mathematics 59, in Economics and 55 in Statistic. Calculate weight mean of the marks if weight are respectively 2,1,5,5 and 3.

Solution:

Subject	Marks (X)	Weight (W)	WX
English	60	2	120
Hindi	75	1	75
Mathematics	63	5	315
Economics	59	5	295
Statistics	55	3	165
		$\sum W = 16$	$\sum WX = 970$

$$\bar{X}_w = \frac{\Sigma WX}{\Sigma W}$$

$$\bar{X}_w = \frac{970}{16}$$

$$\bar{X}_w = 60.625$$

Q.22. A housewife uses 10 kg of Wheat, 20 kg of fuel, 5 kg of Sugar, and 2 kg of oil. Prices (per kg) of these items are Rs 1.50, 150 paise, Rs 2.80 and Rs 10 respectively. Taking quantities used as weight find out the weighted arithmetic average of the prices.

Solution:

Price (X)	Quantity (W)	WX
1.5	10	15
0.5	20	10
2.8	5	14
10	2	20
	$\sum W = 37$	$\sum WX = 59$

$$\bar{X}_w = \frac{\Sigma WX}{\Sigma W}$$

$$\bar{X}_w = \frac{59}{37}$$

$$\therefore \boxed{\bar{X}_w = 1.59}$$

Q.23. Calculate weighted mean of the following data by using Direct and Short-cut Methods:

Items	81	76	74	58	70	73
Weight	2	3	6	7	3	7

Solution:

Items (X)	Weight (W)	WX	Deviation $d = X - A$	Wd
81	2	162	7	14
76	3	228	2	6
74 = *A*	6	444	0	0
58	7	406	−16	−112
70	3	210	−4	−12
73	7	511	−1	−7
	$\sum W = 28$	$\sum WX = 1961$		$\sum Wd = -111$

Direct Method :

$$\bar{X}_w = \frac{\Sigma WX}{\Sigma W}$$

$$\bar{X}_w = \frac{1961}{28}$$

$$\therefore \boxed{\bar{X}_w = 70.04}$$

Shortcut Method :

$$\bar{X}_w = A + \frac{\Sigma Wd}{\Sigma W}$$

$$\bar{X}_w = 74 + \left(\frac{-111}{28}\right)$$

$$\bar{X}_w = 74 - 3.964$$

$$\therefore \boxed{\bar{X}_w = 70.04}$$

Chapter – 10

Measures of Central Tendency- Median and Mode

Essential Practical:

Q.1. Given below is the data of the age of 9 children of a street. Find the median.
5,8,7,3,4,6,2,9,1

Solution:

Arranging data in ascending order:

1,2,3,4,5,6,7,8,9

$N = 9$

$$\text{Median} = \text{size of } \left(\frac{N+1}{2}\right)^{th} \text{ item} = \text{size of } \left(\frac{9+1}{2}\right)^{th} \text{ item}$$

$$\text{Median} = \text{size of } 5^{th} \text{ item} = 5$$

$\therefore$ Median $= 5$

Q.2. Find the median of the following values:
30,20,15,10,25,35,18,21,28,40,36

Solution:

Arranging data in ascending order:

$$10, 15, 18, 20, 21, 25, 28, 30, 35, 36, 40$$

$N = 11$

$$\text{Median} = \text{size of } \left(\frac{N+1}{2}\right)^{th} \text{ item}$$

$$\text{Median} = \text{size of } \left(\frac{11+1}{2}\right)^{th} \text{ item}$$

$$\text{Median} = \text{size of } 6^{th} \text{ item}$$

$\therefore$ Median $= 25$

Q.3. Find out median of the series of the following table:

Item	3	4	5	6	7	8
Frequency	6	9	11	14	23	10

Solution:

Items	Frequency (*f*)	Cumulative Frequency (*c.f.*)
3	6	6
4	9	15
5	11	26
6	14	40
7	23	63
8	10	73
	$N = 73$	

$$\text{Median} = \text{size of } \left(\frac{N+1}{2}\right)^{th} \text{ item} = \text{size of } \left(\frac{73+1}{2}\right)^{th} \text{ item}$$

$$\text{Median} = \text{size of } \left(\frac{74}{4}\right)^{th} \text{ item}$$

$$\text{Median} = \text{size of } 37^{th} \text{ item}$$

Median value corresponds to the 37^{th} item in the series. Thus, median value is 6 as it corresponds to cumulative frequency 40.

Q.4. Data relating to wages of some workers are given below. Find out median wage.

Wages (Rs)	20-30	30-40	40-50	50-60	60-70

Number of Workers	25	12	15	13	5

Solution:

Wages	No. of Workers (*F*)	Cumulative Frequency (*c.f.*)
20 – 30	25	**25**
30 – 40	**12 (*f*)**	37
40 – 50	15	52
50 – 60	13	65
60 – 70	5	70
	$N = \sum f = 70$	

$$\text{Median} = \text{size of } \left(\frac{N}{2}\right)^{th} \text{ item}$$

$$\text{Median} = \text{size of } \left(\frac{70}{2}\right)^{th} \text{ item} = \text{size of } 35^{th} \text{ item}$$

35th item lies in cumulative frequency 37 which corresponds to the class interval 30-40.
Thus, median class is 30-40.

$$\text{Median} = l_1 + \frac{\frac{N}{2} - c.f.}{f} \times i$$

$$\text{Median} = 30 + \frac{35 - 25}{12} \times 10$$

$$\therefore \boxed{\text{Median} = 38.33}$$

Q.5. The following table expresses the age of eight students. Find the median age.

S.No	1	2	3	4	5	6	7	8
Age (years)	18	16	14	11	13	10	9	2

Solution:

S. No	Ages
1	2
2	9
3	10
4	11
5	13
6	14
7	16
8	18

N = 8

$$\text{Median} = \frac{\text{size of } \left(\frac{N}{2}\right)^{th} \text{ item} + \text{size of } \left(\frac{N}{2}+1\right)^{th} \text{ item}}{2}$$

$$\text{Median} = \frac{\text{size of } \left(\frac{8}{2}\right)^{th} \text{ item} + \text{size of } \left(\frac{8}{2}+1\right)^{th} \text{ item}}{2}$$

$$\text{Median} = \frac{\text{size of } 4^{th} \text{item} + \text{size of } 5^{th} \text{item}}{2}$$

$$\text{Median} = \frac{11 + 13}{2}$$

$$\therefore \boxed{\text{Median} = 12 \text{ years}}$$

Q.6. Number of person living in a house is reported to be a under for 500 house in a village. Find the median number of persons in a house in the village.

Number of persons in a house	1	2	3	4	5	6	7	8	9	10

Number of Houses	26	113	120	95	60	42	21	14	5	4

Solution:

No. of person in a House	No. of House (f)	Cumulative Frequency (c.f.)
1	26	26
2	113	139
3	120	259
4	95	354
5	60	414
6	42	456
7	21	477
8	14	491
9	5	496
10	4	500
	∑f =500	

$$\text{Medain} = \text{size of } \left(\frac{N+1}{2}\right)^{th} \text{ item}$$

$$\text{Median} = \text{size of } \left(\frac{500+1}{2}\right)^{th} \text{ item}$$

$$\text{Median} = \text{size of } 250.5^{th} \text{ item}$$

Median value corresponds to the 250.5^{th} item in the series. Thus, median value is 3 as it corresponds to cumulative frequency 259.

Q.7. Find out median of the following series:

Size	15	20	25	30	35	40
Frequency	10	15	25	5	5	20

Solution:

Size	Frequency (f)	Cumulative Frequency (c.f.)
15	10	10
20	15	25
25	25	50
30	5	55
35	5	60
40	20	80
	N = 80	

$$\text{Medain} = \text{size of } \left(\frac{N+1}{2}\right)^{th} \text{ item}$$

$$\text{Median} = \text{size of } \left(\frac{80+1}{2}\right)^{th} \text{ item}$$

$$\text{Median} = \text{size of } 40.5^{th} \text{item}$$

Median value corresponds to the 40.5^{th} item in the series. Thus, median value is 25 as it corresponds to cumulative frequency 50.

Q.8. Distribution of marks obtained by 100 students of a class in given below. Find out the median marks.

Marks	0	5	10	15	20	25	30	35	40	45
Number of students	4	6	15	5	8	12	28	14	3	5

Solution:

Marks	No. of Student	Cumulative Frequency

	(f)	(c.f.)
0	4	4
5	6	10
10	15	25
15	5	30
20	8	38
25	12	50
30	28	78
35	14	92
40	3	95
45	5	100
	N = 100	

$$\text{Medain} = \text{size of } \left(\frac{N+1}{2}\right)^{th} \text{item}$$

$$\text{Median} = \text{size of } \left(\frac{100+1}{2}\right)^{th} \text{item}$$

$$\text{Median} = \text{size of } 50.5^{th} \text{item}$$

Median value corresponds to the 50.5^{th} item in the series. Thus, median value is 30 as it corresponds to cumulative frequency 78.

Q.9. Find out median wage rate from the following data-set:

Wage Rate (Rs)	5-15	15-25	25-35	35-45	45-55	55-65
Number of Workers	4	6	10	5	3	2

Solution:

Wages	**No. of Worker (*f*)**	**Cumulative Frequency (*c.f.*)**
5 – 15	4	4
15 – 25	6	10
25 – 35	10	20
35 – 45	5	25
45 – 55	3	28
55 – 65	2	30
	$N = \sum f = 30$	

$$\text{Median} = \text{size of } \left(\frac{N}{2}\right)^{th} \text{item}$$

$$\text{Median} = \text{size of } \left(\frac{30}{2}\right)^{th} \text{item}$$

$$\text{Median} = \text{size of } 15^{th} \text{item}$$

15^{th} item lies in cumulative frequency 20 which corresponds to the class interval 25-35.
Thus, median class is 25-35.

$$\text{Median} = l_1 + \frac{\frac{N}{2} - \text{c.f.}}{f} \times i$$

$$\text{Median} = 25 + \frac{15-10}{10} \times 10$$

$$\therefore \boxed{\text{Median} = 30}$$

Q.10. Find out median of the following series:

Wage Rate (Rs)	25-30	20-25	15-20	10-15	5-10	0-5
Number of Workers	5	10	20	5	8	2

Solution:

Wages	**No. of Worker (*f*)**	**Cumulative Frequency (*c.f.*)**

0 – 5	2	2
5 – 10	8	10
10 – 15	5	15 (*c.f.*)
15 – 20	20 (*f*)	35
20 – 25	10	45
25 – 30	5	50
	$N = \sum f = 50$	

$$\text{Median} = \text{size of } \left(\frac{N}{2}\right)^{th} \text{ item}$$

$$\text{Median} = \text{size of } \left(\frac{50}{2}\right)^{th} \text{ item}$$

$$\text{Median} = \text{size of } 25^{th} \text{item}$$

15^{th} item lies in cumulative frequency 20 which corresponds to the class interval 15-20.

Thus, median class is 15-20.

$$\text{Median} = l_1 + \frac{\frac{N}{2} - \text{c.f.}}{f} \times i$$

$$\text{Median} = 15 + \frac{25 - 15}{20} \times 5$$

$\therefore$ **Median = 17.5**

Q.11. Calculate the median from the following series:

Age (Year)	55-60	50-55	45-50	40-45	35-40	30-35	25-30	20-25
Number of Students	7	13	10	15	30	33	28	14

Solution:

Age	**No. of students (*f*)**	**Cumulative Frequency (*c.f.*)**
20 – 25	14	14
25 – 30	28	**42 (*c.f.*)**
30 – 35	**33 (*f*)**	75
35 – 40	30	105
40 – 45	15	120
45 – 50	10	130
50 – 55	13	143
55 – 60	7	150
	$N = \sum f = 150$	

$$\text{Median} = \text{size of } \left(\frac{N}{2}\right)^{th} \text{ item}$$

$$\text{Median} = \text{size of } \left(\frac{150}{2}\right)^{th} \text{ item}$$

$$\text{Median} = \text{size of } 75^{th} \text{item}$$

75^{th} item lies in cumulative frequency 75 which corresponds to the class interval 30-35.

Thus, median class is 30-35.

$$\text{Median} = l_1 + \frac{\frac{N}{2} - \text{c.f.}}{f} \times i$$

$$\text{or, Median} = 30 + \frac{75 - 42}{33} \times 5$$

$\therefore$ **Medain = 35**

Q.12. 50 Students of economics, secured the following marks in an examination:

Marks	20-25	25-30	30-35	35-40	40-45	45-50	50-55	55-60	60-65	65-70
Students	6	3	7	4	6	4	2	8	3	7

Calculate median.

Solution:

Marks	Student (f)	Cumulative Frequency ($c.f.$)
20 – 25	6	6
25 – 30	3	9
30 – 35	7	16
35 – 40	4	**20 ($c.f.$)**
40 – 45	**6(f)**	26
45 – 50	4	30
50 – 55	2	32
55 – 60	8	40
60 – 65	3	43
65 – 70	7	50
	$N = \sum f = 50$	

$$\text{Median} = \text{size of } \left(\frac{N}{2}\right)^{th} \text{ item}$$

$$\text{Median} = \text{size of } \left(\frac{50}{2}\right)^{th} \text{ item}$$

$$\text{Median} = \text{size of } 25^{th} \text{item}$$

25^{th} item lies in cumulative frequency 26 which corresponds to the class interval 40-45.

Thus, median class is 40-45.

$$\text{Median} = l_1 + \frac{\frac{N}{2} - \text{c.f.}}{f} \times i$$

$$\text{Median} = 40 + \frac{25 - 20}{6} \times 5$$

$$\text{Median} = 40 + \frac{25}{6} \times 5$$

$\therefore$ $\boxed{\text{Median} = 44.16}$

Q.13. Given the following data, find out median:

Age	20-25	25-30	30-35	35-40	40-45	45-50	50-55	55-60
Number of Students	50	70	100	180	150	120	70	60

Solution:

Age	No. of Students (f)	Cumulative Frequency ($c.f.$)
20 – 25	50	50
25 – 30	70	120
30 – 35	100	**220 ($c.f.$)**
35 – 40	**180 (f)**	400
40 – 45	150	550
45 – 50	120	670
50 – 55	70	740
55 – 60	60	800
	$N = \sum f = 800$	

Median = size of $\left(\frac{N}{2}\right)^{th}$ item

Median = size of $\left(\frac{800}{2}\right)^{th}$ item

Median = size of 400^{th} item

400^{th} item lies in cumulative frequency 400 which corresponds to the class interval 35-40.

Thus, median class is 35-40.

$$\text{Median} = l_1 + \frac{\frac{N}{2} - \text{c.f.}}{f} \times i$$

$$\text{Median} = 35 + \frac{400 - 220}{180} \times 5$$

$$\text{Median} = 35 + \frac{180}{180} \times 5$$

$\therefore$ Median = 40

Q.14. Find out median, with the help of the following data:

Price Level	10-20	20-30	30-40	40-50	50-60	60-70
Number of Commodity	2	5	8	4	6	3

Solution:

Price Level	**No. of Commodity (*f*)**	**Cumulative Frequency (*c.f.*)**
10 – 20	2	2
20 – 30	5	**7(*c.f*)**
30 – 40	**8(*f*)**	15
40 – 50	4	19
50 – 60	6	25
60 – 70	3	28
	$N = \sum f = 28$	

Median = size of $\left(\frac{N}{2}\right)^{th}$ item

Median = size of $\left(\frac{28}{2}\right)^{th}$ item

Median = size of 14^{th} item

14^{th} item lies in cumulative frequency 15 which corresponds to the class interval 30-40.

Thus, median class is 30-40.

$$\text{Median} = l_1 + \frac{\frac{N}{2} - \text{c.f.}}{f} \times i$$

$$\text{Median} = 30 + \frac{14 - 7}{8} \times 10$$

$\therefore$ Median = 38.75

Q.15. Calculate median, given the following data:

Mid-Value	20	30	40	50	60	70
Male (c.f)	12	25	42	46	48	50

Solution:

Lower limits and upper limits of class intervals are calculated using the following formula.

$$\text{Lower limit } (l_1) = m - \frac{1}{2}i$$

$$\text{Upper limit } (l_2) = m + \frac{1}{2}i$$

Where m is mid value and i is the difference between mid-values.

Mid Value	Class Interval	Cumulative Frequency (*c.f.*)	Frequency (*f*)
20	15 – 25	**12 (*c.f.*)**	12
30	25 – 35	25	**25 – 12 = 13 (*f*)**
40	35 – 45	42	42 – 25 = 17
50	45 – 55	46	46 – 42 = 4
60	55 – 65	48	48 – 46 = 2
70	65 – 75	50	50 – 48 = 2
			$N = \sum f = 50$

$$\text{Median} = \text{size of } \left(\frac{50}{2}\right)^{\text{th}} \text{ item}$$

Median = size of 25^{th} item

25^{th} item lies in cumulative frequency 25 which corresponds to the class interval 25-35.

Thus, median class is 25-35.

$$\text{Median} = l_1 + \frac{\frac{N}{2} - \text{c.f.}}{f} \times i$$

$$\text{Median} = 25 + \frac{25 - 12}{13} \times 10$$

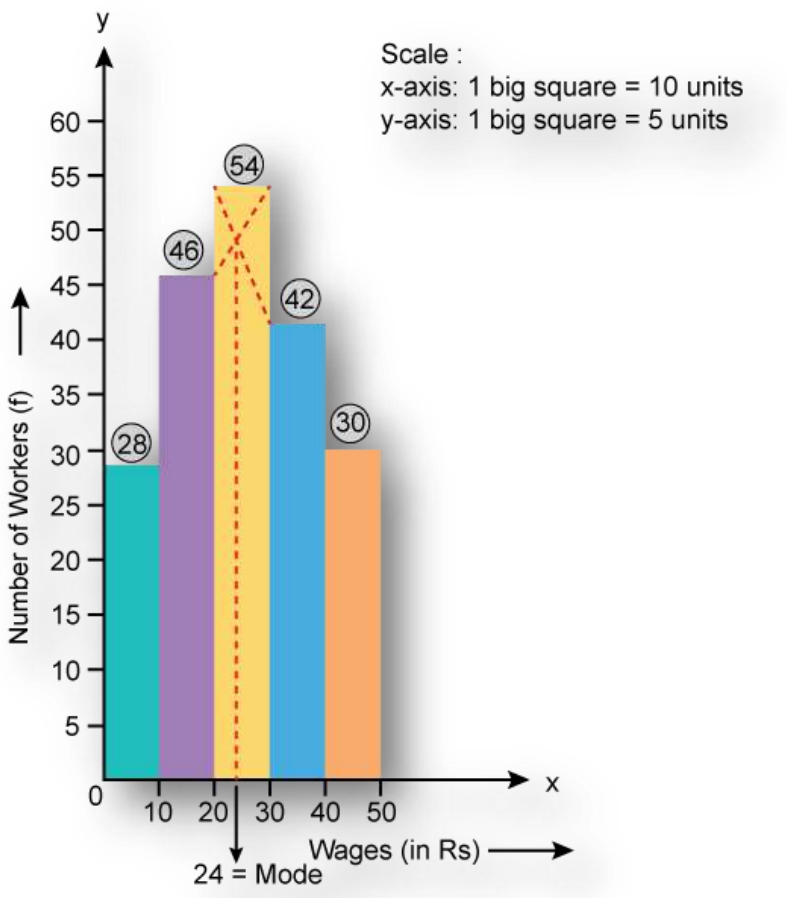
$\therefore$ **Median = 35**

Q.16. Calculate mode of the following series using the graphic technique. Counter check the modal value with the formula.

Wage	0-10	10-20	20-30	30-40	40-50
Number of Workers	28	46	54	42	30

Solution:

Wages (in Rs)	No. of Workers (*f*)
0 – 10	28
10 – 20	46 = f_0
20 – 30	54 = f_1
30 – 40	42 = f_2
40 – 50	30



Modal class is 20-30 as it has the highest frequency.

$$\text{Mode } (Z) = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i$$

$$Z = 20 + \frac{54 - 46}{2(54) - 46 - 42} \times 10$$

$$Z = 20 + \frac{8}{20} \times 10$$

$$\therefore \boxed{Z = 24}$$

Q.17. Calculate mode from the following data:

Wage	25	50	75	80	85	90
Number of workers	4	6	9	3	2	1

Solution:

Mode of the given series is **75** as it has the highest frequency of **9**.

Q.18. Find out mode from the following data:

Class Interval	5-10	10-15	15-20	20-25	25-30	30-35	35-40
Number of children	4	5	3	2	6	7	3

Solution:

Class Interval	**No. of Children (*f*)**
5 – 10	4
10 – 15	5
15 – 20	3
20 – 25	2
25 – 30	$6 = f_0$
30 – 35	$7 = f_1$
35 – 40	$3 = f_2$

Modal class is (30-35) as it has the highest frequency of 7.

$$\text{Mode } (Z) = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i$$

$$Z = 30 + \frac{7 - 6}{2(7) - 6 - 3} \times 5$$

$$\therefore \boxed{Z = 31}$$

Q.19. Calculate mode of the following series, using grouping method:

Size	40	44	48	52	56	60	64	68	72	76
Frequency	10	12	14	20	15	20	18	10	8	4

Solution:

Size	(I) Frequency	(II) 1 + 2	(III) 2 + 3	(IV) 1 + 2 + 3	(V) 2 + 3 + 4	(VI) 3 + 4 + 5
40	10	10 + 12 = 22				
44	12		12 + 14 = 26	10 + 12 + 14 = 36		
48	14	14 + 20 = 34			12 + 14 + 20 = 46	
52	20		20 + 15 = 35	20 + 15 + 20 = 55		14 + 20 + 15 = 49
56	15	15 + 20 = 35			15 + 20 + 18 = 53	
60	20		20 + 18 = 38			20 + 18 + 10 = 48
64	18	18 + 10 = 28		18 + 20 + 8 = 36		
68	10		10 + 8 = 18		10 + 8 + 4 = 22	
72	8	8 + 4 = 12				
76	4					

Analysis Table

Column	Size of items containing maximum frequency									
	40	44	48	52	56	60	64	68	72	76
I				✓		✓				
II					✓	✓				
III						✓	✓			
IV				✓	✓	✓				
V					✓	✓	✓			
VI			✓	✓	✓					
Total	-	-	1	3	4	5	2	-	-	-

Mode is **60** as it repeats itself maximum number of times.

Q.20. Calculate mode of the following distribution:

Marks	10-19	20-29	30-39	40-49	50-59	60-69	70-79	80-89	90-99
Number of students	29	87	181	247	263	133	40	9	2

Solution:

Class Interval	Exclusive Class Interval	Frequency (f)
10 – 19	9.5 – 19.5	29
20 – 29	19.5 – 29.5	87
30 – 39	29.5 – 39.5	181
40 – 49	39.5 – 49.5	**247 = f_0**
50 – 59	49.5 – 59.5	**263 = f_1**
60 – 69	59.5 – 69.5	**133 = f_2**
70 – 79	69.5 – 79.5	40
80 – 89	79.5 – 89.5	9
90 – 99	89.5 – 99.5	2

$$\text{Mode } (Z) = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i$$

$$Z = 49.5 + \frac{263 - 247}{2(263) - 247 - 133} \times 10$$

$$Z = 49.5 + \frac{160}{146}$$

$$\therefore \boxed{Z = 50.59}$$

Q.21. Find out mode, given the following information:

Size	6-10	11-15	16-20	21-25	26-30
Frequency	20	30	50	40	10

Solution:

Size	Exclusive Class Interval	Frequency (f)
6 – 10	5.5 – 10.5	20
11 – 15	10.5 – 15.5	30
16 – 20	15.5 – 20.5	50
21 – 25	20.5 – 25.5	40
26 – 30	25.5 – 30.5	10

Modal class is (15.5-20.5)as it has the hightes frequency of 50.

$$\text{Mode } (Z) = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i$$

$$Z = 15.5 + \frac{50 - 30}{2(50) - 30 - 40} \times 5$$

$$Z = 15.5 + \frac{100}{30}$$

$$\therefore \boxed{Z = 18.33}$$

Q.22. Calculate mode from the following data:

Wages (Rs)	**Number of Workers**
Less than 10	15
Less than 20	35
Less than 30	60
Less than 40	84
Less than 50	96
Less than 60	127
Less than 70	198
Less than 80	250

Solution:

Wages	**No. of Wages (f)**
0 – 10	15
10 – 20	35 – 15 = 20
20 – 30	60 – 35 = 25
30 – 40	84 – 60 = 24
40 – 50	97 – 84 = 12
50 – 60	**127 – 96 = 31(f_0)**
60 – 70	**198 – 127 = 71 (f_1)**
70 – 80	**250 – 198 = 52 (f_2)**

By inspection, we can say that the modal class is 60 - 70 as it has the highest frequency of 71.

$$\text{Mode } (Z) = l_1 + \frac{f_1 + f_0}{2f_1 - f_0 - f_2} \times i$$

$$Z = 60 + \frac{71 - 31}{2(71) - 31 - 52} \times 10$$

$$Z = 60 + \frac{400}{59}$$

$$\therefore \boxed{Z = 66.78}$$

Q.23. Calculate mode from the following series:

Size	1	2	3	4	5	6	7	8	9	10
Frequency	8	6	10	12	20	12	5	3	2	4

Solution:

Mode of the given series is **5** as it has the highest frequency of **20** times.

Q.24. Calculate the mean, median and mode of the number of persons per house in a village with the help of the following information:

Number of Persons per Houses	1	2	3	4	5	6	7	8	9	10
Number of Houses	26	113	120	95	60	42	21	14	5	4

Solution:

No. of person per House (X)	No. of House (f)	fx	Cumulative Frequency (c.f.)
1	26	26	26
2	113	226	139
3	120	360	259
4	95	380	354
5	60	300	414
6	42	252	456
7	21	147	477
8	14	112	491
9	5	45	496
10	4	40	500
	N=∑f = 500	∑fx = 1888	

$$\text{Mean } (\bar{X}) = \frac{\sum fx}{\sum f} = \frac{1888}{500}$$

$$\therefore \boxed{\text{Mean } (\bar{X}) = 3.78}$$

$$\text{Median} = \text{size of } \left(\frac{N+1}{2}\right)^{th} \text{ item}$$

$$\text{Median} = \text{size of } \left(\frac{500+1}{2}\right)^{th} \text{ item}$$

Median = size of 250.5^{th} item

Median value corresponds to the 250.5^{th} item in the series.

Thus, median value is 3 as it corresponds to cumulative frequency 259.

$$\therefore \boxed{\text{Median} = 3}$$

Mode is 3 as it has the highest frequency of 120 times.

$$\therefore \boxed{\text{Mode} = 3}$$

Q.25. Calculate the median and mode from the following data:

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Number of Students	2	18	30	45	35	20	6	3

Solution:

Marks	No. of Workers (*f*)	Cumulative Frequency (*c.f.*)
0 – 10	2	2
10 – 20	18	20
20 – 30	**30 = f_0**	50
30 – 40	**45 = f_1**	95
40 – 50	**35 = f_2**	130
50 – 60	20	150
60 – 70	6	156
70 – 80	3	159
	$N = \sum f = 159$	

$$\text{Median} = \text{size of } \left(\frac{N}{2}\right)^{th} \text{ item}$$

$$\text{Median} = \text{size of } \left(\frac{159}{2}\right)^{th} \text{ item}$$

Median = size of 79.5^{th} item

79.5^{th} item lies in cumulative frequency 95 which corresponds to the class interval 30-40.
Thus, median class is 30-40.

$$\text{Median} = l_1 + \frac{\frac{N}{2} - \text{c.f.}}{f} \times i$$

$$\text{Median} = 30 + \frac{29.5}{45} \times 10$$

$\therefore$ $\boxed{\text{Median} = 36.55}$

Modal class is 60-70 as it has the highest frequency of 45.

$$\text{Mode } (Z) = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i$$

$$Z = 30 + \frac{45 - 30}{2(45) - 30 - 35} \times 10$$

$$Z = 30 + \frac{150}{25}$$

$\therefore$ $\boxed{Z = 36}$

Q.26. Calculate the median value, given the following statistical information:

Age	20-25	25-30	30-35	35-40	40-45	45-50	50-55	55-60
Number of Students	50	70	100	180	150	120	70	60

Solution:

Age	**No. of Students (*f*)**	**Cumulative Frequency (*c.f.*)**
20 – 25	50	50
25 – 30	70	120
30 – 35	100	**220 (*c.f.*)**
35 – 40	**180 (*f*)**	400
40 – 45	150	550
45 – 50	120	670
50 – 55	70	740
55 – 60	60	800
	$\sum f = 800$	

$$\text{Median} = \text{size of } \left(\frac{N}{2}\right)^{th} \text{ item}$$

$$\text{Median} = \text{size of } \left(\frac{800}{2}\right)^{th} \text{ item}$$

Median = size of 400^{th} item

400^{th} item lies in cumulative frequency 400 which corresponds to the class interval 35-45.
Thus, median class is 35-45.

$$\text{Median} = l_1 + \frac{\frac{N}{2} - \text{c.f.}}{f} \times i$$

$$\text{Median} = 35 + \frac{400 - 220}{180} \times 5$$

$$\text{Median} = 35 + \frac{180}{180} \times 5$$

$\therefore$ $\boxed{\text{Median} = 40}$

Q.27. Obtain the mean, median and mode of the following data:

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Number of Students	5	7	15	25	20	15	8	5

Solution:

Marks	Mid Value (m)	No. of Workers (f)	Cumulative Frequency (c.f.)	fm
0 - 10	5	5	5	25
10 - 20	15	7	12	105
20 - 30	25	15	27	375
30 - 40	35	25	52	875
40 - 50	45	20	72	900
50 - 60	55	15	87	825
60 - 70	65	8	95	520
70 - 80	75	5	100	375
		N = ∑f =100		∑fm = 4000

$$\text{Mean } (\bar{X}) = \frac{\Sigma fm}{\Sigma f} = \frac{4000}{100} = 40$$

$$\therefore \boxed{\text{Mean } (\bar{X}) = 40}$$

$$\text{Median} = \text{size of } \left(\frac{N}{2}\right)^{th} \text{ item}$$

$$\text{Median} = \text{size of } \left(\frac{100}{2}\right)^{th} \text{ item}$$

$$\text{Median} = \text{size of } 50^{th} \text{ item}$$

50^{th} item lies in cumulative frequency 52 which corresponds to the class interval 30-40.
Thus, median class is 30-40.

$$\text{Median} = l_1 + \frac{\frac{N}{2} - c.f.}{f} \times i$$

$$\text{Median} = 30 + \frac{50 - 27}{25} \times 10$$

$$\therefore \boxed{\text{Median} = 39.2}$$

Modal class is (30-40) as it has the highest frequency of 25.

$$\text{Mode } (Z) = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i$$

$$Z = 30 + \frac{25 - 15}{2(25) - 15 - 20} \times 10$$

$$Z = 30 + \frac{10}{50 - 35} \times 10$$

$$\therefore \boxed{Z = 36.6}$$

Q.28. Calculate median in an asymmetric distribution if mode is 83 and arithmetic mean is 92.

Solution:

Given:
Mode = 83
Mean = 92
Median =?

We know:
Mode = 3(Median) - 2(Mean)
83 = 3 (Median) - 2(92)
3 (Median) = 83 + 184

$$\therefore \boxed{\text{Median} = 89}$$

Q.29. Calculate mode when arithmetic mean is 146 and median is 130.

Solution:

Given:

Mean = 146

Median = 130

Mode =?

Mode = 3(Median) - 2(Mean)

Mode = 3(130) - 2(146)

Mode = 390 - 292

∴ Mode = 98

Q.30. If mode is 63 and median is 77, calculate arithmetic mean.

Solution:

Given:

Mode = 63

Median = 77

Mean =?

We know:

Mode = 3(Median) - 2(Mean)

63 = 3 (77) - 2 (Mean)

2 (Mean) = 231 - 63

∴ Mean = 84

Q.31. Calculate arithmetic mean, median and mode of the following series:

Marks	Number of Students
Less than 10	12
Less than 20	26
Less than 30	40
Less than 40	58
Less than 50	80
Less than 60	110
Less than 70	138
Less than 80	150

Solution:

Marks	Mid Point (m)	Cumulative Frequency	Frequency	fm
0 - 10	5	12	12	60
10 - 20	15	26	14	210
20 - 30	25	40	14	350
30 - 40	35	58	18	630
40 - 50	45	80	22	990
50 - 60	55	110	30	1650
60 - 70	65	138	28	1820
70 - 80	75	150	12	900
			∑f = 150	∑fm = 6610

$$\text{Mean } (\bar{X}) = \frac{\sum fm}{\sum f} = \frac{6610}{150}$$

$$\therefore \boxed{\text{Mean } (\bar{X}) = 44.07}$$

$$\text{Median} = \text{size of } \left(\frac{N}{2}\right)^{th} \text{ item}$$

$$\text{Median} = \text{size of } \left(\frac{150}{2}\right)^{th} \text{ item}$$

$$\text{Median} = \text{size of } 75^{th} \text{ item}$$

75^{th} item lies in cumulative frequency 80 which corresponds to the class interval 40-50.
Thus, median class is 40-50.

$$\text{Median} = l_1 + \frac{\frac{N}{2} - \text{c.f.}}{f} \times i$$

$$\text{Median} = 40 + \frac{75 - 58}{22} \times 10$$

$\therefore$ Median = 47.73

Modal class is (50-30) as it has the highest frequency of 30.

$$\text{Mode } (Z) = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i$$

$$Z = 50 + \frac{30 - 22}{2(30) - 22 - 28} \times 10$$

$\therefore$ Z = 58

Chapter – 11
Measures of Dispersion

Essential Practical:

Q.1. Calculate range and coefficient of range from the following data:

4, 7, 8, 46, 53, 77, 8, 1, 5, 13

Solution:

Highest value (H) = 77

Lowest value (L) =1

Range = H - L

Range = 77 - 1

$\therefore$ Range = 76

$$\text{Coefficient of Range} = \frac{H-L}{H+L}$$

$$\text{Coefficient of Range} = \frac{77-1}{77+1} = \frac{76}{78}$$

$\therefore$ Coefficient of Range = 0.97

Q.2. Given the following data-set, calculate range and the coefficient of range:

Size	4.5	5.5	6.5	7.5	8.5	9.5	10.5	11.5
Frequency	4	5	6	3	2	1	3	5

Solution:

Hightest value (H) = 11.5

Lowest value (L) = 4.5

Range = H - L

Range = 11.5 - 4.5

$\therefore$ Range = 7

$$\text{Coefficient of Range} = \frac{H-L}{H+L}$$

$$\text{Coefficient of Range} = \frac{11.5-4.5}{11.5+4.5} = \frac{7}{16}$$

$\therefore$ Coefficient of Range = 0.44

Q.3. Find out the range and the coefficient of range, given the following data-set:

Class Interval	1-5	6-10	11-15	16-20	21-25	26-30	31-35
Frequency	2	8	15	35	20	10	14

Solution:

Range = Highest mid-value – Lowest mid-value

$$\text{Range} = \frac{31+35}{2} - \frac{1+5}{2}$$

Range = 33 – 3

$\therefore$ Range = 30

$$\text{Coefficient of Range} = \frac{H-L}{H-L}$$

$$\text{Coefficient of Range} = \frac{33-3}{33+3}$$

$\therefore$ Coefficient of Range = 0.833

Q.4. Find out quartile deviation and the coefficient of quartile deviation of the following series.

Wages of 9 Workers in Rupees:

170, 82, 110, 100, 150, 150, 200, 116, 250

Solution:

Arranging data in ascending order:

82, 100, 110, 116, 150, 150, 170, 200, 250

N = 9

Q_1 = size of $\left(\frac{9+1}{4}\right)^{th}$ = size of 2.5^{th} item

Q_1 = size of 2^{nd} item + 0.5 (size of 3^{rd} item - size of 2^{nd} item)

Q_1=100 + 0.5 (110-100)

$\therefore Q_1 = 105$

Q_3 = size of $\left(\frac{9+1}{4}\right)^{th}$ = size of 7.5^{th} item

Q_3 = size of 7^{th} item + 0.5 (size of 8^{th} item - size of 7^{th} item)

Q_3=170 + 0.5 (200-170)

$\therefore Q_3 = 185$

$$QD=\frac{Q_3-Q_1}{2}$$

$$QD=\frac{185-105}{2}$$

$\therefore$ $\boxed{QD = 40}$

$$\text{Coefficient of QD}=\frac{Q_3-Q_1}{Q_3+Q_1}$$

$$\text{Coefficient of QD}=\frac{185-105}{185+105}$$

$\therefore$ $\boxed{\text{Coefficient of QD} = 0.276}$

Q.5. Given the following data, estimate the coefficient of QD:

15, 20, 23, 23, 25, 25, 27, 40.

Solution:

Q_1 = size of $\left(\frac{8+1}{4}\right)^{th}$ item

Q_1 = size of 2.25^{th} item

Q_1 = size of 2^{nd} item + 0.25 (size of 3^{rd} item - size of 2^{nd} item)

Q_1=20 + 0.25 (23-20)

$\therefore$ $\boxed{Q_1 = 20.75}$

Q_3 = size of $\left(\frac{8+1}{4}\right)^{th}$ item

Q_3 = size of 6.75^{th} item

Q_3 = size of 6^{th} item + 0.75 (size of 7^{th} item - size of 6^{th} item)

Q_3=25 + 0.25 (27 - 25)

$\therefore$ $\boxed{Q_3=26.5}$

$$\text{Coefficient of Q.D.} =\frac{Q_3-Q_1}{Q_3+Q_1}$$

$$\text{Coefficient of Q.D.} =\frac{26.5-20.75}{26.5+20.75}$$

$\therefore$ $\boxed{\text{Coefficient of QD} = 0.12}$

Q.6. Find out mean deviation of the following series from mean and median:

Size	4	6	8	10	12	14	16
Frequency	2	4	5	31	2	1	4

Solution:

X	f	fX	$\lvert d_{\bar{x}}\rvert = \lvert X - \bar{X}\rvert$	$fd_{\bar{x}}$
4	2	8	5.87	11.74
6	4	24	3.87	15.48
8	5	40	1.87	9.35
10	31	310	0.13	4.03
12	2	24	2.13	4.26
14	1	14	4.13	4.13
16	4	64	6.13	24.52
	$\Sigma f = 49$	$\Sigma fX = 484$		$\Sigma fd_{\bar{x}} = 73.51$

Mean $(\bar{X}) = \frac{\Sigma fx}{\Sigma f} = \frac{484}{49} = 9.87$

MD from mean $(MD_{\bar{x}}) = \frac{\Sigma fd_{\bar{x}}}{\Sigma f} = \frac{73.51}{49}$

$\therefore$ MD from mean $(MD_{\bar{x}}) = 1.50$

X	f	c. f.	$\lvert d_m\rvert = \lvert x - M\rvert$	$f\lvert d_m\rvert$
4	2	2	6	12
6	4	6	4	16
8	5	11	2	10
10	31	42	0	0
12	2	44	2	4
14	1	45	4	4
16	4	49	6	24
	$\Sigma f = 49$			$\Sigma f\lvert d_M\rvert = 70$

Medain = size of $\left(\frac{N+1}{2}\right)^{th}$ item

Median = size of $\left(\frac{49+1}{2}\right)^{th}$ item

Median= size of 25^{th} item

25^{th} item lies in cumulative frequency 42 which corresponds to 10.

Thus, median is 10.

MD from median = $(MD_m) = \frac{\Sigma f\lvert d_m\rvert}{\Sigma f} = \frac{70}{49}$

$\therefore$ MD from median = 1.43

Q.7. Calculate mean deviation and coefficient of mean deviation with the help of median:

Class Interval	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	15	19	14	20	18	14

Solution:

X	Mid Value (m)	f	c.f.	$\lvert d_m\rvert = \lvert m - M\rvert$	$f\lvert d_m\rvert$
0 – 10	5	15	15	26	390
10 – 20	15	19	34	16	304
20 – 30	25	14	48	6	80
30 – 40	35	20	68	4	252
40 – 50	45	18	86	4	252
50 – 60	55	14	100	14	336
		$\Sigma fx = 100$			$\Sigma f\lvert d_m\rvert = 1446$

Median class =size of $\left(\frac{100}{2}\right)^{th}$ item

Median class =size of 50^{th} item

50^{th} item lies in cumulative frequency 68 which corresponds to 30-40 class.

$$\text{Median} = l_1 + \frac{\frac{N}{2} - \text{c.f.}}{f} \times i$$

$$\text{Median} = 30 + \frac{50 - 48}{20} \times 10$$

$$\text{Median} = 30 + \frac{20}{20}$$

$\therefore$ $\boxed{\text{Median} = 31}$

$$\text{MD from median}(MD_m) = \frac{\Sigma f|d_m|}{\Sigma f} = \frac{1446}{100}$$

$\therefore$ $\boxed{\text{MD from median}(MD_m) = 14.46}$

$$\text{Coefficient of MD} = \frac{\text{Mean Deviation}}{\text{Median}} = \frac{14.46}{31}$$

$\therefore$ $\boxed{\text{Coefficient of MD} = 0.466}$

Q.8. Calculate mean deviation from mean of the following series:

Size of Items	3-4	4-5	5-6	6-7	7-8	8-9	9-10
Frequency	3	7	22	60	85	32	9

Solution:

Class Interval	Mid Value (m)	f	fm	$\lvert d_{\bar{x}}\rvert = \lvert m - \bar{X}\rvert$	$f\lvert d_{\bar{x}}\rvert$
3 – 4	3.5	3	10.5	3.6	10.8
4 – 5	4.5	7	31.5	2.6	18.2
5 – 6	5.5	22	121	1.6	35.2
6 – 7	6.5	60	390	0.6	36
7 – 8	7.5	85	637.5	0.4	34
8 – 9	8.5	32	272	1.4	44.8
9 – 10	9.5	9	85.5	2.4	21.6
		$\Sigma f = 218$	$\Sigma fm = 1548$		$\Sigma f\lvert d_{\bar{x}}\rvert = 200.6$

$$\text{Mean}(\bar{X}) = \frac{\Sigma fm}{\Sigma f} = \frac{1548}{218}$$

$\therefore$ $\boxed{\text{Mean}(\bar{X}) = 7.10}$

$$\text{MD from } (MD_{\bar{x}}) = \frac{\Sigma f|d_{\bar{x}}|}{\Sigma f} = \frac{200.6}{218}$$

$\therefore$ $\boxed{\text{MD from } (MD_{\bar{x}}) = 0.92}$

Q.9. Given below are the marks obtained by the students of a class. Calculate mean deviation, and its coefficient, using median of data:

Marks	17	35	38	16	42	27	19	11	40	25

Solution:

Arranging the data in ascending order:

11, 16, 17, 19, 25, 27, 35, 38, 40, 42

$N = 10$

$$\text{Median} = \frac{\text{size of } \left(\frac{N}{2}\right)^{th} \text{ item} + \text{size of } \left(\frac{N}{2} + 1\right)^{th} \text{ item}}{2}$$

$$\text{Median} = \frac{\text{size of } 5^{th} \text{ item} + \text{size of } 6^{th} \text{ item}}{2}$$

$$\text{Median} = \frac{25 + 27}{2}$$

$\therefore$ $\boxed{\text{Median} = 26}$

X	$\lvert d_m \rvert = \lvert x - M \rvert$
11	15
16	10
17	9
19	7
25	1
27	1
35	9
38	12
40	14
42	16
	$\Sigma \lvert d_m \rvert = 94$

$$\text{MD from Median } (MD_m) = \frac{\Sigma |d_m|}{n} = \frac{94}{10}$$

$\therefore$ $\boxed{\text{MD from Median } (MD_m) = 9.4}$

$$\text{Coefficient of } MD_m = \frac{MD_m}{\text{median}} = \frac{9.4}{26}$$

$\therefore$ $\boxed{\text{Coefficient of } MD_m = 0.36}$

Q.10. Nine students of a class obtained following marks. Calculate mean deviation from median.

S.No.	1	2	3	4	5	6	7	8	9
Marks	68	49	32	21	54	38	59	66	41

Solution:

Arranging data in ascending order:

21, 32, 38, 41, 49, 54, 59, 66, 68

N = 9 (odd)

$$\text{Median} = \text{size of } \left(\frac{9+1}{2}\right)^{th} \text{ item}$$

Median = size of 5^{th} item

$\therefore$ Median = 49

Marks	Arranged Marks	$\lvert d_m \rvert = \lvert x - M \rvert$
68	21	28
49	32	17
32	38	11
21	41	8
54	49	0
38	54	5
59	59	10
66	66	17
41	68	19
		$\Sigma \lvert d_m \rvert = 115$

$$\text{MD from Median } (MD_m) = \frac{\Sigma |d_m|}{n} = \frac{115}{9}$$

$\therefore$ $\boxed{\text{MD from Median } (MD_m) = 12.77}$

Q.11. Following data relate to the age- difference of husbands and wives of a particular community. Find out mean deviation from mean.

Age-difference	0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40
Frequency	449	705	507	281	109	52	16	4

Solution:

X	Mid Value m	f	fm	$\lvert d_{\bar{x}} \rvert = \lvert m - \bar{X} \rvert$	$f\lvert d_{\bar{x}} \rvert$
0 – 5	2.5	449	1122.5	7.96	3574.04
5 – 10	7.5	705	5287.5	2.96	2086.8
10 – 15	12.5	507	6337.5	2.04	1034.28
15 – 20	17.5	281	4917.5	7.04	1978.24
20 – 25	22.5	109	2452.5	12.02	1312.36
25 – 30	27.5	52	1430	17.04	886.08
30 – 35	32.5	16	520	22.04	352.64
35 – 40	37.5	4	150	27.4	108.16
		$\Sigma f = 2123$	$\Sigma fm = 22217.5$		$\Sigma f\lvert d_{\bar{x}} \rvert = 11332.6$

$$\text{Mean}\left(\bar{X}\right) = \frac{\Sigma fm}{\Sigma f} = \frac{22217.5}{2123}$$

$$\therefore \boxed{\text{Mean}\left(\bar{X}\right) = 10.46}$$

$$\text{MD from Mean } (MD_{\bar{x}}) = \frac{\Sigma f\lvert d_{\bar{x}} \rvert}{\bar{X}} = \frac{11332.6}{2123}$$

$$\therefore \boxed{\text{MD from Mean } (MD_{\bar{x}}) = 5.34}$$

Q.12. Find out the mean deviation and its coefficient using median of the following data:

Number of Accidents	1	2	3	4	5	6	7	8	9	10	11	12
Number of Victims of accidents	16	21	10	17	8	4	2	1	2	2	2	2

Solution:

Arranging data in ascending order:

1,2,2,2,2,2,4,8,10,16,17,21

$$\text{Median} = \frac{\text{size of } \left(\frac{N}{2}\right)^{th} \text{ item} + \text{size of } \left(\frac{N}{2}+1\right)^{th} \text{ item}}{2}$$

$$\text{Median} = \frac{\text{size of } 6^{th} \text{ item} + \text{size of } 7^{th} \text{ item}}{2}$$

$$\therefore \text{Median} = \frac{2+4}{2} = 3$$

No. of Victims	Arranged data (X)	$\lvert d_m \rvert = \lvert X - M \rvert$
16	1	2
21	2	1
10	2	1
17	2	1
8	2	1
4	2	1
2	4	1
1	8	5
2	10	7
2	16	13
2	17	14
2	21	18
		$\Sigma\lvert d_m \rvert = 65$

$$\text{MD from Median } (MD_m) = \frac{\Sigma\lvert d_m \rvert}{n} = \frac{65}{12}$$

$$\therefore \boxed{\text{MD from Median } (MD_m) = 5.42}$$

$$\text{Coefficient of } MD_m = \frac{MD_m}{\text{median}} = \frac{5.42}{3}$$

$$\therefore \boxed{\text{Coefficient of } MD_m = 1.81}$$

Q.13. Calculate standard deviation, given the following data:

10, 12, 14, 16, 18, 22, 24, 26, 28

Solution:

X	$x = (x - \bar{X})$	x^2
10	-8.88	78.85
12	-6.88	47.33
14	-4.88	23.81
16	-2.88	8.29
18	-0.88	0.77
22	3.12	9.73
24	5.12	26.21
26	7.12	50.69
28	9.12	83.17
$\Sigma X = 170$		$\Sigma x^2 = 328.85$

$$\text{Mean } (\bar{X}) = \frac{\Sigma X}{N} = \frac{170}{9}$$

$$\therefore \boxed{\text{Mean } (\bar{X}) = 18.88}$$

$$\text{Standard Deviation } (\sigma) = \sqrt{\frac{\Sigma x^2}{N}} = \sqrt{\frac{328.85}{9}} =$$

$$\therefore \boxed{\text{Standard Deviation } (\sigma) = 6.05}$$

Q.14. Calculate standard deviation and the coefficient of standard deviation, given the following data:

Income (Rs)	5	10	15	20	25	30	35	40
Number of Workers	26	29	40	35	26	18	14	12

Solution:

Income (X)	No. of workers (f)	fx	$x = X - \bar{x}$	x^2	fx^2
5	26	130	-14.4	207.36	5391.36
10	29	290	-9.4	88.36	2562.44
15	40	600	-4.4	19.36	774.4
20	35	700	0.6	0.36	12.6
25	26	650	5.6	31.36	815.36
30	18	540	10.6	112.36	2022.48
35	14	490	15.6	243.36	3407.04
40	12	280	20.6	424.36	5092.32
	$\Sigma f = 200$	$\Sigma fx = 3880$			$\Sigma fx^2 = 20078$

$$\bar{X} = \frac{\Sigma fx}{\Sigma f} = \frac{3880}{200}$$

$$\therefore \boxed{\bar{X} = 19.4}$$

$$\text{Standard Deviation } (\sigma) = \sqrt{\frac{\Sigma fx^2}{\Sigma f}} = \sqrt{\frac{20078}{200}}$$

$$\therefore \boxed{\text{Standard Deviation } (\sigma) = 10.02}$$

$$\text{Coefficient of SD} = \frac{\sigma}{\bar{X}} = \frac{10.02}{19.4}$$

$$\therefore \boxed{\text{Coefficient of SD} = 0.516}$$

Q.15. Of the two sets of income distribution of five and seven persons respectively, as given below calculate standard deviation:

(i) Income (Rs)	4,000	4,200	4,400	4,600	4,800		
(ii) Income (Rs)	3,000	4,000	4,200	4,400	4,600	4,800	5,800

Solution:

Income distribution I:

X_1	$x_1 = X_1 - \overline{X_1}$	x_1^2
4000	−400	160000
4200	−200	40000
4400	0	0
4600	200	40000
4800	400	160000
$\Sigma X_1 = 22000$		$\Sigma x_1^2 = 400000$

$$\overline{X_1} = \frac{\Sigma x_1}{n_1} = \frac{22000}{5}$$

$\therefore \overline{X_1} = 4400$

$$\text{Standard Deviation } (\sigma_1) = \sqrt{\frac{\Sigma x^2_I}{n_1}} = \sqrt{\frac{400000}{5}}$$

$\therefore$ Standard Deviation $(\sigma_1) = 282.84$

Income distribution II:

X_2	$x_2 = X_2 - \overline{X_2}$	x_2^2
3000	−1400	1960000
4000	−400	160000
4200	−200	40000
4400	0	0
4600	200	40000
4800	400	160000
5800	1400	1960000
30800		$\Sigma x_2^2 = 4320000$

$$\overline{X_2} = \frac{\Sigma x_2}{n_2} = \frac{30800}{7}$$

$\therefore \overline{X_2} = 4400$

$$\text{Standard Deviation } (\sigma_1) = \sqrt{\frac{\Sigma x^2_2}{n_2}} = \sqrt{\frac{4320000}{7}}$$

$\therefore$ Standard Deviation $(\sigma_1) = 785.58$

Q.16. Find out the standard deviation of the marks secured by 10 students:

S.No	1	2	3	4	5	6	7	8	9	10
Marks	43	48	65	57	31	60	37	48	78	59

Solution:

S.No	Marks (X)	$x = (X - \overline{X})$	X^2
1	43	−9.6	92.16
2	48	−4.6	21.16
3	65	12.4	153.76
4	57	4.4	19.36
5	31	−21.6	466.56
6	60	7.4	54.76
7	37	−15.6	243.36
8	48	−4.6	21.16
9	78	25.4	645.16
10	59	6.4	40.96
	$\Sigma X = 526$		$\Sigma x^2 = 1758.4$

$$\text{Mean } (\bar{X}) = \frac{\Sigma X}{N} = \frac{526}{10}$$

$$\therefore \boxed{\text{Mean } (\bar{X}) = 52.6}$$

$$\text{Standard Deviation } (\sigma) = \sqrt{\frac{\Sigma x^2}{N}} = \sqrt{\frac{1758.4}{10}}$$

$$\therefore \boxed{\text{Standard Deviation } (\sigma) = 13.26}$$

Q.17. Data of daily sale proceeds of a shop are as below. Calculate mean deviation and standard deviation:

Daily Sales	102	100	110	114	118	122	126
Days	3	9	25	35	14	10	1

Solution:

Daily Sales (X)	Days (f)	fX	$\lvert d_{\bar{X}} \rvert = \lvert X - \bar{X} \rvert$	$f\lvert d_{\bar{X}} \rvert$	$f\lvert d_{\bar{X}} \rvert^2$
102	3	306	10.98	32.94	361.68
100	9	900	12.98	116.82	1516.32
110	25	2750	2.98	745	222
114	35	3990	1.02	35.7	36
118	17	2006	5.02	85.34	428.4
122	10	1220	9.02	90.2	813.6
126	1	126	13.02	13.02	169.52
	$\Sigma f = 100$	$\Sigma fx = 11298$		$\Sigma f\lvert d_{\bar{X}} \rvert = 1119.02$	$\Sigma f\lvert d_{\bar{X}} \rvert^2 = 3547.92$

$$\text{Mean}(\bar{X}) = \frac{\Sigma fx}{N} = \frac{11298}{100}$$

$$\therefore \text{Mean}(\bar{X}) = 112.98$$

$$\text{MD from mean } (MD_{\bar{X}}) = \frac{\Sigma f d_{\bar{X}}}{\Sigma f} = \frac{1119.02}{100}$$

$$\therefore \text{MD from mean } (MD_{\bar{X}}) = 11.19$$

$$\text{Standard Deviation } (\sigma) = \sqrt{\frac{\Sigma f\lvert d_{\bar{X}} \rvert^2}{\Sigma f}} = \sqrt{\frac{3547.92}{100}}$$

$$\therefore \text{Standard Deviation } (\sigma) = 5.96$$

Q.18. Calculate range, standard deviation and coefficient of variation of marks secured by students:

50	55	57	49	54	61	64	59	58	56

Solution:

X	$x = (X - \bar{X})$	X^2
50	-6.3	39.69
55	-1.3	1.69
57	0.7	0.49
49	-7.3	53.29
54	-2.3	5.29
61	4.7	22.09
64	7.7	59.29
59	2.7	7.29
58	1.7	2.89
56	-0.3	0.09
$\Sigma X = 563$		$\Sigma x^2 = 192.1$

$$\text{Range } (R) = H - L = 64 - 49$$

$$\therefore \boxed{\text{Range } (R) = 15}$$

$$\text{Mean } (\bar{X}) = \frac{\Sigma X}{N} = \frac{563}{10}$$

$$\therefore \boxed{\text{Mean } (\bar{X}) = 56.3}$$

$$\text{Standard deviation } (\sigma) = \sqrt{\frac{\Sigma x^2}{N}} = \sqrt{\frac{192.1}{10}}$$

$$\therefore \boxed{\text{Standard deviation } (\sigma) = 4.38}$$

$$\text{Coefficient of Variation} = \frac{\sigma}{\bar{X}} \times 100 = \frac{4.38}{56.3} \times 100$$

$$\therefore \boxed{\text{Coefficient of Variation} = 7.78}$$

Q.19. Following data show the number of runs made by Sachin and Sourabh in different innings. Find out who is a good scorer and who is a consistent player?

Sachin	92	17	83	56	72	76	64	45	40	32
Sourabh	28	70	31	00	59	108	82	14	3	95

Solution:

For Sachin:

X_1	x_1	x_1^2
92	34.3	1176.49
17	-40.7	1656.49
83	25.3	640.09
56	-1.7	2.89
72	14.3	204.49
76	18.3	334.89
64	6.3	39.69
45	-12.7	161.29
40	-17.7	313.29
32	-25.7	660.49
ΣX_1=577		**Σx_1^2=5190.1**

$$\bar{X} = \frac{\sum X_1}{n_1} = \frac{577}{10}$$

$$\therefore \boxed{\bar{X} = 57.7}$$

$$\sigma = \sqrt{\frac{\sum x_1^2}{n_1}} = \sqrt{\frac{5190.1}{10}}$$

$$\therefore \boxed{\sigma = 22.78}$$

$$\text{Coefficient of Variation} = \frac{\sigma}{\bar{X}} \times 100 = \frac{22.78}{57.7} \times 100$$

$$\therefore \boxed{\text{Coefficient of Variation} = 39.48}$$

For Sourabh:

X_2	x_2	x_2^2
28	-27	729
70	25	625
31	-24	576
00	-55	3025
59	4	16
108	53	2809
82	27	729
14	-41	1681
3	-52	2704
95	40	1600
ΣX_2=550		**Σx_2^2=14494**

$$\bar{X}=\frac{\sum X_2}{n_2}=\frac{550}{10}$$

$$\therefore \boxed{\bar{X}=55}$$

$$\sigma=\sqrt{\frac{\sum x_2^2}{n_2}}=\sqrt{\frac{14494}{10}}$$

$$\therefore \boxed{\sigma=38.07}$$

$$\text{Coefficient of Variation}=\frac{\sigma}{\bar{X}}\times 100=\frac{38.07}{55}\times 100$$

$$\therefore \boxed{\text{Coefficient of Variation}=69.21}$$

Good scorer- Sachin is a good scorer as his mean runs is 57.7 which is higher than Saurabh whose mean runs is 55.

Consistent player- Sachin is a consistent player as his coefficient of variation is 39.48 which is lower than Saurabh whose coefficient of variation is 69.21.

Q.20. Calculate standard deviation of marks secured by 100 examinees in the examination:

Marks	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90
Number of Examination	19	3	2	49	24	2	0	1

Solution:

X	m	f	fm	$x=X-\bar{X}$	x^2	fx^2
10 – 20	15	19	285	-26.70	712.89	13544.91
20 – 30	25	3	75	-16.70	278.89	836.67
30 – 40	35	2	70	-6.7	44.89	89.78
40 – 50	45	49	2205	3.3	10.89	533.61
50 – 60	55	24	1320	13.3	176.89	4245.36
60 – 70	65	2	130	23.3	542.89	1085.78
70 – 80	75	0	0	33.3	1108.89	0
80 – 90	85	1	85	43.3	1874.89	1874.89
		$\sum f = 100$	$\sum fx = 4170$			$\sum fx^2 = 22211$

$$\bar{X}=\frac{\sum fX}{\sum f}=\frac{4170}{100}$$

$$\therefore \boxed{\bar{X} = 41.70}$$

$$\text{Standard Deviation } (\sigma)=\sqrt{\frac{\sum fx^2}{\sum f}}=\sqrt{\frac{22211}{100}}$$

$$\therefore \boxed{\text{Standard Deviation } (\sigma) = 14.90}$$

Q.21. Calculate coefficient of variation from the following data:

Variables	10	20	30	40	50	60	70
Frequencies	6	8	16	15	32	1	12

Solution:

X	f	fx	$x=X-\bar{X}$	x^2	fx^2
10	6	60	-34	1156	6936
20	8	160	-24	576	4608
30	16	480	-14	196	3136
40	15	600	-4	16	240
50	32	1600	6	36	1152
60	11	660	16	256	2816
70	12	840	26	676	8112
	$\sum f=100$	$\sum fX=4400$			$\sum fx^2=27000$

$$\bar{X}=\frac{\sum fX}{\sum f}=\frac{4400}{100}$$

$\therefore \boxed{\bar{X}=44}$

$$\text{Standard Deviation } (\sigma)=\sqrt{\frac{\sum fx^2}{\sum f}}=\sqrt{\frac{27000}{100}}$$

$\therefore \boxed{\text{Standard Deviation } (\sigma)=16.43}$

$$\text{Coefficient of Variation}=\frac{\sigma}{\bar{X}}\times 100=\frac{16.43}{44}\times 100$$

$\therefore \boxed{\text{Coefficient of Variation}=37.34}$

Q.22. Estimate coefficient of variation of the following data:

Weight (k.g)	0-20	20-40	40-60	60-80	80-100
Number of persons	81	40	66	49	14

Solution:

Weight	m	f	fm	$x=X-\bar{X}$	x^2	fx^2
0-20	10	81	810	-30	900	72900
20-40	30	40	1200	-10	100	4000
40-60	50	66	3300	10	100	6600
60-80	70	49	3430	30	900	44100
80-100	90	14	1260	50	2500	35000
		$\sum f=250$	$\sum fm=10000$			$\sum fx^2=162600$

$$\bar{X}=\frac{\sum fm}{\sum f}=\frac{10000}{250}$$

$\therefore \boxed{\bar{X}=40}$

$$\text{StandardDeviation } (\sigma)=\sqrt{\frac{\sum fx^2}{\sum f}}=\sqrt{\frac{162600}{250}}$$

$\therefore \boxed{\text{Standard deviation } (\sigma)=25.50}$

$$\text{Coefficient of Variation}=\frac{\sigma}{\bar{X}}\times 100=\frac{25.50}{40}\times 100$$

$\therefore \boxed{\text{Coefficient of Variation}=63.75}$

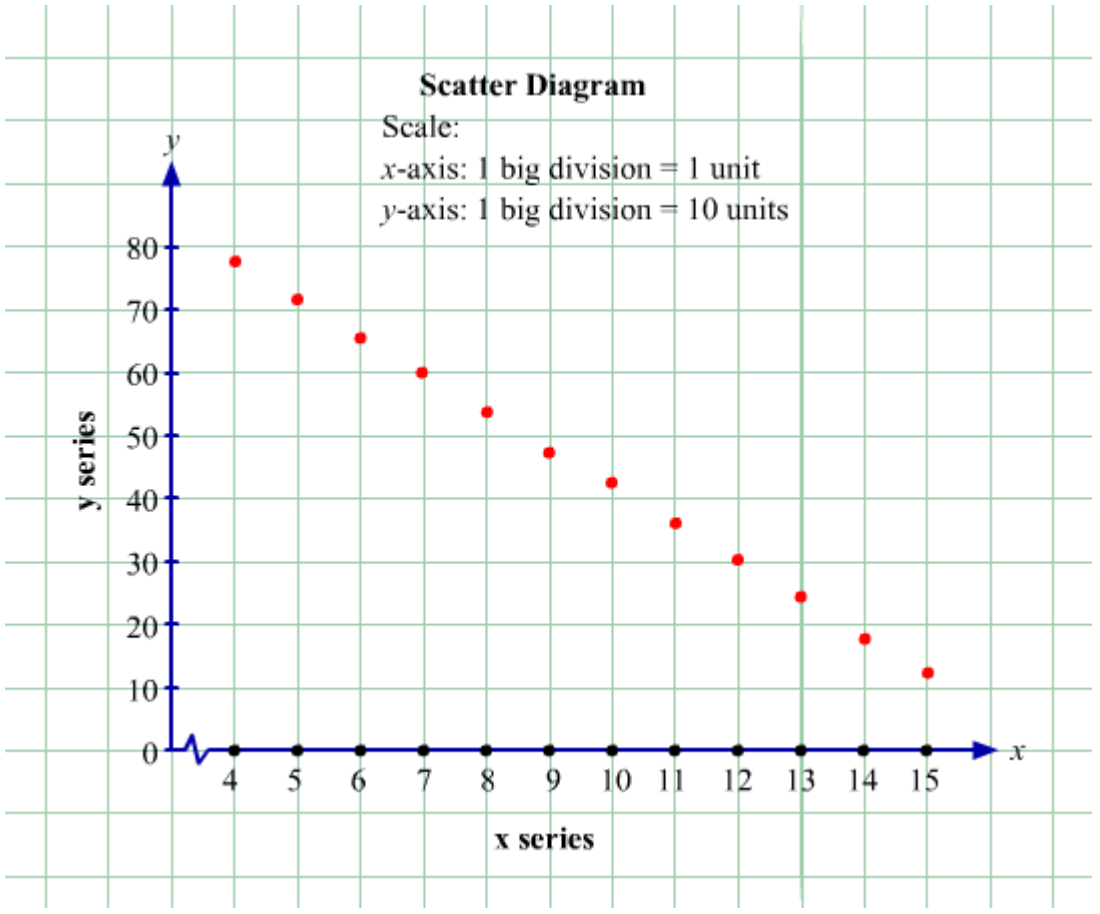
Chapter – 12
Correlation

Essential Practical:

Q.1. Make a scattered diagram of the data given below. Does any relationship exist between the two?

X	4	5	6	7	8	9	10	11	12	13	14	15
Y	78	72	66	60	54	48	42	36	30	24	18	12

Solution:



X and Y series show a ***perfect negative relationship*** between each other.

Q.2. Calculate coefficient of correlation of the age of husband and wife using Karl Pearson's method.

Husband (Age)	23	27	28	29	30	31	33	35	36
Wife (Age)	18	20	22	27	29	27	29	28	29

Solution:

h	$h=H-\bar{H}$	h^2	w	$w=W-\bar{W}$	w^2	hw
23	-7.22	52.12	18	-7.44	55.35	53.71
27	-3.22	10.36	20	-5.44	29.59	17.51
28	-2.22	4.92	22	-3.44	11.83	7.63
29	-1.22	1.48	27	1.56	2.43	-1.90
30	-0.22	0.04	29	3.56	12.67	-0.78
31	0.78	0.60	27	1.56	2.43	1.21
33	2.78	7.72	29	3.56	12.67	9.89
35	4.78	22.84	28	2.56	6.55	12.23
36	5.78	33.40	29	3.56	12.67	20.57
$\sum h=272$		$\sum h^2=133.48$	$\sum w=229$		$\sum w^2=146.19$	$\sum wh=120.07$

Husbands' mean age $(\bar{H}) = \frac{\sum h}{n} = \frac{272}{9}$

$\therefore \boxed{\bar{H} = 30.22}$

Wifes' mean age $(\bar{W}) = \frac{\sum w}{n} = \frac{229}{9}$

$\therefore \boxed{\bar{W} = 25.44}$

$$r = \frac{\sum wh}{\sqrt{\sum h^2 \times \sum w^2}}$$

$$r = \frac{120.17}{\sqrt{133.48 \times 146.19}}$$

$\therefore \boxed{r = 0.86}$

Q.3. Calculate correlation of the following data using Karl Pearson's method.

Series A	112	114	108	124	145	150	119	125	147	150
Series B	200	190	214	187	170	170	210	190	180	181

Solution:

A	$a=A-\bar{A}$	a^2	B	$b=B-\bar{B}$	b^2	ab
112	-17.4	302.76	200	10.8	116.64	-187.92
114	-15.4	237.16	190	0.8	0.64	-12.32
108	-21.4	457.96	214	24.8	615.04	-530.72
124	-5.4	29.16	187	-2.2	4.84	11.88
145	15.6	243.36	170	-19.2	368.64	-299.52
150	20.6	424.36	170	-19.2	368.64	-395.52
119	-10.4	108.16	210	20.8	432.64	-216.32
125	-4.4	19.36	190	0.8	0.64	-3.52
147	17.6	309.76	180	-9.2	84.64	-161.92
150	20.6	424.36	181	-8.2	67.24	-168.92
$\sum A=1294$		$\sum a^2=2556.4$	$\sum B=1892$		$\sum b^2=2059.6$	$\sum ab=-1964.8$

Mean of series A $(\bar{A}) = \frac{\sum A}{n} = \frac{1294}{10}$

$\therefore \boxed{\bar{A} = 129.4}$

Mean of Series B $(\bar{B}) = \frac{\sum B}{n} = \frac{1892}{10}$

$\therefore \boxed{\bar{B} = 189.2}$

$$r = \frac{\sum ab}{\sqrt{\sum a^2 \times \sum b^2}}$$

$$r = \frac{-1964.8}{\sqrt{2556.4 \times 2059.6}}$$

$\therefore \boxed{r = -0.85}$

Q.4. Using assumed average in Karl Pearson's formula, calculate coefficient of correlation, given the following data :

X	78	89	97	69	59	79	68	61
Y	125	137	156	112	107	106	123	138

Solution:

X	$d_x = X-69$	d_x^2	Y	$d_y = Y-125$	d_y^2	$d_x d_y$
78	9	81	125	0	0	0
89	20	400	137	12	144	240
97	28	784	156	31	961	868
69	0	0	112	-13	169	0
59	-10	100	107	-18	324	180
79	10	100	106	-19	361	-190
68	-1	1	123	-2	4	2
61	-8	64	138	13	169	-104
N=8	$\sum d_x = 48$	$\sum d_x^2 = 1530$	N=8	$\sum d_y = 4$	$\sum d_y^2 = 2132$	$\sum d_x d_y = 996$

$$r = \frac{\sum d_x d_y - \frac{(\sum d_x) \times (\sum d_y)}{N}}{\sqrt{\sum d_x^2 - \frac{(\sum d_x)^2}{N}} \times \sqrt{\sum d_y^2 - \frac{(\sum d_y)^2}{N}}}$$

$$r = \frac{996 - \frac{48 \times 4}{8}}{\sqrt{1530 - \frac{(48)^2}{8}} \times \sqrt{2132 - \frac{(4)^2}{8}}}$$

$\therefore$ $\boxed{r = 0.597}$

Q.5. Find out Karl Pearson's coefficient of correlation :

Capital Units (in '000)	10	20	30	40	50	60	70	80	90	100
Profit Receipt	2	4	8	5	10	15	14	20	22	30

Solution:

X	$d_x = X-50$	d_y^2	Y	$d_y = Y-15$	d_y^2	$d_x d_y$
10	-40	1600	2	-13	169	520
20	-30	900	4	-11	121	330
30	-20	400	8	-7	49	140
40	-10	100	5	-10	100	100
50	0	0	10	-5	25	0
60	10	100	15	0	0	0
70	20	400	14	-1	1	-20
80	30	900	20	5	25	150
90	40	1600	22	7	49	280
100	50	2500	30	15	225	750
N=10	$\sum d_x = 50$	$\sum d_x^2 = 8500$	N=10	$\sum d_y = -20$	$\sum d_y^2 = 764$	$\sum d_x d_y = 2250$

$$r = \frac{\sum d_x d_y - \frac{(\sum d_x) \times (\sum d_y)}{N}}{\sqrt{\sum d_x^2 - \frac{(\sum d_x)^2}{N}} \times \sqrt{\sum d_y^2 - \frac{(\sum d_y)^2}{N}}}$$

$$r = \frac{2250 - \frac{50 \times (-20)}{10}}{\sqrt{8500 - \frac{(50)^2}{8}} \times \sqrt{764 - \frac{(-20)^2}{10}}}$$

$\therefore$ $\boxed{r = 0.96}$

Q.6. Seven students of a class secured following marks in Economics and History. Calculate coefficient of correlation with the help of these data.

Economics	66	90	89	55	58	44	42
History	58	76	65	58	53	49	56

Solution:

Economics (E)	Rank R_1	History (H)	Rank R_2	D= R_1 - R_2	D^2
66	3	58	3.5	-5	0.25
90	1	76	1	0	0
89	2	65	2	0	0
55	5	58	3.5	1.5	2.25
58	4	53	6	-2	4
44	6	49	7	-1	1
42	7	56	5	2	4
N = 7					$\Sigma D^2 = 11.50$

58 is repeated two times in series 2. Thus, $m_1 = 2$ and following formula is used to calculate correlation.

$$r_k = 1 - \frac{6\left[\sum D^2 + \frac{1}{12}(m_1^3 - m_1)\right]}{N^3 - N}$$

$$r_k = 1 - \frac{6\left[11.50 + \frac{1}{12}(8-2)\right]}{343-7}$$

$$\therefore \boxed{r_k = 0.79}$$

Q.7. Find out rank difference correlation of X and Y :

X	80	78	75	75	58	67	60	59
Y	12	13	14	14	14	16	15	17

Solution:

X	R_1	Y	R_2	D = R_1 - R_2	D^2
80	1	12	8	-7	49
78	2	13	7	-5	25
75	3.5	14	5	-1.5	2.25
75	3.5	14	5	-1.5	2.25
58	8	14	5	3	9
67	5	16	2	3	9
60	6	15	3	3	9
59	7	17	1	6	36
N = 8					$\Sigma D^2 = 141.5$

75 is repeated two times in series 1 and 14 is repeated three times in series 2. Thus, $m_1 = 2$ and $m_2 = 3$ and following formula is used to calculate correlation.

$$r_k = 1 - \frac{6\left[\sum D^2 + \frac{1}{12}(m_1^3 - m_1) + \frac{1}{12}(m_2^3 - m_2)\right]}{N^3 - N}$$

$$r_k = 1 - \frac{6\left[141.50 + \frac{1}{12}(2^3-2) + \frac{1}{12}(3^3-3)\right]}{512-8}$$

$$\therefore \boxed{r_k = -0.714}$$

Q.8. Calculate coefficient of correlation of the following data with rank difference and Karl Pearson's methods :

Economics (Marks)	77	54	27	52	14	35	90	25	56	60
Hindi (Marks)	35	58	60	46	50	40	35	56	44	42

Solution:

Karl Pearson's Method:

Economics (X)	$d_x = X - 35$	d_x^2	History (Y)	$d_y = Y - 50$	dy^2	$d_x d_y$
77	42	1764	35	-15	225	-630
54	19	361	58	8	64	152
27	-8	64	60	10	100	-80
52	17	289	46	-4	16	-68
14	-21	441	50	0	0	0
35	0	0	40	-10	100	0
90	55	3025	35	-15	225	-825
25	-10	100	56	6	36	-60
56	21	441	44	-6	36	-126
60	25	625	42	-8	64	-200
N = 10	$\Sigma d_x = 140$	$\Sigma d_x^2 = 7110$	N = 10	$\Sigma d_y = -34$	$\Sigma dy^2 = 860$	$\Sigma d_x d_y = -1837$

$$r = \frac{\sum d_x d_y - \frac{(\sum d_x) \times (\sum d_y)}{N}}{\sqrt{\sum d_x^2 - \frac{(\sum d_x)^2}{N}} \times \sqrt{\sum d_y^2 - \frac{(\sum d_y)^2}{N}}}$$

$$r \frac{-1831 - \frac{140 \times (-34)}{10}}{\sqrt{7110 - \frac{(140)^2}{10}} \times \sqrt{860 - \frac{(-34)^2}{10}}}$$

$$\therefore \boxed{r = -0.69}$$

Rank Difference Method:

Economics	R_1	History	R_2	$D = R_1 - R_2$	D^2
77	2	35	9.5	-7.5	56.25
54	5	58	2	3	9
27	8	60	1	7	49
52	6	46	5	1	1
14	10	50	4	6	36
35	7	40	8	-1	1
90	1	35	9.5	-8.5	72.25
25	9	56	3	6	36
56	4	44	6	-2	4
60	3	42	7	-4	16
N = 10					$\Sigma D^2 = 280.5$

35 is repeated two times in series 2. Thus, $m_1 = 2$ and following formula is used to calculate correlation.

$$r_k = 1 - \frac{6\left[\sum D^2 + \frac{1}{12}(m_1^3 - m_1)\right]}{N^3 - N}$$

$$r_k = 1 - \frac{6\left[280.5 + \frac{1}{12}(2^3 - 2)\right]}{1000 - 10}$$

$$\therefore \boxed{r_k = -0.703}$$

Q.9. Seven methods of teaching Economics in two universities are shown below. Calculate rank difference correlation.

Teaching Methods	I	II	III	IV	V	VI	VII
Rank of 'A's Students	2	1	5	3	4	7	6
Rank of 'B's Students	1	3	2	4	7	5	6

Solution:

Teaching Method	Rank of A R_A	Rank of B R_B	$D = R_A - R_B$	D^2
I	2	1	1	1
II	1	3	-2	4
III	5	2	3	9
IV	3	4	-1	1
V	4	7	-3	9
VI	7	5	2	4
VII	6	6	0	0
N=7				$\Sigma D^2 = 28$

$$r_k = 1 - \frac{6\sum D^2}{N^3 - N}$$

$$r_k = 1 - \frac{6 \times 28}{343 - 7}$$

$$\therefore \boxed{r_k = 0.5}$$

Q.10. Give three examples of perfect correlation. Find out rank difference correlation with the help of the following data:

X	48	33	40	9	16	65	26	15	57
Y	13	13	22	6	14	20	9	6	15

Solution:

Examples of perfect correlation:

1. Relationship between study hours and marks
2. Relationship between consumption and saving from fixed income
3. Relationship between amount of loan taken from bank and interest paid

X	R_1	Y	R_2	$D = R_1 - R_2$	D^2
48	3	13	5.5	-2.5	6.25
33	5	13	5.5	-0.5	0.25
40	4	22	1	3	9
9	9	6	8.5	0.5	0.25
16	7	14	4	3	9
65	1	20	2	-1	1
26	6	9	7	-1	1
15	8	6	8.5	-0.5	0.25
57	2	15	3	0.1	1
N = 9					$\Sigma D^2 = 28$

13 and 6 are repeated two times in series 2. Thus, $m_1 = 2$ and $m_2 = 2$ and following formula is used to calculate correlation.

$$r_k = 1 - \frac{6\left[\sum D^2 + \frac{1}{12}(m_1^3 - m_1) + \frac{1}{12}(m_2^3 - m_2)\right]}{N_3 - N}$$

$$r_k = 1 - \frac{6\left[28 + \frac{1}{12}(2^3 - 2) + \frac{1}{12}(2^3 - 2)\right]}{729 - 9}$$

$$\therefore \boxed{r_k = 0.758}$$

Q.11. Calculate coefficient of correlation of the following data :

X	10	6	9	10	12	13	11	9
Y	9	4	6	9	11	13	8	4

Solution:

X	$d_x = X - 10$	d_x^2	Y	$d_y = Y - 11$	d_y^2	$d_x d_y$

10	0	0	9	-2	4	0
6	-4	16	4	-7	49	28
9	-1	1	6	-5	25	5
10	0	0	9	-2	4	0
12	2	9	11	0	0	0
13	3	4	13	2	4	6
11	1	1	8	-3	9	-3
9	-1	1	4	-7	49	7
N = 8	$\Sigma d_x = 0$	$\Sigma d_x^2 = 32$	N = 8	$\Sigma d_y = -24$	$\Sigma d_y^2 = 144$	$\Sigma d_x d_y = 43$

$$r=\frac{\sum d_x d_y-\frac{(\sum d_x)\times(\sum d_y)}{N}}{\sqrt{\sum d_x^2-\frac{(\sum d_x)^2}{N}}\times\sqrt{\sum d_y^2-\frac{(\sum d_y)^2}{N}}}$$

$$r=\frac{43-\frac{0\times(-24)}{8}}{\sqrt{32-\frac{(0)^2}{8}}\times\sqrt{144-\frac{(-24)^2}{8}}}$$

$\therefore \boxed{r=0.9}$

Q.12. Deviation of two series X and Y are shown. Calculate coefficient of correlation.

X	+5	-4	-2	+20	-10	0	+3	0	-15	-5
Y	+5	-12	-7	+25	-10	-3	0	+2	-9	-15

Solution:

d_x	d_x^2	d_y	d_y^2	$d_x d_y$
5	25	5	25	25
-4	16	-12	144	48
-2	4	-7	49	14
20	400	25	625	500
-10	100	-10	100	100
0	0	-3	9	0
3	9	0	0	0
0	0	2	4	0
-15	225	-9	81	135
-5	25	-15	225	75
Σ **d_x = -8**	$\Sigma d_x^2 = 804$	$\Sigma d_y = -24$	$\Sigma d_y^2 = 1262$	$\Sigma d_x d_y = 897$

$$r=\frac{\sum d_x d_y-\frac{(\sum d_x)\times(\sum d_y)}{N}}{\sqrt{\sum d_x^2-\frac{(\sum d_x)^2}{N}}\times\sqrt{\sum d_y^2-\frac{(\sum d_y)^2}{N}}}$$

$$r=\frac{897-\frac{(-8)\times(-24)}{10}}{\sqrt{804-\frac{(-8)^2}{10}}\times\sqrt{1262-\frac{(-24)^2}{10}}}$$

$\therefore \boxed{r=0.89}$

Q.13. In a baby competition, two judges accorded following ranks to 12 competitors. Find the coefficient of rank correlation.

Entry	A	B	C	D	E	F	G	H	I	J	K	L
Judge X	1	2	3	4	5	6	7	8	9	10	11	12
Judge Y	12	9	6	10	3	5	4	7	8	2	11	1

Solution:

Entry	Judge X (R_X)	Judge Y (R_Y)	D = R_X - R_Y	D^2
A	1	12	-11	121
B	2	9	-7	49
C	3	6	-3	9
D	4	10	-6	36
E	5	3	2	4
F	6	5	1	1
G	7	4	3	9
H	8	7	1	1
I	9	8	1	1
J	10	2	8	64
K	11	11	0	0
L	12	1	11	121
N = 12				$\Sigma D^2 = 416$

$$r_k = 1 - \frac{6\sum D^2}{N^3 - N}$$

$$r_k = 1 - \frac{6 \times 416}{1728 - 12}$$

$$\therefore \quad \boxed{r_k = -0.455}$$

Q.14. In a Fancy-dress competition, two judges accorded the following ranks to eight participants :

Judge X	8	7	6	3	2	1	5	4
Judge Y	7	5	4	1	3	2	6	8

Calculate coefficient of rank correlation.

Solution:

R_X	R_Y	D = R_X - R_Y	D^2
8	7	1	1
7	5	2	4
6	4	2	4
3	1	2	4
2	3	-1	1
1	2	-1	1
5	6	-1	1
4	8	-4	16
			$\Sigma D^2 = 32$

$$r_k = 1 - \frac{6\sum D^2}{N^3 - N}$$

$$r_k = 1 - \frac{6 \times 32}{512 - 8}$$

$$\therefore \quad \boxed{r_k = 0.619}$$

Q.15. In a beauty contest, three judges accorded following ranks to 10 participants :

Judge I	1	6	5	10	3	2	4	9	7	8
Judge II	3	5	8	4	7	10	2	1	6	9
Judge III	6	4	9	8	1	2	3	10	5	7

Find out by Spearman's Rank Difference Method which pair of judges has a common taste in respect of beauty.

Solution:

R_1	R_2	R_3	$D_1 = R_1 - R_2$	$D_2 = R_1 - R_3$	$D_3 = R_2 - R_3$	D_1^2	D_2^2	D_3^2
1	3	6	-2	-5	-3	4	25	9
6	5	4	1	2	1	1	4	1

5	8	9	-3	-4	-1	9	16	1
10	4	8	6	2	-4	36	4	16
3	7	1	-4	2	6	16	4	36
2	10	2	-8	0	8	64	0	64
4	2	3	2	1	-1	4	1	1
9	1	10	8	-1	-9	64	1	81
7	6	5	1	2	1	1	4	1
8	9	7	-1	1	8	1	1	64
						$\Sigma D_1^2 = 200$	$\Sigma D_2^2 = 60$	$\Sigma D_3^2 = 214$

$$r_k(\text{Judge I \& Judge II}) = 1 - \frac{6\sum D_1^2}{N^3 - N}$$

$$r_k(\text{Judge I \& Judge II}) = 1 - \frac{6 \times 200}{1000 - 10}$$

$$\therefore \boxed{r_k(\text{Judge I \& Judge II}) = -0.212}$$

$$r_k(\text{Judge I \& Judge III}) = 1 - \frac{6 \times 60}{1000 - 10}$$

$$\therefore \boxed{r_k(\text{Judge I \& Judge II}) = 0.636}$$

$$r_k(\text{Judge II \& Judge III}) = 1 - \frac{6 \times 214}{1000 - 10}$$

$$\therefore \boxed{r_k(\text{Judge I \& Judge II}) = -0.296}$$

Judges I and III have a common taste in respect of beauty as they have the highest positive rank correlation coefficient.

Q.16. Following data relates to age group and percentage of regular players. Calculate Karl Pearson's coefficient of correlation.

Age Group	20-25	25-30	30-35	35-40	40-45	45-50
% of Regular Players	40	35	28	20	15	5

Solution:

Age Group	Mid value (X)	% of Regular Players (Y)	$d_x = \frac{X-37.5}{5}$	$d_y = \frac{Y-28}{5}$	$(d_x)(d_y)$	$(d_x)^2$	$(d_y)^2$
20-25	22.5	40	-3	2.4	-7.2	9	5.76
25-30	27.5	35	-2	1.4	-2.8	4	1.96
30-35	32.5	28	-1	0	0	1	.0
35-40	37.5	20	0	-1.6	0	0	2.56
40-45	42.5	15	1	-2.6	-2.6	1	6.76
45-50	47.5	5	2	-4.6	-9.2	4	21.16
			$\sum d_x = -3$	$\sum d_y = -5$	$\sum (d_x)(d_y) = -21.8$	$\sum (d_x)^2 = 19$	$\sum (d_y)^2 = 38.2$

$$r = \frac{\sum d_x d_y - \frac{(\sum d_x) \times (\sum d_y)}{N}}{\sqrt{\left[\sum d_x^2 - \frac{(\sum d_x)^2}{N}\right] \times \left[\sum d_y^2 - \frac{(\sum d_y)^2}{N}\right]}}$$

$$r = \frac{(-21.8) - \frac{(-3) \times (-5)}{6}}{\sqrt{\left(19 - \frac{(-3)^2}{6}\right) \times \left(38.2 - \frac{(-5)^2}{6}\right)}}$$

$$r = -\frac{-24.3}{\sqrt{17.5 \times 34.03}}$$

$$\therefore \boxed{r = -0.996}$$

Q.17. From the following data, relating to playing habits in various age group of 900 students. Calculate coefficient of correlation between age group and playing habits.

age group	15-16	16-17	17-18	18-19	19-20	20-21
Number of Students	250	200	150	120	100	80
Regular Players	200	150	90	48	30	12

Solution:

Age Group	Mid Value (X)	Percentage of Players (%) (Y)	d_X = X-17.5	d_Y = Y-40	d_Xd_Y	d_X^2	d_Y^2
15-16	15.5	200/250×100 = 80	-2	40	-80	4	1600
16-17	16.5	150/2-00×100 = 75	-1	35	-35	1	1225
17-18	17.5	90/150×100 = 60	0	20	0	0	400
18-19	18.5	48/120×100 = 40	1	0	0	1	0
19-20	19.5	30/100×100 = 30	2	-10	-20	4	100
20-21	20.5	12/80×100 = 15	3	-25	-75	9	625
	N = 6		$\Sigma d_X = 3$	$\Sigma d_Y = 60$	$\Sigma d_Xd_Y = -210$	$\Sigma d_X^2 = 19$	$\Sigma d_Y^2 = \Sigma 3950$

$$r=\frac{\sum d_Xd_Y-\frac{(\sum d_X)\times(\sum d_Y)}{N}}{\sqrt{\sum d_X^2-\frac{(\sum d_X)^2}{N}}\times\sqrt{\sum d_Y^2-\frac{(\sum d_Y)^2}{N}}}$$

$$r=\frac{-210-\frac{3\times(60)}{6}}{\sqrt{19-\frac{(3)^2}{6}}\times\sqrt{3950-\frac{(60)^2}{6}}}$$

$\therefore$ $\boxed{r=-0.992}$

Q.18. Following data relates to density of population, number of deaths and population of various cities. Calculate death rate and Karl Pearson coefficient between density of population and death rate.

Cities	P	Q	R	S	T	U
Density of Population	200	500	700	500	600	900
Number of Deaths	840	300	312	560	1,440	1,224
Population	42,000	30,000	24,000	40,000	90,000	72,000

Solution:

Density (X)	dx = X - 500	dx^2	Death rate (Y)= $\frac{\text{No. of deaths}}{\text{Population}}\times 100$	dy = Y-1	dy^2	dxdy
200	-300	90000	2	1	1	-300
500	0	0	1	0	0	0
700	200	40000	1.3	0.3	0.09	60
500	0	0	1.4	0.4	0.16	0
600	100	10000	1.6	0.6	0.36	60
900	400	160000	1.7	0.7	0.49	280
	$\Sigma dx=400$	$\Sigma dx^2=300000$		$\Sigma dy=3$	$\Sigma dy^2= 2.1$	$\Sigma dxdy = 100$

$$r=\frac{\sum dxdy-\frac{(\sum d_x)(\sum d_y)}{n}}{\sqrt{\sum d_{x^2}-\frac{(\sum d_x)^2}{n}}\sqrt{\sum d_{y^2}-\frac{(\sum d_y)^2}{n}}}$$

$$r=\frac{100-\frac{400\times 3}{6}}{\sqrt{300000-\frac{(400)^2}{6}}\sqrt{2.1-\frac{(3)^2}{6}}}$$

$\therefore$ $\boxed{r = -0.248}$

Q.19. From the following data, determine Karl Pearson's coefficient of correlation between X and Y series for 15 pairs :

	X-series	Y-series
Mean	80	120
Sum of Squares of deviation from Arithmetic Mean	56	156
Sum of product of deviations of X and Y from their respective Means	92	

Solution:

Given:

$N = 15$

$\bar{X} = 80,\ \bar{Y} = 120$

$\sum x^2 = 56,\ \sum y^2 = 156$

$\sum xy = 92$

$$\sigma_x = \sqrt{\frac{\sum x^2}{N}} = \sqrt{\frac{56}{15}} = 1.93$$

$$\sigma_y = \sqrt{\frac{\sum y^2}{N}} = \sqrt{\frac{156}{15}} = 3.22$$

$$r = \frac{\sum xy}{N\sigma_x\sigma_y}$$

$$r = \frac{92}{15 \times 1.93 \times 3.22}$$

$\therefore \boxed{r = 0.98}$

Q.20.

	X - Series	Y - Series
Number of Items	15	15
Mean	25	18
SD	3.01	3.03
Sum of Squares of deviation from Mean	136	138
Sum of product of deviation of X and Y from their respective Means	122	

Solution:

Given:

$N = 15$

$\bar{X} = 25,\ \bar{Y} = 18$

$\sigma_x = 3.01,\ \sigma_y = 3.03$

$\sum x^2 = 136$

$\sum y^2 = 138$

$\sum xy = 122$

$$r = \frac{\sum xy}{N\sigma_x\sigma_y}$$

$$r = \frac{122}{15 \times 3.01 \times 3.03}$$

$\therefore \boxed{r = 0.89}$

Chapter – 13
Index Numbers

Essential Practical:

Q.1. Taking 2004 as base year, construct the index numbers of the years 2005 and 2009.

Year	2004	2005	2006	2007	2008	2009
Price	10	14	16	20	22	24

Solution:

Year	Price
2004	10
2005	14
2006	16
2007	20
2008	22
2009	24

Given:

$P_0 = 10$

Index number for year 2005:

$$P_{01} = \frac{P_1}{P_0} \times 100 = \frac{14}{10} \times 100$$

$$\therefore \boxed{P_{01} = 140}$$

Index number for year 2009:

$$P_{01} = \frac{24}{10} \times 100$$

$$\therefore \boxed{P_{01} = 240}$$

Q.2. Construct index number by Price Relative Method taking 2004 as base year.

Price per Unit in Rs

Year	A	B	C	D
2004	25	18	16	21
2012	20	22	24	22
2013	25	20	25	25
2014	28	24	30	26

Solution:

	2004 (P_0)	2012 (P_1)	Price Relative $= \frac{P_1}{P_0} \times 100$
A	25	20	$\frac{20}{25} \times 100 = 80$
B	18	22	$\frac{22}{18} \times 100 = 122.22$
C	16	24	$\frac{24}{16} \times 100 = 150$
D	21	22	$\frac{22}{21} \times 100 = 104.76$
Total			456.98

$$P_{01} = \frac{\sum\left(\frac{P_1}{P_0} \times 100\right)}{N}$$

$$P_{01} = \frac{456.98}{4}$$

$$\therefore \boxed{P_{01} = 114.245}$$

	2004 (P_0)	2013 (P_1)	Price Relative $= \frac{P_1}{P_0} \times 100$
A	25	25	$\frac{25}{25} \times 100 = 100$
B	18	20	$\frac{20}{18} \times 100 = 111.11$
C	16	25	$\frac{25}{16} \times 100 = 156.25$
D	21	25	$\frac{25}{21} \times 100 = 119.04$
Total			486.4

$$P_{01} = \frac{486.4}{4}$$

$\therefore$ $\boxed{P_{01} = 121.60}$

	2004 (P_0)	2014 (P_1)	Price Relative $= \frac{P_1}{P_0} \times 100$
A	25	28	$\frac{28}{25} \times 100 = 280$
B	18	24	$\frac{24}{18} \times 100 = 133.33$
C	16	30	$\frac{30}{16} \times 100 = 187.5$
D	21	26	$\frac{26}{21} \times 100 = 123.80$
Total			556.63

$$P_{01} = \frac{556.63}{4}$$

$\therefore$ $\boxed{P_{01} = 139.16}$

Q.3. Compute a Price Index for the following by (i) Simple Aggregative Method, and (ii) Average of Price Relative Method :

Commodities	A	B	C	D	E	F
Price in 2009 (Rs.)	20	30	10	25	40	50
Price in 2014 (Rs.)	25	30	15	35	45	55

Solution:

(i) Simple Aggregate Method:

	2009 (P_0)	2014 (P_1)
A	20	25
B	30	30
C	10	15
D	25	35
E	40	45
F	50	55
	$\sum P_0 = 175$	$\sum P_1 = 205$

$$P_{01} = \frac{\sum P_1}{\sum P_0} \times 100 = \frac{205}{175} \times 100$$

$\therefore$ $\boxed{P_{01} = 117.14}$

(ii) Price Relative Method:

	2009 (P_0)	2014 (P_1)	$R = \frac{P_0}{P_1} \times 100$
A	20	25	$\frac{25}{20} \times 100 = 125$
B	30	30	$\frac{30}{30} \times 100 = 100$
C	10	15	$\frac{15}{10} \times 100 = 150$
D	25	35	$\frac{35}{25} \times 100 = 140$
E	40	45	$\frac{45}{40} \times 100 = 112.5$
F	50	55	$\frac{55}{50} \times 100 = 110$
			$\sum R = 737.5$

$$P_{01} = \frac{\sum R}{N} = \frac{737.5}{6}$$

$$\therefore \boxed{P_{01} = 122.92}$$

Q.4. Construct price index number of the following data by using : (i) Laspeyre's Method, (ii) Paasche's Method, and (iii) Fisher's Method.

Items	Base Year		Current Year	
	Quantity	Price	Quantity	Price
A	3	5	2	8
B	7	4	5	6
C	4	7	3	10
D	6	6	5	7

Solution:

	q_0	p_0	p_0q_0	p_1	q_1	p_1q_1	p_1q_0	p_0q_1
A	3	5	15	8	2	16	24	10
B	7	4	28	6	5	30	42	20
C	4	7	28	10	3	30	40	21
D	6	6	36	7	5	35	42	30
			$\Sigma p_0q_0 = 107$			$\Sigma p_1q_1 = 111$	$\Sigma p_1q_0 = 148$	$\Sigma p_0q_1 = 81$

(i) Laspeyre's Price index:

$$P_{01} = \frac{\sum P_1q_0}{\sum P_0q_0} \times 100 = \frac{148}{107} \times 100$$

$$\therefore \boxed{P_{01} = 138.32}$$

(ii) Paasche's Price index :

$$P_{01} = \frac{\sum P_1q_1}{\sum P_0q_1} \times 100 = \frac{111}{81} \times 100$$

$$\therefore \boxed{P_{01} = 137.04}$$

(iii) Fisher's Price index:

$$P_{01} = \sqrt{\frac{\sum P_1q_0}{\sum P_0q_0} \times \frac{\sum P_1q_1}{\sum P_0q_1}} \times 100$$

$$P_{01} = \sqrt{\frac{148}{107} \times \frac{111}{81}} \times 100$$

$$\therefore \boxed{P_{01} = 137.66}$$

Q.5. Construct an index number for the year 2014, taking 2004 as base year by any method you deem ideal:

Year	Good I		Good II		Good III	
	Price	Quantity	Price	Quantity	Price	Quantity
2004	5	10	8	6	6	3
2014	4	12	7	7	5	4

Solution:

	p_0	q_0	p_1	q_1	p_0q_0	p_1q_1	p_0q_1	p_1q_0
Good I	5	10	4	12	50	48	60	40
Good II	8	6	7	7	48	49	56	42
Good III	6	3	5	4	18	20	24	15
					$\Sigma p_0q_0 = 116$	$\Sigma p_1q_1 = 117$	$\Sigma p_0q_1 = 140$	$\Sigma p_1q_0 = 97$

Using Fisher's method:

$$P_{01}=\sqrt{\frac{\sum P_1q_0}{\sum P_0q_0}\times\frac{\sum P_1q_1}{\sum P_0q_1}}\times 100$$

$$P_{01}=\sqrt{\frac{97}{116}\times\frac{117}{140}}\times 100$$

$$\therefore \boxed{P_{01}=83.59}$$

Q.6. Given the following data and taking 2004 as the base year, construct index of prices using:
(i) Laspeyre's Method, (ii) Paasche's Method, and (iii) Fisher's Method.

Year	Commodities							
	A		B		C		D	
	Price	Quantity	Price	Quantity	Price	Quantity	Price	Quantity
2004	24	8	9	3	16	5	10	3
2014	30	10	10	4	20	8	9	4

Solution:

	q_0	p_0	p_0q_0	p_1	q_1	p_1q_1	p_1q_0	p_0q_1
A	24	8	192	30	10	300	240	240
B	9	3	27	10	4	40	30	36
C	16	5	80	20	8	160	100	128
D	10	3	30	9	4	36	27	40
			$\Sigma p_0q_0 = 329$			$\Sigma p_1q_1 = 536$	$\Sigma p_1q_0 = 397$	$\Sigma p_0q_1 = 444$

Laspeyre's Price index:

$$P_{01}=\frac{\sum P_1q_0}{\sum P_0q_0}\times 100$$

$$P_{01}=\frac{397}{329}\times 100$$

$$\therefore \boxed{P_{01}=120.67}$$

Paasche's Price index:

$$P_{01}=\frac{\sum P_1q_1}{\sum P_0q_1}\times 100$$

$$P_{01}=\frac{536}{444}\times 100$$

$$\therefore \boxed{P_{01}=120.72}$$

Fisher's Price index:

$$P_{01}=\sqrt{\frac{\sum P_1q_0}{\sum P_0q_0}\times\frac{\sum P_1q_1}{\sum P_0q_1}}\times 100$$

$$P_{01}=\sqrt{\frac{397}{329}\times\frac{536}{444}}\times 100$$

$$\therefore \boxed{P_{01}=120.69}$$

Q.7. Construct a weighted index number of the following data using price relative method :

Item	A	B	C	D	E
Base Year (Quantity)	24	14	8	4	8
Base Year (Price)	2	4	6	10	5
Current Year (Price)	3	5	9	12	5

Solution:

	P_0	q_0	P_1	$P_0q_0=W$	$R=\frac{P_1}{P_0}\times 100$	RW
A	2	24	3	48	$\frac{3}{2}\times 100=150$	7200
B	4	14	5	56	$\frac{5}{4}\times 100=125$	7000
C	6	8	9	48	$\frac{9}{6}\times 100=150$	7200
D	10	4	12	40	$\frac{12}{10}\times 100=120$	4800
E	5	8	5	40	$\frac{5}{5}\times 100=100$	4000
				$\sum W=232$		$\sum RW=30200$

Weighted index number:

$$P_{01}=\frac{\sum RW}{\sum W}=\frac{30200}{232}$$

$$\therefore \boxed{P_{01}=130.17}$$

Q.8. Find out the index number of the following data with Laspeyre's Method :

Commodity	2013		2014	
	Price	Quantity	Price	Quantity
A	70	7	80	6
B	62	3	74	2

Solution:

	P_0	q_0	P_0q_0	P_1	q_1	P_1q_0
A	70	7	490	80	6	560
B	62	3	186	74	2	222
			$\Sigma P_0q_0 = 676$			$\Sigma P_1q_0 = 782$

Laspeyre's Price index

$$P_{01}=\frac{\sum P_1q_0}{\sum P_0q_0}\times 100$$

$$P_{01}=\frac{782}{676}\times 100$$

$$\therefore \boxed{P_{01}=115.68}$$

Q.9. Construct index numbers of the following data with Laspeyre's and Paasche's Methods :

Commodity	Base Year		Current Year	
	Price	Quantity	Price	Quantity
A	10	0.80	11	0.70
B	8	0.85	9	0.90
C	5	1.30	5.5	0.80

Solution:

	p_0	q_0	p_0q_0	p_1	q_1	p_1q_1	p_0q_1	p_1q_0
A	0.8	10	8	0.7	11	7.7	8.8	7
B	0.85	8	6.8	0.9	9	8.1	7.65	7.2
C	1.35	5	6.5	0.8	5.5	4.4	7.15	4
			$\Sigma p_0q_0 = 21.3$			$\Sigma p_1q_1 = 20.2$	$\Sigma p_0q_1 = 23.60$	$\Sigma p_0q_1 = 18.2$

Laspeyre's price index:

$$P_{01} = \frac{\sum P_1q_0}{\sum P_0q_0} \times 100$$

$$P_{01} = \frac{18.2}{21.3} \times 100$$

$$\therefore \boxed{P_{01} = 85.45}$$

Paasche's price index:

$$P_{01} = \frac{\sum P_1q_1}{\sum P_0q_1} \times 100$$

$$P_{01} = \frac{20.2}{23.60} \times 100$$

$$\therefore \boxed{P_{01} = 85.59}$$

Q.10. Construct index numbers of the following data by Fisher's Method :

Commodity	Base Year		Current Year	
	Price	Value	Price	Value
A	3	18	7	14
B	5	35	10	100
C	6	42	11	55
D	4	32	6	60
E	8	24	9	36

Solution:

	P_0	Base year value	$q_0 = \frac{\text{value}}{P_0}$	P_0q_0	P_1	Current year value	$q_1 = \frac{\text{value}}{P_1}$	p_1q_1	p_1q_0	p_0q_1
A	3	18	6	18	7	14	2	14	42	6
B	5	35	7	35	10	100	10	100	70	50
C	6	42	7	42	11	55	5	55	77	30
D	4	32	8	32	6	60	10	60	48	40
E	8	24	3	24	9	36	4	36	27	32
				$\sum P_0q_0 = 151$				$\sum p_1q_1 = 265$	$\sum p_1q_0 = 264$	$\sum p_0q_1 = 158$

Fisher'sPrice Index:

$$P_{01} = \sqrt{\frac{\sum P_1q_0}{\sum P_0q_0} \times \frac{\sum P_1q_1}{\sum P_0q_1}} \times 100$$

$$P_{01} = \sqrt{\frac{264}{151} \times \frac{265}{158}} \times 100$$

$$\therefore \boxed{P_{01} = 171.24}$$

Q.11. Construct Cost of Living Index on the basis of the following data :

Items	Price	Weight
Wheat	241	10
Rice	150	4
Maida	200	2
Pulses	170	2
Oil	125	2

Solution:

Items	Price (P)	Weights (W)	PW
Wheat	241	10	2410
Rice	150	4	600
Maida	200	2	400
Pulses	170	2	340
Oil	125	2	250
		$\Sigma W = 20$	$\Sigma PW = 4000$

$$\text{Cost of living Index} = \frac{\sum PW}{\sum W}$$

$$\text{Cost of living Index} = \frac{4000}{620}$$

$$\therefore \boxed{\text{Cost of living Index} = 200}$$

Q.12. Construct Cost of Price Index Number with the help of the following data :

Items	Price	Weight
Food	125	40
Fuel	120	10
Cloth	66.67	25
House Rent	120	15
Miscellaneous	150	10

Solution:

Items	Price (P)	Weights (W)	PW
Food	125	40	5000
Fuel	120	10	1200
Cloth	66.67	25	16 66.75
House rent	120	15	1800
Miscellaneous	150	10	1500
		$\Sigma W = 100$	$\Sigma PW = 11166.75$

$$\text{Cost of living Index} = \frac{\sum PW}{\sum W}$$

$$\text{Cost of living Index} = \frac{11166.75}{100}$$

$$\therefore \boxed{\text{Cost of living Index} = 111.67}$$

Q.13. From the following data find Consumer Price Index or Cost of Living Index :

Items	Quantity Consumed in Current Year	Price in Base Year	Price in Current Year
Rice	30 qt	12	25
Pulses	36 kg	0.4	0.6
Oil	24 l	1.5	2.2
Clothing	72 metres	0.75	10
Housing	per month	20	30
Miscellaneous	per month	20	15

Solution:

Items	q_1	P_0	P_1	P_1q_1	P_0q_1
Rice	30	12	25	750	360
Pulses	36	0.4	0.6	21.6	14.4
Oil	24	1.5	2.2	52.8	36
Clothing	72	0.75	10	720	54
Housing	1	20	30	30	20
Miscellaneous	1	20	15	15	20
				$\Sigma P_1q_1 = 1589.4$	$\Sigma P_0q_1=504.4$

$$CPI = \frac{\sum P_1q_1}{\sum P_0q_1} \times 100$$

$$CPI = \frac{1589.4}{504.4} \times 100$$

$$\therefore \boxed{CPI = 315.1}$$

Q.14. Construct Cost of Living Index Number for the year 2014 from the following statistics :

Commodity	2004 Price	2004 Quantity	2014 Price
A	25	16.0	35
B	36	7.0	48
C	12	3.5	16
D	6	2.5	10
E	28	4.0	28

Solution:

	q_1	q_0	P_1	P_1q_0	P_0q_0
A	25	16	35	560	400
B	36	7	48	336	252
C	12	3.5	16	56	42
D	6	2.5	10	25	15
E	28	4	28	112	112
				$\Sigma P_1q_0 = 821$	$\Sigma P_0q_0=1089$

$$CPI = \frac{\sum P_0q_0}{\sum P_1q_0} \times 100$$

$$CPI = \frac{1089}{821} \times 100$$

$$\therefore \boxed{CPI = 132.64}$$

Q.15. Find the Consumer Price Index from the following data. Using

i. Aggregative Expenditure Method, and

ii. Family Budget Method.

Is there any difference between the two results?

Commodity	Quantity Consumed in the Year 2004	Unit	Price in 2004 (Rs.)	Price in 2014 (Rs.)
Rice	6	Quintal	100	120
Wheat	8	Quintal	80	90
Bajra	1	Quintal	70	70
Arhar	2	Quintal	120	115
Desi Ghee	20	kg	12	15
Sugar	1	Quintal	160	170

Solution:

	q_0	P_0	P_1	$W=P_0q_0$	P_1q_0	$R=\frac{P_1}{P_0}\times100$	WR
Rice	6	100	120	600	720	$\frac{120}{100}\times100=120$	72,000
Wheat	8	80	90	640	720	$\frac{90}{80}\times100=112.5$	72,000
Bajra	1	70	70	70	70	$\frac{70}{70}\times100=100$	70,000
Arhar	2	120	115	240	230	$\frac{115}{120}\times100=95.83$	22,999.2
Ghee	20	12	15	240	300	$\frac{15}{12}\times100=125$	30,000
Sugar	1	160	170	160	170	$\frac{170}{160}\times100=106.25$	17,000
				$\sum P_0q_0=1950$	$\sum P_1q_0=2210$		283999.2

(i) Aggregate expenditure method:

$$CPI = \frac{\sum P_1q_0}{\sum P_0q_0}\times100$$

$$CPI=\frac{2210}{1950}\times100$$

$$\therefore \boxed{CPI=113.33}$$

(ii) Family Budged Method

$$CPI = \frac{\sum WR}{\sum W}$$

$$CPI=\frac{283999.2}{1950}$$

$$\therefore \boxed{CPI=145.64}$$

Yes, as per aggregate expenditure method, CPI is 113.33. On the contrary, family budged method gives CPI as 145.64. The **difference** is equal to **32.31.**

Q.16. Construct index number of induction production in the year 2014 from the following data on the basis of 2005's production :

Industry	Unit	2005	2014	Weight
Electrical and Electronics	Mill. Nos.	12	70	36
Metallurgical	Th. Tonnes	22	37	12
Mechanical	Th. Tonnes	72	105	10
Mining	Th. Tonnes	100	123	22
Textiles	Mill. Mtrs.	60	130	8
Miscellaneous	Th. Tonnes	123	270	12

Solution:

Industry	q_0	q_1	W	$R=\frac{q_1}{q_0}\times100$	WR
Electronics	12	70	36	583.33	20999.88
Metallurgical	22	37	12	168.18	2018.16
Mechanical	72	105	10	145.83	1458.3
Mining	100	123	22	123	2706
Textiles	60	130	8	216.67	1733.36
Misce	123	270	12	219.51	2634.12
			$\sum W=100$		$\sum WR=31549.82$

$$IIP = \frac{\sum WR}{\sum W}$$

$$IIP = \frac{31549.82}{100}$$

$$\therefore \boxed{IIP=315.5}$$

Q.17. Construct index number of industrial production from the following data :

Item	Unit	Production or Output		Weight
		Base Year	Current Year	
Mechanical	Mill. Nos.	237	400	5
Sugar and Tea	Th. Tonnes	62	150	10
Textiles	Mill. Mtrs.	572	820	35
Mining	Th. Tonnes	165	200	15
Transportation	Mill. Nos.	335	727	20
Electricity and Electronics	Mill. Nos.	87	323	15

Solution:

Item	q_0	q_1	$R=\frac{q_1}{q_0}\times 100$	W	RW
Mechanical	237	400	168.78	5	843.85
Sugar and Tea	62	150	241.19	10	2419.35
Textile	572	820	143.35	35	5017.48
Mining	165	200	121.21	15	1818.15
Transportation	335	727	217.01	20	4340.2
Electricity and electronics	87	323	371.26	15	5568.9
				$\sum W=100$	$\sum RW=20008.3$

$$IIP=\frac{\sum RW}{\sum W}$$

$$IIP=\frac{20008.3}{100}$$

$$\therefore \boxed{IIP=200.08}$$

Q.18. Construct index number of industrial production from the following data :

Industry	Number of Items		Weight
	Base Year	Current Year	
Mining and Quarrying	35	107	10
Manufacturing	413	1225	85
Electricity and Electronics	10	27	5

Solution:

Item	q_0	q_1	$R=\frac{q_1}{q_0}\times 100$	W	RW
Mining and quarrying	35	107	305.71	10	3057.1
Manufacturing	413	1225	296.61	85	25211.85
Electricity and electronics	10	27	270	5	1350
				$\sum W=100$	$\sum RW=29618.95$

$$IIP=\frac{\sum RW}{\sum W}$$

$$IIP=\frac{29618.95}{100}$$

$$\therefore \boxed{IIP=296.19}$$