Ex 20.1

Definite Integrals Ex 20.1 Q1

We know that $\int x^n dx = \frac{x^{n+1}}{n+1} + C$

Now,

$$\int_4^9 \frac{1}{\sqrt{x}} dx$$

$$= \left[\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\right]_4^9$$

$$= \left[\frac{\sqrt{x}}{\frac{1}{2}}\right]_4^9$$

$$= 2[\sqrt{9} - \sqrt{4}]$$

$$= 2[3 - 2]$$

$$= 2$$

$$\therefore \int_4^9 \frac{1}{\sqrt{x}} dx = 2$$

Definite Integrals Ex 20.1 Q2

We know that $\int \frac{dx}{x} = \log x + C$

Now,

$$\int_{-2}^3 \frac{1}{x+7} dx$$

$$= \left[\log(x+7)\right]_{-2}^3$$

$$= \left[\log 10 - \log 5\right]_{-2}^3$$

$$= \log \frac{10}{5} \qquad \left[\because \log a - \log b = \log \frac{a}{b}\right]$$

$$= \log 2$$

$$\therefore \int_{-2}^3 \frac{1}{x+7} dx = \log 2$$

Definite Integrals Ex 20.1 Q3

Let $x = \sin\theta$

$\Rightarrow dx = \cos\theta d\theta$

Now,

$x = 0 \Rightarrow \theta = 0$

$x = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$

$$\therefore \int_0^{\frac{\pi}{6}} \frac{1}{\sqrt{1-\sin^2\theta}} \cos\theta d\theta$$

$$= \int_0^{\frac{\pi}{6}} \frac{\cos\theta d\theta}{\cos\theta}$$

$$= \int_0^{\frac{\pi}{6}} d\theta$$

$$= [\theta]_0^{\frac{\pi}{6}}$$

$$= \left[\frac{\pi}{6} - 0\right]$$

$$= \frac{\pi}{6}$$

$$\therefore \int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}} = \frac{\pi}{6}$$

Definite Integrals Ex 20.1 Q4

We have,

$$I = \int_0^1 \frac{1}{1+x^2} dx$$

$$= \left[\tan^{-1} x\right]_0^1$$

$$= \left[\tan^{-1} 1 - \tan^{-1} 0\right]$$

$$= \left[\frac{\pi}{4} - 0\right] \qquad \left[\because \tan^{-1} 1 = \frac{\pi}{4}\right]$$

$$= \frac{\pi}{4}$$

$$\therefore \int_0^1 \frac{1}{1+x^2} dx = \frac{\pi}{4}$$

Definite Integrals Ex 20.1 Q5

Let $x^2+1=t$

$\Rightarrow 2x\,dx = dt$

$\Rightarrow x\,dx = \dfrac{dt}{2}$

Now,

$x = 2 \Rightarrow t = 5$

$x = 3 \Rightarrow t = 10$

$$\therefore \int_2^3 \frac{x}{x^2+1}dx = \frac{1}{2}\int_5^{10}\frac{dt}{t} = \frac{1}{2}\left[\log t\right]_5^{10}$$
$$= \frac{1}{2}\left[\log 10 - \log 5\right]$$
$$= \frac{1}{2}\left[\log \frac{10}{5}\right]$$
$$= \frac{1}{2}\left[\log 2\right]$$
$$= \log\sqrt{2}$$

$$\therefore \int_2^3 \frac{x}{x^2+1} = \log\sqrt{2}$$

Definite Integrals Ex 20.1 Q6

We have,

$$\int_0^\infty \frac{1}{a^2+b^2x^2}dx = \frac{1}{b^2}\int_0^\infty \frac{1}{\left(\frac{a}{b}\right)^2+x^2}dx$$

We know that $\int \dfrac{1}{a^2+x^2} = \dfrac{1}{a}\tan^{-1}\dfrac{x}{a}$

$$\therefore \frac{1}{b^2}\int_0^\infty \frac{1}{\left(\frac{a}{b}\right)^2+x^2}dx = \frac{1}{b^2}\left[\frac{b}{a}\tan^{-1}\left(\frac{bx}{a}\right)\right]_0^\infty$$
$$= \frac{1}{ab}\left[\tan^{-1}\left(\frac{bx}{a}\right)\right]_0^\infty$$
$$= \frac{1}{ab}\left[\tan^{-1}\infty - \tan^{-1}0\right]$$
$$= \frac{1}{ab}\left[\frac{\pi}{2} - 0\right]$$
$$= \frac{\pi}{2ab}$$
$$\Rightarrow \int_0^\infty \frac{1}{a^2+b^2x^2}dx = \frac{\pi}{2ab}$$

Definite Integrals Ex 20.1 Q7

We have,

$$\int_{-1}^{1} \frac{1}{1+x^2}\,dx$$

We know that $\int \frac{1}{1+x^2}\,dx = \tan^{-1} x$

Now,

$$\int_{-1}^{1} \frac{1}{1+x^2}\,dx$$

$$= \left[\tan^{-1} x\right]_{-1}^{1}$$

$$= \left[\tan^{-1} 1 - \tan^{-1}(-1)\right]$$

$$= \left[\frac{\pi}{4} - \left(\frac{-\pi}{4}\right)\right] \qquad \left[\because \tan^{-1}(-1) = \frac{-\pi}{4}\right]$$

$$= \left[\frac{\pi}{4} + \frac{\pi}{4}\right]$$

$$= \frac{2\pi}{4}$$

$$\therefore \int_{-1}^{1} \frac{1}{1+x^2}\,dx = \frac{\pi}{2}$$

Definite Integrals Ex 20.1 Q8

We have,

$$\int_{0}^{\infty} e^{-x}\,dx$$

We know that $\int e^{-x}\,dx = -e^{-x}$

Now,

$$\int_{0}^{\infty} e^{-x}\,dx$$

$$= \left[-e^{-x}\right]_{0}^{\infty}$$

$$= \left[-e^{-\infty} + e^{-0}\right] \qquad \left[\because e^{\infty} = 0, \quad e^{0} = 1\right]$$

$$= \left[-0 + 1\right]$$

$$\therefore \int_{0}^{\infty} e^{-x}\,dx = 1$$

Definite Integrals Ex 20.1 Q9

We have,

$$\int_{0}^{1} \frac{x}{x+1}\,dx \qquad \left[\text{Add and subtract 1 in numerator}\right]$$

$$= \int_{0}^{1} \frac{(x+1)-1}{x+1}\,dx$$

$$= \int_{0}^{1} 1\,dx - \int_{0}^{1} \frac{1}{x+1}\,dx$$

$$= \left[x\right]_{0}^{1} - \left[\log(x+1)\right]_{0}^{1}$$

$$= 1 - \left[\log 2 - \log 1\right]$$

$$= 1 - \log\frac{2}{1}$$

$$= 1 - \log 2$$

$$= \log e - \log 2 \qquad \left[\because \log e = 1\right]$$

$$= \log\frac{e}{2}$$

$$\therefore \int_{0}^{1} \frac{x}{x+1}\,dx = \log\frac{e}{2}$$

Definite Integrals Ex 20.1 Q10

We have,

$$\int_0^{\frac{\pi}{2}} (\sin x + \cos x)dx$$

$$= \int_0^{\frac{\pi}{2}} \sin x dx + \int_0^{\frac{\pi}{2}} \cos x dx$$

$$= \left[-cos x\right]_0^{\frac{\pi}{2}} + \left[\sin x\right]_0^{\frac{\pi}{2}}$$

$$= \left[\cos\frac{\pi}{2} + \cos 0\right] + \left[\sin\frac{\pi}{2} - \sin 0\right]$$

$$= \left[-0 + 1\right] + 1$$

$$= 1 + 1$$

$$= 2$$

$$\therefore \int_0^{\frac{\pi}{2}} (\sin x + \cos x)dx = 2$$

Definite Integrals Ex 20.1 Q11

We have,

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot x dx$$

We know that $\int \cot x dx = \log(\sin x)$

Now,

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot x dx$$

$$= \left[\log(\sin x)\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= \left[\log\left(\sin\frac{\pi}{2}\right) - \log\left(\sin\frac{\pi}{4}\right)\right]$$

$$= \left[\log 1 - \log\frac{1}{\sqrt{2}}\right]$$

$$= \left[0 - (\log 1 - \log\sqrt{2})\right]$$

$$= \log\sqrt{2} \qquad \left[\because \log a^n = n\log a\right]$$

$$= \frac{1}{2}\log 2$$

$$\therefore \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot x dx = \frac{1}{2}\log 2$$

Definite Integrals Ex 20.1 Q12

We have,

$$\int_0^{\frac{\pi}{4}} \sec x dx$$

We know that $\int \sec x dx = \log(\sec x + \tan x)$

$$\therefore \int_0^{\frac{\pi}{4}} \sec x dx$$

$$= \left[\log(\sec x + \tan x)\right]_0^{\frac{\pi}{4}}$$

$$= \left[\log(\sqrt{2} + 1) - \log(1 + 0)\right]$$

$$= \log(\sqrt{2} + 1) \qquad \left[\because \log 1 = 0\right]$$

$$\therefore \int_0^{\frac{\pi}{4}} \sec x dx = \log(\sqrt{2} + 1)$$

Definite Integrals Ex 20.1 Q13

$$\text{Let } I = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \operatorname{cosec} x \, dx$$

$$\int \operatorname{cosec} x \, dx = \log\left|\operatorname{cosec} x - \cot x\right| = \mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$I = \mathrm{F}\left(\frac{\pi}{4}\right) - \mathrm{F}\left(\frac{\pi}{6}\right)$$
$$= \log\left|\operatorname{cosec}\frac{\pi}{4} - \cot\frac{\pi}{4}\right| - \log\left|\operatorname{cosec}\frac{\pi}{6} - \cot\frac{\pi}{6}\right|$$
$$= \log\left|\sqrt{2} - 1\right| - \log\left|2 - \sqrt{3}\right|$$
$$= \log\left(\frac{\sqrt{2} - 1}{2 - \sqrt{3}}\right)$$

Definite Integrals Ex 20.1 Q14

We have,

$$\int_0^1 \frac{1 - x}{1 + x} dx$$

Let $x = \cos 2\theta \Rightarrow dx = -2\sin 2\theta d\theta$

Now,

$x = 0 \Rightarrow \theta = \frac{\pi}{4}$

$x = 1 \Rightarrow \theta = 0$

Now,

$$\int_0^1 \frac{1 - x}{1 + x} dx$$
$$= \int_{\frac{\pi}{4}}^0 \frac{1 - \cos 2\theta}{1 + \cos 2\theta} \times (-2\sin 2\theta) d\theta$$
$$= \int_0^{\frac{\pi}{4}} \frac{2\sin^2\theta}{2\cos^2\theta} \times 2\sin 2\theta d\theta \qquad \left[\because -\int_a^b f(x)dx = \int_b^a f(x)dx\right]$$
$$= \int_0^{\frac{\pi}{4}} \frac{4\sin^3\theta}{\cos\theta} d\theta$$

Let $\cos\theta = t$

$\Rightarrow -\sin\theta d\theta = dt$

Now,

$\theta = 0 \Rightarrow t = 1$

$\theta = \frac{\pi}{4} \Rightarrow t = \frac{1}{\sqrt{2}}$

$$\therefore \int_0^{\frac{\pi}{4}} \frac{4\sin^3\theta}{\cos\theta} d\theta$$
$$= -4\int_1^{\frac{1}{\sqrt{2}}} \frac{(1 - t^2)}{t} dt$$
$$= -4\left[\log t - \frac{t^2}{2}\right]_1^{\frac{1}{\sqrt{2}}}$$
$$= -4\left[\log\left(\frac{1}{\sqrt{2}}\right) - \frac{1}{4} - 0 + \frac{1}{2}\right]$$
$$= -4\left[-\log\sqrt{2} + \frac{1}{4}\right]$$

$$\therefore \int_0^1 \frac{1 - x}{1 + x} dx = 2\log 2 - 1$$

Definite Integrals Ex 20.1 Q15

$$I = \int_0^{\pi} \frac{1}{1+\sin x}\,dx$$

Multiplying Numerator and Denominator by $(1-\sin x)$

$$I = \int_0^{\pi} \frac{1}{1+\sin x} \times \frac{1-\sin x}{1-\sin x}\,dx$$
$$= \int_0^{\pi} \frac{(1-\sin x)}{(1^2-\sin^2 x)}\,dx$$
$$= \int_0^{\pi} \frac{1-\sin x}{(\cos^2 x)}\,dx$$
$$= \int_0^{\pi} \frac{1}{\cos^2 x}\,dx - \int_0^{\pi} \frac{\sin x}{\cos^2 x}\,dx$$
$$= \int_0^{\pi} \sec^2 x\,dx - \int_0^{\pi} \tan x.\sec x\,dx$$
$$= [\tan x]_0^{\pi} - [\sec x]_0^{\pi}$$
$$= [\tan\pi - \tan 0] - [\sec\pi - \sec 0]$$
$$= [0-0] - [-1-1]$$
$$= 2$$
$$I = 2$$

$$\therefore \int_0^{\pi} \frac{1}{1+\sin x}\,dx = 2$$

Definite Integrals Ex 20.1 Q16

We have,

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{1+\sin x}\,dx$$

We know,

$$\sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$$

$$\therefore \frac{1}{1+\sin x} = \frac{1}{1+\left(\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)} = \frac{1+\tan^2\frac{x}{2}}{\left(1+\tan\frac{x}{2}\right)^2} = \frac{\sec^2\frac{x}{2}}{\left(1+\tan\frac{x}{2}\right)^2}$$

$$\Rightarrow \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{1+\sin x}\,dx = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\sec^2\frac{x}{2}}{\left(1+\tan\frac{x}{2}\right)^2}\,dx$$

If $f(x)$ is an even function $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$

So,

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\sec^2\frac{x}{2}}{\left(1+\tan\frac{x}{2}\right)^2}\,dx = 2\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\sec^2\frac{x}{2}}{\left(1+\tan\frac{x}{2}\right)^2}\,dx$$

let $1+\tan\frac{x}{2}=t$

$\Rightarrow \frac{1}{2}\sec^2\frac{x}{2}dx = dt$

Now,

$x=-\frac{\pi}{4} \Rightarrow t = 1-\tan\frac{\pi}{8}$

$x=\frac{\pi}{4} \Rightarrow t = 1+\tan\frac{\pi}{8}$

$$\therefore 2\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\sec^2\frac{x}{2}}{\left(1+\tan\frac{x}{2}\right)^2}dx = 2\int_{1-\tan\frac{\pi}{8}}^{1+\tan\frac{\pi}{8}} \frac{8dt}{t^2}$$

$$= 2\left[\frac{-1}{t}\right]_{1-\tan\frac{\pi}{8}}^{1+\tan\frac{\pi}{8}}$$

$$= 2\left[\frac{1}{1-\tan\frac{\pi}{8}} - \frac{1}{1+\tan\frac{\pi}{8}}\right]$$

$$= 2\left[\frac{2\tan\frac{\pi}{8}}{1-\tan^2\frac{\pi}{8}}\right]$$

$$= 2\tan\frac{\pi}{4} \qquad \left[\because \tan 2A = \frac{2\tan A}{1-\tan^2 A}\right]$$

$$= 2$$

$$\therefore \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{1+\sin x}dx = 2$$

Definite Integrals Ex 20.1 Q17

Let $I = \int_0^{\frac{\pi}{2}} \cos^2 x\, dx$

$$\int \cos^2 x\, dx = \int\left(\frac{1+\cos 2x}{2}\right)dx = \frac{x}{2}+\frac{\sin 2x}{4} = \frac{1}{2}\left(x+\frac{\sin 2x}{2}\right) = \mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$I = \left[\mathrm{F}\left(\frac{\pi}{2}\right) - \mathrm{F}(0)\right]$$

$$= \frac{1}{2}\left[\left(\frac{\pi}{2}-\frac{\sin\pi}{2}\right)-\left(0+\frac{\sin 0}{2}\right)\right]$$

$$= \frac{1}{2}\left[\frac{\pi}{2}+0-0-0\right]$$

$$= \frac{\pi}{4}$$

Definite Integrals Ex 20.1 Q18

We have,

$$\int_0^{\frac{\pi}{2}} \cos^3 x dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\cos 3x + 3\cos x}{4} dx \qquad \left[\because \cos 3x = 4\cos^3 x - 3\cos x\right]$$

$$= \frac{1}{4}\int_0^{\frac{\pi}{2}} \left(\cos 3x + 3\cos x\right) dx$$

$$= \frac{1}{4}\left[\frac{\sin 3x}{3} + 3\sin x\right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{4}\left[\left(\frac{\sin 3\frac{\pi}{2}}{3} + 3\sin\frac{\pi}{2}\right) - \left(\frac{\sin 0}{3} + 3\sin 0\right)\right]$$

$$= \frac{1}{4}\left[\left(\frac{-1}{3} + 3\right) - (0+0)\right] = \frac{2}{3}$$

$$= \frac{1}{4}\left[\frac{8}{3}\right]$$

$$= \frac{2}{3}$$

$$\therefore \int_0^{\frac{\pi}{2}} \cos^3 x dx = \frac{2}{3}$$

Definite Integrals Ex 20.1 Q19

We have,

$$\int_0^{\frac{\pi}{6}} \cos x \cos 2x dx \qquad \left[\because 2\cos C\cos D = \cos(C+D) - \cos(C-D)\right]$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{6}} 2\cos x \cos 2x dx$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{6}} \left(\cos 3x + \cos x\right) dx$$

$$= \frac{1}{2}\int\left[\frac{\sin 3x}{3} + \sin x\right]_0^{\frac{\pi}{6}}$$

$$= \frac{1}{2}\left[\left(\frac{\sin 3\frac{\pi}{6}}{3} + \sin\frac{\pi}{6}\right) - (\sin 0 - \sin 0)\right]$$

$$= \frac{1}{2}\left[\frac{\sin\frac{\pi}{2}}{3} + \sin\frac{\pi}{6}\right]$$

$$= \frac{1}{2}\left(\frac{1}{3} + \frac{1}{2}\right)$$

$$= \frac{1}{2}\left(\frac{5}{6}\right)$$

$$= \frac{5}{12}$$

$$\therefore \int_0^{\frac{\pi}{6}} \cos x \cos 2x dx = \frac{5}{12}$$

Definite Integrals Ex 20.1 Q20

We have,

$$\int_0^{\frac{\pi}{2}} \sin x \sin 2x dx$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} 2\sin x \sin 2x dx \qquad [\because 2\sin C \times \sin D = \cos(D-C) - \cos(D+C)]$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} (\cos x - \cos 3x)dx$$

$$= \frac{1}{2}\left[\sin x - \frac{\sin 3x}{3}\right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{2}\left[\left(\sin\frac{\pi}{2} - \sin 0\right) - \left(\frac{\sin 3\frac{\pi}{2}}{3} - \frac{\sin 0}{3}\right)\right]$$

$$= \frac{1}{2}\left[(1-0) - \left(\frac{-1}{3} - 0\right)\right] \qquad \left[\because \sin 3\frac{\pi}{2} = -1\right]$$

$$= \frac{1}{2} \times \frac{4}{3}$$

$$= \frac{2}{3}$$

$$\therefore \int_0^{\frac{\pi}{2}} \sin x \sin 2x dx = \frac{2}{3}$$

Definite Integrals Ex 20.1 Q21

We have,

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{4}} (\tan x + \cot x)^2 dx$$

$$= \int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \left(\frac{\sin^2 x + \cot^2 x}{\sin x \cos x}\right)^2 dx$$

$$= \int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \left(\frac{1}{\sin x \cos x}\right)^2 dx$$

Multiplying numerator and denominator by 2

$$= \int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \left(\frac{2}{2\sin x \cos x}\right)^2 dx$$

$$= \int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \left(\frac{2}{\sin 2x}\right)^2 dx \qquad [\because 2\sin x \cos x = \sin 2x]$$

$$= 4\int_{\frac{\pi}{3}}^{\frac{\pi}{4}} cosec^2 x dx$$

$$= 4\left[-\frac{\cot 2x}{2}\right]_{\frac{\pi}{3}}^{\frac{\pi}{4}}$$

$$= 2\left[-\cot\frac{\pi}{2} + \cot 2\frac{\pi}{3}\right]$$

$$= 2\left[\frac{-1}{\sqrt{3}} - 0\right]$$

$$= \frac{-2}{\sqrt{3}}$$

$$\therefore \int_{\frac{\pi}{3}}^{\frac{\pi}{4}} (\tan x + \cot x)^2 dx = \frac{-2}{\sqrt{3}}$$

Definite Integrals Ex 20.1 Q22

We have,

$$\int_0^{\frac{\pi}{2}} \cos^4 x dx$$

$$= \frac{1}{4}\int_0^{\frac{\pi}{2}} (1+\cos 2x)^2 dx \qquad [\because 2\cos^2 x = 1+\cos 2x]$$

$$= \frac{1}{4}\int_0^{\frac{\pi}{2}} \left(1+\cos^2 2x + 2\cos 2x\right) dx$$

$$= \frac{1}{4}\int_0^{\frac{\pi}{2}} \left(1+\frac{1+\cos 4x}{2} + 2\cos 2x\right) dx$$

$$= \frac{1}{4}\left[x+\frac{1}{2}x+\frac{\sin 4x}{8}+\sin 2x\right]_0^{\frac{\pi}{2}} \qquad \left[\because \int \cos 4x dx = \frac{\sin 4x}{4}\right]$$

$$= \frac{1}{4}\left[\frac{\pi}{2}+\frac{\pi}{4}+0+0-0-0-0-0\right]$$

$$= \frac{1}{4}\times\frac{3\pi}{4}$$

$$= \frac{3\pi}{16}$$

$$\therefore \int_0^{\frac{\pi}{2}} \cos^4 x dx = \frac{3\pi}{16}$$

Definite Integrals Ex 20.1 Q23

We have,

$$\int_0^{\frac{\pi}{2}} \left(a^2\cos^2 x + b^2\left(1-\cos^2 x\right)\right) dx$$

$$= \int_0^{\frac{\pi}{2}} \left(\left(a^2-b^2\right)\cos^2 x + b^2\right) dx$$

$$= \frac{a^2-b^2}{2}\int_0^{\frac{\pi}{2}} (1+\cos 2x) dx + b^2 \int_0^{\frac{\pi}{2}} dx$$

$$= \frac{a^2-b^2}{2}\left[x+\frac{\sin 2x}{2}\right]_0^{\frac{\pi}{2}} + b^2[x]_0^{\frac{\pi}{2}}$$

$$= \frac{a^2-b^2}{2}\left[\frac{\pi}{2}+0-0-0\right] + b^2\left[\frac{\pi}{2}-0\right]$$

$$= \frac{a^2-b^2}{2}\left[\frac{\pi}{2}\right] + b^2\left[\frac{\pi}{2}\right]$$

$$= a^2\frac{\pi}{4} + b^2\left[\frac{\pi}{2}-\frac{\pi}{4}\right]$$

$$= \frac{\pi a^2}{4} + \frac{\pi b^2}{4}$$

$$= \frac{\pi}{4}\left(a^2+b^2\right)$$

$$\therefore \int_0^{\frac{\pi}{2}} (a^2\cos^2 x + b^2\sin^2 x) dx = \frac{\pi}{4}\left(a^2+b^2\right)$$

Definite Integrals Ex 20.1 Q24

We have,

$$\int_0^{\frac{\pi}{2}} \sqrt{1+\sin x}\, dx$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{1+\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}}\, dx \qquad \text{We use } \sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$$

$$\int_0^{\frac{\pi}{2}} \sqrt{1+\sin x}\, dx \quad = \int_0^{\frac{\pi}{2}} \sqrt{\frac{\left(1+\tan\frac{x}{2}\right)^2}{1+\tan^2\frac{x}{2}}}\, dx$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{\frac{\left(1+\tan\frac{x}{2}\right)^2}{\sec^2\frac{x}{2}}}\, dx$$

$$= \int_0^{\frac{\pi}{2}} \left(\frac{1+\tan\frac{x}{2}}{\sec\frac{x}{2}}\right) dx$$

$$= \int_0^{\frac{\pi}{2}} \left(\cos\frac{x}{2}+\sin\frac{x}{2}\right) dx$$

$$= \left[2\sin\frac{x}{2}-2\cos\frac{x}{2}\right]_0^{\frac{\pi}{2}}$$

$$= 2\left[\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}-0+1\right]$$

$$\therefore \int_0^{\frac{\pi}{2}} \sqrt{1+\sin x}\, dx = 2$$

Definite Integrals Ex 20.1 Q25

We have,

$$\int_0^{\frac{\pi}{2}} \sqrt{1+\cos x}$$

We use $1+\cos x = 2\cos^2\frac{x}{2}$

$$= \int_0^{\frac{\pi}{2}} \sqrt{2\cos^2\frac{x}{2}}\, dx$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{2}\cos\frac{x}{2} dx$$

$$= \sqrt{2}\left[2\sin\frac{x}{2}\right]_0^{\frac{\pi}{2}}$$

$$= 2\sqrt{2}\left[\frac{1}{\sqrt{2}}\right]$$

$$= 2$$

$$\therefore \int_0^{\frac{\pi}{2}} \sqrt{1+\cos x} = 2$$

Definite Integrals Ex 20.1 Q26

We have,

$$\int x\sin x\, dx = x\int \sin x dx - \int\left(\int \sin x dx\right)\left(\frac{dx}{dx}\right) dx$$

$$= -x\cos x + \int \cos x dx$$

$$\therefore \int_0^{\frac{\pi}{2}} x\sin x\, dx = \left[-x\cos x+\sin x\right]_0^{\frac{\pi}{2}} = \left(-\frac{\pi}{2}\times 0\right)+1+0-0 = 1$$

$$\therefore \int_0^{\frac{\pi}{2}} x\sin x dx = 1$$

Definite Integrals Ex 20.1 Q27

We have,

$$\int x\cos x\,dx = x\int \cos x dx - \int\left(\int \cos x dx\right)\frac{dx}{dx}dx = x\sin x - \int \sin x dx$$

$$\therefore \int_0^{\frac{\pi}{2}} x\cos x\,dx = \left[x\sin x + \cos x\right]_0^{\frac{\pi}{2}} = \left[\frac{\pi}{2} + 0 - 0 - 1\right] = \frac{\pi}{2} - 1$$

$$\therefore \int_0^{\frac{\pi}{2}} x\cos x dx = \frac{\pi}{2} - 1$$

Definite Integrals Ex 20.1 Q28

We have,

$$\int x^2\cos x\,dx = x^2\int \cos x dx - \int(2x)\left(\int \cos x dx\right)dx = x^2\sin x - \int \sin x.2x dx$$

$$= x^2\sin x - 2\left[x\int \sin x - \int\left(\int \sin x dx\right)dx\right]$$

$$= x^2\sin x - 2\left[-x\cos x + \int \cos x dx\right]$$

$$\therefore \int_0^{\frac{\pi}{2}} x^2\cos x\,dx = \left[x^2\sin x + 2x\cos x - 2\sin x\right]_0^{\frac{\pi}{2}}$$

$$= \left[\frac{\pi^2}{4} + 0 - 2 - 0 - 0 + 0\right]$$

$$= \frac{\pi^2}{4} - 2$$

$$\therefore \int_0^{\frac{\pi}{2}} x^2\cos x dx = \frac{\pi^2}{4} - 2$$

Definite Integrals Ex 20.1 Q29

We have,

$$\int x^2\sin x\,dx = x^2\int \sin x dx - \int 2x\left(\int \sin x dx\right)dx = x^2\cos x + \int 2x\cos x dx$$

$$= x^2\cos x + 2\left[x\int \cos x dx - \int\left(\int \cos x dx\right)dx\right]$$

$$= -x^2\cos x + 2\left[x\sin x - \int \sin x dx\right]$$

$$\therefore \int_0^{\frac{\pi}{4}} x^2\sin x\,dx = \left[-x^2\cos x + 2x\sin x + 2\cos x\right]_0^{\frac{\pi}{4}}$$

$$= \frac{-\pi^2}{16}.\frac{1}{\sqrt{2}} + \frac{\pi}{2}.\frac{1}{\sqrt{2}} + 2.\frac{1}{\sqrt{2}} + 0 - 0 - 2$$

$$= \frac{1}{\sqrt{2}}\left[\frac{-\pi^2}{16} + \frac{\pi}{2} + 2\right] - 2$$

$$= \sqrt{2} + \frac{\pi}{2\sqrt{2}} - \frac{\pi^2}{16\sqrt{2}} - 2$$

$$\therefore \int_0^{\frac{\pi}{4}} x^2\sin x dx = \sqrt{2} + \frac{\pi}{2\sqrt{2}} - \frac{\pi^2}{16\sqrt{2}} - 2$$

Definite Integrals Ex 20.1 Q30

We have,

$$\int x^2 \cos 2x\,dx = x^2\int \cos 2x dx - \int 2x\left(\int \cos 2x dx\right)dx$$

$$= \frac{x^2 \sin 2x}{2} - \int 2x \times \frac{\sin 2x}{2}dx$$

$$= \frac{x^2 \sin 2x}{2} - \left[x\int \sin 2x dx - \int\left(\int \sin 2x dx\right)dx\right]$$

$$= \frac{x^2 \sin 2x}{2} + \left[\frac{x\cos 2x}{2} - \int\frac{x\cos 2x}{2}\right]$$

$$\therefore \quad \int_0^{\frac{\pi}{2}} x^2 \cos 2x\,dx = \left[\frac{x^2 \sin 2x}{2} + \frac{x\cos 2x}{2} - \frac{\sin 2x}{4}\right]_0^{\frac{\pi}{2}}$$

$$= \left[\frac{\pi^2}{8}\times 0 + \frac{\pi}{4}(-1) - 0 - 0 - 0 + 0\right]$$

$$= \frac{-\pi}{4}$$

$$\therefore \int_0^{\frac{\pi}{2}} x^2 \cos 2x dx = \frac{-\pi}{4}$$

Definite Integrals Ex 20.1 Q31

We have,

$$\int x^2 \cos^2 x dx = \int x^2\left(\frac{1+\cos 2x}{2}\right)dx = \frac{1}{2}\int\left(x^2 + x^2\cos 2x\right)dx = \frac{1}{2}\left[\int x^2 dx + \int x^2 \cos 2x dx\right] \quad ..(A)$$

Now,

$$\int_0^{\frac{\pi}{2}} x^2 dx = \left[\frac{x^3}{3}\right]_0^{\frac{\pi}{2}} = \frac{\pi^3}{24} \quad ...(B)$$

$$\int x^2 \cos 2x\,dx = x^2\int \cos 2x dx - \int 2x\left(\int \cos 2x dx\right)dx = \frac{x^2 \sin 2x}{2} - \int \frac{\sin 2x}{2}.2x dx$$

$$= \frac{x^2 \sin 2x}{2} - \left[x\int \sin 2x - \int\left(\int \sin 2x dx\right)dx\right]$$

$$= \frac{x^2 \sin 2x}{2} + \frac{x\cos 2x}{2} - \int\frac{\cos 2x}{2}dx$$

$$\therefore \quad \int_0^{\frac{\pi}{2}} x^2 \cos 2x\,dx = \left[\frac{x^2 \sin 2x}{2} + \frac{x\cos 2x}{2} - \frac{\sin 2x}{4}\right]_0^{\frac{\pi}{2}} = \frac{-\pi}{4} \quad -(C)$$

Now, Put (B) & (C) in (A), we get,

$$\int_0^{\frac{\pi}{2}} x^2 \cos^2 x dx = \int_0^{\frac{\pi}{2}} x^2 dx + \int_0^{\frac{\pi}{2}} x^2 \cos 2x\,dx = \frac{1}{2}\left[\frac{\pi^3}{24} - \frac{\pi}{4}\right] = \frac{\pi^3}{48} - \frac{\pi}{8}$$

Definite Integrals Ex 20.1 Q32

We have,

$$\int \log x dx = \int 1.\log x dx = \log x\int dx - \int\left(\int dx\right).\frac{1}{x}dx = x\log x - \int x.\frac{1}{x}dx = x\log x - \int dx$$

$$\therefore \quad \int_1^2 \log x dx = \left[x\log x - x\right]_1^2 = 2\log 2 - 2 - 0 + 1 = 2\log 2 - 1$$

Definite Integrals Ex 20.1 Q33

We have,

$$\int \frac{\log x}{(x+1)^2}dx = \int \frac{1}{(x+1)^2}\log x dx = \log x\int\frac{1}{(x+1)^2}dx - \int\left(\int\frac{1}{(x+1)^2}dx\right)\frac{1}{x}dx$$

$$= \frac{-\log x}{(x+1)} + \int\frac{1}{x(x+1)}dx$$

$$= \frac{-\log x}{(x+1)} + \int\left(\frac{1}{x} - \frac{1}{x+1}\right)dx$$

$$\therefore \quad \int_1^3 \frac{\log x}{(x+1)^2}dx = \left[\frac{-\log x}{x+1} + \log x - \log(x+1)\right]_1^3 = \frac{3}{4}\log 3 - \log 2$$

Definite Integrals Ex 20.1 Q34

Let $I = \int_1^e \frac{e^x}{x}\left(1 + x\log x\right)dx$

$I = \int_1^e \frac{e^x}{x}dx + \int_1^e e^x \log x dx$

$I = \left[e^x \log x\right]_1^e - \int_1^e e^x . \log x + \int_1^e e^x \log x$

$I = \left[e^x \log x\right]_1^e$

$I = \left[e^x \log e - e^1 . \log 1\right]$

$I = \left[e^e . 1 - 0\right]$

$I = e^e$

$\therefore \int_1^e \frac{e^x}{x}\left(1 + x\log x\right)dx = e^e$

Definite Integrals Ex 20.1 Q35

We have,

$\int_1^e \frac{\log x}{x}dx$

Let $\log x = t$

$\Rightarrow \frac{1}{x}dx = dt$

Now,

$x = 1 \Rightarrow t = 0$

$x = e \Rightarrow t = 1$

$\therefore \int_1^e \frac{\log x}{x}dx = \int_0^1 t\,dt$

$= \left[\frac{t^2}{2}\right]_0^1$

$= \frac{1}{2}$

$\therefore \int_1^e \frac{\log x}{x}dx = \frac{1}{2}$

Definite Integrals Ex 20.1 Q36

We have,

$\int_e^{e^2} \left\{\frac{1}{\log x} - \frac{1}{(\log x)^2}dx\right\}$

$I = \int \frac{1}{\log x}.1dx = \frac{1}{\log x}\int dx - \int\left(\int dx\right).\frac{d}{dx}\left(\frac{1}{\log x}\right)dx = \frac{x}{\log x} + \int \frac{1}{(\log x)^2}.x.\frac{1}{x}dx$

$= \frac{x}{\log x} + \int \frac{dx}{(\log x)^2}$

$\int_e^{e^2} \left\{\frac{1}{\log x} - \frac{1}{(\log x)^2}dx\right\} = \left[\frac{x}{\log x}\right]_e^{e^2} + \int_e^{e^2} \frac{dx}{(\log x)^2} - \int_e^{e^2} \frac{dx}{(\log x)^2}$

$= \left[\frac{x}{\log x}\right]_e^{e^2}$

$= \frac{e^2}{2} - e$

Definite Integrals Ex 20.1 Q37

We have,

$$\int_1^2 \frac{x+3}{x(x+2)}dx$$

$$= \int_1^2 \frac{x}{x(x+2)}dx + \int_1^2 \frac{3}{x(x+2)}dx$$

$$= \int_1^2 \frac{dx}{(x+2)} + \int_1^2 \frac{3}{x(x+2)}dx$$

$$= \left[\log(x+2)\right]_1^2 + \frac{3}{2}\int_1^2 \frac{1}{x} - \frac{1}{x+2}dx \qquad \left[\text{using partial fraction}\right]$$

$$= \left[\log(x+2)\right]_1^2 + \left[\frac{3}{2}\log x - \frac{3}{2}\log(x+2)\right]_1^2$$

$$= \left[\frac{3}{2}\log x - \frac{1}{2}\log(x+2)\right]_1^2$$

$$= \frac{1}{2}\left[3\log 2 - \log 4 + \log 3\right]$$

$$= \frac{1}{2}\left[3\log 2 - 2\log 2 + \log 3\right] \qquad \left[\because \log 4 = 2\log 2\right]$$

$$= \frac{1}{2}\left[\log 2 + \log 3\right]$$

$$= \frac{1}{2}\left[\log 6\right]$$

$$= \frac{1}{2}\log 6$$

$$\therefore \int_1^2 \frac{x+3}{x(x+2)}dx = \frac{1}{2}\log 6$$

Definite Integrals Ex 20.1 Q38

$$\text{Let } I = \int_0^1 \frac{2x+3}{5x^2+1}dx$$

$$\int \frac{2x+3}{5x^2+1}dx = \frac{1}{5}\int \frac{5(2x+3)}{5x^2+1}dx$$

$$= \frac{1}{5}\int \frac{10x+15}{5x^2+1}dx$$

$$= \frac{1}{5}\int \frac{10x}{5x^2+1}dx + 3\int \frac{1}{5x^2+1}dx$$

$$= \frac{1}{5}\int \frac{10x}{5x^2+1}dx + 3\int \frac{1}{5\left(x^2+\frac{1}{5}\right)}dx$$

$$= \frac{1}{5}\log\left(5x^2+1\right) + \frac{3}{5}\cdot\frac{1}{\frac{1}{\sqrt{5}}}\tan^{-1}\frac{x}{\frac{1}{\sqrt{5}}}$$

$$= \frac{1}{5}\log\left(5x^2+1\right) + \frac{3}{\sqrt{5}}\tan^{-1}\left(\sqrt{5}x\right)$$

$$= F(x)$$

Definite Integrals Ex 20.1 Q39

$$\int_0^2 \frac{dx}{x+4-x^2} = \int_0^2 \frac{dx}{-\left(x^2-x-4\right)}$$

$$= \int_0^2 \frac{dx}{-\left(x^2-x+\frac{1}{4}-\frac{1}{4}-4\right)}$$

$$= \int_0^2 \frac{dx}{-\left[\left(x-\frac{1}{2}\right)^2-\frac{17}{4}\right]}$$

$$= \int_0^2 \frac{dx}{\left(\frac{\sqrt{17}}{2}\right)^2-\left(x-\frac{1}{2}\right)^2}$$

Let $x-\frac{1}{2}=t \Rightarrow dx = dt$

When $x=0$, $t=-\frac{1}{2}$ and when $x=2$, $t=\frac{3}{2}$

$$\therefore \int_0^2 \frac{dx}{\left(\frac{\sqrt{17}}{2}\right)^2-\left(x-\frac{1}{2}\right)^2} = \int_{-\frac{1}{2}}^{\frac{3}{2}} \frac{dt}{\left(\frac{\sqrt{17}}{2}\right)^2-t^2}$$

$$= \left[\frac{1}{2\left(\frac{\sqrt{17}}{2}\right)} \log \frac{\frac{\sqrt{17}}{2}+t}{\frac{\sqrt{17}}{2}-t}\right]_{-\frac{1}{2}}^{\frac{3}{2}}$$

$$= \frac{1}{\sqrt{17}}\left[\log \frac{\frac{\sqrt{17}}{2}+\frac{3}{2}}{\frac{\sqrt{17}}{2}-\frac{3}{2}} - \frac{\log \frac{\sqrt{17}}{2}-\frac{1}{2}}{\log \frac{\sqrt{17}}{2}+\frac{1}{2}}\right]$$

$$= \frac{1}{\sqrt{17}}\left[\log \frac{\sqrt{17}+3}{\sqrt{17}-3} - \log \frac{\sqrt{17}-1}{\sqrt{17}+1}\right]$$

$$= \frac{1}{\sqrt{17}} \log \frac{\sqrt{17}+3}{\sqrt{17}-3} \times \frac{\sqrt{17}+1}{\sqrt{17}-1}$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{17+3+4\sqrt{17}}{17+3-4\sqrt{17}}\right]$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{20+4\sqrt{17}}{20-4\sqrt{17}}\right]$$

$$= \frac{1}{\sqrt{17}} \log \left(\frac{5+\sqrt{17}}{5-\sqrt{17}}\right)$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{\left(5+\sqrt{17}\right)\left(5+\sqrt{17}\right)}{25-17}\right]$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{25+17+10\sqrt{17}}{8}\right]$$

$$= \frac{1}{\sqrt{17}} \log \left(\frac{42+10\sqrt{17}}{8}\right)$$

$$= \frac{1}{\sqrt{17}} \log \left(\frac{21+5\sqrt{17}}{4}\right)$$

Definite Integrals Ex 20.1 Q40

We have,

$$\int_0^1 \frac{1}{2x^2+x+1}dx$$

$$= \frac{1}{2}\int_0^1 \frac{1dx}{\left(x^2+\frac{1}{2}x+\frac{1}{2}\right)}$$

$$= \frac{1}{2}\int_0^1 \frac{dx}{\left(x+\frac{1}{4}\right)^2+\frac{1}{2}-\frac{1}{16}} \qquad \left[\text{Adding } \frac{1}{16} \text{ \& substracting} \frac{1}{16} \text{ in numerator}\right]$$

$$= \frac{1}{2}\int_0^1 \frac{dx}{\left(x+\frac{1}{4}\right)^2+\frac{7}{16}}$$

$$= \frac{1}{2}\int_0^1 \frac{dx}{\left(x+\frac{1}{4}\right)^2+\left(\frac{\sqrt{7}}{4}\right)^2}$$

$$= \frac{1}{2}\cdot\frac{4}{\sqrt{7}}\left[\tan^{-1}\left(\frac{x+\frac{1}{4}}{\frac{\sqrt{7}}{4}}\right)\right]_0^1$$

$$= \frac{2}{\sqrt{7}}\left\{\tan^{-1}\frac{5}{\sqrt{7}}-\tan^{-1}\left(\frac{1}{\sqrt{7}}\right)\right\}$$

$$\therefore \int_0^1 \frac{1}{2x^2+x+1}dx = \frac{2}{\sqrt{7}}\left\{\tan^{-1}\frac{5}{\sqrt{7}}-\tan^{-1}\left(\frac{1}{\sqrt{7}}\right)\right\}$$

Definite Integrals Ex 20.1 Q41

Let $I = \int_0^1 \sqrt{x(1-x)}\,dx$

let $x = \sin^2\theta$
$\Rightarrow dx = 2\sin\theta.\cos\theta d\theta$

Now,
$x = 0 \Rightarrow \theta = 0$
$x = 1 \Rightarrow \theta = \frac{\pi}{2}$

$\therefore\ I = \int_0^{\frac{\pi}{2}} \sqrt{\sin^2\theta\left(1-\sin^2\theta\right)}.2\sin\theta.\cos\theta d\theta$

$= \int_0^{\frac{\pi}{2}} 2\sin^2\theta.\cos^2\theta d\theta$

$= \frac{1}{2}\int_0^{\frac{\pi}{2}} 4\sin^2\theta.\cos^2\theta d\theta$

$= \frac{1}{2}\int_0^{\frac{\pi}{2}} \left(\sin^2 2\theta\right) d\theta$

$= \frac{1}{2}\int_0^{\frac{\pi}{2}} \left(\frac{1-\cos 4\theta}{2}\right) d\theta$

$= \frac{1}{4}\int_0^{\frac{\pi}{2}} \left(1-\cos 4\theta\right) d\theta$

$= \frac{1}{4}\int_0^{\frac{\pi}{2}} d\theta - \frac{1}{4}\int_0^{\frac{\pi}{2}} \cos 4\theta d\theta$

$= \frac{1}{4}\left[\theta\right]_0^{\frac{\pi}{2}} - \frac{1}{4}\left[\frac{\sin 4\theta}{4}\right]_0^{\frac{\pi}{2}}$

$= \frac{1}{4}\left[\frac{\pi}{2} - 0\right] - \frac{1}{16}\left[\sin\pi - \sin 0\right]$

$= \frac{\pi}{8} - \frac{1}{16}\left[0-0\right]$

$= \frac{\pi}{8}$

$I = \frac{\pi}{8}$

$\therefore\ \int_0^1 \sqrt{x(1-x)}\,dx = \frac{\pi}{8}$

Definite Integrals Ex 20.1 Q42

We have,

$\int_0^2 \frac{dx}{\sqrt{3+2x-x^2}}$

$\int_0^2 \frac{dx}{\sqrt{3+1-\left(x^2-2x+1\right)}}$ [Add and subtract 1 in denominator]

$= \int_0^2 \frac{dx}{\sqrt{(2)^2(x-1)^2}}$ $\left[\because \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a}\right]$

$= \left[\sin^{-1}\left(\frac{x-1}{2}\right)\right]_0^2$

$= \sin^{-1}\frac{1}{2} - \sin^{-1}\left(\frac{-1}{2}\right)$

$= \sin^{-1}\left(\sin\frac{\pi}{6}\right) - \sin^{-1}\left[\sin\left(\frac{-\pi}{6}\right)\right]$

$= \frac{\pi}{6} + \frac{\pi}{6}$

$= \frac{\pi}{3}$

$\therefore\ \int_0^2 \frac{dx}{\sqrt{3+2x-x^2}} = \frac{\pi}{3}$

Definite Integrals Ex 20.1 Q43

We have,

$$\int_0^4 \frac{dx}{\sqrt{4x - x^2}}$$

$$= \int_0^4 \frac{dx}{\sqrt{4 - 4 + 4x - x^2}} \qquad [\text{Add and subtract 4 in denominator}]$$

$$= \int_0^4 \frac{dx}{\sqrt{4 - (x^2 - 4x + 4)}}$$

$$= \int_0^4 \frac{dx}{\sqrt{(2)^2 - (x-2)^2}}$$

$$= \left[\sin^{-1}\left(\frac{x-2}{2}\right)\right]_0^4 \qquad \left[\because \int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a}\right]$$

$$= \sin^{-1}(1) - \sin^{-1}(-1)$$

$$= \frac{\pi}{2} - \left(-\frac{\pi}{2}\right)$$

$$= \frac{2\pi}{2} = \pi$$

$$\therefore \int_0^4 \frac{dx}{\sqrt{4x - x^2}} = \pi$$

Definite Integrals Ex 20.1 Q44

$$\int_{-1}^{1} \frac{dx}{x^2 + 2x + 5} = \int_{-1}^{1} \frac{dx}{(x^2 + 2x + 1) + 4} = \int_{-1}^{1} \frac{dx}{(x+1)^2 + (2)^2}$$

Let $x + 1 = t \Rightarrow dx = dt$

When $x = -1$, $t = 0$ and when $x = 1$, $t = 2$

$$\therefore \int_{-1}^{1} \frac{dx}{(x+1)^2 + (2)^2} = \int_0^2 \frac{dt}{t^2 + 2^2}$$

$$= \left[\frac{1}{2}\tan^{-1}\frac{t}{2}\right]_0^2$$

$$= \frac{1}{2}\tan^{-1}1 - \frac{1}{2}\tan^{-1}0$$

$$= \frac{1}{2}\left(\frac{\pi}{4}\right) = \frac{\pi}{8}$$

Definite Integrals Ex 20.1 Q45

We have,

$$\int_1^4 \frac{x^2+x}{\sqrt{2x+1}}\,dx$$

Let $2x+1=t^2$

$\Rightarrow 2dx = 2t\,dt$

Now,

$x=1 \Rightarrow t=\sqrt{3}$

$x=4 \Rightarrow t=3$

$$\therefore \int_1^4 \frac{x^2+x}{\sqrt{2x+1}}\,dx = \int_{\sqrt{3}}^{3} \frac{\left(\frac{t^2-1}{2}\right)^2+\left(\frac{t^2-1}{2}\right)}{t}\,t\,dt$$

$$= \frac{1}{4}\int_{\sqrt{3}}^{3}\left(t^4-2t^2+1+2t^2-2\right)dt$$

$$= \frac{1}{4}\int_{\sqrt{3}}^{3} t^4-1$$

$$= \frac{1}{4}\left[\frac{t^5}{5}-t\right]_{\sqrt{3}}^{3}$$

$$= \frac{1}{4}\left[\frac{243-9\sqrt{3}}{5}-3+\sqrt{3}\right]$$

$$= \frac{1}{4}\left[\frac{228}{5}-\sqrt{3}\,(4)\right]$$

$$= \frac{57-\sqrt{3}}{5}$$

$$\therefore \int_1^4 \frac{x^2+x}{\sqrt{2x+1}}\,dx = \frac{57-\sqrt{3}}{5}$$

Definite Integrals Ex 20.1 Q46

We have,

$$\int_0^1 x\,(1-x)^5\,dx$$

Expanding $(1-x)^5$ by Binomial theorem

$$\therefore (1-x)^5 = 1^5 + {}^5C_1(-x) + {}^5C_2(-x)^2 + {}^5C_3(-x)^3 + {}^5C_4(-x)^4 + {}^5C_5(-x)^5$$

$$= 1-5x+10x^2-10x^3+5x^4-x^5$$

$$= \int_0^1 x\left(1-5x+10x^2-10x^3+5x^4-x^5\right)dx$$

$$= \left[\frac{x^2}{2}-\frac{5x^3}{3}+\frac{10x^4}{4}-\frac{10x^5}{5}+\frac{5x^6}{6}-\frac{x^7}{7}\right]_0^1$$

$$= \frac{1}{2}-\frac{5}{3}+\frac{10}{4}-\frac{10}{5}+\frac{5}{6}-\frac{1}{7}$$

$$= \frac{1}{42}$$

$$\therefore \int_0^1 x\,(1-x)^5\,dx = \frac{1}{42}$$

Definite Integrals Ex 20.1 Q47

We have,

$$\int_1^2 \left(\frac{x-1}{x^2}\right) e^x dx = \int_1^2 \frac{xe^x}{x^2} - \int_1^2 \frac{e^x}{x^2} dx = \int_1^2 \frac{e^x dx}{x} - \int_1^2 \frac{e^x}{x^2} dx$$

Expanding 1^{st} integral by by parts we get

$$= \frac{1}{x}\int_1^2 e^x dx - \int_1^2 \left(\int e^x . \frac{d\left(\frac{1}{x}\right)}{dx} dx\right) - \int_1^2 \frac{e^x}{x^2}$$

$$= \left[\frac{e^x}{x}\right]_1^2 + \int_1^2 \frac{e^x}{x^2} dx - \int_1^2 \frac{e^x}{x^2}$$

$$= \left[\frac{e^x}{x}\right]_1^2$$

$$= \frac{e^2}{2} - e$$

$$\therefore \int_1^2 \left(\frac{x-1}{x^2}\right) e^x dx = \frac{e^2}{2} - e$$

Definite Integrals Ex 20.1 Q48

We have,

$$\int_0^1 \left(xe^{2x} + \sin\frac{\pi x}{2}\right) dx = \int_0^1 \underset{\mathrm{I}}{x}\, \underset{\mathrm{II}}{e^{2x}}\, dx + \int_0^1 \sin\frac{\pi x}{2} dx$$

Applying by parts in first integral

$$= x\int_0^1 e^{2x} dx - \int_0^1 \left(\int e^{2x} dx.\right) \frac{dx}{dx} dx + \left[\frac{-\cos\frac{\pi x}{2}}{\frac{\pi}{2}}\right]_0^1$$

$$= \frac{xe^{2x}}{2} - \frac{1}{2}\int_0^1 e^{2x} dx + \frac{2}{\pi}[1-0]$$

$$= \frac{xe^{2x}}{2} - \frac{1}{2}\int_0^1 e^{2x} dx + \frac{2}{\pi}[1-0]$$

$$= \left[\frac{xe^{2x}}{2} - \frac{1}{4}e^{2x}\right]_0^1 + \frac{2}{\pi}[1-0]$$

$$= \frac{e^2}{2} - \frac{1}{4}e^2 + \frac{1}{4} + \frac{2}{\pi}[1-0]$$

$$= \frac{e^2}{4} + \frac{2}{\pi} + \frac{1}{4}$$

$$= \frac{e^2}{4} + \frac{1}{4} + \frac{2}{\pi}$$

$$\therefore \int_0^1 \left(xe^{2x} + \sin\frac{\pi x}{2}\right) dx = \frac{e^2}{4} + \frac{1}{4} + \frac{2}{\pi}$$

Definite Integrals Ex 20.1 Q49

We have,

$$\int_0^1 \left(xe^x + \cos\frac{\pi x}{4}\right)dx$$

$$= \int_0^1 \underset{\mathrm{I}}{x}\,\underset{\mathrm{II}}{e^x}\,dx + \int_0^1 \cos\frac{\pi x}{4}\,dx$$

Applying by by parts in 1st integral we get,

$$= x\int_0^1 e^x dx - \int_0^1 \left(\int e^x dx\right)\frac{dx}{dx}.dx + \int_0^1 \cos\frac{\pi x}{4}\,dx$$

$$= \left[xe^x\right]_0^1 - \int_0^1 e^x dx + \left[\frac{\sin\frac{\pi x}{4}}{\frac{\pi}{4}}\right]_0^1$$

$$= \left[xe^x - e^x\right]_0^1 + \frac{4}{\pi}\left[\frac{1}{\sqrt{2}} - 0\right]$$

$$= \left[e^x(x-1)\right]_0^1 + \frac{4}{\pi}\left[\frac{1}{\sqrt{2}}\right]$$

$$= 0 + 1 + \frac{4}{\pi\sqrt{2}}$$

$$= 1 + \frac{2\sqrt{2}}{\pi}$$

$$\therefore \int_0^1 \left(xe^x + \cos\frac{\pi x}{4}\right)dx = 1 + \frac{2\sqrt{2}}{\pi}$$

Definite Integrals Ex 20.1 Q50

$$\int_{\frac{\pi}{2}}^{\pi} e^x \frac{1-\sin x}{1-\cos x}dx = \int_{\frac{\pi}{2}}^{\pi} e^x \frac{1-2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin^2\frac{x}{2}}dx \qquad \left[1-\cos x = 2\sin^2\frac{x}{2}\right]$$

$$= -\int_{\frac{\pi}{2}}^{\pi} e^x\left(-\frac{1}{2}\csc^2\frac{x}{2} + \cot\frac{x}{2}\right)dx$$

$$= -e^x\cot\frac{x}{2}\Big|_{\frac{\pi}{2}}^{\pi} \qquad \left[\int e^x\left(F(x)+F'(x)\right)dx = e^x F(x)\right]$$

$$= e^{\frac{\pi}{2}}$$

Definite Integrals Ex 20.1 Q51

We have,

$$\int_0^{2\pi} e^{x/2}\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)dx = \int_0^{2\pi} e^{x/2}\left(\sin\frac{x}{2}\cos\frac{\pi}{4} + \cos\frac{x}{2}\sin\frac{\pi}{4}\right)dx$$

$$= \int_0^{2\pi} e^{x/2}\sin\frac{x}{2}.\frac{1}{\sqrt{2}}dx + \int_0^{2\pi} e^{x/2}\cos\frac{x}{2}.\frac{1}{\sqrt{2}}dx$$

Expanding 1st part by parts, we get,

$$\int_0^{2\pi} e^{x/2}\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)dx = \frac{1}{\sqrt{2}}\left\{\sin\frac{x}{2}\int_0^{2\pi} e^{x/2}dx - \int_0^{2\pi}\left(\int_0^{2\pi} e^{x/2}dx\right).\frac{d\left(\sin\frac{x}{2}\right)}{dx}dx\right\} + \frac{1}{\sqrt{2}}\int_0^{2\pi} e^{x/2}.\cos\frac{x}{2}dx$$

$$= \frac{1}{\sqrt{2}}\left\{\sin\frac{x}{2}.2e^{x/2}\right\}_0^{2\pi} - \frac{1}{\sqrt{2}}\int_0^{2\pi} e^{x/2}2.\frac{1}{2}\cos\frac{x}{2}dx + \frac{1}{\sqrt{2}}\int_0^{2\pi} e^{x/2}\cos\frac{x}{2}dx$$

$$= \frac{1}{\sqrt{2}}\left\{\sin\frac{x}{2}.2e^{x/2}\right\}_0^{2\pi} = \frac{1}{\sqrt{2}}\{0-0\} = 0$$

$$\therefore \int_0^{2\pi} e^{x/2}\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)dx = 0$$

Definite Integrals Ex 20.1 Q52

$$\text{Let } I = \int_0^{2\pi} e^x \cos\left(\frac{\pi}{4}+\frac{x}{2}\right)dx = \left[\cos\left(\frac{\pi}{4}+\frac{x}{2}\right).e^x\right]_0^{2\pi} + \frac{1}{2}\int_0^{2\pi} e^x \sin\left(\frac{\pi}{4}+\frac{x}{2}\right)dx$$

$$\Rightarrow I = \left[\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)e^x\right]_0^{2\pi} + \frac{1}{2}\left[\left\{\sin\left(\frac{\pi}{4}+\frac{x}{2}\right).e^x\right\}_0^{2\pi} - \frac{1}{2}\int_0^{2\pi} e^x \cos\left(\frac{\pi}{4}+\frac{x}{2}\right)\right]dx$$

$$I = \left[\cos\left(\pi+\frac{\pi}{4}\right)e^{2\pi} - \cos\frac{\pi}{4}\right] + \frac{1}{2}\left[\sin\left(\pi+\frac{\pi}{4}\right).e^{2\pi} - \sin\frac{\pi}{4} - \frac{1}{2}I\right]$$

$$I = \left[-\cos\frac{\pi}{4}.e^{2\pi} - \cos\frac{\pi}{4}\right] + \frac{1}{2}\left[-\sin\frac{\pi}{4}.e^{2\pi} - \sin\frac{\pi}{4}\right] - \frac{I}{4}$$

$$\frac{5I}{4} = -\frac{1}{\sqrt{2}}\left(e^{2\pi}+1\right) - \frac{1}{2}.\frac{1}{\sqrt{2}}\left(e^{2\pi}+1\right) = \frac{-3}{2\sqrt{2}}\left(e^{2\pi}+1\right)$$

$$I = \frac{-3\sqrt{2}}{5}\left(e^{2\pi}+1\right)$$

$$\therefore \int_0^{2\pi} e^x \cos\left(\frac{\pi}{4}+\frac{x}{2}\right)dx = \frac{-3\sqrt{2}}{5}\left(e^{2\pi}+1\right)$$

Definite Integrals Ex 20.1 Q53

$$\text{Let } I = \int_0^1 \frac{dx}{\sqrt{1+x}-\sqrt{x}}$$

$$I = \int_0^1 \frac{1}{\left(\sqrt{1+x}-\sqrt{x}\right)} \times \frac{\left(\sqrt{1+x}+\sqrt{x}\right)}{\left(\sqrt{1+x}+\sqrt{x}\right)}dx$$

$$= \int_0^1 \frac{\sqrt{1+x}+\sqrt{x}}{1+x-x}dx$$

$$= \int_0^1 \sqrt{1+x}\,dx + \int_0^1 \sqrt{x}\,dx$$

$$= \left[\frac{2}{3}(1+x)^{\frac{3}{2}}\right]_0^1 + \left[\frac{2}{3}(x)^{\frac{3}{2}}\right]_0^1$$

$$= \frac{2}{3}\left[(2)^{\frac{3}{2}}-1\right] + \frac{2}{3}[1]$$

$$= \frac{2}{3}(2)^{\frac{3}{2}}$$

$$= \frac{2\cdot 2\sqrt{2}}{3}$$

$$= \frac{4\sqrt{2}}{3}$$

Definite Integrals Ex 20.1 Q54

$$\int_1^2 \frac{x}{(x+1)(x+2)}dx = -\int_1^2 \frac{1}{x+1}dx + \int_1^2 \frac{2}{x+2}dx \qquad \text{[Using Partial Fraction]}$$

$$= -\log(x+1)\Big|_1^2 + 2\log(x+2)\Big|_1^2$$

$$= -(\log 3 - \log 2) + 2(\log 4 - \log 3)$$

$$= -3\log 3 + 5\log 2$$

$$= \log\frac{32}{27}$$

Definite Integrals Ex 20.1 Q55

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \sin^3 x\, dx$$

$$I = \int_0^{\frac{\pi}{2}} \sin^2 x \cdot \sin x\, dx$$

$$= \int_0^{\frac{\pi}{2}} \left(1 - \cos^2 x\right) \sin x\, dx$$

$$= \int_0^{\frac{\pi}{2}} \sin x\, dx - \int_0^{\frac{\pi}{2}} \cos^2 x \cdot \sin x\, dx$$

$$= \left[-\cos x\right]_0^{\frac{\pi}{2}} + \left[\frac{\cos^3 x}{3}\right]_0^{\frac{\pi}{2}}$$

$$= 1 + \frac{1}{3}[-1] = 1 - \frac{1}{3} = \frac{2}{3}$$

Hence, the given result is proved.

Definite Integrals Ex 20.1 Q56

$$\text{Let } I = \int_0^{\pi} \left(\sin^2 \frac{x}{2} - \cos^2 \frac{x}{2}\right) dx$$

$$= -\int_0^{\pi} \left(\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}\right) dx$$

$$= -\int_0^{\pi} \cos x\ dx$$

$$\int \cos x\ dx = \sin x = \mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$I = \mathrm{F}(\pi) - \mathrm{F}(0)$$

$$= \sin\pi - \sin 0$$

$$= 0$$

Definite Integrals Ex 20.1 Q57

$$\int_1^2 \left(\frac{1}{x} - \frac{1}{2x^2}\right) e^{2x} dx$$

Let $2x = t \Rightarrow 2dx = dt$

When $x = 1$, $t = 2$ and when $x = 2$, $t = 4$

$$\therefore \int_1^2 \left(\frac{1}{x} - \frac{1}{2x^2}\right) e^{2x} dx = \frac{1}{2} \int_2^4 \left(\frac{2}{t} - \frac{2}{t^2}\right) e^t dt$$

$$= \int_2^4 \left(\frac{1}{t} - \frac{1}{t^2}\right) e^t dt$$

$$\text{Let } \frac{1}{t} = f(t)$$

$$\text{Then, } f'(t) = -\frac{1}{t^2}$$

$$\Rightarrow \int_2^4 \left(\frac{1}{t} - \frac{1}{t^2}\right) e^t dt = \int_2^4 e^t \left[f(t) + f'(t)\right] dt$$

$$= \left[e^t f(t)\right]_2^4$$

$$= \left[e^t \cdot \frac{2}{t}\right]_2^4$$

$$= \left[\frac{e^t}{t}\right]_2^4$$

$$= \frac{e^4}{4} - \frac{e^2}{2}$$

$$= \frac{e^4 - 2e^2}{4}$$

Definite Integrals Ex 20.1 Q58

$$\int_1^2 \frac{1}{\sqrt{(x-1)(2-x)}}dx$$
$$= \int_1^2 \frac{1}{\sqrt{-\left(x-\frac{3}{2}\right)^2+\left(\frac{1}{4}\right)}}dx$$
$$= \int_1^2 \frac{1}{\sqrt{\left(\frac{1}{2}\right)^2-\left(x-\frac{3}{2}\right)^2}}dx$$
$$= \left[\sin^{-1}(2x-3)\right]_1^2$$
$$= \sin^{-1}(1)-\sin^{-1}(-1)$$
$$= \pi$$

Definite Integrals Ex 20.1 Q59

We have,

$$\int_0^k \frac{dx}{2+8x^2} = \frac{\pi}{16}$$

$$\Rightarrow \frac{1}{8}\int_0^k \frac{dx}{\left(\frac{1}{2}\right)^2+x^2} = \frac{\pi}{16}$$

$$\Rightarrow \frac{1}{8}\left[2\tan^{-1}2x\right]_0^k = \frac{\pi}{16} \qquad \left[\because \int \frac{dx}{a^2-x^2} = 2\tan^{-1}\frac{x}{a}\right]$$

$$\Rightarrow \frac{1}{4}\left[\tan^{-1}2k-\tan^{-1}0\right] = \frac{\pi}{16}$$

$$\Rightarrow \tan^{-1}2k - 0 = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}2k = \frac{\pi}{4}$$

$$\Rightarrow 2k = 1$$

$$k = \frac{1}{2}$$

Definite Integrals Ex 20.1 Q60

We have,

$$\int_0^a 3x^2 dx = 8$$

$$\Rightarrow \left[x^3\right]_0^a = 8$$

$$\Rightarrow a^3 = 8$$

$$\Rightarrow a = 2$$

Definite Integrals Ex 20.1 Q61

$$\int_{\pi}^{\frac{3\pi}{2}} \sqrt{1-(1-2\sin^2 x)}dx$$

$$\int_{\pi}^{\frac{3\pi}{2}} \sqrt{2\sin^2 x}dx$$

$$\sqrt{2}\int_{\pi}^{\frac{3\pi}{2}} \sin x dx$$

$$\sqrt{2}(-\cos x)_{\pi}^{\frac{3\pi}{2}}$$

$$= \sqrt{2}$$

Definite Integrals Ex 20.1 Q62

$$I = \int_0^{2\pi} \sqrt{1+\sin\frac{x}{2}}\,dx$$

$$\Rightarrow I = \int_0^{2\pi} \sqrt{\sin^2\frac{x}{4}+\cos^2\frac{x}{4}+2\sin\frac{x}{4}\cos\frac{x}{4}}\,dx$$

$$\Rightarrow I = \int_0^{2\pi} \sqrt{\left(\sin\frac{x}{4}+\cos\frac{x}{4}\right)^2}\,dx$$

$$\Rightarrow I = \int_0^{2\pi} \left(\sin\frac{x}{4}+\cos\frac{x}{4}\right)dx$$

$$\Rightarrow I = \left[\frac{-\cos\frac{x}{4}}{\frac{1}{4}}+\frac{\sin\frac{x}{4}}{\frac{1}{4}}\right]_0^{2\pi}$$

$$\Rightarrow I = 4(0+1+1-0)$$

$$\Rightarrow I = 8$$

Definite Integrals Ex 20.1 Q63

$$I = \int_0^{\pi/4} (\tan x+\cot x)^{-2}\,dx$$

$$I = \int_0^{\pi/4} \frac{1}{(\tan x+\cot x)^2}\,dx$$

$$I = \int_0^{\pi/4} \frac{1}{\left(\frac{\sin^2 x+\cos^2 x}{\sin x\cos x}\right)^2}\,dx$$

$$I = \int_0^{\pi/4} (\sin x\cos x)^2\,dx$$

$$I = \int_0^{\pi/4} \sin^2 x\left(1-\sin^2 x\right)dx$$

$$I = \int_0^{\pi/4} \sin^2 x\,dx - \int_0^{\pi/4} \sin^4 x\,dx$$

We know that by reduction formula,

$$\int \sin^n x\ dx = \frac{n-1}{n}\int \sin^{n-2} x\ dx - \frac{\cos x\sin^{n-1} x}{n}$$

For $n = 2$

$$\int \sin^2 x\ dx = \frac{2-1}{2}\int 1\ dx - \frac{\cos x\sin x}{2}$$

$$\int \sin^2 x\ dx = \frac{1}{2}x - \frac{\cos x\sin x}{2}$$

For $n = 4$

$$\int \sin^4 x\ dx = \frac{4-1}{4}\int \sin^2 x\ dx - \frac{\cos x\sin^3 x}{4}$$

$$\int \sin^4 x\ dx = \frac{3}{4}\left\{\frac{1}{2}x - \frac{\cos x\sin x}{2}\right\} - \frac{\cos x\sin^3 x}{4}$$

Hence,

$$I = \left\{\frac{1}{2}x - \frac{\cos x\sin x}{2}\right\}_0^{\pi/4} - \left\{\frac{3}{4}\left\{\frac{1}{2}x - \frac{\cos x\sin x}{2}\right\} - \frac{\cos x\sin^3 x}{4}\right\}_0^{\pi/4}$$

$$= \left\{\frac{\pi}{8}-\frac{1}{4}\right\} - \left\{\frac{3}{4}\left(\frac{\pi}{8}-\frac{1}{4}\right)-\frac{1}{16}\right\}$$

$$= \frac{\pi}{32}$$

$$\int_0^{\frac{\pi}{2}} (\sin x \cos x)^2 dx$$

$$\int_0^{\frac{\pi}{2}} \sin^2 x(1-\sin^2 x)dx$$

$$\int_0^{\frac{\pi}{2}} \sin^2 x - \sin^4 xdx$$

$$\int_0^{\frac{\pi}{2}} \sin^2 xdx - \int_0^{\frac{\pi}{2}} \sin^4 xdx$$

We know , By reduction formula

$$\int \sin^n xdx = \frac{n-1}{n}\int \sin^{n-2} xdx - \frac{\cos x \sin^{n-1} x}{n}$$

For n=2

$$\int \sin^2 xdx = \frac{2-1}{2}\int 1dx - \frac{\cos x \sin x}{2}$$

$$\int \sin^2 xdx = \frac{1}{2}x - \frac{\cos x \sin x}{2}$$

For n=4

$$\int \sin^4 xdx = \frac{4-1}{4}\int \sin^2 xdx - \frac{\cos x \sin^3 x}{4}$$

$$\int \sin^4 xdx = \frac{3}{4}\left\{\frac{1}{2}x - \frac{\cos x \sin x}{2}\right\} - \frac{\cos x \sin^3 x}{4}$$

Hence

$$\left\{\frac{1}{2}x - \frac{\cos x \sin x}{2}\right\}_0^{\frac{\pi}{2}} - \left\{\frac{3}{4}\left\{\frac{1}{2}x - \frac{\cos x \sin x}{2}\right\} - \frac{\cos x \sin^3 x}{4}\right\}_0^{\frac{\pi}{2}}$$

$$\frac{\Pi}{4} - \frac{3}{4}\left\{\frac{\Pi}{4}\right\}$$

$$\frac{\Pi}{16}$$

Definite Integrals Ex 20.1 Q64

Using Integration By parts

$$\int f'g = fg - \int fg'$$

$$f' = x, g = \log(2x+1)$$

$$f = \frac{x^2}{2}, g' = \frac{2}{2x+1}$$

$$\int_0^1 x\log(1+2x)\,dx$$

$$= \left[\frac{x^2\log(1+2x)}{2}\right]_0^1 - \int_0^1 \frac{2x^2}{2(2x+1)}dx$$

$$= \frac{\log(3)}{2} - \int_0^1 \frac{x}{2} - \frac{1}{4} + \frac{1}{4(2x+1)}dx$$

$$= \frac{\log(3)}{2} - \left[\frac{x^2}{4} - \frac{x}{4} + \frac{1}{8}\log|2x+1|\right]_0^1$$

$$= \frac{\log(3)}{2} - \frac{1}{8}\log(3)$$

$$= \frac{3}{8}\log_e(3)$$

Definite Integrals Ex 20.1 Q65

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (\tan^2 x + 2\tan x \cot x + \cot^2 x) dx$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \left\{\left(\sec^2 x - 1\right) + 2 + \left(\operatorname{cosec}^2 x - 1\right)\right\} dx$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \left\{\sec^2 x + \operatorname{cosec}^2 x\right\} dx$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^2 x dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \operatorname{cosec}^2 x dx$$

$$\left(\tan x\right)_{\frac{\pi}{6}}^{\frac{\pi}{3}} + \left(-\cot x\right)_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$

$$\left\{\sqrt{3} - \frac{1}{\sqrt{3}}\right\} - \left\{\frac{1}{\sqrt{3}} - \sqrt{3}\right\}$$

$$2\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)$$

$$\frac{4}{\sqrt{3}}$$

Definite Integrals Ex 20.1 Q66

$$I = \int_0^{\pi/4} \left(a^2\cos^2 x + b^2 \sin^2 x\right) dx$$

$$I = \int_0^{\pi/4} \left(a^2\left(1 - \sin^2 x\right) + b^2 \sin^2 x\right) dx$$

$$I = \int_0^{\pi/4} \left(a^2 - a^2\sin^2 x + b^2\sin^2 x\right) dx$$

$$I = \int_0^{\pi/4} a^2 + \left(b^2 - a^2\right)\sin^2 x\, dx$$

$$I = \int_0^{\pi/4} a^2 + \left(b^2 - a^2\right)\frac{(1 + \cos 2x)}{2} dx$$

$$I = \left[a^2 x + \frac{\left(b^2 - a^2\right)}{2}\left(x + \frac{\sin 2x}{2}\right)\right]_0^{\pi/4}$$

$$I = \left[\frac{a^2\pi}{4} + \frac{\left(b^2 - a^2\right)}{2}\left(\frac{\pi}{4} + \frac{1}{2}\right)\right]$$

$$I = \frac{\left(b^2 + a^2\right)\pi}{8} + \frac{\left(b^2 - a^2\right)}{4}$$

Definite Integrals Ex 20.1 Q67

$$\int_0^1 \frac{1}{x^4+2x^3+2x^2+2x+1}dx$$

$$\int_0^1 \frac{1}{(x+1)^2(x^2+1)}dx$$

$$\int_0^1 \left\{-\frac{x}{2(x^2+1)}+\frac{1}{2(x+1)}+\frac{1}{2(x+1)^2}\right\}dx$$

$$-\int_0^1 \frac{x}{2(x^2+1)}dx+\int_0^1 \frac{1}{2(x+1)}dx+\int_0^1 \frac{1}{2(x+1)^2}dx$$

$$-\left\{\frac{\log(x^2+1)}{4}\right\}_0^1+\left\{\frac{\log(x+1)}{2}\right\}_0^1-\left\{\frac{1}{2(x+1)}\right\}_0^1$$

$$-\frac{\log 2}{4}+\frac{\log 2}{2}-\frac{1}{4}+\frac{1}{2}$$

$$\frac{\log 2}{4}+\frac{1}{4}$$

=(1/4)log(2e)

Ex 20.2

Definite Integrals Ex 20.2 Q1

Let $I = \int_2^4 \frac{x}{x^2+1} dx$

$\int \frac{x}{x^2+1} dx = \frac{1}{2}\int \frac{2x}{x^2+1} dx = \frac{1}{2}\log\left(1+x^2\right) = F(x)$

By the second funamental theorem of calculus, we obtain

$I = F(4) - F(2)$

$= \frac{1}{2}\left[\log\left(1+4^2\right) - \log\left(1+2^2\right)\right]$

$= \frac{1}{2}[\log 17 - \log 5]$

$= \frac{1}{2}\log\left(\frac{17}{5}\right)$

Definite Integrals Ex 20.2 Q2

Let $1+\log x = t$

Differentiating w.r.t. x, we get

$$\frac{1}{x}dx = dt$$

Now, $x = 1 \Rightarrow t = 1$

$x = 2 \Rightarrow t = 1+\log 2$

$$\therefore \int_1^2 \frac{1}{x\left(1+\log x\right)^2} dx = \int_1^{1+\log 2} \frac{dt}{t^2}$$

$$= \left[\frac{-1}{t}\right]_1^{1+\log 2}$$

$$= \left[\frac{-1}{1+\log 2} + 1\right]$$

$$= \left[\frac{-1+1+\log 2}{1+\log 2}\right]$$

$$= \left[\frac{\log 2}{1+\log 2}\right] \qquad [\because \log e = 1]$$

$$= \frac{\log 2}{\log e + \log 2} \qquad [\log a + \log b = \log ab]$$

$$= \frac{\log 2}{\log 2e}$$

$$\therefore \int_1^2 \frac{1}{x\left(1+\log x\right)^2} dx = \frac{\log 2}{\log 2e}$$

Definite Integrals Ex 20.2 Q3

Let $9x^2 - 1 = t$

Differentiating w.r.t. x, we get

$$18x\,dx = dt$$

$$3x\,dx = \frac{dt}{6}$$

Now, $x = 1 \Rightarrow t = 8$

$x = 2 \Rightarrow t = 35$

$$\therefore \int_1^2 \frac{3x}{9x^2-1} dx = \int_8^{35} \frac{dt}{6t}$$

$$= \frac{1}{6}\left[\log t\right]_8^{35}$$

$$= \frac{1}{6}\left(\log 35 - \log 8\right)$$

$$\therefore \int_1^2 \frac{3x}{9x^2-1} dx = \frac{1}{6}\left(\log 35 - \log 8\right)$$

Definite Integrals Ex 20.2 Q4

Put $\sin x = \dfrac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

$\cos x = \dfrac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

$$\int_0^{\frac{\pi}{2}} \frac{dx}{5\cos x + 3\sin x} = \int_0^{\frac{\pi}{2}} \frac{\sec^2\frac{x}{2}\,dx}{5\left(1-\tan^2\frac{x}{2}\right)+6\tan\frac{x}{2}}$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sec^2\frac{x}{2}\,dx}{5-5\tan^2\frac{x}{2}+6\tan\frac{x}{2}}$$

Let $\tan\frac{x}{2} = t$

Differentiating w.r.t. x, we get

$$\frac{1}{2}\sec^2\frac{x}{2}\,dx = dt$$

Now, $x = 0 \Rightarrow t = 0$

$x = \frac{\pi}{2} \Rightarrow t = 1$

$$\int_0^{\frac{\pi}{2}} \frac{\sec^2\frac{x}{2}\,dx}{5-5\tan^2\frac{x}{2}+6\tan\frac{x}{2}} = \int_0^1 \frac{2dt}{5-5t^2+6t} = \frac{2}{5}\int \frac{dt}{1-t^2+\frac{6}{5}t}$$

Forming perfect square by adding and subtracting $\frac{9}{25}$

$$\frac{2}{5}\int_0^1 \frac{dt}{1-t^2+\frac{6}{5}t}$$

$$= \frac{2}{5}\int_0^1 \frac{dt}{\frac{34}{25}-\left(t-\frac{3}{5}\right)^2}$$

$$= \frac{2}{5}\cdot\frac{1}{2}\sqrt{\frac{25}{34}}\log\left(\frac{\sqrt{\frac{34}{25}}+t-\frac{3}{5}}{\sqrt{\frac{34}{25}}-t+\frac{3}{5}}\right)_0^1 \qquad \left[\because \int \frac{dx}{a^2-x^2} = \frac{1}{2a}\log\left(\frac{x+a}{x-a}\right)\right]$$

$$= \frac{1}{\sqrt{34}}\left\{\log\left(\frac{\sqrt{34}+2}{\sqrt{34}-2}\right) - \log\left(\frac{\sqrt{34}-3}{\sqrt{34}+3}\right)\right\}$$

$$= \frac{1}{\sqrt{34}}\log\left(\frac{\left(\sqrt{34}+2\right)\left(\sqrt{34}-3\right)}{\left(\sqrt{34}-2\right)\left(\sqrt{34}-3\right)}\right)$$

$$= \frac{1}{\sqrt{34}}\log\left(\frac{40+5\sqrt{34}}{40-5\sqrt{34}}\right)$$

$$= \frac{1}{\sqrt{34}}\log\left(\frac{8+\sqrt{34}}{8-\sqrt{34}}\right)$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{dx}{5\cos x + 3\sin x} = \frac{1}{\sqrt{34}}\log\left(\frac{8+\sqrt{34}}{8-\sqrt{34}}\right)$$

Definite Integrals Ex 20.2 Q5

Let $a^2 + x^2 = t^2$

Differentiating w.r.t. x, we get

$$2x\,dx = 2t\,dt$$
$$x\,dx = t\,dt$$

Now, $x = 0 \Rightarrow t = 0$

$x = a \Rightarrow t = \sqrt{2}a$

$$\therefore \int_0^a \frac{x\,dx}{\sqrt{a^2 + x^2}}dx = \int_a^{\sqrt{2}a} \frac{t\,dt}{t}$$
$$= \int_a^{\sqrt{2}a} dt$$
$$= \left[t\right]_a^{\sqrt{2}a}$$
$$= \left[\sqrt{2}a - a\right]$$
$$= a\left(\sqrt{2} - 1\right)$$

$$\therefore \int_0^a \frac{x}{\sqrt{a^2 + x^2}}dx = a\left(\sqrt{2} - 1\right)$$

Definite Integrals Ex 20.2 Q6

Let $e^x = t$

Differentiating w.r.t. x, we get

$$e^x\,dx = dt$$

Now, $x = 0 \Rightarrow t = 1$

$x = 1 \Rightarrow t = e$

$$\therefore \int_0^1 \frac{e^x}{1 + e^{2x}}dx = \int_1^e \frac{dt}{1 + t^2}$$
$$= \left[\tan^{-1} t\right]_1^e \qquad \left[\because \int \frac{dt}{1 + t^2} = \tan^{-1} t\right]$$
$$= \left[\tan^{-1} e - \tan^{-1} 1\right] \qquad \left[\because \tan\frac{\pi}{4} = 1\right]$$
$$= \tan^{-1} e - \frac{\pi}{4}$$

$$\therefore \int_0^1 \frac{e^x}{1 + e^{2x}}dx = \tan^{-1} e - \frac{\pi}{4}$$

Definite Integrals Ex 20.2 Q7

Let $x^2 = t$

Differentiating w.r.t. x, we get

$$2x\,dx = dt$$

Now,

$x = 0 \Rightarrow t = 0$

$x = 1 \Rightarrow t = 1$

$$\therefore \int_0^1 xe^{x^2}dx = \int_0^1 \frac{e^t dt}{2}$$
$$= \frac{1}{2}\int_0^1 e^t dt$$
$$= \frac{1}{2}\left[e^t\right]_0^1$$
$$= \frac{1}{2}\left[e^1 - e^0\right] \qquad \left[\because e^0 = 1\right]$$
$$= \frac{1}{2}(e - 1)$$

$$\therefore \int_0^1 xe^{x^2}dx = \frac{1}{2}(e - 1)$$

Definite Integrals Ex 20.2 Q8

Let $\log x = t$

Differentiating w.r.t. x, we get

$$\frac{1}{x}dx = dt$$

Now,

$x = 0 \Rightarrow t = 0$

$x = 3 \Rightarrow t = \log 3$

$$\int_1^3 \frac{\cos(\log x)}{x}dx$$

$$= \int_0^{\log 3} \cos t\, dt \qquad \left[\because \int \cos t = \sin t\right]$$

$$= \left[\sin t\right]_0^{\log 3}$$

$$= \sin(\log 3) - \sin 0$$

$$= \sin(\log 3)$$

$$\int_1^3 \frac{\cos(\log x)}{x}dx = \sin(\log 3)$$

Definite Integrals Ex 20.2 Q9

Let $x^2 = t$

Differentiating w.r.t. x, we get

$$2x\, dx = dt$$

Now,

$x = 0 \Rightarrow t = 0$

$x = 1 \Rightarrow t = 1$

$$\therefore \int_0^1 \frac{2x}{1+x^4}dx$$

$$= \int_0^1 \frac{dt}{1+t^2}$$

$$= \left[\tan^{-1} t\right]_0^1$$

$$= \left[\tan^{-1} 1 - \tan^{-1} 0\right] \qquad \left[\because \tan\frac{\pi}{4} = 1\right]$$

$$= \frac{\pi}{4}$$

$$\therefore \int_0^1 \frac{2x}{1+x^4}dx = \frac{\pi}{4}$$

Definite Integrals Ex 20.2 Q10

Let $x = a\sin\theta$

Differentiating w.r.t. x, we get

$dx = a\cos\theta d\theta$

Now,

$x = 0 \Rightarrow \theta = 0$

$x = a \Rightarrow \theta = \frac{\pi}{2}$

$\therefore \int_0^a \sqrt{a^2 - x^2}\,dx$

$= \int_0^{\frac{\pi}{2}} \sqrt{a^2\left(1 - \sin^2\theta\right)}\, a\cos\theta d\theta$

$= a^2 \int_0^{\frac{\pi}{2}} \cos^2\theta d\theta \qquad \left[\because \left(1 - \sin^2\theta\right) = \cos^2\theta \text{ and } \frac{1 + \cos 2\theta}{2} = \cos 2\theta\right]$

$= \frac{a^2}{2} \int_0^{\frac{\pi}{2}} \left(1 + \cos 2\theta\right) d\theta$

$= \frac{a^2}{2}\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{2}}$

$= \frac{a^2}{2}\left[\frac{\pi}{2} + 0 - 0 - 0\right]$

$= \frac{\pi a^2}{4}$

$\therefore \int_0^a \sqrt{a^2 - x^2}\,dx = \frac{\pi a^2}{4}$

Definite Integrals Ex 20.2 Q11

Let $I = \int_0^{\frac{\pi}{2}} \sqrt{\sin\phi}\cos^5\phi\, d\phi = \int_0^{\frac{\pi}{2}} \sqrt{\sin\phi}\cos^4\phi \cos\phi\, d\phi$

Also, let $\sin\phi = t \Rightarrow \cos\phi\, d\phi = dt$

When $\phi = 0$, $t = 0$ and when $\phi = \frac{\pi}{2}$, $t = 1$

$\therefore I = \int_0^1 \sqrt{t}\left(1 - t^2\right)^2 dt$

$= \int_0^1 t^{\frac{1}{2}}\left(1 + t^4 - 2t^2\right) dt$

$= \int_0^1 \left[t^{\frac{1}{2}} + t^{\frac{9}{2}} - 2t^{\frac{5}{2}}\right] dt$

$= \left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \frac{t^{\frac{11}{2}}}{\frac{11}{2}} - \frac{2t^{\frac{7}{2}}}{\frac{7}{2}}\right]_0^1$

$= \frac{2}{3} + \frac{2}{11} - \frac{4}{7}$

$= \frac{154 + 42 - 132}{231}$

$= \frac{64}{231}$

Definite Integrals Ex 20.2 Q12

Let $\sin x = t$

Differentiating w.r.t. x, we get

$$\cos x\, dx = dt$$

Now,

$x = 0 \Rightarrow t = 0$

$x = \frac{\pi}{2} \Rightarrow t = 1$

$$\int_0^{\frac{\pi}{2}} \frac{\cos x}{1+\sin^2 x} dx$$

$$= \int_0^1 \frac{dt}{1+t^2}$$

$$= \left[\tan^{-1} t\right]_0^1$$

$$= \left[\tan^{-1} 1 - \tan^{-1} 0\right] \qquad \left[\because \tan\frac{\pi}{4} = 1\right]$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{\cos x}{1+\sin^2 x} dx = \frac{\pi}{4}$$

Definite Integrals Ex 20.2 Q13

Let $1 + \cos\theta = t^2$

Differentiating w.r.t. x, we get

$$-\sin\theta\, d\theta = 2t\, dt$$

$$\sin\theta\, d\theta = -2t\, dt$$

Now,

$x = 0 \Rightarrow t = \sqrt{2}$

$x = \frac{\pi}{2} \Rightarrow t = 1$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{\sin\theta\, d\theta}{\sqrt{1+\cos\theta}}$$

$$= \int_{\sqrt{2}}^1 \frac{-2t dt}{t}$$

$$= -2\int_{\sqrt{2}}^1 dt$$

$$= -2\left[t\right]_{\sqrt{2}}^1$$

$$= -2\left[1-\sqrt{2}\right]$$

$$= 2\left[\sqrt{2}-1\right]$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{\sin\theta\, d\theta}{\sqrt{1+\cos\theta}} = 2\left[\sqrt{2}-1\right]$$

Definite Integrals Ex 20.2 Q14

Let $3+4\sin x = t$
Differentiating w.r.t. x, we get

$$4\cos x dx = dt$$
$$\cos x dx = \frac{dt}{4}$$

Now,
$x = 0 \Rightarrow t = 3$
$x = \frac{\pi}{3} \Rightarrow t = 3+2\sqrt{3}$

$$\therefore \int_0^{\frac{\pi}{3}} \frac{\cos x}{3+4\sin x} dx$$
$$= \int_3^{3+2\sqrt{3}} \frac{dt}{4t}$$
$$= \frac{1}{4}\left[\log t\right]_3^{3+2\sqrt{3}}$$
$$= \frac{1}{4}\left[\log\left(3+2\sqrt{3}\right) - \log 3\right]$$
$$= \frac{1}{4}\log\left(\frac{3+2\sqrt{3}}{3}\right)$$

$$\therefore \int_0^{\frac{\pi}{3}} \frac{\cos x}{3+4\sin x} dx = \frac{1}{4}\log\left(\frac{3+2\sqrt{3}}{3}\right)$$

Definite Integrals Ex 20.2 Q15

Let $\tan^{-1} x = t$
Differentiating w.r.t. x, we get

$$\frac{1}{1+x^2} dx = dt$$

Now,
$x = 0 \Rightarrow t = 0$
$x = 1 \Rightarrow t = \frac{\pi}{4}$

$$\therefore \int_0^1 \frac{\sqrt{\tan^{-1} x}}{1+x^2} dx$$
$$= \int_0^{\frac{\pi}{4}} t^{1/2} dt$$
$$= \left[\frac{t^{3/2}}{3/2}\right]_0^{\frac{\pi}{4}}$$
$$= \frac{2}{3}\left[t^{3/2}\right]_0^{\frac{\pi}{4}}$$
$$= \frac{2}{3}\left[\left(\frac{\pi}{4}\right)^{3/2} - 0\right]$$
$$= \frac{1}{12}\pi^{3/2}$$

$$\therefore \int_0^1 \frac{\sqrt{\tan^{-1} x}}{1+x^2} dx = \frac{1}{12}\pi^{3/2}$$

Definite Integrals Ex 20.2 Q16

$$\int_0^2 x\sqrt{x+2}\,dx$$

Let $x + 2 = t^2 \Rightarrow dx = 2tdt$

When $x = 0$, $t = \sqrt{2}$ and when $x = 2$, $t = 2$

$$\therefore \int_0^2 x\sqrt{x+2}\,dx = \int_{\sqrt{2}}^2 \left(t^2-2\right)\sqrt{t^2}\,2tdt$$
$$= 2\int_{\sqrt{2}}^2 \left(t^2-2\right)t^2 dt$$
$$= 2\int_{\sqrt{2}}^2 \left(t^4-2t^2\right)dt$$
$$= 2\left[\frac{t^5}{5}-\frac{2t^3}{3}\right]_{\sqrt{2}}^2$$
$$= 2\left[\frac{32}{5}-\frac{16}{3}-\frac{4\sqrt{2}}{5}+\frac{4\sqrt{2}}{3}\right]$$
$$= 2\left[\frac{96-80-12\sqrt{2}+20\sqrt{2}}{15}\right]$$
$$= 2\left[\frac{16+8\sqrt{2}}{15}\right]$$
$$= \frac{16\left(2+\sqrt{2}\right)}{15}$$
$$= \frac{16\sqrt{2}\left(\sqrt{2}+1\right)}{15}$$

Definite Integrals Ex 20.2 Q17

Let $x = \tan\theta$

Differentiating w.r.t. x, we get

$dx = \sec^2\theta d\theta$

Now,

$x = 0 \Rightarrow \theta = 0$

$x = 1 \Rightarrow \theta = \frac{\pi}{4}$

$$\int_0^1 \tan^{-1}\left(\frac{2x}{1-x^2}\right)dx$$
$$= \int_0^{\frac{\pi}{4}} \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)\sec^2\theta d\theta \qquad \left[\because \tan^2\theta = \frac{2\tan\theta}{1-\tan^2\theta}\right]$$
$$= \int_0^{\frac{\pi}{4}} \tan^{-1}\left(\tan 2\theta\right)\sec^2\theta d\theta$$
$$= \int_0^{\frac{\pi}{4}} 2\theta\sec^2\theta d\theta$$

Applying by parts, we get

$$= 2\left[\theta\int_0^{\frac{\pi}{4}}\sec^2\theta d\theta - \int_0^{\frac{\pi}{4}}\left(\sec^2\theta d\theta\right)\frac{d\theta}{d\theta}d\theta\right]$$
$$=2\left[\theta\tan\theta\right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}}\tan\theta d\theta]$$
$$= 2\left[\theta\tan\theta+\log(\cos\theta)\right]_0^{\frac{\pi}{4}}$$
$$= 2\left[\frac{\pi}{4}+\log\left(\frac{1}{\sqrt{2}}\right)-0-0\right]$$
$$= 2\left[\frac{\pi}{4}+\frac{1}{2}\log 2\right]$$
$$= \frac{\pi}{2}-\log 2$$

$$\therefore \int_0^1 \tan^{-1}\left(\frac{2x}{1-x^2}\right)dx = \frac{\pi}{2}-\log 2$$

Definite Integrals Ex 20.2 Q18

Let $\sin^2 x = t$

Differentiating w.r.t. x, we get

$$2\sin x\cos x dx = dt$$

Now,

$x = 0 \Rightarrow t = 0$

$x = \frac{\pi}{2} \Rightarrow t = 1$

$$\int_0^{\frac{\pi}{2}} \frac{\sin x\cos x}{1+\sin^4 x}dx$$

$$= \frac{1}{2}\int_0^1 \frac{dt}{1+t^2}$$

$$= \frac{1}{2}\left[\tan^{-1} t\right]_0^1$$

$$= \frac{1}{2}\left[\tan^{-1}(1) - \tan^{-1}(0)\right]$$

$$= \frac{1}{2}\left[\tan^{-1}\left(\tan\frac{\pi}{4}\right) - \tan^{-1}(\tan 0)\right]$$

$$= \frac{1}{2}\times\frac{\pi}{4}$$

$$= \frac{\pi}{8}$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{\sin x\cos x}{1+\sin^4 x}dx = \frac{\pi}{8}$$

Definite Integrals Ex 20.2 Q19

Putting $\cos x = \dfrac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}} = \dfrac{1-\tan^2\frac{x}{2}}{\sec^2\frac{x}{2}}$

$$\sin x = \frac{2\tan\frac{x}{2}}{\sec^2\frac{x}{2}}$$

$$\therefore \quad I = \int_0^{\frac{\pi}{2}} \frac{1}{a\cos x + b\sin x}dx = \int_0^{\frac{\pi}{2}} \frac{\sec^2\frac{x}{2}}{a\left(1-\tan^2\frac{x}{2}\right)+2b\tan^2\frac{x}{2}}dx$$

Put $\tan\frac{x}{2} = t$

$$\frac{1}{2}\sec^2\frac{x}{2}dx = dt$$

If $x = 0,\ t = 0$ and if $x = \frac{\pi}{2},\ t = 1$

$$\Rightarrow \quad I = 2\int_0^1 \frac{dt}{a\left(1-t^2\right)+2bt}$$

$$= 2\int_0^1 \frac{dt}{-at^2+2bt+a}$$

$$= 2\int_0^1 \frac{dt}{-a\left[t^2-\frac{2b}{a}t-1\right]}$$

$$= \frac{2}{a}\int_0^1 \frac{dt}{-\left[\left(t-\frac{b}{a}\right)^2-1-\frac{b^2}{a^2}\right]}$$

$$= \frac{2}{a}\int_0^1 \frac{dt}{\left(\frac{b^2}{a^2}+1\right)-\left(t-\frac{b}{a}\right)^2}$$

$$= \frac{2}{a}\left[\frac{1}{2\sqrt{\frac{b^2+a^2}{a^2}}}\left(\log\left|\frac{\frac{\sqrt{b^2+a^2}}{a^2}+\left(t-\frac{b}{a}\right)}{\frac{\sqrt{b^2+a^2}}{a^2}-\left(t-\frac{b}{a}\right)}\right|\right)\right]_0^1$$

$$= \frac{1}{\sqrt{b^2+a^2}}\log\left(\frac{a+b+\sqrt{a^2+b^2}}{a+b-\sqrt{a^2+b^2}}\right)$$

Definite Integrals Ex 20.2 Q20

We know that $\sin x = \dfrac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{1}{5+4\sin x}\,dx = \int_0^{\frac{\pi}{2}} \frac{1}{5+4\sin\left(\dfrac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)}\,dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{1}{\dfrac{5\left(1+\tan^2\frac{x}{2}\right)+4\left(2\tan\frac{x}{2}\right)}{1+\tan^2\frac{x}{2}}}\,dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{1+\tan^2\frac{x}{2}}{\left(5+5\tan^2\frac{x}{2}+8\tan\frac{x}{2}\right)}\,dx$$

$$\int_0^{\frac{\pi}{2}} \frac{\sec^2\frac{x}{2}}{5+5\tan^2\frac{x}{2}+8\tan\frac{x}{2}}\,dx$$

Let $\tan\frac{x}{2} = t$

Differentiating w.r.t. x, we get

$$\frac{1}{2}\sec^2\frac{x}{2}\,dx = dt$$

Now,

$x = 0 \Rightarrow t = 0$

$x = \frac{\pi}{2} \Rightarrow t = 1$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{\sec^2\frac{x}{2}}{5+5\tan^2\frac{x}{2}+8\tan\frac{x}{2}}\,dx$$

$$= \int_0^1 \frac{2dt}{5 + 5t^2 + 8t}$$
$$= \frac{2}{5}\int_0^1 \frac{dt}{1 + t^2 + \frac{8}{5}t}$$
$$= \frac{2}{5}\int_0^1 \frac{dt}{1 - \frac{16}{25} + \frac{16}{25} + t^2 + \frac{8}{5}t} \qquad \left[\text{Adding and subtracting } \frac{16}{25}\right]$$
$$= \frac{2}{5}\int_0^1 \frac{dt}{\left(\frac{3}{2}\right)^2 + \left(t + \frac{4}{5}\right)^2}$$
$$= \frac{2}{5}\left[\frac{5}{3}\tan^{-1}\left(t+\frac{4}{5}\right)\times\frac{5}{3}\right]_0^1$$
$$= \frac{2}{3}\left[\tan^{-1}\left(1+\frac{4}{5}\right)\times\frac{5}{3} - \tan^{-1}\frac{4}{5}\times\frac{5}{3}\right]_0^1$$
$$= \frac{2}{3}\left[\tan^{-1} 3 - \tan^{-1}\frac{4}{3}\right]_0^1$$
$$= \frac{2}{3}\left[\tan^{-1}\left(\frac{3-\frac{4}{3}}{1+3\times\frac{4}{3}}\right)\right]_0^1 \qquad \left[\because \tan^{-1} A - \tan^{-1} B = \tan^{-1}\left(\frac{A-B}{1+AB}\right)\right]$$
$$= \frac{2}{3}\left[\tan^{-1}\frac{\frac{5}{3}}{5}\right]$$
$$= \frac{2}{3}\tan^{-1}\frac{1}{3}$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{1}{5 + 4\sin x}dx = \frac{2}{3}\tan^{-1}\frac{1}{3}$$

Definite Integrals Ex 20.2 Q21

We have,

$$\int_0^{\pi} \frac{\sin x}{\sin x + \cos x}dx$$

$$\text{Let } \sin x = K(\sin x + \cos x) + L\frac{d}{dx}(\sin x + \cos x)$$
$$= K(\sin x + \cos x) + L(\cos x - \sin x)$$
$$= \sin x(K - L) + \cos x(K + L)$$

Equating similar terms

$$K - L = 1$$
$$K + L = 0$$

$$\Rightarrow K = \frac{1}{2} \text{ and } L = -\frac{1}{2}$$

$$\therefore \int_0^{\pi} \frac{\sin x}{\sin x + \cos x}dx = \frac{1}{2}\int_0^{\pi} dx + \left(\frac{-1}{2}\right)\int_0^{\pi} \frac{\cos x - \sin x}{\sin x + \cos x}dx$$
$$= \frac{1}{2}[x]_0^{\pi} - \frac{1}{2}\left(\log|\sin x + \cos x|\right)_0^{\pi} = \frac{\pi}{2} - \frac{1}{2}(0) = \frac{\pi}{2}$$

$$\therefore \int_0^{\pi} \frac{\sin x}{\sin x + \cos x}dx = \frac{\pi}{2}$$

Definite Integrals Ex 20.2 Q22

We know,

$$\sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}},\ \cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$$

$$\therefore \frac{1}{3+2\sin x+\cos x}$$

$$= \frac{1}{3+2\left(\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)+\left(\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)}$$

$$= \frac{\left(1+\tan^2\frac{x}{2}\right)}{3\left(1+\tan^2\frac{x}{2}\right)+4\tan\frac{x}{2}+\left(1-\tan^2\frac{x}{2}\right)}$$

$$= \frac{\sec^2\frac{x}{2}dx}{3+3\tan^2\frac{x}{2}+4\tan\frac{x}{2}+1-\tan^2\frac{x}{2}}$$

$$\therefore \int_0^{\pi}\frac{1}{3+2\sin x+\cos x}dx = \int_0^{\pi}\frac{\sec^2\frac{x}{2}dx}{2\tan^2\frac{x}{2}+4\tan\frac{x}{2}+4}$$

Let $\tan\frac{x}{2} = t$

Differentiating w.r.t. x, we get

$$\frac{1}{2}\sec^2\frac{x}{2}dx = dt$$

Now,

$x = 0 \Rightarrow t = 0$

$x = \pi \Rightarrow t = \infty$

$$\therefore \int_0^{\pi}\frac{\sec^2\frac{x}{2}dx}{2\tan^2\frac{x}{2}+4\tan\frac{x}{2}+4}$$

$$= \int_0^{\infty}\frac{dt}{t^2+2t+2}$$

$$= \int_0^{\infty}\frac{dt}{(t+1)^2+1}$$

$$= \left[\tan^{-1}(t+1)\right]_0^{\infty}$$

$$= \tan^{-1}(\infty) - \tan^{-1}(0+1)$$

$$= \tan^{-1}(\infty) - \tan^{-1}(1)$$

$$= \tan^{-1}\left(\tan\frac{\pi}{2}\right) - \tan^{-1}\left(\tan\frac{\pi}{4}\right)$$

$$= \frac{\pi}{2} - \frac{\pi}{4}$$

$$= \frac{2\pi-\pi}{4}$$

$$= \frac{\pi}{4}$$

$$\therefore \int_0^{\pi}\frac{1}{3+2\sin x+\cos x}dx = \frac{\pi}{4}$$

Definite Integrals Ex 20.2 Q23

We have,

$$\int_0^1 1 \cdot \tan^{-1} x\, dx = \tan^{-1} x \int_0^1 dx - \int_0^1 \left(\int dx.\right) \frac{d}{dx}\left(\tan^{-1} x\right) dx$$

$$= \left[x \tan^{-1} x\right]_0^1 - \int_0^1 \frac{x}{1+x^2} dx$$

$$= \left[x \tan^{-1} x - \frac{1}{2}\log\left(1+x^2\right)\right]_0^1$$

$$= \frac{\pi}{4} - \frac{1}{2}\left(\log 2 - 0\right)$$

$$= \frac{\pi}{4} - \frac{1}{2}\log 2$$

$$\therefore \int_0^1 \tan^{-1} x\, dx = \frac{\pi}{4} - \frac{1}{2}\log 2$$

Definite Integrals Ex 20.2 Q24

Using Integration By parts

$$\int f'g = fg - \int fg'$$

$$f' = \frac{x}{\sqrt{1-x^2}}, g = \sin^{-1} x$$

$$f = -\sqrt{1-x^2}, g' = \frac{1}{\sqrt{1-x^2}}$$

$$\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx = -\sqrt{1-x^2}\, \sin^{-1} x - \int (-1) dx$$

$$\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx = -\sqrt{1-x^2}\, \sin^{-1} x + x$$

Hence

$$\int_0^{\frac{1}{2}} \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx = \left\{x - \sqrt{1-x^2}\, \sin^{-1} x\right\}_0^{\frac{1}{2}}$$

$$\int_0^{\frac{1}{2}} \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx = \left\{\frac{1}{2} - \sqrt{1-(\frac{1}{2})^2}\, \sin^{-1} \frac{1}{2}\right\}$$

$$\int_0^{\frac{1}{2}} \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx = \left\{\frac{1}{2} - \frac{\sqrt{3}}{2}\frac{\Pi}{6}\right\}$$

Definite Integrals Ex 20.2 Q25

$$I = \int_0^{\pi/4} \left(\sqrt{\tan x} + \sqrt{\cot x}\right) dx$$

$$I = \int_0^{\pi/4} \left(\frac{\sqrt{\sin x}}{\sqrt{\cos x}} + \frac{\sqrt{\cos x}}{\sqrt{\sin x}}\right) dx$$

$$I = \int_0^{\pi/4} \left(\frac{\sin x + \cos x}{\sqrt{\sin x \cos x}}\right) dx$$

$$I = \sqrt{2}\int_0^{\pi/4} \left(\frac{\sin x + \cos x}{\sqrt{2 \sin x \cos x}}\right) dx$$

$$I = \sqrt{2}\int_0^{\pi/4} \left(\frac{\sin x + \cos x}{\sqrt{1 - (\sin x - \cos x)^2}}\right) dx$$

Let $\sin x - \cos x = t$

$(\cos x + \sin x)dx = dt$

$x = 0 \Rightarrow t = -1$ and $x = \frac{\pi}{4} \Rightarrow t = 0$

$$I = \sqrt{2}\int_{-1}^{0} \left(\frac{1}{\sqrt{1 - t^2}}\right) dt$$

$$I = \sqrt{2}\left[\sin^{-1} t\right]_{-1}^{0}$$

$$I = \sqrt{2}\left[\sin^{-1}(0) - \sin^{-1}(-1)\right]$$

$$I = \frac{\pi}{\sqrt{2}}$$

Definite Integrals Ex 20.2 Q26

We have,

$$\int_0^{\frac{\pi}{4}} \frac{\tan^3 x}{1 + \cos 2x} dx = \int_0^{\frac{\pi}{4}} \frac{\tan^3 x}{2\cos^2 x} dx = \frac{1}{2}\int_0^{\frac{\pi}{4}} \tan^3 x \sec^2 x \, dx$$

Let $\tan x = t \Rightarrow \sec^2 x \, dx = dt$

Now,

$x = 0 \Rightarrow t = 0$

$x = \frac{\pi}{4} \Rightarrow t = 1$

$$\therefore \frac{1}{2}\int_0^{\frac{\pi}{4}} \sec^2 x \tan^3 x \, dx = \frac{1}{2}\int_0^1 t^3 \, dt = \frac{1}{2}\left[\frac{t^4}{4}\right]_0^1 = \frac{1}{8}$$

$$\therefore \int_0^{\frac{\pi}{4}} \frac{\tan^3 x}{1 + \cos 2x} dx = \frac{1}{8}$$

Definite Integrals Ex 20.2 Q27

We know that,

$$\cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$$

$$\frac{1}{5 + 3\cos x} = \frac{1}{5 + 3\left(\frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}\right)} = \frac{1 + \tan^2 \frac{x}{2}}{5\left(1 + \tan^2 \frac{x}{2}\right) + 3\left(1 - \tan^2 \frac{x}{2}\right)} = \frac{\sec^2 \frac{x}{2} dx}{8 + 2\tan^2 \frac{x}{2}}$$

$$\therefore \int_0^{\pi} \frac{dx}{5 + 3\cos x} dx = \frac{1}{2}\int_0^{\pi} \frac{\sec^2 \frac{x}{2}}{2^2 + \tan^2 \frac{x}{2}} dx$$

Let $\tan \frac{x}{2} = t$

Differentiating w.r.t. x, we get

$$\frac{1}{2}\sec^2 \frac{x}{2} dx = dt$$

Now,

$x = 0 \Rightarrow t = 0$

$x = \pi \Rightarrow t = \infty$

$$\therefore \frac{1}{2}\int_0^{\pi}\left(\frac{\sec^2 \frac{x}{2} dx}{2^2 + \tan^2 \frac{x}{2}}\right) dx$$

$$= \int_0^{\infty} \frac{dt}{2^2 + t^2}$$

$$= \left[\frac{1}{2}\tan^{-1}\left(\frac{t}{2}\right)\right]_0^{\infty}$$

$$= \frac{1}{2}\left[\tan^{-1}(\infty) - \tan^{-1}(0)\right]$$

$$= \frac{1}{2}\left[\tan^{-1}\left(\tan \frac{\pi}{2}\right) - \tan^{-1}(\tan 0)\right]$$

$$= \frac{1}{2}\left[\frac{\pi}{2} - 0\right]$$

$$= \frac{\pi}{4}$$

$$\therefore \int_0^{\pi} \frac{dx}{5 + 3\cos x} dx = \frac{\pi}{4}$$

Definite Integrals Ex 20.2 Q28

We have,

$$\int_0^{\frac{\pi}{2}} \frac{1}{a^2 \sin^2 x + b^2 \cos^2 x} dx$$

Dividing numerator and denominator by $\cos^2 x$

$$\int_0^{\frac{\pi}{2}} \left(\frac{\frac{1}{\cos^2 x}}{a^2 \frac{\sin^2 x}{\cos^2 x} + b^2 \frac{\cos^2 x}{\cos^2 x}} \right) dx$$

$$= \int_0^{\frac{\pi}{2}} \left(\frac{\sec^2 x}{a^2 \tan^2 x + b^2} \right) dx$$

$$= \frac{1}{a^2} \int_0^{\frac{\pi}{2}} \left(\frac{\sec^2 x}{\tan^2 x + \left(\frac{b}{a}\right)^2} \right) dx$$

Let $\tan x = t$

Differentiating w.r.t. x, we get

$$\sec^2 x \, dx = dt$$

When $x = 0 \Rightarrow t = 0$

$$x = \frac{\pi}{2} \Rightarrow t = \infty$$

$$\therefore \frac{1}{a^2} \int_0^{\frac{\pi}{2}} \left(\frac{\sec^2 x}{\tan^2 x + \left(\frac{b}{a}\right)^2} \right) dx$$

$$= \frac{1}{a^2} \int_0^{\infty} \frac{dt}{\left(\frac{b}{a}\right)^2 + t^2}$$

$$= \frac{1}{a^2} \left[\frac{a}{b} \tan^{-1} \frac{at}{b} \right]_0^{\infty}$$

$$= \frac{1}{a^2} \frac{a}{b} \left[\tan^{-1} \infty - \tan^{-1} 0 \right]$$

$$= \frac{1}{ab} \left[\tan^{-1} \tan \frac{\pi}{2} \right] \quad = \frac{\pi}{2ab}$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{1}{a^2 \sin^2 x + b^2 \cos^2 x} dx = \frac{\pi}{2ab}$$

Definite Integrals Ex 20.2 Q29

$$I = \int_0^{\frac{\pi}{2}} \frac{x + \sin x}{1 + \cos x} dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{x + 2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}} dx$$

$$= \int_0^{\frac{\pi}{2}} \left(\frac{x\sec^2\frac{x}{2}}{2} + \tan\frac{x}{2} \right) dx$$

$$= \left[x\tan\left(\frac{x}{2}\right) - \int_0^{\frac{\pi}{2}} \tan\frac{x}{2} \, dx + \int_0^{\frac{\pi}{2}} \tan\frac{x}{2} \, dx \right]_0^{\frac{\pi}{2}}$$

$$= \frac{\pi}{2}$$

$$\therefore I = \int_0^{\frac{\pi}{2}} \frac{x + \sin x}{1 + \cos x} dx = \frac{\pi}{2}$$

Definite Integrals Ex 20.2 Q30

$$I=\int_0^1 \frac{\tan^{-1} x}{1+x^2}dx$$

Let $t=\tan^{-1} x$

$$dt=\frac{1}{1+x^2}dx$$

$$x=0, t=0$$

$$x=1, t=\frac{\pi}{4}$$

$$I=\int_0^{\frac{\pi}{4}} t\,dt$$

$$=\left[\frac{t^2}{2}\right]_0^{\frac{\pi}{4}}$$

$$=\frac{1}{2}\frac{\pi^2}{16}$$

$$=\frac{\pi^2}{32}$$

Definite Integrals Ex 20.2 Q31

$$I=\int_0^{\pi/4}\frac{\sin x+\cos x}{3+\sin 2x}\,dx$$

$$I=\int_0^{\pi/4}\left(\frac{\sin x+\cos x}{3+1-(\cos x-\sin x)^2}\right)dx$$

$$I=\int_0^{\pi/4}\left(\frac{\sin x+\cos x}{4-(\cos x-\sin x)^2}\right)dx$$

$$I=\frac{1}{4}\left[\log\left|\frac{2+\sin x-\cos x}{2-\sin x+\cos x}\right|\right]_0^{\pi/4}$$

$$I=-\frac{1}{4}\log\left(\frac{1}{3}\right)$$

$$I=\frac{1}{4}\log_e 3$$

Definite Integrals Ex 20.2 Q32

We have,

$$\int_0^1 x\tan^{-1} x\,dx=\tan^{-1} x\int_0^1 x\,dx-\int_0^1\left(\int x\,dx\right)\frac{d}{dx}\left(\tan^{-1} x\right)dx$$

$$=\left[\frac{x^2}{2}\tan^{-1} x\right]_0^1-\frac{1}{2}\int_0^1\frac{x^2}{1+x^2}dx$$

$$=\left[\frac{x^2}{2}\tan^{-1} x\right]_0^1-\frac{1}{2}\int_0^1\frac{1+x^2-1}{1+x^2}dx$$

$$=\frac{1}{2}\left(\frac{\pi}{4}\right)-\frac{1}{2}\left[\int_0^1 dx-\int_0^1\frac{dx}{1+x^2}\right]$$

$$=\frac{\pi}{8}-\frac{1}{2}\left[x-\tan^{-1} x\right]_0^1$$

$$=\frac{\pi}{8}-\frac{1}{2}\left[1-\frac{\pi}{4}\right]$$

$$=\frac{\pi}{8}-\frac{1}{2}+\frac{\pi}{8}$$

$$=\frac{\pi}{4}-\frac{1}{2}$$

$$\therefore \int_0^1 x\tan^{-1} x\,dx=\frac{\pi}{4}-\frac{1}{2}$$

Definite Integrals Ex 20.2 Q33

Let $\quad I = \int \frac{1-x^2}{x^4+x^2+1}dx = -\int \frac{x^2-1}{x^4+x^2+1}dx.$

Then,

$$I = -\int \frac{1-\frac{1}{x^2}}{x^2+1+\frac{1}{x^2}}dx \qquad \left[\begin{array}{l}\text{Dividing the numerator and}\\ \text{denominator by } x^2\end{array}\right]$$

$$\Rightarrow \quad I = -\int \frac{1-\frac{1}{x^2}}{\left(x+\frac{1}{x}\right)^2-1^2}dx$$

Let, $x+\frac{1}{x} = u$. Then, $d\left(x+\frac{1}{x}\right) = du \Rightarrow \left(1-\frac{1}{x^2}\right)dx = du$

$$\therefore \quad I = -\int \frac{du}{u^2-1^2}$$

$$\Rightarrow \quad I = -\frac{1}{2(1)}\log\left|\frac{u-1}{u+1}\right| + C$$

$$\Rightarrow \quad I = -\frac{1}{2}\log\left|\frac{x+\frac{1}{x}-1}{x+\frac{1}{x}+1}\right| + C = -\frac{1}{2}\log\left|\frac{x^2-x+1}{x^2+x+1}\right| + C$$

$$\therefore \quad \int_0^1 \frac{1-x^2}{x^4+x^2+1}dx = \left[-\frac{1}{2}\log\left|\frac{x^2-x+1}{x^2+x+1}\right|\right]_0^1 = \left(-\frac{1}{2}\log\left|\frac{1}{3}\right|\right) - \left(-\frac{1}{2}\log|1|\right) = \log\sqrt{3}$$

$$= \log 3^{\frac{1}{2}}$$

$$= \frac{1}{2}\log 3$$

Definite Integrals Ex 20.2 Q34

Let $1+x^2 = t$

Differentiating w.r.t. x, we get

$2x\,dx = dt$

Now, $x = 0 \Rightarrow t = 1$

$x = 1 \Rightarrow t = 2$

$$\int_0^1 \frac{24x^3}{\left(1+x^2\right)^4}dx = \int_1^2 \frac{12\left(t-1\right)}{t^4}dt$$

$$= 12\int_1^2\left(\frac{1}{t^3}-\frac{1}{t^4}\right)dt$$

$$= 12\left[-\frac{1}{2t^2}-\frac{1}{3t^3}\right]_1^2$$

$$= 12\left[-\frac{1}{8}+\frac{1}{24}+\frac{1}{2}-\frac{1}{3}\right]$$

$$= 12\left[\frac{-3+1+12-8}{24}\right]$$

$$= \frac{12\times 2}{24} = 1$$

$$\therefore \int_0^1 \frac{24x^3}{\left(1+x^2\right)^4}dx = 1$$

Definite Integrals Ex 20.2 Q35

Let $x - 4 = t^3$
Differentiating w.r.t. x, we get
$$dx = 3t^2 dt$$

Now, $x = 4 \Rightarrow t = 0$
$x = 12 \Rightarrow t = 2$

$$\therefore \int_4^{12} x(x-4)^{\frac{1}{3}} dx = \int_0^2 \left(t^3 + 1\right) t.3t^2 dt$$
$$= 3\int_0^2 \left(t^6 + 4t^3\right) dt$$
$$= 3\left[\frac{t^7}{7} + t^4\right]_0^2$$
$$= 3\left[\frac{128}{7} + 16\right]$$
$$= \frac{720}{7}$$

$$\therefore \int_4^{12} x(x-4)^{\frac{1}{3}} dx = \frac{720}{7}$$

Definite Integrals Ex 20.2 Q36

We have,
$$\int_0^{\frac{\pi}{2}} \underset{\mathrm{I}}{x^2} \underset{\mathrm{II}}{\sin x}\, dx$$

Using by parts, we get

$$x^2 \int \sin x dx - \int \left(\int \sin x dx\right) \frac{dx^2}{dx}.dx$$
$$= x^2 \cos x + \int \cos x.2x dx$$

Again applying by parts

$$= x^2 \cos x + 2\left[x \int \cos x dx - \int \left(\int \cos x dx\right). \frac{dx}{dx}.dx\right]$$
$$= x^2 \cos x + 2\left[x \sin x - \int \sin x dx\right]$$
$$= \left[x^2 \cos x + 2x \sin x + 2 \cos x\right]_0^{\frac{\pi}{2}}$$
$$= \pi + 0 - 0 - 0 - 2$$
$$= \pi - 2$$

$$\therefore \int_0^{\frac{\pi}{2}} x^2 \sin x dx = \pi - 2$$

Definite Integrals Ex 20.2 Q37

Let $x = \cos 2\theta$

Differentiating w.r.t. x, we get

$dx = -2\sin 2\theta d\theta$

Now, $x = 0 \Rightarrow \theta = \frac{\pi}{4}$

$x = 1 \Rightarrow \theta = 0$

$$\therefore \int_0^1 \sqrt{\frac{1-x}{1+x}}dx = \int_{\frac{\pi}{4}}^{0} \sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}\left(-2\sin 2\theta\right)d\theta$$

$$= \int_0^{\frac{\pi}{4}} \sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}\left(\ 2\sin 2\theta\right)d\theta \qquad \left[\because \sin 2\theta = 2\sin\theta\cos\theta;\ \text{and}\ \sin^2\theta = \frac{1-\cos 2\theta}{2}\right]$$

$$= 2\int_0^{\frac{\pi}{4}} \frac{\sin\theta}{\cos\theta}.\sin 2\theta d\theta$$

$$= 4\int_0^{\frac{\pi}{4}} \sin^2\theta d\theta$$

$$= 2\int_0^{\frac{\pi}{4}} \left(1-\cos 2\theta\right)d\theta$$

$$= 2\left[\theta - \frac{\sin^2\theta}{2}\right]_0^{\frac{\pi}{4}}$$

$$= 2\left[\frac{\pi}{4} - \frac{1}{2}\right]$$

$$= \frac{\pi}{2} - 1$$

$$\therefore \int_0^1 \sqrt{\frac{1-x}{1+x}}dx = \frac{\pi}{2} - 1$$

Definite Integrals Ex 20.2 Q38

We have,

$$\int_0^1 \frac{1-x^2}{\left(1+x^2\right)^2}dx = \int_0^1 \frac{-x^2\left(1-\frac{1}{x^2}\right)dx}{x^2\left(x+\frac{1}{x}\right)^2} = -\int_0^1 \frac{\left(1-\frac{1}{x^2}\right)dx}{\left(x+\frac{1}{x}\right)^2}$$

Let $x + \frac{1}{x} = t \Rightarrow 1 - \frac{1}{x^2}dx = dt$

When $x = 0 \Rightarrow t = \infty$

$x = 1 \Rightarrow t = 2$

$$\therefore \int_0^1 \frac{1-x^2}{\left(1+x^2\right)^2}dx = -\int_\infty^2 \frac{dt}{t^2} = \int_2^\infty \frac{dt}{t^2} = \left[-\frac{1}{t}\right]_2^\infty = \left(\frac{1}{2} - 0\right) = \frac{1}{2}$$

Definite Integrals Ex 20.2 Q39

Put $t = x^5 + 1$, then $dt = 5x^4\ dx$.

Therefore, $\int 5x^4\sqrt{x^5+1}\ dx = \int \sqrt{t}\ dt = \frac{2}{3}dt = \frac{2}{3}t^{\frac{3}{2}} = \frac{2}{3}\left(x^5+1\right)^{\frac{3}{2}}$

Hence, $\int_{-1}^{1} 5x^4\sqrt{x^4+1}\ dx = \frac{2}{3}\left[\left(x^5+1\right)^{\frac{3}{2}}\right]_{-1}^{1}$

$$= \frac{2}{3}\left[\left(1^5+1\right)^{\frac{3}{2}} - \left(\left(-1\right)^5+1\right)^{\frac{3}{2}}\right]$$

$$= \frac{2}{3}\left[2^{\frac{3}{2}} - 0^{\frac{3}{2}}\right] = \frac{2}{3}\left(2\sqrt{2}\right) = \frac{4\sqrt{2}}{3}$$

Alternatively, first we transform the integral and then evaluate the transformed integral with new limits.

Let $t = x^5 + 1$. Then $dt = 5x^4\ dx$.

Note that, when $x = -1$, $t = 0$ and when $x = 1$, $t = 2$

Thus, as x varies from -1 to 1, t varies from 0 to 2

Therefore $\int_{-1}^{1} 5x^4\sqrt{x^5+1}\ dx = \int_0^2 \sqrt{t}\ dt$

$$= \frac{2}{3}\left[t^{\frac{3}{2}}\right]_0^2 = \frac{2}{3}\left[2^{\frac{3}{2}} - 0^{\frac{3}{2}}\right] = \frac{2}{3}\left(2\sqrt{2}\right) = \frac{4\sqrt{2}}{3}$$

Definite Integrals Ex 20.2 Q40

$$I = \int_0^{\pi/2} \frac{\cos^2 x}{1+3\sin^2 x}\,dx$$

$$I = \int_0^{\pi/2} \frac{\sec^2 x}{\sec^2 x\left(\sec^2 x + 3\tan^2 x\right)}\,dx$$

Put $\tan x = t$

$\sec^2 x\,dx = dt$

$x = 0 \Rightarrow t = 0$ and $x = \frac{\pi}{2} \Rightarrow t = \infty$

$$I = \int_0^{\infty} \frac{1}{\left(1+t^2\right)\left(1+4t^2\right)}\,dt$$

$$I = -\frac{1}{3}\int_0^{\infty}\left[\frac{1}{\left(1+t^2\right)} - \frac{1}{\left(1+4t^2\right)}\right]dt$$

$$I = -\frac{1}{3}\left[\tan^{-1} t - 2\tan^{-1} 2t\right]_0^{\infty}$$

$$I = \frac{\pi}{6}$$

Definite Integrals Ex 20.2 Q41

Let $I = \int_0^{\frac{\pi}{4}} \sin^3 2t \cos 2t\,dt$. consider $\int \sin^3 2t \cos 2t\,dt$

Put $\sin 2t = u$ so that $2\cos 2t\,dt = du$ or $\cos 2t\,dt = \frac{1}{2}du$

So $\int \sin^3 2t \cos 2t\,dt = \frac{1}{2}\int u^3\,du$

$$= \frac{1}{8}\left[u^4\right] = \frac{1}{8}\sin^4 2t = F(t) \text{ say}$$

Therefore, by the second fundamental theorem of integrals calculus

$$I = F\left(\frac{\pi}{4}\right) - F(0) = \frac{1}{8}\left[\sin^4 \frac{\pi}{2} - \sin^4 0\right] = \frac{1}{8}$$

Definite Integrals Ex 20.2 Q42

Let $5 - 4\cos\theta = t$

Differentiating w.r.t. x, we get

$4\sin\theta d\theta = dt$

Now, $\theta = 0 \Rightarrow t = 1$

$\theta = \pi \Rightarrow t = 9$

$$\therefore \int_0^{\pi} 5\left(5-4\cos\theta\right)^{\frac{1}{4}}\sin\theta d\theta$$

$$= \frac{5}{4}\int_1^9 t^{\frac{1}{4}}dt$$

$$= \frac{5}{4}\left[\frac{4}{5}.t^{\frac{5}{4}}\right]_1^9$$

$$= 3^{\frac{5}{2}} - 1$$

$$= 9\sqrt{3} - 1$$

$$\therefore \int_0^{\pi} 5\left(5-4\cos\theta\right)^{\frac{1}{4}}\sin\theta d\theta = 9\sqrt{3} - 1$$

Definite Integrals Ex 20.2 Q43

We have,

$$\int_0^{\frac{\pi}{6}} \cos^{-3} 2\theta \sin 2\theta d\theta$$

$$= \int_0^{\frac{\pi}{6}} \frac{\sin 2\theta}{\cos^3 2\theta} d\theta$$

$$= \int_0^{\frac{\pi}{6}} \tan 2\theta . \sec^2 2\theta d\theta$$

Let $\tan 2\theta = t$

Differentiating w.r.t. x, we get

$$2\sec^2 2\theta d\theta = dt$$

Now, $\theta = 0 \Rightarrow t = 0$

$$\theta = \frac{\pi}{6} \Rightarrow t = \sqrt{3}$$

$$\therefore \int_0^{\frac{\pi}{6}} \tan 2\theta . \sec^2 2\theta d\theta = \frac{1}{2}\int_0^{\sqrt{3}} t dt = \frac{1}{2}\left[\frac{t^2}{2}\right]_0^{\sqrt{3}}$$

$$= \frac{3}{4}$$

$$\therefore \int_0^{\frac{\pi}{6}} \cos^{-3} 2\theta \sin 2\theta d\theta = \frac{3}{4}$$

Definite Integrals Ex 20.2 Q44

Let $x^{\frac{2}{3}} = t$

Differentiating w.r.t. x, we get

$$\frac{3}{2}\sqrt{x} dx = dt$$

Now, $x = 0 \Rightarrow t = 0$

$$x = \pi^{\frac{2}{3}} \Rightarrow t = \pi$$

$$\therefore \int_0^{\pi^{\frac{2}{3}}} \sqrt{x} \cos^2 x^{\frac{3}{2}} dx$$

$$= \frac{2}{3}\int_0^{\pi} \cos^2 t \, dt$$

$$= \frac{1}{3}\int_0^{\pi} 1 + \cos 2t \, dt \qquad \left[\because 2\cos^2 t = t + \cos 2t\right]$$

$$= \frac{1}{3}\left[t + \frac{\sin 2t}{t}\right]_0^{\pi}$$

$$= \frac{1}{3}\left[\pi + 0 - 0 - 0\right] = \frac{\pi}{3}$$

$$\therefore \int_0^{\pi^{\frac{2}{3}}} \sqrt{x} \cos^2 x^{\frac{3}{2}} dx = \frac{\pi}{3}$$

Definite Integrals Ex 20.2 Q45

Let $1+\log x = t$
Differentiating w.r.t. x, we get

$$\frac{1}{x}dx = dt$$

When $x = 1 \Rightarrow t = 1$
$x = 2 \Rightarrow t = 1+\log 2$

$$\therefore \int_1^2 \frac{dx}{x\left(1+\log x\right)^2}$$
$$= \int_1^{1+\log 2} \frac{dt}{t^2}$$
$$= \left[-\frac{1}{t}\right]_1^{1+\log 2}$$
$$= 1 - \frac{1}{1+\log 2}$$
$$= \frac{\log 2}{1+\log 2}$$

$$\therefore \int_1^2 \frac{dx}{x\left(1+\log x\right)^2} = \frac{\log 2}{1+\log 2}$$

Definite Integrals Ex 20.2 Q46

We have,

$$\int_0^{\frac{\pi}{2}} \cos^5 x\,dx \quad = \int_0^{\frac{\pi}{2}} \left(1-\sin^2 x\right)^2 \cos x\,dx$$

Let $\sin x = t$
Differentiating w.r.t. x, we get
$\cos x dx = dt$

When $x = 0 \Rightarrow t = 0$
$x = \frac{\pi}{2} \Rightarrow t = 1$

$$\int_0^{\frac{\pi}{2}} \left(1-\sin^2 x\right)^2 \cos x\,dx$$
$$= \int_0^1 \left(1-t^2\right)^2 dt$$
$$= \int_0^1 \left(1-2t^2+t^4\right)dt$$
$$= \left[t-\frac{2}{3}t^3+\frac{t^5}{5}\right]_0^1$$
$$= 1-\frac{2}{3}+\frac{1}{5}$$
$$= \frac{8}{15}$$

$$\therefore \int_0^{\frac{\pi}{2}} \cos^5 x\,dx = \frac{8}{15}$$

Definite Integrals Ex 20.2 Q47

Let $I = \int \frac{\sqrt{x}}{\left(30 - x^{\frac{3}{2}}\right)} dx$. We first find the anti derivative of the integrand.

Pu $30 - x^{\frac{3}{2}} = t$. Then $-\frac{3}{2}\sqrt{x}\, dx = dt$ or $\sqrt{x}\, dx = -\frac{2}{3} dt$

Thus, $\int \frac{\sqrt{x}}{\left(30 - x^{\frac{3}{2}}\right)^2} dx = -\frac{2}{3}\int \frac{dt}{t^2} = \frac{2}{3}\left[\frac{1}{t}\right] = \frac{2}{3}\left[\frac{1}{\left(30 - x^{\frac{3}{2}}\right)}\right] = f(x)$

Therefore, by the second fundamental theorem of calculus, we have

$$I = F(9) - F(4) = \frac{2}{3}\left[\frac{1}{\left(30 - x^{\frac{3}{2}}\right)}\right]_4^9$$

$$= \frac{2}{3}\left[\frac{1}{(30 - 27)} - \frac{1}{30 - 8}\right] = \frac{2}{3}\left[\frac{1}{3} - \frac{1}{22}\right] = \frac{19}{99}$$

Definite Integrals Ex 20.2 Q48

Let $\cos x = t$

Differentiating w.r.t. x, we get

$-\sin x dx = dt$

When $x = 0 \Rightarrow t = 1$

$x = \pi \Rightarrow t = -1$

Now,

$$\int_0^\pi \sin^3 x\,(1 + 2\cos x)(1 + \cos x)^2 dx$$

$$= \int_0^\pi \sin^2 x\,(1 + 2\cos x)(1 + \cos x)^2 . \sin x dx$$

$$= -\int_{-1}^{1} (1 - t^2)(1 + 2t)(1 + t)^2 dt \qquad \left[\sin^2 x = 1 - \cos^2 x\right]$$

$$= \int_{-1}^{1} (1 + 2t - t^2 - 2t^3)(1 + t^2 + 2t) dt$$

$$= \int_{-1}^{1} (1 - t^2 + 2t + 2t + 2t^3 + 4t^2 - t^2 - t^4 - 2t^3 - 2t^5 - 4t^4) dt$$

$$= \int_{-1}^{1} (1 + 4t + 4t^2 - 2t^3 - 5t^4 - 2t^5) dt$$

$$= \left[t + 2t^2 + \frac{4}{3}t^3 - \frac{t^4}{2} - t^5 - \frac{t6}{3}\right]_{-1}^{1}$$

$$= \left[2 + 0 + \frac{8}{3} - 0 - 2 - 0\right] = \frac{8}{3}$$

$$\therefore \int_0^\pi \sin^3 x\,(1 + 2\cos x)(1 + \cos x)^2 dx = \frac{8}{3}$$

Definite Integrals Ex 20.2 Q49

$$I = \int_0^{\frac{\pi}{2}} 2\sin x\cos x\tan^{-1}(\sin x)dx$$

Let $t = \sin x$

$dt = \cos x dx$

$x = 0, t = 0$

$x = \frac{\pi}{2}, t = 1$

$$I = \int_0^1 2t\tan^{-1}(t)dt$$

$$= 2\left[\frac{1}{2}t^2\tan^{-1}t - \frac{t}{2} + \frac{1}{2}\tan^{-1}t\right]_0^1$$

$$= 2\left[\frac{\pi}{4} - \frac{1}{2}\right]$$

$$= \frac{\pi}{2} - 1$$

$$\therefore I = \int_0^{\frac{\pi}{2}} 2\sin x\cos x\tan^{-1}(\sin x)dx = \frac{\pi}{2} - 1$$

Definite Integrals Ex 20.2 Q50

Let $\sin x = t$

Differentiating w.r.t. x, we get

$\cos x\, dx = dt$

Now,

$x = 0 \Rightarrow t = 0$

$x = \frac{\pi}{2} \Rightarrow t = 1$

$$\therefore \int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x)dx = 2\int_0^1 t\tan^{-1}t\,dt \qquad [\because \sin 2x = 2\sin x\cos x]$$

Using by parts

$$= 2\left\{\tan^{-1}t\int t\,dt - \int\left(\int t\,dt\right)\frac{d\tan^{-1}t}{dt}dt\right\}$$

$$= 2\left\{\frac{t^2}{2}\tan^{-1} - \frac{1}{2}\int\frac{t^2}{1+t^2}dt\right\}$$

$$= 2\left\{\frac{t^2}{2}\tan^{-1}t - \frac{1}{2}\left(\int dt - \int\frac{dt}{1+t^2}\right)\right\}$$

$$= 2\left[\frac{t^2}{2}\tan^{-1}t - \frac{1}{2}\left(t - \tan^{-1}t\right)\right]_0^1$$

$$= 2\left\{\frac{1}{2}\frac{\pi}{4} - \frac{1}{2}\left(1 - \frac{\pi}{4}\right)\right\}$$

$$= 2\left\{\frac{\pi}{8} - \frac{1}{2} + \frac{\pi}{8}\right\}$$

$$= 2\left(\frac{\pi}{4} - \frac{1}{2}\right) = \frac{\pi}{2} - 1$$

$$\therefore \int_0^{\frac{\pi}{2}} \sin 2x\tan^{-1}(\sin x)dx = \frac{\pi}{2} - 1$$

Definite Integrals Ex 20.2 Q51

We have,

$$\int_0^1 \left(\cos^{-1} x\right)^2 dx = \left(\cos^{-1} x\right)^2 \int_0^1 dx - \int_0^1 \left(\int dx\right) \frac{d\left(\cos^{-1} x\right)^2}{dx} dx$$

$$= \left[x\left(\cos^{-1} x\right)^2\right]_0^1 + \int_0^1 \frac{x.2\cos^{-1}}{\sqrt{1-x^2}} dx$$

Now,

Let $\cos^{-1} x = t \Rightarrow -\frac{1}{\sqrt{1-x^2}} dx = dt$

When $x = 0 \Rightarrow t = \frac{\pi}{2}$

$x = 1 \Rightarrow t = 0$

$$\therefore \int_0^1 \frac{2x\cos^{-1} x}{\sqrt{1-x^2}} dx = -2\int_{\frac{\pi}{2}}^0 t\cos t\, dt = 2\int_0^{\frac{\pi}{2}} t\cos t\, dt$$

$$= 2\left[t\int \cos t\, dt - \int \left(\cos t\, dt\right)\frac{dt}{dt}.dt\right]_0^{\frac{\pi}{2}}$$

$$= 2\left[t\sin t - \int \sin t\, dt\right]_0^{\frac{\pi}{2}}$$

$$= 2\left[t\sin t + \cos t\right]_0^{\frac{\pi}{2}}$$

$$= 2\left[\frac{\pi}{2} - 1\right]$$

$$\int_0^1 \left(\cos^{-1} x\right)^2 dx = \left[x\left(\cos^{-1} x\right)^2\right]_0^1 + \int_0^1 \frac{x.2\cos^{-1}}{\sqrt{1-x^2}} dx = \left[x\left(\cos^{-1} x\right)^2\right]_0^1 + 2\left(\frac{\pi}{2} - 1\right)$$

$$= 0 - 0 + 2\left(\frac{\pi}{2} - 1\right)$$

$$= (\pi - 2)$$

$$\therefore \int_0^1 \left(\cos^{-1} x\right)^2 dx = (\pi - 2)$$

Definite Integrals Ex 20.2 Q53

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sqrt{1+\cos x}}{(1-\cos x)^{\frac{3}{2}}} dx$$

$$= \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sqrt{2\cos^2 \frac{x}{2}}}{\left(2\sin^2 \frac{x}{2}\right)^{\frac{3}{2}}} dx \qquad \left[\begin{array}{l} \because 1+\cos x = 2\cos^2 \frac{x}{2} \\ 1-\cos x = 2\sin^2 \frac{x}{2} \end{array}\right]$$

$$= \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sqrt{2}\cos \frac{x}{2}}{2\sqrt{2}\sin^3 \frac{x}{2}} dx$$

$$= \frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cot \frac{x}{2} \operatorname{cosec}^2 \frac{x}{2} dx \qquad \left[\begin{array}{l} \because \operatorname{cosec}^2 \frac{x}{2} = \frac{1}{\sin^2 \frac{x}{2}} \\ \cot \frac{x}{2} = \frac{\cos \frac{x}{2}}{\sin \frac{x}{2}} \end{array}\right]$$

Let $\cot \frac{x}{2} = t$

Differentiating w.r.t. x, we get

$$\frac{-1}{2}\operatorname{cosec}^2 \frac{x}{2} = dt$$

Now, $x = \frac{\pi}{3} \Rightarrow t = \sqrt{3}$

$x = \frac{\pi}{2} \Rightarrow t = 1$

$$\therefore \frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cot \frac{x}{2} \operatorname{cosec}^2 \frac{x}{2} dx = -\int_{\sqrt{3}}^{1} t\,dt = -\left[\frac{t^2}{2}\right]_{\sqrt{3}}^{1} = \frac{-1}{2}[1-3]$$

$= 1$

$$\therefore \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sqrt{1+\cos x}}{(1-\cos x)^{\frac{3}{2}}} dx = 1$$

Definite Integrals Ex 20.2 Q54

Substitute $x^2 = a^2\cos 2\theta$

Differentiating w.r.t. x, we get

$$2x\,dx = -2a^2\sin 2\theta\,d\theta$$

Now, $x = 0 \Rightarrow \theta = \frac{\pi}{4}$

$x = a \Rightarrow \theta = 0$

$$\therefore \int_0^a x\sqrt{\frac{a^2-x^2}{a^2+x^2}}dx = \int_{\frac{\pi}{4}}^{0} \sqrt{\frac{a^2(1-\cos 2\theta)}{a^2-(1-\cos 2\theta)}}\left(-a^2\sin 2\theta\right)d\theta$$

$$= -a^2\int_{\frac{\pi}{4}}^{0} \frac{\sin\theta}{\cos\theta}\sin 2\theta\,d\theta$$

$$= a^2\int_0^{\frac{\pi}{4}} 2\sin^2\theta\,d\theta$$

$$= a^2\int_0^{\frac{\pi}{4}} (1-\cos 2\theta)\,d\theta$$

$$= a^2\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{4}}$$

$$= a^2\left[\frac{\pi}{4} - \frac{1}{2}\right]$$

$$\therefore \int_0^a x\sqrt{\frac{a^2-x^2}{a^2+x^2}}dx = a^2\left[\frac{\pi}{4} - \frac{1}{2}\right]$$

Definite Integrals Ex 20.2 Q55

Let $x = a\cos 2\theta$

Differentiating w.r.t. x, we get

$dx = -2a\sin 2\theta$

Now, $x = -a \Rightarrow \theta = \frac{\pi}{2}$

$x = a \Rightarrow \theta = 0$

$$\therefore \int_{-a}^{a}\sqrt{\frac{a-x}{a+x}}dx = \int_{\frac{\pi}{2}}^{0}\sqrt{\frac{a(1-\cos 2\theta)}{a(1+\cos 2\theta)}}(-2\sin 2\theta)d\theta$$

$$= 2a\int_{0}^{\frac{\pi}{2}}\frac{\sin\theta}{\cos\theta}.\sin 2\theta d\theta \qquad \left[\begin{array}{l}\because 1-\cos 2\theta = 2\sin^2\theta \\ 1+\cos 2\theta = 2\cos^2\theta \\ -\int_a^b f(x)dx = \int_b^a f(x)dx\end{array}\right]$$

$$= 2a\int_{0}^{\frac{\pi}{2}}\frac{\sin\theta.2\sin\theta\cos\theta}{\cos\theta}$$

$$= 4a\int_{0}^{\frac{\pi}{2}}s\ in^2\theta d\theta$$

$$= 2a\int_{0}^{\frac{\pi}{2}}(1-\cos 2\theta)d\theta$$

$$= 2a\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{2}}$$

$$= 2a\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{2}}$$

$$= 2a\left[\frac{\pi}{2} - 0 - 0 + 0\right] = \pi a$$

$$\therefore \int_{-a}^{a}\sqrt{\frac{a-x}{a+x}}dx = \pi a$$

Definite Integrals Ex 20.2 Q56

Let $\cos x = t$

Differentiating w.r.t. x, we get

$-\sin x dx = dt$

Now, $x = 0 \Rightarrow t = 1$

$x = \frac{\pi}{2} \Rightarrow t = 0$

$$\therefore \int_{0}^{\frac{\pi}{2}}\frac{\sin x\cos x dx}{\cos^2 x + 3\cos x + 2}$$

$$= -\int_{1}^{0}\frac{tdt}{t^2+3t+2}$$

$$= \int_{0}^{1}\frac{tdt}{(t+2)(t+1)} \qquad \left[\because -\int_a^b f(x) = \int_b^a f(x)\right]$$

$$= \int_{0}^{1}\left(-\frac{1}{t+1}+\frac{2}{t+2}\right)dt \qquad [\text{Applying partial fraction}]$$

$$= \left[-\log|1+t| + 2\log|t+2|\right]_0^1$$

$$= -\log 2 + 2\log 3 + 0 - 2\log 2$$

$$= 2\log 3 - 3\log 2$$

$$= \log\frac{9}{8}$$

$$\therefore \int_{0}^{\frac{\pi}{2}}\frac{\sin x\cos x dx}{\cos^2 x + 3\cos x + 2} = \log\frac{9}{8}$$

Definite Integrals Ex 20.2 Q57

$$I = \int_0^{\pi/2} \frac{\tan x}{1 + m^2 \tan^2 x} dx$$

$$I = \int_0^{\pi/2} \frac{\sin x \cos x}{\cos^2 x + m^2 \sin^2 x} dx$$

Put $\sin^2 x = t$ then $2\sin x\cos x \, dx = dt$

$x = 0 \Rightarrow t = 0$ and $x = \frac{\pi}{2} \Rightarrow t = 1$

$$I = \frac{1}{2}\int_0^1 \frac{1}{(1-t) + m^2 t} dt$$

$$I = \frac{1}{2}\int_0^1 \frac{1}{\left(m^2 - 1\right)t + 1} dt$$

$$I = \frac{1}{2}\left[\frac{1}{m^2 - 1}\log\left|\left(m^2 - 1\right)t + 1\right|\right]_0^1$$

$$I = \frac{1}{2}\left[\frac{1}{m^2 - 1}\log\left|m^2\right| - \frac{1}{m^2 - 1}\ln|1|\right]$$

$$I = \frac{1}{2}\left[\frac{\log\left|m^2\right|}{m^2 - 1}\right]$$

$$I = \frac{1}{2}\left[\frac{2\log|m|}{m^2 - 1}\right]$$

$$I = \frac{\log|m|}{m^2 - 1}$$

Definite Integrals Ex 20.2 Q58

$$I = \int_0^{1/2} \frac{1}{\left(1 + x^2\right)\sqrt{1 - x^2}} dx$$

Let $x = \sin u$

$dx = \cos u \, du$

$$I = \int_0^{\pi/6} \frac{1}{\left(1 + \sin^2 u\right)} du$$

$$I = \int_0^{\pi/6} \frac{\sec^2 u}{\left(1 + 2\tan^2 u\right)} du$$

Let $\tan u = v$

$dv = \sec^2 u \, du$

$$I = \int_0^{1/\sqrt{3}} \frac{1}{\left(1 + 2v^2\right)} dv$$

$$I = \frac{1}{\sqrt{2}}\left[\tan^{-1}\left(\sqrt{2}v\right)\right]_0^{1/\sqrt{3}}$$

$$I = \frac{1}{\sqrt{2}}\left[\tan^{-1}\left(\sqrt{\frac{2}{3}}\right)\right]$$

Definite Integrals Ex 20.2 Q59

$$I = \int_{1/3}^{1} \frac{\left(x - x^3\right)^{1/3}}{x^4} dx$$

$$I = \int_{1/3}^{1} \frac{\left(\frac{1}{x^2} - 1\right)^{1/3}}{x^3} dx$$

Let $\frac{1}{x^2} - 1 = t$

$$\frac{-2}{x^3} dx = dt$$

$x = \frac{1}{3} \Rightarrow t = 8$ and $x = 1 \Rightarrow t = 0$

$$I = -\frac{1}{2}\int_{8}^{0} (t)^{1/3} dt$$

$$I = -\frac{1}{2}\left[\frac{t^{4/3}}{4/3}\right]_8^0$$

$$I = -\frac{1}{2}[0 - 12]$$

$$I = 6$$

Definite Integrals Ex 20.2 Q60

$$\int \sec^2 x \frac{\tan^2 x}{\tan^6 x + 2\tan^3 x + 1} dx$$

$$u = \tan x \to \frac{du}{dx} = \sec^2 x$$

$$\int \frac{u^2}{u^6 + 2u^3 + 1} du$$

$$v = u^3 \to \frac{dv}{du} = 3u^2$$

$$\frac{1}{3}\int \frac{1}{v^2 + 2v + 1} dv$$

$$\frac{1}{3}\int \frac{1}{(v+1)^2} dv$$

$$-\frac{1}{3(v+1)}$$

$$-\frac{1}{3(u^3+1)}$$

$$-\frac{1}{3(\tan^3 x + 1)}$$

$$\left\{-\frac{1}{3(\tan^3 x + 1)}\right\}_0^{\frac{\pi}{4}}$$

$$\left\{-\frac{1}{6} + \frac{1}{3}\right\}$$

$$\frac{1}{6}$$

Definite Integrals Ex 20.2 Q61

$$\int_0^{\frac{\pi}{2}} \sqrt{\cos x(1-\cos^2 x)}\tan^2 x\cos^2 x dx$$

$$\int_0^{\frac{\pi}{2}} \sqrt{\cos x\sin^2 x}\sin^2 x dx$$

$$\int_0^{\frac{\pi}{2}} \sqrt{\cos x}\sin^3 x dx$$

$$\cos x = t \rightarrow -\sin x = \frac{dt}{dx}$$

$$-\int_1^0 \sqrt{t}(1-t^2)dt$$

$$\int_0^1 (\sqrt{t}-t^{\frac{5}{2}})dt$$

$$\left\{\frac{2t^{\frac{3}{2}}}{3}-\frac{2t^{\frac{7}{2}}}{7}\right\}_0^1$$

$$\frac{2}{3}-\frac{2}{7}$$

$$\frac{8}{21}$$

Definite Integrals Ex 20.2 Q62

$$I = \int_0^{\pi/2} \frac{\cos x}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^n} dx$$

$$I = \int_0^{\pi/2} \frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^n} dx$$

$$I = \int_0^{\pi/2} \frac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^{n-1}} dx$$

Let $\cos\frac{x}{2}+\sin\frac{x}{2} = t$

$$\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)dx = 2dt$$

$x = 0 \Rightarrow t = 1$ and $x = \frac{\pi}{2} \Rightarrow t = \sqrt{2}$

$$I = \int_1^{\sqrt{2}} \frac{2}{(t)^{n-1}} dt$$

$$I = \left[\frac{2t^{-n+2}}{-n+2}\right]_1^{\sqrt{2}}$$

$$I = \frac{2}{2-n}\left[\left(\sqrt{2}\right)^{2-n}-1\right]$$

$$I = \frac{2}{2-n}\left[2^{1-\frac{n}{2}}-1\right]$$

Ex 20.3

Definite Integrals Ex 20.3 Q1(i)

We have,

$$\int_1^4 f(x)\,dx$$

$$= \int_1^2 (4x+3)\,dx + \int_2^4 (3x+5)\,dx$$

$$= \left[\frac{4x^2}{2}+3x\right]_1^2 + \left[\frac{3x^2}{2}+5x\right]_2^4$$

$$= \left[\left(\frac{16}{2}+6\right)-\left(\frac{4}{2}+3\right)\right] + \left[\left(\frac{48}{2}+20\right)-\left(\frac{12}{2}+10\right)\right]$$

$$= \left[(14-5)\right] + \left[(44-16)\right]$$

$$= 9+28$$

$$= 37$$

Definite Integrals Ex 20.3 Q1(ii)

We have,

$$\int_0^9 f(x)\,dx$$

$$= \int_0^{\frac{\pi}{2}} \sin x\,dx + \int_{\frac{\pi}{2}}^3 1\,dx + \int_3^9 e^{x-3}\,dx$$

$$= \left[-\cos x\right]_0^{\frac{\pi}{2}} + \left[x\right]_{\frac{\pi}{2}}^3 + \left[e^{x-3}\right]_3^9$$

$$= \left[-\cos\frac{\pi}{2}+\cos 0\right] + \left[3-\frac{\pi}{2}\right] + \left[e^{9-2}-e^{3-3}\right]$$

$$= \left[0+1\right] + \left[3-\frac{\pi}{2}\right] + \left[e^6-e^0\right]$$

$$= 0+1+3-\frac{\pi}{2}+e^6-e^0$$

$$= 1+3-\frac{\pi}{2}+e^6-1$$

$$= 3-\frac{\pi}{2}+e^6$$

Definite Integrals Ex 20.3 Q1(iii)

We have,

$$\int_1^4 f(x)\,dx$$

$$= \int_1^3 (7x+3)\,dx + \int_3^4 8x\,dx$$

$$= \left[\frac{7x^2}{2}+3x\right]_1^3 + \left[\frac{8x^2}{2}\right]_3^4$$

$$= \left[\left(\frac{7\times 9}{2}+3\times 3\right)-\left(\frac{7\times 1}{2}+3\times 1\right)\right]+\left[\left(\frac{8\times 16}{2}-\frac{8\times 9}{2}\right)\right]$$

$$= \left[\frac{63}{2}+9-\frac{7}{2}-3\right]+\left[64-36\right]$$

$$= 34+28$$

$$= 62$$

Definite Integrals Ex 20.3 Q2

We have,

$$\int_{-4}^4 |x+2|\,dx$$

$$= \int_{-4}^{-2} -(x+2)\,dx + \int_{-2}^{4}(x+2)\,dx$$

$$= -\left[\frac{x^2}{2}+2x\right]_{-4}^{-2} + \left[\frac{x^2}{2}+2x\right]_{-2}^{4}$$

$$= -\left[\left(\frac{4}{2}-4\right)-\left(\frac{16}{2}-8\right)\right]+\left[\left(\frac{16}{2}+8\right)-\left(\frac{4}{2}-4\right)\right]$$

$$= -\left[(-2)-(0)\right]+\left[(16)-(-2)\right]$$

$$= -\left[-2\right]+\left[16+2\right]$$

$$= 2-18$$

$$= 20$$

$$\therefore \int_{-4}^{4} |x+2|\,dx = 20$$

Definite Integrals Ex 20.3 Q3

We have,

$$\int_{-3}^{3} |x+1|\,dx$$

$$= \int_{-3}^{-1} -(x+1)\,dx + \int_{-1}^{3} (x+1)\,dx$$

$$= -\left[\frac{x^2}{2}+x\right]_{-3}^{-1} + \left[\frac{x^2}{2}+x\right]_{-1}^{3}$$

$$= -\left[\left(\frac{1}{2}-1\right)-\left(\frac{9}{2}-3\right)\right]+\left[\left(\frac{9}{2}+3\right)-\left(\frac{1}{2}-1\right)\right]$$

$$= -\left[\left(-\frac{1}{2}\right)-\left(1\frac{1}{2}\right)\right]+\left[\left(7\frac{1}{2}\right)-\left(-\frac{1}{2}\right)\right]$$

$$= -\left[-\frac{1}{2}-1\frac{1}{2}\right]+\left[7\frac{1}{2}+\frac{1}{2}\right]$$

$$= [-2]+[8]$$

$$= 2+8$$

$$= 10$$

$$\therefore \int_{-3}^{3} |x+1|\,dx = 10$$

Definite Integrals Ex 20.3 Q4

We have,

$$\int_{-1}^{1} |2x+1|\,dx$$

$$= \int_{-1}^{-\frac{1}{2}} -(2x+1)\,dx + \int_{-\frac{1}{2}}^{1} (2x+1)\,dx$$

$$= -\left[\frac{2x^2}{2}+x\right]_{-1}^{-\frac{1}{2}} + \left[\frac{2x^2}{2}+x\right]_{-\frac{1}{2}}^{1}$$

$$= -\left[\left(\frac{2}{8}-\frac{1}{2}\right)-\left(\frac{2}{2}-1\right)\right]+\left[\left(\frac{2}{2}+1\right)-\left(\frac{2}{8}-\frac{1}{2}\right)\right]$$

$$= -\left[\left(\frac{1}{4}-\frac{1}{2}\right)-(1-1)\right]+\left[(1+1)-\left(\frac{1}{4}-\frac{1}{2}\right)\right]$$

$$= -\left[-\frac{1}{4}\right]+\left[2+\frac{1}{4}\right]$$

$$= \frac{1}{4}+2+\frac{1}{4}$$

$$= 2\frac{1}{2}$$

$$\therefore \int_{-1}^{1} |2x+1|\,dx = \frac{5}{2}$$

Definite Integrals Ex 20.3 Q5

(i)

$$\int_{-2}^{2}|2x+3|dx$$

$$= \int_{-2}^{-\frac{3}{2}} -(2x+3)\,dx + \int_{-\frac{3}{2}}^{2}(2x+3)\,dx$$

$$= -\left[\frac{2x^2}{2}+3x\right]_{-2}^{-\frac{3}{2}} + \left[\frac{2x^2}{2}+3x\right]_{-\frac{3}{2}}^{2}$$

$$= -\left[\left(\frac{2\times 9}{2\times 4}-\frac{9}{2}\right)-\left(\frac{2\times 4}{2}-6\right)\right]+\left[\left(\frac{2\times 4}{2}+6\right)-\left(\frac{2\times 9}{2\times 4}-\frac{9}{2}\right)\right]$$

$$= -\left[\left(\frac{18}{8}-\frac{9}{2}\right)-\left(\frac{8}{2}-6\right)\right]+\left[\left(\frac{8}{2}+6\right)-\left(\frac{18}{8}-\frac{9}{2}\right)\right]$$

$$= -\left[\left(\frac{9}{4}-\frac{9}{2}\right)-(-2)\right]+\left[(10)-\left(\frac{9}{4}-\frac{9}{2}\right)\right]$$

$$= \left[-\frac{9}{4}+2\right]+\left[10+\frac{9}{4}\right]$$

$$= \frac{9}{4}-2+10+\frac{9}{4}$$

$$\Rightarrow 8\frac{9}{2}$$

$$= 12\frac{1}{2}$$

$$\therefore \int_{-2}^{2}|2x+3|dx = \frac{25}{2}$$

Definite Integrals Ex 20.3 Q6

(ii)

We have,

$$\begin{aligned} f(x) &= |x^2-3x+2| \\ &= |(x-1)(x-2)| \\ &= \begin{cases} x^2-3x+2 & 0 \le x \le 1 \\ -(x^2-3x+2) & 1 \le x \le 2 \end{cases} \end{aligned}$$

Hence,

$$\int_{0}^{2}|x^2-3x+2|dx$$

$$= \int_{0}^{1}(x^2-3x+2)\,dx + \int_{1}^{2} -(x^2-3x+2)\,dx$$

$$= \left[\frac{x^3}{3}-\frac{3x^2}{2}+2x\right]_0^1 - \left[\frac{x^3}{3}-\frac{3x^2}{2}+2x\right]_1^2$$

$$= \left[\frac{1}{3}-\frac{3}{2}+2-0\right]-\left[\frac{8}{3}-\frac{12}{2}+4-\frac{1}{3}+\frac{3}{2}+2\right]$$

$$= \left[\frac{1}{6}\right]-\left[-\frac{5}{6}\right]$$

$$= \frac{1}{6}+\frac{5}{6}$$

$$= 1$$

$$\therefore \int_{0}^{2}|x^2-3x+2|dx = 1$$

Definite Integrals Ex 20.3 Q7

$$\int_0^3 |3x-1|\,dx = \int_0^{\frac{1}{3}} -(3x-1)\,dx + \int_{\frac{1}{3}}^3 (3x-1)\,dx$$

$$= -\left[\frac{3x^2}{2} - x\right]_0^{\frac{1}{3}} + \left[\frac{3x^2}{2} - x\right]_{\frac{1}{3}}^3$$

$$= -\left[\left(\frac{3}{9\times 2} - \frac{1}{3}\right) - (0)\right] + \left[\left(\frac{3\times 9}{2} - 3\right) - \left(\frac{3}{9\times 2} - \frac{1}{3}\right)\right]$$

$$= -\left[\left(\frac{1}{6} - \frac{1}{3}\right)\right] + \left[\left(\frac{27}{2} - 3\right) - \left(\frac{1}{6} - \frac{1}{3}\right)\right]$$

$$= -\left[\left(-\frac{1}{6}\right)\right] + \left[\left(10\frac{1}{2}\right) - \left(\frac{-1}{6}\right)\right]$$

$$= -\left[\left(-\frac{1}{6}\right)\right] + \left[10\frac{1}{2} + \frac{1}{6}\right]$$

$$= \frac{1}{6} + 10\frac{1}{2} + \frac{1}{6}$$

$$= \frac{1}{3} + \frac{21}{2} = \frac{2+63}{6} = \frac{65}{6}$$

$$= \frac{65}{6}$$

$$\therefore \int_0^3 |3x-1|\,dx = \frac{65}{6}$$

Definite Integrals Ex 20.3 Q8

$$\int_{-6}^{6} |x+2|\,dx$$

$$= \int_{-6}^{-2} -(x+2)\,dx + \int_{-2}^{6} (x+2)\,dx$$

$$= -\left[\frac{x^2}{2} + 2x\right]_{-6}^{-2} + \left[\frac{x^2}{2} + 2x\right]_{-2}^{6}$$

$$= -\left[\left(\frac{4}{2} + 2(\cdot 2)\right) - \left(\frac{36}{2} - 12\right)\right] + \left[\left(\frac{36}{2} + 12\right) - \left(\frac{4}{2} - 4\right)\right]$$

$$= -\left[(2-4) - (18-12)\right] + \left[(18+12) - (2-4)\right]$$

$$= -[-8] + [30+2]$$

$$= 8 + 32$$

$$= 40$$

$$\therefore \int_{-6}^{6} |x+2|\,dx = 40$$

Definite Integrals Ex 20.3 Q9

$$\int_{-2}^{2} |x+1|\,dx = \int_{-2}^{-1} -(x+1)\,dx + \int_{-1}^{2} (x+1)\,dx$$

$$= -\left[\frac{x^2}{2} + x\right]_{-2}^{-1} + \left[\frac{x^2}{2} + x\right]_{-1}^{2}$$

$$= -\left[\left(\frac{1}{2} - 1\right) - \left(\frac{4}{2} - 2\right)\right] + \left[\left(\frac{4}{2} + 2\right) - \left(\frac{1}{2} - 1\right)\right]$$

$$= -\left[\left(-\frac{1}{2}\right) - 0\right] + \left[4 + \frac{1}{2}\right]$$

$$= \frac{1}{2} + 4\frac{1}{2}$$

$$= 5$$

$$\therefore \int_{-2}^{2} |x+1|\,dx = 5$$

Definite Integrals Ex 20.3 Q10

$$\int_1^2 |x-3|dx = \int_1^2 -(x-3)\,dx \qquad [x-3<0 \text{ for } 1>x>2]$$

$$= -\left[\frac{x^2}{2}-3x\right]_1^2$$

$$= -\left[\left(\frac{4}{2}-6\right)-\left(\frac{1}{2}-3\right)\right]$$

$$= -\left[(-4)-\left(-2\frac{1}{2}\right)\right]$$

$$= -\left[-4+2\frac{1}{2}\right]$$

$$= -\left[-\frac{3}{2}\right]$$

$$= \frac{3}{2}$$

$$\therefore \int_1^2 |x-3|dx = \frac{3}{2}$$

Definite Integrals Ex 20.3 Q11

$$\int_0^{\frac{\pi}{2}} |\cos 2x|\,dx$$

$$= \int_0^{\frac{\pi}{4}} -\cos 2x\,dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} +\cos 2x\,dx$$

$$= \left[\frac{+\sin 2x}{2}\right]_0^{\frac{\pi}{4}} + \left[\frac{-\sin 2x}{2}\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= \frac{1}{2}\left[\sin\frac{\pi}{2}-\sin 0\right]+\frac{1}{2}\left[\sin\pi+\sin\frac{\pi}{2}\right]$$

$$= \frac{1}{2}[1]+\frac{1}{2}[1]$$

$$= \frac{1}{2}+\frac{1}{2}$$

$$= 1$$

$$\therefore \int_0^{\frac{\pi}{2}} |\cos 2x|\,dx = 1$$

Definite Integrals Ex 20.3 Q12

$$\int_0^{2\pi} |\sin x|dx = \int_0^{\pi} \sin x dx + \int_{\pi}^{2\pi} -\sin x dx$$

$$= [-\cos x]_0^{\pi} + [\cos x]_{\pi}^{2\pi}$$

$$= [1+1]+[1+1]$$

$$\int_0^{2\pi} |\sin x|dx = 4$$

Definite Integrals Ex 20.3 Q13

$$\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} |\sin x|\,dx$$

$$= \int_{\frac{-\pi}{4}}^{0} -\sin x\,dx + \int_0^{\frac{\pi}{4}} \sin x\,dx$$

$$= [\cos x]_{\frac{-\pi}{4}}^{0} + [-\cos x]_0^{\frac{\pi}{4}}$$

$$= \left(1-\frac{1}{\sqrt{2}}\right)-\left(\frac{1}{\sqrt{2}}-1\right)$$

$$= \left(2-\sqrt{2}\right)$$

$$\therefore \int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} |\sin x|\,dx = 2-\sqrt{2}$$

Definite Integrals Ex 20.3 Q14

We have,

$$I = \int_2^8 |x - 5| dx$$

We have,

$$|x - 5| = \begin{cases} x - 5 & \text{if } x \in (5, 8) \\ -(x - 5) & \text{if } x \in (2, 5) \end{cases}$$

Hence,

$$I = \int_2^5 -(x - 5) dx + \int_5^8 (x - 5) dx$$

$$= -\left[\frac{x^2}{2} - 5x\right]_2^5 + \left[\frac{x^2}{2} - 5x\right]_5^8$$

$$= -\left[\left(\frac{25}{2} - 25\right) - \left(\frac{4}{2} - 10\right)\right] + \left[\left(\frac{64}{2} - 40\right) - \left(\frac{25}{2} - 25\right)\right]$$

$$= -\left[-\frac{25}{2} + 8\right] + \left[(-8) + \left(\frac{25}{2}\right)\right]$$

$$= \frac{25}{2} - 8 - 8 + \frac{25}{2}$$

$$= 25 - 16 = 9$$

$$\therefore \int_2^8 |x - 5| dx = 9$$

Definite Integrals Ex 20.3 Q15

We have,

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \{\sin|x| + \cos|x|\} dx$$

Let $f(x) = \sin|x| + \cos|x|$

Then, $f(x) = f(-x)$

$\therefore$ f(x) is an even function.

So, $$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \{\sin|x| + \cos|x|\} dx = 2\int_0^{\frac{\pi}{2}} (\sin x + \cos x) dx = 2[-\cos x + \sin x]_0^{\frac{\pi}{2}} = 4$$

$$\therefore \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \{\sin|x| + \cos|x|\} dx = 4$$

Definite Integrals Ex 20.3 Q16

$$I = \int_0^4 |x - 1| dx$$

It can be seen that, $(x - 1) \leq 0$ when $0 \leq x \leq 1$ and $(x - 1) \geq 0$ when $1 \leq x \leq 4$

$$I = \int_0^1 |x-1| dx + \int_1^4 |x-1| dx \qquad \left(\int_a^b f(x) = \int_a^c f(x) + \int_c^b f(x)\right)$$

$$= \int_0^1 -(x-1) dx + \int_1^4 (x-1) dx$$

$$= \left[x - \frac{x^2}{2}\right]_0^1 + \left[\frac{x^2}{2} - x\right]_1^4$$

$$= 1 - \frac{1}{2} + \frac{(4)^2}{2} - 4 - \frac{1}{2} + 1$$

$$= 1 - \frac{1}{2} + 8 - 4 - \frac{1}{2} + 1$$

$$= 5$$

Definite Integrals Ex 20.3 Q17

$$Let\ I=\int_1^4\{|x-1|+|x-2|+|x-4|\}dx$$

$$=\int_1^2\{(x-1)-(x-2)-(x-4)\}dx+\int_2^4\{(x-1)+(x-2)-(x-4)\}dx$$

$$=\int_1^2\{(x-1-x+2-x+4)\}dx+\int_2^4\{(x-1+x-2-x+4)\}dx$$

$$=\int_1^2(5-x)dx+\int_2^4(x+1)dx$$

$$=\left[5x-\frac{x^2}{2}\right]_1^2+\left[\frac{x^2}{2}+x\right]_2^4$$

$$=\left[10-2-5+\frac{1}{2}\right]+[8+4-2-2]$$

$$=\frac{7}{2}+8$$

$$I=\frac{23}{2}$$

Definite Integrals Ex 20.3 Q18

We have,

$$I=\int_{-5}^{0}(|x|+|x+2|+|x+5|)dx=\int_{-5}^{0}|x|dx+\int_{-5}^{0}|x+2|dx+\int_{-5}^{0}|x+5|dx$$

$$\Rightarrow I=\int_{-5}^{0}-x\,dx+\int_{-5}^{-2}-(x+2)dx+\int_{-2}^{0}(x+2)dx+\int_{-5}^{0}(x+5)dx$$

$$=\left[\frac{-x^2}{2}\right]_{-5}^{0}+\left[\frac{-x^2}{2}-2x\right]_{-5}^{-2}+\left[\frac{x^2}{2}+2x\right]_{-2}^{0}+\left[\frac{x^2}{2}+5x\right]_{-5}^{0}$$

$$=\left[+\frac{25}{2}\right]-\left[\frac{4}{2}-4-\frac{25}{2}+10\right]+\left[0+0-\frac{4}{2}+4\right]+\left[0+0-\frac{25}{2}+25\right]$$

$$=\frac{25}{2}-\left[8-\frac{25}{2}\right]+[2]+\left[25-\frac{25}{2}\right]$$

$$=\frac{25}{2}-8+\frac{25}{2}+2+25-\frac{25}{2}$$

$$=19+\frac{25}{2}=31\frac{1}{2}$$

$$I=\frac{63}{2}$$

Definite Integrals Ex 20.3 Q19

$$|x|=\begin{cases}x, & x\geq 0\\ -x, & x<0\end{cases}$$

$$|x-2|=\begin{cases}x-2, & x\geq 2\\ 2-x, & x<2\end{cases}$$

$$|x-4|=\begin{cases}x-4, & x\geq 4\\ 4-x, & x<4\end{cases}$$

Splitting the limits of the integral, we get

$$\int_0^4(|x|+|x-2|+|x-4|)dx$$

$$=\int_0^2(|x|+|x-2|+|x-4|)dx+\int_2^4(|x|+|x-2|+|x-4|)dx$$

$$=\int_0^2(x+2-x+4-x)dx+\int_2^4(x+x-2+4-x)dx$$

$$=\int_0^2(6-x)dx+\int_2^4(2+x)dx$$

$$=\left[6x-\frac{x^2}{2}\right]_0^2+\left[2x+\frac{x^2}{2}\right]_2^4$$

$$=[12-2]+[16-6]$$

$$=10+10$$

$$=20$$

Definite Integrals Ex 20.3 Q20

$$\int_{-1}^{2}|x+1|dx+\int_{-1}^{2}|x|dx+\int_{-1}^{2}|x-1|dx$$
$$\int_{-1}^{2}(x+1)dx-\int_{-1}^{0}xdx+\int_{0}^{2}xdx-\int_{-1}^{1}(x-1)dx+\int_{1}^{2}(x-1)dx$$
$$\left\{\frac{x^2}{2}+x\right\}_{-1}^{2}-\left\{\frac{x^2}{2}\right\}_{-1}^{0}+\left\{\frac{x^2}{2}\right\}_{0}^{2}-\left\{\frac{x^2}{2}-x\right\}_{-1}^{1}+\left\{\frac{x^2}{2}-x\right\}_{1}^{2}$$
$$\left\{(4)-(-\frac{1}{2})\right\}-\left\{-\frac{1}{2}\right\}+\{2\}-\left\{(-\frac{1}{2})-(\frac{3}{2})\right\}+\left\{(0)-(-\frac{1}{2})\right\}$$
$$\left\{4+\frac{1}{2}\right\}+\left\{\frac{1}{2}\right\}+\{2\}+\{2\}+\left\{\frac{1}{2}\right\}$$
$$\frac{19}{2}$$

Definite Integrals Ex 20.3 Q21

$$\int_{-2}^{0}xe^{-x}dx+\int_{0}^{2}xe^{x}dx$$

For

$$\int_{-2}^{0}xe^{-x}dx$$

Using Integration By parts

$$\int f'g=fg-\int fg'$$
$$f'=e^{-x},g=x$$
$$f=-e^{-x},g'=1$$
$$\int_{-2}^{0}xe^{-x}dx=\left\{-xe^{-x}\right\}_{-2}^{0}+\int_{-2}^{0}e^{-x}dx$$
$$\int_{-2}^{0}xe^{-x}dx=\left\{-xe^{-x}-e^{-x}\right\}_{-2}^{0}$$
$$\int_{-2}^{0}xe^{-x}dx=\left\{(-1)-(2e^2-e^2)\right\}$$
$$\int_{-2}^{0}xe^{-x}dx=\left\{-1-e^2\right\}$$

For

$$\int_{0}^{2}xe^{x}dx$$

Using Integration By parts

$$\int f'g=fg-\int fg'$$
$$f'=e^{x},g=x$$
$$f=e^{x},g'=1$$
$$\int_0^2 xe^x dx=\left\{xe^x\right\}_0^2-\int_0^2 e^x dx$$
$$\int_0^2 xe^x dx=\left\{xe^x-e^x\right\}_0^2$$
$$\int_0^2 xe^x dx=2e^2-e^2+1$$
$$\int_0^2 xe^x dx=e^2+1$$

Hence answer is,

$$\int_{-2}^{2}xe^{|x|}dx=-1-e^2+e^2+1=0$$

Definite Integrals Ex 20.3 Q22

$$-\int_{-\frac{\Pi}{4}}^{0} \sin^2 x dx + \int_{0}^{\frac{\Pi}{2}} \sin^2 x dx$$

$$\sin^2 x = \frac{1-\cos 2x}{2}$$

$$-\int_{-\frac{\Pi}{4}}^{0} \frac{1-\cos 2x}{2} dx + \int_{0}^{\frac{\Pi}{2}} \frac{1-\cos 2x}{2} dx$$

$$-\frac{1}{2}\left\{x - \frac{\sin 2x}{2}\right\}_{-\frac{\Pi}{4}}^{0} + \frac{1}{2}\left\{x - \frac{\sin 2x}{2}\right\}_{0}^{\frac{\Pi}{2}}$$

$$-\frac{1}{2}\left\{-(-\frac{\Pi}{4}+\frac{1}{2})\right\} + \frac{1}{2}\left\{\frac{\Pi}{2}\right\}$$

$$\left\{-\frac{\Pi}{8}+\frac{1}{4}\right\} + \left\{\frac{\Pi}{4}\right\}$$

$$\frac{\Pi}{8}+\frac{1}{4}$$

$$\frac{\Pi+2}{8}$$

Definite Integrals Ex 20.3 Q23

$$\int_{0}^{\frac{\Pi}{2}} \cos^2 x dx - \int_{\frac{\Pi}{2}}^{\Pi} \cos^2 x dx$$

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

$$\int_{0}^{\frac{\Pi}{2}} \frac{1+\cos 2x}{2} dx - \int_{\frac{\Pi}{2}}^{\Pi} \frac{1+\cos 2x}{2} dx$$

$$\frac{1}{2}\left\{x + \frac{\sin 2x}{2}\right\}_{0}^{\frac{\Pi}{2}} - \frac{1}{2}\left\{x + \frac{\sin 2x}{2}\right\}_{\frac{\Pi}{2}}^{\Pi}$$

$$\frac{\Pi}{4} - \frac{\Pi}{4}$$

$$0$$

Definite Integrals Ex 20.3 Q24

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{2}} \left(2\sin|x| + \cos|x|\right) dx$$

$$= \int_{-\frac{\pi}{4}}^{0} (-2\sin x + \cos x) dx + \int_{0}^{\frac{\pi}{2}} (2\sin x + \cos x) dx$$

$$= \left[2\cos x + \sin x\right]_{-\frac{\pi}{4}}^{0} + \left[-2\cos x + \sin x\right]_{0}^{\frac{\pi}{2}}$$

$$= 2 + 0 - 0 + 1 + 0 + 1 + 2 - 0$$

$$= 6$$

Definite Integrals Ex 20.3 Q25

$$\int_{\frac{\pi}{2}}^{\pi} \sin^{-1}(\sin x)dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} xdx + \int_{\frac{\pi}{2}}^{\pi} (\Pi - x)dx$$

$$\Rightarrow \left\{\frac{x^2}{2}\right\}_{-\frac{\pi}{2}}^{\frac{\pi}{2}} + \left\{\Pi x - \frac{x^2}{2}\right\}_{\frac{\pi}{2}}^{\pi}$$

$$\Rightarrow \left\{(\Pi^2 - \frac{\Pi^2}{2}) - \left(\frac{\Pi^2}{2} - \frac{\Pi^2}{8}\right)\right\}$$

$$\Rightarrow \left\{\frac{\Pi^2}{2} - \frac{3\Pi^2}{8}\right\}$$

$$\Rightarrow \frac{\Pi^2}{8}$$

Definite Integrals Ex 20.3 Q27

[x]=0 for 0
and [x]=1 for 1
Hence

$$\int_0^1 0 + \int_1^2 2xdx$$

$$\left\{x^2\right\}_1^2$$

3

Definite Integrals Ex 20.3 Q18

$$\int_0^{2\pi} \cos^{-1}(\cos x)dx$$

$$= -\int_0^{\pi} \cos^{-1}(\cos x)dx + \int_{\pi}^{2\pi} \cos^{-1}(\cos x)dx$$

$$= -\int_0^{\pi} xdx + \int_{\pi}^{2\pi} xdx$$

$$= -\left[\frac{x^2}{2}\right]_0^{\pi} + \left[\frac{x^2}{2}\right]_{\pi}^{2\pi}$$

$$= -\frac{\pi^2}{2} + \frac{4\pi^2}{2} - \frac{\pi^2}{2}$$

$$= \pi^2$$

Definite Integrals Ex 20.3 Q33

$$\text{Let } I = \int_a^b \frac{f(x)}{f(x) + f(a+b-x)}dx \qquad --(i)$$

We know that $\int_a^b f(x) = \int_a^b f(a+b-x)dx$

Then

$$I = \int_a^b \frac{f(a+b-x)}{f(a+b-x) + f\{a+b-(a+b-x)\}}dx$$

$$I = \int_a^b \frac{f(a+b-x)}{f(a+b-x)f(x)}dx \qquad --(ii)$$

Adding (i) & (ii)

$$2I = \int_a^b \frac{f(x) + f(a+b-x)}{f(x) + f(a+b-x)}dx$$

$$2I = \int_a^b dx$$

$$I = [x]_a^b$$

$$I = \frac{1}{2}[b-a]$$

$$I = \frac{b-a}{2}$$

Ex 20.4A

Definite Integrals Ex 20.4A Q1

We know

$$\int_0^{2\Pi} f(x)dx = \int_0^{2\Pi} f(2\Pi - x)dx$$

Hence

$$\int_0^{2\Pi} \frac{e^{\sin x}}{e^{\sin x}+e^{-\sin x}}dx = \int_0^{2\Pi} \frac{e^{\sin(2\Pi - x)}}{e^{\sin(2\Pi - x)}+e^{-\sin(2\Pi - x)}}dx$$

We know

$$\sin(2\Pi - x) = -\sin x$$

$$\int_0^{2\Pi} \frac{e^{\sin x}}{e^{\sin x}+e^{-\sin x}}dx = \int_0^{2\Pi} \frac{e^{-\sin x}}{e^{-\sin x}+e^{\sin x}}dx$$

If

$$I = \int_0^{2\Pi} \frac{e^{\sin x}}{e^{\sin x}+e^{-\sin x}}dx$$

Then also

$$I = \int_0^{2\Pi} \frac{e^{-\sin x}}{e^{-\sin x}+e^{\sin x}}dx$$

Hence

$$2I = \int_0^{2\Pi} \frac{e^{-\sin x}}{e^{-\sin x}+e^{\sin x}}dx + \int_0^{2\Pi} \frac{e^{\sin x}}{e^{-\sin x}+e^{\sin x}}dx$$

$$2I = \int_0^{2\Pi} \frac{e^{-\sin x}}{e^{-\sin x}+e^{\sin x}} + \frac{e^{\sin x}}{e^{-\sin x}+e^{\sin x}}dx$$

$$2I = \int_0^{2\Pi} dx$$

$$2I = 2\Pi$$

$$I = \Pi$$

Definite Integrals Ex 20.4A Q2

We know

$$\int_0^{2\Pi} f(x)dx = \int_0^{2\Pi} f(2\Pi - x)dx$$

Hence

$$\int_0^{2\Pi} \log(\sec x + \tan x)dx = \int_0^{2\Pi} \log(\sec(2\Pi - x) + \tan(2\Pi - x))dx$$

$$\int_0^{2\Pi} \log(\sec x + \tan x)dx = \int_0^{2\Pi} \log(\sec x - \tan x)dx$$

If

$$I = \int_0^{2\Pi} \log(\sec x + \tan x)dx$$

Then

$$I = \int_0^{2\Pi} \log(\sec x - \tan x)dx$$

$$2I = \int_0^{2\Pi} \log(\sec x + \tan x)dx + \int_0^{2\Pi} \log(\sec x - \tan x)dx$$

$$2I = \int_0^{2\Pi} \log(\sec x + \tan x) + \log(\sec x - \tan x)dx$$

$$2I = \int_0^{2\Pi} \log(\sec^2 x - \tan^2 x)dx$$

$$2I = \int_0^{2\Pi} \log(1)dx$$

$$2I = 0$$

$$I = 0$$

Definite Integrals Ex 20.4A Q3

We know

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

Hence

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\tan x}}{\sqrt{\tan x}+\sqrt{\cot x}}dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\tan(\frac{\Pi}{2}-x)}}{\sqrt{\tan(\frac{\Pi}{2}-x)}+\sqrt{\cot(\frac{\Pi}{2}-x)}}dx$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\tan x}}{\sqrt{\tan x}+\sqrt{\cot x}}dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cot x}}{\sqrt{\tan x}+\sqrt{\cot x}}dx$$

If

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\tan x}}{\sqrt{\tan x}+\sqrt{\cot x}}dx$$

Then

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cot x}}{\sqrt{\tan x}+\sqrt{\cot x}}dx$$

So

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\tan x}}{\sqrt{\tan x}+\sqrt{\cot x}}dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cot x}}{\sqrt{\tan x}+\sqrt{\cot x}}dx$$

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\tan x}}{\sqrt{\tan x}+\sqrt{\cot x}} + \frac{\sqrt{\cot x}}{\sqrt{\tan x}+\sqrt{\cot x}}dx$$

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 1dx$$

$$2I = \frac{\Pi}{6}$$

$$I = \frac{\Pi}{12}$$

Definite Integrals Ex 20.4A Q4

We know

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

Hence

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin(\frac{\Pi}{2}-x)}}{\sqrt{\sin(\frac{\Pi}{2}-x)}+\sqrt{\cos(\frac{\Pi}{2}-x)}}dx$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$$

If

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$$

Then

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$$

Hence

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} + \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$$

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 1dx$$

$$2I = \frac{\Pi}{6}$$

$$I = \frac{\Pi}{12}$$

Definite Integrals Ex 20.4A Q5

We know

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

Hence

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x}{1+e^x}dx = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2(-x)}{1+e^{-x}}dx$$

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x}{1+e^x}dx = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x}{1+e^{-x}}dx$$

If

$$I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x}{1+e^x}dx$$

Then

$$I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x}{1+e^{-x}}dx$$

So

$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x}{1+e^x} + \frac{\tan^2 x}{1+e^{-x}}dx$$

$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x}{1+e^x} + \frac{\tan^2 x}{1+e^{-x}}dx$$

$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x}{1+e^x} + \frac{e^x \tan^2 x}{1+e^x}dx$$

$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x + e^x \tan^2 x}{1+e^x}dx$$

$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{(1+e^x)\tan^2 x}{1+e^x}dx$$

$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x + e^x \tan^2 x}{1+e^x} dx$$

$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{(1+e^x)\tan^2 x}{1+e^x} dx$$

$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan^2 x dx$$

$$I = \frac{1}{2}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan^2 x dx$$

We know

If f(x) is even

$$\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$$

If f(x) is odd

$$\int_{-a}^{a} f(x)dx = 0$$

Here

$$f(x) = \tan^2 x$$

f(x) is even, hence

$$I = \int_{0}^{\frac{\pi}{4}} \tan^2 x dx$$

$$I = \int_{0}^{\frac{\pi}{4}} \sec^2 x - 1 dx$$

$$I = \{\tan x - x\}_0^{\frac{\pi}{4}}$$

$$I = 1 - \frac{\pi}{4}$$

Note: Answer given in the book is incorrect.

Definite Integrals Ex 20.4A Q6

We know

$$\int_{a}^{b} f(x)dx = \int_{a}^{b} f(a+b-x)dx$$

Hence

$$\int_{-a}^{a} \frac{1}{1+a^x} dx = \int_{-a}^{a} \frac{1}{1+a^{-x}} dx$$

If

$$I = \int_{-a}^{a} \frac{1}{1+a^x} dx$$

Then

$$I = \int_{-a}^{a} \frac{1}{1+a^{-x}} dx$$

So

$$2I = \int_{-a}^{a} \frac{1}{1+a^x} + \frac{1}{1+a^{-x}} dx$$

$$2I = \int_{-a}^{a} \frac{1}{1+a^x} + \frac{a^x}{1+a^x} dx$$

$$2I = \int_{-a}^{a} 1 dx$$

$$2I = 2a$$

$$I = a$$

Definite Integrals Ex 20.4A Q7

We know

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

Hence

$$\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{1}{1+e^{\tan x}} dx = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{1}{1+e^{-\tan x}} dx$$

If

$$I = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{1}{1+e^{\tan x}} dx$$

Then

$$I = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{1}{1+e^{-\tan x}} dx$$

So

$$2I = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{1}{1+e^{\tan x}} + \frac{1}{1+e^{-\tan x}} dx$$

$$2I = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{1}{1+e^{\tan x}} + \frac{e^{\tan x}}{1+e^{\tan x}} dx$$

$$2I = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} 1dx$$

$$2I = \frac{2\Pi}{3}$$

$$I = \frac{\Pi}{3}$$

Definite Integrals Ex 20.4A Q8

We know

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

Hence

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2 x}{1+e^x} dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2(-x)}{1+e^{-x}} dx$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2 x}{1+e^x} dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2 x}{1+e^{-x}} dx$$

If

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2 x}{1+e^x} dx$$

Then

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2 x}{1+e^{-x}} dx$$

So

$$2I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{\cos^2 x}{1+e^x}+\frac{\cos^2 x}{1+e^{-x}}dx$$

$$2I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{\cos^2 x}{1+e^x}+\frac{e^x\cos^2 x}{1+e^x}dx$$

$$2I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{(1+e^x)\cos^2 x}{1+e^x}dx$$

$$2I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos^2 x dx$$

$$2I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{1+\cos 2x}{2}dx$$

$$I=\frac{1}{4}\left\{x+\frac{\sin 2x}{2}\right\}_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$I=\frac{1}{4}\left\{\left(\frac{\Pi}{2}\right)-\left(-\frac{\Pi}{2}\right)\right\}$$

$$I=\frac{\Pi}{4}$$

Note: Answer given in the book is incorrect.

Definite Integrals Ex 20.4A Q9

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{x^{11}-3x^9+5x^7-x^5+1}{\cos^2 x}dx$$

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{x^{11}-3x^9+5x^7-x^5}{\cos^2 x}+\frac{1}{\cos^2 x}dx$$

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{x^{11}-3x^9+5x^7-x^5}{\cos^2 x}dx+\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\sec^2 x dx$$

If f(x) is even

$$\int_{-a}^{a}f(x)dx=2\int_{0}^{a}f(x)dx$$

If f(x) is odd

$$\int_{-a}^{a}f(x)dx=0$$

Here

$\frac{x^{11}-3x^9+5x^7-x^5}{\cos^2 x}$ is odd and

$\sec^2 x$ is even. Hence

$$0+2\int_{0}^{\frac{\pi}{4}}\sec^2 x dx$$

$$2\{\tan x\}_0^{\frac{\pi}{4}}$$

$$2$$

Definite Integrals Ex 20.4A Q10

$$I=\int_a^b \frac{x^{\frac{1}{n}}}{x^{\frac{1}{n}}+(a+b-x)^{\frac{1}{n}}}dx$$

$$I=\int_a^b \frac{(a+b-x)^{\frac{1}{n}}}{(a+b-x)^{\frac{1}{n}}+x^{\frac{1}{n}}}dx$$

$$2I=\int_a^b \frac{x^{\frac{1}{n}}}{x^{\frac{1}{n}}+(a+b-x)^{\frac{1}{n}}}dx+\int_a^b \frac{(a+b-x)^{\frac{1}{n}}}{(a+b-x)^{\frac{1}{n}}+x^{\frac{1}{n}}}dx$$

$$2I=\int_a^b \frac{x^{\frac{1}{n}}+(a+b-x)^{\frac{1}{n}}}{x^{\frac{1}{n}}+(a+b-x)^{\frac{1}{n}}}dx$$

$$I=\frac{1}{2}\int_a^b dx$$

$$I=\frac{b-a}{2}$$

Definite Integrals Ex 20.4A Q11

We have,

$$I=\int_0^{\frac{\pi}{2}}\left(2\log\cos x-\log\sin 2x\right)dx$$

$$=\int_0^{\frac{\pi}{2}}\left(\log\cos^2 x-\log\sin 2x\right)dx$$

$$=\int_0^{\frac{\pi}{2}}\log\frac{\cos^2 x}{\sin x}dx$$

$$=\int_0^{\frac{\pi}{2}}\log\frac{\cos^2 x}{2\sin x.\cos x}dx$$

$$=\int_0^{\frac{\pi}{2}}\log\frac{\cos x}{2\sin x}dx$$

$$=\int_0^{\frac{\pi}{2}}\left(\log\cos x-\log\sin x-\log 2\right)dx$$

$$=\int_0^{\frac{\pi}{2}}\log\cos x\,dx-\int_0^{\frac{\pi}{2}}\log\sin x\,dx-\int_0^{\frac{\pi}{2}}\log 2$$

We know that $\int_0^{\frac{\pi}{2}}\log\cos x\,dx=\int_0^{\frac{\pi}{2}}\log\sin x\,dx$ — (i)

Hence from equation (i)

$$I=-\int_0^{\frac{\pi}{2}}\log 2=-\frac{\pi}{2}\log 2$$

Definite Integrals Ex 20.4A Q12

Let $I = \int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}dx$ ...(1)

It is known that, $\left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$

$I = \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{x}}dx$ ...(2)

Adding (1) and (2), we obtain

$$2I = \int_0^a \frac{\sqrt{x}+\sqrt{a-x}}{\sqrt{x}+\sqrt{a-x}}dx$$
$$\Rightarrow 2I = \int_0^a 1\,dx$$
$$\Rightarrow 2I = [x]_0^a$$
$$\Rightarrow 2I = a$$
$$\Rightarrow I = \frac{a}{2}$$

Definite Integrals Ex 20.4A Q13

Let $I = \int_0^5 \frac{\sqrt[4]{x+4}}{\sqrt[4]{x+4}+\sqrt[4]{9-x}}dx$ --(i)

We know that $\int_0^a f(x) = \int_0^a f(a-x)$

So,

$$I = \int_0^5 \frac{\sqrt[4]{(5-x)+4}}{\sqrt[4]{(5-x)+4}+\sqrt[4]{9-(5-x)}}dx$$
$$I = \int_0^5 \frac{\sqrt[4]{9-x}}{\sqrt[4]{9-x}+\sqrt[4]{4+x}}dx \quad \text{--(ii)}$$

Adding (i) & (ii)

$$2I = \int_0^5 \frac{\sqrt[4]{x+4}}{\sqrt[4]{x+4}+\sqrt[4]{9-x}}dx + \int_0^5 \frac{\sqrt[4]{9-x}}{\sqrt[4]{9-x}+\sqrt[4]{4+x}}dx$$
$$2I = \int_0^5 \frac{\sqrt[4]{x+4}+\sqrt[4]{9-x}}{\sqrt[4]{x+4}+\sqrt[4]{9-x}}dx$$
$$2I = \int_0^5 dx$$
$$2I = [x]_0^5$$
$$I = \frac{1}{2}[5-0] = \frac{5}{2}$$

$$\therefore \int_0^5 \frac{\sqrt[4]{x+4}}{\sqrt[4]{x+4}+\sqrt[4]{9-x}}dx = \frac{5}{2}$$

Definite Integrals Ex 20.4A Q14

Let $I = \int_0^7 \frac{\sqrt[3]{x}}{\sqrt[3]{x} + \sqrt[3]{7-x}} dx$ --(i)

We know that $\int_0^a f(x) = \int_0^a f(a-x)$

Hence,

$I = \int_0^7 \frac{\sqrt[3]{7-x}}{\sqrt[3]{7-x} + \sqrt[3]{x}} dx$ --(ii)

Adding (i) & (ii)

$$2I = \int_0^7 \frac{\sqrt[3]{x}}{\sqrt[3]{x} + \sqrt[3]{7-x}} dx + \int_0^7 \frac{\sqrt[3]{7-x}}{\sqrt[3]{7-x} + \sqrt[3]{x}} dx$$

$$2I = \int_0^7 \frac{\sqrt[3]{x} + \sqrt[3]{7-x}}{\sqrt[3]{x} + \sqrt[3]{7-x}} dx$$

$$2I = \int_0^7 dx$$

$$2I = [x]_0^7$$

$$I = \frac{7}{2}$$

Definite Integrals Ex 20.4A Q15

Let $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1 + \sqrt{\tan x}} dx$

$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$ --(i)

We know that $\int_a^b f(x) = \int_a^b f(a+b-x) dx$

Hence,

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos\left(\frac{\pi}{2} - x\right)}}{\sqrt{\cos\left(\frac{\pi}{2} - x\right)} + \sqrt{\sin\left(\frac{\pi}{2} - x\right)}} dx$$

$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$ --(ii)

Adding (i) & (ii)

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$$

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x} + \sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$$

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} dx$$

$$2I = [x]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$

$$I = \frac{\pi}{12}$$

Definite Integrals Ex 20.4A Q16

$$I = \int_a^b x f(x)\,dx$$

$$I = \int_a^b (a+b-x) f(a+b-x)\,dx$$

$$I = \int_a^b (a+b-x) f(x)\,dx \ldots\ldots\ldots \left[\because f(a+b-x) = f(x)\right]$$

$$I = \int_a^b (a+b) f(x)\,dx - \int_a^b f(x)\,dx$$

$$I = (a+b)\int_a^b f(x)\,dx - I$$

$$2I = (a+b)\int_a^b f(x)\,dx$$

$$I = \frac{(a+b)}{2}\int_a^b f(x)\,dx$$

$$\therefore \int_a^b x f(x)\,dx = \frac{(a+b)}{2}\int_a^b f(x)\,dx$$

Ex 20.4B

Definite Integrals Ex 20.4B Q1

We have,

$$\frac{1}{1+\tan x} = \frac{1}{1+\frac{\sin x}{\cos x}} = \frac{\cos x}{\cos x + \sin x}$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{dx}{1+\tan x} = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\cos x + \sin x}\,dx$$

Let

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\cos x + \sin x}\,dx \qquad --(\mathrm{I})$$

So,

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos\left(\frac{\pi}{2} - x\right)}{\cos\left(\frac{\pi}{2} - x\right) + \sin\left(\frac{\pi}{2} - x\right)}\,dx \qquad \left[\because \int_0^a f(x)dx = \int_0^a f(a-x)\,dx\right]$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin x}{\cos x + \sin x}\,dx \qquad --(\mathrm{II})$$

Hence, adding (I) & (II)

$$2I = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\cos x + \sin x}\,dx + \int_0^{\frac{\pi}{2}} \frac{\sin x}{\cos x + \sin x}\,dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\cos x + \sin x}{\cos x + \sin x}\,dx$$

$$= \int_0^{\frac{\pi}{2}} dx$$

$$2I = [x]_0^{\frac{\pi}{2}}$$

$$2I = \left[\frac{\pi}{2} - 0\right] \quad \Rightarrow \quad I = \frac{\pi}{4}$$

Definite Integrals Ex 20.4B Q2

We have,

$$\frac{1}{1+\cot x} = \frac{1}{1+\frac{\cos x}{\sin x}} = \frac{\sin x}{\sin x + \cos x}$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{1}{1+\cot x}\,dx = \int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x}\,dx$$

Let

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x}\,dx \qquad --(\text{I})$$

So,

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)}\,dx \qquad \left[\left[\because \int_0^a f(x)dx = \int_0^a f(a-x)\,dx\right]\right]$$

$$= \int_0^{\frac{\pi}{2}} \frac{\cos x}{\sin x + \cos x}\,dx \qquad --(\text{II})$$

Adding (I) & (II)

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x}\,dx + \int_0^{\frac{\pi}{2}} \frac{\cos x}{\sin x + \cos x}\,dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin x + \cos x}{\sin x + \cos x}\,dx$$

$$2I = \int_0^{\frac{\pi}{2}} dx$$

$$= [x]_0^{\frac{\pi}{2}}$$

$$2I = \left[\frac{\pi}{2} - 0\right]$$

$$I = \frac{\pi}{4}$$

Definite Integrals Ex 20.4B Q3

We have,

$$\frac{\sqrt{\cot x}}{\sqrt{\cot x}+\sqrt{\tan x}}=\frac{\sqrt{\frac{\cos x}{\sin x}}}{\sqrt{\frac{\cos x}{\sin x}}+\sqrt{\frac{\sin x}{\cos x}}}=\frac{\sqrt{\frac{\cos x}{\sin x}}}{\frac{\cos x+\sin x}{\sqrt{\sin x}.\sqrt{\cos x}}}=\sqrt{\frac{\cos x}{\sin x}}\times\frac{\sqrt{\sin x}.\sqrt{\cos x}}{\cos x+\sin x}$$

$$=\frac{\cos x}{\cos x+\sin x}$$

$$\therefore \int_0^{\frac{\pi}{2}}\frac{\sqrt{\cot x}}{\sqrt{\cot x}+\sqrt{\tan x}}dx=\int_0^{\frac{\pi}{2}}\frac{\cos x}{\cos x+\sin x}dx$$

Let

$$I=\int_0^{\frac{\pi}{2}}\frac{\cos x}{\cos x+\sin x}dx \qquad --(\mathrm{I})$$

So,

B $$I=\int_0^{\frac{\pi}{2}}\frac{\cos\left(\frac{\pi}{2}-x\right)}{\cos\left(\frac{\pi}{2}-x\right)+\sin\left(\frac{\pi}{2}-x\right)}dx \qquad \left[\because \int_0^a f(x)dx=\int_0^a f(a-x)dx\right]$$

$$=\int_0^{\frac{\pi}{2}}\frac{\sin x}{\cos x+\sin x}dx \qquad --(\mathrm{II})$$

Adding (I) & (II)

$$2I=\int_0^{\frac{\pi}{2}}\frac{\cos x}{\cos x+\sin x}dx+\int_0^{\frac{\pi}{2}}\frac{\sin x}{\cos x+\sin x}dx$$

$$2I=\int_0^{\frac{\pi}{2}}\frac{\cos x+\sin x}{\cos x+\sin x}dx$$

$$2I=\int_0^{\frac{\pi}{2}}dx$$

$$2I=\left[x\right]_0^{\frac{\pi}{2}}$$

$$2I=\left[\frac{\pi}{2}-0\right]$$

$$I=\frac{\pi}{4}$$

Definite Integrals Ex 20.4B Q4

$$\text{Let } I=\int_0^{\frac{\pi}{2}}\frac{\sin^{\frac{3}{2}}x}{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}dx \qquad \ldots(1)$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\frac{\sin^{\frac{3}{2}}\left(\frac{\pi}{2}-x\right)}{\sin^{\frac{3}{2}}\left(\frac{\pi}{2}-x\right)+\cos^{\frac{3}{2}}\left(\frac{\pi}{2}-x\right)}dx \qquad \left(\int_0^a f(x)dx=\int_0^a f(a-x)dx\right)$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\frac{\cos^{\frac{3}{2}}x}{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}dx \qquad \ldots(2)$$

Adding (1) and (2), we obtain

$$2I=\int_0^{\frac{\pi}{2}}\frac{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}dx$$

$$\Rightarrow 2I=\int_0^{\frac{\pi}{2}}1\cdot dx$$

$$\Rightarrow 2I=\left[x\right]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I=\frac{\pi}{2}$$

$$\Rightarrow I=\frac{\pi}{4}$$

Definite Integrals Ex 20.4B Q5

$$\int_0^{\frac{\pi}{2}} \frac{\sin^n x}{\sin^n x + \cos^n x} dx$$

Let $I = \int_0^{\frac{\pi}{2}} \frac{\sin^n x}{\sin^n x + \cos^n x} dx \qquad --(i)$

So,

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin^n\left(\frac{\pi}{2} - x\right)}{\sin^n\left(\frac{\pi}{2} - x\right) + \cos^n\left(\frac{\pi}{2} - x\right)} dx \qquad \left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$$

$$= \int_0^{\frac{\pi}{2}} \frac{\cos^n x}{\sin^n x + \cos^n x} \qquad --(II)$$

Adding (I) & (II)

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^n x}{\sin^n x + \cos^n x} dx + \int_0^{\frac{\pi}{2}} \frac{\cos^n x}{\sin^n x + \cos^n x}$$

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^n + \cos^n x}{\sin^n + \cos^n x} dx$$

$$2I = \int_0^{\frac{\pi}{2}} dx$$

$$2I = [x]_0^{\frac{\pi}{2}}$$

$$2I = \left[\frac{\pi}{2} - 0\right]$$

$$I = \frac{\pi}{4}$$

Definite Integrals Ex 20.4B Q6

We have,

$$\int_0^{\frac{\pi}{2}} \frac{1}{1 + \sqrt{\tan x}} dx = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$$

Let

$$I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx \qquad --(i)$$

So

$$= \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos\left(\frac{\pi}{2} - x\right)}}{\sqrt{\cos\left(\frac{\pi}{2} - x\right)} + \sqrt{\sin\left(\frac{\pi}{2} - x\right)}} dx \qquad \left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} \qquad --(ii)$$

Adding (i) & (ii)

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx + \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x} + \sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$$

$$2I = \int_0^{\frac{\pi}{2}} dx$$

$$2I = [x]_0^{\frac{\pi}{2}}$$

$$I = \frac{\pi}{4}$$

Definite Integrals Ex 20.4B Q7

$$Let\ I = \int_0^a \frac{dx}{x + \sqrt{a^2 - x^2}}$$

Let $x = a\sin\theta$

$dx = a\cos\theta\, d\theta$

Now, $x = 0 \Rightarrow \theta = 0$

$x = a \Rightarrow \theta = \frac{\pi}{2}$

$$I = \int_0^{\frac{\pi}{2}} \frac{a\cos\theta\, d\theta}{a\sin\theta + a\cos\theta}$$

$$= \int_0^{\frac{\pi}{2}} \frac{\cos\theta}{\sin\theta + \cos\theta} \qquad --(i)$$

So,

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos\left(\frac{\pi}{2} - \theta\right)}{\sin\left(\frac{\pi}{2} - \theta\right) + \cos\left(\frac{\pi}{2} - \theta\right)} d\theta \qquad \left[\because \int_0^a f(x)dx = \int_0^a f(a-x)\, dx\right]$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin\theta}{\cos\theta + \sin\theta} \qquad --(ii)$$

Adding (i) & (ii) we get

$$2I = \int_0^{\frac{\pi}{2}} \frac{\cos\theta + \sin\theta}{\cos\theta + \sin\theta} d\theta$$

$$2I = \int_0^{\frac{\pi}{2}} d\theta$$

$$2I = \frac{1}{2}\left[\theta\right]_0^{\frac{\pi}{2}}$$

$$I = \frac{\pi}{4}$$

Definite Integrals Ex 20.4B Q8

Put $x = \tan\theta$

$\Rightarrow$ $dx = \sec^2\theta d\theta$

If $x = 0,\ \theta = 0$

If $x = \infty,\ \theta = \frac{\pi}{2}$

$\therefore$ $I = \int_0^{\infty} \frac{\log x}{1 + x^2} dx$

$$= \int_0^{\frac{\pi}{2}} \frac{\log(\tan\theta)\sec^2\theta d\theta}{1 + \tan^2\theta}$$

$\Rightarrow$ $I = \int_0^{\frac{\pi}{2}} \log(\tan\theta) d\theta \qquad ---(i)$

$\Rightarrow$ $I = \int_0^{\frac{\pi}{2}} \log\tan\left(\frac{\pi}{2} - \theta\right) d\theta$

$\Rightarrow$ $I = \int_0^{\frac{\pi}{2}} \log\cot(\theta) d\theta \qquad ---(ii)$

Adding (i) and (ii), we get

$$2I = \int_0^{\frac{\pi}{2}} (\log\tan\theta + \log\cot\theta) d\theta$$

$\Rightarrow$ $2I = \int_0^{\frac{\pi}{2}} \log 1 \times dx = \int_0^{\frac{\pi}{2}} 0 \times dx = 0$

$\Rightarrow$ $I = 0$

Definite Integrals Ex 20.4B Q9

Let $x = \tan\theta$

$\Rightarrow \quad dx = \sec^2\theta d\theta$

If $\quad x = 0,\ \theta = 0$

If $\quad x = 1,\ \theta = \frac{\pi}{4}$

$\therefore \quad \int_0^1 \frac{\log(1+x)}{1+x^2}dx$

$\Rightarrow \quad I = \int_0^{\frac{\pi}{4}} \log(1+\tan\theta)d\theta$

$\Rightarrow \quad I = \int_0^{\frac{\pi}{4}} \log\left\{1+\tan\left(\frac{\pi}{4}-\theta\right)\right\}d\theta$

$\Rightarrow \quad I = \int_0^{\frac{\pi}{4}} \log\left\{1+\frac{1-\tan\theta}{1+\tan\theta}\right\}d\theta$

$\Rightarrow \quad I = \int_0^{\frac{\pi}{4}} \log\left(\frac{2}{1+\tan\theta}\right)d\theta$

$\Rightarrow \quad I = \int_0^{\frac{\pi}{4}} \left(\log 2 - \log(1+\tan\theta)\right)d\theta$

$\Rightarrow \quad 2I = \int_0^{\frac{\pi}{4}} \log 2 \times d\theta = \frac{\pi}{4}\log 2$

$\Rightarrow \quad I = \frac{\pi}{8}\log 2$

Definite Integrals Ex 20.4B Q10

$$I = \int_0^{\infty} \frac{x}{(1+x)\left(1+x^2\right)}dx$$

Let,

$$\frac{x}{(1+x)\left(1+x^2\right)} = \frac{A}{1+x} + \frac{Bx+C}{1+x^2}$$

$$\Rightarrow x = A\left(1+x^2\right) + (Bx+C)(1+x)$$

Equating coeffcients, we get

$A + B = 0 \Rightarrow A = -B$

$B + C = 1 \Rightarrow -2A = 1$

$A + C = 0 \Rightarrow A = -C$

$$\therefore\ A = -\frac{1}{2},\ B = \frac{1}{2},\ C = \frac{1}{2}$$

So,

$$I = \int_0^{\infty}\left(\frac{-\frac{1}{2}}{1+x} + \frac{1}{2}\,\frac{x+1}{x^2+1}\right)dx$$

$$= \int_0^{\infty} -\frac{1}{2}\frac{dx}{1+x} + \frac{1}{2}\int_0^{\infty}\frac{x}{x^2+1}dx + \frac{1}{2}\int_0^{\infty}\frac{dx}{1+x^2}$$

$$= \left[-\frac{1}{2}\log|1+x| + \frac{1}{4}\log\left|x^2+1\right| + \frac{1}{2}\tan^{-1}x\right]_0^{\infty}$$

$$= 0 + 0 + \frac{\pi}{4} + 0 - 0 - 0$$

$$= \frac{\pi}{4}$$

$$\because \int_0^{\infty} \frac{x}{(1+x)\left(1+x^2\right)}dx = \frac{\pi}{4}$$

Definite Integrals Ex 20.4B Q11

We have,

$$I=\int_0^{\pi}\frac{x\tan x}{\sec x\ \mathrm{cosec}\,x}dx$$

$$I=\int_0^{\pi}\frac{x\left(\frac{\sin x}{\cos x}\right)}{\left(\frac{1}{\cos x}\right)\left(\frac{1}{\sin x}\right)}dx$$

$$I=\int_0^{\pi}x\sin^2 x\,dx \qquad \ldots(i)$$

$$I=\int_0^{\pi}(\pi-x)\sin^2(\pi-x)dx \qquad \left[\because \int_0^a f(x)dx=\int_0^a f(a-x)dx\right]$$

$$I=\int_0^{\pi}(\pi-x)\sin^2 x\,dx \qquad --(ii)$$

Add (i) and (ii), we get

$$2I=\int_0^{\pi}(\pi)\sin^2 x\,dx=\pi\int_0^{\pi}\frac{1-\cos 2x}{2}dx=\frac{\pi}{2}\left[x-\frac{\sin 2x}{2}\right]_0^{\pi}=\frac{\pi}{2}\left[\pi-0-0+0\right]=\frac{\pi^2}{2}$$

$$\therefore \int_0^{\pi}\frac{x\tan x}{\sec x\ \mathrm{cosec}\,x}dx=\frac{\pi^2}{4}$$

Definite Integrals Ex 20.4B Q12

Let $I=\int_0^{\pi}x\sin x.\cos^4 x\,dx \qquad --(i)$

So,

$$I=\int_0^{\pi}(\pi-x)\sin(\pi-x).\cos^4(\pi-x)dx \qquad \left[\because \int_0^a f(x)dx=\int_0^a f(a-x)dx\right]$$

$$=\int_0^{\pi}(\pi-x)\sin x.\cos^4 x\,dx$$

$$=\int_0^{\pi}\pi\sin x.\cos^4 x\,dx-\int_0^{\pi}x\sin x.\cos^4 x\,dx$$

So from equation (i)

$$I=\int_0^{\pi}\pi\sin x.\cos^4 x\,dx-I$$

$$2I=\pi\int_0^{\pi}\sin x.\cos^4 x\,dx$$

Let $t=\cos x\,dx$

$dt=-\sin\ x\,dx$

As,

$x=0 \quad t=1$

$x=\pi \quad t=-1$

Hence

$$2I=\pi\int_{-1}^{+1}t^4\,dt=\pi\left[\frac{t^5}{5}\right]_{-1}^{1}=\pi\left[\frac{1}{5}+\frac{1}{5}\right]$$

$$I=\frac{\pi}{5}$$

Definite Integrals Ex 20.4B Q13

Let $I = \int_0^{\pi} x \sin^3 x \, dx$

$= \int_0^{\pi} (\pi - x) \sin^3 (\pi - x) dx \qquad \left[\because \int_0^a f(x) dx = \int_0^a f(a-x) dx\right]$

$= \int_0^{\pi} \pi \sin^3 x \, dx - \int_0^{\pi} x \sin^3 x \, dx$

$\therefore I = \int_0^{\pi} \pi \sin^3 x \, dx - I$

$\Rightarrow 2I = \pi \int_0^{\pi} \sin^3 x \, dx$

$\Rightarrow 2I = \pi \int_0^{\pi} \frac{3 \sin x - \sin 3x}{4} dx$

$= \frac{\pi}{4} \int_0^{\pi} (3 \sin x - \sin 3x) dx$

$= \frac{\pi}{4} \left[-3 \cos x + \frac{\cos 3x}{3}\right]_0^{\pi}$

$= \frac{\pi}{4} \left[\left(-3 \cos \pi + \frac{\cos 3\pi}{3}\right) - \left(-3 \cos 0 + \frac{\cos 0}{3}\right)\right]$

$= \frac{\pi}{4} \left[\left(3 - \frac{1}{3}\right) - \left(-3 + \frac{1}{3}\right)\right]$

$= \frac{\pi}{4} \left[3 - \frac{1}{3} + 3 - \frac{1}{3}\right]$

$\frac{\pi}{4} \left[6 - \frac{2}{3}\right]$

$= \frac{\pi}{4} \times \frac{16}{3} \quad = \quad \frac{4\pi}{3}$

$\therefore I = \frac{2\pi}{3}$

Definite Integrals Ex 20.4B Q14

We have,

$$I = \int_0^{\pi} x \log \sin x \, dx = \int_0^{\pi} (\pi - x) \log \sin(\pi - x) \, dx$$

$$I = \pi \int_0^{\pi} \log \sin(x) \, dx - \int_0^{\pi} x \log \sin x \, dx$$

$$2I = \pi \int_0^{\pi} \log \sin x \, dx$$

Since $f(x) = f(-x)$, f(x) is an even function.

$$\therefore \ 2I = 2\pi \int_0^{\frac{\pi}{2}} \log \sin x \, dx$$

$$I = \pi \int_0^{\frac{\pi}{2}} \log \sin x \, dx \qquad \ldots(i)$$

$$\Rightarrow I = \pi \int_0^{\frac{\pi}{2}} \log \sin\left(\frac{\pi}{2} - x\right) dx = \pi \int_0^{\frac{\pi}{2}} \log \cos x \, dx \quad \ldots(ii)$$

Now adding (i) & (ii) we get

$$2I = \pi \int_0^{\frac{\pi}{2}} \log \sin x \, dx + \pi \int_0^{\frac{\pi}{2}} \log \cos x \, dx = \pi \int_0^{\frac{\pi}{2}} (\log \sin x + \log \cos x) \, dx = \pi \int_0^{\frac{\pi}{2}} \log \sin x . \cos x \, dx$$

$$\Rightarrow 2I = \pi \int_0^{\frac{\pi}{2}} \log\left(\frac{2 \sin x . \cos x}{2}\right) dx = \pi \int_0^{\frac{\pi}{2}} \log\left(\frac{\sin 2x}{2}\right) dx = \pi \int_0^{\frac{\pi}{2}} \log \sin 2x \, dx - \pi \int_0^{\frac{\pi}{2}} \log 2 \, dx \qquad \ldots(iii)$$

Now let $I = \int_0^{\frac{\pi}{2}} \log \sin 2x \, dx$

Putting $2x = t$ we get

$$I_1 = \int_0^{\pi} \log \sin t \frac{dt}{2} = \frac{1}{2} \int_0^{\pi} \log \sin t . dt = \frac{1}{2} \times 2\pi \int_0^{\frac{\pi}{2}} \log \sin t \, dt = \pi \int_0^{\frac{\pi}{2}} \log \sin x \, dx = I$$

So from (iii) we get

$$2I = I - \pi \frac{\pi}{2} \log 2$$

$$I = -\frac{\pi^2}{2} \log 2$$

Definite Integrals Ex 20.4B Q15

Let $I = \int_0^{\pi} \frac{x \sin x}{1+\sin x} dx$

$= \int_0^{\pi} \frac{(\pi - x)\sin(\pi - x)}{1+\sin x} dx \qquad \left[\because \int_0^a f(x)dx = \int_0^a f(a-x)\,dx\right]$

$I = \int_0^{\pi} \frac{\pi \sin x}{1+\sin x} dx - \int_0^{\pi} \frac{x \sin x}{1+\sin x} dx$

$2I = \pi \int_0^{\pi} \frac{\sin x}{1+\sin x} dx$

$2I = \pi \int_0^{\pi} \frac{\sin x}{1+\sin x} \times \frac{(1-\sin x)}{(1-\sin x)} dx$

$2I = \pi \int_0^{\pi} \frac{\sin x - \sin^2 x}{1+\sin^2 x} dx$

$2I = \pi \int_0^{\pi} \frac{(\sin x - \sin^2 x)}{\cos^2 x} dx$

$2I = \pi \int_0^{\pi} (\tan x.\sec x - \tan^2 x) dx$

$2I = \pi \int_0^{\pi} \left[\tan x.\sec x - (\sec^2 x - 1)\right] dx$

$2I = \pi \int_0^{\pi} (\sec x.\tan x - \sec^2 x + 1) dx$

$2I = \pi \int_0^{\pi} (\sec x.\tan x - \sec^2 x + 1) dx$

$2I = \pi \left[\sec x - \tan x + x\right]_0^{\pi}$

$2I = \pi \left[(\sec \pi - \tan \pi + \pi) - (\sec 0 - \tan 0 + 0)\right]$

$2I = \pi \left[(-1 - 0 + \pi) - (1 - 0 + 0)\right]$

$2I = \pi (\pi - 1 - 1)$

$I = \frac{\pi}{2} (\pi - 2)$

$\therefore \int_0^{\pi} \frac{x \sin x}{1+\sin x} dx = \pi \left(\frac{\pi}{2} - 1\right)$

Definite Integrals Ex 20.4B Q16

We have

$$I=\int_0^{\pi}\frac{x\,dx}{1+\cos\alpha\sin x}\qquad --(i)$$

$$\therefore\int_0^{a}f(x)\,dx=\int_0^{a}f(a-x)\,dx$$

$$I=\int_0^{\pi}\frac{(\pi-x)\,dx}{1+\cos\alpha\sin(\pi-x)}=\int_0^{\pi}\frac{(\pi-x)\,dx}{1+\cos\alpha\sin x}\qquad --(ii)$$

Adding (i) & (ii) we get

$$2I=\pi\int_0^{\pi}\frac{\pi}{1+\cos\alpha.\sin x}\,dx$$

Substituting $\sin x=\dfrac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

$$2I=\pi\int_0^{\pi}\frac{\sec^2\frac{x}{2}}{1+\tan^2\frac{x}{2}\,2\cos\alpha.\tan\frac{x}{2}}\,dx=\pi\int_0^{\pi}\frac{\sec^2\frac{x}{2}dx}{1-\cos^2\alpha+\left(\cos\alpha.\tan\frac{x}{2}\right)^2}\,dx$$

Let $\tan\frac{x}{2}=t\Rightarrow\frac{1}{2}\sec^2\frac{x}{2}dx=dt$

When $x=0\quad t=0$

$\pi\ \Rightarrow t=\alpha$

$$2I=\int_0^{\alpha}\frac{dt}{\left(1+\cos^2\alpha\right)+\left(\cos\alpha+t\right)^2}\,dx=2\pi.\frac{1}{\sqrt{1+\cos^2\alpha}}.\left[\tan^{-1}\left(\frac{\cos\alpha+1}{\sqrt{1+\cos^2\alpha}}\right)\right]_0^{\alpha}$$

$$=\frac{2\pi}{\sin\alpha}\left[\frac{\pi}{2}-\tan^{-1}\cot\alpha\right]$$

$$=\frac{2\pi}{\sin\alpha}\left[\cot^{-1}(\cot\alpha)\right]$$

$$=\frac{2\pi}{\sin\alpha}.\alpha$$

$$\Rightarrow I=\frac{\pi\alpha}{\sin\alpha}$$

Definite Integrals Ex 20.4B Q17

Let $I=\int_0^{\pi}x\cos^2x\,dx$

$$I=\int_0^{\pi}(\pi-x)\cos^2(\pi-x)\,dx\qquad\left[\because\int_0^{a}f(x)dx=\int_0^{a}f(a-x)\,dx\right]$$

$$I=\pi\int_0^{\pi}\cos^2x\,dx-\int_0^{\pi}x\cos^2x\,dx$$

$$2I=\pi\int_0^{\pi}\cos^2x\,dx$$

$$=\pi\int_0^{\pi}\left(\frac{1+\cos2x}{2}\right)dx\qquad\text{Since }\cos^2x=\frac{1+\cos2x}{2}$$

$$=\frac{\pi}{2}\int_0^{\pi}(1+\cos2x)\,dx$$

$$=\frac{\pi}{2}\left[x+\left(-\frac{\sin2x}{2}\right)\right]_0^{\pi}$$

$$\therefore 2I=\frac{\pi}{2}\left[\pi-\frac{\sin2.\pi}{2}-0+\frac{\sin0}{2}\right]$$

$$\Rightarrow 2I=\frac{\pi}{2}\left[\pi-0-0+0\right]$$

$$I=\frac{\pi^2}{4}$$

Definite Integrals Ex 20.4B Q18

$$I = \int_{\pi/6}^{\pi/3} \frac{1}{1+\cot^{3/2}x}\,dx$$

$$I = \int_{\pi/6}^{\pi/3} \frac{\sin^{3/2}x}{\sin^{3/2}x+\cos^{3/2}x}\,dx$$

$$I = \int_{\pi/6}^{\pi/3} \frac{\sin^{3/2}\left(\frac{\pi}{2}-x\right)}{\sin^{3/2}\left(\frac{\pi}{2}-x\right)+\cos^{3/2}\left(\frac{\pi}{2}-x\right)}\,dx = \int_{\pi/6}^{\pi/3} \frac{\cos^{3/2}(x)}{\cos^{3/2}(x)+\sin^{3/2}(x)}\,dx$$

$$\therefore\ 2I = \int_{\pi/6}^{\pi/3} \frac{\sin^{3/2}x}{\sin^{3/2}x+\cos^{3/2}x}\,dx + \int_{\pi/6}^{\pi/3} \frac{\cos^{3/2}(x)}{\cos^{3/2}(x)+\sin^{3/2}(x)}\,dx$$

$$2I = \int_{\pi/6}^{\pi/3} \frac{\sin^{3/2}x+\cos^{3/2}x}{\sin^{3/2}x+\cos^{3/2}x}\,dx$$

$$I = \frac{1}{2}\int_{\pi/6}^{\pi/3} dx$$

$$I = \frac{\pi}{12}$$

Definite Integrals Ex 20.4B Q19

$$I = \int_0^{\frac{\pi}{2}} \frac{\tan^7 x}{\tan^7 x+\cot^7 x}dx$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\tan^7\left(\frac{\Pi}{2}-x\right)}{\tan^7\left(\frac{\Pi}{2}-x\right)+\cot^7\left(\frac{\Pi}{2}-x\right)}dx$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\cot^7 x}{\tan^7 x+\cot^7 x}dx$$

Hence

$$2I = \int_0^{\frac{\pi}{2}} \frac{\tan^7 x}{\tan^7 x+\cot^7 x}+\frac{\cot^7 x}{\tan^7 x+\cot^7 x}dx$$

$$2I = \int_0^{\frac{\pi}{2}} 1dx$$

$$2I = \frac{\Pi}{2}$$

$$I = \frac{\Pi}{4}$$

Definite Integrals Ex 20.4B Q20

$$I = \int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}dx$$

$$I = \int_2^8 \frac{\sqrt{10-(8+2-x)}}{\sqrt{(8+2-x)}+\sqrt{10-(8+2-x)}}dx$$

$$I = \int_2^8 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{10-x}}dx$$

$$2I = \int_2^8 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{10-x}}+\frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}dx$$

$$2I = \int_2^8 1dx$$

$$2I = 6$$

$$I = 3$$

Definite Integrals Ex 20.4B Q21

$$\int_0^{\pi} x\sin x\cos^2 x dx = \int_0^{\pi} (\Pi - x)\sin(\Pi - x)\cos^2(\Pi - x)dx$$

$$\int_0^{\pi} x\sin x\cos^2 x dx = \int_0^{\pi} (\Pi - x)\sin x\cos^2 x dx$$

$$\int_0^{\pi} x\sin x\cos^2 x dx = \int_0^{\pi} \Pi\sin x\cos^2 x dx - \int_0^{\pi} x\sin x\cos^2 x dx$$

$$2\int_0^{\pi} x\sin x\cos^2 x dx = \int_0^{\pi} \Pi\sin x\cos^2 x dx$$

$$\int_0^{\pi} x\sin x\cos^2 x dx = \frac{\Pi}{2}\int_0^{\pi} \sin x\cos^2 x dx$$

Now

$$\int_0^{\pi} \sin x\cos^2 x dx$$

Let cosx=t

$$\sin x dx = -dt$$

$$-\int_1^{-1} t^2 dt$$

$$\int_{-1}^{1} t^2 dt$$

$$\left\{\frac{t^3}{3}\right\}_{-1}^{1}$$

$$\frac{2}{3}$$

$$\therefore \int_0^{\pi} x\sin x\cos^2 x dx = \frac{\pi}{2}\times\frac{2}{3} = \frac{\pi}{3}$$

Definite Integrals Ex 20.4B Q22

We have,

$$I = \int_0^{\frac{\pi}{2}} \frac{x\sin x.\cos x}{\sin^4 x + \cos^4 x}dx \qquad --(i)$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\left(\frac{\pi}{2} - x\right)\cos x.\sin x}{\cos^4 x + \sin^4 x}dx \qquad --(ii)$$

Adding (i) & (ii)

$$2I = \frac{\pi}{2}\int_0^{\frac{\pi}{2}} \frac{\sin x.\cos x}{\cos^4 x + \sin^4 x}dx$$

$$2I = \frac{\pi}{4}\int_0^{\frac{\pi}{2}} \frac{2\sin x.\cos x}{\cos^4 x + \sin^4 x}dx$$

Let $t = \sin^2 x$

$$\Rightarrow 2I = \frac{\pi}{4}\int_0^1 \frac{1}{(1-t)^2 + t^2}dt$$

$$\Rightarrow 2I = \frac{\pi}{8}\int_0^1 \frac{1}{\left(t - \frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2}dt$$

$$\Rightarrow 2I = \frac{\pi}{8}\times 2\left[\tan^{-1}(2t-1)\right]_0^1$$

$$\Rightarrow I = \frac{\pi}{8}\left[\frac{\pi}{4} + \frac{\pi}{4}\right] = \frac{\pi^2}{16}$$

Definite Integrals Ex 20.4B Q23

$$\text{Let } I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^3 x\, dx$$

$$f(-x) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^3(-x)\, dx$$

$$= -\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^3 x\, dx$$

Here $f(x) = -f(+x)$

Hence $f(x)$ is odd function.

So,
$I = 0$

Definite Integrals Ex 20.4B Q24

We have,

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^4 x\, dx = 2\int_{0}^{\frac{\pi}{2}} \sin^4 x\, dx \qquad [\because \sin^4 x \text{ is an even function}]$$

$$= 2\int_{0}^{\frac{\pi}{2}} \left(\sin^2 x\right)^2 dx$$

$$= 2\int_{0}^{\frac{\pi}{2}} \left(\frac{1-\cos 2x}{2}\right)^2 dx$$

$$= \frac{1}{2}\int_{0}^{\frac{\pi}{2}} (1-\cos 2x)^2 dx$$

$$= \frac{1}{2}\left[\int_{0}^{\frac{\pi}{2}} \left(1+\cos^2 2x - 2\cos 2x\right) dx\right]$$

$$= \frac{1}{2}\left[\int_{0}^{\frac{\pi}{2}} \left(1-2\cos 2x + \frac{1+\cos 4x}{2}\right) dx\right]$$

$$= \frac{1}{4}\left[\int_{0}^{\frac{\pi}{2}} \left(3-4\cos 2x + \cos 4x\right) dx\right]$$

$$= \frac{1}{4}\left[3x - \frac{4\sin 2x}{2} + \frac{\sin 4x}{4}\right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{4}\left[\left\{\frac{3\pi}{2} - 2\sin\pi + \frac{1}{4}\sin 2\pi\right\} - \{0-0+0\}\right]$$

$$= \frac{1}{4}\left[\frac{3\pi}{2} - 0 + 0\right] = \frac{1}{4}\times\frac{3\pi}{2}$$

$$= \frac{3\pi}{8}$$

$$\therefore \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^4 x\, dx = \frac{3\pi}{8}$$

Definite Integrals Ex 20.4B Q25

We have,

$$I = \int_{-1}^{1} \log\left(\frac{2-x}{2+x}\right) dx$$

$$\textit{Since,} \quad \log\left\{\frac{2-(-x)}{2+(-x)}\right\} = -\log\left(\frac{2-x}{2+x}\right) \quad \therefore \text{ This is an odd function.}$$

Hence,
$I = 0$

Definite Integrals Ex 20.4B Q26

We have,

$$I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin^2 x\, dx$$

$\sin^2 x$ is even function.

Hence,

$$I = 2\int_0^{\frac{\pi}{4}} \sin^2 x\, dx = 2\int_0^{\frac{\pi}{4}} \left(\frac{1-\cos 2x}{2}\right) dx = \frac{2}{2}\left[x - \frac{\sin 2x}{2}\right]_0^{\frac{\pi}{4}} = \frac{1}{2}\left[\frac{2\pi}{4} - \sin\frac{\pi}{2} - 0 + \sin 0\right]$$

$$= \frac{1}{2}\left[\frac{2\pi}{4} - 1\right]$$

$$= \frac{\pi}{4} - \frac{1}{2}$$

$$\therefore \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin^2 x\, dx = \frac{\pi}{4} - \frac{1}{2}$$

Definite Integrals Ex 20.4B Q27

$$I = \int_0^{\pi} \log(1-\cos x)\, dx$$

$$= \int_0^{\pi} \log\left(2\sin^2\frac{x}{2}\right) dx$$

$$= \int_0^{\pi} \log 2\, dx + \int_0^{\pi} \log \sin^2\frac{x}{2}\, dx$$

$$= \int_0^{\pi} \log 2\, dx + 2\int_0^{\pi} \log \sin\frac{x}{2}\, dx$$

$$I = \log 2[x]_0^{\pi} + 4\int_0^{\frac{\pi}{2}} \log \sin t\, dt \qquad \left[\text{Put} \quad t = \frac{x}{2} \Rightarrow dt = \frac{1}{2}dx\right]$$

$$I = \pi \log 2 + 4I_1 \qquad \ldots(i)$$

$$I_1 = \int_0^{\frac{\pi}{2}} \log \sin t\, dt \qquad \ldots(ii)$$

$$= \int_0^{\frac{\pi}{2}} \log \cos t\, dt \qquad \ldots(iii)$$

Adding (ii) & (iii) we get

$$2I_1 = \int_0^{\frac{\pi}{2}} \log \sin t.\cos t\, dt = \int_0^{\frac{\pi}{2}} \log\left(\frac{\sin 2t}{2}\right) dt = \int_0^{\frac{\pi}{2}} \log \sin 2t\, dt - \frac{\pi}{2}\log 2$$

We know the property $\int_a^b f(x) = \int_a^b f(t)$

$$2I_1 = I_1 - \frac{\pi}{2}\log 2$$

$$\Rightarrow I_1 = -\frac{\pi}{2}\log 2 \qquad \ldots(iv)$$

Putting the value from (iv) to (i)

$$I = \pi \log 2 + 4\left(-\frac{\pi}{2}\log 2\right) = \pi\log 2 - 2\pi\log 2 = -\pi\log 2$$

$$I = -\pi \log 2$$

Definite Integrals Ex 20.4B Q28

We have,

$$I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \log\left(\frac{2-\sin x}{2+\sin x}\right)dx$$

Let $f(x) = \log\left(\frac{2-\sin x}{2+\sin x}\right)$

Then,

$$f(-x) = \log\left(\frac{2-\sin(-x)}{2+\sin(-x)}\right) = -\log\left(\frac{2-\sin x}{2+\sin x}\right) = -f(x)$$

Thus, f(x) is an odd function.

$$\therefore I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \log\left(\frac{2-\sin x}{2+\sin x}\right)dx = 0$$

Definite Integrals Ex 20.4B Q29

$$I = \int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x}\,dx$$

$$I = \int_{-\pi}^{\pi} \frac{2x}{1+\cos^2 x}\,dx + \int_{-\pi}^{\pi} \frac{2x\sin x}{1+\cos^2 x}\,dx$$

$$I = 0 + \int_{-\pi}^{\pi} \frac{2x\sin x}{1+\cos^2 x}\,dx \ldots\ldots\ldots\left[\because \frac{2x}{1+\cos^2 x} \text{ is an odd function}\right]$$

$$I = 2\int_{0}^{\pi} \frac{2x\sin x}{1+\cos^2 x}\,dx \ldots\ldots\ldots\left[\because \frac{2x\sin x}{1+\cos^2 x} \text{ is an even function}\right]$$

$$I = 4\int_{0}^{\pi} \frac{x\sin x}{1+\cos^2 x}\,dx$$

$$I = 2\pi\int_{0}^{\pi} \frac{\sin x}{1+\cos^2 x}\,dx \ldots\ldots\ldots\left[\because \int_{0}^{a} xf(x)dx = \frac{a}{2}\int_{0}^{a} f(x)dx\right]$$

Put $\cos x = t$ then $-\sin x\, dx = dt$

$$I = -2\pi\int_{1}^{-1} \frac{1}{1+t^2}\,dt$$

$$I = -2\pi\left[\tan^{-1} t\right]_{1}^{-1}$$

$$I = \pi^2$$

Definite Integrals Ex 20.4B Q30

$$I = \int_{-a}^{a} \log\left(\frac{a-\sin\theta}{a+\sin\theta}\right)d\theta$$

Let $f(\theta) = \log\left(\frac{a-\sin\theta}{a+\sin\theta}\right)$

$$f(-\theta) = \log\left(\frac{a-\sin(-\theta)}{a+\sin(-\theta)}\right) = -\log\left(\frac{a-\sin\theta}{a+\sin\theta}\right) = -f(\theta)$$

$\therefore$ $f(\theta) = \log\left(\frac{a-\sin\theta}{a+\sin\theta}\right)$ is an odd function.

$$\therefore I = \int_{-a}^{a} \log\left(\frac{a-\sin\theta}{a+\sin\theta}\right)d\theta = 0$$

Definite Integrals Ex 20.4B Q31

$$I = \int_{-2}^{2} \frac{3x^3 + 2|x|+1}{x^2+|x|+1}\, dx$$

$$I = \int_{-2}^{2} \frac{3x^3}{x^2+|x|+1}\, dx + \int_{-2}^{2} \frac{2|x|+1}{x^2+|x|+1}\, dx$$

$$I = 0 + \int_{-2}^{2} \frac{2|x|+1}{x^2+|x|+1}\, dx \ldots\ldots\ldots \left[\because \frac{3x^3}{x^2+|x|+1} \text{ is an odd function}\right]$$

$$I = 2\int_{0}^{2} \frac{2|x|+1}{x^2+|x|+1}\, dx \ldots\ldots\ldots \left[\because \frac{2|x|+1}{x^2+|x|+1} \text{ is an even function}\right]$$

$$I = 2\left[\log\left(x^2+|x|+1\right)\right]_0^2$$

$$I = 2\left[\log(4+2+1) - \log(1)\right]$$

$$I = 2\log_e(7)$$

Definite Integrals Ex 20.4B Q32

$$I = \int_{-3\pi/2}^{-\pi/2} \left\{\sin^2(3\pi + x) + (\pi + x)^3\right\}\, dx$$

Substitute $\pi + x = u$ then $dx = du$

$$I = \int_{-\pi/2}^{\pi/2} \left\{\sin^2(2\pi + u) + (u)^3\right\}\, du$$

$$I = \int_{-\pi/2}^{\pi/2} \left\{\sin^2(u) + (u)^3\right\}\, du$$

$$I = \left[\frac{1}{2}\left(u - \frac{1}{2}\sin(2u)\right) + \frac{u^4}{4}\right]_{-\pi/2}^{\pi/2}$$

$$I = \frac{\pi}{2}$$

Definite Integrals Ex 20.4B Q33

Let $I = \int_0^2 x\sqrt{2-x}\,dx$

$$I = \int_0^2 (2-x)\sqrt{x}\,dx \qquad \left(\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right)$$

$$= \int_0^2 \left\{2x^{\frac{1}{2}} - x^{\frac{3}{2}}\right\} dx$$

$$= \left[2\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right) - \frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right]_0^2$$

$$= \left[\frac{4}{3}x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}}\right]_0^2$$

$$= \frac{4}{3}(2)^{\frac{3}{2}} - \frac{2}{5}(2)^{\frac{5}{2}}$$

$$= \frac{4\times 2\sqrt{2}}{3} - \frac{2}{5}\times 4\sqrt{2}$$

$$= \frac{8\sqrt{2}}{3} - \frac{8\sqrt{2}}{5}$$

$$= \frac{40\sqrt{2} - 24\sqrt{2}}{15}$$

$$= \frac{16\sqrt{2}}{15}$$

Definite Integrals Ex 20.4B Q34

$$Let\ I=\int_0^1 \log\left(\frac{1}{x}-1\right)dx$$

$$=\int_0^1 \log\left(\frac{1-x}{x}\right)dx$$

$$=\int_0^1 \log(1-x)dx-\int_0^1 \log(x)dx$$

$$Applying\ the\ property,\ \int_0^a f(x)dx=\int_0^a f(a-x)dx$$

$$Thus,\ I=\int_0^1 \log(1-(1-x))dx-\int_0^1 \log(x)dx$$

$$=\int_0^1 \log(1-1+x)dx-\int_0^1 \log(x)dx$$

$$=\int_0^1 \log(x)dx-\int_0^1 \log(x)dx$$

$$=0$$

Definite Integrals Ex 20.4B Q35

$$I=\int_{-1}^{1}|x\cos\pi x|dx$$

Let $f(x)=|x\cos\pi x|$

$$f(-x)=|-x\cos(-\pi x)|=|-x\cos(\pi x)|=|x\cos\pi x|=f(x)$$

$$\therefore\ I=\int_{-1}^{1}|x\cos\pi x|dx=2\int_0^1|x\cos\pi x|dx$$

Now,

$$f(x)=|x\cos\pi x|=\begin{cases}x\cos\pi x\ ,\ \text{if } 0\le x\le\frac{1}{2}\\ -x\cos\pi x\ ,\ \text{if } \frac{1}{2}<x<1\end{cases}$$

$$\therefore\ I=2\int_0^1|x\cos\pi x|dx$$

$$\Rightarrow I=2\left[\int_0^{\frac{1}{2}}x\cos\pi x\ dx+\int_{\frac{1}{2}}^{1}-x\cos\pi x\,dx\right]$$

$$\Rightarrow I=2\left\{\left[\frac{x}{\pi}\sin\pi x+\frac{1}{\pi^2}\cos\pi x\right]_0^{\frac{1}{2}}-\left[\frac{x}{\pi}\sin\pi x+\frac{1}{\pi^2}\cos\pi x\right]_{\frac{1}{2}}^{1}\right\}$$

$$\Rightarrow I=2\left\{\left[\frac{1}{2\pi}-\frac{1}{\pi^2}\right]-\left[-\frac{1}{\pi^2}-\frac{1}{2\pi}\right]\right\}$$

$$\Rightarrow I=\frac{2}{\pi}$$

Definite Integrals Ex 20.4B Q36

$$I=\int_0^{\Pi}\left(\frac{x}{1+\sin^2 x}+\cos^7 x\right)dx$$

$$I=\int_0^{\Pi}\left(\frac{\Pi-x}{1+\sin^2(\Pi-x)}+\cos^7(\Pi-x)\right)dx$$

$$I=\int_0^{\Pi}\left(\frac{\Pi-x}{1+\sin^2 x}-\cos^7 x\right)dx$$

$$2I=\int_0^{\Pi}\left(\frac{\Pi}{1+\sin^2 x}\right)dx$$

$$2I=\Pi\int_0^{\Pi}\frac{1}{1+\sin^2 x}dx$$

$$2I=\Pi\int_0^{\Pi}\frac{1}{1+2\tan^2 x}\sec^2 x dx$$

$$\mathrm{I}=\pi\int_0^{\frac{\pi}{2}}\frac{1}{1+2\tan^2 x}\sec^2 x dx \ldots\ldots\ldots\left[\because \int_0^{2a} f(x)dx=2\int_0^{a} f(x)dx, \text{ if } f(2a-x)=f(x)\right]$$

Let $\tan x = v$

$dv = \sec^2 x\ dx$

$$\Rightarrow \mathrm{I}=\pi\int_0^{\infty}\frac{1}{1+2v^2}dv$$

$$\Rightarrow \mathrm{I}=\pi\left[\frac{\tan^{-1}\left(\sqrt{2}v\right)}{\sqrt{2}}\right]_0^{\infty}$$

$$\Rightarrow \mathrm{I}=\pi\left[\frac{\pi}{2\sqrt{2}}\right]$$

$$\Rightarrow \mathrm{I}=\frac{\pi^2}{2\sqrt{2}}$$

Definite Integrals Ex 20.4B Q37

$$I = \int_0^{\pi} \frac{x}{1+\cos\alpha \sin x}dx$$

Then,

$$I = \int_0^{\pi} \frac{(\pi - x)}{1+\cos\alpha \sin(\pi - x)}dx$$

$$I = \int_0^{\pi} \frac{(\pi - x)}{1+\cos\alpha \sin x}dx$$

$$2I = \pi\int_0^{\pi} \frac{1}{1+\cos\alpha \sin x}dx$$

$$2I = \pi\int_0^{\pi} \frac{1+\tan^2\left(\frac{x}{2}\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)+2\cos\alpha\tan\left(\frac{x}{2}\right)}dx$$

$$I = \frac{\pi}{2}\int_0^{\pi} \frac{\sec^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+2\cos\alpha\tan\left(\frac{x}{2}\right)+1}dx$$

Put $\tan\left(\frac{x}{2}\right) = t$ then $\sec^2\left(\frac{x}{2}\right)dx = 2dt$

$x = 0 \Rightarrow t = 0$ and $x = \pi \Rightarrow t = \infty$

$$I = \frac{\pi}{2}\int_0^{\infty} \frac{2}{t^2+2t\cos\alpha+1}dt$$

$$I = \pi\int_0^{\infty} \frac{1}{(t+\cos\alpha)^2+(1-\cos^2\alpha)}dt$$

$$I = \pi\int_0^{\infty} \frac{1}{(t+\cos\alpha)^2+\sin^2\alpha}dt$$

$$I = \frac{\pi}{\sin\alpha}\left[\tan^{-1}\left(\frac{t+\cos\alpha}{\sin\alpha}\right)\right]_0^{\infty}$$

$$I = \frac{\pi\alpha}{\sin\alpha}$$

Definite Integrals Ex 20.4B Q38

We know

$$\int_0^{2a} f(x)dx = \int_0^{a} f(x) + \int_0^{a} f(2a-x)dx$$

Also here

$$f(x) = f(2\Pi - x)$$

So

$$I = \int_0^{2\Pi} \sin^{100} x \cos^{101} x dx = 2\int_0^{\Pi} \sin^{100} x \cos^{101} x dx$$

$$I = 2\int_0^{\Pi} \sin^{100}(\Pi - x)\cos^{101}(\Pi - x)dx$$

$$I = -2\int_0^{\Pi} \sin^{100} x \cos^{101} x dx$$

Hence

$$2I = 0$$

$$I = 0$$

Definite Integrals Ex 20.4B Q39

$$I = \int_0^{\pi/2} \frac{a\sin x + b\cos x}{\sin x + \cos x}\,dx$$

Then,

$$I = \int_0^{\pi/2} \frac{a\sin\left(\frac{\pi}{2} - x\right) + b\cos\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)}\,dx$$

$$I = \int_0^{\pi/2} \frac{a\cos x + b\sin x}{\cos x + \sin x}\,dx$$

$$2I = \int_0^{\pi/2} \frac{a\sin x + b\cos x}{\sin x + \cos x}\,dx + \int_0^{\pi/2} \frac{a\cos x + b\sin x}{\cos x + \sin x}\,dx$$

$$2I = (a+b)\int_0^{\pi/2} \frac{\sin x + \cos x}{\sin x + \cos x}\,dx$$

$$I = \frac{(a+b)}{2}\int_0^{\pi/2} 1\,dx$$

$$I = \frac{(a+b)\pi}{4}$$

Definite Integrals Ex 20.4B Q40

We have,

$$I = \int_0^{2a} f(x)\,dx$$

Then

$$I = \int_0^{a} f(x)\,dx + \int_a^{2a} f(x)\,dx$$

$$I = \int_0^{a} f(x)\,dx + I_1$$

where, $I_1 = \int_a^{2a} f(x)\,dx$

Let $2a - t = x$ then $dx = -dt$

If $t = a \Rightarrow x = a$

If $t = 2a \Rightarrow x = 0$

$$I_1 = \int_0^{2a} f(x)\,dx = \int_a^{0} f(2a - t)(-dt) = -\int_a^{0} f(2a - t)\,dt$$

$$I_1 = \int_0^{a} f(2a - t)\,dt = \int_0^{a} f(2a - x)\,dx$$

$$\therefore\ I = \int_0^{a} f(x)\,dx + \int_0^{a} f(2a - x)\,dx$$

$$I = \int_0^{a} f(x)\,dx + \int_0^{a} f(x)\,dx = 2\int_0^{a} f(x)\,dx \quad \left[f(2a - x) = f(x)\right]$$

Hence Proved.

Definite Integrals Ex 20.4B Q41

We have,

$$I = \int_0^{2a} f(x)\,dx = \int_0^{a} f(x)\,dx + \int_a^{2a} f(x)\,dx$$

$$I = \int_0^{a} f(x)\,dx + I_1$$

Let $2a - t = x$ then $dx = -dt$

$t = a,\ x = a$

$t = 2a \quad x = 0$

$$I_1 = \int_0^{2a} f(x) = \int_a^{0} f(2a - t)(-dt)$$

$$= -\int_a^{0} f(2a - t)\,dt$$

$$I_1 = \int_0^{a} f(2a - t)\,dt = \int_0^{a} f(2a - x)\,dx$$

$$I = \int_0^{a} f(x)\,dx + \int_0^{a} f(2a - x)\,dx$$

$$I = \int_0^{a} f(x)\,dx - \int_0^{a} f(x) \qquad [\because f(2a - x) = -f(x)]$$

$$I = 0$$

Hence,

$$\int_0^{2a} f(x)\,dx = 0$$

Definite Integrals Ex 20.4B Q42

(i) We have,

$$I = \int_{-a}^{a} f(x^2)\,dx$$

Clearly $f(x^2)$ is an even function.

So,

$$\int_{-a}^{a} f(t) = 2\int_0^{a} f(t)$$

$$I = 2\int_0^{a} f(x^2)\,dx$$

(ii) We have,

$$I = \int_{-a}^{a} xf(x^2)\,dx$$

Clearly, $xf(x^2)$ is odd function.

So, $I = 0$

$$\therefore \int_{-a}^{a} xf(x^2)\,dx = 0$$

Definite Integrals Ex 20.4B Q43

We have from LHS,

$$I = \int_0^{2a} f(x)\,dx = \int_0^a f(x)\,dx + \int_a^{2a} f(x)\,dx \qquad \ldots(i)$$

Let $x = 2a - t$, then $dx = -dt$

$x = a \Rightarrow t = a$, and $x = 2a \Rightarrow t = 0$

$$\therefore \int_0^{2a} f(x)\,dx = -\int_a^0 f(2a-t)\,dt$$

$$\Rightarrow \int_0^{2a} f(x)\,dx = \int_0^a f(2a-t)\,dt$$

$$\Rightarrow \int_0^{2a} f(x)\,dx = \int_0^a f(2a-x)\,dx$$

Substituting $\int_0^{2a} f(x)\,dx = \int_0^a f(2a-x)\,dx$ in (i)

we get,

$$\int_0^{2a} f(x)\,dx = \int_0^a f(x)\,dx + \int_0^a f(2a-x)\,dx$$

$$\Rightarrow \int_0^{2a} f(x)\,dx = \int_0^a \{f(x) + f(2a-x)\}\,dx$$

Definite Integrals Ex 20.4B Q44

$$I = \int_a^b x f(x)\,dx$$

$$\Rightarrow I = \int_a^b (a+b-x) f(a+b-x)\,dx$$

$$\Rightarrow I = \int_a^b (a+b-x) f(x)\,dx \ldots\ldots\ldots\ldots [\text{Given that } f(a+b-x) = f(x)]$$

$$\Rightarrow I = \int_a^b (a+b) f(x)\,dx - \int_a^b x f(x)\,dx$$

$$\Rightarrow I = \int_a^b (a+b) f(x)\,dx - I$$

$$\Rightarrow 2I = \int_a^b (a+b) f(x)\,dx$$

$$\Rightarrow I = \frac{a+b}{2}\int_a^b f(x)\,dx$$

Definite Integrals Ex 20.4B Q45

We have,

$$I = \int_{-a}^a f(x)\,dx = \int_{-a}^0 f(x)\,dx + \int_0^a f(x)\,dx$$

Let $x = -t$ then $dx = -dt$

$x = -a \Rightarrow t = a$

$x = 0 \Rightarrow t = 0$

$$\therefore \int_{-a}^a f(x)\,dx = \int_a^0 f(-t)(-dt) = -\int_a^0 f(-t)\,dt$$

$$\Rightarrow \int_{-a}^a f(x)\,dx = \int_0^a f(-t)\,dt$$

$$\Rightarrow \int_{-a}^0 f(x)\,dx = \int_0^a f(-x)\,dx$$

$$\therefore \int_{-a}^a f(x)\,dx = \int_0^a f(-x)\,dx + \int_0^a f(x)\,dx$$

Hence,

$$\int_{-a}^a f(x)\,dx = \int_0^a \{f(-x) + f(x)\}\,dx$$

Proved

Definite Integrals Ex 20.4B Q46

$$I = \int_0^{\Pi} xf(\sin x)dx$$

$$I = \int_0^{\Pi} (\Pi - x)f(\sin(\Pi - x))dx$$

$$I = \int_0^{\Pi} (\Pi - x)f(\sin x)dx$$

$$2I = \int_0^{\Pi} \Pi f(\sin x)dx$$

$$I = \frac{\Pi}{2}\int_0^{\Pi} f(\sin x)dx$$

Ex 20.5

Definite Integrals Ex 20.5 Q1

We have,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----+f(a+(n-1)h)\right], \text{ where } h=\frac{b-a}{n}$$

Here, $a=0$, $b=3$ and $f(x)=(x+4)$

$$h=\frac{3}{n} \Rightarrow nh=3$$

Thus, we have,

$$\Rightarrow I = \int_0^3 (x+4)\,dx$$

$$\Rightarrow I = \lim_{h\to 0} h\left[f(0)+f(h)+f(2h)+----f((n-1)h)\right]$$

$$\Rightarrow I = \lim_{h\to 0} h\left[4+(h+4)+(2h+4)+----+((n-1)h+4)\right]$$

$$\Rightarrow I = \lim_{h\to 0} h\left[4n+h(1+2+3+----+(n-1))\right]$$

$$\Rightarrow I = \lim_{h\to 0} h\left[4n+h\left(\frac{n(n-1)}{2}\right)\right] \qquad \left[\because h\to 0 \ \& \ h=\frac{3}{n} \Rightarrow n\to\infty\right]$$

$$\Rightarrow I = \lim_{n\to\infty} \frac{3}{n}\left[4n+\frac{3}{n}\frac{(n^2-1)}{2}\right]$$

$$\Rightarrow I = \lim_{n\to\infty} 12+\frac{9}{2}\left(1-\frac{1}{n}\right)$$

$$= 12+\frac{9}{2} = \frac{33}{2}$$

$$\therefore \int_0^3 (x+4)\,dx = \frac{33}{2}$$

Definite Integrals Ex 20.5 Q2

We have,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----+f(a+(n-1)h)\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here $a = 0$, $b = 2$

$$\Rightarrow h = \frac{2}{n} \ \& \ f(x) = x+3$$

Thus, we have,

$$I = \int_0^2 (x+3)\,dx$$

$$\Rightarrow I = \lim_{h\to 0} h\left[f(0)+f(h)+f(2h)+----f((n-1)h)\right]$$

$$\Rightarrow I = \lim_{h\to 0} h\left[3+(h+3)+(2h+3)+(3h+3)----+(n-1)h+3\right]$$

$$= \lim_{h\to 0} h\left[3n+h(1+2+3+----(n-1))\right]$$

$$= \lim_{h\to 0} h\left[3n+h\frac{n(n-1)}{2}\right]$$

$$\because h = \frac{2}{n} \ \& \text{if } h\to 0 \Rightarrow n\to\infty$$

$$= \lim_{n\to\infty} \frac{2}{n}\left[3n+\frac{2}{n}\frac{n(n-1)}{2}\right]$$

$$= \lim_{n\to\infty} \left[6+\frac{2}{n}n^2\left(1-\frac{1}{n}\right)\right]$$

$$= 6+2 = 8$$

$$\therefore \int_0^2 (x+3)\,dx = 8$$

Definite Integrals Ex 20.5 Q3

We have,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----+f(a+(n-1)h)\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here $a = 1$, $b = 3$ and $f(x) = 3x-2$

$$h = \frac{2}{n} \Rightarrow nh = 2$$

Thus, we have,

$$I = \int_1^3 (3x-2)\,dx$$

$$\Rightarrow I = \lim_{h\to 0} h\left[f(1)+f(1+h)+f(1+2h)+----f(1+(n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[1+\{3(1+h)-2\}+\{3(1+2h)-2\}+-----+\ \{3(1+(n-1)h)-2\}\right]$$

$$= \lim_{h\to 0} h\left[n+3h(1+2+3+----(n-1))\right]$$

$$= \lim_{h\to 0} h\left[n+3h\frac{n(n-1)}{2}\right]$$

$$\because h = \frac{2}{n} \qquad \therefore \text{if } h\to 0 \Rightarrow n\to\infty$$

$$\therefore \lim_{n\to\infty} \frac{2}{n}\left[n+\frac{6}{n}\frac{n(n-1)}{2}\right]$$

$$= \lim_{n\to\infty} 2+\frac{6}{n^2}n^2\left(1-\frac{1}{n}\right)$$

$$= \lim_{n\to\infty} 2+6 = 8$$

$$\therefore \int_1^3 (3x-2)\,dx = 8$$

Definite Integrals Ex 20.5 Q4

We have,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\left[f(a) + f(a+h) + f(a+2h) + ---- + f(a+(n-1)h)\right]$$

where $h = \dfrac{b-a}{n}$

Here $a = -1,\ b = 1$ and $f(x) = x + 3$

$\therefore h = \dfrac{2}{n} \Rightarrow nh = 2$

Thus, we have,

$$I = \int_{-1}^{1}(x+3)\,dx$$

$$I = \lim_{h\to 0} h\left[f(-1) + f(-1+h) + f(-1+2h) + ---- + f(-1+(n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[2 + (2+h) + (2+2h) + ---- + \{(n-1)h + 2\}\right]$$

$$= \lim_{h\to 0} h\left[2n + h(1+2+3+----)\right]$$

$$= \lim_{h\to 0} h\left[2n + h\frac{n(n-1)}{2}\right] \qquad \left[\because h = \frac{2}{n}\ \&\ \text{if } h\to 0 \Rightarrow n\to\infty\right]$$

$$= \lim_{n\to\infty} \frac{2}{n}\left[2n + \frac{2}{n}\frac{n(n-1)}{2}\right]$$

$$= \lim_{n\to\infty}\ 4 + \frac{2n^2}{n^2}\left(1 - \frac{1}{n}\right)$$

$$= 4 + 2 = 6$$

$$\therefore \int_{-1}^{1}(x+3)\,dx = 6$$

Definite Integrals Ex 20.5 Q5

We have,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\left[f(a) + f(a+h) + f(a+2h) + ---- f(a+(n-1)h)\right]$$

where $h = \dfrac{b-a}{n}$

Here $a = 0,\ b = 5$

and $f(x) = (x+1)$

$\therefore h = \dfrac{5}{n} \Rightarrow nh = 5$

Thus, we have,

$$I = \int_0^5 (x+1)\,dx$$

$$I = \lim_{h\to 0} h\left[f(0) + f(h) + f(2h) + ---- f\{(n-1)h\}\right]$$

$$= \lim_{h\to 0} h\left[1 + (h+1) + (2h+1) + ----- + ((n-1)h + 1)\right]$$

$$= \lim_{h\to 0} h\left[n + h(1+2+3+----(n-1))\right]$$

$\because h = \dfrac{5}{n}$ and if $h\to 0,\ n\to\infty$

$$= \lim_{n\to\infty} \frac{5}{n}\left[n + \frac{5}{n}\frac{n(n-1)}{2}\right]$$

$$= \lim_{n\to\infty}\ 5 + \frac{25}{2n^2}\, n^2\left(1 - \frac{1}{n}\right)$$

$$= 5 + \frac{25}{2}$$

$$\therefore \int_0^5 (x+1)\,dx = \frac{35}{2}$$

Definite Integrals Ex 20.5 Q6

We have,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

where $h = \dfrac{b-a}{n}$

Here, $a = 1$, $b = 3$

and $f(x) = (2x+3)$

$\therefore\ h = \dfrac{2}{n} \Rightarrow nh = 2$

Thus, we have,

$I = \int_1^3 (2x+3)\,dx$

$= \lim_{h\to 0} h\left[f(1)+f(1+h)+f(1+2h)+----f(1+(n-1)h)\right]$

$= \lim_{h\to 0} h\left[2+3+\{2(1+h)+3\}+\{2(1+2h)+3\}----+2\{1+(n-1)+3\}\right]$

$= \lim_{h\to 0} h\left[5+(5+2h)+(5+4h)+----5+2(n-1)h\right]$

$= \lim_{h\to 0} h\left[5n+2h(1+2+3+----(n-1))\right]$

$\because h = \dfrac{2}{n}$ and if $h \to 0 \Rightarrow n \to \infty$

$\therefore\ \lim_{n\to\infty} \dfrac{2}{n}\left[5n+\dfrac{4}{n}\dfrac{n(n-1)}{2}\right]$

$= \lim_{n\to\infty} \left[10+\dfrac{4}{n^2}\dfrac{n(n-1)}{2}\right] = 14$

$\therefore\ \int_1^3 (2x+3)\,dx = 14$

Definite Integrals Ex 20.5 Q7

We have,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

where $h = \dfrac{b-a}{n}$

Here, $a = 3$, $b = 5$

and $f(x) = (2-x)$

$\therefore\ h = \dfrac{2}{n} \Rightarrow nh = 2$

Thus, we have,

$I = \int_3^5 (2-x)\,dx$

$= \lim_{h\to 0} h\left[f(3)+f(3+h)+f(3+2h)+----f(3+(n-1)h)\right]$

$= \lim_{h\to 0} h\left[(2-3)+\{2-(3+h)\}+\{2-(3+2h)\}+----\{2-(3+(n-1)h\}\right]$

$= \lim_{h\to 0} h\left[-1+(-1-h)+(-1-2h)+----\{-1-(n-1)h\}\right]$

$= \lim_{h\to 0} h\left[-n-h(1+2+----(n-1)h\right]$

$\because h = \dfrac{2}{n}$ & if $h \to 0 \Rightarrow n \to \infty$

$\therefore\ \lim_{n\to\infty} \dfrac{2}{n}\left[-n-\dfrac{2}{n}\dfrac{n(n-1)}{2}\right]$

$= \lim_{n\to\infty} -2-\dfrac{2}{n^2}n^2\left(1-\dfrac{1}{n}\right) = -2-2 = -4$

$\therefore\ \int_3^5 (2-x)\,dx = -4$

Definite Integrals Ex 20.5 Q8

We have,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

$$\text{where } h=\frac{b-a}{n}$$

Here $a=0,\ b=2$ and $f(x)=\left(x^2+1\right)$

$$\therefore\ h=\frac{2}{n}\Rightarrow nh=2$$

Thus, we have,

$$I=\int_0^2\left(x^2+1\right)dx$$

$$=\lim_{h\to 0} h\left[f(0)+f(h)+f(2h)+----f\{(n-1)h\}\right]$$

$$=\lim_{h\to 0} h\left[1+\left(h^2+1\right)+\left\{(2h)^2+1\right\}+----\left\{\left((n-1)h\right)^2+1\right\}\right]$$

$$=\lim_{h\to 0} h\left[n+h^2\left(1+2^2+3^2+----+(n-1)^2\right)\right]$$

$$\because h=\frac{2}{n}\ \&\text{ if } h\Rightarrow 0\Rightarrow n\to\infty$$

$$\therefore\ \lim_{n\to\infty}\frac{2}{n}\left[n+\frac{4}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$$

$$=\lim_{n\to\infty} 2+\frac{4}{3n^3}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)$$

$$=2+\frac{4}{3}\times 2=\frac{14}{3}$$

$$\therefore\ \int_0^2\left(x^2+1\right)dx=\frac{14}{3}$$

Definite Integrals Ex 20.5 Q9

We have,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

$$\text{where } h=\frac{b-a}{n}$$

Here $a=1,\ b=2$ and $f(x)=x^2$

$$\therefore\ h=\frac{1}{n}\Rightarrow nh=1$$

Thus, we have,

$$I=\int_1^2 x^2\,dx$$

$$=\lim_{h\to 0} h\left[f(1)+f(1+h)+f(1+2h)+----f(1+(n-1)h)\right]$$

$$=\lim_{h\to 0} h\left[1+(1+h)^2+(1+2h)^2+-----+\left(1+(n-1)h\ \right)^2\right]$$

$$=\lim_{h\to 0} h\left[1+\left(1+2h+h^2\right)+\left(1+2\times 2h+2\times 2h^2\right)+----\left(1+2\times(n-1)h+(1-n)^2h^2\right)\right]$$

$$=\lim_{h\to 0} h\left[n+2h\{1+2+3---(n-1)\}+h^2\left\{1^2+2^2+3^2+---(n-1)^2\right\}\right]$$

$$\because h=\frac{1}{n}\ \&\text{ if } h\to 0\Rightarrow n\to\infty$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[n+\frac{2}{n}\frac{n(n-1)}{2}+\frac{1}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$$

$$=\lim_{n\to\infty} 1+\frac{n^2}{n^2}\left(1-\frac{1}{n}\right)+\frac{1}{6n^3}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)$$

$$=1+1+\frac{2}{6}=\frac{7}{3}$$

$$\therefore\ \int_1^2 x^2\,dx=\frac{7}{3}$$

Definite Integrals Ex 20.5 Q10

We have,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h\right]$$

where $h = \dfrac{b-a}{n}$

Here $a = 2,\ b = 3$ and $f(x) = 2x^2+1$

$\therefore\ h = \dfrac{1}{n} \Rightarrow nh = 1$

Thus, we have,

$$I = \int_2^3 \left(2x^2+1\right)dx$$

$$= \lim_{h\to 0} h\left[f(2)+f(2+h)+f(2+2h)+----f(2+(n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[\left(2\times 2^2+1\right)\left\{2(2+h)^2+1\right\}+\left\{2(2+2h)^2+1\right\}+-----+\left\{2\left(2+(n-1)h\right)^2+1\right\}\right]$$

$$= \lim_{h\to 0} h\left[9n+8h(1+2+3---)+2h^2\left(1^2+2^2+3^2+--\right)\right]$$

$\because h = \dfrac{1}{n}$ & if $h\to 0 \Rightarrow n\to\infty$

$$= \lim_{n\to\infty} \frac{1}{n}\left[9n+\frac{8}{n}\frac{n(n-1)}{2}+\frac{2}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$$

$$= \lim_{n\to\infty}\ 9+\frac{4}{n^2}n^2\left(1-\frac{1}{n}\right)+\frac{1}{3n^3}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)$$

$$= 9+4+\frac{2}{3} = \frac{41}{3}$$

$$\therefore \int_2^3 \left(2x^2+1\right)dx = \frac{41}{3}$$

Definite Integrals Ex 20.5 Q11

We have,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h\right]$$

where $h = \dfrac{b-a}{n}$

Here $a = 1,\ b = 2$ and $f(x) = x^2-1$

$\therefore\ h = \dfrac{1}{n} \Rightarrow nh = 1$

Thus, we have,

$$I = \int_1^2 \left(x^2-1\right)dx$$

$$= \lim_{h\to 0} h\left[f(1)+f(1+h)+f(1+2h)+----f(1+(n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[\left(1^2-1\right)\left\{(1+h)^2-1\right\}+\left\{(1+2h)^2-1\right\}+-----+\left\{\left(1+(n-1)h\right)^2-1\right\}\right]$$

$$= \lim_{h\to 0} h\left[0+2h(1+2+3+---)+h^2\left(1+2^2+3^2+--\right)\right]$$

$\because h = \dfrac{1}{n}$ & if $h\to 0 \Rightarrow n\to\infty$

$$= \lim_{n\to\infty} \frac{1}{n}\left[\frac{2}{n}\frac{n(n-1)}{2}+\frac{1}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$$

$$= \lim_{n\to\infty}\ \frac{1}{n^2}n^2\left(1-\frac{1}{n}\right)+\frac{1}{6n^3}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)$$

$$= 1+\frac{2}{6} = \frac{4}{3}$$

$$\therefore \int_1^2 \left(x^2-1\right)dx = \frac{4}{3}$$

Definite Integrals Ex 20.5 Q12

We have,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

where $h = \dfrac{b-a}{n}$

Here $a = 0,\ b = 2$ and $f(x) = x^2 + 4$

$\therefore\ h = \dfrac{2}{n} \Rightarrow nh = 2$

Thus, we have,

$$I = \int_0^2 (x^2+4)\,dx$$

$$= \lim_{h\to 0} h\left[f(0)+f(h)+f(2h)+----f(0+(n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[4(h^2+4)+\{(2h)^2+4\}+----\{(n-1)h^2+4\}\right]$$

$\because h = \dfrac{2}{n}$ & if $h \to 0 \Rightarrow n \to \infty$

$$= \lim_{n\to\infty} \frac{2}{n}\left[4n + \frac{4}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$$

$$= \lim_{n\to\infty} 8 + \frac{4}{3n^2}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)$$

$$= 8 + \frac{4\times 2}{3} = \frac{32}{3}$$

$$\therefore\ \int_0^2 (x^2+4)\,dx = \frac{32}{3}$$

Definite Integrals Ex 20.5 Q13

We have,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

where $h = \dfrac{b-a}{n}$

Here $a = 1,\ b = 4$ and $f(x) = x^2 - x$

$h = \dfrac{3}{n} \Rightarrow nh = 3$

Thus, we have,

$$I = \int_1^4 (x^2-x)\,dx$$

$$= \lim_{h\to 0} h\left[f(1)+f(1+h)+f(1+2h)+----f(1+(n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[(1^2-1)+\{(1+h)^2-(1+h)\}+\{(1+2h)^2-(1+h)\}+----\right]$$

$$= \lim_{h\to 0} h\left[0+(h+h^2)+\{2h+(2h)^2\}+----\right]$$

$$= \lim_{h\to 0} h\left[h+(1+2+3+---(n-1))+h^2\{(1+2^2+3^2+---(n-1)^2\}\right]$$

$\because h = \dfrac{3}{n}$ & if $h \to 0 \Rightarrow n \to \infty$

$$= \lim_{n\to\infty} \frac{3}{n}\left[\frac{3}{n}\frac{n(n-1)}{2} + \frac{9}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$$

$$= \lim_{n\to\infty} \frac{9}{n^2}n^2\left(1-\frac{1}{n}\right) + \frac{3}{2n^3}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)$$

$$= \frac{9}{2} + 3 = \frac{27}{2}$$

$$\therefore\ \int_1^4 (x^2-x)\,dx = \frac{27}{2}$$

Definite Integrals Ex 20.5 Q14

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

where $h = \dfrac{b-a}{n}$

Here, $a = 0$, $b = 1$ and $f(x) = 3x^2 + 5x$

$$h = \frac{1}{n} \quad \Rightarrow nh = 1$$

Thus, we have,

$$I = \int_0^1 \left(3x^2 + 5x\right)dx$$

$$= \lim_{h\to 0} h\left[f(0)+f(0+h)+f(0+2h)+----f(0+(n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[\left\{0+\left(3h^2+5h\right)+\left\{3(2h)^2+5(2h)\right\}+----\right]$$

$$= \lim_{h\to 0} h\left[\left\{3h^2\left(1+2^2+3^2+---(n-1)^2\right)\right\}+5h\left\{1+2+3+---(n-1)\right\}\right]$$

$$\because h = \frac{1}{n} \quad \text{if } h\to 0 \Rightarrow n\to\infty$$

$$= \lim_{n\to\infty} \frac{1}{n}\left[\frac{3}{n^2}\frac{n(n-1)(2n-1)}{6}+\frac{5}{n}\frac{n(n-1)}{2}\right]$$

$$= \lim_{n\to\infty} \frac{3}{n^3}\frac{n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)}{6}+\frac{5}{2n^2}n^2\left(1-\frac{1}{n}\right)$$

$$= \frac{3\times 2}{6}+\frac{5}{2} = \frac{7}{2}$$

$$\therefore \int_0^1 \left(3x^2+5x\right)dx = \frac{7}{2}$$

Definite Integrals Ex 20.5 Q15

We have

$$\int_a^b f(x) = dx \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+...+f(a+(n-1)h)\right]$$

Where $h = \dfrac{b-a}{n}$

Here

$$a = 0, b = 2 \text{ and } f(x) = e^x$$

Now

$$h = \frac{2}{n}$$

$$nh = 2$$

Thus, we have

$$I = \int_0^2 e^x dx$$

$$= \lim_{h\to 0} h\left[f(0)+f(h)+f(2h)+...+f((n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[1+e^h+e^{2h}+...+e^{(n-1)h}\right]$$

$$= \lim_{h\to 0} h\left\{\frac{\left(e^h\right)^n-1}{e^h-1}\right\}$$

$$= \lim_{h\to 0} h\left\{\frac{e^{nh}-1}{e^h-1}\right\}$$

$$= \lim_{h\to 0} h\left\{\frac{e^2-1}{e^h-1}\right\} \quad [nh = 2]$$

$$= \lim_{h\to 0} \left\{\frac{e^2-1}{\frac{e^h-1}{h}}\right\}$$

$$= e^2 - 1$$

Definite Integrals Ex 20.5 Q16

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h\right]$$

where $h = \dfrac{b-a}{n}$

Here, $a = a,\ b = b$ and $f(x) = e^x$

$\therefore\quad h = \dfrac{b-a}{n} \Rightarrow nh = b-a$

Thus, we have,

$I = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$

$= \lim_{h\to 0} h\left[e^a + e^{a+h} + e^{a+2h} + ----e^{a+(n-1)h}\right]$

$= \lim_{h\to 0} he^a\left[1+e^h+e^{2h}+e^{3h}+----e^{(n-1)h}\right]$

$= \lim_{h\to 0} he^a\left[1+e^h+\left(e^h\right)^2+\left(e^h\right)^3+----\left(e^h\right)^{n-1}\right]$

$= \lim_{h\to 0} he^a\left\{\dfrac{\left(e^h\right)^n-1}{e^h-1}\right\}$ $\left[\because a+ar+ar^2+----ar^{n-1} = a\left\{\dfrac{r^n-1}{r-1}\right\} \text{ if } r>1\right]$

$= \lim_{h\to 0} he^a n\left\{\dfrac{e^{nh}-1}{nh}\right\}\times\left(\dfrac{h}{e^{h-1}}\right)$ $\left[\because \lim_{\theta\to 0}\dfrac{e^\theta-1}{\theta} = 1 \quad \& \quad nh = b-a\right]$

$\therefore \lim_{h\to 0}\left(e^{b-a}-1\right) = e^b - e^a$

$\therefore \int_a^b e^x dx = e^b - e^a$

Definite Integrals Ex 20.5 Q17

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+\ldots+f(a+(n-1)h)\right], \text{ where } h = \frac{b-a}{n}.$$

Since we have to find $\int_a^b \cos x dx$

We have, $\quad f(x) = \cos x$

$\therefore\quad I = \int_a^b \cos x dx$

$\Rightarrow\quad I = \lim_{h\to 0} h\left[\cos a + \cos(a+h) + \cos(a+2h) + \ldots + \cos(a+(n-1)h)\right]$

$\Rightarrow\quad I = \lim_{h\to 0} h\left[\dfrac{\cos\left(a+(n-1)\frac{h}{2}\right)\sin\frac{nh}{2}}{\sin\frac{h}{2}}\right] = \lim_{h\to 0} h\left[\dfrac{\cos\left(a+\frac{nh}{2}-\frac{h}{2}\right)\sin\frac{nh}{2}}{\sin\frac{h}{2}}\right]$

$\Rightarrow\quad I = \lim_{h\to 0} h\left[\dfrac{\cos\left(a+\frac{b-a}{2}-\frac{h}{2}\right)\sin\left(\frac{b-a}{2}\right)}{\sin\frac{h}{2}}\right]$ $\quad[\because nh = b-a]$

$\Rightarrow\quad I = \lim_{h\to 0}\left[\dfrac{\frac{h}{2}}{\sin\frac{h}{2}}\times 2\cos\left(\dfrac{a+b}{2}-\dfrac{h}{2}\right)\sin\left(\dfrac{b-a}{2}\right)\right]$

$\Rightarrow\quad I = \lim_{h\to 0}\left(\dfrac{\frac{h}{2}}{\sin\frac{h}{2}}\right)\times\lim_{h\to 0} 2\cos\left(\dfrac{a+b}{2}-\dfrac{h}{2}\right)\sin\left(\dfrac{b-a}{2}\right) = 2\cos\left(\dfrac{a+b}{2}\right)\sin\left(\dfrac{b-a}{2}\right)$

$\Rightarrow\quad I = \sin b - \sin a$ $\quad[\because 2\cos A\sin B = \sin(A-B) - \sin(A+B)]$

Definite Integrals Ex 20.5 Q18

We have,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here, $a = 0,\ b = \frac{\pi}{2}$ and $f(x) = \sin x$

$$\therefore \quad h = \frac{\frac{\pi}{2}-0}{n} = \frac{\pi}{2n} \qquad nh = \frac{2}{\pi}$$

Thus, we have,

$$I = \int_0^{\frac{\pi}{2}} \sin x\,dx$$

$$= \lim_{h\to 0} h\left[f(0)+f(0+h)+f(0+2h)+----f\{0+(n-1)h\}\right]$$

$$= \lim_{h\to 0} h\left[\sin 0+\sin h+\sin 2h+----\sin(n-1)h\right]$$

$$= \lim_{h\to 0} h\left[\frac{\sin\left(\frac{nh}{2}-\frac{h}{2}\right)\times\sin\frac{nh}{2}}{\sin\frac{h}{2}}\right]$$

$$= \lim_{h\to 0} h\left[\frac{\sin\left(\frac{\pi}{4}-\frac{h}{2}\right)\times\sin\frac{\pi}{4}}{\sin\frac{h}{2}}\right]$$

$$\left[\therefore \lim_{h\to 0}\frac{\sin\theta}{\theta} = 1\right] \qquad \therefore \lim_{h\to 0}\frac{h}{\sin\frac{h}{2}}\left[\frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}}\right]$$

$$= 2\times\frac{1}{2} = 1$$

$$\therefore \int_0^{\frac{\pi}{2}} \sin x\,dx = 1$$

Definite Integrals Ex 20.5 Q19

We have,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

where $h = \dfrac{b-a}{n}$

Here, $a = 0,\ b = \dfrac{\pi}{2}$ and $f(x) = \cos x$

$$\therefore \quad h = \frac{\frac{\pi}{2}-0}{n} = \frac{\pi}{2n} \qquad nh = \frac{2}{\pi}$$

Thus, we have,

$$I = \int_0^{\frac{\pi}{2}} \cos x\,dx$$

$$= \lim_{h\to 0} h\left[f(0)+f(0+h)+f(0+2h)+----f\{0+(n-1)h\}\right]$$

$$= \lim_{h\to 0} h\left[\cos 0+\cos h+\cos 2h+----\cos(n-1)h\right]$$

$$= \lim_{h\to 0} h\left[\frac{\cos\left(\frac{nh}{2}-\frac{h}{2}\right)\times\cos\frac{nh}{2}}{\cos\frac{h}{2}}\right]$$

$$= \lim_{h\to 0} h\left[\frac{\cos\left(\frac{\pi}{4}-\frac{h}{2}\right)\times\cos\frac{\pi}{4}}{\cos\frac{h}{2}}\right]$$

$$\left[\therefore \lim_{h\to 0}\frac{\cos\theta}{\theta} = 1\right] \qquad \therefore \lim_{h\to 0}\frac{h}{\cos\frac{h}{2}}\left[\frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}}\right]$$

$$= 2\times\frac{1}{2} = 1$$

$$\therefore \int_0^{\frac{\pi}{2}} \cos x\,dx = 1$$

Definite Integrals Ex 20.5 Q20

We have,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

where $h = \dfrac{b-a}{n}$

Here, $a = 1,\ b = 4$ and $f(x) = 3x^2+2x$

$$I = \lim_{h\to 0} h\left[f(1)+f(1+h)+f(1+2h)+----f(a+(n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[(3+2)+\left\{3(1+h)^2+2(1-+h)+\left\{3(1+2h)^2+2(1+2h)\right\}+----\right.\right]$$

$$= \lim_{h\to 0} h\left[5+8h(1+2+3+--)+3h^2\left(1+2^2+3^2+---\right)\right]$$

$$\because h = \frac{3}{n} \text{ \& if } h\to 0 \Rightarrow n\to\infty$$

$$= \lim_{n\to\infty} \frac{3}{n}\left[5n+\frac{24}{n}\frac{n(n-1)}{2}+\frac{27}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$$

$$= \lim_{n\to\infty} 15+\frac{36}{n^2}n^2\left(1-\frac{1}{n}\right)+\frac{27}{2n^3}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)$$

$$= 15+36+27 \quad = 78$$

$$\therefore \int_1^4 \left(3x^2+2x\right)dx = 78$$

Definite Integrals Ex 20.5 Q21

We have,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here, $a=0,\ b=2$ and $f(x)=3x^2-2$

$$\therefore\quad h=\frac{2}{n} \Rightarrow nh=2$$

Thus, we have,

$$I=\int_0^2 \left(3x^2-2\right)dx$$

$$=\lim_{h\to 0} h\left[f(0)+f(0+h)+f(0+2h)+----f\{0+(n-1)h\}\right]$$

$$=\lim_{h\to 0} h\left[-2+\left(3h^2-2\right)+\left\{3(2h)^2-2\right\}+----\right]$$

$$=\lim_{h\to 0} h\left[-2h+3h^2\left(1+2^2+3^2+----\right)\right]$$

$$\because h=\frac{2}{n} \quad \text{if } h\to 0 \Rightarrow n\to\infty$$

$$=\lim_{n\to\infty} \frac{2}{n}\left[-2n+\frac{12}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$$

$$=\lim_{n\to\infty} -4+\frac{4}{n^3}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right) = -4+8 \ = 4$$

$$\therefore\quad \int_0^2 \left(3x^2-2\right)dx = 4$$

Definite Integrals Ex 20.5 Q22

We have,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here, $a=0,\ b=2$ and $f(x)=x^2+2$

$$\therefore\quad h=\frac{2}{n} \Rightarrow nh=2$$

Thus, we have,

$$I=\int_0^2 \left(x^2+2\right)dx$$

$$=\lim_{h\to 0} h\left[f(0)+f(0+h)+f(2h)+----f(0+(n-1)h)\right]$$

$$=\lim_{h\to 0} h\left[2+\left(h^2+2\right)+\left\{(2h)^2+2\right\}+----\right]$$

$$=\lim_{h\to 0} h\left[2h+h^2\left(1+2^2+3^2+----(n-1)^2\right]$$

$$\because h=\frac{2}{n} \quad \& \text{ if } h\to 0 \Rightarrow n\to\infty$$

$$=\lim_{n\to\infty} \frac{2}{n}\left[2n+\frac{4}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$$

$$=\lim_{n\to\infty} 4+\frac{4}{3n^3}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)$$

$$= \ 4+\frac{8}{3} \ = \frac{20}{3}$$

$$\therefore\quad \int_0^2 \left(x^2+2\right)dx = \frac{20}{3}$$

Definite Integrals Ex 20.5 Q23

It is known that,

$$\int_a^b f(x)\,dx=(b-a)\lim_{n\to\infty}\frac{1}{n}\left[f(a)+f(a+h)+\ldots+f(a+(n-1)h)\right],\text{ where } h=\frac{b-a}{n}$$

Here, $a=0,\ b=4,$ and $f(x)=x+e^{2x}$

$$\therefore h=\frac{4-0}{n}=\frac{4}{n}$$

$$\Rightarrow \int_0^4\left(x+e^{2x}\right)dx=(4-0)\lim_{n\to\infty}\frac{1}{n}\left[f(0)+f(h)+f(2h)+\ldots f((n-1)h)\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[\left(0+e^0\right)+\left(h+e^{2h}\right)+\left(2h+e^{2\cdot 2h}\right)+\ldots+\left\{(n-1)h+e^{2(n-1)h}\right\}\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[1+\left(h+e^{2h}\right)+\left(2h+e^{4h}\right)+\ldots+\left\{(n-1)h+e^{2(n-1)h}\right\}\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[\left\{h+2h+3h+\ldots+(n-1)h\right\}+\left(1+e^{2h}+e^{4h}+\ldots+e^{2(n-1)h}\right)\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[h\left\{1+2+\ldots(n-1)\right\}+\left(\frac{e^{2hn}-1}{e^{2h}-1}\right)\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[\frac{(h(n-1)n)}{2}+\left(\frac{e^{2hn}-1}{e^{2h}-1}\right)\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[\frac{4}{n}\cdot\frac{(n-1)n}{2}+\left(\frac{e^8-1}{e^{\frac{8}{n}}-1}\right)\right]$$

$$=4(2)+4\lim_{n\to\infty}\frac{\left(e^8-1\right)}{\left(\dfrac{e^{\frac{8}{n}}-1}{\frac{8}{n}}\right)8}$$

$$=8+\frac{4\cdot\left(e^8-1\right)}{8} \qquad \left(\lim_{x\to 0}\frac{e^x-1}{x}=1\right)$$

$$=8+\frac{e^8-1}{2}$$

$$=\frac{15+e^8}{2}$$

Definite Integrals Ex 20.5 Q24

We have,

$$\int_a^b f(x)\,dx=\lim_{h\to 0}h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

where $h=\dfrac{b-a}{n}$

Thus, we have,

$$I=\int_0^2\left(x^2+x\right)dx$$

$$=\lim_{h\to 0}h\left[f(0)+f(0+h)+f(0+2h)+----f\{(n-1)h\}\right]$$

$$=\lim_{h\to 0}h\left[0+\left(h^2+h\right)+\left\{(2h)^2+2h\right\}+----\right]$$

$$=\lim_{h\to 0}h\left[\left\{h^2\left(1+2^2+3^2+----(n-1)^2\right\}+h\left\{1+2+3----(n-1)\right\}\right]$$

$$\because h=\frac{2}{n}\ \ \&\text{ if } h\to 0\Rightarrow n\to\infty$$

$$=\lim_{n\to\infty}\frac{2}{n}\left[\frac{4}{n^2}\frac{n(n-1)(2n-1)}{6}+\frac{2}{n}\frac{n(n-1)}{2}\right]$$

$$=\lim_{n\to\infty}\frac{4}{3n^3}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)+\frac{2}{n^2}n^2\left(1-\frac{1}{n}\right)$$

$$=\frac{8}{3}+2=\frac{14}{3}$$

$$\therefore \int_0^2\left(x^2+x\right)dx=\frac{14}{3}$$

Definite Integrals Ex 20.5 Q25

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

where $h = \dfrac{b-a}{n}$

Here, $a = 0,\ b = 2$ and $f(x) = x^2+2x+1$

$\therefore\ h = \dfrac{2}{n} \Rightarrow nh = 2$

Thus, we have,

$$I = \int_0^2 \left(x^2+2x+1\right)dx$$

$$= \lim_{h\to 0} h\left[f(0)+f(h)+f(2h)+----f(0+(n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[1+\left(h^2+2h+1\right)+\left\{(2h)^2+2\times 2h+1\right\}+----\right]$$

$$= \lim_{h\to 0} h\left[n+h^2\left(1+2^2+3^2+----(n-1)^2+2h\left(1+2+3----(n-1)\right)\right]$$

$\because h = \dfrac{2}{n}$ if $h \to 0 \Rightarrow n \to \infty$

$$= \lim_{n\to\infty} \frac{2}{n}\left[n+\frac{4}{n^2}\frac{n(n-1)(2n-1)}{6}+\frac{4}{n}\frac{n(n-1)}{2}\right]$$

$$= \lim_{n\to\infty} 2+\frac{4}{3n^3}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)+\frac{4}{n^2}n^2\left(1-\frac{1}{n}\right)$$

$$= 2+\frac{8}{3}+4 = \frac{26}{3}$$

$$\therefore \int_0^2 \left(x^2+2x+1\right)dx = \frac{26}{3}$$

Definite Integrals Ex 20.5 Q26

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

where $h = \dfrac{b-a}{n}$

Here, $a = 0,\ b = 3$ and $f(x) = 2x^2+3x+5$

$\therefore\ h = \dfrac{3}{n} \Rightarrow nh = 3$

Thus, we have,

$$I = \int_0^3 \left(2x^2+3x+5\right)dx$$

$$= \lim_{h\to 0} h\left[f(0)+f(h)+f(2h)+----f((n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[5+\left(2h^2+3h+5\right)+\left\{2(2h)^2+3\times 2h+5\right\}+----\right]$$

$$= \lim_{h\to 0} h\left[5n+2h^2\left(1+2^2+3^2+----(n-1)^2+3h\left(1+2+3----(n-1)\right)\right)\right]$$

$\because h = \dfrac{3}{n}$ & if $h \to 0 \Rightarrow n \to \infty$

$$= \lim_{n\to\infty} \frac{2}{n}\left[5n+\frac{18}{n^2}\frac{n(n-1)(2n-1)}{6}+\frac{9}{n}\frac{n(n-1)}{2}\right]$$

$$= \lim_{n\to\infty} 15+\frac{9}{n^3}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)+\frac{27}{2n^2}n^2\left(1-\frac{1}{n}\right)$$

$$= 15+18+\frac{27}{2} = \frac{93}{2}$$

$$\therefore \int_0^3 \left(2x^2+3x+5\right)dx = \frac{93}{2}$$

Definite Integrals Ex 20.5 Q27

It is known that,

$$\int_a^b f(x)\,dx = (b-a)\lim_{n\to\infty}\frac{1}{n}\left[f(a)+f(a+h)+\ldots+f(a+(n-1)h)\right], \text{ where } h=\frac{b-a}{n}$$

Here, $a=a$, $b=b$, and $f(x)=x$

$$\therefore \int_a^b x\,dx = (b-a)\lim_{n\to\infty}\frac{1}{n}\left[a+(a+h)\ldots(a+2h)\ldots a+(n-1)h\right]$$

$$=(b-a)\lim_{n\to\infty}\frac{1}{n}\left[\left(\underbrace{a+a+a+\ldots+a}_{n\text{ times}}\right)+\left(h+2h+3h+\ldots+(n-1)h\right)\right]$$

$$=(b-a)\lim_{n\to\infty}\frac{1}{n}\left[na+h\left(1+2+3+\ldots+(n-1)\right)\right]$$

$$=(b-a)\lim_{n\to\infty}\frac{1}{n}\left[na+h\left\{\frac{(n-1)(n)}{2}\right\}\right]$$

$$=(b-a)\lim_{n\to\infty}\frac{1}{n}\left[na+\frac{n(n-1)h}{2}\right]$$

$$=(b-a)\lim_{n\to\infty}\frac{n}{n}\left[a+\frac{(n-1)h}{2}\right]$$

$$=(b-a)\lim_{n\to\infty}\left[a+\frac{(n-1)h}{2}\right]$$

$$=(b-a)\lim_{n\to\infty}\left[a+\frac{(n-1)(b-a)}{2n}\right]$$

$$=(b-a)\lim_{n\to\infty}\left[a+\frac{\left(1-\frac{1}{n}\right)(b-a)}{2}\right]$$

$$=(b-a)\left[a+\frac{(b-a)}{2}\right]$$

$$=(b-a)\left[\frac{2a+b-a}{2}\right]$$

$$=\frac{(b-a)(b+a)}{2}$$

$$=\frac{1}{2}\left(b^2-a^2\right)$$

Definite Integrals Ex 20.5 Q28

Let $I = \int_0^5 (x+1)\,dx$

It is known that,

$$\int_a^b f(x)\,dx = (b-a)\lim_{n\to\infty}\frac{1}{n}\left[f(a)+f(a+h)\ldots f(a+(n-1)h)\right], \text{ where } h=\frac{b-a}{n}$$

Here, $a=0$, $b=5$, and $f(x)=(x+1)$

$$\Rightarrow h=\frac{5-0}{n}=\frac{5}{n}$$

$$\begin{aligned}
\therefore \int_0^5 (x+1)\,dx &= (5-0)\lim_{n\to\infty}\frac{1}{n}\left[f(0)+f\left(\frac{5}{n}\right)+\ldots+f\left((n-1)\frac{5}{n}\right)\right]\\
&= 5\lim_{n\to\infty}\frac{1}{n}\left[1+\left(\frac{5}{n}+1\right)+\ldots\left\{1+\left(\frac{5(n-1)}{n}\right)\right\}\right]\\
&= 5\lim_{n\to\infty}\frac{1}{n}\left[\left(\underbrace{1+1+1\ldots1}_{n\text{ times}}\right)+\left[\frac{5}{n}+2\cdot\frac{5}{n}+3\cdot\frac{5}{n}+\ldots(n-1)\frac{5}{n}\right]\right]\\
&= 5\lim_{n\to\infty}\frac{1}{n}\left[n+\frac{5}{n}\{1+2+3\ldots(n-1)\}\right]\\
&= 5\lim_{n\to\infty}\frac{1}{n}\left[n+\frac{5}{n}\cdot\frac{(n-1)n}{2}\right]\\
&= 5\lim_{n\to\infty}\frac{1}{n}\left[n+\frac{5(n-1)}{2}\right]\\
&= 5\lim_{n\to\infty}\left[1+\frac{5}{2}\left(1-\frac{1}{n}\right)\right]\\
&= 5\left[1+\frac{5}{2}\right]\\
&= 5\left[\frac{7}{2}\right]\\
&= \frac{35}{2}
\end{aligned}$$

Definite Integrals Ex 20.5 Q29

It is known that,

$$\int_a^b f(x)\,dx = (b-a)\lim_{n\to\infty}\frac{1}{n}\left[f(a)+f(a+h)+f(a+2h)\ldots f\{a+(n-1)h\}\right], \text{ where } h=\frac{b-a}{n}$$

Here, $a=2$, $b=3$, and $f(x)=x^2$

$$\Rightarrow h=\frac{3-2}{n}=\frac{1}{n}$$

$$\therefore \int_2^3 x^2dx=(3-2)\lim_{n\to\infty}\frac{1}{n}\left[f(2)+f\left(2+\frac{1}{n}\right)+f\left(2+\frac{2}{n}\right)\ldots f\left\{2+(n-1)\frac{1}{n}\right\}\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[(2)^2+\left(2+\frac{1}{n}\right)^2+\left(2+\frac{2}{n}\right)^2+\ldots\left(2+\frac{(n-1)}{n}\right)^2\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[2^2+\left\{2^2+\left(\frac{1}{n}\right)^2+2\cdot2\cdot\frac{1}{n}\right\}+\ldots+\left\{(2)^2+\frac{(n-1)^2}{n^2}+2\cdot2\cdot\frac{(n-1)}{n}\right\}\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[\left(\underbrace{2^2+\ldots+2^2}_{n \text{ times}}\right)+\left\{\left(\frac{1}{n}\right)^2+\left(\frac{2}{n}\right)^2+\ldots+\left(\frac{n-1}{n}\right)^2\right\}+2\cdot2\cdot\left\{\frac{1}{n}+\frac{2}{n}+\frac{3}{n}+\ldots+\frac{(n-1)}{n}\right\}\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[4n+\frac{1}{n^2}\left\{1^2+2^2+3^2\ldots+(n-1)^2\right\}+\frac{4}{n}\left\{1+2+\ldots+(n-1)\right\}\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[4n+\frac{1}{n^2}\left\{\frac{n(n-1)(2n-1)}{6}\right\}+\frac{4}{n}\left\{\frac{n(n-1)}{2}\right\}\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[4n+\frac{n\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)}{6}+\frac{4n-4}{2}\right]$$

$$=\lim_{n\to\infty}\left[4+\frac{1}{6}\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)+2-\frac{2}{n}\right]$$

$$=4+\frac{2}{6}+2$$

$$=\frac{19}{3}$$

Definite Integrals Ex 20.5 Q30

We have

$$\int_a^b f(x)=dx\lim_{h\to0}h\left[f(a)+f(a+h)+f(a+2h)+\ldots+f(a+(n-1)h)\right]$$

Where $h=\frac{b-a}{n}$

Here

$a=1, b=3$ and $f(x)=x^2+x$

Now

$h=\frac{2}{n}$

$nh=2$

Thus, we have

$$I=\int_1^3(x^2+x)dx$$

$$=\lim_{h\to0}h\left[f(1)+f(1+h)+f(1+2h)+\ldots+f(1+(n-1)h)\right]$$

$$=\lim_{h\to0}h\left[(1^2+1)+\left\{(1+h)^2+(1+h)\right\}+\left\{(1+2h)^2+(1+2h)\right\}+\ldots\right]$$

$$=\lim_{h\to0}h\left[\left(1^2+(1+h)^2+(1+2h)^2+\ldots\right)+\left\{1+(1+h)+(1+2h)+\ldots\right\}\right]$$

$$=\lim_{h\to0}h\left[\left(n+2h(1+2+3+\ldots)+h^2(1+2^2+3^3+\ldots)\right)+\left(n+h(1+2+3+\ldots)\right)\right]$$

$$=\lim_{h\to0}h\left[\left(2n+3h(1+2+3+\ldots+(n-1))+h^2\left(1+2^2+3^3+\ldots+(n-1)^2\right)\right)\right]$$

$\because h=\frac{2}{n}$ & if $h\to0\Rightarrow n\to\infty$

$$=\lim_{h\to\infty}\frac{2}{n}\left[2n+\frac{9}{n}\frac{n(n-1)}{2}+\frac{9}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$$

$$=\frac{38}{3}$$

Definite Integrals Ex 20.5 Q31

We have

$$\int_a^b f(x) = dx \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+...+f(a+(n-1)h)\right]$$

Where $h=\frac{b-a}{n}$

Here

$$a=0, b=2 \text{ and } f(x)=x^2-x$$

Now

$$h=\frac{2}{n}$$
$$nh=2$$

Thus, we have

$$I=\int_0^2 (x^2-x)\,dx$$
$$=\lim_{h\to 0} h\left[f(0)+f(h)+f(2h)+...+f((n-1)h)\right]$$
$$=\lim_{h\to 0} h\left[\left\{(0)^2-(0)\right\}+\left\{(h)^2-(h)\right\}+\left\{(2h)^2-(2h)\right\}+...\right]$$
$$=\lim_{h\to 0} h\left[\left((h)^2+(2h)^2+...\right)-\left\{(h)+(2h)+...\right\}\right]$$
$$=\lim_{h\to 0} h\left[h^2\left(1+2^2+3^2+...+(n-1)^2\right)-h\left\{1+2+3+...+(n-1)\right\}\right]$$
$$\because h=\frac{2}{n} \text{ \& if } h\to 0 \Rightarrow n\to\infty$$
$$=\lim_{h\to\infty}\frac{2}{n}\left[\frac{9}{n^2}\frac{n(n-1)(2n-1)}{6}-\frac{9}{n}\frac{n(n-1)}{2}\right]$$
$$=\frac{2}{3}$$

Definite Integrals Ex 20.5 Q32

We have

$$\int_a^b f(x) = dx \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+...+f(a+(n-1)h)\right]$$

Where $h=\frac{b-a}{n}$

Here

$$a=1, b=3 \text{ and } f(x)=2x^2+5x$$

Now

$$h=\frac{2}{n}$$
$$nh=2$$

Thus, we have

$$I=\int_1^3 (2x^2+5x)\,dx$$
$$=\lim_{h\to 0} h\left[f(1)+f(1+h)+f(1+2h)+...+f(1+(n-1)h)\right]$$
$$=\lim_{h\to 0} h\left[(2+5)+\left\{2(1+h)^2+5(1+h)\right\}+\left\{2(1+2h)^2+5(1+2h)\right\}+...\right]$$
$$=\lim_{h\to 0} h\left[\left(7n+9h(1+2+3+...)+2h^2\left(1+2^2+3^2+...\right)\right)\right]$$
$$\because h=\frac{2}{n} \text{ \& if } h\to 0 \Rightarrow n\to\infty$$
$$=\lim_{h\to\infty}\frac{2}{n}\left[7n+\frac{18}{n}\frac{n(n-1)}{2}+\frac{8}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$$
$$=\frac{112}{3}$$

Definite Integrals Ex 20.5 Q33

Given,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a) + f(a+h) + f(a+2h) + \ldots + f(a+(n-1)h)\right],$$

where $h = \dfrac{b-a}{n}$

Here, $f(x) = 3x^2 + 1$, $a = 1$, $b = 3$. Therefore, $h = \dfrac{3-1}{n} = \dfrac{2}{n}$

$$\therefore I = \int_1^3 (3x^2 + 1)dx$$

$$\Rightarrow I = \lim_{h\to 0} h\left[f(1) + f(1+h) + f(1+2h) + \ldots + f(1+(n-1)h)\right]$$

$$\Rightarrow I = \lim_{h\to 0} h\left[3(1)^2 + 1 + 3(1+h)^2 + 1 + 3(1+2h)^2 + 1 + \ldots + 3(1+(n-1)h)^2 + 1\right]$$

$$\Rightarrow I = \lim_{h\to 0} h\left[3n + n + 6h(1+2+3+\ldots+(n-1)) + 3h^2(1^2 + 2^2 + \ldots + (n-1)^2)\right]$$

$$\Rightarrow I = \lim_{n\to\infty} \frac{2}{n}\left[4n + \frac{12}{n}(1+2+3+\ldots+(n-1)) + 3\times\frac{4}{n^2}(1^2 + 2^2 + \ldots + (n-1)^2)\right]$$

$$\Rightarrow I = \lim_{n\to\infty}\left[8 + \frac{24}{n^2}\times\frac{n(n-1)}{2} + \frac{24}{n^3}\times\frac{(n-1)(n)(2n-1)}{6}\right]$$

$$\Rightarrow I = \lim_{n\to\infty}\left[8 + 12\left(1-\frac{1}{n}\right) + 4\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)\right]$$

$$\Rightarrow I = 8 + 12 + 4\times 2 = 28$$