Ex 11.1

Differentiation Ex 11.1 Q1

Let $\quad f(x) = e^{-x}$

$\Rightarrow \quad f(x+h) = e^{-(x+h)}$

$$\frac{d}{dx}\big(f(x)\big) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{e^{-(x+h)} - e^{-x}}{h}$$

$$= \lim_{h \to 0} \frac{e^{-x} \times e^{-h} - e^{-x}}{h}$$

$$= \lim_{h \to 0} e^{-x} \left\{ \frac{\left(e^{-h} - 1\right)}{-h} \right\} \times (-1)$$

$$= -e^{-x} \qquad\qquad \left[\text{Since, } \lim_{h \to 0} \frac{e^h - 1}{h} = 1 \right]$$

So,

$$\frac{d}{dx}\big(e^{-x}\big) = -e^{-x}$$

Differentiation Ex 11.1 Q2

Let $\quad f(x) = e^{3x}$

$\Rightarrow \quad f(x+h) = e^{3(x+h)}$

$$\frac{d}{dx}\big(f(x)\big) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{e^{3(x+h)} - e^{3x}}{h}$$

$$= \lim_{h \to 0} \frac{e^{3x} e^{3h} - e^{3x}}{h}$$

$$= \lim_{h \to 0} e^{3x} \left\{ \frac{\left(e^{3h} - 1\right)}{3h} \right\} \times 3$$

$$= 3e^{3x} \qquad\qquad \left[\text{Since, } \lim_{x \to 0} \frac{e^x - 1}{x} = 1 \right]$$

Hence,

$$\frac{d}{dx}\big(e^{3x}\big) = 3e^{3x}$$

Differentiation Ex 11.1 Q3

Let $\quad f(x) = e^{ax+b}$

$\Rightarrow \quad f(x+h) = e^{a(x+h)+b}$

$\therefore \quad \dfrac{d}{dx}(f(x)) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$\qquad = \lim\limits_{h \to 0} \dfrac{e^{a(x+h)+b} - e^{(ax+b)}}{h}$

$\qquad = \lim\limits_{h \to 0} \dfrac{e^{ax+b}e^{ax} - e^{ax+b}}{h}$

$\qquad = \lim\limits_{h \to 0} e^{ax+b}\left\{\dfrac{\left(e^{ah} - 1\right)}{ah}\right\} \times a$

$\qquad = ae^{ax+b} \qquad\qquad\qquad \left[\text{Since, } \lim\limits_{x \to 0} \dfrac{e^x - 1}{x} = 1\right]$

So,

$\dfrac{d}{dx}\left(e^{ax+b}\right) = ae^{ax+b}$

Differentiation Ex 11.1 Q4

Let $\quad f(x) = e^{\cos x}$

$\Rightarrow \quad f(x+h) = e^{\cos(x+h)}$

$\therefore \quad \dfrac{d}{dx}(f(x)) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$\qquad = \lim\limits_{h \to 0} \dfrac{e^{\cos(x+h)} - e^{\cos x}}{h}$

$\qquad = \lim\limits_{h \to 0} e^{\cos x}\left[\dfrac{e^{\cos(x+h)-\cos x} - 1}{h}\right]$

$\qquad = \lim\limits_{h \to 0} e^{\cos x}\left[\dfrac{e^{\cos(x+h)-\cos x} - 1}{\cos(x+h) - \cos x}\right] \times \dfrac{\cos(x+h) - \cos x}{h}$

$\qquad = \lim\limits_{h \to 0} e^{\cos x} \times \left(\dfrac{\cos(x+h) - \cos x}{h}\right) \qquad \left[\text{Since, } \lim\limits_{h \to 0} \dfrac{e^x - 1}{x} = 1\right]$

$\qquad = \lim\limits_{h \to 0} e^{\cos x} \times \left(\dfrac{-2\sin\dfrac{x+h+x}{2} \times \sin\dfrac{x+h-x}{2}}{h}\right) \quad \left[\text{Since, } \cos A - \cos B = -2\sin\dfrac{A+B}{2}\right.$
$\left.\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \sin\dfrac{A-B}{2}\right]$

$\qquad = e^{\cos x} \lim\limits_{h \to 0} \dfrac{-2\sin\left(\dfrac{2x+h}{2}\right)}{2} \times \dfrac{\sin\left(\dfrac{h}{2}\right)}{\dfrac{h}{2}}$

$\qquad = e^{\cos x} \lim\limits_{h \to 0} -2\sin\left(\dfrac{2x+h}{2}\right) \times \dfrac{1}{2} \qquad \left[\text{since, } \lim\limits_{x \to 0} \dfrac{\sin x}{x} = 1\right]$

$\qquad = e^{\cos x}(-\sin x)$

$\qquad = -\sin x \, e^{\cos x}$

Hence,

$\dfrac{d}{dx}\left(e^{\cos x}\right) = -\sin x \, e^{\cos x}$

Differentiation Ex 11.1 Q5

Let $f(x) = e^{\sqrt{2x}}$

$\Rightarrow \quad f(x+h) = e^{\sqrt{2(x+h)}}$

$\dfrac{d}{dx}\{f(x)\} = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$

$= \lim\limits_{h\to 0} \dfrac{e^{\sqrt{2(x+h)}} - e^{\sqrt{2x}}}{h}$

$= \lim\limits_{h\to 0} e^{\sqrt{2x}} \dfrac{e^{\sqrt{2(x+h)} - \sqrt{2x}} - 1}{h}$

$= e^{\sqrt{2x}} \lim\limits_{h\to 0} \left(\dfrac{\left(e^{\sqrt{2(x+h)} - \sqrt{2x}} - 1\right)}{\sqrt{2(x+h)} - \sqrt{2x}}\right)\left(\dfrac{\sqrt{2(x+h)} - \sqrt{2x}}{h}\right)$

$= e^{\sqrt{2x}} \lim\limits_{h\to 0} \dfrac{\sqrt{2(x+h)} - \sqrt{2x}}{h}$ $\left[\text{Since, } \lim\limits_{h\to 0} \dfrac{e^h - 1}{h} = 1\right]$

$= e^{\sqrt{2x}} \lim\limits_{h\to 0} \dfrac{\sqrt{2(x+h)} - \sqrt{2x}}{h} \times \dfrac{\sqrt{2(x+h)} + \sqrt{2x}}{\sqrt{2(x+h)} + \sqrt{2x}}$ [Rationalizing the numerator]

$= de^{\sqrt{2x}} \lim\limits_{h\to 0} \dfrac{2(x+h) - 2x}{h\left(\sqrt{2(x+h)} + \sqrt{2x}\right)}$

$= e^{\sqrt{2x}} \lim\limits_{h\to 0} \dfrac{2x + 2h - 2x}{h\left(\sqrt{2(x+h)} + \sqrt{2x}\right)}$

$= e^{\sqrt{2x}} \lim\limits_{h\to 0} \dfrac{2h}{h\left(\sqrt{2(x+h)} + \sqrt{2x}\right)}$

$= e^{\sqrt{2x}} \lim\limits_{h\to 0} \dfrac{2}{\left(\sqrt{2(x+h)} + \sqrt{2x}\right)}$

$= \dfrac{e^{\sqrt{2x}}}{\sqrt{2x}}$

So,

$\dfrac{d}{dx}\left(e^{\sqrt{2x}}\right) = \dfrac{e^{\sqrt{2x}}}{\sqrt{2x}}$

Differentiation Ex 11.1 Q6

Let $f(x) = \log \cos x$

$\Rightarrow$ $f(x+h) = \log \cos(x+h)$

$\therefore$ $\dfrac{d}{dx}(f(x)) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$= \lim\limits_{h \to 0} \dfrac{\log \cos(x+h) - \log \cos x}{h}$

$= \lim\limits_{h \to 0} \dfrac{\log \dfrac{\cos(x+h)}{\cos x}}{h}$ $\left[\text{Since, } \log A - \log B = \log \dfrac{A}{B} \right]$

$= \lim\limits_{h \to 0} \dfrac{\log \left\{ 1 + \dfrac{\cos(x+h)}{\cos x} - 1 \right\}}{h}$

$= \lim\limits_{h \to 0} \dfrac{\log \left\{ 1 + \dfrac{\cos(x+h)}{\cos x} \right\}}{\left(\dfrac{\cos(x+h)}{\cos x} \right) h \times \left(\dfrac{\cos x}{\cos(x+h) - \cos x} \right)}$

$= \lim\limits_{h \to 0} \dfrac{\cos(x+h) - \cos x}{\cos x \times h}$ $\left[\text{Since, } \lim\limits_{x \to 0} \dfrac{\log(1+x)}{x} = 1 \right]$

$= \lim\limits_{h \to 0} \dfrac{-2 \sin\left(\dfrac{x+h+x}{2} \right) \sin\left(\dfrac{x+h-x}{2} \right)}{\cos x \times h}$

$= -2 \lim\limits_{h \to 0} \dfrac{\sin\left(\dfrac{2x+h}{2} \right) \times \left(\sin \dfrac{h}{2} \right)}{2 \cos x \left(\dfrac{h}{2} \right)}$

$= \dfrac{-2 \sin x}{2 \cos x}$ $\left[\text{Since, } \lim\limits_{x \to 0} \dfrac{\sin x}{x} = 1 \right]$

$= -\tan x$

So,

$\dfrac{d}{dx}(\log \cos x) = -\tan x$

Differentiation Ex 11.1 Q7

Let $f(x) = e^{\sqrt{\cot x}}$

$\Rightarrow \quad f(x+h) = e^{\sqrt{\cot(x+h)}}$

$\dfrac{d}{dx}(f(x)) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$= \lim\limits_{h \to 0} \dfrac{e^{\sqrt{\cot(x+h)}} - e^{\sqrt{\cot x}}}{h}$

$= \lim\limits_{h \to 0} \dfrac{e^{\sqrt{\cot x}}\left(e^{\sqrt{\cot(x+h)} - \sqrt{\cot x}} - 1\right)}{h}$

$= e^{\sqrt{\cot x}} \lim\limits_{h \to 0} \left(\dfrac{e^{\sqrt{\cot(x+h)} - \sqrt{\cot x}} - 1}{\sqrt{\cot(x+h)} - \sqrt{\cot x}}\right) \times \left(\dfrac{\sqrt{\cot(x+h)} - \sqrt{\cot x}}{h}\right)$

$= e^{\sqrt{\cot x}} \lim\limits_{h \to 0} \dfrac{\left(\sqrt{\cot(x+h)} - \sqrt{\cot x}\right)}{h} \times \dfrac{\sqrt{\cot(x+h)} + \sqrt{\cot x}}{\sqrt{\cot(x+h)} + \sqrt{\cot x}}$

$\left[\text{Since, } \lim\limits_{x \to 0} \dfrac{e^x - 1}{x} = 1 \text{ and rationalizing numerator}\right]$

$= e^{\sqrt{\cot x}} \lim\limits_{h \to 0} \dfrac{\cot(x+h) - \cot x}{h\left(\sqrt{\cot(x+h)} + \sqrt{\cot x}\right)}$

$= e^{\sqrt{\cot x}} \lim\limits_{h \to 0} \dfrac{\dfrac{\cot(x+h)\cot x + 1}{\cot(x+h-x)}}{h\left(\sqrt{\cot(x+h)} + \sqrt{\cot x}\right)}$

$\left[\text{Since, } \cot(A-B) = \dfrac{\cot A \cot B + 1}{\cot A - \cot B}\right]$

$= e^{\sqrt{\cot x}} \lim\limits_{h \to 0} \dfrac{\cot(x+h)\cot x + 1}{\cot h\, h\left(\sqrt{\cot(x+h)} + \sqrt{\cot x}\right)}$

$= e^{\sqrt{\cot x}} \lim\limits_{h \to 0} \dfrac{(\cot(x+h)\cot x + 1)}{\left(\dfrac{h}{\tanh}\right)\left(\sqrt{\cot(x+h)} + \sqrt{\cot x}\right)}$

$= \dfrac{e^{\sqrt{\cot x}} \times (\cot^2 x + 1)}{2\sqrt{\cot x}}$

$\left[\text{Since, } \lim\limits_{x \to 0} \dfrac{\tan x}{x} = 1\right]$

$= \dfrac{e^{\sqrt{\cot x}} \times \cosec^2 x}{2\sqrt{\cot x}}$

$\left[\text{Since, } (1 + \cot^2 x) = \cosec^2 x\right]$

So,

$\dfrac{d}{dx}\left(e^{\sqrt{\cot x}}\right) = \dfrac{e^{\sqrt{\cot x}} \times \cosec^2 x}{2\sqrt{\cot x}}$

Differentiation Ex 11.1 Q8

Let $f(x) = x^2 e^x$

$\Rightarrow \quad f(x+h) = (x+h)^2 e^{(x+h)}$

$\therefore \quad \dfrac{d}{dx}(f(x)) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$= \lim\limits_{h \to 0} \dfrac{(x+h)^2 e^{(x+h)} - x^2 e^x}{h}$

$= \lim\limits_{h \to 0} \left(\dfrac{x^2 e^{(x+h)} - x^2 e^x}{h} + \dfrac{2xh e^{(x+h)}}{h} + \dfrac{h^2 e^{(x+h)}}{h}\right)$

$= \lim\limits_{h \to 0} \left(\dfrac{x^2 e^x \left(e^{(x+h)-x} - 1\right)}{h} + 2x e^{(x+h)} + h e^{(x+h)}\right)$

$= \lim\limits_{h \to 0} \left[x^2 e^x \dfrac{\left(e^h - 1\right)}{h} + 2x e^{(x+h)} + h e(x+h)\right]$

$= x^2 e^x + 2x e^x + 0 \times e^x$

$\left[\text{Since, } \lim\limits_{x \to 0} \dfrac{e^x - 1}{x} = 1\right]$

So,

$\dfrac{d}{dx}\left(x^2 e^x\right) = e^x\left(x^2 + 2x\right)$

Differentiation Ex 11.1 Q9

Let $\quad f(x) = \log \cosec x$

$\Rightarrow \quad f(x+h) = \log \cosec (x+h)$

$$\frac{d}{dx}(f(x)) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\log \cosec (x+h) - \log \cosec x}{h}$$

$$= \lim_{h \to 0} \frac{\log \left(\dfrac{\cosec (x+h)}{\cosec x} \right)}{h}$$

$$= \lim_{h \to 0} \frac{\log \left(1 + \left(\dfrac{\sin x}{\sin (x+h)} - 1 \right) \right)}{h}$$

$$= \lim_{h \to 0} \left\{ \frac{\log \left(1 + \left(\dfrac{\sin x - \sin (x+h)}{\sin (x+h)} \right) \right)}{\left(\dfrac{\sin x - \sin (x+h)}{\sin (x+h)} \right)} \right\} \frac{\left(\dfrac{\sin x - \sin (x+h)}{\sin (x+h)} \right)}{h}$$

$$= \lim_{h \to 0} \frac{2 \cos \left(\dfrac{x + x + h}{2} \right) \sin \left(\dfrac{x - x - h}{2} \right)}{\sin (x+h) h}$$

$$\left[\text{Since, } \lim_{x \to 0} \frac{\log (1+x)}{x} = 1 \text{ and } \sin A - \sin B = 2 \cos \left(\frac{A+B}{2} \right) \sin \left(\frac{A-B}{2} \right) \right]$$

$$= \lim_{h \to 0} \frac{2 \cos \left(\dfrac{2x+h}{2} \right)}{\sin (x+h)(-2)} \left\{ \frac{\sin \left(-\dfrac{h}{2} \right)}{-\dfrac{h}{2}} \right\} \qquad \left[\text{Since, } \lim_{x \to 0} \frac{\sin x}{x} = 1 \right]$$

$$= -\cot x$$

So,

$$\frac{d}{dx}(\log \cosec x) = -\cot x.$$

Differentiation Ex 11.1 Q10

Let $\quad f(x) = \sin^{-1}(2x + 3)$

$\Rightarrow \quad f(x + h) = \sin^{-1}\big(2(x + h) + 3\big)$

$\Rightarrow \quad f(x + h) = \sin^{-1}(2x + 2h + 3)$

$\therefore \quad \dfrac{d}{dx}\big(f(x)\big) = \lim\limits_{h \to 0} \dfrac{f(x + h) - f(x)}{h}$

$\qquad\qquad\qquad = \lim\limits_{h \to 0} \dfrac{\sin^{-1}(2x + 2h + 3) - \sin^{-1}(2x + 3)}{h}$

$\qquad\qquad\qquad = \lim\limits_{h \to 0} \dfrac{\sin^{-1}\left[(2x + 2h + 3)\sqrt{1 - (2x + 3)^2} - (2x + 3)\sqrt{1 - (2x + 2h + 3)^2}\right]}{h}$

$$\left[\text{Since, } \sin^{-1} x - \sin^{-1} y = \sin^{-1}\left[x\sqrt{1 - y^2} - y\sqrt{1 - x^2}\right]\right]$$

$\qquad\qquad\qquad = \lim\limits_{h \to 0} \dfrac{\sin^{-1} z}{z} \times \dfrac{z}{h}$

Where, $z = (2x + 2h + 3)\sqrt{1 - (2x + 3)^2} - (2x + 3)\sqrt{1 - (2x + 2h + 3)^2}$ and $\lim\limits_{h \to 0} \dfrac{\sin^{-1} h}{h} = 1$

$\qquad\qquad\qquad = \lim\limits_{h \to 0} \dfrac{z}{h}$

$\qquad\qquad\qquad = \lim\limits_{h \to 0} \dfrac{(2x + 2h + 3)\sqrt{1 - (2x + 3)^2} - (2x + 3)\sqrt{1 - (2x + 2h + 3)^2}}{h}$

$\qquad\qquad\qquad = \lim\limits_{h \to 0} \dfrac{(2x + 2h + 3)^2 - (2x + 3)^2 - (2x + 3)^2\left(1 - (2x + 2h + 3)^2\right)}{h\left\{(2x + 2h + 3)\sqrt{1 - (2x + 3)^2} + (2x + 3)\sqrt{1 - (2x + 2h + 3)^2}\right\}}$

$$[\text{Since, rationalizing numerator}]$$

$\qquad\qquad\qquad = \lim\limits_{h \to 0} \dfrac{\left[(2x + 3)^2 + 4h^2 + 4h(2x + 3)\right]\left(1 - (2x + 3)^2\right) - (2x + 3)^2 \left[1 - (2x + 3)^2 - 4h^2 - 4h(2x + 3)\right]}{h\left\{(2x + 2h + 3)\sqrt{1 - (2x + 3)^2} + (2x + 3)\sqrt{1 - (2x + 2h + 3)^2}\right\}}$

$\qquad\qquad\qquad = \lim\limits_{h \to 0} \dfrac{\left[\begin{array}{l}(2x + 3)^2 + 4h^2 + 4h(2x + 3) - (2x + 3)^4 - 4h^2(2x + 3)^2 - 4h(2x + 3)^3 - (2x + 3)^2 \\ \qquad + (2x + 3)^4 + 4h^2(2x + 3)^2 + 4h(2x + 3)^3\end{array}\right]}{h\left\{(2x + 2h + 3)\sqrt{1 - (2x + 3)^2} + (2x + 3)\sqrt{1 - (2x + 2h + 3)^2}\right\}}$

$\qquad\qquad\qquad = \lim\limits_{h \to 0} \dfrac{4h\left[h + (2x + 3)\right]}{h\left\{(2x + 2h + 3)\sqrt{1 - (2x + 3)^2} + (2x + 3)\sqrt{1 - (2x + 2h + 3)^2}\right\}}$

$\qquad\qquad\qquad = \dfrac{4(2x + 3)}{(2x + 3)\sqrt{1 - (2x + 3)^2} + (2x + 3)\sqrt{1 - (2x + 3)^2}}$

$\qquad\qquad\qquad = \dfrac{4(2x + 3)}{2(2x + 3)\sqrt{1 - (2x + 3)^2}}$

$\qquad\qquad\qquad = \dfrac{2}{\sqrt{1 - (2x + 3)^2}}$

So,

$\qquad\qquad \dfrac{d}{dx}\big(\sin^{-1}(2x + 3)\big) = \dfrac{2}{\sqrt{1 - (2x + 3)^2}}$

Ex 11.2

Differentiation Ex 11.2 Q1

Let,

$$y = \sin(3x + 5)$$

Differentiate y with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\big(\sin(3x + 5)\big)$$

$$= \cos(3x + 5)\frac{d}{dx}(3x + 5) \qquad\qquad [\text{using chain rule}]$$

$$= \cos(3x + 5) \times \big[3(1) + 0\big]$$

$$= 3\cos(3x + 5)$$

So,

$$\frac{d}{dx}\big(\sin(3x + 5)\big) = 3\cos(3x + 5).$$

Differentiation Ex 11.2 Q2

Let,

$$y = \tan^2 x$$

Differentiate it with respect to x,

$$\frac{dy}{dx} = 2\tan x \frac{d}{dx}(\tan x) \qquad\qquad [\text{using chain rule}]$$

$$= 2\tan x \times \sec^2 x$$

So,

$$\frac{d}{dx} = \big(\tan^2 x\big) = 2\tan x \, \sec^2 x.$$

Differentiation Ex 11.2 Q3

Let,

$$y = \tan(x° + 45°)$$

$$y = \tan\left\{(x° + 45°)\frac{\pi}{180°}\right\}$$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\tan\left\{(x° + 45°)\frac{\pi}{180°}\right\}$$

$$= \sec^2\left\{(x° + 45°)\frac{\pi}{180°}\right\} \times \frac{d}{dx}(x° + 45°)\frac{\pi}{180°} \qquad \text{[Using chain rule]}$$

$$= \frac{\pi}{180°}\sec^2(x° + 45°)$$

So,

$$\frac{d}{dx} = \left(\tan(x° + 45°)\right) = \frac{\pi}{180°}\sec^2(x° + 45°).$$

Differentiation Ex 11.2 Q4

Let,

$$y = \sin(\log x)$$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\sin(\log x)$$

$$= \cos(\log x)\frac{d}{dx}(\log x) \qquad \text{[Using chain rule]}$$

$$= \frac{1}{x}\cos(\log x)$$

So,

$$\frac{d}{dx} = \left(\sin(\log x)\right) = \frac{1}{x}\cos(\log x).$$

Differentiation Ex 11.2 Q5

Let,

$$y = e^{\sin\sqrt{x}}$$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{\sin\sqrt{x}}\right)$$

$$= e^{\sin\sqrt{x}}\frac{d}{dx}\left(\sin\sqrt{x}\right) \qquad \text{[Using chain rule]}$$

$$= e^{\sin\sqrt{x}} \times \cos\sqrt{x}\frac{d}{dx}\sqrt{x} \qquad \text{[Using chain rule]}$$

$$= e^{\sin\sqrt{x}} \times \cos\sqrt{x} \times \frac{1}{2\sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}}\cos\sqrt{x} \times e^{\sin\sqrt{x}}$$

So,

$$\frac{d}{dx} = \left(e^{\sin\sqrt{x}}\right) = \frac{1}{2\sqrt{x}}\cos\sqrt{x} \times e^{\sin\sqrt{x}}.$$

Differentiation Ex 11.2 Q6

Let,

$$y = e^{\tan x}$$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{\tan x}\right)$$

$$= e^{\tan x}\frac{d}{dx}(\tan x) \qquad \text{[Using chain rule]}$$

$$= e^{\tan x} \times \sec^2 x$$

So,

$$\frac{d}{dx} = \left(e^{\tan x}\right) = \sec^2 x \times e^{\tan x}.$$

Differentiation Ex 11.2 Q7

Let,

$$y = \sin^2(2x + 1)$$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left[\sin^2(2x + 1)\right]$$

$$= 2\sin(2x + 1)\frac{d}{dx}\sin(2x + 1) \qquad \text{[Using chain rule]}$$

$$= 2\sin(2x + 1)\cos(2x + 1)\frac{d}{dx}(2x + 1) \qquad \text{[Using chain rule]}$$

$$= 4\sin(2x + 1)\cos(2x + 1)$$

$$= 2\sin 2(2x + 1) \qquad \left[\text{Since, } \sin^2 A = 2\sin A \cos A\right]$$

$$= 2\sin(4x + 2)$$

So,

$$\frac{d}{dx}\left(\sin^2(2x + 1)\right) = 2\sin(4x + 2).$$

Differentiation Ex 11.2 Q8

Let,

$$y = \log_7(2x - 3)$$

$$\Rightarrow \quad y = \frac{\log(2x - 3)}{\log 7} \qquad \left[\text{Since, } \log_a{}^b = \frac{\log b}{\log a}\right]$$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{1}{\log 7}\frac{d}{dx}\left(\log(2x - 3)\right)$$

$$= \frac{1}{\log 7} \times \frac{1}{(2x - 3)}\frac{d}{dx}(2x - 3) \qquad \text{[Using chain rule]}$$

$$= \frac{2}{(2x - 3)\log 7}$$

Hence,

$$\frac{d}{dx}\left(\log_7(2x - 3)\right) = \frac{2}{(2x - 3)\log 7}.$$

Differentiation Ex 11.2 Q9

Let,

$$y = \tan 5x°$$

$$\Rightarrow \quad y = \tan\left(5x° \times \frac{\pi}{180°}\right)$$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\tan\left(5x° \times \frac{\pi}{180°}\right)$$

$$= \sec^2 x\left(5x° \times \frac{\pi}{180°}\right)\frac{d}{dx}\left(5x° \frac{\pi}{180°}\right) \qquad \text{[Using chain rule]}$$

$$= \left(\frac{5\pi}{180°}\right)\sec^2\left(5x° \frac{\pi}{180°}\right)$$

$$= \frac{5\pi}{180°}\sec^2(5x°)$$

Hence,

$$\frac{d}{dx}\left(\tan(5x°)\right) = \frac{5\pi}{180°}\sec^2(5x°).$$

Differentiation Ex 11.2 Q10

Let,

$$y = 2^{x^3}$$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(2^{x^3}\right)$$

$$= 2^{x^3} \times \log 2 \frac{d}{dx}\left(x^3\right) \qquad \text{[Using chain rule]}$$

$$= 3x^2 \times 2^{x^3} \times \log 2$$

So,

$$\frac{d}{dx}\left(2^{x^3}\right) = 3x^2 \times 2^{x^3} \log 2 .$$

Differentiation Ex 11.2 Q11

Let, $y = 3^{e^x}$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(3^{e^x}\right)$$

$$= 3^{e^x} \log 3 \frac{d}{dx}\left(e^x\right) \qquad \text{[Using chain rule]}$$

$$= e^x \times 3^{e^x} \log 3$$

So,

$$\frac{d}{dx}\left(3^{e^x}\right) = e^x \times 3^{e^x} \log 3 .$$

Differentiation Ex 11.2 Q12

Let $\quad y = \log_x 3$

$$\Rightarrow \quad y = \frac{\log 3}{\log x} \qquad \left[\text{Since, } \log_a{}^b = \frac{\log b}{\log a}\right]$$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\log 3}{\log x}\right)$$

$$= \log 3 \frac{d}{dx}(\log x)^{-1}$$

$$= \log 3 \times \left[-1(\log x)^{-2}\right]\frac{d}{dx}(\log x) \qquad \text{[Using chain rule]}$$

$$= -\frac{\log 3}{(\log x)^2} \times \frac{1}{x}$$

$$= -\left(\frac{\log 3}{\log x}\right)^2 \times \frac{1}{x} \times \frac{1}{\log 3}$$

$$= -\frac{1}{x \log 3 \left(\log_3 x\right)^2} \qquad \left[\text{Since, } \frac{\log b}{\log a} = \log_a{}^b\right]$$

So,

$$\frac{d}{dx}\left(\log_x 3\right) = -\frac{1}{x \log 3 \left(\log_3 x\right)^2} .$$

Differentiation Ex 11.2 Q13

Let $y = 3^{x^2+2x}$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(3^{x^2+2x}\right)$$

$$= 3^{x^2+2x} \times \log 3 \frac{d}{dx}\left(x^2 + 2x\right) \qquad \text{[Using chain rule]}$$

$$= (2x + 2)\log 3 \times 3^{x^2+2x}$$

So,

$$\frac{d}{dx}\left(3^{x^2+2x}\right) = (2x + 2)\log 3 \times 3^{x^2+2x}.$$

Differentiation Ex 11.2 Q14

Let $\quad y = \sqrt{\dfrac{a^2 - x^2}{a^2 + x^2}}$

$\Rightarrow \quad y = \left(\dfrac{a^2 - x^2}{a^2 + x^2}\right)^{\frac{1}{2}}$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{a^2 - x^2}{a^2 + x^2}\right)^{\frac{1}{2}}$$

$$= \frac{1}{2}\left(\frac{a^2 - x^2}{a^2 + x^2}\right)^{\frac{1}{2}-1} \times \frac{d}{dx}\left(\frac{a^2 - x^2}{a^2 + x^2}\right) \qquad \text{[Using chain rule]}$$

$$= \frac{1}{2}\left(\frac{a^2 - x^2}{a^2 + x^2}\right)^{\frac{-1}{2}} \times \left\{\frac{\left(a^2 + x^2\right)\frac{d}{dx}\left(a^2 - x^2\right) - \left(a^2 - x^2\right)\frac{d}{dx}\left(a^2 + x^2\right)}{\left(a^2 + x^2\right)^2}\right\} \qquad \begin{bmatrix}\text{Using chain} \\ \text{rule}\end{bmatrix}$$

$$= \frac{1}{2}\left(\frac{a^2 + x^2}{a^2 - x^2}\right)^{\frac{1}{2}}\left\{\frac{-2x\left(a^2 + x^2\right) - 2x\left(a^2 - x^2\right)}{\left(a^2 + x^2\right)^2}\right\}$$

$$= \frac{1}{2}\left(\frac{a^2 + x^2}{a^2 - x^2}\right)^{\frac{1}{2}}\left\{\frac{-2xa^2 - 2x^3 - 2xa^2 + 2x^3}{\left(a^2 + x^2\right)^2}\right\}$$

$$= \frac{1}{2}\left(\frac{a^2 + x^2}{a^2 - x^2}\right)^{\frac{1}{2}}\left(\frac{-4xa^2}{\left(a^2 + x^2\right)^2}\right)$$

$$= \frac{-2xa^2}{\sqrt{a^2 - x^2}\left(a^2 + x^2\right)^{\frac{3}{2}}}$$

So,

$$\frac{d}{dx}\left(\sqrt{\frac{a^2 - x^2}{a^2 + x^2}}\right) = \frac{-2a^2x}{\sqrt{a^2 - x^2}\left(a^2 + x^2\right)^{\frac{3}{2}}}.$$

Differentiation Ex 11.2 Q15

Let $y = 3^{x\log x}$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(3^{x\log x}\right)$$

$$= 3^{x\log x} \times \log 3 \frac{d}{dx}(x\log x) \qquad [\text{Using chain rule}]$$

$$= 3^{x\log x} \times \log 3\left[x\frac{d}{dx}(\log x) + \log x\frac{d}{dx}(x)\right] \qquad [\text{Using chain rule}]$$

$$= 3^{x\log x} \times \log 3\left[\frac{x}{x} + \log x\right]$$

$$= 3^{x\log x}(1 + \log x) \times \log 3$$

So,

$$\frac{d}{dx}\left(3^{x}\log x\right) = \log 3 \times 3^{x\log x}(1 + \log x).$$

Differentiation Ex 11.2 Q16

Let $y = \sqrt{\dfrac{1 + \sin x}{1 - \sin x}}$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1 + \sin x}{1 - \sin x}\right)^{\frac{1}{2}}$$

$$= \frac{1}{2}\left(\frac{1 + \sin x}{1 - \sin x}\right)^{\frac{1}{2}-1} \frac{d}{dx}\left(\frac{1 + \sin x}{1 - \sin x}\right)$$

$$= \frac{1}{2}\left(\frac{1 - \sin x}{1 + \sin x}\right)^{\frac{1}{2}}\left[\frac{(1 - \sin x)(\cos x) - (1 + \sin x)(-\cos x)}{(1 - \sin x)^2}\right]$$

$$= \frac{1}{2}\frac{(1 - \sin x)^{\frac{1}{2}}}{(1 + \sin x)^{\frac{1}{2}}}\left[\frac{\cos x - \cos x \sin x + \cos x + \sin x \cos x}{(1 - \sin x)^2}\right]$$

$$= \frac{1}{2} \times \frac{2\cos x}{\sqrt{1 + \sin x}\,(1 - \sin x)^{\frac{3}{2}}}$$

$$= \frac{\cos x}{\sqrt{1 + \sin x}\,(1 - \sin x)^{\frac{3}{2}}}$$

$$= \frac{\cos x}{\sqrt{1 + \sin x}\sqrt{1 - \sin x}\,(1 - \sin x)}$$

$$= \frac{\cos x}{\sqrt{1 - \sin^2 x}\,(1 - \sin x)}$$

$$= \frac{\cos x}{\cos x\,(1 - \sin x)} \qquad \left[\text{Using } 1 - \sin^2 x = \cos^2 x\right]$$

$$= \frac{1}{(1 - \sin x)} \times \frac{(1 + \sin x)}{(1 + \sin x)}$$

$$= \frac{(1 + \sin x)}{(1 - \sin^2 x)}$$

$$= \frac{1 + \sin x}{\cos^2 x}$$

Thus, $\dfrac{dy}{dx} = \dfrac{1}{\cos^2 x} + \dfrac{\sin x}{\cos^2 x}$

$\Rightarrow \dfrac{dy}{dx} = \sec^2 x + \tan x \sec x$

$\Rightarrow \dfrac{dy}{dx} = \sec x[\tan x + \sec x]$

Differentiation Ex 11.2 Q17

Let $y = \sqrt{\dfrac{1-x^2}{1+x^2}}$

$y = \left(\dfrac{1-x^2}{1+x^2}\right)^{\frac{1}{2}}$

Differentiate it with respect to x,

$\dfrac{dy}{dx} = \dfrac{d}{dx}\left(\dfrac{1-x^2}{1+x^2}\right)^{\frac{1}{2}}$

$= \dfrac{1}{2}\left(\dfrac{1-x^2}{1+x^2}\right)^{\frac{1}{2}-1}\dfrac{d}{dx}\left(\dfrac{1-x^2}{1+x^2}\right)$ [Using chain rule]

$= \dfrac{1}{2}\left(\dfrac{1-x^2}{1+x^2}\right)^{\frac{-1}{2}}\left[\dfrac{\left(1+x^2\right)\dfrac{d}{dx}\left(1-x^2\right)-\left(1-x^2\right)\dfrac{d}{dx}\left(1+x^2\right)}{\left(1+x^2\right)^2}\right]$ [Using quotient rule]

$= \dfrac{1}{2}\left(\dfrac{1+x^2}{1-x^2}\right)^{\frac{1}{2}}\left[\dfrac{\left(1+x^2\right)(-2x)-\left(1-x^2\right)(2x)}{\left(1+x^2\right)^2}\right]$

$= \dfrac{1}{2}\left(\dfrac{1+x^2}{1-x^2}\right)^{\frac{1}{2}}\left[\dfrac{-2x-2x^3-2x+2x^3}{\left(1+x^2\right)^2}\right]$

$= \dfrac{1}{2}\dfrac{-4x}{\sqrt{1-x^2}\left(1+x^2\right)^{\frac{3}{2}}}$

So,

$\dfrac{d}{dx}\left(\sqrt{\dfrac{1-x^2}{1+x^2}}\right) = \dfrac{-2x}{\sqrt{1-x^2}\left(1+x^2\right)^{\frac{3}{2}}}$.

Differentiation Ex 11.2 Q18

Let $y = \left(\log\sin x\right)^2$

Differentiate with respect to x,

$\dfrac{dy}{dx} = \dfrac{d}{dx}\left(\log\sin x\right)^2$

$= 2\left(\log\sin x\right)\dfrac{d}{dx}\left(\log\sin x\right)$ [Using chain rule]

$= 2\left(\log\sin x\right)\times\dfrac{1}{\sin x}\dfrac{d}{dx}\left(\log x\right)$

$= 2\left(\log\sin x\right)\times\dfrac{1}{\sin x}\times\dfrac{1}{x}$

$= \dfrac{2\log\sin x}{x\sin x}$

So,

$\dfrac{d}{dx}\left(\log\sin x\right)^2 = \dfrac{2\log\sin x}{x\sin x}$

Differentiation Ex 11.2 Q19

Let $y = \sqrt{\dfrac{1+x}{1-x}}$

$\Rightarrow \quad y = \left(\dfrac{1+x}{1-x}\right)^{\frac{1}{2}}$

Differentiate it with respect to x,

$$\dfrac{dy}{dx} = \dfrac{d}{dx}\left(\dfrac{1+x}{1-x}\right)^{\frac{1}{2}}$$

$$= \dfrac{1}{2}\left(\dfrac{1+x}{1-x}\right)^{\frac{1}{2}-1}\dfrac{d}{dx}\left(\dfrac{1+x}{1-x}\right) \qquad \text{[Using chain rule]}$$

$$= \dfrac{1}{2}\left(\dfrac{1+x}{1-x}\right)^{\frac{-1}{2}}\left[\dfrac{(1-x)\dfrac{d}{dx}(1+x)-(1+x)\dfrac{d}{dx}(1-x)}{(1-x)^2}\right] \qquad \text{[Using chain rule]}$$

$$= \dfrac{1}{2}\left(\dfrac{1-x}{1+x}\right)^{\frac{1}{2}}\left[\dfrac{(1-x)(1)-(1+x)(-1)}{(1-x)^2}\right]$$

$$= \dfrac{1}{2}\left(\dfrac{1-x}{1+x}\right)^{\frac{1}{2}}\left[\dfrac{1-x+1+x}{(1-x)^2}\right]$$

$$= \dfrac{1}{2}\dfrac{(1-x)^{\frac{1}{2}}}{(1+x)^{\frac{1}{2}}}\times\dfrac{2}{(1-x)^2}$$

$$= \dfrac{1}{\sqrt{1+x}\,(1-x)^{3/2}}$$

So,

$$\dfrac{d}{dx}\left(\sqrt{\dfrac{1+x}{1-x}}\right) = \dfrac{1}{\sqrt{1+x}\,(1-x)^{3/2}}$$

Differentiation Ex 11.2 Q20

Let $y = \sin\left(\dfrac{1+x^2}{1-x^2}\right)$

Differentiate it with respect to x,

$$\dfrac{dy}{dx} = \dfrac{d}{dx}\left(\sin\left(\dfrac{1+x^2}{1-x^2}\right)\right)$$

$$= \cos\left(\dfrac{1+x^2}{1-x^2}\right)\dfrac{d}{dx}\left(\dfrac{1+x^2}{1-x^2}\right) \qquad \text{[Using chain rule]}$$

$$= \cos\left(\dfrac{1+x^2}{1-x^2}\right)\left[\dfrac{(1-x^2)\dfrac{d}{dx}(1+x^2)-(1+x^2)\dfrac{d}{dx}(1-x^2)}{(1-x)^2}\right] \qquad \text{[Using quotient rule]}$$

$$= \cos\left(\dfrac{1+x^2}{1-x^2}\right)\left[\dfrac{(1-x^2)(2x)-(1+x^2)(-2x)}{(1-x^2)^2}\right]$$

$$= \cos\left(\dfrac{1+x^2}{1-x^2}\right)\left[\dfrac{2x-2x^3+2x+2x^3}{(1-x^2)^2}\right]$$

$$= \dfrac{4x}{(1-x^2)^2}\cos\left(\dfrac{1+x^2}{1-x^2}\right)$$

So,

$$\dfrac{d}{dx}\left(\sin\left(\dfrac{1+x^2}{1-x^2}\right)\right) = \dfrac{4x}{(1-x^2)^2}\cos\left(\dfrac{1+x^2}{1-x^2}\right).$$

Differentiation Ex 11.2 Q21

Let $y = e^{3x}\cos 2x$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{3x}\cos 2x\right)$$

$$= e^{3x}\times\frac{d}{dx}(\cos 2x) + \cos 2x\,\frac{d}{dx}\left(e^{3x}\right) \qquad \text{[Using product rule]}$$

$$= e^{3x}\times(-\sin 2x)\frac{d}{dx}(2x) + \cos 2x\,e^{3x}\,\frac{d}{dx}(3x) \qquad \text{[Using chain rule]}$$

$$= -2e^{3x}\sin 2x + 3e^{3x}\cos 2x$$

$$= e^{3x}(3\cos 2x - 2\sin 2x)$$

so,

$$\frac{d}{dx}\left(e^{3x}\cos 2x\right) = e^{3x}(3\cos 2x - 2\sin 2x).$$

Differentiation Ex 11.2 Q22

Let $y = \sin(\log\sin x)$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\sin(\log\sin x)$$

$$= \cos(\log\sin x)\frac{d}{dx}(\log\sin x) \qquad \text{[Using chain rule]}$$

$$= \cos(\log\sin x)\times\frac{1}{\sin x}\frac{d}{dx}0\sin x$$

$$= \cos(\log\sin x)\frac{\cos x}{\sin x}$$

$$= \cos(\log s\sin x)\times\cot x$$

Hence,

$$\frac{d}{dx}\left(\sin(\log\sin x)\right) = \cos(\log\sin x)\times\cot x.$$

Differentiation Ex 11.2 Q23

Let $y = e^{\tan 3x}$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{\tan 3x}\right)$$

$$= e^{\tan 3x}\frac{d}{dx}(\tan 3x) \qquad \text{[Using chain rule]}$$

$$= e^{\tan 3x}\times\sec^2 3x\times\frac{d}{dx}(3x)$$

$$=$$

So,

$$\frac{d}{dx}\left(e^{\tan 3x}\right) = 3e^{\tan 3x}\times\sec^2 3x$$

Differentiation Ex 11.2 Q24

Let $\quad y = e^{\sqrt{\cot x}}$

$\Rightarrow \quad y = e^{(\cot x)^{\frac{1}{2}}}$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{(\cot x)^{\frac{1}{2}}}\right)$$

$$= e^{(\cot x)^{\frac{1}{2}}} \times \frac{d}{dx}(\cot x)^{\frac{1}{2}} \qquad \text{[Using chain rule]}$$

$$= e^{\sqrt{\cot x}} \times \frac{1}{2}(\cot x)^{\frac{1}{2}-1}\frac{d}{dx}(\cot x)$$

$$= -\frac{e^{\sqrt{\cot x}} \times \cosec^2 x}{2\sqrt{\cot x}}$$

So,

$$\frac{d}{dx}\left(e^{\sqrt{\cot x}}\right) = -\frac{e^{\sqrt{\cot x}} \times \cosec^2 x}{2\sqrt{\cot x}}$$

Differentiation Ex 11.2 Q25

Let $\quad y = \log\left(\dfrac{\sin x}{1+\cos x}\right)$

Differentiating with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\log\left(\frac{\sin x}{1+\cos x}\right)$$

$$= \frac{1}{\left(\dfrac{\sin x}{1+\cos x}\right)} \times \frac{d}{dx}\left(\frac{\sin x}{1+\cos x}\right) \qquad \text{[Using chain rule]}$$

$$= \left(\frac{1+\cos x}{\sin x}\right)\left[\frac{(1+\cos x)\dfrac{d}{dx}(\sin x) - \sin x \dfrac{d}{dx}(1+\cos x)}{(1+\cos x)^2}\right] \qquad \text{[Using quotient rule]}$$

$$= \frac{(1+\cos x)}{\sin x}\left[\frac{(1+\cos x)(\cos x) - \sin x(-\sin x)}{(1+\cos x)^2}\right]$$

$$= \frac{(1+\cos x)}{\sin x}\left[\frac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2}\right]$$

$$= \frac{(1+\cos x)}{\sin x}\left[\frac{(1+\cos x)}{(1+\cos x)^2}\right]$$

$$= \frac{1}{\sin x}$$

$$= \cosec x$$

So,

$$\frac{d}{dx}\left(\log\left(\frac{\sin x}{1+\cos x}\right)\right) = \cosec x.$$

Differentiation Ex 11.2 Q26

Let $\quad y = \log\sqrt{\dfrac{1-\cos x}{1+\cos x}}$

$\Rightarrow \quad y = \log\left(\dfrac{1-\cos x}{1+\cos x}\right)^{\frac{1}{2}}$

$\Rightarrow \quad y = \dfrac{1}{2}\log\left(\dfrac{1-\cos x}{1+\cos x}\right)$ $\qquad\qquad\qquad\qquad\qquad\qquad\left[\text{Using } \log a^b = b\log a\right]$

Differentiate it with respect to x,

$\dfrac{dy}{dx} = \dfrac{d}{dx}\left\{\dfrac{1}{2}\log\left(\dfrac{1-\cos x}{1+\cos x}\right)\right\}$

$\quad = \dfrac{1}{2}\times\dfrac{1}{\left(\dfrac{1-\cos x}{1+\cos x}\right)}\times\dfrac{d}{dx}\left(\dfrac{1-\cos x}{1+\cos x}\right)$ $\qquad\qquad$ [Using chain rule]

$\quad = \dfrac{1}{2}\left(\dfrac{1+\cos x}{1-\cos x}\right)\left[\dfrac{(1+\cos x)\dfrac{d}{dx}(1-\cos x)-(1-\cos x)\dfrac{d}{dx}(1+\cos x)}{(1+\cos x)^2}\right]$ $\qquad\left[\begin{array}{l}\text{Using}\\\text{quotient}\\\text{rule}\end{array}\right]$

$\quad = \dfrac{1}{2}\left(\dfrac{1+\cos x}{1-\cos x}\right)\left[\dfrac{(1+\cos x)(\sin x)-(1-\cos x)(-\sin x)}{(1+\cos x)^2}\right]$

$\quad = \dfrac{1}{2}\dfrac{(1+\cos x)}{(1-\cos x)}\left[\dfrac{\sin x+\sin x\cos x+\sin x-\sin x\cos x}{(1+\cos x)^2}\right]$

$\quad = \dfrac{1}{2}\dfrac{(1+\cos x)}{(1-\cos x)}\left[\dfrac{2\sin x}{(1+\cos x)^2}\right]$

$\quad = \dfrac{\sin x}{(1-\cos x)(1+\cos x)}$

$\quad = \dfrac{\sin x}{1-\cos^2 x}$

$\quad = \dfrac{\sin x}{\sin^2 x}$ $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\left[\text{Sicne } 1-\cos^2 x = \sin^2 x\right]$

$\quad = \dfrac{1}{\sin x}$

$\quad = \operatorname{cosec} x$

So,

$\dfrac{d}{dx}\left(\log\sqrt{\dfrac{1-\cos x}{1+\cos x}}\right) = \operatorname{cosec} x.$

Differentiation Ex 11.2 Q27

Let $\quad y = \tan\left(e^{\sin x}\right)$

Differentiate it with respect to x,

$\dfrac{dy}{dx} = \dfrac{d}{dx}\left[\tan e^{\sin x}\right]$

$\quad = \sec^2\left(e^{\sin x}\right)\dfrac{d}{dx}\left(e^{\sin x}\right)$ $\qquad\qquad$ [Using chain rule]

$\quad = \sec^2\left(e^{\sin x}\right)\times e^{\sin x}\times\dfrac{d}{dx}(\sin x)$

$\quad = \cos x\,\sec^2\left(e^{\sin x}\right)\times e^{\sin x}$

So,

$\dfrac{d}{dx}\left(\tan e^{\sin x}\right) = \sec^2\left(e^{\sin x}\right)\times e^{\sin x}\times\cos x.$

Differentiation Ex 11.2 Q28

Let $y = \log\left(x + \sqrt{x^2 + 1}\right)$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\log\left(x + \sqrt{x^2 + 1}\right)$$

$$= \frac{1}{x + \sqrt{x^2 + 1}}\frac{d}{dx}\left(x + \left(x^2 + 1\right)^{\frac{1}{2}}\right) \qquad \text{[Using chain rule]}$$

$$= \frac{1}{x + \sqrt{x^2 + 1}}\left[1 + \frac{1}{2}\left(x^2 + 1\right)^{\frac{1}{2}-1}\frac{d}{dx}\left(x^2 + 1\right)\right]$$

$$= \frac{1}{x + \sqrt{x^2 + 1}}\left[1 + \frac{1}{2\sqrt{x^2 + 1}} \times 2x\right]$$

$$= \frac{1}{x + \sqrt{x^2 + 1}}\left[\frac{\sqrt{x^2 + 1} + x}{\sqrt{x^2 + 1}}\right]$$

$$= \frac{1}{\sqrt{x^2 + 1}}$$

So,

$$\frac{d}{dx}\left(\log\left(x + \sqrt{x^2 + 1}\right)\right) = \frac{1}{\sqrt{x^2 + 1}}.$$

Differentiation Ex 11.2 Q29

Let $y = \dfrac{e^x \log x}{x^2}$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{x^2\frac{d}{dx}\left(e^x \log x\right) - \left(e^x \log x\right)\frac{d}{dx}\left(x^2\right)}{\left(x^2\right)^2} \qquad \text{[Using quotient rule]}$$

$$= \frac{x^2\left[e^x\frac{d}{dx}\left(\log x\right) + \log x\frac{d}{dx}\left(e^x\right) - e^x \log x \times 2x\right]}{x^4} \qquad \text{[Using product rule]}$$

$$= \frac{x^2\left[\frac{e^x}{x} + e^x \log x\right] - 2xe^x \log x}{x^4}$$

$$= \frac{\frac{x^2 e^x\left(1 + x \log x\right)}{x} - 2xe^x \log x}{x^4}$$

$$= \frac{xe^x\left[1 + x \log x - 2\log x\right]}{x^4}$$

$$= \frac{xe^x}{x^3}\left[\frac{1}{x} + \frac{x \log x}{x} - \frac{2\log x}{x}\right]$$

$$= e^x x^{-2}\left[\frac{1}{x} + \log x - \frac{2}{x}\log x\right]$$

So,

$$\frac{d}{dx}\left[\frac{e^x \log x}{x^2}\right] = e^x x^{-2}\left[\frac{1}{x} + \log x - \frac{2}{x}\log x\right].$$

Differentiation Ex 11.2 Q30

Let $y = \log(\operatorname{cosec} x - \cot x)$

Differentiating with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\log(\operatorname{cosec} x - \cot x)$$

$$= \frac{1}{(\operatorname{cosec} x - \cot x)}\frac{d}{dx}(\operatorname{cosec} x - \cot x) \qquad \text{[Using chain rule]}$$

$$= \frac{1}{(\operatorname{cosec} x - \cot x)} \times \left(-\operatorname{cosec} x \cot x + \operatorname{cosec}^2 x\right)$$

$$= \frac{\operatorname{cosec} x\,(\operatorname{cosec} x - \cot x)}{(\operatorname{cosec} x - \cot x)}$$

$$= \operatorname{cosec} x$$

So,

$$\frac{d}{dx}\left(\log(\operatorname{cosec} x - \cot x)\right) = \operatorname{cosec} x.$$

Differentiation Ex 11.2 Q31

Let $y = \dfrac{e^{2x} + e^{-2x}}{e^{2x} - e^{-2x}}$

Differentiating with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{e^{2x} + e^{-2x}}{e^{2x} - e^{-2x}}\right]$$

$$= \left[\frac{\left(e^{2x} - e^{-2x}\right)\frac{d}{dx}\left(e^{2x} + e^{-2x}\right) - \left(e^{2x} + e^{-2x}\right)\frac{d}{dx}\left(e^{2x} - e^{-2x}\right)}{\left(e^{2x} - e^{-2x}\right)^2}\right] \qquad \begin{bmatrix}\text{Using quotient rule} \\ \text{and chain rule}\end{bmatrix}$$

$$= \frac{\left(e^{2x} - e^{-2x}\right)\left[e^{2x}\frac{d}{dx}(2x) + e^{-2x}\frac{d}{dx}(-2x)\right] - \left(e^{2x} + e^{-2x}\right)\left[e^{2x}\frac{d}{dx}(2x) - e^{-2x}\frac{d}{dx}(-2x)\right]}{\left(e^{2x} - e^{-2x}\right)^2}$$

$$= \frac{\left(e^{2x} - e^{-2x}\right)\left(2e^{2x} - 2e^{-2x}\right) - \left(e^{2x} + e^{-2x}\right)\left(2e^{2x} + 2e^{-2x}\right)}{\left(e^{2x} - e^{-2x}\right)^2}.$$

$$= \frac{2\left(e^{2x} - e^{-2x}\right)^2 - 2\left(e^{2x} + e^{-2x}\right)^2}{\left(e^{2x} - e^{-2x}\right)^2}$$

$$= \frac{2\left[e^{4x} + e^{-4x} - 2e^{2x}e^{-2x} - e^{4x} - e^{-4x} - 2e^{2x}e^{-2x}\right]}{\left(e^{2x} - e^{-2x}\right)^2}$$

$$= \frac{-8}{\left(e^{2x} - e^{-2x}\right)^2}$$

So,

$$\frac{d}{dx}\left(\frac{e^{2x} + e^{-2x}}{e^{2x} - e^{-2x}}\right) = \frac{-8}{\left(e^{2x} - e^{-2x}\right)^2}.$$

Differentiation Ex 11.2 Q32

Let $y = \log\left(\dfrac{x^2 + x + 1}{x^2 - x + 1}\right)$

Differentiating with respect to x,.

$$\frac{dy}{dx} = \frac{d}{dx}\left[\log\left(\frac{x^2 + x + 1}{x^2 - x + 1}\right)\right]$$

$$= \frac{1}{\left(\dfrac{x^2 + x + 1}{x^2 - x + 1}\right)} \frac{d}{dx}\left(\frac{x^2 + x + 1}{x^2 - x + 1}\right) \qquad \text{[Using chain rule and quotient rule]}$$

$$= \left(\frac{x^2 - x + 1}{x^2 + x + 1}\right)\left[\frac{\left(x^2 - x + 1\right)\dfrac{d}{dx}\left(x^2 + x + 1\right) - \left(x^2 + x + 1\right)\dfrac{d}{dx}\left(x^2 - x + 1\right)}{\left(x^2 - x + 1\right)^2}\right]$$

$$= \left(\frac{x^2 - x + 1}{x^2 + x + 1}\right)\left[\frac{\left(x^2 - x + 1\right)(2x + 1) - \left(x^2 + x + 1\right)(2x - 1)}{\left(x^2 - x + 1\right)^2}\right]$$

$$= \frac{\left(x^2 - x + 1\right)}{\left(x^2 + x + 1\right)}\left[\frac{2x^3 - 2x^2 + 2x + x^2 - x + 1 - 2x^3 - 2x^2 - 2x + x^2 + x + 1}{\left(x^2 - x + 1\right)^2}\right]$$

$$= \frac{-4x^2 + 2x^2 + 2}{\left(x^2 + x + 1\right)\left(x^2 - x + 1\right)}$$

$$= \frac{-2\left(x^2 - 1\right)}{x^4 + 1 + 2x^2 - x^2}$$

$$= \frac{-2\left(x^2 - 1\right)}{x^4 + x^2 + 1}$$

So,

$$\frac{d}{dx}\left(\log\frac{x^2 + x + 1}{x^2 - x + 1}\right) = \frac{-2\left(x^2 - 1\right)}{x^4 + x^2 + 1}$$

Differentiation Ex 11.2 Q33

Let $y = \tan^{-1}\left(e^x\right)$

Differentiate it with respect to x,.

$$\frac{dy}{dx} = \frac{d}{dx}\left(\tan^{-1} e^x\right)$$

$$= \frac{1}{1 + \left(e^x\right)^2}\frac{d}{dx}\left(e^x\right) \qquad \text{[Using chain rule]}$$

$$= \frac{1}{1 + e^{2x}} \times e^x$$

$$= \frac{e^x}{1 + e^{2x}}$$

So,

$$\frac{d}{dx}\left(\tan^{-1} e^x\right) = \frac{e^x}{1 + e^{2x}}.$$

Differentiation Ex 11.2 Q34

Let $\quad y = e^{\sin^{-1} 2x}$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{\sin^{-1} 2x}\right)$$

$$= e^{\sin^{-1} 2x} \times \frac{d}{dx}\left(\sin^{-1} 2x\right) \qquad \text{[Using chain rule]}$$

$$= e^{\sin^{-1} 2x} \times \frac{1}{\sqrt{1 - (2x)^2}} \frac{d}{dx}(2x)$$

$$= \frac{2e^{\sin^{-1} 2x}}{\sqrt{1 - 4x^2}}$$

So,

$$\frac{d}{dx}\left(e^{\sin^{-1} 2x}\right) = \frac{2e^{\sin^{-1} 2x}}{\sqrt{1 - 4x^2}}.$$

Differentiation Ex 11.2 Q35

Let $\quad y = \sin\left(2\sin^{-1} x\right)$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sin\left(2\sin^{-1} x\right)\right)$$

$$= \cos\left(2\sin^{-1} x\right)\frac{d}{dx}\left(2\sin^{-1} x\right) \qquad \text{[Using chain rule]}$$

$$= \cos\left(2\sin^{-1} x\right) \times 2\frac{1}{\sqrt{1 - x^2}}$$

$$= \frac{2\cos\left(2\sin^{-1} x\right)}{\sqrt{1 - x^2}}$$

So,

$$\frac{d}{dx}\left(\sin\left(2\sin^{-1} x\right)\right) = \frac{2\cos\left(2\sin^{-1} x\right)}{\sqrt{1 - x^2}}.$$

Differentiation Ex 11.2 Q36

Let $\quad y = e^{\tan^{-1} \sqrt{x}}$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{\tan^{-1} \sqrt{x}}\right)$$

$$= e^{\tan^{-1} \sqrt{x}} \frac{d}{dx}\left(\tan^{-1} \sqrt{x}\right) \qquad \text{[Using chain rule]}$$

$$= e^{\tan^{-1} \sqrt{x}} \times \frac{1}{1 + \left(\sqrt{x}\right)^2}\frac{d}{dx}\left(\sqrt{x}\right)$$

$$= \frac{e^{\tan^{-1} \sqrt{x}}}{1 + x} \times \frac{1}{2\sqrt{x}}$$

$$= \frac{e^{\tan^{-1} \sqrt{x}}}{2\sqrt{x}\,(1 + x)}$$

So,

$$\frac{d}{dx}\left(e^{\tan^{-1} \sqrt{x}}\right) = \frac{e^{\tan^{-1} \sqrt{x}}}{2\sqrt{x}\,(1 + x)}.$$

Differentiation Ex 11.2 Q37

Let $y = \sqrt{\tan^{-1}\left(\dfrac{x}{2}\right)}$

$\Rightarrow \quad y = \left(\tan^{-1}\left(\dfrac{x}{2}\right)\right)^{\frac{1}{2}}$

Differentiate with respect to x,

$$\dfrac{dy}{dx} = \dfrac{d}{dx}\left(\tan^{-1}\left(\dfrac{x}{2}\right)\right)^{\frac{1}{2}}$$

$$= \dfrac{1}{2}\left(\tan^{-1}\dfrac{x}{2}\right)^{\frac{1}{2}-1}\dfrac{d}{dx}\left(\tan^{-1}\dfrac{x}{2}\right) \qquad \text{[Using chain rule]}$$

$$= \dfrac{1}{2}\left(\tan^{-1}\dfrac{x}{2}\right)^{\frac{-1}{2}} \times \dfrac{1}{1+\left(\dfrac{x}{2}\right)^2} \times \dfrac{d}{dx}\left(\dfrac{x}{2}\right)$$

$$= \dfrac{4}{4\sqrt{\tan^{-1}\left(\dfrac{x}{2}\right)}\left(4+x^2\right)}$$

$$= \dfrac{1}{\left(4+x^2\right)\sqrt{\tan^{-1}\left(\dfrac{x}{2}\right)}}$$

So,

$$\dfrac{d}{dx}\left(\sqrt{\tan^{-1}\left(\dfrac{x}{2}\right)}\right) = \dfrac{1}{\left(4+x^2\right)\sqrt{\tan^{-1}\left(\dfrac{x}{2}\right)}}.$$

Differentiation Ex 11.2 Q38

Let $y = \log\left(\tan^{-1}x\right)$

Differentiate with respect to x,.

$$\dfrac{dy}{dx} = \dfrac{d}{dx}\log\left(\tan^{-1}x\right)$$

$$= \dfrac{1}{\tan^{-1}x} \times \dfrac{d}{dx}\left(\tan^{-1}x\right) \qquad \text{[Using chain rule]}$$

$$= \dfrac{1}{\left(1+x^2\right)\tan^{-1}x}$$

So,

$$\dfrac{d}{dx}\left(\log\tan^{-1}x\right) = \dfrac{1}{\left(1+x^2\right)\tan^{-1}x}.$$

Differentiation Ex 11.2 Q39

Let $y = \dfrac{2^x \cos x}{\left(x^2 + 3\right)^2}$

Differentiating with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{2^x \cos x}{\left(x^2 + 3\right)^2}\right]$$

$$= \left[\frac{\left(x^2 + 3\right)^2 \dfrac{d}{dx}\left(2^x \cos x\right) - \left(2^x \cos x\right)\dfrac{d}{dx}\left(x^2 + 3\right)^2}{\left[\left(x^2 + 3\right)^2\right]^2}\right]$$

[Using quotient rule, product rule and chain rule]

$$= \left[\frac{\left(x^2 + 3\right)^2\left[2^x \dfrac{d}{dx}\cos x + \cos x \dfrac{d}{dx}2^x\right] - \left(2^x \cos x\right)2\left(x^2 + 3\right)\dfrac{d}{dx}\left(x^2 + 3\right)}{\left(x^2 + 3\right)^4}\right]$$

$$= \left[\frac{\left(x^2 + 3\right)^2\left[-2^x \sin x + \cos x \, 2^x \log 2\right] - 2\left(2^x \cos x\right)\left(x^2 + 3\right)(2x)}{\left(x^2 + 3\right)^4}\right]$$

$$= \left[\frac{2^x\left(x^2 + 3\right)\left[\left(x^2 + 3\right)\{\cos x \log 2 - \sin x\}\right] - 4x\cos x}{\left(x^2 + 3\right)^4}\right]$$

$$= \frac{2^x}{\left(x^2 3\right)^2}\left[\cos x \log 2 - \sin x - \frac{4x \cos x}{\left(x^2 + 3\right)}\right]$$

So,

$$\frac{d}{dx}\left(\frac{2^x \cos x}{\left(x^2 + 3\right)^2}\right) = \frac{2^x}{\left(x^2 + 3\right)^2}\left[\cos x \log 2 - \sin x - \frac{4x \cos x}{\left(x^2 + 3\right)}\right].$$

Differentiation Ex 11.2 Q40

Let $y = x \sin 2x + 5^x + k^k + \left(\tan^2 x\right)^3$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left[x \sin 2x + 5^x + k^k + \left(\tan^6 x\right)\right]$$

$$= \frac{d}{dx}\left(x \sin 2x\right) + \frac{d}{dx}\left(5^x\right) + \frac{d}{dx}\left(k^k\right) + \frac{d}{dx}\left(\tan^6 x\right)$$

$$= \left[x \frac{d}{dx}\left(\sin 2x\right) + \sin 2x \frac{d}{dx}(x)\right] + 5^x \log 5 + 0 + 6\tan^5 x \frac{d}{dx}\left(\tan x\right)$$

[Using product rule and chain rule]

$$= \left[x \cos 2x \frac{d}{dx}\left(2x\right) + \sin 2x\right] + 5^x \log 5 + 6\tan^5 x \sec^2 x$$

$$= 2x \cos 2x + \sin 2x + 5^x \log 5 + 6\tan^5 x \sec^2 x$$

so,

$$\frac{d}{dx}\left(x \sin 2x + 5^x + k^k + \left(\tan^2 x\right)^3\right) = 2x \cos 2x + \sin 2x + 5^x \log 5 + 6\tan^5 x \sec^2 x.$$

Differentiation Ex 11.2 Q41

Let $\quad y = \log(3x + 2) - x^2 \log(2x - 1)$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\Big[\log(3x + 2) - x^2 \log(2x - 1)\Big]$$

$$= \frac{d}{dx}\log(3x + 2) - \frac{d}{dx}\big(x^2 \log(2x - 1)\big)$$

$$= \frac{1}{(3x + 2)}\frac{d}{dx}(3x + 2) - \Big[x^2 \frac{d}{dx}\log(2x - 1) + \log(2x - 1)\frac{d}{dx}(x^2)\Big]$$

[Using product rule and chain rule]

$$= \frac{3}{3x + 2} - \Big[x^2 \times \frac{1}{(2x - 1)}\frac{d}{dx}(2x - 1) + \log(2x - 1) \times 2x\Big]$$

$$= \frac{3}{3x + 2} - \frac{2x^2}{(2x - 1)} - 2x \log(2x - 1)$$

So,

$$\frac{d}{dx}\big(\log(3x + 2) - x^2\log(2x - 1)\big) = \frac{3}{3x + 2} - \frac{2x^2}{(2x - 1)} - 2x\log(2x - 1).$$

Differentiation Ex 11.2 Q42

Let $\quad y = \dfrac{3x^2 \sin x}{\sqrt{7 - x^2}}$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{3x^2 \sin x}{\left(7 - x^2\right)^{\frac{1}{2}}}\right)$$

$$= \frac{\left(7 - x^2\right)^{\frac{1}{2}} \times \frac{d}{dx}\left(3x^2 \sin x\right) - 3x^2 \sin x \frac{d}{dx}\left(7 - x^2\right)^{\frac{1}{2}}}{\left[\left(7 - x^2\right)^{\frac{1}{2}}\right]^2}$$

[Using quotient rule, chain and product rule]

$$= \left[\frac{\left(7 - x^2\right)^{\frac{1}{2}} \times 3 \times \left[x^2 \frac{d}{dx}\sin x + \sin x \frac{d}{dx}x^2\right] - 3x^2 \sin x \times \frac{1}{2}\left(7 - x^2\right)^{\frac{-1}{2}}\frac{d}{dx}\left(7 - x^2\right)}{\left(7 - x^2\right)}\right]$$

$$= \left[\frac{\left(7 - x^2\right)^{\frac{1}{2}} 3\left(x^2 \cos x + 2x \sin x\right) - 3x^2 \sin x \times \frac{1}{2}\left(7 - x^2\right)^{\frac{-1}{2}}(-2x)}{\left(7 - x^2\right)}\right]$$

$$= \left[\frac{\left(7 - x^2\right)^{\frac{1}{2}} \times 3\left(x^2 \cos x + 2x \sin x\right)}{\left(7 - x^2\right)} + \frac{3x^3 \sin x \left(7 - x^2\right)^{\frac{-1}{2}}}{\left(7 - x^2\right)}\right]$$

$$= \left[\frac{6x \sin x + 3x^2 \cos x}{\sqrt{\left(7 - x^2\right)}} + \frac{3x^3 \sin x}{\left(7 - x^2\right)^{\frac{3}{2}}}\right]$$

So,

$$\frac{d}{dx}\left(\frac{3x^2 \sin x}{\sqrt{7 - x^2}}\right) = \left[\frac{6x \sin x + 3x^2 \cos x}{\sqrt{7 - x^2}} + \frac{3x^3 \sin x}{\left(7 - x^2\right)^{\frac{3}{2}}}\right].$$

Differentiation Ex 11.2 Q43

Let $y = \sin^2\left[\log(2x + 3)\right]$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left[\sin^2\left(\log(2x + 3)\right)\right]$$

$$= 2\sin\left(\log(2x + 3)\right)\frac{d}{dx}\sin\left(\log(2x + 3)\right) \quad \textbf{Using chain rule}$$

$$= 2\sin\left(\log(2x + 3)\right)\cos\left(\log(2x + 3)\right)\frac{d}{dx}\log(2x + 3)$$

$$= \sin\left(2\log(2x + 3)\right) \times \frac{1}{(2x + 3)}\frac{d}{dx}(2x + 3)$$

$$\left[\text{Since, } 2\sin A \cos A = \sin^2 A\right]$$

$$= \sin\left(2\log(2x + 3)\right) \times \frac{2}{(2x + 3)}$$

So,

$$\frac{d}{dx}\left(\sin^2\log(2x + 3)\right) = \sin\left(2\log(2x + 3)\right) \times \frac{2}{(2x + 3)}.$$

Differentiation Ex 11.2 Q44

Let $y = e^x \log\sin 2x$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left[e^x \log\sin 2x\right]$$

$$= e^x \frac{d}{dx}\log\sin 2x + \log\sin 2x \frac{d}{dx}\left(e^x\right)$$

$$\left[\text{Using product rule and chain rule}\right]$$

$$= e^x \frac{1}{\sin 2x}\frac{d}{dx}(\sin 2x) + \log\sin 2x \left(e^x\right)$$

$$= \frac{e^x}{\sin 2x}\cos 2x \frac{d}{dx}(2x) + e^x \log\sin 2x$$

$$= \frac{2\cos 2x e^x}{\sin 2x} + e^x \log\sin 2x$$

$$= e^x \left(2\cot 2x + \log\sin 2x\right)$$

so,

$$\frac{d}{dx}\left(e^x \log\sin 2x\right) = e^x \left(2\cot 2x + \log\sin 2x\right).$$

Differentiation Ex 11.2 Q45

Let $\quad y = \dfrac{\sqrt{x^2+1} + \sqrt{x^2-1}}{\sqrt{x^2+1} - \sqrt{x^2-1}}$

$\Rightarrow \quad y = \dfrac{\left(x^2+1\right)^{\frac{1}{2}} + \left(x^2-1\right)^{\frac{1}{2}}}{\left(x^2+1\right)^{\frac{1}{2}} - \left(x^2-1\right)^{\frac{1}{2}}}$

Differentiate it with respect to x,

$$\dfrac{dy}{dx} = \dfrac{d}{dx}\left[\dfrac{\left(x^2+1\right)^{\frac{1}{2}} + \left(x^2-1\right)^{\frac{1}{2}}}{\left(x^2+1\right)^{\frac{1}{2}} - \left(x^2-1\right)^{\frac{1}{2}}}\right]$$

$$= \left[\dfrac{\left\{\left(x^2+1\right)^{\frac{1}{2}} - \left(x^2-1\right)^{\frac{1}{2}}\right\}\dfrac{d}{dx}\left\{\left(x^2+1\right)^{\frac{1}{2}} + \left(x^2-1\right)^{\frac{1}{2}}\right\} - \left\{\left(x^2+1\right)^{\frac{1}{2}} + \left(x^2-1\right)^{\frac{1}{2}}\right\}}{\left\{\left(x^2+1\right)^{\frac{1}{2}} - \left(x^2-1\right)^{\frac{1}{2}}\right\}^2} \dfrac{d}{dx}\left\{\left(x^2+1\right)^{\frac{1}{2}} - \left(x^2-1\right)^{\frac{1}{2}}\right\}\right]$$

$\left[\text{Using quotient rule and chian rule}\right]$

$$= \left[\dfrac{\left\{\left(x^2+1\right)^{\frac{1}{2}} - \left(x^2-1\right)^{\frac{1}{2}}\right\}\left[\dfrac{1}{2}\left(x^2+1\right)^{\frac{-1}{2}}\dfrac{d}{dx}\left(x^2+1\right) + \dfrac{1}{2}\left(x^2-1\right)^{\frac{-1}{2}}\dfrac{d}{dx}\left(x^2-1\right)\right]}{\left[\left(x^2+1\right) + \left(x^2-1\right) - 2\sqrt{x^4-1}\right]}\right]$$

$$- \left[\dfrac{\left\{\left(x^2+1\right)^{\frac{1}{2}} + \left(x^2-1\right)^{\frac{1}{2}}\right\}\dfrac{1}{2}\left[\left(x^2+1\right)^{\frac{-1}{2}}\dfrac{d}{dx}\left(x^2+1\right) - \dfrac{1}{2}\left(x^2-1\right)^{\frac{-1}{2}}\dfrac{d}{dx}\left(x^2-1\right)\right]}{\left[\left(x^2+1\right)\left(x^2-1\right) - 2\sqrt{x^4-1}\right]}\right]$$

$$= \left[\frac{\left(\sqrt{x^2+1} - \sqrt{x^2-1}\right)\left(\frac{2x}{2\sqrt{x^2+1}} + \frac{2x}{2\sqrt{x^2-1}}\right)}{\left[2x^2 - 2\sqrt{x^4-1}\right]} \right] -$$

$$\left[\frac{\left(\sqrt{x^2+1} + \sqrt{x^2-1}\right)\left(\frac{2x}{2\sqrt{x^2+1}} - \frac{2x}{2\sqrt{x^2-1}}\right)}{\left[2x^2 - 2\sqrt{x^4-1}\right]} \right]$$

$$= \left[\frac{x\left(\sqrt{x^2+1} - \sqrt{x^2-1}\right)\left(\sqrt{x^2-1} + \sqrt{x^2+1}\right)}{2\left[x^2 - \sqrt{x^4-1}\right]\left(\sqrt{x^2+1}\sqrt{x^2-1}\right)} \right] -$$

$$\left[\frac{x\left(\sqrt{x^2+1} + \sqrt{x^2-1}\right)\left(\sqrt{x^2-1} - \sqrt{x^2+1}\right)}{2\left[x^2 - \sqrt{x^4-1}\right]\left(\sqrt{x^2+1}\sqrt{x^2-1}\right)} \right]$$

$$= \left[\frac{x\left(x^2+1-x^2+1\right) - x\left(x^2-1-x^2-1\right)}{2\left[x^2 - \sqrt{x^4-1}\right]\sqrt{x^4-1}} \right]$$

$$= \left[\frac{4x}{2\left(x^2 - \sqrt{x^4-1}\right)\sqrt{x^4-1}} \right]$$

$$= 2x \left[\frac{1 \times \left(x^2 + \sqrt{x^4-1}\right)}{\left(x^2 - \sqrt{x^4-1}\sqrt{x^4-1} \times \left(x^2 + \sqrt{x^4-1}\right)\right)} \right]$$

Multiplying and divide by $\left\{x^2 + \sqrt{x^4-1}\right\}$,

$$= 2x \left[\frac{x^2 + \sqrt{x^4-1}}{\left(x^4 - x^4 + 1\right)\sqrt{x^4-1}} \right]$$

$$= 2x \left[\frac{x^2 + \sqrt{x^4-1}}{\sqrt{x^4-1}} \right]$$

$$= \frac{2x^3}{\sqrt{x^4-1}} + 2x$$

So,

$$\frac{d}{dx}\left[\frac{\sqrt{x^2+1} + \sqrt{x^2-1}}{\sqrt{x^2+1} - \sqrt{x^2-1}} \right] = \frac{2x^3}{\sqrt{x^4-1}} + 2x.$$

Differentiation Ex 11.2 Q46

Let $y = \log\left[x + 2 + \sqrt{x^2 + 4x + 1}\right]$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\log\left[x + 2 + \sqrt{x^2 + 4x + 1}\right]$$

$$= \frac{1}{\left[x + 2 + \sqrt{x^4 + 4x + 1}\right]}\frac{d}{dx}\left[x + 2 + \left(x^2 + 4x + 1\right)^{\frac{1}{2}}\right]$$

[using chain rule]

$$= \frac{1}{\left[x + 2 + \sqrt{x^4 + 4x + 1}\right]} \times \left[1 + 0 + \frac{1}{2}\left(x^2 + 4x + 1\right)^{\frac{-1}{2}}\frac{d}{dx}\left(x^2 + 4x + 1\right)\right]$$

$$= \frac{1 + \dfrac{(2x + 4)}{2\left(\sqrt{x^2 + 4x + 1}\right)}}{\left[x + 2 + \sqrt{x^4 + 4x + 1}\right]}$$

$$= \frac{\sqrt{x^2 + 4x + 1} + x + 2}{\left[x + 2 + \sqrt{x^2 + 4x + 1}\right] \times \sqrt{x^2 + 4x + 1}}$$

$$= \frac{1}{\sqrt{x^2 + 4x + 1}}$$

So,

$$\frac{d}{dx}\left[\log\left\{x + 2 + \sqrt{x^2 + 4x + 1}\right\}\right] = \frac{1}{\sqrt{x^2 + 4x + 1}}.$$

Differentiation Ex 11.2 Q47

Let $y = \left(\sin^{-1} x^4\right)^4$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sin^{-1} x^4\right)^4$$

$$= 4\left(\sin^{-1} x^4\right)\frac{d}{dx}\left(\sin^{-1} x^4\right) \qquad \text{[Using chain rule]}$$

$$= 4\left(\sin^{-1} x^4\right)^3 \frac{1}{\sqrt{1 - \left(x^4\right)^2}}\frac{d}{dx}\left(x^4\right)$$

$$= 4\left(\sin^{-1} x^4\right)^3 \frac{4x^3}{\sqrt{1 - x^8}}$$

$$= \frac{16x^3\left(\sin^{-1} x^4\right)^3}{\sqrt{1 - x^8}}$$

So,

$$\frac{d}{dx}\left(\sin^{-1} x^4\right) = \frac{16x^3\left(\sin^{-1} x^4\right)^3}{\sqrt{1 - x^8}}.$$

Differentiation Ex 11.2 Q48

Let $y = \sin^{-1}\left(\dfrac{x}{\sqrt{x^2 + a^2}}\right)$

Differentiating with respect to x,

$\dfrac{dy}{dx} = \dfrac{d}{dx}\sin^{-1}\left(\dfrac{x}{\sqrt{x^2 + a^2}}\right)$

$= \dfrac{1}{\sqrt{1 - \left(\dfrac{x}{\sqrt{x^2 + a^2}}\right)^2}} \times \dfrac{d}{dx}\left(\dfrac{x}{\sqrt{x^2 + a^2}}\right)$ [Using chain rule and quotient rule]

$= \dfrac{1}{\sqrt{1 - \left(\dfrac{x}{\sqrt{x^2 + a^2}}\right)^2}} \times \left[\dfrac{(x^2 + a^2)^{\frac{1}{2}}\dfrac{d}{dx}(x) - \dfrac{d}{dx}(x^2 + a^2)^{\frac{1}{2}}}{\left[(x^2 + a^2)^{\frac{1}{2}}\right]^2}\right]$

$= \dfrac{\sqrt{x^2 + a^2}}{\sqrt{x^2 + a^2 - x^2}}\left[\dfrac{\sqrt{x^2 + a^2} - x \times \dfrac{1}{2\sqrt{x^2 + a^2}}\dfrac{d}{dx}(x^2 + a^2)}{(x^2 + a^2)}\right]$

$= \dfrac{\sqrt{x^2 + a^2}}{a(x^2 + a^2)}\left[\sqrt{x^2 + a^2} - \dfrac{x}{2\sqrt{x^2 + a^2}} \times 2x\right]$

$= \dfrac{\sqrt{x^2 + a^2}}{a(x^2 + a^2)}\left[\dfrac{x^2 + a^2 - x^2}{\sqrt{x^2 + a^2}}\right]$

$= \dfrac{a^2}{a(x^2 + a^2)}$

$= \dfrac{a}{(a^2 + x^2)}$

So,

$\dfrac{d}{dx}\left(\sin^{-1}\dfrac{x}{\sqrt{x^2 + a^2}}\right) = \dfrac{a}{a^2 + x^2}$

Differentiation Ex 11.2 Q49

Consider

$y = \dfrac{e^x \sin x}{(x^2 + 2)^3}$

Differentiating it with respect to x and applying the chain and product rule, we get

$\dfrac{dy}{dx} = \dfrac{(x^2 + 2)^3 \dfrac{d}{dx}(e^x \sin x) - e^x \sin x \dfrac{d}{dx}(x^2 + 2)^3}{\left[(x^2 + 2)^3\right]^2}$

$= \dfrac{(x^2 + 2)^3\left[e^x \cos x + \sin x e^x\right] - e^x \sin x\, 3(x^2 + 2)^2(2x)}{(x^2 + 2)^6}$

$= \dfrac{(x^2 + 2)^3\left[e^x \cos x + \sin x e^x\right] - 6xe^x \sin x(x^2 + 2)^2}{(x^2 + 2)^6}$

$= \dfrac{(x^2 + 2)^2\left[(x^2 + 2)(e^x \cos x + \sin x e^x) - 6xe^x \sin x\right]}{(x^2 + 2)^6}$

$= \dfrac{x^2 e^x \cos x + x^2 \sin x e^x + 2e^x \cos x + 2\sin x e^x - 6xe^x \sin x}{(x^2 + 2)^4}$

$= \dfrac{e^x \sin x}{(x^2 + 2)^3} + \dfrac{e^x \cos x}{(x^2 + 2)^3} - \dfrac{6xe^x \sin x}{(x^2 + 2)^4}$

Therefore,

$\dfrac{dy}{dx} = \dfrac{e^x \sin x}{(x^2 + 2)^3} + \dfrac{e^x \cos x}{(x^2 + 2)^3} - \dfrac{6xe^x \sin x}{(x^2 + 2)^4}$

Differentiation Ex 11.2 Q50

Consider

$$y = 3e^{-3x} \log (1+x)$$

Differentiating it with respect to x and applying the chain and product rule, we get

$$\frac{dy}{dx} = 3\frac{d}{dx}\left[e^{-3x} \log (1+x)\right]$$

$$\frac{dy}{dx} = 3\left(e^{-3x}\frac{1}{1+x} + \log(1+x)\left(-3e^{-3x}\right)\right)$$

$$= 3\left(\frac{e^{-3x}}{1+x} - 3e^{-3x} \log (1+x)\right)$$

$$= 3e^{-3x}\left(\frac{1}{1+x} - 3\log (1+x)\right)$$

Differentiation Ex 11.2 Q51

Consider

$$y = \frac{x^2 + 2}{\sqrt{\cos x}}$$

Differentiating it with respect to x and applying the chain and product rule, we get

$$\frac{dy}{dx} = \frac{\sqrt{\cos x}\frac{d}{dx}(x^2+2) - (x^2+2)\frac{d}{dx}\sqrt{\cos x}}{\left(\sqrt{\cos x}\right)^2}$$

$$= \frac{2x\sqrt{\cos x} - (x^2+2)\left(-\frac{1}{2}\frac{\sin x}{\sqrt{\cos x}}\right)}{\cos x}$$

$$= \frac{2x\sqrt{\cos x} + \frac{(x^2+2)\sin x}{2\sqrt{\cos x}}}{\cos x}$$

$$= \frac{4x\cos x + (x^2+2)\sin x}{2(\cos x)^{\frac{3}{2}}}$$

$$= \frac{2x}{\sqrt{\cos x}} + \frac{1}{2}\frac{(x^2+2)\sin x}{(\cos x)^{\frac{3}{2}}}$$

Therefore,

$$\frac{dy}{dx} = \frac{2x}{\sqrt{\cos x}} + \frac{1}{2}\frac{(x^2+2)\sin x}{(\cos x)^{\frac{3}{2}}}$$

Differentiation Ex 11.2 Q52

Consider

$$y = \frac{x^2(1-x^2)^3}{\cos 2x}$$

Differentiating it with respect to x and applying the chain and product rule, we get

$$\frac{dy}{dx} = \frac{\cos 2x\frac{d}{dx}x^2(1-x^2)^3 - x^2(1-x^2)^3\frac{d}{dx}\cos 2x}{\cos^2 2x}$$

$$= \frac{\cos 2x\left[x^2\frac{d}{dx}(1-x^2)^3 + (1-x^2)^3\frac{d}{dx}x^2 - x^2(1-x^2)^3(-2\sin 2x)\right]}{\cos^2 2x}$$

$$= \frac{\cos 2x\left[-6x^3(1-x^2)^2 + (1-x^2)^3 2x + 2x^2(1-x^2)^3 \sin 2x\right]}{\cos^2 2x}$$

$$= \frac{2x(1-x^2)^2}{\cos 2x} - \frac{6x^3(1-x^2)^2}{\cos 2x} + \frac{2x^2(1-x^2)^3 \sin 2x}{\cos^2 2x}$$

$$= 2x(1-x^2)\sec 2x\{1-4x^2 + x(1-x^2)\tan 2x\}$$

Therefore,

$$\frac{dy}{dx} = 2x(1-x^2)\sec 2x\{1-4x^2 + x(1-x^2)\tan 2x\}$$

Differentiation Ex 11.2 Q53

Consider

$$y = \log(3x+2) - x^2 \log(2x-1)$$

Differentiating it with respect to x and applying the chain and product rule, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left[\log(3x+2) - x^2 \log(2x-1)\right]$$

$$\frac{dy}{dx} = \frac{3}{3x+2} - \left[x^2\frac{d}{dx}\log(2x-1) + \log(2x-1)\frac{d}{dx}x^2\right]$$

$$\frac{dy}{dx} = \frac{3}{3x+2} - \left(\frac{2x^2}{2x-1} + 2x\log(2x-1)\right)$$

$$\frac{dy}{dx} = \frac{3}{3x+2} - \frac{2x^2}{2x-1} - 2x\log(2x-1)$$

Therefore,

$$\frac{dy}{dx} = \frac{3}{3x+2} - \frac{2x^2}{2x-1} - 2x\log(2x-1)$$

Differentiation Ex 11.2 Q54

Consider
$$y = e^{ax} \sec x \tan 2x$$
Differentiating it with respect to x and applying the chain and product rule, we get
$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{ax} \sec x \tan 2x\right)$$
$$= e^{ax} \frac{d}{dx} \sec x \tan 2x + \sec x \tan 2x \frac{d}{dx} e^{ax}$$
$$= e^{ax}\left[\sec x \tan x \tan 2x + \left(2 + 2\tan^2 2x\right)\sec x\right] + ae^{ax} \sec x \tan 2x$$
$$= e^{ax}\left[\sec x \tan x \tan 2x + 2\sec x + 2\tan^2 2x \sec x\right] + ae^{ax} \sec x \tan 2x$$
$$= ae^{ax} \sec x \tan 2x + e^{ax} \sec x \tan x \tan 2x + e^{ax} \sec x \left(2 + 2\tan^2 2x\right)$$
$$\frac{dy}{dx} = e^{ax} \sec x \left\{a \tan 2x + \tan x \tan 2x + 2\sec^2 2x\right\}$$
Therefore,
$$\frac{dy}{dx} = e^{ax} \sec x \left\{a \tan 2x + \tan x \tan 2x + 2\sec^2 2x\right\}$$

Differentiation Ex 11.2 Q55

Consider
$$y = \log\left(\cos x^2\right)$$
Differentiating it with respect to x and applying the chain and product rule, we get
$$\frac{dy}{dx} = \frac{d}{dx} \log\left(\cos x^2\right)$$
$$= \frac{-2x \sin x^2}{\cos x^2}$$
$$\frac{dy}{dx} = -2x \tan x^2$$
Therefore,
$$\frac{dy}{dx} = -2x \tan x^2$$

Differentiation Ex 11.2 Q56

Consider
$$y = \cos\left(\log x\right)^2$$
Differentiating it with respect to x and applying the chain and product rule, we get
$$\frac{dy}{dx} = \frac{d}{dx} \cos\left(\log x\right)^2$$
$$= -\sin\left(\log x\right)^2 \frac{d}{dx}\left(\log x\right)^2$$
$$= -\sin\left(\log x\right)^2 \frac{2\log x}{x}$$
$$\frac{dy}{dx} = \frac{-2\log x \sin\left(\log x\right)^2}{x}$$
Therefore,
$$\frac{dy}{dx} = \frac{-2\log x \sin\left(\log x\right)^2}{x}$$

Differentiation Ex 11.2 Q57

Consider
$$y = \log \sqrt{\frac{x-1}{x+1}}$$
Differentiating it with respect to x and applying the chain and product rule, we get
$$y = \log\left(\frac{x-1}{x+1}\right)^{\frac{1}{2}}$$
$$y = \frac{1}{2} \log\left(\frac{x-1}{x+1}\right)$$
$$y = \frac{1}{2}\left[\log(x-1) - \log(x+1)\right]$$
$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{d}{dx}\log(x-1) - \frac{d}{dx}\log(x+1)\right]$$
$$= \frac{1}{2}\left(\frac{1}{x-1} - \frac{1}{x+1}\right)$$
$$= \frac{1}{2}\left(\frac{2}{x^2-1}\right)$$
$$\frac{dy}{dx} = \frac{1}{x^2-1}$$
Therefore,
$$\frac{dy}{dx} = \frac{1}{x^2-1}$$

Differentiation Ex 11.2 Q58

Here $y = \log\left\{\sqrt{x-1} - \sqrt{x+1}\right\}$

Differentiating it with respect to x and applying the chain and product rule, we get

$$\frac{dy}{dx} = \frac{d}{dx}\log\left\{\sqrt{x-1} - \sqrt{x+1}\right\}$$

$$\frac{dy}{dx} = \frac{1}{\left\{\sqrt{x-1} - \sqrt{x+1}\right\}}\frac{d}{dx}\left(\sqrt{x-1} - \sqrt{x+1}\right)$$

$$= \frac{1}{\left\{\sqrt{x-1} - \sqrt{x+1}\right\}}\left[\frac{d}{dx}\sqrt{x-1} - \frac{d}{dx}\sqrt{x+1}\right]$$

$$= \frac{1}{\left\{\sqrt{x-1} - \sqrt{x+1}\right\}}\left[\frac{1}{2}(x-1)^{-\frac{1}{2}} - \frac{1}{2}(x+1)^{-\frac{1}{2}}\right]$$

$$= \frac{1}{2}\frac{1}{\left\{\sqrt{x-1} - \sqrt{x+1}\right\}}\left(\frac{1}{\sqrt{x-1}} - \frac{1}{\sqrt{x+1}}\right)$$

$$= \frac{1}{2}\frac{1}{\left\{\sqrt{x-1} - \sqrt{x+1}\right\}}\left(\frac{-\left\{\sqrt{x-1} - \sqrt{x+1}\right\}}{\left(\sqrt{x-1}\right)\left(\sqrt{x+1}\right)}\right)$$

$$= \frac{-1}{2}\left(\frac{1}{\left(\sqrt{x-1}\right)\left(\sqrt{x+1}\right)}\right)$$

$$\frac{dy}{dx} = \frac{-1}{2\sqrt{x^2-1}}$$

Therefore,

$$\frac{dy}{dx} = \frac{-1}{2\sqrt{x^2-1}}$$

Differentiation Ex 11.2 Q59

Here $y = \sqrt{x+1} + \sqrt{x-1}$

Differentiating it with respect to x and applying the chain and product rule, we get

$$\frac{dy}{dx} = \frac{d}{dx}\sqrt{x+1} + \frac{d}{dx}\sqrt{x-1}$$

$$= \frac{1}{2}(x+1)^{-\frac{1}{2}} + \frac{1}{2}(x-1)^{-\frac{1}{2}}$$

$$= \frac{1}{2}\left(\frac{1}{\sqrt{x+1}} + \frac{1}{\sqrt{x-1}}\right)$$

$$= \frac{1}{2}\left(\frac{\sqrt{x-1} + \sqrt{x+1}}{\left(\sqrt{x+1}\right)\left(\sqrt{x-1}\right)}\right)$$

$$\frac{dy}{dx} = \frac{1}{2}\left(\frac{y}{\left(\sqrt{x^2-1}\right)}\right)$$

$$\sqrt{x^2-1}\frac{dy}{dx} = \frac{1}{2}y$$

Differentiation Ex 11.2 Q60

Here $y = \frac{x}{x+2}$

Differentiating it with respect to x and applying the chain and product rule, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x}{x+2}\right)$$

$$= \frac{(x+2)\frac{dx}{dx} - x\frac{d}{dx}(x+2)}{(x+2)^2}$$

$$= \frac{x+2-x}{(x+2)^2}$$

$$= \frac{x+2}{(x+2)^2} - \frac{x}{(x+2)^2}$$

$$= \frac{1}{x+2} - \frac{xy^2}{x^2} \qquad \left[\text{Since } x+2 = \frac{x}{y}\right]$$

$$= \frac{y}{x} - \frac{y^2}{x}$$

$$\frac{dy}{dx} = \frac{1}{x}y(1-y)$$

$$x\frac{dy}{dx} = (1-y)y$$

Differentiation Ex 11.2 Q61

Here $y = \log\left(\sqrt{x} + \dfrac{1}{\sqrt{x}}\right)$

Differentiating it with respect to x and applying the chain and product rule, we get

$$\frac{dy}{dx} = \frac{d}{dx}\log\left(x^{\frac{1}{2}} + x^{-\frac{1}{2}}\right)$$

$$= \frac{1}{x^{\frac{1}{2}} + x^{-\frac{1}{2}}} \frac{d}{dx}\left(x^{\frac{1}{2}} + x^{-\frac{1}{2}}\right)$$

$$= \frac{1}{\sqrt{x} + \dfrac{1}{\sqrt{x}}}\left(\frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2}x^{-\frac{1}{2}}x^{-1}\right)$$

$$= \frac{1}{2}\frac{\sqrt{x}}{x+1}\left(\frac{1}{\sqrt{x}} - \frac{1}{x\sqrt{x}}\right)$$

$$= \frac{1}{2}\frac{\sqrt{x}}{x+1}\left(\frac{x-1}{x\sqrt{x}}\right)$$

$$\frac{dy}{dx} = \frac{x-1}{2x(x+1)}$$

Differentiation Ex 11.2 Q62

Given, $y = \sqrt{\dfrac{1+e^x}{1-e^x}}$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sqrt{\frac{1+e^x}{1-e^x}}\right)$$

$$= \frac{1}{2\sqrt{\dfrac{1+e^x}{1-e^x}}} \times \frac{d}{dx}\left(\frac{1+e^x}{1-e^x}\right) \qquad \text{[Using chain rule, quotient rule]}$$

$$= \frac{1}{2} \times \sqrt{\frac{1-e^x}{1+e^x}}\left[\frac{\left(1-e^x\dfrac{d}{dx}\left(1+e^x\right) - \left(1+e^x\right)\dfrac{d}{dx}\left(1-e^x\right)\right)}{\left(1-e^x\right)^2}\right]$$

$$= \frac{1}{2}\sqrt{\frac{1-e^x}{1+e^x}}\left[\frac{\left(1-e^x\right)e^x + \left(1+e^x\right)e^x}{\left(1-e^x\right)2}\right]$$

$$= \frac{1}{2}\sqrt{\frac{1-e^x}{1+e^x}} \times \left[\frac{2e^x}{\left(1-e^x\right)^2}\right]$$

$$= \frac{e^x}{\sqrt{\left(1+e^x\right)}\sqrt{\left(1-e^x\right)}}\frac{1}{\left(1-e^x\right)}$$

$$\frac{dy}{dx} = \frac{e^x}{\left(1-e^x\right)\sqrt{1-e^{2x}}}.$$

Differentiation Ex 11.2 Q63

Given, $y = \sqrt{x} + \dfrac{1}{\sqrt{x}}$

Differentiate with respect to x,

$$\dfrac{dy}{dx} = \dfrac{d}{dx}\left(\sqrt{x} + \dfrac{1}{\sqrt{x}}\right)$$

$$= \dfrac{d}{dx}\left(\sqrt{x}\right) + \dfrac{d}{dx}\left(x^{-\frac{1}{2}}\right)$$

$$= \dfrac{1}{2\sqrt{x}} + \left(-\dfrac{1}{2} \times x^{-\frac{1}{2}-1}\right)$$

$$= \dfrac{1}{2\sqrt{x}} - \dfrac{1}{2\sqrt[x]{x}}$$

$$\dfrac{dy}{dx} = \dfrac{x-1}{2x\sqrt{x}}$$

$$2x\dfrac{dy}{dx} = \dfrac{x-1}{\sqrt{x}}$$

$$\Rightarrow \quad 2x\dfrac{dy}{dx} = \dfrac{x}{\sqrt{x}} - \dfrac{1}{\sqrt{x}}$$

$$\Rightarrow \quad 2x\dfrac{dy}{dx} = \sqrt{x} - \dfrac{1}{\sqrt{x}}.$$

Differentiation Ex 11.2 Q64

Given, $y = \dfrac{x\sin^{-1}x}{\sqrt{1-x^2}}$

Differentiate with respect to x,

$$\dfrac{dy}{dx} = \dfrac{d}{dx}\left(\dfrac{x\sin^{-1}x}{\sqrt{1-x^2}}\right)$$

$$= \left[\dfrac{\sqrt{1-x^2}\,\dfrac{d}{dx}\left(x\sin^{-1}x\right) - \left(x\sin^{-1}x\right)\dfrac{d}{dx}\left(\sqrt{1-x^2}\right)}{\left(\sqrt{1-x^2}\right)^2}\right]$$

[Using quotient rule, product rule, chain rule]

$$= \left[\dfrac{\sqrt{1-x^2}\left\{x\dfrac{d}{dx}\sin^{-1}x + \sin^{-1}x\dfrac{d}{dx}(x)\right\} - \left(x\sin^{-1}x\right)\dfrac{1}{2\sqrt{1-x^2}}\dfrac{d}{dx}\left(1-x^2\right)}{\left(1-x^2\right)}\right]$$

$$= \left[\dfrac{\sqrt{1-x^2}\left\{\dfrac{x}{\sqrt{1-x^2}} + \sin^{-1}x\right\} - \dfrac{x\sin^{-1}x\,(-2x)}{2\sqrt{1-x^2}}}{\left(1-x^2\right)}\right]$$

$$= \left[\dfrac{x + \sqrt{1-x^2}\sin^{-1}x + \dfrac{x^2\sin^{-1}x}{\sqrt{1-x^2}}}{\left(1-x^2\right)}\right]$$

$$\left(1-x^2\right)\dfrac{dy}{dx} = x + \dfrac{\sqrt{1-x^2}}{1}\sin^{-1}x + \dfrac{x^2\sin^{-1}x}{\sqrt{1-x^2}}$$

$$\Rightarrow \quad \left(1-x^2\right)\dfrac{dy}{dx} = x + \left(\dfrac{\left(1-x^2\right)\sin^{-1}x + x^2\sin^{-1}x}{\sqrt{1-x^2}}\right)$$

$$\Rightarrow \quad \left(1-x^2\right)\dfrac{dy}{dx} = x + \left(\dfrac{\sin^{-1}x - x^2\sin^{-1}x + x^2\sin^{-1}x}{\sqrt{1-x^2}}\right)$$

$$\Rightarrow \quad \left(1-x^2\right)\dfrac{dy}{dx} = x + \left(\dfrac{\sin^{-1}x}{\sqrt{1-x^2}}\right)$$

$$\left(1-x^2\right)\dfrac{dy}{dx} = x + \dfrac{y}{x} \qquad \left\{\text{Since, given } y = \dfrac{x\sin^{-1}x}{\sqrt{1-x^2}}\right\}.$$

Differentiation Ex 11.2 Q65

Given, $y = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

Differentiating with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right)$$

$$= \left[\frac{\left(e^x + e^{-x}\right)\frac{d}{dx}\left(e^x - e^{-x}\right) - \left(e^x - e^{-x}\right)\frac{d}{dx}\left(e^x + e^{-x}\right)}{\left(e^x + e^{-x}\right)^2}\right]$$

[Using quotient rule and chain rule]

$$= \left[\frac{\left(e^x + e^{-x}\right)\left[e^x - e^{-x}\frac{d}{dx}(-x)\right] - \left(e^x - e^{-x}\right)\left[e^x + e^{-x}\frac{d}{dx}(-x)\right]}{\left(e^x + e^{-x}\right)^2}\right]$$

$$= \left[\frac{\left(e^x + e^{-x}\right)\left(e^x + e^{-x}\right) - \left(e^x - e^{-x}\right)\left(e^x - e^{-x}\right)}{\left(e^x + e^{-x}\right)^2}\right]$$

$$= \left[\frac{e^{2x} + e^{-2x} + 2e^x \times e^{-x} - e^{2x} - e^{-2x} + 2e^x e^{-x}}{\left(e^x + e^{-x}\right)2}\right]$$

$$\frac{dy}{dx} = \left[\frac{4}{\left(e^x + e^{-x}\right)2}\right] \qquad\qquad ---(i)$$

Now,

$$1 - y^2 = 1 - \left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right)^2$$

$$= 1 - \frac{\left(e^x - e^{-x}\right)^2}{\left(e^x + e^{-x}\right)^2}$$

$$= \frac{\left(e^x + e^{-x}\right)^2 - \left(e^x - e^{-x}\right)^2}{\left(e^x + e^{-x}\right)^2}$$

$$= \frac{4}{\left(e^x + e^{-x}\right)2}$$

Differentiation Ex 11.2 Q66

Given, $y = (x - 1)\log(x - 1) - (x + 1)\log(x + 1)$

Differentiating with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left[(x - 1)\log(x - 1) - (x + 1)\log(x + 1)\right]$$

$$= \left[(x - 1)\frac{d}{dx}\log(x - 1) + \log(x - 1)\frac{d}{dx}(x - 1)\right] -$$

$$\left[(x + 1)\frac{d}{dx}\log(x + 1) + \log(x + 1)\frac{d}{dx}(x + 1)\right]$$

[Using product rule, chain rule]

$$= \left[(x - 1) \times \frac{1}{(x - 1)}\frac{d}{dx}(x - 1) + \log(x - 1) \times (1)\right] -$$

$$\left[(x + 1)\frac{1}{(x + 1)} \times \frac{d}{dx}(x + 1) + \log(x + 1)(1)\right]$$

$$= \left[(1) + \log(x - 1)\right] - \left[1 + \log(x + 1)\right]$$

$$= \log(x - 1) - \log(x + 1)$$

$$\frac{dy}{dx} = \log\frac{(x - 1)}{(x + 1)} \qquad\qquad \left[\text{Since, } \log\left(\frac{a}{b}\right) = \log a - \log b\right].$$

Differentiation Ex 11.2 Q67

Given, $y = e^x \cos x$

Differentiating with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^x \cos x\right)$$

$$= e^x \frac{d}{dx}\cos x + \cos x \frac{d}{dx}e^x \qquad \text{[Using product rule]}$$

$$= e^x\left(-\sin x\right) + e^x \cos x$$

$$= e^x\left(\cos x - \sin x\right)$$

$$= \sqrt{2}e^x\left(\frac{\cos x}{\sqrt{2}} - \frac{\sin x}{\sqrt{2}}\right) \qquad \left[\text{Multiplying and dividing by } \sqrt{2}\right]$$

$$= \sqrt{2}e^x\left(\cos\frac{\pi}{4}\cos x - \sin\frac{\pi}{4}\sin x\right)$$

$$\frac{dy}{dx} = \sqrt{2}e^x \cos\left(x + \frac{\pi}{4}\right).$$

Differentiation Ex 11.2 Q68

Given, $y = \frac{1}{2}\log\left(\frac{1 - \cos 2x}{1 + \cos 2x}\right)$

$$\Rightarrow \quad y = \frac{1}{2}\log\left(\frac{2\sin^2 x}{2\cos^2 x}\right) \qquad \left[\begin{array}{l}\text{Since, } 1 - \cos 2x = 2\sin^2 x, \\ 1 + co2x = 2\cos^2 x\end{array}\right]$$

$$\Rightarrow \quad y = \frac{1}{2}\log\left(\tan^2 x\right)$$

$$\Rightarrow \quad y = \frac{2}{2}\log\tan x \qquad \left[\text{Since, } \log a^b = b\log a\right]$$

$$\Rightarrow \quad y = \log\tan x$$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \left(\log\tan x\right)$$

$$= \frac{1}{\tan x} \times \frac{d}{dx}\left(\tan x\right) \qquad \text{[Using chain rule]}$$

$$= \frac{\sec^2 x}{\tan x}$$

$$= \frac{1}{\cos^2 x \times \dfrac{\sin x}{\cos x}}$$

$$= \frac{1}{\sin x \cos x}$$

$$= \frac{2}{2\sin x \cos x}$$

$$= \frac{2}{\sin 2x} \qquad \left[\text{Since, } \frac{1}{\sin x} = \cos ecx\right]$$

So,

$$\frac{dy}{dx} = 2\cos ec\ 2x.$$

Differentiation Ex 11.2 Q69

Here, $y = x\sin^{-1}x + \sqrt{1-x^2}$

Differentiating with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left[x\sin^{-1}x + \sqrt{1-x^2}\right]$$

$$= \frac{d}{dx}\left(x\sin^{-1}x\right) + \frac{d}{dx}\left(\sqrt{1-x^2}\right)$$

$$= \left[x\frac{d}{dx}\sin^{-1}x + \sin^{-1}x\frac{d}{dx}(x)\right] + \frac{1}{2\sqrt{1-x^2}}\frac{d}{dx}\left(1-x^2\right)$$

[Using product rule and chain rule]

$$= \left[\frac{x}{\sqrt{1-x^2}} + \sin^{-1}x\right] - \frac{2x}{2\sqrt{1-x^2}}$$

$$= \frac{x}{\sqrt{1-x^2}} + \sin^{-1}x - \frac{x}{\sqrt{1-x^2}}$$

$$= \sin^{-1}x$$

So,

$$\frac{dy}{dx} = \sin^{-1}x.$$

Differentiation Ex 11.2 Q70

Here, $y = \sqrt{x^2 + a^2}$

Differentiating with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sqrt{x^2 + a^2}\right)$$

$$= \frac{1}{2\sqrt{x^2 + a^2}}\frac{d}{dx}\left(x^2 + a^2\right) \qquad \text{[Using chain rule]}$$

$$= \frac{1}{2\sqrt{x^2 + a^2}} \times (2x)$$

$$= \frac{x}{\sqrt{x^2 + a^2}}$$

$$\frac{dy}{dx} = \frac{x}{y} \qquad \left[\text{Since } \sqrt{x^2 + a^2} = y\right]$$

$$\Rightarrow \quad y\frac{dy}{dx} = x$$

$$\Rightarrow \quad y\frac{dy}{dx} - x = 0.$$

Differentiation Ex 11.2 Q71

Here, $y = e^x + e^{-x}$

Differentiating with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^x + e^{-x}\right)$$

$$= \frac{d}{dx}e^x + \frac{d}{dx}e^{-x}$$

$$= e^x + e^{-x}\frac{d}{dx}(-x) \qquad \text{[Using chian rule]}$$

$$= e^x + e^{-x}(-1)$$

$$= \left(e^x - e^{-x}\right)$$

$$= \sqrt{\left(e^x + e^{-x}\right)^2 - 4e^x \times e^{-x}} \qquad \left[\text{Since } (a - b) = \sqrt{(a+b)^2 - 4ab}\right]$$

$$= \sqrt{y^2 - 4} \qquad \left[\text{Since } e^x + e^{-x} = y\right]$$

So,

$$\frac{dy}{dx} = \sqrt{y^2 - 4}.$$

Differentiation Ex 11.2 Q72

Given, $y = \sqrt{a^2 - x^2}$

Differentiating with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sqrt{a^2 - x^2}\right)$$

$$= \frac{1}{2\sqrt{a^2 - x^2}} \frac{d}{dx}\left(a^2 - x^2\right) \qquad \text{[Using chain rule]}$$

$$= \frac{1}{2\sqrt{a^2 - x^2}}(-2x)$$

$$= \frac{-x}{\sqrt{a^2 - x^2}}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-x}{y} \qquad \left[\text{Since } \sqrt{a^2 - x^2} = y\right]$$

$$\Rightarrow \quad y\frac{dy}{dx} = -x$$

$$y\frac{dy}{dx} + x = 0$$

Differentiation Ex 11.2 Q73

Here, $xy = 4$

$$\Rightarrow \quad y = \frac{4}{x}$$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{4}{x}\right)$$

$$= 4\frac{d}{dx}\left(x^{-1}\right)$$

$$= 4\left(-1 \times x^{-1-1}\right)$$

$$= 4\left(-\frac{1}{x^2}\right)$$

$$= \frac{-4}{x^2}$$

$$= -\frac{4}{\left(\frac{x}{y}\right)^2} \qquad \left[\text{Since } x = \frac{4}{y}\right]$$

$$= -\frac{4y^2}{16}$$

$$\frac{dy}{dx} = -\frac{y^2}{4}$$

$$\Rightarrow \quad 4\frac{dy}{dx} = -y^2$$

$$\Rightarrow \quad 4\frac{dy}{dx} = 3y^2 - 4y^2$$

$$\Rightarrow \quad 4\frac{dy}{dx} + 4y^2 = 3y^2$$

$$\Rightarrow \quad 4\left(\frac{dy}{dx} + y^2\right) = 3y^2$$

Dividing both the sides by x,

$$\Rightarrow \quad \frac{4}{x}\left(\frac{dy}{dx} + y^2\right) = \frac{3y^2}{x}$$

$$\Rightarrow \quad y\left(\frac{dy}{dx} + y^2\right) = \frac{3y^2}{x} \qquad \left[\text{Since } \frac{4}{x} = y\right]$$

$$\Rightarrow \quad x\left(\frac{dy}{dx} + y^2\right) = \frac{3y^2}{y}$$

$$\Rightarrow \quad x\left(\frac{dy}{dx} + y^2\right) = 3y.$$

Differentiation Ex 11.2 Q74

$$\frac{d}{dx}\left\{\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right\}=\sqrt{a^2-x^2}$$

LHS $= \dfrac{d}{dx}\left\{\dfrac{x}{2}\sqrt{a^2-x^2}+\dfrac{a^2}{2}\sin^{-1}\dfrac{x}{a}\right\}$

$\quad = \dfrac{d}{dx}\left(\dfrac{x}{2}\sqrt{a^2-x^2}\right)+\dfrac{d}{dx}\left(\dfrac{a^2}{2}\sin^{-1}\dfrac{x}{a}\right)$

$\quad = \dfrac{1}{2}\left[x\,\dfrac{d}{dx}\sqrt{a^2-x^2}+\sqrt{a^2-x^2}\,\dfrac{d}{dx}(x)\right]+\dfrac{a^2}{2}\times\dfrac{1}{\sqrt{1-\left(\dfrac{x}{a}\right)^2}}\times\dfrac{d}{dx}\left(\dfrac{x}{a}\right)$

[Using product rule, chain rule]

$\quad = \dfrac{1}{2}\left[x\times\dfrac{1}{2\sqrt{a^2-x^2}}\dfrac{d}{dx}\left(a^2-x^2\right)+\sqrt{a^2-x^2}\right]+\left(\dfrac{a^2}{2}\right)\times\dfrac{1}{\sqrt{\dfrac{a^2-x^2}{a^2}}}\times\left(\dfrac{1}{a}\right)$

$\quad = \dfrac{1}{2}\left[\dfrac{x\,(-2x)}{2\sqrt{a^2-x^2}}+\sqrt{a^2-x^2}\right]+\left(\dfrac{a^2}{2}\right)\dfrac{a}{\sqrt{a^2-x^2}}\times\left(\dfrac{1}{a}\right)$

$\quad = \dfrac{1}{2}\left[\dfrac{-2x^2+2\left(a^2-x^2\right)}{2\sqrt{a^2-x^2}}\right]+\dfrac{a^2}{2\sqrt{a^2-x^2}}$

$\quad = \dfrac{1}{2}\left[\dfrac{2\left(a^2-2x^2\right)}{2\sqrt{a^2-x^2}}\right]+\dfrac{a^2}{2\sqrt{a^2-x^2}}$

$\quad = \dfrac{a^2-2x^2}{2\sqrt{a^2-x^2}}+\dfrac{a^2}{2\sqrt{a^2-x^2}}$

$\quad = \dfrac{a^2-2x^2+a^2}{2\sqrt{a^2-x^2}}$

$\quad = \dfrac{2a^2-2x^2}{2\sqrt{a^2-x^2}}$

$\quad = \dfrac{2\left(a^2-x^2\right)}{2\sqrt{a^2-x^2}}$

$\quad = \dfrac{\left(a^2-x^2\right)}{\sqrt{a^2-x^2}}$

$\quad = \sqrt{a^2-x^2}$

$\quad = $ RHS

Differentiation Ex 11.3 Q1

Let $y = \cos^{-1}\left\{2x\sqrt{1-x^2}\right\}$

Put $\quad x = \cos\theta$

$\quad y = \cos^{-1}\left\{2\cos\theta\sqrt{1-\cos^2\theta}\right\}$

$\quad\quad = \cos^{-1}\left\{2\cos\theta\sin\theta\right\}$

$\quad y = \cos^{-1}\left\{\sin 2\theta\right\}$ $\qquad\qquad\left[\text{Since } \sin 2\theta = 2\sin\theta\cos\theta, \sin^2\theta + \cos^2\theta = 1\right]$

$\quad y = \cos^{-1}\left[\cos\left(\dfrac{\pi}{2} - \theta\right)\right]$ $\qquad$ ---(i)

Now,

$\qquad \dfrac{1}{\sqrt{2}} < x < 1$

$\Rightarrow \qquad \dfrac{1}{\sqrt{2}} < \cos\theta < 1$

$\Rightarrow \qquad 0 < \theta < \dfrac{\pi}{4}$

$\Rightarrow \qquad 0 < 2\theta < \dfrac{\pi}{2}$

$\Rightarrow \qquad 0 > -2\theta > -\dfrac{\pi}{2}$

$\Rightarrow \qquad \dfrac{\pi}{2} > \left(\dfrac{\pi}{2} - 2\theta\right) > 0$

Hence, from equation (i),

$\qquad y = \dfrac{\pi}{2} - 2\theta$ $\qquad\qquad\left[\text{Since } \cos^{-1}(\cos\theta) = \theta, \text{if } \theta \in [0, \pi]\right]$

$\qquad y = \dfrac{\pi}{2} - 2\cos^{-1}x$ $\qquad\qquad\left[\text{Since } x = \cos\theta\right]$

Differentiating it with respect to x,

$\qquad \dfrac{dy}{dx} = \dfrac{d}{dx}\left(\dfrac{\pi}{2}\right) - 2\dfrac{d}{dx}\left(\cos^{-1}x\right)$

$\qquad\quad = 0 - 2\left(\dfrac{-1}{\sqrt{1-x^2}}\right)$

$\qquad \dfrac{dy}{dx} = \dfrac{2}{\sqrt{1-x^2}}.$

Differentiation Ex 11.3 Q2

Let $\quad y = \cos^{-1}\left\{\sqrt{\dfrac{1+x}{2}}\right\}$

Put $\quad x = \cos 2\theta$

$\quad y = \cos^{-1}\left\{\sqrt{\dfrac{1+\cos 2\theta}{2}}\right\}$

$\quad\quad = \cos^{-1}\left\{\sqrt{\dfrac{2\cos^2\theta}{2}}\right\}$

$\quad y = \cos^{-1}\left\{\cos\theta\right\}$ $\qquad\qquad$ ---(i)

Here, $-1 < x < 1$

$\rightarrow \qquad -1 < \cos 2\theta < 1$

$\Rightarrow \qquad 0 < 2\theta < \pi$

$\Rightarrow \qquad 0 < \theta < \dfrac{\pi}{2}$

So, from equation (i),

$\qquad y = \theta$ $\qquad\qquad\left[\text{Since } \cos^{-1}(\cos\theta) = \theta \text{ if } \theta \in [0, \pi]\right]$

$\qquad y = \dfrac{1}{2}\cos^{-1}x$ $\qquad\qquad\left[\text{Since } x = \cos 2\theta\right]$

Differentiating it with respect to x,

$\qquad \dfrac{dy}{dx} = -\dfrac{1}{2\sqrt{1-x^2}}.$

Differentiation Ex 11.3 Q3

Let $y = \sin^{-1}\left\{\sqrt{\dfrac{1-x}{2}}\right\}$

Let $x = \cos 2\theta$

$y = \sin^{-1}\left\{\sqrt{\left(\dfrac{1-\cos 2\theta}{2}\right)}\right\}$

$= \sin^{-1}\left\{\sqrt{\dfrac{2\sin^2\theta}{2}}\right\}$

$y = \sin^{-1}(\sin\theta)$ ---(i)

Here, $0 < x < 1$

$\Rightarrow$ $0 < \cos 2\theta < 1$

$\Rightarrow$ $0 < 2\theta < \dfrac{\pi}{2}$

$\Rightarrow$ $0 < \theta < \dfrac{\pi}{4}$

so, from equation (i),

$y = \theta$ $\left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]\right]$

$y = \dfrac{1}{2}\cos^{-1} x$ $\left[\text{Since, } x = \cos 2\theta\right]$

Differentiating it with respect to x,

$\dfrac{dy}{dx} = -\dfrac{1}{2\sqrt{1-x^2}}$.

Differentiation Ex 11.3 Q4

Let $y = \sin^{-1}\left\{\sqrt{1-x^2}\right\}$

Let $x = \cos\theta$

$y = \sin^{-1}\left\{\sqrt{1-\cos^2\theta}\right\}$

$y = \sin^{-1}(\sin\theta)$ ---(i)

Here, $0 < x < 1$

$\Rightarrow$ $0 < \cos\theta < 1$

$\Rightarrow$ $0 < \theta < \dfrac{\pi}{2}$

From equatoin(i),

$y = \theta$ $\left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]\right]$

$y = \cos^{-1} x$ $\left[\text{Since } x = \cos\theta\right]$

Differentiating with respect to x,

$\dfrac{dy}{dx} = -\dfrac{1}{\sqrt{1-x^2}}$.

Differentiation Ex 11.3 Q5

Let $\quad y = \tan^{-1}\left\{ \dfrac{x}{\sqrt{a^2-x^2}} \right\}$

Let $\quad x = a\sin\theta$

$$y = \tan^{-1}\left\{ \frac{a\sin\theta}{\sqrt{a^2 - a^2\sin^2\theta}} \right\}$$

$$y = \tan^{-1}\left\{ \frac{a\sin\theta}{\sqrt{a^2\left(1 - \sin^2\theta\right)}} \right\}$$

$$y = \tan^{-1}\left\{ \frac{a\sin\theta}{a\cos\theta} \right\}$$

$$y = \tan^{-1}\left(\tan\theta\right) \qquad\qquad \text{---(i)}$$

Here, $\quad -a < x < a$

$\Rightarrow \quad -1 < \dfrac{x}{a} < 1$

$\Rightarrow \quad -\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2}$

From equatoin (i),

$$y = \theta \qquad\qquad \left[\text{Since, } \tan^{-1}\left(\tan\theta\right) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

$$y = \sin^{-1}\left(\frac{x}{a}\right) \qquad\qquad \left[\text{Since } x = a\sin\theta\right]$$

Differentiating it with respect to x,

Using chain rule,

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 - \left(\frac{x}{a}\right)^2}} \frac{d}{dx}\left(\frac{x}{a}\right)$$

$$= \frac{a}{\sqrt{a^2 - x^2}} \times \left(\frac{1}{a}\right)$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{a^2 - x^2}}.$$

Differentiation Ex 11.3 Q6

Let $\quad y = \sin^{-1}\left\{ \dfrac{x}{\sqrt{x^2 + a^2}} \right\}$

Put $\quad x = a\tan\theta$

$$y = \sin^{-1}\left\{ \frac{a\tan\theta}{\sqrt{a^2 + \tan^2\theta + a^2}} \right\}$$

$$= \sin^{-1}\left\{ \frac{a\tan\theta}{\sqrt{a^2\left(\tan^2\theta + 1\right)}} \right\}$$

$$= \sin^{-1}\left\{ \frac{a\tan\theta}{a\sec\theta} \right\}$$

$$= \sin^{-1}\left\{\sin\theta\right\}$$

$$= \theta$$

$$y = \tan^{-1}\left(\frac{x}{a}\right) \qquad\qquad \left[x = a\tan\theta\right]$$

Differentiating it with respect to x using chain rule,

$$\frac{dy}{dx} = \frac{1}{1 + \left(\frac{x}{a}\right)^2} \frac{d}{dx}\left(\frac{x}{a}\right)$$

$$= \frac{a^2}{a^2 + x^2} \times \left(\frac{1}{a}\right)$$

$$\frac{dy}{dx} = \frac{a}{a^2 + x^2}.$$

Differentiation Ex 11.3 Q7

Let $\quad y = \sin^{-1}\left\{2x^2 - 1\right\}$

Let $\quad x = \cos\theta$

$\qquad y = \sin^{-1}\left(2\cos^2\theta - 1\right)$

$\qquad\quad = \sin^{-1}(\cos 2\theta)$

$\qquad y = \sin^{-1}\left\{\sin\left(\dfrac{\pi}{2} - 2\theta\right)\right\}$ $\qquad\qquad\qquad$ ---(i)

Here, $\quad 0 < x < 1$

$\Rightarrow \qquad 0 < \cos\theta < 1$

$\Rightarrow \qquad 0 < \theta < \dfrac{\pi}{2}$

$\Rightarrow \qquad 0 < 2\theta < \pi$

$\Rightarrow \qquad 0 > -2\theta > -\pi$

$\Rightarrow \qquad \dfrac{\pi}{2} > \left(\dfrac{\pi}{2} - 2\theta\right) > -\dfrac{\pi}{2}$

So, from equatoin (i),

$\qquad y = \dfrac{\pi}{2} - 2\theta$ $\qquad\qquad$ $\left[\text{Since, } \sin^{-1}(\cos\theta) = \theta, \text{ if } \theta \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]\right]$

$\qquad y = \dfrac{\pi}{2} - 2\cos^{-1}x$ $\qquad\quad$ $\left[\text{Since } x = \cos\theta\right]$

$\qquad \dfrac{dy}{dx} = 0 - 2\dfrac{d}{dx}\left(\cos^{-1}x\right)$

$\qquad\qquad = -2\left(-\dfrac{1}{\sqrt{1-x^2}}\right)$

$\qquad \dfrac{dy}{dx} = \dfrac{2}{\sqrt{1-x^2}}.$

Differentiation Ex 11.3 Q8

Let $\quad y = \sin^{-1}\left\{1 - 2x^2\right\}$

Let $\quad x = \sin\theta$, So,

$\qquad y = \sin^{-1}\left(1 - 2\sin^2\theta\right)$

$\qquad\quad = \sin^{-1}(\cos 2\theta)$

$\qquad y = \sin^{-1}\left\{\sin\left(\dfrac{\pi}{2} - 2\theta\right)\right\}$ $\qquad\qquad\qquad$ ---(i)

Here, $\quad 0 < x < 1$

$\Rightarrow \qquad 0 < \sin\theta < 1$

$\Rightarrow \qquad 0 < \theta < \dfrac{\pi}{2}$

$\Rightarrow \qquad 0 < 2\theta < \pi$

$\Rightarrow \qquad 0 > -2\theta > -\pi$

$\Rightarrow \qquad \dfrac{\pi}{2} > \left(\dfrac{\pi}{2} - 2\theta\right) > \dfrac{\pi}{2} - \pi$

$\Rightarrow \qquad \dfrac{\pi}{2} > \left(\dfrac{\pi}{2} - 2\theta\right) > \left(-\dfrac{\pi}{2}\right)$

So, from equatoin (i),

$\qquad y = \dfrac{\pi}{2} - 2\theta$ $\qquad\qquad$ $\left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]\right]$

$\qquad y = \dfrac{\pi}{2} - 2\sin^{-1}x$ $\qquad\quad$ $\left[\text{Since } x = \sin\theta\right]$

Differentiating with respect to x,

$\qquad \dfrac{dy}{dx} = 0 - 2\left(\dfrac{1}{\sqrt{1-x^2}}\right)$

$\qquad \dfrac{dy}{dx} = -\dfrac{2}{\sqrt{1-x^2}}.$

Differentiation Ex 11.3 Q9

Let $\quad y = \cos^{-1}\left\{\dfrac{x}{\sqrt{x^2 + a^2}}\right\}$

Put $\quad x = a\cot\theta,$

$$y = \cos^{-1}\left\{\dfrac{a\cot\theta}{\sqrt{a^2\cot^2\theta + a^2}}\right\}$$

$$= \cos^{-1}\left\{\dfrac{a\cot\theta}{a\,\mathrm{cosec}\,\theta}\right\}$$

$$= \cos^{-1}\left\{\dfrac{\dfrac{\cos\theta}{\sin\theta}}{\dfrac{1}{\sin\theta}}\right\}$$

$$= \cos^{-1}(\cos\theta)$$

$$= \theta$$

$$y = \cot^{-1}\left(\dfrac{x}{a}\right) \qquad\qquad [\text{Since, } a\cot\theta = x]$$

Differentiating it with respect to x using chain rule,

$$\dfrac{dy}{dx} = \dfrac{-1}{1 + \left(\dfrac{x}{a}\right)^2}\dfrac{d}{dx}\left(\dfrac{x}{a}\right)$$

$$= \dfrac{-a^2}{a^2 + x^2} \times \left(\dfrac{1}{a}\right)$$

$$\dfrac{dy}{dx} = \dfrac{-a}{a^2 + x^2}.$$

Differentiation Ex 11.3 Q10

Let $\quad y = \sin^{-1}\left\{\dfrac{\sin x + \cos x}{\sqrt{2}}\right\}$

$$= \sin^{-1}\left\{\sin x\left(\dfrac{1}{\sqrt{2}}\right) + \cos x \times \left(\dfrac{1}{\sqrt{2}}\right)\right\}$$

$$= \sin^{-1}\left\{\sin x \cos\dfrac{\pi}{4} + \cos x \times \sin\dfrac{\pi}{4}\right\}$$

$$y = \sin^{-1}\left\{\sin\left(x + \dfrac{\pi}{4}\right)\right\}$$

Here, $\quad \dfrac{-3\pi}{4} < x < \dfrac{\pi}{4}$

$\Rightarrow \quad \left(\dfrac{-3\pi}{4} + \dfrac{\pi}{4}\right) \qquad\qquad \left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{if } \theta \in \left[\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right]\right]$

Differentiating it with respect to x,

$$\dfrac{dy}{dx} = 1 + 0$$

$$\dfrac{dy}{dx} = 1$$

Differentiation Ex 11.3 Q11

Let $\quad y = \cos^{-1}\left\{\dfrac{\cos x + \sin x}{\sqrt{2}}\right\}$

$\qquad y = \cos^{-1}\left\{\cos x\left(\dfrac{1}{\sqrt{2}}\right) + \sin x\left(\dfrac{1}{\sqrt{2}}\right)\right\}$

$\qquad = \cos^{-1}\left\{\cos x \cos\left(\dfrac{\pi}{4}\right) + \sin x \sin\left(\dfrac{\pi}{4}\right)\right\}$

$\qquad y = \cos^{-1}\left[\cos\left(x - \dfrac{\pi}{4}\right)\right]$ ---(i)

Here, $\quad -\dfrac{\pi}{4} < x < \dfrac{\pi}{4}$

$\Rightarrow \qquad \left(-\dfrac{\pi}{4} - \dfrac{\pi}{4}\right) < \left(x - \dfrac{\pi}{4}\right) < \left(\dfrac{\pi}{4} - \dfrac{\pi}{4}\right)$

$\Rightarrow \qquad -\dfrac{\pi}{2} < \left(x - \dfrac{\pi}{4}\right) < 0$

So, from equation (i),

$\qquad y = -\left(x - \dfrac{\pi}{4}\right)$ $\qquad \left[\text{Since, } \cos^{-1}(\cos\theta) = -\theta, \text{ if } \theta \in [-\pi, 0]\right]$

$\qquad y = -x + \dfrac{\pi}{4}$

Differentiating it with respect to x,

$\qquad \dfrac{dy}{dx} = -1.$

Differentiation Ex 11.3 Q12

Let $\quad y = \tan^{-1}\left\{\dfrac{x}{1 + \sqrt{1 - x^2}}\right\}$

Put $\quad x = \sin\theta$, so

$\qquad y = \tan^{-1}\left\{\dfrac{\sin\theta}{1 + \sqrt{1 - \sin^2\theta}}\right\}$

$\qquad = \tan^{-1}\left\{\dfrac{\sin\theta}{1 + \cos\theta}\right\}$

$\qquad = \tan^{-1}\left\{\dfrac{\dfrac{2\sin\theta}{2}\dfrac{\cos\theta}{2}}{\dfrac{2\cos^2\theta}{2}}\right\}$

$\qquad y = \tan^{-1}\left\{\dfrac{\tan\theta}{2}\right\}$ ---(i)

Here, $\quad -1 < x < 1$

$\Rightarrow \qquad -1 < \sin\theta < 1$

$\Rightarrow \qquad -\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2}$

$\Rightarrow \qquad -\dfrac{\pi}{4} < \dfrac{\theta}{2} < \dfrac{\pi}{4}$

So, from equation (i),

$\qquad y = \dfrac{\theta}{2}$ $\qquad \left[\text{Since, } \tan^{-1}(\tan\theta) = \theta, \text{ if } \theta \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]\right]$

$\qquad y = \dfrac{1}{2}\sin^{-1} x$ $\qquad \left[\text{Since, } x = \sin\theta\right]$

Differentiating it with respect to x,

$\qquad \dfrac{dy}{dx} = \dfrac{1}{2\sqrt{1 - x^2}}.$

Differentiation Ex 11.3 Q13

Let $\quad y = \tan^{-1}\left\{\dfrac{x}{a + \sqrt{a^2 - x^2}}\right\}$

Put $\quad x = a\sin\theta$, so

$$y = \tan^{-1}\left\{\dfrac{a\sin\theta}{a + \sqrt{a^2 - a^2\sin^2\theta}}\right\}$$

$$= \tan^{-1}\left\{\dfrac{a\sin\theta}{a + \sqrt{a^2\left(1 - \sin^2\theta\right)}}\right\}$$

$$= \tan^{-1}\left\{\dfrac{a\sin\theta}{a + a\cos\theta}\right\}$$

$$= \tan^{-1}\left\{\dfrac{a\sin\theta}{a\left(1 + \cos\theta\right)}\right\}$$

$$= \tan^{-1}\left(\dfrac{\sin\theta}{1 + \cos\theta}\right)$$

$$= \tan^{-1}\left(\dfrac{2\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2}}{2\cos^2\dfrac{\theta}{2}}\right)$$

$$y = \tan^{-1}\left(\tan\dfrac{\theta}{2}\right) \qquad\qquad\qquad \text{--- (i)}$$

Here, $\quad -a < x < a$

$\Rightarrow \qquad -1 < \dfrac{x}{a} < 1$

$\Rightarrow \qquad -\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2}$

$\Rightarrow \qquad -\dfrac{\pi}{4} < \dfrac{\theta}{2} < \dfrac{\pi}{4}$

So, from equation (i),

$$y = \dfrac{\theta}{2} \qquad\qquad \left[\text{Since, } \tan^{-1}\left(\tan\theta\right) = \theta, \text{ if } \theta \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]\right]$$

$$y = \dfrac{1}{2} + \sin^{-1}\left(\dfrac{x}{a}\right) \qquad\qquad \left[\text{Since, } x = a\sin\theta\right]$$

Differentiating it with respect to x using chain rule,

$$\dfrac{dy}{dx} = \dfrac{1}{2} \times \dfrac{1}{\sqrt{1 - \left(\dfrac{x}{a}\right)^2}} \dfrac{d}{dx}\left(\dfrac{x}{a}\right)$$

$$= \dfrac{a}{2\sqrt{a^2 - x^2}} \times \left(\dfrac{1}{a}\right)$$

$$\dfrac{dy}{dx} = \dfrac{1}{2\sqrt{a^2 - x^2}}.$$

Differentiation Ex 11.3 Q14

Let $\quad y = \sin^{-1}\left\{\dfrac{x + \sqrt{1 - x^2}}{\sqrt{2}}\right\}$

Put $\quad x = \sin\theta,$ so

$\quad\quad = \sin^{-1}\left\{\dfrac{\sin\theta + \sqrt{1 - \sin^2\theta}}{\sqrt{2}}\right\}$

$\quad\quad = \sin^{-1}\left\{\dfrac{\sin\theta + \cos\theta}{\sqrt{2}}\right\}$

$\quad\quad = \sin^{-1}\left\{\sin\theta\left(\dfrac{1}{\sqrt{2}}\right) + \cos\theta\left(\dfrac{1}{\sqrt{2}}\right)\right\}$

$\quad\quad = \sin^{-1}\left\{\sin\theta\cos\dfrac{\pi}{4} + \cos\theta\sin\dfrac{\pi}{4}\right\}$

$\quad y = \sin^{-1}\left\{\sin\left(\theta + \dfrac{\pi}{4}\right)\right\}$ $\quad\quad\quad\quad$ ---(i)

Here, $\quad -1 < x < 1$

$\Rightarrow \quad\quad -1 < \sin\theta < 1$

$\Rightarrow \quad\quad -\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2}$

$\Rightarrow \quad\quad \left(-\dfrac{\pi}{2} + \dfrac{\pi}{4}\right) < \left(\dfrac{\pi}{4} + \theta\right) < \dfrac{3\pi}{4}$

So, from equation (i),

$\quad\quad y = \theta + \dfrac{\pi}{4}$ $\quad\quad\quad\quad\quad\quad\quad$ $\left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ as } \theta \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]\right]$

$\quad\quad y = \sin^{-1}x + \dfrac{\pi}{4}$ $\quad\quad\quad\quad\quad$ $\left[\text{Since, } \sin\theta = x\right]$

Differentiating it with respect to x,

$\quad\quad \dfrac{dy}{dx} = \dfrac{1}{\sqrt{1 - x^2}} + 0$

$\quad\quad \dfrac{dy}{dx} = \dfrac{1}{\sqrt{1 - x^2}}.$

Differentiation Ex 11.3 Q15

Let $\quad y = \cos^{-1}\left\{\dfrac{x + \sqrt{1 - x^2}}{\sqrt{2}}\right\}$

Put $\quad x = \sin\theta$, so

$$y = \cos^{-1}\left\{\dfrac{\sin\theta + \sqrt{1 - \sin^2\theta}}{\sqrt{2}}\right\}$$

$$= \cos^{-1}\left\{\dfrac{\sin\theta + \cos\theta}{\sqrt{2}}\right\}$$

$$= \cos^{-1}\left\{\sin\theta\left(\dfrac{1}{\sqrt{2}}\right) + \cos\theta\left(\dfrac{1}{\sqrt{2}}\right)\right\}$$

$$= \cos^{-1}\left\{\sin\theta \times \sin\dfrac{\pi}{4} + \cos\theta \times \cos\dfrac{\pi}{4}\right\}$$

$$y = \cos^{-1}\left\{\cos\left(\theta - \dfrac{\pi}{4}\right)\right\} \qquad\qquad ---(i)$$

Here, $\quad -1 < x < 1$

$\Rightarrow \qquad -1 < \sin\theta < 1$

$\Rightarrow \qquad -\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2}$

$\Rightarrow \qquad -\dfrac{\pi}{2} + \dfrac{\pi}{4} < \left(\theta - \dfrac{\pi}{4}\right) < \dfrac{\pi}{2} - \dfrac{\pi}{4}$

$\Rightarrow \qquad \left(-\dfrac{3\pi}{4}\right) < \left(\theta - \dfrac{\pi}{4}\right) < \left(\dfrac{\pi}{4}\right)$

So, from equation (i),

$$y = -\left(\theta - \dfrac{\pi}{4}\right) \qquad\qquad \left[\text{Since, } \cos^{-1}(\cos\theta) = -\theta, \text{ if } \theta \in [-\pi, 0]\right]$$

$$y = -\theta + \dfrac{\pi}{4}$$

$$y = -\sin^{-1}x + \dfrac{\pi}{4} \qquad\qquad \left[\text{Since, } x = \sin\theta\right]$$

Differentiating it with respect to x,

$$\dfrac{dy}{dx} = -\dfrac{1}{\sqrt{1 - x^2}} + 0$$

$$\dfrac{dy}{dx} = -\dfrac{1}{\sqrt{1 - x^2}}.$$

Differentiation Ex 11.3 Q16

Let $\quad y = \tan^{-1}\left\{\dfrac{4x}{1-4x^2}\right\}$

Put $\quad 2x = \tan\theta$, so

$\quad y = \tan^{-1}\left\{\dfrac{2\tan\theta}{1-\tan^2\theta}\right\}$

$\quad y = \tan^{-1}\{\tan 2\theta\}$ ---(i)

Here, $\quad -\dfrac{1}{2} < x < \dfrac{1}{2}$

$\Rightarrow \quad -1 < 2x < 1$

$\Rightarrow \quad -1 < \tan\theta < 1$

$\Rightarrow \quad -\dfrac{\pi}{4} < \theta < \dfrac{\pi}{4}$

$\Rightarrow \quad -\dfrac{\pi}{2} < (2\theta) < \dfrac{\pi}{2}$

So, from equation (i),

$\quad y = 2\theta$ $\left[\text{Sicne, } \tan^{-1}(\tan\theta) = \theta, \text{ if } \theta \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]\right]$

$\quad y = 2\tan^{-1}(2x)$ $\left[\text{Since, } 2x = \tan\theta\right]$

Differentiating it with respect to x using chain rule,

$$\frac{dy}{dx} = 2\left(\frac{1}{1+(2x)^2}\right)\frac{d}{dx}(2x)$$

$$\frac{dy}{dx} = \frac{4}{1+4x^2}.$$

Differentiation Ex 11.3 Q17

Let $\quad y = \tan^{-1}\left\{\dfrac{2^{x+1}}{1-4^x}\right\}$

Put $\quad 2^x = \tan\theta$, so,

$\quad = \tan^{-1}\left\{\dfrac{2^x \times 2}{1-\left(2^x\right)^2}\right\}$

$\quad = \tan^{-1}\left\{\dfrac{2\tan\theta}{1-\tan^2\theta}\right\}$

$\quad y = \tan^{-1}\{\tan(2\theta)\}$ ---(i)

Here, $\quad -\infty < x < 0$

$\Rightarrow \quad 2^{-\infty} < 2^x < 2^0$

$\Rightarrow \quad 0 < 2^x < 1$

$\Rightarrow \quad 0 < \theta < \dfrac{\pi}{4}$

$\Rightarrow \quad 0 < (2\theta) < \dfrac{\pi}{2}$

From equatoin (i),

$\quad y = 2\theta$ $\left[\text{Since, } \tan^{-1}(\tan\theta) = \theta, \text{ if } \theta \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)\right]$

$\quad y = 2\tan^{-1}\left(2^x\right)$

Differentiate it with respect to x using chain rule,

$$\frac{dy}{dx} = \frac{2}{1+\left(2^x\right)^2}\frac{d}{dx}\left(2^x\right)$$

$$= \frac{2 \times 2^x \log 2}{1+4^x}$$

$$\frac{dy}{dx} = \frac{2^{x+1}\log 2}{1+4^x}.$$

Differentiation Ex 11.3 Q18

Let $\quad y = \tan^{-1}\left\{\dfrac{2a^x}{1 - a^{2x}}\right\}$

Put $\quad a^x = \tan\theta$,

$$y = \tan^{-1}\left\{\dfrac{2\tan\theta}{1 - \tan^2\theta}\right\}$$

$$y = \tan^{-1}\left\{\tan(2\theta)\right\} \qquad\qquad \text{--- (i)}$$

Here, $\quad -\infty < x < 0$

$\Rightarrow \quad a^{-\infty} < a^x < 2^0$

$\Rightarrow \quad 0 < \tan\theta < 1$

$\Rightarrow \quad 0 < \theta < \dfrac{\pi}{4}$

$\Rightarrow \quad 0 < (2\theta) < \dfrac{\pi}{2}$

From equatoin (i),

$$y = 2\theta \qquad\qquad \left[\text{Since, } \tan^{-1}(\tan\theta) = \theta, \text{if } \theta \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)\right]$$

$$y = 2\tan^{-1}\left(a^x\right)$$

Differentiate it with respect to x using chain rule,

$$\dfrac{dy}{dx} = \dfrac{2}{1 + \left(a^x\right)^2}\dfrac{d}{dx}\left(a^x\right)$$

$$\dfrac{dy}{dx} = \dfrac{2a^x \log a}{1 + a^{2x}}.$$

Differentiation Ex 11.3 Q19

Let $\quad y = \sin^{-1}\left\{\dfrac{\sqrt{1+x} + \sqrt{1-x}}{2}\right\}$

Put $\quad x = \cos 2\theta$, so,

$$= \sin^{-1}\left\{\dfrac{\sqrt{1+\cos 2\theta} + \sqrt{1-\cos 2\theta}}{2}\right\}$$

$$= \sin^{-1}\left\{\dfrac{\sqrt{2\cos^2\theta} + \sqrt{2\sin^2\theta}}{2}\right\}$$

$$= \sin^{-1}\left\{\dfrac{\sqrt{2}\cos\theta + \sqrt{2}\sin\theta}{2}\right\}$$

$$= \sin^{-1}\left\{\cos\theta\left(\dfrac{1}{\sqrt{2}}\right) + \left(\dfrac{1}{\sqrt{2}}\right)\sin\theta\right\}$$

$$= \sin^{-1}\left\{\cos\theta\sin\left(\dfrac{\pi}{4}\right) + \cos\dfrac{\pi}{4}\sin\theta\right\}$$

$$y = \sin^{-1}\left\{\sin\left(\theta + \dfrac{\pi}{4}\right)\right\} \qquad\qquad\qquad ---(i)$$

Here, $\quad 0 < x < 1$

$\Rightarrow \qquad 0 < \cos 2\theta < 1$

$\Rightarrow \qquad 0 < 2\theta < \dfrac{\pi}{2}$

$\Rightarrow \qquad 0 < \theta < \dfrac{\pi}{4}$

$\Rightarrow \qquad \dfrac{\pi}{4} < \left(\theta + \dfrac{\pi}{4}\right) < \dfrac{\pi}{2}$

So, from equatoin (i),

$$y = \theta + \dfrac{\pi}{4} \qquad\qquad\qquad \left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)\right]$$

$$y = \dfrac{1}{2}\cos^{-1}x + \dfrac{\pi}{4}$$

Differentiate it with respect to x,

$$\dfrac{dy}{dx} = \dfrac{1}{2}\left(\dfrac{-1}{\sqrt{1-x^2}}\right) + 0$$

$$\dfrac{dy}{dx} = \dfrac{-1}{2\sqrt{1-x^2}}.$$

Differentiation Ex 11.3 Q20

Let $\quad y = \tan^{-1}\left(\dfrac{\sqrt{1+a^2x^2}-1}{ax}\right)$

Put $\quad ax = \tan\theta$

$\quad y = \tan^{-1}\left(\dfrac{\sqrt{1+a^2x^2}-1}{ax}\right)$

$\qquad = \tan^{-1}\left(\dfrac{\sec\theta - 1}{\tan\theta}\right)$

$\qquad = \tan^{-1}\left(\dfrac{1-\cos\theta}{\sin\theta}\right)$

$\qquad = \tan^{-1}\left(\dfrac{\dfrac{2\sin^2\theta}{2}}{\dfrac{2\sin\theta}{2}\dfrac{\cos\theta}{2}}\right)$

$\quad y = \tan^{-1}\left(\dfrac{\tan\theta}{2}\right)$

$\qquad = \dfrac{\theta}{2}$

$\quad y = \dfrac{1}{2}\tan^{-1}(ax)$

Differentiating it with respect to x using chain rule,

$\qquad \dfrac{dy}{dx} = \dfrac{1}{2}\times\left(\dfrac{1}{1+(ax)^2}\right)\dfrac{d}{dx}(ax)$

$\qquad \dfrac{dy}{dx} = \dfrac{1}{2\left(1+a^2x^2\right)}(a)$

$\qquad \dfrac{dy}{dx} = \dfrac{a}{2\left(1+a^2x^2\right)}.$

Differentiation Ex 11.3 Q21

Let $f(x) = \tan^{-1}\left(\dfrac{\sin x}{1+\cos x}\right)$

This function is defined for all real numbers where $\cos x \neq 1$
i.e at all odd multiples of π

$\quad f(x) = \tan^{-1}\left(\dfrac{\sin x}{1+\cos x}\right)$

$\qquad = \tan^{-1}\left[\dfrac{2\sin\left(\dfrac{x}{2}\right)\cos\left(\dfrac{x}{2}\right)}{2\cos^2\left(\dfrac{x}{2}\right)}\right]$

$\qquad = \tan^{-1}\left[\tan\left(\dfrac{x}{2}\right)\right] = \dfrac{x}{2}$

Thus, $f'(x) = \dfrac{d}{dx}\left(\dfrac{x}{2}\right) = \dfrac{1}{2}$

Differentiation Ex 11.3 Q22

Let $\quad y = \sin^{-1}\left(\dfrac{1}{\sqrt{1+x^2}}\right)$

Put $\quad x = \cot\theta$

$\quad y = \sin^{-1}\left(\dfrac{1}{\sqrt{1+\cot^2\theta}}\right)$

$\qquad = \sin^{-1}\left(\dfrac{1}{\sqrt{\cosec^2\theta}}\right)$

$\qquad = \sin^{-1}(\sin\theta)$

$\qquad = \theta$

$\quad y = \cot^{-1}x$ $\qquad\qquad\qquad$ [Since, $\cot\theta = x$]

Differentiating it with respect to x,

$\qquad \dfrac{dy}{dx} = -\dfrac{1}{\left(1+x^2\right)}.$

Differentiation Ex 11.3 Q23

Let $\quad y = \cos^{-1}\left(\dfrac{1-x^{2n}}{1+x^{2n}}\right)$

Put $\quad x^n = \tan\theta, so,$

$$y = \cos^{-1}\left(\dfrac{1-\left(x^n\right)^2}{1+\left(x^n\right)^2}\right)$$

$$= \cos^{-1}\left(\dfrac{1-\tan^2\theta}{1+\tan^2\theta}\right)$$

$$y = \cos^{-1}\left(\cos 2\theta\right) \qquad\qquad ---(i)$$

Here, $\quad 0 < x < \infty$

$\Rightarrow \qquad 0 < x^n < \infty$

$\Rightarrow \qquad 0 < \theta < \dfrac{\pi}{2}$

$\Rightarrow \qquad 0 < (2\theta) < \pi$

So, from equation (i),

$$y = 2\theta \qquad\qquad \left[\text{Sicne, } \cos^{-1}\left(\cos\theta\right) = \theta, \text{ if } \theta \in \left[0,\pi\right]\right]$$

$$y = 2\tan^{-1}\left(x^n\right)$$

Differentiating it with respect to x using chain rule,

$$\dfrac{dy}{dx} = 2\left(\dfrac{1}{1+\left(x^n\right)^2}\right)\dfrac{d}{dx}\left(x^n\right)$$

$$= \dfrac{2}{1+x^{2n}} \times \left(nx^{n-1}\right)$$

$$\dfrac{dy}{dx} = \dfrac{2nx^{n-1}}{1+x^{2n}}.$$

Differentiation Ex 11.3 Q24

Let $\quad y = \sin^{-1}\left(\dfrac{1-x^2}{1+x^2}\right) + \sec^{-1}\left(\dfrac{1+x^2}{1-x^2}\right)$

$$= \sin^{-1}\left(\dfrac{1-x^2}{1+x^2}\right) + \cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right) \qquad\qquad \left[\text{Since, } \sec^{-1}x = \cos^{-1}\left(\dfrac{1}{x}\right)\right]$$

$$y = \dfrac{\pi}{2} \qquad\qquad \left[\text{Since, } \sin^{-1}x + \cos^{-1}x = \dfrac{\pi}{2}\right]$$

Differentiating it with respect to x,

$$\dfrac{dy}{dx} = 0.$$

Differentiation Ex 11.3 Q25

Let $\quad y = \tan^{-1}\left(\dfrac{a+x}{1-ax}\right)$

$$y = \tan^{-1}a + \tan^{-1}x \qquad\qquad \left[\text{Since, } \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\dfrac{x+y}{1-xy}\right)\right]$$

Differentiating it with respect to x,

$$\dfrac{dy}{dx} = \dfrac{d}{dx}\left(\tan^{-1}a\right) + \dfrac{d}{dx}\left(\tan^{-}x\right)$$

$$= 0 + \dfrac{1}{1+x^2}$$

$$\dfrac{dy}{dx} = \dfrac{1}{1+x^2}.$$

Differentiation Ex 11.3 Q26

Let $\quad y = \tan^{-1}\left(\dfrac{\sqrt{x} + \sqrt{a}}{1 - \sqrt{xa}}\right)$

$y = \tan^{-1}\sqrt{x} + \tan^{-1}\sqrt{a}$ $\qquad\left[\text{Since, } \tan^{-1}x + \tan^{-1}y = \tan^{-1}\dfrac{x+y}{1-xy}\right]$

Differentiating it with respect to x using chain rule,

$$\dfrac{dy}{dx} = \dfrac{d}{dx}\left(\tan^{-1}\sqrt{x}\right) + \dfrac{d}{dx}\left(\tan^{-1}\sqrt{a}\right)$$

$$= \dfrac{1}{1+\left(\sqrt{x}\right)^2}\dfrac{d}{dx}\left(\sqrt{x}\right) + 0$$

$$= \left(\dfrac{1}{1+x}\right)\left(\dfrac{1}{2\sqrt{x}}\right)$$

$$\dfrac{dy}{dx} = \dfrac{1}{2\sqrt{x}\,(1+x)}.$$

Differentiation Ex 11.3 Q27

Let $\quad y = \tan^{-1}\left[\dfrac{a + b\tan x}{b - a\tan x}\right]$

$$= \tan^{-1}\left[\dfrac{\dfrac{a + b\tan x}{b}}{\dfrac{b - a\tan x}{b}}\right]$$

$$= \tan^{-1}\left[\dfrac{\dfrac{a}{b} + \tan x}{1 + \dfrac{a}{b}\tan x}\right]$$

$$= \tan^{-1}\left[\dfrac{\tan\left(\tan^{-1}\dfrac{a}{b}\right) + \tan x}{1 - \tan\left(\tan^{-1}\dfrac{a}{b}\right) + \tan x}\right]$$

$$= \tan^{-1}\left[\tan\left(\tan^{-1}\dfrac{a}{b} + x.\right)\right]$$

$$y = \tan^{-1}\left(\dfrac{a}{b}\right) + x$$

Differentiate it with respect to x,

$$\dfrac{dy}{dx} = 0 + 1$$

$$\dfrac{dy}{dx} = 1.$$

Differentiation Ex 11.3 Q28

Let $\quad y = \tan^{-1}\left(\dfrac{a + bx}{b - ax}\right)$

$$= \tan^{-1}\left(\dfrac{\dfrac{a + bx}{b}}{\dfrac{b - ax}{b}}\right)$$

$$= \tan^{-}\left(\dfrac{\dfrac{a}{b} + \dfrac{bx}{b}}{\dfrac{b}{a} - \dfrac{ax}{b}}\right)$$

$$= \tan^{-1}\left(\dfrac{\dfrac{a}{b} + x}{1 - \left(\dfrac{a}{b}\right)x}\right)$$

$$y = \tan^{-1}\left(\dfrac{a}{b}\right) + \tan^{-1}x \qquad \left[\text{Since, } \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\dfrac{x+y}{1-xy}\right)\right]$$

Differentiating it with respect to x,

$$\dfrac{dy}{dx} = 0 + \dfrac{1}{1+x^2}$$

$$\dfrac{dy}{dx} = \dfrac{1}{1+x^2}.$$

Differentiation Ex 11.3 Q29

Let $\quad y = \tan^{-1}\left(\dfrac{x-a}{x+a}\right)$

$$= \tan^{-1}\left(\dfrac{\frac{x-a}{x}}{\frac{x+a}{x}}\right)$$

$$= \tan^{-1}\left(\dfrac{\frac{x}{x}-\frac{a}{x}}{\frac{x}{x}+\frac{a}{x}}\right)$$

$$= \tan^{-1}\left(\dfrac{1-\frac{a}{x}}{1+1\times\frac{a}{x}}\right)$$

$$y = \tan^{-1}(1) - \tan^{-1}\left(\dfrac{a}{x}\right)$$

Differentiating it with respect to x using chain rule,

$$\dfrac{dy}{dx} = 0 - \dfrac{1}{1+\left(\frac{a}{x}\right)^2}\dfrac{d}{dx}\left(\dfrac{a}{x}\right)$$

$$= -\dfrac{x^2}{x^2+a^2}\left(\dfrac{-a}{x^2}\right)$$

$$\dfrac{dy}{dx} = \dfrac{a}{a^2+x^2}.$$

Differentiation Ex 11.3 Q30

Let $\quad y = \tan^{-1}\left(\dfrac{x}{1+6x^2}\right)$

$$= \tan^{-1}\left(\dfrac{3x-2x}{1+(3x)(2x)}\right)$$

$$y = \tan^{-1}3x - \tan^{-1}2x \qquad \left[\text{Since, } \tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\dfrac{x-y}{1+xy}\right)\right]$$

Differentiating it with respect to x using chain rule,

$$\dfrac{dy}{dx} = \dfrac{1}{1+(3x)^2}\dfrac{d}{dx}(3x) - \dfrac{1}{1+(2x)^2}\dfrac{d}{dx}(2x)$$

$$= \dfrac{1}{1+9x^2}(3) - \dfrac{1}{1+4x^2}(2)$$

$$\dfrac{dy}{dx} = \dfrac{3}{1+9x^2} - \dfrac{2}{1+4x^2}.$$

Differentiation Ex 11.3 Q31

Let $\quad y = \tan^{-1}\left(\dfrac{5x}{1-6x^2}\right)$

$$= \tan^{-1}\left(\dfrac{3x+2x}{1-(3x)(2x)}\right)$$

$$y = \tan^{-1}(3x) + \tan^{-1}(2x) \qquad \left[\text{Since, } \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\dfrac{x+y}{1-xy}\right)\right]$$

Differentiating it with respect to x using chain rule,

$$\dfrac{dy}{dx} = \dfrac{1}{1+(3x)^2}\dfrac{d}{dx}(3x) + \dfrac{1}{1+(2x)^2}\dfrac{d}{dx}(2x)$$

$$= \dfrac{1}{1+9x^2}(3) + \dfrac{1}{1+4x^2}(2)$$

$$\dfrac{dy}{dx} = \dfrac{3}{1+9x^2} + \dfrac{2}{1+4x^2}.$$

Differentiation Ex 11.3 Q32

Let $y = \tan^{-1}\left[\dfrac{\cos x + \sin x}{\cos x - \sin x}\right]$

$= \tan^{-1}\left[\dfrac{\dfrac{\cos x + \sin x}{\cos x}}{\dfrac{\cos x - \sin x}{\cos x}}\right]$

$= \tan^{-1}\left[\dfrac{\dfrac{\cos x}{\cos x} + \dfrac{\sin x}{\cos x}}{\dfrac{\cos x}{\cos x} - \dfrac{\sin x}{\cos x}}\right]$

$= \tan^{-1}\left[\dfrac{1 + \tan x}{1 - \tan x}\right]$

$= \tan^{-1}\left[\dfrac{\dfrac{\tan \pi}{4} + \tan x}{1 - \dfrac{\tan \pi}{4}\tan x}\right]$

$= \tan^{-1}\left[\tan\left(\dfrac{\pi}{4} + x\right)\right]$

$y = \dfrac{\pi}{4} + x$

Differentiating it with respect to x,

$\dfrac{dy}{dx} = 0 + 1$

$\dfrac{dy}{dx} = 1.$

Differentiation Ex 11.3 Q33

Let $y = \tan^{-1}\left[\dfrac{x^{\frac{1}{3}} + a^{\frac{1}{3}}}{1 - (ax)^{\frac{1}{3}}}\right]$

$y = \tan^{-1}\left(x^{\frac{1}{3}}\right) + \tan^{-1}\left(a^{\frac{1}{3}}\right)$ $\qquad \left[\text{Since, } \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\dfrac{x+y}{1-xy}\right)\right]$

Differentiating it with respect to x using chain rule,

$\dfrac{dy}{dx} = \dfrac{1}{1 + \left(x^{\frac{1}{3}}\right)^2} \times \dfrac{d}{dx}\left(x^{\frac{1}{3}}\right) + 0$

$= \dfrac{\left(\dfrac{1}{3} \times x^{\frac{1}{3}-1}\right)}{1 + x^{\frac{2}{3}}}$

$\dfrac{dy}{dx} = \dfrac{1}{3x^{\frac{2}{3}}\left(1 + x^{\frac{2}{3}}\right)}.$

Differentiation Ex 11.3 Q34

Let $f(x) = \sin^{-1}\left(\dfrac{2^{x+1}}{1+4^x}\right)$

To find the domain, we need to find all x such that

$$-1 \le \dfrac{2^{x+1}}{1+4^x} \le 1$$

Since the quantity in the middle is always positive, we need

to find all x such that $\dfrac{2^{x+1}}{1+4^x} \le 1$

i.e all x such that $2^{x+1} \le 1+4^x$

We may rewrite as $2 \le \dfrac{1}{2^x} + 2^x$, which is true for all x

Hence the function is defined at all real numbers.

Putting $2^x = \tan\theta$

$$f(x) = \sin^{-1}\left(\dfrac{2^{x+1}}{1+4^x}\right) = \sin^{-1}\left(\dfrac{2^x \cdot 2}{1+(2^x)^2}\right)$$

$$= \sin^{-1}\left[\dfrac{2\tan\theta}{1+\tan^2\theta}\right] = \sin^{-1}(\sin 2\theta) = 2\theta = 2\tan^{-1}(2^x)$$

Thus, $f'(x) = 2 \cdot \dfrac{1}{1+(2^x)^2} \dfrac{d}{dx}(2^x)$

$$= \dfrac{2}{1+4^x} \cdot (2^x)\log 2 = \dfrac{2^{x+1}\log 2}{1+4^x}$$

Differentiation Ex 11.3 Q35

Let $\quad y = \sin^{-1}\left(\dfrac{2x}{1+x^2}\right) + \sec^{-1}\left(\dfrac{1+x^2}{1-x^2}\right)$

$$y = \sin^{-1}\left(\dfrac{2x}{1+x^2}\right) + \cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)$$

Put, $\quad x = \tan\theta$

$$y = \sin^{-1}\left(\dfrac{2\tan\theta}{1+\tan^2\theta}\right) + \cos^{-1}\left(\dfrac{1-\tan^2\theta}{1+\tan^2\theta}\right)$$

$$y = \sin^{-1}(\sin 2\theta) + \cos^{-1}(\cos 2\theta) \qquad\qquad ---(i)$$

Here, $\quad 0 < x < 1$

$\Rightarrow \qquad 0 < \tan\theta < 1$

$\Rightarrow \qquad 0 < \theta < \dfrac{\pi}{4}$

$\Rightarrow \qquad 0 < (2\theta) < \dfrac{\pi}{2}$

So, from eqauation (i),

$$y = 2\theta + 2\theta$$

$\left[\begin{array}{l} \text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right] \\ \quad \cos^{-1}(\cos\theta) = \theta, \text{ if } \theta \in [0, \pi] \end{array}\right]$

$$y = 4\theta$$

$$y = 4\tan^{-1}x \qquad\qquad [\text{Since, } x = \tan\theta]$$

Differentiating it with respect to x,

$$\dfrac{dy}{dx} = \dfrac{4}{1+x^2}.$$

Differentiation Ex 11.3 Q36

Here, $y = \sin^{-1}\left(\dfrac{x}{\sqrt{1+x^2}}\right) + \cos^{-1}\left(\dfrac{1}{\sqrt{1+x^2}}\right)$

Put $x = \tan\theta$

$y = \sin^{-1}\left(\dfrac{\tan\theta}{\sqrt{1+\tan^2\theta}}\right) + \cos^{-1}\left(\dfrac{1}{\sqrt{1+\tan^2\theta}}\right)$

$= \sin^{-1}\left(\dfrac{\frac{\sin\theta}{\cos\theta}}{\sec\theta}\right) + \cos^{-1}\left(\dfrac{1}{\sec\theta}\right)$

$= \sin^{-1}\left(\dfrac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}}\right) + \cos^{-1}\left(\cos\theta\right)$

$y = \sin^{-1}\left(\sin\theta\right) + \cos^{-1}\left(\cos\theta\right)$ \hfill ---(i)

Here, $0 < x < \infty$

$\Rightarrow \quad 0 < \tan\theta < \infty$

$\Rightarrow \quad 0 < \theta < \dfrac{\pi}{2}$

So, from eqation (i),

$y = \theta + \theta$

$\left[\begin{array}{l} \text{Since, } \sin^{-1}(\sin\theta)\theta, \text{ if } \theta \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right] \\ \text{and } \cos^{-1}(\cos\theta) = \theta, \text{ if } \theta \in [0,\pi] \end{array}\right]$

$= 2\theta$

$y = 2\tan^{-1}x$ \hfill $\left[\text{Since, } x = \tan\theta\right]$

Differentiating it with respect to x,

$\dfrac{dy}{dx} = \dfrac{2}{1+x^2}.$

Differentiation Ex 11.3 Q37

Let $f(x) = \cos^{-1}(\sin x)$

We observe that this function is defined for all real numbers.

$f(x) = \cos^{-1}(\sin x)$

$= \cos^{-1}\left[\cos\left(\dfrac{\pi}{2} - x\right)\right] = \dfrac{\pi}{2} - x$

Thus, $f'(x) = \dfrac{d}{dx}\left(\dfrac{\pi}{2} - x\right) = -1$

Let $\quad y = \cot^{-1}\left(\dfrac{1-x}{1+x}\right)$

Put $\quad x = \tan\theta$, so,

$y = \cot^{-1}\left(\dfrac{1-\tan\theta}{1+\tan\theta}\right)$

$= \cot^{-1}\left(\dfrac{\tan\frac{\pi}{4} - \tan\theta}{1 + \tan\frac{\pi}{4}\tan\theta}\right)$

$= \cot^{-1}\left[\tan\left(\dfrac{\pi}{4} - \theta\right)\right]$

$= \cot^{-1}\left[\cot\left(\dfrac{\pi}{2} - \dfrac{\pi}{4} + \theta\right)\right]$

$= \dfrac{\pi}{4} + \theta$

$y = \dfrac{\pi}{4} + \tan^{-1}x$ \hfill $\left[\text{Since } x = \tan\theta\right]$

Differentiating it with respect do x,

$\dfrac{dy}{dx} = 0 + \dfrac{1}{1+x^2}$

$\dfrac{dy}{dx} = \dfrac{1}{1+x^2}.$

Differentiation Ex 11.3 Q38

$$\text{Let } y = \cot^{-1}\left[\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right] \qquad \ldots(1)$$

$$\text{Then, } \frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}$$

$$= \frac{\left(\sqrt{1+\sin x} + \sqrt{1-\sin x}\right)^2}{\left(\sqrt{1+\sin x} - \sqrt{1-\sin x}\right)\left(\sqrt{1+\sin x} + \sqrt{1-\sin x}\right)}$$

$$= \frac{(1+\sin x) + (1-\sin x) + 2\sqrt{(1-\sin x)(1+\sin x)}}{(1+\sin x) - (1-\sin x)}$$

$$= \frac{2 + 2\sqrt{1-\sin^2 x}}{2\sin x}$$

$$= \frac{1+\cos x}{\sin x}$$

$$= \frac{2\cos^2 \frac{x}{2}}{2\sin \frac{x}{2}\cos \frac{x}{2}}$$

$$= \cot \frac{x}{2}$$

Therefore, equation (1) becomes

$$y = \cot^{-1}\left(\cot \frac{x}{2}\right)$$

$$\Rightarrow y = \frac{x}{2}$$

$$\therefore \frac{dy}{dx} = \frac{1}{2}\frac{d}{dx}(x)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}$$

Differentiation Ex 11.3 Q39

$$\text{Here, } y = \tan^{-1}\left(\frac{2x}{1-x^2}\right) + \sec^{-1}\left(\frac{1+x^2}{1-x^2}\right)$$

$$y = \tan^{-1}\left(\frac{2x}{1-x^2}\right) + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

$$\text{Put } \quad x = \tan\theta,$$

$$y = \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right) + \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$$

$$y = \tan^{-1}(\tan 2\theta) + \cos^{-1}(\cos 2\theta) \qquad ---(i)$$

Here, $< x < \infty$

$$\Rightarrow \quad 0 < \tan\theta < \infty$$

$$\Rightarrow \quad 0 < \theta < \frac{\pi}{2}$$

$$\Rightarrow \quad 0 < 2\theta < \pi$$

So, from equation (i),

$$y = 2\theta + 2\theta$$

$$\left[\begin{array}{l}\text{Since, } \tan^{-1}(\tan\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \\ \text{and } \cos^{-1}(\cos\theta) = \theta, \text{ if } \theta \in [0, \pi]\end{array}\right]$$

$$y = 4\theta$$

$$y = 4\tan^{-1}x \qquad [\text{Using } x = \tan\theta]$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = \frac{4}{1+x^2}$$

Differentiation Ex 11.3 Q40

Here, $y = \sec^{-1}\left(\dfrac{x+1}{x-1}\right) + \sin^{-1}\left(\dfrac{x-1}{x+1}\right)$

$y = \cos^{-1}\left(\dfrac{x-1}{x+1}\right) + \sin^{-1}\left(\dfrac{x-1}{x+1}\right)$ $\left[\text{Since, } \sec^{-1}(x) = \cos^{-1}\left(\dfrac{1}{x}\right)\right]$

$y = \dfrac{\pi}{2}$ $\left[\text{Since, } \cos^{-1}x + \sin^{-1}x = \dfrac{\pi}{2}\right]$

Differentiating with respect to x,

$\dfrac{dy}{dx} = 0$

Differentiation Ex 11.3 Q41

Here, $y = \sin\left[2\tan^{-1}\left[\sqrt{\dfrac{1-x}{1+x}}\right]\right]$

Put $x = \cos 2\theta$, so,

$y = \sin\left[2\tan^{-1}\sqrt{\dfrac{1-\cos 2\theta}{1+\cos 2\theta}}\right]$

$= \sin\left[2\tan^{-1}\sqrt{\dfrac{2\sin^2\theta}{2\cos^2\theta}}\right]$

$= \sin\left[2\tan^{-1}\sqrt{\tan^2\theta}\right]$

$= \sin\left[2\tan^{-1}(\tan\theta)\right]$

$= \sin(2\theta)$

$= \sin\left[2\times\dfrac{1}{2}\cos^{-1}x\right]$ $[\text{Since, } x = \cos 2\theta]$

$= \sin\left(\sin^{-1}\sqrt{1-x^2}\right)$

$y = \sqrt{1-x^2}$

Differentiating with respect to x using chain rule,

$\dfrac{dy}{dx} = \dfrac{1}{2\sqrt{1-x^2}}\dfrac{d}{dx}\left(1-x^2\right).$

Differentiation Ex 11.3 Q42

Here, $y = \cos^{-1}(2x) + 2\cos^{-1}\sqrt{1-4x^2}$

Put $2x = \cos\theta$, so

$$y = \cos^{-1}(\cos\theta) + 2\cos^{-1}\sqrt{1-\cos^2\theta}$$
$$= \cos^{-1}(\cos\theta) + 2\cos^{-1}(\sin\theta)$$
$$= \cos^{-1}(\cos\theta) + 2\cos^{-1}\left(\cos\left(\frac{\pi}{2} - \theta\right)\right) \qquad \text{---(i)}$$

Here, $0 < x < \frac{1}{2}$

$\Rightarrow \qquad 0 < 2x < 1$

$\Rightarrow \qquad 0 < \cos\theta < 1$

$\Rightarrow \qquad 0 < \theta < \frac{\pi}{2}$

and

$\Rightarrow \qquad 0 > -\theta > -\frac{\pi}{2}$

$\Rightarrow \qquad \frac{\pi}{2} > \left(\frac{\pi}{2} - \theta\right) > 0$

So, from equation (i),

$$y = \theta + 2\left(\frac{\pi}{2} - \theta\right) \qquad\qquad \left[\text{Since, } \cos^{-1}(\cos(\theta)) = \theta, \text{ if } \theta \in [0, \pi]\right]$$
$$= \theta + \pi - 2\theta$$
$$y = \pi - \theta$$
$$y = \pi - \cos^{-1}(2x) \qquad\qquad [\text{Since, } 2x = \cos\theta]$$

Differentiating it with respect to x using chain rule,

$$\frac{dy}{dx} = 0 - \left[\frac{-1}{\sqrt{1-(2x)^2}}\right]\frac{d}{dx}(2x)$$
$$= \frac{1}{\sqrt{1-4x^2}}(2)$$

$$\frac{dy}{dx} = \frac{2}{\sqrt{1-4x^2}}.$$

Differentiation Ex 11.3 Q43

Here, $\dfrac{d}{dx}\left[\tan^{-1}(a+bx)\right] = 1$ at $x = 0$

So, using chain rule,

$$\left[\left\{\frac{1}{1+(a+bx)^2}\right\}\frac{d}{dx}(a+bx)\right]_{x=0} = 1$$

$$\left[\frac{1}{1+(a+bx)^2} \times (b)\right]_{x=0} = 1$$

$\Rightarrow \qquad \dfrac{b}{1+(a+0)^2} = 1$

$\Rightarrow \qquad b = 1 + a^2.$

Differentiation Ex 11.3 Q44

Here, $y = \cos^{-1}(2x) + 2\cos^{-1}\sqrt{1 - 4x^2}$

Put $2x = \cos\theta$, so,

$$y = \cos^{-1}(\cos\theta) + 2\cos^{-1}\sqrt{1 - \cos^2\theta}$$

$$= \cos^{-1}(\cos\theta) + 2\cos^{-1}(\sin\theta)$$

$$y = \cos^{-1}(\cos\theta) + 2\cos^{-1}\left(\cos\left(\frac{\pi}{2} - \theta\right)\right) \qquad\qquad \text{---(i)}$$

Now, $-\dfrac{1}{2} < x < 0$

$\Rightarrow \quad -1 < 2x < 0$

$\Rightarrow \quad -1 < \cos\theta < 0$

$\Rightarrow \quad \dfrac{\pi}{2} < \theta < \pi$

And

$\Rightarrow \quad -\dfrac{\pi}{2} > -\theta > -\pi$

$\Rightarrow \quad \left(\dfrac{\pi}{2} - \dfrac{\pi}{2}\right) > \left(\dfrac{\pi}{2} - \theta\right) > \left(\dfrac{\pi}{2} - \pi\right)$

$\Rightarrow \quad 0 > \left(\dfrac{\pi}{2} - \theta\right) > -\dfrac{\pi}{2}$

So, from equation (i),

$$y = \theta + 2\left[-\left(\dfrac{\pi}{2} - \theta\right)\right] \qquad\qquad \begin{bmatrix} \text{Since, } \cos^{-1}\cos(\theta) = \theta, \text{if } \theta \in [0, \pi] \\ \cos^{-1}\cos(\theta) = -\theta, \text{ if } \theta \in [-\pi, 0] \end{bmatrix}$$

$$y = \theta - 2 \times \dfrac{\pi}{2} + 2\theta$$

$$y = -\pi + 3\theta$$

$$y = -\pi + 3\cos^{-1}(2x) \qquad\qquad [\text{Since, } 2x = \cos\theta]$$

Differentiating it with respect to x using chain rule,

$$\frac{dy}{dx} = 0 + 3\left(\frac{-1}{\sqrt{1 - (2x)^2}}\right)\frac{d}{dx}(2x)$$

$$= \frac{-3}{\sqrt{1 - 4x^2}}(2)$$

$$\frac{dy}{dx} = -\frac{6}{\sqrt{1 - 4x^2}}$$

Differentiation Ex 11.3 Q45

Here, $y = \tan^{-1}\left(\dfrac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right)$

Put $x = \cos 2\theta$, so

$y = \tan^{-1}\left(\dfrac{\sqrt{1+\cos 2\theta} - \sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta} + \sqrt{1-\cos 2\theta}}\right)$

$= \tan^{-1}\left(\dfrac{\sqrt{2\cos^2\theta} - \sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta} + \sqrt{2\sin^2\theta}}\right)$

$= \tan^{-1}\left(\dfrac{\sqrt{2}\,(\cos\theta - \sin\theta)}{\sqrt{2}\,(\cos\theta + \sin\theta)}\right)$

$= \tan^{\{1\}}\left(\dfrac{\dfrac{\cos\theta - \sin\theta}{\cos\theta}}{\dfrac{\cos\theta + \sin\theta}{\cos\theta}}\right)$

$\left[\text{Dividing numerator and denominator by } \cos\theta\right]$

$= \tan^{-}1\left(\dfrac{\dfrac{\cos\theta}{\cos\theta} - \dfrac{\sin\theta}{\cos\theta}}{\dfrac{\cos\theta}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}}\right)$

$= \tan^{-1}\left(\dfrac{1 - \tan\theta}{1 + \tan\theta}\right)$

$= \tan^{-1}\left(\dfrac{\dfrac{\tan\pi}{4} - \tan\theta}{1 + \dfrac{\tan\pi}{4} + \tan\theta}\right)$

$= \tan^{-1}\left[\tan\left(\dfrac{\pi}{4} - \theta\right)\right]$

$= \dfrac{\pi}{4} - \theta$

$y = \dfrac{\pi}{4} - \dfrac{1}{2}\cos^{-1}x$ \qquad\qquad $\left[\text{Using } x = \cos 2\theta\right]$

Differentiating it with respect to x,

$\dfrac{dy}{dx} = 0 - \dfrac{1}{2}\left(\dfrac{-1}{\sqrt{1-x^2}}\right)$

$\dfrac{dy}{dx} = \dfrac{1}{2\sqrt{1-x^2}}$.

Differentiation Ex 11.3 Q46

Here, $y = \cos^{-1}\left\{\dfrac{2x - 3\sqrt{1 - x^2}}{\sqrt{13}}\right\}$

Let $x = \cos\theta$, so,

$$y = \cos^{-1}\left\{\dfrac{2\cos\theta - 3\sqrt{1 - \cos^2\theta}}{\sqrt{13}}\right\}$$

$$= \cos^{-1}\left\{\dfrac{2}{\sqrt{13}}\cos\theta - \dfrac{3}{\sqrt{13}}\sin\theta\right\}$$

Let $\cos\phi = \dfrac{2}{\sqrt{13}}$

$\Rightarrow \quad \sin\phi = \sqrt{1 - \cos^2\phi}$

$$= \sqrt{1 - \left(\dfrac{2}{\sqrt{13}}\right)^2}$$

$$= \sqrt{\dfrac{13 - 4}{13}}$$

$$= \sqrt{\dfrac{9}{13}}$$

$$\sin\phi = \dfrac{3}{\sqrt{13}}$$

So,

$$y = \cos^{-1}\left\{\cos\phi\,\cos\theta - \sin\phi\,\sin\theta\right\}$$

$$= \cos^{-1}\left[\cos(\theta + \phi)\right]$$

$$y = \phi + \theta$$

$$y = \cos^{-1}\left(\dfrac{2}{\sqrt{13}}\right) + \cos^{-1}x \qquad \left[\text{Since, } x = \cos\theta, \cos\phi = \dfrac{2}{\sqrt{13}}\right]$$

Differentiating it with respect to x,

$$\dfrac{dy}{dx} = 0 + \left(-\dfrac{1}{\sqrt{1 - x^2}}\right)$$

$$\dfrac{dy}{dx} = -\dfrac{1}{\sqrt{1 - x^2}}.$$

Differentiation Ex 11.3 Q47

Consider the given expression:

$$y = \sin^{-1}\left\{\dfrac{2^{x+1} \times 3^x}{1 + (36)^x}\right\}$$

$$= \sin^{-1}\left\{\dfrac{2 \times 2^x \times 3^x}{1 + (6^2)^x}\right\}$$

$$y = \sin^{-1}\left\{\dfrac{2 \times 6^x}{1 + 6^{2x}}\right\} \quad ....(1)$$

Substituting $6^x = \tan\theta$ in the above equation, we get,

$$y = \sin^{-1}\left\{\dfrac{2 \times 6^x}{1 + 6^{2x}}\right\}$$

$$= \sin^{-1}\left\{\dfrac{2 \times \tan\theta}{1 + \tan^2\theta}\right\}$$

$$= \sin^{-1}(\sin 2\theta)$$

$$= 2\theta$$

$$= 2\tan^{-1}(6^x)$$

Differentiating the above function with respect to x, we have,

$$\dfrac{d}{dx}\left[\sin^{-1}\left\{\dfrac{2^{x+1} \times 3^x}{1 + (36)^x}\right\}\right] = \dfrac{d}{dx}\left[2\tan^{-1}(6^x)\right]$$

$$= 2 \times \dfrac{1}{1 + (6^x)^2} \times 6^x \log 6$$

$$= \dfrac{2 \times 6^x \log 6}{1 + 6^{2x}}$$

Ex 11.4

Differentiation Ex 11.4 Q1

Given,

$$xy = c^2$$

Differentiate with respect to x,

$$\frac{d}{dx}(xy) = \frac{d}{dx}(c^2)$$

$\Rightarrow \quad x\frac{dy}{dx} + y\frac{d}{dx}(x) = 0$ \qquad\qquad [Using product rule]

$\Rightarrow \quad x\frac{dy}{dx} + y = 0$

$\Rightarrow \quad x\frac{dy}{dx} = -y$

$\Rightarrow \quad \frac{dy}{dx} = -\frac{y}{x}$

Differentiation Ex 11.4 Q2

Here, $\quad y^3 - 3xy^2 = x^3 + 3x^2y$

Differentiating with respect to x,

$\Rightarrow \quad \frac{d}{dx}(y^3) - \frac{d}{dx}(3xy^2) = \frac{d}{dx}(x^3) + \frac{d}{dx}(3x^2y)$

$\Rightarrow \quad 3y^2\frac{dy}{dx} - 3\left[x\frac{d}{dx}y^2 \frac{d}{dx}(x)\right] = 3x^2 + 3\left[x^2\frac{d}{dx}(y) + y\frac{d}{dx}(x^2)\right]$ \qquad [Using product rule]

$\Rightarrow \quad 3y^2\frac{dy}{dx} - 3\left[x(2y)\frac{dy}{dx} + y^2\right] = 3x^2 + 3\left[x^2\frac{dy}{dx} + y(2x)\right]$

$\Rightarrow \quad 3y^2\frac{dy}{dx} - 6xy\frac{dy}{dx} - 3y^2 = 3x^2 + 3x^2\frac{dy}{dx} + 6xy$

$\Rightarrow \quad 3y^2\frac{dy}{dx} - 6xy\frac{dy}{dx} - 3x^2\frac{dy}{dx} = 3x^2 + 6xy + 3y^2$

$\Rightarrow \quad 3\frac{dy}{dx}\left(y^2 - 2xy - x^2\right) = 3\left(x^2 + 2xy + y^2\right)$

$\Rightarrow \quad \frac{dy}{dx} = \frac{3(x+y)^2}{3\left(y^2 - 2xy - x^2\right)}$

$\Rightarrow \quad \frac{dy}{dx} = \frac{(x+y)^2}{y^2 - 2xy - x^2}$

Differentiation Ex 11.4 Q3

Here, $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$

Differentiate it with respect to x,

$$\frac{d}{dx}\left(x^{\frac{2}{3}}\right) + \frac{d}{dx}\left(y^{\frac{2}{3}}\right) = \frac{d}{dx}\left(a^{\frac{2}{3}}\right)$$

$\Rightarrow \quad \frac{2}{3}x^{\left(\frac{2}{3}-1\right)} + \frac{2}{3}y^{\left(\frac{2}{3}-1\right)}\frac{dy}{dx} = 0$

$\Rightarrow \quad \frac{2}{3}x^{\frac{-1}{3}} + \frac{2}{3}y^{\frac{-1}{3}}\frac{dy}{dx} = 0$

$\Rightarrow \quad \frac{2}{3}y^{\frac{-1}{3}}\frac{dy}{dx} = -\frac{2}{3}x^{\frac{-1}{3}}$

$\Rightarrow \quad \frac{dy}{dx} = -\frac{2}{3}x^{\frac{-1}{3}} \times \frac{3}{2y^{\frac{-1}{3}}}$

$\Rightarrow \quad \frac{dy}{dx} = -\frac{x^{\frac{-1}{3}}}{y^{\frac{-1}{3}}}$

$\Rightarrow \quad \frac{dy}{dx} = -\frac{y^{\frac{1}{3}}}{x^{\frac{1}{3}}}$

$\Rightarrow \quad \frac{dy}{dx} = -\left(\frac{y}{x}\right)^{\frac{1}{3}}$

Differentiation Ex 11.4 Q4

Given, $4x + 3y = \log(4x - 3y)$

Differentiating with respect to x,

$$\frac{d}{dx}(4x) + \frac{d}{dx}(3y) = \frac{d}{dx}\left(\log(4x - 3y)\right)$$

$$\Rightarrow \quad 4 + 3\frac{dy}{dx} = \frac{1}{(4x - 3y)}\frac{d}{dx}(4x - 3y) \qquad \text{[Using chain rule]}$$

$$\Rightarrow \quad 4 + 3\frac{dy}{dx} = \frac{1}{(4x - 3y)}\left(4 - 3\frac{dy}{dx}\right)$$

$$\Rightarrow \quad 4 + 3\frac{dy}{dx} = \frac{4}{(4x - 3y)} - \frac{3}{(4x - 3y)}\frac{dy}{dx}$$

$$\Rightarrow \quad 3\frac{dy}{dx} + \frac{3}{(4x - 3y)}\frac{dy}{dx} = \frac{4}{(4x - 3y)} - 4$$

$$\Rightarrow \quad 3\frac{dy}{dx}\left(1 + \frac{1}{(4x - 3y)}\right) = 4\left(\frac{1}{(4x - 3y)} - 1\right)$$

$$\Rightarrow \quad 3\frac{dy}{dx}\left[\frac{4x - 3y + 1}{(4x - 3y)}\right] = 4\left[\frac{1 - 4x + 3y}{(4x - 3y)}\right]$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{4}{3}\left[\frac{1 - 4x + 3y}{(4x - 3y)}\right]\left[\frac{4x - 3y}{4x - 3y + 1}\right]$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{4}{3}\left(\frac{1 - 4x + 3y}{4x - 3y + 1}\right)$$

Differentiation Ex 11.4 Q5

Given,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Differentiating with respect to x,

$$\frac{d}{dx}\left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right) = \frac{d}{dx}(1)$$

$$\Rightarrow \quad \frac{d}{dx}\left(\frac{x^2}{a^2}\right) + \frac{d}{dx}\left(\frac{y^2}{b^2}\right) = 0$$

$$\Rightarrow \quad \frac{1}{a^2}(2x) + \frac{1}{b^2}(2y)\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{2y}{b^2}\frac{dy}{dx} = -\frac{2x}{a^2}$$

$$\Rightarrow \quad \frac{dy}{dx} = -\left(\frac{2x}{a^2}\right)\left(\frac{b^2}{2y}\right)$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{b^2 x}{a^2 y}$$

Differentiation Ex 11.4 Q6

Given,

$$x^5 + y^5 = 5xy$$

Differentiating with respect to x,

$$\frac{d}{dx}\left(x^5\right) + \frac{d}{dx}\left(y^5\right) = \frac{d}{dx}(5xy)$$

$$\Rightarrow \quad 5x^4 + 5y^4\frac{dy}{dx} = 5\left[x\frac{dy}{dx} + y\frac{dy}{dx}(x)\right] \qquad \text{[Using product rule]}$$

$$\Rightarrow \quad 5x^4 + 5y^4\frac{dy}{dx} = 5\left[x\frac{dy}{dx} + y(1)\right]$$

$$\Rightarrow \quad 5x^4 + 5y^4\frac{dy}{dx} = 5x\frac{dy}{dx} + 5y$$

$$\Rightarrow \quad 5y^4\frac{dy}{dx} - 5x\frac{dy}{dx} = 5y - 5x^4$$

$$\Rightarrow \quad 5\frac{dy}{dx}\left(y^4 - x\right) = 5\left(y - x^4\right)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{5\left(y - x^4\right)}{5\left(y^4 - x\right)}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y - x^4}{y^4 - x}$$

Differentiation Ex 11.4 Q7

Given,

$$(x + y)^2 = 2axy$$

Differentiating with respect to x,

$$\Rightarrow \quad \frac{d}{dx}(x + y)^2 = \frac{d}{dx}(2axy)$$

$$\Rightarrow \quad 2(x + y)\frac{d}{dx}(x + y) = 2a\left[x\frac{dy}{dx} + y\frac{d}{dx}(x)\right] \qquad \text{[Using chain rule and product rule]}$$

$$\Rightarrow \quad 2(x + y)\left[1 + \frac{dy}{dx}\right] = 2a\left[x\frac{dy}{dx} + y(1)\right]$$

$$\Rightarrow \quad 2(x + y) + 2(x + y)\frac{dy}{dx} = 2ax\frac{dy}{dx} + 2ay$$

$$\Rightarrow \quad \frac{dy}{dx}\left[2(x + y) - 2ax\right] = 2ay - 2(x + y)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{2[ay - x - y]}{2[x + y - ax]}$$

$$\Rightarrow \quad \frac{dy}{dx} = \left(\frac{ay - x - y}{x + y - ax}\right)$$

Differentiation Ex 11.4 Q8

Given,

$$(x^2 + y^2)^2 = xy$$

Differentiating with respect to x,

$$\frac{d}{dx}\left((x^2 + y^2)^2\right) = \frac{d}{dx}(xy)$$

$$\Rightarrow \quad 2(x^2 + y^2)\frac{d}{dx}(x^2 + y^2) = x\frac{dy}{dx} + y\frac{d}{dx}(x) \qquad \text{[Using chain rule and product rule]}$$

$$\Rightarrow \quad 2(x^2 + y^2)\left(2x + 2y\frac{dy}{dx}\right) = x\frac{dy}{dx} + y(1)$$

$$\Rightarrow \quad 4x(x^2 + y^2) + 4y(x^2 + y^2)\frac{dy}{dx} = x\frac{dy}{dx} + y$$

$$\Rightarrow \quad 4y(x^2 + y^2)\frac{dy}{dx} - x\frac{dy}{dx} = y - 4x(x^2 + y^2)$$

$$\Rightarrow \quad \frac{dy}{dx}\left[4yx^2 + 4y^3 - x\right] = y - 4x^3 - 4xy^2$$

$$\Rightarrow \quad \frac{dy}{dx} = \left(\frac{y - 4x^3 - 4xy^2}{4yx^2 + 4y^3 - x}\right)$$

Differentiation Ex 11.4 Q9

Here,

$$\tan^{-1}(x^2 + y^2) = a$$

Differentiating with respect to x,

$$\frac{d}{dx}\left(\tan^{-1}(x^2 + y^2)\right) = \frac{d}{dx}(a)$$

$$\Rightarrow \quad \frac{1}{1 + (x^2 + y^2)^2} \times \frac{d}{dx}(x^2 + y^2) = 0 \qquad \text{[Using chain rule]}$$

$$\Rightarrow \quad \left[\frac{1}{1 + (x^2 + y^2)^2}\right]\left(2x + 2y\frac{dy}{dx}\right) = 0$$

$$\Rightarrow \quad \left\{\frac{2x}{1 + (x^2 + y^2)^2}\right\} + \left\{\frac{2y}{1 + (x^2 + y^2)^2}\right\}\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{2y}{1 + (x^2 + y^2)^2}\frac{dy}{dx} = -\frac{2x}{1 + (x^2 + y^2)^2}$$

$$\Rightarrow \quad \frac{dy}{dx} = -\left(\frac{2x}{1 + (x^2 + y^2)^2}\right)\left(\frac{1 + (x^2 + y^2)^2}{2y}\right)$$

$$\Rightarrow \quad \frac{dy}{dx} = -\left(\frac{x}{y}\right)$$

Differentiation Ex 11.4 Q10

Given,

$$e^{x-y} = \log\left(\frac{x}{y}\right)$$

Differentiating with respect to x,

$$\frac{d}{dx}\left(e^{x-y}\right) = \frac{d}{dx}\log\left(\frac{x}{y}\right)$$

$$\Rightarrow \quad e^{(x-y)}\frac{d}{dx}(x-y) = \frac{1}{\left(\frac{x}{y}\right)} \times \frac{d}{dx}\left(\frac{x}{y}\right) \qquad \text{[Using chain rule and quotient rule]}$$

$$\Rightarrow \quad e^{(x-y)}\left(1 - \frac{dy}{dx}\right) = \frac{y}{x}\left[\frac{y\frac{d}{dx}(x) - x\frac{dy}{dx}}{y^2}\right]$$

$$\Rightarrow \quad e^{(x-y)} - e^{(x-y)}\frac{dy}{dx} = \frac{1}{xy}\left[y(1) - x\frac{dy}{dx}\right]$$

$$\Rightarrow \quad e^{(x-y)} - e^{(x-y)}\frac{dy}{dx} = \frac{y}{xy} - \frac{x}{xy}\frac{dy}{dx}$$

$$\Rightarrow \quad e^{(x-y)} - e(x-y)\frac{dy}{dx} = \frac{1}{x} - \frac{1}{y}\frac{dy}{dx}$$

$$\Rightarrow \quad \frac{1}{y}\frac{dy}{dx} - e^{(x-y)}\frac{dy}{dx} = \frac{1}{x} - e^{(x-y)}$$

$$\Rightarrow \quad \frac{dy}{dx}\left[\frac{1}{y} - \frac{e^{(x-y)}}{1}\right] = \frac{1}{x} - \frac{e^{(x-y)}}{1}$$

$$\Rightarrow \quad \frac{dy}{dx}\left[\frac{1 - ye^{(x-y)}}{y}\right] = \frac{\left(1 - xe^{(x-y)}\right)}{x}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y}{x}\left[\frac{1 - xe^{(x-y)}}{1 - ye^{(x-y)}}\right]$$

$$= \frac{-y}{-x}\left[\frac{xe^{(x-y)} - 1}{ye^{(x-y)} - 1}\right]$$

$$\frac{dy}{dx} = \frac{y}{x}\left[\frac{xe^{(x-y)} - 1}{ye^{(x-y)} - 1}\right]$$

Differentiation Ex 11.4 Q11

Given,

$$\sin xy + \cos(x+y) = 1$$

Differentiating with respect to x,

$$\frac{d}{dx}\sin xy + \frac{d}{dx}\cos(x+y) = \frac{d}{dx}(1)$$

$$\Rightarrow \quad \cos xy\frac{d}{dx}(xy) - \sin(x+y)\frac{d}{dx}(x+y) = 0 \qquad \text{[Using chain rule and product rule]}$$

$$\Rightarrow \quad \cos(xy)\left[x\frac{dy}{dx} + y\frac{d}{dx}(x)\right] - \sin(x+y)\left[1 + \frac{dy}{dx}\right] = 0$$

$$\Rightarrow \quad \cos(xy)\left[x\frac{dy}{dx} + y(1)\right] - \sin(x+y) + \sin(x+y)\frac{dy}{dx} = 0$$

$$\Rightarrow \quad x\cos(xy)\frac{dy}{dx} + y\cos(xy) - \sin(x+y) + \sin(x+y)\frac{dy}{dx} = 0$$

$$\Rightarrow \quad [x\cos(xy) + \sin(x+y)]\frac{dy}{dx} = [\sin(x+y) - y\cos xy]$$

$$\Rightarrow \quad \frac{dy}{dx} = \left[\frac{\sin(x+y) - y\cos xy}{x\cos xy + \sin(x+y)}\right]$$

Differentiation Ex 11.4 Q12

Here,

$$\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$$

Let $\quad x = \sin A, y = \sin B$, so

$\Rightarrow \quad \sqrt{1 - \sin^2 A} + \sqrt{1 - \sin^2 B} = a(\sin A - \sin B)$

$\Rightarrow \quad \cos A + \cos B = a(\sin A - \sin B)$ $\qquad \left[\text{Since } \left(1 - \sin^2\theta\right) = \cos^2\theta\right]$

$\Rightarrow \quad a = \dfrac{\cos A + \cos B}{\sin A - \sin B}$

$\Rightarrow \quad a = \dfrac{2\cos\dfrac{A+B}{2} \times \cos\dfrac{A-B}{2}}{2\cos\dfrac{A+B}{2} \times \sin\dfrac{A-B}{2}}$ $\qquad \left[\begin{array}{l}\text{Since, } \sin A - \sin B = 2\cos\left(\dfrac{A+B}{2}\right)\sin\left(\dfrac{A-B}{2}\right) \\ \cos A + \cos B = 2\cos\left(\dfrac{A+B}{2}\right)\cos\left(\dfrac{A-B}{2}\right)\end{array}\right]$

$\Rightarrow \quad a = \cot\left(\dfrac{A-B}{2}\right)$

$\Rightarrow \quad \cot^{-1} a = \dfrac{A-B}{2}$

$\Rightarrow \quad 2\cot^{-1} a = A - B$

$\Rightarrow \quad 2\cot^{-1} a = \sin^{-1} x - \sin^{-1} y$ $\qquad \left[\text{Since } x = \sin A, y = \sin B\right]$

Differentiating with respect to x,

$$\dfrac{d}{dx}\left(2\cot^{-1} a\right) = \dfrac{d}{dx}\left(\sin^{-1} x\right) - \dfrac{d}{dx}\left(\sin^{-1} y\right)$$

$\Rightarrow \quad 0 = \dfrac{1}{\sqrt{1-x^2}} - \dfrac{1}{\sqrt{1-y^2}}\dfrac{dy}{dx}$

$\Rightarrow \quad \dfrac{1}{\sqrt{1-y^2}}\dfrac{dy}{dx} = \dfrac{1}{\sqrt{1-x^2}}$

$\Rightarrow \quad \dfrac{dy}{dx} = \dfrac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

$$\dfrac{dy}{dx} = \sqrt{\dfrac{1-y^2}{1-x^2}}$$

Differentiation Ex 11.4 Q13

Here,

$$y\sqrt{1-x^2} + x\sqrt{1-y^2} = 1$$

Let $\quad x = \sin A, y = \sin B$

$\Rightarrow \quad \sin B\sqrt{1 - \sin^2 A} + \sin A\sqrt{1 - \sin^2 B} = 1$

$\Rightarrow \quad \sin B\cos A + \sin A\cos B = 1$ $\qquad \left[\begin{array}{l}\text{since } 1 - \sin^2\theta = \cos^2\theta \text{ and} \\ \sin(x+y) = \sin x\cos y + \cos x\sin y\end{array}\right]$

$\Rightarrow \quad \sin(A+B) = 1$

$\Rightarrow \quad A + B = \sin^{-1}(1)$

$\Rightarrow \quad \sin^{-1} x + \sin^{-1} y = \dfrac{\pi}{2}$ $\qquad \left[\text{Since } x = \sin A, y = \sin B\right]$

Differentiating with respect to x,

$\Rightarrow \quad \dfrac{d}{dx}\left(\sin^{-1} x\right) + \dfrac{d}{dx}\left(\sin^{-1} y\right) = \dfrac{d}{dx}\left(\dfrac{\pi}{2}\right)$

$\Rightarrow \quad \dfrac{1}{\sqrt{1-x^2}} + \dfrac{1}{\sqrt{1-y^2}}\dfrac{dy}{dx} = 0$

$\Rightarrow \quad \dfrac{dy}{dx} = -\sqrt{\dfrac{1-y^2}{1-x^2}}$

Differentiation Ex 11.4 Q14

Here,

$$xy = 1 \qquad\qquad ---(i)$$

Differentiating with respect to x,

$$\frac{d}{dx}(xy) = \frac{d}{dx}(1)$$

$$\Rightarrow \quad x\frac{dy}{dx} + y\frac{d}{dx}(x) = 0 \qquad\qquad \text{[Using product rule]}$$

$$\Rightarrow \quad x\frac{dy}{dx} + y(1) = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{y}{x} \qquad\qquad \left[\text{Put } x = \frac{1}{y} \text{ from equation (i)}\right]$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{y}{\frac{1}{y}}$$

$$\Rightarrow \quad \frac{dy}{dx} = -y^2$$

$$\Rightarrow \quad \frac{dy}{dx} + y^2 = 0$$

Differentiation Ex 11.4 Q15

Here,

$$xy^2 = 1$$

Differentiating with respect to x,

$$\frac{d}{dx}(xy^2) = \frac{d}{dx}(1)$$

$$\Rightarrow \quad x\frac{d}{dx}(y^2) + y^2\frac{d}{dx}(x) = 0 \qquad\qquad \text{[Using product rule]}$$

$$\Rightarrow \quad x(2y)\frac{dy}{dx} + y^2(1) = 0$$

$$\Rightarrow \quad 2xy\frac{dy}{dx} = -y^2$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-y^2}{2xy}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-y}{2x}$$

Put $x = \dfrac{1}{y^2}$ from equation (i)

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-y}{2\left(\dfrac{1}{y^2}\right)}$$

$$\Rightarrow \quad 2\frac{dy}{dx} = -y^3$$

$$2\frac{dy}{dx} + y^3 = 0$$

Differentiation Ex 11.4 Q16

Given,

$$x\sqrt{1+y} + y\sqrt{1+x} = 0$$
$$\Rightarrow \quad x\sqrt{1+y} = -y\sqrt{1+x}$$

Squaring both the sides,

$$\Rightarrow \quad \left(x\sqrt{1+y}\right)^2 = \left(-y\sqrt{1+x}\right)^2$$
$$\Rightarrow \quad x^2(1+y) = y^2(1+x)$$
$$\Rightarrow \quad x^2 + x^2y = y^2 + y^2x$$
$$\Rightarrow \quad x^2 - y^2 = y^2x - x^2y$$
$$\Rightarrow \quad (x-y)(x+y) = xy(y-x)$$
$$\Rightarrow \quad (x+y) = -xy$$
$$\Rightarrow \quad y + xy = -x$$
$$\Rightarrow \quad y(1+x) = -x$$
$$\Rightarrow \quad y = \frac{-x}{(1+x)}$$

Differentiating with respect to x using quotient rule,

$$\Rightarrow \quad \frac{dy}{dx} = \left[\frac{-(1+x)\frac{d}{dx}(x) + (-x)\frac{d}{dx}(x+1)}{(1+x)^2}\right]$$

$$\Rightarrow \quad \frac{dy}{dx} = \left[\frac{-(1+x)(1) + x(1)}{(1+x)^2}\right]$$

$$\Rightarrow \quad \frac{dy}{dx} = \left[\frac{-1-x+x}{(1+x)^2}\right]$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-1}{(1+x)^2}$$

$$\Rightarrow \quad (1+x)^2\frac{dy}{dx} = -1$$

$$\Rightarrow \quad (1+x)^2\frac{dy}{dx} + 1 = 0$$

Differentiation Ex 11.4 Q17

Here,

$$\log\sqrt{x^2+y^2} = \tan^{-1}\left(\frac{x}{y}\right)$$

$$\Rightarrow \quad \log\left(x^2+y^2\right)^{\frac{1}{2}} = \tan^{-1}\left(\frac{y}{x}\right)$$

$$\Rightarrow \quad \frac{1}{2}\log\left(x^2+y^2\right) = \tan^{-1}\left(\frac{y}{x}\right)$$

Differentiating with respect to x,

$$\Rightarrow \quad \frac{1}{2}\frac{d}{dx}\log\left(x^2+y^2\right) = \frac{d}{dx}\tan^{-1}\left(\frac{y}{x}\right)$$

$$\Rightarrow \quad \frac{1}{2}\times\left(\frac{1}{x^2+y^2}\right)\frac{d}{dx}\left(x^2+y^2\right) = \frac{1}{1+\left(\frac{y}{x}\right)^2}\frac{d}{dx}\left(\frac{y}{x}\right) \qquad \text{[Using chain rule, quotient rule]}$$

$$\Rightarrow \quad \frac{1}{2}\left(\frac{1}{x^2+y^2}\right)\left[2x + 2y\frac{dy}{dx}\right] = \frac{x^2}{\left(x^2+y^2\right)}\left[\frac{x\frac{dy}{dx} - y\frac{d}{dx}(x)}{x^2}\right]$$

$$\Rightarrow \quad \frac{1}{2}\left(\frac{1}{x^2+y^2}\right)\times 2\left(x + y\frac{dy}{dx}\right) = \frac{x^2}{\left(x^2+y^2\right)}\left[\frac{x\frac{dy}{dx} - y(1)}{x^2}\right]$$

$$\Rightarrow \quad x + y\frac{dy}{dx} = x\frac{dy}{dx} - y$$

$$\Rightarrow \quad y\frac{dy}{dx} - x\frac{dy}{dx} = -y - x$$

$$\Rightarrow \quad \frac{dy}{dx}(y-x) = -(y+x)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-(y+x)}{y-x}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{x+y}{x-y}$$

Differentiation Ex 11.4 Q18

Here,

$$\sec\left(\frac{x+y}{x-y}\right) = a$$

$$\Rightarrow \quad \frac{x+y}{x-y} = \sec^{-1}(a)$$

Differentiating with respect to x,

$$\Rightarrow \quad \left[\frac{(x-y)\frac{d}{dx}(x+y) - (x+y)\frac{d}{dx}(x-y)}{(x-y)^2}\right] = 0 \qquad \text{[Using quotient rule]}$$

$$\Rightarrow \quad (x-y)\left(1+\frac{dy}{dx}\right) - (x+y)\left(1-\frac{dy}{dx}\right) = 0$$

$$\Rightarrow \quad (x-y) + (x-y)\frac{dy}{dx} - (x+y) + (x+y)\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx}[x-y+x+y] = x+y-x+y$$

$$\Rightarrow \quad \frac{dy}{dx}(2x) = 2y$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y}{x}$$

Differentiation Ex 11.4 Q19

Here,

$$\tan^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right) = a$$

$$\Rightarrow \quad \frac{x^2-y^2}{x^2+y^2} = \tan a$$

$$\Rightarrow \quad x^2-y^2 = \tan a\left(x^2+y^2\right)$$

Differentiating with respect to x,

$$\Rightarrow \quad \frac{d}{dx}\left(x^2-y^2\right) = \tan a\frac{d}{dx}\left(x^2+y^2\right)$$

$$\Rightarrow \quad \left(2x-2y\frac{dy}{dx}\right) = \tan a\left(2x+2y\frac{dy}{dx}\right)$$

$$\Rightarrow \quad 2x-2y\frac{dy}{dx} = 2x\tan a + 2t\tan a\frac{dy}{dx}$$

$$\Rightarrow \quad 2y\tan a\frac{dy}{dx} + 2y\frac{dy}{dx} = 2x - 2x\tan a$$

$$\Rightarrow \quad 2y\frac{dy}{dx}(1+\tan a) = 2x(1-\tan a)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{x}{y}\left(\frac{1-\tan a}{1+\tan a}\right)$$

Differentiation Ex 11.4 Q20

Here,
$$xy \log(x + y) = 1$$

Differentiating it with respect to x,

$$\Rightarrow \quad \frac{d}{dx}\left[xy \log(x + y)\right] = \frac{d}{dx}(1)$$

$$\Rightarrow \quad xy \frac{d}{dx}\log(x + y) + x \log(x + y)\frac{dy}{dx} + y \log(x + y)\frac{d}{dx}(x) = 0$$

[Using chain rule and product rule]

$$\Rightarrow \quad xy \times \left(\frac{1}{x + y}\right)\frac{d}{dx}(x + y) + x \log(x + y)\frac{dy}{dx} + y \log(x + y)(1) = 0$$

$$\Rightarrow \quad \left(\frac{xy}{x + y}\right)\left(1 + \frac{dy}{dx}\right) + x \log(x + y)\frac{dy}{dx} + y \log(x + y) = 0$$

$$\Rightarrow \quad \left(\frac{xy}{x + y}\right)\frac{dy}{dx} + \left(\frac{xy}{x + y}\right) + x\left(\frac{1}{xy}\right)\frac{dy}{dx} + y\left(\frac{1}{xy}\right) = 0$$

$$\left[\text{Sicne from equation (i)} \quad \log(x + y) = \frac{1}{xy}\right]$$

$$\Rightarrow \quad \frac{dy}{dx}\left[\frac{xy}{x + y} + \frac{1}{y}\right] = -\left[\frac{1}{x} + \frac{xy}{x + y}\right]$$

$$\frac{dy}{dx}\left[\frac{xy^2 + x + y}{(x + y)y}\right] = -\left[\frac{x + y + x^2y}{x(x + y)}\right]$$

$$\frac{dy}{dx} = -\left(\frac{x + y + x^2y}{x(x + y)}\right)\left(\frac{y(x + y)}{xy^2 + x + y}\right)$$

$$= -\frac{y}{x}\left(\frac{x + y + x^2y}{x + y + xy^2}\right)$$

So,

$$\frac{dy}{dx} = -\frac{y}{x}\left(\frac{x^2y + x + y}{xy^2 + x + y}\right)$$

Differentiation Ex 11.4 Q21

Here,
$$y = x \sin(a + y) \qquad\qquad ---(i)$$

Differentiating with respect to y,

$$\Rightarrow \quad \frac{dy}{dx} = \frac{d}{dx}\left[x \sin(a + y)\right]$$

$$\Rightarrow \quad \frac{dy}{dx} = x \frac{d}{dx}\sin(a + y) + \sin(a + y)\frac{d}{dx}(x) \qquad \text{[Using product rule, chain ruel]}$$

$$\Rightarrow \quad \frac{dy}{dx} = x \cos(a + y)\frac{d}{dx}(a + y) + \sin(a + y)(1)$$

$$= x \cos(a + y)\left(0 + \frac{dy}{dx}\right) + \sin(a + y)$$

$$\Rightarrow \quad \frac{dy}{dx}\left(1 - x \cos(a + y)\right) = \sin(a + y)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{\sin(a + y)}{1 - x \cos(a + y)}$$

Put x from equation (i), $x = \dfrac{y}{\sin(a + y)}$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{\sin(a + y)}{1 - \dfrac{y}{\sin(a + y)}\cos(a + y)}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{\sin^2(a + y)}{\sin(a + y) - y \cos(a + y)}$$

Differentiation Ex 11.4 Q22

Here,
$$x \sin(a+y) + \sin a \cos(a+y) = 0 \qquad\qquad \text{---(i)}$$

Differentiating with respect to x,

$$\Rightarrow \quad \frac{d}{dx}\left[x\sin(a+y)\right] + \frac{d}{dx}\left[\sin a \cos(a+y)\right] = 0$$

$$\Rightarrow \quad \left[x\frac{d}{dx}\sin(a+y) + \sin(a+y)\frac{d}{dx}(x)\right] + \sin a\frac{d}{dx}\cos(a+y) = 0$$

[Using product rule and chain rule]

$$\Rightarrow \quad \left[x\cos(a+y)\frac{d}{dx}(a+y) + \sin(a+y)(1)\right] + \sin a\left[-\sin(a+y)\frac{d}{dx}(a+y)\right] = 0$$

$$\Rightarrow \quad \left[x\cos(a+y)\left(0+\frac{dy}{dx}\right) + \sin(a+y)\right] - \sin a\sin(a+y)\left(0+\frac{dy}{dx}\right) = 0$$

$$\Rightarrow \quad x\cos(a+y)\frac{dy}{dx} + \sin(a+y) - \sin a\sin(a+y)\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx}\left[x\cos(a+y) - \sin a\sin(a+y)\right] = -\sin(a+y)$$

Put $x = -\sin a\dfrac{\cos(a+y)}{\sin(a+y)}$ from equation (i),

$$\Rightarrow \quad \frac{dy}{dx}\left[-\sin a\frac{\cos^2(a+y)}{\sin(a+y)} - \sin a\sin(a+y)\right] = -\sin(a+y)$$

$$\Rightarrow \quad -\frac{dy}{dx}\left[\frac{\sin a\cos^2(a+y) + \sin a\sin^2(a+y)}{\sin(a+y)}\right] = -\sin(a+y)$$

$$\Rightarrow \quad \frac{dy}{dx} = \sin(a+y)^2\left[\frac{\sin(a+y)}{\sin a\left(\cos^2(a+y) + \sin^2(a+y)\right)}\right]$$

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a} \qquad\qquad \left[\text{Since } \sin^2\theta + \cos^2\theta = 1\right]$$

Differentiation Ex 11.4 Q23

Here,
$$y = x\sin y$$

Differentiating with respect to x,

$$\Rightarrow \quad \frac{dy}{dx} = \frac{d}{dx}(x\sin y)$$

$$\Rightarrow \quad \frac{dy}{dx} = x\frac{d}{dx}(\sin y) + \sin y\frac{d}{dx}(x) \qquad\qquad \text{[Using product rule]}$$

$$\Rightarrow \quad \frac{dy}{dx} = x\cos y\frac{dy}{dx} + \sin y\,(1)$$

$$\Rightarrow \quad \frac{dy}{dx}(1 - x\cos y) = \sin y$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{\sin y}{1 - x\cos y}$$

Differentiation Ex 11.4 Q24

Here,

$$y\sqrt{x^2+1} = \log\left(\sqrt{x^2+1}-x\right)$$

Differentiating with respect to x,

$\Rightarrow \quad \dfrac{d}{dx}\left(y\sqrt{x^2+1}\right) = \dfrac{d}{dx}\log\left(\sqrt{x^2+1}-x\right)$ [Using product rule and chain rule]

$\Rightarrow \quad y\dfrac{d}{dx}\left(\sqrt{x^2+1}\right)+\sqrt{x^2+1}\dfrac{dy}{dx} = \dfrac{1}{\left(\sqrt{x^2+1}-x\right)}\times\dfrac{d}{dx}\left(\sqrt{x^2+1}-x\right)$

$\Rightarrow \quad y\dfrac{1}{2\sqrt{x^2+1}}\times\dfrac{d}{dx}\left(x^2+1\right)+\sqrt{x^2+1}\dfrac{dy}{dx} = \dfrac{1}{\left(\sqrt{x^2+1}-x\right)}\times\left[\dfrac{1}{2\sqrt{x^2+1}}\dfrac{d}{dx}\left(x^2+1\right)-1\right]$

$\Rightarrow \quad \dfrac{2xy}{2\sqrt{x^2+1}}+\sqrt{x^2+1}\dfrac{dy}{dx} = \dfrac{1}{\left(\sqrt{x^2+1}-x\right)}\left[\dfrac{2x}{2\sqrt{x^2+1}}-1\right]$

$\Rightarrow \quad \sqrt{x^2+1}\dfrac{dy}{dx} = \left[\dfrac{1}{\sqrt{x^2+1}-x}\right]\left[\dfrac{x-\sqrt{x^2+1}}{\sqrt{x^2+1}}\right]-\dfrac{xy}{\sqrt{x^2+1}}$

$\Rightarrow \quad \sqrt{x^2+1}\dfrac{dy}{dx} = \dfrac{-1}{\sqrt{x^2+1}}-\dfrac{xy}{\sqrt{x^2+1}}$

$\Rightarrow \quad \sqrt{x^2+1}\dfrac{dy}{dx} = \dfrac{-(1+xy)}{\sqrt{x^2+1}}$

$\Rightarrow \quad \left(x^2+1\right)\dfrac{dy}{dx} = -(1+xy)$

$\Rightarrow \quad \left(x^2+1\right)\dfrac{dy}{dx}+1+xy = 0$

Differentiation Ex 11.4 Q25

Here,

$$y = \left[\log_{\cos x}\sin x\right]\left[\log_{\sin x}\cos x\right]^{-1}+\sin^{-1}\left(\dfrac{2x}{1+x^2}\right)$$

$$y = \left[\log_{\cos x}\sin x\right]\left[\log_{\cos x}\sin x\right]+\sin^{-1}\left(\dfrac{2x}{1+x^2}\right)$$

$$\left[\text{Since, } \log_a a = \left(\log_b a\right)^{-1}\right]$$

$$y = \left[\dfrac{\log\sin x}{\log\cos x}\right]^2+\sin^{-1}\left(\dfrac{2x}{1+x^2}\right)$$

$$\left[\text{Sicne, } \log_a b = \dfrac{\log b}{\log a}\right]$$

Differentiating with respect to x,

$$\dfrac{dy}{dx} = \dfrac{d}{dx}\left[\dfrac{\log\sin x}{\log\cos x}\right]^2+\dfrac{d}{dx}\left(\sin^{-1}\left(\dfrac{2x}{1+x^2}\right)\right)$$

$$= 2\left[\dfrac{\log\sin x}{\log\cos x}\right]\dfrac{d}{dx}\left(\dfrac{\log\sin x}{\log\cos x}\right)+\dfrac{1}{\sqrt{1-\left(\dfrac{2x}{1+x^2}\right)^2}}\times\dfrac{d}{dx}\left[\dfrac{2x}{1+x^2}\right]$$

$$\dfrac{dy}{dx} = 2\left[\dfrac{\log\sin x}{\log\cos x}\right]\left(\dfrac{(\log\cos x)\dfrac{d}{dx}(\log\sin x)-\log\sin x\dfrac{d}{dx}(\log\cos x)}{(\log\cos x)^2}\right)+$$

[Using chain rule, quotient rule] $\left(\dfrac{\left(1+x^2\right)}{\sqrt{1+x^4-2x^2}}\right)\left(\dfrac{\left(1+x^2\right)(2)-(2x)(2x)}{\left(1+x^2\right)^2}\right)$

$$= 2\left(\dfrac{\log\sin x}{\log\cos x}\right)\left(\dfrac{\log\cos x\times\dfrac{1}{\sin x}\dfrac{d}{dx}(\sin x)-\log\sin x\times\dfrac{1}{\cos x}\dfrac{d}{dx}(\cos x)}{(\log\cos x)^2}\right)+$$

$$\left(\dfrac{\left(1+x^2\right)}{\sqrt{1+x^4-2x^2}}\right)\left(\dfrac{\left(1+x^2\right)(2)-(2x)(2x)}{\left(1+x^2\right)^2}\right)$$

$$= 2\left(\frac{\log\sin x}{\log\cos x}\right)\left(\frac{\log\cos x\left(\frac{\cos x}{\sin x}\right)+\log\sin x\left(\frac{\sin x}{\cos x}\right)}{(\log\cos x)^2}\right)+$$

$$\left(\frac{1+x^2}{\sqrt{(1-x^2)^2}}\right)\left(\frac{2+2x^2-4x^2}{(1+x^2)^2}\right)$$

$$\frac{dy}{dx}=2\frac{\log\sin x}{(\log\cos x)^3}(\cot x\log\cos x+\tan x\log\sin x)+\frac{2}{1+x^2}$$

Put $x=\frac{\pi}{4}$

$$\frac{dy}{dx}=2\left(\frac{\log\sin\frac{\pi}{4}}{\left(\log\cos\frac{\pi}{4}\right)^3}\right)\left(\cot\frac{\pi}{4}\log\cos\frac{\pi}{4}+\tan\frac{\pi}{4}\log\sin\frac{\pi}{4}\right)+2\left(\frac{1}{1+\left(\frac{\pi}{4}\right)^2}\right)$$

$$=2\left(\frac{1}{\left(\log\frac{1}{\sqrt{2}}\right)^2}\right)\left(1\times\log\frac{1}{\sqrt{2}}+1\times\log\frac{1}{\sqrt{2}}\right)+2\left(\frac{16}{16+\pi}\right)$$

$$=2\times\frac{2\log\left(\frac{1}{\sqrt{2}}\right)}{\left(\log\left(\frac{1}{\sqrt{2}}\right)\right)^2}+\frac{32}{16+\pi^2}$$

$$=4\frac{1}{\log\left(\frac{1}{\sqrt{2}}\right)}+\frac{32}{16+\pi^2}$$

$$=4\frac{1}{-\frac{1}{2}\log2}+\frac{32}{16+\pi^2}$$

$$=-\frac{8}{\log2}+\frac{32}{16+\pi^2}$$

$$\left(\frac{dy}{dx}\right)_{x=\frac{\pi}{4}}=8\left[\frac{4}{16+\pi^2}-\frac{1}{\log2}\right]$$

Differentiation Ex 11.4 Q26

Here,

$$\sin(xy)+\frac{y}{x}=x^2-y^2$$

Differentiating with respect to x,

$$\Rightarrow\quad\frac{d}{dx}(\sin xy)+\frac{d}{dx}\left(\frac{y}{x}\right)=\frac{d}{dx}(x^2)-\frac{d}{dx}(y^2)$$

$$\Rightarrow\quad\cos(xy)\frac{d}{dx}(xy)+\left[\frac{x\frac{dy}{dx}-y\frac{d}{dx}(x)}{x^2}\right]=2x-2y\frac{dy}{dx}\qquad\begin{bmatrix}\text{Using chain rule, quotient ruel,}\\\text{product rule}\end{bmatrix}$$

$$\Rightarrow\quad\cos(xy)\left[x\frac{dy}{dx}+y\frac{d}{dx}(x)\right]+\left[\frac{x\frac{dy}{dx}-y(1)}{x^2}\right]=2x-2y\frac{dy}{dx}$$

$$\Rightarrow\quad\cos(xy)\left[x\frac{dy}{dx}+y(1)\right]+\frac{1}{x^2}\left(x\frac{dy}{dx}-y\right)=2x-2y\frac{dy}{dx}$$

$$\Rightarrow\quad x\cos(xy)\frac{dy}{dx}+y\cos(xy)+\frac{1}{x}\frac{dy}{dx}-\frac{y}{x^2}=2x-2y\frac{dy}{dx}$$

$$\Rightarrow\quad\frac{dy}{dx}\left[x\cos(xy)+\frac{1}{x}+2y\right]=\frac{y}{x^2}-y\cos(xy)+2x$$

$$\Rightarrow\quad\frac{dy}{dx}\left[\frac{x^2\cos(xy)+1+2xy}{x}\right]=\frac{1}{x^2}\left(y-x^2y\cos xy+2x^3\right)$$

$$\Rightarrow\quad\frac{dy}{dx}=\frac{2x^3+y-x^2y\cos(xy)}{x\left(x^2\cos xy+1+2xy\right)}$$

Differentiation Ex 11.4 Q27

Here,

$$\sqrt{y+x} + \sqrt{y-x} = c$$

Differentiating with respect to x,

$$\Rightarrow \quad \frac{d}{dx}\left(\sqrt{y+x}\right) + \frac{d}{dx}\sqrt{y-x} = \frac{d}{dx}(c)$$

$$\Rightarrow \quad \frac{1}{2\sqrt{y+x}}\frac{d}{dx}(y+x) + \frac{1}{2\sqrt{y-x}}\frac{d}{dx}(y-x) = 0$$

Using chain rule

$$\Rightarrow \quad \frac{1}{2\sqrt{y+x}}\left[\frac{dy}{dx}+1\right] + \frac{1}{2\sqrt{y-x}}\left[\frac{dy}{dx}-1\right] = 0$$

$$\Rightarrow \quad \frac{dy}{dx}\left(\frac{1}{2\sqrt{y+x}}\right) + \frac{dy}{dx}\left(\frac{1}{2\sqrt{y-x}}\right) = \frac{1}{2\sqrt{y-x}} - \frac{1}{2\sqrt{y+x}}$$

$$\Rightarrow \quad \frac{dy}{dx}\times\frac{1}{2}\left[\frac{1}{\sqrt{y+x}}+\frac{1}{\sqrt{y-x}}\right] = \frac{1}{2}\left[\frac{\sqrt{y+x}-\sqrt{y-x}}{\sqrt{y-x}\sqrt{y+x}}\right]$$

$$\Rightarrow \quad \frac{dy}{dx}\left[\frac{\sqrt{y-x}+\sqrt{y+x}}{\sqrt{y+x}\sqrt{y-x}}\right] = \left[\frac{\sqrt{y+x}-\sqrt{y-x}}{\sqrt{y-x}\sqrt{y+x}}\right]$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{\sqrt{y+x}-\sqrt{y-x}}{\sqrt{y-x}+\sqrt{y+x}}\times\frac{\left(\sqrt{y+x}-\sqrt{y-x}\right)}{\left(\sqrt{y+x}-\sqrt{y-x}\right)}$$

[rationalizing the denominator]

$$\Rightarrow \quad \frac{dy}{dx} = \frac{(y+x)+(y-x)-2\sqrt{y+x}\sqrt{y-x}}{y+x-y+x}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{2y-2\sqrt{y^2-x^2}}{2x}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{2y}{2x} - \frac{2\sqrt{y^2-x^2}}{2x}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y}{x} - \sqrt{\frac{y^2-x^2}{x^2}}$$

$$\frac{dy}{dx} = \frac{y}{x} - \sqrt{\frac{y^2}{x^2}-1}$$

Differentiation Ex 11.4 Q28

Here,

$$\tan(x+y) + \tan(x-y) = 1$$

Differentiating with respect to x,

$$\Rightarrow \quad \frac{d}{dx}\tan(x+y) + \frac{d}{dx}\tan(x-y) = \frac{d}{dx}(1)$$

$$\Rightarrow \quad \sec^2(x+y)\frac{d}{dx}(x+y) + \sec^2(x-y)\frac{d}{dx}(x-y) = 0 \qquad\qquad \text{[Using chain rule]}$$

$$\Rightarrow \quad \sec^2(x+y)\left[1+\frac{dy}{dx}\right] + \sec^2(x-y)\left[1-\frac{dy}{dx}\right] = 0$$

$$\Rightarrow \quad \sec^2(x+y)\frac{dy}{dx} - \sec^2(x-y)\frac{dy}{dx} = -\left[\sec^2(x+y)+\sec^2(x-y)\right]$$

$$\Rightarrow \quad \frac{dy}{dx}\left[\sec^2(x+y)-\sec^2(x-y)\right] = -\left[\sec^2(x+y)+\sec^2(x-y)\right]$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{\sec^2(x+y)+\sec^2(x-y)}{\sec^2(x-y)-\sec^2(x+y)}$$

Differentiation Ex 11.4 Q29

Here,

$$e^x + e^y = e^{x+y}$$

Differentiating with respect to x using chain rule,

$$\Rightarrow \quad \frac{d}{dx}\left(e^x\right) + \frac{d}{dx}e^y = \frac{d}{dx}\left(e^{x+y}\right)$$

$$\Rightarrow \quad e^x + e^y \frac{dy}{dx} = e^{x+y}\frac{d}{dx}(x+y)$$

$$\Rightarrow \quad e^x + e^y \frac{dy}{dx} = e^{x+y}\left[1 + \frac{dy}{dx}\right]$$

$$\Rightarrow \quad e^y\frac{dy}{dx} - e^{x+y}\frac{dy}{dx} = e^{x+y} - e^x$$

$$\Rightarrow \quad \frac{dy}{dx}\left(e^y - e^{x+y}\right) = e^{x+y} - e^x$$

$$\Rightarrow \quad \frac{dy}{dx} = \left(\frac{e^x \times e^y - e^x}{e^y - e^x \times e^y}\right)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{e^x\left(e^y - 1\right)}{e^y\left(1 - e^x\right)}$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{e^x\left(e^y - 1\right)}{e^y\left(e^x - 1\right)}$$

Differentiation Ex 11.4 Q30

It is given that, $\cos y = x\cos(a+y)$

$$\therefore \frac{d}{dx}[\cos y] = \frac{d}{dx}\left[x\cos(a+y)\right]$$

$$\Rightarrow -\sin y\frac{dy}{dx} = \cos(a+y)\cdot\frac{d}{dx}(x) + x\cdot\frac{d}{dx}\left[\cos(a+y)\right]$$

$$\Rightarrow -\sin y\frac{dy}{dx} = \cos(a+y) + x\cdot\left[-\sin(a+y)\right]\frac{dy}{dx}$$

$$\Rightarrow \left[x\sin(a+y) - \sin y\right]\frac{dy}{dx} = \cos(a+y) \qquad \ldots(1)$$

Since $\cos y = x\cos(a+y)$, $x = \dfrac{\cos y}{\cos(a+y)}$

Then, equation (1) reduces to

$$\left[\frac{\cos y}{\cos(a+y)}\cdot\sin(a+y) - \sin y\right]\frac{dy}{dx} = \cos(a+y)$$

$$\Rightarrow \left[\cos y\cdot\sin(a+y) - \sin y\cdot\cos(a+y)\right]\cdot\frac{dy}{dx} = \cos^2(a+y)$$

$$\Rightarrow \sin(a+y-y)\frac{dy}{dx} = \cos^2(a+b)$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos^2(a+b)}{\sin a}$$

Hence, proved.

Ex 11.5

Differentiation Ex 11.5 Q1

Let $\quad y = x^{\frac{1}{x}}$ $\qquad\qquad$ ---(i)

Taking log on both the sides,

$\Rightarrow \quad \log y = \log x^{\frac{1}{x}}$

$\Rightarrow \quad \log y = \dfrac{1}{x}\log x$ $\qquad\qquad$ $\left[\text{Since, } \log a^b = b\log a\right]$

Differentiating with respect to x,

$\Rightarrow \quad \dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{1}{x}\dfrac{d}{dx}(\log x) + \log x \dfrac{d}{dx}\left(x^{-1}\right)$ $\qquad$ $\left[\text{Using product rule}\right]$

$\Rightarrow \quad \dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{1}{x}\times\dfrac{1}{x} + (\log x)\times\left(-\dfrac{1}{x^2}\right)$

$\Rightarrow \quad \dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{1}{x^2} - \dfrac{\log x}{x^2}$

$\Rightarrow \quad \dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{(1-\log x)}{x^2}$

$\Rightarrow \quad \dfrac{dy}{dx} = y\left[\dfrac{1-\log x}{x^2}\right]$

Put the value of y from equation (i),

$\dfrac{dy}{dx} = x^{\frac{1}{x}}\left[\dfrac{1-\log x}{x}\right]$

Differentiation Ex 11.5 Q2

Let $\quad y = x^{\sin x}$ $\qquad\qquad$ ---(i)

Taking log on both the sides,

$\log y = \log x^{\sin x}$

$\log y = \sin x \log x$ $\qquad\qquad$ $\left[\text{Since, } \log a^b = b\log a\right]$

Differentiating with respect to x,

$\dfrac{1}{y}\dfrac{dy}{dx} = \sin x \dfrac{d}{dx}\log x + \log x \dfrac{d}{dx}\sin x$ $\qquad$ $\left[\text{Using product rule}\right]$

$\dfrac{1}{y}\dfrac{dy}{dx} = \sin x\left(\dfrac{1}{x}\right) + \log x (\cos x)$

$\dfrac{dy}{dx} = y\left[\dfrac{\sin x}{x} + (\log x)(\cos x)\right]$

Put the value of y,

$\dfrac{dy}{dx} = x^{\sin x}\left[\dfrac{\sin x}{x} + (\log x)(\cos x)\right]$

Differentiation Ex 11.5 Q3

Let $y = (1 + \cos x)^x$ ---(i)

Taking log on both the sides,

$\log y = \log (1 + \cos x)^x$

$\log y = x \log (1 + \cos x)$

Differentiating with respect to x,

$\dfrac{1}{y}\dfrac{dy}{dx} = x\dfrac{d}{dx}\log(1+\cos x) + \log(1+\cos x)\dfrac{d}{dx}(x)$ [Using product rule and chain rule]

$\dfrac{1}{y}\dfrac{dy}{dx} = x\dfrac{1}{(1+\cos x)}\dfrac{d}{dx}(1+\cos x) + \log(1+\cos x)(1)$

$\dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{x}{(1+\cos x)}(0-\sin x) + \log(1+\cos x)$

$\dfrac{1}{y}\dfrac{dy}{dx} = \log(1+\cos x) - \dfrac{x\sin x}{(1+\cos x)}$

$\dfrac{dy}{dx} = y\left[\log(1+\cos x) - \dfrac{x\sin x}{1+\cos x}\right]$

$\dfrac{dy}{dx} = (1+\cos x)^x\left[\log(1+\cos x) - \dfrac{x\sin x}{(1+\cos x)}\right]$ [Using equation (i)]

Differentiation Ex 11.5 Q4

Let $y = x^{\cos^{-1}x}$ ---(i)

Taking log on both the sides,

$\log y = \log x^{\cos^{-1}x}$

$\log y = \cos^{-1}x \log x$ $\left[\text{Since, } \log a^b = b\log a\right]$

Differentiating it with respect to x using product rule,

$\dfrac{1}{y}\dfrac{dy}{dx} = \cos^{-1}x\dfrac{d}{dx}(\log x) + \log x\dfrac{d}{dx}\left(\cos^{-1}x\right)$

$\qquad = \cos^{-1}x\left(\dfrac{1}{x}\right) + \log x\left(\dfrac{-1}{\sqrt{1-x^2}}\right)$

$\dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{\cos^{-1}x}{x} - \dfrac{\log x}{\sqrt{1-x^2}}$

$\dfrac{dy}{dx} = y\left[\dfrac{\cos^{-1}x}{x} - \dfrac{\log x}{\sqrt{1-x^2}}\right]$

$\dfrac{dy}{dx} = x^{\cos^{-1}x}\left[\dfrac{\cos^{-1}x}{x} - \dfrac{\log x}{\sqrt{1-x^2}}\right]$ [Using equation (i)]

Differentiation Ex 11.5 Q5

Let $y = (\log x)^x$ ---(i)

Taking log on both the sides,

$\log y = \log(\log x)^x$

$\log y = x\log(\log x)$ $\left[\text{Since, } \log a^b = b\log a\right]$

Differentiating with respect to x, using product rule, chain rule,

$\dfrac{1}{y}\dfrac{dy}{dx} = x\dfrac{d}{dx}\log(\log x) + \log\log x\dfrac{d}{dx}(x)$

$\qquad = x\dfrac{1}{\log x}\dfrac{d}{dx}(\log x) + \log\log x\,(1)$

$\qquad = \dfrac{x}{\log x}\left(\dfrac{1}{x}\right) + \log\log x$

$\dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{1}{\log x} + \log\log x$

$\dfrac{dy}{dx} = y\left[\dfrac{1}{\log x} + \log\log x\right]$

$\dfrac{dy}{dx} = (\log x)^x\left[\dfrac{1}{\log x} + \log\log x\right]$ [Using equation (i)]

Differentiation Ex 11.5 Q6

Let $\quad y = (\log x)^{\cos x} \qquad \text{---(i)}$

Taking log on both the sides,

$\log y = \log(\log x)^{\cos x}$

$\log y = \cos x \log(\log x) \qquad\qquad \left[\text{Since, } \log a^b = b \log a\right]$

Differentiating with respect to x, using product rule, chain rule,

$\dfrac{1}{y}\dfrac{dy}{dx} = \cos x \dfrac{d}{dx}\log(\log x) + \log\log x \dfrac{d}{dx}(\cos x)$

$\qquad = \dfrac{\cos x}{\log x}\dfrac{d}{dx}(\log x) + \log\log x \times (-\sin x)$

$\dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{\cos x}{\log x} \times \left(\dfrac{1}{x}\right) - \sin x \log\log x$

$\dfrac{dy}{dx} = y\left[\dfrac{\cos x}{x \log x} - \sin x \log\log x\right]$

$\dfrac{dy}{dx} = (\log x)^{\cos x}\left[\dfrac{\cos x}{x \log x} - \sin x \log\log x\right] \qquad\qquad \left[\text{Using equation (i)}\right]$

Differentiation Ex 11.5 Q7

Let $\quad y = (\sin x)^{\cos x} \qquad \text{--- (i)}$

Taking log on both the sides,

$\log y = \log(\sin x)^{\cos x}$

$\log y = \cos x \log \sin x \qquad\qquad \left[\text{Since, } \log a^b = b \log a\right]$

Differentiating with respect to x, using product rule, chain rule,

$\dfrac{1}{y}\dfrac{dy}{dx} = \cos x \dfrac{d}{dx}\log \sin x + \log \sin x \dfrac{d}{dx}\cos x$

$\qquad = \cos x \dfrac{1}{\sin x}\dfrac{d}{dx}(\sin x) + \log \sin x (-\sin x)$

$\dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{\cos x}{\sin x}(\cos x) - \sin x \log \sin x$

$\dfrac{dy}{dx} = y\left[\cos x \cot x - \sin x \log \sin x\right]$

$\dfrac{dy}{dx} = (\sin x)^{\cos x}\left[\cos x \cot x - \sin x \log \sin x\right]$

Differentiation Ex 11.5 Q8

Let $\quad y = e^{x \log x}$

$\Rightarrow \quad y = e^{\log x^x} \qquad\qquad\qquad \left[\text{Since, } \log a^b = b \log a\right]$

$\Rightarrow \quad y = x^x \qquad\qquad \text{--- (i)} \qquad \left[\text{Since, } e^{\log a} = a\right]$

Taking log both the sides,

$\log y = \log x^x$

$\log y = x \log x$

Differentiating with respect to x, using product rule,

$\dfrac{1}{y}\dfrac{dy}{dx} = x \dfrac{d}{dx}(\log x) + \log x \dfrac{d}{dx}(x)$

$\qquad = x\left(\dfrac{1}{x}\right) + \log x (1)$

$\dfrac{1}{y}\dfrac{dy}{dx} = 1 + \log x$

$\dfrac{dy}{dx} = y\left[1 + \log x\right]$

$\dfrac{dy}{dx} = x^x (1 + \log x) \qquad\qquad \left[\text{Using equation (i)}\right]$

Differentiation Ex 11.5 Q9

Let $y = (\sin x)^{\log x}$ ---(i)

Taking log on both the sides,

$\log y = \log(\sin x)^{\log x}$

$\log y = \log x \log(\sin x)$ $\qquad$ $\left[\text{Using } \log a^b = b \log a\right]$

Differentiating with respect to x, using product rule and chain rule,

$\dfrac{1}{y}\dfrac{dy}{dx} = \log x \dfrac{d}{dx}(\log \sin x) + \log \sin x \dfrac{d}{dx}(\log x)$

$\qquad = \log x \left(\dfrac{1}{\sin x}\right)\dfrac{d}{dx}(\sin x) + \log \sin x \left(\dfrac{1}{x}\right)$

$\qquad = \dfrac{\log x}{\sin x} \times \cos x + \dfrac{\log \sin x}{x}$

$\dfrac{1}{y}\dfrac{dy}{dx} = \log x \cot x + \dfrac{\log \sin x}{x}$

$\dfrac{dy}{dx} = y\left[\log x \cot x + \dfrac{\log \sin x}{x}\right]$

$\dfrac{dy}{dx} = (\sin x)^{\log x}\left[\log x \cot x + \dfrac{\log \sin x}{x}\right]$ $\qquad$ $\left[\text{Using equation (i)}\right]$

Differentiation Ex 11.5 Q10

Let $y = 10^{\log \sin x}$ ---(i)

Taking log on both the sides,

$\log y = \log 10^{\log \sin x}$

$\log y = \log \sin x \log 10$ $\qquad$ $\left[\text{Since, } \log a^b = b \log a\right]$

Differentiating with respect to x, using chain rule,

$\dfrac{1}{y}\dfrac{dy}{dx} = \log 10 \dfrac{d}{dx}(\log \sin x)$

$\qquad = \log 10 \left(\dfrac{1}{\sin x}\right)\dfrac{d}{dx}(\sin x)$

$\dfrac{1}{y}\dfrac{dy}{dx} = \log 10 \left(\dfrac{1}{\sin x}\right)(\cos x)$

$\dfrac{dy}{dx} = y\left[\log 10 \cot x\right]$

$\dfrac{dy}{dx} = 10^{\log \sin x}\left[\log 10 \times \cot x\right]$ $\qquad$ $\left[\text{Using equation (i)}\right]$

Differentiation Ex 11.5 Q11

Let $y = (\log x)^{\log x}$

Taking logarithm on both the sides, we obtain

$\log y = \log x \cdot \log(\log x)$

Differentiating both sides with respect to x, we obtain

$\dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{d}{dx}\left[\log x \cdot \log(\log x)\right]$

$\Rightarrow \dfrac{1}{y}\dfrac{dy}{dx} = \log(\log x).\dfrac{d}{dx}(\log x) + \log x \cdot \dfrac{d}{dx}\left[\log(\log x)\right]$

$\Rightarrow \dfrac{dy}{dx} = y\left[\log(\log x).\dfrac{1}{x} + \log x \cdot \dfrac{1}{\log x}\cdot\dfrac{d}{dx}(\log x)\right]$

$\Rightarrow \dfrac{dy}{dx} = y\left[\dfrac{1}{x}\log(\log x) + \dfrac{1}{x}\right]$

$\therefore \dfrac{dy}{dx} = (\log x)^{\log x}\left[\dfrac{1}{x} + \dfrac{\log(\log x)}{x}\right]$

Differentiation Ex 11.5 Q12

Let $y = 10^{(10^x)}$ ---(i)

Taking log on both the siedes,

$$\log y = \log 10^{(10^x)}$$
$$\log y = 10^x \log 10$$

Differentiating it with respect to x,

$$\frac{1}{y}\frac{dy}{dx} = \log 10 \times 10^x \log 10$$
$$\frac{1}{y}\frac{dy}{dx} = 10^x \times (\log 10)^2$$
$$\frac{dy}{dx} = 10^{(10^x)} \times 10^x (\log 10)^2$$ [Using equation (i)]

Differentiation Ex 11.5 Q13

Let $y = \sin x^x$
$\Rightarrow$ $\sin^{-1} y = x^x$

Taking log on both the siedes,

$$\log\left(\sin^{-1} y\right) = \log x^x$$
$$\log\left(\sin^{-1} y\right) = x \log x$$ $\left[\text{Since, } \log a^b = b \log a\right]$

Differentiating it with respect to x using chain rule and product rule,

$$\frac{1}{\sin^{-1} y}\frac{dy}{dx} = \left(\sin^{-1} y\right) = x\frac{d}{dx}(\log x) + \log x \frac{d}{dx}(x)$$
$$\frac{1}{\sin^1 y} \times \left(\frac{1}{\sqrt{1-y^2}}\right)\frac{dy}{dx} = x\left(\frac{1}{x}\right) + \log x\,(1)$$
$$\frac{dy}{dx} = \sin^{-1} y \sqrt{1-y^2}\,(1+\log x)$$
$$= \sin^{-1}\left(\sin x^x\right)\sqrt{1 - \left(\sin x^x\right)^2}\,(1+\log x)$$
$$= x^x \sqrt{\cos^2 x^x}\,(1+\log x)$$ [Using equation (i)]

$$\frac{dy}{dx} = x^x \cos x^x\,(1+\log x)$$

Differentiation Ex 11.5 Q14

Let $y = \left(\sin^{-1} x\right)^x$

Taking log on both the sides,

$$\log y = \log\left(\sin^{-1} x\right)^x$$
$$\log y = x \log\left(\sin^{-1} x\right)$$ $\left[\text{Since, } \log a^b = b \log a\right]$

Differentiating it with respect to x using product rule and chain rule,

$$\frac{1}{y}\frac{dy}{dx} = x\frac{d}{dx}\left(\log \sin^{-1} x\right) + \log \sin^{-1} x \frac{d}{dx}(x)$$
$$= x \times \frac{1}{\sin^{-1} x}\frac{d}{dx}\left(\sin^{-1} x\right) + \log \sin^{-1} x\,(1)$$
$$\frac{1}{y}\frac{dy}{dx} = \frac{x}{\sin^{-1} x}\left(\frac{1}{\sqrt{1-x^2}}\right) + \log \sin^{-1} x$$
$$\frac{dy}{dx} = y\left[\log \sin^{-1} x + \frac{x}{\sin^{-1} x\left(\sqrt{1-x^2}\right)}\right]$$
$$\frac{dy}{dx} = \left(\sin^{-1} x\right)^x\left[\log \sin^{-1} x + \frac{x}{\sin^{-1} x\sqrt{1-x^2}}\right]$$ [Using equation (i)]

Differentiation Ex 11.5 Q15

Let $\quad y = x^{\sin^{-1} x}$ $\qquad\qquad$ ---(i)

Taking log on both the sides,

$\quad \log y = \log x^{\sin^{-1} x}$

$\quad \log y = \sin^{-1} x \log x$ $\qquad\qquad\qquad$ $\left[\text{Since, } \log a^b = b \log a \right]$

Differentiating it with respect to x using product rule,

$\quad \dfrac{1}{y}\dfrac{dy}{dx} = \sin^{-1} x \dfrac{d}{dx}(\log x) + (\log x)\dfrac{d}{dx}\left(\sin^{-1} x\right)$

$\quad \dfrac{1}{y}\dfrac{dy}{dx} = \sin^{-1} x \left(\dfrac{1}{x}\right) + (\log x)\left(\dfrac{1}{\sqrt{1-x^2}}\right)$

$\quad \dfrac{dy}{dx} = y\left[\dfrac{\sin^{-1} x}{x} + \dfrac{\log x}{\sqrt{1-x^2}}\right]$

$\quad \dfrac{dy}{dx} = x^{\sin^{-1} x}\left[\dfrac{\sin^{-1} x}{x} + \dfrac{\log x}{\sqrt{1-x^2}}\right]$ $\qquad\qquad$ [Using equation (i)]

Differentiation Ex 11.5 Q16

Let $\quad y = (\tan x)^{\frac{1}{x}}$ $\qquad\qquad$ ---(i)

Taking log on both the sides,

$\quad \log y = \log (\tan x)^{\frac{1}{x}}$

$\quad \log y = \dfrac{1}{x}\log (\tan x)$ $\qquad\qquad\qquad$ $\left[\text{Since, } \log a^b = b \log a \right]$

Differentiating it with respect to x using product rule and chain rule,

$\quad \dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{1}{x}\dfrac{d}{dx}\log (\tan x) + \log (\tan x)\dfrac{d}{dx}\left(\dfrac{1}{x}\right)$

$\qquad\qquad = \dfrac{1}{x} \times \dfrac{1}{\tan x}\dfrac{d}{dx}(\tan x) + \log (\tan x)\left(-\dfrac{1}{x^2}\right)$

$\quad \dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{1}{x \tan x}\left(\sec^2 x\right) - \dfrac{\log (\tan x)}{x^2}$

$\quad \dfrac{dy}{dx} = y\left[\dfrac{\sec^2 x}{x \tan x} - \dfrac{\log (\tan x)}{x^2}\right]$

$\quad \dfrac{dy}{dx} = (\tan x)^{\frac{1}{x}}\left[\dfrac{\sec^2 x}{x \tan x} - \dfrac{\log \tan x}{x^2}\right]$ $\qquad\qquad$ [Using equation (i)]

Differentiation Ex 11.5 Q17

Let $y = x^{\tan^{-1}x}$ ---(i)

Taking log on both the sides,

$\log y = \log x^{\tan^{-1}x}$

$\log y = \tan^{-1}x \log x$ $\left[\text{Since, } \log a^{b} = b \log a\right]$

Differentiating it with respect to x using product rule,

$\dfrac{1}{y}\dfrac{dy}{dx} = \tan^{-1}x \dfrac{d}{dx}(\log x) + \log x \dfrac{d}{dx}(\tan^{-1}x)$

$\dfrac{1}{y}\dfrac{dy}{dx} = \tan^{-1}x \left(\dfrac{1}{x}\right) + \log x \left(\dfrac{1}{1+x^2}\right)$

$\dfrac{dy}{dx} = y\left[\dfrac{\tan^{-1}x}{x} + \dfrac{\log x}{1+x^2}\right]$

$\dfrac{dy}{dx} = x^{\tan^{-1}x}\left[\dfrac{\tan^{-1}x}{x} + \dfrac{\log x}{1+x^2}\right]$ [Using equation (i)]

Differentiation Ex 11.5 Q18(i)

Let $y = x^x \sqrt{x}$ ---(i)

Taking log on both the sides,

$\log y = \log\left(x^x \sqrt{x}\right)$

$= \log x^x + \log x^{\frac{1}{2}}$ $\left[\text{Since, } \log^{(ab)} = \log a + \log b\right]$

$\log y = x\log x + \dfrac{1}{2}\log x$ $\left[\text{Since, } \log a^{b} = b\log a\right]$

Differentiating it with respect to x using product rule,

$\dfrac{1}{y}\dfrac{dy}{dx} = x\dfrac{d}{dx}(\log x) + \log x\dfrac{d}{dx}(x) + \dfrac{1}{2}\dfrac{d}{dx}(\log x)$

$= x\left(\dfrac{1}{x}\right) + \log x\,(1) + \dfrac{1}{2}\left(\dfrac{1}{x}\right)$

$\dfrac{1}{y}\dfrac{dy}{dx} = 1 + \log x + \dfrac{1}{2x}$

$\dfrac{dy}{dx} = y\left(1 + \log x + \dfrac{1}{2x}\right)$

$\dfrac{dy}{dx} = x^x \sqrt{x}\left(1 + \log x + \dfrac{1}{2x}\right)$ [Using equation (i)]

Differentiation Ex 11.5 Q18(ii)

Let $y = x^{(\sin x - \cos x)} + \left(\dfrac{x^2 - 1}{x^2 + 1}\right)$

$y = e^{\log x^{(\sin x - \cos x)}} + \left(\dfrac{x^2 - 1}{x^2 + 1}\right)$

$y = e^{(\sin x - \cos x)\log x} + \left(\dfrac{x^2 - 1}{x^2 + 1}\right)$ $\qquad$ $\left[\text{Since, } e^{\log a} = a, \ \log a^b = b\log a\right]$

Differentiating it with respect to x using chain rule and quotient rule,

$$\frac{dy}{dx} = \frac{d}{dx}\left[e^{(\sin x - \cos x)\log x}\right] + \frac{d}{dx}\left[\frac{x^2 - 1}{x^2 + 1}\right]$$

$$= e^{(\sin x - \cos x)\log x}\frac{d}{dx}\{(\sin x - \cos x)\log x\} + \left[\frac{(x^2 + 1)\frac{d}{dx}(x^2 - 1) - (x^2 - 1)\frac{d}{dx}(x^2 + 1)}{(x^2 + 1)^2}\right]$$

$$= e^{\log x^{(\sin x - \cos x)}}\left[(\sin x - \cos x)\frac{d}{dx}(\log x) + (\log x)\frac{d}{dx}(\sin x - \cos x)\right] + \left[\frac{(x^2 + 1)(2x) - (x^2 - 1)(2x)}{(x^2 + 1)^2}\right]$$

$$= x^{(\sin x - \cos x)}\left[(\sin x - \cos x)\left(\frac{1}{x}\right) + \log x (\sin x + \cos x)\right] + \left[\frac{2x^3 + 2x - 2x^3 + 2x}{(x^2 + 1)^2}\right]$$

$$\frac{dy}{dx} = x^{(\sin x - \cos x)}\left[\frac{(\sin x - \cos x)}{x} + \log x (\sin x + \cos x)\right] + \frac{4x}{(x^2 + 1)^2}$$

Differentiation Ex 11.5 Q18(iii)

Let $y = x^{x\cos x} + \dfrac{x^2 + 1}{x^2 - 1}$

Also, let $u = x^{x\cos x}$ and $v = \dfrac{x^2 + 1}{x^2 - 1}$

$\therefore y = u + v$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{du}{dx} + \dfrac{dv}{dx}$ $\qquad\qquad$ ...(1)

$u = x^{x\cos x}$

$\Rightarrow \log u = \log\left(x^{x\cos x}\right)$

$\Rightarrow \log u = x\cos x \log x$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{u}\frac{du}{dx} = \frac{d}{dx}(x)\cdot\cos x\cdot\log x + x\cdot\frac{d}{dx}(\cos x)\cdot\log x + x\cos x\cdot\frac{d}{dx}(\log x)$$

$$\Rightarrow \frac{du}{dx} = u\left[1\cdot\cos x\cdot\log x + x\cdot(-\sin x)\log x + x\cos x\cdot\frac{1}{x}\right]$$

$$\Rightarrow \frac{du}{dx} = x^{x\cos x}(\cos x\log x - x\sin x\log x + \cos x)$$

$$\Rightarrow \frac{du}{dx} = x^{x\cos x}\left[\cos x(1 + \log x) - x\sin x\log x\right] \qquad\qquad ...(2)$$

$v = \dfrac{x^2 + 1}{x^2 - 1}$

$\Rightarrow \log v = \log\left(x^2 + 1\right) - \log\left(x^2 - 1\right)$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{v}\frac{dv}{dx} = \frac{2x}{x^2+1} - \frac{2x}{x^2-1}$$

$$\Rightarrow \frac{dv}{dx} = v\left[\frac{2x(x^2-1)-2x(x^2+1)}{(x^2+1)(x^2-1)}\right]$$

$$\Rightarrow \frac{dv}{dx} = \frac{x^2+1}{x^2-1} \times \left[\frac{-4x}{(x^2+1)(x^2-1)}\right]$$

$$\Rightarrow \frac{dv}{dx} = \frac{-4x}{(x^2-1)^2} \qquad \qquad \ldots(3)$$

From (1), (2), and (3), we obtain

$$\frac{dy}{dx} = x^{x\cos x}\left[\cos x(1+\log x)-x\sin x\log x\right]-\frac{4x}{(x^2-1)^2}$$

Differentiation Ex 11.5 Q18(iv)

Let $y = (x\cos x)^x + (x\sin x)^{\frac{1}{x}}$

Also, let $u = (x\cos x)^x$ and $v = (x\sin x)^{\frac{1}{x}}$

$\therefore y = u + v$

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \qquad \qquad \ldots(1)$$

$u = (x\cos x)^x$

$\Rightarrow \log u = \log(x\cos x)^x$

$\Rightarrow \log u = x\log(x\cos x)$

$\Rightarrow \log u = x\left[\log x + \log\cos x\right]$

$\Rightarrow \log u = x\log x + x\log\cos x$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{u}\frac{du}{dx} = \frac{d}{dx}(x\log x) + \frac{d}{dx}(x\log\cos x)$$

$$\Rightarrow \frac{du}{dx} = u\left[\left\{\log x \cdot \frac{d}{dx}(x)+x\cdot\frac{d}{dx}(\log x)\right\}+\left\{\log\cos x\cdot\frac{d}{dx}(x)+x\cdot\frac{d}{dx}(\log\cos x)\right\}\right]$$

$$\Rightarrow \frac{du}{dx} = (x\cos x)^x\left[\left(\log x\cdot 1+x\cdot\frac{1}{x}\right)+\left\{\log\cos x\cdot 1+x\cdot\frac{1}{\cos x}\cdot\frac{d}{dx}(\cos x)\right\}\right]$$

$$\Rightarrow \frac{du}{dx} = (x\cos x)^x\left[(\log x+1)+\left\{\log\cos x+\frac{x}{\cos x}\cdot(-\sin x)\right\}\right]$$

$$\Rightarrow \frac{du}{dx} = (x\cos x)^x\left[(1+\log x)+(\log\cos x-x\tan x)\right]$$

$$\Rightarrow \frac{du}{dx} = (x\cos x)^x\left[1-x\tan x+(\log x+\log\cos x)\right]$$

$$\Rightarrow \frac{du}{dx} = (x\cos x)^x\left[1-x\tan x+\log(x\cos x)\right] \qquad \qquad \ldots(2)$$

$$v = (x\sin x)^{\frac{1}{x}}$$

$$\Rightarrow \log v = \log (x\sin x)^{\frac{1}{x}}$$

$$\Rightarrow \log v = \frac{1}{x}\log (x\sin x)$$

$$\Rightarrow \log v = \frac{1}{x}(\log x + \log \sin x)$$

$$\Rightarrow \log v = \frac{1}{x}\log x + \frac{1}{x}\log \sin x$$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{v}\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{x}\log x\right) + \frac{d}{dx}\left[\frac{1}{x}\log (\sin x)\right]$$

$$\Rightarrow \frac{1}{v}\frac{dv}{dx} = \left[\log x \cdot \frac{d}{dx}\left(\frac{1}{x}\right) + \frac{1}{x}\cdot\frac{d}{dx}(\log x)\right] + \left[\log (\sin x)\cdot \frac{d}{dx}\left(\frac{1}{x}\right) + \frac{1}{x}\cdot\frac{d}{dx}\{\log (\sin x)\}\right]$$

$$\Rightarrow \frac{1}{v}\frac{dv}{dx} = \left[\log x \cdot \left(-\frac{1}{x^2}\right) + \frac{1}{x}\cdot\frac{1}{x}\right] + \left[\log (\sin x)\cdot \left(-\frac{1}{x^2}\right) + \frac{1}{x}\cdot\frac{1}{\sin x}\cdot\frac{d}{dx}(\sin x)\right]$$

$$\Rightarrow \frac{1}{v}\frac{dv}{dx} = \frac{1}{x^2}(1-\log x) + \left[-\frac{\log (\sin x)}{x^2} + \frac{1}{x\sin x}\cdot \cos x\right]$$

$$\Rightarrow \frac{dv}{dx} = (x\sin x)^{\frac{1}{x}}\left[\frac{1-\log x}{x^2} + \frac{-\log (\sin x) + x\cot x}{x^2}\right]$$

$$\Rightarrow \frac{dv}{dx} = (x\sin x)^{\frac{1}{x}}\left[\frac{1-\log x - \log (\sin x) + x\cot x}{x^2}\right]$$

$$\Rightarrow \frac{dv}{dx} = (x\sin x)^{\frac{1}{x}}\left[\frac{1-\log (x\sin x) + x\cot x}{x^2}\right] \qquad \ldots(3)$$

From (1), (2), and (3), we obtain

$$\frac{dy}{dx} = (x\cos x)^x\left[1 - x\tan x + \log (x\cos x)\right] + (x\sin x)^{\frac{1}{x}}\left[\frac{x\cot x + 1 - \log (x\sin x)}{x^2}\right]$$

Differentiation Ex 11.5 Q18(v)

Let $y = \left(x + \dfrac{1}{x}\right)^x + x^{\left(1+\frac{1}{x}\right)}$

Also, let $u = \left(x + \dfrac{1}{x}\right)^x$ and $v = x^{\left(1+\frac{1}{x}\right)}$

$\therefore y = u + v$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{du}{dx} + \dfrac{dv}{dx}$ ...(1)

Then, $u = \left(x + \dfrac{1}{x}\right)^x$

$\Rightarrow \log u = \log\left(x + \dfrac{1}{x}\right)^x$

$\Rightarrow \log u = x\log\left(x + \dfrac{1}{x}\right)$

Differentiating both sides with respect to x, we obtain

$\dfrac{1}{u} \cdot \dfrac{du}{dx} = \dfrac{d}{dx}(x) \times \log\left(x + \dfrac{1}{x}\right) + x \times \dfrac{d}{dx}\left[\log\left(x + \dfrac{1}{x}\right)\right]$

$\Rightarrow \dfrac{1}{u}\dfrac{du}{dx} = 1 \times \log\left(x + \dfrac{1}{x}\right) + x \times \dfrac{1}{\left(x + \dfrac{1}{x}\right)} \cdot \dfrac{d}{dx}\left(x + \dfrac{1}{x}\right)$

$\Rightarrow \dfrac{du}{dx} = u\left[\log\left(x + \dfrac{1}{x}\right) + \dfrac{x}{\left(x + \dfrac{1}{x}\right)} \times \left(1 - \dfrac{1}{x^2}\right)\right]$

$\Rightarrow \dfrac{du}{dx} = \left(x + \dfrac{1}{x}\right)^x\left[\log\left(x + \dfrac{1}{x}\right) + \dfrac{\left(x - \dfrac{1}{x}\right)}{\left(x + \dfrac{1}{x}\right)}\right]$

$\Rightarrow \dfrac{du}{dx} = \left(x + \dfrac{1}{x}\right)^x\left[\log\left(x + \dfrac{1}{x}\right) + \dfrac{x^2 - 1}{x^2 + 1}\right]$

$\Rightarrow \dfrac{du}{dx} = \left(x + \dfrac{1}{x}\right)^x\left[\dfrac{x^2 - 1}{x^2 + 1} + \log\left(x + \dfrac{1}{x}\right)\right]$

$$v = x^{\left(1+\frac{1}{x}\right)}$$

$$\Rightarrow \log v = \log\left[x^{\left(1+\frac{1}{x}\right)}\right]$$

$$\Rightarrow \log v = \left(1+\frac{1}{x}\right)\log x$$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{v}\cdot\frac{dv}{dx} = \left[\frac{d}{dx}\left(1+\frac{1}{x}\right)\right]\times\log x + \left(1+\frac{1}{x}\right)\cdot\frac{d}{dx}\log x$$

$$\Rightarrow \frac{1}{v}\frac{dv}{dx} = \left(-\frac{1}{x^2}\right)\log x + \left(1+\frac{1}{x}\right)\cdot\frac{1}{x}$$

$$\Rightarrow \frac{1}{v}\frac{dv}{dx} = -\frac{\log x}{x^2} + \frac{1}{x} + \frac{1}{x^2}$$

$$\Rightarrow \frac{dv}{dx} = v\left[\frac{-\log x + x + 1}{x^2}\right]$$

$$\Rightarrow \frac{dv}{dx} = x^{\left(1+\frac{1}{x}\right)}\left(\frac{x+1-\log x}{x^2}\right) \qquad \text{...(3)}$$

Therefore, from (1), (2), and (3), we obtain

$$\frac{dy}{dx} = \left(x+\frac{1}{x}\right)^x\left[\frac{x^2-1}{x^2+1}+\log\left(x+\frac{1}{x}\right)\right] + x^{\left(1+\frac{1}{x}\right)}\left(\frac{x+1-\log x}{x^2}\right)$$

Differentiation Ex 11.5 Q18(vi)

Let $y = e^{\sin x} + (\tan x)^x$

$y = e^{\sin x} + e^{\log(\tan x)^x}$

$y = e^{\sin x} + e^{x\log(\tan x)}$ $\qquad \left[\text{Since, } \log a^b = b\log a, e^{\log a} = a\right]$

Differentiating it with respect to x using chain rule and product rule,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{\sin x}\right) + \frac{d}{dx}\left(e^{x\log(\tan x)}\right)$$

$$= e^{\sin x}\frac{d}{dx}(\sin x) + e^{x\log(\tan x)}\times\frac{d}{dx}(x\log\tan x)$$

$$= e^{\sin x}(\cos x) + e^{\log(\tan x)^x}\left[x\frac{d}{dx}\log\tan x + \log\tan x\frac{d}{dx}(x)\right]$$

$$= e^{\sin x}(\cos x) + (\tan x)^x\left[\frac{x}{\tan x}\frac{d}{dx}(\tan x) + \log\tan x(1)\right]$$

$$\frac{dy}{dx} = \cos x\, e^{\sin x} + (\tan x)^x\left[\frac{x}{\tan x}(\sec^2 x) + \log\tan x\right]$$

Differentiation Ex 11.5 Q18(vii)

Let $y = (\cos x)^{x} + (\sin x)^{\frac{1}{x}}$

$y = e^{\log(\cos x)^{x}} + e^{\log(\sin x)^{\frac{1}{x}}}$ $\left[\text{Since, } \log a^{b} = b \log a, e^{\log a} = a\right]$

$y = e^{x \log(\cos x)} + e^{\frac{1}{x} \log \sin x}$

Differentiating it with respect to x using chain rule and product rule,

$$\frac{dy}{dx} = \frac{d}{dx} e^{x \log \cos x} + \frac{d}{dx} e^{\frac{1}{x} \log \sin x}$$

$$= e^{x \log \cos x} \times \frac{d}{dx}(x \log x) + e^{\frac{1}{x} \log \sin x} \frac{d}{dx}\left(\frac{1}{x} \log \sin x\right)$$

$$= e^{\log(\cos x)} \times \left[x \frac{d}{dx} \log \cos x + \log \cos x \times \frac{d}{dx}(x)\right] + e^{\log \sin x^{\frac{1}{x}}} \times \left[\frac{1}{x} \frac{d}{dx} \log \sin x + \log \sin x \frac{d}{dx}\left(\frac{1}{x}\right)\right]$$

$$= (\cos x)^{x} \left[x \times \left(\frac{1}{\cos x}\right)\frac{d}{dx} \cos x + \log \cos x (1)\right] + (\sin)^{\frac{1}{x}} \left[\frac{1}{x} \times \frac{1}{\sin x} \times \frac{d}{dx}(\sin x) + \log \sin x \left(-\frac{1}{x^2}\right)\right]$$

$$= (\cos x)^{x} \left[x \left(\frac{1}{\cos x}\right)(-\sin x) + \log \cos x\right] + (\sin x)^{\frac{1}{x}} \left[\frac{1}{x} \times \frac{1}{\sin x}(\cos x) - \frac{1}{x^2} \log \sin x\right]$$

$$\frac{dy}{dx} = (\cos x)^{x} \left[\log \cos x - x \tan x\right] + (\sin x)^{\frac{1}{x}} \left[\frac{\cot x}{x} - \frac{1}{x^2} \log \sin x\right]$$

Differentiation Ex 11.5 Q18(viii)

Let $y = x^{x^2 - 3} + (x - 3)^{x^2}$

Also, let $u = x^{x^2 - 3}$ and $v = (x - 3)^{x^2}$

$\therefore y = u + v$

Differentiating both sides with respect to x, we obtain

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \qquad \ldots(1)$$

$u = x^{x^2 - 3}$

$\therefore \log u = \log\left(x^{x^2 - 3}\right)$

$\log u = \left(x^2 - 3\right) \log x$

Differentiating with respect to x, we obtain

$$\frac{1}{u} \cdot \frac{du}{dx} = \log x \cdot \frac{d}{dx}\left(x^2 - 3\right) + \left(x^2 - 3\right) \cdot \frac{d}{dx}(\log x)$$

$$\Rightarrow \frac{1}{u} \frac{du}{dx} = \log x \cdot 2x + \left(x^2 - 3\right) \cdot \frac{1}{x}$$

$$\Rightarrow \frac{du}{dx} = x^{x^2 - 3} \cdot \left[\frac{x^2 - 3}{x} + 2x \log x\right]$$

Also,

$v = (x - 3)^{x^2}$

$\therefore \log v = \log\left(x - 3\right)^{x^2}$

$\Rightarrow \log v = x^2 \log(x - 3)$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{v}\cdot\frac{dv}{dx} = \log(x-3)\cdot\frac{d}{dx}(x^2) + x^2\cdot\frac{d}{dx}\left[\log(x-3)\right]$$

$$\Rightarrow \frac{1}{v}\frac{dv}{dx} = \log(x-3)\cdot 2x + x^2\cdot\frac{1}{x-3}\cdot\frac{d}{dx}(x-3)$$

$$\Rightarrow \frac{dv}{dx} = v\left[2x\log(x-3) + \frac{x^2}{x-3}\cdot 1\right]$$

$$\Rightarrow \frac{dv}{dx} = (x-3)^{x^2}\left[\frac{x^2}{x-3} + 2x\log(x-3)\right]$$

Substituting the expressions of $\dfrac{du}{dx}$ and $\dfrac{dv}{dx}$ in equation (1), we obtain

$$\frac{dy}{dx} = x^{x^2-3}\left[\frac{x^2-3}{x} + 2x\log x\right] + (x-3)^{x^2}\left[\frac{x^2}{x-3} + 2x\log(x-3)\right]$$

Differentiation Ex 11.5 Q19

Here,
$$y = e^x + 10^x + x^x$$
$$= e^x + 10^x + e^{\log x^x} \qquad \left[\text{Since, } e^{\log_e a} = a, \log a^b = b\log a\right]$$
$$y = e^x + 10^x + e^{x\log x}$$

Differentiating it with respect to x using product rule, chain rule,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^x\right) + \frac{d}{dx}\left(10^x\right) + \frac{d}{dx}\left(e^{x\log x}\right)$$

$$= e^x + 10^x\log 10 + e^{x\log x}\frac{d}{dx}(x\log x)$$

$$= e^x + 10^x\log 10 + e^{x\log x}\left[x\times\frac{d}{dx}(\log x) + \log x\frac{d}{dx}(x)\right]$$

$$= e^x + 10^x\log 10 + e^{\log x^x}\left[x\left(\frac{1}{x}\right) + \log x\,(1)\right]$$

$$= e^x + 10^x\log 10 + x^x\left[1 + \log x\right]$$

$$= e^x + 10^x\log 10 + x^x\left[\log e + \log x\right] \qquad \left[\text{Since, } \log_e e = 1\right]$$

$$\frac{dy}{dx} = e^x + 10^x\log 10 + x^x\left(\log ex\right) \qquad \left[\text{Sicne } \log A + \log B = \log AB\right]$$

Differentiation Ex 11.5 Q20

Here,
$$y = x^n + n^x + x^x + n^n$$
$$y = x^n + n^n + e^{\log x^x} + n^n \qquad \left[\text{Since, } e^{\log_e a} = a \text{ and } \log a^b = b\log a\right]$$
$$y = x^n + n^x + e^{x\log x} + n^n$$

Differentiating it with respect to x using chain rule and product rule,

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^n\right) + \frac{d}{dx}\left(n^x\right) + \frac{d}{dx}\left(e^{x\log x}\right) + \frac{d}{dx}\left(n^n\right)$$

$$= nx^{n-1} + n^x\log n + e^{\log x^x}\left[d\,\frac{d}{dx}\log x + \log x\frac{d}{dx}(1)\right]$$

$$= nx^{n-1} + n^x\log n + x^x\left[x\left(\frac{1}{x}\right) + \log x\right]$$

$$= nx^{n-1} + n^x\log n + x^x\left[1 + \log x\right]$$

$$= nx^{n-1} + n^x\log n + x^x\left[\log e + \log x\right] \qquad \left[\text{Since, } \log_e e = 1 \text{ and } \log A + \log B = \log(AB)\right]$$

$$\frac{dy}{dx} = nx^{n-1} + n^x\log n + x^x\log(ex)$$

Differentiation Ex 11.5 Q21

Here,

$$y = \frac{(x^2 - 1)^3 (2x - 1)}{\sqrt{(x - 3)(4x - 1)}} \qquad \text{---(i)}$$

$$y = \frac{(x^2 - 1)^3 (2x - 1)}{(x - 3)^{\frac{1}{2}} (4x -)^{\frac{1}{2}}}$$

Taking log on both the sides,

$$\log y = \log \left[\frac{(x^2 - 1)^3 (2x - 1)}{(x - 3)^{\frac{1}{2}} (4x - 1)^{\frac{1}{2}}} \right]$$

$$= \log(x^2 - 1)^3 + \log(2x - 1) - \log(x - 3)^{\frac{1}{2}} - \log(4x - 1)^{\frac{1}{2}}$$

$$\left[\text{Since, } \log(AB) = \log A + \log B, \log\left(\frac{A}{B}\right) = \log A - \log B \right]$$

$$= 3\log(x^2 - 1) + \log(2x - 1) - \frac{1}{2}\log(x - 3) - \frac{1}{2}\log(4x - 1)$$

Differentiating it with respect to x using chain rule,

$$\frac{1}{y}\frac{dy}{dx} = 3\frac{d}{dx}\log(x^2 - 1) + \frac{d}{dx}\log(2x - 1) - \frac{1}{2}\frac{d}{dx}\log(x - 3) - \frac{1}{2}\log(4x - 1)$$

$$= 3\left(\frac{1}{x^2 - 1}\right)\frac{d}{dx}(x^2 - 1) + \frac{1}{(2x - 1)}\frac{d}{dx}(2x - 1) - \frac{1}{2}\left(\frac{1}{x - 3}\right)\frac{d}{dx}(x - 3) - \frac{1}{2}\frac{1}{(4x - 1)}\frac{d}{dx}(4x - 1)$$

$$= 3\left(\frac{1}{x^2 - 1}\right)(2x) + \frac{1}{(2x - 1)}(2) - \frac{1}{2}\left(\frac{1}{x - 3}\right)(1) - \frac{1}{2}\left(\frac{1}{4x - 1}\right)(4)$$

$$\frac{1}{y}\frac{dy}{dx} = \left[\frac{6x}{x^2 - 1} + \frac{2}{2x - 1} - \frac{1}{2(x - 3)} - \frac{2}{4x - 1} \right]$$

$$\frac{dy}{dx} = y\left[\frac{6x}{x^2 - 1} + \frac{2}{2x - 1} - \frac{1}{2(x - 3)} - \frac{2}{4x - 1} \right]$$

$$\frac{dy}{dx} = \frac{(x^2 - 1)^3 (2x - 1)}{\sqrt{(x - 3)(4x - 1)}}\left[\frac{6x}{x^2 - 1} + \frac{2}{2x - 1} - \frac{1}{2(x - 3)} - \frac{2}{4x - 1} \right] \qquad \text{[Using equaiton (i)]}$$

Differentiation Ex 11.5 Q22

Here,

$$y = \frac{e^{ax} \sec^x \log x}{\sqrt{1 - 2x}} \qquad \text{---(i)}$$

$$\Rightarrow \quad y = \frac{e^{ax} \cdot \sec^x \log x}{(1 - 2x)^{\frac{1}{2}}}$$

Taking log on both the sides,

$$\log y = \log e^{ax} + \log^{\sec x} + \log\log x - \frac{1}{2}\log(1 - 2x) \qquad \left[\begin{array}{l} \text{Since, } \log\left(\frac{A}{B}\right) = \log A - \log B, \\ \log(AB) = \log A + \log B \end{array} \right]$$

$$\log y = ax + \log^{\sec x} + \log\log x - \frac{1}{2}\log(1 - 2x) \qquad \left[\text{Since, } \log a^b = b\log a \text{ and } \log_e e = 1 \right]$$

Differentiating it with respect to x using chain rule,

$$\frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}(ax) + \frac{d}{dx}(\log\sec x) + \frac{d}{dx}(\log\log x) - \frac{1}{2}\log(1 - 2x)$$

$$\frac{1}{y}\frac{dy}{dx} = a + \frac{1}{\sec x}\frac{d}{dx}(\sec x) + \frac{1}{\log x}\frac{d}{dx}(\log x) - \frac{1}{2}\left(\frac{1}{1 - 2x}\right)\frac{d}{dx}(1 - 2x)$$

$$\frac{1}{y}\frac{dy}{dx} = a + \frac{\sec x \tan x}{\sec x} + \frac{1}{(\log x)}\left(\frac{1}{x}\right) - \frac{1}{2}\left(\frac{1}{1 - 2x}\right)(-2)$$

$$\frac{dy}{dx} = y\left[a + \tan x + \frac{1}{x\log x} + \frac{1}{1 - 2x} \right]$$

$$\frac{dy}{dx} = \frac{e^{ax}\sec x\log x}{\sqrt{1 - 2x}}\left[a + \tan x + \frac{1}{x\log x} + \frac{1}{1 - 2x} \right] \qquad \text{[Using equaton (i)]}$$

Differentiation Ex 11.5 Q23

Here,

$$y = e^{3x} \times \sin 4x \times 2^x \qquad \text{--- (i)}$$

Taking log on both the sides,

$$\log y = \log e^{3x} + \log \sin 4x + \log 2^x \qquad \left[\text{Since, } \log(AB) = \log A + \log B\right]$$
$$\log y = 3x \log e + \log \sin 4x + x \log 2 \qquad \left[\text{Since, } \log_e e = 1, \log a^b = b \log a\right]$$
$$\log y = 3x + \log \sin 4x + x \log 2$$

Differentiating it with respect to x,

$$\frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}(3x) + \frac{d}{dx}(\log \sin 4x) + \frac{d}{dx}(x \log 2)$$
$$= 3 + \frac{1}{\sin 4x}\frac{d}{dx}(\sin 4x) + \log 2(1)$$
$$= 3 + \frac{1}{\sin 4x}(\cos 4x)\frac{d}{dx}(4x) + \log 2$$
$$= 3 + \cot x (4) + \log 2$$
$$\frac{1}{y}\frac{dy}{dx} = 3 + 4\cot 4x + \log 2$$
$$\frac{dy}{dx} = y\left[3 + 4\cot 4x + \log 2\right]$$

$$\frac{dy}{dx} = e^{3x} \times \sin 4x \times 2^x \left[3 + 4\cot 4x + \log 2\right]$$

Differentiation Ex 11.5 Q24

Here,

$$y = \sin x \sin 2x \sin 3x \sin 4x \qquad \text{--- (i)}$$

Taking log on both the sides,

$$\log y = \log(\sin x \sin 2x \sin 3x \sin 4x)$$
$$\log y = \log \sin x + \log \sin 2x + \log \sin 3x + \log \sin 4x$$

Differentiating it with respect to x using chain rule,

$$\frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}\log \sin x + \frac{d}{dx}\log \sin 2x + \frac{d}{dx}\log \sin 3x + \frac{d}{dx}\log \sin 4x$$
$$= \frac{1}{\sin x}\frac{d}{dx}(\sin x) + \frac{1}{\sin 2x}\frac{d}{dx}(\sin 2x) + \frac{1}{\sin 3x}\frac{d}{dx}(\sin 3x) + \frac{1}{\sin 4x}\frac{d}{dx}(\sin 4x)$$
$$= \frac{1}{\sin x}(\cos x) + \frac{1}{\sin 2x}(\cos 2x)\frac{d}{dx}(2x) + \frac{1}{\sin 3x}(\cos 3x)\frac{d}{dx}(3x) + \frac{1}{\sin 4x}(\cos 4x)\frac{d}{dx}(4x)$$
$$\frac{1}{y}\frac{dy}{dx} = \left[\cot x + \cot 2x (2) + \cot 3x (3) + \cot 4x (4)\right]$$
$$\frac{dy}{dx} = y\left[\cot x + 2\cot 2x + 3\cot x 3x + 4\cot 4x\right]$$

$$\frac{dy}{d} = (\sin x \sin 2x \sin 3x \sin 4x)\left[\cot x + 2\cot 2x + 3\cot x 3x + 4\cot 4x\right] \qquad \text{[Using equation (i)]}$$

Differentiation Ex 11.5 Q25

Let $y = x^{\sin x} + (\sin x)^x$

Also, let $u = x^{\sin x}$ and $v = (\sin x)^x$

$\therefore y = u + v$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{du}{dx} + \dfrac{dv}{dx}$ ...(1)

$u = x^{\sin x}$

$\Rightarrow \log u = \log\left(x^{\sin x}\right)$

$\Rightarrow \log u = \sin x \log x$

Differentiating both sides with respect to x, we obtain

$\dfrac{1}{u}\dfrac{du}{dx} = \dfrac{d}{dx}(\sin x)\cdot \log x + \sin x \cdot \dfrac{d}{dx}(\log x)$

$\Rightarrow \dfrac{du}{dx} = u\left[\cos x \log x + \sin x \cdot \dfrac{1}{x}\right]$

$\Rightarrow \dfrac{du}{dx} = x^{\sin x}\left[\cos x \log x + \dfrac{\sin x}{x}\right]$ ...(2)

$v = (\sin x)^x$

$\Rightarrow \log v = \log(\sin x)^x$

$\Rightarrow \log v = x \log(\sin x)$

Differentiating both sides with respect to x, we obtain

$\dfrac{1}{v}\dfrac{dv}{dx} = \dfrac{d}{dx}(x)\times \log(\sin x) + x \times \dfrac{d}{dx}\left[\log(\sin x)\right]$

$\Rightarrow \dfrac{dv}{dx} = v\left[\log(\sin x) + x \cdot \dfrac{1}{\sin x}\cdot \dfrac{d}{dx}(\sin x)\right]$

$\Rightarrow \dfrac{dv}{dx} = (\sin x)^x\left[\log \sin x + \dfrac{x}{\sin x}\cos x\right]$

$\Rightarrow \dfrac{dv}{dx} = (\sin x)^x\left[\log \sin x + x \cot x\right]$

$\Rightarrow \dfrac{dv}{dx} = (\sin x)^x\left[\log \sin x + x \cot x\right]$ ...(3)

From (1), (2), and (3), we obtain

$\dfrac{dy}{dx} = x^{\sin x}\left(\cos x \log x + \dfrac{\sin x}{x}\right) + (\sin x)^x\left[\log \sin x + x \cot x\right]$

Differentiation Ex 11.5 Q26

Here,

$$y = (\sin x)^{\cos x} + (\cos x)^{\sin x}$$

$$y = e^{\log(\sin x)^{\cos x}} + e^{\log(\cos x)^{\sin x}}$$

$$y = e^{\cos x \log \sin x} + e^{\sin x \log \cos x} \qquad \left[\text{Since, } \log_e e = 1 \text{ and } \log a^b = b \log a\right]$$

Differentiating it with respect to x using chain rule and product rule,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{\cos x \log \sin x}\right) + \frac{d}{dx}\left(e^{\sin x \log \cos x}\right)$$

$$= e^{\cos x \log \sin x} \frac{d}{dx}(\cos x \log \sin x) + e^{\sin x \log \cos x} \frac{d}{dx}(\sin x \log \cos x)$$

$$= e^{\log(\sin x)^{\cos x}}\left[\cos x \frac{d}{dx}\log \sin x + \log \sin x \frac{d}{dx}(\cos x)\right] + e^{\log(\cos x)^{\sin x}}\left[\sin x \frac{d}{dx}\log \cos x + \log \cos x \frac{d}{dx}(\sin x)\right]$$

$$= (\sin)^{\cos x}\left[\cos x\left(\frac{1}{\sin x}\right)\frac{d}{dx}(\sin x) + \log \sin x \times (-\sin x)\right] + (\cos x)^{\sin x}\left[\sin x\left(\frac{1}{\cos x}\right)\frac{d}{dx}(\cos x) + \log \cos x (\cos x)\right]$$

$$= (\sin x)^{\cos x}\left[\cot x \times \cos x - \sin x \log \sin x\right] + (\cos x)^{\sin x}\left[\tan x(-\sin x) + \cos x \log \cos x\right]$$

$$\frac{dy}{dx} = (\sin x)^{\cos x}\left[\cot x \times \cos x - \sin x \log \sin x\right] + (\cos x)^{\sin x}\left[\cos x \log \cos x - \sin x \tan x\right]$$

Differentiation Ex 11.5 Q27

Here,

$$y = (\tan x)^{\cot x} + (\cot x)^{\tan x}$$

$$y = e^{\log(\tan x)^{\cot x}} + e^{\log(\cot x)^{\tan x}} \qquad \left[\text{Since, } \log_e e = 1, \log a^b = b \log a\right]$$

$$y = e^{\cot x \log \tan x} + e^{\tan x \log(\cot x)}$$

Differentiating it with respect to x using chain rule and product rule,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{\cot x \log \tan x}\right) + \frac{d}{dx}\left(e^{\tan x \log \cot x}\right)$$

$$= e^{\cot x \log \tan x}\frac{d}{dx}(\cot x \log \tan x) + e^{\tan x \log \cot x}\frac{d}{dx}(\tan x \log \cot x)$$

$$= e^{\log(\tan x)^{\cot x}}\left[\cot x \frac{d}{dx}\log \tan x + \log \tan x \frac{d}{dx}\cot x\right] + e^{\log(\cot x)^{\tan x}}\left[\begin{array}{l}\tan x \frac{d}{dx}\log \cot x + \\ \log \cot x \frac{d}{dx}(\tan x)\end{array}\right]$$

$$= (\tan x)^{\cot x}\left[\cot x \times \left(\frac{1}{\tan x}\right)\frac{d}{dx}(\tan x) + \log \tan x\left(-\cos ec^2 x\right)\right] + (\cot x)^{\tan x}\left[\begin{array}{l}\tan x\left(\frac{1}{\cot x}\right)\frac{d}{dx}(\cot x) \\ + \log \cot x\left(\sec^2 x\right)\end{array}\right]$$

$$= \tan x^{\cot x}\left[(1)\left(\sec^2 x\right) - \cos ec^2 x \log \tan x\right] + (\cot)^{\tan x}\left[(1)\left(-\cos ec^2 x\right) + \sec^2 x \log \cot x\right]$$

$$\frac{dy}{dx} = (\tan)^{\cot x}\left[\sec^2 x - \cos ec^2 x \log \tan x\right] + (\cot x)^{\tan x}\left[\sec^2 x \log \cot x - \cos ec^2 x\right]$$

Differentiation Ex 11.5 Q28

Here,

$$y = (\sin x)^x + \sin^{-1}\sqrt{x}$$

$$= e^{\log(\sin x)^x} + \sin^{-1}\sqrt{x}$$

$$y = e^{x \log \sin x} + \sin^{-1}\sqrt{x} \qquad \left[\text{Since, } \log_e e = 1, \log a^b = b \log a\right]$$

Differentiating it with respect to x using chain rule and product rule,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{x \log \sin x}\right) + \frac{d}{dx}\sin^{-1}\left(\sqrt{x}\right)$$

$$= e^{x \log \sin x}\frac{d}{dx}(x \log \sin x) + \frac{1}{\sqrt{1 - \left(\sqrt{x}\right)^2}}\frac{d}{dx}\left(\sqrt{x}\right)$$

$$= e^{\log(\sin x)^x}\left[x \frac{d}{dx}\log \sin x + \log \sin x \frac{d}{dx}(x) + \frac{1}{\sqrt{1 - x}} \times \frac{1}{2\sqrt{x}}\right]$$

$$= (\sin x)^x\left[x \times \frac{1}{\sin x}\frac{d}{dx}(\sin x) + \log \sin x(1)\right] + \frac{1}{2\sqrt{x - x^2}}$$

$$= (\sin x)^x\left[\frac{x}{\sin x}(\cos x) + \log \sin x\right] + \frac{1}{2\sqrt{x - x^2}}$$

$$\frac{dy}{dx} = (\sin x)^x\left[x \cot x + \log \sin x\right] + \frac{1}{2\sqrt{x - x^2}}$$

Differentiation Ex 11.5 Q29

Here,

$$y = x^{\cos x} + (\sin x)^{\tan x}$$

$$y = e^{\log x^{\cos x}} + e^{\log(\sin x)\tan x} \qquad \left[\text{Since, } e^{\log_a a} = a \text{ and } \log a^b = b\log a\right]$$

$$y = e^{\cos x \log x} + e^{\tan x \log \sin x}$$

Differentiating it with respect to x using chain rule and product rule,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{\cos x \log x}\right) + \frac{d}{dx}\left(e^{\tan x \log \sin x}\right)$$

$$= e^{\cos x \log x}\frac{d}{dx}(\cos x \log x) + e^{\tan x \log \sin x} \times \frac{d}{dx}(\tan x \log \sin x)$$

$$= e^{\log x^{\cos x}}\left[\cos x \frac{d}{dx}(\log x) + \log x \frac{d}{dx}(\cos x)\right] + e^{\log(\sin x)^{\tan x}}\left[\tan x \frac{d}{dx}\log \sin x + \log \sin x \frac{d}{dx}(\tan x)\right]$$

$$= x^{\cos x}\left[\cos x \left(\frac{1}{x}\right) + \log x (-\sin x)\right] + (\sin x)^{\tan x}\left[\tan x \left(\frac{1}{\sin x}\right)\frac{d}{dx}(\sin x) + \log \sin x \left(\sec^2 x\right)\right]$$

$$= x^{\cos x}\left[\frac{\cos x}{x} - \sin x \log x\right] + (\sin x)^{\tan x}\left[\tan x \left(\frac{1}{\sin x}\right)(\cos x) + \sec^2 x \log \sin x\right]$$

$$\frac{dy}{dx} = x^{\cos x}\left[\frac{\cos x}{x} - \sin x \log x\right] + (\sin x)^{\tan x}\left[1 + \sec^2 x \log \sin x\right]$$

Here,

$$y = x^x + (\sin x)^x$$

$$= e^{\log x^x} + e^{\log(\sin x)^x}$$

$$y = e^{x\log x} + e^{x\log \sin x} \qquad \left[\text{Using } e^{\log a} = a \text{ and } \log a^b = b\log a\right]$$

Differentiating with respect to x using chain rule and product rule,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{x\log x}\right) + \frac{d}{dx}\left(e^{x\log \sin x}\right)$$

$$= e^{x\log x}\frac{d}{dx}(x\log x) + e^{x\log \sin x}\frac{d}{dx}(x\log \sin x)$$

$$= e^{\log x^x}\left[x\frac{d}{dx}(\log x) + \log x \frac{d}{dx}(x)\right] + e^{\log(\sin x)^x}\left[x\frac{d}{dx}(\log \sin x) + \log \sin x \frac{d}{dx}(x)\right]$$

$$= x^x\left[x\left(\frac{1}{x}\right) + \log x (1)\right] + (\sin x)^x\left[x \times \left(\frac{1}{\sin x}\right)\frac{d}{dx}(\sin x) + \log \sin x (1)\right]$$

$$= x^x\left[1 + \log x\right] + (\sin x)^x\left[x\left(\frac{1}{\sin x}\right)(\cos x) + \log \sin x\right]$$

$$\frac{dy}{dx} = x^x(1 + \log x) + (\sin x)^x\left[x\cot x + \log \sin x\right]$$

Differentiation Ex 11.5 Q30

Here,

$$y = (\tan x)^{\log x} + \cos^2\left(\frac{\pi}{4}\right)$$

$$y = e^{\log(\tan x)^{\log x}} + \cos^2\left(\frac{\pi}{4}\right)$$

$$y = e^{\log x \log \tan x} + \cos^2\left(\frac{\pi}{4}\right) \qquad \left[\text{Since, } e^{\log a} = a \text{ and } \log a^b = b\log a\right]$$

Differentiating it using chain rule and product rule,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{\log x \log \tan x}\right) + \frac{d}{dx}\cos^2\left(\frac{\pi}{4}\right)$$

$$= e^{\log x \log \tan x}\frac{d}{dx}(\log x \log \tan x) + 0$$

$$= e^{\log(\tan x)^{\log x}}\left[\log x \frac{d}{dx}(\log \tan x) + \log \tan x \frac{d}{dx}(\log x)\right]$$

$$= (\tan x)^{\log x}\left[\log x \left(\frac{1}{\tan x}\right)\frac{d}{dx}(\tan x) + \log \tan x \left(\frac{1}{x}\right)\right]$$

$$= (\tan x)^{\log x}\left[\log x \left(\frac{1}{\tan x}\right)\left(\sec^2 x\right) + \frac{\log \tan x}{x}\right]$$

$$\frac{dy}{dx} = (\tan x)^{\log x}\left[\log x \left(\frac{\sec^2 x}{\tan x}\right) + \frac{\log \tan x}{x}\right]$$

Differentiation Ex 11.5 Q31

Here,

$$y = x^x + x^{\frac{1}{x}}$$

$$= e^{\log x^x} + e^{\log x^{\frac{1}{x}}}$$

$$y = e^{x \log x} + e^{\left(\frac{1}{x}\log x\right)} \qquad \left[\text{Since, } e^{\log a} = a, \log a^b = b \log a\right]$$

Differentiating it with respect to x using chain rule and product rule,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{x \log x}\right) + \frac{d}{dx}\left(e^{\frac{1}{x}\log x}\right)$$

$$= e^{x \log x} + \frac{d}{dx}(x \log x) + e^{\frac{1}{x}\log x} \frac{d}{dx}\left(\frac{1}{x}\log x\right)$$

$$= e^{\log x^x}\left[x \frac{d}{dx}(\log x) + \log x \frac{d}{dx}(x)\right] + e^{\log x^{\frac{1}{x}}}\left[\frac{1}{x}\frac{d}{dx}(\log x) + \log x \frac{d}{dx}\left(\frac{1}{x}\right)\right]$$

$$= x^x\left[x\left(\frac{1}{x}\right) + \log x (1)\right] + x^{\frac{1}{x}}\left[\left(\frac{1}{x}\right)\left(\frac{1}{x}\right) + \log x\left(-\frac{1}{x^2}\right)\right]$$

$$= x^x[1 + \log x] + x^{\frac{1}{x}}\left(\frac{1}{x^2} - \frac{1}{x^2}\log x\right)$$

$$\frac{dy}{dx} = x^x[1 + \log x] + x^{\frac{1}{x}}\frac{(1 - \log x)}{x^2}$$

Differentiation Ex 11.5 Q32

Let $y = (\log x)^x + x^{\log x}$

Also, let $u = (\log x)^x$ and $v = x^{\log x}$

$\therefore y = u + v$

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \qquad \ldots(1)$$

$u = (\log x)^x$

$$\Rightarrow \log u = \log\left[(\log x)^x\right]$$

$$\Rightarrow \log u = x \log(\log x)$$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{u}\frac{du}{dx} = \frac{d}{dx}(x) \times \log(\log x) + x \cdot \frac{d}{dx}\left[\log(\log x)\right]$$

$$\Rightarrow \frac{du}{dx} = u\left[1 \times \log(\log x) + x \cdot \frac{1}{\log x} \cdot \frac{d}{dx}(\log x)\right]$$

$$\Rightarrow \frac{du}{dx} = (\log x)^x\left[\log(\log x) + \frac{x}{\log x} \cdot \frac{1}{x}\right]$$

$$\Rightarrow \frac{du}{dx} = (\log x)^x\left[\log(\log x) + \frac{1}{\log x}\right]$$

$$\Rightarrow \frac{du}{dx} = (\log x)^x\left[\frac{\log(\log x)\cdot \log x + 1}{\log x}\right]$$

$$\frac{1}{v} \cdot \frac{dv}{dx} = \frac{d}{dx}\left[(\log x)^2\right]$$

$$\Rightarrow \frac{1}{v} \cdot \frac{dv}{dx} = 2(\log x) \cdot \frac{d}{dx}(\log x)$$

$$\Rightarrow \frac{dv}{dx} = 2v(\log x) \cdot \frac{1}{x}$$

$$\Rightarrow \frac{dv}{dx} = 2x^{\log x} \frac{\log x}{x}$$

$$\Rightarrow \frac{dv}{dx} = 2x^{\log x - 1} \cdot \log x \qquad \qquad ...(3)$$

Therefore, from (1), (2), and (3), we obtain

$$\frac{dy}{dx} = (\log x)^{x-1}\left[1 + \log x . \log(\log x)\right] + 2x^{\log x - 1} \cdot \log x$$

Differentiation Ex 11.5 Q33

Here,

$$x^{13}y^7 = (x + y)^{20}$$

Taking log on both the sides,

$$\log\left(x^{13}y^7\right) = \log(x + y)^{20}$$

$$13\log x + 7\log y = 20\log(x + y) \qquad \left[\text{Since, } \log(AB) = \log A + \log B, \log a^b = b\log a\right]$$

Differentiating it with respect to x using chain rule,

$$13\frac{d}{dx}(\log x) + 7\frac{d}{dx}(\log y) = 20\frac{d}{dx}\log(x + y)$$

$$\frac{13}{x} + \frac{7}{y}\frac{dy}{dx} = \frac{20}{x+y}\frac{d}{dx}(x + y)$$

$$\frac{13}{x} + \frac{7}{y}\frac{dy}{dx} = \frac{20}{(x+y)}\left[1 + \frac{dy}{dx}\right]$$

$$\frac{7}{y}\frac{dy}{dx} - \frac{20}{(x+y)} = \frac{20}{(x+y)} - \frac{13}{x}$$

$$\frac{dy}{dx}\left[\frac{7}{y} - \frac{20}{(x+y)}\right] = \frac{20}{(x+y)} - \frac{13}{x}$$

$$\frac{dy}{dx}\left[\frac{7(x+y) - 20y}{y(x+y)}\right] = \left[\frac{20x - 13(x+y)}{x(x+y)}\right]$$

$$\frac{dy}{dx} = \left[\frac{20x - 13x - 13y}{x(x+y)}\right]\left(\frac{y(x+y)}{7x + 7y - 20y}\right)$$

$$= \frac{y}{x}\left(\frac{7x - 13y}{7x - 13y}\right)$$

$$\frac{dy}{dx} = \frac{y}{x}$$

Differentiation Ex 11.5 Q34

Here,

$$x^{16}y^9 = \left(x^2 + y\right)^{17}$$

Taking log on both the sides,

$$\log\left(x^{16} \times y^9\right) = \log\left(x^2 + y\right)^{17} \qquad \left[\text{Since, } \log(AB) = \log A + \log B, \log a^b = b \log a\right]$$

$$16\log x + 9\log y = 17\log\left(x^2 + y\right)$$

Differentiating it with respect to x using chain rule,

$$16\frac{d}{dx}\left(\log x\right) + 9\frac{d}{dx}\left(\log y\right) = 17\frac{d}{dx}\log\left(x^2 + y\right)$$

$$\frac{16}{x} + \frac{9}{y}\frac{dy}{dx} = 17\frac{1}{\left(x^2 + y\right)}\frac{d}{dx}\left(x^2 + y\right)$$

$$\frac{16}{x} + \frac{9}{y}\frac{dy}{dx} = \frac{17}{x^2 + y}\left[2x + \frac{dy}{dx}\right]$$

$$\frac{9}{y}\frac{dy}{dx} - \frac{17}{\left(x^2 + y\right)}\frac{dy}{dx} = \left(\frac{34x}{x^2 + y}\right) - \frac{16}{x}$$

$$\frac{dy}{dx}\left[\frac{9}{y} - \frac{17}{\left(x^2 + y\right)}\right] = \frac{34x^2 - 16x^2 - 16y}{x\left(x^2 + y\right)}$$

$$\frac{dy}{dx}\left[\frac{9x^2 + 9y - 17y}{y\left(x^2 + y\right)}\right] = \frac{18x^2 - 16y}{x\left(x^2 + y\right)}$$

$$\frac{dy}{dx} = \frac{y}{x}\left(\frac{2\left(9x^2 - 8y\right)}{9x^2 - 8y}\right)$$

$$\frac{dy}{dx} = \frac{2y}{x}$$

$$x\frac{dy}{dx} = 2y$$

Differentiation Ex 11.5 Q35

Here,

$$y = \sin\left(x^x\right) \qquad\qquad \text{---(i)}$$

Let $\quad u = x^x \qquad\qquad\qquad \text{---(ii)}$

Taking log on both the sides,

$$\log u = \log x^x$$
$$\log u = x \log x$$

Differentiating it with respect to x,

$$\frac{1}{u}\frac{du}{dx} = \frac{d}{dx}\left(x \log x\right)$$
$$= x\frac{d}{dx}\left(\log x\right) + \log x \frac{d}{dx}\left(x\right)$$
$$= x\left(\frac{1}{x}\right) + \log x\,(1)$$
$$\frac{1}{u}\frac{du}{dx} = 1 + \log x$$
$$\frac{du}{dx} = u\left(1 + \log x\right)$$
$$\frac{du}{dx} = x^x\left(1 + \log x\right) \qquad\qquad \text{---(iii)} \left[\text{Using equation (ii)}\right]$$

Now, using equation (ii) in equation (i),

$$y = \sin u$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sin u\right)$$
$$= \cos u \frac{du}{dx}$$

Using equation (ii) and (iii),

$$\frac{dy}{dx} = \cos\left(x^x\right) \times x^x\left(1 + \log x\right)$$

Differentiation Ex 11.5 Q36

Here,

$$x^x + y^x = 1$$
$$e^{\log x^x} + e^{\log y^x} = 1$$
$$e^{x\log x} + e^{x\log y} = 1 \qquad\qquad \left[\text{Since, } e^{\log a} = a, \log a^b = b\log a\right]$$

Differentiating it with respect to x using product rule and chain rule,

$$\frac{d}{dx}\left(e^{x\log x}\right) + \frac{d}{dx}\left(e^{x\log y}\right) = \frac{d}{dx}(1)$$
$$e^{x\log x}\frac{d}{dx}\left(x\log x\right) + e^{x\log y}\frac{d}{dx}\left(x\log y\right) = 0$$
$$e^{\log x^x}\left[x\frac{d}{dx}\left(\log x\right) + \log x\frac{d}{dx}\left(x\right)\right] + e^{\log y^x}\left[x\frac{d}{dx}\left(\log y\right) + \log y\frac{d}{dx}\left(x\right)\right] = 0$$
$$x^x\left[x\left(\frac{1}{x}\right) + \log x\,(1)\right] + y^x\left[x\left(\frac{1}{y}\right)\frac{dy}{dx} + \log y\,(1)\right] = 0$$
$$x^x\left[1 + \log x\right] + y^x\left(\frac{x}{y}\frac{dy}{dx} + \log y\right) = 0$$
$$y^x \times \frac{x}{y}\frac{dy}{dx} = -\left[x^x\left(1 + \log x\right) + y^x\log y\right]$$
$$\left(xy^{x-1}\right)\frac{dy}{dx} = -\left[x^x\left(1 + \log x\right) + y^x\log y\right]$$

$$\frac{dy}{dx} = -\left[\frac{x^x\left(1 + \log x\right) + y^x\log y}{xy^{x-1}}\right]$$

Differentiation Ex 11.5 Q37

Here,

$$x^y \times y^x = 1$$

Taking on both sides,

$$\log\left(x^y \times y^x\right) = \log(1)$$

$$y = \log x + x \log y = \log 1 \qquad\qquad \left[\text{Since, } \log(AB) = \log A + \log B, \log a^b = b \log a\right]$$

Differentiating it with respect to x using product rule,

$$\frac{d}{dx}(y \log x) + \frac{d}{dx}(x \log y) = \frac{d}{dx}(\log 1)$$

$$\left[y \frac{d}{dx}(\log x) + \log x \frac{dy}{dx}\right] + \left[x \frac{d}{dx}(\log y) + \log y \frac{d}{dx}(x)\right] = 0$$

$$\left[y\left(\frac{1}{x}\right) + \log x \frac{dy}{dx}\right] + \left[x\left(\frac{1}{y}\frac{dy}{dx}\right) + \log y (1)\right] = 0$$

$$\frac{y}{x} + \log x \frac{dy}{dx} + \frac{x}{y}\frac{dy}{dx} + \log y = 0$$

$$\frac{dy}{dx}\left(\log x + \frac{x}{y}\right) = -\left[\log y + \frac{y}{x}\right]$$

$$\frac{dy}{dx}\left[\frac{y \log x + x}{y}\right] = -\left[\frac{x \log y + y}{x}\right]$$

$$\frac{dy}{dx} = -\frac{y}{x}\left[\frac{x \log y + y}{y \log x + x}\right]$$

Differentiation Ex 11.5 Q38

Here,

$$x^y + y^x = (x+y)^{x+y}$$

$$e^{\log x^y} + e^{\log y^x} = e^{\log(x+y)^{(x+y)}}$$

$$e^{y \log x} + e^{x \log y} = e^{(x+y)\log(x+y)} \qquad\qquad \left[\text{Since, } e^{\log a} = a, \log a^b = b \log a\right]$$

Differentiating it with respect to x using chain rule, product rule,

$$\Rightarrow \quad \frac{d}{dx}\left(e^{y \log x}\right) + \frac{d}{dx}\left(e^{x \log y}\right) = \frac{d}{dx}e^{(x+y)\log(x+y)}$$

$$\Rightarrow \quad e^{y \log x}\left[y \frac{d}{dx}(\log x) + \log x \frac{dy}{dx}\right] + e^{x \log y}\left[x \frac{d}{dx}\log y + \log y \frac{d}{dx}(x)\right] = e^{(x+y)\log(x+y)}\frac{d}{dx}\left[(x+y)\log(x+y)\right]$$

$$\Rightarrow \quad e^{\log x^y}\left[y\left(\frac{1}{x}\right) + \log x \frac{dy}{dx}\right] + e^{\log x}\left[\frac{x}{y}\frac{dy}{dx} + \log y (1)\right] = e^{\log(x+y)^{(x+y)}}\left[\begin{array}{c}(x+y)\frac{d}{dx}\log(x+y) + \log(x+y) \\ \frac{d}{dx}(x+y)\end{array}\right]$$

$$\Rightarrow \quad x^y\left[\frac{y}{x} + \log x \frac{dy}{dx}\right] + y^x\left[\frac{x}{y}\frac{dy}{dx} + \log y\right] = (x+y)^{(x+y)}\left[(x+y)\frac{1}{(x+y)}\frac{d}{dx}(x+y) + \log(x+y)\left(1 + \frac{dy}{dx}\right)\right]$$

$$\Rightarrow \quad x^y \times \frac{y}{x} + x^y \log x \frac{dy}{dx} + y^x \times \frac{x}{y}\frac{dy}{dx} + y^x \log y = (x+y)^{(x+y)}\left[1 \times\left(1 + \frac{dy}{dx}\right) + \log(x+y)\left(1 + \frac{dy}{dx}\right)\right]$$

$$\Rightarrow \quad x^{y-1} \times xy + x^y \log x \frac{dy}{dx} + y^{x-1} \times x \frac{dy}{dx} + y^x \log y = (x+y)^{(x+y)} + (x+y)^{(x+y)}\frac{dy}{dx} + (x+y)^{(x+y)}\log(x+y)$$

$$+ (x+y)^{(x+y)}\log(x+y)\frac{dy}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx}\left[x^y \log x + xy^{x-1} - (x+y)^{(x+y)}\left(1 + \log(x+y)\right)\right] = (x+y)^{(x+y)}\left(1 + \log(x+y)\right) - x^{y-1}xy - y^x \log y$$

$$\Rightarrow \quad \frac{dy}{dx} = \left[\frac{(x+y)^{(x+y)}\left(1 + \log(x+y)\right) - x^{y-1}xy - y^x \log y}{x^y \log x + xy^{x-1} - (x+y)^{(x+y)}\left(1 + \log(x+y)\right)}\right]$$

Differentiation Ex 11.5 Q39

Here,

$$x^m y^n = 1$$

Taking log on both the side,

$$\log\left(x^m y^n\right) = \log(1)$$
$$m\log x + n\log y = \log(1)$$

Differentiating it with respect to x,

$$\frac{dy}{dx}(m\log x) + \frac{d}{dx}(n\log y) = \frac{d}{dx}\left(\log(1)\right)$$
$$\frac{m}{x} + \frac{n}{y}\frac{dy}{dx} = 0$$
$$\frac{dy}{dx} = -\frac{m}{x} \times \frac{y}{n}$$

$$\frac{dy}{dx} = -\frac{my}{nx}$$

Differentiation Ex 11.5 Q40

Here,

$$y^x = e^{y-x}$$

Taking log on both the sides,

$$\log y^x = \log e^{(y-x)}$$ $\left[\text{Since, } \log a^b = b\log a \text{ and } \log_e e = 1\right]$
$$x\log y = (y - x)\log e$$
$$x\log y = y - x \qquad\qquad \text{---(i)}$$

Differentiating it with respect to x using product rule,

$$\frac{d}{dx}(x\log y) = \frac{d}{dx}(y - x)$$
$$\left[x\frac{d}{dx}(\log y) + \log y\frac{d}{dx}(x)\right] = \frac{dy}{dx} - 1$$
$$x\left(\frac{1}{y}\right)\frac{dy}{dx} + \log y(1) = \frac{dy}{dx} - 1$$
$$\frac{dy}{dx}\left(\frac{x}{y} - 1\right) = -1 - \log y$$
$$\frac{dy}{dx}\left(\frac{y}{(1+\log y)y}\right) = -(1+\log y)$$ $\left[\text{Since, from equation (i), } x = \frac{y}{(1+\log y)}\right]$

$$\frac{dy}{dx}\left[\frac{1 - 1 - \log y}{(1+\log y)}\right] = -(1+\log y)$$
$$\frac{dy}{dx} = -\frac{(1+\log y)^2}{-\log y}$$

$$\frac{dy}{dx} = \frac{(1+\log y)^2}{\log y}$$

Differentiation Ex 11.5 Q41

Here,

$$(\sin x)^y = (\cos y)^x$$

Taking log on both the sides,

$$\log(\sin x)^y = \log(\cos y)^x \qquad \left[\text{Using } \log a^b = b \log a\right]$$

$$y \log(\sin x) = x \log(\cos y)$$

Differentiating it with respect to x using product rule and chain rule,

$$\frac{d}{dx}\left[y \log \sin x\right] = \frac{d}{dx}\left[x \log \cos y\right]$$

$$y \frac{d}{dx}(\log \sin x) + \log \sin x \frac{dy}{dx} = x \frac{dy}{dx} \log \cos y + \log \cos y \frac{d}{dx}(x)$$

$$y \left(\frac{1}{\sin x}\right)\frac{d}{dx}(\sin x) + \log \sin x \frac{dy}{dx} = \frac{x}{\cos y}\frac{d}{dx}(\cos y) + \log \cos y (1)$$

$$\frac{y}{\sin x}(\cos x) + \log \sin x \frac{dy}{dx} = \frac{x}{\cos y}(-\sin y)\frac{dy}{dx} + \log \cos y$$

$$y \cot x + \log \sin x \frac{dy}{dx} = -x \tan y \frac{dy}{dx} + \log \cos y$$

$$\frac{dy}{dx}(\log \sin x + x \tan y) = \log \cos y - y \cot x$$

$$\frac{dy}{dx} = \frac{\log \cos y - y \cot x}{\log \sin x + x \tan y}$$

Differentiation Ex 11.5 Q42

Here,

$$(\cos x)^y = (\tan y)^x$$

Taking log on both the sides,

$$\log(\cos x)^y = \log(\tan y)^x$$

$$y \log \cos x = x \log \tan y \qquad \left[\text{Since, } \log a^b = b \log a\right]$$

Differentiating it with respect to x using chain rule and product rule,

$$\frac{d}{dx}(y \log \cos x) = \frac{d}{dx}(x \log \tan y)$$

$$\left(y \frac{d}{dx}\log \cos x + \log \cos x \frac{dy}{dx}\right) = \left(x \frac{d}{dx}\log \tan y + \log \tan y \frac{d}{dx}(x)\right)$$

$$\left(y \left(\frac{1}{\cos x}\right)\frac{d}{dx}(\cos x) + \log \cos x \frac{dy}{dx}\right) = \left(x \frac{1}{\tan y}\frac{d}{dx}(\tan y) + \log \tan y (1)\right)$$

$$\left(\frac{y}{\cos x}(-\sin x) + \log \cos x \frac{dy}{dx}\right) = \left(\frac{x}{\tan y}(\sec^2 y)\right)\frac{dy}{dx} + \log \tan y - y \tan x + \log \cos x \frac{dy}{dx}$$

$$= \left(\sec y \, \text{cosec} y \times x \frac{dy}{dx} + \log \tan y\right)$$

$$\frac{dy}{dx}\left[\log \cos x - x \sec y \, \text{cosec} y\right] = \log \tan y + y \tan x$$

$$\frac{dy}{dx} = \left[\frac{\log \tan y + y \tan x}{\log \cos x - x \sec y \, \text{cosec} y}\right]$$

Differentiation Ex 11.5 Q43

Here,
$$e^x + e^y = e^{x+y} \qquad\qquad ---(i)$$

Differentiating both the sides using chain rule,

$$\frac{d}{dx}\left(e^x\right) + \frac{d}{dx}\left(e^y\right) = \frac{d}{dx}\left(e^{x+y}\right)$$

$$e^x + e^y \frac{dy}{dx} = e^{x+y}\frac{d}{dx}(x+y)$$

$$e^x + e^y \frac{dy}{dx} = e^{x+y}\left[1+\frac{dy}{dx}\right]$$

$$e^y \frac{dy}{dx} - e^{x+y}\frac{dy}{dx} = e^{x+y} - e^x$$

$$\frac{dy}{dx} = \frac{e^{x+y} - e^x}{e^y - e^{x+y}}$$

$$= \left(\frac{e^x + e^y - e^x}{e^y - e^x - e^y}\right) \qquad\qquad\qquad \text{[Using equation (i)]}$$

$$\frac{dy}{dx} = -e^{y-x}$$

$$\frac{dy}{dx} + e^{y-x} = 0$$

Differentiation Ex 11.5 Q44

Here,
$$e^y = y^x$$

Taking log on both the sides,

$$\log e^y = \log y^x$$

$$y\log e = x\log y \qquad\qquad \left[\text{Since, } \log a^b = b\log a, \log_e e = 1\right]$$

$$y = x\log y \qquad\qquad ---(i)$$

Differentiating it with respect to x using product rule,

$$\frac{dy}{dx} = \frac{d}{dx}(x\log y)$$

$$= x\frac{dy}{dx}(\log y) + \log y \frac{d}{dx}(x)$$

$$\frac{dy}{dx} = \frac{x}{y}\frac{dy}{dx} + \log y\,(1)$$

$$\frac{dy}{dx}\left(1 - \frac{x}{y}\right) = \log y$$

$$\frac{dy}{dx}\left(\frac{y-x}{y}\right) = \log y$$

$$\frac{dy}{dx} = \frac{y\log y}{y-x}$$

$$\frac{dy}{dx} = \frac{y\log y}{\left(y - \frac{y}{\log y}\right)} \qquad\qquad \text{[Since, using equation (i)]}$$

$$= \frac{y\log y \times \log y}{y\log y - y}$$

$$= \frac{y(\log y)^2}{y(\log y - 1)}$$

$$\frac{dy}{dx} = \frac{(\log y)^2}{(\log y - 1)}$$

Differentiation Ex 11.5 Q45

Here,

$$e^{x+y} - x = 0$$
$$e^{x+y} = x \qquad\qquad ---(i)$$

Differentiating it with respect to x using chain rule,

$$\frac{d}{dx}\left(e^{x+y}\right) = \frac{d}{dx}(x)$$

$$e^{x+y}\frac{d}{dx}(x+y) = 1$$

$$x\left[1 + \frac{dy}{dx}\right] = 1 \qquad\qquad \left[\text{Using equatoin (i)}\right]$$

$$1 + \frac{dy}{dx} = \frac{1}{x}$$

$$\frac{dy}{dx} = \frac{1}{x} - 1$$

$$\frac{dy}{dx} = \frac{1-x}{x}$$

Differentiation Ex 11.5 Q46

Here $y = x\sin(a+y)$

Differentiating it with respect to x using the chain rule and product rule,

$$\frac{dy}{dx} = x\frac{d}{dx}\sin(a+y) + \sin(a+y)\frac{dx}{dx}$$

$$\frac{dy}{dx} = x\cos(a+y)\frac{dy}{dx} + \sin(a+y)$$

$$(1 - x\cos(a+y))\frac{dy}{dx} = \sin(a+y)$$

$$\frac{dy}{dx} = \frac{\sin(a+y)}{(1 - x\cos(a+y))}$$

$$\frac{dy}{dx} = \frac{\sin(a+y)}{\left(1 - \dfrac{y}{\sin(a+y)}\cos(a+y)\right)} \qquad \left[\text{Since } \frac{y}{\sin(a+y)} = x\right]$$

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin(a+y) - y\cos(a+y)}$$

Differentiation Ex 11.5 Q47

Here $x\sin(a+y) + \sin a\cos(a+y) = 0$

Differentiating it with respect to x using the chain rule and product rule,

$$\frac{d}{dx}\left[x\sin(a+y) + \sin a\cos(a+y)\right] = 0$$

$$x\frac{d}{dx}\sin(a+y) + \sin(a+y)\frac{dx}{dx} + \sin a\frac{d}{dx}\cos(a+y) + \cos(a+y)\frac{d}{dx}\sin a = 0$$

$$x\cos(a+y)\left(0 + \frac{dy}{dx}\right) + \sin(a+y) + \sin a\left(-\sin(a+y)\frac{dy}{dx}\right) + 0 = 0$$

$$\left[x\cos(a+y) - \sin a\sin(a+y)\right]\frac{dy}{dx} + \sin(a+y) = 0$$

$$\frac{dy}{dx} = -\frac{\sin(a+y)}{x\cos(a+y) - \sin a\sin(a+y)}$$

$$\frac{dy}{dx} = \frac{-\sin(a+y)}{\left(-\dfrac{\sin a\cos(a+y)}{\sin(a+y)}\right)\cos(a+y) - \sin a\sin(a+y)} \qquad \left[\text{Since } x = -\frac{\sin a\cos(a+y)}{\sin(a+y)}\right]$$

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{(\sin a)\cos^2(a+y) + \sin a\sin^2(a+y)}$$

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{(\sin a)\left[\cos^2(a+y) + \sin^2(a+y)\right]}$$

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{(\sin a)} \qquad \left[\text{Since } \cos^2(a+y) + \sin^2(a+y) = 1\right]$$

Differentiation Ex 11.5 Q48

Here,

$$(\sin x)^y = x + y$$

Taking log on both the sides,

$$\log(\sin x)^y = \log(x + y)$$
$$y \log(\sin x) = \log(x + y) \qquad\qquad \left[\text{Since, } \log a^b = b \log a\right]$$

Differentiating it with respect to x using chain rule, product rule,

$$\frac{d}{dx}\left(y \log(\sin x)\right) = \frac{d}{dx}\log(x + y)$$

$$y \frac{d}{dx}\log \sin x + \log \sin x \frac{dy}{dx} = \frac{1}{x+y}\frac{d}{dx}(x+y)$$

$$\frac{y}{\sin x}\frac{d}{dx}(\sin x) + \log \sin x \frac{dy}{dx} = \frac{1}{(x+y)}\left[1 + \frac{dy}{dx}\right]$$

$$\frac{y(\cos x)}{(\sin x)} + \log \sin x \frac{dy}{dx} = \frac{1}{(x+y)} + \frac{1}{(x+y)}\frac{dy}{dx}$$

$$\frac{dy}{dx}\left(\log \sin x - \frac{1}{x+y}\right) = \frac{1}{(x+y)} - y\cot x$$

$$\frac{dy}{dx}\left(\frac{(x+y)\log \sin x - 1}{(x+y)}\right) = \left(\frac{1 - y(x+y)\cot x}{x+y}\right)$$

$$\frac{dy}{dx} = \left(\frac{1 - y(x+y)\cot x}{(x+y)\log \sin x - 1}\right)$$

Differentiation Ex 11.5 Q49

Here,

$$xy \log(x + y) = 1 \qquad\qquad\qquad ---(i)$$

Differentiating with respect to x using chain rule, product rule,

$$\frac{dy}{dx}\left(xy \log(x+y)\right) = \frac{d}{dx}(1)$$

$$xy \frac{d}{dx}\log(x+y) + x \log(x+y)\frac{dy}{dx} + y \log(x+y)\frac{d}{dx}(x) = 0$$

$$\frac{xy}{(x+y)}\left(1 + \frac{dy}{dx}\right) + x \log(x+y)\frac{dy}{dx} + y \log(x+y)(1) = 0$$

$$\left(\frac{xy}{x+y}\right)\left(1 + \frac{dy}{dx}\right) + x \log(x+y)\frac{dy}{dx} + y \log(x+y) = 0$$

$$\left(\frac{xy}{x+y}\right)\frac{dy}{dx} + \frac{xy}{x+y} + x\left(\frac{1}{xy}\right)\frac{dy}{dx} + y\left(\frac{1}{xy}\right) = 0 \qquad\qquad \left[\text{Using equation }(i)\right]$$

$$\frac{dy}{dx}\left[\frac{xy}{x+y} + \frac{1}{y}\right] = -\left[\frac{1}{x} + \frac{xy}{x+y}\right]$$

$$\frac{dy}{dx}\left[\frac{xy^2 + x + y}{(x+y)y}\right] = -\left[\frac{x + y + x^2 y}{x(x+y)}\right]$$

$$\frac{dy}{dx} = -\frac{y}{x}\left(\frac{x + y + x^2 y}{x + y + xy^2}\right)$$

Differentiation Ex 11.5 Q50

Here,

$$y = x \sin y \qquad \text{--- (i)}$$

Differentiating it with respect to x using product rule,

$$\frac{dy}{dx} = \frac{d}{dx}(x \sin y)$$

$$= x \frac{d}{dx}(\sin y) + \sin y \frac{d}{dx}(x)$$

$$= x \cos y \frac{dy}{dx} + \sin y (1)$$

$$\frac{dy}{dx} - x \cos y \frac{dy}{dx} = \sin y$$

$$\frac{dy}{dx}(1 - x \cos y) = \sin y$$

$$\frac{dy}{dx} = \frac{\sin y}{(1 - x \cos y)}$$

Put the value of $\sin y = \dfrac{y}{x}$ form equation (i),

$$\frac{dy}{dx} = \frac{y}{x(1 - x \cos y)}$$

Differentiation Ex 11.5 Q51

Here,

$$f(x) = (1+x)\left(1+x^2\right)\left(1+x^4\right)\left(1+x^8\right)$$

Differentiating with respect to x using product rule and chain rule,

$$\Rightarrow \quad f'(x) = (1+x)\left(1+x^2\right)\frac{d}{dx}\left(1+x^8\right) + (1+x)\left(1+x^2\right)\left(1+x^8\right)\frac{d}{dx}\left(1+x^4\right) + (1+x)\left(1+x^4\right)\left(1+x^8\right)$$

$$\frac{d}{dx}\left(1+x^2\right) + \left(1+x^2\right)\left(1+x^4\right)\left(1+x^8\right)\frac{d}{dx}(1+x)$$

$$\Rightarrow \quad f'(x) = (1+x)\left(1+x^2\right)\left(1+x^4\right)8x^7 + (1+x)\left(1+x^2\right)\left(1+x^8\right)\left(4x^3\right) + (1+x)\left(1+x^4\right)\left(1+x^8\right)(2x)$$

$$+\left(1+x^2\right)\left(1+x^4\right)\left(1+x^8\right)(1)$$

$$f'(1) = (1+1)(1+1)(8) + (1+1)(1+1)(1+1)(4) + (1+1)(1+1)(1+1)(2) + (1+1)(1+1)(1+1)$$

$$f'(1) = (2)(2)(2)(8) + (2)(2)(2)(4) + (2)(2)(2)(2) + (2)(2)(2)$$

$$= 64 + 32 + 16 + 8$$

$$= 120$$

So,

$$f'(1) = 120$$

Differentiation Ex 11.5 Q52

Here,

$$y = \log\left(\frac{x^2 + x + }{x^2 - x + 1}\right) + \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{\sqrt{3}x}{1 - x^2}\right)$$

Differentiating it with respect to x using chain rule and quotient rule,

$$\frac{dy}{dx} = \frac{d}{dx}\log\left(\frac{x^2 + x + 1}{x^2 - x + 1}\right) + \frac{2}{\sqrt{3}}\frac{d}{dx}\tan^{-1}\left(\frac{\sqrt{3}x}{1 - x^2}\right)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{\left(\frac{x^2 + x + 1}{x^2 - x + 1}\right)}\frac{d}{dx}\left(\frac{x^2 + x + 1}{x^2 - x + 1}\right) + \frac{2}{\sqrt{3}}\left\{\frac{1}{1 + \left(\frac{\sqrt{3}x}{1 - x^2}\right)}\right\}\frac{d}{dx}\left(\frac{\sqrt{3}x}{1 - x^2}\right)$$

$$\Rightarrow \quad \frac{dy}{dx} = \left(\frac{x^2 - x + 1}{x^2 + x + 1}\right)\left(\frac{(x^2 - x + 1)\frac{d}{dx}(x^2 + x + 1) - (x^2 + x + 1)\frac{d}{dx}(x^2 - x + 1)}{(x^2 - x + 1)^2}\right) + \frac{2}{\sqrt{3}}\left\{\frac{(1 - x)^2}{1 + x^4 - 2x^2 + 3x^2}\right\}$$

$$\left\{\frac{(1 - x^2)^2\frac{d}{dx}(\sqrt{3}x) - \sqrt{3}x\frac{d}{dx}(1 - x)^2}{(1 - x^2)^2}\right\}$$

$$\Rightarrow \quad \frac{dy}{dx} = \left(\frac{1}{x^2 + x + 1}\right)\left(\frac{(x^2 - x + 1)(2x + 1) - (x^2 + x + 1)(2x - 1)}{(x^2 - x + 1)}\right) + \frac{2}{\sqrt{3}}\left(\frac{(1 - x^2)^2}{1 + x^2 + x^4}\right)\left|\frac{(1 - x^2)(\sqrt{3}) - \sqrt{3}x(-2x)}{(1 - x^2)^2}\right|$$

$$\Rightarrow \quad \frac{dy}{dx} = \left(\frac{2x^3 - 2x^2 + 2x + x^2 - x + 1 - 2x^3 - 2x^2 - 2x + x^2 + x + 1}{x^4 + 2x^2 + 1 - x^2}\right) + \frac{2}{\sqrt{3}}\left(\frac{\sqrt{3} - \sqrt{3}x^2 + 2\sqrt{3}x^2}{1 + x^2 + x^4}\right)$$

$$= \left(\frac{-2x^2 + 2}{x^4 + x^2 + 1}\right) + \frac{2\sqrt{3}(x^2 + 1)}{\sqrt{3}(1 + x^2 + x^4)}$$

$$= \frac{2(1 - x^2)}{(x^4 + x^2 + 1)} + \frac{2(x^2 + 1)}{1 + x^2 + x^4}$$

$$= \frac{2(1 - x^2 + x^2 + 1)}{1 + x^2 + x^4}$$

$$\frac{dy}{dx} = \frac{4}{1 + x^2 + x^4}$$

Differentiation Ex 11.5 Q53

Here,

$$y = (\sin x - \cos x)^{(\sin x - \cos x)}$$

Takig log on both the sides,

$$\Rightarrow \quad \log y = \log(\sin x - \cos x)^{(\sin x - \cos x)}$$

$$\Rightarrow \quad \log y = (\sin x - \cos x)\log(\sin x - \cos x)$$

Differentiating it with respect to x using product rule, chain rule,

$$\Rightarrow \quad \frac{1}{y}\frac{dy}{dx} = \log(\sin x - \cos x)\frac{d}{dx}(\sin x - \cos x) + (\sin x - \cos x)\frac{d}{dx}\log(\sin x - \cos x)$$

$$\Rightarrow \quad \frac{1}{y}\frac{dy}{dx} = \log(\sin x - \cos x) \times (\cos x + \sin x) + \frac{(\sin x - \cos x)}{(\sin x - \cos x)}\frac{d}{dx}(\sin x - \cos x)$$

$$\Rightarrow \quad \frac{1}{y}\frac{dy}{dx} = (\cos x + \sin x)\log(\sin x - \cos x) + (\cos x + \sin x)$$

$$\Rightarrow \quad \frac{1}{y}\frac{dy}{dx} = (\cos x + \sin x)(1 + \log(\sin x - \cos x))$$

$$\Rightarrow \quad \frac{dy}{dx} = y\left[(\cos x + \sin x)(1 + \log(\sin x - \cos x))\right]$$

Using equation (i),

$$\frac{dy}{dx} = (\sin x - \cos x)^{(\sin x - \cos x)}\left[(\cos x + \sin x)(1 + \log(\sin x - \cos x))\right]$$

Differentiation Ex 11.5 Q54

The given function is $xy = e^{(x-y)}$

Taking logarithm on both the sides, we obtain

$$\log(xy) = \log\left(e^{x-y}\right)$$
$$\Rightarrow \log x + \log y = (x - y)\log e$$
$$\Rightarrow \log x + \log y = (x - y) \times 1$$
$$\Rightarrow \log x + \log y = x - y$$

Differentiating both sides with respect to x, we obtain

$$\frac{d}{dx}(\log x) + \frac{d}{dx}(\log y) = \frac{d}{dx}(x) - \frac{dy}{dx}$$
$$\Rightarrow \frac{1}{x} + \frac{1}{y}\frac{dy}{dx} = 1 - \frac{dy}{dx}$$
$$\Rightarrow \left(1 + \frac{1}{y}\right)\frac{dy}{dx} = 1 - \frac{1}{x}$$
$$\Rightarrow \left(\frac{y+1}{y}\right)\frac{dy}{dx} = \frac{x-1}{x}$$
$$\therefore \frac{dy}{dx} = \frac{y(x-1)}{x(y+1)}$$

Differentiation Ex 11.5 Q55

Given that $y^x + x^y + x^x = a^b$.
Putting $u = y^x$, $v = x^y$ and $w = x^x$, we get $u + v + w = a^b$
Therefore $\dfrac{du}{dx} + \dfrac{dv}{dx} + \dfrac{dw}{dx} = 0$ ...(1)
Now, $u = y^x$. Taking logrithm on both sides, we have
$$\log u = x \log y$$
Differentiating both sides w.r.t. x, we have
$$\frac{1}{u}\cdot\frac{du}{dx} = x\frac{d}{dx}(\log y) + \log y\frac{d}{dx}(x)$$
$$= x\frac{1}{y}\cdot\frac{dy}{dx} + \log y.1$$
So $\dfrac{du}{dx} = u\left(\dfrac{x}{y}\dfrac{dy}{dx} + \log y\right) = y^x\left[\dfrac{x}{y}\dfrac{dy}{dx} + \log y\right]$ ...(2)
Also $v = x^y$
Taking logarithm on both sides, we have
$$\log v = y \log x$$
Differentiating both sides w.r.t. x, we have
$$\frac{1}{v}\cdot\frac{dv}{dx} = y\frac{d}{dx}(\log x) + \log x\frac{dy}{dx}$$
$$= y.\frac{1}{x} + \log x.\frac{dy}{dx}$$
So $\dfrac{dv}{dx} = v\left[\dfrac{y}{x} + \log x\dfrac{dy}{dx}\right]$
$$= x^y\left[\frac{y}{x} + \log x\frac{dy}{dx}\right]$$ ...(3)
Again $w = x^x$
Taking logarithm on both sides, we have
$$\log w = x \log x.$$
Differentiating both sides w.r.t. x, we have
$$\frac{1}{w}\cdot\frac{dw}{dx} = x\frac{d}{dx}(\log x) + \log x.\frac{d}{dx}(x)$$
$$= x.\frac{1}{x} + \log x . 1$$
i.e. $\dfrac{dw}{dx} = w(1 + \log x)$
$$= x^x(1 + \log x)$$ ...(4)
From (1), (2), (3), (4), we have
$$y^x\left(\frac{x}{y}\frac{dy}{dx} + \log y\right) + x^y\left(\frac{y}{x} + \log x\frac{dy}{dx}\right) + x^x(1 + \log x) = 0$$
or $\left(x.y^{x-1} + x^y . \log x\right)\dfrac{dy}{dx} = -x^x(1 + \log x) - y. x^{y-1} - y^x \log y$
Therefore $\dfrac{dy}{dx} = \dfrac{-\left[y^x \log y + y.x^{y-1} + x^x(1 + \log x)\right]}{x.y^{x-1} + x^y \log x}$

Differentiation Ex 11.5 Q56

Here $(\cos x)^y = (\cos y)^x$

Taking log on both sides,

$$\log(\cos x)^y = \log(\cos y)^x$$
$$y\log\cos x = x\log\cos y$$

Differentiating it with respect to x using the chain rule and product rule,

$$\frac{d}{dx}(y\log\cos x) = \frac{d}{dx}(x\log\cos y)$$

$$y\frac{d}{dx}\log\cos x + \log\cos x\frac{dy}{dx} = x\frac{d}{dx}\log\cos y + \log\cos y\frac{d}{dx}x$$

$$y\frac{1}{\cos x}(-\sin x) + \log\cos x\frac{dy}{dx} = x\frac{1}{\cos y}(-\sin y)\frac{dy}{dx} + \log\cos y$$

$$\left(\log\cos x + \frac{x\sin y}{\cos y}\right)\frac{dy}{dx} = \log\cos y + y\frac{\sin y}{\cos y}$$

$$(\log\cos x + x\tan y)\frac{dy}{dx} = \log\cos y + y\tan y$$

$$\frac{dy}{dx} = \frac{\log\cos y + y\tan y}{(\log\cos x + x\tan y)}$$

Differentiation Ex 11.5 Q57

Consider the given function,

$\cos y = x\cos(a+y)$, where $\cos a \neq \pm 1$

Differentiating both sides w.r.t. 'x' we get

$$-\sin y\frac{dy}{dx} = x\left(-\sin(a+y)\frac{dy}{dx}\right) + \cos(a+y)$$

$$\Rightarrow \frac{dy}{dx}[x\sin(a+y) - \sin y] = \cos(a+y)$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos(a+y)}{x\sin(a+y) - \sin y}$$

Multiplying the numerator and the denominator by $\cos(a+y)$ on the R.H.S., we have,

$$\frac{dy}{dx} = \frac{\cos^2(a+y)}{x\cos(a+y)\sin(a+y) - \cos(a+y)\sin y}$$

$$= \frac{\cos^2(a+y)}{\cos y\sin(a+y) - \cos(a+y)\sin y} \quad [\because \cos y = x\cos(a+y),\text{ given function}]$$

$$= \frac{\cos^2(a+y)}{\sin[(a+y) - y]} = \frac{\cos^2(a+y)}{\sin a}$$

Differentiation Ex 11.5 Q58

Consider the given function, $(x-y)e^{\frac{x}{x-y}} = a$.

We need to prove that $y\frac{dy}{dx} + x = 2y$.

Differentiating the given equation w.r.t. 'x' we get

$$(x-y)\left[e^{\frac{x}{x-y}}\left(\frac{(x-y) - x\left(1 - \frac{dy}{dx}\right)}{(x-y)^2}\right)\right] + e^{\frac{x}{x-y}}\left(1 - \frac{dy}{dx}\right) = 0$$

$$\Rightarrow \frac{(x-y) - x\left(1 - \frac{dy}{dx}\right)}{(x-y)} + \left(1 - \frac{dy}{dx}\right) = 0$$

$$\Rightarrow \left(1 - \frac{dy}{dx}\right)\left(1 - \frac{x}{x-y}\right) + 1 = 0$$

$$\Rightarrow \left(1 - \frac{dy}{dx}\right)\left(\frac{-y}{x-y}\right) + 1 = 0$$

$$\Rightarrow -y + y\frac{dy}{dx} + x - y = 0$$

$$\Rightarrow y\frac{dy}{dx} + x = 2y$$

Differentiation Ex 11.5 Q59

$x = e^{x/y}$

$\log x = \dfrac{x}{y} \quad \ldots\ldots(i)$

$y = \dfrac{x}{\log x}$

$\dfrac{dy}{dx} = \dfrac{\log x \dfrac{d}{dx}(x) - x\dfrac{d}{dx}(\log x)}{(\log x)^2}$

$\dfrac{dy}{dx} = \dfrac{\log x - x \times \dfrac{1}{x}}{(\log x)^2}$

$\dfrac{dy}{dx} = \dfrac{\log x - 1}{(\log x)^2}$

$\dfrac{dy}{dx} = \dfrac{\dfrac{x}{y} - 1}{(\log x)^2} \quad \ldots\ldots[\text{from (i)}]$

$\dfrac{dy}{dx} = \dfrac{x - y}{y(\log x)^2}$

$\dfrac{dy}{dx} = \dfrac{x - y}{x(\log x)} \quad \ldots\ldots[\text{from (i)}]$

Differentiation Ex 11.5 Q60

$y = x^{\tan x} + \sqrt{\dfrac{x^2 + 1}{2}}$

$y = e^{\tan x \log x} + e^{\frac{1}{2}\log\left(\frac{x^2+1}{2}\right)}$

$\dfrac{dy}{dx} = e^{\tan x \log x}\dfrac{d}{dx}(\tan x \log x) + e^{\frac{1}{2}\log\left(\frac{x^2+1}{2}\right)}\dfrac{d}{dx}\left(\dfrac{1}{2}\log\left(\dfrac{x^2+1}{2}\right)\right)$

$\dfrac{dy}{dx} = x^{\tan x}\left[\dfrac{\tan x}{x} + \sec^2 x \log x\right] + \sqrt{\dfrac{x^2+1}{2}}\left(\dfrac{1}{2} \times \dfrac{2}{x^2+1} \times (x)\right)$

$\dfrac{dy}{dx} = x^{\tan x}\left[\dfrac{\tan x}{x} + \sec^2 x \log x\right] + \sqrt{\dfrac{x^2+1}{2}}\left(\dfrac{x}{x^2+1}\right)$

$\dfrac{dy}{dx} = x^{\tan x}\left[\dfrac{\tan x}{x} + \sec^2 x \log x\right] + \dfrac{x}{\sqrt{2(x^2+1)}}$

Differentiation Ex 11.5 Q61

$y = 1 + \dfrac{\alpha}{\left(\dfrac{1}{x} - \alpha\right)} + \dfrac{\beta/x}{\left(\dfrac{1}{x} - \alpha\right)\left(\dfrac{1}{x} - \beta\right)} + \dfrac{\gamma/x^2}{\left(\dfrac{1}{x} - \alpha\right)\left(\dfrac{1}{x} - \beta\right)\left(\dfrac{1}{x} - \gamma\right)}$

$\left\{\begin{array}{l}\text{Using the theorem,}\\[2mm] \text{If } y = 1 + \dfrac{ax^2}{(x-a)(x-b)(x-c)} + \dfrac{bx}{(x-b)(x-c)} + \dfrac{c}{(x-c)} \text{ then,}\\[2mm] \dfrac{dy}{dx} = \dfrac{y}{x}\left\{\dfrac{a}{a-x} + \dfrac{b}{b-x} + \dfrac{c}{c-x}\right\}\end{array}\right\}$

Here we have $\dfrac{1}{x}$ instead of x.

So using above theorem we get,

$\dfrac{dy}{dx} = \dfrac{\alpha}{\left(\dfrac{1}{x} - \alpha\right)} + \dfrac{\beta}{\left(\dfrac{1}{x} - \beta\right)} + \dfrac{\gamma}{\left(\dfrac{1}{x} - \gamma\right)}$

Ex 11.6

Differentiation Ex 11.6 Q1

Here,

$$y = \sqrt{x + \sqrt{x + \sqrt{x + \ldots \text{ to } \infty}}}$$
$$y = \sqrt{x + y}$$

Squaring both the sides,

$$y^2 = x + y$$

Differentiating it with respect to x,

$$2y\,\frac{dy}{dx} = 1 + \frac{dy}{dx}$$
$$\frac{dy}{dx}(2y - 1) = 1$$
$$\frac{dy}{dx} = \frac{1}{2y - 1}$$

Differentiation Ex 11.6 Q2

Here,

$$y = \sqrt{\cos x + \sqrt{\cos x + \sqrt{\cos x + \ldots \text{to } \infty}}}$$
$$y = \sqrt{\cos x + y}$$

squaring both the sides,

$$y^2 = \cos x + y$$

Differntiating it with respect to x,

$$2y\frac{dy}{dx} = -\sin x + \frac{dy}{dx}$$
$$\frac{dy}{dx}(2y - 1) = -\sin x$$
$$\frac{dy}{dx} = \frac{-\sin x}{(2y - 1)}$$
$$\frac{dy}{dx} = \frac{\sin x}{1 - 2y}$$

Differentiation Ex 11.6 Q3

Here,

$$y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \ldots \text{to } \infty}}}$$

$$y = \sqrt{\log x + y}$$

Squaring both sides,

$$y^2 = \log x + y$$

Differentiating it with respect to x,

$$2y \frac{dy}{dx} = \frac{1}{x} + \frac{dy}{dx}$$

$$\frac{dy}{dx}(2y - 1) = \frac{1}{x}$$

Differentiation Ex 11.6 Q4

Here,

$$y = \sqrt{\tan x + \sqrt{\tan x + \sqrt{\tan x + \ldots \text{to } \infty}}}$$

$$y = \sqrt{\tan x + y}$$

Squaring both the sides,

$$y^2 = \tan x + y$$

Differentiating it with respect to x,

$$2y \frac{dy}{dx} = \sec^2 x + \frac{dy}{dx}$$

$$\frac{dy}{dx}(2y - 1) = \sec^2 x$$

$$\frac{dy}{dx} = \frac{\sec^2 x}{2y - 1}$$

Differentiation Ex 11.6 Q5

Here,

$$y = (\sin x)^{(\sin x)^{(\sin x)^{\cdots \infty}}}$$

$$\Rightarrow \quad y = (\sin x)^y$$

Taking log on both the sides,

$$\log y = \log(\sin x)^y$$
$$\log y = y(\log \sin x)$$

Differentiating it with respect to x, using product rule,

$$\frac{1}{y}\frac{dy}{dx} = y\frac{d}{dx}(\log \sin x) + \log \sin x \frac{dy}{dx}$$

$$\frac{1}{y}\frac{dy}{dx} = y\frac{1}{\sin x}\frac{d}{dx}(\sin x) + \log \sin x \frac{dy}{dx}$$

$$\frac{dy}{dx}\left(\frac{1}{y} - \log \sin x\right) = \frac{y}{\sin x}(\cot x)$$

$$\frac{dy}{dx}\left(\frac{1 - y\log \sin x}{y}\right) = y\cot x$$

$$\frac{dy}{dx} = \frac{y^2 \cot x}{(1 - y\log \sin x)}$$

Differentiation Ex 11.6 Q6

Here,

$$y = (\tan x)^{(\tan x)^{(\tan x)^{\cdots \infty}}}$$

$$y = (\tan x)^y$$

Taking log on both the sides,

$$\log y = \log(\tan x)^y$$
$$\log y = y\log \tan x$$

Differentiating with respect to x using product rule and chain rule,

$$\frac{1}{y}\frac{dy}{dx} = y\frac{d}{dx}\log \tan x + \log \tan \frac{dy}{dx}$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{y}{\tan x}\frac{d}{dx}(\tan x) + \log \tan x \frac{dy}{dx}$$

$$\frac{dy}{dx}\left(\frac{1}{y} - \log \tan x\right) = \frac{y}{\tan x}\sec^2 x$$

$$\left(\frac{dy}{dx}\right)_{x=\frac{\pi}{4}} = \frac{y\sec^2\left(\frac{\pi}{4}\right)}{\tan\left(\frac{\pi}{4}\right)} * \frac{y}{1 - y\log \tan\left(\frac{\pi}{4}\right)}$$

$$\left(\frac{dy}{dx}\right)_{\frac{\pi}{4}} = \frac{y^2\left(\sqrt{2}\right)^2}{1\left(1 - y\log \tan 1\right)}$$

$$= \frac{2(1)^2}{(1 - 0)}$$

$$\left(\frac{dy}{dx}\right)_{\frac{\pi}{4}} = 2$$

$$\begin{cases} \text{since,} \\ \\ (y)_{\frac{\pi}{4}} = \left(\tan\frac{\pi}{4}\right)^{\left(\tan\frac{\pi}{4}\right)^{\left(\tan\frac{\pi}{4}\right)^{\cdots \infty}}} \\ \Rightarrow y = (1)^\infty \\ \Rightarrow y = 1 \end{cases}$$

Differentiation Ex 11.6 Q7

Here,

$$y = e^{x^{e^x}} + x^{e^{e^x}} + e^{x^{x^e}}$$

$$y = u + v + w$$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} + \frac{dw}{dx} \qquad ---(i)$$

Were $u = e^{x^{e^x}}, v = x^{e^{e^x}}, w = e^{x^{x^e}}$

Now, $u = e^{x^{e^x}}$ $\qquad ---(ii)$

Taking log on both the sides,

$$\log x = \log e^{x^{e^x}}$$

$$\log x = x^{e^x} \log e$$

$$\log x = x^{e^x} \qquad ---(iii) \qquad \begin{cases} \text{since } \log e - 1, \\ \log a^b = b \log a \end{cases}$$

Taking log on both the sides,

$$\log \log x = \log x^{e^x}$$

$$\log \log x = e^x \log x$$

Differentiating it with respect to x,

$$\frac{1}{\log x} \frac{d}{dx} (\log x) = e^x \frac{d}{dx} (\log x) + \log x \frac{d}{dx} \left(e^x \right)$$

$$\frac{1}{\log x} \frac{1}{4} \frac{du}{dx} = \frac{e^x}{x} + e^x \log x$$

$$\frac{du}{dx} = 4 \log x \left[\frac{e^x}{x} + e^x \log x \right]$$

$$\frac{du}{dx} = e^{x^{e^x}} * x^{e^x} \left[\frac{e^x}{x} + e^x \log x \right] \qquad ---(A)$$

Using equqtion (ii) and (iii)
Now

$$v = x^{e^{e^x}} \qquad ---(iv)$$

Taking log on both the sides,

$$\log v = \log x^{e^{e^x}}$$

$$\log v = e^{e^x} \log x$$

Differentiating it with respect to x,

$$\frac{1}{v} \frac{dv}{dx} = e^{e^x} \frac{d}{dx} (\log x) + \log x \frac{d}{dx} \left(e^{e^x} \right)$$

$$\frac{1}{v} \frac{dv}{dx} = e^{e^x} \left(\frac{1}{x} \right) + \log x e^{e^x} \frac{d}{dx} \left(e^x \right)$$

$$\frac{dv}{dx} = v [e^{e^x} \left(\frac{1}{x} \right) + \log x e^{e^x} e^x]$$

$$\frac{dv}{dx} = x^{e^{e^x}} * e^{e^x} [\frac{1}{x} + e^x \log x] \qquad ---(B)$$

$$\{ \text{sinx using equation} (4) \}$$

Now, $w = e^{x^{x^e}}$ $\qquad ---(v)$

Taking log on both the sides,

$$\log w = \log e^{x^{x^e}}$$

$$\log w = x^{x^e} \log e$$

$$\log w = x^{x^e} \qquad ---(vi)$$

Taking log on both the sides,

$$\log \log w = \log x^{x^e}$$

$$\log \log w = x^e \log x$$

Differentiating it with respect to x,

$$\frac{1}{\log w}\frac{d}{dx}(\log w) = x^e \frac{d}{dx}(\log x) + \log x \frac{d}{dx}(x^e)$$

$$\frac{1}{\log w}\left(\frac{1}{w}\right)\frac{dw}{dx} = x^e\left(\frac{1}{x}\right) + \log e x^{e-1}$$

$$\frac{dw}{dx} = w \log w[x^{e-1} + e \log x x^{e-1}]$$

$$\frac{dw}{dx} = e^{x^e} x^{x^e} x^{e-1}(1 + e \log x) \qquad\qquad ---(C) \ \{\text{Using equation (v),(vi)}\}$$

Using equation $(A),(B)$ and (C) in equation (i),

$$\frac{dy}{dx} = e^{x^{e^x}} x^{e^x}[\frac{e^x}{x} + e^x \log x] + x^{e^x} e^{e^x}[\frac{1}{x} + e^x \log x]$$

$$+ e^{x^e} x^{x^e} x^{e-1}(1 + e \log x)$$

Differentiation Ex 11.6 Q8

Here,

$$y = (\cos x)^{(\cos x)^{(\cos x)^{-\infty}}}$$

$$y = (\cos x)^y$$

Taking log on both the sides,

$$\log y = \log(\cos x)^y$$

$$\log y = y \log(\cos x), \{\text{since } \log a^b = b \log a\}$$

Differentiating it with respect to x using product rule and chain rule,

$$\frac{1}{y}\frac{dy}{dx} = y \frac{d}{dx}\log(\cos x) + \log \cos x \frac{dy}{dx}$$

$$\frac{1}{y}\frac{dy}{dx} = y\left(\frac{1}{\cos x}\right)\frac{d}{dx}(\cos x) + \log \cos x \frac{dy}{dx}$$

$$\frac{dy}{dx}\left(\frac{1}{y} - \log \cos x\right) = \frac{y}{\cos x}(-\sin x)$$

$$\frac{dy}{dx}\left(\frac{1 - y \log \cos x}{y}\right) = -y \tan x$$

$$\frac{dy}{dx} = -\frac{y^2 \tan x}{(1 - y \log \cos x)}$$

Ex 11.7

Differentiation Ex 11.7 Q1

Given that $x = at^2$, $y = 2at$

So, $\dfrac{dx}{dt} = \dfrac{d}{dt}\left(at^2\right) = 2at$

$\dfrac{dy}{dt} = \dfrac{d}{dt}\left(2at\right) = 2a$

Therefore, $\dfrac{dy}{dx} = \dfrac{\dfrac{dy}{dt}}{\dfrac{dx}{dt}} = \dfrac{2a}{2at} = \dfrac{1}{t}$

Differentiation Ex 11.7 Q2

Here,

$x = a\left(\theta + \sin\theta\right)$

Differentiating it with respect to θ,

$\dfrac{dx}{d\theta} = a\left(1 + \cos\theta\right)$ ---(i)

And,

$y = a\left(1 - \cos\theta\right)$

Differentiating it with respect to θ,

$\dfrac{dy}{d\theta} = a\left(\theta + \sin\theta\right)$

$\dfrac{dy}{d\theta} = a\sin\theta$ ---(ii)

Using equation (i) and (ii),

$\dfrac{dy}{dx} = \dfrac{\dfrac{dy}{d\theta}}{\dfrac{dx}{d\theta}}$

$= \dfrac{a\sin\theta}{a\left(1 - \cos\theta\right)}$

$= \dfrac{2\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2}}{2\sin^2\dfrac{\theta}{2}}$,

$\left\{\begin{array}{l} \text{Since, } 1 - \cos\theta = \dfrac{2\sin^{2\frac{\theta}{2}}}{2}, \\ \dfrac{2\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2}}{2} = \sin\theta \end{array}\right\}$

$= \dfrac{dy}{dx} = \dfrac{\tan\theta}{2}$

Differentiation Ex 11.7 Q3

Here $x = a\cos\theta$ and $y = b\sin\theta$

Then,

$\dfrac{dx}{d\theta} = \dfrac{d}{d\theta}\left(a\cos\theta\right) = -a\sin\theta$

$\dfrac{dy}{d\theta} = \dfrac{d}{d\theta}\left(b\sin\theta\right) = b\cos\theta$

$\therefore \dfrac{dy}{dx} = \dfrac{\dfrac{dy}{d\theta}}{\dfrac{dx}{d\theta}} = \dfrac{b\cos\theta}{-a\sin\theta} = -\dfrac{b}{a}\cot\theta$

Differentiation Ex 11.7 Q4

Here,

$$x = ae^\theta \left(\sin\theta - \cos\theta\right)$$

Differentiating it with respect to θ,

$$\frac{dx}{d\theta} = a\left[e^\theta \frac{d}{d\theta}\left(\sin\theta - \cos\theta\right) + \left(\sin\theta - \cos\theta\right)\frac{d}{d\theta}\left(e^\theta\right)\right]$$

$$= a\left[e^\theta \left(\cos\theta + \sin\theta\right) + \left(\sin\theta - \cos\theta\right)e^\theta\right]$$

$$\frac{dx}{d\theta} = a\left[2e^\theta \sin\theta\right] \qquad\qquad\text{---(i)}$$

And, $y = ae^\theta \left(\sin\theta + \cos\theta\right)$

Differentiating it with respect to θ,

$$\frac{dy}{d\theta} = a\left[e^\theta \frac{d}{d\theta}\left(\sin\theta + \cos\theta\right) + \left(\sin\theta + \cos\theta\right)\frac{d}{d\theta}\left(e^\theta\right)\right]$$

$$= a\left[e^\theta \left(\cos\theta - \sin\theta\right) + \left(\sin\theta + \cos\theta\right)e^\theta\right]$$

$$\frac{dy}{d\theta} = a\left[2e^\theta \cos\theta\right] \qquad\qquad\text{---(ii)}$$

Dividing equation (ii) by equation (i),

$$\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a\left(2e^\theta \cos\theta\right)}{a\left(2e^\theta \sin\theta\right)}$$

$$\frac{dy}{dx} = \cot\theta$$

Differentiation Ex 11.7 Q5

Here $x = b\sin^2\theta$ and $y = a\cos^2\theta$

Then,

$$\frac{dx}{d\theta} = \frac{d}{d\theta}\left(b\sin^2\theta\right) = 2b\sin\theta\cos\theta$$

$$\frac{dy}{d\theta} = \frac{d}{d\theta}\left(a\cos^2\theta\right) = -2a\cos\theta\sin\theta$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-2a\cos\theta\sin\theta}{2b\sin\theta\cos\theta} = -\frac{a}{b}$$

Differentiation Ex 11.7 Q6

Here $x = a(1 - \cos\theta)$ and $y = a(\theta + \sin\theta)$

Then,

$$\frac{dx}{d\theta} = \frac{d}{d\theta}\left[a(1 - \cos\theta)\right] = a(\sin\theta)$$

$$\frac{dy}{d\theta} = \frac{d}{d\theta}\left[a(\theta + \sin\theta)\right] = a(1 + \cos\theta)$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a(1 + \cos\theta)}{a(\sin\theta)}\Bigg|_{\theta=\frac{\pi}{2}} = \frac{a(1 + 0)}{a} = 1$$

Differentiation Ex 11.7 Q7

Here,

$$x = \frac{e^t + e^{-t}}{2}$$

Differentiating it with respect to t,

$$\frac{dx}{dt} = \frac{1}{2}\left[\frac{d}{dt}\left(e^t\right) + \frac{d}{dt}\left(e^{-t}\right)\right]$$

$$= \frac{1}{2}\left[e^t + e^{-t}\frac{d}{dt}(-t)\right]$$

$$\frac{dx}{dt} = \frac{1}{2}\left(e^t - e^{-t}\right) = y \qquad ---(i)$$

And, $\qquad y = \dfrac{e^t - e^{-t}}{2}$

Differentiating it with respect to t,

$$\frac{dy}{dt} = \frac{1}{2}\left[\frac{d}{dt}\left(e^t\right) - \frac{d}{dt}e^{-t}\right]$$

$$= \frac{1}{2}\left[e^t - e^{-t}\frac{d}{dt}\left(e^{-t}\right)\right]$$

$$= \frac{1}{2}\left(e^t - e^{-t}(-1)\right)$$

$$\frac{dy}{dt} = \frac{1}{2}\left(e^t + e^{-t}\right) = x \qquad ---(ii)$$

Dividing equation (ii) by (i),

$$\frac{\dfrac{dy}{dt}}{\dfrac{dx}{dt}} = \frac{x}{y}$$

$$\frac{dy}{dt} = \frac{x}{y}$$

Differentiation Ex 11.7 Q8

Here,

$$x = \frac{3at}{1+t^2}$$

Differentiating it with respect to t using quotient rule,

$$\frac{dx}{dt} = \left[\frac{\left(1+t^2\right)\frac{d}{dt}\left(3at\right) - 3at\frac{d}{dt}\left(1+t^2\right)}{\left(1+t^2\right)^2}\right]$$

$$= \left[\frac{\left(1+t^2\right)\left(3a\right) - 3at\left(2t\right)}{\left(1+t^2\right)^2}\right]$$

$$= \left[\frac{3a + 3at^2 - 6at^2}{\left(1+t^2\right)^2}\right]$$

$$= \left[\frac{3a - 3at^2}{\left(1-t^2\right)^2}\right]$$

$$\frac{dx}{dt} = \frac{3a\left(1-t^2\right)}{\left(1+t^2\right)^2} \qquad ---(i)$$

And, $\quad y = \frac{3at^2}{1+t^2}$

Differentiating it with respect to t using quotient rule,

$$\frac{dy}{dt} = \left[\frac{\left(1+t^2\right)\frac{d}{dt}\left(3at^2\right) - 3at^2\frac{d}{dt}\left(1+t^2\right)}{\left(1+t^2\right)^2}\right]$$

$$\frac{dy}{dt} = \left[\frac{\left(1+t^2\right)\left(6at\right) - \left(3at^2\right)\left(2t\right)}{\left(1+t^2\right)^2}\right]$$

$$= \left[\frac{6at + 6at^3 - 6at^3}{\left(1+t^2\right)^2}\right]$$

$$\frac{dy}{dt} = \frac{6at}{\left(1+t^2\right)^2} \qquad ---(ii)$$

Dividing equation (ii) by (i),

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{6at}{\left(1+t^2\right)^2} \times \frac{\left(1+t^2\right)^2}{3a\left(1-t^2\right)}$$

$$\frac{dy}{dx} = \frac{2t}{1-t^2}$$

Differentiation Ex 11.7 Q9

The given equations are $x = a\left(\cos\theta + \theta\sin\theta\right)$ and $y = a\left(\sin\theta - \theta\cos\theta\right)$

Then, $\frac{dx}{d\theta} = a\left[\frac{d}{d\theta}\cos\theta + \frac{d}{d\theta}\left(\theta\sin\theta\right)\right] = a\left[-\sin\theta + \theta\frac{d}{d\theta}\left(\sin\theta\right) + \sin\theta\frac{d}{d\theta}\left(\theta\right)\right]$

$$= a\left[-\sin\theta + \theta\cos\theta + \sin\theta\right] = a\theta\cos\theta$$

$$\frac{dy}{d\theta} = a\left[\frac{d}{d\theta}\left(\sin\theta\right) - \frac{d}{d\theta}\left(\theta\cos\theta\right)\right] = a\left[\cos\theta - \left\{\theta\frac{d}{d\theta}\left(\cos\theta\right) + \cos\theta \cdot \frac{d}{d\theta}\left(\theta\right)\right\}\right]$$

$$= a\left[\cos\theta + \theta\sin\theta - \cos\theta\right]$$

$$= a\theta\sin\theta$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \frac{a\theta\sin\theta}{a\theta\cos\theta} = \tan\theta$$

Differentiation Ex 11.7 Q10

Here,

$$x = e^\theta \left(\theta + \frac{1}{\theta}\right)$$

Differentiating it with respect to θ using product rule,

$$\frac{dx}{d\theta} = e^\theta \frac{d}{d\theta}\left(\theta + \frac{1}{\theta}\right) + \left(\theta + \frac{1}{\theta}\right)\frac{d}{d\theta}\left(e^\theta\right)$$

$$= e^\theta \left(1 - \frac{1}{\theta^2}\right) + \left(\frac{\theta^2 + 1}{\theta}\right)e^\theta$$

$$= e^\theta \left(1 - \frac{1}{\theta^2} + \frac{\theta^2 + 1}{\theta}\right)$$

$$= e^\theta \left(\frac{\theta^2 - 1 + \theta^3 + \theta}{\theta^2}\right)$$

$$\frac{dx}{d\theta} = \frac{e^\theta \left(\theta^3 + \theta^2 + \theta - 1\right)}{\theta^2} \qquad\qquad ---(i)$$

And, $y = e^\theta \left(\theta - \frac{1}{\theta}\right)$

Differentiating it with respect to θ using product rule and chain rule,

$$\frac{dy}{d\theta} = e^{-\theta} \frac{d}{d\theta}\left(\theta - \frac{1}{\theta}\right) + \left(\theta - \frac{1}{\theta}\right)\frac{d}{d\theta}\left(e^{-\theta}\right)$$

$$= e^{-\theta}\left(1 + \frac{1}{\theta^2}\right) + \left(\theta - \frac{1}{\theta}\right)e^{-\theta}\frac{d}{d\theta}(-\theta)$$

$$= e^{-\theta}\left(1 + \frac{1}{\theta^2}\right) + \left(\theta - \frac{1}{\theta}\right)e^{-\theta}(-1)$$

$$\frac{dy}{d\theta} = e^{-\theta}\left[1 + \frac{1}{\theta^2} - \theta + \frac{1}{\theta}\right]$$

$$= e^{-\theta}\left[\frac{\theta^2 + 1 - \theta^3 + \theta}{\theta^2}\right]$$

$$\frac{dy}{d\theta} = e^{-\theta}\left[\frac{-\theta^3 + \theta^2 + \theta + 1}{\theta^2}\right] \qquad\qquad ---(ii)$$

Differentiation Ex 11.7 Q11

Here,

$$x = \frac{2t}{1+t^2}$$

Differentiating it with respect to t using quotient rule,

$$\frac{dy}{dx} = \left[\frac{\left(1+t^2\right)\frac{d}{dt}(2t) - 2t\frac{d}{dt}\left(1+t^2\right)}{\left(1+t^2\right)^2} \right]$$

$$= \left[\frac{\left(1+t^2\right)(2) - 2t(2t)}{\left(1+t^2\right)^2} \right]$$

$$= \left[\frac{2+2t^2-4t^2}{\left(1+t^2\right)^2} \right]$$

$$= \left[\frac{2-2t^2}{\left(1+t^2\right)^2} \right]$$

$$\frac{dx}{dt} = \frac{2\left(1-t^2\right)}{\left(1+t^2\right)^2} \qquad\qquad ---(i)$$

And, $\quad y = \dfrac{1-t^2}{1+t^2}$

Differentiating it with respect to t using quotient rule,

$$\frac{dy}{dt} = \left[\frac{\left(1+t^2\right)\frac{d}{dt}\left(1-t^2\right) - \left(1-t^2\right)\frac{d}{dt}\left(1+t^2\right)}{\left(1+t^2\right)^2} \right]$$

$$= \left[\frac{\left(1+t^2\right)(-2t) - \left(1-t^2\right)(2t)}{\left(1+t^2\right)^2} \right]$$

$$= \left[\frac{-2t-2t^3-2t+2t^3}{\left(1+t^2\right)^2} \right]$$

$$\frac{dy}{dt} = \left[\frac{-4t}{\left(1+t^2\right)^2} \right] \qquad\qquad ---(ii)$$

Dividing equation (ii) by (i),

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-4t}{\left(1+t^2\right)^2} \times \frac{\left(1+t^2\right)^2}{2\left(1-t^2\right)}$$

$$= \frac{-2t}{1-t^2}$$

$$\frac{dy}{dx} = -\frac{x}{y} \qquad\qquad \left[\text{Sicne, } \frac{x}{y} = \frac{2t}{1+t^2} \times \frac{1+t^2}{1-t^2} = \frac{2t}{1-t^2} \right]$$

Differentiation Ex 11.7 Q12

Here,

$$x = \cos^{-1}\left(\frac{1}{\sqrt{1+t^2}}\right)$$

Differentiating it with respect to t using chain rule,

$$\frac{dx}{dt} = \frac{-1}{\sqrt{1-\left(\frac{1}{1+t^2}\right)^2}}\frac{d}{dt}\left(\frac{1}{\sqrt{1+t^2}}\right)$$

$$= \frac{-1}{\sqrt{1-\frac{1}{(1+t^2)}}}\left\{-\frac{1}{2(1+t^2)^{\frac{3}{2}}}\right\}\frac{d}{dt}(1+t^2)$$

$$= \frac{(1+t^2)^{\frac{1}{2}}}{\sqrt{1+t^2-1}} \times \frac{-1}{2(1+t^2)^{\frac{3}{2}}}(2t)$$

$$= \frac{-t}{\sqrt{t^2}\times(1+t^2)}$$

$$\frac{dx}{dt} = \frac{-1}{1+t^2} \qquad\qquad ---(i)$$

Now, $\quad y = \sin^{-1}\left(\frac{1}{\sqrt{1+t^2}}\right)$

Differentiating it with respect to t using chain rule,

$$\frac{dy}{dt} = \frac{1}{\sqrt{1-\frac{1}{\left(\sqrt{1+t^2}\right)^2}}} \times \frac{d}{dt}\left(\frac{1}{\sqrt{1+t^2}}\right)$$

$$= \frac{(1+t^2)^{\frac{1}{2}}}{\sqrt{1+t^2-1}} \times \left(\frac{-1}{2(1+t^2)^{\frac{3}{2}}}\right)\frac{d}{dt}(1+t^2)$$

$$= \frac{-1}{2\sqrt{t^2}(1+t^2)} \times (2t)$$

$$\frac{dy}{dt} = \frac{-1}{(1+t^2)} \qquad\qquad ---(ii)$$

Dividing equation (ii) by (i),

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = -\frac{1}{(1+t^2)} \times \frac{(1+t^2)}{-1}$$

$$\frac{dy}{dx} = 1$$

Differentiation Ex 11.7 Q13

Here,

$$x = \frac{1 - t^2}{1 + t^2}$$

Differentiating it with respect to t using quotient rule,

$$\frac{dx}{dt} = \left[\frac{(1 + t^2)\frac{d}{dt}(1 - t^2) - (1 - t^2)\frac{d}{dt}(1 + t^2)}{(1 + t^2)^2} \right]$$

$$= \left[\frac{(1 + t^2)(-2t) - (1 - t^2)(2t)}{(1 + t^2)^2} \right]$$

$$= \left[\frac{-2t - 2t^3 - 2t + 2t^3}{(1 + t^2)^2} \right]$$

$$\frac{dx}{dt} = \left(\frac{-4t}{(1 + t^2)^2} \right) \qquad \text{---(i)}$$

And, $y = \frac{2t}{1 + t^2}$

Differentiating it with respect to t using quotient rule,

$$\frac{dy}{dt} = \left[\frac{(1 + t^2)\frac{d}{dt}(2t) - (2t)\frac{d}{dt}(1 + t^2)}{(1 + t^2)^2} \right]$$

$$\frac{dy}{dt} = \left[\frac{(1 + t^2)(2) - (2t)(2t)}{(1 + t^2)^2} \right]$$

$$= \left[\frac{2 + 2t^2 - 4t^2}{(1 + t^2)^2} \right]$$

$$\frac{dy}{dt} = \frac{2(1 - t^2)}{(1 + t^2)^2} \qquad \text{---(ii)}$$

Differentiation Ex 11.7 Q14

Here, x= 2cosθ − cos2θ

Differentiating it with respect to θ using chain rule,

$$\frac{dx}{d\theta} = 2\left(-\sin\theta\right) - \left(-\sin2\theta\right)\frac{d}{d\theta}(2\theta)$$
$$= -2\sin\theta + 2\sin2\theta$$
$$\frac{dx}{d\theta} = 2\left(\sin2\theta - \sin\theta\right) \qquad \text{---(i)}$$

And, $y = 2\sin\theta - \sin2\theta$

Differentiating it with respect to θ using chain rule,

$$\frac{dy}{d\theta} = 2\cos\theta - \cos2\theta\frac{d}{d\theta}(2\theta)$$
$$= 2\cos\theta - \cos2\theta\,(2)$$
$$= 2\cos\theta - 2\cos2\theta$$
$$\frac{dy}{d\theta} = 2\left(\cos\theta - \cos2\theta\right) \qquad \text{---(ii)}$$

Dividing equation (ii) by equation (i),

$$\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{2\left(\cos\theta - \cos2\theta\right)}{2\left(\sin2\theta - \sin\theta\right)}$$

$$= \frac{\cos\theta - \cos2\theta}{\sin2\theta - \sin\theta}$$

$$\frac{dy}{dx} = \frac{-2\sin\left(\frac{\theta+2\theta}{2}\right)\sin\left(\frac{\theta-2\theta}{2}\right)}{2\cos\left(\frac{2\theta+\theta}{2}\right)\sin\left(\frac{2\theta-\theta}{2}\right)}$$

$$\left[\begin{array}{l} \text{Since, } \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right) \\[2mm] \cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right) \end{array}\right]$$

$$= \frac{-\sin\left(\frac{3\theta}{2}\right)\left(\sin\left(\frac{-\theta}{2}\right)\right)}{\cos\left(\frac{3\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)}$$

$$= \frac{-\sin\left(\frac{3\theta}{2}\right)\left(-\sin\frac{\theta}{2}\right)}{\cos\left(\frac{3\theta}{2}\right)\left(\sin\frac{\theta}{2}\right)}$$

$$= \frac{\sin\left(\frac{3\theta}{2}\right)}{\cos\left(\frac{3\theta}{2}\right)}$$

$$\frac{dy}{dx} = \tan\left(\frac{3\theta}{2}\right)$$

Differentiation Ex 11.7 Q15

Here,

$$x = e^{\cos 2t}$$

Differentiating it with respect to t using chain rule,

$$\frac{dx}{dt} = \frac{d}{dt}\left(e^{\cos 2t}\right)$$

$$= e^{\cos 2t}\frac{d}{dt}(\cos 2t)$$

$$= e^{\cos 2t}(-\sin 2t)\frac{d}{dt}(2t)$$

$$= -\sin 2t\, e^{\cos 2t}(2)$$

$$\frac{dx}{dt} = -2\sin 2t\, e^{\cos 2t} \qquad\qquad ---(i)$$

And, $y = e^{\sin 2t}$

Differentiating it with respect to t using chain rule,

$$\frac{dy}{dt} = \frac{d}{dt}\left(e^{\sin 2t}\right)$$

$$= e^{\sin 2t}\frac{d}{dt}(\sin 2t)$$

$$= e^{\sin 2t}(\cos 2t)\frac{d}{dt}(2t)$$

$$= e^{\sin 2t}(\cos 2t)(2)$$

$$\frac{dy}{dt} = 2\cos 2t\, e^{\sin 2t} \qquad\qquad ---(ii)$$

Dividing equation (ii) by (i),

$$\frac{\dfrac{dy}{dt}}{\dfrac{dx}{dt}} = \frac{2\cos 2t\, e^{\sin 2t}}{-2\sin 2t\, e^{\cos 2t}}$$

$$\frac{dy}{dx} = -\frac{y\log x}{x\log y} \qquad\qquad \left[\begin{array}{l}\text{Since, } x = e^{\cos 2t} \Rightarrow \log x = \cos 2t \\ \qquad\quad y = e^{\sin 2t} \Rightarrow \log y = \sin 2t\end{array}\right]$$

Differentiation Ex 11.7 Q16

Here,

$$x = \cos t$$

Differentiating it with respect to t,

$$\frac{dx}{dt} = \frac{d}{dt}(\cos t)$$

$$\frac{dx}{dt} = -\sin t \qquad\qquad ---(i)$$

and, $y = \sin t$

Differentiating it with respect to t,

$$\frac{dy}{dt} = \frac{d}{dt}(\sin t)$$

$$\frac{dy}{dt} = \cos t \qquad\qquad ---(ii)$$

Dividing equation (ii) by (i),

$$\frac{\dfrac{dy}{dt}}{\dfrac{dx}{dt}} = \frac{\cos t}{-\sin t}$$

$$\frac{dy}{dx} = -\cot t$$

$$\left(\frac{dy}{dx}\right) = -\cot\left(\frac{2\pi}{3}\right)$$

$$= -\cot\left(\pi - \frac{\pi}{3}\right)$$

$$= -\left[-\cot\left(\frac{\pi}{3}\right)\right]$$

$$= \cot\left(\frac{\pi}{3}\right)$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{3}}$$

Differentiation Ex 11.7 Q17

Here,

$$x = a\left(t + \frac{1}{t}\right)$$

Differentiating it with respect to t,

$$\frac{dx}{dt} = a\frac{d}{dt}\left(t + \frac{1}{t}\right)$$

$$= a\left(1 - \frac{1}{t^2}\right)$$

$$\frac{dx}{dt} = a\left(\frac{t^2 - 1}{t^2}\right) \qquad\qquad ---(i)$$

And, $y = a\left(t - \frac{1}{t}\right)$

Differentiating it with respect to t,

$$\frac{dy}{dt} = a\frac{d}{dt}\left(t - \frac{1}{t}\right)$$

$$= a\left(1 + \frac{1}{t^2}\right)$$

$$\frac{dy}{dt} = a\left(\frac{t^2 + 1}{t^2}\right) \qquad\qquad ---(ii)$$

Dividing equation (ii) by (i),

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = a\frac{\left(t^2 + 1\right)}{t^2} \times \frac{t^2}{a\left(t^2 - 1\right)}$$

$$\frac{dy}{dx} = \frac{t^2 + 1}{t^2 - 1}$$

$$\frac{dy}{dx} = \frac{x}{y} \qquad\qquad \left[\text{Since, } \frac{x}{y} = \frac{a\left(t^2 + 1\right)}{t} \times \frac{t}{a\left(t^2 - 1\right)} = \left(\frac{t^2 + 1}{t^2 - 1}\right)\right]$$

Differentiation Ex 11.7 Q18

Here,

$$x = \sin^{-1}\left(\frac{2t}{1+t^2}\right)$$

Put $t = \tan\theta$

$$x = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$$

$$= \sin^{(1)}(\sin 2\theta)$$

$$= 2\theta \qquad\qquad \left[\text{Since, } \sin 2x = \frac{2\tan x}{1+\tan^2 x}\right]$$

$$x = 2\left(\tan^{-1}t\right) \qquad\qquad \left[\text{Since, } t = \sin\theta\right]$$

Differentiating it with respect to t,

$$\frac{dx}{dt} = \frac{2}{1+t^2} \qquad\qquad ---(i)$$

Now,

$$y = \tan^{-1}\left(\frac{2t}{1-t^2}\right)$$

Put $t = \tan\theta$

$$y = \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)$$

$$= \tan^{-1}(\tan 2\theta) \qquad\qquad \left[\text{Sicne, } \tan 2x = \frac{2\tan x}{1-\tan^2 x}\right]$$

$$= 2\theta$$

$$y = 2\tan^{-1}t \qquad\qquad \left[\text{Since, } t = \tan\theta\right]$$

Differentiating it with respect to t,

$$\frac{dy}{dt} = \frac{2}{1+t^2} \qquad\qquad ---(ii)$$

Dividing equation (ii) by (i),

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2}{1+t^2} \times \frac{1+t^2}{2}$$

$$\frac{dy}{dx} = 1$$

Differentiation Ex 11.7 Q19

The given equations are $x = \dfrac{\sin^3 t}{\sqrt{\cos 2t}}$ and $y = \dfrac{\cos^3 t}{\sqrt{\cos 2t}}$

Then, $\dfrac{dx}{dt} = \dfrac{d}{dt}\left[\dfrac{\sin^3 t}{\sqrt{\cos 2t}}\right]$

$$= \frac{\sqrt{\cos 2t}\cdot\dfrac{d}{dt}\left(\sin^3 t\right) - \sin^3 t\cdot\dfrac{d}{dt}\sqrt{\cos 2t}}{\cos 2t}$$

$$= \frac{\sqrt{\cos 2t}\cdot 3\sin^2 t\cdot\dfrac{d}{dt}\left(\sin t\right) - \sin^3 t\times\dfrac{1}{2\sqrt{\cos 2t}}\cdot\dfrac{d}{dt}\left(\cos 2t\right)}{\cos 2t}$$

$$= \frac{3\sqrt{\cos 2t}\cdot\sin^2 t\cos t - \dfrac{\sin^3 t}{2\sqrt{\cos 2t}}\cdot(-2\sin 2t)}{\cos 2t}$$

$$= \frac{3\cos 2t\sin^2 t\cos t + \sin^3 t\sin 2t}{\cos 2t\sqrt{\cos 2t}}$$

$$\frac{dy}{dt} = \frac{d}{dt}\left[\frac{\cos^3 t}{\sqrt{\cos 2t}}\right]$$

$$= \frac{\sqrt{\cos 2t}.\frac{d}{dt}\left(\cos^3 t\right) - \cos^3 t \cdot \frac{d}{dt}\left(\sqrt{\cos 2t}\right)}{\cos 2t}$$

$$= \frac{\sqrt{\cos 2t}.3\cos^2 t \cdot \frac{d}{dt}\left(\cos t\right) - \cos^3 t.\frac{1}{2\sqrt{\cos 2t}} \cdot \frac{d}{dt}\left(\cos 2t\right)}{\cos 2t}$$

$$= \frac{3\sqrt{\cos 2t}.\cos^2 t\left(-\sin t\right) - \cos^3 t \cdot \frac{1}{2\sqrt{\cos 2t}}.\left(-2\sin 2t\right)}{\cos 2t}$$

$$= \frac{-3\cos 2t.\cos^2 t.\sin t + \cos^3 t \sin 2t}{\cos 2t \cdot \sqrt{\cos 2t}}$$

$$\therefore \frac{dy}{dx} = \frac{\left(\dfrac{dy}{dt}\right)}{\left(\dfrac{dx}{dt}\right)} = \frac{-3\cos 2t.\cos^2 t.\sin t + \cos^3 t \sin 2t}{3\cos 2t \sin^2 t \cos t + \sin^3 t \sin 2t}$$

$$= \frac{-3\cos 2t.\cos^2 t.\sin t + \cos^3 t\left(2\sin t \cos t\right)}{3\cos 2t \sin^2 t \cos t + \sin^3 t\left(2\sin t \cos t\right)}$$

$$= \frac{\sin t \cos t\left[-3\cos 2t.\cos t + 2\cos^3 t\right]}{\sin t \cos t\left[3\cos 2t \sin t + 2\sin^3 t\right]}$$

$$= \frac{\left[-3\left(2\cos^2 t - 1\right)\cos t + 2\cos^3 t\right]}{\left[3\left(1 - 2\sin^2 t\right)\sin t + 2\sin^3 t\right]} \qquad \begin{bmatrix} \cos 2t = \left(2\cos^2 t - 1\right), \\ \cos 2t = \left(1 - 2\sin^2 t\right) \end{bmatrix}$$

$$= \frac{-4\cos^3 t + 3\cos t}{3\sin t - 4\sin^3 t}$$

$$= \frac{-\cos 3t}{\sin 3t} \qquad \begin{bmatrix} \cos 3t = 4\cos^3 t - 3\cos t, \\ \sin 3t = 3\sin t - 4\sin^3 t \end{bmatrix}$$

$$= -\cot 3t$$

Differentiation Ex 11.7 Q20

Here,

$$x = \left(t + \frac{1}{t}\right)^a$$

Differentiating it with respect to t using chain rule,

$$\frac{dx}{dt} = \frac{d}{dt}\left(\left(t + \frac{1}{t}\right)^a\right)$$

$$= a\left(t + \frac{1}{t}\right)^{a-1} \frac{d}{dt}\left(t + \frac{1}{t}\right)$$

$$\frac{dx}{dt} = a\left(t + \frac{1}{t}\right)^{a-1}\left(1 - \frac{1}{t^2}\right) \qquad\qquad \text{--- (i)}$$

And, $\quad y = a^{\left(t + \frac{1}{t}\right)}$

Differentiating it with respect to t using chain rule,

$$\frac{dy}{dt} = \frac{d}{dt}\left(a^{\left(t + \frac{1}{t}\right)}\right)$$

$$= a^{\left(t + \frac{1}{t}\right)} \times \log a \frac{d}{dt}\left(t + \frac{1}{t}\right)$$

$$\frac{dy}{dt} = a^{\left(t + \frac{1}{t}\right)} \times \log a\left(1 - \frac{1}{t^2}\right) \qquad\qquad \text{--- (ii)}$$

Dividing equation (ii) by (i),

$$\frac{\dfrac{dy}{dt}}{\dfrac{dx}{dt}} = \frac{a^{\left(t + \frac{1}{t}\right)} \times \log a\left(1 - \frac{1}{t^2}\right)}{a\left(t + \frac{1}{t}\right)^{a-1}\left(1 - \frac{1}{t^2}\right)}$$

$$\frac{dy}{dx} = \frac{a^{\left(t + \frac{1}{t}\right)} \times \log a}{a\left(t + \frac{1}{t}\right)^{a-1}}$$

Differentiation Ex 11.7 Q21

Here,

$$x = a\left(\dfrac{1+t^2}{1-t^2}\right)$$

Differentiating it with respect to t using chain rule,

$$\dfrac{dx}{dt} = a\left[\dfrac{\left(1+t^2\right)\dfrac{d}{dt}\left(1+t^2\right)-\left(1+t^2\right)\dfrac{d}{dt}\left(1-t^2\right)}{\left(1-t^2\right)^2}\right]$$

$$= a\left[\dfrac{\left(1-t^2\right)(2t)-\left(1+t^2\right)(-2t)}{\left(1-t^2\right)^2}\right]$$

$$= a\left[\dfrac{2t-2t^2+2t+2t^3}{\left(1-t^2\right)^2}\right]$$

$$\dfrac{dy}{dt} = \dfrac{4at}{\left(1-t^2\right)^2} \qquad \text{---(i)}$$

And, $\quad y = \dfrac{2t}{1-t^2}$

Differentiating it with respect to t using quotient rule,

$$\dfrac{dy}{dt} = 2\left[\dfrac{\left(1-t^2\right)\dfrac{d}{dt}(t)-t\dfrac{d}{dt}\left(1-t^2\right)}{\left(1-t^2\right)^2}\right]$$

$$= 2\left[\dfrac{\left(1-t^2\right)(1)-t(-2t)}{\left(1-t^2\right)^2}\right]$$

$$= 2\left[\dfrac{1-t^2+2t^2}{\left(1-t^2\right)^2}\right]$$

$$\dfrac{dy}{dt} = \dfrac{2\left(1+t^2\right)}{\left(1-t^2\right)} \qquad \text{---(ii)}$$

Differentiation Ex 11.7 Q22

It is given that, $y = 12(1-\cos t), x = 10(t-\sin t)$

$$\therefore \dfrac{dx}{dt} = \dfrac{d}{dt}\left[10(t-\sin t)\right] = 10\cdot\dfrac{d}{dt}(t-\sin t) = 10(1-\cos t)$$

$$\dfrac{dy}{dt} = \dfrac{d}{dt}\left[12(1-\cos t)\right] = 12\cdot\dfrac{d}{dt}(1-\cos t) = 12\cdot\left[0-(-\sin t)\right] = 12\sin t$$

$$\therefore \dfrac{dy}{dx} = \dfrac{\left(\dfrac{dy}{dt}\right)}{\left(\dfrac{dx}{dt}\right)} = \dfrac{12\sin t}{10(1-\cos t)} = \dfrac{12\cdot 2\sin\dfrac{t}{2}\cdot\cos\dfrac{t}{2}}{10\cdot 2\sin^2\dfrac{t}{2}} = \dfrac{6}{5}\cot\dfrac{t}{2}$$

Differentiation Ex 11.7 Q23

Here $x = a(\theta-\sin\theta)$ and $y = a(1+\cos\theta)$

Then,

$$\dfrac{dx}{d\theta} = \dfrac{d}{d\theta}\left[a(\theta-\sin\theta)\right] = a(1-\cos\theta)$$

$$\dfrac{dy}{d\theta} = \dfrac{d}{d\theta}\left[a(1+\cos\theta)\right] = a(-\sin\theta)$$

$$\therefore \dfrac{dy}{dx} = \dfrac{\dfrac{dy}{d\theta}}{\dfrac{dx}{d\theta}} = \dfrac{-a\sin\theta}{a(1-\cos\theta)}\Bigg|_{\theta=\frac{\pi}{3}} = -\dfrac{\sin\dfrac{\pi}{3}}{1-\cos\dfrac{\pi}{3}} = \dfrac{\dfrac{\sqrt{3}}{2}}{1-\dfrac{1}{2}} = -\sqrt{3}$$

Differentiation Ex 11.7 Q24

Consider the given functions,

$x = a \sin 2t \ (1 + \cos 2t)$ and $y = b \cos 2t \ (1 - \cos 2t)$

Rewriting the above function, we have,

$x = a \sin 2t + \dfrac{a}{2}\sin 4t$

Differentiating the above function w.r.t. 't', we have,

$\dfrac{dx}{dt} = 2a\cos 2t + 2a\cos 4t \dots (1)$

$y = b \cos 2t \ (1 - \cos 2t)$

$y = b \cos 2t - b\cos^2 2t$

$\dfrac{dy}{dt} = -2b\sin 2t + 2b\cos 2t \sin 2t = -2b\sin 2t + b\sin 4t \dots (2)$

From (1) and (2),

$\dfrac{dy}{dx} = \dfrac{\dfrac{dy}{dt}}{\dfrac{dx}{dt}} = \dfrac{-2b\sin 2t + b\sin 4t}{2a\cos 2t + 2a\cos 4t}$

$\therefore \dfrac{dy}{dx}\bigg|_{\pi/4} = \dfrac{\dfrac{dy}{dt}}{\dfrac{dx}{dt}}\bigg|_{t=\pi/4} = \dfrac{-2b}{-2a} = \dfrac{b}{a}$

Differentiation Ex 11.7 Q25

Consider the given functions,

$x = \cos t \left(3 - 2\cos^2 t\right)$

$x = 3\cos t - 2\cos^3 t$

$\dfrac{dx}{dt} = -3\sin t + 6\cos^2 t \sin t \dots (1)$

$y = \sin t \ \left(3 - 2\sin^2 t\right)$

$y = 3\sin t - 2\sin^3 t$

$\dfrac{dy}{dt} = 3\cos t - 6\sin^2 t \cos t \dots (2)$

$\dfrac{dy}{dx} = \left(\dfrac{dy}{dt}\right)\bigg/\left(\dfrac{dx}{dt}\right)$ ....[From equations (1) and (2)]

$\quad = \dfrac{3\cos t - 6\sin^2 t \cos t}{-3\sin t + 6\cos^2 t \sin t}$

$\quad = \dfrac{3\cos t \left(1 - 2\sin^2 t\right)}{3\sin t \left(2\cos^2 t - 1\right)}$

$\quad = \cot t \dfrac{\left(1 - 2\left(1 - \cos^2 t\right)\right)}{\left(2\cos^2 t - 1\right)}$

$\quad = \cot t$

$\dfrac{dy}{dx}\bigg|_{\pi/4} = \cot\dfrac{\pi}{4} = 1$

Differentiation Ex 11.7 Q26

$x = \dfrac{1 + \log t}{t^2}, y = \dfrac{3 + 2\log t}{t}$

$\dfrac{dx}{dt} = \dfrac{t^2\left(\dfrac{1}{t}\right) - (1 + \log t)(2t)}{t^4} = \dfrac{t - 2t - 2t\log t}{t^4} = \dfrac{-2\log t - 1}{t^3}$

$\dfrac{dy}{dt} = \dfrac{t\left(\dfrac{2}{t}\right) - (3 + 2\log t)(1)}{t^2} = \dfrac{2 - 3 - 2\log t}{t^2} = \dfrac{-2\log t - 1}{t^2}$

$\dfrac{dy}{dx} = \dfrac{\dfrac{dy}{dt}}{\dfrac{dx}{dt}} = \dfrac{\dfrac{-2\log t - 1}{t^2}}{\dfrac{-2\log t - 1}{t^3}} = t$

Differentiation Ex 11.7 Q27

$$x = 3\sin t - \sin 3t, \; y = 3\cos t - \cos 3t$$

$$\frac{dx}{dt} = 3\cos t - 3\cos 3t$$

$$\frac{dy}{dt} = -3\sin t + 3\sin 3t$$

$$\frac{dy}{dx} = \frac{\dfrac{dy}{dt}}{\dfrac{dx}{dt}} = \frac{-3\sin t + 3\sin 3t}{3\cos t - 3\cos 3t}$$

When $t = \dfrac{\pi}{3}$

$$\frac{dy}{dx} = \frac{-3\sin\left(\dfrac{\pi}{3}\right) + 3\sin(\pi)}{3\cos\left(\dfrac{\pi}{3}\right) - 3\cos(\pi)} = \frac{-3 \times \dfrac{\sqrt{3}}{2} + 0}{3 \times \dfrac{1}{2} - 3(-1)} = -\frac{1}{\sqrt{3}}$$

Differentiation Ex 11.7 Q28

$$\sin x = \frac{2t}{1 + t^2}, \; \tan y = \frac{2t}{1 - t^2}$$

$$\Rightarrow x = \sin^{-1}\left(\frac{2t}{1 + t^2}\right) \text{ and } y = \tan^{-1}\left(\frac{2t}{1 - t^2}\right)$$

$$\frac{dx}{dt} = \frac{1}{\sqrt{1 - \left(\dfrac{2t}{1 + t^2}\right)^2}} \times \frac{2\left(1 + t^2\right) - (2t)(2t)}{\left(1 + t^2\right)^2}$$

$$\frac{dx}{dt} = \frac{2}{\left(1 + t^2\right)}$$

$$\frac{dy}{dt} = \frac{1}{\left(\dfrac{2t}{1 - t^2}\right)^2 + 1} \times \frac{2\left(1 - t^2\right) - (2t)(-2t)}{\left(1 - t^2\right)^2}$$

$$\frac{dy}{dt} = \frac{2}{\left(1 + t^2\right)}$$

$$\frac{dy}{dx} = \frac{\dfrac{dy}{dt}}{\dfrac{dx}{dt}} = \frac{\dfrac{2}{\left(1 + t^2\right)}}{\dfrac{2}{\left(1 + t^2\right)}} = 1$$

Ex 11.8

Differentiation Ex 11.8 Q1

Let $\quad u = x^2, \ v = x^3$

Differentiating u with respect to x,

$$\frac{du}{dx} = 2x \qquad\qquad ---(i)$$

Differentiating v with respect to x,

$$\frac{dv}{dx} = 3x^2 \qquad\qquad ---(ii)$$

Dividing equation (i) by (ii),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2x}{3x^2}$$

$$\frac{du}{dv} = \frac{2}{3x}$$

Differentiation Ex 11.8 Q2

Let $\quad u = \log\left(1 + x^2\right)$

Differentiating it with respect to x using chain rule,

$$\frac{du}{dx} = \frac{1}{\left(1 + x^2\right)} \frac{d}{dx}\left(1 + x^2\right)$$

$$= \frac{1}{\left(1 + x^2\right)}(2x)$$

$$\frac{du}{dx} = \frac{2x}{\left(1 + x^2\right)} \qquad\qquad ---(i)$$

Let $\quad v = \tan^{-1} x$

Differentiating it with respect to x,

$$\frac{dv}{dx} = \frac{1}{1 + x^2} \qquad\qquad ---(ii)$$

Dividing equation (i) by (ii),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2x}{\left(1 + x^2\right)} \times \frac{\left(1 + x^2\right)}{1}$$

$$\frac{du}{dv} = 2x$$

Differentiation Ex 11.8 Q3

Let $u = (\log x)^x$

Taking log on both the sides,

$\log u = \log(\log x)^x$

$\log u = x \log(\log x)$ $\qquad\qquad$ $\left[\text{Since, } \log a^b = b \log a\right]$

Differentiating it with respect to x using chain rule, product rule,

$\dfrac{1}{u}\dfrac{du}{dx} = x \dfrac{d}{dx}\log(\log x) + \log(\log x)\dfrac{d}{dx}(x)$

$\dfrac{1}{u}\dfrac{du}{dx} = x\left(\dfrac{1}{\log x}\right)\dfrac{d}{dx}(\log x) + \log\log x\,(1)$

$\dfrac{du}{dx} = u\left[\dfrac{x}{\log x}\left(\dfrac{1}{x}\right) + \log\log x\right]$

$\dfrac{du}{dx} = (\log x)^x\left[\dfrac{1}{\log x} + \log\log x\right]$ $\qquad\qquad$ ---(i)

Again, let $\quad v = \log x$

Differentiating it with respect to x,

$\dfrac{dv}{dx} = \dfrac{1}{x}$ $\qquad\qquad$ ---(ii)

Dividing equation (i) by (ii),

$\dfrac{\frac{du}{dx}}{\frac{dv}{dx}} = \dfrac{(\log x)^x\left[\frac{1}{\log x} + \log\log x\right]}{\frac{1}{x}}$

$\dfrac{du}{dv} = \dfrac{(\log x)^x\left[\frac{1 + \log x \times \log\log x}{\log x}\right]}{\frac{1}{x}}$

$\dfrac{du}{dv} = (\log x)^{x-1}\,(1 + \log x \times \log\log x) \times x$

Differentiation Ex 11.8 Q4(i)

Let $u = \sin^{-1}\sqrt{1 - x^2}$

Put $x = \cos\theta,$ so,

$$u = \sin^{-1}\sqrt{1 - \cos^2\theta}$$
$$u = \sin^{-1}(\sin\theta) \hspace{3cm} \text{---(i)}$$

And, $v = \cos^{-1}x \hspace{3cm} \text{---(ii)}$

Now, $x \in (0, 1)$

$\Rightarrow \hspace{1cm} \cos\theta \in (0, 1)$

$\Rightarrow \hspace{1cm} \theta \in \left(0, \dfrac{\pi}{2}\right)$

So, from equation (i),

$$u = \theta \hspace{2cm} \left[\text{Since, } \sin^{-1}(\sin\theta) = \theta \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right]$$
$$u = \cos^{-1}x \hspace{2cm} \left[\text{Since, } \cos\theta = x\right]$$

Differentiating it with respect to x,

$$\frac{du}{dx} = \frac{-1}{\sqrt{1 - x^2}} \hspace{3cm} \text{---(ii)}$$

From equation (ii),

$$v = \cos^{-1}x$$

Differentiating it with respect to x,

$$\frac{dv}{dx} = \frac{-1}{\sqrt{1 - x^2}} \hspace{3cm} \text{---(iv)}$$

Dividing equation (iii) by (iv),

$$\frac{\dfrac{du}{dx}}{\dfrac{dv}{dx}} = \frac{-1}{\sqrt{1 - x^2}} \times \frac{\sqrt{1 - x^2}}{-1}$$
$$\frac{du}{dv} = 1$$

Differentiation Ex 11.8 Q4(ii)

Let $u = \sin^{-1}\sqrt{1-x^2}$

Put $x = \cos\theta$, so,

$u = \sin^{-1}\sqrt{1-\cos^2\theta}$

$u = \sin^{-1}(\sin\theta)$ ---(i)

And, $v = \cos^{-1}x$ ---(ii)

Here,

$x \in (-1, 0)$

$\Rightarrow$ $\cos\theta \in (-,10)$

$\Rightarrow$ $\theta \in \left(\dfrac{\pi}{2}, \pi\right]$

So, from equation (i),

$u = \pi - \theta$ $\left[\text{Since, } \sin^{-1}(\sin\theta) = \pi - \theta, \theta \in \left(\dfrac{\pi}{2}, \dfrac{3\pi}{2}\right)\right]$

$u = \pi - \cos^{-1}x$ $[\text{Since, } x = \cos\theta]$

Differentiating it with respect to x,

$\dfrac{du}{dx} = 0 - \left(\dfrac{-1}{\sqrt{1-x^2}}\right)$

$\dfrac{du}{dx} = \dfrac{1}{\sqrt{1-x^2}}$ ---(v)

And, from equation (ii),

$v = \cos^{-1}x$

Differentiating it with respect to x,

$\dfrac{dv}{dx} = \dfrac{-1}{\sqrt{1-x^2}}$ ---(vi)

Dividing equation (v) by (vi)

$\dfrac{\dfrac{du}{dx}}{\dfrac{dv}{dx}} = \dfrac{1}{\sqrt{1-x^2}} \times \dfrac{\sqrt{1-x^2}}{-1}$

$\dfrac{du}{dv} = -1$

Differentiation Ex 11.8 Q5(i)

Let $\quad u = \sin^{-1}\left(4x\sqrt{1-4x^2}\right)$

Put $\quad 2x = \cos\theta$, so

$\qquad u = \sin^{-1}\left(2 \times \cos\theta\sqrt{1-\cos^2\theta}\right)$

$\qquad\quad = \sin^{-1}\left(2\cos\theta\sin\theta\right)$

$\qquad u = \sin^{-1}\left(\sin 2\theta\right)$ $\qquad\qquad$ ---(i)

Let $\quad v = \sqrt{1-4x^2}$ $\qquad\qquad\qquad$ ---(ii)

Here,

$\qquad x \in \left(-\dfrac{1}{2}, -\dfrac{1}{2\sqrt{2}}\right)$

$\Rightarrow \qquad 2x \in \left(-1, -\dfrac{1}{\sqrt{2}}\right)$

$\Rightarrow \qquad \theta \in \left(\dfrac{3\pi}{4}, \pi\right)$

So, from equation (i),

$\qquad u = \pi - 2\theta$ $\qquad\qquad\qquad$ $\left[\text{Since, } \sin^{-1}(\sin\theta) = \pi - \theta \text{ if } \theta \in \left[\dfrac{\pi}{2}, \dfrac{3\pi}{2}\right]\right]$

$\qquad u = \pi - 2\cos^{-1}(2x)$ $\qquad\qquad$ $\left[\text{Since, } 2x = \cos\theta\right]$

Differentiating it with respect to x using chain rule,

$\qquad \dfrac{du}{dx} = 0 - 2\left(\dfrac{-1}{\sqrt{1-(2x)^2}}\right)\dfrac{d}{dx}(2x)$

$\qquad\qquad = \dfrac{2}{\sqrt{1-4x^2}}(2)$

$\qquad \dfrac{du}{dx} = \dfrac{4}{\sqrt{1-4x^2}}$ $\qquad\qquad$ ---(vi)

From equation (iv)

$\qquad \dfrac{dv}{dx} = \dfrac{-4x}{\sqrt{1-4x^2}}$

but, $\quad x \in \left(-\dfrac{1}{2}, -\dfrac{1}{2\sqrt{2}}\right)$

$\qquad \dfrac{dv}{dx} = \dfrac{-4(-x)}{\sqrt{1-4(-x)^2}}$

$\qquad \dfrac{dv}{dx} = \dfrac{4x}{\sqrt{1-4x^2}}$ $\qquad\qquad$ ---(vii)

Differentiating equation (ii) with respect to x using chain rule,

$\qquad \dfrac{dv}{dx} = \dfrac{1}{2\sqrt{1-4x^2}}\dfrac{d}{dx}\left(1-4x^2\right)$

$\qquad\qquad = \dfrac{1}{2\sqrt{1-4x^2}}(-8x)$

$\qquad \dfrac{dv}{dx} = \dfrac{-4x}{\sqrt{1-4x^2}}$ $\qquad\qquad\qquad$ ---(iv)

Divide equation (iii) by (iv),

$\qquad \dfrac{\dfrac{du}{dx}}{\dfrac{dv}{dx}} = \dfrac{4}{\sqrt{1-4x^2}} \times \dfrac{\sqrt{1-4x^2}}{-4x}$

$\qquad \dfrac{du}{dv} = -\dfrac{1}{x}$

Differentiation Ex 11.8 Q5(ii)

Let $\quad u = \sin^{-1}\left(4x\sqrt{1-4x^2}\right)$

Put $\quad 2x = \cos\theta$, so

$$u = \sin^{-1}\left(2 \times \cos\theta\sqrt{1-\cos^2\theta}\right)$$

$$= \sin^{-1}\left(2\cos\theta\sin\theta\right)$$

$$u = \sin^{-1}\left(\sin 2\theta\right) \qquad\qquad \text{---(i)}$$

Let $\quad v = \sqrt{1-4x^2} \qquad\qquad\qquad \text{---(ii)}$

Here,

$$x \in \left(\frac{1}{2\sqrt{2}}, \frac{1}{2}\right)$$

$$\Rightarrow \quad 2x \in \left(\frac{1}{2\sqrt{2}}, 1\right)$$

$$\Rightarrow \quad \cos\theta \in \left(\frac{1}{\sqrt{2}}, 1\right)$$

$$\Rightarrow \quad \theta \in \left(0, \frac{\pi}{4}\right)$$

So, from equation (i)

$$u = 2\theta \qquad\qquad \left[\text{Since, } \sin^{-1}\left(\sin\theta\right) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

$$u = 2\cos^{-1}\left(2x\right) \qquad\qquad \left[\text{Since, } 2x = \cos\theta\right]$$

Differentiate it with respect to x using chain rule,

$$\frac{du}{dx} = 2\left(\frac{-1}{\sqrt{1-(2x)^2}}\right)\frac{d}{dx}(2x)$$

$$= \left(\frac{-2}{\sqrt{1-4x^2}}(2)\right)$$

$$\frac{du}{dx} = \frac{-4}{\sqrt{1-4x^2}} \qquad\qquad \text{---(v)}$$

Dividing equation (v) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{-4}{\sqrt{1-4x^2}} \times \frac{\sqrt{1-4x^2}}{-4x}$$

$$\frac{du}{dv} = \frac{1}{x}$$

Differentiation Ex 11.8 Q5(iii)

Let $u = \sin^{-1}\left(4x\sqrt{1-4x^2}\right)$

Put $2x = \cos\theta$, so

$$u = \sin^{-1}\left(2 \times \cos\theta\sqrt{1-\cos^2\theta}\right)$$

$$= \sin^{-1}\left(2\cos\theta\sin\theta\right)$$

$$u = \sin^{-1}\left(\sin 2\theta\right) \qquad\qquad \text{---(i)}$$

Let $v = \sqrt{1-4x^2} \qquad\qquad \text{---(ii)}$

Here,

$$x \in \left(-\frac{1}{2}, -\frac{1}{2\sqrt{2}}\right)$$

$$\Rightarrow \quad 2x \in \left(-1, -\frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow \quad \theta \in \left(\frac{3\pi}{4}, \pi\right)$$

So, from equation (i),

$$u = \pi - 2\theta \qquad\qquad \left[\text{Since, } \sin^{-1}\left(\sin\theta\right) = \pi - \theta \text{ if } \theta \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]\right]$$

$$u = \pi - 2\cos^{-1}\left(2x\right) \qquad\qquad \left[\text{Since, } 2x = \cos\theta\right]$$

Differentiating it with respect to x using chain rule,

$$\frac{du}{dx} = 0 - 2\left(\frac{-1}{\sqrt{1-(2x)^2}}\right)\frac{d}{dx}(2x)$$

$$= \frac{2}{\sqrt{1-4x^2}}(2)$$

$$\frac{du}{dx} = \frac{4}{\sqrt{1-4x^2}} \qquad\qquad \text{---(vi)}$$

From equation (iv)

$$\frac{dv}{dx} = \frac{-4x}{\sqrt{1-4x^2}}$$

but, $x \in \left(-\frac{1}{2}, -\frac{1}{2\sqrt{2}}\right)$

$$\frac{dv}{dx} = \frac{-4(-x)}{\sqrt{1-4(-x)^2}}$$

$$\frac{dv}{dx} = \frac{4x}{\sqrt{1-4x^2}} \qquad\qquad \text{---(vii)}$$

Dividing equation (vi) by (vii),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{4}{\sqrt{1-4x^2}} \times \frac{\sqrt{1-4x^2}}{4x}$$

$$\frac{du}{dv} = \frac{1}{x}$$

Differentiation Ex 11.8 Q6

Let $u = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$

Put $x = \tan\theta$, so

$$u = \tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right)$$

$$= \tan^{-1}\left(\frac{\sec\theta-1}{\tan\theta}\right)$$

$$= \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$$

$$= \tan^{-1}\left(\frac{\frac{2\sin^2\theta}{2}}{\frac{2\sin\theta}{2}\frac{\cos\theta}{2}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{\sin\theta}{2}}{\frac{\cos\theta}{2}}\right)$$

$$u = \tan^{-1}\left(\frac{\tan\theta}{2}\right) \qquad\qquad ---(i)$$

And,

Let $\quad v = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

$$= \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$$

$$v = \sin^{-1}(\sin 2\theta) \qquad\qquad ---(ii)$$

Here,

$$-1 < x < 1$$

$$\Rightarrow \quad -1 < \tan\theta < 1$$

$$\Rightarrow \quad -\frac{\pi}{4} < \theta < \frac{\pi}{4} \qquad\qquad ---(A)$$

So, from equation (i),

$$u = \frac{\theta}{2} \qquad\qquad \left[\text{Since, } \tan^{-1}(\tan\theta) = \theta \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

$$u = \frac{1}{2}\tan^{-1}x \qquad\qquad [\text{Since, } x = \tan\theta]$$

Differentiating it with respect to x,

$$\frac{du}{dx} = \frac{1}{2}\left(\frac{1}{1+x^2}\right)$$

$$\frac{du}{dx} = \frac{1}{2\left(1+x^2\right)} \qquad\qquad ---(iii)$$

Now, from equation (ii) and (A)

$$v = 2\theta \qquad\qquad \left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

$$v = 2\tan^{-1}x \qquad\qquad [\text{Since, } x = \tan\theta]$$

Differentiating it with respect to x,

$$\frac{dv}{dx} = 2\left(\frac{1}{1+x^2}\right) \qquad\qquad ---(iv)$$

Dividing equation (iii) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{1}{2\left(1+x^2\right)} \times \frac{\left(1+x^2\right)}{2}$$

$$\frac{du}{dv} = \frac{1}{4}$$

Differentiation Ex 11.8 Q7(i)

Let $u = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$

Put $x = \sin\theta$, so
$$u = \sin^{-1}\left(2\sin\theta\sqrt{1-\sin^2\theta}\right)$$
$$= \sin^{-1}\left(2\sin\theta\cos\theta\right)$$
$$u = \sin^{-1}\left(\sin2\theta\right) \qquad \text{---(i)}$$

And,

Let $v = \sec^{-1}\left(\dfrac{1}{\sqrt{1-x^2}}\right)$
$$= \sec^{-1}\left(\dfrac{1}{\sqrt{1-\sin^2\theta}}\right)$$
$$= \sec^{-1}\left(\dfrac{1}{\cos\theta}\right)$$
$$= \sec^{-1}\left(\sec\theta\right)$$
$$= \cos^{-1}\left(\dfrac{1}{\dfrac{1}{\cos\theta}}\right) \qquad \left[\text{Since, } \sec^{-1}x = \cos^{-1}\left(\dfrac{1}{x}\right)\right]$$
$$v = \cos^{-1}\left(\cos\theta\right) \qquad \text{---(ii)}$$

Here,
$$x \in \left(0, \dfrac{1}{\sqrt{2}}\right)$$
$$\Rightarrow \quad \sin\theta \in \left(0, \dfrac{1}{\sqrt{2}}\right)$$
$$\Rightarrow \quad \theta \in \left(0, \dfrac{\pi}{4}\right)$$

So, from equation (i),

$$u = 2\theta \qquad \left[\text{Since, } \sin^{-1}\left(\sin\theta\right) = \theta, \text{ if } \theta \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]\right]$$

Let $u = 2\sin^{-1}x \qquad [\text{Since, } x = \sin\theta]$

Differentiating it with respect to x,

$$\frac{du}{dx} = 2\left(\frac{1}{\sqrt{1-x^2}}\right)$$
$$\frac{du}{dx} = \frac{2}{\sqrt{1-x^2}} \qquad \text{---(iii)}$$

And, from equation (ii),

$$v = \theta \qquad \left[\text{Since, } \cos^{-1}\left(\cos\theta\right) = \theta, \text{ if } \theta \in [0, \pi]\right]$$
$$v = \sin^{-1}x \qquad [\text{Since, } x = \sin\theta]$$

Differentiating it with respect to x,

$$\frac{dv}{dx} = \frac{1}{\sqrt{1-x^2}} \qquad \text{---(iv)}$$

Dividing equation (iii) by (iv), 3

$$\frac{\dfrac{du}{dx}}{\dfrac{dv}{dx}} = \frac{2}{\sqrt{1-x^2}} \times \frac{\sqrt{1-x^2}}{1}$$

$$\frac{du}{dv} = 2$$

Differentiation Ex 11.8 Q7(ii)

Let $u = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$

Put $x = \sin\theta$, so

$u = \sin^{-1}\left(2\sin\theta\sqrt{1-\sin^2\theta}\right)$

$\quad = \sin^{-1}\left(2\sin\theta\cos\theta\right)$

$u = \sin^{-1}\left(\sin 2\theta\right)$ \hspace{2cm} ---(i)

And,

Let $v = \sec^{-1}\left(\dfrac{1}{\sqrt{1-x^2}}\right)$

$\quad = \sec^{-1}\left(\dfrac{1}{\sqrt{1-\sin^2\theta}}\right)$

$\quad = \sec^{-1}\left(\dfrac{1}{\cos\theta}\right)$

$\quad = \sec^{-1}\left(\sec\theta\right)$

$\quad = \cos^{-1}\left(\dfrac{1}{\sec\theta}\right)$ \hspace{2cm} $\left[\text{Since, } \sec^{-1}x = \cos^{-1}\left(\dfrac{1}{x}\right)\right]$

$v = \cos^{-1}\left(\cos\theta\right)$ \hspace{2cm} ---(ii)

Here,

$x \in \left(\dfrac{1}{\sqrt{2}}, 1\right)$

$\Rightarrow \quad \sin\theta \in \left(\dfrac{1}{\sqrt{2}}, 1\right)$

$\Rightarrow \quad \theta \in \left(\dfrac{\pi}{4}, \dfrac{\pi}{2}\right)$

So, from equation (i),

$u = 2\theta$ \hspace{2cm} $\left[\text{Since, } \sin^{-1}\left(\sin\theta\right) = \theta, \text{ if } \theta \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]\right]$

$u = 2\sin^{-1}x$ \hspace{2cm} $\left[\text{Since, } x = \sin\theta\right]$

Differentiating it with respect to x,

$\dfrac{du}{dx} = 2\left(\dfrac{1}{\sqrt{1-x^2}}\right)$ \hspace{2cm} ---(iv)

From equation (ii)

$v = \theta$ \hspace{2cm} $\left[\text{Since, } \cos^{-1}\left(\cos\theta\right) = \theta, \text{ if } \theta \in \left[0, \pi\right]\right]$

$v = \sin^{-1}x$

Differentiating it with respect to x,

$\dfrac{dv}{dx} = \dfrac{1}{\sqrt{1-x^2}}$ \hspace{2cm} ---(v)

Dividing equation (iv) by (v),

$\dfrac{\dfrac{du}{dx}}{\dfrac{dv}{dx}} = \dfrac{2}{\sqrt{1-x^2}} \times \dfrac{\sqrt{1-x^2}}{1}$

$\dfrac{du}{dv} = 2$

Differentiation Ex 11.8 Q8

Let $\quad u = (\cos x)^{\sin x}$

Taking on both the sides,

$$\log u = \log (\cos x)^{\sin x}$$
$$\log u = \sin x \log (\cos x)$$

Differentiating it with respect to x using product and chain rule,

$$\frac{1}{u}\frac{du}{dx} = \sin x \frac{d}{dx}(\log \cos x) + \log \cos x \frac{d}{dx}(\sin x)$$

$$\frac{1}{u}\frac{du}{dx} = \sin x \left(\frac{1}{\cos x}\right)\frac{d}{dx}(\cos x) + \log \cos x (\cos x)$$

$$\frac{du}{dx} = 4\big[(\tan x) \times (-\sin x) + \log \log x \times (\cos x)\big]$$

$$\frac{du}{dx} = (\cos x)^{\sin x}\big[\cos x \log \cos x - \sin x \tan x\big] \qquad\qquad ---(i)$$

Let $\quad v = (\sin x)^{\cos x}$

Taking log on both the sides,

$$\log v = \log (\sin x)^{\cos x}$$
$$\log v = \cos x \log (\sin x)$$

Differentiating it with respect to x using product rule and chain rule,

$$\frac{1}{v}\frac{dv}{dx} = \cos x \frac{d}{dx}(\log \sin x) + \log \sin x \frac{d}{dx}(\cos x)$$

$$\frac{1}{v}\frac{dv}{dx} = \cos x \left(\frac{1}{\sin x}\right)\frac{d}{dx}(\sin x) + \log \sin x (-\sin x)$$

$$\frac{dv}{dx} = v\big[\cot x (\cos x) - \sin x \log \sin x\big]$$

$$\frac{dv}{dx} = (\sin x)^{\cos x}\big[\cot x (\cos x) - \sin x \log \sin x\big] \qquad\qquad ---(ii)$$

Dividing equation (i) by (ii),

$$\frac{du}{dv} = \frac{(\cos x)^{\sin x}\big[\cos x \log \cos x - \sin x \tan x\big]}{(\sin x)^{\cos x}\big[\cot x (\cos x) - \sin x \log \sin x\big]}$$

Differentiation Ex 11.8 Q9

Let $\quad u = \sin^{-1}\left(\dfrac{2x}{1+x^2}\right)$

Put $\quad x = \tan\theta,$

$\qquad u = \sin^{-1}\left(\dfrac{2\tan\theta}{1+\tan^2\theta}\right)$

$\qquad u = \sin^{-1}\left(\sin 2\theta\right) \qquad\qquad ---(i)$

Let $\quad v = \cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)$

$\qquad = \cos^{-1}\left(\dfrac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$\qquad v = \cos^{-1}\left(\cos 2\theta\right) \qquad\qquad ---(ii)$

Here, $\quad 0 < x < 1$

$\Rightarrow \qquad 0 < \tan\theta < 1$

$\Rightarrow \qquad 0 < \theta < \dfrac{\pi}{4}$

So, from equation (i),

$\qquad u = 2\theta \qquad\qquad \left[\text{Since, } \sin^{-1}\left(\sin\theta\right) = \theta,\ \text{if } \theta \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]\right]$

$\qquad u = 2\tan^{-1}x \qquad\qquad \left[\text{Since, } x = \tan\dfrac{\pi}{2}\right]$

Differentiating it with respect to x,

$\qquad \dfrac{du}{dx} = \dfrac{2}{1+x^2} \qquad\qquad ---(iii)$

From equation (ii),

$\qquad v = 2\theta \qquad\qquad \left[\text{Since, } \cos^{-1}\left(\cos\theta\right) = \theta,\ \text{if } \theta \in \left[0, \pi\right]\right]$

$\qquad v = 2\tan^{-1}x \qquad\qquad \left[\text{Since, } x = \tan\theta\right]$

Differentiating it with respect to x,

$\qquad \dfrac{dv}{dx} = \dfrac{2}{1+x^2} \qquad\qquad ---(iv)$

Differentiation Ex 11.8 Q10

Let $\quad u = \tan^{-1}\left(\dfrac{1+ax}{1-ax}\right)$

Put $\quad ax = \tan\theta$

$$u = \tan^{-1}\left(\dfrac{1+\tan\theta}{1-\tan\theta}\right)$$

$$= \tan^{-1}\left(\dfrac{\dfrac{\tan\pi}{4} + \tan\theta}{1 - \dfrac{\tan\pi}{4}\tan\theta}\right)$$

$$= \tan^{-1}\left(\tan\left(\dfrac{\pi}{4} + \theta\right)\right)$$

$$= \dfrac{\pi}{4} + \theta$$

$$u = \dfrac{\pi}{4} + \tan^{-1}(ax) \qquad\qquad\qquad \left[\text{Since, } \tan\theta = ax\right]$$

Differentiate it with respect to x using chain rule,

$$\dfrac{du}{dx} = 0 + \dfrac{1}{1+(ax)^2}\dfrac{d}{dx}(ax)$$

$$\dfrac{du}{dx} = \dfrac{a}{1+a^2x^2} \qquad\qquad\qquad \text{---(i)}$$

Now,

Let $\quad v = \sqrt{1+a^2x^2}$

Differentiating it with respect to x using chain rule,

$$\dfrac{dv}{dx} = \dfrac{1}{2\sqrt{1+a^2x^2}}\dfrac{d}{dx}\left(1+a^2x^2\right)$$

$$= \dfrac{1}{2\sqrt{1+a^2x^2}}\left(2a^2x\right)$$

$$\dfrac{dv}{dx} = \dfrac{a^2x}{\sqrt{1+a^2x^2}} \qquad\qquad\qquad \text{---(ii)}$$

Differentiation Ex 11.8 Q11

Let $\quad u = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$

Put $\quad x = \sin\theta$,

$\qquad u = \sin^{-1}\left(2\sin\theta\sqrt{1-\sin^2\theta}\right)$

$\qquad\quad = \sin^{-1}\left(2\sin\theta\cos\theta\right)$

$\qquad u = \sin^{-1}\left(\sin 2\theta\right)$ $\qquad\qquad$ ---(i)

Let $\quad v = \tan^{-1}\left(\dfrac{x}{\sqrt{1-x^2}}\right)$

$\qquad\quad = \tan^{-1}\left(\dfrac{\sin\theta}{\sqrt{1-\sin^2\theta}}\right)$

$\qquad\quad = \tan^{-1}\left(\dfrac{\sin\theta}{\cos\theta}\right)$

$\qquad v = \tan^{-1}\left(\tan\theta\right)$ $\qquad\qquad$ ---(ii)

Here, $\quad -\dfrac{1}{\sqrt{2}} < x < \dfrac{1}{\sqrt{2}}$

$\Rightarrow \qquad -\dfrac{1}{\sqrt{2}} < \sin\theta < \dfrac{1}{\sqrt{2}}$

$\Rightarrow \qquad \left(-\dfrac{\pi}{4}\right) < \theta < \left(\dfrac{\pi}{4}\right)$

So, from equation (i),

$\qquad u = 2\theta$ $\qquad$ $\left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]\right]$

$\qquad u = 2\sin^{-1}x$

Differentiating it with respect to x,

$\qquad \dfrac{du}{dx} = \dfrac{2}{\sqrt{1-x^2}}$ $\qquad$ ---(iii)

From equation (ii),

$\qquad v = \theta$ $\qquad$ $\left[\text{Since, } \tan^{-1}(\tan\theta) = \theta, \text{ if } \theta \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]\right]$

$\qquad v = \sin^{-1}x$ $\qquad$ $\left[\text{Since, } x = \sin\theta\right]$

Differentiating it with respect to x,

$\qquad \dfrac{dv}{dx} = \dfrac{1}{\sqrt{1-x^2}}$ $\qquad$ ---(iv)

Dividing equation (iii) by (iv)

$\qquad \dfrac{\dfrac{du}{dx}}{\dfrac{dv}{dx}} = \dfrac{2}{\sqrt{1-x^2}} \times \dfrac{\sqrt{1-x^2}}{1}$

$\qquad \dfrac{du}{dv} = 2$

Differentiation Ex 11.8 Q12

Let $\quad u = \tan^{-1}\left(\dfrac{2x}{1-x^2}\right)$

Put $\quad x = \tan\theta, \text{so}$

$$u = \tan^{-1}\left(\dfrac{2\tan\theta}{1-\tan^2\theta}\right)$$

$$u = \tan^{-1}(\tan2\theta) \qquad\qquad\text{---(i)}$$

Let $\quad v = \cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)$

$$= \cos^{-1}\left(\dfrac{1-\tan^2\theta}{1+\tan^2\theta}\right)$$

$$v = \cos^{-1}(\cos2\theta) \qquad\qquad\text{---(ii)}$$

Here, $\quad 0 < x < 1$

$\Rightarrow \qquad 0 < \tan\theta < 1$

$\Rightarrow \qquad 0 < \theta < \dfrac{\pi}{4}$

So, from equation (i),

$$u = 2\theta \qquad\qquad \left[\text{Since, } tab^{-1}(\tan\theta) = \theta, \text{ if } \theta \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]\right]$$

$$u = 2\tan^{-1}x \qquad\qquad \left[\text{Since, } x = \tan\theta\right]$$

Differentiating it with respect to x,

$$\dfrac{du}{dx} = \dfrac{2}{1+x^2} \qquad\qquad\text{---(iii)}$$

From equation (ii),

$$v = \theta \qquad\qquad \left[\text{Since, } \cos^{-1}(\cos\theta) = \theta, \text{ if } \theta \in \left[0, \pi\right]\right]$$

$$v = 2\tan^{-1}x \qquad\qquad \left[\text{Since, } x = \tan\theta\right]$$

Differentiating it with respect to x,

$$\dfrac{dv}{dx} = \dfrac{2}{1+x^2} \qquad\qquad\text{---(iv)}$$

Dividing equation (iii) by (iv),

$$\dfrac{\frac{du}{dx}}{\frac{dv}{dx}} = \dfrac{2}{1+x^2} \times \dfrac{1+x^2}{2}$$

$$\dfrac{du}{dv} = 1$$

Differentiation Ex 11.8 Q13

Let $\quad u = \tan^{-1}\left(\dfrac{x-1}{x+1}\right)$

Put $\quad x = \tan\theta$, so

$$u = \tan^{-1}\left(\frac{\tan\theta - 1}{\tan\theta + 1}\right)$$

$$= \tan^{-1}\left(\frac{\tan\theta - \dfrac{\tan\pi}{4}}{1 + \tan\theta\dfrac{\tan\pi}{4}}\right)$$

$$u = \tan^{-1}\left(\tan\left(\theta - \frac{\pi}{4}\right)\right) \qquad\qquad ---(i)$$

Here, $\quad -\dfrac{1}{2} < x < \dfrac{1}{2}$

$\Rightarrow \qquad -\dfrac{1}{2} < \tan\theta < \dfrac{1}{2}$

$\Rightarrow \qquad -\tan^{-1}\left(\dfrac{1}{2}\right) < \theta < \tan^{-1}\left(\dfrac{1}{2}\right)$

So,

$$u = \theta - \frac{\pi}{4} \qquad\qquad \left[\text{Since, } \tan^{-1}(\tan\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

$$u = \tan^{-1}x - \frac{\pi}{4}$$

Differentiating it with respect to x,

$$\frac{du}{dx} = \frac{1}{1+x^2} - 0$$

$$\frac{du}{dx} = \frac{1}{1+x^2} \qquad\qquad ---(ii)$$

And,

Let $\quad v = \sin^{-1}\left(3x - 4x^3\right)$

Put $\quad x = \sin\theta$, so

$$v = \sin^{-1}\left(3\sin\theta - 4\sin^3\theta\right)$$

$$v = \sin^{-1}(\sin 3\theta) \qquad\qquad ---(iii)$$

Now, $\quad -\dfrac{1}{2} < x < \dfrac{1}{2}$

$\Rightarrow \qquad -\dfrac{1}{2} < \sin\theta < \dfrac{1}{2}$

$\Rightarrow \qquad -\dfrac{1}{6} < \theta < \dfrac{\pi}{6}$

So, from equation (iii),

$$v = 3\theta \qquad\qquad \left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

$$v = 3\sin^{-1}x \qquad\qquad [\text{Since, } x = \sin\theta]$$

Differentiating it with respect to x,

$$\frac{dv}{dx} = \frac{3}{\sqrt{1-x^2}} \qquad\qquad ---(iv)$$

Dividing equation (iii) by (iv),

$$\frac{\dfrac{du}{dx}}{\dfrac{dv}{dx}} = \frac{1}{1+x^2} \times \frac{\sqrt{1-x^2}}{3}$$

$$\frac{du}{dv} = \frac{\sqrt{1-x^2}}{3\left(1+x^2\right)}$$

Differentiation Ex 11.8 Q14

Let $\quad u = \tan^{-1}\left(\dfrac{\cos x}{1 + \sin x}\right)$

$= \tan^{-1}\left(\dfrac{\dfrac{\cos^2 x}{2} - \dfrac{\sin^2 x}{2}}{\dfrac{\cos^2 x}{2} + \dfrac{\sin^2 x}{2} + \dfrac{2\sin x}{2}\dfrac{\cos x}{2}}\right)$

$= \tan^{-1}\left(\dfrac{\left(\dfrac{\cos x}{2} + \dfrac{\sin x}{2}\right)\left(\dfrac{\cos x}{2} - \dfrac{\sin x}{2}\right)}{\left(\dfrac{\cos x}{2} + \dfrac{\sin x}{2}\right)^2}\right)$

$= \tan^{-1}\left(\dfrac{\dfrac{\cos x}{2} - \dfrac{\sin x}{2}}{\dfrac{\cos x}{2} + \dfrac{\sin x}{2}}\right)$

$= \tan^{-1}\left[\dfrac{\dfrac{\dfrac{\cos x}{2}}{\dfrac{\cos x}{2}} - \dfrac{\dfrac{\sin x}{2}}{\dfrac{\cos x}{2}}}{\dfrac{\dfrac{\cos x}{2}}{\dfrac{\cos x}{2}} + \dfrac{\dfrac{\sin x}{2}}{\dfrac{\cos x}{2}}}\right]$

$= \tan^{-1}\left[\dfrac{1 - \dfrac{\tan x}{2}}{1 + \dfrac{\tan x}{2}}\right]$

$= \tan^{-1}\left[\dfrac{\dfrac{\tan \pi}{4} - \dfrac{\tan x}{2}}{1 + \dfrac{\tan \pi}{4} \times \dfrac{\tan x}{2}}\right]$

$= \tan^{-1}\left[\tan\left(\dfrac{\pi}{4} - \dfrac{x}{2}\right)\right]$

$u = \dfrac{\pi}{4} - \dfrac{x}{2}$

Differentiating it with respect to x,

$\dfrac{du}{dx} = 0 - \left(\dfrac{1}{2}\right)$

$\dfrac{du}{dx} = -\dfrac{1}{2}$ $\qquad\qquad$ ---(i)

Let $\quad v = \sec^{-1} x$

Differentiating it with respect to x,

$\dfrac{dv}{dx} = \dfrac{1}{x\sqrt{x^2 - 1}}$ $\qquad\qquad$ ---(ii)

Dividing equation (i) by (ii),

$\dfrac{\dfrac{du}{dx}}{\dfrac{dv}{dx}} = -\dfrac{1}{2} \times \dfrac{x\sqrt{x^2 - 1}}{1}$

$\dfrac{du}{dv} = \dfrac{-x\sqrt{x^2 - 1}}{2}$

Differentiation Ex 11.8 Q15

Let $\quad u = \sin^{-1}\left(\dfrac{2x}{1+x^2}\right)$

Put $\quad x = \tan\theta \Rightarrow \theta = \tan^{-1}x$, so

$$u = \sin^{-1}\left(\dfrac{2\tan\theta}{1+\tan^2\theta}\right)$$

$$u = \sin^{-1}(\sin 2\theta) \qquad\qquad \text{---(i)}$$

Let $\quad v = \tan^{-1}\left(\dfrac{2x}{1-x^2}\right)$

$$= \tan^{-1}\left(\dfrac{2\tan\theta}{1-\tan^2\theta}\right)$$

$$v = \tan^{-1}(\tan 2\theta) \qquad\qquad \text{---(ii)}$$

Here, $\quad -1 < x < 1$

$\Rightarrow \qquad -1 < \tan\theta < 1$

$\Rightarrow \qquad -\dfrac{\pi}{4} < \theta < \dfrac{\pi}{4}$

So, from equation (i),

$$u = 2\theta \qquad\qquad \left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]\right]$$

$$u = 2\tan^{-1}x$$

Differentiating it with respect to x,

$$\dfrac{du}{dx} = \dfrac{2}{\left(1+x^2\right)} \qquad\qquad \text{---(iii)}$$

From equation (ii),

$$v = 2\theta \qquad\qquad \left[\text{Since, } \tan^{-1}(\tan\theta) = \theta, \text{ if } \theta \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)\right]$$

$$v = 2\tan^{-1}x$$

Differentiating it with respect to x,

$$\dfrac{dv}{dx} = \dfrac{2}{1+x^2} \qquad\qquad \text{---(iv)}$$

Dividing equation (iii) by (iv),

$$\dfrac{\dfrac{du}{dx}}{\dfrac{dv}{dx}} = \dfrac{2}{1+x^2} \times \dfrac{1+x^2}{2}$$

$$\dfrac{du}{dv} = 1$$

Differentiation Ex 11.8 Q16

Let $\quad u = \cos^{-1}\left(4x^3 - 3x\right)$

Put $\quad x = \cos\theta \Rightarrow \theta = \cos^{-1}x$, so

$\qquad u = \cos^{-1}\left(4\cos^3\theta - 3\cos\theta\right)$

$\qquad u = \cos^{-1}\left(\cos 3\theta\right)$ $\qquad$ ---(i)

Let $\quad v = \tan^{-1}\left(\dfrac{\sqrt{1 - x^2}}{x}\right)$

$\qquad = \tan^{-1}\left(\dfrac{\sqrt{1 - \cos^2\theta}}{\cos\theta}\right)$

$\qquad = \tan^{-1}\left(\dfrac{\sin\theta}{\cot\theta}\right)$

$\qquad v = \tan^{-1}\left(\tan\theta\right)$ $\qquad$ ---(ii)

Here,

$\qquad \dfrac{1}{2} < x < 1$

$\Rightarrow \qquad \dfrac{1}{2} < \cos\theta < 1$

$\Rightarrow \qquad 0 < \theta < \dfrac{\pi}{3}$

So, from equation (i),

$\qquad u = 3\theta$ $\qquad \left[\text{Since, } \cos^{-1}\left(\cos\theta\right) = \theta, \text{ if } \theta \in [0, \pi]\right]$

$\qquad u = 3\cos^{-1}x$

Differentiating it with respect to x,

$\qquad \dfrac{du}{dx} = \dfrac{-3}{\sqrt{1 - x^2}}$ $\qquad$ ---(iii)

From equation (ii),

$\qquad v = \theta$ $\qquad \left[\text{Since, } \tan^{-1}\left(\tan\theta\right) = \theta, \text{ if } \theta \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)\right]$

$\qquad v = \cos^{-1}x$

Differentiating it with respect to x,

$\qquad \dfrac{dv}{dx} = \dfrac{-1}{\sqrt{1 - x^2}}$ $\qquad$ ---(iv)

Dividing equation (iii) by (iv),

$\qquad \dfrac{\frac{du}{dx}}{\frac{dv}{dx}} = \left(\dfrac{-3}{\sqrt{1 - x^2}}\right)\left(-\dfrac{\sqrt{1 - x^2}}{1}\right)$

$\qquad \dfrac{du}{dv} = 3$

Differentiation Ex 11.8 Q17

Let $\quad u = \tan^{-1}\left(\dfrac{x}{\sqrt{1-x^2}}\right)$

Put $\quad x = \cos\theta \Rightarrow \theta = \sin^{-1}x$, so

$$u = \tan^{-1}\left(\dfrac{\sin\theta}{\sqrt{1-\sin^2\theta}}\right)$$

$$= \tan^{-1}\left(\dfrac{\sin\theta}{\cos\theta}\right)$$

$$u = \tan^{-1}(\tan\theta) \qquad\qquad ---(i)$$

And,

Let $\quad v = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$

$$v = \sin^{-1}\left(2\sin\theta\sqrt{1-\sin^2\theta}\right)$$

$$= \sin^{-1}(2\sin\theta\cos\theta)$$

$$v = \sin^{-1}(\sin2\theta) \qquad\qquad ---(ii)$$

Here, $\quad -\dfrac{1}{\sqrt{2}} < x < \dfrac{1}{\sqrt{2}}$

$$\Rightarrow \quad -\dfrac{1}{\sqrt{2}} < \sin\theta < \dfrac{1}{\sqrt{2}}$$

$$\Rightarrow \quad -\dfrac{\pi}{4} < \theta < \dfrac{\pi}{4}$$

So, from equation (i),

$$u = \theta \qquad\qquad \left[\text{Since, } \tan^{-1}(\tan\theta) = \theta, \text{ if } \theta \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)\right]$$

$$u = \sin^{-1}x$$

Differentiation Ex 11.8 Q18

Let $\quad u = \sin^{-1}\left(\sqrt{1-x^2}\right)$

Put $\quad x = \cos\theta \Rightarrow \theta = \cos^{-1}x$, so

$$u = \sin^{-1}(\sin\theta) \qquad\qquad ---(i)$$

And,

Let $\quad v = \cot^{-1}\left(\dfrac{x}{\sqrt{1-x^2}}\right)$

$$= \cot^{-1}\left(\dfrac{\cos\theta}{\sqrt{1-\cos^2\theta}}\right)$$

$$= \cot^{-1}\left(\dfrac{\cos\theta}{\sin\theta}\right)$$

$$v = \cot^{-1}(\cot\theta) \qquad\qquad ---(ii)$$

Here, $\quad 0 < x < 1$

$$\Rightarrow \quad 0 < \cos\theta < 1$$

$$\Rightarrow \quad 0 < \theta < \dfrac{\pi}{2}$$

So, from equation (i),

$$u = \theta \qquad\qquad \left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]\right]$$

$$u = \cos^{-1}x$$

Differentiation Ex 11.8 Q19

Let $\quad u = \sin^{-1}\left\{2ax\sqrt{1 - a^2x^2}\right\}$

Put $\quad ax = \sin\theta \Rightarrow \theta = \sin^{-1}(ax)$

$$u = \sin^{-1}\left\{2\sin\theta\sqrt{1 - \sin^2\theta}\right\}$$

$$= \sin^{-1}\{2\sin\theta\cos\theta\}$$

$$u = \sin^{-1}(\sin 2\theta) \qquad\qquad ---(i)$$

And,

Let $\quad v = \sqrt{1 - a^2x^2}$

Differentiating it with respect to x using chain rule,

$$\frac{dv}{dx} = \frac{1}{2\sqrt{1 - a^2x^2}} \times \frac{d}{dx}\left(1 - a^2x^2\right)$$

$$= \left(\frac{0 - 2a^2x}{2\sqrt{1 - a^2x^2}}\right)$$

$$\frac{dv}{dx} = \frac{-a^2x}{\sqrt{1 - a^2x^2}} \qquad\qquad ---(ii)$$

Here, $\quad -\dfrac{1}{\sqrt{2}} < ax < \dfrac{1}{\sqrt{2}}$

$\Rightarrow \quad -\dfrac{1}{\sqrt{2}} < \sin\theta < \dfrac{1}{\sqrt{2}}$

$\Rightarrow \quad -\dfrac{\pi}{4} < \theta < \dfrac{\pi}{4}$

So, from equation (i),

$$u = 2\theta \qquad\qquad \left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

$$u = 2\sin^{-1}ax$$

Differentiation Ex 11.8 Q20

Let $\quad u = \tan^{-1}\left(\dfrac{1-x}{1+x}\right)$

Put $\quad x = \tan\theta \Rightarrow \theta = \tan^{-1}x$, so

$$u = \tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)$$

$$= \tan^{-1}\left(\frac{\dfrac{\tan\pi}{4} - \tan\theta}{1 + \dfrac{\tan\pi}{4}\tan\theta}\right)$$

$$u = \tan^{-1}\left(\tan\left(\frac{\pi}{4} - \theta\right)\right) \qquad\qquad \text{---(i)}$$

Here, $\quad -1 < x < 1$

$\Rightarrow \qquad -1 < \tan\theta < 1$

$\Rightarrow \qquad -\dfrac{\pi}{4} < \theta < \dfrac{\pi}{4}$

So, from equation (i),

$$u = \left(\frac{\pi}{4} - \theta\right) \qquad\qquad \left[\text{Since, } \tan^{-1}(\tan\theta) = \theta, \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right]$$

$$u = \frac{\pi}{4} - \tan^{-1}x$$

Differentiating it with respect to x,

$$\frac{du}{dx} = 0 - \left(\frac{1}{1+x^2}\right)$$

$$\frac{du}{dx} = -\frac{1}{1+x^2} \qquad\qquad \text{---(ii)}$$

And,

Let $\quad v = \sqrt{1-x^2}$

Differentiating it with respect to x using chain rule,

$$\frac{dv}{dx} = \frac{1}{2\sqrt{1-x^2}} \times \frac{d}{dx}\left(1-x^2\right)$$

$$= \frac{1}{2\sqrt{1-x^2}}(-2x)$$

$$\frac{dv}{dx} = \frac{-x}{\sqrt{1-x^2}} \qquad\qquad \text{---(iii)}$$

Dividing equation (ii) by (iii),

$$\frac{\dfrac{du}{dx}}{\dfrac{dv}{dx}} = -\frac{1}{\left(1+x^2\right)} \times \frac{\sqrt{1-x^2}}{-x}$$

$$\frac{du}{dv} = \frac{\sqrt{1-x^2}}{x\left(1+x^2\right)}$$