Ex 3.1

Binary Operations Ex 3.1 Q1(i)

We have,

$$a * b = a^b \text{ for all } a, b \in N$$

Let $a \in N$ and $b \in N$

$\Rightarrow \quad a^b \in N$

$\Rightarrow \quad a * b \in N$

The operation $*$ defines a binary operation on N

Binary Operations Ex 3.1 Q1(ii)

We have,

$$a \circ b = a^b \text{ for all } a, b \in Z$$

Let $a \in Z$ and $b \in Z$

$\Rightarrow \quad a^b \notin Z \qquad \Rightarrow a \circ b \notin Z$

For example, if $\quad a = 2, \; b = -2$

$\Rightarrow \quad a^b = 2^{-2} = \dfrac{1}{4} \notin Z$

$\therefore$ The operation '$\circ$' does not define a binary operation on Z.

Binary Operations Ex 3.1 Q1(iii)

We have,

$a * b = a + b - 2$ for all $a, b \in N$

Let $a \in N$ and $b \in N$

Then, $a + b - 2 \notin N$ for all $a, b \in N$

$\Rightarrow \quad a * b \notin N$

For example $a = 1$, $b = 1$

$\Rightarrow \quad a + b - 2 = 0 \notin N$

$\therefore$ The operation $*$ does not define a binary operation on N

Binary Operations Ex 3.1 Q1(iv)

We have,

$S = \{1, 2, 3, 4, 5\}$

and, $a \times_6 b$ = Remainder when ab is divided by 6

Let $a \in S$ and $b \in S$

$\Rightarrow \quad a \times_6 b \notin S$ for all $a, b \in S$

For example, $a = 2$, $b = 3$

$\Rightarrow \quad 2 \times_6 3$ = Remainder when 6 is divided by 6 = $0 \notin S$

$\therefore \quad \times_6$ does not define a binary oparation on S

Binary Operations Ex 3.1 Q1(v)

We have,

$S = \{0, 1, 2, 3, 4, 5\}$

and, $a +_6 b = \begin{cases} a + b; & \text{if } a + b < 6 \\ a + b - 6; & \text{if } a + b \geq 6 \end{cases}$

Let $a \in S$ and $b \in S$ such that $a + b < 6$

Then $a +_6 b = a + b \in S \quad [\because a + b < 6 = 0, 1, 2, 3, 4, 5]$

Let $a \in S$ and $b \in S$ such that $a + b > 6$

Then $a +_6 b = a + b - 6 \in S \quad [\because \text{if } a + b \geq 6 \text{ then } a + b - 6 \geq 0 = 0, 1, 2, 3, 4, 5]$

$\therefore \quad a +_6 b \in S$ for $a, b \in S$

$\therefore \quad +_6$ defines a binary oparation on S

Binary Operations Ex 3.1 Q1(vi)

We have,

$a \circ b = a^b + b^a$ for all $a, b \in N$

Let $a \in N$ and $b \in N$

$\Rightarrow \quad a^b \in N$ and $b^a \in N$

$\Rightarrow \quad a^b + b^a \in N$

$\Rightarrow \quad a \circ b \in N$

Thus, the operation '$\circ$' defines a binary relation on N

Binary Operations Ex 3.1 Q1(vii)

We have,

$$a*b = \frac{a-1}{b+1} \text{ for all } a,b \in Q$$

Let $a \in Q$ and $b \in Q$

Then $\frac{a-1}{b+1} \notin Q$ for $b = -1$

$\Rightarrow$ $a*b \notin Q$ for all $a,b \in Q$

Thus, the operation $*$ does not define a binary operation on Q

Binary Operations Ex 3.1 Q2

(i) On $\mathbf{Z}^+$, * is defined by $a * b = a - b$.
It is not a binary operation as the image of (1, 2) under * is $1 * 2 = 1 - 2 = -1 \notin \mathbf{Z}^+$.

(ii) On $\mathbf{Z}^+$, * is defined by $a * b = ab$.
It is seen that for each $a, b \in \mathbf{Z}^+$, there is a unique element ab in $\mathbf{Z}^+$.
This means that * carries each pair (a, b) to a unique element $a * b = ab$ in $\mathbf{Z}^+$.
Therefore, * is a binary operation.

(iii) On $\mathbf{R}$, * is defined by $a * b = ab^2$.
It is seen that for each $a, b \in \mathbf{R}$, there is a unique element ab^2 in $\mathbf{R}$.
This means that * carries each pair (a, b) to a unique element $a * b = ab^2$ in $\mathbf{R}$.
Therefore, * is a binary operation.

(iv) On $\mathbf{Z}^+$, * is defined by $a * b = |a - b|$.
It is seen that for each $a, b \in \mathbf{Z}^+$, there is a unique element $|a - b|$ in $\mathbf{Z}^+$.
This means that * carries each pair (a, b) to a unique element $a * b = |a - b|$ in $\mathbf{Z}^+$.
Therefore, * is a binary operation.

(v) On $\mathbf{Z}^+$, * is defined by $a * b = a$.
* carries each pair (a, b) to a unique element $a * b = a$ in $\mathbf{Z}^+$.
Therefore, * is a binary operation.

(vi) on R, * is defined by $a * b = a + 4b^2$
it is seen that for each element $a, b \in R$, there is unique element $a + 4b^2$ in R
This means that * carries each pair (a, b) to a unique element $a * b = a + 4b^2$ in R.
Therefore, * is a binary operation.

Binary Operations Ex 3.1 Q3

It is given that, $a*b = 2a+b-3$
Now

$$\begin{aligned} 3*4 &= 2\times 3+4-3 \\ &= 10-3 \\ &= 7 \end{aligned}$$

Binary Operations Ex 3.1 Q4

The operation * on the set A = {1, 2, 3, 4, 5} is defined as
$a * b$ = L.C.M. of a and b.
2*3 = L.C.M of 2 and 3 = 6. But 6 does not belong to the given set.
Hence, the given operation * is not a binary operation.

Binary Operations Ex 3.1 Q5

We have,

$$S = \{a,b,c\}$$

We know that the total number of binary operation on a set S with n element is n^{n^2}

$\Rightarrow$ Total number of binary operation on $S = \{a,b,c\} = 3^{3^2} = 3^9$

Binary Operations Ex 3.1 Q6

We have,

$$S = \{a,b\}$$

The total number of binary operation on $S = \{a,b\}$ in $2^{2^2} = 2^4 = 16$

Binary Operations Ex 3.1 Q7

We have,

$$M = \left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} : \ a, b \in R - \{0\} \right\} \text{ and}$$

$$A * B = AB \text{ for all } A, B \in M$$

Let $A = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \in M$ and $B = \begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix} \in M$

Now, $AB = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}\begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix} = \begin{bmatrix} ac & 0 \\ 0 & bd \end{bmatrix}$

$\therefore \quad a \in R, \ b \in R, \ c \in R, \ \& \ d \in R$

$\Rightarrow \quad ac \in R$ and $bd \in R$

$\Rightarrow \quad \begin{bmatrix} ac & 0 \\ 0 & bd \end{bmatrix} \in M$

$\Rightarrow \quad A * B \in M$

Thus, the operater $*$ difines a binary operation on M

Binary Operations Ex 3.1 Q8

S = set of rational numbers of the form $\frac{m}{n}$ where $m \in Z$ and $n = 1, 2, 3$

Also, $a * b = ab$

Let $a \in S$ and $b \in S$

$\Rightarrow \quad ab \notin S$

For example $a = \frac{7}{3}$ and $b = \frac{5}{2}$

$\Rightarrow \quad ab = \frac{35}{6} \notin S$

$\therefore \quad a * b \notin S$

Hence, the operater $*$ does not define a binary operation on S

Binary Operations Ex 3.1 Q9

It is given that, a*b = 2a + b

Now

(2*3) = 2× 2 + 3

= 4 + 3

= 7

(2*3)*4 = 7*4 = 2 × 7 + 4

= 14 + 4

= 18

Binary Operations Ex 3.1 Q10

It is given that, a*b = LCM (a, b)

Now

5*7 = LCM (5, 7)

= 35

Ex 3.2

Binary Operations Ex 3.2 Q1

We have,

$$a * b = l.c.m.(a, b) \text{ for all } a, b \in N$$

(1)
Now,

$$2 * 4 = l.c.m\ (2,4) = 4$$
$$3 * 5 = l.c.m\ (3,5) = 15$$
$$1 * 6 = l.c.m\ (1,6) = 6$$

(ii)
Commutativity:
Let $a, b \in N$ then,

$$a * b = l.c.m(a, b)$$
$$= l.c.m(b, a)$$
$$= b * a$$

$\Rightarrow \quad a * b = b * a$

$\therefore \quad *$ is commutative on N.

Associativity:
Let $a, b, c \in N$ then,

$$(a * b) * c = l.c.m(a, b) * c$$
$$= l.c.m(a, b, c) \qquad ---(i)$$

and, $a * (b * c) = a * l.c.m(b, c)$

$$= l.c.m(a, b, c) \qquad ---(ii)$$

From (i) and (ii)

$$(a * b) * c = a * (b * c)$$

$\therefore \quad *$ is associative on N.

Binary Operations Ex 3.2 Q2

(i) Clearly, by definition $a * b = 1 = b * a$, $\forall a, b \in N$
Also, $(a * b) * c = (1 * c) = 1$
and $a * (b * c) = (a * 1) = 1 \qquad \forall a, b, c \in N$
Hence, N is both associative and commutative.

(ii) $a * b = \dfrac{a+b}{2} = \dfrac{b+a}{2} = b * a$,
which shows * is commutative.

Further, $(a * b) * c = \left(\dfrac{a+b}{2}\right) * c = \dfrac{\left(\dfrac{a+b}{2}\right) + c}{2} = \dfrac{a+b+2c}{4}$

$a * (b * c) = a * \left(\dfrac{b+c}{2}\right) = \dfrac{a + \left(\dfrac{b+c}{2}\right)}{2} = \dfrac{2a+b+c}{2} \neq \dfrac{a+b+2c}{4}$

Hence, * is not associative.

Binary Operations Ex 3.2 Q3

We have, binary operator $*$ defined on A and is given by

$$a*b=b \text{ for all } a,b\in A$$

Commutativity: Let $a,b\in A$, then

$$a*b=b\neq a=b*a$$

$\Rightarrow \quad a*b\neq b*a$

$\therefore \quad$ '$*$' is not commutative on A.

Associativity: Let $a,b,c\in A$, then

$$(a*b)*c=b*c=c \qquad ---(\text{i})$$

and, $\quad a*(b*c)=a*c=c \qquad ---(\text{ii})$

From (i) and (ii)

$$(a*b)*c=a*(b*c)$$

$\Rightarrow \quad$ '$*$' is associative on A.

Binary Operations Ex 3.2 Q4(i)

'$*$' is a binary operator on Z defined by $a*b=a+b+ab$ for all $a,b\in Z$.

Commutativity of '$*$':

Let $a,b\in Z$, then

$$a*b=a+b+ab=b+a+ba=b*a$$

$\therefore \quad a*b=b*a$

Associative of '$*$':

Let $a,b\in Z$, then

$$(a*b)*c=(a+b+ab)*c=a+b+ab+c+ac+bc+abc$$
$$=a+b+c+ab+bc+ac+abc \qquad ---(\text{i})$$

Again, $a*(b*c)=a*(b+c+bc)$

$$=a+b+c+bc+ab+ac+abc \qquad ---(\text{ii})$$

From (i) & (ii), we get

$$(a*b)*c=a*(b*c)$$

$\therefore \quad *$ is commutative and associative on Z

Binary Operations Ex 3.2 Q4(ii)

Commutative:

Let $a, b \in N$, then

$$a * b = 2^{ab} = 2^{ba} = b * a$$

$\therefore \quad a * b = b * a$

$\therefore \quad *$ is commutative on N

Associative:

Let $a, b, c \in N$, then

$$(a * b) * c = 2^{ab} * c = 2^{2^{ab}.c} \qquad ---(i)$$

and, $\quad a * (b * c) = a * 2^{bc} = 2^{a.2^{bc}} \qquad ---(ii)$

From (i) & (ii), we get

$$(a * b) * c \neq a * (b * c)$$

$\therefore \quad *$ is not associative on N

Binary Operations Ex 3.2 Q4(iii)

Commutativity:

Let $a, b \in Q$, then

$$a * b = a - b \neq b - a = b * a$$

$\therefore \quad a * b \neq b * a$

$\Rightarrow \quad *$ is not commutative on Q

Associative:

Let $a, b, c \in Q$, then

$$(a * b) * c = (a - b) * c = a - b - c \qquad ---(i)$$

and, $\quad a * (b * c) = a * (b - c) = a - b + c \qquad ---(ii)$

From (i) & (ii), we get

$$(a * b) * c \neq a * (b * c)$$

$\therefore \quad *$ is not associative on Q

Binary Operations Ex 3.2 Q4(iv)

Commutative:

Let $a, b \in Q$, then

$$a \text{ e } b = a^2 + b^2 = b^2 + a^2 = b \text{ e } a$$

$\Rightarrow \quad a \text{ e } b = b \text{ e } a$

$\therefore \quad$ e is commutative on Q.

Associative:

Let $a, b, c \in Q$, then

$$(a \text{ e } b) \text{ e } c = (a^2 + b^2) \text{ e } c = (a^2 + b^2)^2 + c^2 \qquad ---(i)$$

and, $\quad a \text{ e } (b \text{ e } c) = a \text{ e } (a^2 + b^2) = a^2 + (b^2 + c^2)^2 \qquad ---(ii)$

From (i) & (ii),

$$(a \text{ e } b) \text{ e } c \neq a \text{ e } (b \text{ e } c)$$

$\therefore \quad$ e is not associative on Q.

Binary Operations Ex 3.2 Q4(v)

Binary operation '$\circ$' defined on Q, given by $aob = \frac{ab}{2}$ for all $a, b \in Q$

Commutative:

Let $a, b \in Q$, then

$$a \circ b = \frac{ab}{2} = \frac{ba}{2} = b \circ a$$

$\Rightarrow \quad a \circ b = b \circ a$

$\therefore \quad \circ$ is commutative on Q.

Associativity:

Let $a, b, c \in Q$, then

$$(a \circ b) \circ c = \left(\frac{ab}{2}\right) \circ c = \frac{abc}{4} \qquad ---(i)$$

$$a \circ (b \circ c) = a \circ \left(\frac{bc}{2}\right) = \frac{abc}{4} \qquad ---(ii)$$

From (i) & (ii) we get

$$(a \circ b) \circ c = a \circ (b \circ c)$$

$\therefore \quad$ '$\circ$' is associative on Q.

Binary Operations Ex 3.2 Q4(vi)

Commutative:

Let $a, b \in Q$, then

$$a * b = ab^2 \neq ba^2 = b * a$$

$\Rightarrow \quad a * b \neq b * a$

$\therefore \quad *$ is not commutative on Q

Associativity:

Let $a, b, c \in Q$, then

$$(a * b) * c = ab^2 * c = ab^2c^2 \qquad ---(i)$$

& $\quad a * (b * c) = a * bc^2 = a(bc^2)^2 \qquad ---(ii)$

From (i) and (ii)

$$(a * b) * c \neq a * (b * c)$$

$\therefore \quad *$ is not associative on Q

Binary Operations Ex 3.2 Q4(vii)

Commutativity:

Let $a, b \in Q$, then

$$a * b = a + ab \qquad ---(i)$$
$$b * a = b + ab \qquad ---(ii)$$

From (i) & (ii)

$$a * b \neq b * a$$

$\Rightarrow$ $*$ is not commutative on Q

Associativity:

Let $a, b, c \in Q$, then

$$(a * b) * c = (a + ab) * c = a + ab + ac + abc \qquad ---(i)$$
$$a * (b * c) = a * (b + bc)$$
$$= a + ab + abc \qquad ---(ii)$$

From (i) and (ii)

$$(a * b) * c \neq a * (b * c)$$

$\Rightarrow$ $*$ is not associative on Q

Binary Operations Ex 3.2 Q4(viii)

Commutativity: Let $a, b \in R$, then

$$a * b = a + b - 7$$
$$= b + a - 7$$
$$= b * a$$

$\Rightarrow$ $a * b = b * a$

$\Rightarrow$ $*$ is commutative on R

Associativity: Let $a, b, c \in Q$, then

$$(a * b) * c = (a + b - 7) * c$$
$$= a + b - 7 + c - 7$$
$$= a + b + c - 17 \qquad ---(i)$$

and, $a * (b * c) = a * (b + c - 7)$

$$= a + b + c - 7 - 7$$
$$= a + b + c - 17 \qquad ---(ii)$$

From (i) & (ii)

$$(a * b) * c = a * (b * c)$$

$\Rightarrow$ $*$ is associative on R

Binary Operations Ex 3.2 Q4(ix)

Commutativity:
Let $a, b \in R - \{-1\}$, then

$$a * b = \frac{a}{b+1} \neq \frac{b}{a+1} = b * a$$

$\Rightarrow \quad a * b \neq b * a$

$\Rightarrow \quad *$ is not commutative on $R - \{-1\}$

Associativity:
Let $a, b, c \in R - \{-1\}$, then

$$(a * b) * c = \left(\frac{a}{b+1}\right) * c$$

$$= \frac{\frac{a}{b+1}}{c+1} = \frac{a}{(b+1)(c+1)} \qquad ---(i)$$

& $\quad a * (b * c) = a * \left(\frac{b}{c+1}\right)$

$$= \frac{a}{\frac{b}{c+1} + 1} = \frac{a(c+1)}{b+c+1} \qquad ---(ii)$$

From (i) and (ii)

$$(a * b) * c \neq a * (b * c)$$

$\Rightarrow \quad *$ is not associative on $R - \{-1\}$

Binary Operations Ex 3.2 Q4(x)

Commutativity:
Let $a, b \in Q$, then

$$a * b = ab + 1 = ba + 1 = b * a$$

$\Rightarrow \quad a * b = b * a$

$\Rightarrow \quad *$ is commutative on Q

Associativity:
Let $a, b, c \in Q$, then

$$(a * b) * c = (ab + 1) * c$$
$$= abc + c + 1 \qquad ---(i)$$

$$a * (b * c) = a * (bc + 1)$$
$$= abc + a + 1 \qquad ---(ii)$$

From (i) and (ii)

$$(a * b) * c \neq a * (b * c)$$

$\Rightarrow \quad *$ is not associative on Q.

Binary Operations Ex 3.2 Q4(xi)

Commutativity:
Let $a, b \in N$, then

$$a * b = a^b \neq b^a = b * a$$

$\Rightarrow \quad a * b \neq b * a$

$\Rightarrow$ '$*$' is not commutative on N

Associativity:
Let $a, b, c \in N$, then

$$(a * b) * c = a^b * c = \left(a^b\right)^c = a^{bc} \qquad ---(i)$$

$$a * (b * c) = a * b^c = (a)^{b^c} \qquad ---(ii)$$

From (i) and (ii)

$$a^{bc} \neq (a)^{b^c}$$

$\Rightarrow (a * b) * c \neq a * (b * c)$

$\Rightarrow$ '$*$' is not associative on N.

Binary Operations Ex 3.2 Q4(xii)

Commutativity:
Let $a, b \in N$, then

$$a * b = a^b \neq b^a = b * a$$

$\Rightarrow \quad a * b \neq b * a$

$\Rightarrow$ '$*$' is not commutative on N

Associativity:
Let $a, b, c \in N$, then

$$(a * b) * c = a^b * c = \left(a^b\right)^c = a^{bc} \qquad ---(i)$$

$$a * (b * c) = a * b^c = (a)^{b^c} \qquad ---(ii)$$

From (i) and (ii)

$$a^{bc} \neq (a)^{b^c}$$

$\Rightarrow (a * b) * c \neq a * (b * c)$

$\Rightarrow$ '$*$' is not associative on N.

Binary Operations Ex 3.2 Q4(xiii)

Commutativity:
Let $a, b \in Z$ then,

$$a * b = a - b \neq b - a = b * a$$

$\Rightarrow$ $a * b \neq b * a$

$\Rightarrow$ $*$ is not commutative on Z

Associativity:
Let $a, b, c \in Z$, then

$$(a * b) * c = (a - b) * c = (a - b - c) \quad ---(i)$$

& $$a * (b * c) = a * (b - c) = (a - b + c) \quad ---(ii)$$

From (i) & (ii)

$$(a * b) * c \neq a * (b * c)$$

$\Rightarrow$ '$*$' is not associative on Z.

Binary Operations Ex 3.2 Q4(xiv)

Commutativity:
Let $a, b \in Q$ then,

$$a * b = \frac{ab}{4} = \frac{ba}{4} = b * a$$

$\Rightarrow$ $a * b = b * a$

$\therefore$ $*$ is commutative on Q

Associativity:
Let $a, b, c \in Q$ then,

$$(a * b) * c = \frac{ab}{4} * c = \frac{abc}{16} \quad ---(i)$$

and, $$a * (b * c) = a * \frac{bc}{4} = \frac{abc}{16} \quad ---(ii)$$

From (i) and (ii)

$$(a * b) * c = a * (b * c)$$

$\therefore$ '$*$' is associative on Q.

Binary Operations Ex 3.2 Q4(xv)

Commutativity:
Let $a, b \in Q$ then,

$$a * b = (a - b)^2 = (b - a)^2 = b * a$$

$\Rightarrow$ $a * b = b * a$

$\therefore$ '$*$' is commutative on Q.

Associativity:
Let $a, b, c \in Q$ then,

$$(a * b) * c = (a - b)^2 * c = \left[(a - b)^2 - c\right]^2 \quad ---(i)$$

and, $$a * (b * c) = a * (b - c)^2 = \left[a - (b - c)^2\right]^2 \quad ---(ii)$$

From (i) and (ii)

$$(a * b) * c \neq a * (b * c)$$

$\therefore$ $*$ is not associative on Q.

Binary Operations Ex 3.2 Q5

The binary operator o defined on $Q-\{-1\}$ is given by

$$a \circ b = a + b - ab \text{ for all } a, b \in Q - \{-1\}$$

Commutativity:

Let $a, b \in Q-\{-1\}$, then

$$a \circ b = a + b - ab = b + a - ba = b \circ a$$

$\Rightarrow \quad a \circ b = b \circ a$

$\Rightarrow \quad$ '$\circ$' is commutative on $Q-\{-1\}$.

Binary Operations Ex 3.2 Q6

The binary operator $*$ defined on Z and is given by

$$a * b = 3a + 7b$$

Commutativity: Let $a, b \in Z$, then

$$a * b = 1a + 7b \text{ and}$$
$$b * a = 3b + 7a$$

$\therefore \quad a * b \neq b * a$

Hence, '$*$' is not commutative on Z.

Binary Operations Ex 3.2 Q7

We have, $*$ is a binary operator defined on Z is given by

$$a * b = ab + 1 \text{ for all } a, b \in Z$$

Associativity: Let $a, b, c \in Z$, then

$$(a * b) * c = (ab + 1) * c$$
$$= abc + c + 1 \qquad ---(i)$$

and, $\quad a * (b * c) = a * (bc + 1)$
$$= abc + a + 1 \qquad ---(ii)$$

From (i) & (ii)

$\therefore \quad (a * b) * c \neq a * (b * c)$

Hence, '$*$' is not associative on Z.

Binary Operations Ex 3.2 Q8

We have, set of real numbers except -1 and $*$ is an operator given by

$$a * b = a + b + ab \text{ for all } a, b \in S = R - \{-1\}$$

Now, $\forall\ a, b \in S$

$$a * b = a + b + ab \in S$$

$\because$ if $\quad a + b + ab = -1$

$\Rightarrow \quad a + b(1 + a) + 1 = 0$

$\Rightarrow \quad (a + 1)(b + 1) = 0$

$\Rightarrow \quad a = -1 \text{ or } b = -1$

but $a \neq -1$ and $b \neq -1$ (given)

$\therefore \quad a + b + ab \neq -1$

$\Rightarrow \quad a * b \in S$ for $ab \in S$

$\Rightarrow \quad$ '$*$' is a binary operator on S

Commutativity: Let $a, b \in S$

$\Rightarrow \quad a * b = a + b + ab = b + a + ba = b * a$

$\Rightarrow \quad a * b = b * a$

and, $a*(b*c) = a*(b+c+bc)$
$$= a+b+c+bc+ab+ac+abc \qquad ---(ii)$$

From (i) and (ii)
$$(a*b)*c = a*(b*c)$$

$\therefore$ '*' is associative on S.

Now, $(2*x)*3 = 7$

$\Rightarrow \quad (2+x+2x)*3 = 7$

$\Rightarrow \quad 2+x+2x+3+6+3x+6x = 7$

$\Rightarrow \quad 11+12x = 7$

$\Rightarrow \quad 12x = -4$

$\Rightarrow \quad x = \frac{-4}{12} \qquad \Rightarrow x = \frac{-1}{3}$

Binary Operations Ex 3.2 Q9

The binary operator $*$ defined as
$$a*b = \frac{a-b}{2} \text{ for all } a,b \in Q.$$

Now,

Associativity: Let $a,b,c \in Q$, then
$$(a*b)*c = \frac{a-b}{2}*c = \frac{\frac{a-b}{2}-c}{2}$$
$$= \frac{a-b-2c}{4} \qquad ---(i)$$

and, $a*(b*c) = a*\frac{b-c}{2} = \frac{a-\frac{b-c}{2}}{2}$
$$= \frac{2a-b+c}{4} = \qquad ---(ii)$$

From (i) & (ii)
$$(a*b)*c \neq a*(b*c)$$

Hence, '*' is not associative on Q.

Binary Operations Ex 3.2 Q10

The binary operator $*$ defined as
$$a*b = a+3b-4 \text{ for all } a,b \in Z$$

Now,

Commutativity: Let $a,b \in Z$, then
$$a*b = a+3b-4 \neq b+3a-4 = b*a$$

$\Rightarrow \quad a*b \neq b*a$

$\Rightarrow \quad$ '*' is not commutative on Z.

Associativity: Let $a,b,c \in Z$, then
$$(a*b)*c = (a+3b-4)*c = a+3b-4+3c-4$$
$$= a+3b+3c-8 \qquad ---(i)$$

and, $a*(b*c) = a*(b+3c-4) = a+3(b+3c-4)-4$
$$= a+3b+9c-16 \qquad ---(ii)$$

From (i) & (ii)
$$(a*b)*c \neq a*(b*c)$$

Hence, '*' is not associative on Z.

Binary Operations Ex 3.2 Q11

Q be the set of rational numbers and $*$ be a binary operation defined as

$$a*b=\frac{ab}{5} \text{ for all } a,b\in Q$$

Now,

Associativity: Let $a,b,c\in Q$, then

$$(a*b)*c=\frac{ab}{5}*c=\frac{abc}{25} \qquad ---(i)$$

and, $$a*(b*c)=a*\frac{bc}{5}=\frac{abc}{25} \qquad ---(ii)$$

From (i) & (ii)

$\therefore$ $(a*b)*c=a*(b*c)$

$\Rightarrow$ $*$ is associative on Q.

Binary Operations Ex 3.2 Q12

The binary operator $*$ is defined as

$$a*b=\frac{ab}{7} \text{ for all } a,b\in Q$$

Now,

Associativity: Let $a,b,c\in Q$, then

$$(a*b)*c=\frac{ab}{7}*c=\frac{abc}{49} \qquad ---(i)$$

and, $$a*(b*c)=a*\frac{bc}{7}=\frac{abc}{49} \qquad ---(ii)$$

From (i) & (ii)

$$(a*b)*c=a*(b*c)$$

$\Rightarrow$ '$*$' is associative on Q.

Binary Operations Ex 3.2 Q13

The binary operator $*$ defined as

$$a*b=\frac{a+b}{2} \text{ for all } a,b\in Q.$$

Now,

Associativity: Let $a,b,c\in Q$, then

$$(a*b)*c=\frac{a+b}{2}*c=\frac{\frac{a+b}{2}+c}{2}$$
$$=\frac{a+b+2c}{4} \qquad ---(i)$$

and, $$a*(b*c)=a*\frac{b+c}{2}$$
$$=\frac{a+\frac{b+c}{2}}{2}$$
$$=\frac{2a+b+c}{4}= \qquad ---(ii)$$

From (i) & (ii)

$$(a*b)*c\neq a*(b*c)$$

Hence, '$*$' is not associative on Q.

Ex 3.3

Binary Operations Ex 3.3 Q1

The binary operator $*$ is defined on I^+
and is given by,

$$a * b = a + b \text{ for all } a, b \in I^+$$

Let $a \in I^+$ and $e \in I^+$ be the identity element with respect to $*$.
by identity property, we have,

$$a * e = e * a = a$$

$\Rightarrow \quad a + e = a$
$\Rightarrow \quad e = 0$

Thus the required identity element is 0.

Binary Operations Ex 3.3 Q2

Let $R - \{-1\}$ be the set and $*$ be a binary operator, given by

$$a * b = a + b + ab \text{ for all } a, b \in R - \{-1\}$$

Now,
Let $a \in R - \{-1\}$ and $e \in R - \{-1\}$ be the identity element with respect to $*$.
by identity property, we have,

$$a * e = e * a = a$$

$\Rightarrow \quad a + e + ae = a$
$\Rightarrow \quad e(1 + a) = 0$
$\Rightarrow \quad e = 0 \qquad [\because 1 + a \neq 0 \text{ as } a \neq -1]$

$\therefore$ The required identity element is 0.

Binary Operations Ex 3.3 Q3

We are given the binary operator $*$ defined on Z as

$$a * b = a + b - 5 \text{ for all } a, b \in Q.$$

Let e be the identity element with respect to $*$

Then, $\quad a * e = e * a = a \qquad$ [By identity property]

$\Rightarrow \quad a + e - 5 = a$

$\Rightarrow \quad e = 5$

Hence, the required identity element with respect to $*$ is 5.

Binary Operations Ex 3.3 Q4

The binary operator $*$ is defined on Z, and is given by

$$a * b = a + b + 2 \text{ for all } a, b \in Z.$$

Let $a \in Z$ and $e \in Z$ be the identity element with respect to $*$, then

$$a * e = e * a = a \qquad \text{[By identity property]}$$

$\Rightarrow \quad a + e + 2 = a$

$\Rightarrow \quad e = -2 \in Z$

Hence, the identity element with respect to $*$ is -2.

Ex 3.4

Binary Operations Ex 3.4 Q1

Given,

$a * b = a + b - 4$ for all $a, b \in Z$

(i)

Commutative: Let $a, b \in Z$, then

$\Rightarrow \quad a * b = a + b - 4 = b + a - 4 = b * a$

$\Rightarrow \quad a * b = b * a$

So, '$*$' is commutative on Z.

Associativity: Let $a, b, c \in Z$, then

$$(a * b) * c = (a + b - 4) * c = a + b - 4 + c - 4$$
$$= a + b + c - 8 \qquad ---(i)$$

and, $\quad a * (b * c) = a * (b + c - 4) = a + b + c - 8 \qquad ---(ii)$

From (i) & (ii)

$$(a * b) * c = a * (b * c)$$

So, '$*$' is associative on Z.

(ii)

Let $e \in Z$ be the identity element with respect to $*$.

By identity property, we have

$a * e = e * a = a$ for all $a \in Z$

$\Rightarrow \quad a + e - 4 = a$

$\Rightarrow \quad e = 4$

So, $e = 4$ will be the identity element with respect to $*$

(iii)

Let $b \in Z$ be the inverse element of $a \in Z$

Then, $\quad a * b = b * a = e$

$\Rightarrow \quad a + b - 4 = e$

$\Rightarrow \quad a + b - 4 = 4 \qquad [\because e = 4]$

$\Rightarrow \quad b = 8 - a$

Thus, $\quad b = 8 - a$ will be the inverse element of $a \in Z$.

Binary Operations Ex 3.4 Q2

We have,

$$a * b = \frac{3ab}{5} \text{ for all } a,b \in Q_0$$

(i)

Commutative: Let $a,b \in Q_0$, then

$$a * b = \frac{3ab}{5} = \frac{3ba}{5} = b * a$$

$\Rightarrow \quad a * b = b * a$

So, '$*$' is commutative on Q_0

Associativity: Let $a,b,c \in Q_0$, then

$$(a * b) * c = \frac{3ab}{5} * c$$

$$= \frac{9abc}{25} \qquad ---(i)$$

and, $\quad a * (b * c) = a * \frac{3bc}{5}$

$$= \frac{9abc}{25} \qquad ---(ii)$$

From (i) & (ii)

$$(a * b) * c = a * (b * c)$$

So, '$*$' is associative on Q_0

(ii)

Let $e \in Q_0$ be the identity element with respect to $*$, then

$$a * e = e * a = a \text{ for all } a \in Q_0$$

$\Rightarrow \quad \frac{3ae}{5} = a$

$\Rightarrow \quad e = \frac{5}{3}$

will be the identity element with respect to $*$.

(iii)

Let $b \in Q_0$ be the inverse element of $a \in Q_0$, then

$$a * b = b * a = e$$

$\Rightarrow \quad \frac{3}{5}ab = e$

$\Rightarrow \quad \frac{3}{5}ab = \frac{5}{3} \qquad \left[\because e = \frac{5}{3}\right]$

$\Rightarrow \quad b = \frac{25}{9a}$

$\therefore \quad b = \frac{25}{9a}$ is the inverse of $a \in Q_0$.

Binary Operations Ex 3.4 Q3

We have,

$$a * b = a + b + ab \text{ for all } a, b \in Q - \{-1\}$$

(i)
Commutativity: Let $a, b \in Q - \{-1\}$

$\Rightarrow \quad a * b = a + b + ab = b + a + ba = b * a$
$\Rightarrow \quad a * b = b * a$

$\Rightarrow \quad$ '$*$' is commutative on $Q - \{-1\}$

Associativity: Let $a, b, c \in Q - \{-1\}$, then

$$\Rightarrow \quad (a * b) * c = (a + b + ab) * c$$
$$= a + b + ab + c + ac + bc + abc \qquad ---(i)$$

and, $\quad a * (b * c) = a * (b + c + bc)$
$$= a + b + c + bc + ab + ac + abc \qquad ---(ii)$$

From (i) & (ii)
$$(a * b) * c = a * (b * c)$$

$\Rightarrow \quad *$ is associative on $Q - \{-1\}$

(ii)
Let e be identity element with respect to $*$.

By identity property,
$$a * e = a = e * a \text{ for all } a \in Q - \{-1\}$$

$\Rightarrow \quad a + e + ae = a$
$\Rightarrow \quad e(1 + a) = 0 \quad \Rightarrow e = 0 \qquad [\because 1 + a \neq 0 \text{ as } a \neq -1]$

$\therefore \quad e = 0$ is the identity element with respect to $*$

(iii)
Let b be the inverse of $a \in Q - \{-1\}$

Then, $\quad a * b = b * a = e \qquad [e \text{ is the identity element}]$

$\Rightarrow \quad a + b + ab = e$
$\Rightarrow \quad a + b + ab = 0$
$\Rightarrow \quad b(1 + a) = -a$
$\Rightarrow \quad b = \frac{-a}{1 + a} \qquad \left[\begin{array}{l}\because \frac{-a}{1+a} \neq -1, \text{ because if } \frac{-a}{1+a} = -1 \\ \Rightarrow a = 1 + a \Rightarrow 1 = 0 \text{ Not possible}\end{array}\right]$

$\therefore \quad b = \frac{-a}{1 + a}$ is the inverse of a with respect to $*$

Binary Operations Ex 3.4 Q4

We have,

$$(a,b)\odot(c,d)=(ac,bc+d) \text{ for all } (a,b),(c,d)\in R_0\times R$$

(i)

Commutativity: Let $(a,b),(c,d)\in R_0\times R$, then

$\Rightarrow \quad (a,b)\odot(c,d)=(ac,bc+d) \qquad ---(i)$

and, $\quad (c,d)\odot(a,b)=(ca,da+b) \qquad ---(ii)$

From (i) & (ii)

$$(a,b)\odot(c,d)\neq(c,d)\odot(a,b)$$

$\Rightarrow$ '$\odot$' is not commutative on $R_0\times R$.

Associativity: Let $(a,b),(c,d),(e,f)\in R_0\times R$, then

$$\Rightarrow \quad ((a,b)\odot(c,d))\odot(e,f)=(ac,bc+d)\odot(e,f)$$
$$=(ace,bce,de+f) \qquad ---(i)$$

$$\text{and,} \quad (a,b)\odot(c,d\odot(e,f))=(a,b)\odot(ce,de+f)$$
$$=(ace,bce+de+f) \qquad ---(ii)$$

$$\Rightarrow \quad ((a,b)\odot(c,d))\odot(e,f)=(a,b)\odot((c,d)\odot(e,f))$$

$\Rightarrow$ '$\odot$' is associative on $R_0\times R$.

(ii)

Let $(x,y)\in R_0\times R$ be the identity element with respect to $\odot$, then

$$(a,b)\odot(x,y)=(x,y)\odot(a,b)=(a,b) \text{ for all } (a,b)\in R_0\times R$$

$\Rightarrow \quad (ax,bx+y)=(a,b)$

$\Rightarrow \quad ax=a$ and $bx+y=b$

$\Rightarrow \quad x=1$, and $y=0$

$\therefore \quad (1,0)$ will be the identity element with respect to $\odot$.

(iii)

Let $(c,d)\in R_0\times R$ be the inverse of $(a,b)\in R_0\times R$, then

$$(a,b)\odot(c,d)=(c,d)\odot(a,b)=e$$

$\Rightarrow \quad (ac,bc+d)=(1,0) \qquad [\because e=(1,0)]$

$\Rightarrow \quad ac=1$ and $bc+d=0$

$\Rightarrow \quad c=\frac{1}{a}$ and $d=-\frac{b}{a}$

$\therefore \quad \left(\frac{1}{a},-\frac{b}{a}\right)$ will be the inverse of (a,b).

Binary Operations Ex 3.4 Q5

We have,

$$a * b = \frac{ab}{2} \text{ for all } a, b \in Q_0$$

(i)

Commutativity: Let $a, b \in Q_0$, then

$$\Rightarrow \quad a * b = \frac{ab}{2} = \frac{ba}{2} = b * a$$

$$\Rightarrow \quad a * b = b * a$$

Hence, '$*$' is commutative on Q_0.

Associativity: Let $a, b, c \in Q_0$, then

$$\Rightarrow \quad (a * b) * c = \frac{ab}{2} * c = \frac{abc}{4} \qquad ---(i)$$

$$\text{and,} \quad a * (b * c) = a * \frac{bc}{2} = \frac{abc}{4} \qquad ---(ii)$$

From (i) & (ii)

$$(a * b) * c = a * (b * c)$$

$\Rightarrow \quad *$ is associative on Q_0.

(ii)

Let $e \in Q_0$ be the identity element with respect to $*$.

By identity property, we have,

$$a * e = e * a = a \text{ for all } a \in Q_0$$

$$\Rightarrow \quad \frac{ae}{2} = a \qquad \Rightarrow e = 2$$

Thus, the required identity element is 2.

(iii)

Let $b \in Q_0$ be the inverse of $a \in Q_0$ with respect to $*$, then,

$$a * b = b * a = e \text{ for all } a \in Q_0$$

$$\Rightarrow \quad \frac{ab}{2} = e \qquad \Rightarrow \frac{ab}{2} = 2$$

$$\Rightarrow b = \frac{4}{a}$$

Thus, $b = \frac{4}{a}$ is the inverse of a with respect to $*$.

Binary Operations Ex 3.4 Q6

We have,

$$a * b = a + b - ab \text{ for all } a, b \in R - \{+1\}$$

(i)

Commutative: Let $a, b \in R - \{+1\}$, then,

$\Rightarrow \quad a * b = a + b - ab = b + a - ba = b * a$

$\Rightarrow \quad a * b = b * a$

So, '$*$' is commutative on $R - \{+1\}$.

Associativity: Let $a, b, c \in R - \{+1\}$, then

$$\begin{aligned}(a * b) * c &= (a + b - ab) * c \\ &= a + b - ab + c - ac - bc + abc \\ &= a + b + c - ab - ac - bc + abc \qquad ---(i)\end{aligned}$$

and,

$$\begin{aligned}a * (b * c) &= a * (b + c - bc) \\ &= a + b + c - bc - ab - ac + abc \qquad ---(ii)\end{aligned}$$

From (i) & (ii)

$$(a * b) * c = a * (b * c)$$

So, '$*$' is associative on $R - \{+1\}$.

(ii)

Let $e \in R - \{+1\}$ be the identity element with respect to $*$, then

$$a * e = e * a = a \text{ for all } a \in R - \{+1\}$$

$\Rightarrow \quad a + e - ae = a$

$\Rightarrow \quad e(1 - a) = 0$

$\Rightarrow \quad e = 0 \qquad [\because a \neq 1 \Rightarrow 1 - a \neq 0]$

$\therefore \quad e = 0$ will be the identity element with respect to $*$.

(iii)

Let $b \in R - \{1\}$ be the inverse element of $a \in R - \{1\}$, then

$$a * b = b * a = e$$

$\Rightarrow \quad a + b - ab = 0 \qquad [\because e = 0]$

$\Rightarrow \quad b(1 - a) = -a$

$\Rightarrow \quad b = \frac{-a}{1 - a} \neq 1 \qquad \left[\begin{array}{l}\because \text{if } \frac{-a}{1-a} = 1 \\ \Rightarrow -a = 1 - a \quad \Rightarrow 1 = 0 \\ \quad \text{Not possible}\end{array}\right]$

$\therefore \quad b = \frac{-a}{1 - a}$ is the inverse of $a \in R - \{1\}$ with respect to $*$.

Binary Operations Ex 3.4 Q7

We have,

$$(a,b)*(c,d)=(ac,bd) \text{ for all } (a,b),(c,d)\in A$$

(i)

Let $(a,b),(c,d)\in A$, then

$$\begin{aligned}(a,b)*(c,d)&=(ac,bd)\\&=(ca,db) \qquad [\because ac=ca \text{ and } bd=db]\\&=(c,d)*(a,b)\end{aligned}$$

$\Rightarrow \quad (a,b)*(c,d)=(c,d)*(a,b)$

So, '$*$' is commutative on A

Associativity: Let $(a,b),(c,d),(e,f)\in A$, then

$$\begin{aligned}\Rightarrow \quad ((a,b)*(c,d))*(e,f)&=(ac,bd)*(e,f)\\&=(ace,bdf) \qquad ---(i)\end{aligned}$$

$$\begin{aligned}\text{and,} \quad (a,b)*((c,d)*(e,f))&=(a,b)*(ce,df)\\&=(ace,bdf) \qquad ---(ii)\end{aligned}$$

From (i) & (ii)

$\Rightarrow \quad ((a,b)*(c,d))*(e,f)=(a,b)*((c,d)*(e,f))$

So, '$*$' is associative on A.

(ii)

Let $(x,y)\in A$ be the identity element with respect to $*$.

$$(a,b)*(x,y)=(x,y)*(a,b)=(a,b) \text{ for all } (a,b)\in A$$

$\Rightarrow \quad (ax,by)=(a,b)$

$\Rightarrow \quad ax=a$ and $by=b$

$\Rightarrow \quad x=1$, and $y=1$

$\therefore \quad (1,1)$ will be the identity element

(iii)

Let $(c,d)\in A$ be the inverse of $(a,b)\in A$, then

$$(a,b)*(c,d)=(c,d)*(a,b)=e$$

$\Rightarrow \quad (ac,bd)=(1,1) \qquad [\because e=(1,1)]$

$\Rightarrow \quad ac=1$ and $bd=1$

$\Rightarrow \quad c=\frac{1}{a}$ and $d=\frac{1}{b}$

$\therefore \quad \left(\frac{1}{a},\frac{1}{b}\right)$ will be the inverse of (a,b) with respect to $*$.

Binary Operations Ex 3.4 Q8

The binary operation * on **N** is defined as:

$a * b$ = H.C.F. of a and b

It is known that:

H.C.F. of a and b = H.C.F. of b and a, $a, b \in$ **N**.

Therefore, $a * b = b * a$

Thus, the operation * is commutative.

For $a, b, c \in$ **N**, we have:

$(a * b) * c$ = (H.C.F. of a and b) * c = H.C.F. of a, b, and c

$a * (b * c) = a$ *(H.C.F. of b and c) = H.C.F. of a, b, and c

Therefore, $(a * b) * c = a * (b * c)$

Thus, the operation * is associative.

Now, an element $e \in$ **N** will be the identity for the operation * if $a * e = a = e * a$, $\forall\ a \in$ **N**.

But this relation is not true for any $a \in$ **N**.

Thus, the operation * does not have any identity in **N**.

Ex 3.5

Binary Operations Ex 3.5 Q1

$a \times_4 b$ = the remainder when ab is divided by 4.

eg. (i) $2 \times 3 = 6 \Rightarrow 2 \times_4 3 = 2$

[When 6 is divided by 4 we get 2 as remainder]

(ii) $2 \times 3 = 4 \Rightarrow 2 \times_4 2 = 0$

[When 4 is divided by 4 we get 0 as remainder]

The composition table for $\times_4$ on set $S = \{0, 1, 2, 3\}$ *is* :

$\times_4$	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

Binary Operations Ex 3.5 Q 2

$a +_5 b =$ the remainder when $a + b$ is divided by 5.

eg. $2 + 4 = 6 \Rightarrow 2 +_5 4 = 1$ $\because$ [we get 1 as remainder when 6 is divided by 5]

$2 + 4 = 7 \Rightarrow 3 +_5 4 = 2$ $\because$ [we get 2 as remainder when 7 is divided by 5]

The composition table for $+_5$ on set $S = \{0, 1, 2, 3, 4\}$.

$+_5$	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

Binary Operations Ex 3.5 Q3

$a \times_6 b$ = the remainder when the product of ab is divided by 6.

The composition table for $\times_6$ on set $S = \{0, 1, 2, 3, 4, 5\}$.

$\times_6$	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	0	2	4
3	0	3	0	3	0	3
4	0	4	2	0	4	2
5	0	5	4	3	2	1

Binary Operations Ex 3.5 Q4

$a \times_5 b$ = the remainder when the product of ab is divided by 5.

The composition table for $\times_5$ on $Z_5 = \{0, 1, 2, 3, 4\}$.

$\times_5$	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

Binary Operations Ex 3.5 Q5

$a \times_{10} b$ = the remainder when the product of ab is divided by 10.

The composition table for $\times_{10}$ on set $S = \{1,3,7,9\}$

$\times_{10}$	1	3	7	9
1	1	3	7	9
3	3	9	1	7
7	7	1	9	3
9	9	7	3	1

We know that an element $b \in S$ will be the inverse of $a \in S$

if $a \times_{10} b = 1$ $\left[\begin{array}{l}\because 1 \text{ is the identity element with} \\ \text{respect to multiplication}\end{array}\right]$

$\Rightarrow$ $3 \times_{10} b = 1$

From the above table $b = 7$

$\therefore$ Inverse of 3 is 7.

Binary Operations Ex 3.5 Q6

$a \times_7 b$ = the remainder when the product of ab is divided by 7.

The composition table for $\times_7$ on $S = \{1,2,3,4,5,6\}$

$\times_7$	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	1	3	5
3	3	6	2	5	1	4
4	4	1	5	2	6	3
5	5	3	1	6	4	2
6	6	5	4	3	2	1

We know that 1 is the identity element with respect to multiplication

Also, b will be the inverse of a
if, $a \times_7 b = e = 1$

$\Rightarrow$ $3 \times_7 b = 1$

From the above table $3 \times_7 5 = 1$

$\therefore$ $b = 3^{-1} = 5$

Now, $3^{-1} \times_7 4 = 5 \times_7 4 = 6$

Binary Operations Ex 3.5 Q7

$a \times_{11} b$ = the remainder when the product of ab is divided by 11.

The composition table for $\times_{11}$ on Z_{11}

$\times_{11}$	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	1	3	5	7	9
3	3	6	9	1	4	7	10	3	5	8
4	4	8	1	5	9	2	6	10	3	7
5	5	10	4	9	3	8	2	7	1	6
6	6	1	7	2	8	3	9	4	10	5
7	7	3	10	6	2	9	5	1	8	4
8	8	5	3	10	7	4	1	9	6	3
9	9	7	5	3	1	10	8	6	4	2
10	10	9	8	7	6	5	4	3	2	1

From the above table

$5 \times_{11} 9 = 1$ $\qquad$ $[\because 1 \text{ is the identity element}]$

$\therefore$ Inverse of 5 is 9.

Binary Operations Ex 3.5 Q8

$Z_5 = \{0, 1, 2, 3, 4\}$

$a \times_5 b$ = the remainder when the product of ab is divided by 5.

The composition table for $\times_5$ on $Z_5 = \{0, 1, 2, 3, 4\}$

$\times_5$	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

Binary Operations Ex 3.5 Q9

(i)

From the above table we can say that

$a*b=b*a=b$

$a*c=c*a=c$

$a*d=d*a=d$

$b*c=c*b=d$

$b*d=d*b=c$

$c*d=d*c=b$

$\therefore$ '*' is commutative

Again, $a,b,c \in S$

$\Rightarrow$ $(a*b)*c=b*c=d$ and

$a*(b*c)=a*d=d$

$\therefore$ $(a*b)*c=a*(b*c)$

$\therefore$ * is associative

We know that e will be identity element with respect to * if

$a*e=e*a=a$ for all $a \in S$

$\Rightarrow$ $a*a=a,\ a*b=b,\ a*c=c,\ a*d=d$

$\therefore$ a will be the identity element

Again,

b will be the inverse of a if

$b*a=a*b=e$

From the above table

$a*a=a,\quad b*b=b, c*c=c$ and $d*d=d$

$\therefore$ Inverse of $a=a$

$b=b$

$c=c$

$d=d$

(ii)

From the above table, we can observe

$aob = boa, \quad boc = cob$

$aoc = coa, \quad bod = dob$

$aod = doa, \quad cod = doc$

$\therefore$ "o" is commutative on S

Again, for $a, b, c \in S$

$(aob)oc = aoc = a \quad ---(i)$

$ao(boc) = aoc = a \quad ---(ii)$

From (i) & (ii)

$(aob)oc = ao(boc)$

So, "o" is associative on S

Now, we have,

$aob = a$

$bob = b$

$cob = c$

$dob = d$

$\Rightarrow$ b is the identity element with respect to "o"

We know that x will be inverse of y

If $xoy = yox = e$

$\Rightarrow$ $xoy = yox = b \quad [\because e = b]$

Now, from the above table we find that

$bob = b$

$cod = b$

$doc = b$

$\therefore$ $b^{-1} = b$, $c^{-1} = d$, and $d^{-1} = c$

Not: a^{-1} does ont exist.

Binary Operations Ex 3.5 Q10

Let $X = \{0, 1, 2, 3, 4, 5\}$.

The operation * on X is defined as:

$$a * b = \begin{cases} a+b & \text{if } a+b<6 \\ a+b-6 & \text{if } a+b \geq 6 \end{cases}$$

An element $e \in X$ is the identity element for the operation *, if

$a * e = a = e * a \ \forall a \in X$.

For $a \in X$, we observed that:

$a * 0 = a + 0 = a \quad [a \in X \Rightarrow a + 0 < 6]$

$0 * a = 0 + a = a \quad [a \in X \Rightarrow 0 + a < 6]$

$\therefore a * 0 = a = 0 * a \ \forall a \in X$

Thus, 0 is the identity element for the given operation *.

An element $a \in X$ is invertible if there exists $b \in X$ such that $a * b = 0 = b * a$.

$$\text{i.e.,} \begin{cases} a+b = 0 = b+a, & \text{if } a+b<6 \\ a+b-6 = 0 = b+a-6, & \text{if } a+b \geq 6 \end{cases}$$

i.e.,

$a = -b$ or $b = 6 - a$

But, $X = \{0, 1, 2, 3, 4, 5\}$ and $a, b \in X$. Then, $a \neq -b$.

Therefore, $b = 6 - a$ is the inverse of a $a \in X$.

Hence, the inverse of an element $a \in X$, $a \neq 0$ is $6 - a$ i.e., $a^{-1} = 6 - a$.