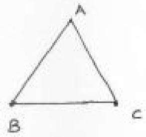
Chapter-15 Properties of Triangles
Exercise-15.1

Solution -01 :-



→ By joining of AB, BC and CA figure obtained is Triangle ABC. where A, B and C are Three non-collinear Points.

(i) the side opposite to ∠B is AC

(ii) the angle opposite to side AB is ∠ACB

(iii) The vertex opposite to side BC is A

(iv) The side opposite to vertex B is AC.

Solution -02 :-

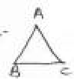
No, By defination of a triangle.

Solution-03 :-

Triangle :-

A plane figure formed by three non parallel Line segments is called a triangle

eg:-



Triangular Region :-

The interior of △ABC together with the △ABC it self, is called the triangular region of △ ABC.

Solution -04 :-

Triangles observed in the figure are △ACD, △ADB and △ABC.

Total no of triangles are '3'

Solution-05:-

Eight triangles observed in the figure are

1. ΔABC
2. ΔABD
3. ΔABO
4. ΔBCO
5. ΔDCO
6. ΔAOD
7. ΔACD
8. ΔBCD.

Solution-06:-

→ A plane of figure formed by three non parallel Line segments is called a triangle where as Triangular Region is the interior of ΔABC together with the ΔABC itself, is called the triangular region ABC.

Solution-07:-

(i) Triangle:-

A Plane figure formed by three non-parallel Line segments is called a triangle.

(ii) Parts or Elements of a Triangle:.

The three sides AB, BC, CA and three angles $\angle A, \angle B, \angle C$ of a ΔABC are together called the six parts of elements of ΔABC

(iii) Scalene Triangle:-

A Triangle whose no two sides are equal, is called Scalene Triangle.

(iv) Isosceles triangle:-

A triangle whose two sides are equal. is called isosceles triangle.

(V) Equilateral triangle:-

A triangle whose all sides are equal. is called Equilateral triangle.

(vi) Acute triangle:-

A triangle whose all the angles are acute is called Acute triangle.

vii) Right triangle:-

A triangle whose one of the angles is right angle is called Right triangle.

viii) obtuse triangle:-

A triangle whose one of the angles is obtuse angle is called Obtuse triangle.

(ix) Enterior of a Triangle:-

The Interior of a triangle is made up of all such points P of the plane, are enclosed by the triangle.

(X) Exterior of a Triangle:-

The Exterior of a triangle is made up of part of the plane which consists of those points Q, which are neither on the triangle nor in its interior.

solution - 08:-

(i) $AB \neq BC \neq CA$

Scalene triangle

(ii) $PQ = PR$; $QR = 5cm$.

isosceles triangle.

(iii) $XY = YZ = ZX$

Equilateral triangle

(iv) $UV \neq VW \neq UW$

Scalene triangle

(v) Two sides ar equal

→ isosceles triangle

solution - 09:-

(i) Angle given is 90°

∴ Right angle Triangle

(ii) Angle given is 120° $[120^\circ > 90^\circ]$

∴ Obtuse triangle

(iii) All the angles are acute $[< 90^\circ \to$ Acute$]$

∴ Acute triangle.

(iv) Right triangle

(v) Obtuse triangle.

Solution-10:-

(i) Three

(ii) Three

(iii) Three

(iv) Six

(v) Scalene

(vi) Isosceles

(vii) Equilateral

(viii) Right triangle

(ix) Acute triangle

(x). Obtuse triangle.

Exercise-15.2

Exercise -15.2 :-

Solution-01:-

Let ABC be a triangle such that $\angle B = 105°$ & $\angle C = 30°$

Then, we have find the measure of the third Angle A.

Now, $\angle B = 105°$ and $\angle C = 30°$

$\Rightarrow \angle B + \angle C = 105° + 30° = 135°$

By the angle sum property of a triangle, we have

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow \angle A + 135° = 180°$

$\Rightarrow \angle A = 180° - 135°$

$\Rightarrow \angle A = 45°$.

Solution-02:-

Let ABC be a triangle such that $\angle A = 130°$ and the other two angles $\angle B = \angle C$

By the angle sum Property of a triangle, we have

$\angle A + \angle B + \angle C = 180°$

$130° + \angle B + \angle C = 180°$ $[\angle A = 130°$ and $\angle C = \angle B]$

$130° + 2\angle C = 180° \Rightarrow 2\angle C = 180° - 130°$

$\therefore \angle C = 25° = \angle B$

$\therefore \angle A = 130°, \angle B = 25°$ & $\angle C = 25°$.

Solution-03:-

Given that,

Three triangles of triangles are equal.

The measured angles be $\angle A = \angle B = \angle C$.

By the angle sum Property of a triangle, we have

$\angle A + \angle B + \angle C = 180°$

$\angle A + \angle A + \angle A = 180°$

$3\angle A = 180°$

$\angle A = \frac{180°}{3} = 60°$

$\therefore \angle A = \angle B = \angle C = 60°$.

Solution-04:-

Given that,

three angles of a triangle in the ratio 1:2:3

Let the measured angles be $x, 2x$ & $3x$.

By the angle sum property of a triangle

we have

$x + 2x + 3x = 180°$

$6x = 180°$

$x = 30°$

1^{st} angle $\rightarrow 30°$

2^{nd} Angle $\rightarrow 2(30°) \longrightarrow 60°$ $[2x]$

3^{rd} Angle $\rightarrow 3(30°) \longrightarrow 90°$. $[3x]$

Solution -05:-

Give angles of a triangle are

$(x-40)^\circ$, $(x-20)^\circ$ & $(\frac{1}{2}x-10)^\circ$.

By the Angle sum property of a triangle,

we have

$$(x-40)^\circ + (x-20)^\circ + (\frac{1}{2}x - 10)^\circ = 180^\circ$$

$$2x + \frac{x}{2} - 70^\circ = 180^\circ \Rightarrow \frac{4x + x - 140^\circ}{2} = 180^\circ$$

$$5x - 140^\circ = 360^\circ$$

$$5x = 360^\circ + 140^\circ$$

$$x = \frac{500^\circ}{5}$$

$$\boxed{x = 100^\circ}$$

Required angles are

$$(x-40)^\circ = (100-40)^\circ = 60^\circ$$

$$(x-20)^\circ = (100-20)^\circ = 80^\circ$$

$$(\frac{x}{2} - 10)^\circ = (\frac{100}{2} - 10)^\circ$$

$$= (50-10)^\circ$$

$$= 40^\circ.$$

$\therefore$ Required angles are 40°, 80° and 60°.

Solution-06:-

Given that,

Angles are arranged in the ascending order say ~~A>B>C~~. $A < B < C$ and

Difference between two angles is 10°.

Then the measured angles be say

x, $x+10$ and $x+20$.

By Angle sum property of a triangle, we have

$$x + x + 10 + x + 20 = 180^\circ$$

$$3x + 30^\circ = 180^\circ$$

$$3x = 180^\circ - 30^\circ$$

$$x = \frac{150^\circ}{3}$$

$$x = 50^\circ$$

$\therefore$ Required angles are x, $x+10$ and $x+20$

$\angle A = x = 50^\circ$

$\angle B = x + 10 = 60^\circ$

$\angle C = x + 10 + 10 = x + 20^\circ = 50^\circ + 20^\circ = 70^\circ$

$\therefore$ Three angles are $50^\circ, 60^\circ, 70^\circ$.

Solution -08:-

Given that,

one angle of a triangle is equal to the sum of the other two

Let the measures of angles be x, y, z

By the Angle sum property of a Δ^{le}, we have

$x+y+z = 180^\circ$

$x+x = 180^\circ$ $\quad [x = y+z]$

$2x = 180^\circ$

$x = 90^\circ$

If one angle is 90° then the given triangle is a right angle triangle.

Solution -09:-

Given that.

each angle of a triangle is less than the sum of the other two.

measure of angles be x, y and z

$x < y+z$

$y < x+z$

$z < x+y$

$\therefore x<90, y<90, z<90$ $\quad$ [By the Angle sum Property of a Δ^{le}]

$\therefore$ The triangle is acute angled.

Solution -10:-

(i) $63^\circ+37^\circ+80^\circ = 180^\circ$

[By the angle sum of properties of a triangle].

Angles form a triangle

(ii) $45^\circ+61^\circ+73^\circ \neq 180^\circ$

(iii) $59+62^\circ+61^\circ \neq 180^\circ$

(iv) $45^\circ+45^\circ+90^\circ = 180^\circ$

Angles form a triangle.

(v) $30^\circ+20^\circ+125^\circ \neq 180^\circ$

Solution -11

Given that,

angles of a triangle in ratio $3:4:5$

measure of Angles be $3x, 4x$ and $5x$

Angle sum property of a triangle, we have

$3x+4x+5x = 180^\circ$

$12x = 180^\circ$

$x = \frac{180^\circ}{12} = 15^\circ$

Smallest angle $= 3x$

$= 3(15)^\circ$

$= 45^\circ$.

Solution-12:-

Given,

Two actue angles of a right angle triangle are equal.

Right triangle:-

Triangle whose one of the angle is right angle.

measured angles be $x, x, 90°$

By Angle sum property of a triangle, we have

$x+x+90° = 180°$

$2x = 180° - 90°$

$x = \frac{90°}{2} \Rightarrow x = 45°$

The two angles are $45°, 45°$.

Solution-13:-

Angle of a triangle is greater than the sum of the other two.

measure of angles be x, y and z.

$x > y+z$ (or)

$y > x+z$ (or)

$z > x+y$.

x (or) y (or) $z > 90°$ which is obtuse

$\therefore$ triangle is a obtuse angle.

Solution-14:-

$\angle FAB + \angle ABC + \angle BCD + \angle CDE + \angle DEF + \angle EFA$.

we know that the sum of angles of a triangle is $180°$.

$\therefore$ In $\triangle ABC$, we have

$\angle CAB + \angle ABC + \angle BCA = 180°$ ---(i)

In $\triangle ACD$, we have

$\angle DAC + \angle ACD + \angle CDA = 180°$ ---(ii)

In $\triangle ADE$, we have

$\angle EAD + \angle ADE + \angle DEA = 180°$ ---(iii)

In $\triangle AEF$, we have

$\angle FAE + \angle AEF + \angle EFA = 180°$ ---(iv)

Adding (i), (ii), (iii) and (iv) we get

$(\angle CAB + \angle ABC + \angle BCA) + (\angle DAC + \angle ACD + \angle CDA)$

$+ (\angle EAD + \angle ADE + \angle DEA) + (\angle EFA + (\angle AEF) + \angle FAE)$

$= 720°$

$\therefore \angle FAB + \angle ABC + \angle BCD + \angle CDE + \angle DEF + \angle EFA = 720°$

$[\ \angle FAB = \angle CAB + \angle DAC + \angle EAD + \angle EAF;$

~~$\angle EAB$~~ $\angle BCD = \angle BCA + \angle ACD;$

$\angle CDE = \angle CDA + \angle EDA;$

$\angle DEF = \angle AEF + \angle DEA;\ \ \angle EFA = \angle EFA\ ]$

Solution-15:-

(i) By Angle sum property of a triangle we have

$\angle 40^\circ + \angle 30^\circ + \angle z = 180^\circ$

$\angle z = 180^\circ - 70^\circ$

$\angle z = 110^\circ$.

$\angle x^\circ$ and 40° are corresponding angles

$\angle x = 40^\circ$

$\angle y$ and 30° are corresponding angles

$\angle y = 30^\circ$.

(ii) By Angle sum property of a Δle, we have

In ΔABC

$x^\circ + 40^\circ + 90^\circ = 180^\circ$

$x = 180^\circ - 130^\circ$

$x = 50^\circ$

In ΔACD

$45^\circ + 90^\circ + y^\circ = 180^\circ$

$y^\circ = 180^\circ - 135^\circ$

$y = 45^\circ$

(iii) By Angle sum property of a ΔABC, we have.

In ΔABC

$50^\circ + 50^\circ + x^\circ = 180^\circ$

$x^\circ = 180^\circ - 100^\circ$

$x^\circ = 80^\circ$

x° and y° is a Linear Pair

$\angle x^\circ + \angle y^\circ = 180^\circ$

$80^\circ + \angle y^\circ = 180^\circ$

$\angle y^\circ = 100^\circ$

(iv) By Angle sum property of triangle, we have

In ΔADE

$x^\circ + 50^\circ + 40^\circ = 180^\circ$

$x^\circ = 180^\circ - 90^\circ$

$x^\circ = 90^\circ$.

$\angle y = 50^\circ$ [corresponding angles]

$\angle z = 48$ [corresponding angles]

Solution-16:-

Given that angle ① = 60°

Let the other angles be x and $2x$

[∵ two angles are in the ratio 1:2]

$$60+2x+x = 180^\circ$$

$$60^\circ + 3x = 180^\circ$$

$$x = \frac{120^\circ}{3}$$

$$x = 40^\circ$$

two angles are $40^\circ, 80^\circ$.

Solution-17:-

Given one angle of a triangle = 100°

the two angles are in the ratio-2:3

measures of angles be $2x$ and $3x$ respectevely

$$2x + 3x + 100^\circ = 180^\circ$$

$$5x + 100^\circ = 180^\circ$$

$$5x = 180 - 100$$

$$x = 16^\circ$$

∴ Two angles are $32^\circ = 2(16)^\circ$

$3(16)^\circ = 48^\circ$.

Solution-18:-

In a △ABC

By Angle sum property of a triangle, we have

$$3\angle A + 4\angle B + 6\angle C = 180^\circ$$

$$3\angle A + 3\angle A + 3\angle A = 180^\circ \quad [\because 3\angle A = 4\angle B = 6\angle C]$$

$$9\angle A = 180^\circ$$

$$\angle A = 20^\circ$$

$$3\angle A = 3(20^\circ) = 60^\circ$$

$$4\angle B = 60^\circ \Rightarrow \angle B = 15^\circ$$

$$6\angle C = 60^\circ \Rightarrow \angle C = \frac{60^\circ}{6} = 10^\circ.$$

Solution-20:-

Given $\angle A = 100^\circ$

$\angle PAB = 50^\circ$, $\angle D = 90^\circ$

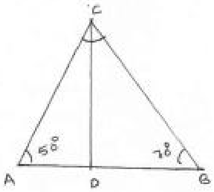
$$\therefore \angle B + \angle DAB + \angle D = 180^\circ$$

$$\angle B = 180^\circ - 140^\circ = 40^\circ$$

Solution - 21:-

Given that,

$\angle A = 50^\circ$, $\angle B = 70^\circ$



By, Angle sum property of a triangle, we have

$\angle A + \angle B + \angle C = 180^\circ$

$50^\circ + 70^\circ + \angle C = 180^\circ$

$\angle C = 180^\circ - 120^\circ$

$\angle C = 60^\circ$

Given that $\angle C$ bisects AB in D.

In $\triangle ACD$

$\angle ACD + \angle ADC + \angle A = 180^\circ$ [A.S.P]

$50^\circ + 30^\circ + \angle ADC = 180^\circ$

$\angle ADC = 100^\circ$

In $\triangle DCB$

$\angle B + \angle C + \angle CDB = 180^\circ$

$70^\circ + 30^\circ + \angle CDB = 180^\circ$

$\angle CDB = 180^\circ - 100^\circ$

$\therefore \angle CDB = 80^\circ$

$\therefore \angle CDB = 80^\circ$, $\angle ADC = 100^\circ$.

Solution - 22:-

Given,

In $\triangle ABC$, $\angle A = 60^\circ$, $\angle B = 80^\circ$

i) By Angle sum property of a triangle, we have

$\angle A + \angle B + \angle C = 180^\circ$

$60^\circ + 80^\circ + \angle C = 180^\circ$

$\angle C = 180^\circ - 140^\circ$

$\angle C = 40^\circ$.

(ii) In $\triangle BOC$, we have

By Angle sum property of a triangle, we have

$\angle B = 40^\circ$, $\angle C = 20^\circ$, $\angle BOC = ?$.

$\angle B + \angle C + \angle BOC = 180^\circ$ [A.S.P]

$40^\circ + 20^\circ + \angle BOC = 180^\circ$

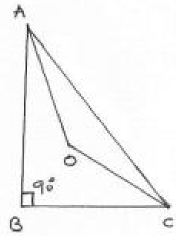
$\angle BOC = 180^\circ - 60^\circ$

$\angle BOC = 120^\circ$

Solution-23 :-

Given that,

The bisectors of the acute angles of a right angle meet at O.



$\angle B = 90°$

$\angle A + \angle C = 90°$

$\angle A + \angle C = 90°$

$\therefore \angle CAO + \angle OCA = \frac{90°}{2} = 45°$ [bisectors]

In $\triangle OAC$, we have

By Angle sum property of a triangle

$\angle CAO + \angle OCA + \angle COA = 180°$

$45° + \angle COA = 180°$

$\angle COA = 180° - 45°$.

$\angle COA = 135°$.

Exercise-15.3

Exercise-15.3

Solution-01:-

(i) The interior adjacent angle of $\angle CBX$ is $\angle ABC$

(ii) $\angle BAC$, $\angle ACB$;
$\angle ABC$, $\angle ACB$.

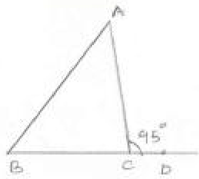
interior opposite angles to exterior $\angle CBX$.

Solution-03:-

Given,

exterior angle of a vertex $= 95^\circ$

interior opposite angle $= 55^\circ$



Let ABC be a triangle whose side BC produced to D to form a exterior angle $\angle ACD$ such that $\angle ACD = 95^\circ$.

Let $\angle B = 55^\circ$. By exterior angle theorem, we have

$\angle ACD = \angle B + \angle A$

$95^\circ = 55^\circ + \angle A$

$\angle A = 95^\circ - 55^\circ$

$\angle A = 40^\circ$

Now, By using Angle sum property of a triangle, we have

$\angle A + \angle B + \angle C = 180^\circ$

$40^\circ + 55^\circ + \angle C = 180^\circ$

$\angle C = 180^\circ - 95^\circ$

$\angle C = 85^\circ$

The angles of a triangle are 40°, 55° and 85°

Solution-06:-

Given

$\angle ACD = 105^\circ$

$\angle EAF = 45^\circ$

$\angle ACD$ and $\angle ACB$ is a Linear pair

$\angle ACD + \angle ACB = 180^\circ$

$105^\circ + \angle ACB = 180^\circ$

$\angle ACB = 180^\circ - 105^\circ$

$\angle ACB = 75^\circ$.

Since $\angle CAB$ and $\angle EAF$ are vertically opposite angles

$\angle CAB = \angle EAF = 45^\circ$

By using Angle sum property of a triangle, we have

$\angle CAB + \angle ACB + \angle ABC = 180^\circ$

$45^\circ + 75^\circ + \angle ABC = 180^\circ$

$\angle ABC = 180^\circ - 120^\circ$

$\angle ABC = 60^\circ$

Solution-07:-

Given

$\angle A : \angle B : \angle C = 3 : 2 : 1$.

By Angle sumproperty of a triangle, we have

measure of angles be $3x, 2x$ and x

$x + 2x + 3x = 180^\circ$

$6x = 180^\circ$

$x = 30^\circ$.

$\angle A = 30^\circ(3) = 90^\circ$

$\angle B = 60^\circ$

$\angle C = 30^\circ$

$\angle ACE + \angle ECD = 180^\circ$ [Linear pair]

$90^\circ + 30^\circ + \angle ECD = 180^\circ$

$\angle ECD = 180^\circ - 120^\circ$

$\angle ECD = 60^\circ$

Solution-08:-

No, since sum of interior angles A and B $= 183^\circ > 180^\circ$.

Solution-09:-

Given $\angle FCD = 50^\circ$.

$\angle BAD = ?$.

In $\triangle ABD$, we have

$\angle ADB = 90^\circ$.

$\angle FCD$ and $\angle BAD$ are vertically opposite angles

$\angle FCD = \angle BAD$

$\therefore \angle BAD = 50^\circ$.

Solution-10:-

Given

$\angle CFD = 60^\circ$,

$\angle FDC = 90^\circ$

$\angle DCF = ?$.

$\angle CFD + \angle FDC + \angle DCF = 180^\circ$ [A.S.P]

$\angle DCF = 180^\circ - 60^\circ - 90^\circ = 30^\circ$

$\angle DCF + x^\circ = 180^\circ \Rightarrow x^\circ = 180^\circ - 30^\circ = 150^\circ$

[Linear Pair] $\therefore x^\circ = 150^\circ$

Here,

$\angle AED + 120^\circ = 180^\circ$ (Linear pair)

$\Rightarrow \angle AED = 180^\circ - 120^\circ = 60^\circ$

We know that the sum of all angles of a triangle is 180°.

Therefore, for $\triangle ADE$, we can say that:

$\angle ADE + \angle AED + \angle DAE = 180^\circ$

$\Rightarrow 60^\circ + \angle ADE + 30^\circ = 180^\circ$

Or,

$\angle ADE = 180^\circ - 60^\circ - 30^\circ = 90^\circ$

From the given figure, we can also say that:

$\angle FDC + 90^\circ = 180^\circ$ (Linear pair)

$\Rightarrow \angle FDC = 180^\circ - 90^\circ = 90^\circ$

Using the above rule for $\triangle CDF$, we can say that:

$\angle CDF + \angle DCF + \angle DFC = 180^\circ$

$\Rightarrow 90^\circ + \angle DCF + 60^\circ = 180^\circ$

$\angle DCF = 180^\circ - 60^\circ - 90^\circ = 30^\circ$

Also,

$\angle DCF + x = 180^\circ$ (Linear pair)

$\Rightarrow 30^\circ + x = 180^\circ$

Or,

$x = 180^\circ - 30^\circ = 150^\circ$

Q11

(i)

Here,

$\angle BAF + \angle FAD = 180^\circ$ (Linear pair)

$\Rightarrow \angle FAD = 180^\circ - \angle BAF = 180^\circ - 90^\circ = 90^\circ$

Also,

$\angle AFE = \angle ADF + \angle FAD$ (Exterior angle property)

$\angle ADF + 90^\circ = 130^\circ$

$\angle ADF = 130^\circ - 90^\circ = 40^\circ$

(ii)

We know that the sum of all the angles of a triangle is 180°.

Therefore, for $\triangle BDE$, we can say that :

$\angle BDE + \angle BED + \angle DBE = 180^\circ$.

$\Rightarrow \angle DBE = 180^\circ - \angle BDE - \angle BED = 180^\circ - 90^\circ - 40^\circ = 50^\circ \quad \ldots(i)$

Also,

$\angle FAD = \angle ABC + \angle ACB$ (Exterior angle property)

$\Rightarrow 90^\circ = 50^\circ + \angle ACB$

Or,

$\angle ACB = 90^\circ - 50^\circ = 40^\circ$

(iii) $\angle ABC = \angle DBE = 50^\circ$ [From (i)]

Solution -11:-

Given $\angle AFE = 130^\circ$

$\angle CAB = 90^\circ$

$\angle DEB = 90^\circ$

(i) $\angle BDE$

$\angle AFE = 130^\circ$

$\angle ACB = 40^\circ$

$\therefore \angle BAE + \angle ACB + \angle ABC = 180^\circ$

$\angle ABC$ ~~$\angle BAC$~~ $= 180^\circ - 40^\circ - 90^\circ$

$\angle ABC = 50^\circ$

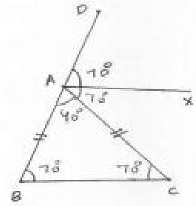
(i) $\angle BDE = 180^\circ - 50^\circ - 90^\circ$

$= 40^\circ$

(ii) $\angle BCA = 40^\circ$

(iii) $\angle ABC = 50^\circ$.

Solution-12:-



Given,

$\angle DAX = 70°$

AX Bisects exterior angle DAC

So, $\angle DAX = \angle DAC$

$\therefore \angle DAC = 70°$

$\angle DAX$, $\angle DAC$ and $\angle BAC$ is Linear pair

$\angle DAX + \angle DAC + \angle BAC = 180°$

$70° + 70° + \angle BAC = 180°$

$\angle BAC = 180° - 140°$

$\angle BAC = 40°$.

Let $\angle B = \angle C = x$.

By using Angle sum property of a triangle

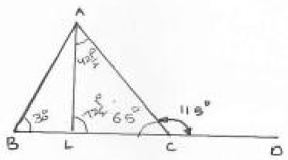
we have, In $\triangle ABC$

$40° + x° + x° = 180° \Rightarrow 2x° = 180 - 40° = 140°$

$\Rightarrow x° = 70°$

$\therefore \angle B = \angle C = x° = 70°$

Solution-13:-



Given,

$\angle ABC = 30°$ & $\angle ACD = 115°$

we have. In $\triangle ABC$

By usingle sum property of a triangle

$30° + 65° + \angle BAC = 180°$

$\angle BAC = 180° - 95°$

$\angle BAC = 42\frac{1}{2}° + 42\frac{1}{2}°$

$\Rightarrow \angle BAL + \angle CAL = 42\frac{1}{2}° + 42\frac{1}{2}°$

$\therefore \angle BAL = 42\frac{1}{2}° = \angle CAL$.

By using Angle sum property of a triangle,

we have,

$65° + 42\frac{1}{2}° + \angle ALC = 180°$

$\angle ALC = -107\frac{1}{2}° + 180°$

$\angle ALC = 72\frac{1}{2}°$.

$\angle APD = ?$

By using Angle Sumproperty of a triangle wehave

In ΔABc,

$$60^\circ + 80^\circ + \angle c = 180^\circ$$

$$\angle c = 180^\circ - 140^\circ$$

$$\angle c = 40^\circ$$

In ΔADc,

$$40^\circ + 40^\circ + \angle ADc = 180^\circ$$

$$\angle ADc = 180^\circ - 40^\circ - 40^\circ$$

$$\angle ADc = 100^\circ$$

$$\angle ADP + \angle PDc = 100^\circ$$

$$\angle ADP = 100^\circ - 15^\circ$$

$$= 85^\circ.$$

By using Angle sum property of a Δ^{le} we have

$$\angle APD + \angle ADP + \angle PAP = 180^\circ$$

[$\angle A = \angle ADc + \angle ADB$]

$$\angle APD = 180^\circ - 85^\circ - 40^\circ$$

[$\angle ADc = \angle ADP$]

$$\angle ADD = 180^\circ - 125^\circ$$

$$\angle APQ = 55^\circ.$$

Solution-15:-

(i) $\angle AcD$ and $\angle AcD$ is a Linear pair

$$75^\circ + \angle AcD = 180^\circ$$

$$\angle AcD = 105^\circ$$

[Bc produced is Let D]

By using Angle sumproperty of a triangle wehave

In ΔABc

$$\angle A + \angle B + \angle AcB = 180^\circ$$

$$\angle B = 180^\circ - 70^\circ - 40^\circ$$

$$\angle B = 180^\circ - 110^\circ$$

$$\angle B = 70^\circ$$

Solution-15 (ii):-

$\angle$ ~~ABc~~ Let AB produced is D.

$\angle DAc$ and $\angle BAc$ is a Linear pair

$$\angle BAc + \angle DAc = 180^\circ$$

$$\angle BAc = 180^\circ - 80^\circ$$

$$\angle BAc = 100^\circ$$

By using Angle sum property of a Δ^{le} we have

$$\angle BAc + \angle ABc + \angle c = 180^\circ$$

$$\angle ABc = 180^\circ - 100^\circ - 30^\circ$$

$$\angle ABc = 50^\circ.$$

Solution-15 -(iii):-

By using Angle sum property of a Δ^{le} we have

In $\angle ACD$,

$\angle CAD + \angle ACD + \angle CDA = 180^\circ$

$\angle CDA = 180^\circ - 100^\circ - 30^\circ$

$\angle CDA = 50^\circ$

ACD and $\angle ACB$ is a Linear pair

$\angle ACD + \angle ACB = 180^\circ$

$\angle ACB = 180^\circ - 50^\circ 100^\circ$

$\angle ACB = 80^\circ$

By using Angle sumproperty of a Δ^{le} we have

$\angle BAC + 45^\circ + 80^\circ = 180^\circ$

$\angle BAC = 180^\circ - 125^\circ$

$\angle BAC = 55^\circ$

$x = 55^\circ$, $y = 50^\circ$.

Solution-15(iv):-

By using Angle sum property of a Δ^{le}, we have

$\angle DBC + \angle DCA + \angle BDC = 180^\circ$

$\angle DBC = 180^\circ - 30^\circ - 50^\circ = 100^\circ$

$\angle DBC$ and $\angle DBA$ is a Linear pair

$\angle DBC + \angle DBA = 180^\circ$

$\angle DBA = 180^\circ - 100^\circ$

$= 80^\circ$.

$x = 80^\circ$

In ΔAEB

$\angle EAB + \angle AEB + \angle ABE = 180^\circ$

$30^\circ + 80^\circ + \angle AEB = 180^\circ$

$\angle AEB = 180^\circ - 110^\circ$

$\angle AEB = 70^\circ$

$\angle AEB$ and $\angle AED$ is a Linear pair

$70^\circ + \angle AED = 180^\circ$

$\angle AED = 180^\circ - 70^\circ$

$= 110^\circ$

$x = 80^\circ$, $y = 110^\circ$

Solution-16:-

(i) $\angle BAE$ and $\angle BAC$ is a Linear pair

$\angle BAE + \angle BAC = 180^\circ$

$\angle BAC = 180^\circ - 120^\circ$

$\angle BAC = 60^\circ$

$\angle ACB$ and $\angle ACD$ is a Linear Pair

$\angle ACB + \angle ACD = 180^\circ$

$\angle ACB = 180^\circ - 112^\circ$

$\angle ACB = 68^\circ$

A.S.P

$\rightarrow \angle BAC + \angle BCA + \angle ABC = 180^\circ$

$\Rightarrow x^\circ = 180^\circ - 60^\circ - 68^\circ = 52^\circ$. $\therefore x = 52^\circ$.

Solution-16 :-

(ii) $\angle ACD$ and $\angle ACB$ is a Linear Pair

$\angle ACD + \angle ACB = 180^\circ$

$110^\circ + \angle ACB = 180^\circ$

$\angle ACB = 180^\circ - 110^\circ$

$\angle ACB = 70^\circ$

$\angle EBA$ and $\angle ABC$ is a Linear pair

$\angle EBA + \angle ABC = 180^\circ$

$\angle ABC = 180^\circ - 120^\circ$

$\angle ABC = 60^\circ$.

By using Angle sum property of a triangle, we have

$\angle ACB + \angle ABC + \angle BAC = 180^\circ$

$60^\circ + 70^\circ + \angle BAC = 180^\circ$

$\angle BAC = 180^\circ - 130^\circ = 50^\circ \quad \therefore x = 50^\circ$

(iii) $\angle BAD$ and $\angle ADC$ are vertically opposite angles

$\angle BAD = \angle ADC = 52^\circ$

By using Angle sum Property of a triangle, we have

$\angle ADC = \angle EDC$.

In $\triangle EDC$ + $\angle ECD + \angle CED = 180^\circ$

$52^\circ + 40^\circ + x = 180^\circ$

$x = 180^\circ - 52 - 40^\circ$

$x = 180 - 92^\circ$

$x^\circ = 88^\circ$.

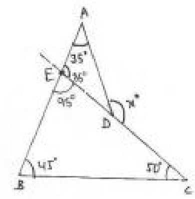
Solution 16 :-

(iv)

Given $\angle ABC = 45^\circ$,

$\angle BCD = 50^\circ$,

$\angle BAD = 35^\circ$.



Construction :- Extend line DC and it intersect AB at 'E'.

Required to find '$\angle ADC$'

In $\triangle BCE$,

$\angle EBC + \angle BCE + \angle CEB = 180^\circ$ (Sum of angles in a triangle is 180°)

$\angle ABC + \angle BCD + \angle CEB = 180^\circ$ ($\because \angle EBC = \angle ABC$, $\angle BCE = \angle BCD$)

$45^\circ + 50^\circ + \angle CEB = 180^\circ$

$\angle CEB = 180^\circ - 95^\circ$

$\angle CEB = 85^\circ$

Consider line AB and line EC.

$\angle BEC$ and $\angle AEC$ form a linear pair.

$\therefore \angle BEC + \angle AEC = 180^\circ$

$85^\circ + \angle AEC = 180^\circ$

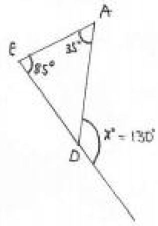
$\angle AEC = 180^\circ - 85^\circ$

$= \cancel{88}\ 95^\circ$

$\therefore AEC = 95^\circ$

In $\triangle AED$, $\angle ADC$ is Exterior angle.

$\therefore$ $\angle ADC$ = Sum of two ~~Int~~opposite interior angle



$$\angle ADC = \angle DAE + \angle AED$$

$$x^\circ = 35^\circ + \angle AEC$$

$$= 35^\circ + 9\cancel{8}5^\circ$$

$$= 130^\circ$$

$$\therefore \angle ADC = 130^\circ$$

$$\therefore x^\circ = 130^\circ$$

Exercise-15.4

Exercise-15.4.

1) Solution:-

i) we have,

$5+7>4,\ 5+9>7,\ 9+7>5$

That is, the sum of any two of the given numbers is greater than the third number.

So, 5cm, 7cm & 9cm can be the lengths of the sides of a triangle

(ii) we have,

$2+10 \not> 15$

So, the given numbers cannot be the lengths of the sides of a triangle

(iii) we have.

$3+4>5,\ 4+5>3,\ 5+3>4$

That is, the sum of any two of the give numbers is greater than the third number

So 3cm, 4cm & 5cm can be the lengths of the sides of a triangle.

(iv) we have

$2+5 \not> 7$

so, the given numbers cannot be the lengths of the sides of a Triangle.

(v) we have

$5+8 \not> 20$

So, the given numbers cannot be the lengths of the sides of a triangle.

Solution-02:-

(i) $<$

(ii) $<$

(iii) $<$

Solution-03:-

(i) False

(ii) True

(iii) False.

Solution-04:-

$OA+OB>AB$ — (1)

Similarly $OB+OC>BC$ — (2)

$OC+OA>CA$ — (3)

Add (1), (2) & (3)

$(OA+OB)+(OB+OC)+(OC+OA) > AB+BC+CA$

$\Rightarrow\ 2(OA+OB+OC) > AB+BC+CA$

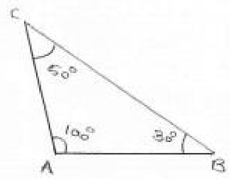
$\Rightarrow\ OA+OB+OC > \frac{1}{2}(AB+BC+CA)$

Solution -05 :-

Given,

In ΔABC

$\angle A = 100^\circ$, $\angle B = 30^\circ$, $\angle C = 50^\circ$.



→ AC is the smallest side which is opposite to the smallest angle $\angle B$

→ BC is the Largest side which is opposite to the Largest angle $\angle A$.

Exercise-15.5

Exercise -15.5.

Solution-01:-

Pythagoras theorem:-

In a right triangle, the square of the hypotenuse equal to the sum of the squares of its remaining two sides.

Converse of Pythagoras theorem:-

If the square of one side of triangle is equal to the sum of the squares of the other two sides, then the triangle is a right triangle with the angle opposite the first side as right angle

In ΔABC, we have

$$AB^2 = CA^2 + BC^2.$$

Solution-2:-

(i) $a = 6cm,\ b = 8cm,\ c = ?$

WKT, by Pythagoras theorem

$$a^2+b^2=c^2$$

$$c^2 = 6^2+8^2$$

$$c^2 = 36+64$$

$$c = \sqrt{100} = 10cm.$$

Given,

(ii) $a = 8cm,\ b = 15cm,\ c = ?$

$a^2+b^2=c^2$ [by Pythagoras theorem]

$$8^2+15^2=c^2$$

$$c^2 = 64+225$$

$$c^2 = 289$$

$$c = \sqrt{289} = 17cm$$

$$c = 17cm$$

(iii) Given,

$a = 3cm,\ b = 4cm$ and $c = ?$

WKT, By Pythagoras theorem

$$a^2+b^2=c^2$$

$$3^2+4^2=c^2$$

$$c^2=25$$

$$c = \sqrt{25} = 5cm$$

$$c = 5cm$$

(iv) Given,

$a = 2cm,\ b = 1.5cm$ and $c = ?$

WKT, By Pythagoras theorem

$$a^2+b^2=c^2$$

$$2^2+1.5^2=c^2$$

$$c = \sqrt{4+1.5\times 1.5}$$

$$c = \sqrt{2.5\times 2.5}$$

$$c = 2.5$$

Solution - 03:-

Given,

Hypotenuse = 2.5 cm

$side = 1.5cm$

$side_2 = ?$

$side^2 + side^2 = Hyp^2$

$2.5^2 - 1.5^2 = side^2$

$side^2 = 4$

$side = \sqrt{4} = 2 cm.$

Solution-04:-

Given Ladder Length = 3.7 m

Ladder Base = 1.2 m

∴ By using Pythagoras theorem

$3.7^2 = 1.2^2 + side^2$

$13.69 - 1.44 = side^2$

$side = \sqrt{12.25}$

$side = 3.5 cm.$

∴ required height = 3.5m

Solution-05:-

Given,

$a = 3cm$, $b = 4cm$ and $c = 6cm$

Here the Largest side is $c = 6cm$.

we have $a^2 + b^2 = c^2$

Clearly, $3^2 + 4^2 \neq 6^2$

$16 \neq 25$

So, the triangle with the given sides is not a right angle.

Solution-06:-

(i) Given

$a = 7cm$, $b = 24cm$ and $c = 25cm$

Here the Larger side is $c = 25cm^2$

we have: $a^2 + b^2 = c^2$

$7^2 + 24^2 = 25^2$

$576 + 49 = 625$

$625 = 625$

So, the triangle with the given sides is a right triangle.

(ii) Here the Largest side $c = 18cm$.

we have: $a^2 + b^2 = c^2$

$18^2 \neq 9^2 + 16^2$

$81 + 256 \neq 324$

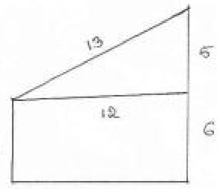
$337 \neq 324$.

So, the triangle with the given sides is not a right triangle

Solution-07:-

Two Poles of heights 6m and 11m

distance between their tops = 12m.



Side = 11m - 6m

= 5cm

∴ By using Pythagoras theorem

$5^2 + 12^2 = Hyp^2$

$Hyp^2 = 25 + 144$

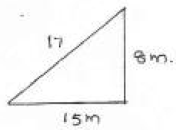
$Hyp = \sqrt{169}$

$Hyp = 13m.$

Solution-08:-

Given

distances of sides are 15m and 8m.



How far is he from the starting point's Hypotenuse

By using Pythagoras theorem

$Hyp^2 = side^2 + side^2$

$Hyp^2 = 15^2 + 8^2 = 225 + 64 = 289$

$Hyp = \sqrt{289}$

$Hyp = 17.$

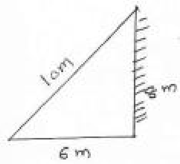
Solution-09:-

Given,

In initial position

Foot of a Ladder = 6m

height = 8m.



By Applying Pythagoras theorem, we get

$6^2 + 8^2 = hyp^2$

$36 + 64 = hyp^2$

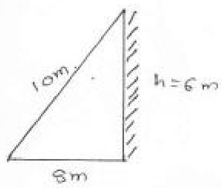
$hyp = \sqrt{100}$

$= 10m$

In Final Position

foot of a Ladder = 8m

$l = hyp = 10m.$



$10^2 = 8^2 + height^2$

$height^2 = 100 - 64$

$h = \sqrt{36}$

height = 6m.

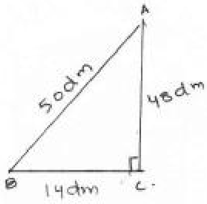
Solution -10:-

Given

Ladder Length = 50dm

height = 48dm

base of the wall = ?.



Ladder Length = 50dm.

By Applying Pythagoras theorem weget

$50^2 = 48^2 + base^2$

$base^2 = 2500 - 2304$

$base^2 = 196$

$base = \sqrt{196}$

base = 14dm

∴ base of the wall = 14dm.

Solution -11:-

The two legs of a right triangle are equal

And $Hyp^2 = 50$

$side_1 = side_2 = side$

$side_1^2 + side_2^2 = Hyp^2$ [∵ Pythagoras theorem]

$2 side^2 = Hyp^2$

$side^2 = \frac{50}{2}$

$side = \sqrt{25}$

side = 5units.

∴ length of each leg = 5units.

Solution - 12:-

(i) Given numbers are 12, 35 & 37

$12^2 + 35^2 = 37^2$

$144 + 1225 = 1369$.

(12, 35, 37) is a Triplet

→ yes.

(ii) Given numbers are 7, 24 and 25

By Pythagoras theorem

$7^2 + 24^2 = \cancel{6}\,25^2$

$49 + 576 = 625$

$625 = 625$

(7, 24, 25) is a Triplet

→ yes.

(iii) Given numbers are, 27, 36 and 45

By using pythagoras theorem

$27^2+36^2 = 45^2$

$729+1296 = 2025$

$2025 = 2025$

(27, 36 & 45) is a triplet

→ yes.

(iv) Given numbers are 15, 36 and 39

By using pythagoras theorem

$15^2+36^2 = 39^2$

$225+1296 = 1521$

$1521 = 1521$

(15, 36, 39) is a triplet

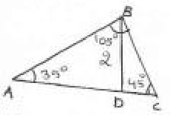
→ yes.

Solution - 13:-

Given,

$\angle A = 35°$

$\angle ABC = 105°$, $\angle BAC = 35°$



By using Angle sum property of a triangle, we have

$100° + 35° + \angle C = 180°$

$\angle C = 180° - 135° = 45°$

BD = 2cm

DC = 2cm.

Solution - 14:-

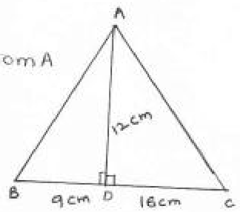
Given.

In $\triangle$ ABC, AD is the altitude from A

AD = 12 cm

DC = 16 cm

DB = 9 cm



By using pythagoras theorem $\triangle^{le}$ ABD, we have

$9^2+12^2 = hyp^2$

$hyp^2 = 144+81$

$hyp^2 = 225$

$hyp = AB = 15$ cm

In $\triangle$ ADC, we have

$12^2+16^2 = hyp^2 = AC^2$

$144+256 = AC^2$

$AC = \sqrt{400}$

$AC = 20$cm

No, $\triangle ABC$ is right angled at A.

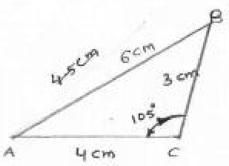
Solution -15:-

Given that

$AC = 4cm$

$BC = 3cm$

$\angle C = 105^\circ$



By construction

$AB = \overset{6}{\cancel{4.5}}\ cm$

$(AB)^2 = AC^2 + BC^2$

$(4.5)^2 \neq 4^2 + 3^2$ $\qquad$ $(6)^2 \neq 3^2 + 4^2$

$20.25 \neq 25$ $\qquad$ $36 \neq 3^2 + 4^2$

$20.25 < 25$ $\qquad$ $36 > 25$ $\qquad$ No, $(AB)^2 \neq AC^2 + BC^2$

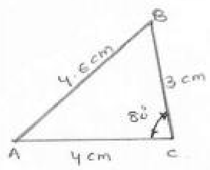
$(AB)^2 > AC^2 + BC^2$

Solution-16:-

Given

$AC = 4cm$,

$BC = 3cm$ and $\angle C = 80^\circ$



By construction, we get

$AB = 4.6$

By Pythagoras theorem

$(AB)^2 = AC^2 + BC^2$

$(4.6)^2 \neq 4^2 + 3^2$

$\Rightarrow$ No.

$(4.6)^2 < 4^2 + 3^2 = 21.16 < 25$

$\therefore (AB)^2 < AC^2 + BC^2$.