Ex 1.1

Q1

Well-defined collections are sets.
Example:
The collection of good teachers in a school is not a set, It is a collection.
Thus, we can say that every set is a collection, but every collection is not necessarily a set.
The collection of vowels in English alphabets is a set.

Q2

Answer :
(i) The collection of all natural numbers less than 50 is a set because it is well defined.

(ii) The collection of good hockey players is not a set because the goodness of a hockey player is not defined here. So, it is not a set.

(iii) The collection of all girls in a class is a set, as it is well defined that all girls of the class are being talked about.

(iv) The collection of the most talented writers of India is a set because it is well defined.

(v) The collection of difficult topics in mathematics is not a set because a topic can be easy for one student while difficult for the other student.

(vi) The collection of all months of a year beginning with the letter J is a set given by {January, June, July}

(vii) A collection of novels written by Munshi Prem Chand is a set because one can determine whether the novel is written by Munshi Prem Chand or not.

(Viii) The collection of all question in this chapter is a set because one can easily check whether it is a question of the chapter or not.

(ix) A collection of most dangerous animals of the world is not a set because we cannot decide whether the animal is dangerous or not.

(x) The collection of prime integers is set given by {2, 3, 5……..}

Q3

(i) $4 \in A$

(ii) $-4 \notin A$

(iii) $12 \notin A$

(iv) $9 \in A$

(v) $0 \in A$

(vi) $-2 \notin A$

Ex 1.2

Q1(i)

In Roster form, we describe a set by listing its elements, reparated by commas and the elements are written within braces $\{\ \}$. If a set has infinitely many elements, them comma is followed by ..., where the dots stand for 'and so on'.

The above set in Roster form can be written as $\{a, b, c, d,\}$. Since the letters $a, b, c,$ and d precedes e in the english alphabet.

Q1(ii)

In Roster form, we describe a set by listing its elements, reparated by commas and the elements are written within braces $\{\ \}$. If a set has infinitely many elements, them comma is followed by ..., where the dots stand for 'and so on'.

$1 \in N \quad \because \; 1^2 = 1 < 25$
$2 \in N \quad \because 2^2 = 4 \; < 25$
$3 \in N \quad \because 3^2 = 9 \; < 25$
$4 \in N \quad \because 4^2 = 16 \; < 25$

Hence, the above set can be written as $\{1, 2, 3, 4\}$

Q1(iii)

In Roster form, we describe a set by listing its elements, reparated by commas and the elements are written within braces $\{\ \}$. If a set has infinitely many elements, them comma is followed by ..., where the dots stand for 'and so on'.

We note that $a < x < b$ means tha x is more than a but less than b.
The prime numbers which are more than 10 fact less than 20 are 11,13,17 and 19.
Hence the above set can be written as $\{11, 13, 17, 19\}$

Q1(iv)

In Roster form, we describe a set by listing its elements, reparated by commas and the elements are written within braces $\{\ \}$. If a set has infinitely many elements, them comma is followed by ..., where the dots stand for 'and so on'.

The above set can be written as $\{2, 4, 6, 8 ...\}$ since all those natural numbers, which can be written as a multiple of 2 are the even natural numbers.

Q1(v)

In Roster form, we describe a set by listing its elements, reparated by commas and the elements are written within braces$\{\ \}$. If a set has infinitely many elements, them comma is followed by ..., where the dots stand for 'and so on'.

We know that given any $x \in R$, x is always less than or equal to itself, i.e $x \leq x$
Hence the above set is empty, i.e ϕ.

Q1(vi)

In Roster form, we describe a set by listing its elements, reparated by commas and the elements are written within braces$\{\ \}$. If a set has infinitely many elements, them comma is followed by ..., where the dots stand for 'and so on'.

The Prime divisors of 60 are 2,3,5.
Hence the above set can be written as $\{2,3,5\}$

Q1(vii)

In Roster form, we describe a set by listing its elements, reparated by commas and the elements are written within braces$\{\ \}$. If a set has infinitely many elements, them comma is followed by ..., where the dots stand for 'and so on'.

The above set can be written as
$\{17,26,35,44,53,62,71,80\}$

Q1(viii)

In Roster form, we describe a set by listing its elements, reparated by commas and the elements are written within braces$\{\ \}$. If a set has infinitely many elements, them comma is followed by ..., where the dots stand for 'and so on'.

As repetition is not allowed in a set, the distinct letters are T,R,I,G,O,N,M,E,Y.
Hence the above set can be written as

$$\{T,R,I,G,O,N,M,E,Y\}$$

Q1(ix)

In Roster form, we describe a set by listing its elements, reparated by commas and the elements are written within braces $\{\ \}$. If a set has infinitely many elements, them comma is followed by ..., where the dots stand for 'and so on'.

The distinct letters are B,E,T,R.

Hence the set can be written as

$$\{B, E, T, R.\}$$

Q2(i)

In set Builder form, a set is described by some characterizing property $P(x)$ of its elements x.

In this case a set can be described as $\{x : P(x) \text{hold}\}$ or $\{x | P(x) \text{holds}\}$ which is read as 'the set of all x such that $P(x)$ holds'.

The symbols ':' or 'I' is read as 'such that'.

So, the above set A in Set-Builder form may be written as

$$A = \{x \in N : x < 7\}$$

i.e A is the set of natural numbers x such that x is less than 7.

or

$$A = \{x \in N | 1 \leq x \leq 6\},$$

i.e A is the set of natural numbers x such that x is greater than or equal 1 and less than or equal to 6.

Q2(ii)

In set Builder form, a set is described by some characterizing property $P(x)$ of its elements x.

In this case a set can be described as $\{x : P(x) \text{hold}\}$ or $\{x | P(x) \text{holds}\}$ which is read as 'the set of all x such that $P(x)$ holds'.

The symbols ':' or 'I' is read as 'such that'.

$$B = \left\{x : x = \frac{1}{n}, n \in N\right\}$$

i.e B is the set of all those x such that $x = \frac{1}{n}$, where $n \in N$

Q2(iii)

In set Builder form, a set is described by some characterizing property $P(x)$ of its elements x.

In this case a set can be described as $\{x : P(x) \text{hold}\}$ or $\{x | P(x) \text{holds}\}$ which is read as 'the set of all x such that $P(x)$ holds'.

The symbols ':' or 'I' is read as 'such that'.

$$C = \left\{x : x = 3k,\ k \in Z^+, \text{the set of positive integers}\right\},$$

i.e C is the set of multiples of 3 including 0

Q2(iv)

In set Builder form, a set is described by some characterizing property $P(x)$ of its elements x.

In this case a set can be described as $\{x : P(x) \text{hold}\}$ or $\{x | P(x) \text{holds}\}$ which is read as 'the set of all x such that $P(x)$ holds'.

The symbols ':' or 'I' is read as 'such that'.

$D = \{x \in N : \ 9 < x < 16\}$,

i.e D is the set of natural numbers which are more than 9 but less than 16.

Q2(v)

In set Builder form, a set is described by some characterizing property $P(x)$ of its elements x.

In this case a set can be described as $\{x : P(x) \text{hold}\}$ or $\{x | P(x) \text{holds}\}$ which is read as 'the set of all x such that $P(x)$ holds'.

The symbols ':' or 'I' is read as 'such that'.

$E = \{x \in Z : -1 < x < 1\}$

or

$E = \{x \in Z : x = 0\}$

Q2(vi)

In set Builder form, a set is described by some characterizing property $P(x)$ of its elements x.

In this case a set can be described as $\{x : P(x)\text{hold}\}$ or $\{x | P(x)\text{holds}\}$ which is read as 'the set of all x such that $P(x)$ holds'.

The symbols ':' or 'I' is read as 'such that'.

As $1^2 = 1$

$2^2 = 4$

$3^2 = 9$

:

:

$10^2 = 100$

$\therefore$ The above set may be described as

$$\{x^2 : x \in N \,\&\, 1 \le x \le 10\}$$

Q2(vii)

In set Builder form, a set is described by some characterizing property $P(x)$ of its elements x.

In this case a set can be described as $\{x : P(x)\text{hold}\}$ or $\{x | P(x)\text{holds}\}$ which is read as 'the set of all x such that $P(x)$ holds'.

The symbols ':' or 'I' is read as 'such that'.

The given set can be described as

$\{x : x = 2n, n \in N\}$ ($\because$ 2, 4, 6, ... are multiples of 2)

Q2(viii)

In set Builder form, a set is described by some characterizing property $P(x)$ of its elements x.

In this case a set can be described as $\{x : P(x) \text{hold}\}$ or $\{x | P(x) \text{holds}\}$ which is read as 'the set of all x such that $P(x)$ holds'.

The symbols ':' or 'I' is read as 'such that'.

$\because 5^1 = 5$
$5^2 = 25$
$5^3 = 125$
$5^4 = 625$

$\therefore$ The above set can be described as

$$\{x : x = 5^n, 1 \le n \le 4\}$$

Q3(i)

The integers whose squares are less than or equal to 10 are:

$$(-3)^2 = 9 < 10$$
$$(-2)^2 = 4 < 10$$
$$(-1)^2 = 1 < 10$$
$$0^2 = 0 < 10$$
$$1^2 = 1 < 10$$
$$2^2 = 4 < 10$$
$$3^2 = 9 < 10$$

The square of other integers are more than 10

Hence $A = \{0, \pm1, \pm2, \pm3\}$
or

$$A = \{0, -1, -2, -3, 1, 2, 3\}$$

Q3(ii)

Let's find the values of $x = \frac{1}{2n-1}$, for $1 \le n \le 5$

for $n = 1, x = \frac{1}{1} = 1$

for $n = 2,\ x = \frac{1}{2 \times 2 - 1} = \frac{1}{4-1} = \frac{1}{3}$

for $n = 3,\ x = \frac{1}{2 \times 3 - 1} = \frac{1}{6-1} = \frac{1}{5}$

for $n = 4,\ x = \frac{1}{2 \times 4 - 1} = \frac{1}{8-1} = \frac{1}{7}$

for $n = 5,\ x = \frac{1}{2 \times 5 - 1} = \frac{1}{10-1} = \frac{1}{9}$

Hence, $B = \left\{1, \frac{1}{3,} \frac{1}{5,} \frac{1}{7,} \frac{1}{9,}\right\}$

Q3(iii)

The integers which lie between $\frac{-1}{2}$ and $\frac{9}{2}$ are $0, 1, 2, 3, 4$

Hence $C = \{0, 1, 3, 4,\}$

Q3(iv)

The vowels in the word EQUATION are E, U, A, I, O.

Since the order in which the elements of a set are written is unmaterial, $D = \{A, E, I, O, U\}$

Q3(v)

A month has either 28, 29, 30 or 31 days.

Out of the 12 months in a year, the months that have 31 days are:
January, March, May, July, August, October, December.

$\therefore E = \{$February, April, June, September, November$\}$

Q3(vi)

The distinct letters of the word 'MISSISSIPPI' are M, I, S, P

Hence $F = \{M, I, S, P\}$

Q4

( ) $\{A,P,L,E\} \leftrightarrow \{x : x$ is a letter of the word "APPLE"$\}$

(i) The solution set of $x^2 - 25 = 0$ is $x = \pm 5$
Hence, $\{-5,5\} \leftrightarrow \{x : x^2 - 25 = 0\}$

(ii) The solution set of $x + 5 = 5$ is $x = 0$
Hence, $\{0\} \leftrightarrow \{x : x + 5 = 5, x \in Z\}$

(v) The natural numbers which are divisor of 10 are 1,2,5,10
Hence, $\{1,2,5,10\} \leftrightarrow \{x : x$ is a natural number and divisor of 10$\}$

(v) The distinct letters of the word "RAJASTHAN" are A,H,J,R,S,T,N
Hence, $\{A,H,J,R,S,T,N\} \leftrightarrow \{x : x$ is a letter of the word "RAJASTHAN"$\}$

(vi) The prime natural numbers which are divisor of 10 are 2,5
Hence, $\{2,5\} \leftrightarrow \{x : x$ is a prime natural number and a divisor of 10$\}$

Q5

The vowels which precede q, that is, come before q are a,e,i,o

Hence the set of vowels in the English alphabet which precede q are
$\{a,e,i,o\}$

Q6

As the cube of an odd integer is odd, and an odd positive integer has the form $2n+1$ for some $n \geq 0$,
Hence the set of all positive integers whose cube is odd may be written in set builder form as $\{x \in Z, x = 2n+1, n \geq 0\}$

Q7

As $2 = 1^2 + 1$
$5 = 2^2 + 1$
$10 = 3^2 + 1$
:
:
$50 = 7^2 + 1$

So, the above set in set builder form can be written as

$$\left\{\frac{n}{n^2+1} : n \in N, 1 \leq n \leq 7\right\}$$

Ex 1.3

Q1

(i) This set is non-empty as 10 is an even natural number divisible by 5.

(ii) As 2 belongs to this set, so it is non-empty.

(iii) $x^2 - 2 = 0 \Rightarrow x^2 = 2 \Rightarrow x = \pm\sqrt{2} \in Q$, the set of rational numbers
So, this set is empty.

(iv) This set is empty as there is no natural number x such that $x < 8$ and simultaneously $x > 12$.

(v) This set is empty as any two parallel lines never intersect each other.

Q2

(i) Infinite, since with a common centre infinitely many circles can be drawn in a plane.

(ii) Finite, as there are only 26 letters of English Alphabet.

(iii) Infinite, $\because$ $\{x \in N : x > 5\} = \{6, 7, 8, \ldots\}$ Which is infinite.

(iv) Finite, $\because \{x \in N : x, 200\} = \{1, 2, 3, \ldots 199\}$ Which is finite.

(v) Infinite, $\because$ $\{x \in Z : x < 5\} = -\{\ldots, -3, -2, -1, 0, 1, 2, 3, 4,\}$ Which is infinite.

(vi) $\{x \in R : 0 < x < 1\}$ is an infinite set $\because$ an interval is an infinite set.

Q3

$A = \{1,2,3\}$

$B = \left\{x \in R : (x-1)^2 = 0\right\}$

$= \{x \in R : x = 1,1\}$

$= \{1\}$

$C = \{1,2,3\}$ ($\because$ repetition is not allowed in a set)

$D = \left\{x \in R : x^3 - 6x^2 + 11x - 6 = 0\right\}$

$= \left\{x \in R : (x-1)\left(x^2 5x + 6\right) = 0\right\}$ $\quad [\because\ x = 1$ satisfies the above equation$]$

$= \{x \in R : (x-1)(x-2)(x-3) = 0\}$

$= \{x \in R : x = 1,2,3\}$

$= \{1,2,3\}$

Hence the set A, C and D are equal.

Q4

$A = \{a,e,p,r\}$

$B = \{a,e,p,r\}$ (repetition of 'p' is not allowed)

$C = \{e,o,p,r\}$

as $A = B \neq C$, $\therefore$ the sets are not equal

Q5

Two finite sets are said to be equivalent if they have the same number of elements.
As A and C have same number of elements, and B and D also have same number of elements.

$\therefore$ A is equivalent to C & B is equivalent to D.

Q6

(i)

Two sets A and B are said to be equal if every elements of A is an elements of B and vice-versa.

We have, $A = \{2, 3\}$

and $B = \{x : x \text{ is a solution of } x^2 + 5x + 6 = 0\}$

$= \{x : x^2 + 3x + 2x + 6 = 0\}$

$= \{x : x(x + 3) + 2(x + 3 = 0)\}$

$= \{x : (x + 3)(x + 2) = 0\}$

$= \{x : x = -2, -3\}$

$= \{-2, -3\}$

Hence $A \neq B$.

(ii)

$A = \{W, O, L, F\}$

$B = \{F, O, L, W\}$ [$\because$ repetition is not allowed]

$= \{W, O, LF\}$ [The order in which the elements are written does not matter.]

Hence $A = B$

Q7

$A = \{0, a\}$

$B = \{1, 2, 3, 4,\}$

$C = \{4, 8, 12\}$

$D = \{3, 1, 2, 4\}$

$\quad = \{1, 2, 3, 4\}$

$E = \{1, 0\}$

$F = \{8, 4, 12\}$

$\quad = \{4, 8, 12\}$

$G = \{1, 5, 7, 11\}$

$H = \{a, b\}$

The sets B and D are equal.

The sets C and F are equal.

As A, E and H have same number of elements so they are equivalent.

As B, D and G have same number of elements, so they are equivalent

Also C and F have same number of elements, so they are equivalent.

Q8

$A = \{1, 2\}$

$B = \{1, 2\}$

$C = \{3, 1\}$

$D = \{1, 3\}$ $\quad$ [$\because$ the odd natural numbers less than 5 are 1 and 3]

$E = \{1, 2\}$ $\quad$ [$\because$ repetition is not allowed]

$F = \{1, 3\}$ $\quad$ [$\because$ repetition is not allowed]

$\therefore$ $\quad$ A, B and E are equal

Aslo, C, D and F are equal

Q9

The set formed by distinct letters of the word "CATARACT" are $\{C, A, T, R\}$.

The set formed by distinct letters of the word "TRACT" are $\{T, R, A, C\}$

Hence the two set are equal.

Ex 1.4

Q1

(i) False, $\because$ the two sets A and B need not be comparable.

(ii) False, $\because$ $\{1\}$ is a finite subset of the infinite set N of natural numbers.

(iii) True, $\because$ the order (or cardinal number) of any subset of a set is less than or equal to the order of the set.
(order (or cardinal number) of a set is the number of elements in the set).

(iv) False, $\because$ the empty set ϕ has no proper subset.

(v) False, $\because$ $\{a,b,a,b,...\} = \{a,b\}$ (repetition is not allowed)

$\therefore$ $\{a,b,a,b,...\}$ is a finite set.

(vi) True, $\because$ equivalent sets have the same cardinal number.

(vii) False,
One knows that if the cardinal number of a set A is n, then the power set of A denoted by $P(A)$, which is the set of all subsets of A, has the cardinal number 2^n.

If the cardinal number of A is infinite, then the cardinal number of $P(A)$ is also infinite.
Hence, the above statement is true provided the set is infinite.

Q2

(i) True, $\because$ 1 is an element of the set $\{1,2,3\}$.

(ii) False, $\because$ a is an element and not a subset of the set $\{b,c,a\}$.

(iii) False, $\because$ $\{a\}$ is a subset of the set $\{a,b,c\}$ and not an element.

(iv) True, $\because$ repetition is not allowed in a set.

(v) False, $\because$ the set $\{x : x + 8 = 8\}$ is the single ton set $\{0\}$ which is not the null set ϕ.

Q3

We have,

$$A = \{x : x \text{ satisfies } x^2 - 8x + 12 = 0\}$$
$$= \{x : x^2 - 6x - 2x + 12 = 0\}$$
$$= \{x : x(x-6) - 2(x-6) = 0\}$$
$$= \{x : (x-6)(x-2) = 0\}$$
$$= \{x : x = 6, 2\}$$
$$= \{6, 2\}$$

$$B = \{2, 4, 6\}$$
$$C = \{2, 4, 6, 8, \ldots\}$$
$$D = \{6\}$$

We know that if E and F are two sets, then E is a subset of F. i.e., $E \subseteq F$ if $x \in E \Rightarrow x \in F$. E is called a proper subset of F if E is strictly contained in F and is denoted by $E \subset F$.

Clearly,

$$D \subset A \{\because 6 \in D \text{ and } 6 \in A\}$$
$$A \subset B \{\because 2, 6 \in A \text{ and they also belong to } B\}$$

Similarly, $B \subset C$

Hence, $D \subset A \subset B \subset C$.

Q4(i)

The given statement is 'True'.

If $m \in z$, then m can be written as $\frac{m}{1}$, which is of the form $\frac{p}{q}$, where p and q are relatively prime integers and $q \neq 0$.

This implies that $m \in Q$, the set of rational numbers.

Thus, $m \in z \Rightarrow m \in Q$

Hence $Z \subseteq Q$

Q4(ii)

The given statement is 'True'.
$\because$ Crows are also Birds.

Q4(iii)

The given statement is 'False'.
$\because$ A rectangle need not be a square.

Q4(iv)

The given statement is 'True'.

If z is a complex number, then it can be written as $z = x + iy$,
where x and y are real numbers and are called the real and imaginary parts of the complex number z.

If x is a real number, then
$x = x + i.0 \in C$,
where C is the set of complex numbers.

Thus $x \in R \Rightarrow x \in C$

Hence, the set of all real numbers is contained in the set of all complex numbers.

Q4(v)

False, $\because$ $a \in P$ but $a \notin B$
Note that $\{a\}$ is an element of B which is different from the element 'a'.

Q4(vi)

$$\begin{aligned} A &= \{L,I,T,E\} && [\because \text{repetition is not allowed}] \\ B &= \{T,I,L,E\} && [\because \text{repetition is not allowed}] \\ &= \{L,I,T,E\} && \left[\begin{array}{l}\because \text{the manner in which the elements are} \\ \text{listed does not matter}\end{array}\right] \end{aligned}$$

$\because$ Each element of A is an element of B and vice-versa

$\therefore$ $A = B$

Hence, the given statement is true.

Q5

(i) False,

The correct statement is $a \in \{a, b, c\}$.

(ii) False, $\because$ $\{a\}$ is a subset and not an element of $\{a, b, c\}$

The correct form is $\{a\} \subset \{a, b, c\}$.

(iii) False, $\because$ a is not an element of $\{\{a\}, b\}$

The correct form is $\{a\} \in \{\{a\}, b\}$

(iv) False, $\because \{a\}$ is not a subset of $\{\{a\}, b\}$ hence it cannot be contained in it.

The correct form is $\{a\} \in \{\{a\}, b\}$. Another correct form could be $\{\{a\}\} \subset \{\{a\}, b\}$.

(v) False, $\because$ $\{b, c\}$ is an element and not a subset of $\{a, \{, bc\}\}$.

The correct form is $\{b, c\} \in \{a, \{b, c\}\}$.

(vi) False, $\because$ $\{a, b\}$ is not a subset of $\{a, \{b, c\}\}$

The correct form is $\{a, b\} \not\subset \{a, \{b, c\}\}$.

(vii) False, $\because$ ϕ is not an element of $\{a, b\}$.

The correct form is $\phi \subset \{a, b\}$.

(viii) True, $\because$ empty set ϕ is a subset of every set.

(ix) False, $\because \{x : x + 3 = 3\} = \{x : x = 0\} = \{0\}$

The correct form is $\{x : x + 3 = 3\} \neq \phi$.

Q6

(i) False, $\{c,d\}$ is an element of A and not a subset of A.

(ii) True, $\because \{c,d\}$ is indeed an element of A.

(iii) True, $\because \{c,d\}$ is a subset of A.

(iv) True,

(v) False, $\because$ a belongs to A and not a subset of A. An element of a set belongs to it whereas a subset of it is contained in it.

(vi) True, $\because \{a,b,e\}$ is a subset of A.

(vii) False, $\because \{a,b,e\}$ is a subset of A, so it does not belong to A.

(viii) False, $\because \{a,b,c\}$ is not a subset of A.

(ix) False, $\because \phi$ is a subset and not an element of A.

(x) False, $\because \phi$ and not $\{\phi\}$ is a subset of A.

Q7

(i) False, $\because$ 1 is not an element of A.

(ii) False, $\because \{1,2,3\}$ is not a subset of A, it is an element of A.

(iii) True, $\because \{6,7,8\}$ is indeed an element of A.

(iv) True, $\because$ $\{\{4,5\}\}$ is indeed a subset of A.

(v) False, $\because$ ϕ is a subset and not an element of A.

(vi) True, $\because$ ϕ is a subset of every set, and hence a subset of A.

Q8

(i) True, $\because$ ϕ indeed belongs to A.

(ii) True, $\because$ $\{\phi\}$ is an element of A.

(iii) False, $\because$ $\{1\}$ is not an element of A.

(iv) True, $\because$ $\{2,\phi\}$ is a subset of A.

(v) False, $\because$ 2 is not a subset of A, it is an element of A.

(vi) True, $\because \{2,\{1\}\}$ is not a subset of A.

(vii) True, $\because$ $\{\{2\},\{1\}\}$ is not a subset of A.

(viii) True, $\because \{\phi,\{\phi\},\{1,\phi\}\}$ is a subset of A.

(ix)True, $\because \{\{\phi\}\}$ is a subset of A.

Q9

(i) We know that, if a set has n elements, then its power set has 2^n elements.

Here, n = 1, so there 2^1=2 subsets of the given set.

The possible subsets are $\phi,\{a\}$.

(ii) The set has two elements, so power set has 2^2 = 4 elements, namely $\phi,\{0\},\{1\},\{0,1\}$.

(iii) The set has 3 elemets , so power set has 2^3 = 8 elements,
namely $\phi,\{a\},\{b\},\{c\},\{a,b\},\{b,c\},\{a,c\},\{a,b,c\}$.

(iv) The set has 2 elements, so power set has 2^2 = 4 elements, namely, $\phi,\{1\},\{\{1\}\},\{1,\{1\}\}$.

(v) The set has 1 element, so power set has ^{1} = 2 elements, namely $\phi,\{\phi\}$.

Q10

(i) We know that if A is a set and B a subset of A, then B is called a proper subset of A if $B \subseteq A$ and $B \neq A$, ϕ and is written as $B \subset A$ or $B \subseteq A$.

Hence, the proper subsets are given by $\{1\},\{2\}$.

(ii) The proper subsets are given by $\{1\},\{2\},\{3\},\{1,2\},\{2,3\},\{1,3\}$.

(iii) The only subsets of the given set are ϕ & $\{1\}$.
Hence, there are no proper subsets.

Q11

We know that, if A is a set having n elements then power set of A, namely $P(A)$ has 2^n elements. Out of this A is not proper subset.

Hence, the total number of proper subsets of a set consisting of n elements in $2^n - 1$

Q12

The symbol '$\Leftrightarrow$' stands for if and only if (in short if).
In order to show that two sets A and B are equal we show that $A \subseteq B$ and $B \subseteq A$.

We have $A \subseteq \phi$. $\because$ ϕ is a subset of every set

$\therefore$ $\phi \subseteq A$

Hence $A = \phi$
To show the backward implication, suppose that $A = \phi$
$\because$ every set is a subset of itself
$\therefore$ $\phi = A \subseteq \phi$

Hence, proved.

Q13

We have $A \subseteq B$, $B \subseteq C$ and $C \subseteq A$, so $A \subseteq B \subseteq C \subseteq A$
Now, A is a subset of B and B is a subset of C, so

A is a subset of C, i.e., $A \subseteq C$

Also, $C \subseteq A$

Hence, $A = C$

Q14

$\because$ an empty set has zero element.

$\therefore$ power set of ϕ has $2^0 = 1$ element.

Q15

(i)
The set of right triangles is a subset of the set of all triangles in the plane. So, the set of all triangles in the plane forms a universal set for the set of right triangles.

(ii)
The set of isosceles triangles forms a subset of the set of all triangles in the plane.

Hence the set of all triangles in the plane forms a universal set for the set of isosceles triangles.

Q16

$$X = \{8^n - 7n - 1 : x \in N\}$$
$$Y = \{4n(n-1) : n \in N\}$$

In order to show that $x \subseteq y$ we show tat every element of X is an element of Y.

So let $x \in X \Rightarrow x = 8^m - 7m - 1$ for some $m \in N$

$\Rightarrow \quad x = (1+7)^m - 7m - 1$

$= \left({}^mC_0 1^m + {}^mC_1 1^{m-1} 7 + \ldots + {}^mC_{m-1} 1^1 7^{m-1} + {}^mC_m 7^m\right) - 7m - 1$

[using binomial expansion]

$= 1 + 7m + {}^mC_2 7^2 + {}^mC_3 7^3 + \ldots + {}^mC_m 7^m - 7m - 1$

$= {}^mC_2 7^2 + {}^mC_3 7^3 + \ldots + {}^mC_m 7^m$

$= 49\left({}^mC_2 + {}^m mC_3 + \ldots + {}^mC_m 7^{m-2}\right), \ m \geq 2$

$= 49t_m, \ m \geq 2, \ \text{where } t_m = {}^mC_2 + {}^mC_3 7 + \ldots + {}^mC_m 7^{m-2}$

Is some positive integer depending on $m \geq 2$

For $m = 1$

$x = 8^1 - 7 \times 1 - 1$

$= 8 - 8$

$= 0$

Hence, X contains all positive integral multiples of 49.

Also, Y consistes of all positive integral multiples of 49, including 0, for $n = 1$.

Thus, we conclude that $X \subseteq Y$.

Ex 1.5

Q1

(i)
$A \cap B$ denotes intersection of the two sets A and B, which consists of elements which are common to both A and B.

Since $A \subset B$, every element of A is already an element of B.

$\therefore\ A \cap B = A$

(ii)
$A \cup B$ denotes the union of the sets A and B which consists of elements which are either in A or B or in both A and B.

Since $A \subset B$, every element of A is already an element of B.

$\therefore\ A \cup B = B$

Q2(i)

$A = \{1, 2, 3, 4, 5\}$
$B = \{4, 5, 6, 7, 8\}$

So, $A \cup B = \{x : x \in A \text{ or } x \in B\}$
$= \{1, 2, 3, 4, 5, 6, 7, 8\}$

Q2(ii)

$A \cup C = \{x : x \in A \text{ or } x \in C\}$
$= \{1, 2, 3, 4, 5, 7, 8, 9, 10, 11\}$

Q2(iii)

$B \cup C = \{x : x \in B \text{ or } x \in C\}$
$= \{4, 5, 6, 7, 8, 9, 10, 11\}$

Q2(iv)

$B \cup D = \{x : x \in B \text{ or } x \in D\}$
$= \{4, 5, 6, 7, 8, 10, 11, 12, 13, 14\}$

Q2(v)

$$A \cup B \cup C = \{x | x \in A \text{ or } x \in B \text{ or } x \in C\}$$
$$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$$

Q2(vi)

$$A \cup B \cup D = \{x : x \in A \text{ or } x \in B \text{ or } x \in D\}$$
$$= \{1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14\}$$

Q2(vii)

$$B \cup C \cup D = \{x | x \in B \text{ or } x \in C \text{ or } x \in D\}$$
$$= \{4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14\}$$

Q2(viii)

$A \cap (B \cup C)$ = all those elements which are common to A and $B \cup C$

$$= \{x | x \in A \text{ and } x \in B \cup C\}$$

Now, $B \cup C = \{4, 5, 6, 7, 8, 9, 10, 11\}$

$$\therefore\ A \cap (B \cup C) = \{1, 2, 3, 4, 5\} \cap \{4, 5, 6, 7, 8, 9, 10, 11\}$$
$$= \{4, 5\}$$

Q2(ix)

$(A \cap B) \cap (B \cap C) = \{x | x \in (A \cap B) \text{ and } x \in (B \cap C)\}$

Now,

$A \cap B = \{x | x \in A \text{ and } x \in B\}$

i.e., elements which are common to A & B

$\therefore\ A \cap B = \{1, 2, 3, 4, 5\} \cap \{4, 5, 6, 7, 8\}$

$= \{4, 5\}$

Also,

$B \cap C = \{4, 5, 6, 7, 8\} \cap \{7, 8, 9, 10, 11\}$

$= \{7, 8\}$

Hence, $(A \cap B) \cap (B \cap C) = \{4, 5\} \cap \{7, 8\}$

$= \phi$ $\quad \left[\begin{array}{l} \because \text{ there is no element common in} \\ \{4,5\} \text{ and } \{7,8\} \end{array}\right]$

Q2(x)

$(A \cup D) \cap (B \cup C) = \{x | x \in (A \cup D) \text{ or } x \in (B \cup C)\}$

Now,

$A \cup D = \{1, 2, 3, 4, 5, 10, 11, 12, 13, 14\}$

and $\quad B \cup C = \{4, 5, 6, 7, 8, 9, 10, 11\}$

$\therefore (A \cup D) \cap (B \cup C) = \{4, 5, 10, 11\}$

Q3(i)

We have,

$$A = \{x : x \in N\}$$
$$= \{1, 2, 3, ...\}, \text{ the set of natrual numbers}$$

$$B = \{x : x = 2n, x \in N\}$$
$$= \{2, 4, 6, 8, ...\}, \text{ the set of even natural numbers}$$

$$\therefore A \cap B = \{x : x \in A \text{ and } x \in B\}$$
$$= \{2, 4, 6, ...\}$$
$$= B \qquad [\because B \subset A]$$

Q3(ii)

We have,

$$A = \{x : x \in N\}$$
$$= \{1, 2, 3, ...\}, \text{ the set of natrual numbers}$$

$$C = \{x : x = 2n - 1, x \in N\}$$
$$= \{1, 3, 5, ...\}, \text{ the set of odd natural numbers}$$

$$A \cap C = \{x : x \in A \text{ and } x \in C\}$$
$$= C \qquad [\because C \subset A]$$

Q3(iii)

We have,

$$A = \{x : x \in N\}$$
$$= \{1, 2, 3, \ldots\}, \text{ the set of natrual numbers}$$

and $D = \{x : x \text{ is a prime natural number}\}$
$$= \{2, 3, 5, 7, \ldots\}$$

$$A \cap D = \{x : x \in A \text{ and } x \in D\}$$
$$= D \qquad [\because D \subset A]$$

Q3(iv)

We have,

$$B = \{x : x = 2n, x \in N\}$$
$$= \{2, 4, 6, 8, \ldots\}, \text{ the set of even natural numbers}$$

and

$$C = \{x : x = 2n - 1, x \in N\}$$
$$= \{1, 3, 5, \ldots\}, \text{ the set of odd natural numbers}$$

$$B \cap C = \{x : x \in B \text{ and } x \in C\}$$
$$= \phi \qquad \left[\begin{array}{l}\because B \text{ and } C \text{ are disjoint sets, i.e.,} \\ \text{have no elements in common}\end{array}\right]$$

Q3(v)

Here,

$$B = \{x : x = 2n, x \in N\}$$
$$= \{2, 4, 6, 8, \ldots\}, \text{ the set of even natural numbers}$$

and $D = \{x : x \text{ is a prime natural number}\}$
$$= \{2, 3, 5, 7, \ldots\}$$

$$B \cap D = \{x : x \in B \text{ and } x \in D\}$$
$$= \{2\}$$

Q3(vi)

Here,

$$C = \{x : x = 2n - 1, x \in N\}$$
$$= \{1, 3, 5, \ldots\}, \text{ the set of odd natural numbers}$$

and $D = \{x : x \text{ is a prime natural number}\}$
$$= \{2, 3, 5, 7, \ldots\}$$

$$C \cap D = \{x : x \in C \text{ and } x \in D\}$$

We observe that except, the element 2, every other element in D is an odd natural number.

Hence, $C \cap D = D - \{2\}$
$$= \{x \in D : x \neq 2\}$$

Q4

We have,

$$A = \{3, 6, 12, 15, 18, 21\}$$
$$B = \{4, 8, 12, 16, 20\}$$
$$C = \{2, 4, 6, 8, 10, 12, 14, 16\}$$
$$D = \{5, 10, 15, 20\}$$

If A and B are two sets, then the set $A - B$ is defined as

$$A - B = \{x \in A : x \notin B\}.$$

(i) $A - B = \{x \in A : x \notin B\} = \{3, 6, 15, 18, 21\}$

(ii) $A - C = \{x \in A : x \notin C\} = \{3, 15, 18, 21\}$

(iii) $A - D = \{x \in A : x \notin D\} = \{3, 6, 12, 18, 21\}$

(iv) $B - A = \{x \in B : x \notin A\} = \{4, 8, 16, 20\}$

(v) $C - A = \{x \in C : x \notin A\} = \{2, 4, 8, 10, 14, 16\}$

(vi) $D - A = \{x \in D : x \notin A\} = \{5, 10, 20\}$

(vii) $B - C = \{x \in B : x \notin C\} = \{20\}$

(viii) $B - D = \{x \in B : x \notin D\} = \{4, 8, 12, 16\}$

Q5

(i)

$U = \{1,2,3,4,5,6,7,8,9\}$, $A = \{1,2,3,4\}$, $B = \{2,4,6,8\}$, $C = \{3,4,5,6\}$

By the complement of a set A, which respect to the universal set U, denoted by A' or A^c or $U - A$, we mean $\{x \in U : x \notin A\}$.

Hence, $A' = \{x \in U : x \notin A\} = \{5,6,7,8,9\}$

(ii) $B' = \{x \in U : x \notin B\} \quad = \{1,3,5,7,9\}$

(iii) $(A \cap C)' = \{x \in U : x \notin A \cap C\}$

Now,

$A \cap C = \{x : x \in A \text{ and } x \in C\} = \{3,4\}$

$\therefore \quad (A \cap C)' = \{1,2,5,6,7,8,9\}$

Q6

(i)

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$A = \{2, 4, 6, 8\}$

$B = \{2, 3, 5, 7\}$

We have,

$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$
$$= \{2, 3, 4, 5, 6, 7, 8\}$$

$$\therefore (A \cup B)' = \{x \in U : x \notin A \cup B\}$$
$$= \{1, 9\}$$

$$A' = \{x \in U : x \notin A\}$$
$$= \{1, 3, 5, 7, 9\}$$
$$B' = \{x \in U : x \notin B\}$$
$$= \{1, 4, 6, 8, 9\}$$

Hence, $A' \cap B' = \{1, 9\}$

Hence, $(A \cup B)' = A' \cap B' = \{1, 9\}$

(ii)

$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$
$$= \{2\}$$
$$\therefore \quad (A \cap B)' = \{x \in U : x \notin A \cap B\}$$
$$= \{1, 3, 4, 5, 6, 7, 8, 9\}$$

Also,

$$A' \cup B' = \{x : x \in A' \text{ or } x \in B'\}$$
$$= \{1, 3, 4, 5, 6, 7, 8, 9\}$$

Hence, $(A \cap B)' = A' \cup B' = \{1, 3, 4, 5, 6, 7, 8, 9\}$

Ex 1.6

Q1

The smallest set A such that

$$A \cup \{1,2\} = \{1,2,3,5,9\} \text{ is } \{3,5,9\}$$

$$\because \quad \{3,5,9\} \cup \{1,2\} = \{1,2,3,5,9\}$$

Any other set B such that $B \cup \{1,2\} = \{1,2,3,5,9\}$ will contain A. For example we contake B to be $\{1,3,5,9\}$ or $\{1,2,3,5,9\}$. Clearly B contains $A = \{3,5,9\}$.

Q2(i)

i. $A = \{1,2,4,5\}, B = \{2,3,5,6\}, C = \{4,5,6,7\}$

$B \cap C = \{5,6\}$

$A \cup (B \cap C) = \{1,2,4,5,6\} \ldots\ldots\ldots (1)$

$(A \cup B) = \{1,2,3,4,5,6\}$

$(A \cup C) = \{1,2,4,5,6,7\}$

$(A \cup B) \cap (A \cup C) = \{1,2,4,5,6\} \ldots\ldots\ldots\ldots\ldots (2)$

From eq^n (1) and eq^n (2), we get

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Q2(ii)

ii. $A = \{1,2,4,5\}, B = \{2,3,5,6\}, C = \{4,5,6,7\}$

$B \cup C = \{2,3,4,5,6,7\}$

$A \cap (B \cup C) = \{2,4,5\} \ldots\ldots\ldots (1)$

$(A \cap B) = \{2,5\}$

$(A \cap C) = \{4,5\}$

$(A \cap B) \cup (A \cap C) = \{2,4,5\} \ldots\ldots\ldots\ldots\ldots (2)$

From eq^n (1) and eq^n (2), we get

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Q2(iii)

iii. $A = \{1, 2, 4, 5\}, B = \{2, 3, 5, 6\}, C = \{4, 5, 6, 7\}$

$B - C = \{2, 3\}$

$A \cap (B - C) = \{2\} \ldots\ldots\ldots (1)$

$(A \cap B) = \{2, 5\}$

$(A \cap C) = \{4, 5\}$

$(A \cap B) - (A \cap C) = \{2\} \ldots\ldots\ldots\ldots\ldots (2)$

From eq^n (1) and eq^n (2), we get

$A \cap (B - C) = (A \cap B) - (A \cap C)$

Q2(iv)

iv. $A = \{1, 2, 4, 5\}, B = \{2, 3, 5, 6\}, C = \{4, 5, 6, 7\}$

$B \cup C = \{2, 3, 4, 5, 6, 7\}$

$A - (B \cup C) = \{1\} \ldots\ldots\ldots (1)$

$(A - B) = \{1, 4\}$

$(A - C) = \{1, 2\}$

$(A - B) \cap (A - C) = \{1\} \ldots\ldots\ldots\ldots\ldots (2)$

From eq^n (1) and eq^n (2), we get

$A - (B \cup C) = (A - B) \cap (A - C)$

Q2(v)

$$\text{v. } A = \{1, 2, 4, 5\}, B = \{2, 3, 5, 6\}, C = \{4, 5, 6, 7\}$$
$$B \cap C = \{5, 6\}$$
$$A - (B \cap C) = \{1, 2, 4\} \ldots\ldots\ldots(1)$$

$$(A - B) = \{1, 4\}$$
$$(A - C) = \{1, 2\}$$

$$(A - B) \cup (A - C) = \{1, 2, 4\} \ldots\ldots\ldots\ldots\ldots(2)$$

From $eq^n (1)$ and $eq^n (2)$, we get

$$A - (B \cap C) = (A - B) \cup (A - C)$$

Q2(vi)

$$\text{vi. } A = \{1, 2, 4, 5\}, B = \{2, 3, 5, 6\}, C = \{4, 5, 6, 7\}$$
$$B \Delta C = (B - C) \cup (C - B) = \{2, 3\} \cup \{4, 7\} = \{2, 3, 4, 7\}$$
$$A \cap (B \Delta C) = \{2, 4\} \ldots\ldots\ldots(1)$$

$$(A \cap B) = \{2, 5\}$$
$$(A \cap C) = \{4, 5\}$$

$$(A \cap B) \Delta (A \cap C) = [(A \cap B) - (A \cap C)] \cup [(A \cap C) - (A \cap B)]$$
$$(A \cap B) \Delta (A \cap C) = \{2\} \cup \{4\} = \{2, 4\} \ldots\ldots\ldots\ldots\ldots(2)$$

From $eq^n (1)$ and $eq^n (2)$, we get

$$A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$$

Q3(i)

$U = \{2, 3, 5, 7, 9\}$ is the universal set

$A = \{3, 7\}$, $B = \{2, 5, 7, 9\}$

$A \cup B = \{x : x \in A \text{ or } x \in B\}$

$= \{2, 3, 5, 7, 9\}$

LHS $= (A \cup B)'$

$= \{2, 3, 5, 7, 9\}'$

$= U - A \cup B$

$= \phi$

RHS $= A' \cap B'$

$A' = \{x \in U : x \notin A\}$

$= \{2, 5, 9\}$

$B' = \{x \in U : x \notin B\}$

$= \{3\}$

$\therefore \; A' \cap B' = \{2, 5, 9\} \cap \{3\}$

$= \phi$ $[\because$ the two sets are disjoint$]$

$\therefore$ LHS = RHS Proved

Q3(ii)

LHS $= (A \cap B)'$

Now,

$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

$= \{7\}$

$\therefore (A \cap B)' = \{7\}'$

$= \{x \in U : x \notin 7\}$

$= \{2, 3, 5, 9\}$

RHS $= A' \cup B'$

Now, $A' = \{2, 5, 9\}$ [form (i)]

and $B' = \{3\}$ [from (i)]

$\therefore$ $A' \cup B' = \{2, 3, 5, 9\}$

Hence, LHS = RHS Proved

Q4(i)

i. Let $x \in B$. Then
$\Rightarrow x \in B \cup A$
$\Rightarrow x \in A \cup B$
$\therefore B \subset (A \cup B)$

Q4(ii)

ii. Let $x \in A \cap B$. Then
$\Rightarrow x \in A$ and $x \in B$
$\Rightarrow x \in B$
$\therefore (A \cap B) \subset B$

Q4(iii)

iii. Let $x \in A \subset B$. Then
$\Rightarrow x \in B$

Let and $x \in A \cap B$
$\Leftrightarrow x \in A$ and $x \in B$
$\Leftrightarrow x \in A$ and $x \in A \quad (\because A \subset B)$
$\therefore (A \cap B) = A$

Q5

(i)

In order to show that the following four statements are equivalent, we need to show that $(1) \Rightarrow (2)$, $(2) \Rightarrow (3)$, $(3) \Rightarrow (4)$ and $(4) \Rightarrow (1)$

We first show that $(1) \Rightarrow (2)$

We assume that $A \subset B$, and use this to show that $A - B = \phi$

Now $A - B = \{x \in A : x \notin B\}$. As $A \subset B$,

$\therefore$ Each element of A is an element of B,

$\therefore$ $A - B = \phi$

Hence, we have proved that $(1) \Rightarrow (2)$.

(ii)

We new show that $(2) \Rightarrow (3)$

So assume that $A - B = \phi$

To show: $A \cup B = B$

$\because$ $A - B = \phi$

$\therefore$ Every element of A is an element of B

$[\because A - B = \phi$ only when ther is some element in A which is not in $B]$

So $A \subset B$ and therefore $A \cup B = B$

So $(2) \Rightarrow (3)$ is true.

(iii)

We new show that $(3) \Rightarrow (4)$

Assume that $A \cup B = B$

To show: $A \cap B = A$

$\because \quad A \cup B = B$

$\therefore \quad A \subset B$ and so $A \cap B = A$

So $(3) \Rightarrow (4)$ is true.

(iv)

Finally we show that $(4) \Rightarrow (1)$, which will prove the equivalence of the four statements.

So, assume that $A \cap B = A$

To show: $A \subset B$

$\because$ $A \cap B = A$, therefore $A \subset B$, and so $(4) \Rightarrow (1)$ is true.

Hence, $(1) \Leftrightarrow (2) \Leftrightarrow (3) \Leftrightarrow (4)$.

Q6(i)

Let $A = \{1, 2, 3\}$, $B = \{2, 4, 6\}$ and $C = \{2, 5, 7\}$

Then,

$A \cap B = \{2\}$

and $\quad A \cap C = \{2\}$

Hence, $A \cap B = A \cap C$, but clearly $B \neq C$.

Q6(ii)

Given $A \subset B$

To show: $C - B \subset C - A$

Let $x \in C - B$

$\Rightarrow \quad x \in C$ and $x \notin B$ $\quad$ [by definition of $C - B$]

$\Rightarrow \quad x \in C$ and $x \notin A$ $\quad$ [$\because A \subset B$]

This can be seen by the venn diagram above

$\Rightarrow \quad x \in C - A$ $\quad$ [by definition of $C - A$]

Thus $x \in C - B \Rightarrow x \in C - A$. This is true for all $x \in C - B$

$\therefore \quad C - B \subset C - A$

Q7

(i)

$$A \cup (A \cap B) = (A \cup A) \cap (A \cup B) \qquad [\because \text{ union } \cup \text{ is distributive over intersection } \cap]$$

$$= A \cap (A \cup B) \qquad [\because A \cup A = A]$$

$$= A \qquad \left[\begin{array}{l}\because A \subset (A \cup B)\text{, as union of two sets is bigger} \\ \text{than each of the individual sets}\end{array}\right]$$

Hence, $A \cup (A \cap B) = A$ Proved.

(ii)

$$A \cap (A \cup B) = (A \cap A) \cup (A \cap B) \qquad [\because \cap \text{ distributes over } \cup]$$

$$= A \cup (A \cap B) \qquad [\because A \cap A = A]$$

$$= A \qquad [\text{using (i)}]$$

Q8

To find sets A, B and C such that $A \cap B \neq \phi$, $A \cap C = \phi$
and $B \cap C = \phi$ and $A \cap B \cap C = \phi$

Take $A = \{1, 2, 3\}$
$B = \{2, 4, 6\}$
and $C = \{3, 4, 7\}$
Then,
$A \cap B = \{2\}$
$\therefore \quad A \cap B \neq \phi$
$A \cap C = \{3\}$
$\therefore \quad A \cap C \neq \phi$
$B \cap C = \{4\}$
$\therefore \quad B \cap C \neq \phi$

However A, B and C have no elements in common,
$\therefore \quad A \cap B \cap C = \phi$

Q9

Given $A \cap B = \phi$, i.e., A and B are disjoint sets this can
represented by venn diagram as follows

To show: $A \subseteq B'$

This is clear from the venn diagram itself
$\because$ A is lying in the complement of B, but we give a proof of it.
So let $x \in A$
$\because \quad A \cap B = \phi$,
$\therefore \quad x \notin B$
and so $x \in B'$ $\quad [\because x \notin B \Rightarrow x \in B']$

Thus $x \in A \Rightarrow x \in B'$. This is true for all $x \in A$

Hence, $A \subseteq B'$

Q10

We need to show that $(A-B)\cap(A\cap B)=\phi$, $(A\cap B)\cap(B-A)=\phi$ and $(A-B)\cap(B-A)=\phi$

The 3 sets $A-B$, $A\cap B$ and $B-A$ may be represented by a venn diagram as follows

It is clear from the diagram that the 3 sets are pairwise disjoint, but we shall give a proff of it.

We first show that $(A-B)\cap(A\cap B)=\phi$

Let $x\in(A-B)$

$\Rightarrow \quad x\in A$ and $x\notin B$ $\qquad$ [by definition of $A-B$]

$\Rightarrow \quad x\notin A\cap B$. This is true for all $x\in(A-B)$

Hence $(A-B)\cap(A\cap B)=\phi$

On a similar lines, it can be seen that $(A\cap B)\cap(B-A)=\phi$

Finally, we show that $(A-B)\cap(B-A)=\phi$

We have,

$A-B=\{x\in A: x\notin B\}$

and $B-A=\{x\in B: x\notin A\}$

Hence, $(A-B)\cap(B-A)=\phi$.

Q11

We need to show $(A\cup B)\cap(A\cap B')=A$

Now,

$(A\cup B)\cap(A\cap B')=((A\cup B)\cap A)\cap B'$ $\qquad$ [Using associative property]

$=((A\cap A)\cup(B\cap A))\cap B'$ $\qquad$ [$\because A\cap A=A$ and $B\cap A=A\cap B$, by commutative law]

$=A\cap B'$ $\qquad$ [$\because A\cup(A\cap B)=A$]

$=A$

Q12(i)

We have $A' \cup B = \cup$, the universal set

To show: $A \subset B$

Let, $x \in A$

$\Rightarrow \quad x \notin A' \qquad [\because A \cap A' = \phi]$

$\because \quad x \in A$ and $A \subset \cup$

$\Rightarrow \quad x \in \cup$

$\Rightarrow \quad x \in (A' \cup B) \qquad [\because \cup = A' \cup B]$

$\Rightarrow \quad x \in A'$ or $x \in B$

But, $x \notin A'$,

$\therefore \quad x \in B$

Thus, $x \in A \Rightarrow x \in B$

This is true for all $x \in A$

$\therefore \quad A \subset B$

Q12(ii)

We have $B' \subset A'$

To show: $A \subset B$

Let, $x \in A$

$\Rightarrow \quad x \notin A' \qquad [\because A \cap A' = \phi]$

$\Rightarrow \quad x \notin B' \qquad [\because B' \subset A']$

$\Rightarrow \quad x \in B \qquad [\because B \cap B' = \phi]$

Thus, $x \in A \Rightarrow x \in B$

This is true for all $x \in A$

$\therefore \quad A \subset B$

Q13

This is a false statement

Let, $A = \{1\}$ and $B = \{2\}$

Then,

$P(A) = \{\phi, \{1\}\}$

and $P(B) = \{\phi, \{2\}\}$

$\therefore$ $P(A) \cup P(B) = \{\phi, \{1\}, \{2\}\}$

Now,

$A \cup B = \{1, 2\}$

and $P(A \cup B) = \{\phi, \{1\}, \{2\}, \{1, 2\}\}$

Hence, $P(A) \cup P(B) \neq P(A \cup B)$

Q14(i)

i. We know that $(A \cap B) \subset A$ and $(A - B) \subset A$

$\Rightarrow (A \cap B) \cap (A - B) \subset A$.........(1)

Let and $x \in (A \cap B) \cap (A - B)$

$\Rightarrow x \in (A \cap B)$ and $x \in (A - B)$

$\Rightarrow x \in A$ and $x \in B$ and $x \in A$ and $x \notin B$

$\Rightarrow x \in A$ and $x \in A$ [$\because x \in B$ and $x \notin B$ are not possible simultaneously]

$\Rightarrow x \in A$

$\therefore (A \cap B) \cap (A - B) \subset A$.......(2)

From (1) and (2), we get

$A = (A \cap B) \cap (A - B)$

Q14(ii)

ii. Let $x \in A \cup (B - A)$

$\Rightarrow x \in A$ or $x \in (B - A)$

$\Rightarrow x \in A$ or $x \in B$ and $x \notin A$

$\Rightarrow x \in A$ or $x \in B$

$\Rightarrow x \in (A \cup B)$

$\therefore A \cup (B - A) \subset (A \cup B) \ldots\ldots\ldots\ldots(1)$

Let and $x \in (A \cup B)$

$\Rightarrow x \in A$ or $x \in B$

$\Rightarrow x \in A$ or $x \in B$ and $x \notin A$

$\Rightarrow x \in A$ or $x \in (B - A)$

$\Rightarrow x \in A \cup (B - A)$

$\therefore (A \cup B) \subset A \cup (B - A) \ldots\ldots\ldots(2)$

From (1) and (2), we get

$A \cup (B - A) = A \cup B$

Q15

Since each X_r has 5 elements and each element of S belongs to exactly 10 of X_r's.

$\therefore S = \bigcup_{r=1}^{20} X_r \Rightarrow \frac{1}{10}\sum_{r=1}^{20} n(X_r) = \frac{1}{10}(5 \times 20) = 10 \ldots\ldots(i)$

Since each Y_r has 2 elements and each element of S belongs to exactly 4 of X_r's.

$\therefore S = \bigcup_{r=1}^{n} X_r \Rightarrow \frac{1}{4}\sum_{r=1}^{n} n(Y_r) = \frac{1}{4}(2n) = \frac{n}{2} \ldots(ii)$

From (i) and (ii), we get

$10 = \frac{n}{2} \Rightarrow n = 20$

Ex 1.7

Q1

To show $A' - B' = B - A$

We show that $A' - B' \subseteq B - A$ and vice versa

Let, $x \in A' - B'$

$\Rightarrow \quad x \in A'$ and $x \notin B'$

$\Rightarrow \quad x \notin A$ and $x \in B \qquad [\because A \cap A' = \phi$ and $B \cap B' = \phi]$

$\Rightarrow \quad x \in B$ and $x \notin A$

$\Rightarrow \quad x \in B - A$

This is true for all $x \in A' - B'$

Hence $A' - B' \subseteq B - A$

Conversely,

Let, $x \in B - A$

$\Rightarrow \quad x \in B$ and $x \notin A$

$\Rightarrow \quad x \notin B'$ and $x \in A' \qquad [\because B \cap B' = \phi$ and $A \cap A' = \phi]$

$\Rightarrow \quad x \in A'$ and x $\notin$ B'

$\Rightarrow \quad x \in A' - B'$

This is true for all $x \in B - A$

Hence $B - A \subseteq A' - B'$

$\therefore\ A' - B' = B - A$ Proved.

Q2(i)

$$
\begin{aligned}
\text{LHS} &= A \cap (A' \cup B) \\
&= (A \cap A') \cup (A \cap B) && [\because \cap \text{ distributes over (i)}] \\
&= \phi \cup (A \cap B) && [\because A \cap A' = \phi] \\
&= A \cap B && [\because \phi \cup x = x \text{ for any set } x] \\
&= \text{RHS}
\end{aligned}
$$

$\therefore$ LHS = RHS Proved.

Q2(ii)

For any sets A and B we have by De-morgan's laws

$$(A \cup B)' = A' \cap B', \quad (A \cap B)' = A' \cup B'$$

Also,

LHS $= A - (A - B)$

$= A \cap (A - B)'$

$= A \cap (A \cap B')'$

$= A \cap (A' \cup (B')')$ [By De-morgan's law]

$= A \cap (A' \cup B)$ $[\because (B')' = B]$

$= (A \cap A') \cup (A \cap B)$

$= \phi \cup (A \cap B)$ $[\because A \cap A' = \phi]$

$= A \cap B$ $[\because \phi \cup x = x,$ for any set $x]$

$=$ RHS

$\therefore$ LHS = RHS Proved.

Q2(iii)

LHS $= A \cap (A \cup B')$

$= A \cap (A' \cap B')$ [By De-morgan's law]

$= (A \cap A') \cap B'$ [By associative law]

$= \phi \cap B'$ $[\because A \cap A' = \phi]$

$= \phi$

$=$ RHS

$\therefore$ LHS = RHS Proved.

Q2(iv)

$$
\begin{aligned}
\text{RHS} &= A\Delta(A\cap B) \\
&= \big(A-(A\cap B)\big)\cup(A\cap B-A) && [\because E\Delta F=(E-F)\cup(F-E)] \\
&= \big(A\cap(A\cap B)'\big)\cup(A\cap B\cap A') && [\because E-F=E\cap F'] \\
&= \big(A\cap(A'\cup B')\big)\cup(A\cap A'\cap B) && \left[\begin{array}{l}\text{By De-morgan's law \&}\\ \text{associative law}\end{array}\right] \\
&= (A\cap A')\cup(A\cap B')\cup(\phi\cap B) && \left[\begin{array}{l}\because \cap \text{ distributes over } \cup \text{ and}\\ A\cap A'=\phi\end{array}\right] \\
&= \phi\cup(A\cap B')\cup\phi && [\because \phi\cap B=\phi] \\
&= A\cap B' && [\because \phi\cup x=x \text{ for any set } x] \\
&= A-B && [\because A\cap B'=A-B] \\
&= \text{LHS}
\end{aligned}
$$

$\therefore$ LHS = RHS Proved.

Q3

We have, ACB

To show: $C-B\subset C-A$

Let, $x\in C-B$

$\Rightarrow \quad x\in C$ and $x\notin B$

$\Rightarrow \quad x\in C$ and $x\notin A \qquad [\because A\subset B]$

$\Rightarrow \quad x\in C-A$

Thus, $x\in C-B\Rightarrow x\in C-A$

This is true for all $x\in C-B$

$\therefore \quad C-B\subset C-A$

Q4(i)

i. $(A\cup B)-B=(A-B)\cup(B-B)$

$$
\begin{aligned}
&= (A-B)\cup\phi \\
&= A-B
\end{aligned}
$$

Q4(ii)

$$\text{ii. } A-(A\cap B)=(A-A)\cap(A-B)$$
$$=\phi\cap(A-B)$$
$$=A-B$$

Q4(iii)

$$\text{iii.Let } x\in A-(A-B)\Leftrightarrow x\in A \text{ and } x\notin(A-B)$$
$$\Leftrightarrow x\in A \text{ and } x\in(A\cap B)$$
$$\Leftrightarrow x\in A\cap(A\cap B)$$
$$\Leftrightarrow x\in(A\cap B)$$
$$\therefore A-(A-B)=(A\cap B)$$

Q4(iv)

$$\text{iv.Let } x\in A\cup(B-A)\Rightarrow x\in A \text{ or } x\in(B-A)$$
$$\Rightarrow x\in A \text{ or } x\in B \text{ and } x\notin A$$
$$\Rightarrow x\in B$$
$$\Rightarrow x\in(A\cup B)\quad[\because B\subset(A\cup B)]$$

This is true for all $x\in A\cup(B - A)$

$\therefore A\cup(B - A)\subset(A\cup B).........(1)$

Conversely,

Let, $x\in(A\cup B)$

$\Rightarrow x\in A \text{ or } x\in B$

$\Rightarrow x\in A \text{ or } x\in(B - A)\quad[\because B\subset(B-A)]$

$\Rightarrow x\in A\cup(B - A)$

$\therefore (A\cup B)\subset A\cup(B - A)..........(2)$

From (1) and (2), we get

$A\cup(B - A)=(A\cup B)$

Q4(v)

v.Let $x \in A$.

Then either $x \in (A-B)$ or $x \in (A \cap B)$

$$\Rightarrow x \in (A-B) \cup (A \cap B)$$

$\therefore A \subset (A-B) \cup (A \cap B)$..........(1)

Conversely,

Let $x \in (A-B) \cup (A \cap B)$

$\Rightarrow x \in (A-B)$ or $x \in (A \cap B)$

$\Rightarrow x \in A$ and $x \notin B$ or $x \in A$ and $x \in B$

$\Rightarrow x \in A$

$\therefore (A-B) \cup (A \cap B) \subset A$..............(2)

$\therefore$ From (1) and (2), we get

$(A-B) \cup (A \cap B) = A$

Ex 1.8

Q1

$n(A \cup B) = 50,\ n(A) = 28,\ n(B) = 32,$ where $n(x)$ doesnotes the cardinal number of the set x.

We know that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\Rightarrow \quad 50 = 28 + 32 - n(A \cap B)$

$\Rightarrow \quad 50 = 60 - n(A \cap B)$

$\Rightarrow \quad n(A \cap B) = 60 - 50$

$= 10$

$\therefore\ n(A \cap B) = 10$

Q2

We have,

$n(P) = 40,\ n(P \cup Q) = 60,\ n(P \cap Q) = 10,$ to find $n(Q)$.

We know $n(P \cup Q) = n(P) + n(Q) - n(P \cap Q)$

$\Rightarrow \quad 60 = 40 + n(Q) - 10$

$\Rightarrow \quad 60 = 30 + n(Q)$

$\Rightarrow \quad n(Q) = 60 - 30$

$= 30$

Hence, Q has 30 elements.

Q3

Let $n(P)$ denote the number of teachers who teach Physics and $n(Q)$ denote the number of teachers who teach Mathematics.

We have,

$$n(P \text{ or } M) = 20$$
$$\text{i.e } n(P \cup M) = 20$$
$$n(M) = 12$$
$$\text{and} \quad n(P \cap M) = 4$$

To find: $n(P)$

We know $n(P \cup M) = n(P) + n(M) - n(P \cap M)$

$\Rightarrow \quad 20 = n(P) + 12 - 4$

$\Rightarrow \quad 20 = n(P) + 8$

$\Rightarrow \quad n(P) = 20 - 8$

$\qquad = 12$

$\therefore$ There are 12 Physics teachers.

Q4

Let,

$n(P)$ denote the total number of people

$n(C)$ denote the number of people who like coffee and

$n(T)$ denote the number of people who like tea.

Then, $n(P) = 70$

$n(C) = 37$

$n(T) = 52$

We are given that each person likes at least one of the two drinks, i.e., $P = C \cup T$

To find: $n(C \cap T)$

We know $n(P) = n(C) + n(T) - n(C \cap T)$

$\Rightarrow \quad 70 = 37 + 52 - n(C \cap T)$

$\Rightarrow \quad 70 = 89 - n(C \cap T)$

$\Rightarrow \quad n(C \cap T) = 89 - 70$

$= 19$

Hence, 19 people like both coffee and tea.

Q5(i)

$n(A) = 20,\ n(A \cup B) = 42$ and $n(A \cap B) = 4,$ to find: $n(B)$

We know $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\Rightarrow \quad 42 = 20 + n(B) - 4$

$\Rightarrow \quad 42 = 16 + n(B)$

$\Rightarrow \quad n(B) = 42 - 16$

$= 26$

$\therefore \ n(B) = 26$

Q5(ii)

To find: $n(A-B)$

We know that if A and B are disjoint sets, then

$$A \cap B = \phi$$

$$\therefore \quad n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$= n(A) + n(B) - n(\phi)$$

$$\Rightarrow \quad n(A \cup B) = n(A) + n(B) \qquad [\because n(\phi) = 0]$$

Now,

$$A = (A-B) \cup (A \cap B)$$

i.e A is the disjoint union of $A-B$ and $A \cap B$

$$\therefore \quad n(A) = n(A-B) \cup (A \cap B)$$

$$= n(A-B) + n(A \cap B) \qquad [\because A-B \text{ and } A \cap B \text{ are disjoint}]$$

$$\Rightarrow \quad 20 = n(A-B) + 4$$

$$\Rightarrow \quad n(A-B) = 20 - 4$$

$$= 16$$

$\therefore \; n(A-B) = 16$

Q5(iii)

To find: $B - A$

On a similar lines we have B is the disjoint union of $B-A$ and $A \cap B$

i.e $B = (B-A) \cup (A \cap B)$

$$\therefore \quad n(B) = n(B-A) + n(A \cap B)$$

$$\Rightarrow \quad 26 = n(B-A) + 4 \qquad [\text{using (i)}]$$

$$\Rightarrow \quad n(B-A) = 26 - 4$$

$$= 22$$

$\therefore \; n(B-A) = 22$

Q6

Let $n(P)$ denote the total percentage of Indians $n(O)$ denotes the percentage of Indians who like oranges, and $n(B)$ denotes the percentage of Indians who like bananas.

Then, $n(P) = 100$, $n(O) = 76$ and $n(B) = 62$

To find: $n(O \cap B)$

Now,

$$n(P) = n(O) + n(B) - n(O \cap B)$$

$$\Rightarrow \quad 100 = 76 + 62 - n(O \cap B)$$

$$\Rightarrow \quad 100 = 138 - n(O \cap B)$$

$$\Rightarrow \quad n(O \cap B) = 138 - 100$$

$$= 38$$

$\therefore$ 38% of Indians like both oranges and bananas.

Q7

(i)

Let,

$n(P)$ denote the total number of persons,

$n(H)$ denote the number of persons who speak Hindi and

$n(E)$ denote the number of persons who speak English.

Then,

$$n(P) = 950,\ n(H) = 750,\ n(E) = 460$$

To find: $n(H \cap E)$

$$n(P) = n(H) + n(E) - n(H \cap E)$$

$\Rightarrow \quad 950 = 750 + 460 - n(H \cap E)$

$\Rightarrow \quad 950 = 2110 - n(H \cap E)$

$\Rightarrow \quad n(H \cap E) = 2110 - 950$

$\quad\quad = 260$

Hence, 260 persons can speak both Hindi and English.

(ii)

Clearly H is the disjoint union of $H - E$ & $H \cap E$

i.e $H = (H - E) \cup (H \cap E)$

$\therefore \quad n(H) = n(H - E) + n(H \cap E)$ $\quad \left[\begin{array}{l} \because \text{ if } A \,\&\, B \text{ are disjoint then} \\ n(A \cup B) = n(A) + n(B) \end{array}\right]$

$\Rightarrow \quad 750 = n(H - E) + 260$

$\Rightarrow \quad n(H - E) = 750 - 260$

$\quad\quad = 490$

Hence, 490 persons can speak Hindi only.

(iii)

On a similar lines we have

$$E = (E - H) \cup (H \cap E)$$

i.e E is the disjoint union of $E - H$ & $H \cap E$

$\therefore \quad n(E) = n(E - H) + n(H \cap E)$

$\Rightarrow \quad 460 = n(E - H) + 260$

$\Rightarrow \quad n(E - H) = 460 - 260$

$\quad\quad = 200$

Hence, 200 persons can speak English only.

Q8

(i)

Let,

$n(P)$ denote the total number of persons,

$n(T)$ denote number of persons who drink tea and

$n(C)$ denote number of persons who drink coffee.

Then, $n(P) = 50$, $n(T - C) = 14$, $n(T) = 30$

To find: $n(T \cap C)$

Clearly T is the disjoint union of $T - C$ and $T \cap C$

$\therefore \quad T = (T - C) \cup (T \cap C)$

$\therefore \quad n(T) = n(T - C) + n(T \cap C)$

$\Rightarrow \quad 30 = 14 + n(T \cap C)$

$\Rightarrow \quad n(T \cap C) = 30 - 14$

$= 16$

Hence, 16 persons drink tea and coffee both.

(ii)

To find: $C - T$

We know $n(P) = n(C) + n(T) - n(T \cap C)$

$\Rightarrow \quad 50 = n(C) + 30 - 16$

$\Rightarrow \quad 50 = n(C) + 14$

$\Rightarrow \quad n(C) = 50 - 14$

$= 36$

New C is the disjoint union of $C - T$ and $T \cap C$

$\therefore \quad C = (C - T) \cup (C \cap T)$

$\Rightarrow \quad n(C) = n(C - T) + n(C \cap T)$

$\Rightarrow \quad 36 = n(C - T) + 16 \qquad [\because n(T \cap C) = n(C \cap T) = 16]$

$\Rightarrow \quad n(C - T) = 36 - 16$

$= 20$

Hence, 20 persons drink coffee but not tea.

Q9

(i)

Let $n(P)$ denote total number of people $n(H)$ denote number of people who read newspaper H $n(T)$ denote number of people who read newspaper T and $n(I)$ denote number of people who read newspaper I

Then, $n(P) = 60,\ n(H) = 25,\ n(T) = 26,\ n(I) = 26$

$$n(H \cap I) = 9,\ n(H \cap T) = 11,\ n(T \cap I) = 8,\ n(H \cap T \cap I) = 3$$

We need to find the number of people who read at least one of the newspaper, i.e., $n(H \text{ or } T \text{ or } I)$, i.e., $n(H \cup T \cup I)$ we know that if A, B, C are 3 sets, then,

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

$$\therefore\ n(H \cup T \cup I) = n(H) + n(T) + n(I) - n(H \cap T) - n(T \cap I) - n(H \cap I) + n(H \cap T \cap I)$$

$$= 25 + 26 + 26 - 9 - 11 - 8 + 3$$

$$= 25 + 52 - 28 + 3$$

$$= 25 + 52 - 25$$

$$= 52$$

Hence, 52 people read at least one of the newspaper.

(ii)

The venn diagram representing people reading newspapers H, T and I is shown above.

The shaded region shows the number of people who read newspaper H only, newspaper T only and newspaer I only respectively.

The number of people who read newspaper H only equals

$$25 - (8 + 3 + 6)$$

$$= 25 - 17$$

$$= 8$$

The number of people who read newspaper T only

$$= 26 - (8 + 3 + 5)$$

$$= 26 - 16$$

$$= 10$$

And, the number of people who read newspaper I only

$$= 26 - (6 + 3 + 5)$$

$$= 26 - 14$$

$$= 12$$

Hence, the number of people, who read exactly one newspaper $= 8 + 10 + 12 = 30$.

Q10

Let,

$n(P)$ denote total number of members,

$n(B)$ denote number of members in the basket ball team

$n(H)$ denote number of members in the hockey team and

$n(F)$ denote number of members in the football team.

Then, $n(B) = 21$, $n(H) = 26$, and $n(F) = 29$

Also, $n(H \cap B) = 14$, $n(H \cap F) = 15$, $n(F \cap B) = 12$, $n(H \cap B \cap F) = 8$

Now,

$$P = B \cup H \cup F$$

$$\begin{aligned}
\therefore \quad n(P) &= n(B \cup H \cup F) \\
&= n(B) + n(H) + n(F) - n(B \cap H) - n(H \cap F) - n(B \cap F) + n(B \cap H \cap F) \\
\Rightarrow \quad n(P) &= 21 + 26 + 29 - 14 - 15 - 12 + 8 \\
&= 76 - 41 + 8 \\
&= 43
\end{aligned}$$

Hence, there are 43 members in all.

Q11

Let,

$n(P)$ denote the total number of people,

$n(H)$ the number of people who speak Hindi and

$n(B)$ the number of people who speak Bengali.

Then, $n(P) = 1000,\ n(H) = 750,\ n(B) = 400$

We have $P = (H \cup B)$

$$\begin{aligned}
\therefore \quad n(P) &= n(H \cup B) \\
&= n(H) + n(B) - n(H \cap B) \\
\Rightarrow \quad 1000 &= 750 + 400 - n(H \cap B) \\
\Rightarrow \quad 1000 &= 1150 - n(H \cap B) \\
\Rightarrow \quad n(H \cap B) &= 1150 - 1000 \\
&= 150
\end{aligned}$$

Hence, 150 people can speak both Hindi and Bengali now $H = (H - B) \cup (H \cap B)$, the union being disjoint

$$\begin{aligned}
\therefore \quad n(H) &= n(H - B) + n(H \cap B) \\
\Rightarrow \quad 750 &= n(H - B) + 150 \\
\Rightarrow \quad n(H - B) &= 750 - 150 \\
&= 600
\end{aligned}$$

Hence, 600 people can speak Hindi only

On a similar lines we have $B = (B - H) \cup (H \cap B)$

$$\begin{aligned}
\Rightarrow \quad n(B) &= n(B - H) + n(H \cap B) \\
\Rightarrow \quad 400 &= n(B - H) + 150 \\
\Rightarrow \quad n(B - H) &= 400 - 150 \\
&= 250
\end{aligned}$$

Hence, 250 people can speak Bengali only.

Q12

Let,

$n(P)$ denote the total number of television vievers,

$n(F)$ be the number of people who watch football,

$n(H)$ be the number of people who watch hockey and

$n(B)$ be the number of people who watch basket ball.

Then, $n(P) = 500$, $n(F) = 285$, $n(H) = 195$, $n(B) = 115$, $n(F \cap B) = 45$, $n(F \cap H) = 70$, $n(H \cap B) = 50$ and $n(F \cup H \cup B) = 50$

Now,

$$n((F \cup H \cup B')) = n(P) - n(F \cup H \cup B)$$

$$\Rightarrow \quad 50 = 500 - \{n(F) + n(H) + n(B) - n(F \cap H) - n(H \cap B) - n(F \cap B) + n(F \cap H \cap B)\}$$

$$\Rightarrow \quad 50 = 500 - \{285 + 195 + 115 - 70 - 50 - 45 + n(F \cap H \cap B)\}$$

$$\Rightarrow \quad 50 = 500 - 430 - n(F \cap H \cap B)$$

$$\Rightarrow \quad 50 = 70 - n(F \cap H \cap B)$$

$$\Rightarrow \quad n(F \cap H \cap B) = 70 - 50 = 20$$

Hence, 20 people watch all the 3 games

Number of people who watch only football

$$= 285 - (50 + 20 + 25)$$
$$= 285 - 95$$
$$= 190$$

Number of people who watch only hockey

$$= 195 - (50 + 20 + 30)$$
$$= 195 - 100$$
$$= 95$$

And, number of people who watch only basket ball

$$= 115 - (25 + 20 + 30)$$
$$= 115 - 75$$
$$= 40$$

Number of people who watch exactly one of the three games

$= $ number of people who watch either football only or hockey only or basket ball only

$= 190 + 95 + 40 \qquad [\because \text{ they are pairwise disjoint}]$

$= 325$

Hence, 325 people watch exactly one of the three games.

Q13

(i)

Let $n(P)$ denote total number of persons

$n(A)$ denote number of people who read magazine A

$n(B)$ denote number of people who read magazine B

and $n(C)$ denote number of people who read magazine C

Then, $n(P) = 100,\ n(A) = 28,\ n(B) = 30,\ n(C) = 42,\ n(A \cap B) = 8,$

$n(A \cap C) = 10, n(B \cap C) = 5,\ n(A \cap B \cap C) = 3$

Now,

$$\begin{aligned} n(A \cup B \cup C) &= n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C) \\ &= 28 + 30 + 42 - 8 - 10 - 5 + 3 \\ &= 100 - 23 + 3 \\ &= 100 - 20 \\ &= 80 \end{aligned}$$

$\therefore$ Number of people who read none of the three magazines

$= n(A \cup B \cup C)'$

$= n(P) - n(A \cup B \cup C)$

$= 100 - 80$

$= 20$

Hence, 20 people read none of the three magazines.

(ii)

$$\begin{aligned} n(C\ only) &= 42 - (7 + 3 + 2) \\ &= 42 - 12 \\ &= 30 \end{aligned}$$

Q14

(i)

Let $n(P)$ denote total number of students

$n(E)$ denote number of students studying English language

$n(H)$ denote number of students studying Hindi language and

$n(S)$ denote number of students studying Sanskrit language

Then, $n(P) = 100,\ n(E - H) = 23,\ n(E \cap S) = 8,\ n(E) = 26,\ n(S) = 48,$
$n(S \cap H) = 8,\ n\{(E \cup H \cup S)'\} = 24$

Number of students studying English only = 18

We have,

$$n\{(E \cup H \cup S)'\} = 24$$
$$\Rightarrow \quad n(P) - n(E \cup H \cup S) = 24$$
$$\Rightarrow \quad 100 - 24 = n(E \cup H \cup S)$$
$$\Rightarrow \quad n(E \cup H \cup S) = 76$$

We have $n(E \cup H \cup S) = n(E) + n(H) + n(S) - n(E \cap H) - n(H \cap S) - n(E \cap S)$
$+ n(E \cap H \cap S)$

$$\Rightarrow \quad 76 = 26 + n(H) + 48 - 3 - 8 - 8 + 3$$
$$\Rightarrow \quad 76 = 26 + n(H) + 48 - 16$$
$$\Rightarrow \quad 76 = 26 + 32 + n(H)$$
$$\Rightarrow \quad n(H) = 76 - 58$$
$$= 18$$

$\therefore$ 18 students were studying Hindi.

(ii)

From (i) we have $n(E \cap H) = 3$

$\therefore$ 3 students were studying both English and Hindi.

Q15

Let $n(P_1)$ be the number of persons liking product P_1
$n(P_2)$ be the number of persons liking product P_2
and $n(P_3)$ be the number of persons liking product P_3

Then, $n(P_1) = 21,\ n(P_2) = 26,\ n(P_3) = 29,\ n(P_1 \cap P_2) = 14,$
$n(P_1 \cap P_3) = 12,\ n(P_2 \cap P_3) = 14,\ n(P_1 \cap P_2 \cap P_3) = 8$

$\therefore$ Number of people liking product P_3 only

$$= 29 - (4 + 8 + 6)$$
$$= 29 - 18$$
$$= 11$$

Hence, 11 persons liked product P_3 only.