Ex 4.1

Q1

(i) $\frac{9\pi}{5}$

We have,

$$\pi \text{ radians} = 180°$$

$$1^c = \left\{\frac{180}{\pi}\right\}^0$$

Now,

$$\left(\frac{9\pi}{5} \times \frac{180}{\pi}\right)^0$$

$$= 324°$$

(ii) $\frac{-5\pi}{6}$

We have,

$$\pi \text{ radians} = 180°$$

$$1^c = \left(\frac{180}{\pi}\right)^0$$

Now,

$$\left(\frac{-5\pi}{6}\right)^c = \left(\frac{-5\pi}{6} \times \frac{180}{\pi}\right)^0 = -150°$$

(iii) $\left(\frac{18\pi}{5}\right)^c$

We have,

$$\pi \text{ radians} = 180°$$

$$1^c = \left(\frac{180}{\pi}\right)^0$$

Now,

$$\left(\frac{18\pi}{5}\right)^c = \left(\frac{18\pi}{5} \times \frac{180}{\pi}\right)^0$$

$$= 648°$$

(iv) We have,

$$\pi \text{ radians} = 180°$$

$$1^c = \left(\frac{180}{\pi}\right)^0$$

Now,

$$(-3)^c = \left(-3 \times \frac{180}{\pi}\right)^0$$

$$= \left(\frac{180}{22} \times 7 \times -3\right)^0$$

$$= \left(-171\frac{9}{11}\right)^0$$

$$= -171^0\left(\frac{9}{11} \times 60\right)^1$$

$$= -171^0 49^1 5^{11}$$

(v) We have,

$$\pi \text{ radians} = 180^0$$

$$1^c = \left(\frac{180}{\pi}\right)^0$$

Now,

$$(11)^c = \left(11 \times \frac{180}{\pi}\right)^0$$

$$= \left(11 \times 180 \times \frac{7}{22}\right)^0$$

$$= 630^0$$

(vi) We have,

$$\pi \text{ radians} = 180^0$$

$$1^e = \left(\frac{180}{\pi}\right)^0$$

Now,

$$1^e = \left(1 \times \frac{180}{\pi}\right)^0$$

$$= 1 \times \frac{180 \times 7}{22}$$

$$= 57^0\left(\frac{3}{11} \times 60\right)$$

$$= 57^0 16^1\left(\frac{4}{11} \times 60\right)^{11}$$

$$= 57^0 16^1 21^{11}$$

Q2

(i) 300°

We have,

$$180° = \pi^c$$

$$\therefore \quad 1° = \left(\frac{\pi}{180}\right)^c$$

Now,

$$300° = 300 \times \frac{\pi}{180} = \frac{5\pi}{3}$$

(ii) 35°

We have,

$$180° = \pi^c$$

$$\therefore \quad 1° = \left(\frac{\pi}{180}\right)^c$$

Now,

$$35° = 35 \times \frac{\pi}{180} = \frac{7\pi}{36}$$

(iii) −56°

We have,

$$180° = \pi^c$$

$$\therefore \quad 1° = \left(\frac{\pi}{180}\right)^c$$

Now,

$$-56° = -56 \times \frac{\pi}{180} = \frac{-14\pi}{45}$$

(iv) 135°

We have,

$$180° = \pi^c$$

$$\therefore \quad 1° = \left(\frac{\pi}{180}\right)^c$$

Now,

$$135° = 135 \times \frac{\pi}{180} = \frac{3\pi}{4}$$

(v) −300°

We have,

$$180° = \pi^c$$

$$\therefore \quad 1° = \left(\frac{\pi}{180}\right)^c$$

Now,

$$-300° = -300 \times \frac{\pi}{180} = \frac{-5\pi}{3}$$

(vi) $7°30^1$

We have,

$$180° = \pi^c$$

$$\therefore \quad 1° = \left(\frac{\pi}{180}\right)^c$$

$$7°30^1 = \left(7 \times \frac{\pi}{180}\right)^c \times \left(\frac{30}{60}\right)^0$$

$$= \left(7\frac{1}{2}\right)^0 \times \left(\frac{\pi}{180}\right)^c$$

$$= \left(\frac{15}{2} \times \frac{\pi}{180}\right)^c$$

$$= \frac{\pi}{24}$$

(vii) $125°30^1$

We have,

$$180° = \pi^c$$

$$\therefore \quad 1° = \left(\frac{\pi}{180}\right)^c$$

$$125°30^1 = 125° \left(\frac{30}{60}\right)^0$$

$$= \left(125\frac{1}{2}\right)^0$$

$$= \left(\frac{251}{2} \times \frac{\pi}{180}\right)^c = \frac{251\pi}{360}$$

(viii) $-47°30^1$

We have,

$$180° = \pi^c$$

$$\therefore \quad 1° = \left(\frac{\pi}{180}\right)^c$$

$$-47°30^1 = -47° \left(\frac{30}{60}\right)^0$$

$$= \left(-47\frac{1}{2}\right)^C$$

$$= \left(\frac{-95}{2}\right)^0$$

$$= \left(\frac{-95}{2} \times \frac{\pi}{180}\right)^c$$

$$= \frac{-19\pi}{72}$$

Q3

Let θ_1 and θ_2 be two acute angles of a right angled triangle.

$\therefore$ difference of acute angles

$$\theta_1 - \theta_2 = \frac{2\pi}{5} \text{ radians}$$

$\because$ in a right angled triangle,

$$\theta_1 + \theta_2 = \frac{\pi}{2}$$

$$\theta_1 - \theta_2 = \frac{2\pi}{5} \qquad \text{---(i)}$$

$$\theta_1 + \theta_2 = \frac{\pi}{2} \qquad \text{---(ii)}$$

On solving

$$2\theta_1 = \frac{2\pi}{5} + \frac{\pi}{2}$$

$$\theta_1 = \frac{9\pi}{20}$$

From equation (ii)

$$\theta_2 = \frac{\pi}{20}$$

So angles in degrees

$$\theta_1 = \frac{9\pi}{20} \times \frac{180}{\pi} = 81°$$

and $\theta_2 = \frac{\pi}{20} \times \frac{180}{\pi} = 9°$

Q4

Let θ_1 and θ_2 and θ_3 be the angle or triangle.

$$\theta_1 = \frac{2}{3}x \text{ gradiants}$$

$$\theta_2 = \frac{3}{2}x \text{ degrees and}$$

$$\theta_3 = \frac{\pi x}{75}x \text{ radians}$$

Now,

we have to express all the angles in degrees

$$\therefore \quad \theta_1 = \left(\frac{3}{2}x \times \frac{90}{100}\right)^0$$

$$= \frac{3}{5}x \qquad \left[1g = \frac{90}{100} \text{ degree}\right]$$

$$\theta_2 = \frac{3}{2}x^0$$

$$\theta_2 = \frac{\pi x}{75} \times \frac{180}{\pi} = \frac{12x}{5}$$

By angleslam property,

$$\theta_1 + \theta_2 + \theta_3 = 180°$$

$$\therefore \quad \frac{3}{5}x° + \frac{3}{2}x^0 + \frac{12x}{5} = 180°$$

$$\Rightarrow \quad \frac{9}{2}x^0 = 180^0$$

$$\Rightarrow \quad x = 40^0$$

$$\therefore \quad \theta_1 = 24^0, \ \theta_2 = 60^0, \ \theta_3 = 96^0$$

Q5

General formula for interior angles of polygon with n side

$$= \left(\frac{2n-4}{n}\right) \times 90^0$$

(i) Pentagon has 5 sides

$\therefore$ magnitude of the interior angle

$$= \frac{2 \times 5 - 4}{5} \times 90^0$$

$$= \frac{6}{5} \times 90 = \quad 180^0$$

Now,

$$\because \quad 1^c = \frac{180}{\pi}$$

And each angle of Pentagon

$$= \frac{2 \times 5 - 4}{5} \times \frac{\pi}{2}$$

$$= \left(\frac{3\pi}{5}\right)^c$$

$$\therefore \quad 108°, \ \left(\frac{3\pi}{5}\right)^c$$

(ii) Octagon

$$n = 8$$

$$\therefore \text{ each angle} = \frac{2 \times 8 - 4}{8} \times 90^0$$

$$= 135^0$$

Again,

$$\text{each angle} = \frac{2 \times 8 - 4}{8} \times \frac{\pi}{2}$$

$$= \left(\frac{3\pi}{4}\right)^c$$

$$\therefore \quad 135^0, \ \left(\frac{3\pi}{4}\right)^c$$

(iii) Heptagon

$$n = 7$$

$$\therefore \text{ each angle} = \frac{2 \times 7 - 4}{7} \times 90^0$$

$$= \frac{10}{7} \times 90^0$$

$$= \frac{900^0}{7}$$

$$= 128^0 34^I 17^{II}$$

Again,

$$\text{each angle} = \frac{2 \times 7 - 4}{7} \times \frac{\pi}{2}$$

$$= \frac{10}{7} \times \frac{\pi}{2}$$

$$= \left(\frac{5\pi}{7}\right)^c$$

$\therefore\ 128^0 34^I 17^{II},\ \left(\frac{5\pi}{7}\right)^c$

(iv) Duodecagon

$n = 12$

$$\therefore\ \text{each angle} = \frac{2 \times 12 - 4}{12} \times 90^0$$

$$= \frac{20}{12} \times 90^0$$

$$= 150^0$$

Agian,

$$\text{each angle} = \frac{2 \times 12 - 4}{12} \times \frac{\pi}{2}$$

$$= \frac{20}{12} \times \frac{\pi}{2}$$

$$= \left(\frac{5\pi}{6}\right)^c$$

$\therefore\ 150^0,\ \left(\frac{5\pi}{6}\right)^c$

Q6

Let the angles in degrees be $a-3d,\ a-d,\ a+d,\ a+3d$

Then,

$$\text{sum of the angles} = 360^0$$

$$\Rightarrow \quad 4a = 360^0$$

$$a = 90^0$$

Also,

$$\text{greatest angle} = 120^0$$

$$a+3d = 120^0$$

$$\Rightarrow \quad 90^0 + 3d = 120^0$$

$$\Rightarrow \quad 3d = 30^0$$

$$\Rightarrow \quad d = 10^0$$

Hence, angles in degrees

$$60^0,\ 80^0,\ 100^0,\ 120^0$$

and in radians, we know that

$$1^0 = \left(\frac{\pi}{180}\right)^c$$

$$\therefore \quad 60 \times \frac{\pi}{180} = \frac{\pi}{3},\ 80 \times \frac{\pi}{180} = \frac{4\pi}{9},$$

$$100 \times \frac{\pi}{180} = \frac{5\pi}{9} \text{ and } 120 \times \frac{\pi}{180} = \frac{2\pi}{3}$$

$$\therefore \ \frac{\pi}{3},\ \frac{4\pi}{9},\ \frac{5\pi}{9},\ \frac{2\pi}{3}$$

Q7

Let A, B & C be the angles of triangle ABC.
We are given that A, B & C are in A.P.

$\therefore$ Let $A = a - d$, $B = a$ and $C = a + d$

According to the question,

$$A + B + C = 180^0 \qquad \text{[By angle sum property]}$$

$$\therefore \quad a - d + a + a + d = 180^0$$

$$\Rightarrow \quad 3a = 180^0 \Rightarrow \quad a = 60^0 \qquad \text{---(i)}$$

Again,

$$\frac{\text{least angle}}{\text{mean angle}} = \frac{1}{120^0}$$

$$\Rightarrow \quad \frac{a - d}{a} = \frac{1}{120}$$

$$\Rightarrow \quad 119a = 120d$$

$$\Rightarrow \quad d = \frac{119a}{120}$$

$$\Rightarrow \quad d = \frac{119}{120} \times 60^0$$

$$= \left(\frac{119}{2}\right)^0$$

$$= \frac{119}{2} \times \frac{\pi}{180} = \frac{119\pi}{360} \text{ radians}$$

Now,

$$1^0 = \frac{\pi}{180} \text{ radians}$$

$$\therefore \quad B = a = 60^0 = \frac{\pi}{3} \text{ radians}$$

$$A = a - d = \frac{\pi}{3} - \frac{119\pi}{360} = \frac{\pi}{360} \text{ radians}$$

$$C = a + d = \frac{\pi}{3} + \frac{119\pi}{360} = \frac{239\pi}{360} \text{ radians.}$$

Q8

Let n & m be the number of sides in two regular polygon respectively.

We know that each angle of n-sided regular polygon is $\frac{(2n-4)}{n}$ right angles.

Now,

According to the question,

$$\frac{\left(\frac{2n-4}{n}\right)\times 90^0}{\left(\frac{2m-4}{m}\right)\times 90^0}=\frac{3}{2}$$

$$\Rightarrow \quad \frac{(2n-4)m}{(2m-4)n}=\frac{3}{2} \qquad \text{---(i)}$$

Also,

$$n = 2m \qquad \text{---(ii)} \qquad [\text{given}]$$

Put (ii) in (i), we get

$$\frac{(4m-4)m}{(2m-4)2m}=\frac{3}{2}$$

$$\Rightarrow \quad 4m-4 = 6m-12$$

$$\Rightarrow \quad 2m = 8$$

$$\therefore \quad m = 4$$

From (ii)

$$n = 2m$$
$$= 2\times 4 = 8$$

$\therefore \; n = 8, \; m = 4$

Q9

According to the question,

A, B & C are in A.P

$\therefore$ Let $A = a - d$, $B = a$ & $C = a + d$

So, $A + B + C = 180^0$ [By angle sum property]

$\Rightarrow \quad a - d + a + a + d = 180^0$

$\Rightarrow \quad 3a = 180^0 \Rightarrow a = 60^0 \quad$ ---(i)

Also,

greatest angle in 5 times the least

$\therefore \quad a + d = 5(a - d)$

$\Rightarrow \quad 4a = 6d$

$\Rightarrow \quad d = \frac{2}{3}a$

$\Rightarrow \quad d = \frac{2}{3} \times 60 = 40^0 \quad$ ---(ii)

$\therefore \quad A = a - d = 20^0$

$B = a \quad = 60^0$

$C = a + d = 100^0$

$\because \quad 1^0 = \left(\frac{\pi}{180^0}\right)$ radians

$\therefore \quad A = 20 \times \frac{\pi}{180} = \frac{\pi}{9}$

$B = 60 \times \frac{\pi}{180} = \frac{\pi}{3}$

$C = 100 \times \frac{\pi}{180} = \frac{5\pi}{9}$

Thus,

$A = \frac{\pi}{9}$, $B = \frac{\pi}{3}$, $C = \frac{5\pi}{9}$

Q10

Let n and m be the number of sides in two regular polygon respectively.

We know that each angle of n-sided regular polygon is

$$\left(\frac{2n-4}{n}\right) \text{ right angles.}$$

Now,

According to the question

$$\frac{n}{m} = \frac{5}{4} \Rightarrow \frac{5m}{4} = n \qquad \text{---(i)}$$

Also,

$$\left(\frac{2n-4}{n}\right)90^0 - \left(\frac{2m-4}{m}\right)90^0 = 9^0$$

$$\Rightarrow \quad \frac{(2n-4)m-(2m-4)n}{mn} = \left(\frac{1}{10}\right)^0 \qquad \text{---(ii)}$$

From (i) and (ii), we get

$$\frac{\left(2\times\frac{5}{4}m-4\right)m-(2m-4)\frac{5}{4}m}{\frac{5}{4}m^2} = \frac{1}{10}$$

$$\Rightarrow \quad \frac{(10m-16)-(10m-20)}{5m} = \frac{1}{10}$$

$$\Rightarrow \quad \frac{4}{m} = \frac{1}{2} \Rightarrow m = 8$$

From (i)

$$n = \frac{5}{4}m = 10$$

Thus,

$$n = 10,\ m = 8$$

Q11

Let AB be the rail road

$$\angle AOB = 25^0 = 25 \times \frac{\pi}{180} = \left(\frac{5\pi}{36}\right)^C \qquad \left[\because 1^0 = \left(\frac{\pi}{180}\right)^C\right]$$

We know that

$$\theta = \frac{\text{arc}}{\text{radius}}$$

$$\Rightarrow \quad \angle AOB = \frac{AB}{OA}$$

$$\Rightarrow \quad \frac{5\pi}{36} = \frac{40}{r}$$

$$\Rightarrow \quad r = \frac{40 \times 36}{5\pi}$$

$$\Rightarrow \quad r = \frac{288}{\pi} \text{ meter} \qquad \left[\because \pi = \frac{22}{7}\right]$$

$$\Rightarrow \quad r = 91.64 \text{ meter}$$

Q12

Let, $\angle AOB = \theta = 1'$

$AB = \text{arc } AB = l$

$OA = OB = r = 5280m$

$$\because \quad 1^0 = 60'$$

$$\Rightarrow \quad 1' = \left(\frac{1}{60}\right)^0 = \left(\frac{1}{60} \times \frac{\pi}{180}\right)^c \qquad \left[\because 1^0 = \left(\frac{\pi}{180}\right)^c\right]$$

Now,

We know that

$$\theta = \frac{\text{arc}}{\text{radius}}$$

$$\Rightarrow \quad \left(\frac{\pi}{180 \times 60}\right)^c = \frac{l}{5280}$$

$$\Rightarrow \quad l = \frac{5280\pi}{180 \times 60} = 1.5365 \text{ m} \qquad \left[\because \pi = \frac{22}{7}\right]$$

Q13

Since A wheel makes 360 revoulation in 1 minutes

$\therefore$ Wheel will make $\frac{360}{60}$ revolution in 1 secons

That is, 6 revoultin in1 second

Now,

In one revolutin the wheel makes 360^0 angle

$\therefore$ In 6 revoulution the wheel will make $360^0 \times 6$ angles

$= 2160^0$

$\because \quad 1^0 = \left(\frac{\pi}{180}\right)^c$

$\therefore \quad 2160^0 = \left(\frac{2160}{180} \times \pi\right)^c$

$= 12\pi$

Q14

(i) We have,

$$OA = \text{length of pendulum} = 75 \text{ cm} = 0.75 \text{ m}$$

$$AB = \text{arc } AB = 10 \text{ cm} = 0.1 \text{ m}$$

Also,

$$\theta = \frac{\text{arc}}{\text{radius}} \quad \text{---(i)}$$

$$\Rightarrow \quad \theta = \frac{0.1}{0.75} = \left(\frac{2}{15}\right)^c$$

$$\theta = \frac{2}{15} \text{ radian}$$

(ii)

$$OA = 75 \text{ cm} = 0.75 \text{ m}$$

$$AB = 15 \text{ cm} = 0.15 \text{ m}$$

From (A)

$$\theta = \frac{0.15}{0.75} = \frac{1}{5} \text{ radian}$$

$$\theta = \frac{1}{5} \text{ radian}$$

(iii)

$$OA = 75 \text{ cm} = 0.75 \text{ m}$$

$$AB = 21 \text{ cm} = 0.21 \text{ m}$$

From (A)

$$\theta = \frac{0.21}{0.75} = \frac{7}{25}$$

$$\therefore \quad \theta = \frac{7}{25} \text{ radian}$$

Q15

We have,

$OA = OB =$ radius of circle $= 30$ cm $= 0.3$ m

$AB =$ chord $AB = 30$ cm $= 0.3$ m

Arc $AB = \overset{\frown}{AB} = l$ (say)

Now,

ΔAOB is equilateral triangle as $OA = OB = AB = 30$ cm

$\therefore \quad \angle AOB = 60^0 = \dfrac{\pi}{3}$ radian.

$\therefore \quad \theta = \dfrac{\text{arc}}{\text{radius}}$

$\Rightarrow \quad \dfrac{\pi}{3} = \dfrac{l}{0.3}$

$\Rightarrow \quad l = \dfrac{0.3}{3}\pi = 0.1\pi$ m

$\therefore \; l =$ arc $AB = 10\pi$ cm.

Q16

We have,

In circular track,

$OA = OB = r = 150$ m

$\angle AOB = \theta =$ angle the train turns in 10 seconds

Speed of train $= 66$ km/hr

$= \dfrac{66 \times 1000}{60 \times 60}$ m/sec

$= \dfrac{110}{6}$ m/sec

$\therefore$ Train will travel in 10 sec $= \dfrac{110}{6} \times 10 = \dfrac{1100}{6}$ m

$\therefore \quad$ arc $AB = \dfrac{1100}{6}$ m

Thus,

$\theta = \dfrac{\text{arc}}{\text{radius}} = \dfrac{1100}{6 \times 1500} = \dfrac{11}{90}$ radian

$\therefore$ The train will turn by $\left(\dfrac{11}{90}\right)^c$ angle in 10 sec.

Q17

Let, r be the distance, at which coin in placed. So that it completely conceals the full moon.

Let, E be the eye of the observer.

Now,

$$\theta = 31' = \left(\frac{31}{60}\right)^0 \qquad \left[\because 60' = 1^0\right]$$

$$= \frac{31}{60} \times \left(\frac{\pi}{180}\right)^c \qquad \left[\because 1^0 = \left(\frac{\pi}{180}\right)^c\right]$$

Also,

$$\widehat{AB} = \text{arc } AB = 2 \text{ cm} = 0.02 \text{ m}.$$

Now,

$$\text{by } \theta = \frac{\text{arc}}{\text{radius}}$$

$$\frac{31\pi}{60 \times 180} = \frac{0.02}{r}$$

$$\Rightarrow \quad r = \frac{0.02 \times 60 \times 180}{31\pi}$$

$$= 2.217 \text{ m} \qquad \left[\because \pi = \frac{22}{7}\right]$$

Thus,

The coin should be placed at a distance of 2.217 m from the eye.

Q18

Let, E be the eye of the observer and S be the sum.

Now,

$$\angle ACB = \theta = 32'$$

$$= \left(\frac{32}{60}\right)^0$$

$$= \left(\frac{32}{60} \times \frac{\pi}{180}\right)^c$$

$$\because \quad \theta = \frac{\text{arc}}{\text{radius}}$$

$$\Rightarrow \quad \frac{32}{60} \times \frac{\pi}{180} = \frac{AB}{91 \times 10^6} \text{ km}$$

$$\Rightarrow \quad AB = \frac{91 \times 10^6 \times 32 \times \pi}{60 \times 180}$$

$$= 8.474074 \times 10^5 \text{ km}$$

$$= 847407.4 \text{ km}$$

$\therefore$ Distance of sun is 847407.4 km.

Q19

Let, C_1 & C_2 are two circles with same Arc length l.
That is $AB = CD = l$

Let, θ_1 adn θ_2 are two angles subtended by arc AB and CD on respective circles.

Let, $OA = OB = r$ [radius of C_1]
and $OC = O\Delta = R$ [radius of C_2]

Also,

$$\theta_1 = 65^0 = \left(\frac{65\pi}{180}\right)^c$$

and $$\theta_2 = 110^0 = \left(\frac{110\pi}{180}\right)^c$$

We know

$$\theta = \frac{\text{arc}}{\text{radius}}$$

$\therefore$ For C_1

$$\theta_1 = \frac{AB}{r}$$

$$\Rightarrow \quad \theta_1 = \frac{l}{r}$$

$$\Rightarrow \quad r = \frac{l}{\theta_1} \qquad \text{---(i)}$$

For C_2

$$\theta_2 = \frac{CD}{R}$$

$$\Rightarrow \quad \theta_2 = \frac{l}{R}$$

$$\Rightarrow \quad R = \frac{l}{\theta_2} \qquad \text{---(i)}$$

From (i) and (ii)

$$\frac{r}{R} = \frac{\frac{l}{\theta_1}}{\frac{l}{\theta_2}} = \frac{\theta_2}{\theta_1} = \frac{\frac{110\pi}{180}}{\frac{65\pi}{180}} = \frac{22}{13}$$

$\therefore \quad r : R = 22 : 13$

Q20

Let, AB = arc AB = 22 cm

$OA = OB = r = 100$ cm

Let θ bet the angle subtanded by arc AB at centre O.

$$\therefore \quad \text{by } \theta = \frac{\text{arc}}{\text{radius}}$$

$$\Rightarrow \quad \theta = \frac{22}{100} \text{ radian}$$

$$\therefore \quad \theta = \left(\frac{22}{100} \times \frac{180}{\pi}\right)^0 \qquad \left[\because 1 \text{ radian} = \left(\frac{180}{\pi}\right)^0\right]$$

$$= 12.6^0$$

$$= 12^0 36^1 \qquad \left[\because 1^0 = 60^1\right]$$

$$\therefore \quad \theta = 12^0 36'$$