

Ex 24.3

Q1

We will use equation in diameter form.

$$(x-2)(x+2) + (y+3)(y-4) = 0$$

$$x^2 - 4 + y^2 - y - 12 = 0$$

$$x^2 + y^2 - y - 16 = 0 \dots\dots\dots (1)$$

$$2g=0, 2f=-1 \dots\dots\dots [\text{From (1)}]$$

$$g = 0, f = -\frac{1}{2}$$

Centre of the circle is $(0, \frac{1}{2})$

$$r = \sqrt{0 + \frac{1}{4} + 16}$$

$$r = \frac{\sqrt{65}}{2}$$

Q2

Centre of the circles

$$x^2 + y^2 + 6x - 14y - 1 = 0$$

is $(-3, 7)$

$$\text{and } x^2 + y^2 - 4x + 10y - 2 = 0$$

is $(2, -5)$

Equation of the circle is

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$$

$$\Rightarrow (x + 3)(x - 2) + (y - 7)(y + 5) = 0$$

$$\Rightarrow x^2 + 3x - 2x - 6 + y^2 - 7y + 5y - 35 = 0$$

$$\Rightarrow x^2 + y^2 + x - 2y - 41 = 0$$

Q3

Let the sides AB , BC , CD and DA of the square $ABCD$ be represented by the equations
 $y = 3$, $x = 6$, $y = 6$ and $x = 9$ respectively.

Then, coordinates are

$$A(6, 3), \quad B(9, 3), \quad C(9, 6) \text{ and } D(6, 6).$$

The equation of the circle with diagonal AC

$$(x - 6)(x - 9) + (y - 3)(y - 6) = 0$$

$$\Rightarrow x^2 - 6x - 9x + 54 + y^2 - 3y - 6y + 18 = 0$$

$$\Rightarrow x^2 + y^2 - 15x - 9y + 72 = 0$$

The equation of the circle with diagonal BD as diameter is

$$(x - 9)(x - 6) + (y - 3)(y - 6) = 0$$

$$\Rightarrow x^2 - 9x - 6x + 54 + y^2 - 3y - 6y + 18 = 0$$

$$\Rightarrow x^2 + y^2 - 15x - 9y + 72 = 0$$

$$x^2 + y^2 - 15x - 9y + 72 = 0$$

Q4

The given equation are

$$x - 3y = 4 \dots\dots\dots (i)$$

$$3x + y = 22 \dots\dots\dots (ii)$$

$$x - 3y = 14 \dots\dots\dots (iii)$$

$$3x + y = 62 \dots\dots\dots (iv)$$

Let A, B, C & D are the points of intersection of the lines (i)&(ii), (i)&(iii), (iii)&(iv) and (iv)&(i)

$$\therefore A = (7, 1), \quad B = (8, -2), \quad C = (20, 2) \quad \& \quad D = (19, 5)$$

AC will be the diameter of the circle

so,

the equation of circle is

$$(x - 7)(x - 20) - (y - 1)(y - 2) = 0$$

$$\Rightarrow x^2 + y^2 - 27x - 3y + 142 = 0$$

Q5

The line $3x + 4y = 12$
will meet the axis at $A(0, 3)$ & $B(4, 0)$

Since the circle passes through origin & A and B
 $\therefore AB$ is a diameter

Thus the equation of circle is

$$(x - 0)(x - 4) + (y - 3)(y - 0) = 0$$

$$\Rightarrow x^2 + y^2 - 4x - 3y = 0$$

Q6

Since the circle is passes through origin and cut intercept a and b on
 x -axis & y -axis

\therefore Coordinate of circle $A = (0, b)$ and $B = (a, 0)$
 AB is the diameter of circle

\therefore The equation of circle is

$$(x - a)(x - 0) + (y - 0)(y - b) = 0$$

$$\Rightarrow x^2 + y^2 \pm ax \pm by = 0$$

Q7

Equation of circle in diameter form is,

$$(x+4)(x-12) + (y-3)(y+1) = 0$$

$$x^2 - 8x - 48 + y^2 - 2y - 3 = 0$$

$$x^2 - 8x - 2y + y^2 - 51 = 0 \dots\dots\dots (1)$$

To find y-intercept, put $x=0$ in (1),

$$y^2 - 2y - 51 = 0$$

$$y = \frac{2 \pm \sqrt{4 + 204}}{2}$$

$$y = \frac{2 \pm \sqrt{208}}{2}$$

$$y \text{ intercepts are } 1 \pm 4\sqrt{13}$$

Q8

The given equations are

$$x^2 + 2ax - b^2 = 0 \dots\dots (i)$$

$$x^2 + 2px - q^2 = 0 \dots\dots (ii)$$

We have, the roots of (i) will give the abscissae and the roots of (ii) will give the ordinate of A & B respectively.

Now roots of (i)

$$x = \frac{-2a \pm \sqrt{4a^2 + 4b^2}}{2} = -a \pm \sqrt{a^2 + b^2}$$

Roots of (ii)

$$x = \frac{-2p \pm \sqrt{4p^2 + 4q^2}}{2} = -p \pm \sqrt{p^2 + q^2}$$

$$\text{Coordinates of A} = \left(-a + \sqrt{a^2 + b^2}, -p + \sqrt{p^2 + q^2} \right)$$

$$B = \left(-a - \sqrt{a^2 + b^2}, -p - \sqrt{p^2 + q^2} \right)$$

so, the equation of circle is

$$\left(x + a - \sqrt{a^2 + b^2} \right) \left(x + a + \sqrt{a^2 + b^2} \right) + \left(y + p - \sqrt{p^2 + q^2} \right) \left(y + p + \sqrt{p^2 + q^2} \right) = 0$$

$$\Rightarrow x^2 + y^2 + 2ax + 2py - (a^2 + b^2 + p^2 + q^2) + a^2 + p^2 = 0$$

$$\Rightarrow x^2 + y^2 + 2ax + 2py - (b^2 + q^2) = 0$$

The radius is

$$\begin{aligned} r &= \sqrt{g^2 + f^2 - c} \\ &= \sqrt{a^2 + p^2 - (b^2 + q^2)} \\ &= \sqrt{a^2 + b^2 + p^2 + q^2} \end{aligned}$$

Q9

Here AB and AD are taken as x – axis and y – axis respectively.

Since $ABCD$ is a square thus the coordinate of

$$A = (0, 0)$$

$$B = (a, 0) \quad C = (a, a)$$

$$D = (0, a)$$

$\therefore BD$ is a diameter

so, the equation of circle is

$$(x - a)(x - 0) + (y - 0)(y - a) = 0$$

$$\Rightarrow x^2 + y^2 - ax - ay = 0$$

so,

$$x^2 + y^2 - a(x + y) = 0$$

Q10

The given equation of line & circle in

$$2x - y + 6 = 0 \dots\dots\dots (i)$$

$$x^2 + y^2 - 2y - 9 = 0 \dots\dots (ii)$$

The point of intersection of (i) & (ii) is

$$x^2 + (2x + 6)^2 - 2(2x + 6) - 9 = 0$$

$$\Rightarrow x^2 + 4x^2 + 24x + 36 - 4x - 12 - 9 = 0$$

$$\Rightarrow 5x^2 + 20x + 15 = 0$$

$$\Rightarrow x^2 + 4x + 3 = 0$$

$$\Rightarrow (x + 3)(x + 1) = 0 \quad \Rightarrow x = (-3, -1)$$

$$\therefore y = (0, 4)$$

$$\therefore A = (-3, 0) \text{ \& \& } B = (-1, 4)$$

$\therefore AB$ is a diameter, so the equation of circle is

$$(x + 3)(x + 1) + (y - 0)(y - 4) = 0$$

$$\Rightarrow x^2 + y^2 + 4x - 4y + 3 = 0$$

Q11

The triangle is formed by

$$x = 0 \dots\dots\dots (i)$$

$$y = 0 \dots\dots\dots (ii) \&$$

$$lx + my = 1 \dots\dots\dots (iii)$$

The line (iii) cuts the axis at

$$A = \left(0, \frac{1}{m}\right) \text{ and at } B = \left(\frac{1}{l}, 0\right)$$

Now, AB will be the diameter of circle.

\therefore equation of circle will be,

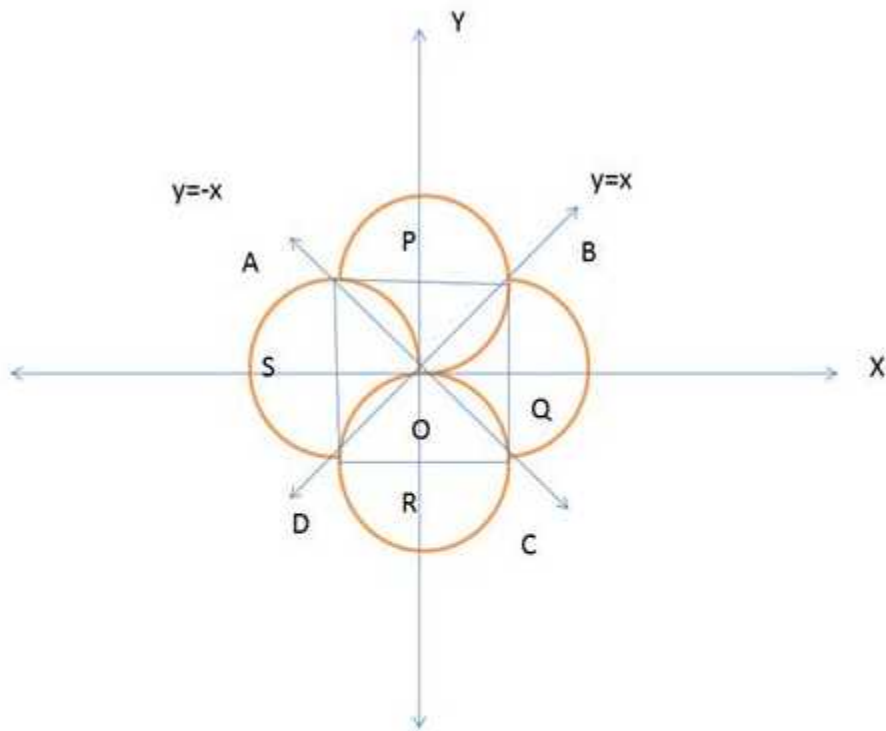
$$\left(x - \frac{1}{l}\right)\left(x - 0\right) + \left(y - 0\right)\left(y - \frac{1}{m}\right) = 0$$

$$\Rightarrow x^2 + y^2 - \frac{x}{l} - \frac{y}{m} = 0$$

$$\Rightarrow x^2 + y^2 - \frac{x}{l} - \frac{y}{m} = 0$$

Q12

Figure shows four such circles.



Angle between $y=x$ and $y=-x$ is $\frac{\pi}{2}$.

\therefore Angle between OB and OA = $\frac{\pi}{2}$

Hence, AB, BC, CD and AD are diameters of circles.

$$\angle BOQ = \frac{\pi}{4}$$

$$\sin \angle BOQ = \frac{BQ}{OB}$$