Ex 28.1

Straight Line in Space Ex 28.1 Q1

Vector equation of a line
is $\vec{r} = \vec{a} + \lambda\vec{b}$
The Cartesian equation of a line is
$$\frac{x - x_1}{a_1} = \frac{y - y_1}{a_2} = \frac{x - x_3}{a_3}$$
Using the above formula,
Vector equation of the line,
$\vec{r} = (5\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(3\hat{i} + 2\hat{j} - 8\hat{k})$
The Cartesian equation of the line
$$\frac{x - 5}{3} = \frac{y - 2}{2} = \frac{z + 4}{-8}$$

Straight Line in Space Ex 28.1 Q2

The direction ratios of the line are
$(3 + 1, 4 - 0, 6 - 2) = (4, 4, 4)$
Since the line passes through $(-1, 0, 2)$
The vector equation of the line,
$\vec{r} = \vec{a} + \lambda\vec{b}$
$\Rightarrow \vec{r} = \vec{a} + \lambda\vec{b}$
$\Rightarrow \vec{r} = (-\vec{i} + 0\vec{j} + 2\vec{k}) + \lambda(4\vec{i} + 4\vec{j} + 4\vec{k})$
$\therefore$ The vector equation of the line,
$\vec{r} = (-\vec{i} + 0\vec{j} + 2\vec{k}) + \lambda(4\vec{i} + 4\vec{j} + 4\vec{k})$

Straight Line in Space Ex 28.1 Q3

We know that, vector equation of line passing through a fixed point $\vec{a}$ and parallel to vector $\vec{b}$ is
$$\vec{r} = \vec{a} + \lambda\vec{b}, \text{ where } \lambda \text{ is scalar}$$

Here, $\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{a} = 5\hat{i} - 2\hat{j} + 4\hat{k}$

So, equation of required line is
$$\vec{r} = \vec{a} + \lambda\vec{b}$$
$$\vec{r} = \left(5\hat{i} - 2\hat{j} + 4\hat{k}\right) + \lambda\left(2\hat{i} - \hat{j} + 3\hat{k}\right)$$

Put $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, so
$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right) = \left(5 + 2\lambda\right)\hat{i} + \left(-2 - \lambda\right)\hat{j} + \left(4 + 3\lambda\right)\hat{k}$$

Comparing the coefficients of $\hat{i}, \hat{j}, \hat{k}$, so
$$x = 5 + 2\lambda,\ y = -2 - \lambda,\ z = 4 + 3\lambda$$

$$\Rightarrow \quad \frac{x - 5}{2} = \lambda,\ \frac{y + 2}{-0} = \lambda,\ \frac{z - 4}{3} = \lambda$$

Cortesian form of equation of the line is,
$$\frac{x - 5}{2} = \frac{y + 2}{-0} = \frac{z - 4}{3}$$

Straight Line in Space Ex 28.1 Q4

We know that, equation of line passing through a vector $\vec{a}$ and parallel to a vector $\vec{b}$ is given by,

$$\vec{r} = \vec{a} + \lambda\vec{b}, \text{ where } \lambda \text{ is scalar,}$$

Here, $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{b} = 3\hat{i} + 4\hat{j} - 5\hat{k}$

Required equation of line is,

$$\vec{r} = \vec{a} + \lambda\vec{b}$$

$$\vec{r} = \left(2\hat{i} - 3\hat{j} + 4\hat{k}\right) + \lambda\left(3\hat{i} + 4\hat{j} - 5\hat{k}\right)$$

Put $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$x\hat{i} + y\hat{j} + z\hat{k} = (2 + 3\lambda)\hat{i} + (-3 + 4\lambda)\hat{j} + (4 - 5\lambda)\hat{k}$$

On equating coefficients of $\hat{i}, \hat{j}$ and $\hat{k}$,

$$\Rightarrow \quad 2 + 3\lambda = x,\ -3 + 4\lambda = y,\ 4 - 5\lambda = z$$

$$\Rightarrow \quad \frac{x-2}{3} = \lambda,\ \frac{y+3}{4} = \lambda,\ \frac{z-4}{-5} = z$$

So, cortesian form of equation of the line is

$$\frac{x-2}{3} = \frac{y+3}{4} = \frac{z-4}{-5}$$

Straight Line in Space Ex 28.1 Q5

$ABCD$ is a parallelogram.

$\Rightarrow$ AC and BD bisect each other at point O (say).

Position vector of point $O = \dfrac{\vec{a}+\vec{c}}{2}$

$$= \frac{\left(4\hat{i} + 5\hat{j} - 10\hat{k}\right) + \left(-\hat{i} + 2\hat{j} + \hat{k}\right)}{2}$$

$$= \frac{3\hat{i} + 7\hat{j} - 9\hat{k}}{2}$$

Let position vector of point O and B are represented by $\vec{o}$ and $\vec{b}$.

Equation of the line BD is the line passing through O and B is given by

$$\vec{r} = \vec{a} + \lambda\left(\vec{b} - \vec{a}\right) \qquad \left[\begin{array}{l}\text{Since equation of the line passing through} \\ \text{two points } \vec{a} \text{ and } \vec{b}\end{array}\right]$$

$$\vec{r} = \vec{b} + \lambda\left(\vec{o} - \vec{b}\right)$$

$$= \left(2\hat{i} - 3\hat{j} + 4\hat{k}\right) + \lambda\left(\frac{3\hat{i} + 7\hat{j} - 9\hat{k}}{2} - 2\hat{i} - 3\hat{j} + 4\hat{k}\right)$$

$$\vec{r} = \left(2\hat{i} - 3\hat{j} + 4\hat{k}\right) + \lambda\left(3\hat{i} + 7\hat{j} - 9\hat{k} - 4\hat{i} + 6\hat{j} - 8\hat{k}\right)$$

$$\vec{r} = \left(2\hat{i} - 3\hat{j} + 4\hat{k}\right) + \lambda\left(-\hat{i} + 13\hat{j} - 17\hat{k}\right)$$

Put $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right) = (2 - \lambda)\hat{i} + (-3 + 13\lambda)\hat{j} + (4 - 17\lambda)\hat{k}$$

Equation the coefficients of $\hat{i}, \hat{j}, \hat{k}$, so

$$\Rightarrow \quad x = 2 - \lambda,\ y = -3 - 13\lambda,\ z = 4 - 17\lambda$$

$$\Rightarrow \quad \frac{x-2}{-1} = \lambda,\ \frac{y+3}{13} = \lambda,\ \frac{z-4}{-17} = \lambda$$

So equation of the line BD in cortesian form,

$$\frac{x-2}{-1} = \frac{y+3}{13} = \frac{z-4}{-17}$$

Straight Line in Space Ex 28.1 Q6

We know that, equation of line passing through two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1} \qquad ---(i)$$

Here, $(x_1, y_1, z_1) = A(1, 2, -1)$
$(x_2, y_2, z_2) = B(2, 1, 1)$

Using equation (i), equation of line AB,

$$\frac{x-1}{2-1} = \frac{y-2}{1-2} = \frac{z+1}{1+1}$$

$$\frac{x-1}{1} = \frac{y-2}{-1} = \frac{z+1}{2} = \lambda \text{ (say)}$$

$$x = \lambda + 1,\ y = -\lambda + 2,\ z = 2\lambda - 1$$

Vector form of equation of line AB is,

$$x\hat{i} + y\hat{j} + z\hat{k} = (\lambda + 1)\hat{i} + (-\lambda + 2)\hat{j} + (2\lambda - 1)\hat{k}$$

$$\vec{r} = (\hat{i} + 2\hat{j} - \hat{k}) + \lambda(\hat{i} - \hat{j} + 2\hat{k})$$

Straight Line in Space Ex 28.1 Q7

We know that vector equation of a line passing through $\vec{a}$ and parallel to vector $\vec{b}$ is given by,

$$\vec{r} = \vec{a} + \lambda\vec{b}$$

Here, $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$

So, required vector equation of line is,

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - 2\hat{j} + 3\hat{k})$$

Now,

$$(x\hat{i} + y\hat{j} + z\hat{k}) = (1 + \lambda)\hat{i} + (2 - 2\lambda)\hat{j} + (3 + 3\lambda)\hat{k}$$

Equating the coefficients of $\hat{i}, \hat{j}, \hat{k}$,

$\Rightarrow \quad x = 1 + \lambda,\ y = 2 - 2\lambda,\ z = 3 + 3\lambda$

$\Rightarrow \quad x - 1 = \lambda,\ \frac{y-2}{2} = \lambda,\ \frac{z-3}{3} = \lambda$

So, required equation of line is cortesian form,

$$\frac{x-1}{1} = \frac{y-2}{-2} = \frac{z-3}{3}$$

Straight Line in Space Ex 28.1 Q8

We know that, equation of a line passing through a point (x_1, y_1, z_1) and having direction ratios proportional to a, b, c is

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c} \qquad ---(i)$$

Here, $(x_1, y_1, z_1) = (2, -1, 1)$ and

Given line $\frac{x-3}{2} = \frac{y+1}{7} = \frac{z-2}{-3}$ is prallel to required line.

$\Rightarrow \quad a = 2\mu,\ b = 7\mu,\ c = -3\mu$

So, equation of required line using equation (i),

$$\frac{x-2}{2\mu} = \frac{y+1}{7\mu} = \frac{z-1}{-3\mu}$$

$$\Rightarrow \quad \frac{x-2}{2} = \frac{y+1}{7} = \frac{z-1}{-3} = \lambda \text{ (say)}$$

$\Rightarrow \quad x = 2\lambda + 2,\ y = 7\lambda - 1,\ z = -3\lambda + 1$

So, $x\hat{i} + y\hat{j} + z\hat{k} = (2\lambda + 2)\hat{i} + (7\lambda - 1)\hat{j} + (-3\lambda + 1)\hat{k}$

$$\vec{r} = \left(2\hat{i} - \hat{j} + \hat{k}\right) + \lambda\left(2\hat{i} + 7\hat{j} - 3\hat{k}\right)$$

Straight Line in Space Ex 28.1 Q9

The Cartesian equation of the line is

$$\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2} \qquad \dots(1)$$

The given line passes through the point (5, −4, 6). The position vector of this point is

$\vec{a} = 5\hat{i} - 4\hat{j} + 6\hat{k}$

Also, the direction ratios of the given line are 3, 7, and 2.

This means that the line is in the direction of vector, $\vec{b} = 3\hat{i} + 7\hat{j} + 2\hat{k}$

It is known that the line through position vector $\vec{a}$ and in the direction of the vector $\vec{b}$ is given by the equation, $\vec{r} = \vec{a} + \lambda\vec{b}, \lambda \in R$

$$\Rightarrow \vec{r} = \left(5\hat{i} - 4\hat{j} + 6\hat{k}\right) + \lambda\left(3\hat{i} + 7\hat{j} + 2\hat{k}\right)$$

This is the required equation of the given line in vector form.

Straight Line in Space Ex 28.1 Q10

We know that, equation of a line passing through a point (x_1, y_1, z_1) and having direction ratios proportional to a, b, c is

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c} \qquad ---(i)$$

Here, $(x_1, y_1, z_1) = (1, -1, 2)$ and

Given line $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z+1}{-2}$ is prallel to required line, so

$\Rightarrow \quad a = \mu,\ b = 2\mu,\ c = -2\mu$

So, equation of required line using equation (i) is,

$$\frac{x-1}{\mu} = \frac{y+1}{2\mu} = \frac{z-2}{-2\mu}$$

$$\Rightarrow \quad \frac{x-1}{1} = \frac{y+1}{2} = \frac{z-2}{-2} = \lambda \text{ (say)}$$

$x = \lambda + 1,\ y = 2\lambda - 1,\ z = -2\lambda + 2$

So, $x\hat{i} + y\hat{j} + z\hat{k} = (\lambda + 1)\hat{i} + (2\lambda - 1)\hat{j} + (-2\lambda + 2)\hat{k}$

$$\vec{r} = \left(\hat{i} - \hat{j} + 2\hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j} - 2\hat{k}\right)$$

Straight Line in Space Ex 28.1 Q11

Given, line is,

$$\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$$

$$\Rightarrow \quad \frac{x-4}{-2}=\frac{y}{6}=\frac{z-1}{-3}=\lambda \text{ (say)}$$

$$x=-2\lambda+4,\ y=6\lambda,\ z=-3\lambda+1$$

So, $x\hat{i}+y\hat{j}+z\hat{k}=(-2\lambda+4)\hat{i}+(6\lambda)\hat{j}+(-3\lambda+1)\hat{k}$

$$\vec{r}=\left(4\hat{i}+\hat{k}\right)+\lambda\left(-2\hat{i}+6\hat{j}-3\hat{k}\right)$$

Direction ratios of the line are $=-2,\ 6,\ -3$

Direction cosines of the line are,

$$\frac{a}{\sqrt{a^2+b^2+c^2}},\ \frac{b}{\sqrt{a^2+b^2+c^2}},\ \frac{c}{\sqrt{a^2+b^2+c^2}}$$

$$\Rightarrow \quad \frac{-2}{\sqrt{(-2)^2+(6)^2+(-3)^2}},\ \frac{6}{\sqrt{(-2)^2+(6)^2+(-3)^2}},\ \frac{-3}{\sqrt{(-2)^2+(6)^2+(-3)^2}}$$

$$\Rightarrow \quad \frac{-2}{7},\ \frac{6}{7},\ \frac{-3}{7}$$

Straight Line in Space Ex 28.1 Q12

$x=ay+b,$

$z=cy+d$

$$\frac{x-b}{a}=\frac{y}{1}=\frac{z-d}{c}=\lambda(say)$$

So DR's of line are (a, 1, c)

From above equation, we can write

$x=a\lambda+b$

$y=\lambda$

$z=c\lambda+d$

So vector equation of line is

$$x\hat{i}+y\hat{j}+z\hat{k}=(b\hat{i}+d\hat{k})+\lambda\left(a\hat{i}+\hat{j}+c\hat{k}\right)$$

Straight Line in Space Ex 28.1 Q13

We know that, equation of a line passing through $\vec{a}$ and parallel to vector $\vec{b}$ is,

$$\vec{r}=\vec{a}+\lambda\vec{b} \qquad ---(i)$$

Here, $\vec{a}=\hat{i}-2\hat{j}-3\hat{k}$

and, $\vec{b}=$ line joining $\left(\hat{i}-\hat{j}+4\hat{k}\right)$ and $\left(2\hat{i}+\hat{j}+2\hat{k}\right)$

$$=\left(2\hat{i}+\hat{j}+2\hat{k}\right)-\left(\hat{i}-\hat{j}+4\hat{k}\right)$$

$$=2\hat{i}-\hat{i}+\hat{j}+\hat{j}+2\hat{k}-4\hat{k}$$

$$=\hat{i}+2\hat{j}-2\hat{k}$$

Equation of the line is

$$\vec{r}=\left(\hat{i}-2\hat{j}-3\hat{k}\right)+\lambda\left(\hat{i}+2\hat{j}-2\hat{k}\right)$$

For cortesion form of equation put $x\hat{i}+y\hat{j}+z\hat{k}$,

$$x\hat{i}+y\hat{j}+z\hat{k}=(1+\lambda)\hat{i}+(-2+2\lambda)\hat{j}+(-3-2\lambda)\hat{k}$$

Equating coefficients of $\hat{i},\hat{j},\hat{k}$, so

$$x=1+\lambda,\ y=-2+2\lambda,\ z=-3-2\lambda$$

$$\Rightarrow \quad \frac{x-1}{1}=\lambda,\ \frac{y+2}{2}=\lambda,\ \frac{z+3}{-2}=\lambda$$

So, $\dfrac{x-1}{1}=\dfrac{y+2}{2}=\dfrac{z+3}{-2}$

Straight Line in Space Ex 28.1 Q14

Distance of point P from $Q = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$

$$PQ = \sqrt{(3\lambda - 2 - 1)^2 + (2\lambda - 1 - 3)^2 + (2\lambda + 3 - 3)^2}$$

$\Rightarrow \quad (5)^2 = (3\lambda - 3)^2 + (2\lambda - 4)^2 + (2\lambda)^2$

$\Rightarrow \quad 25 = 9\lambda^2 + 9 - 18\lambda + 4\lambda^2 + 16 - 16\lambda + 4\lambda^2$

$\Rightarrow \quad 17\lambda^2 - 34\lambda = 0$

$\Rightarrow \quad 17\lambda(\lambda - 2) = 0$

$\Rightarrow \quad \lambda = 0$ or 2

So, points on the line are $(3(0) - 2,\ 2(0) - 1,\ 2(0) + 3)$

$(3(2) - 2,\ 2(2) - 1,\ 2(2) + 3)$

$= (-2, -1, 3),\ (4, 3, 7)$

Straight Line in Space Ex 28.1 Q15

Let the given points are A, B, C with position vectors $\vec{a}, \vec{b}, \vec{c}$ respectively, so

$\vec{a} = -2\hat{i} + 3\hat{j},\ \vec{b} = \hat{i} + 2\hat{j} + 3\hat{k},\ \vec{c} = 7\hat{i} - \hat{k}$

We know that, equation of a line passing through $\vec{a}$ and $\vec{b}$ are,

$$\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$$

$$= (-2\hat{i} + 3\hat{j}) + \lambda((\hat{i} + 2\hat{j} + 3\hat{k}) - (-2\hat{i} + 3\hat{j}))$$

$$= (-2\hat{i} + 3\hat{j}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k} + 2\hat{i} - 3\hat{j})$$

$$\vec{r} = (-2\hat{i} + 3\hat{j}) + \lambda(3\hat{i} - \hat{j} + 3\hat{k}) \quad ---(i)$$

If A, B, C are collinear then $\vec{c}$ must satisfy equation (i),

$$7\hat{i} - \hat{k} = (-2 + 3\lambda)\hat{i} + (3 - \lambda)\hat{j} + (3\lambda)\hat{k}$$

Equation the coefficients of $\hat{i}, \hat{j}, \hat{k}$,

$-2 + 3\lambda = 7 \quad \Rightarrow \lambda = 3$

$3 - \lambda = 0 \quad \Rightarrow \lambda = 3$

$3\lambda = -1 \quad \Rightarrow \lambda = -\frac{1}{3}$

Since, value of λ are not equal, so,

Given points are not collinear.

Straight Line in Space Ex 28.1 Q16

We know that, equation of a line passing through a point (x_1, y_1, z_1) and having direction ratios proportional to a, b, c is

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} \quad ---(i)$$

Here, $(x_1, y_1, z_1) = (1, 2, 3)$ and

Given line $\frac{-x - 2}{1} = \frac{y + 3}{7} = \frac{2z - 6}{3}$

$$\Rightarrow \quad \frac{x + 2}{-1} = \frac{y + 3}{7} = \frac{z - 3}{\frac{3}{2}}$$

It parallel to the required line, so

$$a = \mu,\ b = 7\mu,\ c = \frac{3}{2}\mu$$

So, equation of required line using equation (i) is,

$$\frac{x - 1}{-\mu} = \frac{y - 2}{7\mu} = \frac{z - 3}{\frac{3}{2}\mu}$$

$$\Rightarrow \quad \frac{x - 1}{-1} = \frac{y - 2}{7} = \frac{z - 3}{\frac{3}{2}}$$

Straight Line in Space Ex 28.1 Q17

Given equation of line is,

$$3x+1=6y-2=1-z$$

Dividing all by 6,

$$\frac{3x+1}{6}=\frac{6y-2}{6}=\frac{1-z}{6}$$

$$\Rightarrow \quad \frac{3x}{6}+\frac{1}{6}=\frac{6y}{6}-\frac{2}{6}=\frac{1}{6}-\frac{z}{6}$$

$$\Rightarrow \quad \frac{1}{2}x+\frac{1}{6}=y-\frac{1}{3}=-\frac{z}{6}+\frac{1}{6}$$

$$\Rightarrow \quad \frac{1}{2}\left(x+\frac{1}{3}\right)=1\left(y-\frac{1}{3}\right)=+\frac{1}{6}(z-1)$$

$$\Rightarrow \quad \frac{x+\frac{1}{3}}{2}=\frac{y-\frac{1}{3}}{1}=\frac{z-1}{-6}=\lambda \ (\text{say}) \qquad ---(\text{i})$$

Comparing it with equation of line passing through (x_1, y_1, z_1) and direction ratios a, b, c,

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$$

$$\Rightarrow \quad (x_1, y_1, z_1)=\left(-\frac{1}{3}, \frac{1}{3}, 1\right)$$

$$a=2,\ b=1,\ -6$$

So, direction ratios of the line are $=2, 1, -6$

From equation (i),

$$x=\left(2\lambda-\frac{1}{3}\right),\ y=\left(\lambda+\frac{1}{3}\right),\ z=(-6\lambda+1)$$

So, vector equation of the given line is,

$$x\hat{i}+y\hat{j}+z\hat{k}=\left(2\lambda-\frac{1}{3}\right)\hat{i}+\left(\lambda+\frac{1}{3}\right)\hat{j}+(-6\lambda+1)\hat{k}$$

$$\vec{r}=\left(-\frac{1}{3}\hat{i}+\frac{1}{3}\hat{j}+\hat{k}\right)+\lambda\left(2\hat{i}+\hat{j}-6\hat{k}\right)$$

Ex 28.2

Straight Line in Space Ex 28.2 Q1

Let $l_1 = \frac{12}{13}, m_1 = -\frac{3}{13}, n_1 = -\frac{4}{13}$

$l_2 = \frac{4}{13}, m_2 = \frac{12}{13}, n_1 = \frac{3}{13}$

$l_3 = \frac{3}{13}, m_3 = -\frac{4}{13}, n_3 = \frac{12}{13}$

$l_1 l_2 + m_1 m_2 + n_1 n_2$

$= \frac{12}{13} \times \frac{4}{13} + (-\frac{3}{13}) \times \frac{12}{13} + (-\frac{4}{13}) \times \frac{3}{13}$

$= \frac{48 - 36 - 12}{169} = 0$

$l_2 l_3 + m_2 m_3 + n_2 n_3$

$= \frac{4}{13} \times \frac{3}{13} + \frac{12}{13} \times (-\frac{4}{13}) + \frac{3}{13} \times \frac{12}{13}$

$= \frac{12 - 48 + 36}{169} = 0$

$l_1 l_3 + m_1 m_3 + n_1 n_3$

$= \frac{12}{13} \times \frac{3}{13} + (-\frac{3}{13}) \times (-\frac{4}{13}) + (-\frac{4}{13}) \times \frac{12}{13}$

$= \frac{36 + 12 - 48}{169} = 0$

$\therefore$ The lines are mutually perpendicular.

Straight Line in Space Ex 28.2 Q2

The direction ratios of a line passing through the points $(1, -1, 2)$ and $(3, 4, -2)$ are

$(3 - 1, 4 + 1, -2 - 2)$

$= (2, 5, -4)$

The direction ratios of a line passing through the points $(0, 3, 2)$ and $(3, 5, 6)$ are

$(3 - 0, 5 - 3, 6 - 2)$

$= (3, 2, 4)$

Angle between the lines

$$\cos\theta = \frac{(a_1 a_2 + b_1 b_2 + c_1 c_2)}{\sqrt{a_1{}^2 + b_1{}^2 + c_1{}^2}\sqrt{a_2{}^2 + b_2{}^2 + c_2{}^2}}$$

$$\cos\theta = \frac{[2 \times 3 + 5 \times 2 + (-4) \times 4]}{\sqrt{2^2 + 5^2 + (-4)^2}\sqrt{3^2 + 2^2 + 4^2}}$$

$$\cos\theta = \frac{0}{\sqrt{2^2 + 5^2 + (-4)^2}\sqrt{3^2 + 2^2 + 4^2}}$$

$\cos\theta = 0$

$\theta = \frac{\pi}{2}$

The lines are mutually perpendicular.

Straight Line in Space Ex 28.2 Q3

The direction ratios of a line passing through the points $(4, 7, 8)$ and $(2, 3, 4)$ are

$(4 - 2, 7 - 3, 8 - 4)$

$= (2, 4, 4)$

The direction ratios of a line passing through the points $(-1, -2, 1)$ and $(1, 2, 5)$ are

$(-1 - 1, -2 - 2, 1 - 5)$

$= (-2, -4, -4)$

The direction ratios are proportional.

$\frac{2}{-2} = \frac{4}{-4} = \frac{4}{-4}$

Hence, the lines are mutually parallel.

Straight Line in Space Ex 28.2 Q4

The Cartesian equation of a line passing through (x_1, y_1, z_1)
and with direction ratios (a_1, b_1, c_1)

$$\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$$

The Cartesian equation of a line passing through $(-2, 4, -5)$
and parallel to the line $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$ is

$$\frac{x+2}{3} = \frac{y-4}{5} = \frac{z+5}{6}$$

Straight Line in Space Ex 28.2 Q5

Given equations of lines are $\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1}$ and $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$.

Clearly,

$7 \times 1 + (-5) \times 2 + 1 \times 3$
$= 7 - 10 + 3$
$= 0$

$\therefore$ Lines $\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1}$ and $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ are perpendicular to each other.

Straight Line in Space Ex 28.2 Q6

The direction ratios of a line joining the origin to the point $(2, 1, 1)$
are $(2 - 0, 1 - 0, 1 - 0) = (2, 1, 1)$
The direction ratios of a line joining $(3, 5, -1)$ and $(4, 3, -1)$
are $(4 - 3, 3 - 5, -1 + 1) = (1, -2, 0)$

Angle between the lines

$$\cos\theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$\cos\theta = \frac{2 \times 1 + 1 \times (-2) + 1 \times 0}{\sqrt{2^2 + 1^2 + 1^2}\sqrt{1^2 + (-2)^2 + 0^2}}$$

$$\cos\theta = \frac{0}{\sqrt{6}\sqrt{5}}$$

$$\cos\theta = 0$$

$$\therefore \theta = \frac{\pi}{2}$$

The lines are mutually perpendicular.

Straight Line in Space Ex 28.2 Q7

Vector equation of a line is

$\vec{r} = \vec{a} + \lambda\vec{b}$

The direction cosines of the x – axis are $(1, 0, 0)$. Equation of a line parallel
to the x – axis and passing through the origin is

$\vec{r} = (0\hat{i} + 0\hat{j} + 0\hat{k}) + \lambda(1\hat{i} + 0\hat{j} + 0\hat{k})$

$\vec{r} = \lambda\hat{i}$

Straight Line in Space Ex 28.2 Q8(i)

We know that, If Q be the angle between two lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$, then

$$\cos\theta = \frac{\vec{b_1}.\vec{b_2}}{|\vec{b_1}|.|\vec{b_2}|} \qquad ---(i)$$

Here, $\vec{r} = (4\hat{i} - \hat{j}) + \lambda(\hat{i} + 2\hat{j} - 2\hat{k})$

and, $\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) - \mu(2\hat{i} + 4\hat{j} - 4\hat{k})$

$\Rightarrow \quad \vec{a_1} = 4\hat{i} - \hat{j}, \quad \vec{b_1} = \hat{i} + 2\hat{j} - 2\hat{k}$

$\vec{a_2} = \hat{i} - \hat{j} + 2\hat{k}, \vec{b_2} = 2\hat{i} + 4\hat{j} - 4\hat{k}$

$$|\vec{b_1}| = \sqrt{(1)^2 + (2)^2 + (-2)^2} = 3$$

$$|\vec{b_2}| = \sqrt{(2)^2 + (4)^2 + (-4)^2} = 6$$

Let θ be the angle between given lines. So using equation (i),

$$\cos\theta = \frac{\vec{b_1}.\vec{b_2}}{|\vec{b_1}|.|\vec{b_2}|}$$

$$= \frac{(\hat{i} + 2\hat{j} - 2\hat{k})(2\hat{i} + 4\hat{j} - 4\hat{k})}{3.6}$$

$$= \frac{2 + 8 + 8}{18}$$

$$\cos\theta = 1$$

$$\theta = 0°$$

Straight Line in Space Ex 28.2 Q8(ii)

We know that, angle between two lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$, is given by

$$\cos\theta = \frac{\vec{b_1}.\vec{b_2}}{|\vec{b_1}|.|\vec{b_2}|} \qquad ---(i)$$

Given lines are,

$$\vec{r} = (3\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$$

$$\vec{r} = (5\hat{j} - 2\hat{k}) + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$$

$\Rightarrow \quad \vec{b_1} = \hat{i} + 2\hat{j} + 2\hat{k}, \ \vec{b_2} = 3\hat{i} + 2\hat{j} + 6\hat{k}$

$$|\vec{b_1}| = \sqrt{(1)^2 + (2)^2 + (2)^2} = 3$$

$$|\vec{b_2}| = \sqrt{(3)^2 + (2)^2 + (6)^2} = 7$$

Let θ be the angle between given lines, so using equation (i),

$$\cos\theta = \frac{\vec{b_1}.\vec{b_2}}{|\vec{b_1}|.|\vec{b_2}|}$$

$$= \frac{(\hat{i} + 2\hat{j} + 2\hat{k})(3\hat{i} + 2\hat{j} + 6\hat{k})}{3.7}$$

$$= \frac{3 + 4 + 12}{21}$$

$$= \frac{19}{21}$$

$$\theta = \cos^{-1}\left(\frac{19}{21}\right)$$

Straight Line in Space Ex 28.2 Q8(iii)

We know that, angle between two lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$, is given by

$$\cos\theta = \frac{\vec{b_1}.\vec{b_2}}{|\vec{b_1}||\vec{b_2}|} \qquad ---(i)$$

Equation of given lines are,

$$\vec{r} = \lambda\left(\hat{i} + \hat{j} + 2\hat{k}\right) \text{ and}$$

$$\vec{r} = 2\hat{j} + \mu\left[\left(\sqrt{3}-1\right)\hat{i} - \left(\sqrt{3}+1\right)\hat{j} + 4\hat{k}\right]$$

$$\Rightarrow \qquad \vec{b_1} = \left(\hat{i} + \hat{j} + 2\hat{k}\right),\ \vec{b_2} = \left(\sqrt{3}-1\right)\hat{i} - \left(\sqrt{3}+1\right)\hat{j} + 4\hat{k}$$

Let θ be the angle between given lines, so using equation (i),

$$\cos\theta = \frac{\vec{b_1}.\vec{b_2}}{|\vec{b_1}||\vec{b_2}|}$$

$$= \frac{\left(\hat{i} + \hat{j} + 2\hat{k}\right)\left(\left(\sqrt{3}-1\right)\hat{i} - \left(\sqrt{3}+1\right)\hat{j} + 4\hat{k}\right)}{\sqrt{(1)^2 + (1)^2 + (2)^2}\sqrt{\left(\sqrt{3}-1\right)^2 + \left(-\sqrt{3}-1\right)^2 + (4)^2}}$$

$$= \frac{\sqrt{3} - 1 - \sqrt{3} - 1 + 8}{\sqrt{6}.\sqrt{3 + 1 - 2\sqrt{3} + 3 + 1 + 2\sqrt{3} + 16}}$$

$$= \frac{6}{\sqrt{6}.2\sqrt{6}}$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}$$

Straight Line in Space Ex 28.2 Q9(i)

We know that, angle between two lines

$$\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1} \text{ and } \qquad \frac{x - x_2}{a_2} = \frac{y - y_2}{b_2} = \frac{z - z_2}{c_2}$$

is given by,

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}} \qquad ---(i)$$

Here, given lines are,

$$\frac{x+4}{3} = \frac{y-1}{5} = \frac{z+3}{4} \text{ and } \frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$$

$$\Rightarrow \qquad a_1 = 3,\ b_1 = 5,\ c_1 = 4,\ a_2 = 1,\ b_2 = 1,\ c_2 = 2$$

Let θ be the angle between given lines, so using equation (i),

$$\cos\theta = \frac{(3)(1) + (5)(1) + (4)(2)}{\sqrt{(3)^2 + (5)^2 + (4)^2}\sqrt{(1)^2 + (1)^2 + (2)^2}}$$

$$= \frac{3+5+8}{\sqrt{50}\sqrt{6}}$$

$$= \frac{16}{10\sqrt{3}}$$

$$\cos\theta = \frac{8}{5\sqrt{3}}$$

$$\theta = \cos^{-1}\left(\frac{8}{5\sqrt{3}}\right)$$

Straight Line in Space Ex 28.2 Q9(ii)

We know that, angle between two lines

$$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1} \text{ and } \frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$$

is given by,

$$\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}} \quad ---(i)$$

Given, equation of lines are,

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{-3} \text{ and } \frac{x+3}{-1}=\frac{y-5}{8}=\frac{z-1}{4}$$

$$\Rightarrow \quad a_1=2,\ b_1=3,\ c_1=-3,\ a_2=-1,\ b_2=8,\ c_2=4$$

Let θ be the angle between two given lines, so using equation (i),

$$\cos\theta=\frac{(2)(-1)+(3)(8)+(-3)(4)}{\sqrt{(2)^2+(3)^2+(-3)^2}\sqrt{(-1)^2+(8)^2+(4)^2}}$$

$$=\frac{-2+24-12}{\sqrt{22}\sqrt{81}}$$

$$\cos\theta=\frac{10}{9\sqrt{22}}$$

$$\theta=\cos^{-1}\left(\frac{10}{9\sqrt{22}}\right)$$

Straight Line in Space Ex 28.2 Q9(iii)

We know that, angle between two lines

$$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1} \text{ and } \frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$$

is given by,

$$\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}} \quad ---(i)$$

Given lines are,

$$\frac{5-x}{-2}=\frac{y+3}{1}=\frac{1-z}{3} \text{ and } \frac{x}{3}=\frac{1-y}{-2}=\frac{z+5}{-1}$$

$$\Rightarrow \quad \frac{x-5}{2}=\frac{y+3}{1}=\frac{z-1}{-3} \text{ and } \frac{x}{3}=\frac{y-1}{2}=\frac{z+5}{-1}$$

$$\Rightarrow \quad a_1=2,\ b_1=1,\ c_1=-3,\ a_2=3,\ b_2=2,\ c_2=-1$$

Let θ be the angle between given lines, so using equation (i),

$$\cos\theta=\frac{(2)(3)+(1)(2)+(-3)(-1)}{\sqrt{(2)^2+(1)^2+(-3)^2}\sqrt{(3)^2+(2)^2+(-1)^2}}$$

$$=\frac{6+2+3}{\sqrt{14}\sqrt{14}}$$

$$\cos\theta=\frac{11}{14}$$

$$\theta=\cos^{-1}\left(\frac{11}{14}\right)$$

Straight Line in Space Ex 28.2 Q9(iv)

We know that, angle between two lines

$$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1} \text{ and } \frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$$

is given by,

$$\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

Equation of given lines are,

$$\frac{x-2}{3}=\frac{y+3}{-2}, z=5 \text{ and } \frac{x+1}{1}=\frac{2y-3}{3}=\frac{z-5}{2}$$

$$\Rightarrow \quad \frac{x-2}{3}=\frac{y+3}{-2}, z=5 \text{ and } \frac{x+1}{1}=\frac{\frac{y-3}{3}}{\frac{3}{2}}=\frac{z-5}{2}$$

$$\Rightarrow \quad a_1=3,\ b_1=-2,\ c_1=0,\ a_2=1,\ b_2=\frac{3}{2},\ c_2=2$$

Let θ be the angle between given lines, so from equation (i),

$$\cos\theta=\frac{(3)(1)+(-2)\left(\frac{3}{2}\right)+(0)(2)}{\sqrt{(3)^2+(-2)^2+(0)^2}\sqrt{(1)^2+\left(\frac{3}{2}\right)^2+(2)^2}}$$

$$=\frac{3-3+0}{\sqrt{38}\sqrt{\frac{29}{4}}}$$

$$\cos\theta=0$$

$$\theta=\frac{\pi}{2}$$

Straight Line in Space Ex 28.2 Q9(v)

$$\frac{x-5}{1}=\frac{2y+6}{-2}=\frac{z-3}{1} \text{ and } \frac{x-2}{3}=\frac{y+1}{4}=\frac{z-6}{5}$$

$\hat{a}=\hat{i}-2\hat{j}+\hat{k}, \hat{b}=3\hat{i}+4\hat{j}+5\hat{k}$ are the vectors parallel to above lines

$\therefore$ angle between $\hat{a}$ and $\hat{b} \to \cos\theta=\dfrac{\hat{a}\cdot\hat{b}}{|\hat{a}||\hat{b}|}$

$$\cos\theta=\frac{(\hat{i}-2\hat{j}+\hat{k})\cdot(3\hat{i}+4\hat{j}+5\hat{k})}{|\hat{i}-2\hat{j}+\hat{k}||\hat{i}-2\hat{j}+\hat{k}|}=\frac{3-8+5}{|\hat{i}-2\hat{j}+\hat{k}||\hat{i}-2\hat{j}+\hat{k}|}=0$$

$$\cos\theta=0 \to \theta=90°$$

Straight Line in Space Ex 28.2 Q9(vi)

$$\frac{x-2}{2}=\frac{y-1}{7}=\frac{z+3}{-3} \text{ and } \frac{x+2}{-1}=\frac{y-4}{2}=\frac{z-5}{4}$$

$\hat{a}=2\hat{i}+7\hat{j}-3\hat{k}, \hat{b}=-1\hat{i}+4\hat{j}+4\hat{k}$ are the vectors parallel to above lines

$\therefore$ angle between $\hat{a}$ and $\hat{b} \to \cos\theta=\dfrac{\hat{a}\cdot\hat{b}}{|\hat{a}||\hat{b}|}$

$$\cos\theta=\frac{(2\hat{i}+7\hat{j}-3\hat{k})\cdot(-1\hat{i}+2\hat{j}+4\hat{k})}{|(2\hat{i}+7\hat{j}-3\hat{k})||(-1\hat{i}+2\hat{j}+4\hat{k})|}=0$$

$$\cos\theta=0 \to \theta=90°$$

Straight Line in Space Ex 28.2 Q10(i)

We know that, angle (θ) between two lines

$$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1} \text{ and } \frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$$

is given by,

$$\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}} \quad ---(i)$$

Here, $a_1=5,\ b_1=-12,\ c_1=13$
$a_2=-3,\ b_2=4,\ c_2=5$

Let θ be the required angle, so using equation (i),

$$\cos\theta=\frac{(5)(-3)+(-12)(4)+(13)(5)}{\sqrt{(5)^2+(-12)^2+(13)^2}\sqrt{(-3)^2+(4)^2+(5)^2}}$$

$$=\frac{-15-48+65}{\sqrt{169\times2}\sqrt{25\times2}}$$

$$=\frac{2}{65\times2}$$

$$\cos\theta=\frac{1}{65}$$

$$\theta=\cos^{-1}\left(\frac{1}{65}\right)$$

Straight Line in Space Ex 28.2 Q10(ii)

We know that, angle (θ) between lines

$$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1} \text{ and } \frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$$

is given by,

$$\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}} \quad ---(i)$$

Here, $a_1=2,\ b_1=2,\ c_1=1$
$a_2=4,\ b_2=1,\ c_2=8$

Let θ be required angle, so using equation (i),

$$\cos\theta=\frac{(2)(4)+(2)(1)+(1)(8)}{\sqrt{(2)^2+(2)^2+(1)^2}\sqrt{(4)^2+(1)^2+(8)^2}}$$

$$=\frac{8+2+8}{3.9}$$

$$=\frac{18}{27}$$

$$\cos\theta=\frac{2}{3}$$

$$\theta=\cos^{-1}\left(\frac{2}{3}\right)$$

Straight Line in Space Ex 28.2 Q10(iii)

We know that, angle (θ) between two lines

$$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1} \text{ and } \frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$$

is given by,

$$\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}} \quad ---(i)$$

Here, $a_1=1,\ b_1=2,\ c_1=-2$
$a_2=-2,\ b_2=2,\ c_2=1$

Let θ be the required angle, so using equation (i),

$$\cos\theta=\frac{(1)(-2)+(2)(2)+(-2)(1)}{\sqrt{(1)^2+(2)^2+(-2)^2}\sqrt{(-2)^2+(2)^2+(1)^2}}$$

$$=\frac{-2+4-2}{3.3}$$

$$=\frac{0}{9}$$

$$\cos\theta=0$$

$$\theta=\frac{\pi}{2}$$

Straight Line in Space Ex 28.2 Q10(vi)

a, b, c and $b-c,\ c-a,\ a-b$ are direction ratios

these are the vectors with above direction ratios

$\hat{x} = a\hat{i} + b\hat{j} + c\hat{k},\ \hat{y} = (b-c)\hat{i} + (c-a)\hat{j} + (a-b)\hat{k}$

are the vectors parallel to two given lines

$\therefore$ angle between the lines with above

direction ratios are $\hat{x}$ and $\hat{y} \to \cos\theta = \dfrac{\hat{x}\cdot\hat{y}}{|\hat{x}||\hat{y}|}$

$$\cos\theta = \frac{(a\hat{i} + b\hat{j} + c\hat{k})\cdot((b-c)\hat{i} + (c-a)\hat{j} + (a-b)\hat{k})}{|(a\hat{i} + b\hat{j} + c\hat{k})||(b-c)\hat{i} + (c-a)\hat{j} + (a-b)\hat{k}|}$$

$$= \frac{a(b-c) + b(c-a) + c(a-b)}{\sqrt{a^2+b^2+c^2}\sqrt{(b-c)^2+(c-a)^2+(a-b)^2}}$$

$$= \frac{ab - ac + bc - ba + ca - cb}{\sqrt{a^2+b^2+c^2}\sqrt{(b-c)^2+(c-a)^2+(a-b)^2}} = 0$$

$\cos\theta = 0 \to \theta = 90°$

Straight Line in Space Ex 28.2 Q11

We know that, angle (θ) between two lines

$$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1} \text{ and } \frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$$

is given by,

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

Here, Direction ratios of first line is 2, 2, 1

$\Rightarrow \quad a_1 = 2,\ b_1 = 2,\ c_1 = 1$

Direction ratios of the line joining $(3,1,4)$ and $(7,2,12)$ is given by

$= (7-3), (2-1), (12-4)$

$= 4, 1, 8$

$\Rightarrow \quad a_2 = 4,\ b_2 = 1,\ c_2 = 8$

Let θ be the required angle, so using equation (i),

$$\cos\theta = \frac{(2)(4) + (2)(1) + (1)(8)}{\sqrt{(2)^2+(2)^2+(1)^2}\sqrt{(4)^2+(1)^2+(8)^2}}$$

$$= \frac{8+2+8}{3.9}$$

$$\cos\theta = \frac{2}{3}$$

$$\theta = \cos^{-1}\left(\frac{2}{3}\right)$$

Straight Line in Space Ex 28.2 Q12

We know that equation of a line passing through (x_1, y_1, z_1) and direction ratios are a, b, c is given by

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} \qquad ---(i)$$

Here, $(x_1, y_1, z_1) = (1, 2, -4)$
and required line is parallel to the given line

$$\frac{x-3}{4} = \frac{y-5}{2} = \frac{z+1}{3}$$

$\Rightarrow$ Direction ratios of the required line are proportional to $4, 2, 3$

$\Rightarrow$ $a = 4\lambda,\ b = 2\lambda,\ c = 3\lambda$

So, required equation of the line is

$$\Rightarrow \frac{x-1}{4\lambda} = \frac{y-2}{2\lambda} = \frac{z+4}{3\lambda}$$

$$\Rightarrow \frac{x-1}{4} = \frac{y-2}{2} = \frac{z+4}{3}$$

Straight Line in Space Ex 28.2 Q13

We know that, equation of a line passing through (x_1, y_1, z_1) and direction ratios are a, b, c is given by

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} \qquad ---(i)$$

Here, $(x_1, y_1, z_1) = (-1, 2, 1)$
and required line is parallel to the given line

$$\frac{2x-1}{4} = \frac{3y+5}{2} = \frac{2-z}{3}$$

$$\Rightarrow \frac{x - \frac{1}{2}}{2} = \frac{y + \frac{5}{3}}{\frac{2}{3}} = \frac{z-2}{-3}$$

$\Rightarrow$ Direction ratios of the required line are proportional to $2, \frac{2}{3}, -3$

$\Rightarrow$ $a = 2\lambda,\ b = \frac{2}{3}\lambda,\ c = -3\lambda$

So, required equation of the line using equation (i),

$$\frac{x+1}{2\lambda} = \frac{y-2}{\frac{2}{3}\lambda} = \frac{z-1}{-3\lambda}$$

$$\Rightarrow \frac{x+1}{2} = \frac{y-2}{\frac{2}{3}} = \frac{z-1}{-3}$$

Straight Line in Space Ex 28.2 Q14

We know that equation of a line passing through the point $\vec{a}$ and is the direction of vector $\vec{b}$ is

$$\vec{r} = \vec{a} + \lambda\vec{b} \qquad ---(i)$$

Here, $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$
and given that the required line is parallel to

$$\vec{r} = \left(\hat{i} - 2\hat{j} + \hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} - 5\hat{k}\right)$$

$$\Rightarrow \vec{b} = \left(2\hat{i} + 3\hat{j} - 5\hat{k}\right).\mu$$

So, required equation of the line using equation (i) is

$$\vec{r} = \left(2\hat{i} - \hat{j} + 3\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} - 5\hat{k}\right).\mu$$

$$\vec{r} = \left(2\hat{i} - \hat{j} + 3\hat{k}\right) + \lambda'\left(2\hat{i} + 3\hat{j} - 5\hat{k}\right)$$

where λ' is a scalar such that $\lambda' = \lambda.\mu$

Straight Line in Space Ex 28.2 Q15

We know that, equation of a line passing through (x_1, y_1, z_1) with direction ratios a, b, c is given by

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

So, equation of required line passing through $(2, 1, 3)$ is

$$\frac{x-2}{a} = \frac{y-1}{b} = \frac{z-3}{c} \qquad ---(1)$$

Given that line $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ is perpendicular to line (i), so

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$
$$(a)(1) + (b)(2) + (c)(3) = 0$$
$$a + 2b + 3c = 0 \qquad ---(2)$$

And line $\frac{x}{-3} = \frac{y}{2} = \frac{z}{5}$ is perpendicular to line (i), so

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$
$$(a)(-3) + (b)(2) + (c)(5) = 0$$
$$-3a + 2b + 5c = 0 \qquad ---(3)$$

Solving equation (2) and (3) by cross multiplication,

$$\frac{a}{(2)(5)-(2)(3)} = \frac{b}{(-3)(3)-(1)(5)} = \frac{c}{(1)(2)-(-3)(2)}$$

$$\Rightarrow \quad \frac{a}{10-6} = \frac{b}{-9-5} = \frac{c}{2+6}$$

$$\Rightarrow \quad \frac{a}{4} = \frac{b}{-14} = \frac{c}{8}$$

$$\Rightarrow \quad \frac{a}{2} = \frac{b}{-7} = \frac{c}{4} = \lambda \text{ (Say)}$$

$$\Rightarrow \quad a = 2\lambda,\ b = -7\lambda,\ c = 4\lambda$$

Using a, b, c in equation (i),

$$\frac{x-2}{2\lambda} = \frac{y-1}{-7\lambda} = \frac{z-3}{4\lambda}$$

$$\Rightarrow \quad \frac{x-2}{2} = \frac{y-1}{-7} = \frac{z-3}{4}$$

Straight Line in Space Ex 28.2 Q16

We know that equation of a line passing through a point with position vector $\vec{\alpha}$ and perpendiculat to $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$ is given by

$$\vec{r} = \vec{\alpha} + \lambda\left(\vec{b_1} \times \vec{b_2}\right) \qquad ---(i)$$

Here, $\vec{\alpha} = \left(\hat{i} + \hat{j} - 3\hat{k}\right)$

and required line is perpendicular to

$$\vec{r} = \hat{i} + \lambda\left(2\hat{i} + \hat{j} - 3\hat{k}\right) \text{ and}$$

$$\vec{r} = \left(2\hat{i} + \hat{j} - \hat{k}\right) + \mu\left(\hat{i} + \hat{j} + \hat{k}\right)$$

$$\Rightarrow \quad \vec{b_1} = \left(2\hat{i} + \hat{j} - 3\hat{k}\right),\ \vec{b_2} = \hat{i} + \hat{j} + \hat{k}$$

Now,

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \hat{i}(1+3) - \hat{j}(2+3) + \hat{k}(2-1)$$

$$\vec{b_1} \times \vec{b_2} = 4\hat{i} - 5\hat{j} + \hat{k}$$

Using equation, required equation of line is

$$\vec{r} = \vec{\alpha} + \lambda\left(\vec{b_1} \times \vec{b_2}\right)$$

$$\vec{r} = \left(\hat{i} + \hat{j} - 3\hat{k}\right) + \lambda\left(4\hat{i} - 5\hat{j} + \hat{k}\right)$$

Straight Line in Space Ex 28.2 Q17

We know that equation of a line passing through (x_1, y_1, z_1) and direction ratios as a, b, c is given by

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} \qquad ----(1)$$

So, equation of a line passing through $(1, -1, 1)$ is

$$\frac{x - 1}{a} = \frac{y + 1}{b} = \frac{z - 1}{c} \qquad ---(2)$$

Now, Directions ratios of the line joining $A(4, 3, 2)$ and $B(1, -1, 0)$

$= (1 - 4), (-1 - 3), (0 - 2)$

$\Rightarrow$ Direction ratios of line $AB = -3, -4, -2$

and, Directions ratios of the line joining $C(1, 2, -1)$ and $D(2, 1, 1)$

$= (2 - 1), (1 - 2), (1 + 1)$

$\Rightarrow$ Direction ratios of line $CD = 1, -1, 2$

Given that, line AB is perpendicular to line (2), so

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$
$$(a)(-3) + (b)(-4) + (c)(-2) = 0$$
$$-3a + 4b - 2c = 0$$
$$3a + 4b + 2c = 0 \qquad ---(3)$$

and, line CD is also perpendicular to line (2), so

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$
$$(a)(1) + (b)(-1) + (c)(2) = 0$$
$$a - b + 2c = 0 \qquad ---(4)$$

Solving equation (3) and (4) using cross multiplication,

$$\frac{a}{(4)(2) - (-1)(2)} = \frac{b}{(1)(2) - (3)(2)} = \frac{c}{(3)(-1) - (4)(1)}$$

$$\Rightarrow \quad \frac{a}{8 + 2} = \frac{b}{2 - 6} = \frac{c}{-3 - 4}$$

$$\Rightarrow \quad \frac{a}{10} = \frac{b}{-4} = \frac{c}{-7} = \lambda \text{ (Say)}$$

$$\Rightarrow \quad a = 10\lambda,\ b = -4\lambda,\ c = -7\lambda$$

Straight Line in Space Ex 28.2 Q18

We know that equation of a line passing through a point (x_1, y_1, z_1) and direction ratios a, b, c is given by

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

So, equation of required line passing through $(1, 2, -4)$ is

$$\frac{x-1}{a} = \frac{y-2}{b} = \frac{z+4}{c} \qquad ---(1)$$

Given that, line $\frac{x-8}{8} = \frac{y+9}{-16} = \frac{z-10}{7}$ is perpendicular to line (1), so

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$$\Rightarrow \quad (a)(8) + (b)(-16) + (c)(7) = 0$$

$$\Rightarrow \quad 8a - 16b + 7c = 0 \qquad ----(2)$$

also, line $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$ is perpendicular to line (1), so

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$$\Rightarrow \quad (3)(a) + (8)(b) + (-5)(c) = 0$$

$$\Rightarrow \quad 3a + 8b - 5c = 0 \qquad ----(3)$$

Solving equation (2) and (3) by cross-multiplication,

$$\frac{a}{(-16)(-5)-(8)(7)} = \frac{b}{(3)(7)-(8)(-5)} = \frac{c}{(8)(8)-(3)(-16)}$$

$$\Rightarrow \quad \frac{a}{80-56} = \frac{b}{21+40} = \frac{c}{64+48}$$

$$\Rightarrow \quad \frac{a}{24} = \frac{b}{61} = \frac{c}{112} = \lambda \text{ (Say)}$$

$$\Rightarrow \quad a = 24\lambda,\ b = 61\lambda,\ c = 112\lambda$$

Put a, b, c in equation (1) to get required equation of the line, so

$$\frac{x-1}{24\lambda} = \frac{y-2}{61\lambda} = \frac{z+4}{112\lambda}$$

$$\Rightarrow \quad \frac{x-1}{24} = \frac{y-2}{61} = \frac{z+4}{112}$$

Straight Line in Space Ex 28.2 Q19

Equation of lines are,

$$\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1}$$

and, $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$

Now, $a_1a_2 + b_1b_2 + c_1c_2$

$= (7)(1) + (-5)(2) + (1)(3)$

$= 7 - 10 + 3$

$= 0$

So, given lines are perpendicular.

Straight Line in Space Ex 28.2 Q20

We know that, equation of a line passing through the point (x_1, y_1, z_1) and direction ratios a, b, c is given by

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} \qquad ---(1)$$

So, equation of line passing through $(2, -1, -1)$ is

$$\frac{x - 2}{a} = \frac{y + 1}{b} = \frac{z + 1}{c} \qquad ---(2)$$

Line (2) is parallel to given line,

$$6x - 2 = 3y + 1 = 2z - 2$$

$$\Rightarrow \quad \frac{6x - 2}{6} = \frac{3y + 1}{6} = \frac{2z - 2}{6}$$

$$\Rightarrow \quad \frac{x - \frac{1}{3}}{1} = \frac{y + \frac{1}{2}}{2} = \frac{z - \frac{1}{3}}{3}$$

So, $a = \lambda$, $b = 2\lambda$, $c = 3\lambda$

Using a, b, c in equation (2) to get required equation of line,

$$\frac{x - 2}{\lambda} = \frac{y + 1}{2\lambda} = \frac{z + 1}{3\lambda}$$

$$\Rightarrow \quad \frac{x - 2}{1} = \frac{y + 1}{2} = \frac{z + 1}{3} = \lambda \text{ (Say)}$$

$$\Rightarrow \quad x = \lambda + 2,\ y = 2\lambda - 1,\ z = 3\lambda - 1$$

So,

$$x\hat{i} + y\hat{j} + z\hat{k} = (\lambda + 2)\hat{i} + (2\lambda - 1)\hat{j} + (3\lambda - 1)\hat{k}$$

$$\vec{r} = \left(2\hat{i} - \hat{j} - \hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$$

Straight Line in Space Ex 28.2 Q21

The direction of ratios of the lines, $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$, are -3, $2k$, 2 and $3k$, 1, -5 respectively.

It is known that two lines with direction ratios, a_1, b_1, c_1 and a_2, b_2, c_2, are perpendicular, if $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$$\therefore -3(3k) + 2k \times 1 + 2(-5) = 0$$

$$\Rightarrow -9k + 2k - 10 = 0$$

$$\Rightarrow 7k = -10$$

$$\Rightarrow k = \frac{-10}{7}$$

Therefore, for $k = -\frac{10}{7}$, the given lines are perpendicular to each other.

Straight Line in Space Ex 28.2 Q22

The coordinates of A, B, C, and D are (1, 2, 3), (4, 5, 7), (−4, 3, −6), and (2, 9, 2) respectively.

The direction ratios of AB are (4 − 1) = 3, (5 − 2) = 3, and (7 − 3) = 4

The direction ratios of CD are (2 −(− 4)) = 6, (9 − 3) = 6, and (2 −(−6)) = 8

It can be seen that, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{2}$

Therefore, AB is parallel to CD.

Thus, the angle between AB and CD is either 0° or 180°.

Straight Line in Space Ex 28.2 Q23

Given equation of line are,

$$\frac{x-5}{5\lambda+2}=\frac{2-y}{5}=\frac{1-z}{-1} \text{ and}$$

$$\frac{x}{1}=\frac{2y+1}{4\lambda}=\frac{1-z}{-3}$$

$$\Rightarrow \quad \frac{x-5}{5\lambda+2}=\frac{y-2}{-5}=\frac{z-1}{1} \quad ---(1)$$

$$\text{and,} \quad \frac{x}{1}=\frac{y+\frac{1}{2}}{2\lambda}=\frac{z-1}{3} \quad ---(2)$$

Given that line (1) and (2) are perpendicular,

So,

$$a_1a_2+b_1b_2+c_1c_2=0$$

$$(5\lambda+2)(1)+(-5)(2\lambda)+(1)(3)=0$$

$$5\lambda+2-10\lambda+3=0$$

$$-5\lambda+5=0$$

$$\lambda=\frac{5}{5}$$

$$\lambda=1$$

Straight Line in Space Ex 28.2 Q24

The direction ratios of the line are

$$\frac{x+2}{2}=\frac{2y-7}{6}=\frac{5-z}{6}$$

2,6,6

The direction cosines of the line are

$$l=\frac{2}{\sqrt{2^2+6^2+6^2}}=\frac{2}{\sqrt{76}}$$

$$m=\frac{6}{\sqrt{2^2+6^2+6^2}}=\frac{6}{\sqrt{76}}$$

$$n=\frac{6}{\sqrt{2^2+6^2+6^2}}=\frac{6}{\sqrt{76}}$$

$$\left(\frac{2}{\sqrt{76}},\frac{6}{\sqrt{76}},\frac{6}{\sqrt{76}}\right)$$

$\therefore$ Vector equation of the line is

$$\vec{r}=\vec{a}+\lambda\vec{b}$$

$$\vec{r}=(-\vec{i}+2\vec{j}+3\vec{k})+\lambda(2\vec{i}+6\vec{j}+6\vec{k})$$

Ex 28.3

Straight Line in Space Ex 28.3 Q1

We have equation of first line,

$$\frac{x}{1} = \frac{y-2}{2} = \frac{z+3}{3} = \lambda \ (\text{Say}) \qquad ---(1)$$

General point on line (1) is

$$(\lambda,\ 2\lambda+2,\ 3\lambda-3)$$

Another line is,

$$\frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4} = \mu \ (\text{Say}) \qquad ---(2)$$

General point on line (2) is

$$(2\mu+2,\ 3\mu+6,\ 4\mu+3)$$

If lines (1) and (2) intersect then they have a common point, so for same value of λ and μ, we must have,

$$\lambda = 2\mu + 2 \Rightarrow \lambda - 2\mu = 2 \qquad ---(3)$$
$$2\lambda + 2 = 3\mu + 6 \Rightarrow 2\lambda - 4\mu = 4 \qquad ---(4)$$
$$3\lambda - 3 = 4\mu + 3 \Rightarrow 3\lambda - 4\mu = 6 \qquad ---(5)$$

Now, solving equation (3) and (4) to get λ and μ,

$$\begin{array}{r} 2\lambda - 4\mu = 4 \\ 2\lambda - 3\mu = 4 \\ (-) \quad (+) \quad (-) \\ \hline -\mu = 0 \end{array}$$

$$\Rightarrow \quad \mu = 0$$

Put $\mu = 0$ in equation (3),

$$\lambda - 2\mu = 2$$
$$\lambda - 2(0) = 2$$
$$\lambda = 2$$

Put λ and μ in equation (5),

$$3\lambda - 4\mu = 6$$
$$3(2) - 4(0) = 6$$
$$6 = 6$$

LHS = RHS

Straight Line in Space Ex 28.3 Q2

We have equation of first line,

$$\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-1}{5} = \lambda \ (\text{Say}) \qquad ---(1)$$

General point on line (1) is

$$(3\lambda+1,\ 2\lambda-1,\ 5\lambda+1)$$

Another line is,

$$\frac{x+2}{4} = \frac{y-1}{3} = \frac{z+1}{-2} = \mu \ (\text{Say}) \qquad ---(2)$$

General point on line (2) is,

$$(4\mu-2,\ 3\mu+1,\ -2\mu-1)$$

If lines (1) and (2) intersect, then they have a common point, so for same value of λ and μ, we must have,

$$3\lambda+1 = 4\mu-2 \Rightarrow 3\lambda-4\mu = -3 \qquad ---(3)$$

$$2\lambda-1 = 3\mu+1 \Rightarrow 2\lambda-3\mu = 2 \qquad ---(4)$$

$$5\lambda+1 = -2\mu-1 \Rightarrow 5\lambda+2\mu = -2 \qquad ---(5)$$

Solving equation (3) and (4) to get λ and μ,

$$\begin{array}{r} 6\lambda-8\mu = -6 \\ 6\lambda-9\mu = 6 \\ (-)\quad (+)\quad (-) \\ \hline \mu = -12 \end{array}$$

Put the value of μ in equation (3),

$$3\lambda-4(-12) = -3$$
$$3\lambda+48 = -3$$
$$3\lambda = -3-48$$
$$3\lambda = -51$$
$$\lambda = \frac{-51}{3}$$
$$\lambda = -17$$

Put the value of λ and μ in equation (5),

$$5\lambda+2\mu = -2$$
$$5(-17)+2(-12) = -2$$
$$-85-24 = -2$$
$$-109 \neq -2$$
$$\text{LHS} \neq \text{RHS}$$

Straight Line in Space Ex 28.3 Q3

Given equation of first line is

$$\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7} = \lambda \ (\text{Say}) \qquad ---(1)$$

General point on line (1) is

$$(3\lambda - 1,\ 5\lambda - 3,\ 7\lambda - 5)$$

Another equation of line is

$$\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5} = \mu \ (\text{Say}) \qquad ---(2)$$

General point on line (2) is,

$$(\mu + 2,\ 3\mu + 4,\ 5\mu + 6)$$

If lines (1) and (2) are intersecting then, they have a common point. So for same value of λ and μ, we must have,

$$3\lambda - 1 = \mu + 2 \Rightarrow 3\lambda - \mu = 3 \qquad ---(3)$$

$$5\lambda - 3 = 3\mu + 4 \Rightarrow 5\lambda - 3\mu = 7 \qquad ---(4)$$

$$7\lambda - 5 = 5\mu + 6 \Rightarrow 7\lambda - 5\mu = 11 \qquad ---(5)$$

Solving equation (3) and (4) to get λ and μ,

$$\begin{array}{r} 15\lambda - 5\mu = 15 \\ 15\lambda - 9\mu = 21 \\ (-) \quad (+) \quad (-) \\ \hline 4\mu = -6 \end{array}$$

$$\mu = \frac{-3}{2}$$

Put the value of μ in equation (3),

$$3\lambda - \mu = 3$$

$$3\lambda - \left(-\frac{3}{2}\right) = 3$$

$$3\lambda = 3 - \frac{3}{2}$$

$$\lambda = \frac{1}{2}$$

Put the value of λ and μ in equation (5),

$$7\lambda - 5\mu = 11$$

$$7\left(\frac{1}{2}\right) - 5\left(-\frac{3}{2}\right) = 11$$

$$\frac{7}{2} + \frac{15}{2} = 11$$

$$\frac{22}{2} = 11$$

$$11 = 11$$

$$\text{LHS} \neq \text{RHS}$$

Since, the values of λ and μ obtained by solving (3) and (4) satisfy equation (5), Hence

Given lines intersect each other.

Point of intersection $= (3\lambda - 1,\ 5\lambda - 3,\ 7\lambda - 5)$

$$= \left\{\frac{3}{2} - 1, \left(\frac{5}{2} - 3\right), \left(\frac{7}{2} - 5\right)\right\}$$

$$= \left(\frac{1}{2}, \frac{-1}{2}, \frac{-3}{2}\right)$$

Point of intersection is $\left(\frac{1}{2}, \frac{-1}{2}, \frac{-3}{2}\right)$.

Straight Line in Space Ex 28.3 Q4

Equation of the line passing through $A(0,-1,-1)$ and $B(4,5,1)$ is given by

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$
$$\frac{x-0}{4-0}=\frac{y+1}{5+1}=\frac{z+1}{1+1}$$
$$\frac{x}{4}=\frac{y+1}{6}=\frac{z+1}{2}=\lambda \text{ (say)}$$

So, general point on line AB is

$$(4\lambda,\ 4\lambda,\ 2\lambda-1)$$

Now, equation of the line passing through $C(3,9,4)$ and $D(-4,4,4)$ is

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$
$$\frac{x-3}{-4-3}=\frac{y-9}{4-9}=\frac{z-4}{4-4}$$
$$\frac{x-3}{-7}=\frac{y-9}{-5}=\frac{z-4}{0}=\mu \text{ (say)}$$

So, general point on line CD is

$$(-7\mu+3,\ -5\mu+9,\ 0.\mu+4)$$
$$(-7\mu+3,\ -5\mu+9,\ 4)$$

If lines AB and CD intersect, there must be a common point to them. So we have to find λ and μ such that

$$4\lambda=-7\mu+3 \quad \Rightarrow 4\lambda+7\mu=3 \quad ---(1)$$
$$6\lambda-1=-5\mu+9 \quad \Rightarrow 6\lambda+5\mu=10 \quad ---(2)$$
$$2\lambda-1=4 \quad \Rightarrow 2\lambda-1=4 \quad ---(3)$$

From equation (3),

$$2\lambda=4+1$$
$$\lambda=\frac{5}{2}$$

Put $\lambda=\frac{5}{2}$ in equation (2),

$$6\left(\frac{5}{2}\right)+5\mu=10$$
$$5\mu=10-15$$
$$5\mu=-5$$
$$\mu=-1$$

Now, put values of λ and μ in equation (1),

$$4\lambda+7(\mu)=3$$
$$4\left(\frac{5}{2}\right)+7(-1)=3$$
$$10-7=3$$
$$3=3$$

$$\text{LHS} \ne \text{RHS}$$

Since, the values of λ and μ by solving (2) and (3) satisfy equation (1), so

Line AB and CD are intersecting lines

Point of intersection of AB and CD

$$=(-7\mu+3,\ -5\mu+9,\ 4)$$
$$=(-7(-1)+3,\ -5(-1)+9,\ 4)$$
$$=(7+3,\ 5+9,\ 4)$$
$$=(10,14,4)$$

So, point of intersection of AB and CD $=(10,14,4)$.

Straight Line in Space Ex 28.3 Q5

Given equations of lines are

$$\vec{r} = \left(\hat{i} + \hat{j} - \hat{k}\right) + \lambda\left(3\hat{i} - \hat{j}\right)$$

$$\vec{r} = \left(4\hat{i} - \hat{k}\right) + \mu\left(2\hat{i} + 3\hat{k}\right)$$

If these lines intersect, they must have a common point, so, for some value of λ and μ we must have,

$$\left(\hat{i} + \hat{j} - \hat{k}\right) + \lambda\left(3\hat{i} - \hat{j}\right) = \left(4\hat{i} - \hat{k}\right) + \mu\left(2\hat{i} + 3\hat{k}\right)$$

$$(1 + 3\lambda)\hat{i} + (1 - \lambda)\hat{j} - \hat{k} = (4 + 2\mu)\hat{i} + (-1 + 3\mu)\hat{k}$$

Equation the coefficients of $\hat{i}, \hat{j}, \hat{k}$, we get

$$1 + 3\lambda = 4 + 2\mu \qquad \Rightarrow 3\lambda - 2\mu = 3 \qquad ---(1)$$

$$1 - \lambda = 0 \qquad \Rightarrow \lambda = 1 \qquad ---(2)$$

$$-1 = -1 + 3\mu \qquad \Rightarrow \mu = 0 \qquad ---(3)$$

Put the value of λ and μ in equation (1),

$$3\lambda - 2\mu = 3$$

$$3(1) - 2(0) = 3$$

$$3 = 3$$

$$\text{LHS} = \text{RHS}$$

The value of λ and μ satisfy equation (1), so

Lines are intersecting.

Put value of λ in equation (1) to get point of intersection

$$\vec{r} = \left(\hat{i} + \hat{j} - \hat{k}\right) + (1)\left(3\hat{i} - \hat{j}\right)$$

$$= \hat{i} + \hat{j} - \hat{k} + 3\hat{i} - \hat{j}$$

$$= 4\hat{i} - \hat{k}$$

So, point of intersection is $(4, 0, -1)$.

Straight Line in Space Ex 28.3 Q6(i)

Given equations of lines are

$$\vec{r} = \left(\hat{i} - \hat{j}\right) + \lambda\left(2\hat{i} + \hat{k}\right) \text{ and}$$

$$\vec{r} = \left(2\hat{i} - \hat{j}\right) + \mu\left(\hat{i} + \hat{i} - \hat{k}\right)$$

If these lines intersect each other, there must be some common point, so, we must have λ and μ such that

$$\left(\hat{i} - \hat{j}\right) + \lambda\left(2\hat{i} + \hat{k}\right) = \left(2\hat{i} - \hat{j}\right) + \mu\left(\hat{i} + \hat{i} - \hat{k}\right)$$

$$(1 + 2\lambda)\hat{i} - \hat{j} + \lambda\hat{k} = (2 + \mu)\hat{i} + (-1 + \mu)\hat{j} - \mu\hat{k}$$

Equation the coefficients of $\hat{i}, \hat{j}$ and $\hat{k}$,

$$1 + 2\lambda = 2 + \mu \qquad \Rightarrow 2\lambda - \mu = 1 \qquad ---(1)$$

$$-1 = -1 + \mu \qquad \Rightarrow \mu = 0 \qquad ---(2)$$

$$\lambda = -\mu \qquad \Rightarrow \lambda = 0 \qquad ---(3)$$

Put value of λ and μ in equation (1),

$$2\lambda - \mu = 1$$

$$2(0) - (0) = 1$$

$$0 = 1$$

$$\text{LHS} \neq \text{RHS}$$

Since, the values of λ and μ form equation (2) and (3) does not satisfy equation (1),

Hence, given lines do not intersect each other.

Straight Line in Space Ex 28.3 Q6(ii)

Given, equations of first line is

$$\frac{x-1}{2}=\frac{y+1}{3}=\frac{z}{1}=\lambda \text{ (say)} \qquad ---(1)$$

General point on line (1) is

$$(2\lambda+1,\ 3\lambda-1,\ \lambda)$$

Another equation of line is

$$\frac{x-1}{5}=\frac{y-2}{1},\ z=3 \qquad ---(2)$$

$$\frac{x-1}{5}=\frac{y-2}{1}=\mu, \text{ (say)},\ z=3$$

General point on line (2) is

$$(5\mu+1,\ \mu+2,\ 3)$$

If line (1) and (2) intersect each other then, there is a common point to them, so, we must have value of λ and μ such that

$$2\lambda+1=5\mu+1 \quad \Rightarrow 2\lambda-5\mu=0 \qquad ---(3)$$

$$3\lambda-1=\mu+2 \quad \Rightarrow 3\lambda-\mu=3 \qquad ---(4)$$

$$\lambda=3 \quad \Rightarrow \lambda=3 \qquad ---(5)$$

Put value of λ in equation (4),

$$3\lambda-\mu=3$$
$$3(3)-\mu=3$$
$$-\mu=3-9$$
$$\mu=6$$

Put the value of λ and μ in equation (3), so

$$2\lambda-5\mu=0$$
$$2(3)-5(6)=0$$
$$6-30=0$$
$$-24\neq 0$$
$$\text{LHS}\neq\text{RHS}$$

Since the values of λ and μ obtained from equation (4) and (5) does not satisfy equation (3), so,

Given lines are not intersecting.

Straight Line in Space Ex 28.3 Q6(iii)

Given, equations of first line is,

$$\frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0} = \lambda \ (\text{say}) \qquad ---(1)$$

General point on line (1) is,

$$(3\lambda + 1, -\lambda + 1, -1)$$

Another equation of line is

$$\frac{x-4}{2} = \frac{y-0}{0} = \frac{z+1}{3} = \mu \ (\text{say}) \qquad ---(2)$$

General point on line (2) is,

$$(2\mu + 4, 0, 3\mu - 1)$$

If line (1) and (2) intersecting then there must be a common point, so, we must have the value of λ and μ as

$$3\lambda + 1 = 2\mu + 4 \quad \Rightarrow 3\lambda - 2\mu = 3 \qquad ---(1)$$

$$-\lambda + 1 = 0 \quad \Rightarrow \lambda = 1 \qquad ---(2)$$

$$3\mu - 1 = -1 \quad \Rightarrow \mu = 0 \qquad ---(3)$$

Put the value of λ and μ in equation (1), so

$$3\lambda - 2\mu = 3$$

$$3(1) - 2(0) = 3$$

$$3 = 3$$

$$\text{LHS} \neq \text{RHS}$$

Since the values of λ and μ obtained by equation (2) and (3) satisfy equation (1), so,

Given lines are intersecting.

Straight Line in Space Ex 28.3 Q6(iv)

Given, equation of line is

$$\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5} = \lambda \text{ (say)} \qquad ---(1)$$

General point on line (1) is,

$$(4\lambda + 5,\ 4\lambda + 7,\ -5\lambda - 3)$$

Another equation of line is,

$$\frac{x-8}{7} = \frac{y-4}{1} = \frac{z-5}{3} = \mu \text{ (say)} \qquad ---(2)$$

General point on line (2) is

$$(7\mu + 8,\ \mu + 4,\ 3\mu + 5)$$

If line (1) and (2) intersecting, then there must have some common point to them, so, we must have value of λ and μ such that

$$4\lambda + 5 = 7\mu + 8 \quad \Rightarrow 4\lambda - 7\mu = 3 \qquad ---(3)$$

$$4\lambda + 5 = \mu + 4 \quad \Rightarrow 4\lambda - \mu = -3 \qquad ---(4)$$

$$-5\lambda - 3 = 3\mu + 5 \quad \Rightarrow -5\lambda - 3\mu = 8 \qquad ---(5)$$

Solving equation (3) and (4) to find λ and μ,

$$4\lambda - 7\mu = 3$$

$$\underset{(-)}{4\lambda} \underset{(+)}{-} \mu \underset{(-)}{=} -3$$

$$-6\mu = 6$$

$$\mu = -1$$

Put value of λ in equation (3),

$$4\lambda - 7\mu = 3$$

$$4\lambda - 7(-1) = 3$$

$$4\lambda = 3 - 7$$

$$\lambda = -1$$

Put the value of λ and μ in equation (5),

$$-5\lambda - 3\mu = 8$$

$$-5(-1) - 3(-1) = 8$$

$$5 + 3 = 8$$

$$\text{LHS} = \text{RHS}$$

Straight Line in Space Ex 28.3 Q7

$\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$

$\vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$

If the lines intersect eachother, then the shortest distance between the lines should be zero.

Here,

$\vec{a_1} = 3\hat{i} + 2\hat{j} - 4\hat{k}$

$\vec{a_2} = 5\hat{i} - 2\hat{j}$

$\vec{b_1} = \hat{i} + 2\hat{j} + 2\hat{k}$

$\vec{b_2} = 3\hat{i} + 2\hat{j} + 6\hat{k}$

$$(\vec{b_1} \times \vec{b_2}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 3 & 2 & 6 \end{vmatrix}$$

$= \vec{i}(12 - 4) - \vec{j}(6 - 6) + \vec{k}(2 - 6)$

$= 8\vec{i} - 0\vec{j} - 4\vec{k}$

$(\vec{a_2} - \vec{a_1}) = (5\hat{i} - 2\hat{j} - 3\hat{i} - 2\hat{j} + 4\hat{k}) = (2\hat{i} - 4\hat{j} + 4\hat{k})$

$$\text{Shortest Distance, d} = \left|\frac{(\vec{b_1} \times \vec{b_2}).(\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|}\right|$$

$$= \left|\frac{(8\vec{i} - 0\vec{j} - 4\vec{k}).(2\hat{i} - 4\hat{j} + 4\hat{k})}{|8\vec{i} - 0\vec{j} - 4\vec{k}|}\right|$$

$$= \left|\frac{8 \times 2 - 0 \times 4 + (-4) \times 4}{|8\vec{i} - 0\vec{j} - 4\vec{k}|}\right|$$

$$= \left|\frac{0}{|8\vec{i} - 0\vec{j} - 4\vec{k}|}\right| = 0$$

Since the shortest distance is zero, the lines are intersect each other.

Point of intersection of the lines,

$\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$

$\vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$

Lines in the Cartesian form,

$$\frac{x-3}{1} = \frac{y-2}{2} = \frac{z+4}{2} = \lambda$$

$x = \lambda + 3, y = 2\lambda + 2, z = 2\lambda - 4$

$$\frac{x-5}{3} = \frac{y+2}{2} = \frac{z}{6} = \mu$$

$x = 3\mu + 5, y = 2\mu - 2, z = 6\mu$

From coordinates of x,

$\lambda + 3 = 3\mu + 5$

$\lambda = 3\mu + 2$.....(i)

From coordinates of y,

$2\lambda + 2 = 2\mu - 2$

$\lambda = \mu - 2$.......(ii)

Solving (i) and (ii),

$\lambda = -4, \mu = -2$

Coordinates of the point of intersection,

$x = 3(-2) + 5, y = 2(-2) - 2, z = 6(-2)$

$x = -1, y = -6, z = -12$

$(-1, -6, -12)$

Ex 28.4

Straight Line in Space Ex 28.4 Q1

Let the foot of the perpendicular drawn from $P(3,-1,11)$ to the line

$\frac{x}{2}=\frac{y-2}{-3}=\frac{z-3}{4}$ is Q, so we have to find length of PQ.Q is general point on the line

$$\frac{x}{2}=\frac{y-2}{-3}=\frac{z-3}{4}=\lambda \quad (say)$$

Co-ordinate of $Q=(2\lambda,-3\lambda+2,4\lambda+3)$

Direction ratios of the given line $=2.-3,4$

Since PQ is perpendicular to the given line therefore

$$\begin{aligned}
&a1a2+b1b2+c1c2=0\\
&\Rightarrow 2(2\lambda-3)+(-3)(-3\lambda+3)+4(4\lambda-8)=0\\
&\Rightarrow 4\lambda-6+9\lambda-9+16\lambda-32=0\\
&\Rightarrow 29\lambda-47=0\\
&\Rightarrow \lambda=\frac{47}{29}
\end{aligned}$$

Therefore co-ordinates of Q

$$=2\left(\frac{47}{29}\right),-3\left(\frac{47}{29}\right)+2,4\left(\frac{47}{29}\right)+3$$

$$=\frac{94}{29},\frac{-83}{29},\frac{275}{29}$$

Distance between P and Q is

Straight Line in Space Ex 28.4 Q2

Let foot of the perpendicular drawn from the point $P(1,0,0)$ to the line $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$ is Q. We have to find length of PQ.

Q is a general point on the line,

$$\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}=\lambda \quad (say)$$

Coordinate of Q $=(2\lambda+1,-3\lambda-1,8\lambda-10)$

Direction ratios line PQ are

$$=(2\lambda+1-1),\ (-3\lambda-1-0),\ (8\lambda-10-0)$$

$$\Rightarrow \quad =(2\lambda),\ (-3\lambda-1),\ (8\lambda-10)$$

Since, line PQ is perpendicular to the given line, so

$$\begin{aligned}
&a_1a_2+b_1b_2+c_1c_2=0\\
&(2)(2\lambda)+(-3)(-3\lambda-1)+8(8\lambda-10)=0\\
&4\lambda+9\lambda+3+64\lambda-80=0\\
&77\lambda-77=0\\
&\lambda=1
\end{aligned}$$

Therefore, coordinate of Q is $(2\lambda+1,\ -3\lambda-1,\ 8\lambda-10)$

$$=(2(1)+1,\ -3(1)-1,\ 8(1)-10)$$
$$=(3,-4,-2)$$

Therefore, coordinate of Q is $(2\lambda+1,\ -3\lambda-1,\ 8\lambda-10)$

$$=(2(1)+1,\ -3(1)-1,\ 8(1)-10)$$
$$=(3,-4,-2)$$

$$\begin{aligned}
PQ&=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}\\
&=\sqrt{(1-3)^2+(0+4)^2+(0+2)^2}\\
&=\sqrt{4+16+4}\\
&=\sqrt{24}\\
&=2\sqrt{6}
\end{aligned}$$

So, foot of perpendicular $=(3,-4,-2)$

length of perpendicular $=2\sqrt{6}$ units

Straight Line in Space Ex 28.4 Q3

Let the foot of the perpendicular drawn from $A(1,0,3)$ to the line joining the points $B(4,7,1)$

And $C(3,5,3)$ be D

Equation of line passing through $B(4,7,1)$ and $C(3,5,3)$ is

$$\frac{x-x1}{x2-x1}=\frac{y-y1}{y2-y1}=\frac{z-z1}{z2-z1}$$
$$\Rightarrow \frac{x-4}{3-4}=\frac{y-7}{5-7}=\frac{z-1}{3-1}$$
$$\Rightarrow \frac{x-4}{-1}=\frac{y-7}{-2}=\frac{z-1}{2}=\lambda \text{ (say)}$$

Direction ratio of AD are

$$(-\lambda+4-1),(-2\lambda+7-0),(2\lambda+1-3)$$
$$=(-\lambda+3),(-2\lambda+7),(2\lambda-2)$$

Line AD is perpendicular to BC so

$$a1a2+b1b2+c1c2=0$$
$$\Rightarrow(-1)(-\lambda+3)+(-2)(-2\lambda+7)+2(2\lambda-2)=0$$
$$\Rightarrow \lambda-3+4\lambda-14+4\lambda-4=0$$
$$\Rightarrow 9\lambda-21=0$$
$$\Rightarrow \lambda=\frac{21}{9}$$

Co-ordinates of D are

$$=\left(-\frac{21}{9}+4,(-2)\left(\frac{21}{9}+7\right),2\left(\frac{21}{9}+1\right)\right)$$
$$=\left(\frac{15}{9},\frac{21}{9},\frac{51}{9}\right)$$
$$=\left(\frac{5}{3},\frac{7}{3},\frac{17}{3}\right)$$

Straight Line in Space Ex 28.4 Q4

Given that D is the foot of perpendicular from $A(1,0,4)$ on BC, so

Equation of line passing through B,C is

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$

$$\Rightarrow \quad \frac{x-0}{2-0}=\frac{y+11}{-3+11}=\frac{z-3}{1-3}$$

$$\Rightarrow \quad \frac{x}{2}=\frac{y+11}{8}=\frac{z-3}{-2}=\lambda \text{ (say)}$$

Coordinate of $D=(2\lambda,\ 8\lambda-11,\ -2\lambda+3)$

Direction ratios of $AD = 2\lambda-1,\ 8\lambda-11-0,\ -2\lambda+3-4$

$$=(2\lambda-1),\ (8\lambda-11),\ (-2\lambda-1)$$

Since, line AD is perpendicular on BC, so

$$a_1a_2+b_1b_2+c_1c_2=0$$

$$\Rightarrow \quad (2)(2\lambda-1)+(8)(8\lambda-11)+(-2)(-2\lambda-1)=0$$
$$\Rightarrow \quad 4\lambda-2+64\lambda-88+4\lambda+2=0$$
$$\Rightarrow \quad 72\lambda-88=0$$
$$\Rightarrow \quad \lambda=\frac{88}{72}$$
$$\lambda=\frac{11}{9}$$

Coordinate of $D=(2\lambda,\ 8\lambda-11,\ -2\lambda+3)$

$$=\left(2\left(\frac{11}{9}\right),\ 8\left(\frac{11}{9}\right)-11,\ -2\left(\frac{11}{9}\right)+3\right)$$

Coordinate of $D=\left(\frac{22}{9},\ \frac{-11}{9},\ \frac{5}{9}\right)$

Straight Line in Space Ex 28.4 Q5

Let foot of the perpendicular from $P(2,3,4)$ is θ on the line $\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$, so

Equation of given line is,

$$\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$$

$$\frac{x-4}{-2}=\frac{y}{6}=\frac{z-1}{-3}=\lambda \quad \text{(say)}$$

Coordinate of $Q = (-2\lambda+4,\ 6\lambda,\ -3\lambda+1)$

Direction ratios of $PQ = (-2\lambda+4-2),\ (6\lambda-3),\ (-3\lambda+1-4)$

$$= (-2\lambda+2),\ (6\lambda-3),\ (-3\lambda-3)$$

Line PQ is perpendicular to given line, so

$$a_1a_2+b_1b_2+c_1c_2=0$$

$$(-2)(-2\lambda+2)+(6)(6\lambda-3)+(-3)(-3\lambda-3)=0$$

$$4\lambda-4+36\lambda-18+9\lambda+9=0$$

$$49\lambda-13=0$$

$$\lambda=\frac{13}{49}$$

Coordinate of $Q = (-2\lambda+4,\ 6\lambda,\ -3\lambda+1)$

$$=\left(-2\left(\frac{13}{49}\right)+4,\ 6\left(\frac{13}{49}\right),\ -3\left(\frac{13}{49}\right)+1\right)$$

$$=\left(\frac{-26+196}{49},\ \frac{78}{49},\ \frac{-39+49}{49}\right)$$

Coordinate of $Q=\left(\frac{170}{49},\ \frac{78}{49},\ \frac{10}{49}\right)$

$$PQ=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$$

$$=\sqrt{\left(\frac{170}{49}-2\right)^2+\left(\frac{78}{49}-3\right)^2+\left(\frac{10}{49}-4\right)^2}$$

$$=\sqrt{\left(\frac{72}{49}\right)^2+\left(\frac{69}{49}\right)^2+\left(-\frac{168}{49}\right)^2}$$

$$=\sqrt{\frac{5184+4761+34596}{2401}}$$

$$=\sqrt{\frac{44541}{2401}}$$

$$=\sqrt{\frac{909}{49}}$$

$$=\frac{3\sqrt{101}}{49}$$

Perpendicular distance from $(2,3,4)$ to given line is $\frac{3\sqrt{101}}{49}$ units.

Straight Line in Space Ex 28.4 Q6

Let θ be the foot of the perpendicular drawn from $P(2,4,-1)$ to the line

$$\frac{x+5}{1}=\frac{y+3}{4}=\frac{z-6}{-9}$$

Given line is $\frac{x+5}{1}=\frac{y+3}{4}=\frac{z-6}{-9}=\lambda$ (say)

Coordinate of Q (General point on the line)

$$=(\lambda-5,\ 4\lambda-3,\ -9\lambda+6)$$

Direction ratios of PQ are

$$=(\lambda-5-2),\ (4\lambda-3-4),\ (-9\lambda+6+1)$$
$$=\lambda-7,\ 4\lambda-7,\ -9\lambda+7$$

Line PQ is perpendicular to the given line, so

$$a_1a_2+b_1b_2+c_1c_2=0$$
$$(1)(\lambda-7)+(4)(4\lambda-7)+(-9)(-9\lambda+7)=0$$
$$\lambda-7+16\lambda-28+81\lambda-63=0$$
$$98\lambda-98=0$$
$$\lambda=1$$

$\therefore$ Coordinate of $Q=(\lambda-5,\ 4\lambda-3,\ -9\lambda+6)$

$$=(1-5,\ 4(1)-3,\ -9(1)+6)$$

Coordinate of foot of perpendicular $=(-4,1,-3)$

So, equation of the perpendicular PQ is

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$

$$\Rightarrow\quad \frac{x-2}{-4-2}=\frac{y-4}{1-4}=\frac{z+1}{-3+1}$$

$$\Rightarrow\quad \frac{x-2}{-6}=\frac{y-4}{-3}=\frac{z+1}{-2}$$

Straight Line in Space Ex 28.4 Q7

Let foot of the perpendicular drawn from $P(5,4,-1)$ to the given line is Q, so

Given equation of line is,

$$\vec{r}=\hat{i}+\lambda(2\hat{i}+9\hat{j}+5\hat{k})$$
$$(x\hat{i}+y\hat{j}+z\hat{k})=(1+2\lambda)\hat{i}+(9\lambda)\hat{j}+(5\lambda)\hat{k}$$

Equation the coefficients of $\hat{i},\hat{j}$ and $\hat{k}$

$$\Rightarrow\quad x=1+2\lambda,\ y=9\lambda,\ z=5\lambda$$
$$\Rightarrow\quad \frac{x-1}{2}=\lambda,\ \frac{y}{9}=\lambda,\ \frac{z}{5}=\lambda$$
$$\Rightarrow\quad \frac{x-1}{2}=\frac{y}{9}=\frac{z}{5}=\lambda\ \text{(say)}$$

Coordinate of $Q=(2\lambda+1,\ 9\lambda,\ 5\lambda)$

Direction ratios of line PQ are

$$(2\lambda+1-5),\ 9\lambda-4,\ 5\lambda+1$$

$$\Rightarrow\quad 2\lambda-4,\ 9\lambda-4,\ 5\lambda+1$$

Straight Line in Space Ex 28.4 Q8

Let position vector of foot of perpendicular drown from $P\left(\hat{i}+6\hat{j}+3\hat{k}\right)$ on

$\vec{r}=\left(\hat{j}+2\hat{k}\right)+\lambda\left(\hat{i}+2\hat{j}+3\hat{k}\right)$ be $Q\left(\vec{q}\right)$. So

Q is on the line $\vec{r}=\left(\hat{j}+2\hat{k}\right)+\lambda\left(\hat{i}+2\hat{j}+3\hat{k}\right)$

So, Position vector of Q = $(\lambda)\hat{i}+(1+2\lambda)\hat{j}+(2+3\lambda)\hat{k}$

$$\begin{aligned}
\overrightarrow{PQ} &= \text{Position vector of Q} - \text{Position vector of } P\\
&= \left\{\lambda\hat{i}+(1+2\lambda)\hat{j}+(2+3\lambda)\hat{k}\right\}-\left(\hat{i}+6\hat{j}+3\hat{k}\right)\\
&= (\lambda-1)\hat{i}+(1+2\lambda-6)\hat{j}+(2+3\lambda-3)\hat{k}\\
\overrightarrow{PQ} &= (\lambda-1)\hat{i}+(2\lambda-5)\hat{j}+(3\lambda-1)\hat{k}
\end{aligned}$$

Here, $\overrightarrow{PQ}$ is perpendicular to given line

So,

$$\left\{(\lambda-1)\hat{i}+(2\lambda-5)\hat{j}+(3\lambda-1)\hat{k}\right\}\left(\hat{i}+2\hat{j}+3\hat{k}\right)=0$$

$\Rightarrow \quad (\lambda-1)(1)+(2\lambda-5)(2)+(3\lambda-1)(3)=0$

$\Rightarrow \quad \lambda-1+4\lambda-10+9\lambda-3=0$

$\Rightarrow \quad 14\lambda-14=0$

$\Rightarrow \quad \lambda=1$

$$\begin{aligned}
\text{Position vector of Q} &= \left(\hat{j}+2\hat{k}\right)+\lambda\left(\hat{i}+2\hat{j}+3\hat{k}\right)\\
&= \left(\hat{j}+2\hat{k}\right)+(1)\left(\hat{i}+2\hat{j}+3\hat{k}\right)
\end{aligned}$$

Foot of perpendicular $=\hat{i}+3\hat{j}+5\hat{k}$

$$\begin{aligned}
\overrightarrow{PQ} &= \text{Position vector of Q} - \text{Position vector of } P\\
&= \left(\hat{i}+3\hat{j}+5\hat{k}\right)-\left(\hat{i}+6\hat{j}+3\hat{k}\right)\\
&= \hat{i}+3\hat{j}+5\hat{k}-\hat{i}-6\hat{j}-3\hat{k}\\
&= -3\hat{i}+2\hat{k}
\end{aligned}$$

$$\begin{aligned}
\left|\overrightarrow{PQ}\right| &= \sqrt{(-3)^2+(2)^2}\\
&= \sqrt{13}\text{ units}
\end{aligned}$$

Length of perpendicular $=\sqrt{13}$ units

Straight Line in Space Ex 28.4 Q9

Let Q be the perpendicular drown from $P\left(-\hat{i}+3\hat{j}+2\hat{k}\right)$ on the line

$$\vec{r}=\left(2\hat{j}+3\hat{k}\right)+\lambda\left(2\hat{i}+\hat{j}+3\hat{k}\right)$$

Let the position vector of Q be

$$\left(2\hat{j}+3\hat{k}\right)+\lambda\left(2\hat{i}+\hat{j}+3\hat{k}\right)$$
$$(2\lambda)\hat{i}+(2+\lambda)\hat{j}+(3+3\lambda)\hat{k}$$

$$\overrightarrow{PQ}=\text{Position vector of }Q-\text{Position vector of }P$$
$$=\left\{(2\lambda)\hat{i}+(2+\lambda)\hat{j}+(3+3\lambda)\hat{k}\right\}-\left(-\hat{i}+3\hat{j}+2\hat{k}\right)$$
$$=(2\lambda+1)\hat{i}+(2\lambda-3)\hat{j}+(3+3\lambda-2)\hat{k}$$
$$\overrightarrow{PQ}=(2\lambda+1)\hat{i}+(\lambda-1)\hat{j}+(3\lambda+1)\hat{k}$$

Since, $\overrightarrow{PQ}$ is perpendicular to the given line, so

$$\left\{(2\lambda+1)\hat{i}+(\lambda-1)\hat{j}+(3\lambda+1)\hat{k}\right\}\left(2\hat{i}+\hat{j}+3\hat{k}\right)=0$$
$$(2\lambda+1)(2)+(\lambda-1)(1)+(3\lambda+1)3=0$$
$$4\lambda+2+\lambda-1+9\lambda+3=0$$
$$14\lambda+4=0$$
$$\lambda=-\frac{4}{14}$$
$$\lambda=-\frac{2}{7}$$

Position vector of Q $=(2\lambda)\hat{i}+(2+\lambda)\hat{j}+(3+3\lambda)\hat{k}$

$$=2\left(-\frac{2}{7}\right)\hat{i}+\left(2-\frac{2}{7}\right)\hat{j}+\left(3+3\left(-\frac{2}{7}\right)\right)\hat{k}$$
$$=-\frac{4}{7}\hat{i}+\frac{12}{7}\hat{j}+\frac{15}{7}\hat{k}$$

Coordinates of foot of the perpendicular $=\left(-\frac{4}{7},\frac{12}{7},\frac{15}{7}\right)$

Equation of PQ is

$$\vec{r}=\vec{a}+\lambda\left(\vec{b}-\vec{a}\right)$$

$$\Rightarrow\quad \vec{r}=\left(-\hat{i}+3\hat{j}+2\hat{k}\right)+\lambda\left(\left(-\frac{4}{7}\hat{i}+\frac{12}{7}\hat{j}+\frac{15}{7}\hat{k}\right)-\left(-\hat{i}+3\hat{j}+2\hat{k}\right)\right)$$

Straight Line in Space Ex 28.4 Q10

Let foot of the perpendicular drawn from $(0,2,7)$ to the line $\frac{x+2}{-1}=\frac{y-1}{3}=\frac{z-3}{-2}$ be Q.

Given equation of the line is

$$\frac{x+2}{-1}=\frac{y-1}{3}=\frac{z-3}{-2}=\lambda\ \text{(say)}$$

Coordinate of Q is $(-\lambda-2,\ 3\lambda+1,\ -2\lambda+3)$

Direction ratios of PQ are $(-\lambda-2-0),\ (3\lambda+1-2),\ (-2\lambda+3-7)$

$$=(-\lambda-2),\ (3\lambda-1),\ (-2\lambda-4)$$

Since, PQ is perpendicular to given line, so

$$a_1a_2+b_1b_2+c_1c_2=0$$
$$(-1)(-\lambda-2)+(3)(3\lambda-1)+(-2)(-2\lambda-4)=0$$

$$\Rightarrow\quad \lambda+2+9\lambda-3+4\lambda+8=0$$
$$\Rightarrow\quad 14\lambda+7=0$$
$$\lambda=-\frac{1}{2}$$

Foot of the perpendicular $=(-\lambda-2,\ 3\lambda+1,\ -2\lambda+3)$

$$=\left(-\left(-\frac{1}{2}\right)-2,\ 3\left(-\frac{1}{2}\right)+1,\ -2\left(-\frac{1}{2}\right)+3\right)$$

Foot of the perpendicular $=\left(-\frac{3}{2},-\frac{1}{2},4\right)$

Straight Line in Space Ex 28.4 Q11

Let foot of the perpendicular from $P(1, 2, -3)$ to the line $\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1}$ be Q

Given equation of the line is

$$\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1} = \lambda$$

$\Rightarrow \quad x = 2\lambda - 1,\ y = -2\lambda + 3,\ z = -\lambda$

Coordinate of Q $(2\lambda - 1,\ -2\lambda + 3,\ -\lambda)$

Direction ratios of PQ are

$$(2\lambda - 1 - 1),\ (-2\lambda + 3 - 2),\ (-\lambda + 3)$$

$\Rightarrow \quad (2\lambda - 2),\ (-2\lambda + 1),\ (-\lambda + 3)$

Let PQ is perpendicular to given line, so

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$\Rightarrow \quad (2)(2\lambda - 2) + (-2)(-2\lambda + 1) + (-1)(-\lambda + 3) = 0$
$\Rightarrow \quad 4\lambda - 4 + 4\lambda - 2 + \lambda - 3 = 0$
$\Rightarrow \quad 9\lambda - 9 = 0$
$\Rightarrow \quad \lambda = 1$

Coordinate of foot of perpendicular

$= (2\lambda - 1,\ -2\lambda + 3,\ -\lambda)$
$= (2(1) - 1,\ -2(1) + 3,\ -1)$

$= (1, 1, -1)$

Straight Line in Space Ex 28.4 Q12

Equation of line AB is

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

$\Rightarrow \quad \frac{x - 0}{-3 - 0} = \frac{y - 6}{-6 - 6} = \frac{z + 9}{3 + 9}$
$\Rightarrow \quad \frac{x}{-3} = \frac{y - 6}{-12} = \frac{z + 9}{12} = \lambda$ (say)

Coordinate of point $D = (-3\lambda,\ -12\lambda + 6, 12\lambda - 9)$

Direction ratios of $CD = (-3\lambda - 7),\ (-12\lambda + 6 - 4),\ (12\lambda - 9 + 1)$
$= (-3\lambda - 7),\ (-12\lambda + 2),\ (12\lambda - 8)$

Line CD is perpendicular to line AB, so

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$\Rightarrow \quad (-3)(-3\lambda - 7) + (-12)(-12\lambda + 2) + (12)(12\lambda - 8) = 0$
$\Rightarrow \quad 9\lambda + 21 + 144\lambda - 24 + 144\lambda - 96 = 0$
$\Rightarrow \quad 297\lambda - 99 = 0$
$\Rightarrow \quad \lambda = \frac{1}{3}$

Coordinate of $D = (-3\lambda,\ -12\lambda + 6, 12\lambda - 9)$

$$= \left(-3\left(\frac{1}{3}\right),\ -12\left(\frac{1}{3}\right) + 6, 12\left(\frac{1}{3}\right) - 9\right)$$

Coordinate of $D = (-1, 2, -5)$

Equation of CD is,

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

$$\Rightarrow \quad \frac{x - 7}{-1 - 7} = \frac{y - 4}{2 - 4} = \frac{z + 1}{-5 + 1}$$

$$\Rightarrow \quad \frac{x - 7}{-8} = \frac{y - 4}{-2} = \frac{z + 1}{-4}$$

$$\text{or} \quad \frac{x - 7}{4} = \frac{y - 4}{1} = \frac{z + 1}{2}$$

Straight Line in Space Ex 28.4 Q13

Let P ≡ (2, 4, -1).

In order to find the distance we need to find a point Q on the line.

We see that line is passing through the point Q(-5, -3, 6).

So,let take this point as required point.

Also line is parallel to the vector $\vec{b} = \hat{i} + 4\hat{j} - 9\hat{k}$.

Now, $\overrightarrow{PQ} = \left(-5\hat{i} - 3\hat{j} + 6\hat{k}\right) - \left(2\hat{i} + 4\hat{j} - \hat{k}\right) = -7\hat{i} - 7\hat{j} + 7\hat{k}$

$$\vec{b} \times \overrightarrow{PQ} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & -9 \\ -7 & -7 & 7 \end{vmatrix} = -35\hat{i} + 56\hat{j} + 21\hat{k}$$

$$\left|\vec{b} \times \overrightarrow{PQ}\right| = \sqrt{1225+3136+441} = \sqrt{4802}$$

$$\left|\vec{b}\right| = \sqrt{1 + 16 + 81} = \sqrt{98}$$

$$d = \frac{\left|\vec{b} \times \overrightarrow{PQ}\right|}{\left|\vec{b}\right|} = \frac{\sqrt{4802}}{\sqrt{98}} = 7$$

Straight Line in Space Ex 28.4 Q14

Let L be the foot of the perpendicular drawn from A(1, 8, 4) on the line joining the points B (0, -1, 3) and C (2, -3, -1).

Equation of the line passing through the points B (0, -1, 3) and C (2, -3, -1) is given by,

$$\vec{r} = \vec{b} + \lambda\left(\vec{c} - \vec{b}\right)$$

$$\vec{r} = (0 + 2\lambda)\hat{i} + (-1 - 2\lambda)\hat{j} + (3 - 4\lambda)\hat{k}$$

Let positon vector of L be,

$$\vec{r} = (2\lambda)\hat{i} + (-1 - 2\lambda)\hat{j} + (3 - 4\lambda)\hat{k} \text{ ....(i)}$$

Then, $\overrightarrow{AL}$ = Position vector of L - position vector of A

$$\Rightarrow \overrightarrow{AL} = (2\lambda)\hat{i} + (-1 - 2\lambda)\hat{j} + (3 - 4\lambda)\hat{k} - \left(1\hat{i} + 8\hat{j} + 4\hat{k}\right)$$

$$\Rightarrow \overrightarrow{AL} = (-1 + 2\lambda)\hat{i} + (-9 - 2\lambda)\hat{j} + (-1 - 4\lambda)\hat{k}$$

Since $\overrightarrow{AL}$ is perpendicular to the given line which is parallel to $\vec{b} = 2\hat{i} - 2\hat{j} - 4\hat{k}$

$\therefore \ \overrightarrow{AL} \cdot \vec{b} = 0$

$\Rightarrow 2(-1 + 2\lambda) - 2(-9 - 2\lambda) - 4(-1 - 4\lambda) = 0$

$\Rightarrow -2 + 4\lambda + 18 + 4\lambda + 4 + 16\lambda = 0$

$\Rightarrow 24\lambda = -20$

$\Rightarrow \lambda = \dfrac{-5}{6}$

Putting value of $\lambda = \dfrac{-5}{6}$ in (i) we get

$$\vec{r} = -\frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{19}{3}\hat{k}$$

Coordinates of the foot of the perpendicular are $\left(-\dfrac{5}{3}, \dfrac{2}{3}, \dfrac{19}{3}\right)$.

Ex 28.5

Straight Line in Space Ex 28.5 Q1(i)

We know that, shortest distance between lines

$\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ is given by

$$\text{S.D.} = \left|\frac{\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right)}{\left|\vec{b_1} \times \vec{b_2}\right|}\right| \qquad ---(i)$$

Given equations of lines are.

$$\vec{r} = 3\hat{i} + 8\hat{j} + 3\hat{k} + \lambda\left(3\hat{i} - \hat{j} + \hat{k}\right)$$
$$\vec{r} = \left(-3\hat{i} - 7\hat{j} + 6\hat{k}\right) + \mu\left(-3\hat{i} + 2\hat{j} + 4\hat{k}\right)$$

$$\Rightarrow \quad \vec{a_1} = \left(3\hat{i} + 8\hat{j} + 3\hat{k}\right),\ \vec{b_1} = 3\hat{i} - \hat{j} + \hat{k}$$
$$\vec{a_2} = \left(-3\hat{i} - 7\hat{j} + 6\hat{k}\right),\ \vec{b_2} = \left(-3\hat{i} + 2\hat{j} + 4\hat{k}\right)$$

Now,

$$\vec{a_2} - \vec{a_1} = \left(-3\hat{i} - 7\hat{j} + 6\hat{k}\right) - \left(3\hat{i} + 8\hat{j} + 3\hat{k}\right)$$
$$= -3\hat{i} - 7\hat{j} + 6\hat{k} - 3\hat{i} - 8\hat{j} - 3\hat{k}$$
$$\left(\vec{a_2} - \vec{a_1}\right) = -6\hat{i} - 15\hat{j} + 3\hat{k}$$

$$\left(\vec{b_1} \times \vec{b_2}\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{vmatrix}$$
$$= \hat{i}\left(-4 - 2\right) - \hat{j}\left(12 + 3\right) + \hat{k}\left(6 - 3\right)$$
$$= \left(-6\hat{i} - 15\hat{j} + 3\hat{k}\right)$$

$$\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right) = \left(-6\hat{i} - 15\hat{j} + 3\hat{k}\right).\left(-6\hat{i} - 15\hat{j} + 3\hat{k}\right)$$
$$= \left(-6\right)\left(6\right) + \left(-15\right)\left(-15\right) + \left(3\right)\left(3\right)$$
$$= 36 + 225 + 9$$
$$= 270$$

$$\left|\vec{b_1} \times \vec{b_2}\right| = \sqrt{\left(-6\right)^2 + \left(-15\right)^2 + \left(3\right)^2}$$
$$= \sqrt{36 + 225 + 9}$$
$$= \sqrt{270}$$

Substituting values of $\left|\vec{b_1} \times \vec{b_2}\right|$ and $\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right)$ in equation (i) to get shortest distance between given lines, so

$$\text{S.D.} = \frac{270}{\sqrt{270}}$$
$$= \sqrt{270}$$

$$\text{S.D.} = 3\sqrt{30} \text{ units}$$

Straight Line in Space Ex 28.5 Q1(ii)

We know that, shortest distance between lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ is given by

$$\text{S.D.} = \left|\frac{(\vec{a_2} - \vec{a_1}).(\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|}\right| \qquad ---(i)$$

Given equations of lines are,

$$\vec{r} = (3\hat{i} + 5\hat{j} + 7\hat{k}) + \lambda(\hat{i} - 2\hat{j} + 7\hat{k}) \text{ and}$$
$$\vec{r} = (-\hat{i} - \hat{j} - \hat{k}) + \mu(7\hat{i} - 6\hat{j} + \hat{k})$$

$$\Rightarrow \quad \vec{a_1} = (3\hat{i} + 5\hat{j} + 7\hat{k}),\ \vec{b_1} = (\hat{i} - 2\hat{j} + 7\hat{k})$$
$$\vec{a_2} = (-\hat{i} - \hat{j} - \hat{k}),\ \vec{b_2} = (7\hat{i} - 6\hat{j} + \hat{k})$$

So,
$$\vec{a_2} - \vec{a_1} = (-\hat{i} - \hat{j} - \hat{k}) - (3\hat{i} + 5\hat{j} + 7\hat{k})$$
$$= -\hat{i} - \hat{j} - \hat{k} - 3\hat{i} - 5\hat{j} - 7\hat{k}$$
$$= -4\hat{i} - 6\hat{j} - 8\hat{k} = -2(2\hat{i} + 3\hat{j} + 4\hat{k})$$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 7 \\ 7 & -6 & 1 \end{vmatrix}$$
$$= \hat{i}(-2 + 42) - \hat{j}(1 - 49) + \hat{k}(-6 + 14)$$
$$= 40\hat{i} + 48\hat{j} + 8\hat{k}$$
$$= 8(5\hat{i} + 6\hat{j} + \hat{k})$$

$$(\vec{a_2} - \vec{a_1}).(\vec{b_1} \times \vec{b_2}) = \{-2(2\hat{i} + 3\hat{j} + 4\hat{k})\}\{8(5\hat{i} + 6\hat{j} + \hat{k})\}$$
$$= -16[(2)(5) + (3)(6) + (4)(1)]$$
$$= -16[10 + 18 + 4]$$
$$= -16 \times 32$$
$$(\vec{a_2} - \vec{a_1}).(\vec{b_1} \times \vec{b_2}) = -512$$

$$|\vec{b_1} \times \vec{b_2}| = 8\sqrt{(5)^2 + (6)^2 + (1)^2}$$
$$= 8\sqrt{25 + 36 + 1}$$
$$= 8\sqrt{62}$$

Substituting values of $(\vec{a_2} - \vec{a_1}).(\vec{b_1} \times \vec{b_2})$ and $|\vec{b_1} \times \vec{b_2}|$ in equation (i) to get required shortest distance between given lines, so

$$\text{S.D.} = \left|\frac{-512}{8\sqrt{62}}\right|$$

$$\text{S.D.} = \frac{512}{\sqrt{3968}}$$

Straight Line in Space Ex 28.5 Q1(iii)

We know that, shortest distance between lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ is given by

$$\text{S.D.} = \left|\frac{\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right)}{\left|\vec{b_1} \times \vec{b_2}\right|}\right| \qquad \text{---(i)}$$

Given equations of lines are,

$$\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right) \text{ and}$$

$$\vec{r} = \left(2\hat{i} + 4\hat{j} + 5\hat{k}\right) + \mu\left(3\hat{i} + 4\hat{j} + 5\hat{k}\right)$$

Now, $\left(\vec{a_2} - \vec{a_1}\right) = \left(2\hat{i} + 4\hat{j} + 5\hat{k}\right) - \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$

$$= 2\hat{i} + 4\hat{j} + 5\hat{k} - \hat{i} - 2\hat{j} - 3\hat{k}$$

$$\left(\vec{a_2} - \vec{a_1}\right) = \hat{i} + 2\hat{j} + 2\hat{k}$$

$$\left(\vec{b_1} \times \vec{b_2}\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$$

$$= \hat{i}\,(15 - 16) - \hat{j}\,(10 - 12) + \hat{k}\,(8 - 9)$$

$$\left(\vec{b_1} \times \vec{b_2}\right) = -\hat{i} + 2\hat{j} - \hat{k}$$

$$\left(\vec{a_2} - \vec{a_1}\right)\left(\vec{b_1} \times \vec{b_2}\right) = \left(\hat{i} + 2\hat{j} + 2\hat{k}\right)\left(-\hat{i} + 2\hat{j} - \hat{k}\right)$$

$$= (1)(-1) + (2)(2) + (2)(-1)$$

$$= -1 + 4 - 2$$

$$\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right) = 1$$

$$\left|\vec{b_1} \times \vec{b_2}\right| = \sqrt{(-1)^2 + (2)^2 + (-1)^2}$$

$$= \sqrt{1 + 4 + 1}$$

$$\left|\vec{b_1} \times \vec{b_2}\right| = \sqrt{6}$$

Substituting values of $\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right)$ and $\left|\vec{b_1} \times \vec{b_2}\right|$ in equation (i) to get the shortest distance between given lines, so

$$\text{S.D.} = \left|\frac{1}{\sqrt{6}}\right|$$

$$\text{S.D.} = \frac{1}{\sqrt{6}} \text{ units}$$

Straight Line in Space Ex 28.5 Q1(iv)

$$\vec{r} = (1 - t)\hat{i} + (t - 2)\hat{j} + (3 - t)\hat{k}$$

$$\vec{r} = (s + 1)\hat{i} + (2s - 1)\hat{j} - (2s + 1)\hat{k}$$

Above equations can be rewritten as

$$\vec{r} = (i - 2j + 3k) + t(-i + j - k)$$

$$\vec{r} = (i - j - k) + s(i + 2j - 2k)$$

Shortest distance is given by $\left|\frac{(a_2 - a_1)\cdot(b_1 \times b_2)}{|(b_1 \times b_2)|}\right|$

$$(b_1 \times b_2) = -3j - 3k$$

$$(a_2 - a_1) = j - 4k$$

$$(a_2 - a_1)\cdot(b_1 \times b_2) = 9$$

$$|(b_1 \times b_2)| = 3\sqrt{2}$$

Shortest distance is $\frac{9}{3\sqrt{2}} = \frac{3}{\sqrt{2}}$

Straight Line in Space Ex 28.5 Q1(v)

We know that, the shortest distance between lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ is given by

$$\vec{r} = \left|\frac{(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|}\right| \qquad ---(i)$$

Given equations of lines are,

$$\vec{r} = (\lambda - 1)\hat{i} + (\lambda + 1)\hat{j} - (1 + \lambda)\hat{k}$$

$$\Rightarrow \quad \vec{r} = (-\hat{i} + \hat{j} - \hat{k}) + \lambda(\hat{i} + \hat{j} - \hat{k}) \text{ and}$$

$$\vec{r} = (1 - \mu)\hat{i} + (2\mu - 1)\hat{j} + (\mu + 2)\hat{k}$$

$$\Rightarrow \quad \vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(-\hat{i} + 2\hat{j} + \hat{k})$$

So, $\vec{a_1} = (-\hat{i} + \hat{j} - \hat{k})$, $\vec{b_1} = (\hat{i} + \hat{j} - \hat{k})$ and

$\vec{a_2} = (\hat{i} - \hat{j} + 2\hat{k})$, $\vec{b_2} = (-\hat{i} + 2\hat{j} + \hat{k})$

$$(\vec{a_2} - \vec{a_1}) = (\hat{i} - \hat{j} + 2\hat{k}) - (-\hat{i} + \hat{j} - \hat{k})$$

$$= \hat{i} - \hat{j} + 2\hat{k} + \hat{i} - \hat{j} + \hat{k}$$

$$(\vec{a_2} - \vec{a_1}) = 2\hat{i} - 2\hat{j} + 3\hat{k}$$

$$(\vec{b_1} \times \vec{b_2}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ -1 & 2 & 1 \end{vmatrix}$$

$$= \hat{i}(1 + 2) - \hat{j}(1 - 1) + \hat{k}(2 + 1)$$

$$(\vec{b_1} \times \vec{b_2}) = 3\hat{i} + 3\hat{k}$$

$$(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2}) = (2\hat{i} - 2\hat{j} + 3\hat{k})(3\hat{i} + 3\hat{k})$$

$$= (2)(3) + (-2)(0) + (3)(3)$$

$$= 6 + 0 + 9$$

$$(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2}) = 15$$

$$|\vec{b_1} \times \vec{b_2}| = \sqrt{(3)^2 + (3)^2}$$

$$= \sqrt{18}$$

$$|\vec{b_1} \times \vec{b_2}| = 3\sqrt{2}$$

Substituting values of $(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})$ and $|\vec{b_1} \times \vec{b_2}|$ in equation (i) to get the shortest distance between the given lines, so

$$\text{S.D.} = \left|\frac{15}{3\sqrt{2}}\right|$$

$$\text{S.D.} = \frac{5}{\sqrt{2}} \text{ units}$$

Straight Line in Space Ex 28.5 Q1(vi)

We know that, the shortest distance between lines $\vec{r} = \vec{a_1} + \lambda \vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu \vec{b_2}$ is given by

$$\text{S.D.} = \left| \frac{(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|} \right| \qquad ---(i)$$

Given equations of lines are,

$$\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \lambda(2\hat{i} - 5\hat{j} + 2\hat{k}) \text{ and}$$

$$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \mu(\hat{i} - \hat{j} + \hat{k})$$

$\Rightarrow \quad \vec{a_1} = (2\hat{i} - \hat{j} - \hat{k}),\ \vec{b_1} = (2\hat{i} - 5\hat{j} + 2\hat{k})$ and

$\vec{a_2} = (\hat{i} + 2\hat{j} + \hat{k}),\ \vec{b_2} = (\hat{i} - \hat{j} + \hat{k})$

$$(\vec{a_2} - \vec{a_1}) = (\hat{i} + 2\hat{j} + \hat{k}) - (2\hat{i} - \hat{j} - \hat{k})$$

$$= \hat{i} + 2\hat{j} + \hat{k} - 2\hat{i} + \hat{j} + \hat{k}$$

$$(\vec{a_2} - \vec{a_1}) = -\hat{i} + 3\hat{j} + 2\hat{k}$$

$$|\vec{b_1} \times \vec{b_2}| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -5 & 2 \\ 1 & -1 & 1 \end{vmatrix}$$

$$= \hat{i}(-5+2) - \hat{j}(2-2) + \hat{k}(-2+5)$$

$$= -3\hat{i} + 3\hat{k}$$

$$(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2}) = (-\hat{i} + 3\hat{j} + 2\hat{k})(-3\hat{i} + 3\hat{k})$$

$$= (-1)(-3) + (3)(0) + (2)(3)$$

$$= 3 + 0 + 6$$

$$(\vec{a_2} - \vec{a_1})(\vec{b_1} \times \vec{b_2}) = 9$$

$$|\vec{b_1} \times \vec{b_2}| = \sqrt{(-3)^2 + (3)^2}$$

$$= \sqrt{9+9}$$

$$|\vec{b_1} \times \vec{b_2}| = 3\sqrt{2}$$

Substituting the values of $(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})$ and $|\vec{b_1} \times \vec{b_2}|$ in equation (i) to get the shortest distance between given lines, so

$$\text{S.D.} = \left| \frac{9}{3\sqrt{2}} \right|$$

$$\text{S.D.} = \frac{3}{\sqrt{2}}$$

Straight Line in Space Ex 28.5 Q1(vii)

Given,

$$\vec{r} = \hat{i} + \hat{j} + \lambda\left(2\hat{i} - \hat{j} + \hat{k}\right) \qquad ------(i)$$

and

$$\vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu\left(3\hat{i} - 5\hat{j} + 2\hat{k}\right) \qquad ------(ii)$$

Comparing (i) and (ii) with $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ respectively, we get

$$\vec{a}_1 = \hat{i} + \hat{j}, \qquad \vec{b}_1 = 2\hat{i} - \hat{j} + \hat{k}$$

$$\vec{a}_2 = 2\hat{i} + \hat{j} - \hat{k}, \qquad \vec{b}_2 = 3\hat{i} - 5\hat{j} + 2\hat{k}$$

$\therefore \quad \vec{a}_2 - \vec{a}_1 = \hat{i} - \hat{k}$

and $\quad \vec{b}_1 \times \vec{b}_2 = \left(2\hat{i} - \hat{j} + \hat{k}\right) \times \left(3\hat{i} - 5\hat{j} + 2\hat{k}\right)$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix} = 3\hat{i} - \hat{j} - 7\hat{k}$$

So, $\quad \left|\vec{b}_1 \times \vec{b}_2\right| = \sqrt{9 + 1 + 49} = \sqrt{59}$

Hence, the shortest distance between the lines l_1 and l_2 is given by

$$d = \left|\frac{\left(\vec{b}_1 \times \vec{b}_2\right) \cdot \left(\vec{a}_2 - \vec{a}_1\right)}{\left|\vec{b}_1 \times \vec{b}_2\right|}\right| = \frac{|3 - 0 + 7|}{\sqrt{59}} = \frac{10}{\sqrt{59}}$$

Straight Line in Space Ex 28.5 Q1(viii)

The equation of lines are

$\vec{r} = (8 + 3\lambda)\hat{i} - (9 + 16\lambda)\hat{j} + (10 + 7\lambda)\hat{k}$ and $\vec{r} = 15\hat{i} + 29\hat{j} + 5\hat{k} + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$

The lines pass through $\vec{a_1} = 8\hat{i} - 9\hat{j} + 10\hat{k}$ and $\vec{a_2} = 15\hat{i} + 29\hat{j} + 5\hat{k}$

and parallel to vectors, $\vec{b_1} = 3\lambda\hat{i} - 16\lambda\hat{j} + 7\lambda\hat{k}$ and $\vec{b_2} = 3\mu\hat{i} + 8\mu\hat{j} - 5\mu\hat{k}$

$$\vec{a_1} - \vec{a_2} = -7\hat{i} - 38\hat{j} + 5\hat{k}$$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix} = 24\hat{i} + 36\hat{j} + 72\hat{k}$$

So, $\left(\vec{a_1} - \vec{a_2}\right) \cdot \left(\vec{b_1} \times \vec{b_2}\right) = -168 - 1368 + 360 = -1176$

$\left|\vec{b_1} \times \vec{b_2}\right| = \sqrt{576 + 1296 + 5184} = 84$

$$S.D. = \left|\frac{\left(\vec{a_1} - \vec{a_2}\right) \cdot \left(\vec{b_1} \times \vec{b_2}\right)}{\left|\vec{b_1} \times \vec{b_2}\right|}\right| = \left|\frac{-1176}{84}\right| = 14$$

Straight Line in Space Ex 28.5 Q2(i)

Given lines are,

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda \ \text{(say)}$$

$$x = 2\lambda + 1,\ y = 3\lambda + 2,\ z = 4\lambda + 3$$

$$\Rightarrow \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$
$$= (2\lambda + 1)\hat{i} + (3\lambda + 2)\hat{j} + (4\lambda + 3)\hat{k}$$
$$\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)$$

$$\Rightarrow \quad \vec{a_1} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right),\ \vec{b_1} = \left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)$$

and, $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-5}{5} = \mu \ \text{(say)}$

$$x = 3\mu + 2,\ y = 4\mu + 3,\ z = 5\mu + 5$$

$$\Rightarrow \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$
$$= (3\mu + 2)\hat{i} + (4\mu + 3)\hat{j} + (5\mu + 5)\hat{k}$$
$$\vec{r} = \left(2\hat{i} + 3\hat{j} + 5\hat{k}\right) + \mu\left(3\hat{i} + 4\hat{j} + 5\hat{k}\right)$$

$$\Rightarrow \quad \vec{a_2} = \left(2\hat{i} + 3\hat{j} + 5\hat{k}\right),\ \vec{b_2} = \left(3\hat{i} + 4\hat{j} + 5\hat{k}\right)$$

We know that, the shortest distance between the lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$ is given by,

$$\text{S.D.} = \left|\frac{\left(\vec{a_2} - \vec{a_1}\right)\cdot\left(\vec{b_1} \times \vec{b_2}\right)}{\left|\vec{b_1} \times \vec{b_2}\right|}\right| \qquad ---(i)$$

$$\left(\vec{a_2} - \vec{a_1}\right) = \left(2\hat{i} + 3\hat{j} + 5\hat{k}\right) - \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$$
$$= 2\hat{i} + 3\hat{j} + 5\hat{k} - \hat{i} - 2\hat{j} - 3\hat{k}$$
$$\left(\vec{a_2} - \vec{a_1}\right) = \hat{i} + \hat{j} + 2\hat{k}$$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$$
$$= \hat{i}(15 - 16) - \hat{j}(10 - 12) + \hat{k}(8 - 9)$$

$$\left(\vec{b_1} \times \vec{b_2}\right) = -\hat{i} + 2\hat{j} - \hat{k}$$

$$\left(\vec{a_2} - \vec{a_1}\right)\cdot\left(\vec{b_1} \times \vec{b_2}\right) = \left(\hat{i} + \hat{j} + 2\hat{k}\right)\left(-\hat{i} + 2\hat{j} - \hat{k}\right)$$
$$= (1)(-1) + (1)(2) + (2)(-1)$$
$$= -1 + 2 - 2$$
$$\left(\vec{a_2} - \vec{a_1}\right)\cdot\left(\vec{b_1} \times \vec{b_2}\right) = -1$$

$$\left|\vec{b_1} \times \vec{b_2}\right| = \sqrt{(-1)^2 + (2)^2 + (-1)^2}$$
$$= \sqrt{1 + 4 + 1}$$
$$= \sqrt{6}$$

Using the values of $\left(\vec{a_2} - \vec{a_1}\right)\cdot\left(\vec{b_1} \times \vec{b_2}\right)$ and $\left|\vec{b_1} \times \vec{b_2}\right|$ in equation (i) to get the shortest distance between given lines, so

$$\text{S.D.} = \left|\frac{-1}{\sqrt{3}}\right|$$

$$\text{S.D.} = \frac{1}{\sqrt{6}} \text{ units}$$

Straight Line in Space Ex 28.5 Q2(ii)

Given equations of line are,

$$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z}{1} = \lambda \text{ (say)}$$

$\Rightarrow \quad x = 2\lambda + 1,\ y = 3\lambda - 1,\ z = \lambda$

$$\Rightarrow \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$
$$= (2\lambda + 1)\hat{i} + (3\lambda - 1)\hat{j} + \lambda\hat{k}$$
$$\vec{r} = \left(\hat{i} - \hat{j}\right) + \lambda\left(2\hat{i} + 3\hat{j} + \hat{k}\right)$$

$\Rightarrow \quad \vec{a_1} = \hat{i} - \hat{j},\ \vec{b_1} = 2\hat{i} + 3\hat{j} + \hat{k}$

and, $\quad \frac{x+1}{3} = \frac{y-2}{1} = \mu,\ z = 2$

$\Rightarrow \quad x = 3\mu - 1,\ y = \mu + 2,\ z = 2$

$$\Rightarrow \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$
$$= (3\mu - 1)\hat{i} + (\mu + 2)\hat{j} + 2\hat{k}$$
$$\vec{r} = \left(-\hat{i} + 2\hat{j} + 2\hat{k}\right) + \mu\left(3\hat{i} + \hat{j}\right)$$

$\Rightarrow \quad \vec{a_2} = \left(-\hat{i} + 2\hat{j} + 2\hat{k}\right),\ \vec{b_2} = \left(3\hat{i} + \hat{j}\right)$

We know that, the shortest distance between two lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$ is given by,

$$\text{S.D.} = \left|\frac{\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right)}{\left|\vec{b_1} \times \vec{b_2}\right|}\right| \qquad ---(i)$$

$$\left(\vec{a_2} - \vec{a_1}\right) = \left(\hat{i} - \hat{j}\right) - \left(-\hat{i} + 2\hat{j} + 2\hat{k}\right)$$
$$= \hat{i} - \hat{j} + \hat{i} - 2\hat{j} - 2\hat{k}$$
$$\left(\vec{a_2} - \vec{a_1}\right) = 2\hat{i} - 3\hat{j} - 2\hat{k}$$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 3 & 1 & 0 \end{vmatrix}$$
$$= \hat{i}\,(0-1) - \hat{j}\,(0-3) + \hat{k}\,(2-9)$$

$$\left(\vec{b_1} \times \vec{b_2}\right) = -\hat{i} + 3\hat{j} - 7\hat{k}$$

$$\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right) = \left(2\hat{i} - 3\hat{j} - 2\hat{k}\right)\left(-\hat{i} + 3\hat{j} - 7\hat{k}\right)$$
$$= (2)(-1) + (-3)(3) + (-2)(-7)$$
$$= -2 - 9 + 14$$
$$\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right) = 3$$

$$\left|\vec{b_1} \times \vec{b_2}\right| = \sqrt{(-1)^2 + (3)^2 + (-7)^2} = \sqrt{59}$$

Substitute the value of $\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right)$ and $\left|\vec{b_1} \times \vec{b_2}\right|$ in equation (i) to get the shortest distance between given lines, so

$$\text{S.D.} = \left|\frac{3}{\sqrt{59}}\right|$$

$$\text{S.D.} = \frac{3}{\sqrt{59}} \text{ units}$$

Straight Line in Space Ex 28.5 Q2(iii)

Given equation of lines are,

$$\frac{x-1}{-1} = \frac{y+2}{1} = \frac{z-3}{-2} = \lambda \ (\text{say})$$

$\Rightarrow \quad x = -\lambda + 1,\ y = \lambda - 2,\ z = -2\lambda + 3$

$$\Rightarrow \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$
$$= (-\lambda + 1)\hat{i} + (\lambda - 2)\hat{j} + (-2\lambda + 3)\hat{k}$$
$$\vec{r} = (\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(-\hat{i} + \hat{j} - 2\hat{k})$$

$\Rightarrow \quad \vec{a_1} = (\hat{i} - 2\hat{j} + 3\hat{k}),\ \vec{b_1} = (-\hat{i} + \hat{j} - 2\hat{k})$

and, $\quad \frac{x-1}{1} = \frac{y+1}{2} = \frac{z+1}{-2} = \mu \ (\text{say})$

$\Rightarrow \quad x = \mu + 1,\ y = 2\mu - 1,\ z = -2\mu - 1$

$$\Rightarrow \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$
$$= (\mu + 1)\hat{i} + (2\mu - 1)\hat{j} + (-2\mu - 1)\hat{k}$$
$$\vec{r} = (\hat{i} - \hat{j} - \hat{k}) + \mu(\hat{i} + 2\hat{j} - 2\hat{k})$$

$\Rightarrow \quad \vec{a_2} = (\hat{i} - \hat{j} - \hat{k}),\ \vec{b_2} = (\hat{i} + 2\hat{j} - 2\hat{k})$

$$(\vec{a_2} - \vec{a_1}) = (\hat{i} - \hat{j} - \hat{k}) - (\hat{i} - 2\hat{j} + 3\hat{k})$$
$$= \hat{i} - \hat{j} - \hat{k} - \hat{i} + 2\hat{j} - 3\hat{k}$$
$$= \hat{j} - 4\hat{k}$$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 1 & 2 & -2 \end{vmatrix}$$
$$= \hat{i}(-2+4) - \hat{j}(2+2) + \hat{k}(-2-1)$$
$$(\vec{b_1} \times \vec{b_2}) = 2\hat{i} - 4\hat{j} - 3\hat{k}$$

$$|\vec{b_1} \times \vec{b_2}| = \sqrt{(2)^2 + (-4)^2 + (-3)^2}$$
$$= \sqrt{29}$$

$$(\vec{a_2} - \vec{a_1})(\vec{b_1} \times \vec{b_2}) = (\hat{j} - 4\hat{k})(2\hat{i} - 4\hat{j} - 3\hat{k})$$
$$= (0)(2) + (1)(-4) + (-4)(-3)$$
$$= 0 - 4 + 12$$
$$(\vec{a_2} - \vec{a_1})(\vec{b_1} \times \vec{b_2}) = 8$$

We know that, shortest distance between $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$ is given by,

$$\text{S.D.} = \left| \frac{(\vec{a_2} - \vec{a_1}).(\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|} \right| \qquad \text{---(i)}$$

So, shortest distance between given lines is

$$\text{S.D.} = \left| \frac{8}{\sqrt{29}} \right|$$

$$\text{S.D.} = \frac{8}{\sqrt{29}} \text{ units}$$

Straight Line in Space Ex 28.5 Q2(iv)

Given equation of lines are,

$$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1} = \lambda \text{ (say)}$$

$$\Rightarrow \quad x = \lambda + 3,\ y = -2\lambda + 5,\ z = \lambda + 7$$

$$\Rightarrow \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$= (\lambda + 3)\hat{i} + (-2\lambda + 5)\hat{j} + (\lambda + 7)\hat{k}$$

$$\vec{r} = \left(3\hat{i} + 5\hat{j} + 7\hat{k}\right) + \lambda\left(\hat{i} - 2\hat{j} + \hat{k}\right)$$

$$\Rightarrow \quad \vec{a_1} = \left(3\hat{i} + 5\hat{j} + 7\hat{k}\right),\ \vec{b_1} = \left(\hat{i} - 2\hat{j} + \hat{k}\right)$$

and, $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1} = \mu$ (say)

$$\Rightarrow \quad x = 7\mu - 1,\ y = -6\mu - 1,\ z = \mu - 1$$

$$\Rightarrow \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$= (7\mu - 1)\hat{i} + (-6\mu - 1)\hat{j} + (\mu - 1)\hat{k}$$

$$\vec{r} = \left(-\hat{i} - \hat{j} - \hat{k}\right) + \mu\left(7\hat{i} - 6\hat{j} + \hat{k}\right)$$

$$\Rightarrow \quad \vec{a_2} = \left(-\hat{i} - \hat{j} - \hat{k}\right),\ \vec{b_2} = 7\hat{i} - 6\hat{j} + \hat{k}$$

We know that, shortest distance between $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$ is given by,

$$\text{S.D.} = \left|\frac{\left(\vec{a_2} - \vec{a_1}\right)\cdot\left(\vec{b_1} \times \vec{b_2}\right)}{\left|\vec{b_1} \times \vec{b_2}\right|}\right| \qquad ---(i)$$

$$\left(\vec{a_2} - \vec{a_1}\right) = \left(-\hat{i} - \hat{j} - \hat{k}\right) - \left(3\hat{i} + 5\hat{j} + 7\hat{k}\right)$$

$$= -\hat{i} - \hat{j} - \hat{k} - 3\hat{i} - 5\hat{j} - 7\hat{k}$$

$$\left(\vec{a_2} - \vec{a_1}\right) = -4\hat{i} - 6\hat{j} - 8\hat{k}$$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 1 \\ 7 & -6 & 1 \end{vmatrix}$$

$$= \hat{i}(-2 + 6) - \hat{j}(1 - 7) + \hat{k}(-6 + 14)$$

$$\vec{b_1} \times \vec{b_2} = 4\hat{i} + 6\hat{j} + 8\hat{k}$$

$$\left|\vec{b_1} \times \vec{b_2}\right| = \sqrt{(4)^2 + (6)^2 + (8)^2}$$

$$= \sqrt{16 + 36 + 64}$$

$$= \sqrt{116}$$

$$= 2\sqrt{29}$$

$$\left(\vec{a_2} - \vec{a_1}\right)\left(\vec{b_1} \times \vec{b_2}\right) = \left(-4\hat{i} - 6\hat{j} - 8\hat{k}\right)\left(4\hat{i} + 6\hat{j} + 8\hat{k}\right)$$

$$= (-4)(4) + (-6)(6) + (-8)(8)$$

$$= -16 - 36 - 64$$

$$= -116$$

Substituting the values of $\left(\vec{a_2} - \vec{a_1}\right)\cdot\left(\vec{b_1} \times \vec{b_2}\right)$ and $\left|\vec{b_1} \times \vec{b_2}\right|$ in equation (i) to get the shortest distance between the two given lines, so

$$\text{S.D.} = \left|\frac{-116}{2\sqrt{29}}\right|$$

$$= \frac{58}{\sqrt{29}}$$

Straight Line in Space Ex 28.5 Q3(i)

Given equations of lines are,

$$\vec{r} = \left(\hat{i} - \hat{j}\right) + \lambda\left(2\hat{i} + \hat{k}\right)$$

$$\Rightarrow \quad \overrightarrow{a_1} = \left(\hat{i} - \hat{j}\right),\ \overrightarrow{b_1} = \left(2\hat{i} + \hat{k}\right)$$

and, $\vec{r} = \left(2\hat{i} - \hat{j}\right) + \mu\left(\hat{i} + \hat{j} - \hat{k}\right)$

$$\Rightarrow \quad \overrightarrow{a_2} = \left(2\hat{i} - \hat{j}\right),\ \overrightarrow{b_2} = \left(\hat{i} + \hat{j} - \hat{k}\right)$$

We know that, shortest distance between lines $\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \lambda\overrightarrow{b_2}$ is given by

$$\text{S.D.} = \left|\frac{\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right).\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right)}{\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right|}\right| \qquad ---(i)$$

$$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) = \left(2\hat{i} - \hat{j}\right) - \left(\hat{i} - \hat{j}\right)$$
$$= 2\hat{i} - \hat{j} - \hat{i} + \hat{j}$$
$$= \hat{i}$$

$$\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$
$$= \hat{i}\,(0-1) - \hat{j}\,(-2-1) + \hat{k}\,(2-0)$$
$$= -\hat{i} + 3\hat{j} + 2\hat{k}$$

$$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right)\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right) = \left(\hat{i}\right)\left(-\hat{i} + 3\hat{j} + 2\hat{k}\right)$$
$$= (1)(-1) + (0)(3) + (0)(2)$$
$$= -1 + 0 + 0$$
$$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right)\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right) = -1$$

$$\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right| = \sqrt{(-1)^2 + (3)^2 + (2)^2}$$
$$= \sqrt{1 + 9 + 4}$$
$$\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right| = \sqrt{14}$$

So, shortest distance between the given lines using equation (1) is,

$$\text{S.D.} = \left|\frac{-1}{\sqrt{14}}\right|$$

$$= \frac{1}{\sqrt{14}} \text{ units}$$

$$\text{S.D} \neq 0$$

Since, shortest distance between lines is not zero, so lines are not intersecting.

Straight Line in Space Ex 28.5 Q3(ii)

Given equations of lines are,

$$\vec{r} = \left(\hat{i} + \hat{j} - \hat{k}\right) + \lambda\left(3\hat{i} - \hat{j}\right)$$

$$\Rightarrow \quad \vec{a_1} = \left(\hat{i} + \hat{j} - \hat{k}\right),\ \vec{b_1} = \left(3\hat{i} - \hat{j}\right)$$

and, $\vec{r} = \left(4\hat{i} - \hat{k}\right) + \mu\left(2\hat{i} + 3\hat{k}\right)$

$$\Rightarrow \quad \vec{a_2} = \left(4\hat{i} - \hat{k}\right),\ \vec{b_2} = \left(2\hat{i} + 3\hat{k}\right)$$

We know that, shortest distance between two lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$ is given by

$$\text{S.D.} = \left|\frac{\left(\vec{a_2} - \vec{a_1}\right)\cdot\left(\vec{b_1} \times \vec{b_2}\right)}{\left|\vec{b_1} \times \vec{b_2}\right|}\right| \qquad ---(i)$$

$$\left(\vec{a_2} - \vec{a_1}\right) = \left(4\hat{i} - \hat{k}\right) - \left(\hat{i} + \hat{j} - \hat{k}\right)$$
$$= 4\hat{i} - \hat{k} - \hat{i} - \hat{j} + \hat{k}$$
$$= 3\hat{i} - \hat{j}$$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 0 \\ 2 & 0 & 3 \end{vmatrix}$$
$$= \hat{i}\left(-3 - 0\right) - \hat{j}\left(9 - 0\right) + \hat{k}\left(0 + 2\right)$$
$$= -3\hat{i} - 9\hat{j} + 2\hat{k}$$

$$\left|\vec{b_1} \times \vec{b_2}\right| = \sqrt{(-3)^2 + (-9)^2 + (2)^2}$$
$$\left|\vec{b_1} \times \vec{b_2}\right| = \sqrt{9 + 81 + 4}$$
$$= \sqrt{94}$$

$$\left(\vec{a_2} - \vec{a_1}\right)\cdot\left(\vec{b_1} \times \vec{b_2}\right) = \left(3\hat{i} - \hat{j}\right)\left(-3\hat{i} - 9\hat{j} + 2\hat{k}\right)$$
$$= (3)(-3) + (-1)(-9) + (0)(2)$$
$$= -9 + 9 + 0$$
$$= 0$$

Using $\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right)$ and $\left|\vec{b_1} \times \vec{b_2}\right|$ in equation (i) to get shortest distance between given lines, so

$$\text{S.D.} = \left|\frac{0}{\sqrt{94}}\right|$$

$$\text{S.D} = 0$$

Since, shortest distance between the given lines is zero, so lines are intersecting.

Straight Line in Space Ex 28.5 Q3(iii)

Given equations of lines are,

$$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z}{1} = \lambda \text{ (say)}$$

$\Rightarrow \quad x = 2\lambda + 1,\ y = 3\lambda - 1,\ z = \lambda$

$$\Rightarrow \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$
$$= (2\lambda + 1)\hat{i} + (3\lambda - 1)\hat{j} + (\lambda)\hat{k}$$
$$\vec{r} = (\hat{i} - \hat{j}) + \lambda(2\hat{i} + 3\hat{j} + \hat{k})$$

$$\Rightarrow \quad \vec{a_1} = (\hat{i} - \hat{j}),\ \vec{b_1} = (2\hat{i} + 3\hat{j} + \hat{k})$$

and, $\quad \frac{x+1}{5} = \frac{y-2}{1} = \mu \text{ (say)},\ z = 2$

$\Rightarrow \quad x = 5\mu - 1,\ y = \mu + 2,\ z = 2$

$$\Rightarrow \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$
$$= (5\mu - 1)\hat{i} + (\mu + 2)\hat{j} + 2\hat{k}$$
$$\vec{r} = (-\hat{i} + 2\hat{j} + 2\hat{k}) + \mu(5\hat{i} + \hat{j})$$

$$\Rightarrow \quad \vec{a_2} = (-\hat{i} + 2\hat{j} + 2\hat{k}),\ \vec{b_2} = (5\hat{i} + \hat{j})$$

We know that, the shortest distance between $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$ is given by

$$\text{S.D.} = \left| \frac{(\vec{a_2} - \vec{a_1}).(\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|} \right| \qquad ---(i)$$

$$(\vec{a_2} - \vec{a_1}) = (-\hat{i} + 2\hat{j} + 2\hat{k}) - (\hat{i} - \hat{j})$$
$$= -\hat{i} + 2\hat{j} + 2\hat{k} - \hat{i} + \hat{j}$$
$$= -2\hat{i} + 3\hat{j} + 2\hat{k}$$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 5 & 1 & 0 \end{vmatrix}$$
$$= \hat{i}(0-1) - \hat{j}(0-5) + \hat{k}(2-15)$$
$$\vec{b_1} \times \vec{b_2} = -\hat{i} + 5\hat{j} - 13\hat{k}$$

$$(\vec{a_2} - \vec{a_1})(\vec{b_1} \times \vec{b_2}) = (-2\hat{i} + 3\hat{j} + 2\hat{k})(-\hat{i} + 5\hat{j} - 13\hat{k})$$
$$= (-2)(-1) + (3)(5) + (2)(-13)$$
$$= -9$$

$$|\vec{b_1} \times \vec{b_2}| = \sqrt{(-1)^2 + (5)^2 + (-13)^2}$$
$$= \sqrt{1 + 25 + 169}$$
$$= \sqrt{195}$$

Substituting the value of $(\vec{a_2} - \vec{a_1}).(\vec{b_1} \times \vec{b_2})$ and $|\vec{b_1} \times \vec{b_2}|$ in equation (i) to get shortest distance between given lines, so

$$\text{S.D.} = \left| \frac{-9}{\sqrt{195}} \right|$$
$$= \frac{9}{\sqrt{195}} \text{ units}$$

Since, shortest distance between given lines is not zero, so lines are not intersecting.

Straight Line in Space Ex 28.5 Q3(iv)

Given lines are,

$$\frac{x-5}{4} = \frac{y-7}{-5} = \frac{z+3}{-5} = \lambda \text{ (say)}$$

$\Rightarrow \quad x = 4\lambda + 5,\ y = -5\lambda + 7,\ z = -5\lambda - 3$

$\Rightarrow \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$= (4\lambda + 5)\hat{i} + (-5\lambda + 7)\hat{j} + (-5\lambda - 3)\hat{k}$

$\vec{r} = \left(5\hat{i} + 7\hat{j} - 3\hat{k}\right) + \lambda\left(4\hat{i} - 5\hat{j} - 5\hat{k}\right)$

$\Rightarrow \quad \overrightarrow{a_1} = \left(5\hat{i} + 7\hat{j} - 3\hat{k}\right),\ \overrightarrow{b_1} = \left(4\hat{i} - 5\hat{j} - 5\hat{k}\right)$

and, $\quad \dfrac{x-8}{7} = \dfrac{y-7}{1} = \dfrac{z-5}{3} = \mu \text{ (say)}$

$\Rightarrow \quad x = 7\mu + 8,\ y = \mu + 7,\ 3\mu + 5$

$\Rightarrow \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$= (7\mu + 8)\hat{i} + (\mu + 7)\hat{j} + (3\mu + 5)\hat{k}$

$\vec{r} = \left(8\hat{i} + 7\hat{j} + 5\hat{k}\right) + \mu\left(7\hat{i} + \hat{j} + 3\hat{k}\right)$

$\Rightarrow \quad \overrightarrow{a_2} = \left(8\hat{i} + 7\hat{j} + 5\hat{k}\right),\ \overrightarrow{b_2} = \left(7\hat{i} + \hat{j} + 3\hat{k}\right)$

We know that, shortest distance between lines $\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \lambda\overrightarrow{b_2}$ is given by

$$\text{S.D.} = \left|\frac{\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right).\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right)}{\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right|}\right| \qquad ---(i)$$

$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) = \left(8\hat{i} + 7\hat{j} + 5\hat{k}\right) - \left(5\hat{i} + 7\hat{j} - 3\hat{k}\right)$

$= 8\hat{i} + 7\hat{j} + 5\hat{k} - 5\hat{i} - 7\hat{j} + 3\hat{k}$

$= 3\hat{i} + 8\hat{k}$

$$\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -5 & -5 \\ 7 & 1 & 3 \end{vmatrix}$$

$= \hat{i}(-15 + 5) - \hat{j}(12 + 35) + \hat{k}(4 + 35)$

$= -10\hat{i} - 47\hat{j} + 39\hat{k}$

$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right)\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right) = \left(3\hat{i} + 8\hat{k}\right)\left(-10\hat{i} - 47\hat{j} + 39\hat{k}\right)$

$= (3)(-10) + (0)(-4) + (8)(39)$

$= -30 + 312$

$= 282$

Using equation (i) to get the shortest distance between the given lines, so

$$\text{S.D.} = \left|\frac{282}{\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right|}\right|$$

$\text{S.D.} \neq 0$

Since, the shortest distance between given lines is not equal to zero, so

Given lines are not intersecting.

Straight Line in Space Ex 28.5 Q4(i)

Given, equation of lines are,

$$\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(\hat{i} - \hat{j} + \hat{k}\right) \qquad ---(1)$$

$$\vec{r} = \left(2\hat{i} - \hat{j} - \hat{k}\right) + \mu\left(-\hat{i} + \hat{j} - \hat{k}\right)$$

$$\vec{r} = \left(2\hat{i} - \hat{j} - \hat{k}\right) - \mu\left(\hat{i} - \hat{j} + \hat{k}\right)$$

$$\vec{r} = \left(2\hat{i} - \hat{j} - \hat{k}\right) + \mu'\left(\hat{i} - \hat{j} + \hat{k}\right) \qquad ---(2)$$

These two lines passes through the points having position vectors $\vec{a_1} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{a_2} = 2\hat{i} - \hat{j} - \hat{k}$ respectively and both are parallel to the vector $\vec{b} = \hat{i} - \hat{j} + \hat{k}$

We know that, shortest distance between parallel lines $\vec{r} = \vec{a_1} + \lambda\vec{b}$ and $\vec{r} = \vec{a_2} + \mu\vec{b}$ is given by

$$\text{S.D.} = \left|\frac{\left(\vec{a_2} - \vec{a_1}\right).\vec{b}}{\left|\vec{b}\right|}\right| \qquad ---(i)$$

$$\left(\vec{a_2} - \vec{a_1}\right) = \left(2\hat{i} - \hat{j} - \hat{k}\right) - \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$$

$$= 2\hat{i} - \hat{j} - \hat{k} - \hat{i} - 2\hat{j} - 3\hat{k}$$

$$\left(\vec{a_2} - \vec{a_1}\right) = \hat{i} - 3\hat{j} - 4\hat{k}$$

$$\left(\vec{a_2} - \vec{a_1}\right) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & -4 \\ 1 & -1 & 1 \end{vmatrix}$$

$$= \hat{i}\left(-3 - 4\right) - \hat{j}\left(1 + 4\right) + \hat{k}\left(-1 + 3\right)$$

$$\left(\vec{a_2} - \vec{a_1}\right) \times \vec{b} = -7\hat{i} - 5\hat{j} + 2\hat{k}$$

$$\left|\left(\vec{a_2} - \vec{a_1}\right) \times \vec{b}\right| = \sqrt{\left(-7\right)^2 + \left(-5\right)^2 + \left(2\right)^2}$$

$$= \sqrt{49 + 25 + 4}$$

$$= \sqrt{78}$$

$$\left|\vec{b}\right| = \sqrt{\hat{i} - \hat{j} + \hat{k}}$$

$$= \sqrt{\left(1\right)^2 + \left(-1\right)^2 + \left(1\right)^2}$$

$$\left|\vec{b}\right| = \sqrt{3}$$

Using $\left|\left(\vec{a_2} - \vec{a_1}\right) \times \vec{b}\right|$ and $\left|\vec{b}\right|$ in equation (1) to get the shortest distance between parallel lines, so

$$\text{S.D.} = \frac{\sqrt{78}}{\sqrt{3}}$$

$$\text{S.D.} = \sqrt{\frac{78}{3}}$$

$$\text{S.D.} = \sqrt{26} \text{ units}$$

Straight Line in Space Ex 28.5 Q4(ii)

Given, equation of lines are,

$$\vec{r} = (\hat{i} + \hat{j}) + \lambda(2\hat{i} - \hat{j} + \hat{k}) \quad ---(1)$$

$$\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(4\hat{i} - 2\hat{j} + 2\hat{k})$$

$$\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + 2\mu(2\hat{i} - \hat{j} + \hat{k})$$

$$\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu'(2\hat{i} - \hat{j} + \hat{k}) \quad ---(2)$$

So, $\vec{a_1} = (\hat{i} + \hat{j})$, $\vec{a_2} = 2\hat{i} + \hat{j} - \hat{k}$

$\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$

We know that, the shortest distance between the parallel lines $\vec{r} = \vec{a_1} + \lambda\vec{b}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b}$ is given by

$$\text{S.D.} = \frac{\left|(\vec{a_2} - \vec{a_1})\,\vec{b}\right|}{|\vec{b}|} \quad ---(i)$$

$$(\vec{a_2} - \vec{a_1}) = (2\hat{i} + \hat{j} - \hat{k}) - (\hat{i} + \hat{j})$$

$$= 2\hat{i} + \hat{j} - \hat{k} - \hat{i} - \hat{j}$$

$$(\vec{a_2} - \vec{a_1}) = \hat{i} - \hat{k}$$

$$(\vec{a_2} - \vec{a_1}) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & -1 \\ 2 & -1 & 1 \end{vmatrix}$$

$$= \hat{i}(0-1) - \hat{j}(1+2) + \hat{k}(-1-0)$$

$$(\vec{a_2} - \vec{a_1}) \times \vec{b} = -\hat{i} - 3\hat{j} - \hat{k}$$

$$\left|(\vec{a_2} - \vec{a_1}) \times \vec{b}\right| = \sqrt{(-1)^2 + (-3)^2 + (-1)^2}$$

$$= \sqrt{1+9+1}$$

$$\left|(\vec{a_2} - \vec{a_1}) \times \vec{b}\right| = \sqrt{11}$$

$$|\vec{b}| = \sqrt{(2)^2 + (-1)^2 + (1)^2}$$

$$= \sqrt{4+1+1}$$

$$|\vec{b}| = \sqrt{6}$$

Using $\left|(\vec{a_2} - \vec{a_1}) \times \vec{b}\right|$ and $|\vec{b}|$ in equation (1) to get the shortest distance between the given lines, so

$$\text{S.D.} = \frac{\sqrt{11}}{\sqrt{6}}$$

$$\text{S.D.} = \sqrt{\frac{11}{6}} \text{ units}$$

Straight Line in Space Ex 28.5 Q5

Equation of line passing through $(0,0,0)$ and $(1,0,2)$ is given by $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$

$$\vec{r} = \left(0\hat{i} + 0\hat{j} + 0\hat{k}\right) + \lambda\left((1-0)\hat{i} + (0-0)\hat{j} + (2-0)\hat{k}\right)$$

$$\vec{r} = \left(0\hat{i} + 0\hat{j} + 0\hat{k}\right) + \lambda\left(\hat{i} + 2\hat{k}\right) \qquad ---(1)$$

Equation of another line passing through $(1,3,0)$ and $(0,3,0)$ is

$$\vec{r} = \left(\hat{i} + 3\hat{j} + 0\hat{k}\right) + \mu\left((0-1)\hat{i} + (3-3)\hat{j} + (0-0)\hat{k}\right)$$

$$\vec{r} = \left(\hat{i} + 3\hat{j} + 0\hat{k}\right) + \mu\left(-\hat{i}\right) \qquad ---(2)$$

From equation (1) and (2)

$$\vec{a_1} = \left(0.\hat{i} + 0.\hat{j} + 0.\hat{k}\right), \ \vec{b_1} = \left(\hat{i} + 2\hat{k}\right)$$

$$\vec{a_2} = \left(\hat{i} + 3\hat{j} + 0\hat{k}\right), \quad \vec{b_2} = -\hat{i}$$

We know that, shortest distance between the lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$ is given by

$$\text{S.D.} = \frac{\left|\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right)\right|}{\left|\vec{b_1} \times \vec{b_2}\right|} \qquad ---(3)$$

$$\left(\vec{a_2} - \vec{a_1}\right) = \left(\hat{i} + 3\hat{j} + 0.\hat{k}\right) - \left(0.\hat{i} + 0.\hat{j} + 0.\hat{k}\right)$$

$$\left(\vec{a_2} - \vec{a_1}\right) = \left(\hat{i} + 3\hat{j}\right)$$

$$\left(\vec{b_1} \times \vec{b_2}\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 2 \\ -1 & 0 & 0 \end{vmatrix}$$

$$= \hat{i}(0-0) - \hat{j}(0+2) + \hat{k}(-2)$$

$$\left(\vec{b_1} \times \vec{b_2}\right) = -2\hat{j}$$

$$\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right) = \left(\hat{i} + 3\hat{j}\right)\left(-2\hat{j}\right)$$

$$= (1)(0) + (3)(-2)$$

$$\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right) = -6$$

$$\left|\vec{b_1} \times \vec{b_2}\right| = \sqrt{(-2)^2}$$

$$\left|\vec{b_1} \times \vec{b_2}\right| = 2$$

Using $\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right)$ and $\left|\vec{b_1} \times \vec{b_2}\right|$ in equation (1) to get shortest distance between the lines, so

$$\text{S.D.} = \left|\frac{-6}{2}\right|$$

$$\text{S.D.} = 3 \text{ units}$$

Straight Line in Space Ex 28.5 Q6

Given equations of lines are,

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6} = \lambda \ (\text{say})$$

$$\Rightarrow \quad x = 2\lambda + 1,\ y = 3\lambda + 2,\ z = 6\lambda - 4$$

$$\Rightarrow \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$= (2\lambda + 1)\hat{i} + (3\lambda + 2)\hat{j} + (6\lambda - 4)\hat{k}$$

$$\vec{r} = \left(\hat{i} + 2\hat{j} - 4\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$$

$$\Rightarrow \quad \overrightarrow{a_1} = \hat{i} + 2\hat{j} - 4\hat{k},\ \vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$$

Another equation of line is,

$$\frac{x-3}{4} = \frac{y-3}{6} = \frac{z+5}{12} = \mu \ (\text{say})$$

$$\Rightarrow \quad x = 4\mu + 3,\ y = 6\mu + 3,\ 12\mu - 5$$

$$\Rightarrow \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$= (4\mu + 3)\hat{i} + (6\mu + 3)\hat{j} + (12\mu - 5)\hat{k}$$

$$= \left(3\hat{i} + 3\hat{j} - 5\hat{k}\right) + \mu\left(4\hat{i} + 6\hat{j} + 12\hat{k}\right)$$

$$= \left(3\hat{i} + 3\hat{j} - 5\hat{k}\right) + 2\mu\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$$

$$\vec{r} = \left(3\hat{i} + 3\hat{j} - 5\hat{k}\right) + \mu'\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$$

$$\Rightarrow \quad \overrightarrow{a_2} = \left(3\hat{i} + 3\hat{j} - 5\hat{k}\right),\ \vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$$

We know that, shortest distance between parallel lines $\vec{r} = \overrightarrow{a_1} + \lambda\vec{b}$ and $\vec{r} = \overrightarrow{a_2} + \mu\vec{b}$ is given by

$$\text{S.D.} = \frac{\left|\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) \times \vec{b}\right|}{\left|\vec{b}\right|} \qquad ---(\text{i})$$

$$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) = \left(3\hat{i} + 3\hat{j} - 5\hat{k}\right) - \left(\hat{i} + 2\hat{j} - 4\hat{k}\right)$$

$$= 3\hat{i} + 3\hat{j} - 5\hat{k} - \hat{i} - 2\hat{j} + 4\hat{k}$$

$$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) = 2\hat{i} + \hat{j} - \hat{k}$$

$$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix}$$

$$= \hat{i}(6+3) - \hat{j}(12+2) + \hat{k}(6-2)$$

$$= 9\hat{i} - 14\hat{j} + 4\hat{k}$$

$$\left|\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) \times \vec{b}\right| = \sqrt{(9)^2 + (-14)^2 + (4)^2}$$

$$= \sqrt{81 + 196 + 16}$$

$$= \sqrt{293}$$

$$\left|\vec{b}\right| = \sqrt{(2)^2 + (3)^2 + (6)^2}$$

$$= \sqrt{4 + 9 + 36}$$

$$= \sqrt{49}$$

$$\left|\vec{b}\right| = 7$$

Using $\left|\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) \times \vec{b}\right|$ and $\left|\vec{b}\right|$ in equation (i) to get the shortest distance between given lines, so

$$\text{S.D.} = \frac{\sqrt{293}}{7} \text{ units}$$

Straight Line in Space Ex 28.5 Q7(i)

Here,

$a_1 = \hat{i} + 2\hat{j} + \hat{k}$

$b_1 = \hat{i} - \hat{j} + \hat{k}$

$a_2 = 2\hat{i} - \hat{j} - \hat{k}$

$b_2 = 2\hat{i} + \hat{j} + 2\hat{k}$

$\bar{a}_2 - \bar{a}_1 = 2\hat{i} - \hat{j} - \hat{k} - \hat{i} - 2\hat{j} - \hat{k} = \hat{i} - 3\hat{j} - 2\hat{k}$

$$b_1 \times b_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix} = \hat{i}(-2-1) - \hat{j}(2-2) + \hat{k}(1+2) = -3\hat{i} + 3\hat{k}$$

The shortest distance between the two lines,

$$d = \left| \frac{(\vec{b}_1 \times \vec{b}_2).(\bar{a}_2 - \bar{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

$$d = \left| \frac{(-3\hat{i} + 3\hat{k}).(\hat{i} - 3\hat{j} - 2\hat{k})}{|-3\hat{i} - 3\hat{k}|} \right| = \left| \frac{-3-6}{\sqrt{(-3)^2 + (-3)^2}} \right| = \frac{9}{3\sqrt{2}}$$

The shortest distance between the two lines $= \frac{3}{\sqrt{2}}$ units

Straight Line in Space Ex 28.5 Q7(ii)

Here,

$\bar{a}_1 = -\hat{i} - \hat{j} - \hat{k}$

$\bar{a}_2 = 3\hat{i} + 5\hat{j} + 7\hat{k}$

$\vec{b}_1 = 7\hat{i} - 6\hat{j} + \hat{k}$

$\vec{b}_2 = \hat{i} - 2\hat{j} + \hat{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix}$$

$= \hat{i}(-6+2) - \hat{j}(7-1) + \hat{k}(-14+6)$

$= -4\hat{i} - 6\hat{j} - 8\hat{k}$

$\bar{a}_2 - \bar{a}_1 = \hat{i}(3+1) + \hat{j}(5+1) + \hat{k}(7+1)$

$= 4\hat{i} + 6\hat{j} + 8\hat{k}$

The shortest distance between two lines,

$$d = \left| \frac{(\vec{b}_1 \times \vec{b}_2).(\bar{a}_2 - \bar{a}_1)}{|b_1 \times b_2|} \right|$$

$$= \left| \frac{(-4\hat{i} - 6\hat{j} - 8\hat{k}).(4\hat{i} + 6\hat{j} + 8\hat{k})}{\sqrt{(-4)^2 + (-6)^2 + (-8)^2}} \right|$$

$$= \left| \frac{-16-36-64}{\sqrt{116}} \right|$$

$$= \left| \frac{-116}{\sqrt{116}} \right|$$

$= 2\sqrt{29}$ units

Straight Line in Space Ex 28.5 Q7(iii)

Here,

$\vec{a}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$,

$\vec{b}_1 = \hat{i} - 3\hat{j} + 2\hat{k}$

$\vec{a}_2 = 4\hat{i} + 5\hat{j} + 6\hat{k}$,

$\vec{b}_2 = 2\hat{i} + 3\hat{j} + \hat{k}$

$(\vec{a}_2 - \vec{a}_1) = 4\hat{i} + 5\hat{j} + 6\hat{k} - \hat{i} - 2\hat{j} - 3\hat{k} = 3\hat{i} + 3\hat{j} + 3\hat{k}$

$\vec{b}_1 \times \vec{b}_2$

$= (\hat{i} - 3\hat{j} + 2\hat{k}) \times (2\hat{i} + 3\hat{j} + \hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix}$$

$= \hat{i}(-3-6) - \hat{j}(1-4) + \hat{k}(3+6)$

$= -9\hat{i} + 3\hat{j} + 9\hat{k}$

Shortest distance between the two lines $= \left|\dfrac{(a_2 - a_1).(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right|$

$$= \left|\frac{(3\hat{i} + 3\hat{j} + 3\hat{k}).(-9\hat{i} + 3\hat{j} + 9\hat{k})}{|-9\hat{i} + 3\hat{j} + 9\hat{k}|}\right|$$

$$= \left|\frac{3 \times (-9) + 3 \times 3 + 3 \times 9}{\sqrt{(-9)^2 + 3^2 + 9^2}}\right|$$

$$= \left|\frac{-27 + 9 + 27}{\sqrt{(-9)^2 + 3^2 + 9^2}}\right|$$

$$= \left|\frac{9}{\sqrt{171}}\right| = \frac{3}{\sqrt{19}} \text{units}$$

Straight Line in Space Ex 28.5 Q7(iv)

Here,

$\vec{a}_1 = 6\hat{i} + 2\hat{j} + 2\hat{k}$

$\vec{a}_2 = -4\hat{i} - \hat{k}$

$\vec{b}_1 = \hat{i} - 2\hat{j} + 2\hat{k}$

$\vec{b}_2 = 3\hat{i} - 2\hat{j} - 2\hat{k}$

$\vec{a}_2 - \vec{a}_1 = -4\hat{i} - \hat{k} - 6\hat{i} - 2\hat{j} - 2\hat{k}$

$= -10\hat{i} - 2\hat{j} - 3\hat{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 3 & -2 & -2 \end{vmatrix}$$

$= \hat{i}(4+4) - \hat{j}(-2-6) + \hat{k}(-2+6)$

$= 8\hat{i} + 8\hat{j} + 4\hat{k}$

Shortest Distance $= \left|\dfrac{(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right|$

$$= \left|\frac{(-10\hat{i} - 2\hat{j} - 3\hat{k}).(8\hat{i} + 8\hat{j} + 4\hat{k})}{|8\hat{i} - 8\hat{j} + 4\hat{k}|}\right|$$

$$= \left|\frac{(-10) \times 8 + (-2) \times 8 + (-3) \times 4}{\sqrt{8^2 + (-8)^2 + (-4)^2}}\right|$$

$$= \left|\frac{-80 - 16 - 12}{\sqrt{64 + 64 + 16}}\right| = \left|\frac{-108}{\sqrt{144}}\right| = 9 \text{ units}$$

Straight Line in Space Ex 28.5 Q8

$\vec{a}_1 = \hat{i} + 2\hat{j} - 4\hat{k}$

$\vec{a}_2 = 3\hat{i} + 3\hat{j} - 5\hat{k}$

$\vec{a}_2 - \vec{a}_1 = 3\hat{i} + 3\hat{j} - 5\hat{k} - \hat{i} - 2\hat{j} + 4\hat{k}$

$= 2\hat{i} + \hat{j} - \hat{k}$

$\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

$$(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix}$$

$= \hat{i}(6+3) - \hat{j}(12+2) + \hat{k}(6-2)$

$= 9\hat{i} - 14\hat{j} + 4\hat{k}$

Shortest distance betweeen 2lines

$$= \left| \frac{(\vec{a}_2 - \vec{a}_1) \times \vec{b}}{|\vec{b}|} \right|$$

$$= \left| \frac{9\hat{i} - 14\hat{j} + 4\hat{k}}{\left|\sqrt{2^2 + 3^2 + 6^2}\right|} \right|$$

$$= \left| \frac{9\hat{i} - 14\hat{j} + 4\hat{k}}{\sqrt{49}} \right|$$

$$= \left| \frac{\sqrt{9^2 + (-14)^2 + 4^2}}{\sqrt{49}} \right|$$

$$= \left| \frac{\sqrt{293}}{\sqrt{49}} \right| = \frac{\sqrt{293}}{7} \text{units}$$