Ex 18.1

Q1(i)

The expansion of $(x+y)^n$ has n+1 term so, the expansion of $(2x+3y)^5$ has 6 terms.

Using binomial theorem, we have

$$(2x+3y)^5 = {}^5C_0(2x)^5(3y)^0 + {}^5C_1(2x)^4(3y)^1 + {}^5C_2(2x)^3(3y)^2 + {}^5C_3(2x)^2(3y)^3 + {}^5C_4(2x)(3y)^4 + {}^5C_5(2x)^0(3y)^5$$

$$= 2^5x^5 + 5\times 2^4\times 3\times x^4\times y + 10\times 2^3\times 3^2\times x^3\times y^2 + 10\times 2^2\times 3^3\times x^2\times y^3 + 5\times 2\times 3^4\times x\times y^4 + 3^5y^5$$

$$= 32x^5 + 240x^4y + 720x^3y^2 + 1080x^2y^3 + 810xy^4 + 243y^5$$

Q1(ii)

The expansion of $(x+y)^n$ has $n+1$ terms so the expansion of $(2x-3y)^4$ has 5 terms.

Using binomial theorem, we have

$$(2x-3y)^4 = {}^4C_0(2x)^4(3y)^0 - {}^4C_1(2x)^3(3y)^1 + {}^4C_2(2x)^2(3y)^2 - {}^4C_3(2x)^1(3y)^3 + {}^4C_4(2x)^0(3y)^4$$

$$= 2^4x^4 - 4\times 2^3\times 3x^3y + 6\times 2^2\times 3^2\times x^2y^2 - 4\times 2\times 3^3\times xy^3 + 3^4y^4$$

$$= 16x^4 - 96x^3y + 216x^2y^2 - 216xy^3 + 81y^4$$

Q1(iii)

The expansion of $(x+y)^n$ has $n+1$ terms so the expansion of $\left(x-\frac{1}{x}\right)^6$ has 7 term.

Using binomial theorem, we get

$$\left(x-\frac{1}{x}\right)^6 = {}^6C_0x^6\left(\frac{1}{x}\right)^0 - {}^6C_1x^5\left(\frac{1}{x}\right) + {}^6C_2x^4\left(\frac{1}{x}\right)^2 - {}^6C_3x^3\left(\frac{1}{x}\right)^3 + {}^6C_4x^2\left(\frac{1}{x}\right)^4 - {}^6C_5x\left(\frac{1}{x}\right)^5 + {}^6C_6x^0\left(\frac{1}{x}\right)^6$$

$$= x^6 - 6x^4 + 15x^2 - 20 + \frac{15}{x^2} - \frac{6}{x^4} + \frac{1}{x^6}$$

Q1(iv)

The expansion of $(x+y)^n$ has $n+1$ terms so the expansion of $(1-3x)^7$ has 8 terms.
Using binomial theorem to expand, we get

$$(1-3x)^7 = {}^7C_0(1)^7(3x)^0 - {}^7C_1(3x) + {}^7C_2(3x)^2 - {}^7C_3(3x)^3 + {}^7C_4(3x)^4 - {}^7C_5(3x)^5 + {}^7C_6(3x)^6 - {}^7C_7(3x)^7$$

$$= 1 - 21x + 21\times 9x^2 - 35\times 3^3x^3 + 35\times 3^4x^4 - 21\times 3^5x^5 + 7\times 3^6x^6 - 3^7x^7$$

$$= 1 - 21x + 189x^2 - 945x^3 + 2835x^4 - 5103x^5 + 5103x^6 - 2187x^7$$

Q1(v)

The expansion of $(x+y)^n$ has $n+1$ terms so the expansion of $\left(ax-\frac{b}{x}\right)^6$ has 7 terms.
Using binomial theorem to expand, we get

$$\left(ax-\frac{b}{x}\right)^6 = {}^6C_0(ax)^6\left(\frac{b}{x}\right)^0 - {}^6C_1(ax)^5\left(\frac{b}{x}\right) + {}^6C_2(ax)^4\left(\frac{b}{x}\right)^2 - {}^6C_3(ax)^3\left(\frac{b}{x}\right)^3 + {}^6C_4(ax)^2\left(\frac{b}{x}\right)^4 - {}^6C_5(ax)\left(\frac{b}{x}\right)^5 + {}^6C_6(ax)^0\left(\frac{b}{x}\right)^6$$

$$= a^6x^6 - 6a^5x^5\frac{b}{x} + 15a^4x^4\frac{b^2}{x^2} - 20a^3b^3 + 15a^2\frac{b^4}{x^2} - 6a\frac{b^5}{x^4} + \frac{b^6}{x^6}$$

$$= a^6x^6 - 6a^5x^4b + 15a^4b^2x^2 - 20a^3b^3 + 15\frac{a^2b^4}{x^2} - 6\frac{ab^5}{x^4} + \frac{b^6}{x^6}$$

Q1(vi)

The expansion of $(x+y)^n$ has $n+1$ terms so the expansion of $\left(\sqrt{\frac{x}{a}}-\sqrt{\frac{a}{x}}\right)^6$ has 7 terms.
Using binomial theorem to expand, we get

$$\left(\sqrt{\frac{x}{a}}-\sqrt{\frac{a}{x}}\right)^6 = {}^6C_0\left(\sqrt{\frac{x}{a}}\right)^6\left(\sqrt{\frac{a}{x}}\right)^0 - {}^6C_1\left(\sqrt{\frac{x}{a}}\right)^5\left(\sqrt{\frac{a}{x}}\right)^1 + {}^6C_2\left(\sqrt{\frac{x}{a}}\right)^4\left(\sqrt{\frac{a}{x}}\right)^2 - {}^6C_3\left(\sqrt{\frac{x}{a}}\right)^3\left(\sqrt{\frac{a}{x}}\right)^3 + {}^6C_4\left(\sqrt{\frac{x}{a}}\right)^2\left(\sqrt{\frac{a}{x}}\right)^4 - {}^6C_5\left(\sqrt{\frac{x}{a}}\right)\left(\sqrt{\frac{a}{x}}\right)^5 + {}^6C_6\left(\sqrt{\frac{x}{a}}\right)^0\left(\sqrt{\frac{a}{x}}\right)^6$$

$$= \left(\frac{x}{a}\right)^{\frac{1}{2}\times 6} - 6\left(\frac{x}{a}\right)^{\frac{1}{2}\times 5}\left(\frac{a}{x}\right)^{\frac{1}{2}} + 15\left(\frac{x}{a}\right)^{\frac{1}{2}\times 4}\left(\frac{a}{x}\right)^{2\times\frac{1}{2}} - 20\left(\frac{x}{a}\right)^{3\times\frac{1}{2}}\left(\frac{a}{x}\right)^{3\times\frac{1}{2}} + 15\left(\frac{x}{a}\right)^{2\times\frac{1}{2}}\left(\frac{a}{x}\right)^{4\times\frac{1}{2}} - 6\left(\frac{x}{a}\right)^{\frac{1}{2}}\left(\frac{a}{x}\right)^{5\times\frac{1}{2}} + \left(\frac{a}{x}\right)^{6\times\frac{1}{2}}$$

$$= \frac{x^3}{a^3} - 6\frac{x^{\frac{5}{2}-\frac{1}{2}}}{a^{\frac{5}{2}-\frac{1}{2}}} + 15\times\frac{x^2\times a}{a^2\times x} - 20\times\frac{x^{\frac{3}{2}-\frac{3}{2}}}{a^{\frac{3}{2}-\frac{3}{2}}} + 15\times\frac{x}{a}\times\frac{a^2}{x^2} - 6\times\frac{x^{\frac{1}{2}-\frac{5}{2}}}{a^{\frac{1}{2}-\frac{5}{2}}} + \frac{a^3}{x^3}$$

$$= \frac{x^3}{a^3} - \frac{6x^2}{a^2} + \frac{15x}{a} - 20 + \frac{15a}{x} - \frac{6a^2}{x^2} + \frac{a^3}{x^3}$$

Q1(vii)

$$\left(\sqrt[3]{x}-\sqrt[3]{a}\right)^6$$

$$=\binom{6}{0}\left(\sqrt[3]{x}\right)^5\left(-\sqrt[3]{a}\right)^0+\binom{5}{1}\left(\sqrt[3]{x}\right)^5\left(-\sqrt[3]{a}\right)^1+\binom{6}{2}\left(\sqrt[3]{x}\right)^4\left(-\sqrt[3]{a}\right)^2$$

$$\binom{6}{3}\left(\sqrt[3]{x}\right)^3\left(-\sqrt[3]{a}\right)^3+\binom{5}{4}\left(\sqrt[3]{x}\right)^2\left(-\sqrt[3]{a}\right)^4+\binom{6}{5}\left(\sqrt[3]{x}\right)^1\left(-\sqrt[3]{a}\right)^5$$

$$\binom{6}{6}\left(\sqrt[3]{x}\right)^0\left(-\sqrt[3]{a}\right)^6$$

$$=x^2-6x^{\frac{5}{3}}a^{\frac{1}{3}}+15x^{\frac{4}{3}}a^{\frac{2}{3}}-20xa+15x^{\frac{2}{3}}a^{\frac{4}{3}}-6x^{\frac{1}{3}}a^{\frac{5}{3}}+a^2$$

Q1(viii)

Let $y = 1+2x$, then

$$\left(1+2x-3x^2\right)^5=\left(y-3x^2\right)^5$$

The expansion of $(x+y)^n$ has $n+1$ terms so the expansion of $\left(y-3x^2\right)^5$ has 6 terms.
Using binomial theorem to expand, we get

$$\left(y-3x^2\right)^5={}^5C_0y^5\left(3x^2\right)^0-{}^5C_1y^4\left(3x^2\right)^1+{}^5C_2y^3\left(3x^2\right)^2-{}^5C_3y^2\left(3x^2\right)^3+{}^5C_4y\left(3x^2\right)^4-{}^5C_5y^0\left(3x^2\right)^5$$

$$=y^5-5y^4\ 3x^2\ +10y^3\ 9x\ ^4-10y^2\left(27x^6\right)+5y81x^8-243x^{10}$$

Now,

$$y^5=(1+2x)^5={}^5C_0+{}^5C_1(2x)^1+{}^5C_2(2x)^2+{}^5C_3(2x)^3+{}^5C_4(2x)^4+{}^5C_5(2x)^5$$

$$y^4=(1+2x)^4={}^4C_0+{}^4C_1(2x)^1+{}^4C_2(2x)^2+{}^4C_3(2x)^3+{}^4C_4(2x)^4$$

$$y^3=(1+2x)^3={}^3C_0+{}^3C_1(2x)+{}^3C_2(2x)^2+{}^3C_3(2x)^3$$

$$y^2=(1+2x)^2={}^2C_0+{}^2C_1(2x)+{}^2C_2(2x)^2$$

$$y=(1+2x)$$

Substituting the valus of powers of y in the equation above, we get,

$$\left(1+2x-3x^2\right)^5=\left[{}^5C_0+{}^5C_1(2x)^1+{}^5C_2(2x)^2+{}^5C_3(2x)^3+{}^5C_4(2x)^4+{}^5C_5(2x)^5\right]$$

$$-15x^2\left[{}^4C_0+{}^4C_1(2x)^1+{}^4C_2(2x)^2+{}^4C_3(2x)^3+{}^4C_4(2x)^4\right]$$

$$+90x^4\left[{}^3C_0+{}^3C_1(2x)+{}^3C_2(2x)^2+{}^3C_3(2x)^3\right]-270x^6$$

$$\left[{}^2C_0+{}^2C_1(2x)+{}^2C_2(2x)^2+5\times81x^8(1+2x)-243x^{10}\right]$$

$$=\quad 10+10x+10\times4x^2+10\times8x^3+5\times16x^4+32x^5-15x^2-120x^3$$

$$-180x^4+480x^5-240x^6+90x^4+540x^5+1080x^6+720x^7-270x^6$$

$$-1080x^7-1080x^8+405x^8+810x^9-243x^{10}$$

$$=1+10x+25x^2-40x^3-190x^4+92x^5+570x^6-360x^7-675x^8+810x^9-243x^{10}$$

Q1(ix)

Let $y = x + 1$, then

$$\left(x+1-\frac{1}{x}\right)^3 = \left(y-\frac{1}{x}\right)^3$$

The expansion of $(x+y)^n$ has $n+1$ terms so the expansion of $\left(y-\frac{1}{x}\right)^3$ has 4 terms.

Using binomial theorem to expand, we get

$$\left(y-\frac{1}{x}\right)^3 = {}^3C_0 y^3\left(\frac{1}{x}\right)^0 - {}^3C_1 y^2\left(\frac{1}{x}\right) + {}^3C_2 y\left(\frac{1}{x}\right)^2 - {}^3C_3 y^0\left(\frac{1}{x}\right)^3$$

$$= y^3 - 3y^2 \times \frac{1}{x} + 3y \times \frac{1}{x^2} - \frac{1}{x^3}$$

Putting $y = x + 1$, we get

$$\left(x+1-\frac{1}{x}\right)^3 = (x+1)^3 - 3(x+1)^2 \times \frac{1}{x} + 3(x+1) \times \frac{1}{x^2} - \frac{1}{x^3}$$

$$= x^3 + 1 + 3x^2 + 3x - 3x - \frac{3}{x} - 6 + \frac{3}{x} + \frac{3}{x^2} - \frac{1}{x^3}$$

$$= x^3 + 3x^2 - 5 + \frac{3}{x^2} - \frac{1}{x^3}$$

Q1(x)

Let $y = 1 - 2x$, then

$$\left(1-2x+3x^2\right)^3 = \left(y+3x^2\right)^3$$

The expansion of $(x+y)^n$ has $n+1$ terms so the expansion of $\left(y+3x^2\right)^3$ has 4 terms.

Using binomial theorem to expand, we get

$$\left(y+3x^2\right)^3 = {}^3C_0 y^3\left(3x^2\right)^0 + {}^3C_1 y^2\left(3x^2\right)^1 + {}^3C_2 y\left(3x^2\right)^2 + {}^3C_3 y^0\left(3x^2\right)^3$$

$$= y^3 + 3y^2\left(3x^2\right) + 3y\left(9x^2\right) + \left(27x^6\right)$$

Substituting $y = 1-2x$, we get,

$$\left(1-2x+3x^2\right)^3 = (1-2x)^3 + 3\left(1+4x^2-4x\right)\left(3x^2\right) + 3(1-2x)\left(9x^2\right) + \left(27x^6\right)$$

$$= 1 - 8x^3 - 6x + 12x^2 + 9x^2 + 36x^4 - 36x^3 + 27x^2 - 54x^3 + 27x^6$$

$$= 1 - 6x + 21x^2 - 44x^3 + 63x^4 - 54x^5 + 27x^6$$

Q2(i)

$$\left(\sqrt{x+1}+\sqrt{x-1}\right)^6+\left(\sqrt{x+1}-\sqrt{x-1}\right)^6$$

$$= {}^6C_0\left(\sqrt{x+1}\right)^6+{}^6C_1\left(\sqrt{x+1}\right)^5\left(\sqrt{x-1}\right)+{}^6C_2\left(\sqrt{x+1}\right)^4\left(\sqrt{x-1}\right)^2-{}^6C_3\left(\sqrt{x+1}\right)^3\left(\sqrt{x-1}\right)^3$$
$$+{}^6C_4\left(\sqrt{x+1}\right)^2\left(\sqrt{x-1}\right)^4+{}^6C_5\left(\sqrt{x+1}\right)\left(\sqrt{x-1}\right)^5+{}^6C_6\left(\sqrt{x-1}\right)^6+{}^6C_0\left(\sqrt{x+1}\right)^6-$$
$${}^6C_1\left(\sqrt{x+1}\right)^5\left(\sqrt{x-1}\right)+{}^6C_2\left(\sqrt{x+1}\right)^4\times\left(\sqrt{x-1}\right)^2-{}^6C_3\left(\sqrt{x+1}\right)^3\left(\sqrt{x-1}\right)^3+$$
$${}^6C_4\left(\sqrt{x+1}\right)^2\left(\sqrt{x-1}\right)^4-{}^6C_5\left(\sqrt{x+1}\right)\left(\sqrt{x-1}\right)^5+{}^6C_6\left(\sqrt{x-1}\right)^6$$

$$=2\left[(x+1)^3+15(x+1)^2(x-1)+15(x+1)(x-1)^2+(x-1)^3\right]$$

$$=2\left[\begin{array}{l}x^3+1+3x+3x^2+15x^3-15x^2+15x-15+30x^2-30x\\ \qquad +15x^3+15x^2+15x+15-30x^2-30x+x^3-1-3x^2+3x\end{array}\right]$$

$$=64x^3-48x$$
$$=16x\left(4x^2-3\right)$$

Q2(ii)

$$\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6$$

$$=2\left[{}^6C_0x^6+{}^6C_2x^4\left(\sqrt{x^2-1}\right)^2+{}^6C_4x^2\left(\sqrt{x^2-1}\right)^4+{}^6C_6\left(\sqrt{x^2-1}\right)^6\right]$$

$$=2\left[x^6+15x^4\left(x^2-1\right)+15x^2\left(x^2-1\right)^2+\left(x^2-1\right)^3\right]$$

$$=2\left[x^6+15x^6-15x^4+15x^6+15x^2-30x^4+x^6-1-3x^4+3x^2\right]$$

$$=64x^6-96x^4+36x^2-2$$

Q2(iii)

$$\left(1+2\sqrt{x}\right)^5+\left(1-2\sqrt{x}\right)^5$$

$$=2\left[{}^5C_0+{}^5C_2\left(2\sqrt{x}\right)^2+{}^5C_4\left(2\sqrt{x}\right)^4\right]$$

$$=2\left[1+10\times4\times x+16\times x^2\times5\right]$$

$$=2+80x+160x^2$$

Q2(iv)

$$\left(\sqrt{2}+1\right)^6+\left(\sqrt{2}-1\right)^6$$

$$={}^6C_0\left(\sqrt{2}\right)^6+{}^6C_1\left(\sqrt{2}\right)^5+{}^6C_2\left(\sqrt{2}\right)^4+{}^6C_3\left(\sqrt{2}\right)^3+{}^6C_4\left(\sqrt{2}\right)^2+{}^6C_5\left(\sqrt{2}\right)+{}^6C_6+{}^6C_0\left(\sqrt{2}\right)^6-$$
$${}^6C_1\left(\sqrt{2}\right)^5+{}^6C_2\left(\sqrt{2}\right)^4-{}^6C_3\left(\sqrt{2}\right)^3+{}^6C_4\left(\sqrt{2}\right)^2-{}^6C_5\left(\sqrt{2}\right)+{}^6C_6\left(\sqrt{2}\right)^0$$

$$=2\left[2^3+15\times2^2+15\times2+1\right]$$

$$=2\left[8+60+30+1\right]=2(99)=198$$

Q2(v)

$$\left(3+\sqrt{2}\right)^5-\left(3-\sqrt{2}\right)^5$$

$$=2\left[{}^5C_1(3)^4\left(\sqrt{2}\right)^1+{}^5C_3(3)^2\left(\sqrt{2}\right)^3+{}^5C_5\left(\sqrt{2}\right)^5\right]$$

$$=2\left[5\times81\times\sqrt{2}+10\times9\times2\sqrt{2}+4\sqrt{2}\right]$$

$$=2\left[405\sqrt{2}+180\sqrt{2}+4\sqrt{2}\right]$$

$$=2\left[589\sqrt{2}\right]$$
$$=1178\sqrt{2}$$

Q2(vi)

$$\left(2+\sqrt{3}\right)^7+\left(2-\sqrt{3}\right)^7$$

$$=2\left[{}^7C_0 2^7+{}^7C_2 2^5\left(\sqrt{3}\right)^2+{}^7C_4(2)^4\left(\sqrt{3}\right)^4+{}^7C_6 2\left(\sqrt{3}\right)^6\right]$$

$$=2[128+21\times32\times3+35\times8\times9+7\times2\times27]$$

$$=2[128+2016+2520+378]$$

$$=2[5042]$$

$$=10084$$

Q2(vii)

$$\left(\sqrt{3}+1\right)^5-\left(\sqrt{3}-1\right)^5$$

$$=2\left[{}^5C_1\left(\sqrt{3}\right)^4+{}^5C_3\left(\sqrt{3}\right)^2+{}^5C_5\right]$$

$$=2[5\times9+10\times3+1]$$

$$=2[45+30+1]$$

$$=2[76]$$

$$=152$$

Q2(viii)

$$(0.99)^5+(1.01)^5$$

$$=(1-.01)^5+(1+.01)^5$$

$$=2\left[{}^5C_1+{}^5C_3(.01)^2+{}^5C_5(.01)^5\right]$$

$$=2\left[5+10\times\frac{1}{10^4}+\frac{1}{10^{10}}\right]$$

$$=2\left[5+\frac{1}{1000}+\frac{1}{10^{10}}\right]$$

$$=2.0020001$$

Q2(ix)

$$\left(\sqrt{3}+\sqrt{2}\right)^6-\left(\sqrt{3}-\sqrt{2}\right)^6$$

$$= 2\left[{}^6C_1\left(\sqrt{3}\right)^5\left(\sqrt{2}\right)+{}^6C_3\left(\sqrt{3}\right)^3\left(\sqrt{2}\right)^3+{}^6C_5\left(\sqrt{3}\right)\left(\sqrt{2}\right)^5\right]$$

$$= 2\left[6\times\sqrt{6}\times 9+20\times 3\sqrt{3}\times 2\sqrt{2}+6\times\sqrt{3}\times 4\sqrt{2}\right]$$

$$= 2\left[54\sqrt{6}+120\sqrt{6}+24\sqrt{6}\right]$$

$$= 2\left[198\sqrt{6}\right]$$
$$= 396\sqrt{6}$$

Q2(x)

$$\left(a^2-\sqrt{a^2-1}\right)^4+\left(a^2-\sqrt{a^2-1}\right)^4$$

Let $a^2 = A$, $\sqrt{a^2-1} = B$

$$(A+B)^4+(A-B)^4$$
$$= B^4+{}^4C_1AB^3-{}^4C_2A^2B^2+{}^4C_3A^3B+A^4+B^4-{}^4C_1AB^3+{}^4{}_2A^2B^2-{}^4C_3A^3B+A^4$$
$$= 2\left(A^4+{}^4C_2A^2B^2+B^4\right)$$
$$= 2\left(A^2+6A^2B^2+B^4\right)$$
$$= 2\left(a^8+6a^4\left(a^2-1\right)+\left(a^2-1\right)^2\right)$$
$$= 2\left[a^8+6a^6-6a^4+a^4-1-2a^2\right]$$

$$\left(a^2-\sqrt{a^2-1}\right)^4+\left(a^2-\sqrt{a^2-1}\right)^4 = 2a^8-12a^6-10a^4-4a^4+2$$

Q3

We have,

$$(a+b)^4-(a-b)^4$$
$$= \left[{}^4C_0a^4b^0+{}^4C_1a^3b^1+{}^4C_2a^2b^2+{}^4C_3a^1b^3+{}^4C_4a^0b^4\right]$$
$$\left[{}^4C_0a^4b^0-{}^4C_1a^3b^1+{}^4C_2a^2b^2-{}^4C_3a^1b^3+{}^4C_4a^0b^4\right]$$
$$= \left[{}^4C_0a^4(-b)^0+{}^4C_1a^3(-b)^1+{}^4C_2a^2(-b)^2+{}^4C_3a^1(-b)^3+{}^4C_4a^0(-b)^4\right]$$
$$-\left[{}^4C_0a^4(-b)^0+{}^4C_1a^3(-b)^1+{}^4C_2a^2(-b)^2+{}^4C_3a^1(-b)^3+{}^4C_4a^0(-b)^4\right]$$
$$= \left[{}^4C_0a^4+{}^4C_1a^3b+{}^4C_2a^2b^2+{}^4C_3ab^3+{}^4C_4ab^4\right]-\left[{}^4C_0a^4-{}^4C_1a^3b-{}^4C_2a^2b^2-{}^4C_3ab^3+{}^4C_4b^4\right]$$
$$= {}^4C_0a^4+{}^4C_1a^3b+{}^4C_2a^2b^2+{}^4C_3ab^3+{}^4C_4ab^4-{}^4C_0a^4+{}^4C_1a^3b-{}^4C_2a^2b^2+{}^4C_3ab^3-{}^4C_4b^4$$
$$= 2\left[{}^4C_1a^3b+{}^4C_3ab^3\right]$$
$$= 2\left[4a^3b+4ab^3\right]$$
$$= 8\left[a^3b+ab^3\right]$$
$$\therefore \quad (a+b)^4-(a-b)^4 = 8\left(a^3b+ab^3\right) \qquad \text{---(i)}$$

Putting $a = \sqrt{3}$ and $b = \sqrt{2}$ in equation (i), we get

$$\left(\sqrt{3}+\sqrt{2}\right)^4-\left(\sqrt{3}-\sqrt{2}\right)^4 = 8\left[\left(\sqrt{3}\right)^3\times\sqrt{2}+\left(\sqrt{3}\right)\times\left(\sqrt{2}\right)^3\right]$$
$$= 8\left[3\sqrt{6}+2\sqrt{5}\right]$$
$$= 8\times 5\sqrt{6}$$
$$= 40\sqrt{6}$$

$$\therefore \quad \left(\sqrt{3}+\sqrt{2}\right)^4-\left(\sqrt{3}-\sqrt{2}\right)^4 = 40\sqrt{6}.$$

Q4

We have,

$$(x+1)^6 - (x-1)^6$$

$$= \left[{}^6C_0 x^6 + {}^6C_1 x^5 + {}^6C_2 x^4 + {}^6C_3 x^3 + {}^6C_4 x^2 + {}^6C_5 x^1 + {}^6C_6 x^0\right]$$

$$+\left[{}^6C_0 x^6 (-1)^0 + {}^6C_1 x^5 (-1)^1 + {}^6C_2 x^4 (-1)^2 + {}^6C_3 x^3 (-1)^3 + {}^6C_4 x^2 (-1)^4 + {}^6C_5 x^1 (-1)^5 + {}^6C_6 x^0 (-1)^6\right]$$

$$= \left[{}^6C_0 x^6 + {}^6C_1 x^5 + {}^6C_2 x^4 - {}^6C_3 x^3 + {}^6C_4 x^2 + {}^6C_5 x + {}^6C_6 - {}^6C_0 x^6 - {}^6C_1 x^5 + {}^6C_2 x^4 - {}^6C_3 x^3 + {}^6C_4 x^2 - {}^6C_5 x + {}^6C_6\right]$$

$$= 2\left[{}^6C_0 x^6 - {}^6C_2 x^4 + {}^6C_4 x^2 + {}^6C_6\right]$$

$$= 2\left[x^6 + 15x^4 + 15x^2 + 1\right]$$

$\therefore \quad (x+1)^6 + (x-1)^6 = 2\left[x^6 + 15x^4 + 15x^2 + 1\right] \qquad$ ---(i)

Putting $x = \sqrt{2}$ in equation (i), we get

$$(x+1)^6 + (x-1)^6 = 2\left[\left(\sqrt{2}\right)^6 + 15\left(\sqrt{2}\right)^4 + 15\left(\sqrt{2}\right)^2 + 1\right]$$

$$= 2\left[8 + 60 + 30 + 1\right]$$

$$= 2\left[99\right]$$

$$= 198$$

$\therefore \quad (x+1)^6 + (x-1)^6 = 198$

Q5(i)

We have,

$$(96)^3 = (100-4)^3$$

$$= {}^3C_0 \times 100^3 + {}^3C_1 \times 100^2 \times (-4) + {}^3C_2 \times 100 \times (-4)^2 + {}^3C_3 \times (-4)^3$$

$$= 100^3 - 3 \times 100^2 \times 4 + 3 \times 100 \times 4^2 - 4^3$$

$$= 1000000 - 120000 + 4800 - 64$$

$$= 1004800 - 120064$$

$$= 884736$$

$\therefore \quad (96)^3 = 884736$

Q5(ii)

We have,

$$(102)^5 = (100+2)^5$$
$$= {}^5C_0 \times 100^5 + {}^5C_1 \times 100^4 \times 2 + {}^5C_2 \times 100^3 \times 2^2 + {}^5C_3 \times 100^2 \times 2^3 + {}^5C_4 \times 100 \times 2^4 + {}^5C_5 \times 2^5$$
$$= 100^5 + 5 \times 100^4 \times 2 + 10 \times 100^3 \times 2^2 + 10 \times 100^2 \times 2^3 + 5 \times 100 \times 2^4 + 2^5$$
$$= 10000000000 + 1000000000 + 40000000 + 800000 + 8000 + 32$$
$$= 11040808032$$

$$\therefore \quad (102)^5 = 11040808032$$

Q5(iii)

We have,

$$(101)^4 = (100+1)^4$$
$$= {}^4C_0 \times 100^4 + {}^4C_1 \times 100^3 + {}^4C_2 \times 100^2 + {}^4C_3 \times 100 + {}^4C_4$$
$$= 100^4 + 4 \times 100^3 + 6 \times 100^2 + 4 \times 100 + 1$$
$$= 100000000 + 4000000 + 60000 + 400 + 1$$
$$= 104060401$$

$$\therefore \quad (101)^4 = 104060401$$

Q5(iv)

We have,

$$(98)^5 = (100-2)^5$$
$$= {}^5C_0 \times 100^5 - {}^5C_1 \times 100^4 \times (-2) + {}^5C_2 \times 100^3 \times (-2)^2 - {}^5C_3 \times 100^2 \times (-2)^3 + {}^5C_4 \times 100 \times (-2)^4 - {}^5C_5 \times (-2)^5$$
$$= {}^5C_0 \times 100^5 - {}^5C_1 \times 100^4 \times 2 + {}^5C_2 \times 100^3 \times 4 - {}^5C_3 \times 100^2 \times 8 + {}^5C_4 \times 100 \times 16 - {}^5C_5 \times 32$$
$$= 100^5 - 10 \times 100^4 + 40 \times 100^3 - 80 \times 100^2 + 80 \times 100 - 32$$
$$= 10000000000 - 1000000000 + 40000000 - 800000 + 8000 - 32$$
$$= 10040008000 - 1000800032$$
$$= 9039207968$$

$$\therefore \quad (98)^5 = 9039207968$$

Q6

$$2^{3n}-7n-1$$
$$=2^{3(n)}-7(n)-1$$
$$=8^{n}-7n-1$$
$$=(1+7)^{n}-7n-1$$
$$=\left({}^{n}C_{0}+{}^{n}C_{1}(7)^{1}+{}^{n}C_{2}(7)^{2}+......{}^{n}C_{n}(7)^{n}\right)-7n-1$$
$$=\left(1+7n+49\,{}^{n}C_{2}+......+49(7)^{n-2}\right)-7n-1$$
$$=49\left({}^{n}C_{2}+......+7^{n-2}\right)$$

$\therefore$ $2^{3n}-7n-1$ is divisible by 49

Hence, proved

Q7

$$3^{2n+2}-8n-9$$
$$=3^{2(n+1)}-8n-9$$
$$=9^{n+1}-8n-9$$
$$=(1+8)^{n+1}-8n-9$$
$$=\left({}^{n+1}C_{0}+{}^{n+1}C_{1}8^{1}+{}^{n+1}C_{2}8^{2}+......+{}^{n+1}C_{n+1}8^{n+1}\right)-8n-9$$
$$=\left(1+8(n+1)+64\,{}^{n+1}C_{2}+......+64(8)^{n-1}\right)-8n-9$$
$$=64\left({}^{n+1}C_{2}+......+8^{n-1}\right)$$

Thus, $3^{2n+2}-8n-9$ is divisible by 64.

Q8

$$3^{3n}-26n-1$$
$$=\left(3^3\right)^n-26n-1$$
$$=27^n-26n-1$$
$$=(1+26)^n-26n-1$$
$$=\left({}^nC_0+{}^nC_1(26)^1+{}^nC_2(26)^2+\ldots\ldots+{}^nC_n(26)^n\right)-26n-1$$
$$=\left(1+26n+676\,{}^nC_2+\ldots\ldots+676(26)^{n-2}\right)-26n-1$$
$$=676\left({}^nC_2+\ldots\ldots+(26)^{n-2}\right)$$

$\therefore$ $3^{3n}-26n-1$ **is divisible for** $n\in N$.

Hence, proved

Q9

We have,

$$(1.1)^{10000}=(1+0.1)^{10000}$$
$$={}^{10000}C_0+{}^{10000}C_1(0.1)+{}^{10000}C_2(0.1)^2+\ldots+{}^{10000}C_{10000}(0.1)^{10000}$$
$$=1+10000\times(0.1)+\text{ other positive terms}$$
$$=1+1000+\text{ other positive terms}$$
$$=1001+\text{ other positive terms}>1000$$

$$\therefore\quad(1.1)^{10000}>1000$$

Q10

$$(1.2)^{4000}=(1+0.2)^{4000}$$
$$={}^{4000}C_0(0.2)^0(1)^{4000}+{}^{4000}C_1\times(0.2)^1\times1^{3999}+\ldots\ldots\ldots+{}^{4000}C_{400}(0.2)^{4000}1^0$$
$$=1+4000\times0.2\times1+\ldots\ldots\ldots+(0.2)^{4000}$$
$$=1+800+\ldots\ldots\ldots+(0.2)^{4000}$$

Here, we clearly observe $(1,2)^{4000}$ **is** less **than** (801) **thus,** $(1.2)^{4000}<800$.

Q11

$$(1.01)^{10}+(1-0.01)^{10} = (1+0.01)^{10}+(1-0.01)^{10}$$

$$= \left({}^{10}C_1 + {}^{10}C_2\frac{1}{10^2} + {}^{10}C_3\frac{1}{10^3} \ldots + {}^{10}C_{10}\frac{1}{10^{10}}\right) + \left({}^{10}C_1 - {}^{10}C_2\frac{1}{10^2} + {}^{10}C_3\frac{1}{10^3} - {}^{10}C_4\frac{1}{10^4} + \ldots\right)$$

$$= 2\left({}^{10}C_1 - {}^{10}C_3\frac{1}{10^3} + {}^{10}C_5\frac{1}{10^5} + {}^{10}C_7\frac{1}{10^7} + {}^{10}C_9\frac{1}{10^9}\right)$$

$$= 2\left(10 + \frac{10!}{3!7!}\frac{1}{1000} + \frac{10!}{5!5!}\frac{1}{(10)^5} + \frac{10!}{7!3!}\times\frac{1}{10^7} + \frac{10!}{9!1!}\frac{1}{10^9}\right)$$

$$= 2\left(10 + \frac{9\times 8}{3\times 2\times 1000} + \frac{9\times 8\times 7\times 6}{5\times 4\times 3\times 2\times 10^5} + \frac{9\times 8}{3\times 2\times 10^7} + \frac{1}{10^8}\right)$$

$$= 2.0090042$$

Q12

$$2^{4n+4} - 15n - 16 = 2^{4(n+1)} - 15n - 15 - 1$$

$$= (16)^{(n+1)} - 15(n+1) - 1$$

$$= (1+15)^{n+1} - 15(n+1) - 1$$

$$= \left[{}^{n+1}C_0 + {}^{n+1}C_1(15) + {}^{n+1}C_2(15)^2 + \ldots\ldots + {}^{n+1}C_{n+1}(15)^{n+1}\right] - 15(n+1) - 1$$

$$= \left[1 + 15(n+1) + {}^{n+1}C_2(15)^2 + \ldots\ldots + {}^{n+1}C_{n+1}(15)^{n+1}\right] - 15(n+1) - 1$$

$$= 225\left[{}^{n+1}C_2 + \ldots\ldots + {}^{n+1}C_{n+1}(15)^{n-1}\right]$$

$$= 225 \times \text{natural number}$$

Ex 18.2

Q1

$$T_{r+1} = T_n = (-1)^r\ {}^nC_r x^{n-r} y^r$$

$$T_{11} = T_{10+1} = (-1)^{10}\ {}^{25}C_{10} (2x)^{15}\left(\frac{1}{x^2}\right)^{10} = {}^{25}C_{10}\left(\frac{2^{15}}{x^5}\right) = \frac{25!}{10!5!} 2^{15} x^{15} \times x^{-20}$$

11^{th} term from the end $= (26 - 11 + 1) = 16^{th}$ from beginning.

$$\Rightarrow \quad T_{16} = T_{15+1} = (-1)^{15}\ {}^{25}C_{15} (2x)^{10}\left(\frac{1}{x^2}\right)^{15} = -{}^{25}C_{15}\frac{2^{10}}{x^{20}}$$

Q2

$$T_n = T_{r+1} = (-1)^r\ x^{n-r} y^r \times {}^{10}C_r$$

$$n = 7,\ r = 6,\ x = 3x^2,\ y = \frac{1}{x^3}$$

$$T_7 = T_{6+1} = (-1)^6\ {}^{10}C_6 (3x^2)^4\left(\frac{1}{x^3}\right)^6 = {}^{10}C_6 3^4 x^8 \times \frac{1}{x^{18}} = {}^{10}C_6 \times \frac{81}{x^{10}} = \frac{210 \times 81}{x^{10}} = \frac{17010}{x^{10}}$$

Q3

Fifth term from the end is

$(11 - 5 + 1) = 7^{th}$ term from beginning

$$T_7 = T_{6+1} = (-1)^r\ {}^nC_7 x^{n-r} y^r$$

$$= (-1)^6\ {}^{10}C_6 (3x)^4\left(\frac{1}{x^2}\right)^6 = {}^{10}C_6 \times 3^4 \times \frac{x^4}{x^{12}} = \frac{210 \times 81}{x^8} = \frac{17010}{x^8}$$

Q4

$$T_N = T_{r+1} = (-1)^r\ {}^nC_r x^{n-r} y^r$$

$$N = 8,\ r = 7,\ x = x^{3/2}y^{1/2},\ y = x^{1/2}y^{3/2},\ n = 10$$

$$T_8 = T_{7+1} = (-1)^7\ {}^{10}C_7 \left(x^{3/2}y^{1/2}\right)^3\left(x^{1/2}y^{3/2}\right)^7 = -{}^{10}C_7 x^{9/2} \times x^{7/2} \times y^{3/2} y^{21/2} = -120x^8y^{12}$$

Q5

$$T_N = T_{r+1} = {}^nC_r x^{n-r} y^r$$

$$N = 7,\ r = 6,\ n = 8,\ x = \frac{4x}{5},\ y = \frac{5}{2x}$$

$$T_7 = T_{6+1} = {}^8C_6\left(\frac{4x}{5}\right)^2\left(\frac{5}{2x}\right)^6 = 28 \times \frac{4^2}{5^2} \times x^4 \times \frac{5^6}{2^6 \times x^6} = \frac{28}{4} \times \frac{5^4}{x^4} = \frac{7 \times 5 \times 125}{x^4} = \frac{4375}{x^4}$$

Q6

Term from the beginning

$$T_N = T_{r+1} = {}^nC_r x^{n-r} y^r \qquad \text{---(i)}$$

$$N = 4,\ r = 3,\ n = 9,\ x = x,\ y = \frac{2}{x}$$

$$T_4 = T_{3+1} = {}^9C_3 x^6 \left(\frac{2}{x}\right)^3 = \frac{9 \times 7 \times 8}{3 \times 2} x^3 \times 8 = \quad 672x^3$$

4^{th} term from the end = 7^{th} term from beginning

Using (i)

$$N = 7,\ r = 6,\ n = 9,\ x = x,\ y = \frac{2}{x}$$

$$T_7 = T_{6+1} = {}^9C_6 x^3 \left(\frac{2}{x}\right)^6 = \frac{9 \times 8 \times 7}{3 \times 2} \times \frac{2^6}{x^3} = \frac{5376}{x^3}$$

Q7

$$T_N = T_{r+1} = (-1)^r\ {}^nC_2 x^{n-r} y^r$$

4^{th} term from the end = 7^{th} term from beginning

$$N = 7,\ r = 6,\ n = 9,\ x = \frac{4x}{5},\ y = \frac{5}{2x}$$

$$T_7 = T_{6+1} = (-1)^6\ {}^9C_6 \left(\frac{4x}{5}\right)^3 \left(\frac{5}{2x}\right)^6 = \frac{9 \times 8 \times 7}{3 \times 2} \times \frac{4^3 \times 5^6}{5^3 \times 2^6} \times \frac{x^3}{x^6} = \frac{9 \times 8 \times 7 \times 5^3}{6 \times x^3} = \frac{9 \times 8 \times 7 \times 125}{6 \times x^3} = \frac{10500}{x^3}$$

Q8

7th term from the end = 3^{rd} term from beginning

$$T_N = T_{r+1} = (-1)^r\ {}^nC_2 x^{n-r} y^r$$

$$N = 3,\ r = 2,\ n = 8,\ x = 2x^2,\ y = \frac{3}{2x}$$

$$T_3 = T_{2+1} = (-1)^2\ {}^8C_2 \left(2x^2\right)^6 \left(\frac{3}{2x}\right)^2 = \frac{8 \times 7}{2} \times \frac{2^6 \times 3^2 \times x^{12}}{2^2 \times x^2} = 8 \times 7 \times 9 \times 8 \times x^{10} = 4032x^{10}$$

Q9(i)

x^{10} in $\left(2x^2 - \frac{1}{x}\right)^{20}$

$$T_n = T_{r+1} = (-1)^r \, {}^nC_r x^{n-r} y^r$$

$$(-1)^r \, {}^{20}C_r \left(2x^2\right)^{20-r} \left(\frac{1}{x}\right)^r$$

Coefficient of x^{10} is

$$(-1)^r \, {}^{20}C_r 2^{20-r} x^{40-2r} x^{-r} \qquad \text{—(i)}$$

$\Rightarrow \quad x^{40-3r} = x^{10}$

$\Rightarrow \quad 10 - 3r = 10$

$3r = 30$

$r = 10$

Substituting $r = 10$ in(i)

$$(-1)^{10} \, {}^{20}C_{10} 2^{10}$$

$= \quad {}^{20}C_{10} 2^{10}$

Q9(ii)

x^7 in $\left(x - \frac{1}{x^2}\right)^{40}$

$$T_n = T_{r+1} = (-1)^r \, {}^nC_r x^{n-r} y^r$$

$= (-1)^r \, {}^{40}C_r x^{40-r} \left(\frac{1}{x^2}\right)^r$

$= (-1)^r \, {}^{40}C_r x^{40-r-2r}$

$\Rightarrow \quad x^7 = x^{40-3r}$

$7 = 40 - 3r$

$3r = 33$

$r = 11$

$= \quad (-1)^{11} \, {}^{40}C_{11}$ is coeff of x^7

$= -{}^{40}C_{11}$

Q9(iii)

$$x^{-15} \text{ in } \left(3x^2 - \frac{a}{3x^3}\right)^{10}$$

$$(-1)^r \, {}^{10}C_r \left(3x^2\right)^{10-r} \left(\frac{a}{3x^3}\right)^r$$

$$(-1)^r \, {}^{10}C_r \, \frac{3^{10-r} a^r}{3^r} x^{20-2r-3r}$$

$$\Rightarrow \quad x^{20-5r} = x^{-15}$$

$$20 - 5r = -15$$

$$35 = 5r$$

$$r = 7$$

$$(-1)^7 \, {}^{10}C_7 \, \frac{3^3 a^7}{3^7}$$

$$-\frac{40}{27} a^7$$

Q9(iv)

$$x^9 \text{ in expansion of } \left(x^2 - \frac{1}{3x}\right)^9$$

$$T_n = T_{r+1} = (-1)^r \, {}^nC_r x^{n-r} y^r$$

$$= (-1)^r \, {}^9C_r \left(x^2\right)^{9-r} \left(\frac{1}{3x}\right)^r$$

$$= (-1)^r \, {}^9C_r \times \frac{1}{3^r} \times x^{18-2r-r}$$

$$\Rightarrow \quad x^{18-3r} = x^9$$

$$18 - 3r = 9$$

$$3r = 9$$

$$r = 3$$

$$= (-1)^3 \, {}^9C_3 \, \frac{1}{3^3}$$

$$= -\frac{9 \times 8 \times 7}{3 \times 2 \times 9 \times 3}$$

$$= \frac{-28}{9}$$

Q9(v)

$$x^m \text{ in expansion of } \left(x+\frac{1}{x}\right)^n$$

$$T_n = {}^nC_r x^{n-r} y^r$$

$$= {}^nC_r x^{n-r}\left(\frac{1}{x}\right)^r$$

$$x^{n-2r} = x$$

$$n-2r = m$$

$$r = \frac{n-m}{2}$$

$${}^nC_{\frac{n-m}{2}} = \frac{n!}{\left(\frac{n-m}{2}\right)!\left(\frac{n+m}{2}\right)!}$$

Q9(vi)

$$\left(1-2x^3+3x^4\right)\left(1+\frac{1}{x}\right)^8 = \left(1-2x^3+3x^4\right)\left(\begin{array}{l} {}^8C_0 + {}^8C_1\frac{1}{x} + {}^8C_2\left(\frac{1}{x}\right)^2 + {}^8C_3\left(\frac{1}{x}\right)^3 + {}^8C_4\left(\frac{1}{x}\right)^4 + \\ {}^8C_5\left(\frac{1}{x}\right)^5 + {}^8C_6\left(\frac{1}{x}\right)^6 + {}^8C_7\left(\frac{1}{x}\right)^7 + {}^8C_8\left(\frac{1}{x}\right)^8 \end{array}\right)$$

$$= -\left(2x^3\right)\left({}^8C_3\left(\frac{1}{x}\right)^3\right) + \left(3x^4 \times {}^8C_4\left(\frac{1}{x}\right)^4\right)$$

$$= -(56)+(210)$$

$$= -112+168$$

$$= 154$$

Q9(vii)

$$(a-2b)^{12} = {}^{12}C_0 a^{12} - {}^{12}C_1 a^{11}(2b)^1 + {}^{12}C_2 a^{10}(2b)^2 - {}^{12}C_3 a^9 (2b)^3 + \_\_ - {}^{12}C_7 a^5 (2b)^7 + \_\_$$

$$= -\frac{12!}{7!5!} \times 128$$

$$= -\frac{12\times 11\times 10\times 9\times 8}{5\times 4\times 3\times 2} \times 128$$

$$= -101376$$

Q9(viii)

$$\left(1-3x+7x^2\right)\left(1-x\right)^{16} = \left(1-3x+7x^2\right)\left({}^{16}C_0 - {}^{16}C_1x + {}^{n}C_2x^2 + \ldots\ldots + {}^{16}C_{16}x^{16}\right)$$

$\therefore$ Coefficient of x in $\left(1-3x+7x^2\right)\left(1-x\right)^{16}$

$= 1\times\left(-{}^{16}C_1\right) - 3\times\left(-{}^{16}C_0\right)$

$= -16-3$

$= -19$

Q10

$$T_n = T_{r+1} = {}^{n}C_r x^{n-r} y^r$$

$$= {}^{21}C_r \left(\left(\frac{x}{\sqrt{y}}\right)^{\frac{1}{3}}\right)^{21-r} \left(\left(\frac{y}{x^{\frac{1}{3}}}\right)^{\frac{1}{2}}\right)^r$$

$$= {}^{21}C_r \left(\frac{x^{7-\frac{r}{3}}}{y^{\frac{7}{2}-\frac{r}{6}}}\right)\frac{y^{\frac{r}{2}}}{x^{\frac{r}{6}}}$$

$$\frac{x^{7-\frac{r}{3}-\frac{r}{6}}}{y^{\frac{7}{2}-\frac{r}{6}-\frac{r}{2}}}$$

$$\Rightarrow \quad x\,\frac{42-2r-r}{6} = y\,\frac{21-r-3r}{6}$$

Since x and y have same power

$$\frac{42-3r}{6} = \frac{-(21-4r)}{6}$$

$$42+21 = 4r+3r$$

$$63 = 7r$$

$$r = 9$$

Term is 10^{th}

$$(t_n = t_{r+1})$$

Q11

$$(-1)^r\ {}^{20}C_r\left(2x^2\right)^{20-r}\left(\frac{1}{x}\right)^r$$
$$x^{40-2r}x^{-r}=x^9$$
$$40-3r=9$$
$$31=3r$$
$$r=\frac{31}{3}$$
r can not be in fraction

$\therefore$ There is no term involving x^9.

Q12

Any term in the expansion of $\left(x^2+\frac{1}{x}\right)^{12}$ is

$$t_N=t_{r+1}={}^nC_r x^{n-r}y^r$$
$$={}^{12}C_r\left(x^2\right)^{12-r}\left(\frac{1}{x}\right)^{12}$$
$$={}^{12}C_r x^{24-2r}x^{-12}$$
$$x^{12-2r}=x^{-1}$$
$$12-2r=-1$$
$$2r=13$$
$$r=\frac{13}{2}$$

r can not be a fraction, therefore there is no term in the expansion of $\left(x^2+\frac{1}{x}\right)^{12}$

having the term x^{-1}.

Q13(i)

$$\left(\frac{2}{3}x-\frac{3}{2x}\right)^{20}$$

Here, $n-20$ which is an even number so, $\left(\frac{20}{2}+1\right)^{th}$ i.e., 11^{th} term is the middle term.

We know that,

$$T_n=T_{r+1}=(-1)^r\ {}^nC_r x^{n-r}y^r$$
$$n=20,\ r=10,\ x=\frac{2}{3}x,\ Y=\frac{2}{3X}$$
$$T_{11}=T_{10+1}=(-1)^{10}\ {}^{20}C_{10}\left(\frac{2}{3}x\right)^{10}\left(\frac{3}{2x}\right)^{10}$$
$$=\quad {}^{20}C_{10}\frac{2^{10}}{3^{10}}\times\frac{3^{10}}{2^{10}}\times\frac{x^{10}}{x^{10}}$$
$$=\quad {}^{20}C_{10}$$

Q13(ii)

Here, $n = 12$, which is even number.

SO, $\left(\frac{12}{2}+1\right)$th term i.e., 7th term is the middle term.

Hence, the middle term $= T_7 = T_{6+1}$

$$\therefore \quad T_7 = T_{6+1} = {}^{12}C_6 \times \left(\frac{a}{x}\right)^{12-6} \times (bx)^6$$

$$= {}^{12}C_6 \left(\frac{a}{x}\right)^6 \times (bx)^6$$

$$= \frac{12!}{(12-6)!6!} \times \frac{a^6}{x^6} \times b^6x^6$$

$$= \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6!}{(6 \times 5 \times 4 \times 3 \times 2 \times 1)} \times a^6b^6$$

$$= 924 \times a^6b^6$$

$\therefore$ The middle term $= 924 \times a^6b^6$.

Q13(iii)

$$\left(x^2 - \frac{2}{x}\right)^{10}$$

Here, $n = 10$

$\therefore$ $\left(\frac{n}{2}+1\right)^{th} = \left(\frac{10}{2}+1\right)^{th} = 6^{th}$ **term is the middle term.**

The term formula is

$$T_n = T_{r+1} = (-1)^r \, {}^nC_r x^{n-r} y^r$$

$$T_6 = T_{5+1} = (-1)^5 \, {}^{10}C_5 \left(x^2\right)^{10-5} \left(\frac{2}{x}\right)^5$$

$$= -{}^{10}C_5 x^{20-10} \frac{2^5}{x^5}$$

$$= \frac{-10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2} x 2^5 x^5$$

$$= -8064x^5$$

Q13(iv)

$$\left(\frac{x}{a} - \frac{a}{x}\right)^{10}$$

Here $n = 10$, which is even, therefore it has 11 terms

$\therefore$ middle term is $\left(\frac{n}{2}+1\right) = 6^{th}$ term

$$T_n = T_{r+1} = (-1)^r \, {}^nC_r x^{n-r} y^r$$

$$T_6 = T_{5+1} = (-1)^5 \, {}^{10}C_5 \left(\frac{x}{a}\right)^{10-5} \left(\frac{a}{x}\right)^5$$

$$= -\frac{10!}{5!5!} \times \frac{x^5}{a^5} \times a^5 \times x^{-5}$$

$$= -252$$

Q14(i)

$$\left(3x - \frac{x^3}{6}\right)^9$$

Here, $n = 9$, which is odd number

$\therefore \left(\frac{9+1}{2}\right)^{th}$ and $\left(\frac{9+1}{2}+1\right)^{th}$ i.e., 5^{th}, 6^{th} term are the middle term.

Here, the term formula is:

$$T_5 = T_{4+1} = (-1)^4\, {}^9C_4 (3x)^5 \left(\frac{x^3}{6}\right)^4$$

$$= {}^9C_4 \frac{3^5}{6^4} \times x^5 \times x^{12}$$

$$= \frac{9\times8\times7\times6\times3^5}{4\times3\times2\times3^4\times2^4} x^{17}$$

$$= \frac{189}{8} x^{17}$$

$$T_6 = T_{5+1} = (-1)^5\, {}^9C_5 (3x)^4 \left(\frac{x^3}{6}\right)^5$$

$$= -\frac{9\times8\times7\times6}{5\times4\times3\times2}\times\frac{3^4}{6^5}\times x^4\times x^{15}$$

$$= -\frac{9\times8\times7\times6\times3^4}{5\times4\times3\times2\times3^5\times2^5} x^{19}$$

$$= \frac{-21}{16} x^{19}$$

Q14(ii)

$$\left(3x^2 - \frac{1}{x}\right)^7$$

Here, $n = 7$, which is odd

$\therefore \left(\frac{7+1}{2}\right)^{th}$ and $\left(\frac{7+1}{2}+1\right)^{th} = 4^{th}$, 5^{th} term are middle term or $\left(2x^2 - \frac{1}{x}\right)^7$

$$T_n = T_{r+1} = (-1)^r\, {}^nC_r x^{n-r} y^r$$

$$T_4 = T_{3+1} = (-1)^3\, {}^7C_3 \left(2x^2\right)^{7-3}\left(\frac{1}{x}\right)^3$$

$$= -{}^7C_3 \frac{2^4 x^8}{x^3}$$

$$= -560x^5$$

$$T_5 = T_{4+1} = (-1)^4\, {}^7C_4 \left(2x^2\right)^{7-4}\left(\frac{1}{x}\right)^4$$

$$= {}^7C_4 \frac{2^3 x^6}{x^4}$$

$$= {}^7C_4 \frac{7\times6\times5\times8}{3\times2} x^2$$

$$= 280x^2$$

Q14(iii)

$$\left(3x-\frac{2}{x^2}\right)^{15}$$

7^{th} and 8^{th} terms are middle terms

$$\binom{15}{7}(3x)^8\left(-\frac{2}{x^2}\right)^7, \binom{15}{8}(3x)^7\left(-\frac{2}{x^2}\right)^8$$

$$\frac{-6435\times 3^8\times 2^7}{x^6}, \frac{6437\times 3^7\times 2^8}{x^9}$$

Q14(iv)

$$\left(x^4-\frac{1}{x^3}\right)^{11}$$

Here, $n = 11$, which is odd number

$\therefore \left(\frac{11+1}{2}\right)^{th}$ **and** $\left(\frac{11+1}{2}+1\right)^{th} = 6^{th}, 7^{th}$ **term are the middle terms in** $\left(x^4-\frac{1}{x^3}\right)^{11}$

The term formula is

$$T_n = T_{r+1} = (-1)^r\ {}^nC_r x^{n-r} y^r$$

$$T_6 = T_{5+1} = (-1)^5\ {}^{11}C_5 \left(x^4\right)^{11-5}\left(\frac{1}{x^3}\right)^5$$

$$= -{}^{11}C_5 x^{24}\frac{1}{x^{15}}$$

$$= \frac{-11\text{x}10\text{x}9\text{x}8\text{x}7}{5\text{x}4\text{x}3\text{x}2\text{x}1}x^9$$

$$= -11\text{x}3\text{x}2\text{x}7\ \ x^9$$

$$= -462\ x^9$$

$$T_7 = T_{6+1} = (-1)^6\ {}^{11}C_6 \left(x^4\right)^{11-6}\left(\frac{1}{x^3}\right)^6$$

$$= 462\frac{x^{20}}{x^{18}}$$

$$= 462x^2$$

Q15(i)

$$\left(x-\frac{1}{x}\right)^{10}$$

Here, $n=10$, which is even, $\therefore$ it has 11 terms

$\therefore$ middle term is $\left(\frac{n}{2}+1\right)=6^{th}$ term

$$T_n = T_{r+1} = (-1)^r \, {}^nC_r x^{n-r} y^r$$

$$T_6 = T_{5+1} = (-1)^5 \, {}^{10}C_5 (x)^{10-5}\left(\frac{1}{x}\right)^5$$

$$= \frac{-10\times9\times8\times7\times6}{5\times4\times3\times2}\times\frac{x^5}{x^5}$$

$$=-3\times2\times7\times6$$

$$=-252$$

Q15(ii)

$$\left(1-2x+x^2\right)^n$$

Here, n is odd, $\because$ $\left(1-2x+x^2\right)$ has $n+1$ = even term

$\therefore$ middle term is $\left(\frac{n+1}{2}\right)^{th}$ term

$$T_n = T_{r+1} = {}^nC_r x^{n-r} y^r$$

$$T_{\frac{n+1}{2}} = T_{\frac{n}{2}} = {}^nC_{\frac{n}{2}} (1-2x)^{n-\frac{n}{2}} \left(x^2\right)^{\frac{n}{2}}$$

$$= \frac{n!}{\frac{n}{2}!\frac{n}{2}!}(1-2x)^{\frac{n}{2}} x^{\frac{2n}{2}}$$

$$= \frac{(2n)!}{(n!)^2}(-1)^n x^n \qquad \left[\because (1-x)^n = 1-nx\right]$$

Q15(iii)

$$\left(1+3x+3x^2+x^3\right)^{2n}$$

This expansion is $\left((1+x)^3\right)^{2n} = (1+x)^{6n}$

Since $6n$ is even $\therefore$ it has $6n+1$ = odd terms has middle term is

$$\left(\frac{6n}{2}+1\right)^{th} = (4n)^{th} \text{ term}$$

$$T_n = T_{r+1} = {}^nC_r x^{n-r} y^r$$

$$T_{4n} = T_{3n+1} = {}^{6n}C_{3n} (1)^{6n-3n} (x)^{3n}$$

$$= \frac{(6n)!}{(3n)!(3n)!} x^{3n} \qquad \left[\because 1^{6n-3n} = 1\right]$$

Q15(iv)

$$\left(2x-\frac{x^2}{4}\right)^9$$

4^{th} and 5^{th} terms are middle terms

$$\binom{9}{4}(2x)^5\left(-\frac{x^2}{4}\right)^4+\binom{9}{5}(2x)^4\left(-\frac{x^2}{4}\right)^5$$

$$\frac{63}{4}x^{13},-\frac{63}{32}x^{14}$$

Q15(v)

$$\left(x-\frac{1}{x}\right)^{2n+1}$$

$2n+1$ is odd hence this expansion will have $2n+2$ = even terms.

Hene, middle terms is $\frac{2n+1}{2}=n+1,n+2$

Term formula is

$$T_n=T_{r+1}=(-1)^r\ {}^nC_r x^{n-r}y^r$$

$$T_{n+1}=T_{n+1}=(-1)^n\ {}^{2n+1}C_n(x)^{2n+1-n}\left(\frac{1}{x}\right)^n$$

$$=(-1)^n\ {}^{2n+1}C_n x^{n+1-n}$$

$$=(-1)^n\ {}^{2n+1}C_n x$$

$$T_{n+2}=T_{n+1+1}=(-1)^{n+1}\ {}^{2n+1}C_{n+1}(x)^{2n+1-n-1}\left(\frac{1}{x}\right)^{n+1}$$

$$=(-1)^{n+1}\ {}^{2n+1}C_{n+1}x^{-1}$$

$$=(-1)^{n+1}\ {}^{2n+1}C_{n+1}\frac{1}{x}$$

$$=(-1)^{n+1}\ {}^{2n+1}C_n\frac{1}{x}\qquad\left[\because {}^nC_r={}^nC_{r-1}\right]$$

Q15(vi)

$$\left(3-\frac{x^3}{6}\right)^7$$

Here $n=7$, which is odd

$\therefore$ middle term is $\left(\frac{7+1}{2}\right)$ and $\left(\frac{7+1}{2}+1\right)=4^{th},5^{th}$ terms

$$T_n=T_{r+1}=(-1)^r\,{}^nC_r x^{n-r}y^r$$

$$T_4=T_{3+1}=(-1)^3\,{}^7C_3(3)^{7-3}\left(\frac{x^3}{6}\right)^3$$

$$=-\frac{7!}{3!4!}\times 3^4\times\frac{x^9}{6^3}$$

$$=-\frac{7\times6\times5}{3\times2\times1}\times81\times\frac{x^9}{216}$$

$$=-\frac{105}{8}x^9$$

And

$$T_n=T_{r+1}=(-1)^r\,{}^nC_r x^{n-r}y^r$$

$$T_5=T_{4+1}=(-1)^4\,{}^7C_4(3)^{7-4}\left(\frac{x^3}{6}\right)^4$$

$$=\frac{7!}{4!3!}\times3^3\times\frac{x^{12}}{6^4}$$

$$=\frac{7\times6\times5}{3\times2\times1}\times27\times\frac{x^{12}}{1296}$$

$$=\frac{35}{48}x^{12}$$

Q15(vii)

$$\left(\frac{x}{3}+9y\right)^{10}$$

Here $n=10$, which is even, therefore it has 11 terms

$\therefore$ middle term is $\left(\frac{n}{2}+1\right)=6^{th}$ term

$$T_n=T_{r+1}=(-1)^r\,{}^nC_r x^{n-r}y^r$$

$$T_6=T_{5+1}=(-1)^5\,{}^{10}C_5\left(\frac{x}{3}\right)^{10-5}(9y)^5$$

$$=-\frac{10!}{5!5!}\times\frac{x^5}{3^5}\times9^5\times y^5$$

$$=61236x^5y^5$$

Q15(viii)

For the given binomial expansion n = 12

So middle term is $\left(\frac{12}{2}+1\right) = 7^{th}$ term.

$$T_7 = {}^{12}C_6 (2ax)^{12-6} \left(-\frac{b}{x^2}\right)^6$$

$$T_7 = {}^{12}C_6 (2ax)^6 \left(\frac{b}{x^2}\right)^6$$

$$T_7 = {}^{12}C_6 (2^6 a^6 x^6) \left(\frac{b^6}{x^{12}}\right)$$

$$T_7 = {}^{12}C_6 \left(\frac{2^6 a^6 b^6}{x^6}\right)$$

Middle term is ${}^{12}C_6 \left(\frac{2^6 a^6 b^6}{x^6}\right)$.

Q15(ix)

For the given binomial expansion n = 9.

So middle terms are $\left(\frac{9+1}{2}\right) = 5^{th}$ term and $\left(\frac{9+3}{2}\right) = 6^{th}$ term.

$$T_5 = {}^9C_4 \left(\frac{p}{x}\right)^{9-4} \left(\frac{x}{p}\right)^4$$

$$T_5 = {}^9C_4 \left(\frac{p}{x}\right)^5 \left(\frac{x}{p}\right)^4$$

$$T_5 = {}^9C_4 \left(\frac{p}{x}\right)$$

$$T_6 = {}^9C_5 \left(\frac{p}{x}\right)^{9-5} \left(\frac{x}{p}\right)^5$$

$$T_6 = {}^9C_5 \left(\frac{p}{x}\right)^4 \left(\frac{x}{p}\right)^5$$

$$T_6 = {}^9C_5 \left(\frac{x}{p}\right)$$

The middle terms are ${}^9C_4 \left(\frac{p}{x}\right)$ and ${}^9C_5 \left(\frac{x}{p}\right)$.

Q15(x)

For the given binomial expansion n = 10.

So middle term is $\left(\frac{10}{2}+1\right) = 6^{th}$ term.

$$T_6 = {}^{10}C_5\left(\frac{x}{a}\right)^{10-5}\left(-\frac{a}{x}\right)^5$$

$$T_6 = -\,{}^{10}C_5\left(\frac{x}{a}\right)^5\left(\frac{a}{x}\right)^5$$

$$T_6 = -\,{}^{10}C_5 = -252$$

Middle term is -252.

Q16(i)

$$\left(\frac{3x^2}{2}-\frac{1}{3x}\right)^9$$

In expansion

$$T_{r+1} = {}^9C_r\left(\frac{3x^2}{2}\right)^{9-r}\left(\frac{-1}{3x}\right)^r$$

$$= {}^9C_r\left(\frac{3}{2}\right)^{9-r}\left(x^{18-2r}\right)\left(\frac{-1}{3}\right)^r x^{-r}$$

Let T_{r+1} be independent of x

$$18-3r = 0 \text{ or } r = 6$$

$\therefore$ **Required term**

$$\Rightarrow \quad T_{r+1} = T_{6+1} = T_7 = {}^9C_6\left(\frac{3}{2}\right)^{9-6}\left(\frac{-1}{3}\right)^6 x^{18-3(6)}$$

$$= 84\left(\frac{27}{8}\right)\left(\frac{1}{179}\right)x^0 = \frac{7}{18}$$

Q16(ii)

$$\left(2x+\frac{1}{3x^2}\right)^9$$

4th term is independent of x

$$\binom{9}{3}(2x)^6\left(\frac{1}{3x^2}\right)^3 = \binom{9}{3}\frac{64}{27}$$

Q16(iii)

$$T_{r+1} = (-1)^r \, {}^nC_r \left(2x^2\right)^{25-r} \left(\frac{3}{x^3}\right)^r = (-1)^r \, {}^nC_r 2^{25-r} 3^r x^{50-2r-3r}$$

Term independent of $x = x^0$

$\Rightarrow \quad x^{50-5r} = x^0 \Rightarrow 50 - 5r = 0 \Rightarrow r = 10$

$\therefore \; t_{11} = (-1)^{10} \, {}^{25}C_{10} 2^{15} \times 3^{10} = {}^{25}C_{10} 2^{15} 3^{10}$

Q16(iv)

$$\left(3x - \frac{2}{x^2}\right)^{15}$$

$$T_{r+1} = (-1)^r \, {}^{15}C_r (3x)^{15-r} \left(\frac{2}{x^2}\right)^r$$

$$= (-1)^r \, {}^{15}C_r 3^{15-r} 2^r x^{15-r-2r}$$

Term independent of $x \Rightarrow x^0$

$\Rightarrow \quad x^{15-3r} = x^0$

$15 - 3r = 0 \Rightarrow r = 5$

$\therefore \; t_6 = (-1)^5 \, {}^{15}C_5 3^{10} 2^5$

$$= -\frac{15!}{5!10!} 3^{10} 2^5 = -\frac{15 \times 14 \times 13 \times 12 \times 11}{120} 3^{10} 2^5$$

$$= -3003 \times 3^{10} \times 2^5$$

Q16(v)

$$\left(\sqrt{\frac{x}{3}} + \frac{3}{2x^2}\right)^{10}$$

$$T_{r+1} = {}^{10}C_r \left(\sqrt{\frac{x}{3}}\right)^{10-r} \left(\frac{3}{2x^2}\right)^r$$

$$= {}^{10}C_r x^{5-\frac{r}{2}-2r} 3^r \times 3^{-5+\frac{r}{2}} \times 2^{-r}$$

Independent of $x \Rightarrow x^0$

$$x^{\frac{10-r-4r}{r}} = x^0$$

$10 - 5r = 0$

$r = 2$

$t_3 = {}^{10}C_2 3^{2-5+1} 2^{-2}$

$= {}^{10}C_2 3^{-2} 2^{-2}$

$$= \frac{10!}{2!8!} \times \frac{1}{36} = \frac{10 \times 9}{2 \times 36} = \frac{5}{4}$$

Q16(vi)

$$\left(x-\frac{1}{x^2}\right)^{3n}$$

$$T_{r+1}=(-1)^r\,{}^{3n}C_r x^{3n-r}\left(\frac{1}{x^2}\right)^r$$

$$=(-1)^r\,{}^{3n}C_r x^{3n-r-2r}$$

Independent of $x \Rightarrow x^0$

$$x^{3n-3r}=x^0 \Rightarrow r=n$$

$$=(-1)^n\,{}^{3n}C_r$$

Q16(vii)

We have,

$$\left(\frac{1}{2}x^{\frac{1}{3}}+x^{\frac{-1}{5}}\right)^8$$

Let $(r+1)^{th}$ term be independent of x.

$\therefore \quad T_{r+1}={}^8C_r\left(\frac{1}{2}x^{\frac{1}{3}}\right)^{8-r}\left(x^{\frac{-1}{5}}\right)^r$

$$={}^8C_r\left(\frac{1}{2}\right)^{8-r}\times\left(x^{\frac{1}{3}}\right)^{8-r}\times\left(\frac{1}{x^{\frac{1}{5}}}\right)^r$$

$$={}^8C_r\left(\frac{1}{2}\right)^{8-r}\times(x)^{\frac{8-r}{3}}\times\left(\frac{1}{x^{\frac{r}{5}}}\right)$$

$$={}^8C_r\left(\frac{1}{2}\right)^{8-r}\times(x)^{\frac{8-r}{3}-\frac{r}{5}}$$

$$={}^8C_r\left(\frac{1}{2}\right)^{8-r}\times(x)^{\frac{40-5r-3r}{15}}$$

$$={}^8C_r\left(\frac{1}{2}\right)^{8-r}\times(x)^{\frac{40-8r}{15}}$$

If it is independent of x, we must have

$$\frac{40-8r}{15}=0$$

$\Rightarrow \quad 8r=40$

$\Rightarrow \quad r=5$

$\therefore \quad$ The term independent of $x = T_6$

Now,

$$T_6={}^8C_r\left(\frac{1}{2}x^{\frac{1}{3}}\right)^{8-5}\left(x^{\frac{-1}{5}}\right)^5$$

$$=56\times\left(\frac{1}{2}\right)^3$$

$$=56\times\frac{1}{8}$$

$$=7$$

Hence, required term = 7

Q16(viii)

$$\left(1+x+2x^3\right)\left(\frac{3}{2}x^2-\frac{1}{3x}\right)^9$$

$$=\left(1+x+2x^3\right)\left[\left(\frac{3}{2}x^2\right)^9-{}^9C_1\left(\frac{3}{2}x^2\right)^8\frac{1}{3x}\ \ldots-{}^9C_6\left(\frac{3}{2}x^2\right)^3\left(\frac{1}{3x}\right)^6-{}^9C_7\left(\frac{3}{2}x^2\right)^2\left(\frac{1}{3x}\right)^7\right]$$

In the second bracket, we have to search the term as x^0 and $\frac{1}{x^3}$ which when multiplying by 1 and $2x^3$ is first bracket will give the term independent of x. The term containing $\frac{1}{x}$ will not occur is second bracket.

The term independent of x

$$=1\left[{}^9C_6\,\frac{3^3}{2^3}\times\frac{1}{3^6}\right]-2x^3\left[{}^9C_7\,\frac{3^2}{2^2}\times\frac{1}{3^7}\times\frac{1}{x^3}\right]$$

$$=\left[\frac{9\times8\times7}{1\times2\times3}\times\frac{1}{8\times27}\right]-2\left[\frac{9\times8}{1\times2}\times\frac{1}{4\times243}\right]$$

$$=\frac{7}{18}-\frac{2}{27}$$

$$=\frac{17}{54}$$

Required term $=\frac{17}{54}$

Q16(ix)

We have,

$$\left(\sqrt[3]{x}+\frac{1}{2\sqrt[3]{x}}\right)^{18},\ x>0$$

Let $(r+1)^{th}$ term be independent of x.

$$\therefore\quad T_{r+1}={}^{18}C_r\left(\sqrt[3]{x}\right)^{18-r}\times\left(\frac{1}{2\sqrt[3]{x}}\right)^r$$

$$={}^{18}C_r\left(\left(x\right)^{\frac{1}{3}}\right)^{18-r}\times\left(\frac{1}{2}\right)^r\times\left(\frac{1}{x^{\frac{1}{3}}}\right)^r$$

$$={}^{18}C_r\left(x\right)^{\frac{18-r}{3}}\times\left(\frac{1}{x^{\frac{r}{3}}}\right)\times\left(\frac{1}{2}\right)^r$$

$$={}^{18}C_r\left(x\right)^{\frac{18-r}{3}}\times\left(\frac{1}{2}\right)^r$$

$$={}^{18}C_r\left(x\right)^{\frac{18-r}{3}}\times\left(\frac{1}{2}\right)^r$$

If it is independent of x, we must have

$$\frac{18-2r}{3}=0$$

$\Rightarrow\quad 18=2r$

$\Rightarrow\quad r=9$

$\therefore\quad$ Term independet of $x=T_{9+1}=T_{10}$

Now,

$$T_{10}={}^{18}C_9\left(\sqrt[3]{x}\right)^{18-9}\left(\frac{1}{2\sqrt[3]{x}}\right)^9$$

$$={}^{18}C_9\left(\sqrt[3]{x}\right)^9\times\frac{1}{2^9}\times\left(\frac{1}{\sqrt[3]{x}}\right)^9$$

$$=\frac{{}^{18}C_9}{2^9}$$

Hence, required term $=\frac{{}^{18}C_9}{2^9}$

Q16(x)

$$\left(\frac{3x^2}{2}-\frac{1}{3x}\right)^6$$

In expansion

$$T_{r+1} = {}^6C_r\left(\frac{3x^2}{2}\right)^{6-r}\left(-\frac{1}{3x}\right)^r$$

$$= {}^6C_r\left(\frac{3}{2}\right)^{6-r}\left(x^{12-3r}\right)\left(-\frac{1}{3}\right)^r$$

Let T_{r+1} be independent of x,

$12-3r=0$ or $r=4$

$\therefore$ Required term

$$\Rightarrow T_{r+1}=T_{4+1}=T_5={}^6C_4\left(\frac{3}{2}\right)^{6-4}\left(\frac{-1}{3}\right)^4 x^{12-3(4)}$$

$$=15\left(\frac{9}{4}\right)\left(\frac{1}{81}\right)x^0=\frac{5}{12}$$

Q17

We know that the coefficient of rth term in the expansion of $(1+x)^n$ is ${}^nC_{r-1}$

$\therefore$ Coefficient of $(2r+4)$ th term of the expansion $(1+x)^{18}$ = ${}^{18}C_{2r+4-1}$ = ${}^{18}C_{2r+3}$

and, coefficient of $(r-2)$ th term of the expansion $(1+x)^{18}$ = ${}^{18}C_{r-2-1}$ = ${}^{18}C_{r-3}$

It is given that these coefficients are equal.

$\therefore$ ${}^{18}C_{2r+3} = {}^{18}C_{r-3}$

$\Rightarrow$ $2r+3=r-3$ or, $2r+3+r-3=18$ $\left[\begin{array}{l}\because {}^nC_r = {}^nC_s \\ \Rightarrow r=s \text{ or, } r+s=n\end{array}\right]$

$\Rightarrow$ $r=-6$ or, $3r=18$

$\Rightarrow$ $r=-6$ or, $r=6$

$\Rightarrow$ $r=6$ $[\because r=-6 \text{ is not possible}]$

Q18

$$(1+x)^{43}$$

$$\binom{43}{2r}=\binom{43}{r+1}$$

$$2r+r+1=43$$

$$3r=42$$

$$r=14$$

Q19

Now, Coefficient of $(r+1)$ th term in the expansion of $(4+x)^{n+1} = {}^{n+1}C_{r+1-1} = {}^{n+1}C_r$

and, Coefficient of rth term in $(1+x)^n$ + Coefficient of $(r+1)$ th term in $(1+x)^n$

$$= {}^nC_{r-1} + {}^nC_{r+1-1}$$
$$= {}^nC_{r-1} + {}^nC_r$$
$$= \frac{n!}{\{n-(r-1)\}!\,(r-1)!} + \frac{n!}{(n-r)!\,r!}$$
$$= \frac{n!}{\{n-r+1\}!\,(r-1)!} + \frac{n!}{(n-r)!\,r!}$$
$$= \frac{n!}{(n-r+1)(n-r)!\,(r-1)!} + \frac{n!}{(n-r)!\,r\,(r-1)!}$$
$$= \frac{n!}{(n-r+1)(n-r)!\,(r-1)!} + \frac{n!}{(n-r)!\,(r-1)!\,r}$$
$$= \frac{n!}{(n-r)!\,(r-1)!}\left[\frac{1}{n-r+1} + \frac{1}{r}\right]$$
$$= \frac{n!}{(n-r)!\,(r-1)!}\left[\frac{r+n-r+1}{(n-r+1)r}\right]$$
$$= \frac{n!}{(n-r)!\,(r-1)!}\left[\frac{n+1}{(n-r+1)r}\right]$$
$$= \frac{n!\,(n+1)}{(n-r)!\,(n-r+1)(r-1)!\,r}$$
$$= \frac{(n+1)!}{(n-r+1)!\,r!}$$
$$= \frac{(n+1)!}{(n+1-r)!\,r!}$$
$$= {}^{n+1}C_r$$

$\therefore \quad {}^{n+1}C_r = {}^nC_{r-1} + {}^nC_r$

The coefficient of $(r+1)$ th term in the expansion of $(1+x)^{n+1}$ is equal to the sum of the coefficients of rth and $(r+1)$ th terms in the expansion of $(1+x)^n$.

Q20

We have,

$$\left(x+\frac{1}{x}\right)^{2n}$$

Let $(r+1)^{\text{th}}$ term be independent of x.

$$\therefore \quad T_{r+1} = {}^{2n}C_r\,(x)^{2n-r}\left(\frac{1}{x}\right)^r$$

$$= {}^{2n}C_r\,(x)^{2n-r-r}$$

$$= {}^{2n}C_r x^{2n-2r}$$

If it is independent of x, we must have,

$$2n-2r=0$$

$$\Rightarrow \quad 2n=2r$$

$$\Rightarrow \quad r=n$$

$\therefore$ Term independent of $x = T_{n+1}$

Now,

$$T_{n+1} = {}^{2n}C_n\,(x-1)^{2n-n}\left(\frac{1}{x}\right)^n$$

$$= {}^{2n}C_n$$

$$= \frac{(2n)!}{(2n-n)!\,n!}$$

$$= \frac{(2n)!}{n!\,n!}$$

$$= \frac{(2n)(2n-1)(2n-2)\ldots 5\times 4\times 3\times 2\times 1}{n!\,n!}$$

$$= \frac{\{1\times 3\times 5\times\ldots(2n-1)\}\{2\times 4\times 6\times\ldots 2n\}}{n!\,n!}$$

$$= \frac{\{1\times 3\times 5\times\ldots(2n-1)\}\times 2^n\{1\times 2\times 3\times\ldots n\}}{n!\,n!}$$

$$= \frac{\{1\times 3\times 5\times\ldots(2n-1)\}\times 2^n\times n!}{n!\,n!}$$

$$= 2^n\times\frac{\{1\times 3\times 5\times\ldots(2n-1)\}}{n!}$$

$\therefore$ The term independent to $x = \dfrac{\{1\times 3\times 5\times\ldots(2n-1)\}}{n!}\times 2^n$ Hence proved.

Q21

We have,

$$(1+x)^n$$

Now,

Coefficient of 5th term = ${}^nC_{5-1} = {}^nC_4$

Coefficient of 5th term = ${}^nC_{6-1} = {}^nC_5$

and, Coefficient of 5th term = ${}^nC_{7-1} = {}^nC_6$

It is given that these coefficients are in A.P.

$\therefore \quad 2\,{}^nC_5 = {}^nC_4 + {}^nC_6$

$$\Rightarrow \quad 2\left[\frac{n!}{(n-5)!\,5!}\right] = \frac{n!}{(n-4)!\,4!} + \frac{n!}{(n-6)!\,6!}$$

$$\Rightarrow \quad \frac{2}{(n-5)!\,5!} = \frac{1}{(n-4)!\,4!} + \frac{1}{(n-6)!\,6!}$$

$$\Rightarrow \quad \frac{2}{(n-5)(n-6)!\,5\times 4!} = \frac{1}{(n-4)(n-5)(n-6)!\,4!} + \frac{1}{(n-6)!\,6\times 5\times 4!}$$

$$\Rightarrow \quad \frac{2}{(n-5)\times 5} = \frac{1}{(n-4)(n-5)} + \frac{1}{6\times 5}$$

$$\Rightarrow \quad \frac{2}{5(x-5)} - \frac{1}{30} = \frac{1}{(n-4)(n-5)}$$

$$\Rightarrow \quad \frac{12-(n-5)}{30(n-5)} = \frac{1}{(n-4)(n-5)}$$

$$\Rightarrow \quad \frac{12-n+5}{30} = \frac{1}{(n-4)(n-5)}$$

$$\Rightarrow \quad \frac{17-n}{30} = \frac{1}{n-4}$$

$\Rightarrow \quad 17n - 68 - n^2 + 4n = 30$

$\Rightarrow \quad 21n - 68 - m^2 - 30 = 0$

$\Rightarrow \quad 21n - n^2 - 98 = 0$

$\Rightarrow \quad n^2 - 21n + 98 = 0$

$\Rightarrow \quad n^2 - 7n - 14n + 98 = 0$

$\Rightarrow \quad n(n-7) - 17(n-7) = 0$

$\Rightarrow \quad (n-7)(n-14) = 0$

$\Rightarrow \quad n = 7$ or, $n = 14$

Q22

We have,

$$(1+x)^{2n}$$

Now,

Coefficient 2nd term = ${}^{2n}C_{2-1} = {}^{2n}C_1$

Coefficient 3rd term = ${}^{2n}C_{3-1} = {}^{2n}C_2$

and, Coefficient 4th term = ${}^{2n}C_{4-1} = {}^{2n}C_3$

It is given that these coefficients are in A.P.

$\therefore \quad 2\,{}^{2n}C_2 = {}^{2n}C_1 + {}^{2n}C_3$

$\Rightarrow \quad 2 = \dfrac{{}^{2n}C_1}{{}^{2n}C_2} + \dfrac{{}^{2n}C_3}{{}^{2n}C_2}$

$\Rightarrow \quad 2 = \dfrac{2}{2n-2+1} + \dfrac{2n-3+1}{3} \qquad \left[\because \dfrac{{}^{n}C_r}{{}^{n}C_{r-1}} = \dfrac{n-r+1}{r}\right]$

$\Rightarrow \quad 2 = \dfrac{2}{2n-1} + \dfrac{2n-2}{3}$

$\Rightarrow \quad 2 = \dfrac{6+(2n-1)(2n-2)}{3(2n-1)}$

$\Rightarrow \quad 6(2n-1) = 6 + 4n^2 - 4n - 2n + 2$

$\Rightarrow \quad 12n - 6 = 8 + 4n^2 - 6n$

$\Rightarrow \quad 4n^2 - 6n - 12n + 8 + 6 = 0$

$\Rightarrow \quad 4n^2 - 18n + 14 = 0$

$\Rightarrow \quad 2(2n^2 - 9n + 7) = 0$

$\Rightarrow \quad 2n^2 - 9n + 7 = 0$ Hence proved.

Q23

We have,

$$\left(1+x\right)^n$$

Let the three consecutive terms are rth $\left(r+1\right)^{\text{th}}$ and $\left(r+2\right)^{\text{th}}$ i.e., T_r, T_{r+1} and T_{r+2}

$\therefore$ Coefficients of rth term = ${}^nC_{r-1} = 220$

Coefficients of $\left(r+1\right)^{\text{th}}$ term = ${}^nC_{r+1-1} = {}^nC_r = 495$

and, Coefficients of $\left(r+2\right)^{\text{th}}$ term = ${}^nC_{r+2-1} = {}^nC_{r+1} = 792$

Now,

$$\frac{{}^nC_{r+1}}{{}^nC_r} = \frac{792}{495}$$

$$\Rightarrow \frac{n-\left(r+1\right)+1}{r+1} = \frac{792}{495} \qquad \left[\because \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}\right]$$

$$\Rightarrow \frac{n-r}{r+1} = \frac{792}{495}$$

$$= \frac{72}{45}$$

$$= \frac{8}{5}$$

$$\Rightarrow \frac{n-r}{r+1} = \frac{8}{5}$$

$$\Rightarrow 5n-5r = 8r+8$$

$$\Rightarrow 5n-5r-8r = 8$$

$$\Rightarrow 5n-13r = 8 \qquad \text{---(i)}$$

and,

$$\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{495}{220}$$

$$\Rightarrow \frac{n-r+1}{r} = \frac{495}{220}$$

$$= \frac{45}{20}$$

$$= \frac{9}{4}$$

$$\Rightarrow \frac{n-r+1}{r} = \frac{9}{4}$$

$$\Rightarrow 4n-4r+4 = 9r$$

$$\Rightarrow 4n-4r-9r = -4$$

$$\Rightarrow 4n-13r = -4 \qquad \text{---(ii)}$$

Subtracting equation (ii) from equation (i),

$$n = 8+4$$

$$\Rightarrow n = 12$$

Q24

We have,

$(1+x)^n$

$\therefore$ Coefficients of 2nd term = ${}^nC_{2-1}$ = ${}^nC_1$

Coefficients of 3rd term = ${}^nC_{3-1}$ = ${}^nC_2$

and, Coefficients of 4th term = ${}^nC_{4-1}$ = ${}^nC_3$

It is given that these coeffic̄ents are in A.P.

$\therefore \quad 2\,{}^nC_2 = {}^nC_1 + {}^nC_3$

$\Rightarrow \quad 2 = \dfrac{{}^nC_1}{{}^nC_2} + \dfrac{{}^nC_3}{{}^nC_2}$

$\Rightarrow \quad 2 = \dfrac{2}{n-2+1} + \dfrac{n-3+1}{3} \qquad \left[\because \dfrac{{}^nC_{r+1}}{{}^nC_r} = \dfrac{n-r}{r+1}\right]$

$\Rightarrow \quad 2 = \dfrac{2}{n-1} + \dfrac{n-2}{3}$

$\Rightarrow \quad 2 = \dfrac{6+(n-1)(n-2)}{3(n-1)}$

$\Rightarrow \quad 6(n-1) = 6+n^2-2n-n+2$

$\Rightarrow \quad 6n-6 = 8+n^2-3n$

$\Rightarrow \quad n^2-3n-6n+8+6 = 0$

$\Rightarrow \quad n^2-9n+14 = 0$

$\Rightarrow \quad n^2-7n-2n+14 = 0$

$\Rightarrow \quad n(n-7)-2(n-7) = 0$

$\Rightarrow \quad (n-2)(n-7) = 0 \qquad [\because n-2 \neq 0]$

$\Rightarrow \quad n = 7$

Q25

We have,

$(1+x)^n$

Coefficients of pth term = ${}^nC_{p-1}$

and, Coefficients of qth term = ${}^nC_{q-1}$

It is given that, these coefficients are equal.

$\therefore \quad {}^nC_{p-1} = {}^nC_{q-1}$

$\Rightarrow \quad p-1 = q-1 \text{ or, } p-1+q-1 = n \qquad \left[\begin{array}{l} \because {}^nC_r = {}^nC_s \\ \Rightarrow r = s \text{ or, } r+s = n \end{array}\right]$

$\Rightarrow \quad p-q = 0 \text{ or, } p+q = n+2$

$\therefore \quad p+q = n+2$ Hence proved.

Q26

We have,

$$(1+x)^n$$

Let the three consecutive terms are T_r, T_{r+1} and T_{r+2}

$\therefore$ Coefficients of T_r = ${}^nC_{r-1} = 56$

Coefficients of T_{r+1} = ${}^nC_{r+1-1} = {}^nC_r = 70$

and, Coefficients of T_{r+2} = ${}^nC_{r+2-1} = {}^nC_{r+1} = 56$

Now,

$$\frac{{}^nC_{r+1}}{{}^nC_r} = \frac{56}{70}$$

$$\Rightarrow \quad \frac{n-(r+1)+1}{r+1} = \frac{4}{5} \qquad \left[\because \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}\right]$$

$$\Rightarrow \quad \frac{n-r}{r+1} = \frac{4}{5}$$

$$\Rightarrow \quad 5n - 5r = 4r + 4$$

$$\Rightarrow \quad 5n - 9r = 4 \qquad \text{---(i)}$$

and,

$$\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{70}{56}$$

$$\Rightarrow \quad \frac{n-r+1}{r} = \frac{5}{4}$$

$$\Rightarrow \quad 4n - 4r + 4 = 5r$$

$$\Rightarrow \quad 4n - r = -4 \qquad \text{---(ii)}$$

Subtracting equation (ii) from (i), we get

$$n = 4 + 4 = 8$$

Put $n = 8$ in equation (i), we get

$$5 \times 8 - 9r = 4$$

$$\Rightarrow \quad -9r = 4 - 40$$

$$\Rightarrow \quad r = 4$$

$\therefore$ Three consecutive terms are 4th, 5th and 6th.

Q27

We are given,

$$T_3 = a,\ T_4 = b,\ T_5 = c,\ T_6 = d$$

We have to prove that

$$\frac{b^2 - ac}{c^2 - bd} = \frac{5a}{3c}$$

$$\Rightarrow \quad \frac{b^2 - ac}{a} = \frac{5}{3}\left[\frac{c^2 - bd}{c}\right]$$

$$\Rightarrow \quad \frac{1}{b}\left[\frac{b^2 - ac}{a}\right] = \frac{5}{3}\left[\frac{c^2 - bd}{bc}\right]$$

$$\Rightarrow \quad \frac{b}{a} - \frac{c}{b} = \frac{5}{3}\left[\frac{c}{b} - \frac{d}{c}\right] \qquad \text{---(i)}$$

Now we know,

$$a = {}^nC_2 x^{n-2}\alpha^2$$

$$b = {}^nC_3 x^{n-3}\alpha^3$$

$$c = {}^nC_4 x^{n-4}\alpha^4$$

$$d = {}^nC_5 x^{n-5}\alpha^5$$

Putting these values in equation (i), we get

$$\frac{{}^nC_3 x^{n-3}\alpha^3}{{}^nC_2 x^{n-2}\alpha^2} - \frac{{}^nC_4 x^{n-4}\alpha^4}{{}^nC_3 x^{n-3}\alpha^3} = \frac{5}{3}\left[\frac{{}^nC_4 x^{n-4}\alpha^4}{{}^nC_3 x^{n-3}\alpha^3} - \frac{{}^nC_5 x^{n-5}\alpha^5}{{}^nC_4 x^{n-4}\alpha^4}\right]$$

$$\Rightarrow \quad \left[\frac{{}^nC_3}{{}^nC_2} - \frac{{}^nC_4}{{}^nC_3}\right]\frac{\alpha}{x} = \frac{5\alpha}{3x}\left[\frac{{}^nC_4}{{}^nC_3} - \frac{{}^nC_5}{{}^nC_4}\right]$$

We know that,

$$\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}$$

$\therefore$ The given equation above becomes,

$$\left[\frac{n-2}{3} - \frac{n-3}{4}\right] = \frac{5}{3}\left[\frac{n-3}{4} - \frac{n-4}{5}\right]$$

$$\Rightarrow \quad \frac{4n - 8 - 3n + 9}{3 \times 4} = \frac{5n - 15 - 4n + 16}{3 \times 4}$$

$$\Rightarrow \quad \frac{n+1}{12} = \frac{n+1}{12}$$

Which is true.

Hence proved.

Q28

Suppose the binomial is $(x+\alpha)^n$

We are given,

$$T_6 = a,\ T_7 = b,\ T_8 = c,\ T_9 = d$$

We have to prove that

$$\frac{b^2 - ac}{c^2 - bd} = \frac{4a}{3c}$$

$$\Rightarrow \quad \frac{b^2 - ac}{a} = \frac{4}{3}\left[\frac{c^2 - bd}{c}\right]$$

$$\Rightarrow \quad \frac{1}{b}\left[\frac{b^2 - ac}{a}\right] = \frac{4}{3}\left[\frac{c^2 - bd}{bc}\right]$$

$$\Rightarrow \quad \frac{b}{a} - \frac{c}{b} = \frac{4}{3}\left[\frac{c}{b} - \frac{d}{c}\right] \quad ---(i)$$

Now we know,

$$a = {}^nC_5 x^{n-5}\alpha^5$$

$$b = {}^nC_6 x^{n-6}\alpha^6$$

$$c = {}^nC_7 x^{n-7}\alpha^7$$

$$d = {}^nC_8 x^{n-8}\alpha^8$$

Putting these values in equation (i), we get

$$\frac{{}^nC_6 x^{n-6}\alpha^6}{{}^nC_5 x^{n-5}\alpha^5} - \frac{{}^nC_7 x^{n-7}\alpha^7 v}{{}^nC_6 x^{n-6}\alpha^6} = \frac{4}{3}\left[\frac{{}^nC_7 x^{n-7}\alpha^7}{{}^nC_6 x^{n-6}\alpha^6} - \frac{{}^nC_8 x^{n-8}\alpha^8}{{}^nC_7 x^{n-7}\alpha^7}\right]$$

$$\Rightarrow \quad \left[\frac{{}^nC_6}{{}^nC_5} - \frac{{}^nC_7}{{}^nC_6}\right]\frac{\alpha}{x} = \frac{4\alpha}{3x}\left[\frac{{}^nC_7}{{}^nC_6} - \frac{{}^nC_8}{{}^nC_7}\right]$$

We know that,

$$\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}$$

$\therefore$ The given equation above becomes,

$$\left[\frac{n-5}{6} - \frac{n-6}{7}\right] = \frac{4}{3}\left[\frac{n-6}{7} - \frac{n-7}{8}\right]$$

$$\Rightarrow \quad \frac{7n - 35 - 6n + 36}{6 \times 7} = \frac{8n - 48 - 7n + 49}{3 \times 7 \times 2}$$

$$\Rightarrow \quad \frac{n+1}{42} = \frac{n+1}{42}$$

Which is true.

Hence proved.

Q29

We have,

$$(1+x)^n$$

Let the three consecutive terms are T_r , T_{r+1} and T_{r+2}

$\therefore$ Coefficients of rth term = ${}^nC_{r-1} = 76$

Coefficients of $(r+1)$th term = ${}^nC_{r+1-1} = {}^nC_r = 95$

and, Coefficients of $(r+2)$th term = ${}^nC_{r+2-1} = {}^nC_{r+1} = 76$

Now,

$$\frac{{}^nC_{r+1}}{{}^nC_r} = \frac{76}{95}$$

$$\Rightarrow \quad \frac{n-(r+1)+1}{r+1} = \frac{76}{95} \qquad \left[\because \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}\right]$$

$$\Rightarrow \quad \frac{n-r-1}{r+1} = \frac{4}{5}$$

$$\Rightarrow \quad \frac{n-r}{r+1} = \frac{4}{5}$$

$$\Rightarrow \quad 5n - 5r = 4r + 4$$

$$\Rightarrow \quad 5n - 5r - 4r = 4$$

$$\Rightarrow \quad 5n - 9r = 4 \qquad \text{---(i)}$$

and,

$$\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{95}{76}$$

$$\Rightarrow \quad \frac{n-r+1}{r} = \frac{5}{4}$$

$$\Rightarrow \quad 4n - 4r + 4 = 5r$$

$$\Rightarrow \quad 4n - 9r = -4 \qquad \text{---(ii)}$$

Subtracting equation (ii) from (i), we get

$$n = 4 + 4$$

$$\Rightarrow \quad n = 8$$

Q30

It is given that,

$$T_6 = 112, T_7 = 7, T_8 = \frac{1}{4}$$

$$\therefore \quad T_6 = {}^nC_{n-5}x^{n-5} \times a^5 = 112$$

$$T_7 = {}^nC_{n-6}x^{n-6} \times a^6 = 7$$

and,

$$T_8 = {}^nC_{n-7}x^{n-7} \times a^7 = \frac{1}{4}$$

Now,

$$\frac{T_7}{T_6} = \frac{{}^nC_{n-6}x^{n-6} \times a^6}{{}^nC_{n-5}x^{n-5} \times a^5} = \frac{7}{112}$$

$$\Rightarrow \quad \frac{{}^nC_{n-6}}{{}^nC_{n-5}} \times \frac{a}{x} = \frac{1}{16}$$

$$\Rightarrow \quad \frac{n-6+1}{n-(n-5)+1} \times \frac{a}{x} = \frac{1}{16} \qquad \left[\because \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}\right]$$

$$\Rightarrow \quad \frac{n-5}{6} \times \frac{a}{x} = \frac{1}{16}$$

$$\Rightarrow \quad \frac{a}{x} = \frac{6}{16} \times \frac{1}{n-5}$$

$$\Rightarrow \quad \frac{a}{x} = \frac{3}{8} \times \frac{1}{(n-5)} \qquad \text{---(i)}$$

and,

$$\frac{T_8}{T_7} = \frac{{}^nC_{n-7}x^{n-7} \times a^7}{{}^nC_{n-6}x^{n-6} \times a^6} = \frac{\frac{1}{4}}{7}$$

$$\Rightarrow \quad \frac{T_8}{T_7} = \frac{{}^nC_{n-7}}{{}^nC_{n-6}} \times \frac{a}{x} = \frac{1}{28}$$

$$\Rightarrow \quad \frac{{}^nC_{n-7}}{{}^nC_{n-6}} \times \frac{a}{x} = \frac{1}{28}$$

$$\Rightarrow \quad \frac{n-7+1}{n-(n-6)+1} \times \frac{a}{x} = \frac{1}{28}$$

$$\Rightarrow \quad \frac{n-6}{7} \times \frac{a}{x} = \frac{1}{28}$$

$$\Rightarrow \quad \frac{a}{x} = \frac{1}{4(n-6)} \qquad \text{---(ii)}$$

Comparing equation (i) and (ii), we get

$$\frac{3}{8}\times\frac{1}{(n-5)}=\frac{1}{4(n-6)}$$

$$\Rightarrow \quad \frac{3}{2}\times\frac{1}{(n-5)}=\frac{1}{(n-6)}$$

$$\Rightarrow \quad 3(n-6)=2(n-5)$$

$$\Rightarrow \quad 3n-18=2n-10$$

$$\Rightarrow \quad 3n-2n=18-10$$

$$\Rightarrow \quad n=8$$

Putting $n = 8$ in equation (ii), we get

$$\frac{a}{x}=\frac{1}{4(8-6)}$$

$$\Rightarrow \quad \frac{a}{x}=\frac{1}{8}$$

$$\Rightarrow \quad x=8a$$

Now,

$$T6=112$$

$$\Rightarrow \quad {}^{n}C_{n-5}\times x^{n-5}\times a^{5}=112$$

$$\Rightarrow \quad {}^{8}C_{3}\times x^{3}\times a^{5}=112 \qquad [\because n=8]$$

$$\Rightarrow \quad {}^{8}C_{3}\times(8a)^{3}a^{5}=112 \qquad [\because x=8a]$$

$$\Rightarrow \quad \frac{8!}{(8-3)!3!}\times 8^{3}\times a^{8}=112$$

$$\Rightarrow \quad \frac{8!}{5!3!}\times 512\times a^{8}=112$$

$$\Rightarrow \quad \frac{8\times7\times6\times5!}{5!3!}\times 512\times a^{8}=112$$

$$\Rightarrow \quad a^{8}=\frac{112}{56\times512}$$

$$\Rightarrow \quad a^{8}=\frac{2}{512}$$

$$\Rightarrow \quad a^{8}=\frac{1}{256}$$

$$\Rightarrow \quad a^{8}=\left(\frac{1}{2}\right)^{8}$$

$$\Rightarrow \quad a=\frac{1}{2}$$

Putting $a=\frac{1}{2}$ in $x = 8a$, we get

$$x=8\times\frac{1}{2}=4$$

Hence, $x = 4$, $a=\frac{1}{2}$ and $n = 8$.

Q31

It is given that

$T_2 = 240$

$T_3 = 720$

$T_4 = 1080$

$\therefore \quad T_2 = {}^nC_1 \times x^{n-1} \times a = 240$

$T_3 = {}^nC_2 \times x^{n-2} \times a^2 = 720$

and, $\quad T_4 = {}^nC_3 \times x^{n-3} \times a^3 = 1080$

Now,

$$\frac{T_4}{T_3} = \frac{{}^nC_3 \times x^{n-3} \times a^3}{{}^nC_2 \times x^{n-2} \times a^2} = \frac{1080}{720}$$

$$\Rightarrow \quad \frac{{}^nC_3 a}{{}^nC_2 x} = \frac{3}{2}$$

$$\Rightarrow \quad \frac{n-3+1}{2+1} \times \frac{a}{x} = \frac{3}{2}$$

$$\Rightarrow \quad \frac{n-2}{3} \times \frac{a}{x} = \frac{3}{2}$$

$$\Rightarrow \quad \frac{a}{x} = \frac{9}{2(n-2)} \qquad \text{---(i)}$$

and,

$$\frac{T_3}{T_2} = \frac{{}^nC_2 \times x^{n-2} \times a^2}{{}^nC_1 \times x^{n-1} \times a} = \frac{720}{240}$$

$$\Rightarrow \quad \frac{{}^nC_2}{{}^nC_1} \times \frac{a}{x} = 3$$

$$\Rightarrow \quad \frac{n-2+1}{2} \times \frac{a}{x} = 3$$

$$\Rightarrow \quad \frac{n-1}{2} \times \frac{a}{x} = 3$$

$$\Rightarrow \quad \frac{a}{x} = \frac{6}{n-1} \qquad \text{---(ii)}$$

Comparing equation (i) and equation (ii), we get

$$\frac{6}{n-1} = \frac{9}{2(n-2)}$$

$\Rightarrow \quad 12(n-2) = 9(n-1)$

$\Rightarrow \quad 12n - 24 = 9n - 9$

$\Rightarrow \quad 3n = 24 - 9$

$\Rightarrow \quad 3n = 15$

$\Rightarrow \quad n = 5$

Putting $n = 5$ in equation (ii), we get

$$\frac{a}{x} = \frac{6}{5-1}$$

$\Rightarrow \quad \frac{a}{x} = \frac{6}{4}$

$\Rightarrow \quad \frac{a}{x} = \frac{3}{2}$

$\Rightarrow \quad a = \frac{3}{2}x$

Now,

$$T_2 = {}^nC_1 \times x^{n-1} \times a = 240$$

$\Rightarrow \quad {}^5C_1 \times x^4 \times \left(\frac{3}{2}x\right) = 240 \qquad \left[\because n = 5 \text{ and } a = \frac{3}{2}x\right]$

$\Rightarrow \quad x^5 = \frac{240 \times 2}{5 \times 3}$

$\Rightarrow \quad x^5 = 32$

$\Rightarrow \quad x^5 = 2^5$

$\Rightarrow \quad x = 2$

Putting $x = 2$ in $a = \frac{3}{2}x$, we get

$$a = \frac{3}{2} \times 2 = 3$$

Hence, $x = 2$, $a = 3$ and $n = 5$.

Q32

It is given that

$T_1 = 729$

$T_2 = 7290$

and, $T_3 = 30375$

$\therefore \quad T_1 = {}^nC_0 \times a^n = 729$

$T_2 = {}^nC_{n-1} \times a^{n-1} \times b = 7290$

and, $T_3 = {}^nC_{n-2} \times a^{n-2} \times b^2 = 30375$

Now,

$$\frac{T_2}{T_1} = \frac{{}^nC_{n-1} \times a^{n-1} \times b}{{}^nC_0 \times a^n} = \frac{7290}{729}$$

$$\Rightarrow \quad \frac{{}^nC_{n-1} \times a^{n-1} \times b}{{}^nC_0 \times a^n} = 10$$

$$\Rightarrow \quad \frac{{}^nC_{n-1}}{1} \times \frac{b}{a} = 10$$

$$\Rightarrow \quad \frac{n!}{(n-n+1)!\,(n-1)!} \times \frac{b}{a} = 10$$

$$\Rightarrow \quad \frac{n!}{(n-1)!} \times \frac{b}{a} = 10$$

$$\Rightarrow \quad \frac{n\,(n-1)!}{(n-1)!} \times \frac{b}{a} = 10$$

$$\Rightarrow \quad \frac{b}{a} = \frac{10}{n} \qquad \text{---(i)}$$

and,

$$\frac{T_3}{T_2} = \frac{{}^nC_{n-2} \times a^{n-2} \times b^2}{{}^nC_{n-1} \times a^{n-1} \times b} = \frac{30375}{7290}$$

$$\Rightarrow \quad \frac{{}^nC_{n-2}}{{}^nC_{n-1}} \times \frac{b}{a} = \frac{25}{6}$$

$$\Rightarrow \quad \frac{n-2+1}{n-(n-1)+1} \times \frac{b}{a} = \frac{26}{6} \qquad \left[\because \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}\right]$$

$$\Rightarrow \quad \frac{n-1}{2} \times \frac{b}{a} = \frac{25}{6}$$

$$\Rightarrow \quad \frac{b}{a} = \frac{25}{6} \times \frac{2}{(n-1)}$$

Q33

We have,

$$(3+ax)^9 = {}^9C_0 \times 3^9 + {}^9C_1 \times 3^8 \times (ax)^1 + {}^9C_2 \times 3^7 \times (ax)^2 + {}^9C_3 \times 3^6 \times (ax)^3 + \ldots$$

$\therefore$ Coefficient of $x^2 = {}^9C_2 \times 3^7 \times a^2$

and, Coefficient of $x^3 = {}^9C_3 \times 3^6 \times a^3$

Now, Coefficient of x^2 = Coefficient of x^3

$\Rightarrow \quad {}^9C_2 \times 3^7 \times a^2 = {}^9C_3 \times 3^6 \times a^3$

$\Rightarrow \quad 36 \times 3^7 \times a^2 = 84 \times 3^6 \times a^3$

$\Rightarrow \quad a = \dfrac{36 \times 3^7}{84 \times 3^6} = \dfrac{9}{7}$

Q34

We have,

$$(1+2a)^4(2-a)^5$$

Now,

$$(1+2a)^4 = {}^4C_0 + {}^4C_1 2a + {}^4C_2 (2a)^2 + {}^4C_3 (2a)^3 + {}^4C_4 (2a)^4$$

and,
$$(2-a)^5 = {}^5C_0 \times 2^5 + {}^5C_1 \times 2^4(-a) + {}^5C_2 \times 2^3(-a)^2 + {}^5C_3 \times 2^2(-a)^3 + {}^5C_4 \times 2(-a)^4 + {}^5C_5(-a)^5$$
$$= {}^5C_0 \times 2^5 - {}^5C_1 \times 2^4 \times a + {}^5C_2 \times 2^3 \times a^2 - {}^5C_3 \times 2^2 \times a^3 + {}^5C_4 \times 2 \times a^4 - {}^5C_5 \times a^5$$

$\therefore$
$$(1+2a)^4(2-a)^5 = \left[{}^4C_0 + {}^4C_1 2a + {}^4C_2 (2a)^2 + {}^4C_3 (2a)^3 + {}^4C_4 (2a)^4\right]\begin{bmatrix} {}^5C_0 \times 2^5 - {}^5C_1 \times 2^4 \times a + {}^5C_2 \times 2^3 \times a^2 \\ - {}^5C_3 \times 2^2 \times a^3 + {}^5C_4 \times 2 \times a^4 - {}^5C_5 \times a^5 \end{bmatrix}$$

$\therefore$ Coefficients of $a^4 = 2\,{}^5C_4 - {}^4C_1 \times 2 \times {}^5C_3 \times 2^2 + {}^4C_2 (2)^2 \times {}^5C_2 \times 2^3 - {}^4C_3 (2)^3 \times {}^5C_1 \times 2^4 + {}^4C_4 (2)^4 \times {}^5C_0 \times 2^5$

$= 2 \times 5 - 8 \times 4 \times 10 + 32 \times 6 \times 10 - 128 \times 4 \times 5 + 512 \times 1 \times 1$

$= 10 - 320 + 1920 - 2560 + 512$

$= 2442 - 2880$

$= -438$

$\therefore$ Coefficients of $a^4 = -438$.

Q35

$$\left(\sqrt{x} - \frac{k}{x^2}\right)^{10}$$

$$\binom{10}{2}\left(\sqrt{x}\right)^8\left(-\frac{k}{x^2}\right)^2 = 405$$

$$45k^2 = 405$$

$$k^2 = 9$$

$$k = 3$$

Q36

$$\left(y^{1/2}+x^{1/3}\right)^{n}$$

$$\binom{n}{n-2}\left(y^{1/2}\right)^{2}\left(x^{1/3}\right)^{n-2}$$

$$\binom{n}{n-2}=45$$

$$n(n-1)=90$$

$$n^{2}-10n+9n-90$$

$$n(n-10)+9(n-10)=0$$

$$n=-9 or 10$$

$$n\ cannot\ be\ negative. So, n=10$$

$$6th term\binom{10}{5}\left(y^{1/2}\right)^{5}\left(x^{1/3}\right)^{5}=252y^{\frac{5}{3}}x^{\frac{5}{3}}$$

Q37

$$\left(\frac{p}{2}+2\right)^{8}$$

$$\binom{8}{4}\left(\frac{p}{2}\right)^{4}2^{4}=1120$$

$$70p^{4}=1120$$

$$p^{4}=16$$

$$p=2$$

Q38

$$\left(\sqrt[3]{2}+\frac{1}{\sqrt[3]{3}}\right)^n$$

7th term from begining is

$$\binom{n}{6}\left(\sqrt[3]{2}\right)^{n-6}\left(\frac{1}{\sqrt[3]{3}}\right)^6$$

7th term from end is

$$\binom{n}{n-6}\left(\sqrt[3]{2}\right)^{6}\left(\frac{1}{\sqrt[3]{3}}\right)^{n-6}$$

$$\text{Given}\ \frac{\text{7th term from begining}}{\text{7th term from end}}=\frac{\binom{n}{6}\left(\sqrt[3]{2}\right)^{n-12}\left(\frac{1}{\sqrt[3]{3}}\right)^{12-n}}{\binom{n}{n-6}}$$

$$=\frac{\binom{n}{6}\left(\sqrt[3]{2}\right)^{n-12}\left(\sqrt[3]{3}\right)^{n-12}}{\binom{n}{n-6}}$$

$$=\frac{\binom{n}{6}\left(6\right)^{\frac{n-12}{3}}}{\binom{n}{n-6}}=\frac{1}{6}$$

$$\frac{n-12}{3}=-1$$

$$n=12-3=9$$

Q39

Seventh term from the beginning and end in the binomial expansion of $\left(\sqrt[3]{2}+\frac{1}{\sqrt[3]{2}}\right)^n$ are equal.

$\Rightarrow T_7 = T_{n-6}$

$\Rightarrow {}^nC_6\left(\sqrt[3]{2}\right)^6\left(\frac{1}{\sqrt[3]{2}}\right)^{n-6} = {}^nC_{n-6}\left(\sqrt[3]{2}\right)^{n-6}\left(\frac{1}{\sqrt[3]{2}}\right)^6$

$\Rightarrow\left(\sqrt[3]{2}\right)^6\left(\frac{1}{\sqrt[3]{2}}\right)^{n-6} = \left(\sqrt[3]{2}\right)^{n-6}\left(\frac{1}{\sqrt[3]{2}}\right)^6$

$\Rightarrow\left(\frac{1}{\sqrt[3]{2}}\right)^{2n-12} = \left(\frac{1}{\sqrt[3]{2}}\right)^{12}$

$\Rightarrow 2n - 12 = 12$

$\Rightarrow n = 12$