Mean and Variance of a Random Variable Ex 32.1 Q1

EX - 32.1

(i) Here

$X:$	3	2	1	0	-1
$P(x):$	0.3	0.2	0.4	0.1	0.05

$$p(x=3)+p(x=2)+p(x=1)+p(x=0)+p(x=-1)$$
$$=0.3+0.2+0.4+0.1+0.05$$
$$=1.05 \neq 1$$

So, the given distribution of probabilities is not a probability distribution.

(ii) Here

$X:$	0	1	2
$P(x):$	0.6	0.4	0.2

$$p(x=0)+p(x=1)+p(x=2)$$
$$=0.6+0.4+0.2$$
$$=1.2 \neq 1$$

So, the given distribution of probabilities is not a probability distribution.

(iii) Here

$X:$	0	1	2	3	4
$P(x):$	0.1	0.5	0.2	0.1	0.1

$$p(x=0)+p(x=1)+p(x=2)+p(x=3)+p(x=4)$$
$$=0.1+0.5+0.2+0.1+0.1$$
$$=1$$

So, the given distribution of probabilities is a probability distribution.

(iv) Here

$X:$	0	1	2	3
$P(x):$	0.3	0.2	0.4	0.1

$$p(x=0)+p(x=1)+p(x=2)+p(x=3)$$
$$=0.3+0.2+0.4+0.1$$
$$=1$$

So, the given distribution of probabilities is a probability distribution.

Mean and Variance of a Random Variable Ex 32.1 Q2

Here

$x:$	-2	-1	0	1	2	3
$P(x):$	0.1	k	0.2	$2k$	0.3	k

We know that,

$$p(-2)+p(-1)+p(0)+p(1)+p(2)+p(3)=1$$
$$\Rightarrow \quad 0.1+k+0.2+2k+0.3+k=1$$
$$\Rightarrow \quad 4k+0.6=1$$
$$\Rightarrow \quad 4k=1-0.6$$
$$\Rightarrow \quad 4k=0.4$$
$$\Rightarrow \quad k=\frac{0.4}{4}$$
$$\Rightarrow \quad k=\frac{1}{10}$$
$$\Rightarrow \quad k=0.1$$

Mean and Variance of a Random Variable Ex 32.1 Q3

Here

X:	0	1	2	3	4	5	6	7	8
$P(x)$:	a	$3a$	$5a$	$7a$	$9a$	$11a$	$13a$	$15a$	$17a$

Since $\sum p(x) = 1$

$$P(0) + P(1) + P(2) + P(3) + P(4) + P(5) + P(6) + P(7) + P(8) = 1$$

$\Rightarrow \quad = a + 3a + 5a + 7a + 9a + 11a + 13a + 15a + 17a = 1$

$\Rightarrow \quad 81a = 1$

$\Rightarrow \quad a = \frac{1}{81}$

(ii) $P(x < 3) = P(0) + P(1) + P(2)$

$= a + 3a + 5a$

$= 9a$

$= 9\left(\frac{1}{81}\right)$

$\therefore P(x < 3) = \frac{1}{9}$

$P(x \geq 3) = 1 - P(x < 3) = 1 - \frac{1}{9} = \frac{8}{9}$

$P(0 < x < 5) = P(1) + P(2) + P(3) + P(4)$

$= 3a + 5a + 7a + 9a$

$= 24a$

$= 24\left(\frac{1}{81}\right)$

$\therefore P(0 < x < 5) = \frac{8}{27}$

Mean and Variance of a Random Variable Ex 32.1 Q4

Here:-

x:	0	1	2
$P(x)$:	$3c^2$	$4c - 10c^2$	$5c - 1$

Where $c > 0$

(i) since $\sum P(x) = 1$

$\Rightarrow \quad P(0) + P(1) + P(2) = 1$

$\Rightarrow \quad 3c^3 + 4c - 10c^2 + 5c - 1 = 1$

$\Rightarrow \quad 3c^3 - 10c^2 + 9c - 2 = 0$

$\Rightarrow \quad 3c^3 - 3c^2 - 7c^2 + 7c + 2c - 2 = 0$

$\Rightarrow \quad 3c^2(c - 1) - 7c(c - 1) + 2(c - 1) = 0$

$\Rightarrow \quad (c - 1)(3c^2 - 7c + 2) = 0$

$\Rightarrow \quad (c - 1)(3c^2 - 6c - c + 2) = 0$

$\Rightarrow \quad (c - 1)(3c(c - 2) - 1(c - 2)) = 0$

$(c - 1)(3c - 1)(c - 2) = 0$

$c = 1, \quad c = 2, \quad c = \frac{1}{3}$

Only $c = \frac{1}{3}$ is possible. Because if c = 1, or c = 2 then P(2) will become negative.

(ii) $P(X < 2) = P(0) + P(1)$

$$= 3c^3 + 4c - 10c^2$$
$$= 3\left(\frac{1}{3}\right)^3 + 4\left(\frac{1}{3}\right) - 10\left(\frac{1}{3}\right)^2$$
$$= \frac{3}{27} + \frac{4}{3} - \frac{10}{9}$$
$$= \frac{1}{9} + \frac{4}{3} - \frac{10}{9}$$
$$= \frac{3}{9}$$

$\therefore P(X < 2) = \frac{1}{3}$

(iii) $P(1 < X \leq 2) = P(2)$

$$= 5c - 1$$
$$= 5\left(\frac{1}{3}\right) - 1$$

$\therefore P(1 < X \leq 2) = \frac{2}{3}$

Mean and Variance of a Random Variable Ex 32.1 Q5

Here,

$$2P(x_1) = 3P(x_2) = P(x_3) = 5P(x_4)$$

Let $P(x_3) = a$

$2P(x_1) = P(x_3) \quad \Rightarrow \quad P(x_1) = \frac{a}{2}$

$3P(x_2) = P(x_3) \quad \Rightarrow \quad P(x_2) = \frac{a}{3}$

$5P(x_4) = P(x_3) \quad \Rightarrow \quad P(x_4) = \frac{a}{5}$

Since $\quad P(x_1) + P(x_2) + P(x_3) + P(x_4) = 1$

$\Rightarrow \quad \frac{a}{2} + \frac{a}{3} + \frac{a}{1} + \frac{a}{5} = 1$

$\Rightarrow \quad \frac{15a + 10a + 30a + 6a}{30} = 1$

$\Rightarrow \quad 61a = 30$

$\Rightarrow \quad a = \frac{30}{61}$

so,

X :	x_1	x_2	x_3	x_4
$P(X)$:	$\frac{15}{61}$	$\frac{10}{61}$	$\frac{30}{61}$	$\frac{6}{61}$

Mean and Variance of a Random Variable Ex 32.1 Q6

Here,

$$P(X=0)=P(X>0)=P(X<0)$$

Let $P(X=0)=k$

$\Rightarrow \quad P(X>0)=k=P(X<0)$

Since $\Sigma P(X)=1$

$\Rightarrow \quad P(X<0)+P(X=0)+P(X>0)=1$

$\Rightarrow \quad k+k+k=1$

$\Rightarrow \quad 3k=1$

$\Rightarrow \quad k=\frac{1}{3}$

So, $P(X<0)=$

$$P(X=-1)+P(X=-2)+P(X=-3)=\frac{1}{3}$$

$$3P(X=-1)=\frac{1}{3}, \qquad [\because \quad P(X=-1)=P(X=-2)=P(X=-3)]$$

$$P(X=-1)=\frac{1}{9}$$

$\Rightarrow \quad P(X=-1)=P(X=-2)=P(X=-3)=\frac{1}{9}$ - - - - (i)

$\Rightarrow \quad P(X=0)=\frac{1}{3}$ - - - - - - - - - (ii)

and

$$P(X>0)=k$$

$$P(X=1)+P(X=2)+P(X=3)=\frac{1}{3}$$

$$3P(X=1)=\frac{1}{3}, \qquad [\because \quad P(X=1)=P(X=2)=P(X=3)]$$

$\Rightarrow \quad P(X=1)=\frac{1}{9}$

$\Rightarrow \quad P(X=1)=P(X=2)=P(X=3)=\frac{1}{9}$ - - - (iii)

From equation (i), (ii), (iii),

X :	-3	-2	-1	0	1	2	3
$P(X)$:	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$

Mean and Variance of a Random Variable Ex 32.1 Q7

Let X denote number of aces in a sample of 2 cards drawn.

There are four aces in a pack of 52 cards.

So, X can have values 0, 1, 2

Now,

$$P(X=0)=\frac{48C_2}{52C_2}=\frac{48\times 47}{2}\times\frac{2}{52\times 51}=\frac{188}{221}$$

$$P(X=1)=\frac{48C_1\times 4C_1}{52C_2}=\frac{48\times 4\times 2}{52\times 51}=\frac{32}{221}$$

$$P(X=2)=\frac{4C_2}{52C_2}=\frac{4\times 3}{2}\times\frac{2}{52\times 51}=\frac{1}{221}$$

So,

X :	0	1	2
$P(X)$:	$\frac{188}{221}$	$\frac{32}{221}$	$\frac{1}{221}$

Mean and Variance of a Random Variable Ex 32.1 Q8

Probability of getting a Head in one throw of a coin= $\frac{1}{2}$

$$P(H) = \frac{1}{2}$$

$$\Rightarrow \quad P(T) = 1 - \frac{1}{2}$$

$$\Rightarrow \quad P(T) = \frac{1}{2}$$

Let X denote the number of heads obtained is 3 throws of a coin .

Then $X = 0, 1, 2, 3$

Now,

$$P(x = 0) = P(T)P(T)P(T) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

$$P(x = 1) = P(H)P(T)P(T) + P(T)P(H)P(T) + P(T)P(T)P(H)$$

$$= \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right)$$

$$P(x = 1) = \frac{3}{8}$$

$$P(x = 2) = P(H)P(H)P(T) + P(H)P(T)P(H) + P(T)P(H)P(H)$$

$$= \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right)$$

$$P(x = 2) = \frac{3}{8}$$

$$P(x = 3) = P(H)P(H)P(H)$$

$$= \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right)$$

$$= \frac{1}{8}$$

So,

Required probability distribution is

x :	0	1	2	3
$P(x)$:	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Mean and Variance of a Random Variable Ex 32.1 Q9

Let x denote number of aces drawn out of 4 cards drawn.

There are four ace aces in a pack of 52.

So, $X = 0, 1, 2, 3, 4$

Now,

$$P(x = 0) = \frac{48C_4}{52C_4}$$

$$P(x = 1) = \frac{48C_3 \times 4C_1}{52C_4}$$

$$P(x = 2) = \frac{48C_2 \times 4C_2}{52C_4}$$

$$P(x = 3) = \frac{48C_1 \times 4C_3}{52C_4}$$

$$P(x = 4) = \frac{4C_4}{52C_4}$$

So,

Required probability distribution is

x :	0	1	2	3	4
$P(x)$:	$\frac{48C_4}{52C_4}$	$\frac{48C_3 \times 4C_1}{52C_4}$	$\frac{48C_2 \times 4C_2}{52C_4}$	$\frac{48C_1 \times 4C_3}{52C_4}$	$\frac{4C_4}{52C_4}$

Mean and Variance of a Random Variable Ex 32.1 Q10

A bag has 4 red and 6 black balls. Three balls are drawn.

Let X denote number of red balls out of 3 drawn.

Then X = 0,1,2,3.

So,

$$P\left(\text{no red balls}\right) = P\left(X = 0\right) = \frac{6C_3}{10C_3} = \frac{6\times5\times4}{3\times2}\times\frac{3\times2}{10\times9\times8} = \frac{1}{6}$$

$$P\left(\text{one red balls}\right) = P\left(X = 1\right) = \frac{4C_1\times6C_2}{10C_3} = \frac{4\times6\times5}{2}\times\frac{3\times2}{10\times9\times8} = \frac{1}{2}$$

$$P\left(\text{two red balls}\right) = P\left(X = 1\right) = \frac{4C_2\times6C_1}{10C_3} = \frac{4\times3\times6}{2}\times\frac{3\times2}{10\times9\times8} = \frac{3}{10}$$

$$P\left(\text{all three red}\right) = P\left(X = 3\right) = \frac{4C_3}{10C_3} = \frac{4\times3\times2}{10\times9\times8} = \frac{1}{30}$$

The required probability distribution is

X :	0	1	2	3
$P\left(x\right)$:	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{30}$

Mean and Variance of a Random Variable Ex 32.1 Q11

Here 5 defective and 15 non-defective mangoes. Let X denote the defective mangoes drawn out of 4 mangoes drawn.

So, $X = 0,1,2,3,4.$

$$P(X=0) = \frac{15C_4}{20C_4}$$
$$= \frac{15\times14\times13\times12}{4\times3\times2\times1}\times\frac{4\times3\times2\times1}{20\times19\times18\times17}$$
$$= \frac{91}{323}$$

$$P(X=1) = \frac{5C_1\times15C_3}{20C_4}$$
$$= \frac{5\times15\times14\times13}{3\times2\times1}\times\frac{4\times3\times2\times1}{20\times19\times18\times17}$$
$$= \frac{455}{969}$$

$$P(X=2) = \frac{5C_2\times15C_2}{20C_4}$$
$$= \frac{5\times4}{2}\times\frac{15\times14}{2}\times\frac{4\times3\times2\times1}{20\times19\times18\times17}$$
$$= \frac{70}{323}$$

$$P(X=3) = \frac{5C_3\times15C_1}{20C_4}$$
$$= \frac{5\times4}{2}\times15\times\frac{4\times3\times2\times1}{20\times19\times18\times17}$$
$$= \frac{10}{323}$$

$$P(X=4) = \frac{5C_4}{20C_4}$$
$$= 5\times\frac{4\times3\times2\times1}{20\times19\times18\times17}$$
$$= \frac{1}{969}$$

So, required probability distribution is

X :	0	1	2	3	4
$P(X)$:	$\frac{91}{323}$	$\frac{455}{969}$	$\frac{70}{323}$	$\frac{10}{323}$	$\frac{1}{969}$

Mean and Variance of a Random Variable Ex 32.1 Q12

Here, X denote the number of sum of two number or two dice thrown together

So, $X = 2,3,4,5,6,7,8,9,10,11,12.$

So,

$$P(x=2) = \frac{1}{36} \qquad [\text{Possible pairs: } (1,1)]$$

$$P(x=3) = \frac{2}{36} = \frac{1}{18} \qquad [\text{Possible pairs: } (1,2),(2,1)]$$

$$P(x=4) = \frac{3}{36} = \frac{1}{12} \qquad [\text{Possible pairs: } (1,3),(2,2),(3,1)]$$

$$P(x=5) = \frac{4}{36} = \frac{1}{9} \qquad [\text{Possible pairs: } (1,4),(2,3),(3,2),(4,1)]$$

$P(x=6) = \frac{5}{36}$ [Possible pairs: (1,5), (2,4), (3,3), (4,2), (5,1)]

$P(x=7) = \frac{6}{36} = \frac{1}{6}$ [Possible pairs: (1,6), (2,5), (3,4), (4,3), (5,2), (6,1)]

$P(x=8) = \frac{5}{36}$ [Possible pairs: (2,6), (3,5), (4,4), (5,3), (6,2)]

$P(x=9) = \frac{4}{36} = \frac{1}{9}$ [Possible pairs: (3,6), (4,5), (5,4), (6,3)]

$P(x=10) = \frac{3}{36} = \frac{1}{12}$ [Possible pairs: (4,6), (5,5), (6,4)]

$P(x=11) = \frac{2}{36} = \frac{1}{18}$ [Possible pairs: (5,6), (6,5)]

$P(x=12) = \frac{1}{36}$ [Possible pairs: (6,6)]

So, required probability distribution is

x :	2	3	4	5	6	7	8	9	10	11	12
$P(x)$:	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$

Mean and Variance of a Random Variable Ex 32.1 Q13

There are 15 students in the class. Each student has the same chance to be chosen.

Therefore, the probability of each student to be selected is $\frac{1}{15}$.

The given information can be compiled in the frequency table as follows.

X	14	15	16	17	18	19	20	21
f	2	1	2	3	1	2	3	1

$P(X=14) = \frac{2}{15}$, $P(X=15) = \frac{1}{15}$, $P(X=16) = \frac{2}{15}$, $P(X=16) = \frac{3}{15}$,

$P(X=18) = \frac{1}{15}$, $P(X=19) = \frac{2}{15}$, $P(X=20) = \frac{3}{15}$, $P(X=21) = \frac{1}{15}$

Therefore, the probability distribution of random variable X is as follows.

X	14	15	16	17	18	19	20	21
f	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$

Mean and Variance of a Random Variable Ex 32.1 Q14

Here 5 defective and 20 non-defective bolts. Let X denote the number of defective bolts drawn out of 4 bolts drawn.So, X can have values 0,1,2,3,4.

$$P(X=0)=\frac{20C_4}{25C_4}$$
$$=\frac{20\times19\times18\times17}{25\times24\times23\times22}$$
$$=\frac{969}{2530}$$

$$P(X=1)=\frac{5C_1\times20C_3}{25C_4}$$
$$=\frac{5\times20\times19\times18}{3\times2\times1}\times\frac{4\times3\times2}{25\times24\times23\times22}$$
$$=\frac{114}{253}$$

$$P(X=2)=\frac{5C_2\times20C_2}{25C_4}$$
$$=\frac{5\times4}{2}\times\frac{20\times19}{2}\times\frac{4\times3\times2\times1}{25\times24\times23\times22}$$
$$=\frac{38}{253}$$

$$P(X=3)=\frac{5C_3\times20C_1}{25C_4}$$
$$=\frac{5\times4}{2}\times\frac{20\times4\times3\times2\times1}{25\times24\times23\times22}$$
$$=\frac{4}{253}$$

$$P(X=4)=\frac{5C_4}{25C_4}$$
$$=5\times\frac{4\times3\times2\times1}{25\times24\times23\times22}$$
$$=\frac{1}{2530}$$

So, required probability distribution is

x :	0	1	2	3	4
$P(x)$:	$\frac{969}{2530}$	$\frac{114}{253}$	$\frac{38}{253}$	$\frac{4}{253}$	$\frac{1}{2530}$

Mean and Variance of a Random Variable Ex 32.1 Q15

Two cards are drawn successively with replacement from a pack of 52 cards.

Let X be the number of aces obtained. Then X = 0,1,2.

$$P\left(X=0\right)=P\left(\overline{A}_1\right)\times P\left(\overline{A}_2\right)$$
$$=\frac{48}{52}\times\frac{48}{52}$$
$$=\frac{144}{169}$$

$$P\left(X=1\right)=P\left(A_1\right)P\left(\overline{A}_2\right)+P\left(\overline{A}_1\right)P\left(A_2\right)$$
$$=\frac{4}{52}\times\frac{48}{52}+\frac{48}{52}\times\frac{4}{52}$$
$$=\frac{24}{169}$$

$$P\left(X=2\right)=P\left(A_1\right)P\left(A_2\right)$$
$$=\frac{4}{52}\times\frac{4}{52}$$
$$=\frac{1}{169}$$

So,

Required probability distribution is

X :	0	1	2
$P(X)$:	$\frac{144}{169}$	$\frac{24}{169}$	$\frac{1}{169}$

Mean and Variance of a Random Variable Ex 32.1 Q16

Two cards are drawn successively with replcement from a pack of 52 cards.Let X denote the number of kings drawn out of 2 cards.
So, X = 0,1,2.

$$P\left(X=0\right)=P\left(\overline{K}_1\right)\times P\left(\overline{K}_2\right)$$
$$=\frac{48}{52}\times\frac{48}{52}$$
$$=\frac{144}{169}$$

$$P\left(X=1\right)=P\left(K_1\right)P\left(\overline{K}_2\right)+P\left(\overline{K}_1\right)P\left(K_2\right)$$
$$=\frac{4}{52}\times\frac{48}{52}+\frac{48}{52}\times\frac{4}{52}$$
$$=\frac{24}{169}$$

$$P\left(X=2\right)=P\left(K_1\right)P\left(K_2\right)$$
$$=\frac{4}{52}\times\frac{4}{52}$$
$$=\frac{1}{169}$$

So, required probability distribution is

X :	0	1	2
$P(X)$:	$\frac{144}{169}$	$\frac{24}{169}$	$\frac{1}{169}$

Mean and Variance of a Random Variable Ex 32.1 Q17

Two cards are drawn without replcement from a pack of 52 cards. Let X denote the number of aces drawn from pack out of 2 cards.
So, $X = 0,1,2.$

$$P(X = 0) = \frac{48C_2}{52C_2}$$
$$= \frac{48 \times 47}{2} \times \frac{2 \times 1}{52 \times 51}$$
$$= \frac{188}{221}$$

$$P(X = 1) = \frac{4C_1 \times 48C_1}{52C_2}$$
$$= \frac{4 \times 48 \times 2}{52 \times 51}$$
$$= \frac{32}{221}$$

$$P(X = 2) = \frac{4C_2}{52C_2}$$
$$= \frac{4 \times 3}{2} \times \frac{2}{52 \times 51}$$
$$= \frac{1}{221}$$

So, required probability distribution is

x :	0	1	2
$P(x)$:	$\frac{188}{221}$	$\frac{32}{221}$	$\frac{1}{221}$

Mean and Variance of a Random Variable Ex 32.1 Q18

Given bag have 4 white and 6 red balls.Let X denote the number of white balls out of 3 balls drawn without replcement,
So, $X = 0,1,2,3.$

$$P(\text{No white ball}) = \frac{6C_3}{10C_3}$$
$$= \frac{6 \times 5 \times 4}{3 \times 2} \times \frac{3 \times 2}{10 \times 9 \times 8}$$
$$= \frac{5}{30}$$
$$P(\text{One white ball}) = \frac{4C_1 \times 6C_2}{10C_3}$$
$$= \frac{4 \times 6 \times 5}{2} \times \frac{3 \times 2}{10 \times 9 \times 8}$$
$$= \frac{15}{30}$$
$$P(\text{Two white balls}) = \frac{4C_2 \times 6C_1}{10C_3}$$
$$= \frac{4 \times 3}{2} \times \frac{6 \times 3 \times 2}{10 \times 9 \times 8}$$
$$= \frac{9}{30}$$
$$P(\text{Three white balls}) = \frac{4C_3}{10C_3}$$
$$= \frac{4 \times 3 \times 2 \times 1}{10 \times 9 \times 8}$$
$$= \frac{1}{30}$$

So,

Required probability distribution is

x :	0	1	2	3
$P(x)$:	$\frac{5}{30}$	$\frac{15}{30}$	$\frac{9}{30}$	$\frac{1}{30}$

Mean and Variance of a Random Variable Ex 32.1 Q19

Since total is 9 when dice has $(3,6)(4,5)(5,4)(6,3)$

$\therefore P(\text{A total of 9 appears}) = p(A) = \frac{4}{36}$

Two dice are thrown 2 times.
Here, Y denotes the numbers of times a total of 9 appears.

So, $Y = 0,1,2$

$$P(Y=0) = P(\overline{A_1}) \times P(\overline{A_2})$$
$$= \frac{32}{36} \times \frac{32}{36}$$
$$= \frac{64}{81}$$
$$P(Y=1) = P(A_1)P(\overline{A_2}) + P(\overline{A_1})P(A_2)$$
$$= \frac{4}{36} \times \frac{32}{36} + \frac{32}{36} \times \frac{4}{36}$$
$$= \frac{16}{81}$$
$$P(Y=2) = P(A_1)P(A_2)$$
$$= \frac{4}{36} \times \frac{4}{36}$$
$$= \frac{1}{81}$$

So,

Required probability distribution is

x :	0	1	2
$P(x)$:	$\frac{64}{81}$	$\frac{16}{81}$	$\frac{1}{81}$

Mean and Variance of a Random Variable Ex 32.1 Q20

Given 25 items in the lot. 5 are defective. Good items are 20.
4 items are chosen at random.
Let X be the random variable that denotes the number of defective items in the selected lot.

P (X = 0) = P (4 non-defective and 0 defective) = ${}^{5}C_{0}.{}^{20}C_{4}/{}^{25}C_{4}$

= 4845/12650

P (X = 1) = P (3 non-defective and 1 defective) = ${}^{5}C_{1}.\ {}^{20}C_{3}/{}^{25}C_{4}$

=5x1140/12650

P (X = 2) = P (2 non-defective and 2 defective) = ${}^{5}C_{2}.\ {}^{20}C_{2}/{}^{25}C_{4}$

=10x190/12650

P (X = 3) = P (1 non-defective and 3 defective) = ${}^{5}C_{3}.\ {}^{20}C_{1}/{}^{25}C_{4}$

=10x20/12650

P (X = 4) = P (0 non-defective and 4 defective) = ${}^{5}C_{4}.\ {}^{20}C_{0}/{}^{25}C_{4}$

=5/12650

Mean and Variance of a Random Variable Ex 32.1 Q21

Three cards are thrown with replacement.Let X denote the numbers of hearts if three cards are drawn.
So, X has values 0,1,2,3

$$P(X=0)=P(\overline{H}_1)\times P(\overline{H}_2)\times P(\overline{H}_3)$$
$$=\frac{39}{52}\times\frac{39}{52}\times\frac{39}{52}$$
$$=\frac{27}{26}$$
$$P(X=1)=P(H_1)P(\overline{H}_2)P(\overline{H}_3)+P(\overline{H}_1)P(H_2)P(\overline{H}_3)+P(\overline{H}_1)P(\overline{H}_2)P(H_3)$$
$$=\frac{13}{52}\times\frac{39}{52}\times\frac{39}{52}+\frac{39}{52}\times\frac{13}{52}\times\frac{39}{52}+\frac{39}{52}\times\frac{39}{52}\times\frac{13}{52}$$
$$=\frac{27}{64}$$
$$P(X=2)=P(H_1)P(H_2)P(\overline{H}_3)+P(H_1)P(\overline{H}_2)P(H_3)+P(\overline{H}_1)P(H_2)P(H_3)$$
$$=\frac{13}{52}\times\frac{13}{52}\times\frac{39}{52}+\frac{13}{52}\times\frac{39}{52}\times\frac{13}{52}+\frac{39}{52}\times\frac{13}{52}\times\frac{13}{52}$$
$$=\frac{9}{64}$$
$$P(X=3)=P(H_1)P(H_2)P(H_3)$$
$$=\frac{13}{52}\times\frac{13}{52}\times\frac{13}{52}$$
$$=\frac{1}{64}$$

So,

Required probability distribution is

x :	0	1	2	3
$P(x)$:	$\frac{27}{64}$	$\frac{27}{64}$	$\frac{9}{64}$	$\frac{1}{64}$

Mean and Variance of a Random Variable Ex 32.1 Q22

Urn has 4 red and 3 blue balls.3 balls are drawn with replacement.
Let X denote numbers of blue balls drawn out of 3 drawn.
So, X has values 0,1,2,3

$$P(X=0)=P(\overline{B}_1)\times P(\overline{B}_2)\times P(\overline{B}_3)$$
$$=\frac{4}{7}\times\frac{4}{7}\times\frac{4}{7}$$
$$=\frac{64}{343}$$
$$P(X=1)=P(B_1)P(\overline{B}_2)P(\overline{B}_3)+P(\overline{B}_1)P(B_2)P(\overline{B}_3)+P(\overline{B}_1)P(\overline{B}_2)P(B_3)$$
$$=\frac{3}{7}\times\frac{4}{7}\times\frac{4}{7}+\frac{4}{7}\times\frac{3}{7}\times\frac{4}{7}+\frac{4}{7}\times\frac{4}{7}\times\frac{3}{7}$$
$$=\frac{144}{343}$$
$$P(X=2)=P(B_1)P(B_2)P(\overline{B}_3)+P(B_1)P(\overline{B}_2)P(B_3)+P(\overline{B}_1)P(B_2)P(B_3)$$
$$=\frac{3}{7}\times\frac{3}{7}\times\frac{4}{7}+\frac{3}{7}\times\frac{4}{7}\times\frac{3}{7}+\frac{4}{7}\times\frac{3}{7}\times\frac{3}{7}$$
$$=\frac{108}{343}$$
$$P(X=3)=P(B_1)P(B_2)P(B_3)$$
$$=\frac{3}{7}\times\frac{3}{7}\times\frac{3}{7}$$
$$=\frac{27}{343}$$

So,

Required probability distribution is

x :	0	1	2	3
$P(x)$:	$\frac{64}{343}$	$\frac{144}{343}$	$\frac{108}{343}$	$\frac{27}{343}$

Mean and Variance of a Random Variable Ex 32.1 Q23

Two cards are drawn simultaneously .Let X denote the number of spades obtained.

So, X can have values 0,1,2.

$$P(X=0)=\frac{39C_2}{52C_2}$$
$$=\frac{39\times 38}{52\times 51}$$
$$=\frac{19}{34}$$
$$P(X=1)=\frac{39C_1\times 13C_1}{52C_2}$$
$$=\frac{13\times 39\times 2}{52\times 51}$$
$$=\frac{13}{34}$$
$$P(X=2)=\frac{13C_2}{52C_2}$$
$$=\frac{13\times 12}{52\times 51}$$
$$=\frac{2}{34}$$

So,

Required probability distribution is

x :	0	1	2
$P(x)$:	$\frac{19}{34}$	$\frac{13}{34}$	$\frac{2}{34}$

Mean and Variance of a Random Variable Ex 32.1 Q24

Let A be the event of occurence of a number less than 3.

$$P(A)=\frac{2}{6} \qquad [\because 1,2 \text{ are less than } 3.]$$
$$P(A)=\frac{1}{3}$$

Let X denote the number of success is 2 throws of die.

So, X has value 0,1,2.

$$P(X=0)=P(\overline{A}_1)\times P(\overline{A}_2)$$
$$=\frac{2}{3}\times\frac{2}{3}$$
$$=\frac{4}{9}$$
$$P(X=1)=P(A_1)P(\overline{A}_2)+P(\overline{A}_1)P(A_2)$$
$$=\frac{1}{3}\times\frac{2}{3}+\frac{2}{3}\times\frac{1}{3}$$
$$=\frac{4}{9}$$
$$P(X=2)=P(A_1)P(A_2)$$
$$=\frac{1}{3}\times\frac{1}{3}$$
$$=\frac{1}{9}$$

So,

Required probability distribution is

x :	0	1	2
$P(x)$:	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

Mean and Variance of a Random Variable Ex 32.1 Q25

Urn has 5 red and 2 black balls.2 balls are randomly selected.
Here, X denote the numbers of black balls.
So, possible values of $X = 0,1,2$

$$P(X=0) = P(\bar{B}_1) \times P(\bar{B}_2)$$
$$= \frac{5}{7} \times \frac{5}{7}$$
$$= \frac{25}{49}$$

$$P(X=1) = P(B_1)P(\bar{B}_2) + P(\bar{B}_1)P(B_2)$$
$$= \frac{2}{7} \times \frac{5}{7} + \frac{5}{7} \times \frac{2}{7}$$
$$= \frac{20}{49}$$

$$P(X=2) = P(B_1)P(B_2)$$
$$= \frac{2}{7} \times \frac{2}{7}$$
$$= \frac{4}{49}$$

Now,

$$P(x=0) + P(x=1) + P(x=2)$$
$$= \frac{25}{49} + \frac{20}{49} + \frac{4}{49}$$
$$= \frac{49}{49}$$
$$= 1$$

So, $\quad \Sigma P(x) = 1$

Therefore

X is a random variable

Mean and Variance of a Random Variable Ex 32.1 Q26

Here, coin is tossed 6 times.
So, there can have
$1H\,5T$ or $2H\,4T$ or $3H\,3T$ or
$4H\,2T$ or $5H\,1T$ or $6H$ or
$6T$
Here, X denote the difference between the number of head and number of tails.
So,

$X = 6, 4, 2, 0, -2, -4, -6$

Mean and Variance of a Random Variable Ex 32.1 Q27

It is given that out of 10 bulbs, 3 are defective.
Number of non-defective bulbs $= 10 - 3 = 7$
2 bulbs are drawn from the lot with replacement.
Let X be the random variable that denotes the number of defective bulbs in the selected bulbs.

$$\therefore P(X=0) = \frac{{}^7C_2}{{}^{10}C_2}$$
$$= \frac{7}{15}$$
$$\therefore P(X=1) = \frac{{}^3C_1 \times {}^7C_1}{{}^{10}C_2}$$
$$= \frac{7}{15}$$
$$\therefore P(X=2) = \frac{{}^3C_2}{{}^{10}C_2}$$
$$= \frac{1}{15}$$

Therefore, the required probability distribution is

X	0	1	2
$P(X)$	$\frac{7}{15}$	$\frac{7}{15}$	$\frac{1}{15}$

Mean and Variance of a Random Variable Ex 32.1 Q28

Clearly, X can assume values 0, 1, 2, 3, 4 such that

$$P(X = 0) = (\text{Probablity of getting no red ball}) = \frac{{}^8C_0 \times {}^4C_4}{{}^{12}C_4} = \frac{1 \times 1}{\frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2 \times 1}} = \frac{1}{495}$$

$$P(X = 1) = (\text{Probablity of getting one red ball}) = \frac{{}^8C_1 \times {}^4C_3}{{}^{12}C_4} = \frac{8 \times 4}{\frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2 \times 1}} = \frac{32}{495}$$

$$P(X = 2) = (\text{Probablity of getting two red balls}) = \frac{{}^8C_2 \times {}^4C_2}{{}^{12}C_4} = \frac{\frac{8 \times 7}{2 \times 1} \times \frac{4 \times 3}{2 \times 1}}{\frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2 \times 1}} = \frac{168}{495}$$

$$P(X = 3) = (\text{Probablity of getting three red balls}) = \frac{{}^8C_3 \times {}^4C_1}{{}^{12}C_4} = \frac{\frac{8 \times 7 \times 6}{3 \times 2 \times 1} \times 4}{\frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2 \times 1}} = \frac{224}{495}$$

$$P(X = 4) = (\text{Probablity of getting four red balls}) = \frac{{}^8C_4 \times {}^4C_0}{{}^{12}C_4} = \frac{\frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1} \times 1}{\frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2 \times 1}} = \frac{70}{495}$$

Thus, probablity distribution of random veriable X is,

X	0	1	2	3	4
P(X)	$\frac{1}{495}$	$\frac{32}{495}$	$\frac{168}{495}$	$\frac{224}{495}$	$\frac{70}{495}$

(i) We know that,

$$P(0)+P(1)+P(2)+P(3)=1$$

$$k+\frac{k}{2}+\frac{k}{4}+\frac{k}{8}=1$$

$$15k=8$$

$$k=\frac{8}{15}$$

(ii) $P(X\le 2)$

$$=P(0)+P(1)+P(2)$$

$$=k+\frac{k}{2}+\frac{k}{4}$$

$$=\frac{8}{15}+\frac{8}{30}+\frac{8}{60}$$

$$=\frac{14}{15}$$

$$P(X>2)=P(3)=\frac{k}{8}=\frac{1}{15}$$

(iii) $P(X\le 2)+P(X>2)$

$$=P(0)+P(1)+P(2)+P(3)$$

$$=1$$

EX - 32.2

Mean and Variance of a Random Variable Ex 32.2 Q1(i)

x_i	p_i	$p_i x_i$	$p_i x_i^2$
2	0.2	0.4	0.8
3	0.5	1.5	4.5
4	0.3	1.2	4.8
		$\sum p_i x_i = 3.1$	$\sum p_i x_i^2 = 10.1$

$\text{Mean} = \sum p_i x_i = 3.1$

$\text{Variance} = \sum p_i x_i^2 - \left(\sum p_i x_i\right)^2 = 10.1 - (3.1)^2 = 0.49$

$\text{Standard deviation} = \sqrt{\text{Variance}} = 0.7$

Mean and Variance of a Random Variable Ex 32.2 Q1(ii)

x_i	p_i	$x_i\ p_i$	$x_i^2 p_i$
1	0.4	0.4	0.4
3	0.1	0.3	0.9
4	0.2	0.8	3.2
5	0.3	1.5	7.5
		$\sum xp = 3$	$\sum x^2 p = 12$

$\text{Mean} = \sum xp$

$\text{mean} = 3$

$\text{Standard deviation} = \sqrt{\sum x^2 p - (\text{mean})^2}$

$= \sqrt{12 - (3)^2}$

$= \sqrt{3}$

Standard Deviation = 1.732

Mean and Variance of a Random Variable Ex 32.2 Q1(iii)

x_i	p_i	$x_i\ p_i$	$x_i^2 p_i$
-5	$\frac{1}{4}$	$-\frac{5}{4}$	$\frac{25}{4}$
-4	$\frac{1}{8}$	$-\frac{1}{2}$	2
1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
2	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$
		$\Sigma xp = -1$	$\Sigma x^2 p = \frac{37}{4}$

Mean = Σxp
mean = -1

Standard deviation = $\sqrt{\Sigma x^2 p - (\text{mean})^2}$

$= \sqrt{\frac{37}{4} - (-1)^2}$

$= \sqrt{\frac{33}{4}}$

$= \sqrt{8.25}$

Standard Deviation = 2.9

Mean and Variance of a Random Variable Ex 32.2 Q1(iv)

x_i	p_i	$x_i\ p_i$	$x_i^2 p_i$
-1	0.3	-0.3	0.3
0	0.1	0	0
1	0.1	0.1	0.1
2	0.3	0.6	1.2
3	0.2	0.6	1.8
		$\Sigma xp = 1$	$\Sigma x^2 p = 3.4$

Mean = Σxp
mean = 1

Standard deviation = $\sqrt{\Sigma x^2 p - (\text{mean})^2}$

$= \sqrt{(3.4) - (1)^2}$

$= \sqrt{2.4}$

Standard Deviation = 1.5

Mean and Variance of a Random Variable Ex 32.2 Q1(v)

x_i	p_i	$x_i\ p_i$	$x_i^2 p_i$
1	0.4	0.4	0.4
2	0.3	0.6	1.2
3	0.2	0.6	1.8
4	0.1	0.4	1.6
		$\Sigma xp = 2$	$\Sigma x^2p = 5$

Mean = Σxp
mean = 2

Standard deviation = $\sqrt{\Sigma x^2p - (\text{mean})^2}$
$= \sqrt{5 - (2)^2}$

Standard Deviation = 1

Mean and Variance of a Random Variable Ex 32.2 Q1(vi)

x_i	p_i	$x_i\ p_i$	$x_i^2 p_i$
0	0.2	0	0
1	0.5	0.5	0.5
3	0.2	0.6	1.8
5	0.1	0.5	2.5
		$\Sigma xp = 1.6$	$\Sigma x^2p = 4.8$

Mean = Σxp
mean = 1.6

Standard deviation = $\sqrt{\Sigma x^2p - (\text{mean})^2}$
$= \sqrt{4.8 - (1.6)^2}$
$= \sqrt{4.8 - 2.56}$
$= \sqrt{2.24}$

Standard Deviation = 1.497

Mean and Variance of a Random Variable Ex 32.2 Q1(vii)

x_i	p_i	$x_i\ p_i$	$x_i^2 p_i$
-2	0.1	-0.2	0.4
-1	0.2	-0.2	0.2
0	0.4	0	0
1	0.2	0.2	0.2
2	0.1	0.2	0.4
		$\Sigma xp = 0$	$\Sigma x^2p = 1.2$

Mean = $\sum xp$
mean = 0

Standard deviation = $\sqrt{\sum x^2 p - (\text{mean})^2}$
$= \sqrt{(1.2)^2 - (0)^2}$

Standard Deviation = 1.2

Mean and Variance of a Random Variable Ex 32.2 Q1(viii)

x_i	p_i	$x_i\ p_i$	$x_i^2 p_i$
-3	0.05	-0.15	0.45
-1	0.45	-0.45	0.45
0	0.20	0	0
1	0.25	0.25	0.25
3	0.05	0.15	0.45
		$\sum xp = -0.2$	$\sum x^2 p = 1.6$

Mean = $\sum xp$
mean = -0.2

Standard deviation = $\sqrt{\sum x^2 p - (\text{mean})^2}$
$= \sqrt{1.6 - (-0.2)^2}$
$= \sqrt{1.6 - 0.04}$
$= \sqrt{1.56}$

Standard Deviation = 1.249

Mean and Variance of a Random Variable Ex 32.2 Q1(ix)

X_i	p_i	$p_i X_i$	$p_i X_i^2$
0	$\frac{1}{6}$	0	0
1	$\frac{5}{18}$	$\frac{5}{18}$	$\frac{5}{18}$
2	$\frac{2}{9}$	$\frac{4}{9}$	$\frac{8}{9}$
3	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{2}$
4	$\frac{1}{9}$	$\frac{4}{9}$	$\frac{16}{9}$
5	$\frac{1}{18}$	$\frac{5}{18}$	$\frac{25}{18}$
		$\sum p_i X_i = \frac{35}{18}$	$\sum p_i X_i^2 = \frac{35}{6}$

$$\text{Mean} = \sum p_i X_i = \frac{35}{18}$$

$$\text{Variance} = \sum p_i X_i^2 - \left(\sum p_i X_i\right)^2 = \frac{35}{6} - \left(\frac{35}{18}\right)^2 = \frac{665}{324}$$

$$\text{Standard deviation} = \sqrt{\text{Variance}} = \frac{\sqrt{665}}{18}$$

Mean and Variance of a Random Variable Ex 32.2 Q2

(i) We know that,

$P(0.5)+P(1)+P(1.5)+P(2)=1$

$k+k^2+2k^2+k=1$

$3k^2+2k-1=0$

$3k^2+3k-k-1=0$

$(3k-1)(k+1)=0$

$k=\frac{1}{3}$ or $k=-1$

We know that $0 \le P(X) \le 1$

$\therefore k=\frac{1}{3}$

(ii)

x_i	p_i	$p_i x_i$
0.5	$\frac{1}{3}$	$\frac{1}{6}$
1	$\frac{1}{9}$	$\frac{1}{9}$
1.5	$\frac{2}{9}$	$\frac{1}{3}$
2	$\frac{1}{3}$	$\frac{2}{3}$
		$\Sigma p_i x_i = \frac{23}{18}$

$$\text{Mean} = \Sigma p_i x_i = \frac{23}{18}$$

Mean and Variance of a Random Variable Ex 32.2 Q3

x_i	p_i	$x_i\ p_i$	$x_i^2 p_i$
a	p	ap	a^2p
b	q	bq	b^2q

$\text{Mean} = \Sigma xp$

$\text{mean} = ap+bq$

$$\begin{aligned}\text{Variance} &= \Sigma x^2p-(\text{mean})^2\\ &= (a^2p+b^2q)-(ap+bq)^2\\ &= a^2p+b^2q-a^2p^2-b^2q^2-2abpq\\ &= a^2pq+b^2pq-2abpq \qquad [\because \quad p+q=1]\\ &= pq\,(a^2+b^2-2ab)\end{aligned}$$

$\text{Variance} = pq\,(a-b)^2$

$\text{Standard deviation} = |a-b|\sqrt{pq}$

Mean and Variance of a Random Variable Ex 32.2 Q4

We know that in a throw of coin.

$P(H) = \frac{1}{2}, \qquad P(T) = \frac{1}{2}$

Let X denote the number of heads in three tosses of coin.

So, $X = 0,1,2,3$

$$\begin{aligned}
P(x=0) &= P(T)P(T)P(T) \\
&= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} \\
&= \frac{1}{8} \\
P(x=1) &= P(H)P(T)P(T)+P(T)P(H)P(T)+P(T)P(T)P(H) \\
&= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} \\
&= \frac{3}{8} \\
P(x=2) &= P(H)P(H)P(T)+P(H)P(T)P(H)+P(T)P(H)P(H) \\
&= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} \\
&= \frac{3}{8} \\
P(x=3) &= P(H)P(H)P(H) \\
&= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} \\
&= \frac{1}{8}
\end{aligned}$$

So,

x_i	p_i	$x_i\ p_i$	$x_i^2 p_i$
0	$\frac{1}{8}$	0	0
1	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{3}{8}$
2	$\frac{3}{8}$	$\frac{6}{8}$	$\frac{12}{8}$
3	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{9}{8}$
		$\Sigma xp = \frac{3}{2}$	$\Sigma x^2 p = 3$

Mean $= \Sigma xp$

mean $= \frac{3}{2}$

$$\begin{aligned}
\text{Variance} &= \Sigma x^2 p - (\text{mean})^2 \\
&= 3 - \frac{9}{4}
\end{aligned}$$

Variance $= \frac{3}{4}$

Mean and Variance of a Random Variable Ex 32.2 Q5

Two cards are drawn simultneously from a pack of 52 cards.
Let X denotes the number of kings drawn.

So, $X = 0,1,2$

$$P(x=0) = \frac{48C_2}{52C_2}$$
$$= \frac{48\times 47}{52\times 51}$$
$$= \frac{188}{221}$$
$$P(x=1) = \frac{4C_1 \times 48C_1}{52C_2}$$
$$= \frac{4\times 48\times 2}{52\times 51}$$
$$= \frac{32}{221}$$
$$P(x=2) = \frac{4C_2}{52C_2}$$
$$= \frac{4\times 3}{52\times 51}$$
$$= \frac{1}{221}$$

So,

x_i	p_i	$x_i\ p_i$	$x_i^2 p_i$
0	$\frac{188}{221}$	0	0
1	$\frac{32}{221}$	$\frac{32}{221}$	$\frac{32}{221}$
2	$\frac{1}{221}$	$\frac{2}{221}$	$\frac{4}{221}$
		$\Sigma xp = \frac{34}{221}$	$\Sigma x^2p = \frac{36}{221}$

Mean = Σxp

$$\text{mean} = \frac{34}{221}$$

$$\text{Variance} = \Sigma x^2p - (\text{mean})^2$$
$$= \frac{36}{221} - \left(\frac{34}{221}\right)^2$$
$$= \frac{7956 - 1156}{48841}$$
$$= \frac{6800}{48841}$$

$$\text{Variance} = \frac{400}{2873}$$

Mean and Variance of a Random Variable Ex 32.2 Q6

We know that ,in a throw of coin.

$P(T) = \frac{1}{2}, \qquad P(H) = \frac{1}{2}$

Let X denote the number of tails in three throws of coins.
So, X can take values from 0,1,2,3

$$P(x=0) = P(H)P(H)P(H)$$
$$= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}$$
$$= \frac{1}{8}$$
$$P(x=1) = P(T)P(H)P(H) + P(H)P(T)P(H) + P(H)P(H)P(T)$$
$$= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}$$
$$= \frac{3}{8}$$
$$P(x=2) = P(T)P(T)P(H) + P(T)P(H)P(T) + P(H)P(T)P(T)$$
$$= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}$$
$$= \frac{3}{8}$$
$$P(x=3) = P(T)P(T)P(T)$$
$$= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}$$
$$= \frac{1}{8}$$

So,

$\text{Mean} = \Sigma xp$

$\text{mean} = \frac{3}{2}$

$$\text{Variance} = \Sigma x^2 p - (\text{mean})^2$$
$$= 3 - \left(\frac{3}{2}\right)^2$$
$$= 3 - \frac{9}{4}$$

$\text{Variance} = \frac{3}{4}$

$\text{Standard Deviation} = \sqrt{\text{Variance}}$
$$= \sqrt{\frac{3}{4}}$$

Standard Deviation = 0.87

Mean and Variance of a Random Variable Ex 32.2 Q7

Total 12 good and bad eggs. 10 are good and 2 are bad.
3 eggs are drawn from this lot
Let X be the random variable that denotes the number of bad eggs in the lot.

$P(X = 0) = P(\text{3good and 0 bad}) = {}^{3}C_0 . {}^{10}C_3 / {}^{12}C_3$

$= 1 \times 120/220 = 6/11$

$P(X = 1) = P(\text{2good and 1 bad}) = {}^{2}C_1 . {}^{10}C_2 / {}^{12}C_3$

$=2\times 45/220 = 9/22$

$P(X = 2) = P(\text{1good and 2 bad}) = {}^{2}C_2 . {}^{10}C_1 / {}^{12}C_3$

$= 1\times 10/220 = 1/22$

The probability distribution of X is

X	0	1	2
P(X)	$\frac{6}{11}$	$\frac{9}{22}$	$\frac{1}{22}$

The mean $= 0\times\frac{6}{11} + 1\times\frac{9}{22} + 2\times\frac{1}{22} = \frac{11}{22} = 1/2$

Mean and Variance of a Random Variable Ex 32.2 Q8

A pair of dice is thrown.And X denote minimum of the two number appeared.
So, X can have values 2,3,4,5,6.

$P(x=1)=\frac{11}{36}$ $\left[\begin{array}{l}\text{Possible pairs: } (1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(3,1),\\ (4,1),(5,1),(6,1)\end{array}\right]$

$P(x=2)=\frac{9}{36}$ [Possible pairs: $(2,2),(2,3),(2,4),(2,5),(2,6),(3,2),(4,2),(5,2),(6,2)$]

$P(x=3)=\frac{7}{36}$ [Possible pairs: $(3,3),(3,4),(3,5),(3,6),(4,3),(5,3),(6,3)$]

$P(x=4)=\frac{5}{36}$ [Possible pairs: $(4,4),(4,5),(4,6),(5,4),(6,4)$]

$P(x=5)=\frac{3}{36}$ [Possible pairs: $(5,5),(5,6),(6,5)$]

$P(x=6)=\frac{1}{36}$ [Possible pairs: $(6,6)$]

x_i	p_i	$x_i\, p_i$	$x_i^2 p_i$
1	$\frac{11}{36}$	$\frac{11}{36}$	$\frac{11}{36}$
2	$\frac{9}{36}$	$\frac{18}{36}$	$\frac{36}{36}$
3	$\frac{7}{36}$	$\frac{21}{36}$	$\frac{63}{36}$
4	$\frac{5}{36}$	$\frac{20}{36}$	$\frac{80}{36}$
5	$\frac{3}{36}$	$\frac{15}{36}$	$\frac{75}{36}$
6	$\frac{1}{36}$	$\frac{6}{36}$	$\frac{36}{36}$
		$\Sigma xp=\frac{91}{36}$	$\Sigma x^2p=\frac{301}{36}$

Mean $=\Sigma xp$

Mean $=\frac{91}{36}$

$$\begin{aligned}\text{Variance} &= \Sigma x^2p-(\text{mean})^2\\ &=\frac{301}{36}-\left(\frac{91}{36}\right)^2\\ &=\frac{10836-8281}{1296}\\ &=\frac{2555}{1296}\end{aligned}$$

Variance = 1.97

Probability distribution is

x :	1	2	3	4	5	6
$P(x)$:	$\frac{11}{36}$	$\frac{9}{36}$	$\frac{7}{36}$	$\frac{5}{36}$	$\frac{3}{36}$	$\frac{1}{36}$

Mean and Variance of a Random Variable Ex 32.2 Q9

We know that ,In a toss of coin.

$P(T) = \frac{1}{2}, \quad P(H) = \frac{1}{2}$

Let X denote the number of occuring head in 4 throws of coins.

So, X can take values from $X = 0,1,2,3,4$

$$P(x=0) = P(T)P(T)P(T)P(T)$$
$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$
$$= \frac{1}{16}$$
$$P(x=1) = P(H)P(T)P(T)P(T) \times {}^4C_1$$
$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times 4$$
$$= \frac{4}{16}$$
$$P(x=2) = P(H)P(H)P(T)P(T) \times {}^4C_2$$
$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times 6$$
$$= \frac{6}{16}$$
$$P(x=3) = P(H)P(H)P(H)P(T) \times {}^4C_3$$
$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times 4$$
$$= \frac{4}{16}$$
$$P(x=4) = P(H)P(H)P(H)P(H)$$
$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$
$$= \frac{1}{16}$$

So,

x_i	p_i	$x_i\ p_i$	$x_i^2 p_i$
0	$\frac{1}{16}$	0	0
1	$\frac{4}{16}$	$\frac{4}{16}$	$\frac{4}{16}$
2	$\frac{6}{16}$	$\frac{12}{16}$	$\frac{24}{16}$
3	$\frac{4}{16}$	$\frac{12}{16}$	$\frac{36}{16}$
4	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{16}{16}$
		$\Sigma xp = 2$	$\Sigma x^2 p = 5$

Mean $= \Sigma xp$

mean $= 2$

Variance $= \Sigma x^2 p - (\text{mean})^2$

$= 5 - (2)^2$

Variance $= 1$

Probability distribution is

x :	0	1	2	3	4
$P(x)$:	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{6}{16}$	$\frac{4}{16}$	$\frac{1}{16}$

Mean and Variance of a Random Variable Ex 32.2 Q10

X denotes twice the number appearing on the die.
So, $X = 2,4,6,8,10,12$.

Probability distribution is

$$\begin{array}{lcccccc} X: & 2 & 4 & 6 & 8 & 10 & 12 \\ P(x): & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{array}$$

x_i	p_i	$x_i\, p_i$	$x_i^2 p_i$
2	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{4}{6}$
4	$\frac{1}{6}$	$\frac{4}{6}$	$\frac{16}{6}$
6	$\frac{1}{6}$	$\frac{6}{6}$	$\frac{36}{6}$
8	$\frac{1}{6}$	$\frac{8}{6}$	$\frac{64}{6}$
10	$\frac{1}{6}$	$\frac{10}{6}$	$\frac{100}{6}$
12	$\frac{1}{6}$	$\frac{12}{6}$	$\frac{144}{6}$
		$\Sigma xp = 7$	$\Sigma x^2p = \frac{364}{6}$

Mean = Σxp
mean = 7

$$\begin{aligned} \text{Variance} &= \Sigma x^2p - (\text{mean})^2 \\ &= \left(\frac{364}{6}\right) - (7)^2 \\ &= \frac{364 - 294}{6} \\ &= \frac{70}{6} \end{aligned}$$

Variance = 11.7

Mean and Variance of a Random Variable Ex 32.2 Q11

Probability of even number = $P(E) = \frac{3}{6} = \frac{1}{2}$

$\Rightarrow \quad P(O) = \frac{1}{2}$

Here, X have values 1 or 3 according as an odd or even number.

So,

$$\begin{array}{lcc} X: & 1 & 3 \\ P(x): & \frac{1}{2} & \frac{1}{2} \end{array}$$

x_i	p_i	$x_i\, p_i$	$x_i^2 p_i$
1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
3	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{9}{2}$
		$\Sigma xp = 2$	$\Sigma x^2p = 5$

Mean = Σxp
mean = 2

Variance = $\Sigma x^2 p - (\text{mean})^2$
= 5 − 4

Variance = 1

Mean and Variance of a Random Variable Ex 32.2 Q12

Let the event of getting a head = H and getting a tail = T
Let X denote the variable longest consecutive heads occurring in 4 tosses. The possible values are

X = 0 (no head) {T, T, T, T}
X = 1 (1 heads) {H, T, T, T}
X = 2 (2 heads) {H, H, T, T}
X = 3 (3 heads) {H, H, H, T}
X = 4 (4 heads) {H, H, H, H}

n(S) = {(HHHH),(HHHT),(HHTT),(HTHH),(HTHT),(HTTH),(HTTT),
(THHH),(THTH),(THHT),(THTT),(THTT),(TTHH),(TTHT)
(TTTH),(TTTT)}

$P(X=0) = \frac{1}{16}$

$P(X=1) = \frac{7}{16}$

$P(X=2) = \frac{5}{16}$

$P(X=3) = \frac{2}{16}$

$P(X=4) = \frac{1}{16}$

Probability distribution is

X	0	1	2	3	4
$p_i = P(X)$	$\frac{1}{16}$	$\frac{7}{16}$	$\frac{5}{16}$	$\frac{2}{16}$	$\frac{1}{16}$
$p_i x_i^2$	0	$\frac{7}{16}$	$\frac{20}{16}$	$\frac{18}{16}$	1

Mean = $\sum_{i=1\ to\ n} X_i \times P(X_i)$

Mean, $\mu = 0 \times \frac{1}{16} + 1 \times \frac{7}{16} + 2 \times \frac{5}{16} + 3 \times \frac{2}{16} + 4 \times \frac{1}{16}$
$= 0 + \frac{7}{16} + \frac{10}{16} + \frac{6}{16} + \frac{4}{16}$
$= \frac{27}{16} = 1.7$
Variance Var(X) = $\sum p_i x_i^2 - (\sum p_i x_i)^2$
$= \frac{61}{16} - 1.7^2$
=3.825-2.89
= 0.935

Mean and Variance of a Random Variable Ex 32.2 Q13

Box contains five cards 1,1,2,2,3.

Here,
X denotes the sum of two number on cards drawn.
Y denotes the maximum of the two number.

So, $X = 2,3,4,5$
$Y = 1,2,3$

$$P(X=2) = P(1)P(1)$$
$$= \frac{2}{5} \times \frac{1}{4}$$
$$= 0.1$$
$$P(X=3) = P(1)P(2) + P(2)P(1)$$
$$= \frac{2}{5} \times \frac{2}{4} + \frac{2}{5} \times \frac{2}{4}$$
$$= 0.4$$
$$P(X=4) = P(2)P(2) + P(1)P(3) + P(3)P(1)$$
$$= \frac{2}{5} \times \frac{1}{4} + \frac{2}{5} \times \frac{1}{4} + \frac{1}{5} \times \frac{2}{4}$$
$$= 0.3$$
$$P(X=5) = P(2)P(3) + P(3)P(2)$$
$$= \frac{2}{5} \times \frac{1}{4} + \frac{1}{5} \times \frac{2}{4}$$
$$= 0.2$$

Probability Distribution for X

X :	2	3	4	5
$P(x)$:	0.1	0.4	0.3	0.2

x_i	p_i	$x_i\ p_i$	$x_i^2 p_i$
2	0.1	0.1	0.4
3	0.4	1.2	3.6
4	0.3	1.2	4.8
5	0.2	1.0	5.0
		$\Sigma xp = 3.6$	$\Sigma x^2 p = 13.8$

Mean = $\sum xp$

mean = 3.6

Variance = $\sum x^2p - (\text{mean})^2$

$= 13.8 - (3.6)^2$

$= 13.8 - 12.96$

Variance = 0.84

$P(Y = 1) = P(1)P(1)$

$= \frac{2}{5} \times \frac{1}{4}$

$= \frac{2}{20}$

$= 0.1$

$P(Y = 2) = P(1)P(2) + P(2)P(1) + P(2)P(2)$

$= \frac{2}{5} \times \frac{2}{4} + \frac{2}{5} \times \frac{2}{4} + \frac{2}{5} \times \frac{1}{4}$

$P(Y = 2) = 0.5$

$P(Y = 3) = P(1)P(3) + P(2)P(3) + P(3)P(1) + P(3)P(2)$

$= \frac{2}{5} \times \frac{1}{4} + \frac{2}{5} \times \frac{1}{4} + \frac{1}{5} \times \frac{2}{4} + \frac{1}{5} \times \frac{2}{4}$

$= 0.4$

Probability distribution for Y is

X :	1	2	3
$P(x)$:	0.1	0.5	0.4

Y_i	p_i	$Y_i\, p_i$	$Y_i^2 p_i$
1 2 3	0.1 0.5 0.4	0.1 1.0 1.2	0.1 2.0 3.6
		$\sum xp = 2.6$	$\sum x^2p = 5.7$

Mean = $\sum xp$

mean = 2.3

Variance = $\sum x^2p - (\text{mean})^2$

$= 5.7 - (2.3)^2$

Variance = 0.41

Mean and Variance of a Random Variable Ex 32.2 Q14

Probability of getting an odd number $= P(O) = \frac{1}{2}$

$\Rightarrow \quad P(E) = \frac{1}{2}$

Die is tossed twice. Let X denote the number of times an odd number occurs.

So, $X = 0,1,2$.

$$P(X=0) = P(E)P(E)$$
$$= \frac{1}{2} \times \frac{1}{2}$$
$$= \frac{1}{4}$$
$$P(X=1) = P(O)P(E) + P(E)P(O)$$
$$= \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2}$$
$$= \frac{1}{2}$$
$$P(X=2) = P(O)P(O)$$
$$= \frac{1}{2} \times \frac{1}{2}$$
$$P(X=2) = \frac{1}{4}$$

x_i	p_i	$x_i\ p_i$	$x_i^2 p_i$
0	$\frac{1}{4}$	0	0
1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
2	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{4}{4}$
		$\Sigma xp = 1$	$\Sigma x^2 p = \frac{3}{2}$

Mean $= \Sigma xp = 1$

Variance $= \Sigma x^2 p - (\text{mean})^2$
$$= \frac{3}{2} - 1$$

Variance $= \frac{1}{2}$

Mean and Variance of a Random Variable Ex 32.2 Q15

Out of 13 bulbs 5 are defective $\Rightarrow$ 8 bulbs are good.
3 bulbs are drawn without replacement ,
Let X denote number of defective bulbs,
So, X can have values 0,1,2,3

$$P(X=0) = p(\text{No defective})$$
$$= \frac{8C_3}{13C_3}$$
$$= \frac{8\times7\times6}{13\times12\times11}$$
$$= \frac{28}{143}$$
$$P(X=1) = p(\text{Only one defective})$$
$$= \frac{8C_2\times5C_1}{13C_3}$$
$$= \frac{8\times7\times5}{2}\times\frac{3\times2\times1}{13\times12\times11}$$
$$= \frac{70}{143}$$
$$P(X=2) = p(\text{Only two defective})$$
$$= \frac{8C_1\times5C_2}{13C_3}$$
$$= \frac{8\times5\times4}{2}\times\frac{3\times2\times1}{13\times12\times11}$$
$$= \frac{40}{143}$$
$$P(X=3) = p(\text{all three are defective})$$
$$= \frac{5C_3}{13C_3}$$
$$= \frac{4\times5}{2}\times\frac{3\times2\times1}{13\times12\times11}$$
$$= \frac{5}{143}$$
So, Probability distribution is

X :	0	1	2	3
$P(x)$:	$\frac{28}{143}$	$\frac{70}{143}$	$\frac{40}{143}$	$\frac{5}{143}$

Mean and Variance of a Random Variable Ex 32.2 Q16

$P(\text{win}) = \frac{1}{13} \Rightarrow \qquad P(\text{lose}) = \frac{12}{13}$

He gains Rs 90 if he wins and loses Rs 10 if his number does not appear.

Let X denote total loss or gain, so,

X :	90	-10
$P(x)$:	$\frac{1}{13}$	$\frac{12}{13}$
XP :	$\frac{90}{13}$	$\frac{-120}{13}$

$$E(X) = \Sigma XP$$
$$= \frac{90}{13} - \frac{120}{13}$$
$$E(X) = -\frac{30}{13}$$

Mean and Variance of a Random Variable Ex 32.2 Q17

Let 'X' be the random variable which can assume values from 0 to 3.

$$P(X=0)=\frac{^{26}C_3}{^{52}C_3}=\frac{2600}{22100}=\frac{2}{17}$$

$$P(X=1)=\frac{^{26}C_1\times{}^{26}C_2}{^{52}C_3}=\frac{8450}{22100}=\frac{13}{34}$$

$$P(X=2)=\frac{^{26}C_2\times{}^{26}C_1}{^{52}C_3}=\frac{8450}{22100}=\frac{13}{34}$$

$$P(X=3)=\frac{^{26}C_3}{^{52}C_3}=\frac{2600}{22100}=\frac{2}{17}$$

Probability distribution of X:

$X=x_i$	0	1	2	3
$p(X=x_i)$	$\frac{2}{17}$	$\frac{13}{34}$	$\frac{13}{34}$	$\frac{2}{17}$

$$\text{Mean}=\sum_{i=0}^{3}(x_i\times p_i)$$
$$=x_0p_0+x_1p_1+x_2p_2+x_3p_3$$
$$=0\times\frac{2}{17}+1\times\frac{13}{34}+2\times\frac{13}{34}+3\times\frac{2}{17}$$
$$=\frac{13+26+12}{34}$$
$$=\frac{51}{34}$$
$$=\frac{3}{2}$$
$$=1.5$$

Mean and Variance of a Random Variable Ex 32.2 Q18

X can assume values 0, 1, 2.
Yes X is a random variable.

$$P(X=0)=(\text{Probablity of getting no black ball})=\frac{^2C_0\times{}^5C_2}{^7C_2}=\frac{1\times\frac{5\times4}{2\times1}}{\frac{7\times6}{2\times1}}=\frac{20}{42}$$

$$P(X=1)=(\text{Probablity of getting one black ball})=\frac{^2C_1\times{}^5C_1}{^7C_2}=\frac{2\times5}{\frac{7\times6}{2\times1}}=\frac{20}{42}$$

$$P(X=2)=(\text{Probablity of getting two black balls})=\frac{^2C_2\times{}^5C_0}{^7C_2}=\frac{1\times1}{\frac{7\times6}{2\times1}}=\frac{2}{42}$$

Thus, probablity distribution of random veriable X is,

X	0	1	2
P(X)	$\frac{20}{42}$	$\frac{20}{42}$	$\frac{2}{42}$

X_i	p_i	p_iX_i	$p_iX_i^2$
0	$\frac{20}{42}$	0	0
1	$\frac{20}{42}$	$\frac{20}{42}$	$\frac{20}{42}$
2	$\frac{2}{42}$	$\frac{4}{42}$	$\frac{8}{42}$
		$\Sigma p_iX_i=\frac{4}{7}$	$\Sigma p_iX_i^2=\frac{2}{3}$

$$\text{Mean}=\Sigma p_iX_i=\frac{4}{7}$$

$$\text{Variance}=\Sigma p_iX_i^2-(\Sigma p_iX_i)^2=\frac{2}{3}-\left(\frac{4}{7}\right)^2=\frac{50}{147}$$

Mean and Variance of a Random Variable Ex 32.2 Q19

We can select two positive in $6 \times 5 = 30$ different ways.
X denotes tha larger number so, X can assume values 3, 4, 5, 6 and 7.
Yes X is a random variable.

$$P(X = 3) = P(\text{larger number is } 3) = \{(2,3),(3,2)\} = \frac{2}{30}$$

$$P(X = 4) = P(\text{larger number is } 4) = \{(2,4),(4,2),(3,4),(4,3)\} = \frac{4}{30}$$

$$P(X = 5) = P(\text{larger number is } 5) = \{(2,5),(5,2),(3,5),(5,3),(4,5),(5,4)\} = \frac{6}{30}$$

$$P(X = 6) = P(\text{larger number is } 6) = \{(2,6),(6,2),(3,6),(6,3),(4,6),(6,4),(5,6),(6,5)\} = \frac{8}{30}$$

$$P(X = 7) = P(\text{larger number is } 7) = \{(2,7),(7,2),(3,7),(7,3),(4,7),(7,4),(5,7),(7,5),(6,7),(7,6)\} = \frac{10}{30}$$

Thus, probablity distribution of random veriable X is,

x_i	p_i	$p_i x_i$	$p_i x_i^2$
3	$\frac{2}{30}$	$\frac{6}{30}$	$\frac{18}{30}$
4	$\frac{4}{30}$	$\frac{16}{30}$	$\frac{64}{30}$
5	$\frac{6}{30}$	$\frac{30}{30}$	$\frac{150}{30}$
6	$\frac{8}{30}$	$\frac{48}{30}$	$\frac{288}{30}$
7	$\frac{10}{30}$	$\frac{70}{30}$	$\frac{490}{30}$
		$\Sigma p_i x_i = \frac{17}{3}$	$\Sigma p_i x_i^2 = \frac{101}{3}$

$$\text{Mean} = \Sigma p_i x_i = \frac{17}{3}$$

$$\text{Variance} = \Sigma p_i x_i^2 - (\Sigma p_i x_i)^2 = \frac{101}{3} - \left(\frac{17}{3}\right)^2 = \frac{14}{9}$$