Ex 25.1

Vector or Cross Product Ex 25.1 Q1

If $\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$ and
$\vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$, then

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & -2 \\ -1 & 0 & 3 \end{vmatrix}$$

$$= \hat{i}(9-0) - \hat{j}(3-2) + \hat{k}(0+3)$$

$$\vec{a}\times\vec{b} = 9\hat{i} - \hat{j} + 3\hat{k}$$

$$|\vec{a}\times\vec{b}| = \sqrt{(9)^2 + (-1)^2 + (3)^2}$$
$$= \sqrt{81+1+9}$$

$$|\vec{a}\times\vec{b}| = \sqrt{91}$$

Vector or Cross Product Ex 25.1 Q2(i)

If $\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$ and
$\vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$, then

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 0 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \hat{i}(4-0) - \hat{j}(3-0) + \hat{k}(3-4)$$
$$= 4\hat{i} - 3\hat{j} - \hat{k}$$

$$|\vec{a}\times\vec{b}| = \sqrt{(4)^2 + (-3)^2 + (-1)^2}$$
$$= \sqrt{16+9+1}$$

$$|\vec{a}\times\vec{b}| = \sqrt{26}$$

Vector or Cross Product Ex 25.1 Q2(ii)

If $\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$ and
$\vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$, then

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \hat{i}(0-1) - \hat{j}(2-1) + \hat{k}(2-0)$$
$$= -\hat{i} - \hat{j} + 2\hat{k}$$

$$|\vec{a}\times\vec{b}| = \sqrt{(-1)^2 + (-1)^2 + (2)^2}$$
$$= \sqrt{1+1+4}$$

$$|\vec{a}\times\vec{b}| = \sqrt{6}$$

Magnitude of $\vec{a}\times\vec{b} = \sqrt{6}$.

Vector or Cross Product Ex 25.1 Q3(i)

A vector perpendicular to both $\vec{a}$ and $\vec{b} = \vec{a} \times \vec{b}$.

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$\vec{c}\,(\text{say}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 3 \\ -2 & 1 & -2 \end{vmatrix}$$

$$\vec{c} = \hat{i}\,(2-3) - \hat{j}\,(-8+6) + \hat{k}\,(4-2)$$
$$\vec{c} = -\hat{i} + 2\hat{j} + 2\hat{k}$$

$\vec{c}$ is a vector perpendicular to both $\vec{a}$ and $\vec{b}$.

$$\hat{c} = \frac{\vec{c}}{|\vec{c}|}$$

$$= \frac{-\hat{i} + 2\hat{j} + 2\hat{k}}{\sqrt{(-1)^2 + (2)^2 + (2)^2}}$$
$$= \frac{-\hat{i} + 2\hat{j} + 2\hat{k}}{\sqrt{1+4+4}}$$

$$= \frac{1}{3}\left(-\hat{i} + 2\hat{j} + 2\hat{k}\right)$$

So, unit vector perpendicular to both $\vec{a}$ and $\vec{b} = \frac{1}{3}\left(-\hat{i} + 2\hat{j} + 2\hat{k}\right)$.

Vector or Cross Product Ex 25.1 Q3(ii)

A vector perpendicular to the plane containing the vector $\vec{a}$ and $\vec{b}$ is given by
$\vec{a} \times \vec{b} = \pm\vec{c}$ (Say)

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{vmatrix}$$

$$\vec{c} = \hat{i}\,(1-2) - \hat{j}\,(2-1) + \hat{k}\,(4-1)$$
$$\vec{c} = -\hat{i} - \hat{j} + 3\hat{k}$$

$$\hat{c} = \frac{\vec{c}}{|\vec{c}|}$$

$$= \frac{-\hat{i} - \hat{j} + 3\hat{k}}{\sqrt{(-1)^2 + (-1)^2 + (3)^2}}$$
$$= \frac{-\hat{i} - \hat{j} + 3\hat{k}}{\sqrt{1+1+9}}$$

$$= \frac{1}{\sqrt{11}}\left(-\hat{i} - \hat{j} + 3\hat{k}\right)$$

Unit vector perpendicular to the plane of $\vec{a}$ and $\vec{b} = \pm\frac{1}{\sqrt{11}}\left(-\hat{i} - \hat{j} + 3\hat{k}\right)$.

Vector or Cross Product Ex 25.1 Q4

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 4 & 3 \\ 1 & 1 & -1 \end{vmatrix}$$

$$= \hat{i}\,(-4-3) - \hat{j}\,(0-3) + \hat{k}\,(0-4)$$
$$= -7\hat{i} + 3\hat{j} - 4\hat{k}$$

$$\left|\vec{a} \times \vec{b}\right| = \sqrt{(-7)^2 + (3)^2 + (-4)^2}$$
$$= \sqrt{49 + 9 + 16}$$

$$\left|\vec{a} \times \vec{b}\right| = \sqrt{74}$$

Vector or Cross Product Ex 25.1 Q5

$$\vec{b} = \hat{i} - 2\hat{k}$$
$$\hat{b} = \frac{\vec{b}}{\left|\vec{b}\right|}$$

$$= \frac{\hat{i} - 2\hat{k}}{\sqrt{(1)^2 + (-2)^2}}$$
$$= \frac{\hat{i} - 2\hat{k}}{\sqrt{1+4}}$$

$$= \frac{\hat{i} - 2\hat{k}}{\sqrt{5}}$$

$$2\hat{b} = \frac{2}{\sqrt{5}}\hat{i} - \frac{4}{\sqrt{5}}\hat{k}$$

And, $\vec{a} = 4\hat{i} + 3\hat{j} + \hat{k}$

If $\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$ and
$\vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$,

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$2\hat{b} \times \vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{2}{\sqrt{5}} & 0 & -\frac{4}{\sqrt{5}} \\ 4 & 3 & 1 \end{vmatrix}$$

$$= \hat{i}\left(0 + \frac{12}{\sqrt{5}}\right) - \hat{j}\left(\frac{2}{\sqrt{5}} + \frac{16}{\sqrt{5}}\right) + \hat{k}\left(\frac{6}{\sqrt{5}} - 0\right)$$

$$2\hat{b} \times \vec{a} = \frac{12}{\sqrt{5}}\hat{i} - \frac{18}{\sqrt{5}}\hat{j} + \frac{6}{\sqrt{5}}\hat{k}$$

$$\left|2\hat{b} \times \vec{a}\right| = \sqrt{\left(\frac{12}{\sqrt{5}}\right)^2 + \left(-\frac{18}{\sqrt{5}}\right)^2 + \left(\frac{6}{\sqrt{5}}\right)^2}$$
$$= \sqrt{\frac{144}{5} + \frac{324}{5} + \frac{36}{5}}$$

$$\left|2\hat{b} \times \vec{a}\right| = \sqrt{\frac{504}{5}}$$

Vector or Cross Product Ex 25.1 Q6

$$\vec{a}+2\vec{b}=\left(3\hat{i}-\hat{j}-2\hat{k}\right)+2\left(2\hat{i}+3\hat{j}+\hat{k}\right)$$
$$=3\hat{i}-\hat{j}-2\hat{k}+4\hat{i}+6\hat{j}+2\hat{k}$$

$$\vec{a}+2\vec{b}=7\hat{i}+5\hat{j}$$

$$2\vec{a}-\vec{b}=2\left(3\hat{i}-\hat{j}-2\hat{k}\right)-\left(2\hat{i}+3\hat{j}+\hat{k}\right)$$
$$=6\hat{i}-2\hat{j}-4\hat{k}-2\hat{i}-3\hat{j}-\hat{k}$$

$$=4\hat{i}-5\hat{j}-5\hat{k}$$

We know that if $\vec{a}=a_1\hat{i}+b_1\hat{j}+c_1\hat{k}$ and $\vec{b}=a_2\hat{i}+b_2\hat{j}+c_2\hat{k}$ then,

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\end{vmatrix}$$

Therefore,

$$\left(\vec{a}+2\vec{b}\right)\times\left(2\vec{a}-\vec{b}\right)=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 7 & 5 & 0\\ 4 & -5 & -5\end{vmatrix}$$

$$=\hat{i}\left(-25-0\right)-\hat{j}\left(-35-0\right)+\hat{k}\left(-35-20\right)$$
$$=-25\hat{i}+35\hat{j}-55\hat{k}$$

$$\left(\vec{a}+2\vec{b}\right)\times\left(2\vec{a}-\vec{b}\right)=-25\hat{i}+35\hat{j}-55\hat{k}$$

Vector or Cross Product Ex 25.1 Q7(i)

Let, $\vec{a}=2\hat{i}+3\hat{j}+6\hat{k}$, $\vec{b}=3\hat{i}-6\hat{j}+2\hat{k}$

If $\vec{a}=a_1\hat{i}+b_1\hat{j}+c_1\hat{k}$ and
$\vec{b}=a_2\hat{i}+b_2\hat{j}+c_2\hat{k}$, then,

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\end{vmatrix}$$

$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 6\\ 3 & -6 & 2\end{vmatrix}$$

$$=\hat{i}\left(6+36\right)-\hat{j}\left(4-18\right)+\hat{k}\left(-12-9\right)$$
$$=42\hat{i}+14\hat{j}-21\hat{k}$$

$$=7\left(6\hat{i}+2\hat{j}-3\hat{k}\right)$$

$$\left|\vec{a}\times\vec{b}\right|=7\sqrt{(6)^2+(2)^2+(-3)^2}$$
$$=7\sqrt{36+4+9}$$

$$\left|\vec{a}\times\vec{b}\right|=7\sqrt{49}$$
$$\left|\vec{a}\times\vec{b}\right|=7\times7$$

$$\left|\vec{a}\times\vec{b}\right|=49$$

Vector or Cross Product Ex 25.1 Q7(ii)

Vector perpendicular to $\vec{a}$ and $\vec{b}$

with magnitude 1 $= \dfrac{\vec{a}\times\vec{b}}{\left|\vec{a}\times\vec{b}\right|}$

$$= \frac{1}{49}\left(7\left(6\hat{i}+2\hat{j}-3\hat{k}\right)\right)$$

$$= \frac{1}{7}\left(6\hat{i}+2\hat{j}-3\hat{k}\right)$$

vector of magnitude 49, which is perpendicular to $\vec{a}$ and $\vec{b}$

$$= 49\left(\frac{\vec{a}\times\vec{b}}{\left|\vec{a}\times\vec{b}\right|}\right)$$

$$= 49\left[\frac{1}{7}\left(6\hat{i}+2\hat{j}-3\hat{k}\right)\right]$$

$$= 42\hat{i}+14\hat{j}-21\hat{k}$$

The required vector $= 42\hat{i}+14\hat{j}-21\hat{k}$

Vector or Cross Product Ex 25.1 Q7(iii)

If $\vec{a} = a_1\hat{i}+b_1\hat{j}+c_1\hat{k}$ and

$\vec{b} = a_2\hat{i}+b_2\hat{j}+c_2\hat{k}$, then,

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -4 \\ 6 & 5 & -2 \end{vmatrix}$$

$$= \hat{i}\left(-2+20\right)-\hat{j}\left(-6+24\right)+\hat{k}\left(15-6\right)$$
$$= 18\hat{i}-18\hat{j}+9\hat{k}$$

$$= 9\left(2\hat{i}-2\hat{j}+\hat{k}\right)$$

$$\left|\vec{a}\times\vec{b}\right| = 9\sqrt{2^2+\left(-2\right)^2+\left(1\right)^2}$$
$$= 9\sqrt{4+4+1}$$

$$\left|\vec{a}\times\vec{b}\right| = 9\sqrt{9}$$
$$\left|\vec{a}\times\vec{b}\right| = 9\times 3$$

$$\left|\vec{a}\times\vec{b}\right| = 27$$

Vector or Cross Product Ex 25.1 Q7(iv)

Unit vector perpendicular to the vector

$\vec{a}$ and $\vec{b} = \dfrac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$

$= \dfrac{1}{27}\left(9\left(2\hat{i}-2\hat{j}+\hat{k}\right)\right)$

$= \dfrac{1}{3}\left(2\hat{i}-2\hat{j}+\hat{k}\right)$

vector with length 3 and which is perpendicular to both $\vec{a}$ and $\vec{b}$

$= 3\left(\dfrac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}\right)$

$= 3\left[\dfrac{1}{3}\left(2\hat{i}-2\hat{j}+\hat{k}\right)\right]$

$= 2\hat{i}-2\hat{j}+\hat{k}$

Required vector $= 2\hat{i}-2\hat{j}+\hat{k}$

Vector or Cross Product Ex 25.1 Q8(i)

Here, $\vec{a} = 2\hat{i}+0.\hat{j}+0.\hat{k}$

$\vec{b} = 0.\hat{i}+3\hat{j}+0.\hat{k}$,

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 0 \\ 0 & 3 & 0 \end{vmatrix}$$

$= \hat{i}(0-0)-\hat{j}(0-0)+\hat{k}(6-0)$

$= 6\hat{k}$

Area of parallelogram $= |\vec{a}\times\vec{b}|$

$= \left|0\hat{i}+0.\hat{j}+6\hat{k}\right|$

$= \sqrt{(0)^2+(0)^2+(6)^2}$

Area of parallelogram = 6 sq.unit

Vector or Cross Product Ex 25.1 Q8(ii)

Let, $\vec{a} = 2\hat{i}+\hat{j}+3\hat{k}$

$\vec{b} = \hat{i}-\hat{j}$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{vmatrix}$$

$= \hat{i}(0+3)-\hat{j}(0-3)+\hat{k}(-2-1)$

$= 3\hat{i}+3\hat{j}-3\hat{k}$

$= 3\left(\hat{i}+\hat{j}-\hat{k}\right)$

Area of parallelogram $= |\vec{a}\times\vec{b}|$

$= 3\sqrt{(1)^2+(1)^2+(-1)^2}$

$= 3\sqrt{3}$

Area of parallelogram $= 3\sqrt{3}$ sq.unit

Vector or Cross Product Ex 25.1 Q8(iii)

Let, $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$

$\vec{b} = \hat{i} - 3\hat{j} + 4\hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix}$$

$$= \hat{i}\,(4 - 6) - \hat{j}\,(12 + 2) + \hat{k}\,(-9 - 1)$$
$$= -2\,\hat{i} - 14\hat{j} - 10\hat{k}$$
$$= -2\left(\hat{i} + 7\hat{j} + 5\hat{k}\right)$$

Area of parallelogram $= \left|\vec{a} \times \vec{b}\right|$

$$= 2\sqrt{(1)^2 + (7)^2 + (5)^2}$$

$$= 2\sqrt{1 + 49 + 25}$$
$$= 2\sqrt{75}$$
$$= 10\sqrt{3}$$

Area of parallelogram $= 10\sqrt{3}$ sq.unit

Vector or Cross Product Ex 25.1 Q8(iv)

Let, $\vec{a} = \hat{i} - 3\hat{j} + \hat{k}$

$\vec{b} = \hat{i} + \hat{j} + \hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \hat{i}\,(-3 - 1) - \hat{j}\,(1 - 1) + \hat{k}\,(1 + 3)$$
$$= -4\hat{i} - 0.\hat{j} + 4\hat{k}$$

Area of parallelogram $= \left|\vec{a} \times \vec{b}\right|$

$$= \sqrt{(-4)^2 + (0)^2 + (4)^2}$$

$$= \sqrt{16 + 0 + 16}$$
$$= \sqrt{32}$$

$$= 4\sqrt{2}$$

Area of parallelogram $= 4\sqrt{2}$ sq.unit

Vector or Cross Product Ex 25.1 Q9(i)

Area of parallelogram = $\frac{1}{2}|\vec{d_1} \times \vec{d_2}|$

Here, $d_1 = 4\hat{i} - \hat{j} - 3\hat{k}$

$d_2 = -2\hat{i} + \hat{j} - 2\hat{k}$

$$\vec{d_1} \times \vec{d_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & -3 \\ -2 & 1 & -2 \end{vmatrix}$$

$$= \hat{i}(2+3) - \hat{j}(-8-6) + \hat{k}(4-2)$$
$$= 5\hat{i} + 14\hat{j} + 2\hat{k}$$

$$|\vec{d_1} \times \vec{d_2}| = \sqrt{(5)^2 + (14)^2 + (2)^2}$$

$$= \sqrt{25 + 196 + 4}$$
$$= \sqrt{225}$$

$$= 15$$

Area of parallelogram = $\frac{1}{2}|\vec{d_1} \times \vec{d_2}|$

Area of parallelogram = $\frac{15}{2}$ sq.unit

Vector or Cross Product Ex 25.1 Q9(ii)

Given, $d_1 = 2\hat{i} + \hat{k}$

$d_2 = \hat{i} + \hat{j} + \hat{k}$

$$\vec{d_1} \times \vec{d_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \hat{i}(0-1) - \hat{j}(2-1) + \hat{k}(2-0)$$
$$= -\hat{i} - \hat{j} + 2\hat{k}$$

$$|\vec{d_1} \times \vec{d_2}| = \sqrt{(-1)^2 + (-1)^2 + (2)^2}$$

$$= \sqrt{1 + 1 + 4}$$

$$= \sqrt{6}$$

Area of parallelogram = $\frac{1}{2}|\vec{d_1} \times \vec{d_2}|$

Area of parallelogram = $\frac{1}{2}\sqrt{6}$ sq.unit

Vector or Cross Product Ex 25.1 Q9(iii)

Given, $d_1 = 3\hat{i} + 4\hat{j}$

$d_2 = \hat{i} + \hat{j} + \hat{k}$

$$\overrightarrow{d_1} \times \overrightarrow{d_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 0 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \hat{i}\left(4-0\right) - \hat{j}\left(3-0\right) + \hat{k}\left(3-4\right)$$

$$= 4\hat{i} - 3\hat{j} - \hat{k}$$

$$\left|\overrightarrow{d_1} \times \overrightarrow{d_2}\right| = \sqrt{(4)^2 + (-3)^2 + (-1)^2}$$

$$= \sqrt{16+9+1}$$

$$= \sqrt{26}$$

Area of parallelogram $= \frac{1}{2}\left|\overrightarrow{d_1} \times \overrightarrow{d_2}\right|$

Area of parallelogram $= \frac{\sqrt{26}}{2}$ sq.unit

Vector or Cross Product Ex 25.1 Q9(iv)

Here, $d_1 = 2\hat{i} + 3\hat{j} + 6\hat{k}$

$d_2 = 3\hat{i} - 6\hat{j} + 2\hat{k}$

$$\overrightarrow{d_1} \times \overrightarrow{d_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 3 & -6 & 2 \end{vmatrix}$$

$$= \hat{i}\left(6+36\right) - \hat{j}\left(4-18\right) + \hat{k}\left(-12-9\right)$$

$$= 42\hat{i} + 14\hat{j} - 21\hat{k}$$

$$= 7\left(6\hat{i} + 2\hat{j} - 3\hat{k}\right)$$

$$\left|\overrightarrow{d_1} \times \overrightarrow{d_2}\right| = 7\sqrt{(6)^2 + (2)^2 + (-3)^2}$$

$$= 7\sqrt{36+4+9}$$

$$= 7\sqrt{49}$$

$$= 7 \times 7$$

$$= 49$$

Area of parallelogram $= \frac{1}{2}\left|\overrightarrow{d_1} \times \overrightarrow{d_2}\right|$

Area of parallelogram $= \frac{49}{2}$ sq.unit

Vector or Cross Product Ex 25.1 Q10

Given, $\vec{a} = 2\hat{i} + 5\hat{j} - 7\hat{k}$,

$\vec{b} = -3\hat{i} + 4\hat{j} + \hat{k}$,

$\vec{c} = \hat{i} - 2\hat{j} - 3\hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 5 & -7 \\ -3 & 4 & 1 \end{vmatrix}$$

$$= \hat{i}(5+28) - \hat{j}(2-21) + \hat{k}(8+15)$$
$$= 33\hat{i} + 19\hat{j} + 23\hat{k}$$

$$\left(\vec{a} \times \vec{b}\right) \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 33 & 19 & 23 \\ 1 & -2 & -3 \end{vmatrix}$$

$$= \hat{i}(-57+46) - \hat{j}(-99-23) + \hat{k}(-66-19)$$
$$\left(\vec{a} \times \vec{b}\right) \times \vec{c} = -11\hat{i} + 122\hat{j} - 85\hat{k} \qquad ---(i)$$

$$\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 4 & 1 \\ 1 & -2 & -3 \end{vmatrix}$$

$$= \hat{i}(-12+2) - \hat{j}(9-1) + \hat{k}(6-4)$$
$$= -10\hat{i} - 8\hat{j} + 2\hat{k}$$

$$\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 5 & -7 \\ -10 & -8 & 2 \end{vmatrix}$$

$$= \hat{i}(10+56) - \hat{j}(4-70) + \hat{k}(-16+50)$$

$$\vec{a} \times \left(\vec{b} \times \vec{c}\right) = 66\hat{i} + 66\hat{j} + 36\hat{k} \qquad ---(ii)$$

From equation (i) and (ii)

$$\vec{a} \times \left(\vec{b} \times \vec{c}\right) \neq \left(\vec{a} \times \vec{b}\right) \times \vec{c}$$

Vector or Cross Product Ex 25.1 Q11

We know that, if θ be the angle between $\vec{a}$ and $\vec{b}$, then,

$$\vec{a} \times \vec{b} = |\vec{a}| . |\vec{b}| . \sin\theta . \hat{n}$$
$$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| . |\sin\theta| . |\hat{n}|$$
$$8 = 2.5. \sin\theta . 1 \qquad \left[\text{As } \hat{n} \text{ is a unit vector}\right]$$

$$\sin\theta = \frac{8}{10}$$
$$\sin\theta = \frac{4}{5}$$
$$\cos^2\theta = 1 - \sin^2\theta$$
$$= 1 - \left(\frac{4}{5}\right)^2$$
$$= 1 - \frac{16}{25}$$
$$= \frac{25-16}{25}$$
$$= \frac{9}{25}$$

$$\cos\ \theta = \frac{3}{5}$$

$$\vec{a}.\vec{b} = |\vec{a}| . |\vec{b}| . \cos\theta$$
$$= 2.5. \frac{3}{5}$$

$$\vec{a}.\vec{b} = 6$$

Vector or Cross Product Ex 25.1 Q12

Given, $\vec{a} = \frac{1}{7}\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$

$\vec{b} = \frac{1}{7}\left(3\hat{i} - 6\hat{j} + 2\hat{k}\right)$

$\vec{c} = \frac{1}{7}\left(6\hat{i} + 2\hat{j} - 3\hat{k}\right)$,

$$\vec{a} \times \vec{b} = \frac{1}{7} \times \frac{1}{7}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 3 & -6 & 2 \end{vmatrix}$$

$$= \frac{1}{49}\left[\hat{i}(6+36) - \hat{j}(4-18) + \hat{k}(-12-9)\right]$$

$$= \frac{1}{49}\left[42\hat{i} + 14\hat{j} - 21\hat{k}\right]$$

$$= \frac{7\left(6\hat{i} + 2\hat{j} - 3\hat{k}\right)}{49}$$

$$= \frac{1}{7}\left(6\hat{i} + 2\hat{j} - 3\hat{k}\right)$$

$$\vec{a} \times \vec{b} = \vec{c} \qquad ---(i)$$

$$\vec{b} \times \vec{c} = \frac{1}{7} \times \frac{1}{7}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -6 & 2 \\ 6 & 2 & -3 \end{vmatrix}$$

$$= \frac{1}{49}\left[\hat{i}(18-4) - \hat{j}(-9-12) + \hat{k}(6+36)\right]$$

$$= \frac{1}{49}\left[14\hat{i} + 21\hat{j} + 42\hat{k}\right]$$

$$= \frac{7\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)}{49}$$

$$\vec{c} \times \vec{a} = \frac{1}{7} \times \frac{1}{7}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 2 & -3 \\ 2 & 3 & 6 \end{vmatrix}$$

$$\vec{c} \times \vec{a} = \frac{1}{49}\left[\hat{i}(12+9) - \hat{j}(36+6) + \hat{k}(18-4)\right]$$

$$= \frac{1}{49}\left[21\hat{i} - 42\hat{j} + 14\hat{k}\right]$$

$$= \frac{7\left(3\hat{i} - 6\hat{j} + 2\hat{k}\right)}{49}$$

$$= \frac{1}{7}\left(3\hat{i} - 6\hat{j} + 2\hat{k}\right)$$

$$\vec{c} \times \vec{a} = \vec{b} \qquad ---(iii)$$

From (i), (ii), and (iii),

$$\left.\begin{matrix} \vec{a} \times \vec{b} = \vec{c} \\ \vec{b} \times \vec{c} = \vec{a} \\ \vec{c} \times \vec{a} = \vec{b} \end{matrix}\right\} - (A)$$

$$\left|\vec{a}\right| = \frac{1}{7}\sqrt{(2)^2 + (3)^2 + (6)^2}$$

$$= \frac{1}{7}\sqrt{4+9+36}$$

$$= \frac{1}{7}\sqrt{49}$$

$$= \frac{1}{7} \times 7$$

$$\left|\vec{a}\right| = 1 \qquad ---(iv)$$

$$\left|\vec{b}\right| = \frac{1}{7}\sqrt{(3)^2 + (-6)^2 + (2)^2}$$

$$= \frac{1}{7}\sqrt{9+36+4}$$

$$= \frac{1}{7}\sqrt{49}$$

$$= \frac{7}{7}$$

$$\left|\vec{b}\right| = 1 \qquad ---(v)$$

$$\left|\vec{c}\right| = \frac{1}{7}\sqrt{(6)^2 + (2)^2 + (-3)^2}$$

$$= \frac{1}{7}36+4+9$$

$$= \frac{1}{7}\sqrt{49}$$

$$= \frac{7}{7}$$

$$\left|\vec{c}\right| = 1 \qquad ---(vi)$$

From equation (iv), (v), (vi),

$$\left|\vec{a}\right| = \left|\vec{b}\right| = \left|\vec{c}\right| = 1 \qquad ---(B)$$

From (A) and (B), We can say that

$\vec{a}, \vec{b}, \vec{c}$ is a right handed orthogonal system of unit vectors

Vector or Cross Product Ex 25.1 Q13

We know that, if θ is angle between $\vec{a}$ and $\vec{b}$,

$$\vec{a}\vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

$$60 = 13.5.\cos\theta$$

$$\cos\theta = \frac{60}{65}$$

$$\cos\theta = \frac{12}{13}$$

$$\sin^2\theta = 1 - \cos^2\theta$$

$$= 1 - \left(\frac{12}{13}\right)^2$$

$$= 1 - \frac{144}{169}$$

$$= \frac{169 - 144}{169}$$

$$= \frac{25}{169}$$

$$\sin\ \theta = \pm\sqrt{\frac{25}{169}}$$

$$= \pm\frac{5}{13}$$

$$|\sin\theta| = \frac{5}{13}$$

We know that,

$$\vec{a}\times\vec{b} = |\vec{a}|.|\vec{b}|.\sin\theta.\hat{n}$$

$$|\vec{a}\times\vec{b}| = |\vec{a}|.|\vec{b}|.|\sin\theta|.|\hat{n}|$$

$$= 13.5.\frac{5}{13}.1 \qquad \left[\text{Since, } \hat{n} \text{ is a unit vector}\right]$$

$$|\vec{a}\times\vec{b}| = 25$$

Vector or Cross Product Ex 25.1 Q14

We know that, if θ be the angle between $\vec{a}$ and $\vec{b}$, then

$$\vec{a}.\vec{b} = |\vec{a}|\ |\vec{b}|\cos\theta \qquad ---(i)$$

$$\text{And, } \vec{a}\times\vec{b} = |\vec{a}|\ |\vec{b}|.\sin\theta.\hat{n}$$

$$|\vec{a}\times\vec{b}| = |\vec{a}|.|\vec{b}|.|\sin\theta|.|\hat{n}|$$

$$= |\vec{a}|\ |\vec{b}||\sin\theta|.1 \qquad \left[\text{Since, } \hat{n} \text{ is a unit vector}\right]$$

$$|\vec{a}\times\vec{b}| = |\vec{a}|\ |\vec{b}|\sin\theta \qquad ---(ii)$$

Given that, $|\vec{a}\times\vec{b}| = \vec{a}.\vec{b}$

$$|\vec{a}|\ |\vec{b}|\sin\theta = |\vec{a}|.|\vec{b}|\cos\theta$$

$$\sin\theta = \cos\theta$$

$$\theta = \frac{\pi}{4}$$

Angle between $\vec{a}$ and $\vec{b}$ = $\frac{\pi}{4}$

Vector or Cross Product Ex 25.1 Q15

We have,

$\vec{a} \times \vec{b} = \vec{b} \times \vec{c}$

$\left(\vec{a} \times \vec{b}\right) - \left(\vec{b} \times \vec{c}\right) = \vec{0}$

$\left(\vec{a} \times \vec{b}\right) + \left(\vec{c} \times \vec{b}\right) = \vec{0}$ $\left[\text{Since, } \left(\vec{b} \times \vec{c}\right) = -\left(\vec{c} \times \vec{b}\right)\right]$

$\left(\vec{a} + \vec{c}\right) \times \vec{b} = \vec{0}$ $\left[\text{Using distributive property}\right]$

We know that, if $\vec{a} \times \vec{b} = \vec{0}$, then vector $\vec{a}$ is parallel to vector $\vec{b}$.

Thus, $\left(\vec{a} + \vec{c}\right)$ is parallel to $\vec{b}$

$\left(\vec{a} + \vec{c}\right) = \overline{mb}$

Where m is any scalar

Vector or Cross Product Ex 25.1 Q16

We know that,

$\vec{a} \times \vec{b} = \left|\vec{a}\right| \left|\vec{b}\right| \sin\theta \ \hat{n}$

$\left|\vec{a} \times \vec{b}\right| = \left|\vec{a}\right| \left|\vec{b}\right| \left|\sin\theta\right| \left|\hat{n}\right|$

$= \left|\vec{a}\right| \left|\vec{b}\right| \left|\sin\theta\right| .1$ $\left[\text{as } \hat{n} \text{ is a unit vector}\right]$

$\left|\vec{a} \times \vec{b}\right| = \left|\vec{a}\right| \left|\vec{b}\right| \left|\sin\theta\right|$

$\sqrt{(3)^2 + (2)^2 + (6)^2} = 2.7.\left|\sin\theta\right|$

$\sqrt{9 + 4 + 36} = 14.\left|\sin\theta\right|$

$\sqrt{49} = 14\left|\sin\theta\right|$

$\sin\theta = \frac{7}{14}$

$\sin\theta = \frac{1}{2}$

$\theta = \sin^{-1}\left(\frac{1}{2}\right)$

$\theta = \frac{\pi}{6}$

Angle between $\vec{a}$ and $\vec{b} = \frac{\pi}{6}$

Vector or Cross Product Ex 25.1 Q17

Given that $\vec{a} \times \vec{b} = \vec{0}$

This gives us four conclusions about $\vec{a}$ and $\vec{b}$

(i) $\vec{a} = \vec{0}$ or

(ii) $\vec{b} = \vec{0}$ or

(iii) $\vec{a} = \vec{b} = \vec{0}$ or

(iv) $\vec{a}$ is parallel to $\vec{b}$.

Also, it is given that $\vec{a}.\vec{b} = 0$

This also gives us four canclusions about $\vec{a}$ and $\vec{b}$.

(i) $\vec{a} = \vec{0}$ or

(ii) $\vec{b} = \vec{0}$ or

(iii) $\vec{a} = \vec{b} = \vec{0}$ or

(iv) $\vec{a}$ is perpendicular to $\vec{b}$.

Now,

$\vec{a}$ parallel $\vec{b}$ and $\vec{a}$ is perpendicular to $\vec{b}$ are not possible simultaneously.

So,

$\vec{a} = 0$ or $\vec{b} = 0$ or $\vec{a} = \vec{b} = \vec{0}$

Vector or Cross Product Ex 25.1 Q18

Given that $\vec{a}, \vec{b}, \vec{c}$ are three unit vectors such that

$\vec{a} \times \vec{b} = \vec{c},\ \vec{b} \times \vec{c} = \vec{a},\ \vec{c} \times \vec{a} = \vec{b},$

$\vec{a} \times \vec{b} = \vec{c}$

$\Rightarrow \quad \vec{c}$ is a vector perpendicular to both $\vec{a}$ and $\vec{b}$ $\qquad ---(i)$

$\vec{b} \times \vec{c} = \vec{a}$

$\Rightarrow \quad \vec{a}$ is a vector perpendicular to $\vec{b}$ and $\vec{c}$ $\qquad ---(ii)$

$\vec{c} \times \vec{a} = \vec{b}$

$\Rightarrow \quad \vec{b}$ is a vector perpendicular to $\vec{a}$ and $\vec{c}$ $\qquad ---(iii)$

Using $(i), (ii)$ and (iii), we can see that $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular unit vectors.

Since, $\vec{a} \times \vec{b} = \vec{c}$

$\vec{b} \times \vec{c} = \vec{a}$

$\vec{c} \times \vec{a} = \vec{b}$

Therefore,

$\vec{a}, \vec{b}, \vec{c}$ form an orthonormal right handed traid of unit vectors.

Vector or Cross Product Ex 25.1 Q19

Here, Position vector of $A = \left(3\hat{i} - \hat{j} + 2\hat{k}\right)$

Position vector of $B = \left(\hat{i} - \hat{j} - 3\hat{k}\right)$

Position vector of $C = \left(4\hat{i} - 3\hat{j} + \hat{k}\right)$

$$\begin{aligned}\overrightarrow{AB} &= \vec{B} - \vec{A} \\ &= \left(\hat{i} - \hat{j} - 3\hat{k}\right) - \left(3\hat{i} - \hat{j} + 2\hat{k}\right) \\ &= \hat{i} - \hat{j} - 3\hat{k} - 3\hat{i} + \hat{j} - 2\hat{k} \\ &= -2\hat{i} - 5\hat{k}\end{aligned}$$

$$\begin{aligned}\overrightarrow{AC} &= \vec{C} - \vec{A} \\ &= \left(4\hat{i} - 3\hat{j} + \hat{k}\right) - \left(3\hat{i} - \hat{j} + 2\hat{k}\right) \\ &= 4\hat{i} - 3\hat{j} + \hat{k} - 3\hat{i} + \hat{j} - 2\hat{k} \\ &= \hat{i} - 2\hat{j} - \hat{k}\end{aligned}$$

Vector perpendicular to the plane ABC

$$= \overrightarrow{AC} \times \overrightarrow{AB} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & -1 \\ -2 & 0 & -5 \end{vmatrix}$$

$$\begin{aligned}\overrightarrow{AC} \times \overrightarrow{AB} &= \hat{i}\left(10 - 0\right) - \hat{j}\left(-5 - 2\right) + \hat{k}\left(0 - 4\right) \\ &= 10\hat{i} + 7\hat{j} - 4\hat{k}\end{aligned}$$

$$\begin{aligned}\left|\overrightarrow{AC} \times \overrightarrow{AB}\right| &= \sqrt{(10)^2 + (7)^2 + (-4)^2} \\ &= \sqrt{100 + 49 + 16} \\ &= \sqrt{165}\end{aligned}$$

Therefore, unit vector perpendicular to the plane $ABC = \dfrac{\overrightarrow{AC} \times \overrightarrow{AB}}{\left|\overrightarrow{AC} \times \overrightarrow{AB}\right|}$

$$= \frac{1}{\sqrt{165}}\left(10\hat{i} + 7\hat{j} - 4\hat{k}\right)$$

Unit vector perpendicular to the plane $ABC = \dfrac{1}{\sqrt{165}}\left(10\hat{i} + 7\hat{j} - 4\hat{k}\right)$

Vector or Cross Product Ex 25.1 Q20

Here, It is given that

In $\triangle ABC$

$\overrightarrow{BC} + \overrightarrow{CA} + \overrightarrow{AB}$
$= \overrightarrow{BA} + \overrightarrow{AB}$
$= \overrightarrow{BA} - \overrightarrow{BA}$ $\left[\text{Since, } \overrightarrow{BA} = -\overrightarrow{AB}\right]$

$= \vec{0}$

Given that, $\left|\overrightarrow{BC}\right| = a$
$\left|\overrightarrow{CA}\right| = b$
$\left|\overrightarrow{AB}\right| = c$

Let, $\overrightarrow{BC} = \vec{a}$, $\overrightarrow{CA} = \vec{b}$ and $\overrightarrow{AB} = \vec{c}$

We have,
$\overrightarrow{BC} + \overrightarrow{CA} + \overrightarrow{AB} = \vec{0}$

$\Rightarrow \quad \overrightarrow{BC} + \overrightarrow{CA} = -\overrightarrow{AB}$
$\Rightarrow \quad \overrightarrow{BC} + \overrightarrow{CA} = \overrightarrow{BA}$
$\Rightarrow \quad \vec{a} + \vec{b} = -\vec{c}$
$\Rightarrow \quad \vec{a} + \vec{b} + \vec{c} = \vec{0}$
$\Rightarrow \quad \vec{a} \times \left(\vec{a} + \vec{b} + \vec{c}\right) = \vec{a} \times \vec{0}$
$\Rightarrow \quad \vec{a} \times \vec{a} + \vec{a} \times \vec{b} + \vec{a} \times \vec{c} = \vec{0}$
$\Rightarrow \quad \vec{0} + \vec{a} \times \vec{b} + \vec{a} \times \vec{c} = \vec{0}$ $\left[\text{Since, } \vec{a} \times \vec{a} = \vec{0}\right]$
$\Rightarrow \quad \vec{a} \times \vec{b} = -\left(\vec{a} \times \vec{c}\right)$

$\Rightarrow \quad \vec{a} \times \vec{b} = \vec{c} \times \vec{a}$ $\quad ---(\text{i})$

Again, $\vec{a} + \vec{b} + \vec{c} = \vec{0}$

$\Rightarrow \quad \vec{b} \times \left(\vec{a} + \vec{b} + \vec{c}\right) = \vec{b} \times \vec{0}$
$\Rightarrow \quad \vec{b} \times \vec{a} + \vec{b} \times \vec{b} + \vec{b} \times \vec{c} = \vec{0}$
$\Rightarrow \quad \vec{b} \times \vec{a} + \vec{0} + \vec{b} \times \vec{c} = \vec{0}$ $\left[\text{Since, } \vec{b} \times \vec{b} = \vec{0}\right]$
$\Rightarrow \quad \vec{b} \times \vec{a} + \vec{b} \times \vec{c} = \vec{0}$
$\Rightarrow \quad \vec{b} \times \vec{c} = -\left(\vec{b} \times \vec{a}\right)$

$\Rightarrow \quad \vec{b} \times \vec{c} = \vec{a} \times \vec{b}$ $\quad ---(\text{ii})$

From equation (i) and (ii), we get
$\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$

$\Rightarrow \quad \left|\vec{a} \times \vec{b}\right| = \left|\vec{b} \times \vec{c}\right| = \left|\vec{c} \times \vec{a}\right|$
$\Rightarrow \quad \left|\vec{a}\right| \left|\vec{b}\right| \sin\left(\pi - C\right) = \left|\vec{b}\right| \left|\vec{c}\right| \sin\left(\pi - A\right) = \left|\vec{c}\right| \left|\vec{a}\right| \sin\left(\pi - B\right)$
$\Rightarrow \quad ab \sin C = bc \sin A = ca \sin B$

Dividing by abc

$$\Rightarrow \quad \frac{ab \sin C}{abc} = \frac{bc \sin A}{abc} = \frac{ca \sin B}{abc}$$
$$\Rightarrow \quad \frac{\sin C}{c} = \frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\Rightarrow \quad \frac{c}{\sin C} = \frac{a}{\sin A} = \frac{b}{\sin B}$$

Vector or Cross Product Ex 25.1 Q21

Here, $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} - 5\hat{k}$,

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 2 & 3 & -5 \end{vmatrix}$$

$$= \hat{i}(10-9) - \hat{j}(-5-6) + \hat{k}(3+4)$$

$$= \hat{i} + 11\hat{j} + 7\hat{k}$$

Now, $\vec{a}.(\vec{a} \times \vec{b}) = (\hat{i} - 2\hat{j} + 3\hat{k}).(\hat{i} + 11\hat{j} + 7\hat{k})$

$$\vec{a}.(\vec{a} \times \vec{b}) = (1)(1) + (-2)(11) + (3)(7)$$
$$= 1 - 22 + 21$$
$$= 22 - 22$$

$$\vec{a} \times (\vec{a} \times \vec{b}) = 0$$

Dot product of $\vec{a}$ and $\vec{a} \times \vec{b}$ is zero, then,
$\vec{a}$ is perpendicular to $(\vec{a} \times \vec{b})$

Vector or Cross Product Ex 25.1 Q22

Given $\vec{p}$ and $\vec{q}$ be unit vector with angle $30°$ between then

$$|\vec{p}| = |\vec{q}| = 1$$

$$\vec{p} \times \vec{q} = |\vec{p}|\,|\vec{q}| \sin 30° \hat{n}$$
$$= 1.1.\left(\frac{1}{2}\right)\hat{n}$$

$$|\vec{p} \times \vec{q}| = \left|\frac{\hat{n}}{2}\right|$$

$$|\vec{p} \times \vec{q}| = \frac{1}{2} \qquad ---(i) \qquad \left[\text{Since, } \hat{n} \text{ is a unit vector}\right]$$

Area of parallelogram $= \frac{1}{2}|\vec{a} \times \vec{b}|$

$$= \frac{1}{2}\left|(\vec{p} + 2\vec{q}) \times (2\vec{p} + \vec{q})\right|$$
$$= \frac{1}{2}\left|\vec{p} \times 2\vec{p} + \vec{p} \times \vec{q} + 2\vec{q} \times 2\vec{p} + 2\vec{q} \times \vec{q}\right|$$
$$= \frac{1}{2}\left|\vec{0} + \vec{p} \times \vec{q} + 2\vec{q} \times 2\vec{p} + \vec{0}\right| \qquad \left[\text{Since, } \vec{p} \times 2\vec{q} = \vec{0} \text{ and } 2\vec{q} \times \vec{q} = \vec{0}\right]$$
$$= \frac{1}{2}\left|\vec{p} \times \vec{q} + 4(\vec{q} \times \vec{p})\right|$$
$$= \frac{1}{2}\left|(\vec{p} \times \vec{q}) - 4(\vec{p} \times \vec{q})\right| \qquad \left[\text{Since, } \vec{q} \times \vec{p} = -\vec{p} \times \vec{q}\right]$$
$$= \frac{1}{2}\left|-3(\vec{p} \times \vec{q})\right|$$
$$= \frac{3}{2}\left|\vec{p} \times \vec{q}\right|$$
$$= \frac{3}{2} \times \frac{1}{2} \qquad \left[\text{Using (i)}\right]$$
$$= \frac{3}{4}$$

Area of parallelogram $= \frac{3}{4}$ sq. unit

Vector or Cross Product Ex 25.1 Q23

We know that

$$\vec{a}\times\vec{b} = |\vec{a}|.|\vec{b}|.\sin\theta.\hat{n}$$

$$|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta.|\hat{n}|$$

$$= |\vec{a}||\vec{b}|\sin\theta.1 \qquad \left[\text{Since, } \hat{n} \text{ is unit vector}\right]$$

$$|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta$$

Squaring both the sides,

$$|\vec{a}\times\vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2\sin^2\theta$$

$$= |\vec{a}|^2|\vec{b}|^2\left(1-\cos^2\theta\right)$$

$$= |\vec{a}|^2|\vec{b}|^2 - |\vec{a}|^2|\vec{b}|^2\cos^2\theta$$

$$= |\vec{a}|^2|\vec{b}|^2 - \left(|\vec{a}||\vec{b}|\cos\theta\right)^2$$

$$= \left(\vec{a}\right)^2\left(\vec{b}\right)^2 - \left(\vec{a}.\vec{b}\right)^2 \qquad \left[\text{Since, } |\vec{a}||\vec{b}|\cos\theta = \vec{a}.\vec{b}\right]$$

$$|\vec{a}\times\vec{b}|^2 = \left(\vec{a}.\vec{a}\right)\left(\vec{b}.\vec{b}\right) - \left(\vec{a}.\vec{b}\right)\left(\vec{b}.\vec{a}\right) \qquad \left[\text{Since, } \left(\vec{a}.\vec{b}\right) = \left(\vec{b}.\vec{a}\right)\right]$$

$$|\vec{a}\times\vec{b}|^2 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} \\ \vec{b}.\vec{a} & \vec{b}\vec{b} \end{vmatrix}$$

Vector or Cross Product Ex 25.1 Q24

Define of $\vec{a}\times\vec{b}$: – Let $\vec{a}$, $\vec{b}$ be two non-zero, non-parallel vectors. Then $\vec{a}\times\vec{b}$, in that order, is defined as a vector whose magnitude is $|\vec{a}|\ |\vec{b}|\sin\theta$, where θ is the angle between $\vec{a}$ and $\vec{b}$ and whose direction is perpendicular to the plane of $\vec{a}$ and $\vec{b}$ and this constitute a right handed system.

$$\vec{a}\times\vec{b} = |\vec{a}|\ |\vec{b}|\sin\theta\hat{n}$$

Where $\hat{n}$ is a unit vector perpendicular to the plane of $\vec{a}$ and $\vec{b}$ such that $\vec{a},\vec{b},\hat{n}$ form a right handed system.

Now,

$$\vec{a}\times\vec{b} = |\vec{a}|\ |\vec{b}|\sin\theta\ \hat{n}$$

$$|\vec{a}\times\vec{b}| = |\vec{a}|\ |\vec{b}||\sin\theta|\ |\hat{n}|$$

$$= |\vec{a}|\ |\vec{b}||\sin\theta|.1$$

$$|\vec{a}\times\vec{b}| = |\vec{a}|\ |\vec{b}||\sin\theta|$$

$$= \frac{\vec{a}.\vec{b}}{\cos\theta}.\sin\theta \qquad \left[\text{Since, } \cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}\right]$$

$$|\vec{a}\times\vec{b}| = \vec{a}.\vec{b}.\tan\theta$$

Vector or Cross Product Ex 25.1 Q25

$\vec{a} \times \vec{b} = |\vec{a}|\,|\vec{b}|\sin\theta\,.\hat{n}$

$|\vec{a} \times \vec{b}| = |\vec{a}|\,|\vec{b}|\,|\sin\theta|\,|\hat{n}|$

$35 = \sqrt{26}.7\ |\sin\theta|.1$

$\sin\theta = \dfrac{35}{\sqrt{26}.7}$

$\sin\theta = \dfrac{5}{\sqrt{26}}$

$\cos^2\theta = 1 - \sin^2\theta$

$$= 1 - \left(\frac{5}{\sqrt{26}}\right)^2$$

$$= \frac{1}{1} - \frac{25}{26}$$

$$= \frac{26-25}{26}$$

$$= \frac{1}{26}$$

$\cos\theta = \dfrac{1}{\sqrt{26}}$

$\vec{a}.\vec{b} = |\vec{a}|\,|\vec{b}|\cos\theta$

$$= \sqrt{26}.7.\frac{1}{\sqrt{26}}$$

$\vec{a}.\vec{b} = 7$

Vector or Cross Product Ex 25.1 Q26

Area of triangle $= \dfrac{1}{2}\left|\overrightarrow{OA} \times \overrightarrow{OB}\right|$

$$\left|\overrightarrow{OA} \times \overrightarrow{OB}\right| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ -3 & -2 & 1 \end{vmatrix}$$

$$= \hat{i}\,(2+6) - \hat{j}\,(1+9) + \hat{k}\,(-2+6)$$

$$= 8\hat{i} - 10\hat{j} + 4\hat{k}$$

$$= 2\left(4\hat{i} - 5\hat{j} + 2\hat{k}\right)$$

Area of triangle $= \dfrac{1}{2}\left|\overrightarrow{OA} \times \overrightarrow{OB}\right|$

$$= \frac{1}{2}\left[2\sqrt{(4)^2 + (-5)^2 + (2)^2}\right]$$

$$= \frac{1}{2}\left[2\sqrt{16+25+4}\right]$$

$$= \sqrt{45}$$

$$= 3\sqrt{5}$$

Area of triangle $= 3\sqrt{5}$ Sq.unit

Vector or Cross Product Ex 25.1 Q27

Let $\vec{d} = d_1\hat{i} + d_2\hat{j} + d_3\hat{k}$.

Since $\vec{d}$ is perpendicular to both $\vec{a}$ and $\vec{b}$, we have:

$$\vec{d}\cdot\vec{a} = 0$$
$$\Rightarrow d_1 + 4d_2 + 2d_3 = 0 \qquad \text{...(i)}$$
And,
$$\vec{d}\cdot\vec{b} = 0$$
$$\Rightarrow 3d_1 - 2d_2 + 7d_3 = 0 \qquad \text{...(ii)}$$

Also, it is given that:

$$\vec{c}\cdot\vec{d} = 15$$
$$\Rightarrow 2d_1 - d_2 + 4d_3 = 15 \qquad \text{...(iii)}$$

On solving (i), (ii), and (iii), we get:

$$d_1 = \frac{160}{3}, d_2 = -\frac{5}{3} \text{ and } d_3 = -\frac{70}{3}$$
$$\therefore \vec{d} = \frac{160}{3}\hat{i} - \frac{5}{3}\hat{j} - \frac{70}{3}\hat{k} = \frac{1}{3}\left(160\hat{i} - 5\hat{j} - 70\hat{k}\right)$$

Hence, the required vector is $\frac{1}{3}\left(160\hat{i} - 5\hat{j} - 70\hat{k}\right)$.

Vector or Cross Product Ex 25.1 Q28

Given, $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$
$\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$

Let, $\vec{d} = \vec{a} + \vec{b}$
$= \left(3\hat{i} + 2\hat{j} + 2\hat{k}\right) + \left(\hat{i} + 2\hat{j} - 2\hat{k}\right)$
$\vec{d} = 4\hat{i} + 4\hat{j} - 0\hat{k}$

And, $\vec{e} = \vec{a} - \vec{b}$
$= \left(3\hat{i} + 2\hat{j} + 2\hat{k}\right) - \left(\hat{i} + 2\hat{j} - 2\hat{k}\right)$
$\vec{e} = 2\hat{i} + 4\hat{k}$

Let, $\vec{f}$ be any vector perpendicular to both $\vec{d}$ and $\vec{e}$

$$\vec{f} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 4 & 0 \\ 2 & 0 & 4 \end{vmatrix}$$

$= \hat{i}(16 - 0) - \hat{j}(16 - 0) + \hat{k}(0 - 8)$
$\vec{f} = 16\hat{i} - 16\hat{j} - 8\hat{k}$

$= 8\left(2\hat{i} - 2\hat{j} - \hat{k}\right)$

Let $\vec{g}$ be the required vector, then

$\vec{g} = \lambda\vec{f}$ and $|\vec{g}| = 1$
$\vec{g} = 8\lambda\left(2\hat{i} - 2\hat{j} - \hat{k}\right) \qquad ---(i)$

$|\vec{g}| = 1$

$8\lambda\sqrt{(2)^2 + (-2)^2 + (-1)^2} = 1$
$8\lambda\sqrt{4 + 4 + 1} = 1$
$8\lambda\sqrt{9} = 1$
$24\lambda = 1$

$\lambda = \frac{1}{24}$

Put λ in (i)

$$\vec{g} = 8\left(\frac{1}{24}\right)\left(2\hat{i} - 2\hat{j} - \hat{k}\right)$$

$$\vec{g} = \frac{1}{3}\left(2\hat{i} - 2\hat{j} - \hat{k}\right)$$

Thus,

Unit vector perpendicular to $\left(\vec{a} + \vec{b}\right)$ and $\left(\vec{a} - \vec{b}\right) = \frac{1}{3}\left(2\hat{i} - 2\hat{j} - \hat{k}\right)$

Vector or Cross Product Ex 25.1 Q29

Given, $A = (2,3,5)$

$B = (3,5,8)$

$C = (2,7,8)$

Position vector of $A = 2\hat{i} + 3\hat{j} + 5\hat{k}$

Position vector of $B = 3\hat{i} + 5\hat{j} + 8\hat{k}$

Position vector of $C = 2\hat{i} + 7\hat{j} + 8\hat{k}$

$\overrightarrow{AB}$ = Position vector of B – Position vector of A

$$= \left(3\hat{i} + 5\hat{j} + 8\hat{k}\right) - \left(2\hat{i} + 3\hat{j} + 5\hat{k}\right)$$

$$= 3\hat{i} + 5\hat{j} + 8\hat{k} - 2\hat{i} - 3\hat{j} - 5\hat{k}$$

$$\overrightarrow{AB} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$\overrightarrow{AC}$ = Position vector of C – Position vector of A

$$= \left(2\hat{i} + 7\hat{j} + 8\hat{k}\right) - \left(2\hat{i} + 3\hat{j} + 5\hat{k}\right)$$

$$= 2\hat{i} + 7\hat{j} + 8\hat{k} - 2\hat{i} - 3\hat{j} - 5\hat{k}$$

$$\overrightarrow{AC} = 4\hat{j} + 3\hat{k}$$

Area of triangle $= \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 0 & 4 & 3 \end{vmatrix}$$

$$= \hat{i}\,(6 - 12) - \hat{j}\,(3 - 0) + \hat{k}\,(4 - 0)$$

$$\overrightarrow{AB} \times \overrightarrow{AC} = -6\hat{i} - 3\hat{j} + 4\hat{k}$$

$$\left|\overrightarrow{AB} \times \overrightarrow{AC}\right| = \sqrt{(-6)^2 + (-3)^2 + (4)^2}$$

$$= \sqrt{36 + 9 + 16}$$

$$= \sqrt{61}$$

Area of triangle $= \frac{1}{2}\sqrt{61}$ Sq. unit

Vector or Cross Product Ex 25.1 Q30

Let $\vec{d} = d_1\hat{i} + d_2\hat{j} + d_3\hat{k}$.

Since $\vec{d}$ is perpendicular to both $\vec{a}$ and $\vec{b}$, we have:

$$\vec{d}\cdot\vec{a} = 0$$
$$\Rightarrow d_1 + 4d_2 + 2d_3 = 0 \qquad ...(i)$$

And,

$$\vec{d}\cdot\vec{b} = 0$$
$$\Rightarrow 3d_1 - 2d_2 + 7d_3 = 0 \qquad ...(ii)$$

Also, it is given that:

$$\vec{c}\cdot\vec{d} = 15$$
$$\Rightarrow 2d_1 - d_2 + 4d_3 = 15 \qquad ...(iii)$$

On solving (i), (ii), and (iii), we get:

$$d_1 = \frac{160}{3}, d_2 = -\frac{5}{3} \text{ and } d_3 = -\frac{70}{3}$$
$$\therefore \vec{d} = \frac{160}{3}\hat{i} - \frac{5}{3}\hat{j} - \frac{70}{3}\hat{k} = \frac{1}{3}\left(160\hat{i} - 5\hat{j} - 70\hat{k}\right)$$

Hence, the required vector is $\frac{1}{3}\left(160\hat{i} - 5\hat{j} - 70\hat{k}\right)$.

Vector or Cross Product Ex 25.1 Q31

$$\vec{a}\cdot\vec{b} = 0$$

Then,

(i) Either $|\vec{a}| = 0$ or $|\vec{b}| = 0$, or $\vec{a}\perp\vec{b}$ $\left(\text{in case } \vec{a} \text{ and } \vec{b} \text{ are non-zero}\right)$

$$\vec{a}\times\vec{b} = 0$$

(ii) Either $|\vec{a}| = 0$ or $|\vec{b}| = 0$, or $\vec{a}\parallel\vec{b}$ $\left(\text{in case } \vec{a} \text{ and } \vec{b} \text{ are non-zero}\right)$

But, $\vec{a}$ and $\vec{b}$ cannot be perpendicular and parallel simultaneously.

Hence, $|\vec{a}| = 0$ or $|\vec{b}| = 0$.

Vector or Cross Product Ex 25.1 Q32

Take any parallel non-zero vectors so that $\vec{a}\times\vec{b} = \vec{0}$.

Let $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{b} = 4\hat{i} + 6\hat{j} + 8\hat{k}$.

Then,

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 4 & 6 & 8 \end{vmatrix} = \hat{i}(24-24) - \hat{j}(16-16) + \hat{k}(12-12) = 0\hat{i} + 0\hat{j} + 0\hat{k} = \vec{0}$$

It can now be observed that:

$$|\vec{a}| = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{29}$$
$$\therefore \vec{a} \neq \vec{0}$$
$$|\vec{b}| = \sqrt{4^2 + 6^2 + 8^2} = \sqrt{116}$$
$$\therefore \vec{b} \neq \vec{0}$$

Hence, the converse of the given statement need not be true.

Vector or Cross Product Ex 25.1 Q33

We have,

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k},\ \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k},\ \vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$$

$$\left(\vec{b} + \vec{c}\right) = (b_1 + c_1)\hat{i} + (b_2 + c_2)\hat{j} + (b_3 + c_3)\hat{k}$$

$$\text{Now, } \vec{a} \times \left(\vec{b} + \vec{c}\right) \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 + c_1 & b_2 + c_2 & b_3 + c_3 \end{vmatrix}$$

$$= \hat{i}\left[a_2(b_3 + c_3) - a_3(b_2 + c_2)\right] - \hat{j}\left[a_1(b_3 + c_3) - a_3(b_1 + c_1)\right] + \hat{k}\left[a_1(b_2 + c_2) - a_2(b_1 + c_1)\right]$$

$$= \hat{i}\left[a_2b_3 + a_2c_3 - a_3b_2 - a_3c_2\right] + \hat{j}\left[-a_1b_3 - a_1c_3 + a_3b_1 + a_3c_1\right] + \hat{k}\left[a_1b_2 + a_1c_2 - a_2b_1 - a_2c_1\right] \quad \ldots(1)$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$= \hat{i}\left[a_2b_3 - a_3b_2\right] + \hat{j}\left[b_1a_3 - a_1b_3\right] + \hat{k}\left[a_1b_2 - a_2b_1\right] \quad (2)$$

$$\vec{a} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= \hat{i}\left[a_2c_3 - a_3c_2\right] + \hat{j}\left[a_3c_1 - a_1c_3\right] + \hat{k}\left[a_1c_2 - a_2c_1\right] \quad (3)$$

Vector or Cross Product Ex 25.1 Q34

Given that

$$A = (1,1,2)$$
$$B = (2,3,5)$$
$$C = (1,5,5)$$

Position vector of $A = \hat{i} + \hat{j} + 2\hat{k}$

Position vector of $B = 2\hat{i} + 3\hat{j} + 5\hat{k}$

Position vector of $C = \hat{i} + 5\hat{j} + 5\hat{k}$

$\overrightarrow{AB}$ = Position vector of B − Position vector of A

$$= 2\hat{i} + 3\hat{j} + 5\hat{k} - \left(\hat{i} + \hat{j} + 2\hat{k}\right)$$
$$= \hat{i} + 2\hat{j} + 3\hat{k}$$

$\overrightarrow{AC}$ = Position vector of C − Position vector of A

$$= \hat{i} + 5\hat{j} + 5\hat{k} - \left(\hat{i} + \hat{j} + 2\hat{k}\right)$$
$$= 4\hat{j} + 3\hat{k}$$

Area of triangle $= \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 0 & 4 & 3 \end{vmatrix}$$

$$= \hat{i}(6 - 12) - \hat{j}(3 - 0) + \hat{k}(4 - 0)$$
$$= -6\hat{i} - 3\hat{j} + 4\hat{k}$$

$$\left|\overrightarrow{AB} \times \overrightarrow{AC}\right| = \sqrt{(-6)^2 + (-3)^2 + 4^2}$$
$$= \sqrt{36 + 9 + 16}$$
$$= \sqrt{61}$$

Area of the triangle $= \frac{1}{2}\sqrt{61}$ Sq.unit

Vector or Cross Product Ex 25.1 Q35

Let

$$\vec{a} = 2\hat{i} - 4\hat{j} + 5\hat{k}$$
$$\vec{b} = \hat{i} - 2\hat{j} - 3\hat{k}$$

The unit vector parallel to one of its diagonals is $\dfrac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$.

Now

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -4 & 5 \\ 1 & -2 & -3 \end{vmatrix}$$
$$= \hat{i}(12+10) - \hat{j}(-6-5) + \hat{k}(-4+4)$$
$$= 22\hat{i} + 11\hat{j}$$
$$= 11(2\hat{i} + \hat{j})$$

$$|\vec{a}\times\vec{b}| = 11\sqrt{2^2+1^2}$$
$$= 11\sqrt{5}$$

Therefore

$$\frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|} = \frac{11(2\hat{i}+\hat{j})}{11\sqrt{5}}$$
$$= \frac{1}{\sqrt{5}}(2\hat{i}+\hat{j})$$

The unit vector parallel to one of its diagonals is $\dfrac{1}{\sqrt{5}}(2\hat{i}+\hat{j})$.

Again, the area of the parallelogram is $|\vec{a}\times\vec{b}| = 11\sqrt{5}$ Sq. unit