

EX - 32.1

Q1(i)

First arrange the given numbers in ascending order

write these numbers in ascending order

3011, 2780, 3020, 2354, 3541, 4150, 5000

we get 2354, 2780, 3011, 3020, 3541, 4150, 5000

Clearly, the middle number is median, 3020

Calculation of Mean Deviations

x_i	$ d_i = x_i - 3020 $
3011	9
2780	240
3020	0
2354	666
3541	521
4150	1130
5000	1980
Total	$d_i = \sum x_i - 3020 = 4546$

$$M.D = \frac{\sum d_i}{n} = \frac{4546}{7} = 649.428$$

Q1(ii)

Clearly, the middle observations are 46 and 48. So, median=47

34
38
42
44
46
48
54
55
63
70

We have,

$$\sum |x_i - 47| = \sum d_i = 86$$

$$\therefore M.D = \frac{1}{n} \sum |d_i| = \frac{1}{10} [86] = 8.6$$

Q1(iii)

30
34
38
40
42
44
50
51
60
66

Q1(iv)

22
24
25
27
28
29
30
31
41
42

Q1(v)

Arranging the observations in ascending order of magnitude, we have

34
38
42
44
47
48
53
55
63
70

Q2(i)

$$\text{Mean} = \frac{1}{n} \sum |x_i| = \frac{80}{8} = 10$$

Calculation of Mean Deviation

X-values	Deviation From Mean
4	6
7	3
8	2
9	1
10	0
12	2
13	3
17	7
Total	24

Q2(ii)

$$\text{Mean} = \frac{1}{n} \sum |x_i| = \frac{168}{12} = 14$$

Calculation of Mean Deviation

X-values	Deviation From Mean
13	1
17	3
16	2
14	0
11	3
13	1
10	4
16	2
11	3
18	4
12	2
17	3
Total	28

Q2(iii)

$$\text{Mean} = \frac{1}{n} \sum |x_i| = \frac{500}{10} = 50$$

Calculation of Mean Deviation

X-values	Deviation From Mean
38	12
70	20
48	2
40	10
42	8
55	5
63	13
46	4
54	4
44	6
Total	84

Q2(iv)

Calculation of Mean Deviation

X-values	Deviation From Mean
38	12
70	20
48	2
40	10
42	8
55	5
63	13
46	4
54	4
44	6
Total	84

Q2(v)

First arrange the given numbers in ascending order

write these numbers in ascending order

57, 64, 43, 67, 49, 59, 44, 47, 61, 59

we get 43, 44, 47, 49, 57, 59, 59, 61, 64, 67

Let \bar{X} be the mean of given data, we get

$$\bar{X} = \frac{43+44+47+49+57+59+59+61+64+67}{10} = 55$$

Calculation of Mean Deviations from mean

x_i	$ d_i = x_i - 55 $
43	12
44	11
47	8
49	6
57	2
59	4
59	4
61	6
64	9
67	12
Total	74

$$M.D = \frac{\sum d_i}{n} = \frac{74}{10} = 7.4$$

Q3

Arrange the given data for income group I in ascending order, middle observation is 4400.

So, median = 4400.

Mean deviation for group I

x_i	$ d_i = x_i - 4400 $
4000	400
4200	200
4400	0
4600	200
4800	400
Total	$\sum d_i = 1000$

$$M.D. = \frac{1}{n} \sum |d_i| = \frac{1000}{5} = 200$$

Q4

First arrange the given numbers in ascending order

write these numbers in ascending order

40.0, 52.3, 55.2, 72.9, 52.8, 79.0, 32.5, 15.2, 27.9, 30.2

we get 15.2, 27.9, 30.2, 32.5, 40.0, 52.3, 52.8, 55.2, 72.9, 79.0

$$\text{Clearly, Median} = \frac{40.0+52.3}{2} = 46.15$$

Let \bar{X} be the mean of given data, we get

$$\bar{X} = \frac{15.2+27.9+30.2+32.5+40.0+52.3+52.8+55.2+72.9+79.0}{10} = 45.8$$

Q5(i)

$$\text{Mean} = \frac{1}{n} \sum x_i = \frac{455}{10} = 45.5$$

X-values	Deviation From Mean
34	11.5
66	20.5
30	15.5
38	7.5
44	1.5
50	4.5
40	5.5
60	14.5
42	3.5
51	5.5
Total	90

Q5(ii)

$$\text{Mean} = \frac{1}{n} \sum x_i = \frac{299}{10} = 29.9$$

X-values	Deviation From Mean
22	7.9
24	5.9
30	0.1
27	2.9
29	0.9
31	1.1
25	4.9
28	1.9
41	11.1
42	12.1
Total	48.8

Q5(iii)

$$\text{Mean} = \frac{1}{n} \sum x_i = \frac{494}{10} = 49.4$$

38	11.4
70	20.6
48	1.4
34	15.4
63	13.6
42	7.4
55	5.6
44	5.4
53	3.6
47	2.4
	86.8

Q6

$$\sigma = \sqrt{\frac{1}{n} \sum (x_i - \bar{x})^2}$$

$$\sigma' = \sqrt{\frac{1}{n} \sum x_i^2 - \bar{x}^2}$$

$$(x_i - \bar{x})^2 = x_i^2 + \bar{x}^2 - 2x_i \bar{x}$$

$$\sum 2x_i \bar{x} = 2\bar{x} \sum x_i = 2n\bar{x}^2$$

$$\frac{1}{n} \sum (x_i - \bar{x})^2 = \frac{\sum (x_i^2 + \bar{x}^2 - 2x_i \bar{x})}{n}$$

$$= \frac{\sum x_i^2 + \sum \bar{x}^2 - \sum 2x_i \bar{x}}{n}$$

$$= \frac{1}{n} \sum x_i^2 + \frac{n\bar{x}^2 - 2n\bar{x}^2}{n}$$

$$= \frac{1}{n} \sum x_i^2 - \bar{x}^2$$

EX - 32.2

Q1

x_i	f_i	Cum. Freq	$ d_i = x_i - 61 $	$f_i d_i $
58	15	15	3	45
59	20	35	2	40
60	32	67	1	32
61	35	102	0	0
62	35	137	1	35
63	22	159	2	44
64	20	179	3	60
65	10	189	4	40
66	8	197	5	40
$N = 197$				$Total = 336$

$$N = 197, \frac{N}{2} = 98.5$$

Corresponding value for median is 61

$$\text{Mean Deviation} = \frac{336}{197} = 1.705$$

Q2

We have to calculate mean deviation from the median. So, first we calculate the median.

x	f	cf	d = (x-med)	fd
0	14	14	4	56
1	21	35	3	63
2	25	60	2	50
3	43	103	1	43
4	51	154	0	0
5	40	194	1	40
6	39	233	2	78
7	12	245	3	36
	245			366

Q3

x_i	f_i	Cum. fre	$ d_i = x_i - 13 $	$f_i d_i $
5	2	2	8	16
7	4	6	6	24
9	6	12	4	24
11	8	20	2	16
13	10	30	0	0
15	12	42	2	24
17	8	50	4	32
$N = 50$				$Total = 136$

$$\frac{N}{2} = 25$$

Value corresponding to 25 is Median=13

$$M.D = \frac{136}{50} = 2.72$$

Q4(i)

x_i	f_i	$f_i x_i$	$ d_i = x_i - 9 $	$f_i d_i $
5	8	40	4	32
7	6	42	2	12
9	2	18	0	0
10	2	20	1	2
12	2	24	3	6
15	6	90	6	36
26		$Total = 234$		$Total = 88$

$$\frac{\sum f_i x_i}{26} = 9$$

Mean=9

$$M.D = \frac{88}{26} \approx 3.39$$

Q4(ii)

x	f	xf	d=(x-mean)	fd
5	7	35	9	63
10	4	40	4	16
15	6	90	1	6
20	3	60	6	18
25	5	125	11	55
	25	350		158

Q4(iii)

x	f	xf	d=(x-mean)	fd
10	4	40	40	160
30	24	720	20	480
50	28	1400	0	0
70	16	1120	20	320
90	8	720	40	320
	80	4000		1280

Q4(iv)

x_i	f_i	$f_i x_i$	$ d_i = x_i - 21.65 $	$f_i d_i $
20	6	120	1.65	9.9
21	4	84	0.65	2.6
22	5	110	0.35	1.75
23	1	23	1.35	1.35
24	4	96	2.35	9.40
	20	Total = 433		Total = 25

$$\frac{\sum f_i x_i}{20} = 21.65$$

$$\text{Mean} = 21.65$$

$$\text{M.D} = \frac{25}{20} \approx 1.25$$

Q5

x_i	f_i	<i>Cum Freq</i>	$ d_i = x_i - 30 $	$f_i d_i $
15	3	3	15	45
21	5	8	9	45
27	6	14	3	18
30	7	21	0	0
35	8	29	5	40
	29			<i>Total</i> = 148

$$\frac{N}{2} = 14.5$$

$$\text{Median} = 30$$

$$\text{M.D} = \frac{148}{29} \approx 5.10$$

Q1

We have to calculate mean deviation from the median. So, first we calculate the median.

CI	x	f	cf	d = (x-med)	fd
0-10	5	5	5	20	100
10-20	15	10	15	10	100
20-30	25	20	35	0	0
30-40	35	5	91	10	50
40-50	45	10	101	20	200
		50			450

$$M.D = \frac{1}{n} \sum f_i |d_i| = \frac{1}{50} [450] = 9$$

Q2(i)

CI	x	f	xf	d=(x-mean)	fd
0-100	50	4	200	308	1232
100-200	150	8	1200	208	1664
200-300	250	9	2250	108	972
300-400	350	10	3500	8	80
400-500	450	7	3150	92	644
500-600	550	5	2750	192	960
600-700	650	4	2600	292	1168
700-800	750	3	2250	392	1176
		50	17900		7896

$$\text{Mean} = \frac{1}{n} \sum f_i x_i = \frac{17900}{50} = 358$$

$$\therefore M.D = \frac{1}{n} \sum f_i |d_i| = \frac{1}{50} [7896] = 157.92$$

Q2(ii)

<i>Classes</i>	f_i	x_i	d_i	$f_i d_i$	$ x_i - \bar{X} $	$f_i x_i - \bar{X} $
95–105	9	100	-3	-27	28.58	257.22
105–115	13	110	-2	-26	18.58	241.54
115–125	16	120	-1	-16	8.58	137.28
125–135	26	130	0	0	1.42	36.92
135–145	30	140	1	30	11.42	342.6
145–155	12	150	2	24	21.42	257.04
$N = 106$		$Total = -15$		$Total = 1272.60$		

$$N = 106$$

$$a = 130$$

$$h = 10$$

$$\bar{X} = a + h \left(\frac{\sum f_i d_i}{N} \right) = 128.58$$

$$M.D = \frac{\sum f_i |x_i - \bar{X}|}{N} = \frac{1272.60}{106} = 12.005$$

Q2(iii)

CI	x	f	xf	d=(x-mean)	fd
0-10	5	6	30	22	132
10-20	15	8	120	12	96
20-30	25	14	350	2	28
30-40	35	16	560	8	128
40-50	45	4	180	18	72
50-60	55	2	110	28	56
		50	1350		512
	Mean		27		
	Mean Deviation		10.24		

$$\text{Mean} = \frac{1}{n} \sum f_i x_i = \frac{1350}{50} = 27$$

$$\therefore \text{M.D} = \frac{1}{n} \sum f_i |d_i| = \frac{1}{50} [512] = 10.24$$

Q3

Find the mean deviation from the mean for the data:

Classes	0-10	10-20	20-30	30-40	40-50	50-60
Frequencies	6	8	14	16	4	2

$$\text{Mean} = \frac{1}{n} \sum f_i x_i = \frac{5390}{110} = 49$$

$$\therefore \text{M.D} = \frac{1}{n} \sum f_i |d_i| = \frac{1}{110} [1644] = 14.95$$

Q4

We have to calculate mean deviation from the median. So, first we calculate the median.

CI	x	f	cf	d = (x-med)	fd
17-19.5	18.25	5	5	20	100
20-25.5	22.75	16	21	15.5	248
26-35.5	30.75	12	33	7.5	90
36-40.5	38.25	26	59	0	0
41-50.5	45.75	14	73	7.5	105
51-55.5	53.25	12	85	15	180
56-60.5	58.25	6	91	20	120
61-70.5	65.75	5	96	27.5	137.5
		96			980.5

We have $N = 96 \Rightarrow N/2 = 48$

The cumulative frequency just greater than $N/2$ is 59 and the corresponding value of x is 38.25.

Hence, median = 38.25

$$\therefore \text{M.D} = \frac{1}{n} \sum f_i |d_i| = \frac{1}{96} [980.5] = 10.21$$

Q5

M.D from Median

Marks	Students	x_i	Cum. Freq	$ d_i = \left x_i - \frac{70}{3} \right $	$f_i d_i$
0-10	5	5	5	$\frac{55}{3}$	$\frac{275}{3}$
10-20	8	15	13	$\frac{25}{3}$	$\frac{200}{3}$
20-30	15	25	28	$\frac{5}{3}$	$\frac{75}{3}$
30-40	16	35	44	$\frac{35}{3}$	$\frac{560}{3}$
40-50	6	45	50	$\frac{65}{3}$	$\frac{390}{3}$

$N = 50$

$Total = 500$

$$\text{Median} = l + \frac{\frac{N}{2} - F}{f} \times h = 20 + \frac{30 - 25}{15} \times 10 = 20 + \frac{10}{3} = \frac{70}{3}$$

Q6

Converting the given data into continuous frequency distribution by subtracting 0.5 from the lower limit and adding 0.5 to the upper limit of each class interval.

Age	x_i	f_i	Cumulative frequency	$ d_i = x_i - 38 $	$f_i d_i $
15.5 - 20.5	18	5	5	20	100
20.5 - 25.5	23	6	11	15	90
25.5 - 30.5	28	12	23	10	120
30.5 - 35.5	33	14	37	5	70
35.5 - 40.5	38	26	63	0	0
40.5 - 45.5	43	12	75	5	60
45.5 - 50.5	48	16	91	10	160
50.5 - 55.5	53	9	100	15	135
		$N = \sum f_i = 100$			$\sum f_i d_i = 735$

Clearly, $N = 100 \Rightarrow \frac{N}{2} = 50$.

Cumulative frequency is just greater than $\frac{N}{2}$ is 63 and the corresponding class is 35.5 - 40.5.

$l = 35.5$, $f = 26$, $h = 5$, $F = 37$

Therefore, median = $l + \frac{\frac{N}{2} - F}{f} \times h = 35.5 + \frac{50 - 37}{26} \times 5 = 38$

$$M.D. = \frac{1}{N} \sum f_i |d_i| = \frac{735}{100} = 7.35$$

Q7

Classes	f_i	x_i	$f_i x_i$	$ x_i - 9.2 $	$f_i x_i - 9.2 $
0-4	4	2	8	7.2	28.8
4-8	6	6	36	3.2	19.2
8-12	8	10	80	0.8	6.4
12-16	5	14	70	4.8	24.0
16-20	2	18	36	8.8	17.6
	$N = 25$		$Total = 230$		$Total = 96.0$

$$Mean = \frac{230}{25} = 9.2$$

$$M.D. = \frac{96}{25} = 3.84$$

Q8

<i>Classes</i>	f_i	x_i	$f_i x_i$	$ x_i - 14.1 $	$f_i x_i - 14.1 $
0-6	4	3	12	11.1	44.4
6-12	5	9	45	5.1	25.5
12-18	3	15	45	0.9	2.7
18-24	6	21	126	6.9	41.4
24-30	2	27	54	12.9	25.8
$N = 20$		$Total = 282$		$Total = 139.8$	

$$Mean = \frac{282}{20} = 14.1$$

$$M.D = \frac{139.8}{20} = 6.99$$

EX - 32.4

Q1(i)

x	d=(x- Mean)	d ²
2	-5	25
4	-3	9
5	-2	4
6	-1	1
8	1	1
17	10	100
42		140

$$\bar{x} = \frac{1}{n} \sum x_i = \frac{1}{6} [42] = 7$$

$$\text{var}(x) = \frac{1}{n} \left\{ \sum (x_i - \bar{x})^2 \right\} = \frac{1}{6} \{140\} = 23.33$$

$$S.D(x) = \sqrt{\text{var}(x)} = \sqrt{23.33} = 4.8$$

Q1(ii)

x	d=(x- Mean)	d ²
6	-3	9
7	-2	4
10	1	1
12	3	9
13	4	16
4	-5	25
8	-1	1
12	3	9
72		74

$$\bar{x} = \frac{1}{n} \sum x_i = \frac{1}{8} [72] = 9$$

$$\text{var}(x) = \frac{1}{n} \left\{ \sum (x_i - \bar{x})^2 \right\} = \frac{1}{8} \{74\} = 9.25$$

$$S.D(x) = \sqrt{\text{var}(x)} = \sqrt{9.25} = 3.04$$

Q1(iii)

x_i	$d_i = x_i - 299$	d_i^2
227	-72	5184
235	-64	4096
255	-44	1936
269	-30	900
292	-7	49
299	0	0
312	13	169
321	22	484
333	34	1156
348	49	2401
Total = -99		Total = 16375

$$\bar{X} = 299 + \frac{-99}{10} = 289.1$$

$$Var = \frac{16375}{10} - \left(\frac{-99}{10}\right)^2 = 1637.5 - 98.01 = 1539.49$$

$$SD = \sqrt{1539.49} = 39.24$$

Q1(iv)

x_i	$d_i = x_i - 15$	d_i^2
15	0	0
22	7	49
27	12	144
11	-4	16
9	-6	36
21	6	36
14	-1	1
9	-6	36
Total = 8		Total = 318

$$Mean = 15 + \frac{8}{8} = 16$$

$$Var = \frac{318}{8} - 1 = 38.75$$

$$SD = \sqrt{38.75} = 6.22$$

Q2

We have, $n = 20$, and $\sigma^2 = 5$

Now each observation is multiplied by 2.

Suppose $X = 2x$ be the new data.

$$\therefore \bar{X} = \frac{1}{20} \sum 2x_i = \frac{1}{20} \times 2 \sum x_i = 2\bar{x}$$

$$\Rightarrow \sum X_i^2 = 4 \sum x_i^2$$

Since, $\sigma^2 = 5$

$$\Rightarrow \frac{1}{n} \sum x_i^2 - (\bar{x})^2 = 5$$

Now, for the new data:

$$\sigma^2 = \frac{1}{n} \sum X_i^2 - (\bar{X})^2 = 4 \sum x_i^2 - (2\bar{x})^2 = 4 \left(\sum x_i^2 - (\bar{x})^2 \right) = 4 \times 5 = 20$$

Q3

We have, $n = 15$, and $\sigma^2 = 4$

Now each observation is increased by 9.

Suppose $X = x + 9$ be the new data.

$$\therefore \bar{X} = \frac{1}{15} \sum (x_i + 9) = \left(\frac{1}{15} \times \sum x_i \right) + 9 = \bar{x} + 9$$

$$\Rightarrow \sum x_i^2 = \sum (x_i + 9)^2 = \sum x_i^2 + \sum 18x_i + \sum 9^2$$

Since, $\sigma^2 = 5$

$$\Rightarrow \frac{1}{n} \sum x_i^2 - (\bar{x})^2 = 4$$

Now, for the new data:

$$\begin{aligned} \sigma^2 &= \frac{1}{n} \sum x_i^2 - (\bar{x})^2 = \frac{1}{15} \left(\sum x_i^2 + \sum 18x_i + \sum 9^2 \right) - (\bar{x} + 9)^2 = \\ &= \frac{1}{15} \sum x_i^2 + \frac{1}{15} \sum 18x_i + \frac{1}{15} \sum 9^2 - (9)^2 - (18\bar{x}) - (\bar{x})^2 \\ &= \left[\frac{1}{15} \sum x_i^2 - (\bar{x})^2 \right] + \left[\frac{1}{15} \sum 18x_i - (18\bar{x}) \right] + \left[\frac{1}{15} \sum 9^2 - (9)^2 \right] \\ &= \left[\frac{1}{15} \sum x_i^2 - (\bar{x})^2 \right] + \left[18 \times \frac{1}{15} \sum x_i - (18\bar{x}) \right] + \left[\frac{1}{15} \times 15 \times (9)^2 - (9)^2 \right] \\ &= \frac{1}{15} \sum x_i^2 - (\bar{x})^2 \\ &= 4 \end{aligned}$$

Q4

Let the other two be x and y

$$1 + 2 + 6 + x + y = 5 \times 4.4 \text{ because of the mean}$$
$$x + y = 13$$

$$\text{Variance} = [(1 - 4.4)^2 + (2 - 4.4)^2 + (6 - 4.4)^2 + (x - 4.4)^2 + (y - 4.4)^2]/5$$

Hence

$$11.56 + 5.76 + 2.56 + (x - 4.4)^2 + (y - 4.4)^2 = 41.2$$
$$(x - 4.4)^2 + (y - 4.4)^2 = 21.32$$

Solve simultaneously

$$(x - 4.4)^2 + (13 - x - 4.4)^2 = 21.32$$

$$(x - 4.4)^2 + (8.6 - x)^2 = 21.32$$

$$x^2 - 8.8x + 19.36 + 73.96 - 17.2x + x^2 = 21.32$$

$$2x^2 - 26x + 72 = 0$$

$$x^2 - 13x + 36 = 0$$

$$(x - 4)(x - 9) = 0$$

$$x = 4 \text{ or } x = 9$$

$$\text{If } x = 4, y = 9 \text{ and}$$

The other two observations are 4 and 9.

Q5

If mean and SD of observations are \bar{X} and σ respectively,

then mean and SD of observations multiplied by a constant 'k' are

$$\text{Mean} = k\bar{X}$$

$$\text{SD} = |k|\sigma$$

In this question, it is given that $k=3$

$$\text{So New mean} = 8 \times 3 = 24$$

$$\text{New SD} = 4 \times 3 = 12$$

Q6

Let x and y be the remaining two observations. Then,

$$\text{Mean} = 9$$

$$\Rightarrow \frac{6+7+10+12+12+13+x+y}{8} = 9$$

$$\Rightarrow 60+x+y = 72$$

$$\Rightarrow x+y = 12 \quad \text{----- (i)}$$

$$\text{Variance} = 9.25$$

$$\Rightarrow \frac{1}{8} \{6^2+7^2+10^2+12^2+12^2+13^2+x^2+y^2\} - (\text{Mean})^2 = 9.25$$

$$\Rightarrow \frac{1}{8} \{36+49+100+144+144+169+x^2+y^2\} - 81 = 9.25$$

$$\Rightarrow 642+x^2+y^2 = 722$$

$$\Rightarrow x^2+y^2 = 80 \quad \text{----- (ii)}$$

$$\text{Now, } (x+y)^2 + (x-y)^2 = 2(x^2+y^2)$$

$$\Rightarrow 144 + (x-y)^2 = 2 \times 80$$

$$\Rightarrow (x-y)^2 = 16$$

$$\Rightarrow x-y = \pm 4$$

$$\text{if } x-y = 4, \text{ then } x+y = 12 \text{ and } x-y = 4 \Rightarrow x = 8, y = 4$$

$$\text{if } x-y = -4, \text{ then } x+y = 12 \text{ and } x-y = -4 \Rightarrow x = 4, y = 8$$

Hence, the remaining two observations are 4 and 8.

Q7

We have,

$$n = 200, \bar{X} = 40, \sigma = 15.$$

$$\therefore \bar{X} = \frac{1}{n} \sum x_i = \bar{X} = 200 \times 40 = 8000.$$

$$\begin{aligned} \text{Corrected } \sum x_i &= \text{Incorrect } \sum x_i - (\text{sum of incorrect values}) + (\text{sum of correct values}) \\ &= 8000 - 34 - 53 + 43 + 35 = 7991 \end{aligned}$$

$$\therefore \text{Corrected mean} = \frac{\text{corrected } \sum x_i}{n} = \frac{7991}{200} = 39.955$$

Now, $\sigma = 15$

$$\Rightarrow 15^2 = \frac{1}{200} (\sum x_i^2) - \left(\frac{1}{200} \sum x_i \right)^2$$

$$\Rightarrow 255 = \frac{1}{200} (\sum x_i^2) - \left(\frac{8000}{200} \right)^2$$

$$\Rightarrow 255 = \frac{1}{200} (\sum x_i^2) - 1600$$

$$\Rightarrow \sum x_i^2 = 200 \times 1825 = 365000$$

$$\Rightarrow \text{Incorrect } \sum x_i^2 = 365000.$$

$$\begin{aligned} \text{corrected } \sum x_i^2 &= (\text{incorrect } \sum x_i^2) - (\text{sum of squares of incorrect values}) \\ &\quad + (\text{sum of squares of correct values}) \end{aligned}$$

$$= 365000 - (34)^2 - 53^2 + (43)^2 + 35^2 = 364109$$

891

$$\begin{aligned} \text{so, Corrected } \sigma &= \sqrt{\frac{1}{n} \sum x_i^2 - \left(\frac{1}{n} \sum x_i \right)^2} = \sqrt{\frac{364109}{200} - \left(\frac{7991}{200} \right)^2} \\ &= \sqrt{1820.545 - 1596.402} = 14.97 \end{aligned}$$

Q8

We have,

$$n = 100, \bar{X} = 40, \sigma = 5.1$$

$$\therefore \bar{X} = \frac{1}{n} \sum x_i = \bar{X} = 100 \times 40 = 4000.$$

$$\begin{aligned} \text{Corrected } \sum x_i &= \text{Incorrect } \sum x_i - (\text{sum of incorrect values}) + (\text{sum of correct values}) \\ &= 4000 - 50 + 40 = 3990 \end{aligned}$$

$$\therefore \text{Corrected mean} = \frac{\text{corrected } \sum x_i}{n} = \frac{3990}{100} = 39.9$$

Now, $\sigma = 5.1$

$$\Rightarrow 5.1^2 = \frac{1}{100} \left(\sum x_i^2 \right) - \left(\frac{1}{100} \sum x_i \right)^2$$

$$\Rightarrow 26.01 = \frac{1}{100} \left(\sum x_i^2 \right) - \left(\frac{4000}{100} \right)^2$$

$$\Rightarrow 26.01 = \frac{1}{100} \left(\sum x_i^2 \right) - 1600$$

$$\Rightarrow \sum x_i^2 = 100 \times 1626.01 = 162601$$

$$\Rightarrow \text{Incorrect } \sum x_i^2 = 162601.$$

$$\begin{aligned} \text{corrected } \sum x_i^2 &= \left(\text{incorrect } \sum x_i^2 \right) - (\text{sum of squares of incorrect values}) \\ &\quad + (\text{sum of squares of correct values}) \end{aligned}$$

$$= 162601 - (50)^2 + (40)^2 = 161701$$

$$\begin{aligned} \text{so, Corrected } \sigma &= \sqrt{\frac{1}{n} \sum x_i^2 - \left(\frac{1}{n} \sum x_i \right)^2} = \sqrt{\frac{161701}{100} - \left(\frac{3990}{100} \right)^2} \\ &= \sqrt{1617.01 - 1592.01} = 5 \end{aligned}$$

Q9

We have, $n = 20$, $\bar{x} = 10$ and $\sigma = 2$

$$\therefore \bar{x} = \frac{1}{n} \sum x_i$$

$$\Rightarrow \sum x_i = n\bar{x} = 20 \times 10 = 200$$

$$\Rightarrow \text{Incorrected } \sum x_i = 200$$

and,

$$\sigma = 2$$

$$\Rightarrow \sigma^2 = 4$$

$$\Rightarrow \frac{1}{n} \sum x_i^2 - (\text{Mean})^2 = 4$$

$$\Rightarrow \frac{1}{20} \sum x_i^2 - 100 = 4$$

$$\Rightarrow \sum x_i^2 = 104 \times 20$$

$$\Rightarrow \text{Incorrected } \sum x_i^2 = 2080.$$

Q10

We have, $n = 100$, $\bar{x} = 20$ and $\sigma = 3$

$$\text{Since } \bar{x} = \frac{1}{n} \sum x_i$$

$$\Rightarrow \sum x_i = n\bar{x} = 20 \times 100 = 2000$$

$$\Rightarrow \text{Incorrect } \sum x_i = 2000$$

and,

$$\sigma = 3$$

$$\Rightarrow \sigma^2 = 9$$

$$\Rightarrow \frac{1}{n} \sum x_i^2 - (\text{Mean})^2 = 9$$

$$\Rightarrow \frac{1}{100} \sum x_i^2 - 400 = 9$$

$$\Rightarrow \sum x_i^2 = 409 \times 100$$

$$\Rightarrow \text{Incorrect } \sum x_i^2 = 40900.$$

Q11

$$\begin{aligned}\text{We have } \sum (x_i - \bar{X})^2 &= \sum (x_i^2 - 2x_i\bar{X} + \bar{X}^2) \\&= \sum (x_i^2) + \sum (-2x_i\bar{X}) + \sum (\bar{X})^2 \\&= \sum (x_i^2) - 2\bar{X}\sum (x_i) + (\bar{X})^2 \sum 1 \\&= \sum (x_i^2) - 2\bar{X}(n\bar{X}) + n(\bar{X})^2 \\&= \sum (x_i^2) - n(\bar{X})^2\end{aligned}$$

Dividing both the sides by n we get,

$$\frac{1}{n} \sum (x_i - \bar{X})^2 = \frac{1}{n} \sum (x_i^2) - n(\bar{X})^2$$

Taking square root on both the sides

$$\sqrt{\frac{1}{n} \sum (x_i - \bar{X})^2} = \sqrt{\frac{1}{n} \sum (x_i^2) - n(\bar{X})^2}$$

Q1

x	f	fx	x-mean	(x-mean) ²	f(x-mean) ²
4.5	1	4.5	-33.14	1098.45	1098.45
14.5	5	72.5	-23.14	535.59	2677.96
24.5	12	294	-13.14	172.73	2072.82
34.5	22	759	-3.14	9.88	217.31
44.5	17	756.5	6.86	47.02	799.35
54.5	9	490.5	16.86	284.16	2557.47
64.5	4	258	26.86	721.31	2885.22
	N=70	2635			12308.57

Here, $N = 70$, $\sum f_i x_i = 2635$

$$\therefore \bar{x} = \frac{1}{N} (\sum f_i x_i) = \frac{2635}{70} = 37.64$$

We have, $\sum f_i (x_i - \bar{x})^2 = 12308.57$

$$\therefore \text{var}(x) = \frac{1}{N} \left[\sum f_i (x_i - \bar{x})^2 \right] = \frac{12308.57}{70} = 175.84$$

$$S.D. = \sqrt{\text{var}(x)} = \sqrt{175.84} = 13.26$$

Q2

X	F	Fx	x-mean	F(x-mean)	$(x-mean)^2$	$F(x-mean)^2$
0	51	0	-6	-306	36	1836
1	203	203	-5	-1015	25	5075
2	383	766	-4	-1532	16	6128
3	525	1575	-3	-1575	9	4725
4	532	2128	-2	-1064	4	2128
5	408	2040	-1	-408	1	408
6	273	1638	0	0	0	0
7	139	973	1	139	1	139
8	43	344	2	86	4	172
9	27	243	3	81	9	243
10	10	100	4	40	16	160
11	4	44	5	20	25	100
12	2	24	6	12	36	72
	2600	10078		-5522		21186

Here, $N = 2600$, $\sum f_j x_j = 10078$

$$\therefore \bar{x} = \frac{1}{N} (\sum f_j x_j) = \frac{10078}{2600} = 3.88$$

Since ,

$$\text{var}(x) = h^2 \left(\frac{1}{N} \sum f_i (x - \text{mean})_i^2 - \left\{ \frac{1}{N} \sum f_i (x - \text{mean})_i \right\}^2 \right)$$

$$\sigma^2 = 1 \left(\frac{21186}{2600} - \left(\frac{-5522}{2600} \right)^2 \right)$$

$$\sigma^2 = 8.14846 - 4.51072$$

$$\sigma^2 = \boxed{3.64}$$

Q3

i)

x_i	Cum.Freq	f_i	$f_i x_i$	$f_i x_i^2$
10	15	15	150	1500
20	32	17	340	6800
30	51	19	570	17100
40	78	27	1080	43200
50	97	19	950	47500
60	109	12	720	43200
		$N = 109$	Total = 3810	Total = 159300

$$\text{Mean} = \frac{3810}{109} = 34.95$$

$$\text{Var} = \frac{159300}{109} - (34.95)^2 = 239.96$$

$$\text{SD} = \sqrt{239.96} = 15.49$$

Q4

(i)

x	f	fx	x-mean	(x-mean) ²	f(x-mean) ²
3	7	21	-9.79	95.88	671.13
8	10	80	-4.79	22.96	229.60
13	15	195	0.21	0.04	0.65
18	10	180	5.21	27.13	271.26
23	6	138	10.21	104.21	625.26
	48	614			1797.92

Here, $N = 48$, and $\sum f_i x_i = 614$

$$\therefore \bar{x} = \frac{1}{N} (\sum f_i x_i) = \frac{614}{48} = 12.79$$

$$\sum f_i (x_i - \bar{x})^2 = 1797.92$$

$$\therefore \text{var}(x) = \frac{1}{N} \left[\sum f_i (x_i - \bar{x})^2 \right] = \frac{1797.92}{48} = 37.46$$

$$\text{S.D.} = \sqrt{\text{var}(x)} = \sqrt{37.46} = 6.12$$

Q1

CI	f	x	u=(x-A)/h	fu	u ²	fu ²
0-10	14	5	-2	-28	4	56
10-20	13	15	-1	-13	1	13
20-30	27	25	0	0	0	0
30-40	21	35	1	21	1	21
40-50	15	45	2	30	4	60
	90			10		150

Here, $N = 90$, $A = 25$, $\sum f\mu_i = 10$, $\sum f\mu_i^2 = 150$ and $h = 10$

$$\therefore \text{Mean} = \bar{x} = A + h \left(\frac{1}{N} \sum f\mu_i \right)$$

$$\Rightarrow \bar{x} = 25 + 10 \left(\frac{10}{90} \right) = 26.11$$

$$\text{var}(x) = h^2 \left[\frac{1}{N} \sum f\mu_i^2 - \left(\frac{1}{N} \sum f\mu_i \right)^2 \right] = 100 \left[\frac{150}{90} - \left(\frac{10}{90} \right)^2 \right] = 165.4$$

$$\therefore S.D. = \sqrt{\text{var}(x)} = \sqrt{165.4} = 12.86$$

Q2

CI	f	x	u=(x-A)/h	f*u	u ²	fu ²
0-30	9	15	-3	-27	9	81
30-60	17	45	-2	-34	4	68
60-90	43	75	-1	-43	1	43
90-120	82	105	0	0	0	0
120-150	81	135	1	81	1	81
150-180	44	165	2	88	4	176
180-210	24	195	3	72	9	216
	300			137		665

Here, $N = 300$, $A = 105$, $\sum f_i u_i = 137$, $\sum f_i u_i^2 = 665$ and $h = 30$

$$\therefore \text{Mean} = \bar{x} = A + h \left(\frac{1}{N} \sum f_i u_i \right)$$

$$\Rightarrow \bar{x} = 105 + 30 \left(\frac{137}{300} \right) = 118.7$$

$$\text{var}(x) = h^2 \left[\frac{1}{N} \sum f_i u_i^2 - \left(\frac{1}{N} \sum f_i u_i \right)^2 \right] = 900 \left[\frac{665}{300} - \left(\frac{137}{300} \right)^2 \right] = 1807.31$$

$$\therefore S.D. = \sqrt{\text{var}(x)} = \sqrt{1807.31} = 42.51$$

Q3

CI	f	x	u=(x-A)/h	f*u	u ²	fu ²
0-10	18	5	-3	-54	9	162
10-20	16	15	-2	-32	4	64
20-30	15	25	-1	-15	1	15
30-40	12	35	0	0	0	0
40-50	10	45	1	10	1	10
50-60	5	55	2	10	4	20
60-70	2	65	3	6	9	18
70-80	1	75	4	4	16	16
	79			-71		305

Here, $N = 79$, $A = 35$, $\sum f\mu_i = -71$, $\sum f\mu_i^2 = 305$ and $h = 10$

$$\therefore \text{Mean} = \bar{x} = A + h \left(\frac{1}{N} \sum f\mu_i \right)$$

$$\Rightarrow \bar{x} = 35 + 10 \left(\frac{-71}{79} \right) = 26.01$$

$$\text{var}(x) = h^2 \left[\frac{1}{N} \sum f\mu_i^2 - \left(\frac{1}{N} \sum f\mu_i \right)^2 \right] = 100 \left[\frac{305}{79} - \left(\frac{-71}{79} \right)^2 \right] = 305.30$$

$$\therefore S.D. = \sqrt{\text{var}(x)} = \sqrt{305.30} = 17.47$$

Q4

We have, $n = 100$, $\bar{x} = 40$ and $\sigma = 5.1$

$$\therefore \bar{x} = \frac{1}{n} \sum x_i$$

$$\Rightarrow \sum x_i = n\bar{x} = 100 \times 40 = 4000$$

$$\therefore \text{Incorrect } \sum x_i = 4000$$

and,

$$\sigma = 5.1$$

$$\Rightarrow \sigma^2 = 26.01$$

$$\Rightarrow \frac{1}{n} \sum x_i^2 - (\text{Mean})^2 = 26.01$$

$$\Rightarrow \frac{1}{100} \sum x_i^2 - 1600 = 26.01$$

$$\Rightarrow \sum x_i^2 = 1626.01 \times 100$$

$$\therefore \text{Incorrect } \sum x_i^2 = 162601$$

Q5

CI	Freq	MidValue	u_i	$f_i u_i$	$f_i u_i^2$
31-35	2	33	-4	-8	32
36-40	3	38	-3	-9	27
41-45	8	43	-2	-16	32
46-50	12	48	-1	-12	12
51-55	16	53	0	0	0
56-60	5	58	1	5	5
61-65	2	63	2	4	8
66-70	2	68	3	6	18
$N = 50$			$Total = -30$		
			$Total = 134$		

$$Mean = 53 + 5 \times \frac{-30}{50} = 50$$

$$Var = 25 \times \left(\frac{134}{50} - \frac{9}{25} \right) = 58$$

$$SD = \sqrt{58} = 7.62$$

Q6

Converting the given data into continuous frequency distribution by subtracting 0.5 from the lower limit and adding 0.5 to the upper limit of each class interval.

Class interval	f_i	Mid-value x_i	$u_i = \frac{x_i - 5.5}{1}$	$f_i u_i$	u_i^2	$f_i u_i^2$
1 - 2	6	1.5	-4	-24	16	96
3 - 4	4	3.5	-2	-8	4	16
5 - 6	5	5.5	0	0	0	0
7 - 8	1	7.5	2	2	4	4
	$N = \sum f_i = 16$			$\sum f_i u_i = -30$		$\sum f_i u_i^2 = 116$

$$N = 16, \sum f_i u_i = -30, \sum f_i u_i^2 = 116, A = 5.5 \text{ and } h = 1$$

$$Mean = A + h \left(\frac{1}{N} \sum f_i u_i \right) = 5.5 + 1 \left(\frac{1}{16} \times (-30) \right) = 3.625$$

$$Var(X) = h^2 \left\{ \left(\frac{1}{N} \sum f_i u_i^2 \right) - \left(\frac{1}{N} \sum f_i u_i \right)^2 \right\} = 1 \left\{ \left(\frac{1}{16} \times 116 \right) - \left(\frac{1}{16} \times (-30) \right)^2 \right\} = \{ 7.25 - 3.51 \} = 3.74$$

Note: Answer given in the book is incorrect.

Q7

<i>CI</i>	x_i	f_i	u_i	$f_i u_i$	$f_i u_i^2$
200-201	200.5	13	-1.5	-19.5	29.25
201-202	201.5	27	-1	-27	27
202-203	202.5	18	-0.5	-9	4.5
203-204	203.5	10	0	0	0
204-205	204.5	1	0.5	0.5	0.25
205-206	205.5	1	1	1	1
		$N = 70$	$Total = -54 \quad Total = 62$		

$$Mean = 203.5 + 2 \left(\frac{-54}{70} \right) = 201.9$$

$$Var = 4 \left(\frac{62}{70} - \left(\frac{-54}{70} \right)^2 \right) = 0.98$$

$$SD = \sqrt{0.98} = 0.99$$

Q8

$$Mean = 40$$

$$SD = 10$$

$$n = 100$$

$$\sum x_i = 40 \times 100 = 4000$$

$$Corrected \text{ Sum} = 4000 - 30 - 70 + 3 + 27 = 3930$$

$$Corrected \text{ Mean} = \frac{3930}{100} = 39.3$$

$$Variance = 100$$

$$100 = \frac{\sum x_i^2}{100} - (40)^2$$

$$Incorrect \sum x_i^2 = 170000$$

$$Corrected \sum x_i^2 =$$

$$Incorrect \sum x_i^2 - (\text{Sum of squares of incorrect values}) + (\text{Sum of squares of corrected values})$$

$$Corrected \sum x_i^2 = 170000 - (900 + 4900) + (9 + 729)$$

$$Corrected \sum x_i^2 = 164938$$

$$Corrected \sigma = \sqrt{\frac{Corrected \sum x_i^2}{n} - (Corrected \text{ Mean})^2}$$

$$Corrected \sigma = \sqrt{\frac{164938}{100} - (39.3)^2} = 10.24$$

Q9

$$\text{Mean} = 45$$

$$\text{Variance} = 16$$

$$n = 10$$

$$\sum x_i = 450$$

$$\text{Corrected Sum} = 450 - 52 + 25 = 423$$

$$\text{Corrected Mean} = 42.3$$

$$\text{Variance} = 16$$

$$16 = \frac{\sum x_i^2}{10} - (45)^2$$

$$\text{Incorrect } \sum x_i^2 = 20410$$

$$\text{Corrected } \sum x_i^2 =$$

$$\text{Incorrect } \sum x_i^2 - (\text{Sum of squares of incorrect values}) +$$
$$(\text{Sum of squares of corrected values})$$

$$\text{Corrected } \sum x_i^2 = 20410 - 2704 + 625 = 18331$$

$$\text{Corrected } \sigma = \sqrt{\frac{\text{Corrected } \sum x_i^2}{n} - (\text{Corrected Mean})^2}$$

$$\text{Corrected } \sigma = \sqrt{\frac{18331}{10} - (42.3)^2} = 6.62$$

$$\text{Corrected Variance} = 6.62 * 6.62 = 43.82$$

Q10

Class interval	f_i	Mid-value x_i	$u_i = \frac{x_i - 35}{10}$	$f_i u_i$	u_i^2	$f_i u_i^2$
0 – 10	11	5	-3	-33	9	99
10 – 20	29	15	-2	-58	4	116
20 – 30	18	25	-1	-18	1	18
30 – 40	4	35	0	0	0	0
40 – 50	5	45	1	5	1	5
50 – 60	3	55	2	6	4	12
	$N = \sum f_i = 70$			$\sum f_i u_i = -98$		$\sum f_i u_i^2 = 250$

$$N = 70, \sum f_i u_i = -98, \sum f_i u_i^2 = 250, A = 35 \text{ and } h = 10$$

$$\text{Mean} = A + h \left(\frac{1}{N} \sum f_i u_i \right) = 35 + 10 \left(\frac{-98}{70} \right) = -21$$

$$\text{Var}(X) = h^2 \left\{ \left(\frac{1}{N} \sum f_i u_i^2 \right) - \left(\frac{1}{N} \sum f_i u_i \right)^2 \right\} = 100 \left\{ \left(\frac{1}{70} \times 250 \right) - \left(\frac{1}{70} \times (-98) \right)^2 \right\} = 100 \{ 3.57 - 1.96 \} = 161$$

$$\text{SD} = \sqrt{\text{Var}(X)} = \sqrt{161} = 12.7$$

EX 32.7

Q1

We observe that the average monthly wages in both firms is same i.e. Rs. 2500. Therefore the plant with greater variance will have greater variability. Thus plant B has greater variability in individual wages.

Q2

We observe that the average weights and heights for the 50 students is same i.e. 63.2. Therefore, the parameter with greater variance will have more variability. Thus, *height* has greater variability than weights.

Q3

$$\text{Coefficient of variation} = \frac{\sigma}{\bar{x}} \times 100$$

So, we have:

$$60\% = \frac{21}{\bar{x}} \times 100 \Rightarrow \bar{x} = \frac{21}{.60} \times 100 = 35$$

$$70\% = \frac{16}{\bar{x}} \times 100 \Rightarrow \bar{x} = \frac{16}{.70} \times 100 = 22.85$$

Q4

CI	f	x	u=(x-A)/h	fu	u ²	fu ²
1000-1700	12	1350	-2	-24	4	48
1700-2400	18	2050	-1	-18	1	18
2400-3100	20	2750	0	0	0	0
3100-3800	25	3450	1	25	1	25
3800-4500	35	4150	2	70	4	140
4500-5200	10	4850	3	30	9	90
	120			83		321

Here, $N = 120$, $A = 2750$, $\sum f_i u_i = 83$, $\sum f_i u_i^2 = 321$ and $h = 700$

$$\therefore \text{Mean} = \bar{x} = A + h \left(\frac{1}{N} \sum f_i u_i \right)$$

$$\Rightarrow \bar{x} = 2750 + 700 \left(\frac{83}{120} \right) = 3234.17$$

$$\text{var}(x) = h^2 \left[\frac{1}{N} \sum f_i u_i^2 - \left(\frac{1}{N} \sum f_i u_i \right)^2 \right] = 490000 \left[\frac{321}{120} - \left(\frac{83}{120} \right)^2 \right] = 1076332.64$$

$$\therefore S.D. = \sqrt{\text{var}(x)} = \sqrt{1076332.64} = 1037.46$$

$$\text{Coefficient of variation} = \frac{S.D.}{\bar{x}} \times 100 = \frac{1037.46}{3234.17} \times 100 = 32.08$$

Q5

(i)

$$\begin{aligned}\text{Total wages paid by firm A} &= (\text{Average wages}) \times (\text{Number of employees}) \\ &= 52.5 \times 587 = \text{Rs } 30817.50\end{aligned}$$

$$\begin{aligned}\text{Total wages paid by firm B} &= (\text{Average wages}) \times (\text{Number of employees}) \\ &= 47.5 \times 648 = \text{Rs } 30780\end{aligned}$$

So, firm A pays higher total wages.

(ii)

In order to compare the variability of wages among the two firms, we have to calculate their coefficients of variation.

Let σ_1 and σ_2 denote the standard deviations of Firm A and Firm B respectively. Further, let \bar{X}_1 and \bar{X}_2 be the mean wages in firms A and B respectively.

We have,

$$\bar{X}_1 = 52.5, \quad \bar{X}_2 = 47.5$$

$$\sigma_1^2 = 100 \quad \text{and} \quad \sigma_2^2 = 121$$

$$\Rightarrow \sigma_1 = \sqrt{100} = 10 \quad \text{and} \quad \sigma_2 = \sqrt{121} = 11$$

Now,

$$\begin{aligned}\text{Coefficient of variation in wages in firm A} &= \frac{\sigma_1}{\bar{X}_1} \times 100 \\ &= \frac{10}{52.5} \times 100 = 19.05\end{aligned}$$

and,

$$\begin{aligned}\text{Coefficient of variation in wages in firm B} &= \frac{\sigma_2}{\bar{X}_2} \times 100 \\ &= \frac{11}{47.5} \times 100 = 23.16\end{aligned}$$

Clearly, coefficient of variation in wages is greater for firm B than for firm A.
So, firm B shows more variability in wages.

Q6

In order to compare the variability of weight in boys and girls, we have to calculate their coefficients of variation.

Let σ_1 and σ_2 denote the standard deviations of weight in boys and girls respectively. Further, let \bar{X}_1 and \bar{X}_2 be the mean weight of boys and girls respectively.

We have,

$$\begin{aligned}\bar{X}_1 &= 60, & \bar{X}_2 &= 45 \\ \sigma_1^2 &= 9 \text{ and } \sigma_2^2 = 4 \\ \Rightarrow \sigma_1 &= \sqrt{9} = 3 \text{ and } \sigma_2 = \sqrt{4} = 2\end{aligned}$$

Now,

$$\begin{aligned}\text{Coefficient of variation in weights in boys} &= \frac{\sigma_1}{\bar{X}_1} \times 100 \\ &= \frac{3}{60} \times 100 = 5\end{aligned}$$

and,

$$\begin{aligned}\text{Coefficient of variation in weights in girls} &= \frac{\sigma_2}{\bar{X}_2} \times 100 \\ &= \frac{2}{45} \times 100 = 4.44\end{aligned}$$

Clearly, coefficient of variation in weights is greater in boys than in girls.
So, weights shows more variability in boys.

Q7

In order to compare the variability of marks in Math, Physics, and Chemistry, we have to calculate their coefficients of variation.

Let σ_1, σ_2 and σ_3 denote the standard deviations of marks in Math, Physics and Chemistry respectively. Further, let \bar{X}_1 , \bar{X}_2 and \bar{X}_3 be the mean scores in Math, Physics and Chemistry respectively.

We have,

$$\begin{aligned}\bar{X}_1 &= 42, & \bar{X}_2 &= 32 & \bar{X}_3 &= 40.9 \\ \Rightarrow \sigma_1 &= 12 & \sigma_2 &= 15 & \sigma_3 &= 20\end{aligned}$$

Now,

$$\text{Coefficient of variation in Maths} = \frac{\sigma_1}{\bar{X}_1} \times 100 = \frac{12}{42} \times 100 = 28.57$$

$$\text{Coefficient of variation in Physics} = \frac{\sigma_2}{\bar{X}_2} \times 100 = \frac{15}{32} \times 100 = 46.88$$

$$\text{Coefficient of variation in Chemistry} = \frac{\sigma_3}{\bar{X}_3} \times 100 = \frac{20}{40.9} \times 100 = 48.90$$

Clearly, coefficient of variation in marks is greatest in Chemistry and lowest in Math.
So, marks in chemistry show highest variability and marks in maths show lowest variability.

Q8

Let's first find the coefficient of variable for Group G₁

CI	f	x	u=(x-A)/h	fu	u ²	fu ²
10-20	9	15	-3	-27	9	81
20-30	17	25	-2	-34	4	68
30-40	32	35	-1	-32	1	32
40-50	33	45	0	0	0	0
50-60	40	55	1	40	1	40
60-70	10	65	2	20	4	40
70-80	9	75	3	27	9	81
	150			-6		342

Here, $N = 150$, $A = 45$, $\sum f_i u_i = -6$, $\sum f_i u_i^2 = 342$ and $h = 10$

$$\therefore \text{Mean} = \bar{x} = A + h \left(\frac{1}{N} \sum f_i u_i \right)$$

$$\Rightarrow \bar{x} = 45 + 10 \left(\frac{-6}{150} \right) = 44.6$$

$$\text{var}(x) = h^2 \left[\frac{1}{N} \sum f_i u_i^2 - \left(\frac{1}{N} \sum f_i u_i \right)^2 \right] = 100 \left[\frac{342}{150} - \left(\frac{-6}{150} \right)^2 \right] = 227.84$$

$$\therefore S.D. = \sqrt{\text{var}(x)} = \sqrt{227.84} = 15.09$$

$$\text{Coefficient of variation} = \frac{S.D.}{\bar{x}} \times 100 = \frac{15.09}{44.6} \times 100 = 33.83$$

Q9

CI	f	x	$u=(x-A)/h$	fu	u^2	fu^2
10-15	2	12.5	-2	-4	4	8
15-20	8	17.5	-1	-8	1	8
20-25	20	22.5	0	0	0	0
25-30	35	27.5	1	35	1	35
30-35	20	32.5	2	40	4	80
35-40	15	37.5	3	45	9	135
100				108		266

Here, $N = 100$, $A = 22.5$, $\sum f u_i = 108$, $\sum f u_i^2 = 266$ and $h = 5$

$$\therefore \text{Mean} = \bar{x} = A + h \left(\frac{1}{N} \sum f u_i \right)$$

$$\Rightarrow \bar{x} = 22.5 + 5 \left(\frac{108}{100} \right) = 27.90$$

$$\text{var}(x) = h^2 \left[\frac{1}{N} \sum f u_i^2 - \left(\frac{1}{N} \sum f u_i \right)^2 \right] = 25 \left[\frac{266}{100} - \left(\frac{108}{100} \right)^2 \right] = 37.34$$

$$\therefore S.D. = \sqrt{\text{var}(x)} = \sqrt{37.34} = 6.11$$

$$\text{Coefficient of variation} = \frac{S.D.}{\bar{x}} \times 100 = \frac{6.11}{27.90} \times 100 = 21.9$$

Q10

x	d=(x- Mean)	d ²
35	-13	169
24	-24	576
52	4	16
53	5	25
56	8	64
58	10	100
52	4	16
50	2	4
51	3	9
49	1	1
480		980

$$\bar{x} = \frac{1}{n} \sum x_i = \frac{1}{10} [480] = 48$$

$$\text{var}(x) = \frac{1}{n} \left\{ \sum (x_i - \bar{x})^2 \right\} = \frac{1}{10} \{980\} = 98$$

$$S.D(x) = \sqrt{\text{var}(x)} = \sqrt{98} = 9.9$$

$$\text{Coefficient of variation} = \frac{S.D}{\bar{x}} \times 100 = \frac{9.9}{48} \times 100 = 20.6$$

Q11

Factory A

Length of life	Mid value x_i	f_i	$u_i = \frac{x_i - 800}{100}$	$f_i u_i$	$f_i u_i^2$
550 - 650	600	10	-2	-20	40
650 - 750	700	22	-1	-22	22
750 - 850	800	52	0	0	0
850 - 950	900	20	1	20	20
950 - 1050	1000	16	2	32	64
		$N = \sum f_i = 120$		$\sum f_i u_i = 10$	$\sum f_i u_i^2 = 146$

$$N = 120, \sum f_i u_i = 10, \sum f_i u_i^2 = 146, A = 800 \text{ and } h = 100$$

$$\bar{X}_A = A + h \left(\frac{1}{N} \sum f_i u_i \right) = 800 + 100 \left(\frac{10}{120} \right) = 808.33$$

$$\sigma_A^2 = h^2 \left\{ \left(\frac{1}{N} \sum f_i u_i^2 \right) - \left(\frac{1}{N} \sum f_i u_i \right)^2 \right\} = 10000 \left\{ \left(\frac{1}{120} \times 146 \right) - \left(\frac{1}{120} \times (10) \right)^2 \right\} = 10000 \{1.2166 - 0.0069\} = 12097$$

$$\Rightarrow \sigma_A = \sqrt{12097} = 109.98 \approx 110$$

Q12

Ravi

x_i	f_i	$f_i x_i$	$f_i x_i^2$
1	25	25	25
2	50	100	200
3	45	135	405
4	30	120	480
5	70	350	1750
6	42	252	1512
7	36	252	1764
8	45	360	2880
9	35	315	2835
10	60	600	6000
$N = \sum f_i = 438$		$\sum f_i x_i = 2509$	$\sum f_i x_i^2 = 17851$

$$\text{Variance} = \left(\frac{1}{N} \sum f_i x_i^2 \right) - \left(\frac{1}{N} \sum f_i x_i \right)^2 = \left(\frac{1}{438} \times 17851 \right) - \left(\frac{1}{438} \times (2509) \right)^2 = \{40.75 - 32.81\} = 7.94$$

$$\Rightarrow \text{SD} = \sqrt{7.94} = 2.82$$