EX 30.1

Q1

We have,

$f(x) = 3x$

$$\because f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

$$\therefore f'(a) = \lim_{h\to 0} \frac{f(2+h) - f(2)}{h}$$

$$= \lim_{h\to 0} \frac{3(2+h) - 6}{h}$$

$$= \lim_{h\to 0} \frac{3h}{h}$$

$$= \lim_{h\to 0} 3$$

$$\therefore f'(2) = 3$$

Q2

We have,

$f(x) = x^2 - 2$

$$\because f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

$$f'(10) = \lim_{h\to 0} \frac{f(10+h) - f(10)}{h}$$

$$= \lim_{h\to 0} \frac{(10+h)^2 - 2 - 98}{h}$$

$$= \lim_{h\to 0} \frac{100 + 20h + h^2 - 100}{h}$$

$$= \lim_{h\to 0} \frac{h(20+h)}{h}$$

$$= \lim_{h\to 0} (20+h)$$

$$\therefore f'(10) = 20$$

Q3

We have,
$f(x) = 99x$

$$\because f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$f'(100) = \lim_{h \to 0} \frac{f(100+h) - f(100)}{h}$$
$$= \lim_{h \to 0} \frac{99(100+h) - 9900}{h}$$
$$= \lim_{h \to 0} \frac{9900 + 99h - 9900)}{h}$$
$$= \lim_{h \to 0} 99$$

$$\therefore f'(100) = 99$$

Q4

We have,
$f(x) = x$

$$\because f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$
$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$
$$= \lim_{h \to 0} \frac{(1+h) - 1}{h}$$
$$= \lim_{h \to 0} 1$$

$$\therefore f'(1) = 1$$

Q5

We have,
$f(x) = \cos x$

$$\because f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

$$f'(0) = \lim_{h\to 0} \frac{f(0+h)-f(0)}{h}$$

$$= \lim_{h\to 0} \frac{\cos(0+h)-\cos 0}{h}$$

$$= \lim_{h\to 0} \frac{\cos h - \cos 0}{h}$$

$$= \lim_{h\to 0} \frac{(1-\frac{h^2}{2!}+\frac{h^4}{4!}-\text{---})-1}{h} \qquad \left[\because \cos x = 1-\frac{x^2}{2!}+\frac{x^4}{4!}-\ldots\right]$$

$$= \lim_{h\to 0} \frac{h(-\frac{h}{2!}+\frac{h^3}{4!}-\frac{h^5}{6!}-\ldots)}{h}$$

$$= \lim_{h\to 0} h(-\frac{h}{2!}+\frac{h^3}{4!}-\frac{h^5}{6!}-\ldots)$$

$$= 0$$

$\therefore f'(0) = 0$

Q6

We have,
$f(x) = \tan x$

$$\because f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

$$\therefore f'(0) = \lim_{h\to 0} \frac{f(0+h)-f(0)}{h}$$

$$= \lim_{h\to 0} \frac{\tan h - \tan 0}{h}$$

$$= \lim_{h\to 0} \frac{\tan h}{h}$$

$$= 1 \qquad \left[\because \lim_{\theta\to 0} \frac{\tan\theta}{\theta} = 1\right]$$

$\therefore f'(0) = 1$

Q7(i)

We have,

$f(x) = \sin x$

$$\because f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

$$\therefore f'(\tfrac{\pi}{2}) = \lim_{h\to 0} \frac{f\left(\frac{\pi}{2}+h\right) - f\left(\frac{\pi}{2}\right)}{h}$$

$$= \lim_{h\to 0} \frac{\sin\left(\frac{\pi}{2}+h\right) - \sin\left(\frac{\pi}{2}\right)}{h}$$

$$= \lim_{h\to 0} \frac{\cos h - 1}{h}$$

$$= \lim_{h\to 0} \frac{\left(1 - \frac{h^2}{2!} + \frac{h^4}{4!} - \ldots\right) - 1}{h} \qquad \left[\because \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots\right]$$

$$= \lim_{h\to 0} \frac{h\left(-\frac{h}{2!} + \frac{h^3}{4!} - \frac{h^5}{6!} + \ldots\right)}{h}$$

$$= \lim_{h\to 0} h\left(-\frac{h}{2!} + \frac{h^3}{4!} - \frac{h^5}{6!} + \ldots\right)$$

$$= 0$$

$$\therefore f'\left(\frac{\pi}{2}\right) = 0$$

Q7(ii)

We have,

$f(x) = x$

$$\because f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

$$\therefore f'(1) = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h\to 0} \frac{1+h-1}{h}$$

$$= \lim_{h\to 0} \ 1$$

$$\therefore f'(1) = 1$$

Q7(iii)

We have,

$\because f(x) = 2\cos x$

$$\therefore f'(a) = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

$$= \lim_{h\to 0}\frac{f\left(\frac{\pi}{2}+h\right)-f\left(\frac{\pi}{2}\right)}{h}$$

$$= \lim_{h\to 0}\frac{2\cos\left(\frac{\pi}{2}+h\right)-2\cos\frac{\pi}{2}}{h}$$

$$= \lim_{h\to 0}\frac{-2\sin h - 0}{h}$$

$$= -2 \qquad \left[\because \lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1\right]$$

$$\therefore f'\left(\frac{\pi}{2}\right) = -2$$

Q7(iv)

We have, $f(x) = \sin 2x$

Therefore,

$$f'(a) = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

$$= \lim_{h\to 0}\frac{f\left(\frac{\pi}{2}+h\right)-f\left(\frac{\pi}{2}\right)}{h}$$

$$= \lim_{h\to 0}\frac{\sin 2\left(\frac{\pi}{2}+h\right)-\sin 2\left(\frac{\pi}{2}\right)}{h}$$

$$= \lim_{h\to 0}\frac{\sin\left(\frac{\pi}{2}\times 2+2h\right)-\sin(\pi)}{h}$$

$$= \lim_{h\to 0}\frac{-\cos 2h - 0}{h}$$

$$= -2$$

Therefore $f'\left(\frac{\pi}{2}\right) = -2$

Ex 30.2

Q1(i)

We have,

$$\because f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{f\frac{2}{x+h} - \frac{2}{x}}{h}$$

$$= \lim_{h\to 0} \frac{(2x-2x-2h)}{hx(x+h)}$$

$$= \lim_{h\to 0} \frac{2(x-x-h)}{h.x(x+h)}$$

$$= \lim_{h\to 0} \frac{-2h}{hx(x+h)}$$

$$= \lim_{h\to 0} \frac{-2}{x(x+h)}$$

$$= \frac{-2}{x^2}$$

Q1(ii)

We have,

$$\because f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{x}-\sqrt{x+h}}{\sqrt{x}\sqrt{x+h}.h}$$

$$= \lim_{h\to 0} \frac{\sqrt{x}-\sqrt{x+h}}{\sqrt{x}\sqrt{x+h}.h} \times \frac{\sqrt{x}+\sqrt{x+h}}{\sqrt{x}+\sqrt{x+h}\,h}$$

$$= \lim_{h\to 0} \frac{x-(x+h)}{\sqrt{x}\sqrt{x+h}.h\left(\sqrt{x}+\sqrt{x+h}\right)}$$

$$= \lim_{h\to 0} \frac{-h}{\sqrt{x}\sqrt{x+h}.h\left(\sqrt{x}+\sqrt{x+h}\right)}$$

$$= \lim_{h\to 0} \frac{-1}{x\sqrt{x+h}+\sqrt{x}\left(\sqrt{x+h}\right)}$$

$$= \lim_{h\to 0} \frac{-1}{2x\sqrt{x}}$$

$$= \frac{-1}{2}x^{-\frac{3}{2}}$$

Q1(iii)

We have,

$f(x) = \frac{1}{x^3}$

$$\because f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{(x+h)^3} - \frac{1}{x^3}}{h}$$

$$= \lim_{h\to 0} \frac{x^3 - (x+h)^3}{x^3 h (x+h)^3}$$

$$= \lim_{h\to 0} \frac{x^3 - (x^3 + 3x^2h + 3xh^2 + h^3)}{x^3 h (x+h)^3}$$

$$= \lim_{h\to 0} \frac{x^3 - x^3 - 3x^2 - 3xh - h^2}{x^3 (x+h)^3}$$

$$= \lim_{h\to 0} \frac{-3x^2 - 3xh - h^2}{x^3 (x+h)^3}$$

$$= \frac{-3x^2}{x^6}$$

$$= \frac{-3}{x^4}$$

Q1(iv)

We have,

$$f(x) = \frac{x^2+1}{x}$$

$$\because f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{(x+h)^2+1}{(x+h)} - \frac{x^2+1}{x}}{h}$$

$$= \lim_{h\to 0} \frac{x\left[x^2+h^2+2xh+1\right]-\left(x^2+1\right)(x+h)}{hx(x+h)}$$

$$= \lim_{h\to 0} \frac{x^3+xh^2+2x^2h+x-x^3-x-x^2h-h}{h.x(x+h)}$$

$$= \lim_{h\to 0} \frac{xh+2x^2-x^2-1}{x(x+h)}$$

$$= \frac{x^2-1}{x^2}$$

$$= 1-\frac{1}{x^2}$$

We have,

$$f(x) = \frac{x^2+1}{x}$$

$$\because f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{(x+h)^2+1}{(x+h)} - \frac{x^2+1}{x}}{h}$$

$$= \lim_{h\to 0} \frac{x\left[x^2+h^2+2xh+1\right]-\left(x^2+1\right)(x+h)}{hx(x+h)}$$

$$= \lim_{h\to 0} \frac{x^3+xh^2+2x^2h+x-x^3-x-x^2h-h}{h.x(x+h)}$$

$$= \lim_{h\to 0} \frac{xh+2x^2-x^2-1}{x(x+h)}$$

$$= \frac{x^2-1}{x^2}$$

$$= 1-\frac{1}{x^2}$$

Q1(v)

We have,

$$f(x) = \frac{x^2 - 1}{x}$$

$$\therefore f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{(x+h)^2 - 1}{(x+h)} - \frac{x^2 - 1}{x}}{h}$$

$$= \lim_{h \to 0} \frac{x\left(x^2 + h^2 + 2xh - 1\right) - (x+h)\left(x^2 - 1\right)}{x(x+h)h}$$

$$= \lim_{h \to 0} \frac{xh + 2x^2 - x^2 + 1}{x(x+h)}$$

$$= \frac{x^2 + 1}{x^2}$$

$$= 1 + \frac{1}{x^2}$$

Q1(vi)

We have,

$$f(x) = \frac{x+1}{x+2}$$

$$\because f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{(x+h)+1}{(x+h)+2} - \frac{x+1}{x+2}}{h}$$

$$= \lim_{h \to 0} \frac{(x+2)(x+h+1) - (x+1)(x+h+2)}{(x+h+2)(x+2)h}$$

$$= \lim_{h \to 0} \frac{\left(x^2 - 2x + xh + 2h + 2 + x\right) - \left(x^2 + xh + 2x + x + h + 2\right)}{(x+h+2)(x+2)h}$$

$$= \lim_{h \to 0} \frac{h}{(x+h+2)(x+2)h}$$

$$= \frac{1}{(x+2)^2}$$

Q1(vii)

We have,

$$f(x) = \frac{x+2}{3x+5}$$

$$\because f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{(x+h+2)}{3(x+h)+5} - \frac{x+2}{3x+5}}{h}$$

$$= \lim_{h\to 0} \frac{(3x+5)(x+h+2) - (x+2)(3x+3h+5)}{(3x+5)(3x+3h+5)h}$$

$$= \lim_{h\to 0} \frac{(3x^2+5x+3xh+5h+6x+10) - (3x^2+3xh+5x+6x+6h+10)}{(3x+5)(3x+3h+5)h}$$

$$= \lim_{h\to 0} \frac{-h}{(3x+5)(3x+3h+5)h}$$

$$= \lim_{h\to 0} \frac{-1}{(3x+5)(3x+3h+5)}$$

$$= \frac{-1}{(3x+5)^2}$$

Q1(viii)

We have,

$$f(x) = kx^n$$

$$\because f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{k(x+h)^n - kx^n}{h}$$

$$= k \lim_{h\to 0} \frac{\left(x^n + nx^{n-1}h + \frac{n(n-1)}{2}x^{n-2}h^2 + \dots\right) - x^n}{h} \quad \left[\because (x+y)^n = x^n + nx^{n-1}y\dots\right]$$

$$= k \lim_{h\to 0} nx^{n-1} + \frac{n(n-1)}{2!}x^{n-2}h + \frac{n(n-1)(n-2)}{3!}x^{n-3}h^2\dots$$

$$= k\,nx^{n-1} + 0 + 0\dots$$

$$= k\,nx^{n-1}$$

Q1(ix)

We have,

$$f(x)=\frac{1}{\sqrt{3-x}}$$

$$\because f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{\frac{1}{\sqrt{3-(x+h)}}-\frac{1}{\sqrt{3-x}}}{h}$$

$$=\lim_{h\to 0}\frac{\sqrt{3-x}-\sqrt{3-(x+h)}}{\sqrt{3-x}\sqrt{3-(x+h)}\times h}$$

$$=\lim_{h\to 0}\frac{\sqrt{3-x}-\sqrt{3-(x+h)}}{\sqrt{3-x}\sqrt{3-(x+h)}h}\times\frac{\sqrt{3-x}+\sqrt{3-(x+h)}}{\sqrt{3-x}+\sqrt{3-(x+h)}}$$ [Rationalising the numerator by $\sqrt{3-x}+\sqrt{3-(x+h)}$]

$$=\lim_{h\to 0}\frac{(3-x)-(3-(x+h))}{\sqrt{3-x}\sqrt{3-(x+h)}\times h\left(\sqrt{3-x}+\sqrt{3-(x+h)}\right)}$$

$$=\lim_{h\to 0}\frac{h}{\sqrt{3-x}\sqrt{3-(x+h)}\times h\left(\sqrt{3-x}+\sqrt{3-(x+h)}\right)}$$

$$=\frac{1}{(3-x)\times 2\sqrt{3-x}}$$

$$=\frac{1}{2(3-x)^{\frac{3}{2}}}$$

Q1(x)

We have,

$$f(x)=x^2+x+3$$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{\left\{(x+h)^2+(x+h)+3\right\}-x^2+x+3}{h}$$

$$=\lim_{h\to 0}\frac{x^2+h^2+2xh+x+h+3-x^2-x-3}{h}$$

$$=\lim_{h\to 0}\frac{2xh+h^2+h}{h}$$

$$=\lim_{h\to 0}\frac{h(2x+h+1)}{h}$$

$$=2x+0+1$$

$$=2x+1$$

Q1(xi)

We have,

$f(x) = (x+2)^3$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0}\frac{(x+h+2)^3-(x+2)^3}{h}$$
$$= \lim_{h\to 0}\frac{\{(x+2)+h\}^3-(x+2)^3}{h}$$
$$= \lim_{h\to 0}\frac{(x+2)^3+h^3+3h(x+2)^2+3(x+2)h^2-(x+2)^3}{h}$$
$$= \lim_{h\to 0} 3(x+2)^2+3(x+2)h+h^2$$
$$= 3(x+2)^2$$

Q1(xii)

We have,

$f(x) = x^3+4x^2+3x+2$

$$\because f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0}\frac{(x+h)^3+4(x+h)^2+3(x+h)+2-(x^3+4x^2+3x+2)}{h}$$

On solving we get,

$$= \lim_{h\to 0}\frac{x^3+h^3+3x^2h+3h^2x+4x^2+4h^2+8hx+3x+3h+2-x^3-4x^2-3x-2}{h}$$
$$= \lim_{h\to 0}\frac{3x^2h+3xh^2+h^3+4h^2+8hx+3h}{h}$$
$$= \lim_{h\to 0} 3x^2+3xh+h^2+4h+8x+3$$
$$= 3x^2+8x+3$$

Q1(xiii)

We have,

$f(x) = x^3 - 5x^2 + x - 5$

$$\because f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\left\{(x+h)^3 + (x+h) - 5(x+h)^2 - 5\right\} - \left(x^3 - 5x^2 + x - 5\right)}{h}$$

$$= \lim_{h\to 0} \frac{\left(x^3 + h^3 + 3x^2h + 3h^2x + x + h - 5x^2 - 5h^2 - 10xh - 5\right) - \left(x^3 - 5x^2 + x - 5\right)}{h}$$

$$= \lim_{h\to 0} \frac{\left(3x^2h + 3h^2x + h^3 + h - 5h^2 - 10xh\right)}{h}$$

$$= \lim_{h\to 0} 3x^2 + 3xh + h^2 + 1 - 5h - 10x$$

$$= 3x^2 - 10x + 1$$

Q1(xiv)

We have,

$f(x) = \sqrt{2x^2 + 1}$

$$\because f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{2(x+h)^2 + 1} - \sqrt{2x^2 + 1}}{h}$$

MultiplyingNumerator andDenominator by $\sqrt{2(x+h)^2 + 1} + \sqrt{2x^2 + 1}$

$$= \lim_{h\to 0} \frac{\left\{2(x+h)^2 + 1 - \left(2x^2 + 1\right)\right\}}{h\left(\sqrt{2(x+h)^2 + 1} + \sqrt{2x^2 + 1}\right)}$$

$$= \lim_{h\to 0} \frac{2x^2 + 2h^2 + 4xh + 1 - 2x^2 - 1}{h\left(\sqrt{2(x+h)^2 + 1} + \sqrt{2x^2 + 1}\right)}$$

$$= \lim_{h\to 0} \frac{4xh + 2h^2}{h\sqrt{2(x+h)^2 + 1} + \sqrt{2x^2 + 1}}$$

$$= \frac{4x}{2\sqrt{2x^2 + 1}}$$

$$= \frac{2x}{\sqrt{2x^2 + 1}}$$

Q1(xv)

We have, $f(x)=\dfrac{2x+3}{x-2}$

Therefore,

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}\\
&= \lim_{h\to 0}\frac{\left(\frac{2x+2h+3}{x+h-2}\right)-\left(\frac{2x+3}{x-2}\right)}{h}\\
&= \lim_{h\to 0}\frac{2x^2+2hx+3x-4x-4h-6-2x^2-2hx+4x-3x-3h+6}{h(x+h-2)(x-2)}\\
&= \lim_{h\to 0}\frac{-7}{(x+h-2)(x-2)}\\
&= \boxed{\frac{-7}{(x-2)^2}}
\end{aligned}
$$

Q2(i)

We have,

$f(x)=e^{-x}$

$$
\begin{aligned}
\because f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\
&= \lim_{h\to 0}\frac{e^{-(x+h)}-e^{-x}}{h}\\
&= \lim_{h\to 0}\frac{e^{-x}\left(e^{-h}-1\right)}{h}\\
&= \lim_{h\to 0}\frac{-e^{-x}\left(e^{-h}-1\right)}{-h}\\
&= -e^{-x} \qquad \left[\because \lim_{\theta\to 0}\frac{e^{\theta}-1}{\theta}=1\right]
\end{aligned}
$$

Q2(ii)

We have,

$f(x) = e^{3x}$

$$\because f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{e^{3(x+h)} - e^{3x}}{h}$$

$$= \lim_{h \to 0} \frac{e^{3(x)} . e^{3h} - e^{3x}}{h}$$

$$= \lim_{h \to 0} \frac{e^{3(x)} \left(e^{3h} - 1\right)}{h}$$

Multiplying Numerator and Denominator by 3

$$= \lim_{h \to 0} \; e^{3(x)} \frac{\left(e^{3h} - 1\right)}{3h} \qquad \left[\lim_{h \to 0} \frac{e^{3h} - 1}{3} = 1\right]$$

$$= 3e^{3x}$$

Q2(iii)

We have,

$f(x) = e^{ax+b}$

$$\because f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{e^{a(x+h)+b} - e^{ax+b}}{h}$$

$$= \lim_{h \to 0} \frac{e^{ax} \times e^{ah} \times e^{b} - e^{ax} \times e^{b}}{h}$$

$$= \lim_{h \to 0} \frac{e^{b} \times e^{ax} \left(e^{ah} - 1\right)}{h}$$

$$= \lim_{h \to 0} \; e^{ax+b} \times \frac{a\left(e^{ah} - 1\right)}{a.h}$$

Multiplying Numerator and denominator by a $\qquad \left[\lim_{h \to 0} \frac{\left(e^{ah} - 1\right)}{ah} = 1\right]$

$$= ae^{ax+b}$$

Q2(iv)

We have,

$f(x) = xe^x$

$$\because f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h\to 0} \frac{(x+h)e^{(x+h)} - xe^x}{h}$$
$$= \lim_{h\to 0} \frac{xe^x.e^h + he^x.e^h - xe^x}{h}$$
$$= \lim_{h\to 0} \quad xe^x\left(\frac{e^h - 1}{h}\right) + \frac{he^{x+h}}{h}$$
$$= xe^x + e^x$$
$$= e^x(x+1)$$

Q2(v)

Let $f(x) = -x$. Then, $f(x+h) = -(x+h)$

$$\therefore \frac{d}{dx}(f(x)) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$
$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h\to 0} \frac{-(x+h) + (x)}{h}$$
$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h\to 0} \frac{-h}{h}$$
$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h\to 0} -1$$
$$\Rightarrow \frac{d}{dx}(f(x)) = -1$$

Q2(vi)

Let $f(x) = (-x)^{-1}$. Then, $f(x+h) = (-(x+h))^{-1}$

$$\therefore \frac{d}{dx}(f(x)) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h\to 0}\frac{(-(x+h))^{-1}-(-x)^{-1}}{h}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h\to 0}\frac{\frac{-1}{x+h}+\frac{1}{x}}{h}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h\to 0}\frac{\frac{-x+x+h}{x(x+h)}}{h}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h\to 0}\frac{h}{hx(x+h)}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h\to 0}\frac{1}{x(x+h)}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \frac{1}{x(x+0)}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \frac{1}{x^2}$$

Q2(vii)

Let $f(x) = \sin(x+1)$. Then, $f(x+h) = \sin((x+h)+1)$

$$\therefore \frac{d}{dx}(f(x)) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h\to 0}\frac{\sin((x+h)+1)-\sin(x+1)}{h}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h\to 0}\frac{2\sin\left[\frac{((x+h)+1)-(x+1)}{2}\right]\cos\left[\frac{((x+h)+1)+(x+1)}{2}\right]}{h}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h\to 0}\frac{2\sin\left[\frac{h}{2}\right]\cos\left[\frac{2x+2+h}{2}\right]}{h}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h\to 0}\frac{\sin\left[\frac{h}{2}\right]}{\frac{h}{2}}\times\lim_{h\to 0}\cos\left[\frac{2x+2+h}{2}\right]$$

$$\Rightarrow \frac{d}{dx}(f(x)) = 1\times\cos\left[\frac{2x+2+0}{2}\right]$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \cos(x+1)$$

Q2(viii)

Let $f(x) = \cos\left(x - \frac{\pi}{8}\right)$. Then, $f(x+h) = \cos\left(x + h - \frac{\pi}{8}\right)$

$$\therefore \frac{d}{dx}(f(x)) = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h\to 0}\frac{\cos\left(x+h-\frac{\pi}{8}\right) - \cos\left(x-\frac{\pi}{8}\right)}{h}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h\to 0}\frac{-2\sin\left[\frac{\left(x+h-\frac{\pi}{8}\right)+\left(x-\frac{\pi}{8}\right)}{2}\right]\sin\left[\frac{\left(x+h-\frac{\pi}{8}\right)-\left(x-\frac{\pi}{8}\right)}{2}\right]}{h}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h\to 0}\frac{-2\sin\left[\frac{2x+h-\frac{2\pi}{8}}{2}\right]\sin\left[\frac{h}{2}\right]}{h}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h\to 0} -\sin\left[\frac{2x+h-\frac{2\pi}{8}}{2}\right] \times \lim_{h\to 0}\frac{\sin\left[\frac{h}{2}\right]}{\frac{h}{2}}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = -\sin\left[\frac{2x+0-\frac{2\pi}{8}}{2}\right] \times 1$$

$$\Rightarrow \frac{d}{dx}(f(x)) = -\sin\left(x - \frac{\pi}{8}\right)$$

Q2(ix)

We have,

$f(x) = x \sin x$

$$\because f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{(x+h)\sin(x+h) - x\sin x}{h}$$

$$= \lim_{h\to 0} \frac{x(\sin(x+h) - \sin x)}{h} + \sin(x+h) \qquad \left[\sin c - \sin d = 2\cos\frac{c+d}{2}\sin\frac{c-d}{2}\right]$$

$$= \lim_{h\to 0} \frac{x \times 2\cos\left(x+\frac{h}{2}\right)\sin\frac{h}{2}}{h} + \sin(x+h) \qquad \left[\because \lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1\right]$$

$$= 2x \times \cos x \times \frac{1}{2} + \sin x$$

$$= x \times \cos x + \sin x$$

$$= \sin x + x\cos x$$

Q2(x)

We have,

$f(x) = x \cos x$

$$\because f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{(x+h)\cos(x+h) - x\cos x}{h}$$

$$= \lim_{h\to 0} \frac{x\cos(x+h) + h\cos(x+h) - x\cos x}{h}$$

$$= \lim_{h\to 0} \frac{x\{\cos(x+h) - \cos x\}}{h} + \cos(x+h)$$

$$= \lim_{h\to 0} x.2\sin\left(x - x - \frac{h}{2}\right)\sin\left(x+\frac{h}{2}\right) + \cos(x+h) \qquad \left[\because \cos A - \cos B = 2\sin\frac{B-A}{2}\sin\frac{B+A}{2}\right]$$

$$= \lim_{h\to 0} 2x.\sin\left(\frac{-h}{2}\right)\sin\left(x+\frac{h}{2}\right) + \cos(x+h)$$

$$= -x\sin x + \cos x$$

Q2(xi)

We have,

$f(x) = \sin(2x-3)$

$$\because f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{\sin\{2(x+h)-3\}-\sin(2x-3)}{h}$$

$$= \lim_{h\to 0}\frac{2\cos\frac{(2x+2h-3+2x-3)}{2}\times\sin\frac{(2x+2h-3-2x+3)}{2}}{h} \quad \left[\because \sin C-\sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}\right]$$

$$= \lim_{h\to 0} 2\cos(2x-3+h).\frac{\sinh}{h} \quad \left[\because \lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1\right]$$

$$= 2\cos(2x-3)$$

Q3(i)

We have,

$f(x) = \sqrt{\sin 2x}$

$$\because f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{\sqrt{\sin 2(x+h)}-\sqrt{\sin 2x}}{h}$$

Multiplying Numerator and Denominator by $\left(\sqrt{\sin 2(x+h)}+\sqrt{\sin 2x}\right)$

$$= \lim_{h\to 0}\frac{\sqrt{\sin 2(x+h)}-\sqrt{\sin 2x}}{h}\times\frac{\sqrt{\sin 2(x+h)}+\sqrt{\sin 2x}}{\sqrt{\sin 2(x+h)}+\sqrt{\sin 2x}}$$

$$= \lim_{h\to 0}\frac{\sin(2x+2h)-\sin 2x}{h\left(\sqrt{\sin 2(x+h)}+\sqrt{\sin 2x}\right)}$$

$$= \lim_{h\to 0}\frac{\sin(2x+2h)-\sin 2x}{h\left(\sqrt{\sin(2x+2h)}+\sqrt{\sin 2x}\right)} \quad \left[\sin C-\sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}\right]$$

$$= \lim_{h\to 0}\frac{2\cos(2x+h)\times\sinh}{h}\times\frac{1}{\sqrt{\sin(2x+2h)}+\sqrt{\sin 2x}}$$

$$= \frac{2\cos 2x}{2\sqrt{\sin 2x}}$$

$$= \frac{\cos 2x}{\sqrt{\sin 2x}}$$

Q3(ii)

We have,

$$f(x) = \frac{\sin x}{x}$$

$$\because f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{\sin(x+h)}{x+h} - \frac{\sin x}{x}}{h}$$

$$= \lim_{h \to 0} \frac{x \sin(x+h) - (x+h)\sin x}{xh(x+h)}$$

$$= \lim_{h \to 0} \frac{x(\sin x . \cosh + \cos x . \sinh) - x . \sin x - h . \sin x}{xh(x+h)} \quad \left[\because \sin(A+B) = \sin A . \cos B + \cos A . \sin B\right]$$

$$= \lim_{h \to 0} \frac{x . \sin x (\cosh - 1)}{xh(x+h)} + \frac{x . \cos x . \sinh}{(x+h)xh} - \frac{h \sin x}{(x+h)xh} \quad \left[\because 1 - \cosh = 2\sin^2 \frac{h}{2}\right]$$

$$= \frac{-x \sin x}{x(x+h)} \times \frac{2\sin^2 \frac{h}{2}}{\frac{h^2}{4}} \times \frac{h}{4} + \frac{x \cos x}{x^2} - \frac{\sin x}{x^2}$$

$$\because h \to 0 \Rightarrow \frac{h}{2} \to 0 \quad \text{and} \quad \lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$$

$$= 0 + \frac{x \cos x - \sin x}{x^2}$$

$$= \frac{x \cos x - \sin x}{x^2}$$

Q3(iii)

We have,

$$f(x) = \frac{\cos x}{x}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{\cos(x+h)}{x+h} - \frac{\cos x}{x}}{h}$$

$$= \lim_{h \to 0} \frac{x.\cos(x+h) - (x+h)\cos x}{(x+h) \times h} \qquad \left[\because \cos(A+B) = \cos A.\cos B - \sin A.\sin B\right]$$

$$= \lim_{h \to 0} \frac{x\left[\cos x.\cos h - \sin x \sin h\right] - x.\cos x - h.\cos x}{(x+h) \times h}$$

$$= \lim_{h \to 0} \frac{x \cos x (\cos h - 1)}{(x+h) \times h} - \frac{x.\cos x.\sin h}{(x+h) \times h} - \frac{h.\cos x}{(x+h) \times h}$$

$$= \lim_{h \to 0} \frac{-x \cos x.2\sin^2 \frac{h}{2}}{(x+h) \times \frac{h^2}{4}} \times \frac{h^2}{4} - \frac{x.\sin x}{x(x+h)} - \frac{\cos x}{x(x+h)}$$

$$= 0 - \frac{x \sin x}{x^2} - \frac{\cos x}{x^2}$$

$$= -\frac{x \sin x - \cos x}{x^2}$$

Q3(iv)

We have,

$$f(x) = x^2 \sin x$$

$$\therefore f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 \sin(x+h) - x^2 \sin x}{h}$$

$$= \lim_{h \to 0} \frac{(x^2 + h^2 + 2hx)(\sin x.\cos h + \cos x.\sin h) - x^2 \sin x}{h} \qquad \left[\because \sin(A+B) = \sin A.\cos B + \cos B \sin A\right]$$

$$= \lim_{h \to 0} \frac{x^2 \sin x (\cos h - 1)}{h} + \frac{h(h+2x)\sin x.\cos h}{h} + (x+h)^2 \cos x \frac{\sin h}{h}$$

$$= \lim_{h \to 0} -x^2 \sin x \times \frac{2\sin^2 \frac{h}{2}}{\left(\frac{h}{2}\right)^2} \times \frac{h^2}{4} + (h+2x)\sin x.\cos h + (x+h)^2 \cos x$$

$$= 0 + \left(2x \sin x + x^2 \cos x\right)$$

$$= 2x \sin x + x^2 \cos x$$

Q3(v)

We have,

$f(x) = \sqrt{\sin(3x+1)}$

$$\therefore f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{\sin 3(x+h)+1} - \sqrt{\sin(3x+1)}}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{\sin(3x+3h)+1} - \sqrt{\sin(3x+1)}}{h} \times \frac{\sqrt{\sin(3x+3h)+1} + \sqrt{\sin(3x+1)}}{\sqrt{\sin(3x+3h)+1} + \sqrt{\sin(3x+1)}}$$

$$= \lim_{h\to 0} \frac{\sin(3x+3h+1) - \sin(3x+1)}{h\left(\sqrt{\sin(3x+3h)+1} + \sqrt{\sin(3x+1)}\right)}$$

$$= \lim_{h\to 0} 2\cos\left(3x+1+\frac{3h}{2}\right) \times \frac{\sin\frac{3h}{2}}{\frac{3h}{2}} \times \frac{3}{2} \times \frac{1}{\sqrt{\sin(3x+3h)+1} + \sqrt{\sin(3x+1)}}$$

$$= \frac{3\cos(3x+1)}{2\sqrt{\sin(3x+1)}} \qquad \left[\lim_{h\to 0} \frac{\sin\frac{3h}{2}}{\frac{3h}{2}} = 1\right]$$

Q3(vi)

We have,

$f(x) = \sin x + \cos x$

$$\therefore f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\{\sin(x+h) + \cos(x+h)\} - \sin x + \cos x}{h}$$

$$= \lim_{h \to 0} \frac{\{\sin(x+h) + \cos(x+h) - \sin x - \cos x\}}{h}$$

$$= \lim_{h \to 0} \frac{\{\sin(x+h) - \sin x\} + \{\cos(x+h) - \cos x\}}{h}$$

$$= \lim_{h \to 0} \frac{\left\{2 \sin \frac{(x+h-x)}{2} \cos \frac{(x+h+x)}{2}\right\} + \left\{-2 \sin \frac{x+h+x}{2} \sin \frac{x-h-x}{2}\right\}}{h}$$

$$\left[\because \sin A - \sin B = 2 \sin \frac{A-B}{2} \cos \frac{A+B}{2} \text{ and } \cos A - \cos B = 2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}\right]$$

$$= \lim_{h \to 0} \frac{\sinh . \cos \frac{2x+h}{2} - 2 \sin\left(x + \frac{h}{2}\right) \sinh}{h}$$

$$= \lim_{h \to 0} \frac{\sinh}{h} \left\{\cos \frac{x+h}{2} - \sin\left(x + \frac{h}{2}\right)\right\} \qquad \left[\because \lim_{h \to 0} \frac{\sinh}{h} = 1\right]$$

$$= \cos x - \sin x$$

Q3(vii)

We have,

$f(x) = x^2 e^x$

$$\because f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{x^2 e^x e^h + h^2 . e^x . e^h + 2xhe^x e^h - x^2 e^x}{h}$$

$$= \lim_{h \to 0} x^2 e^x \frac{(e^h - 1)}{h} + e^x e^h \frac{(h^2 + 2xh)}{h} \qquad \left[\because \frac{e^h - 1}{h} = 1\right]$$

$$\therefore x^2 e^x + e^x (0 + 2x)$$

$$= x^2 e^x + 2xe^x$$

$$= e^x (x^2 + 2x)$$

Q3(viii)

We have,

$$f(x) = e^{x^2+1}$$

$$\therefore f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{e^{(x+h)^2} + 1 - e^{x^2+1}}{h}$$

$$= \lim_{h \to 0} \frac{e^{x^2+h^2+2xh+1} - e^{x^2+1}}{h}$$

$$= \lim_{h \to 0} \frac{e^{x^2+1}\left(e^{2xh}.e^{h^2} - 1\right)}{h}$$

$$= \lim_{h \to 0} \frac{e^{x^2+1}\left(e^{2xh+h^2} - 1\right)}{2xh + h^2} \times \frac{2xh + h^2}{h}$$

$\because h \to 0$

$\Rightarrow 2xh + h^2 = 0$

and $\lim_{\theta \to 0} \frac{e^{\theta} - 1}{\theta} = 1$

$= \lim_{h \to 0} \quad e^{x^2+1}.1 \times 2x + h$

$= 2xe^{x^2+1}$

Q3(ix)

We have,

$f(x) = e\sqrt{2x}$

$$\because f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{e\sqrt{2(x+h)} - e\sqrt{2x}}{h}$$

$$= \lim_{h \to 0} \frac{e\sqrt{2x}\left(e^{\sqrt{2(x+h)}-\sqrt{2x}} - 1\right)}{h}$$

$$= \lim_{h \to 0} \ e\sqrt{2x} \ \frac{\left(e^{\sqrt{2(x+h)}-\sqrt{2x}} - 1\right)}{\sqrt{2(x+h)} - \sqrt{2x}} \times \frac{\sqrt{2(x+h)} - \sqrt{2x}}{h}$$

MultiplyingNumerator and Denominator by $\sqrt{2(x+h)} - \sqrt{2x}$

$\because h \to 0 \Rightarrow \sqrt{2(x+h)} - \sqrt{2x} \Rightarrow 0$

and $\lim_{\theta \to 0} \frac{e^{\theta} - 1}{\theta} = 1$

$$\therefore \lim_{h \to 0} \ e^{\sqrt{2x}} \times \frac{\sqrt{2(x+h)} - \sqrt{2x}}{h}$$

AgainMultiplyingNumerator andDenominator by $\sqrt{2(x+h)} + \sqrt{2x}$

$$\therefore \lim_{h \to 0} \ e^{\sqrt{2x}} \times \frac{\sqrt{2(x+h)} - \sqrt{2x}}{h} \times \frac{\sqrt{2(x+h)} + \sqrt{2x}}{\sqrt{2(x+h)} + \sqrt{2x}}$$

$$= e^{\sqrt{2x}} \times \frac{1}{2\sqrt{2x}}$$

Q3(x)

We have,

$$f(x) = e^{\sqrt{ax+b}}$$

$$\because f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{e^{\sqrt{a(x+h)+b}} - e^{\sqrt{ax+b}}}{h}$$

$$= \lim_{h \to 0} e^{\sqrt{a(x+h)+b}} \frac{\left(e^{\sqrt{a(x+h)+b}-\sqrt{ax+b}} - 1\right)}{h}$$

$$= \lim_{h \to 0} e^{\sqrt{ax+b}} \times \frac{e^{\sqrt{a(x+h)+b}-\sqrt{ax+b}} - 1}{\sqrt{a(x+h)+b} - \sqrt{ax+b}} \times \frac{\sqrt{a(x+h)+b} - \sqrt{ax+b}}{h}$$

Multiplying Numerator and Denominator by $\sqrt{a(x+h)+b} - \sqrt{ax+b}$

$\because h \to 0$

$\therefore \sqrt{a(x+h)+b} - \sqrt{ax+b} = 0$

and $\lim_{\theta \to 0} \frac{e^{\theta} - 1}{\theta} = 1$

$$= \lim_{h \to 0} e^{\sqrt{ax+b}} \times 1 \times \frac{\sqrt{a(x+h)+b} - \sqrt{ax+b}}{\sqrt{a(x+h)+b} + \sqrt{ax+b}} \times \frac{\sqrt{a(x+h)+b} + \sqrt{ax+b}}{h}$$

Again multiplied Numerator and Denominator by $\sqrt{a(x+h)+b} + \sqrt{ax+b}$

$$= \lim_{h \to 0} e^{\sqrt{ax+b}} \times \frac{a(x+h)+b-(ax+b)}{h} \times \frac{1}{\left(\sqrt{a(x+h)+b} + \sqrt{ax+b}\right)}$$

$$= \frac{e^{\sqrt{ax+b}} \times a}{2\sqrt{ax+b}}$$

$$= \frac{ae^{\sqrt{ax+b}}}{2\sqrt{ax+b}}$$

Q3(xi)

$$f(x) = a^{\sqrt{x}} = e^{\sqrt{x}\log a}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{e^{\sqrt{x+h}\log a} - e^{\sqrt{x}\log a}}{h}$$

$$= \lim_{h \to 0} e^{\sqrt{x}\log a} \frac{e^{\sqrt{x+h}\log a - \sqrt{x}\log a} - 1}{h}$$

$$= \lim_{h \to 0} e^{\sqrt{x}\log a} \frac{e^{\left(\sqrt{x+h} - \sqrt{x}\right)\log a} - 1}{h}$$

Multiply numerator and denominator by $\left(\sqrt{x+h} - \sqrt{x}\right)\log a$

$$f'(x) = \lim_{h \to 0} e^{\sqrt{x}\log a} \frac{e^{\left(\sqrt{x+h} - \sqrt{x}\right)\log a} - 1}{h\left(\sqrt{x+h} - \sqrt{x}\right)\log a}\left(\sqrt{x+h} - \sqrt{x}\right)\log a$$

$$= e^{\sqrt{x}\log a} \lim_{h \to 0} \frac{e^{\left(\sqrt{x+h} - \sqrt{x}\right)\log a} - 1}{\left(\sqrt{x+h} - \sqrt{x}\right)\log a} \lim_{h \to 0} \log a \frac{\left(\sqrt{x+h} - \sqrt{x}\right)}{h}$$

$$= e^{\sqrt{x}\log a} \lim_{h \to 0} \log a \frac{\left(\sqrt{x+h} - \sqrt{x}\right)}{h}$$

Multiply numerator and denominator by $\left(\sqrt{x+h} + \sqrt{x}\right)$

$$f'(x) = e^{\sqrt{x}\log a} \lim_{h \to 0} \log a \frac{\left(\sqrt{x+h} - \sqrt{x}\right)}{h\left(\sqrt{x+h} + \sqrt{x}\right)}\left(\sqrt{x+h} + \sqrt{x}\right)$$

$$= e^{\sqrt{x}\log a} \lim_{h \to 0} \log a \frac{h}{h\left(\sqrt{x+h} + \sqrt{x}\right)}$$

$$= e^{\sqrt{x}\log a} \frac{\log a}{2\sqrt{x}}$$

$$= \frac{a^{\sqrt{x}}}{2\sqrt{x}} \log_e a$$

Q3(xii)

We have,

$f(x) = 3^{x^2} = e^{x^2 \log 3}$

$$\because f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{e^{(x+h)^2 \log 3} - e^{x^2 \log 3}}{h}$$

$$= \lim_{h \to 0} e^{x^2 \log 3} \frac{\left[e\left(^{(x+h)^2 - x^2} \right)^{\log 3} - 1 \right]}{h}$$

$$= \lim_{h \to 0} e^{x^2 \log 3} \frac{\left[e^{(x+h)^2 - x^2} \right]^{\log 3} - 1}{(x+h)^2 - x^2} \times \frac{(x+h)^2 - x^2}{h}$$

Multiplying Numerator and Denominator by $(x+h)^2 - x^2$

$$\therefore \lim_{h \to 0} e^{x^2 \log 3} \times \frac{(x+h+x)(x+h-x)}{h}$$

$= e^{x^2 \log 3} \times 2x$

$= 2x\, e^{x^2 \log 3}$

$= 2x\, 3^{x^2} \log 3$

Q4(i)

We have,

$f(x) = \tan^2 x$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\tan^2(x+h) - \tan^2 x}{h}$$

$$= \lim_{h \to 0} \frac{\{\tan(x+h) + \tan x\}\{\tan(x+h) - \tan x\}}{h} \qquad \left[\because \tan^2 A - \tan^2 B = (\tan A + \tan B)(\tan A - \tan B)\right]$$

$$= \lim_{h \to 0} \frac{\frac{\sin(x+h+x)}{\cos(x+h)\cos x} \times \frac{\sin(x+h-x)}{\cos(x+h)\cos x}}{h}$$

$$= \lim_{h \to 0} \frac{\sin(2x+h)}{h.\cos(x+h)\cos x} \times \frac{\sin h}{\cos(x+h)\cos x}$$

$$= \lim_{h \to 0} \frac{\sin h}{h} \times \frac{\sin 2x}{\cos^2 x.\cos^2(x+h)}$$

$$= \lim_{h \to 0} \frac{\sin 2x}{\cos^2 x.\cos^2 x} \qquad \left[\because \lim_{h \to 0} \frac{\sin h}{h} = 1\right]$$

$$= \lim_{h \to 0} \frac{2\sin x.\cos x}{\cos^2 x} \times \frac{1}{\cos^2 x} \qquad [\sin 2x = 2\sin x \cos x]$$

$= 2\tan x.\sec^2 x$

Q4(ii)

We have,

$f(x) = \tan(2x+1)$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\tan\{2(x+h)+1\} - \tan(2x+1)}{h}$$

$$= \lim_{h \to 0} \frac{\sin(2x+2h+1-2x-1)}{h.\cos\{2(x+h)+1\}\cos(2x+1)} \qquad \left[\because \tan A - \tan B = \frac{\sin(A-B)}{\cos A.\cos B}\right]$$

$$= \lim_{h \to 0} \frac{2.\sin 2h}{2h.\cos(2x+2h+1)\cos(2x+1)}$$

Multiplying both, Numerator and Denominator by 2.

$$\therefore \lim_{h \to 0} 2\left(\frac{\sin 2h}{2}\right) \times \frac{1}{\cos(2x+2h+1)\cos(2x+1)}$$

$$= \frac{2}{\cos^2(2x+1)} \qquad \left[\because \lim_{h \to 0} \frac{\sin 2h}{2} = 1\right]$$

$$= 2\sec^2(2x+1) \qquad \left[\because \sec^2 x = \frac{1}{\cos^2 x}\right]$$

$$= 2\sec^2(2x+1)$$

Q4(iii)

We have,

$f\left(x\right) = \tan 2x$

$$f'\left(x\right) = \lim_{h\to 0} \frac{f\left(x+h\right) - f\left(x\right)}{h}$$

$$= \lim_{h\to 0} \frac{\tan 2\left(x+h\right) - \tan 2x}{h}$$

$$= \lim_{h\to 0} \frac{\sin\left(2x+2h-2x\right)}{h.\cos\left(2x+2h\right)\cos 2x} \qquad \left[\because \tan A - \tan B = \frac{\sin\left(A-B\right)}{\cos A.\cos B}\right]$$

$$= \lim_{h\to 0} \frac{\sin 2h}{h.\cos\left(2x+2h\right)\cos 2x}$$

$$= \lim_{h\to 0}\left(\frac{\sin 2h}{2h}\right) \times \frac{1\times 2}{\cos\left(2h+2x\right)\cos 2x}$$

$$= \frac{2}{\cos 2x.\cos 2x} \qquad \left[\because \lim_{h\to 0}\frac{\sin 2h}{2h} = 1\right]$$

$$= 2\sec^2 2x \qquad \left[\because \frac{1}{\cos^2 x} = \sec^2 x\right]$$

Q4(iv)

We have,

$f(x) = \sqrt{\tan x}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{\tan(x+h)} - \sqrt{\tan x}}{h}$$

Multiplying Numerator and Denominator by $\sqrt{\tan(x+h)} + \sqrt{\tan x}$

$$= \lim_{h \to 0} \frac{\tan(x+h) - \tan x}{h\left(\sqrt{\tan(x+h)} + \sqrt{\tan x}\right)}$$

$$= \lim_{h \to 0} \frac{\sin(x+h-x)}{h.\cos(x+h)\cos x\left\{\sqrt{\tan(x+h)} + \sqrt{\tan x}\right\}}$$

$$= \lim_{h \to 0} \frac{\sinh}{h} \times \frac{1}{\cos(x+h)\cos x\left\{\sqrt{\tan(x+h)} + \sqrt{\tan x}\right\}}$$

$$= \lim_{h \to 0} \frac{1}{\cos^2 x.2\sqrt{\tan x}} \qquad \left[\lim_{h \to 0} \frac{\sinh}{h} = 1\right]$$

$$= \frac{1}{2} \frac{\sec^2 x}{\sqrt{tanx}} \qquad \left[\because \frac{1}{\cos^2 x} = \sec^2 x\right]$$

Q5(i)

$$\text{let}\, f(x) = \sin\sqrt{2x}.\ \text{Then}\, f(x+h) = \sin\sqrt{2(x+h)}$$

$$\frac{d(f(x))}{dx} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{\sin\sqrt{2(x+h)} - \sin\sqrt{2x}}{h}$$

$$= \lim_{h\to 0}\frac{2\sin\left(\dfrac{\sqrt{2(x+h)}-\sqrt{2x}}{2}\right)\cos\left(\dfrac{\sqrt{2(x+h)}+\sqrt{2x}}{2}\right)}{h}$$

$$= \lim_{h\to 0}\frac{\sin\left(\dfrac{\sqrt{2(x+h)}-\sqrt{2x}}{2}\right)}{\left(\dfrac{\sqrt{2(x+h)}-\sqrt{2x}}{2}\right)}\frac{\left(\sqrt{2(x+h)}-\sqrt{2x}\right)\left(\sqrt{2(x+h)}+\sqrt{2x}\right)}{\left(\sqrt{2(x+h)}+\sqrt{2x}\right)h}\cos\left(\frac{\sqrt{2(x+h)+2x}}{2}\right)$$

$$= \lim_{h\to 0}\frac{\sin\left(\dfrac{\sqrt{2(x+h)}-\sqrt{2x}}{2}\right)}{\left(\dfrac{\sqrt{2(x+h)}-\sqrt{2x}}{2}\right)}\lim_{h\to 0}\frac{2(x+h)-2x}{\left(\sqrt{2(x+h)}+\sqrt{2x}\right)h}\lim_{h\to 0}\cos\left(\frac{\sqrt{2(x+h)+2x}}{2}\right)$$

$$= 1\times\frac{2}{2\sqrt{2x}}\cos\left(\sqrt{2x}\right)$$

$$= \frac{\cos\left(\sqrt{2x}\right)}{\sqrt{2x}}$$

Q5(ii)

We have,

$f\left(x\right) = \cos\sqrt{x}$

$$f'\left(x\right) = \lim_{h\to 0}\frac{f\left(x+h\right) - f\left(x\right)}{h}$$

$$= \lim_{h\to 0}\frac{\cos\sqrt{x+h} - \cos\sqrt{x}}{h}$$

$$= \lim_{h\to 0} - 2\sin\frac{\left(\frac{\sqrt{x+h}+\sqrt{x}}{2}\right)\sin\left(\frac{\sqrt{x+h}-\sqrt{x}}{2}\right)}{h}$$

$$= \lim_{h\to 0} - 2\sin\frac{\left(\frac{\sqrt{x+h}-\sqrt{x}}{2}\right)\left(\sqrt{x+h}-\sqrt{x}\right)\left(\sqrt{x+h}+\sqrt{x}\right)}{\left(\frac{\sqrt{x+h}-\sqrt{x}}{2}\right)\left(\sqrt{x+h}+\sqrt{x}\right)h} \times \sin\left(\frac{\sqrt{x+h}+\sqrt{x}}{2}\right)$$

Multiplied Numerator and Denominator by $\left(\sqrt{x+h}-\sqrt{x}\right)$

$$= \lim_{h\to 0}\frac{-\sin\left(\frac{\sqrt{x+h}-\sqrt{x}}{2}\right)}{\left(\frac{\sqrt{x+h}-\sqrt{x}}{2}\right)} \times \frac{x+h-x}{\left(\sqrt{x+h}+\sqrt{x}\right)h} \times \sin\left(\frac{\sqrt{x+h}+\sqrt{x}}{2}\right)$$

$$= \lim_{h\to 0} -1\,\frac{h}{h\left(\sqrt{x+h}+\sqrt{x}\right)} \times \sin\left(\frac{\sqrt{x+h}+\sqrt{x}}{2}\right) \qquad \left[\because \lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1\right]$$

$$= \lim_{h\to 0}\frac{-1}{\left(\sqrt{x+h}+\sqrt{x}\right)}\sin\frac{\sqrt{x+h}+\sqrt{x}}{2}$$

$$= \frac{-\sin\sqrt{x}}{2\sqrt{x}}$$

Q5(iii)

We have,

$f(x) = \tan\sqrt{x}$

$$\because f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$\therefore f'(x) = \lim_{h\to 0}\frac{\tan\sqrt{(x+h)}-\tan\sqrt{x}}{h}$$

$$= \lim_{h\to 0}\frac{\sin\sqrt{x+h}-\sqrt{x}}{h.\cos\sqrt{x+h}\cos\sqrt{x}} \qquad \left[\because \tan A-\tan B=\frac{\sin(A-B)}{\cos A.\cos B}\right]$$

$$= \lim_{h\to 0}\frac{\sin\sqrt{x+h}-\sqrt{x}}{(x+h-x)\cos\sqrt{x}.\cos\sqrt{x+h}}$$

$$= \lim_{h\to 0}\frac{\sin\left(\sqrt{x+h}-\sqrt{x}\right)}{\left(\sqrt{x+h}-\sqrt{x}\right)\left(\sqrt{x+h}+\sqrt{x}\right)\cos\sqrt{x}.\cos\sqrt{x+h}}$$

$$= \lim_{h\to 0}\frac{\sin\left(\sqrt{x+h}-\sqrt{x}\right)}{\left(\sqrt{x+h}-\sqrt{x}\right)}\times\frac{1}{\left(\sqrt{x+h}+\sqrt{x}\right)\cos\sqrt{x}.\cos\sqrt{x+h}}$$

$$= 1\times\frac{1}{2\sqrt{x}\cos\sqrt{x}\cos\sqrt{x+}} \qquad \left[\because \lim_{h\to 0}=\frac{\sin\sqrt{x+h}-\sqrt{x}}{\sqrt{x+h}-\sqrt{x}}=1\right]$$

$$= \frac{1}{2\sqrt{x}\cos^2 x}$$

$$= \frac{\sec^2 x}{2\sqrt{x}}$$

Q5(iv)

We have,

$f(x) = \tan x^2$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\tan(x+h)^2 - \tan x^2}{h}$$

$$= \lim_{h \to 0} \frac{\frac{\sin(x+h)^2}{\cos(x+h)^2} - \frac{\sin x^2}{\cos x^2}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{\sin(x+h)^2 \cos x^2 - \cos(x+h)^2 \sin x^2}{\cos(x+h)^2 \cos x^2}}{h}$$

$$= \lim_{h \to 0} \frac{\sin\left((x+h)^2 - x^2\right)}{h.\cos(x+h)^2.\cos x^2}$$

$$= \lim_{h \to 0} \frac{\sin\left(x^2 + h^2 + 2hx - x^2\right)}{h.\cos(x+h)^2.\cos x^2}$$

$$= \lim_{h \to 0} \frac{\sin\left(h^2 + 2hx\right)}{h.\cos(x+h)^2.\cos x^2}$$

$$= \lim_{h \to 0} \frac{\sinh}{h} \times \frac{(h+2x)}{\cos(x+h)^2.\cos x^2}$$

$$= 1.\frac{2x}{\cos^2(x)^2} \qquad \left[\because \lim_{h \to 0} \frac{\sinh}{h} = 1\right]$$

$$= 2x \sec^2 x^2$$

Q6(i)

We have,

$f(x) = (-x)$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h\to 0} \frac{-(x+h) + x}{h}$$
$$= \lim_{h\to 0} \frac{-x - h + x}{h}$$
$$= -1$$

Q6(ii)

We have,

$f(x) = (-x)^{-1}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h\to 0} \frac{\frac{-1}{x+h} + \frac{1}{x}}{h}$$
$$= \lim_{h\to 0} \frac{-x + x + h}{hx(x+h)}$$
$$= \lim_{h\to 0} \frac{1}{x^2 + xh}$$
$$= \frac{1}{x^2}$$

Q6(iii)

We have,

$f(x) = \sin(x+1)$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sin(x+h+1) - \sin(x+1)}{h}$$

$$= \lim_{h\to 0} \frac{2\cos\left(\frac{x+h+1+x+1}{2}\right)\sin\left(\frac{x+h+1-x-1}{2}\right)}{h}$$

$$= \lim_{h\to 0} \frac{2\cos\left(\frac{2x+2+h}{2}\right)\sin\frac{h}{2}}{h} \qquad \left[\because \sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}\right]$$

$$= \lim_{h\to 0} \cos\left(\frac{2x+2+h}{2}\right)\left(\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right)$$

$$= \cos\left(\frac{2(x+1)}{2}\right) \qquad \left[\because \lim_{h\to 0}\frac{\sin\frac{h}{2}}{\frac{h}{2}} = 1\right]$$

$$= \cos(x+1)$$

Q6(iv)

We have;

$$f(x) = \cos\left(x - \frac{\pi}{8}\right)$$

$$\because f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\cos\left(x + h - \frac{\pi}{8}\right) - \cos\left(x - \frac{\pi}{8}\right)}{h}$$

$$= \lim_{h \to 0} \frac{-2\sin\left(\frac{2x + h - \frac{2\pi}{8}}{2}\right)\sin\left(\frac{h + x - \frac{\pi}{8} - x + \frac{\pi}{8}}{2}\right)}{h} \qquad \left[\because \cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}\right]$$

$$= \lim_{h \to 0} \frac{-2\sin\left(\frac{2x + h - \frac{2\pi}{8}}{2}\right) \times \sin\left(\frac{h}{2}\right)}{2 \cdot \frac{h}{2}}$$

$$= \lim_{h \to 0} \frac{-2\sin\left(\frac{2x + h - \frac{2\pi}{8}}{2}\right)}{2.} \times \lim_{h \to 0} \frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}$$

$$= \lim_{h \to 0} \sin\left(\frac{2x + h - \frac{2\pi}{8}}{2}\right) \qquad \left[\because \lim_{h \to 0} \frac{\sin\frac{h}{2}}{\frac{h}{2}} = 1\right]$$

$$= \sin\left(\frac{2x - \frac{2\pi}{8}}{2}\right)$$

$$= \sin\left(x - \frac{\pi}{8}\right)$$

EX 30.3

Q1

We have to differentiate $f(x)$ with respect to x

$$\frac{d}{dx}\left(x^4 - 2\sin x + 3\cos x\right)$$
$$= \frac{d(x)^4}{dx} - 2\frac{d}{dx}(\sin x) + 3\frac{d}{dx}(\cos x)$$
$$= 4x^3 - 2\cos x - 3\sin x$$

Q2

We have to differentiate $f[x]$ with respect to x

$$\frac{d}{dx}\left(3^x + x^3 + 3^3\right)$$
$$= \frac{d}{dx}\left(3^x\right) + \frac{d}{dx}\left(x^3\right) + \frac{d}{dx}\left(3^3\right)$$
$$= 3^x \log 3 + 3x^2 + 0 \qquad \left[\because \frac{d}{dx}^{(a^x)} = a^x \log a\right]$$
$$= 3^x \log 3 + 3x^2$$

Q3

We have to differentiate $f(x)$ with respect to x

$$\frac{d}{dx}\left(\frac{x^3}{3} - 2\sqrt{x} + \frac{5}{x^2}\right)$$
$$= \frac{1d}{3dx}^{(x3)} - 2\frac{d}{dx}^{(\sqrt{x})} + 5\frac{d}{dx}^{(x^{-2})}$$
$$= \frac{1}{3}.3x^2 - 2.\frac{1.1}{2\sqrt{x}} + 5.(-2)x^{-3}$$
$$= x^2 - x^{\frac{-1}{2}} - 10x^{-3}$$
$$= x^2 - \frac{1}{\sqrt{x}} - \frac{10}{x^3}$$

Q4

We have,

$$\frac{d}{dx}\left(e^{x\log a}+e^{a\log x}+e^{a\log a}\right)$$

$$=\frac{d}{dx}\left(e^{x\log a}\right)+\frac{d}{dx}\left(e^{a\log x}\right)+\frac{d}{dx}\left(e^{a\log a}\right)$$

$$=e^{x\log a}.\log a+e^{a\log x}.\frac{a}{x}+0 \qquad \left[\because e^{a\log a}\text{is constant}\right]$$

$$=\log a.e^{x\log a}+\frac{a}{x}e^{a\log x}$$

$$=\log a.a^{x}+\frac{a}{x}x^{a} \qquad \left[a^{x}\text{ can be written as }e^{x\log a}\right]$$

$$=a^{x}\log a+ax^{a-1}$$

Q5

We have,

$$\frac{d}{dx}\left(2x^{2}+1\right)\left(3x+2\right)$$

$$=\left(3x+2\right)\frac{d}{dx}\left(2x^{2}+1\right)+\left(2x^{2}+1\right)\frac{d}{dx}\left(3x+2\right) \qquad \left[\text{Using product rule}\right]$$

$$=\left(3x+2\right)\left(4x+0\right)+\left(2x^{2}+1\right)\left(3+0\right)$$

$$=\left(12x^{2}+8x+6x^{2}+3\right)$$

$$=18x^{2}+8x+3$$

Q6

We have,

$$\frac{d}{dx}f\left(x\right)=\frac{d}{dx}\left(\log_{3}x+3\log_{e}x+2\tan x\right)$$

$$=\frac{1}{\log 3}\frac{d}{dx}\left(\log x\right)+3\frac{d}{dx}\left(\log_{e}x\right)+2\frac{d}{dx}\left(\tan x\right) \qquad \left[\because \log_{3}x=\frac{\log x}{\log 3}\right]$$

$$=\frac{1}{\log 3}\times\frac{1}{x}+\frac{3}{x}+2\sec^{2}x$$

$$=\frac{1}{x\log 3}+\frac{3}{x}+2\sec^{2}x$$

Q7

We have,

$$\frac{d}{dx}\left(x+\frac{1}{x}\right)\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)$$

$$=\left(x+\frac{1}{x}\right)\frac{d}{dx}\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)+\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)\frac{d}{dx}\left(x+\frac{1}{x}\right) \quad \text{[Using product rule]}$$

$$=\left(x+\frac{1}{x}\right)\left(\frac{1}{2\sqrt{x}}-\frac{1}{2x^{\frac{3}{2}}}\right)+\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)\left(1-\frac{1}{x^2}\right)$$

$$=\left(\frac{x}{2\sqrt{x}}-\frac{x}{2x^{\frac{3}{2}}}+\frac{1}{2x^{\frac{3}{2}}}-\frac{1}{2x^{\frac{5}{2}}}\right)+\left(\sqrt{x}-\frac{\sqrt{x}}{x^2}+\frac{1}{\sqrt{x}}-\frac{1}{x^{\frac{5}{2}}}\right)$$

$$=\left(\frac{1}{2}\sqrt{x}-\frac{1}{2\sqrt{x}}+\frac{1}{2x^{\frac{3}{2}}}-\frac{1}{2x^{\frac{5}{2}}}+\sqrt{x}-\frac{1}{x^{\frac{3}{2}}}+\frac{1}{\sqrt{x}}-\frac{1}{x^{\frac{5}{2}}}\right)$$

$$=\left(\frac{3}{2}\sqrt{x}+\frac{1}{2}\sqrt{x}-\frac{1}{2x^{\frac{3}{2}}}-\frac{3}{2x^{\frac{5}{2}}}\right)$$

$$=\frac{3}{2}x^{\frac{1}{2}}+\frac{1}{2}x^{\frac{-1}{2}}-\frac{1}{2}x^{\frac{-3}{2}}-\frac{3}{2}x^{\frac{-5}{2}}$$

Q8

We have,

$$\frac{d}{dx}\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^3$$

$$=\frac{d}{dx}\left(x^{3/2}+3x.\frac{1}{\sqrt{x}}+3\sqrt{x}.\frac{1}{x}+\frac{1}{x^{3/2}}\right) \quad \left[(a+b)^3=a^2+3a^2b+3ab^2+b^3\right]$$

$$=\frac{d}{dx}\left(x^{3/2}+3x^{1/2}+3x^{-1/2}+x^{-3/2}\right)$$

$$=\frac{3}{2}x^{1/2}+\frac{3}{2}x^{-1/2}+3.\left(\frac{-1}{2}\right)x^{-3/2}-\frac{3}{2}x^{-5/2}$$

$$=\frac{3}{2}x^{1/2}-\frac{3}{2}x^{-5/2}+\frac{3}{2}x^{-1/2}-\frac{3}{2}x^{-3/2}$$

Q9

We have,

$$\frac{d}{dx}\left(\frac{2x^2+3x+4}{x}\right)$$

$$= \frac{d}{dx}\left(\frac{2x^2}{x}+\frac{3x}{x}+\frac{4}{x}\right)$$

$$= \frac{d}{dx}\left(2x+3+4x^{-1}\right)$$

$$= 2-\frac{4}{x^2}$$

Q10

We have,

$$\frac{d}{dx}\frac{\left\{\left(x^3+1\right)\left(x-2\right)\right\}}{x^2}$$

$$= \frac{d}{dx}\frac{\left\{\left(x^4-2x^3+x-2\right)\right\}}{x^2}$$

$$= \frac{d}{dx}\left(x^2-2x+x^{-1}-2x^{-2}\right)$$

$$= \frac{d}{dx}\left(x^2\right)-2\frac{dx}{dx}+\frac{dx^{-1}}{dx}-2\frac{dx^{-2}}{dx}$$

$$= 2x-2-\frac{1}{x^2}+2.\frac{2}{x^3}$$

$$= 2x-2-\frac{1}{x^2}+\frac{4}{x^3}$$

Q11

We have,

$$\frac{d}{dx}\left(\frac{a\cos x+b\sin x+c}{\sin x}\right)$$

$$= a\frac{d}{dx}\left(\frac{\cos x}{\sin x}\right)+b\frac{d}{dx}(1)+c\frac{d}{dx}\left(\frac{1}{\sin x}\right)$$

$$= a\left(-\operatorname{cosec}^2 x\right)+0+c\left(-\operatorname{cosec} x.\cot x\right)$$

$$= -a\operatorname{cosec}^2 x-c\operatorname{cosec} x.\cot x$$

Q12

We have,

$$\frac{d}{dx}\left(2\sec x + 3\cot x - 4\tan x\right)$$

$$= 2\frac{d}{dx}\left(2\sec x\right) + 3\frac{d}{dx}\left(\cot x\right) - 4\frac{d}{dx}\left(\tan x\right)$$
$$= 2\sec x\tan x - 3\operatorname{cosec}^2 x - 4\sec^2 x$$

Q13

We have,

$$\frac{d}{dx}\left(a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x + a_n\right)$$

$$= a_0\frac{d(x)^n}{dx} + a_1\frac{d(x)^{n-1}}{dx} + a_2\frac{d(x)^{n-2}}{dx} + \ldots + a_{n-1}\frac{d(x)}{dx} + a_n\frac{d(1)}{dx}$$
$$= na_0x^{n-1} + (n-1)a_1x^{n-2} + \ldots + a_{n-1} + 0$$
$$= na_0x^{n-1} + (n-1)a_1x^{n-2} + \ldots + a_{n-1}$$

Q14

We have,

$$\frac{d}{dx}\left(\frac{1}{\sin x} + 2^{x+3} + \frac{4}{\log x^3}\right)$$

$$= \frac{d}{dx}\operatorname{cosec}x + 2^3\frac{d}{dx}\left(2^x\right) + \frac{4}{\log 3}\times\frac{d}{dx}\left(\log x\right) \quad \left[\because\ \log_b a = \frac{\log a}{\log b}\right]$$
$$= -\operatorname{cosec}x.\cot x + 8.2^x\log 2 + \frac{4}{\log 3}\times\frac{1}{x} \qquad \left[\because \frac{d}{dx}\left(a^x\right) = a^x\log a\right]$$

$$= -\operatorname{cosec}x\cot x + 2^{x+3}\log 2 + \frac{4}{x\log 3}$$

Q15

We have,

$$\frac{d}{dx}\left\{\frac{(x+5)(2x^2-1)}{x}\right\}$$

$$= \frac{d}{dx}\left(\frac{2x^3+10x^2-x-5}{x}\right)$$

$$= \frac{d}{dx}\left(2x^2+10x-1-5x^{-1}\right)$$

$$= 2\frac{d}{dx}\left(x^2\right)+10\frac{d}{dx}(x)-\frac{d}{dx}(1)-5\frac{d}{dx}\left(x^{-1}\right)$$

$$= 2\times 2x+10-0+\frac{5}{x^2}$$

$$= 4x+10+\frac{5}{x^2}$$

Q16

$$\frac{d}{ax}\left\{\log\left(\frac{1}{\sqrt{x}}\right)+5x^a-3a^x+\sqrt[3]{x^2}+6\sqrt[4]{x^{-3}}\right\}$$

$$= \frac{d}{ax}\log\left(\frac{1}{\sqrt{x}}\right)+5\frac{d}{ax}\left(x^a\right)-3\left(a^x\right)+\frac{d}{ax}\left(\sqrt[3]{x^2}\right)+6\frac{d}{ax}\left(\sqrt[4]{x^{-3}}\right)$$

$$= \frac{-1}{2}\frac{1}{x}+5ax^{a-1}-3a^x\log a+\frac{2x^{-1/3}}{3}+6x^{-7/4}\left(-3/4\right)$$

$$= \frac{-1}{2x}+5ax^{a-1}-3a^x\log a+\frac{2x^{-1/3}}{3}-\frac{9}{2}x^{-7/4}$$

Q17

We have,

$$\frac{d}{dx}\{\cos(x+a)\}$$

$$= \frac{d}{dx}(\cos x.\cos a-\sin x.\sin a) \qquad [\because \cos(x+a)=\cos x\cos a-\sin x\sin a]$$

$$= \cos a\frac{d}{dx}(\cos x)-\sin a\frac{d}{dx}(\sin x)$$

$$= \cos a(-\sin x)-\sin a(\cos x)$$

$$= \cos x\sin a+\sin x\cos a$$

$$= -(\sin x\cos a+\cos x\sin a)$$

$$= -\sin(x+a)$$

Q18

We have,

$$\frac{d}{dx}\frac{\{\cos(x-2)\}}{\sin x}$$

$$= \frac{d}{dx}\frac{(\cos x.\cos 2 + \sin x.\sin 2)}{\sin x}$$

$$= \cos 2\frac{d}{dx}(\cot x) + \sin 2\frac{d}{dx}(1)$$

$$= -\cos 2.\operatorname{cosec}^2 x + 0$$

$$= -\operatorname{cosec}^2 x \cos 2$$

Q19

We have,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2$$

$$= \frac{d}{dx}\left(\sin^2\frac{x}{2} + \cos^2\frac{x}{2} + 2\sin\frac{x}{2}.\cos\frac{x}{2}\right)$$

$$= \frac{d}{dx}(1+\sin x) \qquad \begin{bmatrix}\because \sin^2\theta + \cos^2\theta = 1 \\ \sin^2\theta = 2\sin\theta.\cos\theta\end{bmatrix}$$

$$= 0 + \cos x$$

$$= \cos x$$

$$\frac{dy}{dx}\, at\, x = \frac{\pi}{6}$$

$$= \cos\frac{\pi}{6}$$

$$= \frac{\sqrt{3}}{2}$$

Q20

We have,

$$y = \frac{2 - 3\cos x}{\sin x}$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{2 - 3\cos x}{\sin x}\right)$$

$$= \frac{d}{dx}\left(2\operatorname{cosec} x - 3\cot x\right)$$

$$= 2\frac{d}{dx}\left(\operatorname{cosec} x\right) - 3\frac{d}{dx}\left(\cot x\right)$$

$$= -2\operatorname{cosec} x . \cot x + 3\operatorname{cosec}^2 x$$

$$\frac{dy}{dx} \; at \; x = \frac{\pi}{4}$$

$$= -2\operatorname{cosec}\frac{\pi}{4} . \cot\frac{\pi}{4} + 3\operatorname{cosec}^2\frac{\pi}{4}$$

$$= -2\sqrt{2} - 1 + 3.2$$

$$= -2\sqrt{2} + 6$$

$$= 6 - 2\sqrt{2}$$

Q21

Slope of the tangent at a point $x = a$ is the value of the derivative at $x = a$.

We have,

$$f(x) = 2x^6 + x^4 - 1$$

$$= \frac{d}{dx}^{(2x^6 + x^4 - 1)}$$

$$= 2\frac{dx^6}{dx} + \frac{dx^4}{dx} - \frac{d.1}{dx}$$

$$= 12x^5 + 4x^3 - 0$$

$$= 12x^5 + 4x^3$$

$$\frac{dy}{dx} \; at \; x = 1$$

$$= 12(1)^5 + 4(1)^3$$

$$= 12 + 4$$

$$= 16$$

$\therefore$ The slope of the tangent to the curve $f(x) = 2x^6 + x^4 - 1$ at $x = 1$ is 16.

Q22

We have,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sqrt{\frac{x}{a}}+\sqrt{\frac{a}{x}}\right)$$

$$= \frac{1}{\sqrt{a}}\frac{d}{dx}\sqrt{x} + \sqrt{a}\frac{d}{dx}\left(\frac{1}{\sqrt{x}}\right)$$

$$= \frac{1}{\sqrt{a}}\frac{1}{2\sqrt{x}} + \sqrt{a}\left(\frac{-1}{2}\right)\times\frac{1}{x\sqrt{x}}$$

$$= \frac{1}{2x}\left(\sqrt{\frac{x}{a}}+\left(-\sqrt{\frac{a}{x}}\right)\right)$$

$$\Rightarrow 2x\frac{dy}{dx} = \sqrt{\frac{x}{a}} - \sqrt{\frac{a}{x}}$$

Multiplying both side by $y = \sqrt{\frac{x}{a}} + \sqrt{\frac{a}{x}}$

$$\Rightarrow 2xy\frac{dy}{dx} = \left(\sqrt{\frac{x}{a}} - \sqrt{\frac{a}{x}}\right)\left(\sqrt{\frac{x}{a}} + \sqrt{\frac{a}{x}}\right)$$

$$= \left(\frac{x}{a} - \frac{a}{x}\right)$$

Hence, proved.

Q23

We have,

$$f(x) = x^4 - 2x^3 + 3x^2 + x + 5$$

Differentiate with respect to x

$$\frac{d f(x)}{dx} = \frac{d}{dx}\left(x^4 - 2x^3 + 3x^2 + x + 5\right)$$

$$= 4x^3 - 6x^2 + 6x + 1$$

Q24

We have,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{2x^9}{3} - \frac{5}{7}x^7 + 6x^3 - x\right)$$

$$= \frac{2}{3}\frac{dx^9}{dx} - \frac{5}{7}\frac{dx^7}{dx} + 6\frac{dx^3}{dx} - \frac{d.x}{dx}$$
$$= \frac{2}{3}.9x^8 - \frac{5}{7}.7x^6 + 18x^2 - 1$$
$$= 6x^8 - 5x^6 + 18x^2 - 1$$

$\therefore \frac{dy}{dx} at\, x = 1$

$$= 6(1)^8 - 5(1)^6 + 18(1)^2 - 1$$
$$= 6 - 5 + 18 - 1$$
$$= 18$$

Q25

We have,

$f(x) = \lambda x^2 + \mu x + 12$

$\Rightarrow f'(x) = 2\lambda x + \mu$ ....(i)

but, $f'(4) = 15$

from (i)

$8\lambda + \mu = 15$ ....(ii)

also, $f'(12) = 11$

$4\lambda + \mu = 11$ ....(iii)

$(ii) - (iii)$ gives

$4\lambda = 4$

$\Rightarrow \lambda = 1$

from (ii)

$8.1 + \mu = 15$

$\Rightarrow \mu = 7$

Hence,

$\lambda = 1$ and $\mu = 7$

Q26

We have,

$$f(x) = \frac{x^{100}}{100} + \frac{x^{99}}{99} + \ldots + \frac{x^2}{2} + x + 1$$

Differentiating with respect to x, we get

$$f'(x) = x^{99} + x^{98} + \ldots + x + 1 + 0 \text{ --- } (i)$$

from (i)

$$f'(1) = 1 + 1 + \ldots (100 \text{ times})$$

$$= 100$$

Again,

$$f'(0) = 0 + 0 + \ldots + 1$$

$$= 1$$

Now,

$$f'(1) = 100 = 100 \times 1 = 100 \times f'(0)$$

Hence,

$$f'(1) = 100 f'(0)$$

Ex 30.4

Q1

We have,

$$\frac{d}{dx}\left(x^3 \sin x\right)$$
$$= \sin x \frac{d\left(x^3\right)}{dx} + x^3 \frac{d\left(\sin x\right)}{dx} \quad \text{[Using product rule]}$$
$$= \sin x . 3x^2 + x^3 . \cos x$$
$$= x^2\left(3 \sin x + x \cos x\right)$$

Q2

We have,

$$\frac{d}{dx}\left(x^3 e^x\right)$$
$$= e^x \frac{d}{dx}\left(x^3\right) + x^3 \frac{d}{dx}\left(e^x\right) \quad \text{[Using product rule]}$$
$$= e^x 3x^2 + x^3 e^x$$
$$= x^2 e^x \left(3 + x\right)$$

Q3

We have,

$$\frac{d}{dx}\left(x^2 e^x \log x\right)$$
$$= e^x \log x \frac{d}{dx}\left(x^2\right) + x^2 \log x \frac{d}{dx}\left(e^x\right) + x^2 e^x \frac{d}{dx}\left(\log x\right) \quad \text{[Usingproductrule]}$$
$$= e^x \log x . 2x + x^2 \log x . e^x + x^2 e^x . \frac{1}{x}$$
$$= x e^x \left(2 . \log x + x \log x + 1\right)$$

Q4

We have,

$$\frac{d}{dx}\left(x^n \tan x\right)$$
$$= \tan x \frac{d}{dx}\left(x^n\right) + x^n \frac{d}{dx}\left(\tan x\right) \quad \text{[Usingproductrule]}$$
$$= \tan x . n x^{n-1} + x^n \sec^2 x$$
$$= x^{n-1}\left(n . \tan x + x . \sec^2 x\right) \quad \left[x^n = x^{n-1} x^1 = x^{n-1+1}\right]$$

Q5

We have,

$$\frac{d}{dx}\left(x^n \log_a x\right)$$

$$= \log_a x \frac{d}{dx}\left(x^n\right) + x^n \frac{d}{dx}\left(\log_a x\right) \qquad \text{[Using product rule]}$$

$$= nx^{n-1}.\log_a x + \frac{x^n}{\log a}.\frac{1}{x} \qquad \left[\because \log_a x = \frac{\log x}{\log a}\right]$$

$$= x^{n-1}\left[n.\log_a x + \frac{1}{\log a}\right]$$

Q6

We have,

$$\frac{d}{dx}\left(x^3 + x^2 + 1\right)\sin x$$

$$= \sin x \frac{d}{dx}\left(x^3 + x^2 + 1\right) + \left(x^3 + x^2 + 1\right)\frac{d}{dx}(\sin x) \qquad \text{[Using product rule]}$$

$$= \sin x\left(3x^2 + 2x\right) + \left(x^3 + x^2 + 1\right)\cos x$$

$$\therefore \left(x^3 + x^2 + 1\right)\cos x + \left(3x^2 + 2x\right)\sin x$$

Q7

We have,

$$\frac{d}{dx}(\sin x \times \cos x)$$

$$\cos x \frac{d}{dx}(\sin x) + \sin x \frac{d}{dx}(\cos x) \qquad \text{[using product rule]}$$

$$= \cos x\,(\cos x) + \sin x\,(-\sin x)$$

$$= \cos^2 x - \sin^2 x \qquad \left[\because \cos 2x = \cos^2 x - \sin^2 x\right]$$

$$= \cos 2x$$

Q8

We have,

$$\frac{d}{dx}\left(2^x \times \cot x \times x^{-1/2}\right)$$

$$= \cot x \times \frac{1}{\sqrt{x}} \times \frac{d}{dx}\left(2^x\right) + 2^x \times \frac{1}{\sqrt{x}} \times \frac{d}{dx}(\cot x) + 2^x \times \cot x \times \frac{d}{dx}\left(x^{-1/2}\right) \qquad \text{[Using procuct rule]}$$

$$= \frac{\cot x}{\sqrt{x}} \times 2^x \times \log 2 + \frac{2^x}{\sqrt{x}}\left(-\operatorname{cosec}^2 x\right) + 2^x \times \cot x \left(-\frac{1}{2}\right)\frac{1}{2^{3/2}}$$

$$= \frac{2x}{\sqrt{x}}\left(\cot x \times \log 2 - \operatorname{cosec}^2 x - \frac{\cot x}{2x}\right)$$

Q9

$$\frac{d}{dx}\left(x^2 \sin x \log x\right)$$

$$= \sin x \log x \frac{d}{dx}\left(x^2\right) + x^2 \log x \frac{d}{dx}(\sin x) + x^2 \sin x \frac{d}{dx}(\log x) \qquad \text{[Using product rule]}$$

$$= \sin x \log x \times 2x + x^2 \log x \times \cos x + x^2 \sin x \times \frac{1}{x}$$

$$= 2x \times \sin x \times \log x + x^2 \times \cos x \times \log x + x \sin x$$

Q10

We have,

$$\frac{d}{dx}\left(x^5 e^x + x^6 \log x\right)$$

$$= \frac{d}{dx}\left(x^5 e^x\right) + \frac{d}{dx}\left(x^6 \log x\right)$$

$$= e^x \frac{dx^5}{dx} + x^5 \frac{de^5}{dx} + \log x \frac{d}{dx}\left(x^6\right) + x^6 \frac{d}{dx}(\log x) \qquad \text{[Using product rule]}$$

$$= e^x \times 5x^4 + x^5 \times e^x + \log x \times 6x^5 + x^6 \times \frac{1}{x}$$

$$= 5x^4 \times e^x + x^5 \times e^x + 6x^5 \times \log x + x^5$$

$$= x^4\left(5e^x + ex^x + 6x \log x + x\right)$$

Q11

We have,

$$\frac{d}{dx}\{(x\sin x + \cos x)(x\cos x - \sin x)\}$$

We will apply product rule,

$$= (x\cos x - \sin x)\frac{d}{dx}(x\sin x + \cos x) + (x\sin x - \cos x)\frac{d}{dx}(x\cos x - \sin x)$$
$$= (x\cos x - \sin x)\left\{\frac{d}{dx}(x\sin x) + \frac{d}{dx}(\cos x)\right\} + (x\sin x + \cos x)\left\{\frac{d}{dx}(x\cos x) - \frac{d}{dx}(\sin x)\right\}$$

Again apply product rule,

$$= (x\cos x - \sin x)\left\{\left(\sin x\frac{dx}{dx} + x\frac{d\sin x}{dx}\right)\right\} + (-\sin x) + (x\cos x + \sin x)\left\{\left(\sin x\frac{dx}{dx} + x\frac{d\cos x}{dx} - \cos x\right)\right\}$$
$$= (x\cos x - \sin x)\{(\sin x + x\cos x) - \sin x\} - (x\sin x + \cos x)\{(\cos x - x\sin x - \cos x)\}$$
$$= (x\cos x - \sin x)\times x\cos x + (x\sin x + \cos x)(-x\sin x)$$
$$= (x^2\cos^2 x - x\sin x\times\cos x) + (-x^2\sin^2 x - x\sin x\times\cos x)$$
$$= x^2(\cos^2 x - \sin^2 x) - x(\sin x\cos x + \sin x\cos x)$$
$$= x^2 - \cos 2x - x\times 2\sin x\cos x$$
$$= x^2\cos x 2x - x\sin 2x$$
$$= x\{x\times\cos 2x - \sin 2x\}$$

Q12

We have,

$$\frac{d}{dx}\{(x\sin x + \cos x)(e^x + x^2\log x)\}$$

We will apply product rule,

$$= (e^x + x^2\log x)\frac{d}{dx}(x\sin x + \cos x) + (x\sin x + \cos x)\frac{d}{dx}(e^x + x^2\log x)$$
$$= (e^x + x^2\log x)\left(\frac{d}{dx}(x\sin x) + \frac{d}{dx}\cos x\right) + (x\sin x + \cos x)\times\left\{\frac{d}{dx}(e^x) + \frac{d}{dx}(x^2\log x)\right\}$$

Again apply product rule,

$$= (e^x + x^2\log x)\left(\sin x\frac{d}{dx}(x) + x\frac{d}{dx}(\sin x)\right) - \sin x + (x\sin x + \cos x)\left\{e^x + \left(\log x\frac{d}{dx}(x^2) + x^2\frac{d}{dx}(\log x)\right)\right\}$$
$$= (e^x + x^2\log x)(\sin x + x\cos x - \sin x) + (x\sin x + \cos x)\left(e^x + \log x\times 2x + x^2\frac{1}{x}\right)$$
$$= (e^x + x^2\log x)x\cos x + (x\sin x + \cos x)(e^x + 2x\times\log x + x)$$
$$= x\cos x e^x + e^x\cos x\log x + xe^x\sin x + e^x\cos x + 2x^2\sin x\times\log x + 2x\cos x\log x + x^2\sin x + x\cos x$$
$$= x\cos x(e^x + x^2\log x) + (x\sin x + \cos x)(e^x + x + 2x\log x)$$

Q13

We have,

$$\frac{d}{dx}\left\{(1-2\tan x)(5+4\sin x)\right\}$$

$$= (5+4\sin x)\frac{d}{dx}(1-2\tan x)+(1-2\tan x)\frac{d}{dx}(5+4\sin x) \quad \text{[Using product rule,]}$$

$$= (5+4\sin x)\left(0-2\sec^2 x\right)+(1-2\tan x)(0+4\cos x)$$

$$= -10\sec^2 x - 8\sin x \times \sec^2 x + 4\cos x - 8\cos x \times \tan x$$

$$= 4\left(\frac{-5}{2}\sec^2 x - 2\sin x \times \frac{1}{\cos^2 x} + \cos x - 2\cos x \times \frac{\sin x}{\cos x}\right)$$

$$= 4\left(\frac{-5}{2}\sec^2 x - 2\tan x \sec x + \cos x - 2\sin x\right)$$

$$= 4\left(\cos x - 2\sin x - 2\tan x \sec x - \frac{5}{2}\sec^2 x\right)$$

Q14

We have,

$$\frac{d}{dx}\left\{\left(1+x^2\right)\cos x\right\}$$

$$= \cos x\frac{d}{dx}\left(1+x^2\right)+\left(1+x^2\right)\frac{d}{dx}(\cos x) \quad \text{(using product rule)}$$

$$= \cos x \times 2x + \left(1+x^2\right)(-\sin x)$$

$$= 2x\cos x - \left(1+x^2\right)\sin x$$

Q15

We have,

$$\frac{d}{dx}\left(\sin^2 x\right)$$

$$= \frac{d}{dx}(\sin x)(\sin x)$$

$$= \sin x\frac{d}{dx}(\sin x) + \sin x\frac{d}{dx}(\sin x) \quad \text{[Using product rule]}$$

$$= \sin x \times \cos x + \sin x \times \cos x$$

$$= 2\sin x\cos x$$

$$= \sin 2x \quad [\because \sin 2A = 2\sin A\cos A]$$

Q16

We have,

$$\frac{d}{dx}\left(\log_{x^2} x\right)$$

$$\log_{x^2} x = \frac{\log x}{\log x^2}$$

$$= \frac{\log x}{2\log x}$$

$$= \frac{1}{2}$$

$$\frac{d}{dx}\left(\frac{1}{2}\right) = 0$$

$$\therefore \frac{d}{dx}\left(\log_{x^2} x\right) = 0$$

Q17

$$\frac{d}{dx}\left(e^x \log\sqrt{x}\ \tan x\right)$$

Apply product rule,

$$= \log\sqrt{x} \times \tan x \frac{d}{dx}\left(e^x\right) + e^x \times \tan x \frac{d}{dx}\left(\log\sqrt{x}\right) + e^x \log\sqrt{x} \frac{d}{dx}\left(\tan x\right)$$

$$= \log\sqrt{x} \times \tan e^x + e^x \tan x \frac{1}{2x} + e^x \log\sqrt{x} \times \sec^2 x$$

$$= \frac{1}{2}\log x \times \tan x \times e^x + \frac{1\tan x}{2x} e^x + e^x \frac{1}{2}\log x \sec^2 x \qquad \left[\because \log\sqrt{x} = \frac{1}{2}\log x\right]$$

$$= \frac{1}{2}e^x \left(\log x \times \tan x + \frac{\tan x}{x} + \log x \sec^2 x\right)$$

Q18

We have,

$$\frac{d}{dx}\left(x^3 e^x \cos x\right)$$

$$= e^x \cos x \frac{d}{dx}\left(x^3\right) + x^3 \cos x \frac{d}{dx}\left(e^x\right) + x^3 e^x \frac{d}{dx}\left(\cos x\right) \qquad \text{[Using product rule]}$$

$$= e^x \cos x \times 3x^2 + x^3 \cos x \times e^x + x^3 e^x \left(-\sin x\right)$$

$$= x^2 e^x \left(3\cos x + x\cos x + x\left(-\sin x\right)\right)$$

$$= x^2 e^x \left(3\cos x + x\cos x - x\sin x\right)$$

Q19

We have,

$$\frac{d}{dx}\left(x^2 \cos\frac{\pi}{4} \times \operatorname{cosec} x\right)$$

$$= \cos\frac{\pi}{4} \operatorname{cosec} x \frac{d}{dx}\left(x^2\right) + x^2 \operatorname{cosec} x \frac{d}{dx}\left(\cos\frac{\pi}{4}\right) + x^2 \cos\frac{\pi}{4} \frac{d}{dx}\left(\operatorname{cosec} x\right) \quad \text{[Using product rule,]}$$

$$= \cos\frac{\pi}{4} \operatorname{cosec} x \times 2x + x^2 \operatorname{cosec} x \times 0 + x^2 \cos\frac{\pi}{4}\left(-\operatorname{cosec} x \cot x\right) \quad \left[\because \frac{d}{dx}\left(\cos\frac{\pi}{4}\right) = 0\right]$$

$$= \left(\frac{2x}{\sin x} - \frac{x^2 \operatorname{cosec} x}{\sin^2 x}\right)\cos\frac{\pi}{4}$$

$$\therefore \cos\frac{\pi}{4}\left(\frac{2x}{\sin x} - \frac{x^2 \cos x}{\sin x}\right)$$

Q20

We have,

$$= \frac{d}{dx}\left\{x^4\left(5\sin x - 3\cos x\right)\right\}$$

$$= \frac{d}{dx} x^4 5\sin x - 3x^4 \cos x$$

$$= 5\frac{d}{dx}\left(x^4 \sin x\right) - 3\frac{d}{dx}\left(x^4 \cos x\right)$$

$$= 5\left(\sin x \frac{d}{dx}\left(x^4\right) + x^4 \frac{d}{dx}(\sin x)\right) - 3\left(\cos x \frac{d}{dx}\left(x^4\right) + x^4 \frac{d}{dx}(\cos x)\right) \quad \text{[Apply product rule,]}$$

$$= 5\left(\sin x \times 4x^3 + x^4 \times \cos x\right) - 3\left(\cos x \times 4x^3 + x^4(-\sin x)\right)$$

$$= 20x^3 \times \sin x + 5x^4 \cos x - 12x^3 \cos x + 3x^4 \sin x$$

Q21

We have,

$$\frac{d}{dx}\left(2x^2 - 3\right)\sin x$$

$$= \sin x \frac{d}{dx}\left(2x^2 - 3\right) + \left(2x^2 - 3\right)\frac{d}{dx}(\sin x) \quad \text{[Using product rule]}$$

$$= \sin x \times 4x + \left(2x^2 - 3\right)\cos x$$

$$= 4x \sin x + \left(2x^2 - 3\right)\cos x$$

Q22

We have,

$$\frac{d}{dx}x^5\left(3-6x^{-9}\right)$$

$$=\left(3-6x^{-9}\right)\frac{d}{dx}\left(x^5\right)+x^5\frac{d}{dx}\left(3-6x^{-9}\right) \qquad \text{[Using product rule]}$$

$$=\left(3-6x^{-9}\right)5x^4+x^5\left(54x^{-10}\right)$$

$$=15x^4-30x^5+54x^{-5}$$

$$=15x^4+24x^{-5}$$

Q23

We have,

$$\frac{d}{dx}\left\{x^{-4}\left(3-4x^{-5}\right)\right\}$$

$$=\left(3-4x^{-5}\right)\frac{d}{dx}\left(x^{-4}\right)+x^{-4}\frac{d}{dx}\left(3-4x^{-5}\right) \qquad \text{[Using product rule]}$$

$$=\left(3-4x^{-5}\right)\left(-4x^{-5}\right)+\left(x^{-4}\right)20x^{-6}$$

$$=-12x^{-5}+16x^{-10}+20x^{-10}$$

$$=-12x^{-5}+36x^{-10}$$

Q24

We have,

$$\frac{d}{dx}\left\{x^{-3}\left(5+3x\right)\right\}$$

Apply product rule,

$$=5+3x\,\frac{d\left(x^{-3}\right)}{dx}+x^{-3}\,\frac{d\left(5+3x\right)}{dx}$$

$$=\left(5+3x\right)\left(-3x^{-4}\right)+x^{-3}\left(3\right)$$

$$=-15x^{-4}-9x^{-3}+3x^{-3}$$

$$=-15x^{-4}-6x^{-3}$$

Q25

$$\frac{d}{dx}\left(\frac{(ax+b)}{(cx+d)}\right) = \left(\frac{1}{cx+d}\right)\frac{d}{dx}(ax+b) + (ax+b)\frac{d}{dx}\left(\frac{1}{cx+d}\right)$$

$$= \left(\frac{1}{cx+d}\right)(a) + (ax+b)\frac{d}{dx}(cx+d)^{-1}$$

$$= \left(\frac{1}{cx+d}\right)(a) + (ax+b)\left(-1 \times (cx+d)^{-2} \times c\right)$$

$$= \left(\frac{a}{cx+d}\right) - \left(\frac{c(ax+b)}{(cx+d)^2}\right)$$

$$= \frac{a(cx+d) - c(ax+b)}{(cx+d)^2}$$

$$= \frac{ad-bc}{(cx+d)^2}$$

Q26

$$\frac{d}{dx}(ax+b)^n (cx+d)^m = (cx+d)^m \frac{d}{dx}(ax+b)^n + (ax+b)^n \frac{d}{dx}(cx+d)^m$$

$$= (cx+d)^m \left[n \times (ax+b)^{n-1} \times a\right] + (ax+b)^n \left[m \times (cx+d)^{m-1} \times c\right]$$

$$= na(cx+d)^m (ax+b)^{n-1} + mc(ax+b)^n (cx+d)^{m-1}$$

$$= na(cx+d)^{m-1} (ax+b)^{n-1} \left[na(cx+d) + mc(ax+b)\right]$$

Q27

Using product rule

$$\frac{d}{dx}(1+2\tan x)(5+4\cos x) = (1+2\tan x)\frac{d}{dx}(5+4\cos x)+(5+4\cos x)\frac{d}{dx}(1+2\tan x)$$
$$= (1+2\tan x)\left[0+4(-\sin x)\right]+(5+4\cos x)\left[0+2\left(\sec^2 x\right)\right]$$
$$= -4(1+2\tan x)\sin x+2(5+4\cos x)\sec^2 x$$
$$= -4\sin x-8\tan x\sin x+10\sec^2 x+8\cos x\sec^2 x$$
$$= -4\sin x-\frac{8\sin^2 x}{\cos x}+10\sec^2 x+\frac{8}{\cos x}$$
$$= -4\sin x+10\sec^2 x+\frac{8\left[1-\sin^2 x\right]}{\cos x}$$
$$= -4\sin x+10\sec^2 x+\frac{8\cos^2 x}{\cos x}$$
$$= -4\sin x+10\sec^2 x+8\cos x$$

Using alternate method

$$\frac{d}{dx}(1+2\tan x)(5+4\cos x) = \frac{d}{dx}(5+4\cos x+10\tan x+8\tan x\cos x)$$
$$= \frac{d}{dx}(5+4\cos x+10\tan x+8\sin x)$$
$$= -0+4(-\sin x)+10\left(\sec^2 x\right)+8\cos x$$
$$= -4\sin x+10\sec^2 x+8\cos x$$

Q28(i)

Using product rule

$$\frac{d}{dx}\left(3x^2+2\right)^2 = \left(3x^2+2\right)\frac{d}{dx}\left(3x^2+2\right)+\left(3x^2+2\right)\frac{d}{dx}\left(3x^2+2\right)$$
$$=\left(3x^2+2\right)(6x+0)+\left(3x^2+2\right)(6x+0)$$
$$= 18x^3+12x+18x^3+12x$$
$$= 36x^3+24x$$

Using alternate method

$$\frac{d}{dx}\left(3x^2+2\right)^2 = \frac{d}{dx}\left(9x^4+12x^2+4\right)$$
$$= \left(36x^3+24x+0\right)$$
$$= 36x^3+24x$$

Q28(ii)

Using product rule

$$\frac{d}{dx}(x+2)(x+3) = (x+2)\frac{d}{dx}(x+3) + (x+3)\frac{d}{dx}(x+2)$$
$$= (x+2)(1+0) + (x+3)(1+0)$$
$$= x+2+x+3$$
$$= 2x+5$$

Using alternate method

$$\frac{d}{dx}(x+2)(x+3) = \frac{d}{dx}\left(x^2+5x+6\right)$$
$$= (2x+5+0)$$
$$= 2x+5$$

Q28(iii)

Using product rule

$$\frac{d}{dx}(3\sec x - 4\operatorname{cosec} x)(-2\sin x - 5\cos x)$$
$$= (3\sec x - 4\operatorname{cosec} x)\frac{d}{dx}(-2\sin x + 5\cos x) + (-2\sin x + 5\cos x)\frac{d}{dx}(3\sec x - 4\operatorname{cosec} x)$$
$$= (3\sec x - 4\operatorname{cosec} x)(-2\cos x + 5(-\sin x)) + (-2\sin x + 5\cos x)(3\sec x\tan x - 4(-\operatorname{cosec} x\cot x))$$
$$= -6\sec x\cos x - 15\sec x\sin x + 8\operatorname{cosec} x\cos x - 20\operatorname{cosec} x\sin x$$
$$-6\sin x\sec x\tan x - 8\sin x\operatorname{cosec} x\cot x + 15\cos x\sec x\tan x + 20\cos x\operatorname{cosec} x\cot x$$
$$= -6 - 15\tan x + 8\cot x + 20 - 6\tan^2 x - 8\cot x + 15\tan x + 20\cot^2 x$$
$$= -6 - 6\tan^2 x + 20 + 20\cot^2 x$$
$$= -6\left(1+\tan^2 x\right) - 20\left(1+\cot^2 x\right)$$
$$= -6\sec^2 x + 20\operatorname{cosec}^2 x$$

Using alternate method

$$\frac{d}{dx}(3\sec x - 4\operatorname{cosec} x)(-2\sin x - 5\cos x)$$
$$= \frac{d}{dx}(-6\sec x\sin x + 15\sec x\cos x + 8\operatorname{cosec} x\sin x - 20\operatorname{cosec} x\cos x)$$
$$= \frac{d}{dx}(-6\tan x + 15 + 8 - 20\cot x)$$
$$= -6\sec^2 x + 0 + 0 - 20\left(-\operatorname{cosec}^2 x\right)$$
$$= -6\sec^2 x + 20\operatorname{cosec}^2 x$$

EX – 30.5

Q1

Using quotient rule, we have

$$\frac{d}{dx}\left(\frac{x^2+1}{x+1}\right) = \frac{(x+1)\frac{d}{dx}(x^2+1)-(x^2+1)\frac{d}{dx}(x+1)}{(x+1)^2}$$

$$= \frac{(x+1)\times 2x-(x^2+1)\times 1}{(x+1)^2}$$

$$= \frac{2x^2+2x-x^2-1}{(x+1)^2}$$

$$= \frac{x^2+2x-1}{(x+1)^2}$$

Q2

Using quotient rule, we have get,

$$\frac{d}{dx}\left(\frac{2x-1}{x^2+1}\right)$$

$$= \frac{(x^2+1)\frac{d}{dx}(2x-1)-(2x-1)\frac{d}{dx}(x^2+1)}{(x^2+1)^2}$$

$$= \frac{(x^2+1)\times 2-(2x-1)\times 2x}{(x^2+1)^2}$$

$$= \frac{2x^2+2-4x^2+2x}{(x^2+1)^2}$$

$$= \frac{-2x^2+2x+2}{(x^2+1)^2}$$

$$= \frac{2(-x^2+x+1)}{(x^2+1)^2}$$

$$= \frac{2(1+x-x^2)}{(1+x^2)^2}$$

Q3

By using quotient rule, we have,

$$\frac{d}{dx}\left(\frac{x+e^x}{1+\log x}\right)$$

$$=\frac{(1+\log x)\frac{d}{dx}(x+e^x)-(x+e^x)\frac{d}{dx}(1+\log x)}{(1+\log x)^2}$$

$$=\frac{(1+\log x)(1+e^x)-(x+e^x)\times\frac{d}{dx}}{(1+\log x)^2}$$

$$=\frac{x(1+\log x+e^x+e^x\log x)-x-e^x}{x(1+\log x)^2}$$

$$=\frac{x+x\log x+xe^x+xe^x\log x-x-e^x}{x(1+\log x)^2}$$

$$=\frac{x\log x(1+e^x)-e^x(1-x)}{x(1+\log x)^2}$$

Q4

Using quotient rule, we have,

$$\frac{d}{dx}\left(\frac{e^x-\tan x}{\cot x-x^n}\right)$$

$$=\frac{(\cot x-x^n)\frac{d}{dx}(e^x-\tan x)-(e^x-\tan x)\frac{d}{dx}(\cot x-x^n)}{(\cot x-x^n)^2}$$

$$=\frac{(\cot x-x^n)(e^x-\sec^2 x)-(e^x-\tan x)(-\operatorname{cosec}^2 x-nx^{n-1})}{(\cot x-x^n)^2}$$

$$=\frac{(\cot x-x^n)(e^x-\sec^2 x)+(e^x-\tan x)(\operatorname{cosec}^2 x+nx^{n-1})}{(\cot x-x^n)^2}$$

Q5

Using quotient rule, we have,

$$\frac{d}{dx}\left(\frac{ax^2+bx+c}{px^2+qx+r}\right)$$

$$=\frac{\left(px^2+qx+r\right)\frac{d}{dx}\left(ax^2+bx+c\right)-\left(ax^2+bx+c\right)\frac{d}{dx}\left(px^2+qx+r\right)}{\left(px^2+qx+r\right)^2}$$

$$=\frac{\left(px^2+qx+r\right)(2ax+b)-\left(ax^2+bx+c\right)(2px+q)}{\left(px^2+qx+r\right)^2}$$

$$=\frac{2apx^3+2aqx^2+2axr+bpx^2+bqx+br-\left(2apx^3+2pbx^2+2pcx+qax^2+bqx+cq\right)}{\left(px^2+qx+r\right)^2}$$

$$=\frac{2apx^3-2apx^3+2aqx^2+bpx^2-2pbx^2-qax^2+2arx+bqx-2pcx-bqx+br-cq}{\left(px^2+qx+r\right)^2}$$

$$=\frac{aqx^2-bpx^2+2arx-2cpx+br-cq}{\left(px^2+qx+r\right)^2}$$

$$=\frac{x^2(aq-bp)+2(ar-cp)x+br-cq}{\left(px^2+qx+r\right)^2}$$

$$=\frac{(aq-bp)x^2+2(ar-cp)x+br-cq}{\left(px^2+qx+r\right)^2}$$

Q6

Using quotient rule, we have,

$$\frac{d}{dx}\left(\frac{x}{1+\tan x}\right)$$

$$=\frac{(1+\tan x)\frac{d}{dx}(x)-x\frac{d}{dx}(1+\tan x)}{(1+\tan x)^2}$$

$$=\frac{(1+\tan x)-x\left(\sec^2 x\right)}{(1+\tan x)^2}$$

$$=\frac{1+\tan x-x\sec^2 x}{(1+\tan x)^2}$$

Q7

Using quotient rule, we have

$$\frac{d}{dx}\left(\frac{1}{ax^2+bx+c}\right)$$

$$=\frac{\left(ax^2+bx+c\right)\frac{d}{dx}(1)-1\times\frac{d}{dx}\left(ax^2+bx+c\right)}{\left(ax^2+bx+c\right)^2}$$

$$=\frac{-(2ax+b)}{\left(ax^2+bx+c\right)^2}$$

$$\therefore \frac{d}{dx}\frac{1}{ax^2+bx+c}=\frac{-(2ax+b)}{\left(ax^2+bx+c\right)^2}$$

Q8

We have,

$$\frac{d}{dx}\left(\frac{e^x}{1+x^2}\right)$$

Using quotient rule,

$$=\frac{\left(1+x^2\right)\frac{d}{dx}\left(e^x\right)-\left(e^x\right)\frac{d}{dx}\left(1+x^2\right)}{\left(1+x^2\right)^2}$$

$$=\frac{\left(1+x^2\right)e^x-e^x\times 2x}{\left(1+x^2\right)^2}$$

$$=\frac{e^x\left(1+x^2-2x\right)}{\left(1+x^2\right)^2}$$

$$=\frac{e^x(1-x)^2}{\left(1+x^2\right)^2}$$

Q9

We have,

$$\frac{d}{dx}\left(\frac{e^x+\sin x}{1+\log x}\right)$$

Using quotient rule, we get

$$=\frac{(1+\log x)\frac{d}{dx}(e^x+\sin x)-(e^x+\sin x)\frac{d}{dx}(1+\log x)}{(1+\log x)^2}$$

$$=\frac{(1+\log x)(e^x+\cos x)-(e^x+\sin x)\frac{1}{x}}{(1+\log x)^2}$$

$$=\frac{x(1+\log x)(e^x+\cos x)-(e^x+\sin x)}{x(1+\log x)^2}$$

Q10

We have,

$$\frac{d}{dx}\left(\frac{x\tan x}{\sec x+\tan x}\right)$$

Using quotient rule, we get

$$=\frac{(\sec x+\tan x)\frac{d}{dx}(x\tan x)-(x\tan x)\frac{d}{dx}(\sec x+\tan x)}{(\sec x+\tan x)^2}$$

$$=\frac{(\sec x+\tan x)(x\sec^2 x+\tan x)-(x\tan x)(\sec x\tan x+\sec^2 x)}{(\sec x+\tan x)^2}\quad\text{[Used product rule]}$$

$$=\frac{(\sec x+\tan x)(x\sec^2 x+\tan x)-x\sec x+\tan^2 x-x\tan x\sec^2 x}{(\sec x+\tan x)^2}$$

$$=\frac{(\sec x+\tan x)(x\sec^2 x+\tan x)-x\tan x(\sec x\tan x+\sec^2 x)}{(\sec x+\tan x)^2}$$

$$=\frac{(\sec x+\tan x)(x\sec^2 x+\tan x)-x\tan x\sec x(\sec x+\tan x)}{(\sec x+\tan x)^2}$$

$$=\frac{(x\sec^2 x+\tan x-x\tan x\sec x)(\sec x+\tan x)}{(\sec x+\tan x)^2}$$

$$=\frac{(x\sec^2 x+\tan x-x\tan x\sec x)}{(\sec x+\tan x)}$$

$$=\frac{x\sec x(\sec x-\tan x)+\tan x}{(\sec x+\tan x)}$$

Q11

We have,

$$\frac{d}{dx}\left(\frac{x\sin x}{1+\cos x}\right)$$

Using quotient rule, we get

$$=\frac{(1+\cos x)\frac{d}{dx}(x\sin x)-(x\sin x)\frac{d}{dx}(1+\cos x)}{(1+\cos x)^2}$$

$$=\frac{(1+\cos x)\left(x\frac{d}{dx}\sin x+\sin x\frac{d}{dx}\right)-x\sin x(-\sin x)}{(1+\cos x)^2}$$ [Used product rule]

$$=\frac{(1+\cos x)(x\cos x+\sin x)+x\sin^2 x}{(1+\cos x)^2}$$

$$=\frac{x\cos x+x\cos^2 x+\sin x+\sin x\cos x+x\sin^2 x}{(1+\cos x)^2}$$

$$=\frac{(x\cos x+\sin x+\sin x\cos x)+x\left(\sin^2 x+\cos^2 x\right)}{(1+\cos x)^2}$$

$$=\frac{\sin x+\sin x\cos x+x\left(\cos x+\sin^2 x+\cos^2 x\right)}{(1+\cos x)^2}$$

$$=\frac{\sin x(1+\cos x)+x(\cos x+1)}{(1+\cos x)^2}$$

$$=\frac{(x+\sin x)(\cos x+1)}{(1+\cos x)^2}$$

Q12

We have,

$$\frac{d}{dx}\left(\frac{2^x \cot x}{\sqrt{x}}\right)$$

Using quotient rule, we get

$$\frac{\sqrt{x}\frac{d}{dx}\left(2^x \cot x\right)-\left(2^x \cot x\right)\frac{d}{dx}\left(\sqrt{x}\right)}{\left(\sqrt{x}\right)^2}$$

$$=\frac{\sqrt{x}\left(2^x \frac{d}{dx}\cot x+\cot x\frac{d}{dx}2^x\right)-2^x \cot x\times\frac{1}{2}x^{-1/2}}{\left(\sqrt{x}\right)^2}$$

$$=\frac{\sqrt{x}\left(2^x\ -\operatorname{cosec}^2 x+\cot x\ \times\log 2\times 2^x\right)-2^x \cot x\times\frac{1}{2\sqrt{x}}}{\left(\sqrt{x}\right)^2}$$

$$=\frac{2^x\left\{-x\operatorname{cosec}^2 x+x\cot x\times\log 2-\left(\frac{1}{2}\right)\cot x\right\}}{\left(\sqrt{x}\right)^2\times\sqrt{x}}$$

$$=\frac{2^x\left(-x\operatorname{cosec}^2 x+x\cot x\times\log 2-\left(\frac{1}{2}\right)\cot x\right)}{x^{3/2}}$$

Q13

We have,

$$\frac{d}{dx}\left(\frac{\sin x - x\cos x}{x\sin x + \cos x}\right)$$

Apply quotient rule, we get

$$\frac{(x\sin x + \cos x)\frac{d}{dx}(\sin x - x\cos x) - (\sin x - x\cos x)\frac{d}{dx}(x\sin x + \cos x)}{(x\sin x + \cos x)^2}$$

$$= \frac{(x\sin x + \cos x)\left\{\cos x - \left(\frac{dx}{dx}\cos x + \cos x\frac{dx}{dx}\right)\right\} - (\sin x - x\cos x)\left(\frac{dx}{dx}\sin x + \sin x\frac{dx}{dx}\right) + \frac{d}{dx}\cos x}{(x\sin x + \cos x)^2}$$

$$= \frac{(x\sin x + \cos x)(\cos x + x\sin x - \cos x) - (\sin x - x\cos x)(x\cos x + \sin x - \sin x)}{(x\sin x + \cos x)^2}$$

$$= \frac{(x\sin x + \cos x)x\sin x - (\sin x - x\cos x)x\cos x}{(x\sin x + \cos x)^2}$$

$$= \frac{x^2\sin^2 x + x\sin x\cos x - x\sin x\cos x + x^2\cos^2 x}{(x\sin x + \cos x)^2}$$

$$= \frac{x^2(\sin^2 x + \cos^2 x)}{(x\sin x + \cos x)^2} \qquad (\because \sin^2 x + \cos^2 x = 1)$$

$$= \frac{x^2}{x\sin x + \cos x}$$

Q14

We have,

$$\frac{d}{dx}\left(\frac{x^2-x+1}{x^2+x+1}\right)$$

Using quotient rule,

$$\frac{\left(x^2+x+1\right)\frac{d}{dx}\left(x^2-x+1\right)-\left(x^2-x+1\right)\frac{d}{dx}\left(x^2+x+1\right)}{\left(x^2+x+1\right)^2}$$

$$=\frac{\left(x^2+x+1\right)(2x-1)-\left(x^2-x+1\right)(2x+1)}{\left(x^2+x+1\right)^2}$$

$$=\frac{\left(x^2+1-x\right)(2x-1)-\left(x^2-x+1\right)(2x+1)}{\left(x^2+x+1\right)^2}$$

$$=\frac{2x^3+2x+2x^2-x^2-1-x-2x^3+2x^2-2x-x^2+x-1}{\left(x^2+x+1\right)^2}$$

$$=\frac{2x^2-2}{\left(x^2+x+1\right)^2}$$

$$=\frac{2\left(x^2-1\right)}{\left(x^2+x+1\right)^2}$$

Q15

We have,

$$\frac{d}{dx}\left(\frac{\sqrt{a}+\sqrt{x}}{\sqrt{a}-\sqrt{x}}\right)$$

Using quotient rule,

$$\frac{\left(\sqrt{a}-\sqrt{x}\right)\frac{d}{dx}\left(\sqrt{a}+\sqrt{x}\right)-\left(\sqrt{a}+\sqrt{x}\right)\frac{d}{dx}\left(\sqrt{a}-\sqrt{x}\right)}{\left(\sqrt{a}-\sqrt{x}\right)^2}$$

$$=\frac{\left(\sqrt{a}-\sqrt{x}\right)\times\frac{1}{2\sqrt{x}}-\left(\sqrt{a}+\sqrt{x}\right)\times\frac{-1}{2\sqrt{x}}}{\left(\sqrt{a}-\sqrt{x}\right)^2}$$

$$=\frac{\sqrt{a}-\sqrt{x}+\sqrt{a}+\sqrt{x}}{2\sqrt{x}\left(\sqrt{a}-\sqrt{x}\right)^2}$$

$$==\frac{\sqrt{a}}{\sqrt{x}\left(\sqrt{a}-\sqrt{x}\right)^2}$$

Q16

We have,

$$\frac{d}{dx}\left(\frac{a+\sin x}{1+a\sin x}\right)$$

Using quotient rule, we get

$$\frac{\left(1+a\sin x\right)\frac{d}{dx}\left(a+\sin x\right)-\left(a+\sin x\right)\frac{d}{dx}\left(1+a\sin x\right)}{\left(1+a\sin x\right)^2}$$

$$=\frac{\left(1+a\sin x\right)\cos x-\left(a+\sin x\right)a\cos x}{\left(1+a\sin x\right)^2}$$

$$=\frac{c\,os\,x+a\sin x\cos x-a^2\cos x-a\sin x\cos x}{\left(1+a\sin x\right)^2}$$

$$=\frac{\left(1-a^2\right)\cos x}{\left(1+a\sin x\right)^2}$$

Q17

We have,

$$\frac{d}{dx}\left(\frac{10^x}{\sin x}\right)$$

Using quotient rule,

$$\frac{(\sin x)\frac{d}{dx}(10^x) - (10^x)\frac{d}{dx}(\sin x)}{(\sin x)^2}$$

$$= \frac{\sin x \times 10^x \log 10 - 10^x \cos x}{(\sin x)^2}$$

$$= 10^x \operatorname{cosec} x \log 10 - 10^x \operatorname{cosec} x \cot x$$

$$= 10^x \operatorname{cosec} x (\log 10 - \cot x)$$

Q18

We have,

$$\frac{d}{dx}\left(\frac{1+3^x}{1-3^x}\right)$$

Using quotient rule,

$$\frac{(1-3^x)\frac{d}{dx}(1+3^x) - (1+3^x)\frac{d}{dx}(1-3^x)}{(1-3^x)^2}$$

$$= \frac{(1-3^x)3^x \log 3 + (1+3^x)3^x \log 3}{(1-3^x)^2}$$

$$= \frac{3^x \log 3 - 3^x \times 3^x \log 3 + 3^x \log 3 + 3^x \times 3^x \log 3}{(1-3^x)^2}$$

$$= \frac{2 \times 3^x \log 3}{(1-3^x)^2}$$

Q19

We have,

$$\frac{d}{dx}\left(\frac{3^x}{x+\tan x}\right)$$

Applying quotient rule,

$$\frac{(x+\tan x)\frac{d}{dx}(3^x) - 3^x\frac{d}{dx}(x+\tan x)}{(x+\tan x)^2}$$

$$= \frac{(x+\tan x)\times 3^x\log 3 - 3^x(1+\sec^2 x)}{(x+\tan x)^2}$$

$$= \frac{3^x\left[(x+\tan x)\log 3 - (1+\sec^2 x)\right]}{(x+\tan x)^2}$$

Q20

We have,

$$\frac{d}{dx}\left(\frac{1+\log x}{1-\log x}\right)$$

Using quotient rule,

$$\frac{(1-\log x)\frac{d}{dx}(1+lox) - (1+\log x)\frac{d}{dx}(1-\log x)}{(1-\log x)^2}$$

$$= \frac{(1-\log x)\times\frac{1}{x} - (1+\log x)\left(-\frac{1}{x}\right)}{(1-\log x)^2}$$

$$= \frac{1-\log x+1+\log x}{x(1-\log x)^2}$$

$$== \frac{2}{x(1-\log x)^2}$$

Q21

We have,

$$\frac{d}{dx}\left(\frac{4x+5\sin x}{3x+7\cos x}\right)$$

Using quotient rule, we get

$$\frac{(3x+7\cos x)\frac{d}{dx}(4x+5\sin x)-(4x+5\sin x)\frac{d}{dx}(3x+7\cos x)}{(3x+7\cos x)^2}$$

$$=\frac{(3x+7\cos x)(4x+5\sin x)-(4x+5\sin x)(3+7(-\sin x))}{(3x+7\cos x)^2}$$

$$=\frac{12x+28\cos x+15x\cos x+13\cos^2 x-12x-15\sin x+28x\sin x+25\sin^2 x}{(3x+7\cos x)^2}$$

$$=\frac{15x\cos x+28x\sin x+28\cos x-15\sin x+35\left(\sin^2 x+\cos^2 x\right)}{(3x+7\cos x)^2} \qquad \left(\because \sin^2 x+\cos^2 x=1\right)$$

$$\therefore \frac{15x\cos x+28x\sin x+28\cos x-15\sin x+35}{(3x+7\cos x)^2}$$

Q22

$$\frac{d}{dx}\left(\frac{1}{ax^2+bx+c}\right)$$

$$=\frac{d}{dx}\left(ax^2+bx+c\right)^{-1}$$

$$=\frac{-(2ax+b)}{\left(ax^2+bx+c\right)^2}$$

Q23

We have,

$$\frac{d}{dx}\left(\frac{a+b\sin x}{c+d\cos x}\right)$$

Using quotient rule, we get

$$\frac{(c+d\cos x)\frac{d}{dx}(a+b\sin x)-(a+b\sin x)\frac{d}{dx}(c+d\cos x)}{(c+d\cos x)^2}$$

$$=\frac{(c+d\cos x)(b\cos x)-(a+b\sin x)(-d\sin x)}{(c+d\cos x)^2}$$

$$=\frac{bc\cos x+bd\cos^2 x+ad\sin x+bd\sin^2 x}{(c+d\cos x)^2}$$

$$=\frac{bc\cos x+ad\sin x+bd\left(\sin^2 x++\cos^2 x\right)}{(c+d\cos x)^2}$$

$$=\frac{bc\cos x+ad\sin x+bd}{(c+d\cos x)^2}$$

Q24

We have,

$$\frac{d}{dx}\left(\frac{px^2+qx+r}{ax+b}\right)$$

Using quotient rule, we get

$$\frac{(ax+b)\frac{d}{dx}\left(px^2+qx+r\right)-\left(px^2+qx+r\right)\frac{d}{dx}(ax+b)}{(ax+b)^2}$$

$$=\frac{(ax+b)(2px+q)-\left(px^2+qx+r\right)a}{(ax+b)^2}$$

$$=\frac{2apx^2+2pbx+aqx+bq-apx^2-aqx-ar}{(ax+b)^2}$$

$$=\frac{apx^2+2pbx+bq-ar}{(ax+b)^2}$$

Q25

We have,

$$\frac{d}{dx}\left(\frac{\sec x - 1}{\sec x + 1}\right)$$

$$\frac{(\sec x + 1)\frac{d}{dx}(\sec x - 1) - (\sec x - 1)\frac{d}{dx}(\sec x + 1)}{(\sec x + 1)^2}$$

$$= \frac{(\sec x + 1)(\sec x \tan x) - (\sec x - 1)(\sec x \tan x)}{(\sec x + 1)^2}$$

$$= \frac{\sec x \tan x (\sec x + 1 - \sec x + 1)}{(\sec x + 1)^2}$$

$$= \frac{2 \sec x \tan x}{(\sec x + 1)^2}$$

Q26

We have,

$$\frac{d}{dx}\left(\frac{x^5 - \cos x}{\sin x}\right)$$

Using quotient rule, we get

$$\frac{(\sin x)\frac{d}{dx}(x^5 - \cos x) - (x^5 - \cos x)\frac{d}{dx}(\sin x)}{(\sin^2 x)}$$

$$= \frac{\sin x (5x^4 \sin x) - (x^5 - \cos x)\cos x}{(\sin^2 x)}$$

$$= \frac{5x^4 \sin x + \sin^2 x - x^5 \cos x + \cos^2 x}{(\sin^2 x)} \qquad (\because \sin^2 x + \cos^2 x = 1)$$

$$= \frac{-x^5 \cos x + 5x^4 \sin x + 1}{(\sin^2 x)}$$

Q27

We have,

$$\frac{d}{dx}\left(\frac{x+\cos x}{\tan x}\right)$$

Using quotient rule, we get

$$\frac{(\tan x)\frac{d}{dx}(x+\cos x)-(x+\cos x)\frac{d}{dx}(\tan x)}{(\tan^2 x)}$$

$$=\frac{\tan x\{1+(-\sin x)\}-(x+\cos x)\sec^2 x}{(\tan^2 x)}$$

$$=\frac{(1-\sin x)\tan x-(x+\cos x)\sec^2 x}{(\tan^2 x)}$$

Q28

$$\frac{d}{dx}\left(\frac{x^n}{\sin x}\right)$$

$$=x^n\frac{d}{dx}(\sin x)^{-1}+\frac{1}{\sin x}\frac{d}{dx}(x^n)$$

$$=x^n\frac{-1}{\sin^2 x}+\frac{1}{\sin x}nx^{n-1}$$

$$=\frac{\sin x(nx^{n-1})-x^n(\cos x)}{\sin^2 x}$$

Q29

$$\frac{d}{dx}\left(\frac{ax+b}{px^2+qx+r}\right)=\frac{(px^2+qx+r)\frac{d}{dx}(ax+b)-(ax+b)\frac{d}{dx}(px^2+qx+r)}{(px^2+qx+r)^2}$$

$$=\frac{(px^2+qx+r)(a)-(ax+b)(2px+q)}{(px^2+qx+r)^2}$$

$$=\frac{(apx^2+aqx+ar)-(2apx^2+aqx+2bpx+bq)}{(px^2+qx+r)^2}$$

$$=\frac{-(apx^2+2bpx+bq-ar)}{(px^2+qx+r)^2}$$

Q30

$$\frac{d}{dx}\left(\frac{1}{ax^2+bx+c}\right)$$

$$=\frac{\left(ax^2+bx+c\right)\frac{d}{dx}(1)-(1)\frac{d}{dx}\left(ax^2+bx+c\right)}{\left(ax^2+bx+c\right)^2}$$

$$=\frac{\left(ax^2+bx+c\right)(0)-(1)(2ax+b)}{\left(ax^2+bx+c\right)^2}$$

$$=\frac{-(2ax+b)}{\left(ax^2+bx+c\right)^2}$$