Ex 6.1

Determinants Ex 6.1 Q1(i)

Let M_{ij} and C_{ij} represents the minor and co-factor respectively of an element which is placed at the i^{th} row and j^{th} column.

Now,

$M_{11} = -1$ $\quad\left[\begin{array}{l}\text{In a } 2\times 2 \text{ matrix, the minor is obtained for a particular element, by}\\ \text{deleting that row and column where the element is present.}\end{array}\right]$

$M_{21} = 20$

$C_{11} = (-1)^{1+1} \times M_{11}$ $\quad\left[\because C_{ij} = (-1)^{i+j} \times M_{ij}\right]$

$= (+1)(-1)$

$= -1$

$C_{21} = (-1)^{2+1} M_{21}$

$= (-1)^3 \times 20$

$= -20$

Also,

$|A| = 5(-1) - (0) \times (20)$ $\quad\left[\text{If } A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \text{ then } |A| = a_{11}a_{22} - a_{21}a_{12}\right]$

$= -5$

Determinants Ex 6.1 Q1(ii)

Let M_{ij} and C_{ij} represents the minor and co-factor respectively of an element which is present at the i^{th} row and j^{th} column.

Now,

$M_{11} = 3$ $\left[\begin{array}{l}\text{In a } 2\times 2 \text{ matrix, the minor of an element is obtained by} \\ \text{deleting that row and that column in which it is present.}\end{array}\right]$

$M_{21} = 4$

$C_{11} = (-1)^{1+1} \times M_{11}$ $\left[C_{ij} = (-1)^{i+j} \times M_{ij}\right]$

$C_{21} = (-1)^{2+1} \times M_{21}$

$= (-1)^3 \times 4$

$= -4$

Also,

$|A| = (-1)\times(3) - (2)\times(4)$

$= -3 - 8$

$= -11$

Determinants Ex 6.1 Q1(iii)

Let M_{ij} and C_{ij} represents the minor and co-factor respectively of an element which is placed at the i^{th} row and j^{th} column.

Now,

$M_{11} = \begin{bmatrix} -1 & 2 \\ 5 & 2 \end{bmatrix}$ $\left[\begin{array}{l}\text{In a } 3\times 3 \text{ matrix, } M_{ij} \text{ equals to the determinant of the } 2\times 2 \\ \text{sub-matrix obtained by leaving the } i^{th} \text{ row and } j^{th} \text{ column of } A.\end{array}\right]$

$= (-1)\times(2) - (5)\times(2)$

$= -2 - 10$

$= -12$

$M_{21} = \begin{bmatrix} -3 & 2 \\ 5 & 2 \end{bmatrix} = (-3)\times(2) - (5)\times(2) = -6 - 10 = -16$

$M_{31} = \begin{bmatrix} -3 & 2 \\ -1 & 2 \end{bmatrix} = (-3)(2) - (-1)(2) = -6 + 2 = -4$

$C_{11} = (-1)^{1+1} M_{11}$ $\left(C_{ij} = (-1)^{i+j} \times M_{ij}\right)$

$= (+)(-12) = -12$

$C_{21} = (-1)^{2+1} M_{21} = (-1)^3 (-16) = 16$

$C_{31} = (-1)^{3+1} M_{31} = (-1)^4 (-4) = -4$

Also, expanding the determinant along the first column.

$|A| = a_{11} \times \left((-1)^{1+1} \times M_{11}\right) + a_{21} \times \left((-1)^{2+1} \times M_{21}\right) + a_{31} \times \left((-1)^{3+1} \times M_{31}\right)$

$= a_{11} \times C_{11} + a_{21} \times C_{21} + a_{31} \times C_{31}$

$= 1 \times (-12) + 4 \times 16 + 3 \times (-4)$

$= -12 + 48 - 12 = 24$

Determinants Ex 6.1 Q1(iv)

Let M_{ij} and C_{ij} are respectively the minor and co-factor of the element a_{ij}.

Now,

$$M_{11} = \begin{bmatrix} b & ca \\ c & ab \end{bmatrix}$$
$$= ab^2 - ac^2$$

$$M_{21} = \begin{bmatrix} a & bc \\ c & ab \end{bmatrix}$$
$$= a^2b - c^2b$$

$$M_{31} = \begin{bmatrix} a & bc \\ b & ca \end{bmatrix}$$
$$= a^2c - b^2c$$

$$C_{11} = (-1)^{1+1} \times M_{11} = +\left(ab^2 - ac^2\right)$$
$$C_{21} = (-1)^{2+1} \times M_{21} = -\left(a^2b - c^2b\right)$$
$$C_{31} = (-1)^{3+1} \times M_{31} = +\left(a^2c - b^2c\right)$$

Also, expanding the determinant, along the first column.

$$\begin{aligned} |A| &= a_{11}C_{11} + a_{21}C_{21} + a_{31}C_{31} \\ &= 1\left(ab^2 - ac^2\right) + 1\left(c^2b - a^2b\right) + 1 \times \left(a^2c - b^2c\right) \\ &= ab^2 - ac^2 + c^2b - a^2b + a^2c - b^2c \end{aligned}$$

Determinants Ex 6.1 Q1(v)

Let M_{ij} and C_{ij} are respectively the minor and co-factor of the element a_{ij}.

Now,

$$M_{11} = \begin{bmatrix} 5 & 0 \\ 7 & 1 \end{bmatrix} = 5 - 0 = 5$$
$$M_{21} = \begin{bmatrix} 2 & 6 \\ 7 & 1 \end{bmatrix} = 2 - 42 = -40$$
$$M_{31} = \begin{bmatrix} 2 & 6 \\ 5 & 0 \end{bmatrix} = 0 - 30 = -30$$

$$C_{11} = (-1)^{1+1} \times M_{11} = +5$$
$$C_{21} = (-1)^{2+1} \times M_{21} = (-)(-40) = 40$$
$$C_{31} = (-1)^{3+1} \times M_{31} = +(-30) = -30$$

Now, expanding the determinant along the first column.

$$\begin{aligned} |A| &= a_{11}C_{11} + a_{21}C_{21} + a_{31}C_{31} \\ &= 0 \times 5 + 1 \times (40) + 3 \times (-30) \\ &= 40 - 90 \\ &= -50 \end{aligned}$$

Determinants Ex 6.1 Q1(vi)

Let M_{ij} and C_{ij} are respectively the minor and co-factor of the element a_{ij}.

Now,

$$M_{11} = \begin{bmatrix} b & f \\ f & c \end{bmatrix} = bc - f^2$$

$$M_{21} = \begin{bmatrix} h & g \\ f & c \end{bmatrix} = hc - gf$$

$$M_{31} = \begin{bmatrix} h & g \\ b & f \end{bmatrix} = hf - bg$$

$$Also\; C_{11} = (-1)^{1+1} M_{11} = bc - f^2$$

$$C_{21} = (-1)^{2+1} M_{21} = -(hc - gf)$$

$$C_{31} = (-1)^{3+1} M_{31} = hf - bg$$

Also, expanding along the first column.

$$\begin{aligned} |A| &= a_{11}C_{11} + a_{21}C_{21} + a_{31}C_{31} \\ &= a\left(bc - f^2\right) + h(-)(hc - gf) + g(hf - bg) \\ &= abc - af^2 + hgf - h^2c + ghf - bg^2 \end{aligned}$$

Determinants Ex 6.1 Q1(vii)

We have,

$$\begin{bmatrix} 2 & -1 & 0 & 1 \\ -3 & 0 & 1 & -2 \\ 1 & 1 & -1 & 1 \\ 2 & -1 & 5 & 0 \end{bmatrix}$$

Here, $M_{11} = \begin{bmatrix} 0 & 1 & -2 \\ 1 & -1 & 1 \\ -1 & 5 & 0 \end{bmatrix} = -1(0+10) - 1(1-2) = -9$

$$M_{21} = \begin{bmatrix} -1 & 0 & 1 \\ 1 & -1 & 1 \\ -1 & 5 & 0 \end{bmatrix} = 9$$

$$M_{31} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & -2 \\ -1 & 5 & 0 \end{bmatrix} = -9$$

$$M_{41} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & -2 \\ 1 & - & 1 \end{bmatrix} = 0$$

$$\therefore \quad C_{11} = (-1)^{1+1} M_{11} = -9$$

$$C_{21} = (-1)^3 M_{21} = -9$$

$$C_{31} = (-4)^4 M_{31} = -9$$

$$C_{41} = (-1)^5 M_{41} = 0$$

Hence,

$$\begin{bmatrix} 2 & -1 & 0 & 1 \\ -3 & 0 & 1 & -2 \\ 1 & 1 & -1 & 1 \\ 2 & -1 & 5 & 0 \end{bmatrix} = 2 \times C_{11} + (-3)C_{21} + 1 \times C_{31} + 2 \times C_{41} = -9[2 - 3 + 1] = 0$$

Determinants Ex 6.1 Q2(i)

Let $A = \begin{vmatrix} x & -7 \\ x & 5x+1 \end{vmatrix}$

$$\begin{aligned} |A| &= x(5x+1) + 7 \times x \\ &= 5x^2 + x + 7x \\ &= 5x^2 + 8x \end{aligned}$$

Hence $|A| = 5x^2 + 8x$

Determinants Ex 6.1 Q2(ii)

Let $A = \begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$

$|A| = \cos\theta \times \cos\theta + \sin\theta \times \sin\theta$
$= \cos^2\theta + \sin^2\theta$
$= 1$

Hence $|A| = 1$

Determinants Ex 6.1 Q2(iii)

Let $A = \begin{vmatrix} \cos 15^\circ & \sin 15^\circ \\ \sin 75^\circ & \cos 75^\circ \end{vmatrix}$

$|A| = \cos 15^\circ \cos 75^\circ - \sin 15^\circ \sin 75^\circ$
$= \cos(75+15)$ $\quad (\because \cos A \cos B - \sin A \sin B = \cos(A+B))$
$= \cos 90^\circ$
$= 0$

Hence $|A| = 0$

Determinants Ex 6.1 Q2(iv)

Let $A = \begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$

$|A| = (a+ib)(a-ib) - (c+id)(-c+id)$
$= (a^2+b^2) + (c+id)(c-id)$ $\quad$ (Taking $(-)$ sign common from $-c+id$)
$\left(Also\ (a+ib)(a-ib) = a^2+b^2\right)$

$= a^2+b^2+c^2+d^2$

Hence $|A| = a^2+b^2+c^2+d^2$

Determinants Ex 6.1 Q3

Since $|AB| = |A| \times |B|$

Hence $|A|^2 = |A| \times |A|$ $\quad ---(1)$

Now let $A = \begin{vmatrix} 2 & 3 & 7 \\ 13 & 17 & 5 \\ 15 & 20 & 12 \end{vmatrix}$

Expanding along the first column, we get

$|A| = 2\begin{vmatrix} 17 & 5 \\ 20 & 12 \end{vmatrix} - 3\begin{vmatrix} 13 & 5 \\ 15 & 12 \end{vmatrix} + 7\begin{vmatrix} 13 & 17 \\ 15 & 20 \end{vmatrix}$
$= 2(204-100) - 3(156-75) + 7(260-255)$
$= 2(104) - 3(81) + 7(5)$
$= 208 - 243 + 35$
$= 243 - 243$
$= 0$

Hence from eq.(1)
$|A|^2 = |A| \times |A| = 0 \times 0 = 0$

Determinants Ex 6.1 Q4

Evaluating the given determinant

$\sin 10^\circ \times \cos 80^\circ + \cos 10^\circ \sin 80^\circ$

$= \sin\left(10^\circ + 80^\circ\right) \qquad \left[\because \sin A \cos B + \cos A \sin B = \sin\left(A+B\right)\right]$

$= \sin 90^\circ$

$= 1$

Hence proved

Determinants Ex 6.1 Q5

We will evaluate the given determinant

(i) Along the first row

(ii) Along the first column

(i) Along the first row

$$|A| = 2\begin{vmatrix} 1 & -2 \\ 4 & 1 \end{vmatrix} - 3\begin{vmatrix} 7 & -2 \\ -3 & 1 \end{vmatrix} - 5\begin{vmatrix} 7 & 1 \\ -3 & 4 \end{vmatrix}$$
$$= 2\left(1+8\right) - 3\left(7-6\right) - 5\left(28+3\right)$$
$$= 2\left(9\right) - 3\left(1\right) - 5\left(31\right)$$
$$= 18 - 3 - 155 = -140$$

(ii) Along the first column

$$|A| = 2\begin{vmatrix} 1 & -2 \\ 4 & 1 \end{vmatrix} - 7\begin{vmatrix} 3 & -5 \\ 4 & 1 \end{vmatrix} - 3\begin{vmatrix} 3 & -5 \\ 1 & -2 \end{vmatrix}$$
$$= 2\left(1+8\right) - 7\left(3+20\right) - 3\left(-6+5\right)$$
$$= 18 - 7\left(23\right) - 3\left(-1\right)$$
$$= 18 - 161 + 3$$
$$= 21 - 161$$
$$= -140$$

We can see, the answer is same with both the methods.

Determinants Ex 6.1 Q6

$$\Delta = \begin{vmatrix} 0 & \sin\alpha & -\cos\alpha \\ -\sin\alpha & 0 & \sin\beta \\ \cos\alpha & -\sin\beta & 0 \end{vmatrix}$$
$$= -\sin\alpha\left(-\sin\beta\cos\alpha\right) - \cos\alpha\left(\sin\alpha\sin\beta\right)$$
$$= 0$$

Determinants Ex 6.1 Q7

$$\Delta = \begin{vmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta & -\sin\alpha \\ -\sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta & \cos\alpha \end{vmatrix}$$

Expanding along C_3, we have:

$$\Delta = -\sin\alpha\left(-\sin\alpha\sin^2\beta - \cos^2\beta\sin\alpha\right) + \cos\alpha\left(\cos\alpha\cos^2\beta + \cos\alpha\sin^2\beta\right)$$
$$= \sin^2\alpha\left(\sin^2\beta + \cos^2\beta\right) + \cos^2\alpha\left(\cos^2\beta + \sin^2\beta\right)$$
$$= \sin^2\alpha\left(1\right) + \cos^2\alpha\left(1\right)$$
$$= 1$$

Determinants Ex 6.1 Q8

Let $A = \begin{bmatrix} 2 & 5 \\ 2 & 1 \end{bmatrix}$

$\Rightarrow |A| = 2 - 10 = -8$

$B = \begin{bmatrix} 4 & -3 \\ 2 & 5 \end{bmatrix}$

$\Rightarrow |B| = 20 + 6 = 26$

$$\begin{aligned} \text{Now } AB &= \begin{bmatrix} 2 & 5 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 4 & -3 \\ 2 & 5 \end{bmatrix} \\ &= \begin{bmatrix} 2\times 4 + 5\times 2 & 2\times(-3) + 5\times 5 \\ 2\times 4 + 1\times 2 & 2\times(-3) + 1\times 5 \end{bmatrix} \\ &= \begin{bmatrix} 8+10 & -6+25 \\ 8+2 & -6+5 \end{bmatrix} \\ &= \begin{bmatrix} 18 & 19 \\ 10 & -1 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \Rightarrow \quad |AB| &= 18\times(-1) - (10)(19) \\ &= -18 - 190 = -208 \end{aligned}$$

Now $|AB| = |A| \times |B|$

$-208 = (-8)\times(26)$

$-208 = -208$

Hence verified.

Determinants Ex 6.1 Q9

Let $A = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{vmatrix}$

Evaluating the determinant along the first column

$$\begin{aligned} |A| &= 1\begin{vmatrix} 1 & 2 \\ 0 & 4 \end{vmatrix} - 0\begin{vmatrix} 0 & 1 \\ 0 & 4 \end{vmatrix} + 0\begin{vmatrix} 0 & 1 \\ 1 & 2 \end{vmatrix} \\ &= 1\times(4-0) - 0 + 0 \\ &= 4 \end{aligned}$$

Again $3A = \begin{bmatrix} 3 & 0 & 3 \\ 0 & 3 & 6 \\ 0 & 0 & 12 \end{bmatrix}$ (every element of A will be multiplied by 3)

Now, evaluating this determinant

$$\begin{aligned} |3A| &= 3\begin{vmatrix} 3 & 6 \\ 0 & 12 \end{vmatrix} - 0\begin{vmatrix} 0 & 3 \\ 0 & 12 \end{vmatrix} + 0\begin{vmatrix} 0 & 3 \\ 3 & 6 \end{vmatrix} \\ &= 3(36-0) - 0 + 0 \\ &= 108 \end{aligned}$$

Now, according to the question

$|3A| = 27|A|$

$108 = 27(4)$ (Substituting values)

$108 = 108$

Hence proved

Determinants Ex 6.1 Q10

(i) $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$

$\Rightarrow 2\times1-5\times4 = 2x\times x-6\times4$
$\Rightarrow 2-20 = 2x^2-24$
$\Rightarrow 2x^2 = 6$
$\Rightarrow x^2 = 3$
$\Rightarrow x = \pm\sqrt{3}$

(ii) $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$

$\Rightarrow 2\times5-3\times4 = x\times5-3\times2x$
$\Rightarrow 10-12 = 5x-6x$
$\Rightarrow -2 = -x$
$\Rightarrow x = 2$

(iii)

$\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$
$3-x^2 = 3-8$
$x^2 = 8$
$x = \pm2\sqrt{2}$

(iv)

$\begin{vmatrix} 3x & 7 \\ 2 & 4 \end{vmatrix} = 10$
$12x-14 = 10$
$12x = 24$
$x = 2$

Determinants Ex 6.1 Q11

Let $A = \begin{vmatrix} x^2 & x & 1 \\ 0 & 2 & 1 \\ 3 & 1 & 4 \end{vmatrix}$

Expanding the given determinant along the first column

$|A| = x^2\begin{vmatrix} 2 & 1 \\ 1 & 4 \end{vmatrix} - 0\begin{vmatrix} x & 1 \\ 1 & 4 \end{vmatrix} + 3\begin{vmatrix} x & 1 \\ 2 & 1 \end{vmatrix}$
$28 = x^2(8-1) - 0(4x-1) + 3(x-2)$
$28 = 7x^2 + 3x - 6$
or
$7x^2 + 3x - 6 = 28$
$7x^2 + 3x - 34 = 0$
Solving using quadratic formula, we get x = 2.

Determinants Ex 6.1 Q12(i)

A matrix A is called singular if $|A| = 0$

Now expanding along the first row $|A|$

$$= (x-1)\begin{vmatrix} x-1 & 1 \\ 1 & x-1 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 1 & x-1 \end{vmatrix} + 1\begin{vmatrix} 1 & 1 \\ x-1 & 1 \end{vmatrix}$$
$$= (x-1)\left[(x-1)^2 - 1\right] - 1\left[x-1-1\right] + 1\left[1-x+1\right]$$
$$= (x-1)\left(x^2+1-2x-1\right) - 1(x-2) + 1(2-x)$$
$$= (x-1)\left(x^2-2x\right) - x + 2 + 2 - x$$
$$= (x-1) \times x \times (x-2) + (4-2x)$$
$$= (x-1) \times x \times (x-2) + 2(2-x)$$
$$= (x-1) \times x \times (x-2) - 2(x-2)$$
$$= (x-2)\left[x(x-1) - 2\right] \qquad \text{(Taking } (x-2) \text{ common)}$$

Since A is a singular matrix, so $|A| = 0$

$i.e\ (x-2)\left(x^2 - x - 2\right) = 0$

either $(x-2) = 0$ $\quad$ or $x^2 - x - 2 = 0$

$x = 2$ $\quad$ or $x^2 - 2x + x - 2 = 0$

$x(x-2) + 1(x-2) = 0$

$(x-2)(x+1) = 0$

$x = 2, -1$

$x = 2$ or -1

Determinants Ex 6.1 Q12(ii)

A matrix A is said to be singular if $|A| = 0$

Now

$$\begin{vmatrix} 1+x & 7 \\ 3-x & 8 \end{vmatrix} = 0$$
$$8 + 8x - 21 + 7x = 0$$
$$15x = 13$$
$$x = \frac{13}{15}$$

Ex 6.2

Chapter 6 Determinants Ex 6.2 Q1-i

$$\begin{vmatrix} 1 & 3 & 5 \\ 2 & 6 & 10 \\ 31 & 11 & 38 \end{vmatrix} = 2\begin{vmatrix} 1 & 3 & 5 \\ 1 & 3 & 5 \\ 31 & 11 & 38 \end{vmatrix} = 0$$

Chapter 6 Determinants Ex 6.2 Q1-ii

Consider the determinant

$$\Delta = \begin{vmatrix} 67 & 19 & 21 \\ 39 & 13 & 14 \\ 81 & 24 & 26 \end{vmatrix}$$

Applying $C_1 \to C_1 - 4C_3$, *we get,*

$$\Delta = \begin{vmatrix} 4 & 19 & 21 \\ -3 & 13 & 14 \\ -3 & 24 & 26 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 4 & 19 & 21 \\ -3 & 13 & 14 \\ -3 & 24 & 26 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 32 & 35 \\ -3 & 13 & 14 \\ 0 & 11 & 12 \end{vmatrix} \quad [\text{Applying } R_3 \to R_3 - R_2 \text{ and } R_1 \to R_1 + R_2]$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 32 & 35 \\ 0 & 109 & 119 \\ 0 & 11 & 12 \end{vmatrix} \quad [\text{Applying } R_2 \to 3R_1 + R_2]$$

$$\Rightarrow \Delta = 1(109 \times 12 - 119 \times 11)$$

$$\Rightarrow \Delta = -1$$

Chapter 6 Determinants Ex 6.2 Q1-iii

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$$

$$= a(bc - f^2) - h(hc - fg) + g(hf - bg)$$

$$= abc - af^2 - h^2c + hfg + ghf - bg^2$$

Chapter 6 Determinants Ex 6.2 Q1-iv

$$\begin{vmatrix} 1 & -3 & 2 \\ 4 & -1 & 2 \\ 3 & 5 & 2 \end{vmatrix} = 2\begin{vmatrix} 1 & -3 & 1 \\ 4 & -1 & 1 \\ 3 & 5 & 1 \end{vmatrix} = 2\begin{vmatrix} 1 & -3 & 1 \\ 3 & 2 & 0 \\ 2 & 8 & 0 \end{vmatrix} = 2(24 - 4) = 40$$

Chapter 6 Determinants Ex 6.2 Q1-v

Let Δ be the determinant.

$$\Delta = \begin{vmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{vmatrix}$$

Applying $R_3 \to R_3 - R_2$, we get,

$$\Delta = \begin{vmatrix} 1 & 4 & 9-4 \\ 4 & 9 & 16-9 \\ 9 & 16 & 25-16 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 4 & 5 \\ 4 & 9 & 7 \\ 9 & 16 & 9 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 5 & 5 \\ 4 & 13 & 7 \\ 9 & 25 & 9 \end{vmatrix} \quad [Applying\ C_2 \to C_1 + C_2]$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 0 & 0 \\ 4 & -7 & -13 \\ 9 & -20 & -36 \end{vmatrix} \quad [Applying\ C_2 \to 5C_1 - C_2\ and\ C_3 \to 5C_1 - C_3]$$

$$\Rightarrow \Delta = 1(7 \times 36 - 13 \times 20) = 252 - 260 = -8$$

Chapter 6 Determinants Ex 6.2 Q1-vi

$$\begin{vmatrix} 6 & -3 & 2 \\ 2 & -1 & 2 \\ -10 & 5 & 2 \end{vmatrix}$$

Apply: $R_1 \to R_1 + (-3)R_2$ and $R_3 \to R_3 + 5R_2$

$$= \begin{vmatrix} 0 & 0 & -4 \\ 2 & -1 & 2 \\ 0 & 0 & 12 \end{vmatrix} = 0$$

Chapter 6 Determinants Ex 6.2 Q1-vii

$$\begin{vmatrix} 1 & 3 & 9 & 27 \\ 3 & 9 & 27 & 1 \\ 9 & 27 & 1 & 3 \\ 27 & 1 & 3 & 9 \end{vmatrix} = \begin{vmatrix} 1 & 3 & 3^2 & 3^3 \\ 3 & 3^2 & 3^3 & 1 \\ 3^2 & 3^3 & 1 & 3 \\ 3^3 & 1 & 3 & 3^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1+3+3^2+3^3 & 3 & 3^2 & 3^3 \\ 1+3+3^2+3^3 & 3^2 & 3^3 & 1 \\ 1+3+3^2+3^3 & 3^3 & 1 & 3 \\ 1+3+3^2+3^3 & 1 & 3 & 3^2 \end{vmatrix}$$

$$= \left(1+3+3^2+3^3\right)\begin{vmatrix} 1 & 3 & 3^2 & 3^3 \\ 1 & 3^2 & 3^3 & 1 \\ 1 & 3^3 & 1 & 3 \\ 1 & 1 & 3 & 3^2 \end{vmatrix}$$

$$= \left(1+3+3^2+3^3\right)\begin{vmatrix} 1 & 3 & 3^2 & 3^3 \\ 0 & 3^2-3 & 3^3-3^2 & 1-3^3 \\ 0 & 3^3-3 & 1-3^2 & 3-3^3 \\ 0 & 1-3 & 3-3^2 & 3^2-3^3 \end{vmatrix}$$

$$= \left(1+3+3^2+3^3\right)\begin{vmatrix} 6 & 18 & -26 \\ 24 & -8 & -24 \\ -2 & -6 & -18 \end{vmatrix}$$

$$= \left(1+3+3^2+3^3\right)2^3\begin{vmatrix} 3 & -9 & 13 \\ 12 & 4 & 12 \\ -1 & 3 & 9 \end{vmatrix}$$

$$= \left(1+3+3^2+3^3\right)2^3\begin{vmatrix} 0 & 0 & 40 \\ 12 & 4 & 12 \\ -1 & 3 & 9 \end{vmatrix}$$

$$= \left(1+3+3^2+3^3\right)2^3 \times 40(36+4) = 512000$$

Chapter 6 Determinants Ex 6.2 Q1-viii

$$Let\ \Delta = \begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$$

Applying $R_3 \to 17R_2 - R_3$, *we get*,

$$\Delta = \begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 0 & 48 & 62 \end{vmatrix}$$

Applying $R_2 \to 102R_2 - R_1$, *we get*,

$$\Delta = \begin{vmatrix} 102 & 18 & 36 \\ 0 & 288 & 372 \\ 0 & 48 & 62 \end{vmatrix}$$

Thus,

$$\Delta = 102(288 \times 62 - 372 \times 48)$$

$$\Rightarrow \Delta = 0$$

Chapter 6 Determinants Ex 6.2 Q2(i)

$$\begin{vmatrix} 8 & 2 & 7 \\ 12 & 3 & 5 \\ 16 & 4 & 3 \end{vmatrix}$$

Apply: $R_3 \to R_3 - R_2$

$$= \begin{vmatrix} 8 & 2 & 7 \\ 12 & 3 & 5 \\ 4 & 1 & -2 \end{vmatrix}$$

Apply: $R_2 \to R_2 - R_1$

$$= \begin{vmatrix} 8 & 2 & 7 \\ 4 & 1 & -2 \\ 4 & 1 & -2 \end{vmatrix}$$

Since, $R_3 = R_2$, the value of the determinant is zero.

Chapter 6 Determinants Ex 6.2 Q2(ii)

$$\begin{vmatrix} 6 & -3 & 2 \\ 2 & -1 & 2 \\ -10 & 5 & 2 \end{vmatrix}$$

Taking (-2) common from C_1, we get

$$= (-2)\begin{vmatrix} -3 & -3 & 2 \\ -1 & -1 & 2 \\ 5 & 5 & 2 \end{vmatrix}$$

$$= 0$$

$\because C_1$ and C_2 are identical.

Chapter 6 Determinants Ex 6.2 Q2(iii)

$$\begin{vmatrix} 2 & 3 & 7 \\ 13 & 17 & 5 \\ 15 & 20 & 12 \end{vmatrix}$$

Use: $R_3 \to R_3 - R_2$

$$= \begin{vmatrix} 2 & 3 & 7 \\ 13 & 17 & 5 \\ 2 & 3 & 7 \end{vmatrix}$$

$$= 0$$

$\because R_3 = R_1$

Chapter 6 Determinants Ex 6.2 Q2(iv)

$$\begin{vmatrix} \frac{1}{a} & a^2 & bc \\ \frac{1}{b} & b^2 & ca \\ \frac{1}{c} & c^2 & ab \end{vmatrix}$$

Multiply: R_1, R_2 and R_3 by a, b and c respectively, we get

$$= \frac{1}{abc}\begin{vmatrix} 1 & a^3 & abc \\ 1 & b^3 & bca \\ 1 & c^3 & cab \end{vmatrix}$$

Take abc common from C_3, we get,

$$= \begin{vmatrix} 1 & a^3 & 1 \\ 1 & b^3 & 1 \\ 1 & c^3 & 1 \end{vmatrix}$$

$$= 0$$

$\because \quad C_1 = C_3$

Chapter 6 Determinants Ex 6.2 Q2(v)

$$\begin{vmatrix} a+b & 2a+b & 3a+b \\ 2a+b & 3a+b & 4a+b \\ 4a+b & 5a+b & 6a+b \end{vmatrix}$$

Apply: $C_3 \to C_3 - C_2$

$$= \begin{vmatrix} a+b & 2a+b & a \\ 2a+b & 3a+b & a \\ 4a+b & 5a+b & a \end{vmatrix}$$

Apply: $C_2 \to C_2 - C_1$

$$= \begin{vmatrix} a+b & a & a \\ 2a+b & a & a \\ 4a+b & a & a \end{vmatrix}$$

$= 0$

$\because \quad C_3 = C_2$

Chapter 6 Determinants Ex 6.2 Q2(vi)

$$\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ac \\ 1 & c & c^2-ab \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \begin{vmatrix} 1 & a & bc \\ 1 & b & ac \\ 1 & c & ab \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{vmatrix} - \begin{vmatrix} 1 & a & bc \\ 0 & b-a & (a-b)c \\ 0 & c-a & (a-c)b \end{vmatrix}$$

$$= (b-a)(c-a)\begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & b+a \\ 0 & 1 & c+a \end{vmatrix} - (b-a)(c-a)\begin{vmatrix} 1 & a & bc \\ 0 & 1 & -c \\ 0 & 1 & -b \end{vmatrix}$$

$= (b-a)(c-a)(c+a-b-a) - (b-a)(c-a)(-b+c)$

$= (b-a)(c-a)(c-b) - (b-a)(c-a)(-b+c)$

$= 0$

Chapter 6 Determinants Ex 6.2 Q2(vii)

$$\begin{vmatrix} 49 & 1 & 6 \\ 39 & 7 & 4 \\ 26 & 2 & 3 \end{vmatrix}$$

Apply: $C_1 \to C_1 + (-8)C_3$

$$= \begin{vmatrix} 1 & 1 & 6 \\ 7 & 7 & 4 \\ 2 & 2 & 3 \end{vmatrix} = 0$$

$\because \quad C_1 = C_2$

Chapter 6 Determinants Ex 6.2 Q2(viii)

$$\begin{vmatrix} 0 & x & y \\ -x & 0 & z \\ -y & -z & 0 \end{vmatrix}$$

Multiply C_1, C_2 and C_3 by z, y, and x respectively

$$= \frac{1}{xyz}\begin{vmatrix} 0 & xy & yx \\ -xz & 0 & zx \\ -yz & -zy & 0 \end{vmatrix}$$

Take y, x, and z common from R_1, R_2 and R_3 respectively

$$= \begin{vmatrix} 0 & x & x \\ -z & 0 & z \\ -y & -y & 0 \end{vmatrix}$$

Apply: $C_2 \to C_2 - C_3$

$$= \begin{vmatrix} 0 & 0 & x \\ -z & -z & z \\ -y & -y & 0 \end{vmatrix}$$

$= 0$

$\because C_1 = C_2$

Chapter 6 Determinants Ex 6.2 Q2(ix)

$$\begin{vmatrix} 1 & 43 & 6 \\ 7 & 35 & 4 \\ 3 & 17 & 2 \end{vmatrix}$$

Apply: $C_2 \to C_2 + (-7)C_3$

$$= \begin{vmatrix} 1 & 1 & 6 \\ 7 & 7 & 4 \\ 3 & 3 & 2 \end{vmatrix}$$

$= 0$

$\because \quad C_1 = C_2$

Chapter 6 Determinants Ex 6.2 Q2(x)

$$\begin{vmatrix} 1^2 & 2^2 & 3^2 & 4^2 \\ 2^2 & 3^2 & 4^2 & 5^2 \\ 3^2 & 4^2 & 5^2 & 6^2 \\ 4^2 & 5^2 & 6^2 & 7^2 \end{vmatrix}$$

$Apply : C3 \to C3 - C2, C4 \to C4 - C1$

$$= \begin{vmatrix} 1^2 & 2^2 & 3^2 - 2^2 & 4^2 - 1^2 \\ 2^2 & 3^2 & 4^2 - 3^2 & 5^2 - 2^2 \\ 3^2 & 4^2 & 5^2 - 4^2 & 6^2 - 3^2 \\ 4^2 & 5^2 & 6^2 - 5^2 & 7^2 - 4^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1^2 & 2^2 & 5 & 15 \\ 2^2 & 3^2 & 7 & 21 \\ 3^2 & 4^2 & 9 & 27 \\ 4^2 & 5^2 & 11 & 33 \end{vmatrix}$$

Take 3 common from C_4

$$= 3\begin{vmatrix} 1^2 & 2^2 & 5 & 5 \\ 2^2 & 3^2 & 7 & 7 \\ 3^2 & 4^2 & 9 & 9 \\ 4^2 & 5^2 & 11 & 11 \end{vmatrix}$$

$= 0$

$\because \quad C_3 = C_4$

Chapter 6 Determinants Ex 6.2 Q2(xi)

$$\begin{vmatrix} a & b & c \\ a+2x & b+2y & c+2z \\ x & y & z \end{vmatrix}$$

$$= \begin{vmatrix} a & b & c \\ 2a+2x & 2b+2y & 2c+2z \\ x+a & y+b & z+c \end{vmatrix}$$

$$= 2\begin{vmatrix} a & b & c \\ a+x & b+y & c+z \\ x+a & y+b & z+c \end{vmatrix}$$

$= 0$

Chapter 6 Determinants Ex 6.2 Q3

$$\begin{vmatrix} a & b+c & a^2 \\ b & c+a & b^2 \\ c & a+b & c^2 \end{vmatrix}$$

Apply: $C_2 \to C_2 + C_1$.

$$= \begin{vmatrix} a & b+c+a & a^2 \\ b & c+a+b & b^2 \\ c & a+b+c & c^2 \end{vmatrix}$$

Take $(a+b+c)$ common from C_2

$$= (b+c+a)\begin{vmatrix} a & 1 & a^2 \\ b & 1 & b^2 \\ c & 1 & c^2 \end{vmatrix}$$

Apply: $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$= (b+c+a)\begin{vmatrix} a & 1 & a^2 \\ b-a & 0 & b^2-a^2 \\ c-a & 0 & c^2-a^2 \end{vmatrix}$$

$$= (b+c+a)(b-a)(c-a)\begin{vmatrix} a & 1 & a^2 \\ 1 & 0 & b+a \\ 1 & 0 & c+a \end{vmatrix}$$

$$= (b+c+a)(b-a)(c-a)(b-c)$$

Chapter 6 Determinants Ex 6.2 Q4

$$Let\ \Delta = \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ *and* $R_3 \to R_3 - R_1$ *we get,*

$$\Delta = \begin{vmatrix} 1 & a & bc \\ 0 & b-a & ca-bc \\ 0 & c-a & ab-ba \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & a & bc \\ 0 & b-a & c(a-b) \\ 0 & c-a & b(a-c) \end{vmatrix}$$

Taking $(a-b)$ *and* $(a-c)$ *common, we have*

$$\Delta = (a-b)(a-c)\begin{vmatrix} 1 & a & bc \\ 0 & -1 & c \\ 0 & -1 & b \end{vmatrix}$$

$$\Rightarrow \Delta = (a-b)(c-a)(b-c)$$

Chapter 6 Determinants Ex 6.2 Q5

$$Let\ \Delta = \begin{vmatrix} x+\lambda & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$ *we get,*

$$\Delta = \begin{vmatrix} 3x+\lambda & x & x \\ 3x+\lambda & x+\lambda & x \\ 3x+\lambda & x & x+\lambda \end{vmatrix}$$

Taking $(3x+\lambda)$ *common, we have*

$$\Delta = (3x+\lambda)\begin{vmatrix} 1 & x & x \\ 1 & x+\lambda & x \\ 1 & x & x+\lambda \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$, *we get,*

$$\Delta = (3x+\lambda)\begin{vmatrix} 1 & x & x \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{vmatrix}$$

$$\Rightarrow \Delta = \lambda^2(3x+\lambda)$$

Chapter 6 Determinants Ex 6.2 Q6

$$Let\ \Delta = \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$$

$Applying\ C_1 \to C_1 + C_2 + C_3\ we\ get,$

$$\Delta = \begin{vmatrix} a+b+c & b & c \\ a+b+c & a & b \\ a+b+c & c & a \end{vmatrix}$$

$Taking\ (a+b+c)\ common,\ we\ have$

$$\Delta = (a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & a & b \\ 1 & c & a \end{vmatrix}$$

$Applying\ R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1,\ we\ get,$

$$\Delta = (a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & a-b & b-c \\ 0 & c-b & a-c \end{vmatrix}$$

$$\Rightarrow \Delta = (a+b+c)[(a-b)(a-c)-(b-c)(c-b)]$$

$$\Rightarrow \Delta = (a+b+c)\left[a^2 - ac - ab + bc + b^2 + c^2 - 2bc\right]$$

$$\Rightarrow \Delta = (a+b+c)\left[a^2 + b^2 + c^2 - ac - ab - bc\right]$$

Chapter 6 Determinants Ex 6.2 Q7

$$\begin{vmatrix} x & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{vmatrix} = \begin{vmatrix} 2+x & 1 & 1 \\ 2+x & x & 1 \\ 2+x & 1 & x \end{vmatrix} = (2+x)\begin{vmatrix} 1 & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{vmatrix}$$

$$= (2+x)\begin{vmatrix} 1 & 1 & 1 \\ 0 & x-1 & 0 \\ 0 & 0 & x-1 \end{vmatrix}$$

$$= (2+x)(x-1)^2$$

Chapter 6 Determinants Ex 6.2 Q8

$$\begin{vmatrix} 0 & xy^2 & xz^2 \\ x^2y & 0 & yz^2 \\ x^2z & zy^2 & 0 \end{vmatrix}$$

$$= 0\left(0 - y^3z^3\right) - xy^2\left(0 - x^2yz^3\right) + xz^2\left(x^2y^3z - 0\right)$$

$$= 0 + x^3y^3z^3 + x^3y^3z^3$$

$$= 2x^3y^3z^3$$

Chapter 6 Determinants Ex 6.2 Q9

$$\text{Let } \Delta = \begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix}$$

Applying $R_1 \to R_1 - R_2$ and $R_3 \to R_3 - R_2$

$$\Delta = \begin{vmatrix} a & -a & 0 \\ x & a+y & z \\ 0 & -a & a \end{vmatrix}$$

Applying $C_2 \to C_2 + C_1$

$$\Delta = \begin{vmatrix} a & 0 & 0 \\ x & a+x+y & z \\ 0 & -a & a \end{vmatrix}$$

$$\Delta = a\left[a(a+x+y) + az\right] + 0 + 0$$

$$\Delta = a^2(a+x+y+z)$$

Chapter 6 Determinants Ex 6.2 Q10

$$\Delta + \Delta_1 = \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} + \begin{vmatrix} 1 & 1 & 1 \\ yz & zx & xy \\ x & y & z \end{vmatrix}$$

$$= \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} + \begin{vmatrix} 1 & yz & x \\ 1 & zx & y \\ 1 & xy & z \end{vmatrix} \ldots\ldots\ldots \left[\because |A| = \left|A^T\right|\right]$$

$$= \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} - \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$$

$$\ldots\ldots \begin{bmatrix} \text{If any two rows (columns) of the determinant are interchanged} \\ \text{then value of the determinant changes in sign.} \end{bmatrix}$$

$$= \begin{vmatrix} 0 & 0 & x^2 - yz \\ 0 & 0 & y^2 - zx \\ 0 & 0 & z^2 - xy \end{vmatrix}$$

$$= 0 \ldots\ldots\ldots [\because C_1 \text{ and } C_2 \text{ are identical}]$$

Chapter 6 Determinants Ex 6.2 Q11

$$\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3 + b^3 + c^3 - 3abc$$

$$\text{LHS} = \begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix}$$

Apply: $C_1 \to C_1 + C_2 + C_3$.

$$= \begin{vmatrix} a+b+c & b & c \\ 0 & b-c & c-a \\ 2(a+b+c) & c+a & a+b \end{vmatrix}$$

Take $(a+b+c)$ common from C_1

$$= (a+b+c) \begin{vmatrix} 1 & b & c \\ 0 & b-c & c-a \\ 2 & c+a & a+b \end{vmatrix}$$

Apply: $R_3 \to R_3 - 2R_1$

$$= (a+b+c) \begin{vmatrix} 1 & b & c \\ 0 & b-c & c-a \\ 0 & c+a-2b & a+b-2c \end{vmatrix}$$

$$= (a+b+c)\left[(b-c)(a+b-2c) - (c-a)(c+a-2b)\right]$$

$$= a^3 + b^3 + c^3 - 3abc$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q12

$$\begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix} = 3abc - a^3 - b^3 - c^3$$

$$\text{LHS} = \begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix}$$

$$= \begin{vmatrix} b+c+a & -b & a \\ c+a+b & -c & b \\ a+b+c & -a & c \end{vmatrix}$$

$$= -(b+c+a)\begin{vmatrix} 1 & b & a \\ 1 & c & b \\ 1 & a & c \end{vmatrix}$$

$$= -(b+c+a)\begin{vmatrix} 1 & b & a \\ 0 & c-b & b-a \\ 0 & a-b & c-a \end{vmatrix}$$

$$= -(b+c+a)\left[(c-b)(c-a) - (b-a)(a-b)\right]$$

$$= 3abc - a^3 - b^3 - c^3$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q13

$$\begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

$$\text{LHS} = \begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix}$$

Apply: $C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} 2(a+b+c) & b+c & c+a \\ 2(a+b+c) & c+a & a+b \\ 2(a+b+c) & a+b & b+c \end{vmatrix}$$

$$= 2\begin{vmatrix} a+b+c & b+c & c+a \\ a+b+c & c+a & a+b \\ a+b+c & a+b & b+c \end{vmatrix}$$

Apply: $C_2 \to C_2 - C_1, C_3 \to C_3 - C_1$

$$= 2\begin{vmatrix} a+b+c & -a & -b \\ a+b+c & -b & -c \\ a+b+c & -c & -a \end{vmatrix}$$

$$= 2\begin{vmatrix} a+b+c & a & b \\ a+b+c & b & c \\ a+b+c & c & a \end{vmatrix}$$

$$= 2\left(\begin{vmatrix} c & a & b \\ a & b & c \\ b & c & a \end{vmatrix} + \begin{vmatrix} a & a & b \\ b & b & c \\ c & c & a \end{vmatrix} + \begin{vmatrix} b & a & b \\ c & b & c \\ a & c & a \end{vmatrix}\right)$$

$$= 2\begin{vmatrix} c & a & b \\ a & b & c \\ b & c & a \end{vmatrix}$$

$$= 2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q14

We need to prove the following identity:

$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$$

Applying $C_1 \to C_1 + C_2 + C_3$, *we have,*

$$L.H.S = \begin{vmatrix} 2a+2b+2c & a & b \\ 2a+2b+2c & b+c+2a & b \\ 2a+2b+2c & a & c+a+2b \end{vmatrix}$$

Taking the term $2a+2b+2$ *as common, we have*

$$L.H.S = (2a+2b+2c)\begin{vmatrix} 1 & a & b \\ 1 & b+c+2a & b \\ 1 & a & c+a+2b \end{vmatrix}$$

$$\Rightarrow L.H.S = 2(a+b+c)\begin{vmatrix} 1 & a & b \\ 1 & b+c+2a & b \\ 1 & a & c+a+2b \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ *and* $R_3 \to R_3 - R_1$ *we have*

$$L.H.S = 2(a+b+c)\begin{vmatrix} 1 & a & b \\ 0 & a+b+c & 0 \\ 0 & 0 & a+b+c \end{vmatrix}$$

Thus, we have,

$$\begin{aligned} L.H.S &= 2(a+b+c)\left[1 \times (a+b+c)^2\right] \\ &= 2(a+b+c)(a+b+c)^2 \\ &= 2(a+b+c)^3 \end{aligned}$$

Chapter 6 Determinants Ex 6.2 Q15

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$

$$\text{LHS} = \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Apply: $R_1 \to R_1 + R_2 + R_3$.

$$= \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Take $(a+b+c)$ common from R_1

$$= (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Apply: $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$

$$= (a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ 2b & -b-c-a & 0 \\ 2c & 0 & -c-a-b \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ 2b & b+c+a & 0 \\ 2c & 0 & b+c+a \end{vmatrix}$$

$$= (a+b+c)^3$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q16

$$LHS = \begin{vmatrix} 1 & b+c & b^2+c^2 \\ 1 & c+a & c^2+a^2 \\ 1 & a+b & a^2+b^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & b+c & b^2+c^2 \\ 0 & a-b & a^2-b^2 \\ 0 & a-c & a^2-c^2 \end{vmatrix}$$

$$= (a-b)(a-c)\begin{vmatrix} 1 & b+c & b^2+c^2 \\ 0 & 1 & a+b \\ 0 & 1 & a+c \end{vmatrix}$$

$$= (a-b)(a-c)\begin{vmatrix} 1 & b+c & b^2+c^2 \\ 0 & 1 & a+b \\ 0 & 0 & c-b \end{vmatrix}$$

$$= (a-b)(b-c)(c-a)$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q17

$$LHS = \begin{vmatrix} a & a+b & a+2b \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix}$$

$$= \begin{vmatrix} 3a+3b & 3a+3b & 3a+3b \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix}$$

$$= (3a+3b)\begin{vmatrix} 1 & 1 & 1 \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix}$$

$$= 3(a+b)\begin{vmatrix} 0 & 1 & 0 \\ 2b & a & b \\ -b & a+2b & -2b \end{vmatrix}$$

$$= 3(a+b)b^2\begin{vmatrix} 0 & 1 & 0 \\ 2 & a & 1 \\ -1 & a+2b & -2 \end{vmatrix}$$

$$= 9(a+b)b^2$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q18

$$\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$$

Apply $R_1 \to R_1 a,\ R_2 \to R_2 b,\ R_3 \to R_3 c$

$$= \frac{1}{abc}\begin{vmatrix} a & a^2 & abc \\ b & b^2 & cab \\ c & c^2 & abc \end{vmatrix}$$

$$= \frac{abc}{abc}\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix}$$

$$= -\begin{vmatrix} a & 1 & a^2 \\ b & 1 & b^2 \\ c & 1 & c^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

Chapter 6 Determinants Ex 6.2 Q19

$$\begin{vmatrix} z & x & y \\ z^2 & x^2 & y^2 \\ z^4 & x^4 & y^4 \end{vmatrix} = \begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^4 & y^4 & z^4 \end{vmatrix} = \begin{vmatrix} x^2 & y^2 & z^2 \\ x^4 & y^4 & z^4 \\ x & y & z \end{vmatrix} = xyz\,(x-y)(y-z)(z-x)(x+y+z)$$

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^4 & y^4 & z^4 \end{vmatrix}$$

$$= xyz\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^3 & y^3 & z^3 \end{vmatrix}$$

$$= xyz\begin{vmatrix} 0 & 1 & 0 \\ x-y & y & z-y \\ x^3-y^3 & y^3 & z^3-y^3 \end{vmatrix}$$

$$= xyz(x-y)(z-y)\begin{vmatrix} 0 & 1 & 0 \\ 1 & y & 1 \\ x^2+y^2+xy & y^3 & z^2+y^2+zy \end{vmatrix}$$

$$= -xyz(x-y)(z-y)\left[z^2+y^2+zy-x^2-y^2-xy\right]$$

$$= -xyz(x-y)(z-y)\left[(z-x)(z+x)+y(z-x)\right]$$

$$= -xyz(x-y)(z-y)(z-x)[z+x+y]$$

$$= xyz(x-y)(y-z)(z-x)(x+y+z)$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q20

$$\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)\left(a^2+b^2+c^2\right)$$

$$\text{LHS} = \begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix}$$

Apply: $C_1 \to C_1 + C_2 - 2C_3$

$$= \begin{vmatrix} (b+c)^2+a^2-2bc & a^2 & bc \\ (c+a)^2+b^2-2ca & b^2 & ca \\ (a+b)^2+c^2-2ab & c^2 & ab \end{vmatrix}$$

$$= \begin{vmatrix} a^2+b^2+c^2 & a^2 & bc \\ a^2+b^2+c^2 & b^2 & ca \\ a^2+b^2+c^2 & c^2 & ab \end{vmatrix}$$

Take $\left(a^2+b^2+c^2\right)$ common from C_1

$$= \left(a^2+b^2+c^2\right)\begin{vmatrix} 1 & a^2 & bc \\ 1 & b^2 & ca \\ 1 & c^2 & ab \end{vmatrix}$$

$$= \left(a^2+b^2+c^2\right)\begin{vmatrix} 1 & a^2 & bc \\ 0 & b^2-a^2 & ca-bc \\ 0 & c^2-a^2 & ab-bc \end{vmatrix}$$

$$= \left(a^2+b^2+c^2\right)(b-a)(c-a)\begin{vmatrix} 1 & a^2 & bc \\ 0 & b+a & -c \\ 0 & c+a & -b \end{vmatrix}$$

$$= \left(a^2+b^2+c^2\right)(b-a)(c-a)\left[(b+a)(-b)-(-c)(c+a)\right]$$

$$= (a-b)(b-c)(c-a)(a+b+c)\left(a^2+b^2+c^2\right)$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q21

$$\begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix} = -2$$

$$\text{LHS} = \begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$$

Apply $R_3 \to R_3 - R_2$

$$= \begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)2 & 1 & 0 \end{vmatrix}$$

Apply $R_2 \to R_2 - R_1$

$$= \begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)2 & 1 & 0 \\ (a+3)2 & 1 & 0 \end{vmatrix}$$

$$= \left[(2a+4)(1) - (1)(2a+6)\right]$$

$$= -2$$

$$= \text{RHS}$$

Chapter 6 Determinants Ex 6.2 Q22

$$\begin{vmatrix} a^2 & a^2-(b-c)^2 & bc \\ b^2 & b^2-(c-a)^2 & ca \\ c^2 & c^2-(a-b)^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)\left(a^2+b^2+c^2\right)$$

$$\text{LHS} = \begin{vmatrix} a^2 & a^2-(b-c)^2 & bc \\ b^2 & b^2-(c-a)^2 & ca \\ c^2 & c^2-(a-b)^2 & ab \end{vmatrix}$$

Apply: $C_2 \to C_2 - 2C_1 - 2C_3$

$$= \begin{vmatrix} a^2 & a^2-(b-c)^2-2a^2-2bc & bc \\ b^2 & b^2-(c-a)^2-2b^2-2ca & ca \\ c^2 & c^2-(a-b)^2-2c^2-2ab & ab \end{vmatrix}$$

$$= \begin{vmatrix} a^2 & -\left(b^2+c^2+a^2\right) & bc \\ b^2 & -\left(b^2+c^2+a^2\right) & ca \\ c^2 & -\left(b^2+c^2+a^2\right) & ab \end{vmatrix}$$

Take $-\left(a^2+b^2+c^2\right)$ common from C_2

$$= -\left(b^2+c^2+a^2\right)\begin{vmatrix} a^2 & 1 & bc \\ b^2 & 1 & ca \\ c^2 & 1 & ab \end{vmatrix}$$

$$= -\left(b^2+c^2+a^2\right)\begin{vmatrix} a^2 & 1 & bc \\ b^2-a^2 & 0 & ca-bc \\ c^2-a^2 & 0 & ab-bc \end{vmatrix}$$

$$= -\left(b^2+c^2+a^2\right)(a-b)(c-a)\begin{vmatrix} a^2 & 1 & bc \\ -(b+a) & 0 & c \\ c+a & 0 & -b \end{vmatrix}$$

$$= -\left(b^2+c^2+a^2\right)(a-b)(c-a)\left[(-(b+a))(-b)-(c)(c+a)\right]$$

$$= (a-b)(b-c)(c-a)(a+b+c)\left(a^2+b^2+c^2\right)$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q23

$$\begin{vmatrix}1 & a^2+bc & a^3\\1 & b^2+ca & b^3\\1 & c^2+ab & c^3\end{vmatrix} = -(a-b)(b-c)(c-a)\left(a^2+b^2+c^2\right)$$

$$\text{LHS} = \begin{vmatrix}1 & a^2+bc & a^3\\1 & b^2+ca & b^3\\1 & c^2+ab & c^3\end{vmatrix}$$

Apply: $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$= \begin{vmatrix}1 & a^2+bc & a^3\\0 & b^2+ca-a^2-bc & b^3-a^3\\0 & c^2+abb^2+ca-a^2-bc & c^3-a^3\end{vmatrix}$$

$$= \begin{vmatrix}1 & a^2+bc & a^3\\0 & \left(b^2-a^2\right)-c(b-a) & b^3-a^3\\0 & \left(c^2-a^2\right)-b(c-a) & c^3-a^3\end{vmatrix}$$

$$= \begin{vmatrix}1 & a^2+bc & a^3\\0 & (b-a)(b+a-c) & b^3-a^3\\0 & (c-a)(c+a-b) & c^3-a^3\end{vmatrix}$$

$$= (b-a)(c-a)\begin{vmatrix}1 & a^2+bc & a^3\\0 & (b+a-c) & b^2+a^2+ab\\0 & (c+a-b) & c^2+a^2+ac\end{vmatrix}$$

$$= (b-a)(c-a)\left[((b+a-c))\left(c^2+a^2+ac\right)-\left(b^2+a^2+ab\right)\left(c^2+a^2+ac\right)\right]$$

$$= -(a-b)(b-c)(c-a)\left(a^2+b^2+c^2\right)$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q24

We need to prove the following identity:

$$\begin{vmatrix}a^2 & bc & ac+c^2\\a^2+ab & b^2 & ac\\ab & b^2+bc & c^2\end{vmatrix} = 4a^2b^2c^2$$

Taking the term a,b,c *common from* C_1, C_2 *and* C_3, *respectively, we have,*

$$L.H.S = abc\begin{vmatrix}a & c & a+c\\a+b & b & a\\b & b+c & c\end{vmatrix}$$

Applying $C_1 \to C_1+C_2+C_3$, *we have,*

$$L.H.S = abc\begin{vmatrix}2a+2c & c & a+c\\2a+2b & b & a\\2b+2c & b+c & c\end{vmatrix}$$

$$\Rightarrow L.H.S = 2abc\begin{vmatrix}a+c & c & a+c\\a+b & b & a\\b+c & b+c & c\end{vmatrix}$$

$Applying\ C_2 \to C_2 - C_1\ and\ C_3 \to C_3 - C_1,\ we\ have,$

$$L.H.S = 2abc\begin{vmatrix} a+c & -a & 0 \\ a+b & -a & -b \\ b+c & 0 & -b \end{vmatrix}$$

$Applying\ C_1 \to C_1 + C_2 + C_3,\ we\ have,$

$$\Rightarrow L.H.S = 2abc\begin{vmatrix} c & -a & 0 \\ 0 & -a & -b \\ c & 0 & -b \end{vmatrix}$$

$Taking\ c,\ a,\ and\ b\ \ from\ C_1,\ C_2\ and\ C_3\ respectively,\ we\ have,$

$$L.H.S = 2a^2b^2c^2\begin{vmatrix} 1 & -1 & 0 \\ 0 & -1 & -1 \\ 1 & 0 & -1 \end{vmatrix}$$

$Applying\ R_3 \to R_3 - R_1,\ we\ have$

$$L.H.S = 2a^2b^2c^2\begin{vmatrix} 1 & -1 & 0 \\ 0 & -1 & -1 \\ 0 & 1 & -1 \end{vmatrix}$$

$$= 4a^2b^2c^2$$

Chapter 6 Determinants Ex 6.2 Q25

$We\ need\ to\ prove\ the\ following\ identity:$

$$\begin{vmatrix} x+4 & x & x \\ x & x+4 & x \\ x & x & x+4 \end{vmatrix} = 16(3x+4)$$

$Let\ us\ consider\ the\ L.H.S\ of\ the\ above\ equation.$

$$\Delta = \begin{vmatrix} x+4 & x & x \\ x & x+4 & x \\ x & x & x+4 \end{vmatrix}$$

$Applying\ C_1 \to C_1 + C_2 + C_3,\ we\ get,$

$$\Delta = \begin{vmatrix} 3x+4 & x & x \\ 3x+4 & x+4 & x \\ 3x+4 & x & x+4 \end{vmatrix}$$

$Taking\ the\ common\ term\ 3x+4,\ we\ get,$

$$\Delta = (3x+4)\begin{vmatrix} 1 & x & x \\ 1 & x+4 & x \\ 1 & x & x+4 \end{vmatrix}$$

$Applying\ R_2 \to R_2 - R_1\ and\ R_3 \to R_3 - R_1,\ we\ get,$

$$\Delta = (3x+4)\begin{vmatrix} 1 & x & x \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{vmatrix}$$

$$\Rightarrow \Delta = 16(3x+4)$$

Chapter 6 Determinants Ex 6.2 Q26

We need to prove the following identity:

$$\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 4+3p+2q \\ 3 & 6+3p & 10+6p+3q \end{vmatrix} = 1$$

Let us consider the L.H.S of the above equation.

Applying $C_2 \to C_2 - pC_1$ *and* $C_3 \to C_3 - qC_1$, *we get*

$$\Delta = \begin{vmatrix} 1 & 1 & 1+p \\ 2 & 3 & 4+3p \\ 3 & 6 & 10+6p \end{vmatrix}$$

Applying $C_3 \to C_3 - pC_2$, *we get*

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 3 & 6 & 10 \end{vmatrix}$$

Applying $C_2 \to C_2 - pC_1$ *and* $C_3 \to C_3 - qC_1$, *we get*

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 0 & 0 \\ 2 & 1 & 2 \\ 3 & 3 & 7 \end{vmatrix}$$

$$\Rightarrow \Delta = 1[7-6] = 1$$

Chapter 6 Determinants Ex 6.2 Q27

$$\begin{vmatrix} a & b-c & c-b \\ a-c & b & c-a \\ a-b & b-a & c \end{vmatrix}$$

$R_1 \to R_1 - R_2 - R_3$

$$= \begin{vmatrix} -a+c+b & -b-c+a & -c-b+a \\ a-c & b & c-a \\ a-b & b-a & c \end{vmatrix}$$

$$= (b+c-a)\begin{vmatrix} 1 & -1 & -1 \\ a-c & b & c-a \\ a-b & b-a & c \end{vmatrix}$$

$$= (b+c-a)\begin{vmatrix} 1 & 0 & 0 \\ a-c & b+a-c & 0 \\ a-b & 0 & c+a-b \end{vmatrix}$$

$$= (a+b-c)(b+c-a)(c+a-b)$$

$= RHS$

Chapter 6 Determinants Ex 6.2 Q28

$$LHS = \begin{vmatrix} a^2 & 2ab & b^2 \\ b^2 & a^2 & 2ab \\ 2ab & b^2 & a^2 \end{vmatrix}$$

$$= \begin{vmatrix} a^2+b^2+2ab & 2ab & b^2 \\ a^2+b^2+2ab & a^2 & 2ab \\ a^2+b^2+2ab & b^2 & a^2 \end{vmatrix}$$

$$= \left(a^2+b^2+2ab\right)\begin{vmatrix} 1 & 2ab & b^2 \\ 1 & a^2 & 2ab \\ 1 & b^2 & a^2 \end{vmatrix}$$

$$= \left(a^2+b^2+2ab\right)\begin{vmatrix} 1 & 2ab & b^2 \\ 0 & a^2-2ab & 2ab-b^2 \\ 0 & b^2-2ab & a^2-b^2 \end{vmatrix}$$

$$= \left(a^2+b^2+2ab\right)\begin{vmatrix} 1 & 2ab & b^2 \\ 0 & a^2-b^2 & 2ab-a^2 \\ 0 & b^2-2ab & a^2-b^2 \end{vmatrix}$$

$$= (a+b)^2\left[\left(a^2-b^2\right)\left(a^2-b^2\right)-\left(2ab-a^2\right)\left(b^2-2ab\right)\right]$$

$$= (a+b)^2\left(a^2+b^2-ab\right)^2$$

$$= \left(a^3+b^3\right)^2$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q29

We need to prove the following identity:

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$$

Let us consider the L.H.S of the above equation.

$$\Delta = \begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix}$$

Applying $R_1 \to R_1(a)$, $R_2 \to R_2(b)$ *and* $R_3 \to R_3(c)$, *we get*

$$\Delta = \frac{1}{abc}\begin{vmatrix} a\left(a^2+1\right) & a^2b & a^2c \\ ab^2 & b\left(b^2+1\right) & b^2c \\ c^2a & c^2b & c\left(c^2+1\right) \end{vmatrix}$$

Taking a,b, and c common from C_1,C_2 *and* C_3, *respectively, we get,*

$$\Delta = \frac{abc}{abc}\begin{vmatrix} \left(a^2+1\right) & a^2 & a^2 \\ b^2 & \left(b^2+1\right) & b^2 \\ c^2 & c^2 & \left(c^2+1\right) \end{vmatrix}$$

Applying $R_1 \to R_1+R_2+R_3$, *we get,*

$$\Delta = \frac{abc}{abc}\begin{vmatrix} \left(a^2+b^2+c^2+1\right) & \left(a^2+b^2+c^2+1\right) & \left(a^2+b^2+c^2+1\right) \\ b^2 & \left(b^2+1\right) & b^2 \\ c^2 & c^2 & \left(c^2+1\right) \end{vmatrix}$$

Taking the term, $(a^2+b^2+c^2+1)$ *common from the above equation, we have,*

$$\Delta=(a^2+b^2+c^2+1)\begin{vmatrix}1 & 1 & 1\\ b^2 & (b^2+1) & b^2\\ c^2 & c^2 & (c^2+1)\end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$, *we get,*

$$\Delta=(a^2+b^2+c^2+1)\begin{vmatrix}1 & 0 & 1\\ b^2 & 1 & 0\\ c^2 & 0 & 1\end{vmatrix}$$

$$\Rightarrow \Delta=(a^2+b^2+c^2+1)$$

Chapter 6 Determinants Ex 6.2 Q30

Let us consider the L.H.S of the given equation.

$$\text{Let } \Delta=\begin{vmatrix}1 & a & a^2\\ a^2 & 1 & a\\ a & a^2 & 1\end{vmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$, *we have,*

$$\Delta=\begin{vmatrix}1+a+a^2 & a & a^2\\ 1+a+a^2 & 1 & a\\ 1+a+a^2 & a^2 & 1\end{vmatrix}$$

Taking the term $(1+a+a^2)$ *common, we have,*

$$\Delta=(1+a+a^2)\begin{vmatrix}1 & a & a^2\\ 1 & 1 & a\\ 1 & a^2 & 1\end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ *and* $R_3 \to R_3 - R_1$, *we have*

$$\Delta=(1+a+a^2)\begin{vmatrix}1 & a & a^2\\ 0 & 1-a & a(1-a)\\ 0 & -a(1-a) & (1-a)(1+a)\end{vmatrix}$$

Taking the term $(1-a)$ *common from* R_2 *and* R_3, *we have*

$$\Rightarrow \Delta=(1+a+a^2)(1-a)^2\begin{vmatrix}1 & a & a^2\\ 0 & 1 & a\\ 0 & -a & (1+a)\end{vmatrix}$$

$$\Rightarrow \Delta=(1+a+a^2)(1-a)^2(1+a+a^2)$$

$$\Rightarrow \Delta=(1+a+a^2)^2(1-a)^2$$

$$\Rightarrow \Delta=[(1+a+a^2)(1-a)]^2$$

$$\Rightarrow \Delta=[(a^3-1)]^2$$

Chapter 6 Determinants Ex 6.2 Q31

$$\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix} = 2(a+b)(b+c)(c+a)$$

$$\text{LHS} = \begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix}$$

Apply: $C_1 \to C_1 + C_3$ and $C_2 \to C_2 + C_3$

$$= \begin{vmatrix} a+c & -(c+b) & -b \\ -(c+a) & b+c & -a \\ a+c & b+c & a+b+c \end{vmatrix}$$

$$=(c+a)(c+b)\begin{vmatrix} 1 & -1 & -b \\ -1 & 1 & -a \\ 1 & 1 & a+b+c \end{vmatrix}$$

$$=(c+a)(c+b)\begin{vmatrix} 1 & -1 & -b \\ 0 & 0 & -a-b \\ 0 & 2 & a+c \end{vmatrix}$$

$$= 2(a+b)(b+c)(c+a)$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q32

We need to prove the following identity:

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$$

Let us consider the L.H.S of the above equation.

$$\Delta = \begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

Applying $R_1 \to R_1 + R_2 + R_3$, *we have,*

$$\Delta = \begin{vmatrix} 2(b+c) & 2(a+c) & 2(a+b) \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

Taking 2 *common from the above equation, we have,*

$$\Delta = 2\begin{vmatrix} (b+c) & (a+c) & (a+b) \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ *and* $R_3 \to R_3 - R_1$, *we have,*

$$\Delta = 2\begin{vmatrix} (b+c) & (a+c) & (a+b) \\ -c & 0 & -a \\ -b & -a & 0 \end{vmatrix}$$

Applying $R_1 \to R_1 + R_2 + R_3$, *we have,*

$$\Delta = 2\begin{vmatrix} 0 & c & b \\ -c & 0 & -a \\ -b & -a & 0 \end{vmatrix}$$

$$\Rightarrow \Delta = 2(0 + 2abc + abc)$$

$$\Rightarrow \Delta = 4abc$$

Chapter 6 Determinants Ex 6.2 Q33

$$\begin{vmatrix} b^2+c^2 & ab & ac \\ ba & c^2+a^2 & bc \\ ca & cb & a^2+b^2 \end{vmatrix} = 4a^2b^2c^2$$

$$LHS = \begin{vmatrix} b^2+c^2 & ab & ac \\ ba & c^2+a^2 & bc \\ ca & cb & a^2+b^2 \end{vmatrix}$$

Multiply R_1, R_2 and R_3 by a, b and c respectively.

$$= \frac{1}{abc}\begin{vmatrix} ab^2+ac^2 & a^2b & a^2c \\ b^2a & bc^2+ba^2 & b^2c \\ c^2a & c^2b & ca^2+cb^2 \end{vmatrix}$$

Take a, b and c common from C_1, C_2 and C_3 respectively.

$$= \frac{abc}{abc}\begin{vmatrix} b^2+c^2 & a^2 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$$

Now apply $R_1 \to R_1 + R_2 + R_3$

$$= \begin{vmatrix} 2\left(b^2+c^2\right) & 2\left(c^2+a^2\right) & 2\left(a^2+b^2\right) \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$$

$$= 2\begin{vmatrix} \left(b^2+c^2\right) & \left(c^2+a^2\right) & \left(a^2+b^2\right) \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$$

$$= 2\begin{vmatrix} c^2 & 0 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$$

$$= 2\left[c^2\left\{\left(c^2+a^2\right)\left(a^2+b^2\right) - b^2c^2\right\} + a^2\left\{b^2c^2 - \left(c^2+a^2\right)c^2\right\}\right]$$

$$= 4a^2b^2c^2$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q34

$$\begin{vmatrix} 0 & b^2a & c^2a \\ a^2b & 0 & c^2b \\ a^2c & b^2c & 0 \end{vmatrix} = 2a^3b^3c^3$$

$$\text{LHS} = \begin{vmatrix} 0 & b^2a & c^2a \\ a^2b & 0 & c^2b \\ a^2c & b^2c & 0 \end{vmatrix}$$

$$= a^2b^2c^2\begin{vmatrix} 0 & a & a \\ b & 0 & b \\ c & c & 0 \end{vmatrix}$$

$$= a^3b^3c^3\begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix}$$

$$= a^3b^3c^3\begin{vmatrix} 0 & 0 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 0 \end{vmatrix}$$

$$= 2a^3b^3c^3$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q35

$$\begin{vmatrix} \frac{a^2+b^2}{c} & c & c \\ a & \frac{b^2+c^2}{a} & a \\ b & b & \frac{c^2+a^2}{b} \end{vmatrix}$$

$$= \frac{1}{abc}\begin{vmatrix} a^2+b^2 & c^2 & c^2 \\ a^2 & c^2+b^2 & a^2 \\ b^2 & b^2 & c^2+a^2 \end{vmatrix}$$

$$= \frac{1}{abc}\begin{vmatrix} 0 & -2b^2 & -2a^2 \\ a^2 & c^2+b^2 & a^2 \\ b^2 & b^2 & c^2+a^2 \end{vmatrix}$$

$$= \frac{-2}{abc}\begin{vmatrix} 0 & b^2 & a^2 \\ a^2 & c^2+b^2 & a^2 \\ b^2 & b^2 & c^2+a^2 \end{vmatrix}$$

$$= \frac{-2}{abc}\begin{vmatrix} 0 & b^2 & a^2 \\ a^2 & c^2+b^2 & a^2 \\ b^2 & 0 & c^2 \end{vmatrix}$$

$$= \frac{-2}{abc}\begin{vmatrix} 0 & b^2 & a^2 \\ a^2 & c^2 & 0 \\ b^2 & 0 & c^2 \end{vmatrix}$$

$$= \frac{-2}{abc}\left[\left(-a^2\right)\left(b^2c^2\right)+\left(b^2\right)\left(-a^2c^2\right)\right]$$

$$= \frac{-2}{abc}\left(-2a^2b^2c^2\right)$$

$$= 4abc$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q36

$$\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix}$$

Multiply R_1, R_2 and R_3 by a, b and c respectively

$$= \frac{1}{abc}\begin{vmatrix} -abc & ab^2+abc & ac^2+abc \\ a^2b+abc & -abc & bc^2+abc \\ a^2c+abc & b^2c+abc & -abc \end{vmatrix}$$

Take a, b and c common from C_1, C_2 and C_3 respectively.

$$= \frac{abc}{abc}\begin{vmatrix} -bc & ab+ac & ac+ab \\ ab+bc & -ac & bc+ab \\ ac+bc & bc+ac & -ab \end{vmatrix}$$

Apply: $R_1 \to R_1 + R_2 + R_3$

$$= \begin{vmatrix} ab+bc+ca & ab+bc+ca & ab+bc+ca \\ ab+bc & -ac & bc+ab \\ ac+bc & bc+ac & -ab \end{vmatrix}$$

$$= \left(ab+bc+ca\right)\begin{vmatrix} 1 & 1 & 1 \\ ab+bc & -ac & bc+ab \\ ac+bc & bc+ac & -ab \end{vmatrix}$$

$$= \left(ab+bc+ca\right)\begin{vmatrix} 0 & 1 & 0 \\ ab+bc+ac & -ac & bc+ab+ac \\ 0 & bc+ac & -ab-bc-ac \end{vmatrix}$$

$$= \left(ab+bc+ca\right)^3\begin{vmatrix} 0 & 1 & 0 \\ 1 & -ac & 1 \\ 0 & bc+ac & -1 \end{vmatrix}$$

$$= \left(ab+bc+ca\right)^3$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q37

L.H.S.,

$$\begin{vmatrix} x+\lambda & 2x & 2x \\ 2x & x+\lambda & 2x \\ 2x & 2x & x+\lambda \end{vmatrix}$$

$$= \begin{vmatrix} x+\lambda & 2x & 2x \\ 2x & x+\lambda & 2x \\ 2x & 2x & x+\lambda \end{vmatrix} [C_1 \to C_1 - C_3, C_2 \to C_2 - C_3]$$

$$= \begin{vmatrix} \lambda - x & 0 & 2x \\ 0 & \lambda - x & 2x \\ x-\lambda & x-\lambda & x+\lambda \end{vmatrix}$$

$$= (\lambda - x)(\lambda - x)\begin{vmatrix} 1 & 0 & 2x \\ 0 & 1 & 2x \\ -1 & -1 & x+\lambda \end{vmatrix}$$

$$= (\lambda - x)^2\begin{vmatrix} 1 & 0 & 2x \\ 0 & 1 & 2x \\ -1 & -1 & x+\lambda \end{vmatrix}$$

$$= (\lambda - x)^2[1(x+\lambda) + 2x + 2x(0+1)]$$

$$= (\lambda - x)^2[x + \lambda + 2x + 2x]$$

$$= (\lambda - x)^2[5x + \lambda]$$

= R.H.S

Hence Proved

Chapter 6 Determinants Ex 6.2 Q38

$$LHS = \begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$$

Apply $C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} 5x+4 & 2x & 2x \\ 5x+4 & x+4 & 2x \\ 5x+4 & 2x & x+4 \end{vmatrix}$$

$$= (5x+4)\begin{vmatrix} 1 & 2x & 2x \\ 1 & x+4 & 2x \\ 1 & 2x & x+4 \end{vmatrix}$$

$$= (5x+4)\begin{vmatrix} 1 & 2x & 2x \\ 0 & -x+4 & 0 \\ 0 & 0 & -x+4 \end{vmatrix}$$

$$= (5x+4)(4-x)^2\begin{vmatrix} 1 & 2x & 2x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= (5x+4)(4-x)^2$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q39

$$\text{Let } \Delta = \begin{vmatrix} y+z & z & y \\ z & z+x & x \\ y & x & x+y \end{vmatrix}$$

Applying $R_1 \to R_1 - R_2$

$$\Delta = \begin{vmatrix} y & -x & y-x \\ z & z+x & x \\ y & x & x+y \end{vmatrix}$$

Applying $R_1 \to R_1 - R_3$

$$\Delta = \begin{vmatrix} 0 & -2x & -2x \\ z & z+x & x \\ y & x & x+y \end{vmatrix}$$

$$\Delta = 2x\left[z(x+y) - xy\right] - 2x\left[zx - y(z+x)\right]$$

$$\Delta = 2x\left[zx + zy - xy - zx + yz + yx\right]$$

$$\Delta = 4xyz$$

Chapter 6 Determinants Ex 6.2 Q40

$$\begin{vmatrix} -a\left(b^2+c^2-a^2\right) & 2b^3 & 2c^3 \\ 2a^3 & -b\left(c^2+a^2-b^2\right) & 2c^3 \\ 2a^3 & 2b^3 & -c\left(a^2+b^2-c^2\right) \end{vmatrix} = abc\left(a^2+b^2+c^2\right)^3$$

$$\text{LHS} = \begin{vmatrix} -a\left(b^2+c^2-a^2\right) & 2b^3 & 2c^3 \\ 2a^3 & -b\left(c^2+a^2-b^2\right) & 2c^3 \\ 2a^3 & 2b^3 & -c\left(a^2+b^2-c^2\right) \end{vmatrix}$$

Take a, b and c common from C_1, C_2 and C_3 respectively.

$$= abc\begin{vmatrix} -\left(b^2+c^2-a^2\right) & 2b^2 & 2c^2 \\ 2a^2 & -\left(c^2+a^2-b^2\right) & 2c^2 \\ 2a^2 & 2b^2 & -\left(a^2+b^2-c^2\right) \end{vmatrix}$$

Apply: $R_1 \to R_1 - R_3, R_2 \to R_2 - R_3$

$$= abc\begin{vmatrix} -\left(b^2+c^2-a^2\right) - 2a^2 & 0 & 2c^2 + \left(a^2+b^2-c^2\right) \\ 0 & -\left(c^2+a^2-b^2\right) - 2b^2 & 2c^2 + \left(a^2+b^2-c^2\right) \\ 2a^2 & 2b^2 & -\left(a^2+b^2-c^2\right) \end{vmatrix}$$

$$= abc\begin{vmatrix} -\left(b^2+c^2+a^2\right) & 0 & \left(a^2+b^2+c^2\right) \\ 0 & -\left(c^2+a^2+b^2\right) & \left(a^2+b^2+c^2\right) \\ 2a^2 & 2b^2 & -\left(a^2+b^2-c^2\right) \end{vmatrix}$$

$$= abc\left(b^2+c^2+a^2\right)^2\begin{vmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 2a^2 & 2b^2 & -\left(a^2+b^2-c^2\right) \end{vmatrix}$$

$$= abc\left(b^2+c^2+a^2\right)^2\begin{vmatrix} -1 & 0 & 0 \\ 0 & -1 & 1 \\ 2a^2 & 2b^2 & -\left(a^2+b^2-c^2\right) + 2a^2 \end{vmatrix}$$

$$= abc\left(b^2+c^2+a^2\right)^2\begin{vmatrix} -1 & 0 & 0 \\ 0 & -1 & 1 \\ 2a^2 & 2b^2 & -b^2+c^2+a^2 \end{vmatrix}$$

$$= -abc\left(b^2+c^2+a^2\right)^2\left[(-1)\left(-b^2+c^2+a^2\right) - (1)\left(2b^2\right)\right]$$

$$abc\left(a^2+b^2+c^2\right)^3$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q41

$$LHS = \begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+a & 1 \\ 1 & 1 & 1+a \end{vmatrix}$$

$$= \begin{vmatrix} 3+a & 3+a & 3+a \\ 1 & 1+a & 1 \\ 1 & 1 & 1+a \end{vmatrix}$$

$$= (3+a)\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+a & 1 \\ 1 & 1 & 1+a \end{vmatrix}$$

$$= (3+a)\begin{vmatrix} 1 & 1 & 1 \\ 0 & a & 0 \\ 0 & 1 & a \end{vmatrix}$$

$$= (3+a)a^2$$
$$= a^3 + 3a^2$$
$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q42

L.H.S.,

$$\begin{vmatrix} 2y & y-z-x & 2y \\ 2z & 2z & z-x-y \\ x-y-z & 2x & 2x \end{vmatrix}$$

$$= \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ 2z & 2z & z-x-y \\ x-y-z & 2x & 2x \end{vmatrix} [R_1 = R_1 + R_2 + R_3]$$

$$= (x+y+z)\begin{vmatrix} 1 & 1 & 1 \\ 2z & 2z & z-x-y \\ x-y-z & 2x & 2x \end{vmatrix}$$

$$= (x+y+z)\begin{vmatrix} 1 & 0 & 0 \\ 2z & 0 & -x-y-z \\ x-y-z & x+y+z & x+y+z \end{vmatrix} [C_2 = C_2 - C_1, C_3 = C_3 - C_1]$$

$$= (x+y+z)[1\{0+(x+y+z)(x+y+z)\}]$$
$$= (x+y+z)^3$$
$$= \text{R.H.S.}$$

Hence Proved

Chapter 6 Determinants Ex 6.2 Q43

$$\begin{vmatrix} y+z & x & y \\ z+x & z & x \\ x+y & y & z \end{vmatrix}$$

$$= \begin{vmatrix} 2(y+z+x) & y+z+x & y+z+x \\ z+x & z & x \\ x+y & y & z \end{vmatrix}$$

$$= (x+y+z)\begin{vmatrix} 2 & 1 & 1 \\ z+x & z & x \\ x+y & y & z \end{vmatrix}$$

$$= (x+y+z)\begin{vmatrix} 0 & 1 & 1 \\ z+x-z-x & z & x \\ x+y-y-z & y & z \end{vmatrix}$$

$$= (x+y+z)\begin{vmatrix} 0 & 1 & 1 \\ 0 & z & x \\ x-z & y & z \end{vmatrix}$$

$$= (x+y+z)(x-z)^2$$
$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q44

L.H.S. =

$$\begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix}$$

$$= \begin{vmatrix} a+x+y+z & y & z \\ a+x+y+z & a+y & z \\ a+x+y+z & x & a+z \end{vmatrix} [C_1 = C_1 + C_2 + C_3]$$

$$= (a+x+y+z)\begin{vmatrix} 1 & y & z \\ 1 & a+y & z \\ 1 & x & a+z \end{vmatrix}$$

$$= (a+x+y+z)\begin{vmatrix} 1 & y & z \\ 0 & a & 0 \\ 0 & x-y & a \end{vmatrix} [R_2 \to R_2 - R_1, R_3 \to R_3 - R_1]$$

$$= (a+x+y+z)[1(a^2 - 0)]$$

$$= a^2(a+x+y+z)$$

= R.H.S.

Hence Pr oved.

Chapter 6 Determinants Ex 6.2 Q45

$$\text{Let } \Delta = \begin{vmatrix} a^3 & 2 & a \\ b^3 & 2 & b \\ c^3 & 2 & c \end{vmatrix}$$

$$\Delta = 2\begin{vmatrix} a^3 & 1 & a \\ b^3 & 1 & b \\ c^3 & 1 & c \end{vmatrix}$$

$$\Delta = 2\left\{a^3(c-b) - 1\left(b^3c - bc^3\right) + a\left(b^3 - c^3\right)\right\}$$

$$\Delta = 2\left\{a^3(c-b) - bc(b-c)(b+c) + a(b-c)\left(b^2 + bc + c^2\right)\right\}$$

$$\Delta = 2(b\text{-}c)\left\{-a^3 - bc(b+c) + a\left(b^2 + bc + c^2\right)\right\}$$

$$\Delta = 2(a\text{-}b)(b\text{-}c)(c-a)(a+b+c)$$

Chapter 6 Determinants Ex 6.2 Q46

$$\begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix} = -\begin{vmatrix} x & y & z \\ a & b & c \\ p & q & r \end{vmatrix} = (-1)^2\begin{vmatrix} x & y & z \\ p & q & r \\ a & b & c \end{vmatrix} = \begin{vmatrix} x & y & z \\ p & q & r \\ a & b & c \end{vmatrix}$$

$$= (-1)\begin{vmatrix} y & x & z \\ q & p & r \\ b & a & c \end{vmatrix}$$

$$= (-1)^2\begin{vmatrix} y & x & z \\ b & a & c \\ q & p & r \end{vmatrix}$$

Taking transpose, we get

$$= \begin{vmatrix} y & b & p \\ x & a & q \\ z & c & r \end{vmatrix}$$

Chapter 6 Determinants Ex 6.2 Q47

$$\text{Consider the determinant } \begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}, \text{ where } a, b, c \text{ are in A.P.}$$

$$\text{Let } \Delta = \begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$, *we have,*

$$\Delta = \begin{vmatrix} 3x+1+2+a & x+2 & x+a \\ 3x+2+3+b & x+3 & x+b \\ 3x+3+4+c & x+4 & x+c \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 3x+3+a & x+2 & x+a \\ 3x+5+b & x+3 & x+b \\ 3x+7+c & x+4 & x+c \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ *and* $R_3 \to R_3 - R_2$, *we have,*

$$\Rightarrow \Delta = \begin{vmatrix} 3x+3+a & x+2 & x+a \\ 2+b-a & 1 & b-a \\ 2+c-b & 1 & c-b \end{vmatrix}$$

Since a, b *and* c *are in arithmetic progression, we have*

$b - a = c - b = k(say)$.

Thus,

$$\Delta = \begin{vmatrix} 3x+3+a & x+2 & x+a \\ 2+k & 1 & k \\ 2+k & 1 & k \end{vmatrix}$$

Since the second row and the third row are identical, we have

$\Delta = 0$

Chapter 6 Determinants Ex 6.2 Q48

Since, α, β, γ are in A.P, $2\beta = \alpha + \gamma$

$$LHS = \begin{vmatrix} x-3 & x-4 & x-\alpha \\ x-2 & x-3 & x-\beta \\ x-1 & x-2 & x-\gamma \end{vmatrix}$$

$R_2 \to R_2 - \frac{R_1}{2} - \frac{R_3}{2}$

$$= \begin{vmatrix} x-3 & x-4 & x-\alpha \\ (x-2) - \frac{x-3}{2} - \frac{x-1}{2} & (x-3) - \frac{x-4}{2} - \frac{x-2}{2} & (x-\beta) - \frac{x-\alpha}{2} - \frac{x-\gamma}{2} \\ x-1 & x-2 & x-\gamma \end{vmatrix}$$

$$= \begin{vmatrix} x-3 & x-4 & x-\alpha \\ 0 & 0 & 0 \\ x-1 & x-2 & x-\gamma \end{vmatrix} \qquad [\because 2\beta = \alpha + \gamma]$$

$= 0$

Chapter 6 Determinants Ex 6.2 Q49

$$\Delta = \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}$$

Applying $R_1 \to R_1 + R_2 + R_3$, we have:

$$\Delta = \begin{vmatrix} 2(a+b+c) & 2(a+b+c) & 2(a+b+c) \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}$$

$$= 2(a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$, we have:

$$\Delta = 2(a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ c+a & b-c & b-a \\ a+b & c-a & c-b \end{vmatrix}$$

Expanding along R_1, we have:

$$\Delta = 2(a+b+c)(1)\left[(b-c)(c-b)-(b-a)(c-a)\right]$$
$$= 2(a+b+c)\left[-b^2-c^2+2bc-bc+ba+ac-a^2\right]$$
$$= 2(a+b+c)\left[ab+bc+ca-a^2-b^2-c^2\right]$$

It is given that $\Delta = 0$.

$$(a+b+c)\left[ab+bc+ca-a^2-b^2-c^2\right]=0$$

$\Rightarrow$ Either $a+b+c=0$, or $ab+bc+ca-a^2-b^2-c^2=0$.

Now,

$$ab+bc+ca-a^2-b^2-c^2=0$$
$$\Rightarrow -2ab-2bc-2ca+2a^2+2b^2+2c^2=0$$
$$\Rightarrow (a-b)^2+(b-c)^2+(c-a)^2=0$$
$$\Rightarrow (a-b)^2=(b-c)^2=(c-a)^2=0 \qquad \left[(a-b)^2,(b-c)^2,(c-a)^2 \text{ are non-negative}\right]$$
$$\Rightarrow (a-b)=(b-c)=(c-a)=0$$
$$\Rightarrow a=b=c$$

Hence, if $\Delta = 0$, then either $a+b+c=0$ or $a=b=c$.

Chapter 6 Determinants Ex 6.2 Q50

$$\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} p-a & 0 & c-r \\ 0 & q-b & c-r \\ a & b & r \end{vmatrix} = 0 [R_1 = R_1 - R_3, R_2 = R_2 - R_3]$$

$$\Rightarrow (p-a)[r(q-b) - b(c-r)] + (c-r)[0 - a(q-b)] = 0$$

$$\Rightarrow (p-a)r(q-b) - (p-a)b(c-r) - (c-r)a(q-b) = 0$$

$$\Rightarrow \frac{(p-a)r(q-b)}{(p-a)(q-b)(r-c)} - \frac{(p-a)b(c-r)}{(p-a)(q-b)(r-c)} - \frac{(c-r)a(q-b)}{(p-a)(q-b)(r-c)} = 0$$

$$\Rightarrow \frac{r}{(r-c)} + \frac{b}{(q-b)} + \frac{a}{(p-a)} = 0$$

$$\Rightarrow \frac{r}{(r-c)} + \frac{b-q+q}{(q-b)} + \frac{a+p-p}{(p-a)} = 0$$

$$\Rightarrow \frac{r}{(r-c)} + \frac{q}{(q-b)} + \frac{(b-q)}{(q-b)} + \frac{(a-p)}{(p-a)} + \frac{p}{(p-a)} = 0$$

$$\Rightarrow \frac{r}{(r-c)} + \frac{q}{(q-b)} - 1 - 1 + \frac{p}{(p-a)} = 0$$

$$\Rightarrow \frac{r}{(r-c)} + \frac{q}{(q-b)} + \frac{p}{(p-a)} = 2$$

$$\therefore \frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} = 2$$

Chapter 6 Determinants Ex 6.2 Q51

Let us show that $x = 2$ is a root of the given equation:
Putting $x = 2$ in the LHS, we get

$$\begin{vmatrix} 2 & -6 & -1 \\ 2 & -6 & -1 \\ -3 & 4 & 4 \end{vmatrix} = 0$$

$\because R_1 = R_2$

Hence, $x = 2$ is a root of the given equation.

Now, we see if there are any other roots. For this we need to solve the equation:

$$\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-1 & -6 & -1 \\ x-1 & -3x & x-3 \\ x-1 & 2x & x+2 \end{vmatrix} = 0$$

$$\Rightarrow (x-1)\begin{vmatrix} 1 & -6 & -1 \\ 1 & -3x & x-3 \\ 1 & 2x & x+2 \end{vmatrix} = 0$$

$$\Rightarrow (x-1)\begin{vmatrix} 1 & -6 & -1 \\ 1 & -3x & x-3 \\ 1 & 2x & x+2 \end{vmatrix} = 0$$

$$\Rightarrow (x-1)\begin{vmatrix} 1 & -6 & -1 \\ 0 & -3x+6 & x-3+1 \\ 0 & 2x+6 & x+2+1 \end{vmatrix} = 0$$

$$\Rightarrow (x-1)\begin{vmatrix} 1 & -6 & -1 \\ 0 & -3(x-2) & x-2 \\ 0 & 2(x+3) & x+3 \end{vmatrix} = 0$$

$$\Rightarrow (x-1)(x-2)(x+3)\begin{vmatrix} 1 & -6 & -1 \\ 0 & -3 & 1 \\ 0 & 2 & 1 \end{vmatrix} = 0$$

$$\Rightarrow (x-1)(x-2)(x+3) = 0$$

$$\Rightarrow (x-1) = 0 \qquad (x-2) = 0 \qquad (x+3) = 0$$

$$\Rightarrow x = 1 \qquad x = 2 \qquad x = -3$$

Chapter 6 Determinants Ex 6.2 Q52-i

$$\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = 0$$

Apply $C_1 \to C_1 + C_2 + C_3$

$$\Rightarrow \begin{vmatrix} x+a+b+c & b & c \\ x+a+b+c & x+b & c \\ x+a+b+c & b & x+c \end{vmatrix} = 0$$

$$\Rightarrow (x+a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & x+b & c \\ 1 & b & x+c \end{vmatrix} = 0$$

$$\Rightarrow (x+a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & x & 0 \\ 0 & 0 & x \end{vmatrix} = 0$$

$$\Rightarrow (x+a+b+c)x^2 = 0$$

$$\Rightarrow x = -(a+b+c) \quad or \quad x = 0$$

Chapter 6 Determinants Ex 6.2 Q52-ii

$$\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$$

Applying $R_1 \to R_1 + R_2 + R_3$, we get:

$$\begin{vmatrix} 3x+a & 3x+a & 3x+a \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$$

$$\Rightarrow (3x+a)\begin{vmatrix} 1 & 1 & 1 \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$$

Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$, we have:

$$(3x+a)\begin{vmatrix} 1 & 0 & 0 \\ x & a & 0 \\ x & 0 & a \end{vmatrix} = 0$$

Expanding along R_1, we have:

$$(3x+a)\left[1 \times a^2\right] = 0$$

$$\Rightarrow a^2(3x+a) = 0$$

But $a \neq 0$.

Therefore, we have:

$$3x + a = 0$$

$$\Rightarrow x = -\frac{a}{3}$$

Chapter 6 Determinants Ex 6.2 Q52-iii

$$\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$$

Apply $C_1 \to C_1 + C_2 + C_3$

$$\Rightarrow \quad \begin{vmatrix} 3x-2 & 3 & 3 \\ 3x-2 & 3x-8 & 3 \\ 3x-2 & 3 & 3x-8 \end{vmatrix} = 0$$

$$\Rightarrow \quad (3x-2)\begin{vmatrix} 1 & 3 & 3 \\ 1 & 3x-8 & 3 \\ 1 & 3 & 3x-8 \end{vmatrix} = 0$$

$$\Rightarrow \quad (3x-2)\begin{vmatrix} 1 & 3 & 3 \\ 0 & 3x-11 & 0 \\ 0 & 0 & 3x-11 \end{vmatrix} = 0$$

$$\Rightarrow \quad (3x-2)(3x-11)^2 = 0$$

$$\Rightarrow \quad (3x-2) = 0 \quad or \quad (3x-11)^2 = 0$$

$$\Rightarrow \quad x = \frac{2}{3} \; or \quad x = \pm\frac{11}{3}$$

Chapter 6 Determinants Ex 6.2 Q52-iv

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & a & a^2 \\ 1 & b & b^2 \end{vmatrix} = 0$$

$$\Rightarrow \quad \begin{vmatrix} 1 & x & x^2 \\ 0 & a-x & a^2-x^2 \\ 0 & b-x & b^2-x^2 \end{vmatrix} = 0$$

$$\Rightarrow \quad (a-x)(b-x)\begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & a+x \\ 0 & 1 & b+x \end{vmatrix} = 0$$

$$\Rightarrow \quad (a-x)(b-x)\begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & a+x \\ 0 & 0 & b-a \end{vmatrix} = 0$$

$$\Rightarrow \quad (a-x)(b-x)(b-a) = 0$$

$$\Rightarrow \quad (a-x) = 0 \quad or \quad (b-x) = 0$$

$$\Rightarrow \quad a = x \quad or \quad b = x$$

Chapter 6 Determinants Ex 6.2 Q52-v

$$\begin{vmatrix} x+1 & 3 & 5 \\ 2 & x+2 & 5 \\ 2 & 3 & x+4 \end{vmatrix} = 0$$

Apply $C_1 \to C_1 + C_2 + C_3$

$$\Rightarrow \quad \begin{vmatrix} x+9 & 3 & 5 \\ x+9 & x+2 & 5 \\ x+9 & 3 & x+4 \end{vmatrix} = 0$$

$$\Rightarrow \quad (x+9)\begin{vmatrix} 1 & 3 & 5 \\ 1 & x+2 & 5 \\ 1 & 3 & x+4 \end{vmatrix} = 0$$

$$\Rightarrow \quad (x+9)\begin{vmatrix} 1 & 3 & 5 \\ 0 & x-1 & 0 \\ 0 & 0 & x-1 \end{vmatrix} = 0$$

$$\Rightarrow \quad (x+9)(x-1)^2 = 0$$

$$\Rightarrow \quad (x+9) = 0 \quad or \quad (x-1)^2 = 0$$

$$\Rightarrow \quad x = -9 \quad or \quad x = 1$$

Chapter 6 Determinants Ex 6.2 Q52-vi

$$\Rightarrow \quad \begin{vmatrix} 1 & x & x^3 \\ 0 & b-x & b^3-x^3 \\ 0 & c-x & c^3-x^3 \end{vmatrix} = 0$$

$$\Rightarrow \quad (b-x)(c-x)\begin{vmatrix} 1 & x & x^3 \\ 0 & 1 & b^2+x^2+bx \\ 0 & 1 & c^2+x^2+cx \end{vmatrix} = 0$$

$$\Rightarrow \quad (b-x)(c-x)\begin{vmatrix} 1 & x & x^3 \\ 0 & 1 & b^2+x^2+bx \\ 0 & 0 & c^2+x^2+cx-\left(b^2+x^2+bx\right) \end{vmatrix} = 0$$

$$\Rightarrow \quad (b-x)(c-x)\begin{vmatrix} 1 & x & x^3 \\ 0 & 1 & b^2+x^2+bx \\ 0 & 0 & c^2-b^2+cx-bx \end{vmatrix} = 0$$

$$\Rightarrow \quad (b-x)(c-x)(c-b)\begin{vmatrix} 1 & x & x^3 \\ 0 & 1 & b^2+x^2+bx \\ 0 & 0 & b+c+x \end{vmatrix} = 0$$

$$\Rightarrow \quad (b-x)(c-x)(c-b)(b+c+x) = 0$$

$$\Rightarrow \quad (b-x) = 0 \quad (c-x) = 0 \quad (b+c+x) = 0$$

$$\Rightarrow \quad x = b \quad x = c \quad x = -(b+c)$$

Chapter 6 Determinants Ex 6.2 Q52-vii

$$\begin{vmatrix} 15-2x & 11-3x & 7-x \\ 11 & 17 & 14 \\ 10 & 16 & 13 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 15-2x & 11-3x & 7-x \\ 1 & 1 & 1 \\ 10 & 16 & 13 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 15-2x & -x-4 & 7-x \\ 1 & 0 & 1 \\ 10 & 6 & 13 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 8-x & -x-4 & 7-x \\ 0 & 0 & 1 \\ -3 & 6 & 13 \end{vmatrix} = 0$$

$$\Rightarrow -\left[(8-x)(6)-(-x-4)(-3)\right] = 0$$

$$\Rightarrow -\left[36-9x\right] = 0$$

$$\Rightarrow x = 4$$

Chapter 6 Determinants Ex 6.2 Q52-viii

$$\begin{vmatrix} 1 & 1 & x \\ p+1 & p+1 & p+x \\ 3 & x+1 & x+2 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & 1 & x \\ p & p & p \\ 2 & x & 2 \end{vmatrix} = 0$$

$$\Rightarrow p\begin{vmatrix} 1 & 1 & x \\ 1 & 1 & 1 \\ 2 & x & 2 \end{vmatrix} = 0$$

$$\Rightarrow p\begin{vmatrix} 1 & 1 & x \\ 0 & 0 & 1-x \\ 2 & x & 2 \end{vmatrix} = 0$$

$$\Rightarrow p(x-1)(x-2) = 0$$

$$\Rightarrow (x-1) = 0 \qquad (x-2) = 0$$

$$\Rightarrow x = 1 \qquad x = 2$$

Chapter 6 Determinants Ex 6.2 Q52-ix

$$\begin{vmatrix} 3 & -2 & \sin 3\theta \\ -7 & 8 & \cos 2\theta \\ -11 & 14 & 2 \end{vmatrix} = 0$$

$\Rightarrow 3(16 - 14\cos 2\theta) + 2(-14 + 11\cos 2\theta) + \sin 3\theta(-98 + 88) = 0$

$\Rightarrow 20(1 - \cos 2\theta) + 10\sin 3\theta = 0$

$\Rightarrow 20\left(2\sin^2\theta\right) + 10\left(3\sin\theta - 4\sin^3\theta\right) = 0$

$\Rightarrow 4\sin^2\theta + 3\sin\theta - 4\sin^3\theta = 0$

$\Rightarrow 4\sin\theta + 3 - 4\sin^2\theta = 0$

$\Rightarrow 4\sin^2\theta - 4\sin\theta - 3 = 0$

$\Rightarrow (2\sin\theta + 1)(2\sin\theta - 3) = 0$

$\Rightarrow \sin\theta = -\dfrac{1}{2}$ or $\sin\theta = \dfrac{3}{2} = 1.5$

As $\sin\theta \in [-1, 1]$

$\therefore \sin\theta = -\dfrac{1}{2}$

$\Rightarrow \theta = n\pi + (-1)^n \dfrac{\pi}{6}, n \in Z$

Ex 6.3

Chapter Determinants Ex 6.3 Q1(i)

If the vertices of a triangle are $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) then the area of the triangle is given by :

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Substituting the values

$$\Delta = \frac{1}{2}\begin{vmatrix} 3 & 8 & 1 \\ -4 & 2 & 1 \\ 5 & -1 & 1 \end{vmatrix}$$

expanding the determinant along R_1

$$= \frac{1}{2}\left[3\begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix} - 8\begin{vmatrix} -4 & 1 \\ 5 & 1 \end{vmatrix} + 1\begin{vmatrix} -4 & 2 \\ 5 & -1 \end{vmatrix}\right]$$

$$= \frac{1}{2}\left[3(3) - 8(-9) + 1(-6)\right]$$

$$= \frac{1}{2}\left[9 + 72 - 6\right] = \frac{75}{2} \text{ sq. units}$$

The area of the Δ is $\frac{75}{2}$ sq. units

Chapter Determinants Ex 6.3 Q1(ii)

The area is given by:

$$\Delta = \frac{1}{2}\begin{vmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 10 & 8 & 1 \end{vmatrix}$$

expanding along R_1

$$= \frac{1}{2}\left[2(-7) - 7(-9) + 1(-2)\right]$$

$$= \frac{1}{2}\left[-14 + 63 - 2\right]$$

$$= \frac{47}{2} \text{ sq. units}$$

The area of the Δ is $\frac{47}{2}$ sq. units

Chapter Determinants Ex 6.3 Q1(iii)

The area is given by:

$$\Delta = \frac{1}{2}\begin{vmatrix} -1 & -8 & 1 \\ -2 & -3 & 1 \\ 3 & 2 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[-1(-5) + 8(-5) + 1(5)\right]$$

$$= \frac{1}{2}\left[5 - 40 + 5\right] = \frac{-30}{2} = 15 \text{ sq. units}$$

$\because$ Area can not be negative, so answer will be 15 sq. units.

The area of the Δ is 15 sq. units.

Chapter Determinants Ex 6.3 Q1(iv)

The area is given by:

$$\Delta = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 6 & 0 & 1 \\ 4 & 3 & 1 \end{vmatrix}$$

Expanding along R_1

$$= \frac{1}{2}\left[0 - 0 + 1(18)\right] = 9 \text{ sq. units}$$

The area is 9 sq. units

Chapter Determinants Ex 6.3 Q2(i)

If 3 points are collinear, then the area of the triangle then form will be zero.

Hence

$$\frac{1}{2}\begin{vmatrix} 5 & 5 & 1 \\ -5 & 1 & 1 \\ 10 & 7 & 1 \end{vmatrix} = 0$$

Expanding along R_1

$$= \frac{1}{2}\left[5(-6) - 5(-15) + 1(-35 - 10)\right]$$

$$= \frac{1}{2}\left[-35 + 75 - 45\right]$$

$$= \frac{1}{2}[0]$$

$$= 0$$

Since the area of the triangle is zero, hence the points are collinear.

Chapter Determinants Ex 6.3 Q2(ii)

If 3 points are collinear, then the area of the triangle then form will be zero.

Hence

$$\frac{1}{2}\begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & 1 \\ 4 & 5 & 1 \end{vmatrix} = 0$$

Expanding along R_1

$= \frac{1}{2}\left[1(-4) + 1(-2) + 1(6)\right]$

$= 0$

Since the area of the triangle is zero, hence the points are collinear.

Chapter Determinants Ex 6.3 Q2(iii)

If the points are collinear, then the area of the triangle will be zero.

So

$$\frac{1}{2}\begin{vmatrix} 3 & -2 & 1 \\ 8 & 8 & 1 \\ 5 & 2 & 1 \end{vmatrix} = 0$$

L.H.S

Expanding along R_1

$= \frac{1}{2}\left[3(6) + 2(3) + 1(-24)\right]$

$= \frac{1}{2}\left[18 + 6 - 24\right]$

$= \frac{1}{2}\left[0\right]$

$= 0$

Since the area of the triangle is zero, hence given points are collinear.

Chapter Determinants Ex 6.3 Q2(iv)

If given points are collinear, then the area of the triangle must be zero.

Hence

$$= \frac{1}{2}\begin{vmatrix} 2 & 3 & 1 \\ -1 & -2 & 1 \\ 5 & 8 & 1 \end{vmatrix}$$

$= \frac{1}{2}\left[2(-10) - 3(-6) + 1(2)\right]$

$= \frac{1}{2}\left[-20 + 18 + 2\right]$

$= \frac{1}{2}\left[0\right]$

$= 0$

Hence the given points are collinear.

Chapter Determinants Ex 6.3 Q3

If the given points are collinear, the area of the triangle must be zero.

Hence

$$\frac{1}{2}\begin{vmatrix} a & 0 & 1 \\ 0 & b & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

Expanding along R_1

$= \frac{1}{2}\left[a(b-1) - 0(0-1) + 1(-b)\right] = 0$

$or\ ab - a - 0 - b = 0$

$or\ ab = a + b$

Hence proved

Chapter Determinants Ex 6.3 Q4

If the given points are collinear, then the area of the triangle must be zero.

Hence

$$\frac{1}{2}\begin{vmatrix} a & b & 1 \\ a' & b' & 1 \\ a-a' & b-b' & 1 \end{vmatrix} = 0$$

or

$$\frac{1}{2}\left[a\left(b'-b+b'\right)-b\left(a'-a+a'\right)+1\left(a'b-a'b'-ab'+a'b'\right)\right]=0$$

or $\frac{1}{2}\left[ab'-ab+ab'-a'b+ab-a'b+a'b-ab'\right]=0$

or $ab'-a'b=0$

$ab'=a'b$

Hence proved

Chapter Determinants Ex 6.3 Q5

If the points are collinear, then the area of the triangle must be zero.

Hence

$$\begin{vmatrix} 1 & -5 & 1 \\ -4 & 5 & 1 \\ \lambda & 7 & 1 \end{vmatrix} = 0$$

Expanding along R_1

$$1(-2)+5(-4-\lambda)+1(-28-5\lambda)=0$$

$$-2-20-5\lambda-28-5\lambda=0$$

$$-50-10\lambda=0$$

$$\lambda=-5$$

Hence $\lambda=-5$

Chapter Determinants Ex 6.3 Q6

$$\text{Area} = \left|\frac{1}{2}\begin{vmatrix} x & 4 & 1 \\ 2 & -6 & 1 \\ 5 & 4 & 1 \end{vmatrix}\right|$$

$$\pm 2\times 35 = \begin{vmatrix} x & 4 & 1 \\ 2 & -6 & 1 \\ 5 & 4 & 1 \end{vmatrix}$$

$$\pm 70 = x(-10)-4(-3)+1(38)$$

$$\pm 70 = -10x+12+38$$

$$\pm 70 = -10x+50 \qquad ---(1)$$

Taking $(+)$ sign

$+70=-10x+50$

$10x=-20$ or $x=-2$

Again taking $(-)$ sign

$-70=-10x+50$

$10x=120$ or $x=12$

Hence $x=-2, 12$

Chapter Determinants Ex 6.3 Q7

$$\text{Area} = \frac{1}{2}\begin{vmatrix} 1 & 4 & 1 \\ 2 & 3 & 1 \\ -5 & -3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[1(6) - 4(7) + 1(-6+15)\right]$$

$$= \frac{1}{2}\left[6 - 28 + 9\right]$$

$$= \frac{1}{2}\left[-13\right]$$

$$= \frac{13}{2}\text{ sq. units} \qquad \left[\because \text{Area can not be negative}\right]$$

Also, since the area of the triangle is non-zero.

Hence these points are non-collinear.

Chapter Determinants Ex 6.3 Q8

$$\text{Area} = \frac{1}{2}\begin{vmatrix} -3 & 5 & 1 \\ 3 & -6 & 1 \\ 7 & 2 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[-3(-8) - 5(-4) + 1(48)\right]$$

$$= \frac{1}{2}\left[24 + 20 + 48\right]$$

$$= 46 \text{ sq. units}$$

Hence the area is 46 sq. units.

Chapter Determinants Ex 6.3 Q9

If the given points are collinear, then the area of the triangle must be zero.

$$\text{so } \frac{1}{2}\begin{vmatrix} k & 2-2k & 1 \\ -k+1 & 2k & 1 \\ -4-k & 6-2k & 1 \end{vmatrix} = 0$$

expanding along R_1

$k(2k - 6 + 2k) - (2 - 2k)(-k + 1 + 4 + k) + 1(1 - k) \times (6 - 2k) - 2k(-4 - k) = 0$

$k(4k - 6) - (2 - 2k)(5) + 1\left[6 - 2k - 6k + 2k^2 + 8k + 2k^2\right] = 0$

$4k^2 - 6k - 10 + 10k + 6 + 4k^2 = 0$

$8k^2 + 4k - 4 = 0$

$8k^2 + 8k - 4k - 4 = 0 \qquad$ (Middle term splitting)

$8k(k + 1) - 4(k + 1) = 0$

$(8k - 4)(k + 1) = 0$

If $8k - 4 = 0$ or if $k + 1 = 0$

$k = \frac{1}{2} \qquad k = -1$

Hence $k = -1, \frac{1}{2}$

Chapter Determinants Ex 6.3 Q10

Since the points are collinear, hence the area of the triangle must be zero.

$$\text{so } \frac{1}{2}\begin{vmatrix} x & -2 & 1 \\ 5 & 2 & 1 \\ 8 & 8 & 1 \end{vmatrix} = 0$$

or $x(-6) + 2(-3) + 1(24) = 0$

or $-6x - 6 + 24 = 0$

$-6x + 18 = 0$

$x = 3$

Hence $x = 3$

Chapter Determinants Ex 6.3 Q11

Since the points are collinear, hence the area of the triangle must be zero.

$$\frac{1}{2}\begin{vmatrix} 3 & -2 & 1 \\ x & 2 & 1 \\ 8 & 8 & 1 \end{vmatrix} = 0$$

$$3(-6) + 2(x - 8) + 1(8x - 16) = 0$$
$$-18 + 2x - 16 + 8x - 16 = 0$$
$$10x = 50$$
$$x = 5$$

Hence $x = 5$

Chapter Determinants Ex 6.3 Q12(i)

Let $A(x, y), B(1, 2)$ and $C(3, 6)$ are 3 points in a line.

Since these points are collinear, hence area of the triangle must be zero.

$$\frac{1}{2}\begin{vmatrix} x & y & 1 \\ 1 & 2 & 1 \\ 3 & 6 & 1 \end{vmatrix} = 0$$

Expanding along R_1

$$x(-4) - y(-2) + 1(0) = 0$$
$$-4x + 2y = 0$$
$$\text{or } 2x - y = 0$$
$$\text{or } y = 2x$$

Hence the equation is $y = 2x$

Chapter Determinants Ex 6.3 Q12(ii)

Let $A(x, y), B(3, 1)$ and $C(9, 3)$ are 3 points in a line.

Since these points are collinear, hence the area of the triangle ABC must be zero.

$$\frac{1}{2}\begin{vmatrix} x & y & 1 \\ 3 & 1 & 1 \\ 9 & 3 & 1 \end{vmatrix} = 0$$

Expanding along R_1

$$x(-2) - y(-6) + 1(0) = 0$$
$$-2x + 6y = 0$$
$$x - 3y = 0$$

Hence the equation of the line is $x - 3y = 0$

Chapter Determinants Ex 6.3 Q13(i)

$$\text{Area} = \frac{1}{2}\left\|\begin{matrix} k & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 2 & 1 \end{matrix}\right\|$$

$$\pm 4 = \frac{1}{2}\begin{vmatrix} k & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix}$$

Expanding along R_1

$$\pm 8 = k(-2) - 0(4 - 0) + 1(8)$$
$$\pm 8 = -2k + 8$$

Taking positive $(+)$ sign

$+8 = -2k + 8$ or $k = 0$

Taking negative $(-)$ sign

$-8 = -2k + 8$ or $k = 8$

Hence $k = 0, 8$

Chapter Determinants Ex 6.3 Q13(ii)

$$4 = \frac{1}{2}\left\|\begin{matrix} -2 & 0 & 1 \\ 0 & 4 & 1 \\ 0 & k & 1 \end{matrix}\right\|$$

$$\pm 8 = \begin{vmatrix} -2 & 0 & 1 \\ 0 & 4 & 1 \\ 0 & k & 1 \end{vmatrix}$$

Expanding along R_1

$\pm 8 = -2(4-k) - 0(0-0) + 1(0)$

$\pm 8 = -8 + 2k$

Taking positive $(+)$ sign

$+8 = -8 + 2k$ or $k = 8$

Taking negative $(-)$ sign

$-8 = -8 + 2k$ or $k = 0$

Hence $k = 0, 8$

Ex 6.4

Chapter 6 Determinants Ex 6.4 Q1

Let $D = \begin{vmatrix} 1 & -2 \\ -3 & 5 \end{vmatrix} = 5 - 6 = -1$

$D_1 = \begin{vmatrix} 4 & -2 \\ -7 & 5 \end{vmatrix} = 20 - 14 = 6$

$D_2 = \begin{vmatrix} 1 & 4 \\ -3 & -7 \end{vmatrix} = -7 + 12 = 5$

by definition $x = \frac{D_1}{D} = \frac{6}{-1} = -6$

$y = \frac{D_2}{D} = \frac{5}{-1} = -5$

Hence $x = -6$

$y = -5$

Chapter 6 Determinants Ex 6.4 Q2

Let $D = \begin{vmatrix} 2 & -1 \\ 7 & -2 \end{vmatrix} = -4 + 7 = 3$

$D_1 = \begin{vmatrix} 1 & -1 \\ -7 & -2 \end{vmatrix} = -9$

$D_2 = \begin{vmatrix} 2 & 1 \\ 7 & -7 \end{vmatrix} = -21$

Now, $x = \frac{D_1}{D} = \frac{-9}{3} = -3$

$y = \frac{+D_2}{D} = \frac{-21}{3} = -7$

Hence $x = -3$

$y = -7$

Chapter 6 Determinants Ex 6.4 Q3

Let $D = \begin{vmatrix} 2 & -1 \\ 3 & 5 \end{vmatrix} = 13$

$D_1 = \begin{vmatrix} 17 & -1 \\ 6 & 5 \end{vmatrix} = 91$

$D_2 = \begin{vmatrix} 2 & 17 \\ 3 & 6 \end{vmatrix} = -39$

$x = \frac{D_1}{D} = \frac{91}{13} = 7$

$y = \frac{D_2}{D} = \frac{-39}{13} = -3$

Hence $x = 7$

$y = -3$

Chapter 6 Determinants Ex 6.4 Q4

Let $D = \begin{vmatrix} 3 & 1 \\ 3 & -1 \end{vmatrix} = -6$

$D_1 = \begin{vmatrix} 19 & 1 \\ 23 & -1 \end{vmatrix} = -42$

$D_2 = \begin{vmatrix} 3 & 19 \\ 3 & 23 \end{vmatrix} = 12$

$x = \frac{D_1}{D} = \frac{-42}{-6} = 7$

$y = \frac{D_2}{D} = \frac{12}{-6} = -2$

Hence $x = 7$

$y = -2$

Chapter 6 Determinants Ex 6.4 Q5

Let $D = \begin{vmatrix} 2 & -1 \\ 3 & 4 \end{vmatrix} = 11$

$D_1 = \begin{vmatrix} -2 & -1 \\ 3 & 4 \end{vmatrix} = -5$

$D_2 = \begin{vmatrix} 2 & -2 \\ 3 & 3 \end{vmatrix} = 12$

$x = \frac{D_1}{D} = \frac{-5}{11}$

$y = \frac{D_2}{D} = \frac{12}{11}$

Chapter 6 Determinants Ex 6.4 Q6

Let $D = \begin{vmatrix} 3 & a \\ 2 & a \end{vmatrix} = a$

$D_1 = \begin{vmatrix} 4 & a \\ 2 & a \end{vmatrix} = 2a$

$D_2 = \begin{vmatrix} 3 & 4 \\ 4 & 2 \end{vmatrix} = -2$

$x = \frac{D_1}{D} = \frac{2a}{a} = 2$

$y = \frac{D_2}{D} = \frac{-2}{a}$

Chapter 6 Determinants Ex 6.4 Q7

$$\text{Let } D = \begin{vmatrix} 2 & 3 \\ 1 & 6 \end{vmatrix} = 9$$
$$D_1 = \begin{vmatrix} 10 & 3 \\ 4 & 6 \end{vmatrix} = 48$$
$$D_2 = \begin{vmatrix} 2 & 10 \\ 1 & 4 \end{vmatrix} = -2$$

$$x = \frac{D_1}{D} = \frac{48}{9} = \frac{16}{3}$$
$$y = \frac{D_2}{D} = \frac{-2}{9}$$

Chapter 6 Determinants Ex 6.4 Q8

$$\text{Let } D = \begin{vmatrix} 5 & 7 \\ 4 & 6 \end{vmatrix} = 2$$
$$D_1 = \begin{vmatrix} -2 & 7 \\ -3 & 6 \end{vmatrix} = 9$$
$$D_2 = \begin{vmatrix} 5 & -2 \\ 4 & -3 \end{vmatrix} = -7$$

$$x = \frac{D_1}{D} = \frac{9}{2}$$
$$y = \frac{D_2}{D} = \frac{-7}{2}$$

Chapter 6 Determinants Ex 6.4 Q9

$$\text{Let } D = \begin{vmatrix} 9 & 5 \\ -2 & 3 \end{vmatrix} = 37$$
$$D_1 = \begin{vmatrix} 10 & 5 \\ 8 & 3 \end{vmatrix} = -10$$
$$D_2 = \begin{vmatrix} 9 & 10 \\ -2 & 8 \end{vmatrix} = 92$$

$$x = \frac{D_1}{D} = \frac{-10}{37}$$
$$y = \frac{D_2}{D} = \frac{92}{37}$$

Chapter 6 Determinants Ex 6.4 Q10

$$\text{Let } D = \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = -5$$
$$D_1 = \begin{vmatrix} 1 & 2 \\ 4 & 1 \end{vmatrix} = -7$$
$$D_2 = \begin{vmatrix} 1 & 1 \\ 3 & 4 \end{vmatrix} = 1$$

$$x = \frac{D_1}{D} = \frac{7}{5}$$
$$y = \frac{D_2}{D} = \frac{-1}{5}$$

Chapter 6 Determinants Ex 6.4 Q11

$$\text{Let } D = \begin{vmatrix} 3 & 1 & 1 \\ 2 & -4 & 3 \\ 4 & 1 & -3 \end{vmatrix}$$

Expanding along R_1

$= 3(9) + (-1)(-18) + 1(18)$

$= 27 + 18 + 18 = 63$

$$\text{Again } D_1 = \begin{vmatrix} 2 & 1 & 1 \\ -1 & -4 & 3 \\ -11 & 1 & -3 \end{vmatrix}$$

Expanding along R_1

$= 2(9) + (-1)(36) + 1(-45)$

$= 18 - 36 - 45 = -63$

$$\text{Again } D_2 = \begin{vmatrix} 3 & 2 & 1 \\ 2 & -1 & 3 \\ 4 & -11 & -3 \end{vmatrix}$$

Expanding along R_1 $= 3(3 + 33) - 2(-18) + 1(-22 + 4)$

$= 108 + 36 - 18 = 126$

$$\text{Also } D_3 = \begin{vmatrix} 3 & 1 & 2 \\ 2 & -4 & -1 \\ 4 & 1 & -11 \end{vmatrix}$$

Expanding along R_1

$= 3(45) - 1(-18) + 2(18) = 135 + 18 + 36 = 189$

Now $x = \dfrac{D_1}{D} = \dfrac{-63}{63} = -1$

$y = \dfrac{D_2}{D} = \dfrac{126}{63} = 2$

$z = \dfrac{D_3}{D} = \dfrac{189}{63} = 3$

Chapter 6 Determinants Ex 6.4 Q12

$$\text{Let } D = \begin{vmatrix} 1 & -4 & -1 \\ 2 & -5 & 2 \\ -3 & 2 & 1 \end{vmatrix}$$

Expanding along R_1

$$= 1(-9) + 4(8) - 1(-11) = -9 + 32 + 11 = 34$$

$$\text{Again } D_1 = \begin{vmatrix} 11 & -4 & -1 \\ 39 & -5 & 2 \\ 1 & 2 & 1 \end{vmatrix}$$

Expanding along R_1

$$= 11(-9) + 4(37) - 1(83) \qquad = -99 + 148 - 83$$
$$= 148 - 182$$
$$= -34$$

$$\text{Also } D_2 = \begin{vmatrix} 1 & 11 & -1 \\ 2 & 39 & 2 \\ -3 & 1 & 1 \end{vmatrix}$$

Expanding along R_1

$$= 1(37) - 11(8) - 1(119)$$
$$= 37 - 88 - 119 = -170$$

$$\text{Also } D_3 = \begin{vmatrix} 1 & -4 & 11 \\ 2 & -5 & -39 \\ -3 & 2 & 1 \end{vmatrix}$$

Expanding along R_1

$$= 1(-5 - 78) + 4(2 + 117) + 11(4 - 15)$$
$$= -83 + 476 - 121 = 272$$

Now $\quad x = \dfrac{D_1}{D} = \dfrac{-34}{34} = -1$

$$y = \frac{D_2}{D} = \frac{-170}{34} = -5$$

$$z = \frac{D_3}{D} = \frac{272}{34} = 8$$

Hence $x = -1, y = -5, z = 8$

Chapter 6 Determinants Ex 6.4 Q13

$$\text{Let } D = \begin{vmatrix} 6 & 1 & -3 \\ 1 & 3 & -2 \\ 2 & 1 & 4 \end{vmatrix}$$

Expanding along R_1

$$= 6(14) - 1(8) - 3(-5)$$
$$= 84 - 8 + 15 = 91$$

$$\text{Also } D_1 = \begin{vmatrix} 5 & 1 & -3 \\ 5 & -3 & -2 \\ 8 & 1 & 4 \end{vmatrix}$$

Expanding along R_1

$$= 5(14) - 1(36) - 3(-19) = 70 - 36 + 57 = 91$$

$$\text{Again } D_2 = \begin{vmatrix} 6 & 5 & -3 \\ 1 & 5 & -2 \\ 2 & 8 & 4 \end{vmatrix}$$

Expanding along R_1

$$= 6(36) - 5(8) - 3(-2) = 216 - 40 + 6 = 182$$

$$\text{Also } D_3 = \begin{vmatrix} 6 & 1 & 5 \\ 1 & 3 & 5 \\ 2 & 1 & 8 \end{vmatrix}$$

Expanding along R_1

$$= 6(19) - 1(-2) + 5(-5) = 114 + 2 - 25 = 91$$

$$\text{Now} \quad x = \frac{D_1}{D} = \frac{91}{91} = 1$$
$$y = \frac{D_2}{D} = \frac{182}{91} = 2$$
$$\text{Also} \quad z = \frac{D_3}{D} = \frac{91}{91} = 1$$

Hence $x = 1, y = 2, z = 1$

Chapter 6 Determinants Ex 6.4 Q14

$$\text{Let } D = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix}$$

Expanding along R_1

$$= 1(1) - 1(-1) + 0(-1) = 1 + 1 + 0 = 2$$

$$\text{Also } D_1 = \begin{vmatrix} 5 & 1 & 0 \\ 3 & 1 & 1 \\ 4 & 0 & 1 \end{vmatrix}$$

Expanding along R_1

$$= 5(1) - 1(-1) + 0(-4) = 5 + 1 + 0 = 6$$

$$\text{Again } D_2 = \begin{vmatrix} 1 & 5 & 0 \\ 0 & 3 & 1 \\ 1 & 4 & 1 \end{vmatrix}$$

Expanding along R_1

$$= 1(-1) - 5(-1) + 0(-3) = -1 + 5 + 0 = 4$$

$$\text{Also } D_3 = \begin{vmatrix} 1 & 1 & 5 \\ 0 & 1 & 3 \\ 1 & 0 & 4 \end{vmatrix}$$

$$= 1(4) - 1(-3) + 5(-1) = 4 + 3 - 5 = 2$$

Now $\quad x = \frac{D_1}{D} = \frac{6}{2} = 3$

$$y = \frac{D_2}{D} = \frac{4}{2} = 2$$

$$z = \frac{D_3}{D} = \frac{2}{2} = 1$$

Hence $x = 3, y = 2, z = 1$

Chapter 6 Determinants Ex 6.4 Q15

$$\text{Let } D = \begin{vmatrix} 0 & 2 & -3 \\ 1 & 3 & 0 \\ 3 & 4 & 0 \end{vmatrix}$$

Expanding along R_1

$$= 0(0) - 2(0) - 3(-5) = 15$$

$$\text{Also } D_1 = \begin{vmatrix} 0 & 2 & -3 \\ -4 & 3 & 0 \\ 3 & 4 & 0 \end{vmatrix}$$

Expanding along R_1

$$= 0(0) - 2(0) - 3(-25) = 75$$

$$\text{Again } D_2 = \begin{vmatrix} 0 & 0 & -3 \\ 1 & -4 & 0 \\ 3 & 3 & 0 \end{vmatrix}$$

Expanding along R_1

$$= 0(0) - 0(0) - 3(15) = -45$$

$$\text{Also } D_3 = \begin{vmatrix} 0 & 2 & 0 \\ 1 & 3 & -4 \\ 3 & 4 & 3 \end{vmatrix}$$

$$= 0(25) - 2(15) + 0(1) = -30$$

Now $x = \dfrac{D_1}{D} = \dfrac{75}{15} = 5$

$$y = \frac{D_2}{D} = \frac{-45}{15} = -3$$

$$z = \frac{D_3}{D} = \frac{-30}{15} = -2$$

Hence $x = 5, y = -3, z = -2$

Chapter 6 Determinants Ex 6.4 Q16

Here $D = \begin{vmatrix} 5 & -7 & 1 \\ 6 & -8 & -1 \\ 3 & 2 & -6 \end{vmatrix} = 5(48+2) + 7(-33) + 1(36)$

$$= 250 - 231 + 36 = 55$$

$$D_1 = \begin{vmatrix} 11 & -7 & 1 \\ 15 & -8 & -1 \\ 7 & 2 & -6 \end{vmatrix} = 11(50) + 7(-83) + 1(86)$$

$$= 550 - 581 + 86 = 55$$

$$D_2 = \begin{vmatrix} 5 & 11 & 1 \\ 6 & 15 & -1 \\ 3 & 7 & -6 \end{vmatrix} = 5(-83) - 11(-33) + 1(-3)$$

$$= -415 + 363 - 3 = -55$$

$$D_3 = \begin{vmatrix} 5 & -7 & 11 \\ 6 & -8 & 15 \\ 3 & 2 & 7 \end{vmatrix} = 5(-86) + 7(-3) + 11(36)$$

$$= -430 - 21 + 396$$
$$= -55$$

Now $x = \dfrac{D_1}{D} = \dfrac{55}{55} = 1$

$$y = \frac{D_2}{D} = \frac{-55}{55} = -1$$

$$z = \frac{D_3}{D} = \frac{-55}{55} = -1$$

Hence $x = 1, y = -1, z = -1$

Chapter 6 Determinants Ex 6.4 Q17

$2x - 3y - 4z = 29$
$-2x + 5y - z = -15$
$3x - y + 5z = -11$

From the given system of equation we have

$$D = \begin{vmatrix} 2 & -3 & 4 \\ -2 & 5 & -1 \\ 3 & -1 & 5 \end{vmatrix} = 2(25-1) + 3(-10+3) + 4(2-15) = 48 - 21 - 52 = -25$$

$$D_1 = \begin{vmatrix} 29 & -3 & 4 \\ -15 & 5 & -1 \\ 11 & -1 & 5 \end{vmatrix} = 29(25-1) + 3(-75+11) + 4(15-55) = 696 - 192 - 160 = 344$$

$$D_2 = \begin{vmatrix} 2 & 29 & 4 \\ -2 & -15 & -1 \\ 3 & 11 & 5 \end{vmatrix} = 2(-75+11) - 29(-10+3) + 4(-22+45) = -128 + 203 + 92 = 167$$

$$D_3 = \begin{vmatrix} 2 & -3 & 29 \\ -2 & 5 & -15 \\ 3 & -1 & 11 \end{vmatrix} = 2(55-15) + 3(-22+45) + 29(2-15) = 80 + 69 - 377 = -228$$

So, by Cramer's Rule, we obtain

$$x = \frac{D_1}{D} = -\frac{344}{25}$$

$$y = \frac{D_2}{D} = -\frac{167}{25}$$

$$z = \frac{D_3}{D} = \frac{228}{25}$$

Note: Answer given in the book is incorrect.

Chapter 6 Determinants Ex 6.4 Q18

Here $D = \begin{vmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & -1 & -2 \end{vmatrix} = 1(1) - 1(-3) = 1 + 3 = 4$

$$D_1 = \begin{vmatrix} 1 & 1 & 0 \\ -6 & 0 & 1 \\ 3 & -1 & -2 \end{vmatrix} = 1(1) - 1(9) = -8$$

$$D_2 = \begin{vmatrix} 1 & 1 & 0 \\ 1 & -6 & 1 \\ 1 & 3 & -2 \end{vmatrix} = 1(9) - 1(-3) = 12$$

$$D_3 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & -6 \\ 1 & -1 & 3 \end{vmatrix} = 1(-6) - 1(9) + 1(-1) = -6 - 9 - 1 = -16$$

Now $x = \frac{D_1}{D} = \frac{-8}{4} = -2$

$$y = \frac{D_2}{D} = \frac{12}{4} = 3$$

$$z = \frac{D_3}{D} = \frac{-16}{4} = -4$$

Hence $x = -2, y = 3, z = -4$

Chapter 6 Determinants Ex 6.4 Q19

$$D = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$$

$c_2 \to c_2 - c_1, c_3 \to c_3 - c_1$

$$= \begin{vmatrix} 1 & 0 & 0 \\ a & b-a & c-a \\ a^2 & b^2-a^2 & c^2-a^2 \end{vmatrix}$$

Now taking $(b-a)$ from c_2, and $(c-a)$ c_3 common

$$= (b-a)(c-a)\begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 1 \\ a^2 & b+a & c+a \end{vmatrix}$$

Expanding along R_1

$= (b-a)(c-a)[c+a-b-a]$

$= (b-a)(c-a)(c-b)$

$= (a-b)(b-c)(c-a)$

Again $D_1 = -\begin{vmatrix} 1 & 1 & 1 \\ d & b & c \\ d^2 & b^2 & c^2 \end{vmatrix}$

$c_2 \to c_2 - c_1, c_3 \to c_3 - c_1$

$$D_1 = -\begin{vmatrix} 1 & 0 & 0 \\ d & b-d & c-d \\ d^2 & b^2-d^2 & c^2-d^2 \end{vmatrix}$$

Taking $(b-d)$ common from c_2 and $(c-d)$ from c_3

$$-(b-d)(c-d)\begin{vmatrix} 1 & 0 & 0 \\ d & 1 & 1 \\ d^2 & b+d & c+d \end{vmatrix}$$

Expanding along R_1

$= -(b-d)(c-d)[1(c+d-b-d)]$

$= -(b-d)(c-d)(c-b)$

$= -(b-c)(c-d)(d-b)$

Again $D_2 = -\begin{vmatrix} 1 & 1 & 1 \\ a & d & c \\ a^2 & b^2 & c^2 \end{vmatrix}$

$c_2 \to c_2 - c_1, c_3 \to c_3 - c_1$

$$= -\begin{vmatrix} 1 & 0 & 0 \\ a & d-a & c-a \\ a^2 & d^2-a^2 & c^2-a^2 \end{vmatrix}$$

Taking $(d-a)$ *common from* c_2 *and* $(c-a)$ *from* c_3

$$= -(d-a)(c-a)\begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 1 \\ a^2 & d+a & c+a \end{vmatrix}$$

Expanding along R_1

$= -(d-a)(c-a)\times 1[c+a-d-a]$

$= -(d-a)(c-a)(c-d)$

$= -(a-d)(d-c)(c-a)$

$$Also \quad D_3 = -\begin{vmatrix} 1 & 1 & 1 \\ a & b & d \\ a^2 & b^2 & d^2 \end{vmatrix}$$

$$c_2 \to c_2 - c_1, c_3 \to c_3 - c_1$$

$$= -\begin{vmatrix} 1 & 0 & 0 \\ a & b-a & d-a \\ a^2 & b^2-a^2 & d^2-a^2 \end{vmatrix}$$

Now, taking $(b-a)$ *common from* c_2 *and* $(d-a)$ *from* c_3

$$= -(b-a)(d-a)\begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 1 \\ a^2 & b+a & d+a \end{vmatrix}$$

Expanding along R_1

$$= -(b-a)(d-a) \times 1[d+a-b-a]$$

$$= -(b-a)(d-a)(d-b)$$

$$= -(a-b)(b-d)(d-a)$$

$$\text{Now} \quad x = \frac{D_1}{D} = -\frac{(b-c)(c-d)(d-b)}{(a-b)(b-c)(c-a)}$$

$$y = \frac{D_2}{D} = -\frac{(a-d)(d-c)(c-a)}{(a-b)(b-c)(c-a)}$$

$$z = \frac{D_3}{D} = -\frac{(a-b)(b-d)(d-a)}{(a-b)(b-c)(c-a)}$$

Chapter 6 Determinants Ex 6.4 Q20

Here $D = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & -2 & 2 & 2 \\ 2 & 1 & -2 & 2 \\ 3 & -1 & 3 & -3 \end{vmatrix}$

$$\therefore D = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & -3 & 1 & 1 \\ 2 & -1 & -4 & 0 \\ 3 & -4 & 0 & -6 \end{vmatrix} = 1\begin{vmatrix} -3 & 1 & 1 \\ -1 & -4 & 0 \\ -4 & 0 & -6 \end{vmatrix} \qquad \begin{bmatrix} C2 \to C2 - C1 \\ C3 \to C3 - C1 \\ C4 \to C4 - C1 \end{bmatrix}$$

$$\therefore \quad D = \begin{vmatrix} 0 & 0 & 1 \\ -1 & -4 & 0 \\ -22 & 6 & -6 \end{vmatrix} \qquad \begin{bmatrix} C1 \to C1 + 3C3 \\ C2 \to C2 - C3 \end{bmatrix}$$

$$= 1(-6 - 88) = -94$$

$$D_1 = \begin{vmatrix} 2 & 1 & 1 & 1 \\ -6 & -2 & 2 & 2 \\ -5 & 1 & -2 & 2 \\ -3 & -1 & 3 & -3 \end{vmatrix} = 188$$

$$D_2 = \begin{vmatrix} 1 & 2 & 1 & 1 \\ 1 & -6 & 2 & 2 \\ 2 & -5 & -2 & 2 \\ 3 & -3 & 3 & -3 \end{vmatrix} = -282$$

$$D_3 = \begin{vmatrix} 1 & 1 & 2 & 1 \\ 1 & -2 & -6 & 2 \\ 2 & 1 & -5 & 2 \\ 3 & -1 & -3 & -3 \end{vmatrix} = -141$$

$$D_4 = \begin{vmatrix} 1 & 1 & 1 & 2 \\ 1 & -2 & 2 & -6 \\ 2 & 1 & -2 & -5 \\ 3 & -1 & 3 & -3 \end{vmatrix} = 47$$

Now $x = \frac{D_1}{D} = \frac{188}{-94} = -2$

$$y = \frac{D_2}{D} = \frac{-282}{-94} = 3$$

$$z = \frac{D_3}{D} = \frac{-141}{-94} = \frac{3}{2}$$

$$w = \frac{D_4}{D} = \frac{47}{-94} = -\frac{1}{2}$$

Hence $x = -2, y = 3, z = \frac{3}{2}, w = -\frac{1}{2}$

Chapter 6 Determinants Ex 6.4 Q21

Here $D = \begin{vmatrix} 2 & 0 & -3 & 1 \\ 1 & -1 & 0 & 2 \\ 0 & -3 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix}$

$\therefore D = \begin{vmatrix} 2 & -2 & -5 & 1 \\ 1 & -2 & -1 & 2 \\ 0 & -3 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{vmatrix} = -1 \begin{vmatrix} -2 & -5 & 1 \\ -2 & -1 & 2 \\ -3 & 1 & 1 \end{vmatrix}$ $\begin{bmatrix} C2 \to C2 - C1 \\ C3 \to C3 - C1 \end{bmatrix}$

$\therefore \quad D = -1 \begin{vmatrix} 1 & -6 & 1 \\ 4 & -3 & 2 \\ 0 & 0 & 1 \end{vmatrix}$ $\begin{bmatrix} C1 \to C1 + 3C3 \\ C2 \to C2 - C3 \end{bmatrix}$

$= -1(-3 + 24) = -21$

$D_1 = \begin{vmatrix} 1 & 0 & -3 & 1 \\ 1 & -1 & 0 & 2 \\ 1 & -3 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix} = -21$

$D_2 = \begin{vmatrix} 2 & 1 & -3 & 1 \\ 1 & 1 & 0 & 2 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix} = -6$

$D_3 = \begin{vmatrix} 2 & 0 & 1 & 1 \\ 1 & -1 & 1 & 2 \\ 0 & -3 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix} = -6$

$D_4 = \begin{vmatrix} 2 & 0 & -3 & 1 \\ 1 & -1 & 0 & 1 \\ 0 & -3 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{vmatrix} = 3$

Now $x = \frac{D_1}{D} = \frac{-21}{-21} = 1$

$y = \frac{D_2}{D} = \frac{-6}{-21} = \frac{2}{7}$

$z = \frac{D_3}{D} = \frac{-6}{-21} = \frac{2}{7}$

$w = \frac{D_4}{D} = \frac{3}{-21} = -\frac{1}{7}$

Chapter 6 Determinants Ex 6.4 Q22

Let $D = \begin{vmatrix} 2 & -1 \\ 4 & -2 \end{vmatrix}$

Expanding along R_1

$= -4 + 4 = 0$

Also $D_1 = \begin{vmatrix} 5 & -1 \\ 7 & -2 \end{vmatrix} = -3$

Also $D_2 = \begin{vmatrix} 2 & 5 \\ 4 & 7 \end{vmatrix} = -6$

And since $D = 0$ and D_1 and D_2 are non-zero, hence the given system of equations is inconsistent.

Hence proved.

Chapter 6 Determinants Ex 6.4 Q23

$$D = \begin{vmatrix} 3 & 1 \\ -6 & -2 \end{vmatrix} = -6 + 6 = 0$$

$$D_1 = \begin{vmatrix} 5 & 1 \\ 9 & -2 \end{vmatrix} = -10 - 9 = -19 \neq 0$$

Since $D = 0$ but $D_1 \neq 0$

Hence the given system of equations is inconsistent.

Chapter 6 Determinants Ex 6.4 Q24

Here $D = \begin{vmatrix} 3 & -1 & 2 \\ 2 & 1 & 3 \\ 1 & -2 & -1 \end{vmatrix}$

Expanding along R_1

$$= 3(5) + 1(-5) + 2(-5)$$
$$= 15 - 5 - 10 = 15 - 15 = 0$$

Also $D_1 = \begin{vmatrix} 3 & -1 & 2 \\ 5 & 1 & 3 \\ 1 & -2 & -1 \end{vmatrix}$

Expanding along R_1

$$= 3(5) + 1(-8) + 2(-11)$$
$$= 15 - 8 - 22$$
$$= -15 \neq 0$$

Since $D = 0$ and $D_1 \neq 0$

Hence the given system of equations is inconsistent.

Chapter 6 Determinants Ex 6.4 Q25

Here $D = \begin{vmatrix} 3 & -1 & 2 \\ 2 & -1 & 1 \\ 3 & 6 & 5 \end{vmatrix} = 3(-11) + 1(7) + 2(15) = -33 + 7 + 30 = 4$

$$D_1 = \begin{vmatrix} 6 & -1 & 2 \\ 2 & -1 & 1 \\ 20 & 6 & 5 \end{vmatrix} = 12$$

$$D_2 = \begin{vmatrix} 3 & 6 & 2 \\ 2 & 2 & 1 \\ 3 & 20 & 5 \end{vmatrix} = -4$$

$$D_3 = \begin{vmatrix} 3 & -1 & 6 \\ 2 & -1 & 2 \\ 3 & 6 & 20 \end{vmatrix} = 28$$

Now $x = \dfrac{D_1}{D} = \dfrac{12}{4} = -3$

$y = \dfrac{D_2}{D} = \dfrac{-4}{4} = -1$

$z = \dfrac{D_3}{D} = \dfrac{28}{4} = 7$

Chapter 6 Determinants Ex 6.4 Q26

We have,

$$D = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ -1 & -2 & 2 \end{vmatrix} = \begin{vmatrix} 0 & -1 & 0 \\ 3 & 1 & 0 \\ -3 & -2 & 0 \end{vmatrix} = 0$$

$$D_1 = \begin{vmatrix} 3 & -1 & 1 \\ 2 & 1 & -1 \\ 1 & -2 & 2 \end{vmatrix} = \begin{vmatrix} 3 & -1 & 0 \\ 2 & 1 & 0 \\ 1 & -2 & 0 \end{vmatrix} = 0$$

$$D_2 = \begin{vmatrix} 1 & 3 & 1 \\ 2 & 2 & -1 \\ -1 & 1 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 2 & -4 & -3 \\ -1 & 4 & 3 \end{vmatrix} = 1(-12+12) = 0$$

$$D_3 = \begin{vmatrix} 1 & -1 & 3 \\ 2 & 1 & 2 \\ -1 & -2 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 2 & 3 & -4 \\ -1 & -3 & 4 \end{vmatrix} = 1(12-12) = 0$$

$\therefore \quad D = D_1 = D_2 = D_3 = 0$

So, either the system is consistent with infinitely many solutions or it is inconsistent.

Consider the first two equations, written as

$$x - y = 3 - z$$
$$2x + y = 2 + z$$

To solve these equations we use Cramer's rule.

Here,

$$D = \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} = 1 + 2 = 3$$

$$D_1 = \begin{vmatrix} 3-z & -1 \\ 2+z & 1 \end{vmatrix} = (3-z) + (2+z) = 5$$

$$D_2 = \begin{vmatrix} 1 & 3-z \\ 2 & 2+z \end{vmatrix} = (2+z) - (6-2z) = -4 + 3z$$

$\therefore \quad x = \dfrac{D_1}{D} = \dfrac{5}{3}$

$$y = \frac{D_2}{D} = \frac{-4+3z}{3}$$

Let $z = k$, then the equations have the solution.

$$x = \frac{5}{3},\ y = \frac{-4+3k}{3},\ z = k$$

Chapter 6 Determinants Ex 6.4 Q27

Here,

$$D = \begin{vmatrix} 1 & 2 \\ 3 & 6 \end{vmatrix} = 6 - 6 = 0$$

$$D_1 = \begin{vmatrix} 5 & 2 \\ 15 & 6 \end{vmatrix} = 30 - 30 = 0$$

$$D_2 = \begin{vmatrix} 1 & 5 \\ 3 & 15 \end{vmatrix} = 15 - 15 = 0$$

So, $D = D_1 = D_2 = 0$

Let $y = k$, then we have,

$$x + 2y = 5$$

$\Rightarrow \quad x = 5 - 2y = 5 - 2k$

$\therefore \quad x = 5 - 2k,\ y = k$ are the infinite solutions of the given system.

Chapter 6 Determinants Ex 6.4 Q28

Here,

$$D = \begin{vmatrix} 1 & 1 & -1 \\ 1 & -2 & 1 \\ 3 & 6 & -5 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 1 & -3 & 2 \\ 3 & 3 & -2 \end{vmatrix} = 1(6-6) = 0$$

$$D_1 = \begin{vmatrix} 0 & 1 & -1 \\ 0 & -2 & 1 \\ 0 & 6 & -5 \end{vmatrix} = 0$$

$$D_2 = \begin{vmatrix} 1 & 0 & -1 \\ 1 & 0 & 1 \\ 3 & 0 & -5 \end{vmatrix} = 0$$

$$D_3 = \begin{vmatrix} 1 & 1 & 0 \\ 1 & -2 & 0 \\ 3 & 6 & 0 \end{vmatrix} = 0$$

So, $D = D_1 = D_2 = D_3 = 0$

The given system either has infinite solutions or it is inconsistent.

Consider the first two equations, written as

$$x + y = z$$
$$x - 2y = -z$$

To solve this we will use Cramer's rule

Here,

$$D = \begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} = -2 - 1 = -3$$

$$D_1 = \begin{vmatrix} z & 1 \\ -z & -2 \end{vmatrix} = -2z - z = -z$$

$$D_2 = \begin{vmatrix} 1 & z \\ 1 & -z \end{vmatrix} = -z - z = -2z$$

$$\therefore \quad x = \frac{D_1}{D} = \frac{-z}{-3} = \frac{z}{3}$$

$$y = \frac{D_2}{D} = \frac{-2z}{-3} = \frac{2z}{3}$$

Let $z = k$, then the solutions of the given system are

$$x = \frac{k}{3}, \; y = \frac{2k}{3}, \; z = k$$

Chapter 6 Determinants Ex 6.4 Q29

Here,

$$D = \begin{vmatrix} 2 & 1 & -2 \\ 1 & -2 & 1 \\ 5 & -5 & 1 \end{vmatrix} = \begin{vmatrix} 12 & 9 & -2 \\ -4 & -3 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 1(-36+36) = 0$$

$$D_1 = \begin{vmatrix} 4 & 1 & -2 \\ -2 & -2 & 1 \\ -2 & -5 & 1 \end{vmatrix} = \begin{vmatrix} 0 & 1 & -2 \\ 0 & -2 & 1 \\ 0 & -5 & 1 \end{vmatrix} = 0$$

$$D_2 = \begin{vmatrix} 2 & 4 & -2 \\ 1 & -2 & 1 \\ 5 & -2 & 1 \end{vmatrix} = \begin{vmatrix} 2 & 0 & -2 \\ 1 & 0 & 1 \\ 5 & 0 & 1 \end{vmatrix} = 0$$

$$D_3 = \begin{vmatrix} 2 & 1 & 4 \\ 1 & -2 & -2 \\ 5 & -5 & -2 \end{vmatrix} = \begin{vmatrix} 4 & -3 & 0 \\ 1 & -2 & -2 \\ 4 & -3 & 0 \end{vmatrix} = 2(-12+12) = 0$$

So, $D = D_1 = D_2 = D_3 = 0$

So, the given system is either inconsistent or has infinite solutions.

Consider the 2nd and 3rd equation, written as

$$x - 2y = -2 - z$$
$$5x - 5y = -2 - z$$

Then,

$$D = \begin{vmatrix} 1 & -2 \\ 5 & -5 \end{vmatrix} = -5 - (-10) = 5$$

$$D_1 = \begin{vmatrix} -2-z & -2 \\ -2-z & -5 \end{vmatrix} = (2+z)(5) - 2(2+z) = 3(2+z) = 6 + 3z$$

$$D_2 = \begin{vmatrix} 1 & -(2+z) \\ 5 & -(2+z) \end{vmatrix} = -(2+z) + 5(2+z) = 4(2+z) = 8 + 4z$$

$$\therefore \quad x = \frac{D_1}{D} = \frac{6+3z}{5}$$

$$y = \frac{D_2}{D} = \frac{8+4z}{5}$$

Let $z = k$, then

$x = \frac{6+3k}{5}$, $y = \frac{8+4k}{5}$, $z = k$ are the infinite solution of the given system of equations.

Chapter 6 Determinants Ex 6.4 Q30

Here,

$$D=\begin{vmatrix}1&-1&3\\1&3&-3\\5&3&3\end{vmatrix}=\begin{vmatrix}2&2&0\\1&3&-3\\6&6&0\end{vmatrix}=3(12-12)=0$$

$$D_1=\begin{vmatrix}6&-1&3\\-4&3&-3\\10&3&3\end{vmatrix}=\begin{vmatrix}2&2&0\\-4&3&-3\\6&6&0\end{vmatrix}=0$$

$$D_2=\begin{vmatrix}1&6&3\\1&-4&-3\\5&10&3\end{vmatrix}=\begin{vmatrix}2&2&0\\1&-4&-3\\6&6&0\end{vmatrix}=0$$

$$D_3=\begin{vmatrix}1&-1&6\\1&3&-4\\5&3&10\end{vmatrix}=\begin{vmatrix}1&-1&6\\0&4&-10\\0&8&-20\end{vmatrix}=1(-80+80)=0$$

So, $D=D_1=D_2=D_3=0$

So, the given system is either inconsistent or has infinite solutions.

Consider the first to equations, written as

$$x-y=6-3z$$
$$x+3y=-4+3z$$

Here,

$$D=\begin{vmatrix}1&-1\\1&3\end{vmatrix}=3+1=4$$

$$D_1=\begin{vmatrix}6-3z&-1\\-4+3z&3\end{vmatrix}=3(6-3z)+(-4+3z)=14-6z$$

$$D_2=\begin{vmatrix}1&6-3z\\1&-4+3z\end{vmatrix}=(-4+3z)-(6-3z)=-10+6z$$

$$\therefore \quad x=\frac{D_1}{D}=\frac{14-6z}{4}=\frac{7-3z}{2}$$

$$y=\frac{D_2}{D}=\frac{6z-10}{4}=\frac{3z-5}{2}$$

Let $z=k$, then

$x=\frac{7-3k}{2}$, $y=\frac{3k-5}{2}$, $z=k$ are the infinite solution of the given system of equations.

Chapter 6 Determinants Ex 6.4 Q31

Let the rates of commissions on items A, B and C be x, y and z respectively.

Then we can express the given model as a system of linear equations

$$90x+100y+20z=800$$
$$130x+50y+40z=900$$
$$60x+100y+30z=850$$

We will solve this using the Cramer's rule.

Here,

$$D=\begin{vmatrix}90&100&20\\130&50&40\\60&100&30\end{vmatrix}=\begin{vmatrix}-170&0&-60\\130&50&40\\-200&0&-50\end{vmatrix}=50(8500-12000)=-175000$$

$$D_1=\begin{vmatrix}800&100&20\\900&50&40\\850&100&30\end{vmatrix}=\begin{vmatrix}-1000&0&-60\\900&50&40\\-950&0&-50\end{vmatrix}=50(50000-57000)=-350000$$

$$D_2=\begin{vmatrix}90&800&20\\130&900&40\\60&850&30\end{vmatrix}=\begin{vmatrix}90&800&20\\-50&-700&0\\-75&-350&0\end{vmatrix}=20(17500-52500)=-700000$$

$$D_3=\begin{vmatrix}90&100&800\\130&50&900\\60&100&850\end{vmatrix}=\begin{vmatrix}-170&0&-1000\\130&50&900\\-200&0&-950\end{vmatrix}=50(161500-200000)=-1925000$$

$$\therefore \quad x=\frac{D_1}{D}=\frac{-350000}{-175000}=2$$

$$y=\frac{D_2}{D}=\frac{-700000}{-175000}=4$$

$$z=\frac{D_3}{D}=\frac{-1925000}{-175000}=11$$

$\therefore$ The rates of commission of items A, B and C are 2%, 4% and 11% respectively.

Chapter 6 Determinants Ex 6.4 Q32

Expressing the given information as a system of linear equations we get

$$2x + 3y + 4z = 29$$
$$x + y + 2z = 13$$
$$3x + 2y + z = 16$$

Where x, y, z is the number of cars C_1, C_2 and C_3 produced.

We use Cramer's rule to solve this system.

Here,

$$D = \begin{vmatrix} 2 & 3 & 4 \\ 1 & 1 & 2 \\ 3 & 2 & 1 \end{vmatrix} = \begin{vmatrix} -10 & -5 & 0 \\ -5 & -3 & 0 \\ 3 & 2 & 1 \end{vmatrix} = 1(30 - 25) = 5$$

$$D_1 = \begin{vmatrix} 29 & 3 & 4 \\ 13 & 1 & 2 \\ 16 & 2 & 1 \end{vmatrix} = \begin{vmatrix} -35 & -5 & 0 \\ -19 & -3 & 0 \\ 16 & 2 & 1 \end{vmatrix} = 1(105 - 95) = 10$$

$$D_2 = \begin{vmatrix} 0 & 29 & 4 \\ 1 & 13 & 2 \\ 3 & 16 & 1 \end{vmatrix} = \begin{vmatrix} -10 & -35 & 0 \\ -5 & -19 & 0 \\ 3 & 16 & 1 \end{vmatrix} = 1(190 - 175) = 15$$

$$D_3 = \begin{vmatrix} 2 & 3 & 29 \\ 1 & 1 & 13 \\ 3 & 2 & 16 \end{vmatrix} = \begin{vmatrix} -2 & 0 & 0 \\ 1 & 1 & 13 \\ 3 & 2 & 16 \end{vmatrix} = -2(16 - 26) = 20$$

$\therefore \quad x = \frac{D_1}{D} = \frac{10}{5} = 2$

$y = \frac{D_2}{D} = \frac{15}{5} = 3$

and $\quad z = \frac{D_3}{D} = \frac{20}{5} = 4$

Hence, the number of cars produced of type C_1, C_2 and C_3 are 2, 3 and 4 respectively.

Ex 6.5

Chapter 6 Determinants Ex 6.5 Q1

Here $D = \begin{vmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{vmatrix}$

$= 1(3) - 1(-3) - 2(3)$

$= 3 + 3 - 6$

$= 0$

Since $D = 0$, so the system has infinite solutions:

Now let $z = k$,

$x + y = 2k$

$2x + y = 3k$

Solving there equations by cramer's Rule

$$x = \frac{D_1}{D} = \frac{\begin{vmatrix} 2k & 1 \\ 3k & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix}} = \frac{-k}{-1} = k$$

$$y = \frac{D_2}{D} = \frac{\begin{vmatrix} 1 & 2k \\ 2 & 3k \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix}} = \frac{-k}{-1} = k$$

thus, we have $x = k, y = k, z = k$

and there values satisfy eq. (3)

Hence $x = k, y = k, z = k$

Chapter 6 Determinants Ex 6.5 Q2

$$\text{Here } D = \begin{vmatrix} 2 & 3 & 4 \\ 1 & 1 & 1 \\ 2 & -1 & 3 \end{vmatrix}$$

$$= 2(4) - 3(1) + 4(-3)$$
$$= 8 - 3 - 7$$
$$= -2$$
$$\neq 0$$

So, the given system of equations has only the trivial solutions i.e $x = 0 = y = z$:

Hence $x = 0$
$y = 0$
$z = 0$

Chapter 6 Determinants Ex 6.5 Q3

$$\text{Here } D = \begin{vmatrix} 3 & 1 & 1 \\ 1 & -4 & 3 \\ 2 & 5 & -2 \end{vmatrix}$$

$$= 3(8 - 15) - 1(-2 - 6) + 1(13)$$
$$= -21 + 8 + 13$$
$$= 0$$

So, the system has infinite solutions:

Let $z = k$,

so, $3x + y = -k$
$x - 4y = -3k$

Now,

$$x = \frac{D_1}{D} = \frac{\begin{vmatrix} -k & 1 \\ -3k & -4 \end{vmatrix}}{\begin{vmatrix} 3 & 1 \\ 1 & -4 \end{vmatrix}} = \frac{7k}{-13}$$

$$y = \frac{D_2}{D} = \frac{\begin{vmatrix} 3 & -k \\ 1 & -3k \end{vmatrix}}{\begin{vmatrix} 3 & 1 \\ 1 & -4 \end{vmatrix}} = \frac{-8k}{-13}$$

$$x = \frac{-7k}{13}, y = \frac{8k}{13}, z = k$$

and there values satisfy eq.(3)

Hence $x = -7k, y = 8k, z = 13k$

Chapter 6 Determinants Ex 6.5 Q4

$$D = \begin{vmatrix} 2\lambda & -2 & 3 \\ 1 & \lambda & 2 \\ 2 & 0 & \lambda \end{vmatrix}$$

$$= 3\lambda^3 + 2\lambda - 8 - 6\lambda$$
$$= 2\lambda^3 - 4\lambda - 8$$

which is satisfied by $\lambda = 2$ $[\because$ for non-trivial solutions $\lambda = 2]$

Now Let $z = k$,

$4x - 2y = -3k$
$x + 2y = -3k$

$$x = \frac{D_1}{D} = \frac{\begin{vmatrix} -3k & -2 \\ -2k & 2 \end{vmatrix}}{\begin{vmatrix} 4 & -2 \\ 1 & 2 \end{vmatrix}} = \frac{-10k}{10} = -k$$

$$y = \frac{D_2}{D} = \frac{\begin{vmatrix} 4 & -3k \\ 1 & -2k \end{vmatrix}}{\begin{vmatrix} 4 & -2 \\ 1 & 2 \end{vmatrix}} = \frac{-5k}{10} = \frac{-k}{2}$$

Hence solution is $x = -k, y = \frac{-k}{2}, z = k$

Chapter 6 Determinants Ex 6.5 Q5

$$D = \begin{vmatrix} (a-1) & -1 & -1 \\ -1 & (b-1) & -1 \\ -1 & -1 & (c-1) \end{vmatrix}$$

Now for non-trivial solution, $D = 0$

$$0 = (a-1)\left[(b-1)(c-1)-1\right] + 1\left[-c+\cancel{1}-\cancel{1}\right] - \left[\cancel{1}+b-\cancel{1}\right]$$

$$0 = (a-1)\left[bc-b-c+\cancel{1}-\cancel{1}\right] - c - b$$

$$0 = abc - ab - ac + \cancel{b} + \cancel{c} - \cancel{c} - \cancel{b}$$

$$ab + bc + ac = abc$$

Hence proved