

Ex 22.1

Q1

It is given that O is the origin.

Then,

$$OQ^2 = x_2^2 + y_2^2,$$

$$OP^2 = x_1^2 + y_1^2$$

$$\text{and, } PQ^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

Using cosine formula in $\triangle OPQ$, we have

$$PQ^2 = OP^2 + OQ^2 - 2(OP)(OQ)\cos \alpha$$

$$\Rightarrow (x_2 - x_1)^2 + (y_2 - y_1)^2 = x_2^2 + y_2^2 + x_1^2 + y_1^2 - 2(OP)(OQ)\cos \alpha$$

$$\Rightarrow x_2^2 + x_1^2 - 2x_2x_1 + y_2^2 + y_1^2 - 2y_2y_1 = x_2^2 + y_2^2 + x_1^2 + y_1^2 - 2(OP)(OQ)\cos \alpha$$

$$\Rightarrow -2x_1x_2 - 2y_1y_2 = -2(OP)(OQ)\cos \alpha$$

$$\Rightarrow x_1x_2 + y_1y_2 = OP \cdot OQ \cos \alpha$$

$$\Rightarrow OP \cdot OQ \cos \alpha = x_1x_2 + y_1y_2$$

Hence, proved.

Q2

We know that

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac},$$

where $a = BC$, $b = CA$ and $c = AB$ are the sides of the triangle ABC .

we have,

$$a = BC = \sqrt{(9-2)^2 + (2+1)^2} = \sqrt{49+9} = \sqrt{58}$$

$$b = CA = \sqrt{(0-9)^2 + (0+2)^2} = \sqrt{81+4} = \sqrt{85}$$

$$\text{and, } c = AB = \sqrt{(2-0)^2 + (-1-0)^2} = \sqrt{4+1} = \sqrt{5}$$

$$\begin{aligned}\therefore \cos B &= \frac{a^2 + c^2 - b^2}{2ac} \\ &= \frac{58 + 5 - 85}{2 \times \sqrt{58} \times \sqrt{5}} \\ &= \frac{63 - 85}{2\sqrt{290}} \\ &= \frac{-22}{2\sqrt{290}} = \frac{-11}{\sqrt{290}}\end{aligned}$$

$$\text{Hence, } \cos B = \frac{-11}{\sqrt{290}}.$$

Q3

$$A(6, 3), B(-3, 5), C(4, -2), D(x, 3x)$$

$$\text{or } (\Delta DBC) = \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2} [-3(-2 - 3x) + 4(3x - 5) + x(5 + 2)]$$

$$= \frac{1}{2} [6 + 9x + 12x - 20 + 5x + 2x]$$

$$= \frac{1}{2} [28x - 14]$$

$$= 7[2x - 1]$$

$$\text{or } (\Delta ABC) = \frac{1}{2} [6(5 + 2) - 3(-2 - 3) + 4(3 - 5)]$$

$$= \frac{1}{2} [42 + 15 - 8]$$

$$= \frac{49}{2}$$

$$\frac{\text{or } (\Delta DBC)}{\text{or } (\Delta ABC)} = \frac{1}{2}$$

$$\frac{7(2x - 1)}{\frac{49}{2}} = \frac{1}{2}$$

$$\frac{14(2x - 1)}{49} = \frac{1}{2}$$

$$\frac{28x - 14}{49} = \frac{1}{2}$$

$$56x - 28 = 49$$

$$56x = 28 + 49$$

$$56x = 77$$

$$x = \frac{11}{8}$$

Q4

It is given that $A(2, 0)$, $B(9, 1)$, $C(11, 6)$ and $D(4, 4)$ are the vertices of a quadrilateral.

Now,

Coordinates of the mid-point of AC are $\left(\frac{2+11}{2}, \frac{0+6}{2}\right) = \left(\frac{13}{2}, 3\right)$

Coordinates of the mid-point of BD are $\left(\frac{9+4}{2}, \frac{1+4}{2}\right) = \left(\frac{13}{2}, \frac{5}{2}\right)$

Thus, AC and BD do not have the same mid-point. Hence $ABCD$ is not a parallelogram.

$\therefore AECD$ is not a rhombus.

Q5

Let $A(-35, 7)$, $B(20, 7)$ and $C(0, -8)$ be the vertices of the triangle ABC .

Now,

$$\begin{aligned} a = BC &= \sqrt{(0-20)^2 + (-8-7)^2} \\ &= \sqrt{400 + 225} \\ &= \sqrt{625} \\ &= 25, \end{aligned}$$

$$\begin{aligned} b = AC &= \sqrt{(0+35)^2 + (-8-7)^2} \\ &= \sqrt{1225 + 225} \\ &= \sqrt{1520} \\ &= 39 \end{aligned}$$

$$\begin{aligned} \text{and, } c = AB &= \sqrt{(20+35)^2 + (7-7)^2} \\ &= \sqrt{55^2} \\ &= 55 \end{aligned}$$

The coordinates of the centre of the circle are

$$\begin{aligned} &\left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c} \right) \\ \text{or, } &\left[\frac{25 \times (-35) + 39 \times 20 + 55 \times 0}{25 + 39 + 55}, \frac{25 \times 7 + 39 \times 7 + 55 \times (-8)}{25 + 39 + 55} \right] \\ \text{or, } &\left[\frac{-500 + 780}{120}, \frac{175 + 273 - 440}{120} \right] \\ \text{or, } &\left[\frac{-120}{120}, \frac{0}{120} \right] \\ \text{or, } &(-1, 0) \end{aligned}$$

Hence, the coordinates of the centre of the circle are $(-1, 0)$.

Q6

It is given that ABC is an equilateral triangle.

$$\therefore AB = BC = AC = 2a$$

Area of equilateral triangle

$$= \frac{\sqrt{3}}{4} (\text{side})^2$$

$$= \frac{\sqrt{3}}{4} \times (2a)^2$$

$$= \frac{\sqrt{3}}{4} \times 4 \times a^2$$

$$= \sqrt{3} a^2$$

But, area of triangle = $\frac{1}{2} \times \text{Base} \times \text{Height}$.

$$\Rightarrow \frac{1}{2} \times \text{Base} \times \text{Height} = \sqrt{3} a^2$$

$$\Rightarrow \frac{1}{2} \times BC \times OA = \sqrt{3} a^2$$

$$\Rightarrow \frac{1}{2} \times 2a \times OA = \sqrt{3} a^2$$

$$\Rightarrow OA = \sqrt{3} a$$

\therefore Coordinates of A are $(\sqrt{3}a, 0)$ or $OA(-\sqrt{3}a, 0)$

Clearly, the coordinates of B and C are $(0, -a)$ and $(0, a)$ respectively.

Hence, the vertices of the triangle are $(0, a)$, $(0, -a)$ and $(-\sqrt{3}a, 0)$ or $(0, a)$, $(0, -a)$ and $(\sqrt{3}a, 0)$.

Q7

It is given that $P(x_1, y_1)$ and $Q(x_2, y_2)$ are two points

(i) PQ is parallel to the y -axis.

$$\therefore x_1 = x_2 \dots\dots\dots (1)$$

$$\begin{aligned}\therefore PQ &= \left| \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \right| \\ &= \left| \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \right| \quad [\text{Using equation 1}] \\ &= \left| \sqrt{(y_2 - y_1)^2} \right| \\ &= |y_2 - y_1|\end{aligned}$$

(ii) PQ is parallel to the x -axis.

$$\therefore y_1 = y_2 \dots\dots\dots (2)$$

$$\begin{aligned}\therefore PQ &= \left| \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \right| \\ &= \left| \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \right| \quad [\text{Using equation 2}] \\ &= \left| \sqrt{(x_2 - x_1)^2} \right| \\ &= \left| \sqrt{x_2 - x_1} \right| \\ \therefore PQ &= |x_2 - x_1|\end{aligned}$$

Q8

It is given that C lie on the x-axis. Let coordinates of C be $(x, 0)$.

Now, C is equidistant from the points A(7, 6) and B(3, 4).

$$\therefore AC = BC \quad [\text{given}]$$

$$\Rightarrow AC^2 = BC^2$$

$$\Rightarrow \left[\sqrt{(x-7)^2 + (0-6)^2} \right]^2 = \left[\sqrt{(x-3)^2 + (0-4)^2} \right]^2$$

$$\Rightarrow (x-7)^2 + (-6)^2 = (x-3)^2 + (-4)^2$$

$$\Rightarrow x^2 + 49 - 14x + 36 = x^2 + 9 - 6x + 16$$

$$\Rightarrow 49 + 36 - 36 - 16 - 9 = x^2 - x^2 - 6x + 14x$$

$$\Rightarrow 85 - 25 = 8x$$

$$\Rightarrow 60 = 8x$$

$$\Rightarrow 8x = 60$$

$$\Rightarrow x = \frac{60}{8} = \frac{15}{2}$$

Hence, coordinates of c are $\left(\frac{15}{2}, 0\right)$.

Ex 22.2

Q1

Let $P(h, k)$ be any point on the locus and let $A(2, 4)$ and $B(0, k)$. Then,

$$PA = PB$$

$$\Rightarrow PA^2 = PB^2$$

$$\Rightarrow \left[\sqrt{(2-h)^2 + (4-k)^2} \right]^2 = \left[\sqrt{(0-h)^2 + (k-k)^2} \right]^2$$

$$\Rightarrow (2-h)^2 + (4-k)^2 = (0-h)^2 + (0)^2$$

$$\Rightarrow 4 + h^2 - 4h + 16 + k^2 - 8k = h^2$$

$$\Rightarrow k^2 - 8k - 4h + 20 = 0$$

Hence, locus of (h, k) is $y^2 - 8y - 4x + 20 = 0$

Let $P(h, k)$ be any point on the locus and let $A(2, 4)$ and $B(0, k)$ be the given points.

Q2

Let $P(h, k)$ be any point on the locus and let $A(2, 0)$ and $B(1, 3)$. Then,

$$\frac{PA}{BP} = \frac{5}{4}$$

$$\Rightarrow \frac{PA^2}{BP^2} = \frac{25}{16}$$

$$\Rightarrow \frac{\left[\sqrt{(h-2)^2 + (k-0)^2}\right]^2}{\left[\sqrt{(h-1)^2 + (k-3)^2}\right]^2} = \frac{25}{16}$$

$$\Rightarrow \frac{(h-2)^2 + k^2}{(h-1)^2 + (k-3)^2} = \frac{25}{16}$$

$$\Rightarrow \frac{h^2 + 4 - 4h + k^2}{h^2 + 1 - 2h + k^2 + 9 - 6k} = \frac{25}{16}$$

$$\Rightarrow \frac{(h^2 - 4h + k^2 + 4)}{h^2 + k^2 - 2h - 6k + 10} = \frac{25}{16}$$

$$\Rightarrow 16(h^2 - 4h + k^2 + 4) = 25(h^2 + k^2 - 2h - 6k + 10)$$

$$\Rightarrow 16h^2 - 64h + 16k^2 + 64 = 25h^2 + 25k^2 - 50h - 150k + 250$$

$$\Rightarrow 25h^2 - 16h^2 + 25k^2 - 16k^2 - 50h + 64h - 150k + 250 - 64 = 0$$

$$\Rightarrow 9h^2 + 9k^2 + 14h - 150k + 186 = 0$$

Hence, locus of (h, k) is $9x^2 + 9y^2 + 14x - 150y + 186 = 0$

Q3

Let $P(h, k)$ be any point on the locus and let $A(ae, 0)$ and $B(-ae, 0)$ be the given points.

By the given condition

$$PA - PB = 2a$$

$$\Rightarrow PA = 2a + PB$$

$$\Rightarrow \sqrt{(ae - h)^2 + (0 - k)^2} = 2a + \sqrt{(-ae - h)^2 + (0 - k)^2}$$

$$\Rightarrow (ae - h)^2 + k^2 = \left(2a + \sqrt{(ae + h)^2 + k^2}\right)^2 \quad [\text{Taking square on both sides}]$$

$$\Rightarrow (ae)^2 + h^2 - 2aeh + k^2 = 4a^2 + (ae + h)^2 + k^2 + 2 \times 2a \times \sqrt{(ae + h)^2 + k^2}$$

$$\Rightarrow h^2 + k^2 + (ae)^2 - 2aeh = 4a^2 + (ae)^2 + h^2 + 2hae + k^2 + 4a\sqrt{(ae + h)^2 + k^2}$$

$$\Rightarrow -4a^2 - 2aeh - 2aeh = 4a\sqrt{(ae + h)^2 + k^2}$$

$$\Rightarrow -4a^2 - 4aeh = 4a\sqrt{(ae + h)^2 + k^2}$$

$$\Rightarrow -4[a^2 + aeh] = 4a\sqrt{(ae + h)^2 + k^2}$$

$$\Rightarrow -[a^2 + aeh] = a\sqrt{(ae + h)^2 + k^2}$$

$$\Rightarrow -a[a + eh] = a\sqrt{(ae + h)^2 + k^2}$$

$$\Rightarrow -[a + eh] = \sqrt{(ae + h)^2 + k^2}$$

Q4

Let $P(h, k)$ be any point on the locus and let $A(0, 2)$ and $B(0, -2)$ be the given points.

By the given condition $PA + PB = 6$

$$\Rightarrow \sqrt{(h-0)^2 + (k-2)^2} + \sqrt{(h-0)^2 + (k+2)^2} = 6$$

$$\Rightarrow \sqrt{h^2 + (k-2)^2} = 6 - \sqrt{h^2 + (k+2)^2}$$

$$\Rightarrow h^2 + (k-2)^2 = 36 - 12\sqrt{h^2 + (k+2)^2} + h^2 + (k+2)^2$$

$$\Rightarrow -8k - 36 = -12\sqrt{h^2 + (k+2)^2}$$

$$\Rightarrow (2k+9) = 3\sqrt{h^2 + (k+2)^2}$$

$$\Rightarrow (2k+9)^2 = 9(h^2 + (k+2)^2)$$

$$\Rightarrow 4k^2 + 36k + 81 = 9h^2 + 9k^2 + 36k + 36$$

$$\Rightarrow 9h^2 + 5k^2 = 45$$

Hence, locus of (h, k) is $9x^2 + 5y^2 = 45$.

Q5

Let $P(h, k)$ be any point on the locus and let $A(1, 3)$ and $B(h, 0)$. Then,

$$PA = PB$$

$$\Rightarrow PA^2 = PB^2$$

$$\Rightarrow (1-h)^2 + (3-k)^2 = (h-h)^2 + (0-k)^2$$

$$\Rightarrow 1 + h^2 - 2h + 9 + k^2 - 6k = 0 + k^2$$

$$\Rightarrow h^2 - 2h - 6k + 10 = 0$$

Hence, locus of (h, k) is $x^2 - 2x - 6y + 10 = 0$

Q6

Let $P(h, k)$ be any point on the locus and let $O(0, 0)$ be the origin.

By the given condition

$$OP = 3k \quad [\because k \text{ is the distance of point from } x\text{-axis}]$$

$$\Rightarrow OP^2 = 9k^2$$

$$\Rightarrow \left(\sqrt{(0-h)^2 + (0-k)^2} \right)^2 = 9k^2$$

$$\Rightarrow h^2 + k^2 = 9k^2$$

$$\Rightarrow h^2 = 9k^2 - k^2$$

$$\Rightarrow h^2 = 8k^2$$

Hence, locus of (h, k) is $x^2 = 8y^2$

Q7

Let $P(h, k)$ be any point on the locus. Then,

Area $(PAB) = 9$ sq units

$$\Rightarrow \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| = 9$$

$$\Rightarrow |5(-2 - k) + 3(k - 3) + h(3 + 2)| = 18$$

$$\Rightarrow |-10 - 5k + 3k - 9 + 5h| = 18$$

$$\Rightarrow |5h - 2k - 19| = 18$$

$$\Rightarrow 5h - 2k - 19 = \pm 18$$

$$\Rightarrow 5h - 2k - 19 \mp 18 = 0$$

$$\Rightarrow 5h - 2k - 37 = 0 \quad \text{or,} \quad 5h - 2k - 1 = 0$$

Hence, the locus of (h, k) is

$$5x - 2y - 37 = 0 \quad \text{or,} \quad 5x - 2y - 1 = 0.$$

Q8

Let $P(h, k)$ be the variable point and let $A(2, 0)$ and $B(-2, 0)$ be the given points.

Then $\angle APB = \pi/2$

$$\Rightarrow AB^2 = PA^2 + PB^2$$

$$\Rightarrow (2+2)^2 + 0 = (2-h)^2 + (0-k)^2 + (-2-h)^2 + (0-k)^2$$

$$\Rightarrow 16 = 4 + h^2 - 4h + k^2 + 4 + h^2 + 4h + k^2$$

$$\Rightarrow 16 = 2h^2 + 2k^2 + 8$$

$$\Rightarrow 2h^2 + 2k^2 + 8 - 16 = 0$$

$$\Rightarrow 2h^2 + 2k^2 - 8 = 0$$

$$\Rightarrow h^2 + k^2 - 4 = 0$$

Hence, the locus of (h, k) is $x^2 + y^2 = 4$.

Q9

Let $P(h, k)$ be any point on the locus. Then,

Area $(PAB) = 8$ sq. units

$$\Rightarrow \frac{1}{2} |x_1(y_2 - y_3) + (y_3 - y_1)x_2 + x_3(y_1 - y_2)| = 8$$

$$\Rightarrow \frac{1}{2} |-1(3-k) + 2(k-1) + h(1-3)| = 8$$

$$\Rightarrow \frac{1}{2} |-3+k+2k-2-2h| = 8$$

$$\Rightarrow \frac{1}{2} |-2h+3k-5| = 8$$

$$\Rightarrow |-2h+3k-5| = 16$$

$$\Rightarrow -2h+3k-5 = \pm 16$$

$$\Rightarrow 2h-3k+5 \pm 16 = 0$$

$$\Rightarrow 2h-3k+21=0 \quad \text{or,} \quad 2h-3k-11=0$$

Hence, the locus of (h, k) is

$$2x-3y+21=0 \quad \text{or,} \quad 2x-3y-11=0$$

Q10

Let the two perpendicular lines be the coordinate axes. Let AB be a rod length l . Let the coordinates of A and B be $(a, 0)$ and $(0, b)$ respectively. As the rod slides the value of a and b change, so, a and b are two variables.

Let $P(h, k)$ be the point on the locus. Then,

$$h = \frac{2 \times a + 1 \times 0}{2 + 1}$$

$$\Rightarrow h = \frac{2a}{3}$$

$$\Rightarrow a = \frac{3h}{2}$$

$$\text{and } k = \frac{2 \times 0 + b \times 1}{2 + 1}$$

$$\Rightarrow k = \frac{b}{3}$$

$$\Rightarrow b = 3k$$

from $\triangle AOB$, we have

$$AB^2 = OA^2 + OB^2$$

$$\Rightarrow l^2 = [(a - 0)^2 + (0 - 0)^2] + [(0 - 0)^2 + (b - 0)^2]$$

$$\Rightarrow l^2 = a^2 + b^2$$

$$\Rightarrow a^2 + b^2 = l^2$$

$$\Rightarrow \left(\frac{3h}{2}\right)^2 + (3k)^2 = l^2$$

$$\Rightarrow \frac{9h^2}{4} + 9k^2 = l^2$$

$$\Rightarrow \frac{h^2}{4} + k^2 = \frac{l^2}{9}$$

Hence, the locus of (h, k) is $\frac{x^2}{4} + y^2 = \frac{l^2}{9}$

Q11

Given, line is

$$x \cos \alpha + y \sin \alpha = p$$

$$\frac{\frac{x}{\frac{p}{\cos \alpha}}}{\frac{p}{\cos \alpha}} + \frac{\frac{y}{\frac{p}{\sin \alpha}}}{\frac{p}{\sin \alpha}} = 1$$

Intercepts on x axis is $\frac{p}{\cos \alpha}$ and y - axis is $\frac{p}{\sin \alpha}$

Let $P(x, y)$ be the mid point of AB .

$$(x, y) = \left(\frac{\frac{p}{\cos \alpha} + 0}{2}, \frac{\frac{p}{\sin \alpha} + 0}{2} \right) = \left(\frac{p}{2 \cos \alpha}, \frac{p}{2 \sin \alpha} \right)$$

$$\therefore x = \frac{p}{2 \cos \alpha}, y = \frac{p}{2 \sin \alpha}$$

$$2 \cos \alpha = \frac{p}{x}, \quad 2 \sin \alpha = \frac{p}{y}$$

Square both sides,

$$4 \cos^2 \alpha = \frac{p^2}{x^2} \text{----- (1)}$$

and

$$4 \sin^2 \alpha = \frac{p^2}{y^2} \text{----- (2)}$$

$[(1) + (2)]$

$$4 \cos^2 \alpha + 4 \sin^2 \alpha = \frac{p^2}{x^2} + \frac{p^2}{y^2}$$

$$4 = \frac{p^2 (x^2 + y^2)}{x^2 y^2}$$

$$4x^2 y^2 = p^2 (x^2 + y^2)$$

Q12

Let $P(h, k)$ be the point on the locus and let the coordinates of a are (a, b) . Then,

$$h = \frac{a+0}{2} \text{ and } \frac{b+0}{2} = k \quad [\because P \text{ is the mid-point of } Q \text{ and the origino}]$$

$$h = \frac{a}{2} \quad \text{and} \quad b = 2k$$

$$\Rightarrow \quad a = 2h \quad \text{and} \quad b = 2k$$

point Q lies on the $y^2 = x$. Then,

$$b^2 = a \quad [\because Q : (a, b)]$$

$$\Rightarrow \quad (2k)^2 = 2h \quad [\because a = 2h \text{ and } b = 2k]$$

$$\Rightarrow \quad 4k^2 = 2h$$

$$\Rightarrow \quad 2k^2 = h$$

Hence, the locus of (h, k) is $2y^2 = x$.

Ex 22.3

Q1

We have,

$$(x - a)^2 + (y - b)^2 = r^2 \dots\dots\dots (i)$$

Substituting $x = X + (a - c)$, $y = Y + b$ in the equation (i), we get

$$[X + a - c - a]^2 + [Y + b - b]^2 = r^2$$

$$\Rightarrow [X - c]^2 + [Y]^2 = r^2$$

$$\Rightarrow X^2 + c^2 - 2Xc + Y^2 = r^2$$

$$\Rightarrow X^2 + Y^2 - 2cX = r^2 - c^2$$

Hence, the required equation is $X^2 + Y^2 - 2cX = r^2 - c^2$

Q2

We have,

$$(a - b)(x^2 - y^2) - 2abx = 0$$

Substituting $x = X + \frac{ab}{a-b}$, $y = Y$

in the given equation, we get

$$(a - b) \left[\left(X + \frac{ab}{a-b} \right)^2 + Y^2 \right] - 2ab \left[X + \frac{ab}{a-b} \right] = 0$$

$$\Rightarrow (a - b) \left[X^2 + \left(\frac{ab}{a-b} \right)^2 + 2 \frac{Xab}{a-b} + Y^2 \right] - 2abX - 2 \frac{(ab)^2}{a-b} = 0$$

$$\Rightarrow (a - b) \left[\frac{X^2(a-b)^2 + (ab)^2 + 2Xab(a-b) + Y^2(a-b)^2}{(a-b)^2} \right] - \frac{2abX(a-b) - 2(ab)^2}{a-b} = 0$$

$$\Rightarrow \frac{X^2(a-b)^2 + (ab)^2 + 2ab(a-b) + Y^2(a-b)^2}{a-b} = \frac{2ab(a-b) + 2(ab)^2}{a-b}$$

$$\Rightarrow X^2(a-b)^2 + Y^2(a-b)^2 + (ab)^2 + 2ab(a-b) = 2ab(a-b) + 2(ab)^2$$

$$\Rightarrow (a-b)^2 (X^2 + Y^2) = (ab)^2$$

$$\Rightarrow (a-b)^2 (X^2 + Y^2) = a^2b^2$$

Q3(i)

We have,

$$x^2 + xy - 3x - y + 2 = 0$$

Substituting $x = X + 1$, $y = Y + 1$ in the equation, we get

$$(X + 1)^2 + (X + 1)(Y + 1) - 3(X + 1) - (Y + 1) + 2 = 0$$

$$\Rightarrow X^2 + 1 + 2X + XY + X + Y + 1 - 3X - 3 - Y - 1 + 2 = 0$$

$$\Rightarrow X^2 + XY = 0$$

Q3(ii)

We have,

$$x^2 - y^2 - 2x + 2y = 0$$

Substituting $x = X + 1$, $y = Y + 1$ in the equation, we get

$$(X + 1)^2 - (Y + 1)^2 - 2(X + 1) + 2(Y + 1) = 0$$

$$\Rightarrow X^2 + 1 + 2X - (Y^2 + 1 + 2Y) - 2X - 2 + 2Y + 2 = 0$$

$$\Rightarrow X^2 + 1 - Y^2 - 1 - 2Y + 2Y = 0$$

$$\Rightarrow X^2 - Y^2 = 0$$

Q3(iii)

We have,

$$xy - x - y + 1 = 0$$

Substituting $x = X + 1$, $y = Y + 1$ in the equation, we get

$$(X + 1)(Y + 1) - (X + 1) - (Y + 1) + 1 = 0$$

$$\Rightarrow XY + X + Y + 1 - X - 1 - Y - 1 + 1 = 0$$

$$\Rightarrow XY = 0$$

Q3(iv)

We have,

$$xy - y^2 - x + y = 0$$

Substituting $x = X + 1$, $y = Y + 1$ in the equation, we get

$$(X + 1)(Y + 1) - (Y + 1) - (X + 1) + (Y + 1) = 0$$

$$\Rightarrow XY + Y + Y + 1 - (Y^2 + 1 + 2Y) - X - 1 + Y + 1 = 0$$

$$\Rightarrow XY + 2Y - Y^2 - 1 - 2Y + 1 = 0$$

$$\Rightarrow XY - Y^2 = 0$$

Q4

We have,

$$x^2 + xy - 3x - y - 2 = 0 \dots\dots (i)$$

Let the origin be shifted to (h, k) . Then $x = X + h$ and $y = Y + k$.

Substituting $x = X + h$, $y = Y + k$ in the equation (i), we get

$$(X + h)^2 + (X + h)(Y + k) - 3(X + h) - (Y + k) + 2 = 0$$

$$\Rightarrow X^2 + h^2 + 2Xh + XY + Xk + Yh + hk - 3X - 3h - Y - k + 2 = 0$$

$$\Rightarrow X^2 + XY + 2Xh + Xk + Yh + Yk - 3X - X + h^2 + hk - 3h - k + 2 = 0$$

$$\Rightarrow X^2 + (2h + k - 3)X + XY + (h + 1)Y + (h^2 + hk - 3h - k + 2) = 0$$

$$\Rightarrow X^2 + (2h + k - 3)X + XY + (h + 1)Y + (h^2 + hk - 3h - k + 2) = 0$$

For this equation to be free from first degree and the constant term, we must have,

$$2h + k - 3 = 0 \dots\dots\dots (i)$$

$$h + 1 = 0$$

$$\Rightarrow h = -1 \dots\dots\dots (ii)$$

and

$$h^2 + hk - 3h - k + 2 = 0 \dots\dots\dots (iv)$$

Putting $h = -1$ in equation (i) we get

$$2 - k - 3 = 0$$

$$\Rightarrow k = -1$$

Putting $h = -1$ and $k = -1$ in equation (iv), we get

$$1 + 1 - 3 - 1 + 2 = 0$$

Hence, the value of h and k satisfies the equation (iv)

∴ The origin is shifted at the point $(-1, -1)$.

Q5

Let the vertices of a triangle be A (2,3), B (5,7) and C (-3,-1).

Then, area of $\triangle ABC$ is given by

$$\begin{aligned} A &= \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| \\ &= \frac{1}{2} |2(7 - 1) + 5(-1 - 3) - 3(-3 - 7)| \\ &= \frac{1}{2} |2 \times 6 + 5 \times (-4) - 3 \times (-4)| \\ &= \frac{1}{2} |12 - 20 + 12| \\ &= \frac{8}{2} \\ &= 4 \end{aligned}$$

$$\Rightarrow A = 4 \text{ sq unit}$$

It is given that the origin is shifted at (-1,3). Then new coordinates of the vertices are

$$A_1 = (2 - 3, 3 + 3) = (-1, 6)$$

$$B_1 = (5 - 1, 7 + 3) = (4, 10)$$

$$\text{and } C_1 = (-3 - 1, -1 + 3) = (-4, 2)$$

Therefore, the area of the triangle in the new coordinate system is given by

$$\begin{aligned} A_1 &= \frac{1}{2} [-1(10 - 2) + 4(2 - 6) - 4(6 - 10)] \\ &= \frac{1}{2} [-1 \times 8 + 4 \times (-4) - 4 \times (-4)] \\ &= \frac{1}{2} [-8 - 16 + 16] \\ &= \frac{1}{2} [-8] \\ &= \frac{8}{2} \end{aligned}$$

$$\Rightarrow A_1 = 4, \dots \dots \dots (2)$$

From (i) and (ii), we get

$$A = A_1$$

Hence, the area of a triangle is invariant under the translation of the axes.

Q6(i)

We have,

$$x^2 + xy - 3y^2 - y + 2 = 0 \dots \dots (1)$$

Substituting $x = X + 1$, $y = Y + 1$

in equation (1), we get

$$(X + 1)^2 + (X + 1)(Y + 1) - 3(Y + 1)^2 - (Y + 1) + 2 = 0$$

$$\Rightarrow X^2 + 1 + 2X + XY + X + Y + 1 - 3(Y^2 + 1 + 2Y) - Y - 1 + 2 = 0$$

$$\Rightarrow X^2 + XY + 3X + 3 - 3Y^2 - 3 - 5Y = 0$$

$$\Rightarrow X^2 - 3Y^2 - XY + 3X - 5Y = 0$$

Q6(ii)

We have,

$$xy - y^2 - x + y = 0 \dots\dots\dots (i)$$

Substituting $x = X + 1$, $y = Y + 1$

in equation (i), we get

$$(X + 1)(Y + 1) - (Y + 1)^2 - (X + 1) + (Y + 1) = 0$$

$$\Rightarrow XY + X + Y + 1 - (Y^2 + 1 + 2Y) - X - 1 + Y + 1 = 0$$

$$\Rightarrow XY + 2Y + 1 - Y^2 - 1 - 2Y = 0$$

$$\Rightarrow XY - Y^2 = 0$$

Q6(iii)

We have,

$$xy - x - y + 1 = 0 \dots\dots\dots (i)$$

Substituting $x = X + 1$, $y = Y + 1$

in equation (i), we get

$$(X + 1)(Y + 1) - (X + 1) - (Y + 1) + 1 = 0$$

$$\Rightarrow XY + X + Y + 1 - X - 1 - Y - 1 + 1 = 0$$

$$\Rightarrow XY = 0$$

Q6(iv)

We have,

$$x^2 - y^2 - 2x - 2y = 0 \dots\dots\dots (i)$$

Substituting $x = X + 1$, $y = Y + 1$

in equation (i), we get

$$(X + 1)^2 - (Y + 1)^2 - 2(X + 1) + 2(Y + 1) = 0$$

$$\Rightarrow X^2 + 1 + 2X - (Y^2 + 1 + 2Y) - 2X - 2 + 2Y + 2 = 0$$

$$\Rightarrow X^2 + 1 - Y^2 - 1 - 2Y + 2Y = 0$$

$$\Rightarrow X^2 + 2X + 1 - (Y^2 + 2Y + 1)$$

$$\Rightarrow (X + 1)^2 - (Y + 1)^2$$

$$\Rightarrow X^2 - Y^2 = 0$$

Q7(i)

Let the origin be shifted to (h, k) . Then, $x = X + h$ and $y = Y + k$.

Substituting $x = X + h$, $y = Y + k$

in the equation $y^2 + x^2 - 4x - 8y + 3 = 0$, we get

$$(Y + k)^2 + (X + h)^2 - 4(X + h) - 8(Y + k) + 3 = 0$$

$$\Rightarrow Y^2 + k^2 + 2Yk + X^2 + h^2 + 2Xh - 4X - 4h - 8Y - 8k + 3 = 0$$

$$\Rightarrow Y^2 + X^2 + 2Yk - 8Y + 2Xh - 4X + k^2 + h^2 - 4h - 8k + 3 = 0$$

$$\Rightarrow Y^2 + X^2 + (2k - 8)Y + (2h - 4)X + (k^2 + h^2 - 4h - 8k + 3) = 0$$

For this equation to be free from the term of first degree, we must have

$$2k - 8 = 0 \text{ and } 2h - 4 = 0$$

$$\Rightarrow k = 4 \text{ and } h = 2$$

Hence, the origin is shifted at the point $(2, 4)$.

Q7(ii)

Let the origin be shifted to (h, k) . Then, $x = X + h$ and $y = Y + k$

Substituting $x = X + h$, $y = Y + k$

in the equation $x^2 + y^2 - 5x + 2y - 5 = 0$, we get

$$(X + h)^2 + (Y + k)^2 - 5(X + h) + 2(Y + k) - 5 = 0$$

$$\Rightarrow X^2 + h^2 + 2Xh + Y^2 + k^2 + 2Yk - 5X - 5h + 2Y + 2k - 5 = 0$$

$$\Rightarrow X^2 + Y^2 + 2Yk + 2Y + 2Xh - 5X + h^2 + k^2 - 5h + 2k - 5 = 0$$

$$\Rightarrow X^2 + Y^2 + (2k + 2)Y + (2h - 5)X + h^2 + k^2 - 5h + 2k - 5 = 0$$

For this equation to be free from the term of first degree, we must have

$$2k + 2 = 0 \text{ and } 2h - 5 = 0$$

$$\Rightarrow k = -1 \text{ and } h = \frac{5}{2}$$

Hence, the origin is shifted at the point $\left(\frac{5}{2}, -1\right)$.

Q7(iii)

Let the origin be shifted to (h, k) . Then, $x = X + h$ and $y = Y + k$

Substituting $x = X + h$, $y = Y + k$

in the equation $x^2 + y^2 - 12x + 4 = 0$, we get

$$(X + h)^2 + (Y + k)^2 - 12(X + h) + 4 = 0$$

$$\Rightarrow X^2 + h^2 + 2Xh + Y^2 + k^2 + 2Yk - 12X - 12h + 4 = 0$$

$$\Rightarrow X^2 + Y^2 + (2h - 12)X + h^2 - 12h + 4 = 0$$

For this equation to be free from term of first degree, we must have

$$2h - 12 = 0$$

$$\Rightarrow h = \frac{12}{2}$$

$$\Rightarrow h = 6$$

Hence, the origin is shifted at the point $(6, k)$, $k \in \mathbb{R}$.

Q8

Let the co-ordinate of the vertex be A(4,6) B(7,10) and C(1,-2)

Now area of the ΔABC is given by

$$\begin{aligned}\Delta &= \frac{1}{2} \left| (x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)) \right| \\ &= \frac{1}{2} \left| 4(10 - 2) + 7(-2 - 6) + 1(6 - 10) \right| \\ &= \frac{1}{2} \left| (48 - 56 - 4) \right| \\ &= 6\end{aligned}$$

After transforming the origin to (-2,1), the co-ordinate of the vertex will be

A(2,7), B(5,11) and C(-1,-1). Now the area will be

$$\begin{aligned}\Delta_1 &= \frac{1}{2} \left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right| \\ &= \frac{1}{2} \left| 2(11 - 1) + 5(-1 - 7) - 1(7 - 11) \right| \\ &= \frac{1}{2} \left| 24 - 40 + 4 \right| \\ &= 6\end{aligned}$$

Here $\Delta = \Delta_1$

Hence proved.