Ex 16.1

Q1

(i)

We have,

$$\frac{30!}{28!} = \frac{30 \times 29 \times 28!}{28!}$$

$$= 30 \times 29$$
$$= 870$$

Hence, $\frac{30!}{28!} = 870$

(ii)

We have,

$$\frac{11! - 10!}{9!} = \frac{11 \times 10 \times 9! - 10 \times 9!}{9!}$$

$$= \frac{9! \times 10\left[11-1\right]}{9!}$$

$$= 10 \times 10$$
$$= 100$$

Hence, $\frac{11! - 10!}{9!} = 100$

(iii)

We have,

$$8! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$
$$7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

and $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$\therefore$ L.C.M.$(6!, 7!, 8!) = 8!$

Q2

L.H.S:

$$\frac{1}{9!}+\frac{1}{10!}+\frac{1}{11!}$$

$$\frac{1}{9!}+\frac{1}{10\times 9!}+\frac{1}{11\times 10\times 9!}$$

$$=\frac{11\times 10+11+1}{11\times 10\times 9!}$$

$$=\frac{110+11+1}{11!}$$

$$=\frac{122}{11!}$$

$$= \text{RHS}$$

Hence, $\frac{1}{9!}+\frac{1}{10!}+\frac{1}{11!}=\frac{122}{11!}$

Q3(i)

We have,

$$\frac{1}{4!}+\frac{1}{5!}=\frac{x}{6!}$$

$$\Rightarrow \quad \frac{1}{4!}+\frac{1}{5\times 4!}=\frac{x}{6\times 5\times 4!}$$

$$\Rightarrow \quad 4!\times\left[\frac{1}{4!}+\frac{1}{5\times 4!}\right]=\frac{x}{30}$$

$$\Rightarrow \quad 1+\frac{1}{5}=\frac{x}{30}$$

$$\Rightarrow \quad \frac{6}{5}=\frac{x}{30}$$

$$\Rightarrow \quad \frac{x}{30}=\frac{6}{5}$$

$$\Rightarrow \quad x=\frac{6\times 30}{5}$$

$$\Rightarrow \quad x=6\times 6$$

$$\Rightarrow \quad x=36$$

Hence, $x = 36$.

Q3(ii)

$$\frac{x}{10!}=\frac{1}{8!}+\frac{1}{9!}$$

$$\Rightarrow x=\frac{10!}{8!}+\frac{10!}{9!}$$

$$\Rightarrow x=\frac{10\times 9\times 8!}{8!}+\frac{10\times 9!}{9!}$$

$$\Rightarrow x=10\times 9+10$$

$$\Rightarrow x=100$$

Q3(iii)

$$\frac{1}{6!}+\frac{1}{7!}=\frac{x}{8!}$$

$$\Rightarrow x=\frac{8!}{6!}+\frac{8!}{7!}$$

$$\Rightarrow x=\frac{8\times 7\times 6!}{6!}+\frac{8\times 7!}{7!}$$

$$\Rightarrow x=8\times 7+8$$

$$\Rightarrow x=64$$

Q4(i)

We have,

$$5\times 6\times 7\times 8\times 9\times 10$$

$$=\frac{10\times 9\times 8\times 7\times 6\times 5\times(4\times 3\times 2\times 1)}{4\times 3\times 2\times 1}$$

$$=\frac{10!}{4!}$$

Hence, $5\times 6\times 7\times 8\times 9\times 10=\frac{10!}{4!}$

Q4(ii)

We have,

$$3 \times 6 \times 9 \times 12 \times 15 \times 18$$
$$= 3 \times (3 \times 2) \times (3 \times 3) \times (3 \times 4) \times (3 \times 5) \times (3 \times 6)$$
$$= 3^6 \times [2 \times 3 \times 4 \times 5 \times 6]$$
$$= 3^6 \times (6!)$$

Q4(iii)

We have,

$$(n+1)(n+2)(n+3)\ldots\ldots\ldots(2n)$$
$$= \frac{[1 \times 2 \times 3 \times 4 \ldots\ldots\ldots\ldots (n-1)n] \times (n+1)(n+2)\ldots(2n-1) \times 2n}{[1 \times 2 \times 3 \times 4 \ldots\ldots\ldots\ldots\ldots (n-1)n]}$$

$$= \frac{(2n!)!}{n!}$$

Q4(iv)

We have,

$$1 \times \times 5 \times 7 \times 9 \ldots\ldots\ldots\ldots (2n-1)$$
$$= \frac{[1.3.5.7.9\ldots\ldots(2n-1)].[2.4.6.8\ldots\ldots\ldots(2n-2)(2n)]}{2.4.6.8\ldots\ldots(2n-2)(2n)}$$

$$= \frac{[1.3.5.7.9\ldots\ldots\ldots(2n-1)].[2.4.6.8\ldots\ldots\ldots(2n-2)(2n)]}{2^n[1.2.3.4\ldots\ldots\ldots((n-1)(n))]}$$

$$= \frac{1.2.3.4.5.6.7.8\ldots\ldots\ldots(2n-2)(2n-1)(2n)}{2^n.n!}$$

$$= \frac{(2n)!}{2^n.n!}$$

$$\therefore \quad 1.3.5.7.9\ldots\ldots\ldots(2n-1) = \frac{(2n)!}{2^n.n!}$$

Q5

(i) $\text{LHS} = (2+3)!$

$= 5!$

$= 5 \times 4 \times 3 \times 2 \times 1$

$= 120$

and, $\text{RHS} = 2! + 3!$

$= 2 \times 1 + 3 \times 2$

$= 2 \times 1 + 3 \times 2 \times 1$

$= 2 + 6$

$= 8$

$\therefore \quad 120 \neq 8$

$\therefore \quad (2+3)! \neq 2! + 3!$

So, it is false.

(ii) $\text{LHS} = (2 \times 3)!$

$= 6!$

$= 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 720$

and, $\text{RHS} = 2! \times 3!$

$= 2 \times 1 \times 3 \times 2$

$= 12$

$\therefore \quad 720 \neq 12$

$\therefore \quad (2 \times 3)! \neq 2! \times 3!$

Hence, it is false.

Q6

$\text{LHS} = n! + (n+1)!$

$= n! + (n+1)(n+1-1)!$

$= n! + (n+1)n!$

$= n!(1+n+1)$

$= n!(n+2)$

$= \text{LHS}$

$\therefore \quad n!(n+2) = n! + (n+1)!$

Hence, proved

Q7

We have,

$$(n+2)! = 60\left[(n-1)!\right]$$

$$(n+2)(n+1)(n)(n-1)! = 60\left[(n-1)!\right]$$

$\Rightarrow \quad (n+2)(n+1)n = 60$

$\Rightarrow \quad (n+2)(n+1)n = 5 \times 4 \times 3$

$\therefore \quad n = 3 \qquad$ [By comparing]

Hence, $n = 3$

Q8

We have,

$$(n+1)! = 90\left[(n-1)!\right]$$

$\Rightarrow \quad (n+1) \times n \times (n-1)! = 90\left[(n-1)!\right]$

$\Rightarrow \quad n(n+1) = 90$

$\Rightarrow \quad n^2 + n = 90$

$\Rightarrow \quad n^2 + n - 90 = 0$

$\Rightarrow \quad n^2 + 10n - 9n - 90 = 0$

$\Rightarrow \quad n(n+10) - 9(n+10) = 0$

$\Rightarrow \quad (n-9)(n+10) = 0$

$\Rightarrow \quad n - 9 = 0 \qquad \left[\because n + 10 \neq 0\right]$

$\Rightarrow \quad n = 9$

Hence, $n = 9$

Q9

We have,

$$(n+3)! = 56\left[(n+1)!\right]$$

$\Rightarrow \quad (n+3) \times (n+2) \times (n+1)! = 56\left[(n+1)!\right]$

$\Rightarrow \quad (n+2)(n+3) = 56$

$\Rightarrow \quad n^2 + 3n + 2n + 6 = 56$

$\Rightarrow \quad n^2 + 5n + 6 - 56 = 0$

$\Rightarrow \quad n^2 + 5n - 50 = 0$

$\Rightarrow \quad n^2 + 10n - 5n - 50 = 0$

$\Rightarrow \quad n(n+10) - 5(n+10) = 0$

$\Rightarrow \quad (n+10)(n-5) = 0$

$\Rightarrow \quad n - 5 = 0 \qquad \left[\because n + 10 \neq 0\right]$

$\Rightarrow \quad n - 5 = 0$

$\Rightarrow \quad n = 5$

Q10

We have,

$$\frac{\dfrac{(2n)!}{3!(2n-3)!}}{\dfrac{n!}{2!(n-2)!}} = \frac{44}{3}$$

$$\Rightarrow \quad \frac{(2n)! \times 2!(n-2)!}{3!(2n-3)! \times n!} = \frac{44}{3}$$

$$\Rightarrow \quad \frac{(2n)(2n-1)(2n-2)(2n-3)! \times 2!(n-2)!}{3 \times 2!(2n-3)! \times n(n-1)(n-2)!} = \frac{44}{3}$$

$$\Rightarrow \quad \frac{2n(2n-1)(2n-2)}{3n(n-1)} = \frac{44}{3}$$

$$\Rightarrow \quad \frac{2(2n-1) \times 2(n-1)}{3(n-1)} = \frac{44}{3}$$

$\Rightarrow \quad 4(2n-1) = 44$

$\Rightarrow \quad 2n - 1 = 11$

$\Rightarrow \quad 2n = 12$

$\Rightarrow \quad n = 6$

$\therefore \quad n = 6$

Q11(i)

We have,

$$\text{LHS} = \frac{n!}{(n-r)!}$$

$$= \frac{n(n-1)(n-2)(n-3)\ldots(n-r+2)(n-r+1)(n-r)!}{(n-r)!}$$

$$= n(n-1)(n-2)(n-3)\ldots(n-r+2)(n-r+1)$$

$$= n(n-1)(n-2)(n-3)\ldots((n-(r-2))(n-(r-1))$$

$$= n(n-1)(n-2)(n-3)\ldots(n-(r-1))$$

$= \text{RHS}$

$\therefore \quad \text{LHS} = \text{RHS}$

Hence proved

Q11(ii)

We have,

$$\text{LHS} = \frac{n!}{(n-r)!\,r!} + \frac{n!}{(n-r+1)!\,(r-1)!}$$

$$= \frac{n!}{(n-r)!\,r \times [(r-1)!]} + \frac{n!}{(n-r+1)[(n-r)!](r-1)!}$$

$$= \frac{n!}{(n-r)! \times (r-1)!}\left[\frac{1}{r} + \frac{1}{n-r+1}\right]$$

$$= \frac{n!}{(n-r)! \times (r-1)!}\left[\frac{n-r+1+r}{r(n-r+1)}\right]$$

$$= \frac{n!}{(n-r)! \times (r-1)!}\left[\frac{n+1}{r(n-r+1)}\right]$$

$$= \frac{(n+1) \times n!}{(n-r+1) \times (n-r)! \times r \times (r-1)!}$$

$$= \frac{(n+1)!}{(n-r+1)! \times r!}$$

$$= \frac{(n+1)!}{r!\,(n-r+1)!}$$

= RHS

$\therefore$ LHS = RHS

Hence proved

Q12

We have,

$$\text{LHS} = \frac{(2n+1)!}{n!}$$

$$= \frac{(2n+1)[1.2.3.4.5.6.7.8\ldots(2n-1)2n]}{n!}$$

$$= \frac{[1.3.5.7\ldots\ldots(2n-1)\times(2n+1)][2.4.6.8\ldots(2n-2)2n]}{n!}$$

$$= \frac{[1.3.5.7\ldots\ldots(2n-1)(2n+1)]\times 2^n[1.2.3.4\ldots\ldots(n-1)n]}{n!}$$

$$= \frac{[1.3.5.7\ldots\ldots(2n-1)(2n+1)]2^n \times n!}{n!}$$

$$= 2^n[1.3.5.7\ldots.(2n-1)(2n+1)]$$

= RHS

Hence proved

Ex 16.2

Q1

Here the teacher is to perform two jobs.
(i) selecting a boy among 27 boys, and
(ii) selecting a girl among 14 girls.

The first of these can be performed in 27 ways and the second in 14 ways.
Therefore by the fundamental principle of multiplication, the required number of ways is

$$27 \times 14 = 378$$

Hence,the teacher can make the selection of a boy a girl in 378 ways.

Q2

Here the person is to perform three jobs.
(i) selecting a ball pen from 12 ball pens
(ii) selecting a fountain pen from 10 fountain pens, and
(iii) selecting a pencil from 5 pencils.

The first of these can be performed in 12 ways, the second in 10 ways and the third in 5 ways.

Therefore by the fundamental principle of multiplication, the required number of ways is

$$12 \times 10 \times 5 = 600$$

Hence, the person can make the selection of a fountain pen, ball pen and pencil in 600 ways.

Q3

From Goa to Bombay there are two roots; air and sea.
From Bombay to Delhi there are three routs; air rail and road.

Therefore by the fundamental principle of multiplication, the required number of ways are $2 \times 3 = 6$

Hence, total number of different kinds routes are 6.

Q4

The mint has to perform two jobs,
(i) selecting the number of days in the february month (there can be 28 days or 29 days), and
(ii) selecting the first day of february.

The first job can be compeleted in 2 ways the second can be performed in 7 ways by selecting any one of the seven days of a week.

Thus, the required number of plates = $2 \times 7 = 14$

Hence, total number of calendars = $7 \times 2 = 14$

Q5

Total number of parcels = 4
Total number of post-offices = 5

Since a parcel can be sent to any one of the five post offices.
So, the required number of ways $= 5 \times 5 \times 5 \times 5$
$= 5^4$
$= 625$

Hence, total number of ways is 625.

Q6

Since toss of each coin can result in 2 ways.

When coin is tossed five times, the total number of outcomes
$= 2 \times 2 \times 2 \times 2 \times 2$
$= 32$
Hence, required number of ways is 32

Q7

The number of ways to examinee answer a true/false type question is 2.

$\therefore$ the number of ways for an examinee to answer a set of ten true/false type questions $= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
$= 1024.$

Hence, the required number of ways is 1024.

Q8

The total number of ways to make attempt to open the lock = $10 \times 10 \times 10 = 1000$.

The number of ways to successfuly open the lock = 1

$\therefore$ The number of ways to make an unsuccessful attempt to open the lock = $1000 - 1 = 999$.

Hence,required number of ways to make an unsuccessfuly attempt to the open the lock is 999.

Q9

Each one of the first three questions can be answered in 4 ways.

$\therefore$ The total number of ways to answered the first three question = $4 \times 4 \times 4$
$= 64$

Each of the next three question can be answered in 2 ways.

$\therefore$ The total number of ways the answered the next three questions = $2 \times 2 \times 2 = 8$

so, total number of sequences at answers = $64 \times 8 = 512$

Q10

There are 5 books on mathematics and 6 books on physics in a book shop.

The number of ways to select a mathematics book = 5
The number of ways to select a physics book = 6

Now,

(i) Number of ways in which a student can buy a mathematics book and a physics book = $5 \times 6 = 30$

(ii) Number of ways in which a student buy either a mathematics book or a physics book = $5 + 6 = 11$

Q11

Since there are 7 flags of different colours, therefore, first flag can be selected in 7 ways.

Now, the second flag can be selected from any one of the remaining flags in 6 ways.

Hence, by the fundamental principle of multiplication, the number of flag is $7 \times 6 = 42$

Q12

A boy can be selected from the first team in 6 ways, and from the second in 5 ways.

so, number of single matches between the boys of two teams = $6 \times 5 = 30$.

similarly, the number of single matches between the girls of two teams = $4 \times 3 = 12$.
so, total number of matches = $30 + 12 = 42$.

Q13

Clearly, the total number of ways to select first three prizes is equal to the 3 students from 12 students.

$\therefore$ number of ways to select the three prizes

$= 12 \times 11 \times 10$

$= 1320$

Q14

There are 3 ways to choose the first form and corresponding to each such way there are 5 ways of selecting the common difference.

So, required number of A.P.'s

$= 3 \times 5$

$= 15$

Q15

Clearly the number of ways to appoint one principal, one vice-principal and the teacher- incharge is equal to the number of ways to select the three teachers from the 36 teachers.

$\therefore$ Number of ways to appointed 3 teachers = $36 \times 35 \times 34 = 42840$

Hence, the number of ways to appoint one principal, one vice-principal and the teacher-incharge is equal to 42840.

Q16

We have to form all possible 3-digit numbers with distinct digits.

we cannot have 0 at the hundred's place. so, the hundred's place can be filled with any of the 9 digits 1,2,3,4....,9.

so, there are 9 ways of filling the hundred's place.

Now, 9 digits are left including 0, so, ten's place can be filled with any of the remaining 9 digits in 9 ways. now, the unit's place can be filled which in any of the remaining 8 digits. so, there are 8 ways of filling the unit's place.

Hence, the total number of required numbers = $9 \times 9 \times 8 = 648$

Q17

We cannot have a 0 at the hundred's place. So, the hundred's place can be filled with any of the 9 digits 1,2,3.....,9.

So, there are 9 ways of filling the hundred's place.

Ten's place can be filled with any 10 digits in 10 ways.

Now, the unit's place can be filled with any 10 digits in 10 ways.

Hence, the total number of required numbers = $9 \times 10 \times 10 = 900$

Q18

The three digit numbers are 100 to 999 inclusive so there are

$$999 - 100 + 1 = 999 - 99 = 900$$

So, 900 three digit numbers

If half of all numbers is odd then half of 900 is 450, there are 450 odd positive 3 digit numbers

Q19(i)

Zero cannot be first digit of the license plates.

This means the first digit can be selected from the 9 digits 1,2,3,4....,9
So, there are 9 ways of filling the first digit of the license plates.

Now, 9 digits are left including 0. So, second place can be filled with any of the remaining 9 digits in 9 ways.

The third place of the license plates can be filled with in any of the remaining 8 digits.
So, there are 8 ways of filling the third place.

The fourth place of the license plates can be filled with in any of the remaining 7 digits.
So, there are 7 ways at filling the fourth place.

The last place of the license plates can be filled with in any of the remaining 6 digits.
So, there are 6 ways of filling the fourth place.

Hence, the total number of ways = $9 \times 9 \times 8 \times 7 \times 6 = 27216$

Q19(ii)

Zero cannot be first digit of the license plates.

$\therefore$ first digit can be selected from the 9 digits 1,2,3.....,9
So, there are 9 ways at filling the first digit of the licence plates.

The repetition of digits is allowed to made a license plates number.

$\therefore$ the number of ways to fill the remaining places of the number plates = $10 \times 10 \times 10 \times 10$.

Hence, the total number of ways = $9 \times 10 \times 10 \times 10 \times 10 = 90{,}000$

Q20

The required numbers are greater than 7000.

$\therefore$ the thousand's place can be filled with any of the 3 digits 7,8,9.

so, there are 3 ways of filling the thousand's place.

Since repetition of digits is not allowed, so the hundred's, ten's and one's places can be filled in 4,3, and 2 ways respectively.

Hence, the required number of numbers = $3 \times 4 \times 3 \times 2 = 72$

Q21

Since the required numbers are greater than 8000.

$\therefore$ the thousand's place can be two digits 8 or 9
So, there are 2 ways of filling the thousand's place.

Since repetition of digits is not allowed, so the hundred's, ten's and one's places can be filled in 4,3 and 2 ways respectively.

Hence, the required number of number = $2 \times 4 \times 3 \times 2 = 48$

Q22

First person can be seated in a row in 6 ways.

Second person can be seated in a row in 5 ways.

Third person can be seated in a row in 4 ways.

Fourth person can be seated in a row in 3 ways.

Fifth person can be seated in a row in 2 ways.

And, sixth person can be seated in a row in 1 ways.

Hence, total number of ways in which six persons can be seated in a row
$= 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$.

Q23

In a nine-digit number 0 cannot appear in the first digit. So, the number of ways of filling up the first-digit = 9.

Now, 9 digits are left including 0. So, second digit can be filled with any of the remaining 9 digits in 9 ways.

Similarly, remaining digits can be filled in 8,7,6,5,4,3 and 2 ways.

Hence, the total number of required numbers

$$= 9 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2$$

$$= 9 \times (9!)$$

Q24

Any number less than 1000 may be any of a number from one-digit number, two-digit number and three-digit number.

One-digit odd number:

3 possible ways are there. These numbers are 3 or 5 or 7.

Two-digit odd number:

Tens place can be filled up by 3 ways (using any of the digit among 3, 5 and 7) and then the ones place can be filled in any of the remaining 2 digits.

So, there are $3 \times 2 = 6$ such 2-digit numbers.

Three-digit odd number:

Ignore the presence of zero at ones place for some instance.

Hundreds place can be filled up in 3 ways (using any of any of the digit among 3, 5 and 7), then tens place in 3 ways by using remaining 3 digits (after using a digit, there will be three digits) and then the ones place in 2 ways.

So, there are a total of $3 \times 3 \times 2 = 18$ numbers of 3-digit numbers which includes both odd and even numbers (ones place digit are zero). In order to get the odd numbers, it is required to ignore the even numbers i.e. numbers ending with zero.

To obtain the even 3-digit numbers, ones place can be filled up in 1 way (only 0 to be filled), hundreds place in 3 ways (using any of the digit among 3, 5, 7) and then tens place in 2 ways (using remaining 2 digits after filling up hundreds place).

So, there are a total of $1 \times 3 \times 2 = 6$ even 3-digit numbers using the digits 0, 3, 5 and 7 (repetition not allowed)

So, number of three-digit odd numbers using the digits 0, 3, 5 and 7 (repetition not allowed) = $18 - 6 = 12$.

Therefore, odd numbers less than 1000 can be formed by using the digits 0, 3, 5, 7 when repetition of digits is not allowed are $3 + 6 + 12 = 21$.

Q25

The odd digits are 1,3,5,7,9

$\therefore$ Total number of odd digits = 5

Clearly, the hundred's place can be filled with any of the 5 digits 1,3,5,7 or 9
So, there are 5 ways of filling the hundred's place.

Now, 4 digits are left. So, ten's place can be filled with any of the remaining 4 digits in 4 ways.

Now, the unit's place can be filled with in any of the remaining 3 digits. So, there are 3 ways of filling the unit's place.

Hence, the total number of required number = $5 \times 4 \times 3 = 60$

Q26

First digit of six-digit numbers can be selected in 6 ways.

Second digit of six-digit numbers can be selected in 5 ways

Third digit of six-digit numbers can be selected in 4 ways.

Fourth digit of six-digit numbers can be selected in 3 ways.

Fifth digit of six-digit numbers can be selected in 2 ways.

Last digit of six-digit numbers can be selected in 1 ways.

Hence, total number of numbers = $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

Q27

We cannot have 0 at the first digit of six-digit numbers.

So, the first digit of six-digit numbers can be selected in 5 ways.

Now, 5 digits are left including 0. So, second digit of six-digit numbers can be selected in 5 ways.

Third digit of six-digit numbers can be selected in 4 ways.

Fourth digit of six-digit numbers can be selected in 3 ways.

Fifth digit of six-digit numbers can be selected in 2 ways.

Last digit of six-digit numbers can be selected in 1 ways.

Hence, total number of numbers = $5 \times 5 \times 4 \times 3 \times 2 \times 1 = 600$

Q28

Since the required numbers are greater than 5000.

$\therefore$ the thousand's place can be filled with any of two digits 5 or 9.

So, there are 2 ways of filling the thousand's place.

Since repetition of digits is not allowed, so the hundred's ten's and one's places can be filled in 4,3 and 2 ways respectively.

Hence, the required number of numbers = $2 \times 4 \times 3 \times 2 = 48$

Q29

Each serial number of the product consists of six components. First two are letters and remaining four are numbers.
So all the serial numbers will look as shown below.

L	L	N	N	N	N

For the first position of serial number we can have one of the 6 letters. As repetition is not allowed first position of serial number we can have one of the 5 letters. For the third position of serial number we can have one of the 10 numbers. Similarly for the remaining position we can have 9, 8 and 7 possible ways.

L	L	N	N	N	N
$\Uparrow$	$\Uparrow$	$\Uparrow$	$\Uparrow$	$\Uparrow$	$\Uparrow$
6	5	10	9	8	7

So the required number of serial number is
$6 \times 5 \times 10 \times 9 \times 8 \times 7$.

Q30

Total number of digits = 10
The digits is not repeats in a sequence of three digits.

$\therefore$ required number of sequences = $10 \times 9 \times 8 = 720$

$\therefore$ total number of unsuccessful attempts = $720 - 1 = 719$

Q31

Total number of digits = 4.

$\therefore$ the largest possible number of trials to obtain the correct code = $4 \times 3 \times 2 \times 1$ $\quad [\because$ digits are not repeated$]$

$= 24$

Q32

Total number of jobs = 3

$\therefore$ the number of ways to assined these job is to three persons = $3 \times 2 \times 1$

$= 6$

Q33

The given digits are 1, 2, 3 and 4. These digits can be repeated while forming the numbers. So, number of required four digit natural numbers can be found as follows.

Consider four digit natural numbers whose digit at thousandths place is 1.

Here, hundredths place can be filled in 4 ways. (Using the digits 1 or 2 or 3 or 4)

Similarly, tens place can be filled in 4 ways. (Using the digits 1 or 2 or 3 or 4)

Ones place can be filled in 4 ways. (Using the digits 1 or 2 or 3 or 4)

Number of four digit natural numbers whose digit at thousandths place is 1 = $4 \times 4 \times 4 = 64$

Similarly, number of four digit natural numbers whose digit at thousandths place is 2 = $4 \times 4 \times 4 = 64$

Now, consider four digit natural numbers whose digit at thousandths place is 4:

Here, if the digit at hundredths place is 1, then tens place can be filled in 4 ways and ones place can also be filled in 4 ways.

If the digit at hundredths place is 2, then tens place can be filled in 4 ways and ones place can also be filled in 4 ways.

If the digit at hundredths place is 3 and the digit at tens place is 1, then ones place can be filled in 4 ways.

If the digit at hundredths place is 3 and the digit at tens place is 2, then ones place can be filled only in 1 way so that the number formed is not exceeding 4321.

Number of four digit natural numbers not exceeding 4321 and digit at thousandths place is 3 = $4 \times 4 + 4 \times 4 + 4 + 1 = 37$

Thus, required number of four digit natural numbers not exceeding 4321 is 64 + 64 + 64 + 37 = 229.

Q34

Total number of digits = 6

we cannot have 0 at the first digit of the required six-digit numbers.

The digits cannot repeat in the six digits number.

$\therefore$ total number of six digit number are = $5 \times 5 \times 4 \times 3 \times 2 \times 1 = 600$

Now, the six digit number can be divided by 10, if its last digit is 0

$\therefore$ Total numbers which are divisible by 10 = $5 \times 4 \times 3 \times 2 \times 1 \times 1 = 120$

Q35

Total numbers of faces in each die = 6

$\therefore$ The total number of possible outcomes of three six faced die

$= 6 \times 6 \times 6$

$= 216$

Q36

Since a toss of a coin can result in a head or a tail.

$\therefore$ Total number of possible outcomes in each tossed = 2

$\therefore$ Total number of possible outcomes in four tossed $= 2 \times 2 \times 2 = 2^3 = 8$

$\therefore$ Total number of possible outcomes in four tossed $= 2 \times 2 \times 2 \times 2 = 2^4 = 16$

$\therefore$ total number of possible outcomes in five tossed $= 2 \times 2 \times 2 \times 2 \times 2 = 2^5 = 32$

$\therefore$ total number of possible outcomes in n tossed $= 2 \times 2 \times 2 \ldots\ldots n$ times $= 2^n$

Q37

Total number of digits = 5

Since, the digits can be repeated in the same number.

$\therefore$ Total numbers of four digits numbers $= 5 \times 5 \times 5 \times 5 = 625$

Q38

Total number of digits = 5

We cannot have 0 at the hundred's place so, the hundred's place can be digits with any of the 4 digits 1, 3, 5 or 7, So, there are 4 ways of filling the hundred's place.

Since, the digit may be repeated in three digit numbers.

$\therefore$ Ten's place can be filled with any of the 5 digits in 5 ways
A
nd unit's place can be filled with any of the 5 digits in 5 ways

Hence, the total number of required numbers = $4 \times 5 \times 5 = 100$

Q39

Total number of digits = 6

Clearly, the natural numbers ten's than 1000 can be 3 digits, 2 digits and 1 digit numbers.

Now, 0 cannot be a first digit of the three digit numbers.
So, the hundred's place can be filled with any of the 5 digits 1,2,3....5. So, there are 5 ways of filling the hundred place.

The ten's place can be filled with in any of the 6 digits 0,1,2.....5. So, there are 6 ways of filling the ten's place.

The unit's place can be filled with in any of the 6 digits 0,1,2.....5. So, there are 6 ways of filling the ten's place.

$\therefore$ The total number of 3 digit numbers = $5 \times 6 \times 6 = 180$
Similarly, the total number of 2 digit numbers = $5 \times 6 = 30$

Now, 0 is not a natural number
$\therefore$ the total number of 1digit numbers = 5

$\therefore$ Total number of natural numbers tens than 1000
$= 180 + 30 + 5 = 215.$

Q40

Total number of digits = 10
each number starts with 67 and no digit appears more than once.

$\therefore$ total number of five digit telephone numbers
$= 1 \times 1 \times 8 \times 7 \times 6 = 336$

Q41

Total numbers of toys = 8
Total number of children = 5

$\therefore$ The total number ways in which 8 distinct toys can be distributed among 5 children.
$= 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^8$

Q42

Total numbers of letters = 5
Total number of letters boxes = 7

$\therefore$ The number ways in which one can post 5 letters in 7 letter boxes
$= 7 \times 7 \times 7 \times 7 \times 7 = 7^5$

Q43

Total numbers of dice = 3

$\therefore$ The number of possible outcomes
$= 6 \times 6 \times 6 = 216$

$\therefore$ Total number of possible outcomes in which 5 dose not appear on any dice
$= 5 \times 5 \times 5 = 125$

$\therefore$ Required number of possible
outcomes $= 216 - 125 = 91$

Q44

Total numbers of balls = 20
Total number of boxes = 5

One ball can be put in first box in 20 ways because we can put any one of the twenty balls in first box.

Now, remaining 19 balls are to be but into remaining 4 boxes.
This can be done in 4^{19} ways; because there are 4 choices for each ball

Hence, the required number of ways $= 20 \times 4^{19}$.

Q45

Total number of balls = 5
Total number of boxes = 3

$\therefore$ Total number of ways to distributed 5 different balls in three boxes
$= 3 \times 3 \times 3 \times 3 \times 3 = 243$

Q46

Total number of ball = n = 5
Number of boxes = r = 3

5 different balls can be distributed among three boxes in $^5P_3$ ways.

$$^5P_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!} = \frac{5 \times 4 \times 3 \times 2 \times 1}{2 \times 1} = 60.$$

In 60 ways 5 different balls can be distributed among three boxes.

Q47(i)

4 prizes be distributed among 5 students so that no student gets more than one prize can be done in

$$^5P_4 = \frac{5!}{(5-4)!} = \frac{5!}{(1)!} = 5! \text{ ways.}$$

Q47(ii)

The first prize can be given away in 5 ways as it may be given to anyone of the 5 students.
The second prize can also be given away in 5 ways, since if may be obtained by the student who has already received a prize. Similarly, third and fourth prize can be given away in 5 ways.

Hence, the number of ways in which all the prize can be given away $= 5 \times 5 \times 5 \times 5 = 625$

Q47(iii)

Since any of the 5 students may get all the prizes. So, the number of ways in which a student gets all the 4 prizes is 5.

So, the number of ways in which a student does not get all the prizes = 625 – 5 = 620

Q48

Each lamps has two possibilities either it can be switched on or off.

There are 10 lamps in the hall.

So the total numbers of possibilities are 2^{10}.

To illuminate the hall we require at least one lamp is to be switched on.

There is one possibility when all the lamps are switched off. If all the bulbs are switched off then hall will not be illuminated.

So the number of ways in which the hall can be illuminated is $2^{10}-1$.

Ex 16.3

Q1(i)

We have,

$$^{8}P_3 = \frac{8!}{(8-3)!} \quad \left[\because\ ^{n}P_r = \frac{n!}{(n-r)!}\right]$$

$$= \frac{8 \times 7 \times 6 \times 5!}{5!}$$

$$= 336$$

Hence, $^{8}P_3 = 336$

Q1(ii)

We have,

$$^{10}P_4 = \frac{10!}{(10-4)!} \qquad \left[\because\ ^{n}P_r = \frac{n!}{(n-r)!}\right]$$

$$= \frac{10!}{6!}$$

$$= \frac{10 \times 9 \times 8 \times 7 \times 6!}{6!}$$

$$= 5040$$

$\therefore\ ^{10}P_4 = 5040$

Q1(iii)

We have,

$$^{6}P_6 = \frac{6!}{(6-6)!} \qquad \left[\because\ ^{n}P_r = \frac{n!}{(n-r)!}\right]$$

$$= \frac{6!}{0!}$$

$$= \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{1} \quad [\because\ 0! = 1]$$

$$= 720$$

Hence, $^{6}P_6 = 720$

Q1(iv)

We have,

$$P(6,4) = \frac{6!}{(6-4)!} \qquad \left[\because \ {}^nP_r = \frac{n!}{(n-r)!}\right]$$

$$= \frac{6!}{2!}$$

$$= \frac{6\times5\times4\times3\times2\times1}{2!}$$

$$= 360$$

Hence, $P(6,4) = 360$

Q2

We have,

$P(5,r) = P(6,r-1)$

$$\Rightarrow \quad \frac{5!}{(5-r)!} = \frac{6!}{[6-(r-1)]!} \qquad \left[\because \ {}^nP_r = \frac{n!}{(n-r)!}\right]$$

$$\Rightarrow \quad \frac{1}{(5-r)!} = \frac{6}{[7-r]!}$$

$$\Rightarrow \quad \frac{1}{(5-r)!} = \frac{6}{(7-r)\times(7-r-1)(7-r-2)!}$$

$$\Rightarrow \quad \frac{1}{(5-r)!} = \frac{6}{(7-r)\times(6-r)(5-r)!}$$

$$\Rightarrow \quad 1 = \frac{6}{(7-r)\times(6-r)}$$

$$\Rightarrow \quad (6-r)\times(7-r) = 6$$

$$\Rightarrow \quad 42-6r-7r+r^2 = 6$$

$$\Rightarrow \quad r^2-12r+42-6 = 0$$

$$\Rightarrow \quad r^2-13r+36 = 0$$

$$\Rightarrow \quad r^2-9r-4r+36 = 0$$

$$\Rightarrow \quad r(r-9)-4(r-9) = 0$$

$$\Rightarrow \quad (r-9)(r-4) = 0$$

$$\Rightarrow \quad r = 4 \qquad \begin{bmatrix}\because \ r \le n \\ \therefore \ r-9 \ne 0\end{bmatrix}$$

Hence, $r = 4$

Q3

We have,

$$5P(4, n) = 6.\ P(5, n-1)$$

$$\Rightarrow \quad 5 \times \frac{4!}{(4-n)!} = 6 \times \frac{5!}{[5-(n-1)]!} \qquad \left[\because\ {}^nP_r = \frac{n!}{(n-r)!}\right]$$

$$\Rightarrow \quad 5 \times \frac{4!}{(4-n)!} = \frac{6 \times 5 \times 4!}{[5-n+1]!}$$

$$\Rightarrow \quad \frac{1}{(4-n)!} = \frac{6}{[6-n]!}$$

$$\Rightarrow \quad \frac{1}{(4-n)!} = \frac{6}{(6-n)(6-n-1)(6-n-2)!}$$

$$\Rightarrow \quad \frac{1}{(4-n)!} = \frac{6}{(6-n)(5-n)(4-n)!}$$

$$\Rightarrow \quad \frac{(6-n)(5-n)(4-n)!}{(4-n)!} = 6$$

$$\Rightarrow \quad (6-n)(5-n) = 6$$

$$\Rightarrow \quad 30 - 6n - 5n + n^2 = 6$$

$$\Rightarrow \quad n^2 - 11n + 30 = 6$$

$$\Rightarrow \quad n^2 - 11n + 24 = 0$$

$$\Rightarrow \quad n^2 - 8n - 3n + 24 = 0$$

$$\Rightarrow \quad n(n-8) - 3(n-8) = 0$$

$$\Rightarrow \quad (n-8)(n-3) = 0$$

$$\Rightarrow \quad n - 3 = 0 \qquad \left[\begin{array}{l} \because n \le 4 \\ \therefore n \ne 8 \end{array}\right]$$

$$\Rightarrow \quad n = 3$$

Hence, $n = 3$

Q4

We have,

$$P(n,5) = 20.\ P(n,3)$$

$$\Rightarrow \quad \frac{n!}{(n-5)!} = 20 \times \frac{n!}{(n-3)!}$$

$$\Rightarrow \quad \frac{1}{(n-5)!} = \frac{20}{(n-3)(n-3-1)(n-3-2)!}$$

$$\Rightarrow \quad \frac{1}{(n-5)!} = \frac{20}{(n-3)(n-4)(n-5)!}$$

$$\Rightarrow \quad \frac{(n-3)(n-4)(n-5)!}{(n-5)!} = 20$$

$$\Rightarrow \quad (n-3)(n-4) = 20$$
$$\Rightarrow \quad n^2 - 4n - 3n + 12 = 20$$
$$\Rightarrow \quad n^2 - 7n - 8 = 0$$
$$\Rightarrow \quad n^2 - 8n + 1n - 8 = 0$$
$$\Rightarrow \quad n(n-8) + 1(n-8) = 0$$
$$\Rightarrow \quad (n-8)(n+1) = 0$$
$$\Rightarrow \quad n - 8 = 0 \qquad [\because\ n \neq -1]$$
$$\Rightarrow \quad n = 8$$

Hence, $n = 8$

Q5

We have,

$${}^{n}P_4 = 360$$

$$\Rightarrow \quad \frac{n!}{(n-4)!} = 360$$

$$\Rightarrow \quad \frac{n(n-1)(n-2)(n-3)(n-4)! = 360}{(n-4)!}$$

$$\Rightarrow \quad n(n-1)(n-2)(n-3) = 6 \times 5 \times 4 \times 3$$
$$\Rightarrow \quad n = 6 \qquad [13y\ \text{comparing}]$$

Hence, $n = 6$

Q6

We have,

$$P(9, r) = 3024$$

$$\Rightarrow \quad \frac{9!}{(9-r)!} = 3024 \qquad \left[\because {}^nP_r = \frac{n!}{(n-r)!}\right]$$

$$\Rightarrow \quad \frac{1}{(9-r)!} = \frac{3024}{9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}$$

$$\Rightarrow \quad \frac{1}{(9-r)!} = \frac{336}{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}$$

$$\Rightarrow \quad \frac{1}{(9-r)!} = \frac{42}{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}$$

$$\Rightarrow \quad \frac{1}{(9-r)!} = \frac{1}{5 \times 4 \times 3 \times 2 \times 1}$$

$$\Rightarrow \quad \frac{1}{(9-r)!} = \frac{1}{5!}$$

$$\Rightarrow \quad (9-r)! = 5!$$

$$\Rightarrow \quad 9 - r = 5$$

$$\Rightarrow \quad 9 - 5 = r$$

$$\Rightarrow \quad 4 = r$$

$$\Rightarrow \quad r = 4$$

Hence, $r = 4$

Q7

We have,

$$P(11, r) = P(12, r-1)$$

$$\Rightarrow \quad \frac{11!}{(11-r)!} = \frac{12!}{[12-(r-1)]!} \qquad \left[\because {}^nP_r = \frac{n!}{(n-r)!}\right]$$

$$\Rightarrow \quad \frac{11!}{(11-r)!} = \frac{12 \times 11!}{[12-r+1]!}$$

$$\Rightarrow \quad \frac{1}{(11-r)!} = \frac{12}{[13-r]!}$$

$$\Rightarrow \quad \frac{1}{(11-r)!} = \frac{12}{(13-r)(13-r-1)(13-r-2)!}$$

$$\Rightarrow \quad \frac{1}{(11-r)!} = \frac{12}{(13-r)(12-r)(11-r)!}$$

$$\Rightarrow \quad \frac{(13-r)(12-r)(11-r)}{(11-r)!} = 12$$

$$\Rightarrow \quad (13-r)(12-r) = 12$$

$$\Rightarrow \quad 156 - 13r - 12r + r^2 = 12$$

$$\Rightarrow \quad r^2 - 25r + 156 - 12 = 0$$

$$\Rightarrow \quad r^2 - 25r + 144 = 0$$

$$\Rightarrow \quad r^2 - 16r - 9r + 144 = 0$$

$$\Rightarrow \quad r(r-16) - 9(r-16) = 0$$

$$\Rightarrow \quad (r-16)(r-9) = 0$$

$$\Rightarrow \quad r - 9 = 0 \qquad \begin{bmatrix} \because r \le 11 \\ \therefore r \ne 16 \end{bmatrix}$$

$$\Rightarrow \quad r = 9$$

Q8

We have,

$$P(n,4) = 12.\quad P(n,2)$$

$$\Rightarrow \quad \frac{n!}{(n-4)!} = 12 \times \frac{n!}{(n-2)!} \qquad \left[\because\ {}^nP_r = \frac{n!}{(n-r)!}\right]$$

$$\Rightarrow \quad \frac{1}{(n-4)!} = \frac{12}{(n-2)!}$$

$$\Rightarrow \quad \frac{1}{(n-4)!} = \frac{12}{(n-2)(n-2-1)(n-2-2)!}$$

$$\Rightarrow \quad \frac{1}{(n-4)!} = \frac{12}{(n-2)(n-3)(n-4)!}$$

$$\Rightarrow \quad \frac{(n-2)(n-3)(n-4)!}{(n-4)!} = 12$$

$$\Rightarrow \quad (n-2)(n-3) = 12$$

$$\Rightarrow \quad n^2 - 3n - 2n + 6 = 12$$

$$\Rightarrow \quad n^2 - 5n + 6 - 12 = 0$$

$$\Rightarrow \quad n^2 - 5n - 6 = 0$$

$$\Rightarrow \quad n^2 - 6n + 1n - 6 = 0$$

$$\Rightarrow \quad n(n-6) + 1(n-6) = 0$$

$$\Rightarrow \quad (n-6)(n+1) = 0$$

$$\Rightarrow \quad n - 6 = 0 \qquad [\because n \neq -1]$$

$$\Rightarrow \quad n = 6$$

Hence, $n = 6$

Q9

We have,

$$P(n-1,3): \quad P(n,4) = 1:9$$

$$\Rightarrow \quad \frac{P(n-1,3)}{P(n,4)} = \frac{1}{9}$$

$$\Rightarrow \quad \frac{\frac{(n-1)!}{(n-1-3)!}}{\frac{n!}{(n-4)!}} = \frac{1}{9}$$

$$\Rightarrow \quad \frac{(n-1)! \times (n-4)!}{(n-4)! \times n!} = \frac{1}{9}$$

$$\Rightarrow \quad \frac{(n-1)}{n!} = \frac{1}{9}$$

$$\Rightarrow \quad \frac{(n-1)!}{n \times (n-1)!} = \frac{1}{9}$$

$$\Rightarrow \quad \frac{1}{n} = \frac{1}{9}$$

$$\Rightarrow \quad n = 9$$

Hence, $n = 9$

Q10

We have,

$$P(2n-1,n) : \quad P(2n+1,n-1) = 22 : 7$$

$$\Rightarrow \quad \frac{P(2n-1,n)}{P(2n+1,n-1)} = \frac{22}{7}$$

$$\Rightarrow \quad \frac{\frac{(2n-1)!}{(2n-1-n)!}}{\frac{(2n+1)!}{[2n+1-(n-1)]!}} = \frac{22}{7}$$

$$\Rightarrow \quad \frac{(2n-1)! \times (n+2)!}{(n-1)!(2n+1)!} = \frac{22}{7}$$

$$\Rightarrow \quad \frac{(2n-1)! \times (n+2)(n+2-1)(n+2-2)(n+2-3)!}{(n-1)!(2n+1)(2n+1-1)(2n+1-2)!} = \frac{22}{7}$$

$$\Rightarrow \quad \frac{(2n-1)! \times (n+2)(n+1).n.(n-1)!}{(n-1)!(2n+1).2n.(2n-1)!} = \frac{22}{7}$$

$$\Rightarrow \quad \frac{n(n+2)(n+1)}{2n(2n+1)} = \frac{22}{7}$$

$$\Rightarrow \quad \frac{(n+2)(n+1)}{2(2n+1)} = \frac{22}{7}$$

$$\Rightarrow \quad \frac{n^2+n+2n+2}{4n+2} = \frac{22}{7}$$

$$\Rightarrow \quad 7(n^2+3n+2) = 22 \times (4n+2)$$

$$\Rightarrow \quad 7n^2+21n+14 = 88n+44$$

$$\Rightarrow \quad 7n^2+21n-88n+14-44 = 0$$

$$\Rightarrow \quad 7n^2-67n-30 = 0$$

$$\Rightarrow \quad 7n^2-70n+3n-30 = 0$$

$$\Rightarrow \quad 7n(n-10)+3(n-10) = 0$$

$$\Rightarrow \quad (n-10)(7n+3) = 0$$

$$\Rightarrow \quad n-10 = 0 \qquad [\because 7n+3 \neq 0]$$

$$\Rightarrow \quad n = 10$$

Q11

We have,

$$P(n,5) : P(n,3) = 2 : 1$$

$$\Rightarrow \quad \frac{P(n,5)}{P(n,3)} = \frac{2}{1}$$

$$\Rightarrow \quad \frac{\dfrac{n!}{(n-5)!}}{\dfrac{n!}{(n-3)!}} = \frac{2}{1}$$

$$\Rightarrow \quad \frac{n! \times (n-3)!}{(n-5)! \times n!} = 2$$

$$\Rightarrow \quad \frac{(n-3)!}{(n-5)!} = 2$$

$$\Rightarrow \quad \frac{(n-3)(n-4)(n-5)!}{(n-5)!} = 2$$

$$\Rightarrow \quad (n-3)(n-4) = 2$$

$$\Rightarrow \quad n^2 - 4n - 3n + 12 = 2$$

$$\Rightarrow \quad n^2 - 7n + 12 - 2 = 0$$

$$\Rightarrow \quad n^2 - 7n + 10 = 0$$

$$\Rightarrow \quad n^2 - 5n - 2n + 10 = 0$$

$$\Rightarrow \quad n(n-5) - 2(n-5) = 0$$

$$\Rightarrow \quad (n-5)(n-2) = 0$$

$$\Rightarrow \quad n = 5 \qquad \begin{bmatrix} \because n \geq 5 \\ \therefore n \neq 2 \end{bmatrix}$$

Hence, $n = 5$

Q12

We have,

$$\text{LHS} = 1.P(1,1) + 2.\ P(2,2) + 3.\ P(3,3)........ + n.\ P(n,n)$$

$$= 1.1 + 2.2! + 3.3!..........n.n! \qquad [\because\ P(n,n) = n!]$$

$$= \sum_{r=1}^{n} r.r!$$

$$= \sum_{r=1}^{n} [(r+1)r! - r!]$$

$$= \sum_{r=1}^{n} [(r+1)! - r!] \qquad [\because\ (r+1)r! = (r+1)!]$$

$$= [(2! - !) + (3! - 2!) + (4! - 3!)......... + (n+1)! - n!]$$

$$= (n+1)! - 1!$$

$$= {}^{n+1}P_{n+1} - 1! \qquad [\because\ {}^{n}P_{n} = n!]$$

$$= P(n+1, n+1) - 1$$

$\Rightarrow$ LHS = RHS

Hence proved.

Q13

We have,

$$P(15, r-1) = P(16, r-2) = 3 : 4$$

$$\Rightarrow \quad \frac{P(15, r-1)}{P(16, r-2)} = \frac{3}{4}$$

$$\Rightarrow \quad \frac{\dfrac{15!}{[15-(r-1)]!}}{\dfrac{16!}{[16-(r-2)]!}} = \frac{3}{4}$$

$$\Rightarrow \quad \frac{\dfrac{15!}{[16-r]!}}{\dfrac{16!}{[18-r]!}} = \frac{3}{4}$$

$$\Rightarrow \quad \frac{15!}{(16-r)!} \times \frac{(18-r)!}{16!} = \frac{3}{4}$$

$$\Rightarrow \quad \frac{15! \times (18-r)(17-r)(16-r)!}{(16-r)! \times 16 \times 15!} = \frac{3}{4}$$

$$\Rightarrow \quad \frac{(18-r)(17-r)}{16} = \frac{3}{4}$$

$$\Rightarrow \quad 306 - 18r - 17r + r^2 = \frac{3}{4} \times 16$$

$\Rightarrow \quad r^2 - 35r + 306 = 12$

$\Rightarrow \quad r^2 - 35r + 306 - 12 = 0$

$\Rightarrow \quad r^2 - 35r + 294 = 0$

$\Rightarrow \quad r^2 - 21r - 14r + 294 = 0$

$\Rightarrow \quad r(r-21) - 14(r-21) = 0$

$\Rightarrow \quad (r-21)(r-14) = 0$

$\Rightarrow \quad r - 14 = 0 \qquad [\because r = 21 \neq 0]$

$\Rightarrow \quad r = 14$

Hence, $r = 14$

Q14

We have,

$$^{n+5}P_{n+1} = \frac{11(n-1)}{2}\ ^{n+3}P_n$$

$$\Rightarrow \quad \frac{(n+5)!}{[n+5-(n+1)]!} = \frac{11(n-1)}{2} \times \frac{(n+3)!}{[n+3-n]!}$$

$$\Rightarrow \quad \frac{(n+5)!}{[n+5-n-1]!} = \frac{11(n-1)}{2} \times \frac{(n+3)!}{3!}$$

$$\Rightarrow \quad \frac{(n+5)!}{4!} = \frac{11(n-1)}{2} \times \frac{(n+3)!}{3!}$$

$$\Rightarrow \quad \frac{(n+5)(n+4)(n+3)!}{4!} = \frac{11(n-1)}{2} \times \frac{(n+3)!}{3!}$$

$$\Rightarrow \quad \frac{(n+5)(n+4)}{4 \times 3!} = \frac{11(n-1)}{2 \times 3!}$$

$$\Rightarrow \quad (n+5)(n+4) = \frac{11(n-1) \times 4}{2}$$

$$\Rightarrow \quad (n+5)(n+4) = 22(n-1)$$

$$\Rightarrow \quad n^2 + 4n + 5n + 20 = 22n - 22$$

$$\Rightarrow \quad n^2 + 9n - 22n + 20 + 22 = 0$$

$$\Rightarrow \quad n^2 - 13n + 42 = 0$$

$$\Rightarrow \quad n^2 - 6n - 7n + 42 = 0$$

$$\Rightarrow \quad n(n-6) - 7(n-6) = 0$$

$$\Rightarrow \quad n = 6 \quad \text{or,} \quad n = 7$$

Hence, $n = 6$ or,7

Q15

The total number of ways

$= \text{Number of arrangements of 5 things, taken all at a time} = {}^{5}P_{5}$

$= \dfrac{5!}{(5-5)!}$

$= \dfrac{5 \times 4 \times 3 \times 2 \times 1}{0!} \qquad [\because \; 0! = 1]$

$= 120$

Hence, the total number of ways in which children stand in a queue is 120.

Q16

The total number of teachers in a school = 36

One principal and one uice-principal are to be appointed.

$\therefore$ Total of ways

= Number of arrangement of 36 things taken two at a time

$= {}^{36}P_{2}$

$= \dfrac{36!}{(36-2)!}$

$= \dfrac{36!}{34!}$

$= \dfrac{36 \times 35 \times 34!}{34!}$

$= 36 \times 35$

$= 1260$

Hence, Total number of ways to oppoint one principal and one vice-principal are 1260.

Q17

Total number of letters = 4

$\therefore$ The total number of ordred

paris = Number of arrangements of 4 letters, taken two at a time

$= {}^4P_2$

$= \frac{4!}{(4-2)!}$

$= \frac{4!}{2!}$

$= \frac{4 \times 3 \times 2!}{2!}$

$= 12$

Hence, the total number of ordered paris = 12

Q18

Total number of books = 4

$\therefore$ Total number of ways

= Number of arrangments of 4 books, taken all at a time

$= {}^4P_4$

$= \frac{4!}{(4-4)!}$ $\left[\because {}^nP_r = \frac{n!}{(n-r)!}\right]$

$= \frac{4!}{0!}$

$= 4!$ $[\because 0! = 1]$

$= 4 \times 3 \times 2 \times 1$

$= 24$

Hence, the total number of ways to arrange the books in a shelf = 24

Q19

Total number of letters = 6

$\therefore$ Total number of words

= Number of arrangements of 6 letters, taken 4 at a time = $^{6}P_{4}$

$$= \frac{6!}{(6-4)!}$$

$$= \frac{6!}{2!}$$

$$= \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{2!}$$

$$= 360$$

Hence, the total number of 4 letter words are 360.

Q20

The odd number digits are 1,3,5,6,9.
Total number of odd digits = 5

$\therefore$ Required number of 3 digit numbers

= number of arrangenments of 5 digits by taking 3 at a time

$$= {}^{5}P_{3}$$

$$= \frac{5!}{(5-3)!}$$

$$= \frac{5!}{2!}$$

$$= \frac{5 \times 4 \times 3 \times 2!}{2!}$$

$$= 60$$

Hence, total number of 3 digit numbers are 60

Q21

Total number of letters = 5

$\therefore$ Total number of words

= Number of arrangement of 5 letters, taken 5 at a time

$= {}^{5}P_{5}$

$= \frac{5!}{(5-5)!}$

$= \frac{5!}{0!}$

$= 5!$ $\quad [\because 0! = 1]$

$= 5 \times 4 \times 3 \times 2 \times 1$

$= 120$

Hence, the number of words are 120

Q22

Total number of letters = 8

$\therefore$ Total number of words

= Number of arrangements of 8 letters, taken 8 at a time

$= {}^{8}P_{8}$

$= \frac{8!}{(8-8)!}$

$= \frac{8!}{0!}$

$= 8!$ $\quad [\because 0! = 1]$

Hence, total number of words are 8!

Q23

Let, w_1, w_2, w_3 and w_4 be 4 words, where w_1, w_2 have 3 volumes each and w_3, w_4 have 2 volume each.

These 4 works can be arranged in 4! ways.

Now,

volumes of w_1 can be arranged in 3! ways.

volumes of w_2 can be arranged in 3! ways.

volumes of w_3 can be arranged in 2! ways.

And volumes of w_4 can be arranged in 2! ways

$\therefore$ Total number of ways to arrange

$$\begin{aligned} \text{all books} &= 4!\left(3!\times 3!\times 2!\times 2!\right) \\ &= 24\times 6\times 6\times 2\times 2 \\ &= 3456. \end{aligned}$$

Q24

There are 6 items in column A and 6 items in column B.

Now,

Each answer to the given question is an arrangement of the 6 items of column B keeping the order of items in column A fixed.

Hence, the total number of answers

= Number of arrangements of 6 items in column B

$$\begin{aligned} &= {}^{6}P_{6} \\ &= \frac{6!}{\left(6-6\right)!} \\ &= \frac{6!}{0!} \\ &= 6\times 5\times 4\times 3\times 2\times 1 \qquad \left[\because\ 0! = 1\right] \\ &= 720 \end{aligned}$$

Q25

Total number of digits = 10

Total number of 3 digit numbers = ${}^{10}P_3$

But these arrangements also include those numbers which have 0 at hundred's place.

such numbers are not 3-digit numbers.

When 0 is fixed at hundred's place, we have to arrange remaining 9 digits by taking 2 at a time.

The number of such arrangements is ${}^{9}P_3$.

So, the total of numbers having 0 at hundred's place = ${}^{9}P_2$

Hence, total number of 3 digit numbers which distinct = ${}^{10}P_3 - {}^{9}P_2$

$$= \frac{10!}{(10-3)!} - \frac{9!}{(9-2)!}$$

$$= \frac{10!}{7!} - \frac{9!}{7!}$$

$$= \frac{10 \times 9 \times 8 \times 7!}{7!} - \frac{9 \times 8 \times 7!}{7!}$$

$$= 720 - 72$$

$$= 648.$$

Q26

Total number of digits = 10

The first two digits of telephone is 35 and no digit appears more than once.

$\therefore$ Total number of remaining digits = 10 – 2 = 8

And, Total number of remaining digits of telephone number = 6 – 2 = 4.

$\therefore$ Required number of telephone numbers $= {}^{8}P_{4}$

$$= \frac{8!}{(8-4)!}$$

$$= \frac{8!}{4!}$$

$$= \frac{8 \times 7 \times 6 \times 5 \times 4!}{4!}$$

$$= 1680$$

Q27

Total number of boys = 6

Total number of girls = 5

Now,

Five girls can sit on chairs in a row in ${}^{5}P_{5} = 5!$ ways.

and 6 boys can stand behind them in a row in ${}^{6}P_{6} = 6!$ ways.

Hence, the total number of ways

$= 5! \times 6!$

$= 5 \times 4 \times 3 \times 2 \times 1 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 120 \times 720$

$= 86400$

Q28

'a' denotes the number of permutations of $(x+2)$ things taken all at a time.

$\therefore\ a = {}^{x+2}P_{x+2}$

'b' is the number of permutations of x things taken 11 at a time.

$\therefore\ b = {}^{x}P_{11}$

and, C is the number of permutations of $x-11$ things taken all at a time.

$\therefore\ C = {}^{x-11}P_{x-11}$

Now,

$a = 182bc$ [given]

$\Rightarrow\ {}^{x+2}P_{x+2} = 182 \times {}^{x}P_{11} \times {}^{x-11}P_{x-11}$

$\Rightarrow\ (x+2)! = 182 \times \dfrac{x!}{(x-11)!} \times (x-11)!$ $\left[\because\ {}^{n}P_{n} = n! \text{ and } {}^{n}P_{r} = \dfrac{n!}{(n-r)!}\right]$

$\Rightarrow\ (x+2)! = 182 \times x!$

$\Rightarrow\ (x+2)(x+1)x! = 182 \times x!$

$\Rightarrow\ (x+2)(x+1) = 182$

$\Rightarrow\ x^2 + x + 2x + 2 = 182$

$\Rightarrow\ x^2 + 3x + 2 - 182 = 0$

$\Rightarrow\ x^2 + 3x - 180 = 0$

$\Rightarrow\ x^2 + 15x - 12x - 180 = 0$

$\Rightarrow\ x(x+15) - 12(x+15) = 0$

$\Rightarrow\ (x-12)(x+15) = 0$

$\Rightarrow\ x - 12 = 0$ $[\because\ x \neq -15]$

$\Rightarrow\ x = 12$

Hence, $x = 12$

Q29

There are 9 ways to pick the 1st digit.

For each of those 9 ways there are 8 ways to choose the second digit.
That's 9×8 or 72 ways to pick the first two digits.

For each of those 72 ways there are 7 ways to choose the third digit.
That's 72×7 ways or 504 ways to pick all three digits.

Q30

The even number so last digit must be even .We can so number patterns are

1)odd, odd, even
2)odd, even, even
3)even, odd, even
4)even, even, even

For the pattern 1 - number of ways of choosing 1st digit is 3
2nd digit (already one is gone) is 2
3rd is 3

Therefore, the no of ways is 3x2x3 .
Similarly for pattern 2, the no. of ways is 3x3x2
for pattern 3, the no. of ways is 3x3x2
for pattern 4, the no. of ways is 3x2x1

Total no of ways is 3x2x3 + 3x3x2 + 3x3x2 + 3x2x1
18x3 + 6 = 60

Q31

We can take the digits one at a time, starting at either end.
Let's start from the right.
d c b a = the digits to be chosen.
For a we have 5 choices (1,2,3,4,5)
For b we only have 4 (having used one for a, and repeats not allowed)
For c we have 3
For d we have 2.
5 * 4 * 3 * 2 = 120 choices overall

If we want the number to be even,
we don't have 5 choices for a, we are limited to the set {2, 4}
there are only two digits available.

But for the remaining digits the calculation is the same.
2/5 of the numbers are even $= \frac{2}{5} \times 120 = 48 = 2 \times 4 \times 3 \times 2$

Q32

There are 6 letters in the word 'EAMCOT'. Out of these letters 'E','A' and 'O' are the three vowels.
The remaining three consonants can be arranged in $^3P_3$ ways. In each of these arrangements 4 places are created, shown by the cross marks.

$\times$ V $\times$ V $\times$ V $\times$

Since no two vowels are to be placed adjacent to each other, so we may arrange 3 vowels in 4 places in $^4P_3$ ways.

The total number of arrangements
$= {}^3P_3 \times {}^4P_3$
$= 3! \times 4!$
$= 144$

Ex 16.4

Q1

There are 4 vowels and 3 consonants in the word 'FAILURE'
We have to arrange 7 letters in a row such that consonants occupy odd places. There are 4 odd places $(1,3,5,7)$. There consonants can be arranged in these 4 odd places in ${}^4P_3$ ways.

Remaining 3 even places $(2,4,6)$ are to be occupied by the 4 vowels. This can be done in ${}^4P_3$ ways.

Hence, the total number of words in which consonants occupy odd places = ${}^4P_3 \times {}^4P_3$

$$= \frac{4!}{(4-3)!} \times \frac{4!}{(4-3)!}$$
$$= 4 \times \times 3 \times 2 \times 1 \times 4 \times 3 \times 2 \times 1$$
$$= 24 \times 24$$
$$= 576.$$

Q2

There are 7 letters in the word ' STRANGE', including 2 vowels (A,E) and 5 consonants (S,T,R,N,G).
(i) Considering 2 vowels as one letter, we have 6 letters which can be arranged in ${}^6p_6 = 6!$ ways
A,E can be put together in $2!$ ways.

Hence, required number of words
$$= 6! \times 2!$$
$$= 6 \times 5 \times 4 \times 3 \times 2 \times 1 \times 2$$
$$= 720 \times 2$$
$$= 1440.$$

(ii) The total number of words formed by using all the letters of the words 'STRANGE'
is ${}^7p_7 = 7!$
$$= 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$
$$= 5040.$$
So, the total number of words in which vowels are never together
= Total number of words − number of words in which vowels are always together
$$= 5040 - 1440$$
$$= 3600$$

(iii) There are 7 letters in the word 'STRANGE'. out of these letters 'A' and 'E' are the vowels.
There are 4 odd places in the word 'STRANGE'. The two vowels can be arranged in ${}^4p_2$ ways.
The remaining 5 consonants can be arranged among themselves in ${}^5p_5$ ways.
The total number of arrangements
$$= {}^4p_2 \times {}^5p_5$$
$$= \frac{4!}{2!} \times 5!$$
$$= 1440$$

Q3

There are 6 letters in the word 'SUNDAY'. The total number of words formed with these 6 letters is the number of arrangements of 6 items, taken all at a time, which is equal to $^6P_6 = 6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$.

If we fix up D in the beginning, then the remaining 5 letters can be arranged in $^5P_5 = 5!$ ways.

so, the total number of words which begin with D = 5!
$= 5 \times 4 \times 3 \times 2 \times 1 = 120$.

Q4

There are 4 vowels and 4 consonants in the word 'ORIENTAL'. We have to arrange 8 leeters in a row such that vowels occupy odd places. There are 4 odd places $(1,3,5,7)$. Four vowels can be arranged in these 4 odd places in 4! ways. Remaining 4 even places $(2,4,6,8)$ are to be occupied by the 4 consonants.

This can be done in 4! ways.

Hence, the total number of words in which vowels occupy odd places = $4! \times 4!$
$= 4 \times 3 \times 2 \times 1 \times 4 \times 3 \times 2 \times 1$
$= 576$.

Q5

There are 6 letters in the word 'SUNDAY'. The total number of words formed with these 6 letters is the number of arrangements of 6 items, taken all at a time, which is equal to $^6P_6 = 6!$
$= 6 \times 5 \times 4 \times 3 \times 2 \times 1$
$= 720$.

If we fix up N in the begining, then the remaining 5 letters can be arranged in $^5P_5 = 5!$ ways
so, the total number of words which begin which $N = 5!$
$= 5 \times 4 \times 3 \times 2 \times 1$
$= 120$

if we fix up N in the begining and Y at the end, then the remaining 4 letters can be arranged in $^4P_4 = 4!$ ways.

So, the total number of words which begin with N and end with $Y = 4! = 4 \times 3 \times 2 \times 1 = 24$.

Q6

There are 10 letters in the word 'GANESHPURI'. The total number of words formed is equal to ${}^{10}P_{10} = 10!$

(i) If we fix up G in the begining, then the remaining 9 letters can be arranged in ${}^{9}P_{9} = 9!$ ways

(ii) If we fix up P in the begining and I at the end, begining 8 letters can be arranged in ${}^{8}P_{8} = 8!$.

(iii) There are 4 vowels and 6 consonants in the word 'GANESHPURI'.
Considening 4 vowels as one letter,
We have 7 letters which can be arranged in ${}^{7}P_{7} = 7!$ ways.
A,E,U,I can be put together in 4! ways.
Hence, required number of words = $7! \times 4!$.

(iv) We have to arrange 10 letters in a row such that vowels occupy even places. There are 5 even places $(2,4,6,8,10)$. 4 vowels can be arranged in these 5 even places in ${}^{5}P_{4}$ ways.
Remaining 5 odd places $(1,3,5,7,9)$ are to be occupied by the 6 consonants.
This can be done in ${}^{6}C_{5}$ ways.

Hence, the total number of words in which vowels occupy even places = ${}^{5}P_{4} \times {}^{6}P_{5}$

$$= \frac{5!}{(5-4)!} \times \frac{6!}{(6-1)!}$$

$$= 5! \times 6!$$

Q7

(i) There are 6 letters in the word 'VOWELS'. The total number of words formed with these 6 letters is the number of arrangements of 6 items, taken all at a time, which is equal to

$${}^{6}P_{6} = 6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$$

(ii) If we fix up E in the begining then the remaining 5 letters can be arranged in ${}^{5}P_{5} = 5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$ ways

(iii) If we fix up 0 in the begining and L at the end, the remaining 4 letters can be arranged in ${}^{4}P_{4}$

$$= 4! = 4 \times 3 \times 2 \times 1 = 24.$$

(iv) There are 2 vowels and 4 consonants in the word 'VOWELS'.
Considering 2 vowels as one letter, we have letters which can be arranged in

$${}^{5}P_{5} = 5! \text{ ways.}$$

O, E can be put together in 2! ways.

Hence, required number of

$$\begin{aligned} \text{words} &= 5! \times 2! \\ &= 5 \times 4 \times 3 \times 2 \times 1 \times 2 \times 1 \\ &= 120 \times 2 \\ &= 240 \end{aligned}$$

(v) There are 2 vowels and 4 consonants in the word 'VOWELS'.
Considering 4 consonants as one letter, we have 3 letters which can be arranged in ${}^{3}P_{3} = 3!$ ways.
U, W, L, S can be put together in 4! ways.

Hence, required number of words in which all consonants come together = $3! \times 4!$

$$= 3 \times 2 \times 4 \times 3 \times 2$$

$$= 144.$$

Q8

We have to arrange 7 letters in a row such that vowels occupy even places.
There are 3 even places $(2,4,6)$. Three vowels can be arranged in these 3 even places in 3! ways.

Remaining 4 odd places $(1,3,5,7)$ are to be occupied by the 4 consonants. This can be done in 4! ways.

Hence, the total number of words in which vowels occupy even places $= 3! \times 4!$

$$= 3 \times 2 \times 4 \times 3 \times 2 = 144$$

Q9

Let two husbands A, B be selected out of seven males in $= {}^7C_2$ ways. excluding their wives, we have to select two ladies C, D out of remaining 5 wives is $= {}^5C_2$ ways. Thus, number of ways of selecting the players for mixed double is $= {}^7C_2 \times {}^5C_2$

$$= 21 \times 10$$
$$= 210$$

Now, suppose A chooses C as partner $(B$ will automatically go to $D)$ or
A chooses D as partner $(B$ will automatically go to $C)$
Thus we have, 4 other ways for teams.

Required number of ways $= 210 \times 4 = 840$

Q10

m men can be seated in a row in ${}^mP_m = m!$ ways.

Now, in the $(m+1)$ gaps n women can be arranged in ${}^{m+1}P_n$ ways.

Hence, the number of ways in which no two women sit together

$$= m! \times {}^{m+1}P_n$$
$$= m! \times \frac{(m+1)!}{(m+1-n)!}$$
$$= m! \times \frac{(m+1)!}{(m-n+1)!}$$

Hence, proved

Q11

(i) M O N D A Y has 6 letters with no repetitions, so

Number of words using 4 letters at a time with no repetitions = ${}^6P_4$

$= \frac{6!}{2!}$

$= 360$

(ii) Number of words using all 6 letters at a time with no repetitions = ${}^6P_6$

$= \frac{6!}{(6-6)!}$

$= 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 720$

(iii) Number of words using all 6 letters, starting with vowels

$= 2.{}^5P_5$

$= 2 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 240$

Q12

There are 8 letters in the word 'ORIENTAL'. The total number of three letter words is the number of arrangements of 8 items, taken 3 at a time, which is equal to

${}^8P_3 = \frac{8!}{(8-3)!}$

$= \frac{8!}{5!}$

$= \frac{8 \times 7 \times 6 \times 5!}{8!}$

$= 336.$

Hence, the total number three letter words are 336.

Ex 16.5

Q1(i)

There are 12 letters in the word 'INDEPENDENCE' out of which 2 are D'S, 3 are N'S, 4 are E'S and the rest are all distinct.

so, the total number of words $= \dfrac{12!}{2!\,3!\,4!}$

$$= \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4!}{2!\,3!\,4!}$$

$$= \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5}{2 \times 3 \times 2}$$

$$= 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5$$

$$= 1663200.$$

Q1(ii)

There are 12 letters in the word 'INTERMEDIATE' out of which 2 are I'S, 2 are T'S, 3 are E'S and the rest are all distinct.

so, the total number of words

$$= \frac{12!}{2!\,2!\,3!}$$

$$= \frac{12 \times 11 \times 10 \times 9 \times 8 \times 6 \times 5 \times 4 \times 3 \times 2!}{2!\,2!\,3!}$$

$$= \frac{12 \times 11 \times 10 \times 9 \times 8 \times 6 \times 5 \times 4 \times 3}{2 \times 3 \times 2}$$

$$= 11 \times 10 \times 9 \times 8 \times 6 \times 5 \times 4 \times 3$$

$$= 19958400$$

Q1(iii)

There are 7 letters in the word 'ARRANGE' out of which 2 are A'S, 2 are R'S, and the rest are all distinct.

So, the total number of words

$$= \frac{7!}{2!\,2!}$$

$$= \frac{7 \times 6 \times 5 \times 4 \times 3 \times 2!}{2!\,2!}$$

$$= \frac{7 \times 6 \times 5 \times 4 \times 3}{2 \times 1}$$

$$= 7 \times 6 \times 5 \times 2 \times 3$$

$$= 1260$$

Q1(iv)

There are 5 letters in the word 'INDIA' out of which 2 are I'S, and the rest are all distinct.

so, the total number of

$$\text{words} = \frac{5!}{2!}$$

$$= \frac{5 \times 4 \times 3 \times 2!}{2!}$$

$$= 60$$

Q1(v)

There are 8 letters in the word 'PAKISTAN' out of which 2 are A'S, and the rest are all distinct.

So, the total number of words

$$= \frac{8!}{2!}$$

$$= \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2!}{2!}$$

$$= 8 \times 7 \times 6 \times 5 \times 4 \times 3$$

$$= 20160$$

Q1(vi)

There are 6 letters in the word 'RUSSIA' out of which 2 are S's, and the rest are all distinct.

So, the total number of words

$$= \frac{6!}{2!}$$

$$= \frac{6 \times 5 \times 4 \times 3 \times 2!}{2!}$$

$$= 6 \times 5 \times 4 \times 3$$

$$= 360$$

Q1(vii)

There are 6 letters in the word 'SERIES' out of which 2 are S's, 2 are E's and the rest are all distinct.

so, the total number of words

$$= \frac{6!}{2!\ 2!}$$

$$= \frac{6 \times 5 \times 4 \times 3 \times 2!}{2!\ 2!}$$

$$= \frac{6 \times 5 \times 4 \times 3}{2 \times 1}$$

$$= 6 \times 5 \times 2 \times 3$$

$$= 180$$

Q1(viii)

There are 9 letters in the word 'EXERCISES' out of which 3 are E's, 2 are S's and the rest are all distinct.

So, the total number of words

$$= \frac{9!}{3!\ 2!}$$

$$= \frac{9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3!}{3! \times 2 \times 1}$$

$$= 9 \times 8 \times 7 \times 6 \times 5 \times 2$$

$$= 30240$$

Q1(ix)

There are 14 letters in the word 'CONSTANTINOPLE' out of which 2 are O's, 3 are N's, 2 are T's and the rest are all distinct.

So, the total number of

$$\text{words} = \frac{14!}{2!\ 3!\ 2!}$$

$$= \frac{14!}{2 \times 3 \times 2 \times 2}$$

$$= \frac{14!}{24}$$

Q2

There are 4 consonants in the word 'ALGEBRA'.
The number of ways to arrange these consonants = 4!

There are 3 vowels in the given word of which 2 are A's

The vowels can be arranged among themselves in $\frac{3!}{2!}$ ways.

Hence, the required number of arrangements $= 4! \times \frac{3!}{2}$

$$= 4 \times 3 \times 2 \times \frac{3 \times 2}{2}$$

$$= 72$$

Q3

In the word 'UNIVERSITY' there are 10 letters of which 2 are I's.
There are 4 vowels in the given word of which 2 are I's.

These vowels can be put together in $\frac{4!}{2!}$ ways.

Considering these 4 vowels as one letter there are 7 letters which can be arranged in 7! ways.

Hence, by fundamental principle of multiplication, the required number of arrangements is

$$= \frac{4!}{2!} \times 7!$$

$$= \frac{4 \times 3 \times 2!}{2!} \times 7 \times 6 \times 5 \times 4 \times 3 \times 2$$

$$= 4 \times 3 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2$$

$$= 60480.$$

Q4

There are $3a's$, $2b's$ and $4c's$.
So, the number of arrangements

$$= \frac{9!}{4!3!2!}$$

$$= \frac{9 \times 8 \times 7 \times 6 \times 5 \times 4!}{4! \times 3 \times 2 \times 2}$$

$$= 9 \times 4 \times 7 \times 5$$

$$= 1260.$$

Hence, the total number of arrangements are 1260.

Q5

There are 8 letters in the word 'PARALLEL' out of which A's and 3 are L's and the rest are all distinct.

So, total number of words = $\frac{8!}{2!\ 3!}$

$= \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3!}{2 \times 1 \times 3!}$

$= 8 \times 7 \times 6 \times 5 \times 2$

$= 3360$

Considering all L's together and treating them as one letter we have 6 letters out of which A repeats 2 times and others are distinct. These 6 letters can be arranged in $\frac{6!}{2!}$ ways.

So, the number of words in which all L's come together

$= \frac{6!}{2!}$

$= \frac{6 \times 5 \times 4 \times 3 \times 2!}{2!}$

$= 6 \times 5 \times 4 \times 3$

$= 360$

Hence, the number of words in which all L's do not come together

$= 3360 - 360$

$= 3000.$

Q6

There are 6 letters in the word 'MUMBAI' out of which 2 are M's and the rest are all distinct.

Considering both M's together and treating as one letter we have 5 letters. These 5 letters can be arranged in 5! ways.

Hence, the total number of arrangement = 5!

$= 5 \times 4 \times 3 \times 2 \times 1$

$= 120$

Q7

Total number of digits are = 7

There are 4 odd digits 1,1,3,3 and 4 odd places $(1,3,5,7)$

So, odd digits can be arranged in odd places in $\frac{4!}{2!\ 2!}$ ways

The remaining 3 even digits 2,2,4 can be arranged in 3 even places in $\frac{3!}{2!}$ ways.

Hence, the total number of Numbers = $\frac{4!}{2!\,2!} \times \frac{3!}{2!} = \frac{4 \times 3 \times 2!}{2!\ \ 2!} \times \frac{3 \times 2!}{2!}$ = 18

Q8

Total number of red flags = 4
Total number of white flags = 2
Total number of green flags = 3

We have to arrange 9 flags, out of which 4 are of red, 2 are white and 3 are green

So, total number of signals = $\frac{9!}{4!\,2!\,3!}$

$$= \frac{9 \times 8 \times 7 \times 6 \times 5 \times 4!}{4! \times 2 \times 3 \times 2} = 9 \times 4 \times 7 \times 5 = 1260$$

Hence, total number of signals = 1260.

Q9

Total number of digits = 4

Total number of 4 digit numbers = $\frac{4!}{2!}$

But, zero cannot be first digit of the four digit numbers.

$\therefore$ Total number of 3 digit numbers = $\frac{3!}{2!}$

$\therefore$ Total number of numbers = $\frac{4!}{2!} - \frac{3!}{2!} = \frac{4 \times 3 \times 2!}{2!} - \frac{(3 \times 2!)}{2!}$

$$= 12 - 3 \quad = 9$$

Hence, total number of four digit numbers = 9

Q10

There are 7 letters in the word 'ARRANGE' out of which 2 are A's 2 are R's and the rest are all distinct.

So, total number of words = $\frac{7!}{2!\,2!}$

$$= \frac{7 \times 6 \times 5 \times 4 \times 3 \times 2!}{2 \times 2!}$$

$$= 7 \times 6 \times 5 \times 2 \times 3$$
$$= 1260.$$

Considering all R's together and treating them as one letter we have 6 letters out of which A repeats 2 times and other are distinct. These 6 letters can be arranged in $\frac{6!}{2!}$ ways.

So, the number of words in which all R's come together = $\frac{6!}{2!}$

$$= \frac{6 \times 5 \times 4 \times 3 \times 2!}{2!}$$
$$= 360.$$

Hence, the number of words in which all R's do not came together

= Total number of words – Number of words in which all R's come together

= 1260 – 360

= 900.

Q11

Total number of digits = 5

Now, numbers greater then 50000 will have either 5 or 9 in the first place and will consist of 5 digits.

Number of numbers of which digit 5 at first place = $\frac{4!}{2!}$ $\left[\because 1 \text{ is repeated}\right]$

$$= \frac{4 \times 3 \times 2!}{2!}$$

$$= 12.$$

Number of numbers with digit 9 at first place = $\frac{4!}{2!} = 12$

Hence, the required number of numbers = 12 + 12 = 24.

Q12

In the word 'SERIES' there are 6 letters of which 2 are S and 2 are E's.

Let us fix 5 at the extreme left and at the extreme right end. Now, we are left with 4 letters of which 2 are E's. These four letters can be arranged in $\frac{4!}{2!}$ ways.

Hence, required number of arrangements = $\frac{4!}{2!} = \frac{4 \times 3 \times 2!}{2!} = 12$.

Q13

MADHUBANI

Total number of words that ends with letter I = $\frac{8!}{2!}$

$= 8 \times 7 \times 5 \times 6 \times 4 \times 3$

$= 56 \times 30 \times 12$

$= 20160$

If the words starts with M and end with I , there are 7 space left for 7 letters.

Number of words that starts with M and end with I = $\frac{7!}{2!}$

$= 7 \times 5 \times 4 \times 3$

$= 42 \times 60$

$= 2520$

Number of words which do not start with M but end with I

$= 20160 - 2520$

Required number of words = 17640

Q14

Total numer of digits = 7

Since, 0 cannot be first digit of the 7 digit numbers.

$\therefore$ Number of 6 – digit

Numbers = $\frac{6!}{2!3!}$ $\left[\begin{array}{l} \because \text{2comes} \\ \text{2 times and 3 comes 3 times} \end{array}\right]$

$= \frac{6 \times 5 \times 4 \times 3!}{2 \times 3!}$

$= 6 \times 5 \times 2$

$= 60.$

Now, number of 7-digit numbers = $\frac{7!}{2!3!} = \frac{7 \times 6 \times 5 \times 4 \times 3!}{2 \times 3!}$

$= 7 \times 6 \times 5 \times 2$

$= 420$

Hence, total number of numbers which is greater then 1 million = 420 – 60

= 360.

Q15

There are three copies each of 4 different books.

$\therefore$ Total number of copies = 12

$\therefore$ The number of ways in which these copies arranged in a shelf

$$= \frac{12!}{3!\,3!\,3!\,3!}$$

$$= \frac{12!}{(3!)^4}$$

Hence, required number of ways

$$= \frac{12!}{(3!)^4}$$

Q16

There are 11 letters in the word 'MATHEMATICS' out of which 2 are M's, 2 are A's, 2 are T's and the rest are all distinct.

so, the requisite number of words = $\frac{11!}{2!\,2!\,2!}$

If we fix C in the beginning, then the remaining 10 letters can be arranged in $\frac{10!}{2!\,2!\,2!}$

If we fix T in the beginning, then the remainning 10 letters can be arranged in $\frac{10!}{2!\,2!}$

Q17

Total number of molecules = 12

Now,

the chain contains 4 different molecules A, c, g, and T, and 3 molecules of each kind.

$\therefore$ the number of different arrangements = $\frac{12!}{3!\,3!\,3!\,3!}$

$$= \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3!}{3 \times 2 \times 3 \times 2 \times 3 \times 2 \times 3!}$$

$$= 369600.$$

Hence, the number of different possible arrangements are = 369600.

Q18

4 red, 3 yellow and 2 green discs.

Total discs = 9

Required number of ways

$$= \frac{9!}{4!3!2!}$$

$$= \frac{9 \times 8 \times 7 \times 6 \times 5}{3 \times 2 \times 2}$$

$$= 1260$$

Required number of ways = 1260

Q19

Total number of digits = 7

Now,

$$\text{number of 7-digit numbers} = \frac{7!}{3!\,2!}$$

$$= \frac{7 \times 6 \times 5 \times 4 \times 3!}{3! \times 2}$$

$$= 7 \times 6 \times 5 \times 2$$

$$= 420$$

And,0 cannot be first digit of the 7-digit numbers

$\therefore$ Number of 6-digit numbers

$$= \frac{6!}{3!\,2!}$$

$$= \frac{6 \times 5 \times 4 \times 3!}{3! \times 2}$$

$$= 6 \times 5 \times 2$$

$$= 60$$

Hence, total number of 7-digit number which are greater than 1000000 = 420 – 60 = 360

Q20

There are 13 letters in the word 'ASSASSINATION' out of which 3 are A's, 4 are S's, 2 are I's, 2 are N's and the rest are all distinct.

Considering all S's together and treating them as one letter we have 10 letters.

These 10 letters can be arranged in $\frac{10!}{3!\,2!\,2!}$

$$= \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3!}{3! \times 2 \times 2}$$

$$= 10 \times 9 \times 8 \times 7 \times 6 \times 5$$
$$= 151200.$$

Hence, the total words are 151200

Q21

There are 9 letters in the word 'INSTITUTE' out of which 2 are I's, 3 are T's and the rest are all distinct.

$\therefore$ The total number of permutations of the letters of the word 'INSTITUTE' = $\frac{9!}{2!\,3!}$

Hence, the total number of words are $\frac{9!}{2!\,3!}$

Q22

In a dictionary the words at each stage are arranged in alphabetical order.

Starting with letter I, and arranging the other 5 letters, we obtain $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$.

Then starting with R, and arranging the other five letters I,I,S,T,U in different ways,

we obtain $\frac{5!}{2!} = \frac{120}{2} = 60$.

Number of words beginning with S is $\frac{5!}{2!}$, but one of these words is the word SURITI itself.

So, we first find the number of words beginning with SI, SR, ST, SUI and SURI.

Number of words starting with SI = $4! = 24$

Number of words starting with SR = $\frac{4!}{2!} = 12$

Number of words starting with ST = $\frac{4!}{2!} = 12$

Number of words starting with SUI = $3! = 6$

Now, the words beginning with 'SUR' must follow.

There are $\frac{3!}{2!} = 3$ words beginning with SUR one of these words is the word SURITI.

The first word beginning which SUR is the word SURIIT and the next word is SURITI.

$\therefore$ Rank of SURITI

$$= 120 + 60 + 24 + 2 \times 12 + 6 + 2$$
$$= 180 + 56$$
$$= 236.$$

Q23

In a dictionary the words at each stage are arranged in alphabetical order. In the given problem we must therefore consider the words beginning with A, E, L, T in order.

'A' will occur in the first place as often as remaining 3 letters all at a time i.e A will occur in the first place the same number of times.

$\therefore$ Number of words starting with A = 3! = 6

Number of words starting with E = 3! = 6

Number of words begining with L is 3!, but one of these words is the word LATE itself.
The first word beginning with L is the word LATE and the next word is LATE.

$\therefore$ Rank of LATE $= 2 \times 6 + 2$
$= 12 + 2$
$= 14.$

Q24

In the dictionary the words at each stage are arranged in alphabetical order. In the given problem we must therefore consider the words beginning with E, H, M, O, R, T in order. E will occur in the first place as often as there are ways of arranging the remaining 5 letters

$\therefore$ Number of words starting with E = $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

Number of words starting with H = 5! = 120.
Number of words beginning with M is 5!, but one of these words is the word MOTHER.
So, we first find the number of words beginning with ME and MH.
Number of words starting with ME = $4! = 4 \times 3 \times 2 \times 1 = 24$.

Now, the words beginning with 'MO' must follow.
There are 4! words beginning with MO, one of these words is the word MOTHER itself.
So, we first find the number of words beginning with MOE, MOH and MOR.

Number of words starting with MOE = 3! = 6
Number of words starting with MOH = 3! = 6
Number of words starting with MOR = 3! = 6

Number of words beginning with MOT is 3! but one of these words is the word MOTHER itself
So, we first find the number of words beginning with MOTE.
Number of words starting with MOTE = 2! = 2

Now, the words beginning with MOTH must follow.
There are 2! words beginning with MOTH, one of these words is word MOTHER itself.
The first word beginning with MOTH is the word MOTHER.

$\therefore$ Rank of MOTHER $= 2 \times 120 + 2 \times 24 + 3 \times 6 + 2 + 1$
$= 240 + 48 + 18 + 3$

$= 309$

Q25

In a dictionary the words at each stage are arranged in alphabetical order. In the given problem we must therefore consider the words beginning with a,b,c,d,e in order. 'a' will occur in the first place as often as there are ways of arranging the remaining 4 letters all at a time i.e 'a' will occur 4! times. similarly b and c will occur in the first place the same number of times

$\therefore$ Number of words starting with'a' = $4! = 4\times3\times2\times1 = 24$
Number of words starting with 'b' = $4! = 4\times3\times2\times1 = 24$
Number of words starting with 'c' = $4! = 4\times3\times2\times1 = 24$

Number of words beginning with 'd' is 4!, but one of these words is the word debac.
So, we first find the number of words beginning with da, db, dc, and dea

Number of words starting with *da* = 3! = 6
Number of words starting with *db* = 3! = 6
Number of words starting with *dc* = 3! = 6
Number of words starting with *dea* = 2! = 2

There are 2! words beginning with deb one of these words is the word debac itself
The first word beginning with deb is the word debac.

$$\begin{aligned}\therefore\ \text{Rank of debac} &= 3\times24+3\times6+2+1\\ &= 72+18+3\\ &= 90+3\\ &= 93\end{aligned}$$

Q26

Total number of '+' signs = 6
Total number of '-' signs = 4

six '+' signs can be arranged in a row in $\frac{6!}{6!} = 1$ way $\quad [\because$ All '+' signs are identical$]$

Now, we are left with seven places in which four different things can be arranged in ${}^7P_4$ ways but all the four '-' signs are identical, therefore, four '-' signs can be arranged in $\frac{{}^7P_4}{4!} = \frac{\frac{7!}{(7-4)!}}{4!} = \frac{7!}{3!\times4!}$

$= \frac{7\times6\times5\times4!}{3\times2\times4!} = 7\times5 = 35$

Hence, the required number of ways = $1\times35 = 35$.

Q27

INTERMEDIATE

I = 2 times, T = 2 times, E = 3 times, N, R, M, D, A

Number of letters = 12

(i) There are 6 vowels. They ocuepy even places 2nd, 4th, 6th, 8th, 10th, 12th.

After there six there are six places and 5 letters, T is 2 times.

So, number of ways for consonants = $\frac{6!}{2!}$

The total number of ways when vowels ocuepy even places

$= \frac{6!}{2!} \times \frac{6!}{2!3!}$

$= \frac{6 \times 5 \times 4 \times 3 \times 2 \times 6 \times 5 \times 4 \times 3 \times 2}{2 \times 2 \times 3 \times 2}$

= 21600

Required number of ways = 21600

(ii) Number of ways such that relative order of vowels and consonents do not alter

$= \frac{6!}{2! \times 3!} \times \frac{6!}{2!}$

= 21600

Required number of ways = 21600

Q28

In a dictionary the words at each stage are arranged in alphabetical order. In the given problem we must therefore consider the words beginning with E, H, I, N, T, Z in order.

'E' will occur in the first place as often as there are ways of arranging the remaining 5 letters all at a time i.e. E will occur 5! times. Similarly H will occur in the first place the same number of times.

$\therefore$ Number of words starting with E = 5! = $5 \times 4 \times 3 \times 2 \times 1$ = 120

Number of words starting with H = 5! = 120

Number of words starting with I = 5! = 120

Number of words starting with N = 5! = 120

Number of words starting with T = 5! = 120

Number of words beginning with Z is 5!, but one of these words is the word ZENITH itself.

So, we first find the number of words beginning with ZEH, ZEI and ZENH

Number of words starting with ZEH = 3! = 6

Number of words starting with ZEI = 3! = 6

Number of words starting with ZENH = 2! = 2.

Now, the words beginning with ZENI must follow.

There are 2! words beginning with ZENI one of these words is the word ZENITH itself.

The first word beginning with ZENI is the word ZENIHT and the next word is ZENITH.

$\therefore$ Rank of ZENITH

$= 5 \times 120 + 2 \times 6 + 2 + 2$

$= 600 + 12 + 4$

$= 600 + 16$

$= 616$

Q29

18 mice can be arranged among themselves in
${}^{18}P_{18} = 18!$ ways.
There are three groups and each group is equally large. So 18 mice are divided in three groups and they can be arranged amongst themselves inside the group.

Therefore the number of ways mice placed into three groups are

$$= \frac{18!}{6!6!6!} = \frac{18!}{(6!)^3}$$