WITH GRAPH PAPER

केन्द्रीय माध्यमिक शिक्षा बोर्ड, दिल्ली

सीनियर स्कूल सर्टिफिकेट परीक्षा (कक्षा बारहवीं)

परीक्षार्थी प्रवेश-पत्र के अनुसार भरें

Mathematics

041 I

Wednesday 18·03·2015

English

Code Number	Set Number
65/2/C	① ● ③ ④

हाँ / नहीं Yes / No — No

B D H S C A

Physically Challenged

हाँ / नहीं Yes / No — No

Each letter be written in one box and one box be left blank between each part of the name. In case Candidate's Name exceeds 24 letters, write first 24 letters.

041-0852 4474974

Section - C

Q:20

$$(1+x^2)\frac{dy}{dx} = e^{m\tan^{-1}x} - y$$

$$(1+x^2)\frac{dy}{dx} + y = e^{m\tan^{-1}x}$$

$$\frac{dy}{dx} + \frac{y}{1+x^2} = \frac{e^{m\tan^{-1}x}}{1+x^2}$$

comparing it with

$$\frac{dy}{dx} + Py = Q \rightarrow \text{linear differential equation}$$

$$P = \frac{1}{1+x^2}$$

$$I.F = e^{\int P dx}$$

$$= e^{\int \frac{1}{1+x^2} dx}$$

$$\left\{ \int \frac{1}{x^2+1} dx = \tan^{-1}x \right\}$$

$$I.F = e^{\tan^{-1}x}$$

~~So diff~~ equation becomes

$$y \; I \cdot F = \int Q \times I \cdot F \, dx + c$$

$$y \times e^{\tan^{-1}x} = \int \frac{e^{m\tan^{-1}x}}{1+x^2} \times e^{\tan^{-1}x} dx + c$$

$$y \times e^{\tan^{-1}x} = \int \frac{e^{(m+1)\tan^{-1}x}}{1+x^2} dx + c$$

put $\tan^{-1}x = t$

$$\frac{1}{1+x^2} dx = dt$$

$$y e^{\tan^{-1}x} = \int e^{(m+1)t} \times dt + c$$

$$y e^{\tan^{-1}x} = \frac{e^{(m+1)t}}{(m+1)} + c$$

~~y to~~ $y e^{\tan^{-1}x} = \dfrac{e^{(m+1)\tan^{-1}x}}{m+1} + c$

when $x = 0$ $y = 1$

$$y e^{\tan^{-1}x} = \frac{e^{(m+1)\tan^{-1}x}}{(m+1)} + c$$

when $x = 0$ $y = 1$

$$1 \times e^{\tan^{-1}0} = \frac{e^{(m+1)\tan^{-1}0}}{(m+1)} + c$$

$$1 \times e^{0} = \frac{e^{(m+1)(0)}}{m+1} + c \qquad e^{0} = 1$$

$$1 = \frac{1}{m+1} + c$$

$$1 - \frac{1}{m+1} = c$$

$$\frac{m + \cancel{1} - \cancel{1}}{m+1} = c$$

$$c = \frac{m}{m+1}$$

So equation is

$$\left[y e^{\tan^{-1}x} = \frac{e^{(m+1)\tan^{-1}x}}{(1+m)} + \frac{m}{m+1} \right]$$

Q: 21.

$f(x) = \sin^2 x - \cos x \qquad x \in [0, \pi]$

$f'(x) = 2\sin x\cos x - (-\sin x)$

$f'(x) = 2\sin x\cos x + \sin x$

put $f'(x) = 0$

then $(2\sin x\cos x + \sin x) = 0$

$\sin x(2\cos x + 1) = 0$

$\sin x = 0$ or $\cos x = -\frac{1}{2}$

$x = 0, \pi$ or $x = \frac{2\pi}{3}$

both the extreme value are automatically included.

So $f(0) = [\sin(0)]^2 - \cos 0$

$= 0 - 1$

$= -1$

$f(\pi) = \sin^2\pi - \cos\pi$

$= 0 - (-1)$

$= \underline{1}$

$f\left(\frac{2\pi}{3}\right) = \sin^2\frac{2\pi}{3} - \cos\frac{2\pi}{3}$

$= \left(\frac{\sqrt{3}}{2}\right)^2 - \left(-\frac{1}{2}\right) = \frac{3}{4} + \frac{1}{2} = \frac{10}{8} = \frac{5}{4}$

$f(0) = -1$

$f(\pi) = 1$

$f\left(\frac{2\pi}{3}\right) = \frac{5}{4}$

So $\boxed{\text{Absolute maxima is } \frac{5}{4} \text{ at } x = \frac{2\pi}{3}}$

$\boxed{\text{Absolute minima is } -1 \text{ at } x = 0}$

Q: 22:

$\vec{r} = \hat{i} + \hat{j} + \hat{k} + \lambda(\hat{i} - \hat{j} + \hat{k})$

$\vec{r} = 4\hat{j} + 2\hat{k} + \mu(2\hat{i} - \hat{j} + 3\hat{k})$

These line are coplaner if they are parallel or they are intersecting

But these lines are not parallel.

So they are coplaner if shortest distance between them is zero:

$a_1 = (1, 1, 1)$ $\qquad$ $(a_2 - a_1) = (-1, 3, 1)$

$a_2 = (0, 4, 2)$

$b_1 = \langle 1, -1, 1 \rangle$

$b_2 = \langle 2, -1, 3 \rangle$

Shortest distance $= \dfrac{\left|(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})\right|}{|\vec{b_1} \times \vec{b_2}|}$

Let us find $\left|(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})\right|$

$$(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2}) = \begin{vmatrix} -1 & 3 & 1 \\ 1 & -1 & 1 \\ 2 & -1 & 3 \end{vmatrix}$$

(expanding by 1st row)

$$= -1(-3+1) - 3(3-2) + 1(-1+2)$$

$$= -1(-2) - 3(1) + 1$$

$$= 2 + 1 - 3$$

$$= 0$$

So shortest distance between the lines is zero

So they are coplaner.

Let $\langle a,b,c\rangle$ be the direction ratio of normal to the plane.

So the dot product of direction ratio of normal to plane and the direction ratio of line is 0.

So $\quad a-b+c=0$

$\quad\quad 2a-b+3c=0$

$$\begin{array}{cccc} a & b & & c \\ -1 & 1 & 1 & -1 \\ -1 & 3 & 2 & -1 \end{array}$$

$$\frac{a}{-3+1} = \frac{b}{2-3} = \frac{c}{-1+2}$$

So $\langle a,b,c\rangle = \langle -2,-1,1\rangle$

So passing point of line is also the passing point of plane

So passing point $= (1,1,1)$

So equation becomes

$$-2(x-1)-1(y-1)+1(z-1)=0$$

$$-2x+2-y+\cancel{1}+z-\cancel{1}=0$$

$$\boxed{-2x-y+z+2=0}$$

equation of plane.

Q:23 $f: W \rightarrow W$

$$f(n) = \begin{cases} n-1 & \text{if } n \text{ is odd} \\ n+1 & \text{if } n \text{ is even} \end{cases}$$

prove that it is one-one:

Let us suppose that $n_1, n_2 \in W$

$n_1 \neq n_2$ but $f(n_1) = f(n_2)$

(i) 1st case: if n_1 & n_2 are odd

$$f(n_1) = f(n_2)$$

$$n_1 - \cancel{1} = n_2 - \cancel{1}$$

$n_1 = n_2$ which is contradiction.

(ii) 2nd case : n_1, n_2 are even

$$f(n_1) = f(n_2)$$

$$n_1 + \cancel{1} = n_2 + \cancel{1}$$

$$n_1 = n_2$$

which is contradiction.

(iii) 3rd case if n_1 is even & n_2 is odd

$$f(n_1) = f(n_2)$$

$$n_1 + 1 = n_2 - 1$$

$$\underset{\text{even}}{\underset{\downarrow}{n_1}} + 2 = \underset{\text{odd}}{\underset{\downarrow}{n_2}}$$

if we add 2 in an even no. then we get an even no. but n_2 is odd which is again contradiction

So from these 3 point we see that

$f(n_1) = f(n_2)$ only if $n_1 = n_2$

$\Rightarrow$ f is one - one.

To prove that f is onto:

Let $y \in W$

and y is even

then $y+1$ is odd and it belongs in whole no.

$f(y+1) = y+1-1$

$= y$

So for every $y \in W$ which is even there exist a preimage $y+1$ which is odd and $(y+1) \in W$

Let $y \in W$

y is odd

then $y-1$ is even and $y-1 \in W$

$f(y-1) = y-1+1$

$= y$

So for every $y \in W$ which is odd there exist a pre image $y-1 \in W$ which is even.

So for all $y \in W$ there exist pre image in whole no.

So f is onto.

f is one-one and onto So it is invertible

To find $f^{-1}(x)$

$y = x - 1$ if x is odd

y here is even

$y + 1 = x$

So $x = y + 1$ if y is even

$f^{-1}(x) = x + 1$ if x is even

$y = x + 1$ if x is even

y is odd here

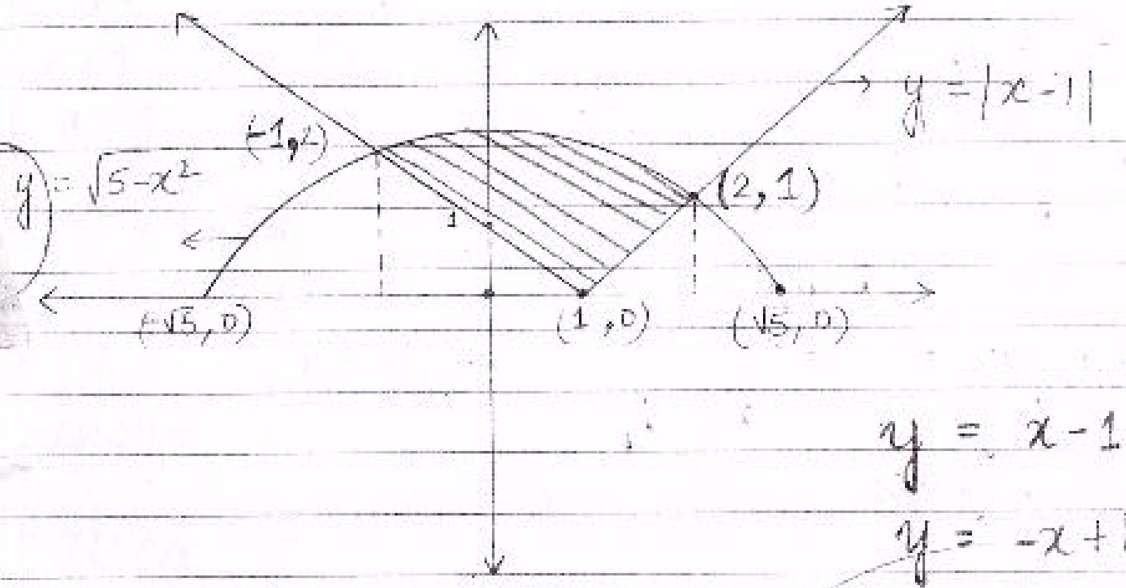
$y - 1 = x$

So $f^{-1}(x) = x - 1$ if x is odd

So $f^{-1}(x) = \begin{cases} x - 1 & \text{if } x \text{ is odd} \\ x + 1 & \text{if } x \text{ is even} \end{cases}$

$\boxed{x \in W}$ So $\boxed{f^{-1} = f}$

<u>24°</u>



$y = x - 1 \quad \text{if} \quad x \geq 1$

$y = -x + 1 \quad \text{if} \quad x < 1$

intersection point :

$y = \sqrt{5-x^2}$

$y = x - 1$

So $\sqrt{5-x^2} = (x-1)$

$5 - x^2 = x^2 + 1 - 2x$

$2x^2 - 2x - 4 = 0$

$\cancel{2}x^2 - x - 2 = 0$

$x^2 - 2x + x - 2 = 0$

$x(x-2) + 1(x-2) = 0$

So $x = 2$ or $x = -1$ ↓ rejected as $x \geq 1$

$x = 2$

$y = \sqrt{5-x^2}$

$y = -x+1$

$y \quad x<1$

$5-x^2 = x^2+1-2x$

so $x = 2, -1$

$x = 2$ $\downarrow$ rejected $\quad \boxed{x=-1}$

Required Area

$$(A) = \int_{-1}^{2} \sqrt{5-x^2}\,dx - \left[\int_{-1}^{1}(-x+1)\,dx + \int_{1}^{2}(x-1)\,dx\right]$$

$$A = \frac{x}{2}\sqrt{5-x^2} + \frac{5}{2}\sin^{-1}\frac{x}{\sqrt{5}}\Bigg|_{-1}^{2} - \left[\frac{-x^2}{2}+x\Bigg|_{-1}^{1} + \frac{x^2}{2}-x\Bigg|_{1}^{2}\right]$$

$$A = 1\times 1 + \frac{5}{2}\sin^{-1}\frac{2}{\sqrt{5}} - \left(\frac{-1\times 2}{2} + \frac{5}{2}\sin^{-1}\left(\frac{-1}{\sqrt{5}}\right)\right) - \left\{\frac{-1}{2}[1-1]+2 + \frac{1}{2}[4-1]-1\right\}$$

$$A = 1 + \frac{5}{2}\sin^{-1}\frac{2}{\sqrt{5}} + 1 + \frac{5}{2}\sin^{-1}\frac{1}{\sqrt{5}} - \left[0 + 2 + \frac{3}{2} - 1\right]$$

$$A = 2 + \frac{5}{2}\left[\sin^{-1}\frac{2}{\sqrt{5}} + \sin^{-1}\frac{1}{\sqrt{5}}\right] - \frac{5}{2}$$

$$A = \frac{5}{2} \times \frac{\pi}{2} + 2 - \frac{5}{2}$$

$$\boxed{A = \frac{5\pi}{4} - \frac{1}{2} \text{ sq units}}$$

$$\left\{\sin^{-1}\frac{2}{\sqrt{5}} + \sin^{-1}\frac{1}{\sqrt{5}} = \sin^{-1}\frac{2}{\sqrt{5}} + \sin^{-1}\cos^{-1}\frac{2}{\sqrt{5}} = \frac{\pi}{2}\right\}$$

$\overline{\theta}1) dx$

$x\Big|_1^2$

2

$[4-1]-1\}$

25. Ist 6 positive integers = (1, 2, 3, 4, 5, 6)

Random Variable = X = larger of 2 numbers.

Sample space S = {(1,2) (1,3) (1,4) (1,5) (1,6)
(2,1) (3,1) (4,1) (5,1) (6,1)
(2,3) (2,4) (2,5) (2,6) (3,2) (4,2) (5,2)
(6,2) (3,4) (3,5) (3,6) (4,3) (5,3) (6,3)
(4,5) (4,6), (5,4), (6,4) (5,6) (6,5)}

X	P(X)	$x_i p_i$	$x_i^2 p_i$
2	$\frac{2}{30} = \frac{1}{15}$	$\frac{2}{15}$	$\frac{4}{15}$
3	$\frac{4}{30} = \frac{2}{15}$	$\frac{6}{15}$	$\frac{18}{15}$
4	$\frac{6}{30} = \frac{3}{15}$	$\frac{12}{15}$	$\frac{48}{15}$
5	$\frac{8}{30} = \frac{4}{15}$	$\frac{20}{15}$	$\frac{100}{15}$
6	$\frac{10}{30} = \frac{5}{15}$	$\frac{30}{15}$	$\frac{180}{15}$

$\Sigma x_i p_i = \frac{70}{15}$ $\quad \Sigma x_i^2 p_i \ \frac{350}{15}$

Mean $= \Sigma x_i p_i^2 = \frac{14}{3} = 4.66$

Variance $= \Sigma x_i^2 p_i - (\mu)^2$

$= \frac{350}{15} - \frac{70}{15} \times \frac{70}{15}$

Rough

2+6

$\begin{array}{r} 36 \\ \times 5 \\ \hline 180 \end{array}$

$\begin{array}{r} 22 \\ 48 \\ \hline 70 \end{array}$

$$\sigma^2 = \frac{70}{15}\left[5 - \frac{70}{15}\right]$$

$$= \frac{70}{15}\left[\frac{75-70}{15}\right]$$

$$= \frac{\cancel{70}^{14} \times \cancel{5}}{{}_3\cancel{15} \; \cancel{15}_3}$$

$$= \frac{14}{9}$$

$$\sigma^2 = 1.55$$

$$\boxed{\text{Variance} = 1.55}$$

Section - B

Q:19

$$x^x + x^y + y^x = a^b$$

$$\downarrow \quad \downarrow \quad \downarrow$$

$$P \quad Q \quad R$$

$$\frac{d}{dx}\left[P + Q + R\right] = 0$$

$$\left[\frac{dP}{dx} + \frac{dQ}{dx} + \frac{dR}{dx} = 0\right]$$

$$P = x^x$$

$$\log P = x \log x$$

$$\frac{1}{P}\frac{dP}{dx} = x\left[\frac{1}{x}\right] + \log x$$

$$\frac{dP}{dx} = x^x(1 + \log x) \quad -(1)$$

$$Q = x^y$$

$$\log Q = y \log x$$

$$\frac{1}{Q}\frac{dQ}{dx} = \left(\frac{y}{x} + \log x \frac{dy}{dx}\right)$$

$$\frac{dQ}{dx} = x^y\left[\frac{y}{x} + \log x \frac{dy}{dx}\right] \quad -(2)$$

$$\cancel{Q = y^x}$$

$$R = y^x$$

$$\log R = x \log y$$

$$\frac{1}{R}\frac{dR}{dx} = \frac{x}{y}\frac{dy}{dx} + \log y$$

$$\frac{dR}{dx} = y^x\left[\frac{x}{y}\frac{dy}{dx} + \log y\right] \quad -③$$

adding ① & ② & ③ we get

$$x^x + x^x\log x + x^{y-1}y + x^y\log x\frac{dy}{dx} + y^{x-1}x\frac{dy}{dx} + y^x\log y = 0$$

$$\frac{dy}{dx} = \frac{-\left[x^x(1+\log x) + x^{y-1}y + y^x\log y\right]}{x^y\log x + y^{x-1}x}$$

Q:18 $\quad y = e^{ax}\cos bx$

$$\frac{dy}{dx} = e^{ax}(-\sin bx)b + \cos bx \times e^{ax} \times a$$

$$\frac{dy}{dx} = -be^{ax}\sin bx + ay \Rightarrow \left[-\frac{1}{b}\left(\frac{dy}{dx} - ay\right) = e^{ax}\sin bx\right] \quad ①$$

$$\frac{d^2y}{dx^2} = -b\left[e^{ax}\cos bx \times b + \sin bx \times e^{ax} \times a\right] + a\frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} = -b\left[by - \frac{a}{b}\left(\frac{dy}{dx} - ay\right)\right] + a\frac{dy}{dx} \quad \text{from (1)}$$

$$\frac{d^2y}{dx^2} = -b^2y + a\frac{dy}{dx} - a^2y + a\frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} + (a^2+b^2)y - 2a\frac{dy}{dx} = 0$$

hence proved.

26.

$Z = 5x+2y$

objective function to be made maximum and minimum

$x - 2y \leq 2$

$3x+2y \leq 12$

$-3x+2y \leq 3$

$x \geq 0$, $y \geq 0$

corresponding equations

$x - 2y = 2$

put $(0,0)$ in it

~~0, 0~~ $0 \leq 2$

which is true so the region is towards origin

$3x + 2y \leq 12 \Rightarrow 3x + 2y = 12$

put $(0,0)$

$0 \leq 12$ which is true

So region towards origin

$-3x + 2y = 3$

put $(0,0)$

$0 \leq 3$

which is true

So region is towards origin

$x \geq 0 \quad y \geq 0$. So it means Ist quadrant.

at A (-2

at B

at C

at

intersection point of	intersection point of:
$-3x+2y=3$	$x-2y=2$
$3x+2y=12$	$3x+2y=12$
$4y=15$	$4x=14$
$y=\frac{15}{4}$	$x=\frac{7}{2}$
$-3x+2\times\frac{15}{\cancel{4}\,2}=3$	$\frac{7}{2}-2y=2$
$-3x=3-\frac{15}{2}$	$\frac{7}{2}-2=2y$
$-x=1-\frac{5}{2}$	$\frac{3}{2}=2y$
$-x=-\frac{3}{2}$	$y=\frac{3}{4}$
$x=\frac{3}{2}$	

$Z = 5x+2y$

at A $(2,0)$ $Z=10$

at B $(0,0)$ $Z=0$

at C $(0,\frac{3}{2})$ $Z=3$

at D $\left(\frac{3}{2},\frac{15}{4}\right)$ $Z=\frac{15}{2}+\frac{15}{2}=15$

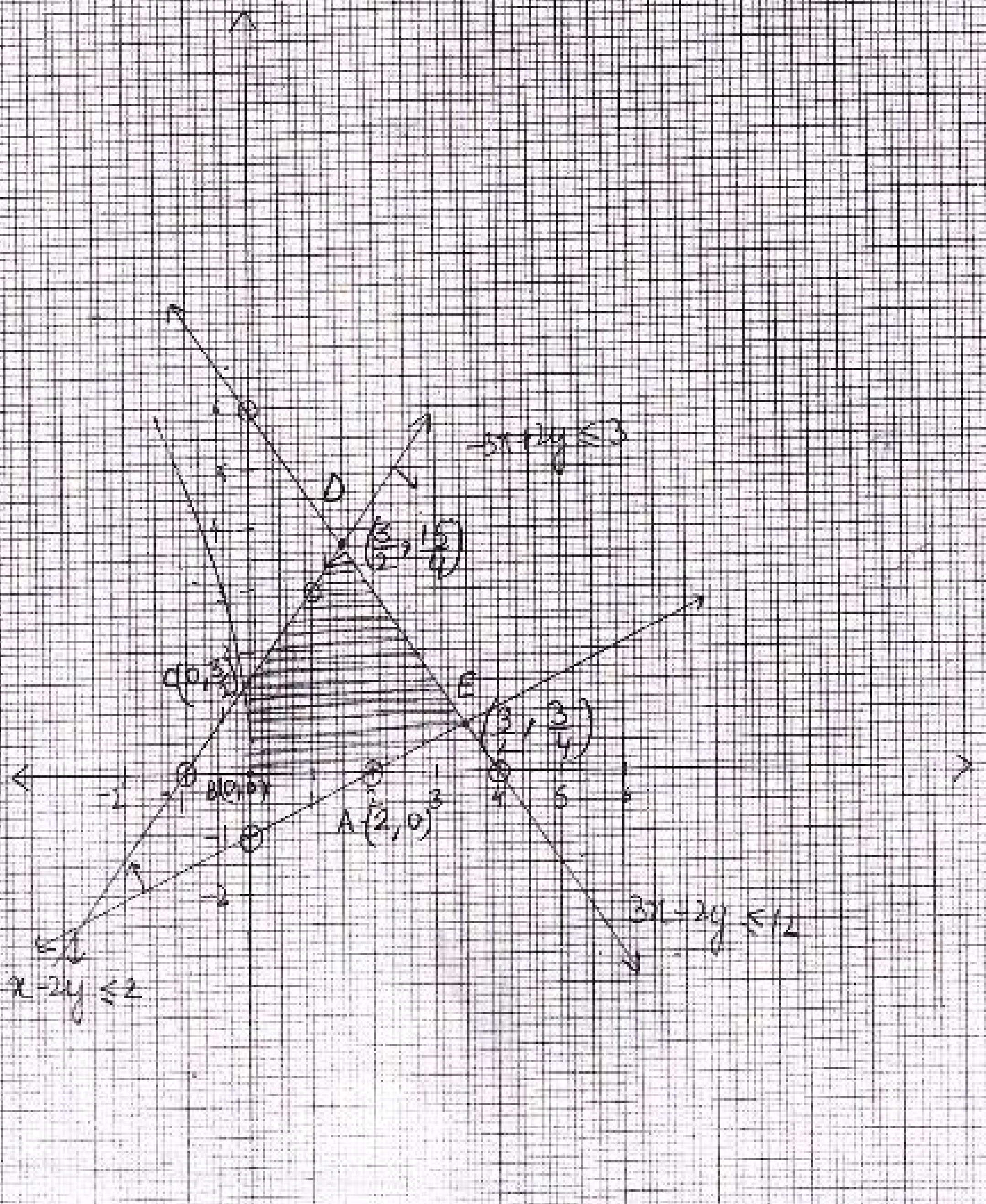
at E $\left(\frac{3}{2},\frac{3}{4}\right)$

$Z=\frac{15}{2}+\frac{3}{2}$

$Z=9$

Q : 26

10 small box = 1 unit



So Z is maximum at $D\left(\frac{3}{2},\frac{15}{4}\right)$

$Z = 15$

Z is minimum at $(0,0)$

$\boxed{Z=0}$

17.

$$A = \begin{array}{c} \\ \\ \\ \\ \end{array}\begin{array}{ccc} \text{Jans} & \text{Mats} & \text{Toys} \\ \end{array}$$

$$A = \begin{bmatrix} 30 & 12 & 70 \\ 40 & 15 & 55 \\ 35 & 20 & 75 \end{bmatrix}_{3\times3} \begin{array}{l} \to \text{school X} \\ \to \text{school Y} \\ \to \text{school Z} \end{array}$$

$$B = \begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix} \begin{array}{l} \to \text{cost of jans} \\ \to \text{cost of Mats} \\ \to \text{cost of toys} \end{array}$$

$$AB = \begin{bmatrix} 30 & 12 & 70 \\ 40 & 15 & 55 \\ 35 & 20 & 75 \end{bmatrix} \begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix}$$

$$X = AB = \begin{bmatrix} 750 + 1200 + 3500 \\ 1000 + 1500 + 2750 \\ 875 + 2000 + 3750 \end{bmatrix}$$

$$X = \begin{bmatrix} 5450 \\ 5250 \\ 6625 \end{bmatrix} \begin{array}{l} \to \text{fund collected by school } X \\ \to \text{fund collected by school } Y \\ \to \text{fund collected by school } Z \end{array}$$

fund by X = Rs. 5450

fund by Y = Rs 5250

fund by Z = Rs 6625

Total fund = Rs 17325

They are helping victims and hence

the value of helping other is generated.

16°
$$I = \int \frac{x+3}{(x+5)^3} e^x \, dx$$

$$I = \int \frac{x+5}{(x+5)^3} e^x dx - \int \frac{2 \cdot e^x}{(x+5)^3} dx$$

$$I = \int \frac{1}{(x+5)^2} e^x dx - \int \frac{2 e^x}{(x+5)^3} dx$$

↓

integrating this by parts $\frac{1}{(x+5)^2}$ is taken as Ist function

$$I = \frac{1}{(x+5)^2} e^x - \int \frac{d}{dx} \frac{1}{(x+5)^2} e^x dx - \int \frac{2}{(x+5)^3} e^x dx$$

$$I = \frac{1}{(x+5)^2} e^x + \int \cancel{\frac{2}{(x+5)^3}} e^x dx - \int \cancel{\frac{2}{(x+5)^3}} e^x dx$$

$$So\ I = \frac{e^x}{(x+5)^2} + c$$

15.

$$x = a\sin 2t\,(1+\cos 2t)$$

$$\frac{dx}{dt} = a\left[(\sin 2t)(-\sin 2t)(2) + (1+\cos 2t)\cos 2t \times 2\right]$$

$$\frac{dx}{dt} = 2a\left[\cos^2 2t + \cos 2t - \sin^2 2t\right]$$

$$\frac{dx}{dt} = 2a\left[\cos 4t + \cos 2t\right] \qquad \left[\cos^2 x - \sin^2 x = \cos 2x\right]$$

$$y = b\cos 2t(1 + \cos 2t)$$

$$y = b\cos 2t + b\cos^2 2t$$

$$\frac{dy}{dt} = -b\sin 2t \times 2 + b \times 2\cos 2t\,\sin 2t \times 2$$

$$\frac{dy}{dt} = 2b\left[-\sin 2t + \sin 4t\right]$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$\frac{dy}{dx} = \frac{2b}{2a}\left[\frac{\sin 4t - \sin 2t}{\cos 2t + \cos 4t}\right].$$

at $t = \frac{\pi}{4}$

$$\frac{dy}{dx} = \frac{b}{a}\left[\frac{\sin \pi - \sin \frac{\pi}{2}}{\cos \frac{\pi}{2} + \cos \pi}\right]$$

$$= \frac{b}{a}\left[\frac{0 - 1}{0 - 1}\right]$$

$$\left.\frac{dy}{dx}\right|_{t=\frac{\pi}{4}} = \frac{b}{a}$$

142

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x + \cos x}\, dx \quad \text{--- (1)}$$

$$\left[\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right]$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin^2\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right) + \cos\left(\frac{\pi}{2}-x\right)}\, dx$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{\cos x + \sin x}\, dx \quad \text{--- (2)}$$

adding (1) & (2) we get

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^2 x + \cos^2 x}{\sin x + \cos x}\, dx$$

$$2I = \int_0^{\frac{\pi}{2}} \frac{1}{(\sin x + \cos x)}\, dx$$

$$2I = \int_0^{\frac{\pi}{2}} \frac{1+\tan^2\frac{x}{2}}{2\tan\frac{x}{2} + 1 - \tan^2\frac{x}{2}}$$

$$\left\{ \begin{array}{l} \text{put } \sin x = \dfrac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} \\ \text{and } \cos x = \left(\dfrac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right) \end{array} \right.$$

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sec^2\frac{x}{2}}{2\tan\frac{x}{2} + 1 - \tan^2\frac{x}{2}}\, dx$$

put $\tan\frac{x}{2} = t$

$$\sec^2\frac{x}{2} \times \frac{1}{2}\, dx = dt$$

$x \to 0 \quad t \to 0$

when $x \to \frac{\pi}{4} \quad t \to 1$

$\cancel{2}$

$$\cancel{2}I = \cancel{2}\int_0^1 \frac{dt}{2t + 1 - t^2}$$

$$I = \int_0^1 \frac{dt}{2t + 1 + 1 - 1 - t^2} \Rightarrow I = \int_0^1 \frac{dt}{2 - (t-1)^2}$$

$$I = \int_0^1 \frac{dt}{(\sqrt{2})^2 - (t-1)^2} \qquad \int \frac{1}{a^2-x^2} = \frac{1}{2a} \log\left|\frac{a+x}{a-x}\right|$$

$$I = \frac{1}{2\sqrt{2}} \log\left|\frac{\sqrt{2}+t-1}{\sqrt{2}-t+1}\right|\Bigg]_0^1$$

$$I = \frac{1}{2\sqrt{2}} \log\left|\frac{\sqrt{2}+\cancel{1}-\cancel{1}}{\sqrt{2}-\cancel{1}+\cancel{1}}\right| - \frac{1}{2\sqrt{2}} \log\left|\frac{\sqrt{2}-1}{\sqrt{2}+1}\right|$$

$$I = \frac{1}{2\sqrt{2}} \log 1 - \frac{1}{2\sqrt{2}} \log\left|\frac{\sqrt{2}-1}{\sqrt{2}+1}\right|$$

$$= 0 - \frac{1}{2\sqrt{2}} \log\left|\frac{\sqrt{2}-1}{\sqrt{2}+1}\right|$$

$$I = \frac{-1}{2\sqrt{2}} \log\left|\frac{\sqrt{2}-1}{\sqrt{2}+1}\right|$$

13°

$$\Delta = \begin{vmatrix} x+2 & x+6 & x-1 \\ x+6 & x-1 & x+2 \\ x-1 & x+2 & x+6 \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$

$$\Delta = \begin{vmatrix} 3x+7 & 3x+7 & 3x+7 \\ x+6 & x-1 & x+2 \\ x-1 & x+2 & x+6 \end{vmatrix}$$

$$\Delta = 3x+7 \begin{vmatrix} 1 & 1 & 1 \\ x+6 & x-1 & x+2 \\ x-1 & x+2 & x+6 \end{vmatrix}$$

$C_1 \to C_1 \to C_2 \qquad C_2 \to C_2 - C_3$

$$\Delta = (3x+7) \begin{vmatrix} 0 & 0 & 1 \\ 7 & -3 & x+2 \\ -3 & -4 & x+6 \end{vmatrix}$$

expanding by R_1

$\Delta = (3x+7)\left[-28 - 9\right]$

$\Delta = 0$

$\Rightarrow 3x+7 = 0$

$\boxed{x = \frac{-7}{3}}$

12.

$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

$A^2 = AA = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1+4+4 & 2+2+4 & 2+4+2 \\ 2+2+4 & 4+1+4 & 4+2+2 \\ 2+4+2 & 4+2+2 & 4+4+1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$$

$$A^2 - 4A - 5I = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 9 \\ 8 & 8 & 9 \end{bmatrix} + \begin{bmatrix} -4 & -8 & -8 \\ -8 & -4 & -8 \\ -8 & -8 & -4 \end{bmatrix} + \begin{bmatrix} -5 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & -5 \end{bmatrix}$$

$$A^2 - 4A - 5I = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O$$

So $A^2 - 4A - 5I = O$

Hence proved.

$A^2 - 4A - 5I = 0$

pre-multiplying by A^{-1}

$A^{-1}AA - 4A^{-1}A - 5A^{-1}I = 0$

$A^{-1}A = I$

$IA - 4I - 5A^{-1} = 0$

$IA = A$

$A - 4I - 5A^{-1} = 0$

$A - 4I = 5A^{-1}$

$$A^{-1} = \frac{A - 4I}{5}$$

$$A^{-1} = \frac{1}{5}\left[A - 4I \right]$$

$$5A^{-1} = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{bmatrix}$$

$$5A^{-1} = \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$$

$$\text{So } A^{-1} = \frac{1}{5}\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$$

11.

$$\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$$

$$\sin^{-1}(1-x) = \frac{\pi}{2} + 2\sin^{-1}x$$

$$(1-x) = \sin\left(\frac{\pi}{2} + 2\sin^{-1}x\right)$$

$$(1-x) = \cos(2\sin^{-1}x)$$

$$\sin^{-1}x = \theta$$

$$x = \sin\theta$$

$$\cos 2\theta = 1 - 2\sin^2\theta$$

$$\cos 2\theta = 1 - 2x^2$$

$$\Rightarrow \quad 1 - x = \cos 2\theta$$

$$\cancel{1} - x = \cancel{1} - 2x^2$$

$$\Rightarrow \quad 2x^2 - x = 0$$

$$x(2x-1) = 0$$

$x = 0$ or $x = \frac{1}{2}$

put $x = \frac{1}{2}$ in equation

$\sin^{-1}\frac{1}{2} - 2\sin^{-1}\frac{1}{2}$

$= \frac{\pi}{6} - 2 \times \frac{\pi}{6}$

$\neq \frac{\pi}{2}$

So $x \neq \frac{1}{2}$

So $\boxed{x = 0}$

10.

passing point of line $= (4, 2, 2)$

since $\vec{b}$ is $\parallel$ to line

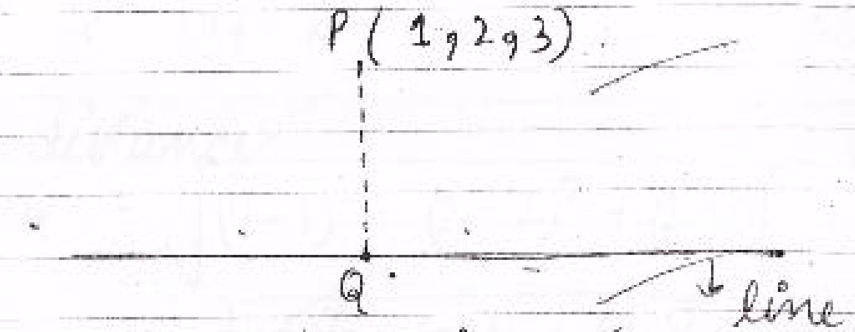
So direction ratio of line $= \langle 2, 3, 6 \rangle$

So equation of line

$$\frac{x-4}{2} = \frac{y-2}{3} = \frac{z-2}{6} = \lambda$$

general point on line

$$(2\lambda+4,\ 3\lambda+2,\ 6\lambda+2)$$



Q be the foot of $\perp$ar

PQ is $\perp$ar to line

So dot product of direction ratio of line and PQ is 0

direction ratio of Q $= \langle 2\lambda+3,\ 3\lambda,\ 6\lambda-1 \rangle$

So according to question :

$$2(2\lambda+3) + (3\lambda)3 + 6(6\lambda-1) = 0$$

$$4\lambda + \cancel{6} + 9\lambda + 36\lambda - \cancel{6} = 0$$

$$\boxed{\lambda = 0}$$

So point $Q = (4, 2, 2)$

So ⊥ar distance

$$PQ = \sqrt{(4-1)^2 + (2-2)^2 + (2-3)^2}$$

$$= \sqrt{(3)^2 + (0)^2 + (-1)^2}$$

$$= \sqrt{9+1}$$

$$\boxed{\text{length of ⊥ar} = \sqrt{10} \text{ unit}}$$

9o ~~AB~~

these A, B, C, D are coplaner

~~So~~ $\overrightarrow{AB} \cdot (\overrightarrow{BC} \times \overrightarrow{CD}) = 0$

triple product is 0.

$\overrightarrow{AB} = 1\hat{i} + (x-1)\hat{j} + 4\hat{k}$

$\overrightarrow{BC} = 0\hat{i} + (1-x)\hat{j} - 7\hat{k}$

$\overrightarrow{CD} = 2\hat{i} + 3\hat{j} + \hat{k}$

$$\overrightarrow{AB} \cdot (\overrightarrow{BC} \times \overrightarrow{CD}) = \begin{vmatrix} 1 & x-1 & 4 \\ 0 & 1-x & -7 \\ 2 & 3 & 1 \end{vmatrix} = 0$$

expanding by R_1

$1(1-x+21) - (x-1)14 + 4(2(x-1)) = 0$

$22 - x - 14x + 14 + 8x - 8 = 0$ $\boxed{x = 4}$

$-7x = -28$

8.

p (probability of success) $= \frac{1}{2}$
i.e that head comes

q (probability of failure) $= \frac{1}{2}$
i.e that tail comes

Let the coin be tossed n times

this event follow the conditions of bernoulli trial

X be the random variable = no of heads

$$P(X \geq 1) = P(1) + \cdots + P(n)$$

$$P(X \geq 1) = 1 - P(0)$$

$$= 1 - {}^{n}C_0 \times \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^n$$

$P(X \geq \cancel{8})$ should be more than 80%

$$P(X \geq 1) > \frac{8}{10}$$

$$\frac{8\cancel{0}}{10\cancel{0}} < 1 - \left(\frac{1}{2}\right)^n$$

$$1-\left(\frac{1}{2}\right)^n \geq \frac{8}{10}$$

$$\left(1-\frac{8}{10}\right) \geq \left(\frac{1}{2}\right)^n$$

$$\frac{1\cancel{2}}{5\cancel{10}} > \left(\frac{1}{2}\right)^n$$

$$5 < 2^n$$

$$n = 3$$

follow the condition

So the coin should be tossed at least 3 times.

7.

$$I = \int \frac{x^2}{x^4+x^2-2}\,dx$$

$$I = \int \frac{x^2}{(x^4+2x^2-x^2-2)}\,dx$$

$$I = \int \frac{x^2}{x^2(x^2+2)-1(x^2+2)}\,dx$$

$$I = \int \frac{x^2 \, dx}{(x^2-1)(x^2+2)}$$

put $x^2 = t$

then $\dfrac{t}{(t-1)(t+2)} = \dfrac{A}{t-1} + \dfrac{B}{t+2}$

$t = A(t+2) + B(t-1)$

put $t = 1$

$1 = A \times 3$

$\boxed{A = \frac{1}{3}}$

put $t = -2$

$-2 = -3B$

$\boxed{B = \frac{2}{3}}$

$$I = \int \frac{1}{3(t-1)}\,dx + \frac{2}{3(t+2)}\,dx$$

$$I = \int \frac{1}{3(x^2-1)}\,dx + \frac{2}{3}\int \frac{dx}{(x^2+2)} \qquad \int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|$$

$$I = \frac{1}{3} \times \frac{1}{2}\log\left|\frac{x-1}{x+1}\right| + \frac{2}{3} \times \frac{1}{\sqrt{2}} \tan^{-1}\frac{x}{\sqrt{2}} + c \qquad \int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}$$

$$I = \frac{1}{6}\log\left|\frac{x-1}{x+1}\right| + \frac{\sqrt{2}}{3} \tan^{-1}\frac{x}{\sqrt{2}} + c$$

Section - A

Q:6 equation of plane

$$6x - 3y + 2z - 4 = 0$$

distance $= \dfrac{|6\times2 - 3\times5 + 2(-3) - 4|}{\sqrt{36+9+4}}$

$$= \frac{|12 - 15 - 6 - 4|}{\sqrt{49}}$$

$$= \frac{|12 - 15 - 10|}{7}$$

$$\boxed{\text{distance} = \frac{13}{7} \text{ units}}$$

Q:5: $\vec{a} = \hat{i} - \hat{j}$ $\quad |\vec{a}| = \sqrt{2}$

$\vec{b} = \hat{j} - \hat{k}$ $\quad |\vec{b}| = \sqrt{2}$

0902

Fictitious Roll No.
(To be entered by Board)
4474874

अपना अनुक्रमांक इस उत्तर-पुस्तिका पर न लिखें
Please do not write your Roll Number on this Answer-Book

अतिरिक्त उत्तर-पुस्तिका (ओं) की संख्या
Supplementary Answer-Book(s) No.

$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$

$(\hat{i} - \hat{j}) \cdot (\hat{j} - \hat{k}) = \sqrt{2} \times \sqrt{2} \cos\theta$

$-\frac{1}{2} = \cos\theta$

$\boxed{\theta = \frac{2\pi}{3}}$

angle between vectors $= \frac{2\pi}{3}$

4.

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 3 \\ 3 & 5 & -2 \end{vmatrix}$$

$\vec{a} \times \vec{b} = \hat{i}(-2-15) - \hat{j}(-4-9) + \hat{k}(10-3)$

$\vec{a} \times \vec{b} = -17\hat{i} + 13\hat{j} + 7\hat{k}$

$$|\vec{a}\times\vec{b}| = \sqrt{(-17)^2+(13)^2+(7)^2}$$

$$= \sqrt{289+169+49}$$

$$= \sqrt{507}$$

$$= \sqrt{3\times169}$$

$$|\vec{a}\times\vec{b}| = 13\sqrt{3}$$

Q: 3.

$$x\log x\frac{dy}{dx} + y = 2\log x$$

$$\frac{dy}{dx} + \frac{y}{x\log x} = 2\log x$$

compare it with $\frac{dy}{dx} + Py = Q$

$$I.F = e^{\int P\,dx}$$

$$= e^{\int \frac{1}{x\log x}dx}$$

put $\log x = t$

$\frac{1}{x}dx = dt$

$$I.F = e^{\int \frac{dt}{t}}$$

$$= e^{\log|t|}$$

$$= t$$

$$\boxed{I.F = \log x}$$

2° general equation of family of lines passing through origin

$y = mx$ $\qquad m = \frac{y}{x}$

$$\frac{dy}{dx} = m$$

$$\frac{dy}{dx} = \frac{y}{x}$$

$$\boxed{x\frac{dy}{dx} - y = 0}$$

$a_{12} = e^{2\times 1\times x} \sin 2x$

$\boxed{a_{12} = e^{2x} \sin 2x}$

Poonam 01/07/2019 (A)

Evaluation done as per CBSE Marking scheme.