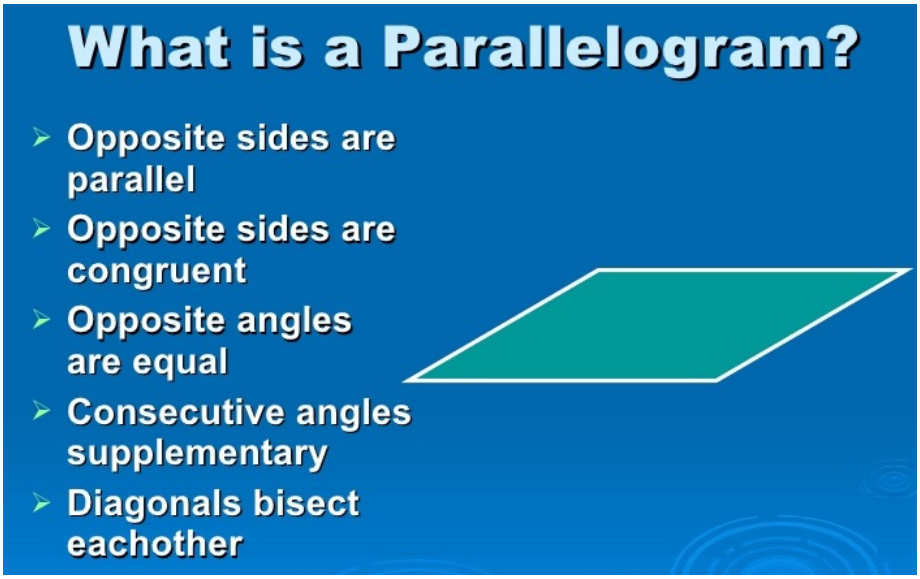
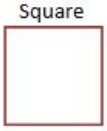
Parallelograms Exercise 16A



Type	Properties
Parallelogram 	• Opposite sides are equal and parallel • Opposite angles are equal
Rectangle 	• Opposite sides are equal and parallel • All angles are right angles (90°)
Square 	• Opposite sides are parallel • All sides are equal • All angles are right angles (90°)
Rhombus 	• Opposite sides are parallel • All sides are equal • Opposite angles are equal • Diagonals bisect each other at right angles (90°)
Trapezoid 	• One pair of opposite sides is parallel
Kite 	• Two pairs of adjacent sides are equal • One pair of opposite sides are equal • One diagonal bisects the other • Diagonals intersect at right angle (90°)

Q1

Answer :

It is given that $ABCD$ is a parallelogram in which $\angle A$ is equal to 110°.
Sum of *the* adjacent angles of a parallelogram is 180°.
$\therefore \angle A + \angle B = 180^\circ$
$\Rightarrow 110^\circ + \angle B = 180^\circ$
$\Rightarrow \angle B = (180^\circ - 110^\circ)$
$\Rightarrow \angle B = 70^\circ$
$\therefore \angle B = 70^\circ$

Also, $\angle B + \angle C = 180^\circ$
$\Rightarrow 70^\circ + \angle C = 180^\circ$
$\Rightarrow \angle C = (180^\circ - 70^\circ)$
$\Rightarrow \angle C = 110^\circ$
$\therefore \angle C = 110^\circ$
Further, $\angle C + \angle D = 180^\circ$
$\Rightarrow 110^\circ + \angle D = 180^\circ$
$\Rightarrow \angle D = (180^\circ - 110^\circ)$
$\Rightarrow \angle D = 70^\circ$
$\therefore \angle D = 70^\circ$

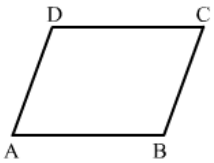
Q2

Answer :

Let the required angle be x°.
As the adjacent angles are equal, we have :
$x + x = 180$ (since the sum of adjacent angles of a parallelogram is 180°)
$\Rightarrow 2x = 180$
$\Rightarrow x = \frac{180}{2}$
$\Rightarrow x = 90^\circ$
Hence, the measure of each of the angles is 90°.

Q3

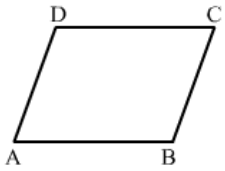
Answer :



Let $ABCD$ be the parallelogram.
Then, $\angle A$ and $\angle B$ are its adjacent angles.
Let $\angle A = (4x)^\circ$
$\angle B = (5x)^\circ$

$\therefore \angle A + \angle B = 180^\circ$ [since sum of *the* adjacent angles of a parallelogram is 180°]
$\Rightarrow 4x + 5x = 180$
$\Rightarrow 9x = 180$
$\Rightarrow x = \frac{180}{9}$
$\Rightarrow x = 20$
$\therefore \angle A = (4 \times 20)^\circ = 80^\circ$
$\angle B = (5 \times 20)^\circ = 100^\circ$
*O*pposite angles of parallelogram are equal.
$\therefore \angle C = \angle A = 80^\circ$
$\angle D = \angle B = 100^\circ$

Q4



Let $ABCD$ be a parallelogram.

$Let\ \angle A = (3x - 4)^\circ$

$\angle B = (3x + 16)^\circ$

$\therefore\ \angle A + \angle B = 180^\circ \quad [since\ the\ sum\ of\ adjacent\ angles\ of\ a\ parallelogram\ is\ 180^\circ]$

$\Rightarrow 3x - 4 + 3x + 16 = 180$

$\Rightarrow 3x - 4 + 3x + 16 = 180$

$\Rightarrow 6x + 12 = 180$

$\Rightarrow 6x = 168$

$\Rightarrow x = \frac{168}{6}$

$\Rightarrow x = 28$

$\therefore\ \angle A = (3 \times 28 - 4)^\circ$

$= (84 - 4)^\circ$

$= 80^\circ$

$\angle B = ((3 \times 28) + 16)^\circ$

$= (84 + 16)^\circ$

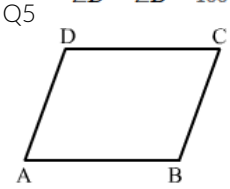
$= 100^\circ$

The opposite angles of a paralleogram are equal.

$\therefore \angle C = \angle A = 80^\circ$

$\angle D = \angle B = 100^\circ$

Q5



Let $ABCD$ be a parallelogram and let the sum of its opposite angles be 130°.

$\angle A + \angle C = 130^\circ$

$The\ opposite\ angles\ are\ equal\ in\ a\ parallelogram.$

$\therefore\ \angle A = \angle C = x^\circ$

$\Rightarrow x + x = 130$

$\Rightarrow 2x = 130$

$\Rightarrow x = \frac{130}{2}$

$\Rightarrow x = 65$

$\therefore \angle A = 65^\circ\ and\ \angle C = 65^\circ$

$\angle A + \angle B = 180^\circ \quad [since\ the\ sum\ of\ adjacent\ angles\ of\ a\ parallelogram\ is\ 180^\circ]$

$\Rightarrow 65^\circ + \angle B = 180^\circ$

$\Rightarrow \angle B = (180 - 65)^\circ$

$\Rightarrow \angle B = 115^\circ$

$\angle D = \angle B = 115^\circ \quad [opposite\ angles\ of\ parallelogram\ are\ equal]$

Q6

Answer :

Let the lengths of two sides of the parallelogram be $5x$ cm and $3x$ cm, respectively.

Then, its perimeter $= 2(5x + 3x)$ cm

$= 16x$ cm

$\therefore\ 16x = 64$

$\Rightarrow x = \frac{64}{16}$

$\Rightarrow x = 4$

$\therefore$ One side $\Rightarrow (5 \times 4)$ cm $= 20$ cm

Other side $\Rightarrow (3 \times 4)$ cm $= 12$ cm

Q7

Answer :

Let the lengths of two sides of the parallelogram be x cm and $(x + 10)$ cm, respectively.

Then, its perimeter $= 2[x + (x + 10)]$ cm

$= 2[x + x + 10]$ cm

$= 2[2x + 10]$ cm

$= 4x + 20$ cm

$4x + 20 = 140$

$\Rightarrow 4x = 140 - 20$

$\Rightarrow 4x = 120$

$\Rightarrow x = \frac{120}{4}$

$\Rightarrow x = 30$

Length of one side $= 30$ cm

Length of the other side $\Rightarrow (30 + 10)$ cm $= 40$ cm

Q8

Answer :

Refer to the figure given in the book.

In $\triangle BMC$ and $\triangle DNA$:
$\angle DNA = \angle BMC = 90^{\circ}$
$\angle BCM = \angle DAN$ (alternate angles)
$BC = DA$ (opposite sides)
By AAS congruency criteria :
$\triangle BMC \cong \triangle DNA$
Yes, it is true that BM *is equal to* DN.
(*by* corresponding parts of congruent triangles BMC and DNA)

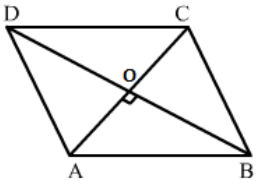
Q9

Refer to the figure of the book.

$\angle A = \angle C$ (opposite angles of *a* parallelogram are equal)
$\Rightarrow \frac{1}{2}\angle A = \frac{1}{2}\angle C$
$=> \angle EAD = \angle FCB$ (AE and CF bisect the angles A and C, respectively)
In $\triangle ADE$ and $\triangle CBF$:
$\angle B = \angle D$ (*o*pposite angles of a parallelogram are equal)
$\angle EAD = \angle FCB$ (*p*roved above)
$AD = BC$ (*o*pposite sides of a parallelogram are equal)
By AAS concruency criteria :
$\triangle ADE \cong \triangle BCF$
$DE = BF$ (*c*orresponding parts of congruent triangles)
$CD = AB$ (*o*pposite sides of a parallelogram are equal)
Also, $CD - DE = AB - BF$
$\Rightarrow CE = AF$
$ABCD$ is a paralleleogram.
$\therefore CD \parallel AB$ (*o*pposite sides of a parallelogram are parallel)
$=> CE \parallel AF$
If one pair of sides of a quadrilateral is parallel and equal, then it is a parallelogram.
Therefore, AECF is a parallelogram.
$\therefore AE \parallel CF$

Q10

Answer :



Let $ABCD$ be a rhombus.
Let AC and BD be the diagonals of the rhombus intersecting at a point O.
Let $AC = 16$ cm
BD $= 12$ cm
We know that the diagonals of a rhombus bisect each other at right angles.
$\therefore AO = \frac{1}{2}AC$
$= \left(\frac{1}{2} \times 16\right)$ cm
$= 8$ cm
$BO = \frac{1}{2}BD$
$= \left(\frac{1}{2} \times 12\right)$ cm
$= 6$ cm

From the right ΔAOB:

$AB^2 = AO^2 + BO^2$

$= \left\{(8)^2 + (6)^2\right\}$ cm^2

$= (64 + 36)$ cm^2

$= 100$ cm^2

$\Rightarrow AB = \sqrt{100}$ cm

$= 10$ cm

Hence, the length of the side AB *is* 10 cm.

$AB = BC = CD = DA = 10$ cm $\quad$ (all sides of a rhombus are equal)

Q11

Answer :

Refer to the figure given in the book.

In ΔADC:

$DA = DC$ $\quad$ (all sides of a square are equal)

$\Rightarrow \angle ACD = \angle CAD$

Let $\angle ACD = \angle CAD = x^\circ$ $\quad$ [Angle opposite to the equal sides are equal]

$x + x + 90 = 180$ $\quad$ [since the sum of the angles of a triangle is 180°]

$\Rightarrow 2x + 90 = 180$

$\Rightarrow 2x = 90$

$\Rightarrow x = \frac{90}{2}$

$\Rightarrow x = 45$

$\therefore \angle CAD = 45^\circ$

Q12

Answer :

Let the length of two sides of the rectangle be $5x$ cm and $4x$ cm, respectively.

Then, its perimeter $= 2(5x + 4x)$ cm

$= 18x$ cm

$\therefore 18x = 90$

$\Rightarrow x = \frac{90}{18}$

$\Rightarrow x = 5$

Length of one side $\Rightarrow (5 \times 5)$ cm $= 25$ cm

Length of the other side $\Rightarrow (4 \times 5)$ cm $= 20$ cm

$\therefore$ Length of the rectangle $= 25$ cm

Breadth $= 20$ cm

Q13

Answer :

(i) The diagonals are equal and the adjacent sides are unequal.
Hence, the given parallelogram is a rectangle.

(ii) The diagonals are equal and the adjacent sides are equal.
Hence, the given parallelogram is a square.

(iii) The diagonals are unequal and the adjacent sides are equal.
Hence, the given parallelogram is a rhombus.

(iv) All the sides are equal and one angle is 60°.
Hence, the given parallelogram is a rhombus.

(v) All the sides are equal and one angle is 90°.
Hence, the given parallelogram is a square.

(vi) All the angles are equal and the adjacent sides are unequal.
Hence, the given parallelogram is a rectangle.

Q14

Answer :

(i) The given statement is false.
The diagonals of a parallelogram bisect each other, but they are not equal in length.
(ii) The given statement is false.
The diagonals of a rectangle are equal and bisect each other, but they are not perpendicular.
(iii) The given statement is false.
All the sides of a rhombus are equal, but the diagonals are not equal.
(iv) The given statement is true.
(v) The given statement is false.
Every square is a rectangle, but every rectangle is not a square.
(vi) The given statement is true.
(vii) The given statement is true.
(viii) The given statement is true.
(ix) The given statement is false.
A rectangle is a special type of parallelogram, but every parallelogram is not a rectangle.
(x) The given statement is true.

Parallelograms Exercise 16B

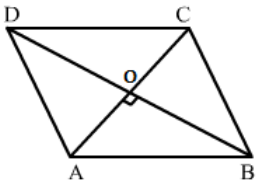
Q1

Answer :

(c) rhombus
In a rhombus, the two diagonals are not necessarily equal.

Q2

(c) 10 cm



Let $ABCD$ be a rhombus.
Let AC and BD be the diagonals of the rhombus intersecting at a point O.
$AC = 16$ cm
$BD = 12$ cm
We know that the diagonals of a rhombus bisect each other at right angles.
$\therefore AO = \frac{1}{2} AC$
$= \left(\frac{1}{2} \times 16\right)$ cm
$= 8$ cm
$BO = \frac{1}{2} BD$
$= \left(\frac{1}{2} \times 12\right)$ cm
$= 6$ cm
From the right ΔAOB :
$AB^2 = AO^2 + BO^2$
$= \left\{(8)^2 + (6)^2\right\}$ cm^2
$= (64 + 36)$ cm^2
$= 100$ cm^2
$\Rightarrow AB = \sqrt{100}$ cm
$= 10$ cm
Hence, the length of the side AB is 10 cm.
Therefore, the length of each side of the rhombus is 10 cm because all the sides of a rhombus are equal.

Q3

Answer :

(b) 32
We know that the sum of adjacent angles of a parallelogram is 180°.
$\Rightarrow 2x + 25 + 3x - 5 = 180$
$\Rightarrow 5x + 20 = 180$
$\Rightarrow 5x = 180 - 20$
$\Rightarrow 5x = 160$
$\Rightarrow x = \frac{160}{5}$
$\Rightarrow x = 32$
Therefore, the value of x is 32.

Q4

Answer :

(a) parallelogram
In a parallelogram, the diagonals do not necessarily intersect at right angles.

Q5

Answer :

(c) 70 cm

Let $ABCD$ be a rectangle and let the diagonal AC be 25 cm, length AB be $4x$ cm and breadth BC be $3x$ cm.
Each angle of a rectangle is a right angle.
$\therefore \angle ABC = 90^\circ$
From the right ΔABC :
$AC^2 = AB^2 + BC^2$
$\Rightarrow (25)^2 = (4x)^2 + (3x)^2$
$\Rightarrow 625 = 16x^2 + 9x^2$
$\Rightarrow 625 = 25x^2$

$x^2 = \frac{625}{25} = 25$
$\Rightarrow x = 5$
$\therefore$ Length $= 4 \times 5 = 20$ cm
Breadth $= 3 \times 5 = 15$ cm

Q6

$\therefore$ Perimeter of the rectangle = 2(20+15) cm
$= 70\ cm$

Answer :

$(d)\ 90^\circ$
The bisectors of any two adjacent angles of a parallelogram intersect at 90°.

Q7

Answer :

$(b)\ 72^\circ$
Let x° be the angle of the parallelogram.
Sum of the adjacent angles of a parallelogram is 180°.
$\therefore x + \left(\frac{2}{3} \times x\right) = 180$
$\Rightarrow x + \frac{2x}{3} = 180$
$\Rightarrow \left(x + \frac{2x}{3}\right) = 180$
$\Rightarrow \frac{5x}{3} = 180$
$\Rightarrow x = \left(180 \times \frac{3}{5}\right)$
$\Rightarrow x = 108$
Hence, one angle of the parallelogram is 108°.
Its adjacent angle $= (180 - 108)^\circ = 72^\circ$
Therefore, the smallest angle of the parallelogram is 72°.

Q8

Answer :

(a) rectangle
In a rectangle, the diagonals do not necessarily bisect the interior angles at the vertices.

Q9

Q10

Answer :

(d) 8
All the sides of a square are equal.
$\therefore AB = BC$
$\Rightarrow 2x + 3 = 3x - 5$
$\Rightarrow 3 + 5 = 3x - 2x$
$\Rightarrow 8 = x$
Therefore, the value of x is 8.

Answer :

$(c)\ 112^\circ$
Let x° be the smallest angle of the parallelogram.
The sum of adjacent angles of a parallelogram is 180°.
$\therefore x + 2x - 24 = 180$
$\Rightarrow 3x - 24 = 180$
$\Rightarrow 3x = 180 + 24$
$\Rightarrow 3x = 204$
$\Rightarrow x = \frac{204}{3}$
$\Rightarrow x = 68$
$\therefore$ Smallest angle $= 68^\circ$
Largest angle $= (180 - 68)^\circ = 112^\circ$