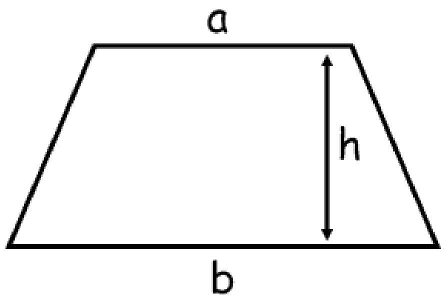
Area of Trapezium and Polygon
Ex 18A



Area of Trapezium = $\frac{1}{2}h(a+b)$

$\therefore$ Area of the trapezium = Area of the rectangle + Area of the triangle

$$= bh + \frac{1}{2}(a-b)h$$
$$= h\left[b + \frac{1}{2}(a-b)\right]$$
$$= h\left[\frac{2b}{2} + \frac{a-b}{2}\right]$$
$$= h\left[\frac{2b+a-b}{2}\right]$$
$$= h\left(\frac{a+b}{2}\right)$$
$$= \begin{pmatrix}\text{Half the sum of} \\ \text{parallel sides}\end{pmatrix} \times \begin{pmatrix}\text{Perpendicular distance} \\ \text{between the parallel sides}\end{pmatrix}$$

Q1.

Answer :

Area of a trapezium $= \frac{1}{2} \times$ (Sum of parallel sides) $\times$ (Distance between them)

$$= \left\{\tfrac{1}{2} \times (24+20) \times 15\right\} \text{ cm}^2$$
$$= \left(\tfrac{1}{2} \times 44 \times 15\right) \text{ cm}^2$$
$$= (22 \times 15) \text{ cm}^2$$
$$= 330 \text{ cm}^2$$

Hence, the area of the trapezium is 330 cm^2.

Q2.

Answer :

Area of a trapezium $= \frac{1}{2} \times$ (Sum of parallel sides) $\times$ (Distance between them)

$$= \left\{\tfrac{1}{2} \times (38.7+22.3) \times 16\right\} \text{ cm}^2$$
$$= \left(\tfrac{1}{2} \times 61 \times 16\right) \text{ cm}^2$$
$$= (61 \times 8) \text{ cm}^2$$
$$= 488 \text{ cm}^2$$

Hence, the area of the trapezium is 488 cm^2.

Q3.

Answer :

Area of a trapezium $= \frac{1}{2} \times$ (Sum of parallel sides) $\times$ (Distance between them)

$$= \left\{\tfrac{1}{2} \times (1+1.4) \times 0.9\right\} \text{ m}^2$$
$$= \left(\tfrac{1}{2} \times 2.4 \times 0.9\right) \text{ m}^2$$
$$= (1.2 \times 0.9) \text{ m}^2$$
$$= 1.08 \text{ m}^2$$

Hence, the area of the top surface of the table is 1.08 m^2.

Q4.

Answer :

Let the distance between the parallel sides be x.
Now,
Area of trapezium $=\left\{\frac{1}{2}\times(55+35)\times x\right\}$ cm^2

$=\left(\frac{1}{2}\times 90\times x\right)cm^2$
$=45x\ cm^2$

Area of the trapezium $=1080$ cm^2 (Given)
$\therefore 45x=1080$
$\Rightarrow x=\frac{1080}{45}$
$\Rightarrow x=24$ cm
Hence, the distance between the parallel sides is 24 cm.

Q5.
Answer :

Let the length of the required side be x cm.
Now,
Area of trapezium $=\left\{\frac{1}{2}\times(84+x)\times 26\right\}$ m^2

$=(1092+13x)$ m^2

Area of trapezium $=1586$ m^2 (Given)
$\therefore 1092+13x=1586$
$\Rightarrow 13x=(1586-1092)$
$\Rightarrow 13x=494$
$\Rightarrow x=\frac{494}{13}$
$\Rightarrow x=38$ m
Hence, the length of the other side is 38 m.

Q6.
Answer :

Let the lengths of the parallel sides of the trapezium be $4x$ cm and $5x$ cm, respectively.
Now,
Area of trapezium $=\left\{\frac{1}{2}\times(4x+5x)\times 18\right\}$ cm^2
$=\left(\frac{1}{2}\times 9x\times 18\right)cm^2$
$=81x\ cm^2$
Area of trapezium $=405$ cm^2 (Given)
$\therefore 81x=405$
$\Rightarrow x=\frac{405}{81}$
$\Rightarrow x=5$ cm
Length of one side $=(4\times 5)$ cm $=20$ cm
Length of the other side $=(5\times 5)$ cm $=25$ cm

Q7.

Answer :

Let the lengths of the parallel sides be x cm and $(x+6)$ cm.
Now,

$$\begin{aligned}\text{Area of trapezium} &= \left\{\tfrac{1}{2}\times(x+x+6)\times 9\right\}\ \text{cm}^2\\ &= \left(\tfrac{1}{2}\times(2x+6)\times 9\right)\ \text{cm}^2\\ &= 4.5(2x+6)\ \text{cm}^2\\ &= (9x+27)\ \text{cm}^2\end{aligned}$$

Area of trapezium $= 180\ \text{cm}^2$ (Given)
$\therefore 9x+27=180$
$\Rightarrow 9x=(180-27)$
$\Rightarrow 9x=153$
$\Rightarrow x=\frac{153}{9}$
$\Rightarrow x=17$
Hence, the lengths of the parallel sides are 17 cm and 23 cm, that is, $(17+6)$ cm.

Q8.

Answer :

Let the lengths of the parallel sides be x cm and $2x$ cm.

$$\begin{aligned}\text{Area of trapezium} &= \left\{\tfrac{1}{2}\times(x+2x)\times 84\right\}\ \text{m}^2\\ &= \left(\tfrac{1}{2}\times 3x\times 84\right)\ \text{m}^2\\ &= (42\times 3x)\ \text{m}^2\\ &= 126x\ \text{m}^2\end{aligned}$$

Area of the trapezium $= 9450\ \text{m}^2$ (Given)
$\therefore 126x=9450$
$\Rightarrow x=\frac{9450}{126}$
$\Rightarrow x=75$
Thus, the length of the parallel sides are 75 m and 150 m, that is, (2×75) m, and the length of the longer side is 150 m.

Q9.

Answer :

$$\begin{aligned}\text{Length of the side AB} &= (130-(54+19+42))\ \text{m}\\ &= 15\ \text{m}\end{aligned}$$

$$\begin{aligned}\text{Area of the trapezium}-\text{shaped field} &= \left\{\tfrac{1}{2}\times(\text{AD}+\text{BC})\times\text{AB}\right\}\\ &= \left\{\tfrac{1}{2}\times(42+54)\times 15\right\}\ \text{m}^2\\ &= \left(\tfrac{1}{2}\times 96\times 15\right)\ \text{m}^2\\ &= (48\times 15)\ \text{m}^2\\ &= 720\ \text{m}^2\end{aligned}$$

Hence, the area of the field is $720\ \text{m}^2$.

Q10.

Answer :

$\angle ABC = 90^\circ$

From the right $\triangle ABC$, we have :

$AB^2 = \left(AC^2 - BC^2\right)$

$\Rightarrow AB^2 = \left\{\left(41^2\right) - \left(40^2\right)\right\}$

$\Rightarrow AB^2 = (1681 - 1600)$

$\Rightarrow AB^2 = 81$

$\Rightarrow AB = \sqrt{81}$

$\Rightarrow AB = 9$ cm

$\therefore$ Length $AB = 9$ cm

Now,

Area of the trapezium $= \left\{\frac{1}{2} \times (AD + BC) \times AB\right\}$

$= \left(\frac{1}{2} \times (16 + 40) \times 9\right)$ cm^2

$= \left(\frac{1}{2} \times 56 \times 9\right)$ cm^2

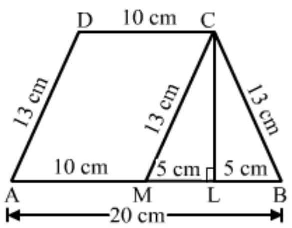
$= (28 \times 9)$ cm^2

$= 252$ cm^2

Hence, the area of the trapezium is 252 cm^2.

Q11.

Answer :



Let ABCD be the given trapezium in which $AB \parallel DC$, $AB = 20$ cm, $DC = 10$ cm and $AD = BC = 13$ cm.

Draw $CL \perp AB$ and $CM \parallel DA$ meeting AB at L and M, respectively.

Clearly, AMCD is a parallelogram.

Now,

$AM = DC = 10$ cm

$MB = (AB - AM)$

$= (20 - 10)$ cm

$= 10$ cm

Also,

$CM = DA = 13$ cm

Therefore, $\triangle CMB$ is an isosceles triangle and $CL \perp MB$.

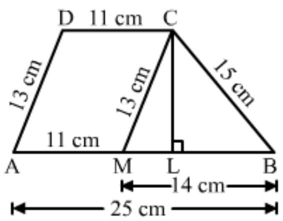
L is the midpoint of B.

$\Rightarrow ML = LB = \left(\frac{1}{2} \times MB\right)$
$= \left(\frac{1}{2} \times 10\right)$ cm
$= 5$ cm

From right $\triangle CLM$, we have :
$CL^2 = \left(CM^2 - ML^2\right)$ cm^2
$\Rightarrow CL^2 = \left\{(13)^2 - (5)^2\right\}$ cm^2
$\Rightarrow CL^2 = (109 - 25)$ cm^2
$\Rightarrow CL^2 = 144$ cm^2
$\Rightarrow CL = \sqrt{144}$ cm
$\Rightarrow CL = 12$ cm
$\therefore$ Length of $CL = 12$ cm
Area of the trapezium $= \left\{\frac{1}{2} \times (AB + DC) \times CL\right\}$
$= \left\{\frac{1}{2} \times (20 + 10) \times 12\right\}$ cm^2
$= \left(\frac{1}{2} \times 30 \times 12\right)$ cm^2
$= (15 \times 12)$ cm^2
$= 180$ cm^2
Hence, the area of the trapezium is 180 cm^2.

Q12

Answer :



Let ABCD be the trapezium in which $AB \parallel DC$, $AB = 25$ cm, $CD = 11$ cm, $AD = 13$ cm and $BC = 15$ cm.
Draw $CL \perp AB$ and $CM \parallel DA$ meeting AB at L and M, respectively.
Clearly, AMCD is a parallelogram.
Now,
$MC = AD = 13$ cm
$AM = DC = 11$ cm
$\Rightarrow MB = (AB - AM)$
$= (25 - 11)$ cm
$= 14$ cm

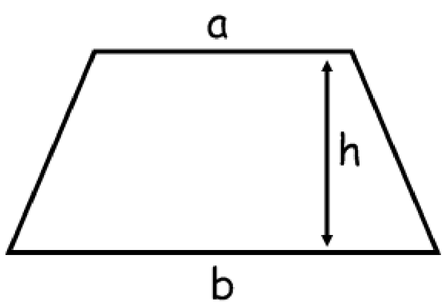
Thus, in $\triangle CMB$, we have :
$CM = 13$ cm
$MB = 14$ cm
$BC = 15$ cm
$\therefore s = \frac{1}{2}(13 + 14 + 15)$ cm
$= \frac{1}{2} 42$ cm
$= 21$ cm
$(s - a) = (21 - 13)$ cm
$= 8$ cm
$(s - b) = (21 - 14)$ cm
$= 7$ cm
$(s - c) = (21 - 15)$ cm
$= 6$ cm
$\therefore$ Area of $\triangle CMB = \sqrt{s(s-a)(s-b)(s-c)}$
$= \sqrt{21 \times 8 \times 7 \times 6}$ cm^2
$= 84$ cm^2

$\therefore \frac{1}{2} \times MB \times CL = 84$ cm^2
$\Rightarrow \frac{1}{2} \times 14 \times CL = 84$ cm^2
$\Rightarrow CL = \frac{84}{7}$
$\Rightarrow CL = 12$ cm

$$\text{Area of the trapezium} = \left\{ \frac{1}{2} \times (\text{AB} + \text{DC}) \times \text{CL} \right\}$$
$$= \left\{ \frac{1}{2} \times (25 + 11) \times 12 \right\} \text{ cm}^2$$
$$= \left(\frac{1}{2} \times 36 \times 12 \right) \text{ cm}^2$$
$$= (18 \times 12) \text{ cm}^2$$
$$= 216 \text{ cm}^2$$

Hence, the area of the trapezium is 216 cm^2.

Area of Trapezium and Polygon
Ex 18B



Area of Trapezium $= \frac{1}{2}h(a+b)$

$\therefore$ Area of the trapezium = Area of the rectangle + Area of the triangle

$$= bh + \frac{1}{2}(a-b)h$$
$$= h\left[b + \frac{1}{2}(a-b)\right]$$
$$= h\left[\frac{2b}{2} + \frac{a-b}{2}\right]$$
$$= h\left[\frac{2b+a-b}{2}\right]$$
$$= h\left(\frac{a+b}{2}\right)$$
$$= \left(\begin{array}{l}\text{Half the sum of}\\ \text{parallel sides}\end{array}\right) \times \left(\begin{array}{l}\text{Perpendicular distance}\\ \text{between the parallel sides}\end{array}\right)$$

Q1.

Answer :

$$\begin{aligned}\text{Area of quadrilateral ABCD} &= (\text{Area of } \triangle\text{ADC}) + (\text{Area of } \triangle\text{ACB})\\ &= \left(\tfrac{1}{2}\times \text{AC}\times\text{DM}\right) + \left(\tfrac{1}{2}\times\text{AC}\times\text{BL}\right)\\ &= \left[\left(\tfrac{1}{2}\times 24\times 7\right) + \left(\tfrac{1}{2}\times 24\times 8\right)\right]\ \text{cm}^2\\ &= (84+96)\ \text{cm}^2\\ &= 180\ \text{cm}^2\end{aligned}$$

Hence, the area of the quadrilateral is $180\ \text{cm}^2$.

Q2.

Answer :

$$\begin{aligned}\text{Area of quadrilateral ABCD} &= (\text{Area of } \triangle\text{ABD}) + (\text{Area of } \triangle\text{BCD})\\ &= \left(\tfrac{1}{2}\times \text{BD}\times\text{AL}\right) + \left(\tfrac{1}{2}\times\text{BD}\times\text{CM}\right)\\ &= \left[\left(\tfrac{1}{2}\times 36\times 19\right) + \left(\tfrac{1}{2}\times 36\times 11\right)\right]\ \text{m}^2\\ &= (342+198)\ \text{m}^2\\ &= 540\ \text{m}^2\end{aligned}$$

Hence, the area of the field is $540\ \text{m}^2$.

Q3.

Answer :

Area of pentagon ABCDE $=$ (Area of $\triangle$AEN) $+$ (Area of trapezium EDMN) $+$ (Area of $\triangle$DMC) $+$ (Area of $\triangle$ACB)

$$= \left(\tfrac{1}{2}\times\text{AN}\times\text{EN}\right) + \left(\tfrac{1}{2}\times(\text{EN}+\text{DM})\times\text{NM}\right) + \left(\tfrac{1}{2}\times\text{MC}\times\text{DM}\right) + \left(\tfrac{1}{2}\times\text{AC}\times\text{BL}\right)$$
$$= \left(\tfrac{1}{2}\times\text{AN}\times\text{EN}\right) + \left(\tfrac{1}{2}\times(\text{EN}+\text{DM})\times(\text{AM}-\text{AN})\right) + \left(\tfrac{1}{2}\times(\text{AC}-\text{AM})\times\text{DM}\right) + \left(\tfrac{1}{2}\times\text{AC}\times\text{BL}\right)$$
$$= \left[\left(\tfrac{1}{2}\times 6\times 9\right) + \left(\tfrac{1}{2}\times(9+12)\times(14-6)\right) + \left(\tfrac{1}{2}\times(18-14)\times 12\right) + \left(\tfrac{1}{2}\times 18\times 4\right)\right]\ \text{cm}^2$$
$$= (27+84+24+36)\ \text{cm}^2$$
$$= 171\ \text{cm}^2$$

Hence, the area of the given pentagon is $171\ \text{cm}^2$.

Q4.

Answer :

Area of hexagon ABCDEF $=$ (Area of $\triangle$ AFP) $+$ (Area of trapezium FENP) $+$ (Area of $\triangle$ ALB)

$$= \left(\tfrac{1}{2} \times AP \times FP\right) + \left(\tfrac{1}{2} \times (FP + EN) \times PN\right) + \left(\tfrac{1}{2} \times ND \times EN\right) + \left(\tfrac{1}{2} \times MD \times CM\right) + \left(\tfrac{1}{2} \times (CM + BL) \times LM\right) + \left(\tfrac{1}{2} \times AL \times BL\right)$$

$$= \left(\tfrac{1}{2} \times AP \times FP\right) + \left(\tfrac{1}{2} \times (FP + EN) \times (PL + LN)\right) + \left(\tfrac{1}{2} \times (NM + MD) \times CM\right) + \left(\tfrac{1}{2} \times MD \times CM\right) + \left(\tfrac{1}{2} \times (CM + BL) \times (LN + NM)\right) + \left(\tfrac{1}{2} \times (AP + PL) \times BL\right)$$

$$= \left[\left(\tfrac{1}{2} \times 6 \times 8\right) + \left(\tfrac{1}{2} \times (8 + 12) \times (2 + 8)\right) + \left(\tfrac{1}{2} \times (2 + 3) \times 12\right) + \left(\tfrac{1}{2} \times 3 \times 6\right) + \left(\tfrac{1}{2} \times (6 + 8) \times (8 + 2)\right) + \left(\tfrac{1}{2} \times (6 + 2) \times 8\right)\right] \text{ cm}^2$$

$$= (24 + 100 + 30 + 9 + 70 + 32) \text{ cm}^2$$

$$= 265 \text{ cm}^2$$

Hence, the area of the hexagon is 265 cm^2.

Q5.

Answer :

Area of pentagon ABCDE $=$ (Area of $\triangle$ ABC) $+$ (Area of $\triangle$ ACD) $+$ (Area of $\triangle$ ADE)

$$= \left(\tfrac{1}{2} \times AC \times BL\right) + \left(\tfrac{1}{2} \times AD \times CM\right) + \left(\tfrac{1}{2} \times AD \times EM\right)$$

$$= \left[\left(\tfrac{1}{2} \times 10 \times 3\right) + \left(\tfrac{1}{2} \times 12 \times 7\right) + \left(\tfrac{1}{2} \times 12 \times 5\right)\right] \text{ cm}^2$$

$$= (15 + 42 + 30) \text{ cm}^2$$

$$= 87 \text{ cm}^2$$

Hence, the area of the pentagon is 87 cm^2.

Q6.

Answer :

Area enclosed by the given figure $=$ (Area of trapezium FEDC) $+$ (Area of square ABCF)

$$= \left[\left\{\tfrac{1}{2} \times (6 + 20) \times 8\right\} + (20 \times 20)\right] \text{cm}^2$$

$$= (104 + 400) \text{cm}^2$$

$$= 504 \text{ cm}^2$$

Hence, the area enclosed by the figure is 504 cm^2.

Q7.

Answer :

We will find the length of AC.

From the right triangles ABC and HGF, we have :

$$AC^2 = HF^2 = \left\{(5)^2 - (4)^2\right\} \text{ cm}$$

$$= (25 - 16) cm$$

$$= 9 \ cm$$

$$AC = HF = \sqrt{9} \ cm$$

$$= 3 \ cm$$

Area of the given figure ABCDEFGH $=$ (Area of rectangle ADEH) $+$ (Area of $\triangle$ ABC) $+$ (Area of $\triangle$ HGF)

$$= (\text{Area of rectangle ADEH}) + 2(\text{Area of } \triangle \text{ ABC})$$

$$= (AD \times DE) + 2(\text{Area of } \triangle \text{ ABC})$$

$$= \{(AC + CD) \times DE\} + 2\left(\tfrac{1}{2} \times BC \times AC\right)$$

$$= \{(3 + 4) \times 8\} + 2\left(\tfrac{1}{2} \times 4 \times 3\right) \text{ cm}^2$$

$$= (56 + 12) \text{ cm}$$

$$= 68 \text{ cm}^2$$

Hence, the area of the given figure is 68 cm^2.

Q8.

Answer :

Let $\mathrm{AL} = \mathrm{DM} = \mathrm{x}$ cm
$\mathrm{LM} = \mathrm{BC} = 13$ cm
$\therefore\ x + 13 + x = 23$
$\Rightarrow 2x + 13 = 23$
$\Rightarrow 2x = (23 - 13)$
$\Rightarrow 2x = 10$
$\Rightarrow x = 5$
$\therefore\ \mathrm{AL} = 5$ cm
From the right $\Delta\,\mathrm{AFL}$, we have :
$FL^2 = AF^2 - AL^2$
$\Rightarrow FL^2 = \left\{\left(13^2\right) - (5)^2\right\}$
$\Rightarrow FL^2 = (169 - 25)$
$\Rightarrow FL^2 = 144$
$\Rightarrow FL = \sqrt{144}$
$\Rightarrow FL = 12\ cm$
$\therefore\ \mathrm{FL} = \mathrm{BL} = 12$ cm
Area of a regular hexagon $=$ ($Area$ of the trapezium ADEF)

Area of a regular hexagon $=$ ($Area$ of the trapezium ADEF)
$+$ ($Area$ of the trapezium ABCD)

$$= 2(Area\ of\ trapezium\ ADEF)$$
$$= 2\left\{\tfrac{1}{2} \times (AD + EF) \times FL\right\}$$
$$= 2\left\{\tfrac{1}{2} \times (23 + 13) \times 12\right\} cm^2$$
$$= 2\left(\tfrac{1}{2} \times 36 \times 12\right) cm^2$$
$$= 432\ cm^2$$

Hence, the area of the given regular hexagon is $432\ \mathrm{cm}^2$.

Area of Trapezium and Polygon
Ex 18C

Q1.

Answer :

(b) 144 cm^2

$$\text{Area of the trapezium} = \left\{\frac{1}{2} \times (14+18) \times 9\right\} \text{ cm}^2$$
$$= \left(\frac{1}{2} \times 32 \times 9\right) cm^2$$
$$= 144\ cm^2$$

Q2.

Answer :

(c) 8 cm

Let the distance between the parallel sides be x cm.

$$\text{Then, area of the trapezium} = \left\{\frac{1}{2} \times (19+13) \times \mathbf{x}\right\} \text{ cm}^2$$
$$= \left(\frac{1}{2} \times 32 \times x\right) \text{ cm}^2$$
$$= 16x \text{ cm}^2$$

But it is given that the area of the trapezium is 128 cm^2.

$\therefore\ 16x = 128$

$\Rightarrow x = \frac{128}{16}$

$\Rightarrow x = 8$ cm

Q3.

Answer :

(a) 45 cm

Let the length of the parallel sides be 3x cm and 4x cm, respectively.

Then, area of the trapezium $= \left\{\frac{1}{2} \times (3x + 4x) \times 12\right\}$ cm^2

$= \left(\frac{1}{2} \times 7x \times 12\right) cm^2$

$= 42\,x\;cm^2$

But it is given that the area of the trapezium is 630 cm^2.

$\therefore\; 42x = 630$

$\Rightarrow x = \frac{630}{42}$

$\Rightarrow x = 15\;cm$

Length of the parallel sides $= (3 \times 15)$cm $= 45$ cm

(4×15) cm $= 60$ cm

Hence, the shorter of the parallel sides is 45 cm.

Q4.

Answer :

(b) 23 cm

Let the length of the parallel sides be x cm and $(x + 6)$ cm, respectively.

Then, area of the trapezium $= \left\{\frac{1}{2} \times (x + x + 6) \times 9\right\}$ cm^2

$= \left\{\frac{1}{2} \times (2x + 6) \times 9\right\}$ cm^2

$= 4.5(2x + 6)$ cm^2

$= (9x + 27)$ cm^2

But it is given that the area of the trapezium is 180 cm^2.

$\therefore\; 9x + 27 = 180$

$\Rightarrow 9x = (180 - 27)$

$\Rightarrow 9x = 153$

$\Rightarrow x = \frac{153}{9}$

$\Rightarrow x = 17$

Therefore, the length of the parallel sides are 17 cm and $(17 + 6)$ cm, which is equal to 23 cm.

Hence, the length of the longer parallel side is 23 cm.

Q5.

Answer :

(c) 80 cm^2

From the given trapezium, we find :

$DC = AL = 7\;cm$ [since $DA \perp AB$ and $CL \perp AB$]

From the right $\triangle$CBL, we have :

$CL^2 = CB^2 - LB^2$

$\Rightarrow CL^2 = (10)^2 - (6)^2$

$\Rightarrow CL^2 = 100 - 36$

$\Rightarrow CL^2 = 64$

$\Rightarrow CL = \sqrt{64}$

$\Rightarrow CL = 8\;cm$

Area of the trapezium $= \left\{\frac{1}{2} \times (7 + 13) \times 8\right\}$ cm^2

$= \left(\frac{1}{2} \times 20 \times 8\right)$ cm^2

$= 80$ cm^2