Exponents Exercise 2A

Q1

Answer :

(i) $4^{-3} = \frac{1}{4^3} = \frac{1}{64}$

(ii) $\left(\frac{1}{2}\right)^{-5} = 2^5 = 32$

(iii) $\left(\frac{4}{3}\right)^{-3} = \left(\frac{3}{4}\right)^{3} = \frac{3^3}{4^3} = \frac{27}{64}$

(iv) $(-3)^{-4} = \left(\frac{-1}{3}\right)^{4} = \frac{(-1)^4}{3^4} = \frac{1}{81}$

(v) $\left(\frac{-2}{3}\right)^{-5} = \left(\frac{-3}{2}\right)^{5} = \frac{(-3)^5}{2^5} = \frac{-243}{32}$

Q2

Answer :

(i) $\left(\frac{5}{3}\right)^{2} \times \left(\frac{5}{3}\right)^{2} = \left(\frac{5}{3}\right)^{4} = \frac{5^4}{3^4} = \frac{625}{81}$

(ii) $\left(\frac{5}{6}\right)^{6} \times \left(\frac{5}{6}\right)^{-4} = \left(\frac{5}{6}\right)^{(6+(-4))} = \left(\frac{5}{6}\right)^{(6-4)} = \left(\frac{5}{6}\right)^{2} = \frac{5^2}{6^2} = \frac{25}{36}$

(iii) $\left(\frac{2}{3}\right)^{-3} \times \left(\frac{2}{3}\right)^{-2} = \left(\frac{2}{3}\right)^{(-3-2)} = \left(\frac{2}{3}\right)^{-5} = \left(\frac{3}{2}\right)^{5} = \frac{3^5}{2^5} = \frac{243}{32}$

(iv) $\left(\frac{9}{8}\right)^{-3} \times \left(\frac{9}{8}\right)^{2} = \left(\frac{9}{8}\right)^{(-3+2)} = \left(\frac{9}{8}\right)^{-1} = \frac{8}{9}$

Q3

Answer :

(i)

$$\left(\frac{5}{9}\right)^{-2} \times \left(\frac{3}{5}\right)^{-3} \times \left(\frac{3}{5}\right)^{0} = \left(\frac{5}{9}\right)^{-2} \times \left(\frac{3}{5}\right)^{-3+0}$$
$$= \left(\frac{5}{9}\right)^{-2} \times \left(\frac{3}{5}\right)^{-3} = \left(\frac{9}{5}\right)^{2} \times \left(\frac{5}{3}\right)^{3}$$
$$= \frac{9^2}{5^2} \times \frac{5^3}{3^3}$$
$$= \frac{\left(3^2\right)^2}{5^2} \times \frac{5^3}{3^3}$$
$$= \frac{3^4}{5^2} \times \frac{5^3}{3^3} = \left(3^{(4-3)}\right) \times \left(5^{(3-2)}\right) = 3 \times 5 = 15$$

(ii)

$$\left(\frac{-3}{5}\right)^{-4} \times \left(\frac{-2}{5}\right)^{2} = \left(\frac{5}{-3}\right)^{4} \times \left(\frac{-2}{5}\right)^{2}$$
$$= \frac{5^4}{-3^4} \times \frac{-2^2}{5^2} = 5^{(4-2)} \times \frac{-2^2}{-3^4} = 5^2 \times \frac{-2^2}{-3^4}$$
$$= 25 \times \frac{4}{81} = \frac{100}{81}$$

(iii)

$$\left(\frac{-2}{3}\right)^{-3} \times \left(\frac{-2}{3}\right)^{-2} = \left(\frac{3}{-2}\right)^{3} \times \left(\frac{3}{-2}\right)^{2}$$
$$= \frac{3^3}{-2^3} \times \frac{3^2}{-2^2} = \frac{3^{(3+2)}}{-2^{(3+2)}} = \frac{3^5}{-2^5} = \frac{-243}{32}$$

Q4

Answer :

(i) $\left\{\left(\frac{-2}{3}\right)^{2}\right\}^{-2} = \left(\frac{-2}{3}\right)^{2\times(-2)} = \left(\frac{-2}{3}\right)^{-4} = \left(\frac{3}{-2}\right)^{4} = \frac{3^4}{(-2)^4} = \frac{3^4}{2^4} = \frac{81}{16}$

(ii)
$$\left[\left\{\left(\frac{-1}{3}\right)^{2}\right\}^{-2}\right]^{-1} = \left[\left(\frac{-1}{3}\right)^{2\times(-2)}\right]^{-1} = \left[\left(\frac{-1}{3}\right)^{-4}\right]^{-1} = \left(\frac{-1}{3}\right)^{-4\times-1} = \left(\frac{-1}{3}\right)^{4} = \frac{-1^4}{3^4} = \frac{1^4}{3^4}$$
$$= \frac{1}{81}$$

(iii) $\left\{\left(\frac{3}{2}\right)^{-2}\right\}^{2} = \left(\frac{3}{2}\right)^{-2\times2} = \left(\frac{3}{2}\right)^{-4} = \left(\frac{2}{3}\right)^{4} = \frac{2^4}{3^4} = \frac{16}{81}$

Q5

Answer :

$$\left\{\left(\frac{1}{3}\right)^{-3} - \left(\frac{1}{2}\right)^{-3}\right\} \div \left(\frac{1}{4}\right)^{-3} = \left\{3^3 - 2^3\right\} \div 4^3 = \{27 - 8\} \div 64 = \frac{19}{64}$$

Q6

Answer :

$$\left\{\left(\frac{4}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1}\right\}^{-1} = \left\{\left(\frac{3}{4}\right)^{1} - \left(\frac{4}{1}\right)^{1}\right\}^{-1} = \left\{\left(\frac{3}{4}\right) - \left(\frac{4}{1}\right)\right\}^{-1}$$
The L.C.M. of 4 and 1 is 4.
$$\therefore \left\{\left(\frac{3\times1}{4\times1}\right) - \left(\frac{4\times4}{1\times4}\right)\right\}^{-1}$$
$$= \left\{\frac{3}{4} - \frac{16}{4}\right\}^{-1} = \left\{\frac{3-16}{4}\right\}^{-1} = \left\{\frac{-13}{4}\right\}^{-1} = \left\{\frac{4}{-13}\right\}^{1} = \frac{4}{-13}$$
$$= \frac{4\times-1}{-13\times-1} = \frac{-4}{13}$$

Q7

Answer :

$$\left[\left(5^{-1} \times 3^{-1}\right)^{-1} \div 6^{-1}\right] = \left[\left(\frac{1}{5} \times \frac{1}{3}\right)^{-1} \div \frac{1}{6}\right] = \left[\left(\frac{1}{15}\right)^{-1} \div \frac{1}{6}\right] = [15 \times 6] = 90$$

Q8

Answer :

(i)

$\left(2^0+3^{-1}\right)\times 3^2=\left(1+\frac{1}{3}\right)\times 3^2\ \ \left(\text{because } 2^0=1 \text{ and } 3^{-1}=\frac{1}{3}\right)$

$=\left(\frac{1\times 3}{1\times 3}+\frac{1\times 1}{3\times 1}\right)\times 3^2=\left(\frac{3}{3}+\frac{1}{3}\right)\times 3^2=\left(\frac{4}{3}\right)\times 3^2=4\times 3^{(2-1)}=4\times 3=12$

(ii)

$\left(2^{-1}\times 3^{-1}\right)\div 2^{-3}=\left(\frac{1}{2}\times\frac{1}{3}\right)\div\left(\frac{1}{2}\right)^3$

$\left(\frac{1}{6}\right)\div\frac{1^3}{2^3}=\left(\frac{1}{6}\right)\div\left(\frac{1}{8}\right)=\frac{1}{6}\times 8=\frac{8}{6}=\frac{4}{3}$

(iii)

$\left(\frac{1}{2}\right)^{-2}+\left(\frac{1}{3}\right)^{-2}+\left(\frac{1}{4}\right)^{-2}=\left(\frac{2}{1}\right)^2+\left(\frac{3}{1}\right)^2+\left(\frac{4}{1}\right)^2=2^2+3^2+4^2=4+9+16=29$

Q9

Answer :

Consider the left side:

$\left(\frac{5}{3}\right)^{-4}\times\left(\frac{5}{3}\right)^{-5}=\left(\frac{5}{3}\right)^{(-4+(-5))}=\left(\frac{5}{3}\right)^{-9}$

Given:

$\left(\frac{5}{3}\right)^{-9}=\left(\frac{5}{3}\right)^{3x}$

Comparing the powers:

$-9=3x\Rightarrow\ x=-3$

Q10

Answer :

Given:

$\left(\frac{4}{9}\right)^4\times\left(\frac{4}{9}\right)^{-7}=\left(\frac{4}{9}\right)^{2x-1}$

$\therefore\left(\frac{4}{9}\right)^{(4-7)}=\left(\frac{4}{9}\right)^{-3}=\left(\frac{4}{9}\right)^{2x-1}$

$\Rightarrow\ 2x-1=-3$

$2x=-3+1=-2$

$\Rightarrow x=-1$

Q11

Answer :

Let the required number be x.

$\therefore x\times\left(-6\right)^{-1}=9^{-1}$

$x\times\frac{1}{-6}=\frac{1}{9}\Rightarrow\frac{x}{-6}=\frac{1}{9}$

$\text{or}\ \ x=\frac{-6}{9}$

The greatest common divisor for the numerator and the denominator is 3.

$\therefore\ x=\frac{-6}{9}=\frac{(-6)\div 3}{9\div 3}=\frac{-2}{3}$

Q12

Answer :

Let the number be x.

$$\therefore \left(\frac{-2}{3}\right)^{-3} \div x = \left(\frac{4}{27}\right)^{-2}$$
$$\Rightarrow \left(\frac{3}{-2}\right)^{3} \div x = \left(\frac{27}{4}\right)^{2}$$
$$\Rightarrow \left(\frac{-3}{2}\right)^{3} \div x = \left(\frac{27}{4}\right)^{2}$$
$$\Rightarrow \left(\frac{-3}{2}\right)^{3} \times \frac{1}{x} = \left(\frac{27}{4}\right)^{2}$$
$$\Rightarrow \frac{-3^3}{2^3} \times \frac{1}{x} = \frac{27^2}{4^2}$$
$$\Rightarrow \frac{-27}{8} \times \frac{1}{x} = \frac{27^2}{4^2} = \frac{27\times27}{4\times4} = \frac{27\times27}{4\times2\times2} = \frac{27\times27}{8\times2}$$
$$\therefore \frac{1}{x} = \left(\frac{27\times27}{8\times2}\right) \Big/ \left(\frac{-27}{8}\right)$$
$$\Rightarrow x = \frac{\left(\frac{-27}{8}\right)}{\left(\frac{27\times27}{8\times2}\right)} = \left(\frac{-27}{8}\right) \times \left(\frac{8\times2}{27\times27}\right) = \frac{-2}{27}$$

Q13

Answer :

Given:

$5^{2x+1} \div 25 = 125$

We know :

$25 = 5 \times 5 = 5^2$

$125 = 5 \times 5 \times 5 = 5^3$

$\therefore \frac{5^{2x+1}}{5^2} = 5^3 \Rightarrow 5^{[(2x+1)-2]} = 5^3$

or $5^{[(2x+1)-2]} = 5^{[2x-1]} = 5^3$

$\Rightarrow 2x - 1 = 3$

$2x = 3 + 1 = 4$

$x = \frac{4}{2} = 2$

$\therefore x = 2$

Exponents Exercise 2B

Q1

Answer :

(i) $57.36 = 5.736 \times 10$
(ii) $3500000 = 3.5 \times 10^6$
(iii) $273000 = 2.73 \times 10^5$
(iv) $168000000 = 1.68 \times 10^8$
(v) $4630000000000 = 4.63 \times 10^{12}$
(vi) $345 \times 10^5 = 3.45 \times 10^7$

Q2

Answer :

(i) $3.74 \times 10^5 = \frac{374}{100} \times 10^5 = \frac{374 \times 10^5}{10^2} = 374 \times 10^{(5-2)} = 374 \times 10^3 = 374000$
(ii) $6.912 \times 10^8 = \frac{6912}{1000} \times 10^8 = \frac{6912 \times 10^8}{10^3} = 6912 \times 10^{(8-3)} = 6912 \times 10^5 = 691200000$
(iii) $4.1253 \times 10^7 = \frac{41253}{10000} \times 10^7 = \frac{41253 \times 10^7}{10^4} = 41253 \times 10^{(7-4)} = 41253 \times 10^3 = 41253000$
(iv) $2.5 \times 10^4 = \frac{25}{10} \times 10^4 = \frac{25 \times 10^4}{10} = 25 \times 10^{(4-1)} = 25 \times 10^3 = 25000$
(v) $5.17 \times 10^6 = \frac{517}{100} \times 10^6 = \frac{517 \times 10^6}{10^2} = 517 \times 10^{(6-2)} = 517 \times 10^4 = 5170000$
(vi) $1.679 \times 10^9 = \frac{1679}{1000} \times 10^9 = \frac{1679 \times 10^9}{10^3} = 1679 \times 10^{(9-3)} = 1679 \times 10^6 = 1679000000$

Q3

Answer :

(i) The height of the Mount Everest is 8848 m.
In standard form, we have:
$8848 = 8.848 \times 1000\ \mathbf{m} = 8.848 \times 10^3 \mathbf{m}$
(ii) The speed of light is 300000000 m/s.
In standard form, we have:
$300000000 = 3 \times 100000000\ \mathbf{m/s} = 3 \times 10^8\ \mathbf{m/s}$
(iii) The Sun—Earth distance is 149600000000 m.
In standard form, we have:
$149600000000 = 1496 \times 100000000 = 1.496 \times 1000 \times 100000000 = 1.496 \times 10^3 \times 10^8 = 1.496 \times 10^{11} \mathbf{m}$

Q4

Answer :

Mass of the Earth = 5.97×10^{24} kg
Now, $5.97 \times 10^{24} = 5.97 \times 10^{(2+22)} = 5.97 \times 10^{2} \times 10^{22} = 597 \times 10^{22}$
So, the mass of the Earth can also be written as 597×10^{22} kg.

Mass of the Moon = 7.35×10^{22} kg
Sum of the masses of the Earth and the Moon:
$= \left(597 \times 10^{22}\right) + \left(7.35 \times 10^{22}\right) = (597 + 7.35) \times 10^{22} = 604.35 \times 10^{22}$ kg
$= 6.0435 \times 100 \times 10^{22} = 6.0435 \times 10^{2} \times 10^{22} = 6.0435 \times 10^{(2+22)} = 6.0435 \times 10^{24}$ kg

Q5

Answer :

(i) $0.0006 = \frac{6}{10^4} = 6 \times 10^{-4}$

(ii) $0.00000083 = \frac{83}{10^8} = \frac{8.3 \times 10}{10^8} = 8.3 \times 10^{(1-8)} = 8.3 \times 10^{-7}$

(iii) $0.0000000534 = \frac{534}{10^{10}} = \frac{5.34 \times 10^2}{10^{10}} = 5.34 \times 10^{(2-10)} = 5.34 \times 10^{-8}$

(iv) $0.0027 = \frac{27}{10^4} = \frac{2.7 \times 10}{10^4} = 2.7 \times 10^{(1-4)} = 2.7 \times 10^{-3}$

(v) 0.00000165 = $\frac{165}{10^8} = \frac{1.65 \times 10^2}{10^8} = 1.65 \times 10^{(2-8)}$ = 1.65×10^{-6}

(vi) 0.00000000689 = $\frac{689}{10^{11}} = \frac{6.89 \times 10^2}{10^{11}} = 6.89 \times 10^{(2-11)}$= 6.89×10^{-9}

Q6

Answer :

(i) 1 micron $= \frac{1}{1000000}$ m $= 1 \times 10^{-6}$ m

(ii) 0.0000004 m $= \frac{4}{10^7}$ m $= \left(4 \times 10^{-7}\right)$ m

(iii) Thickness of paper $= 0.03$ mm $= \frac{3}{10^2}$ mm $= \left(3 \times 10^{-2}\right)$ mm

Q7

Answer :

(i) $2.06 \times 10^{-5} = \frac{206}{100} \times \frac{1}{10^5} = \frac{206}{10^2 \times 10^5} = \frac{206}{10^{(5+2)}} = \frac{206}{10^7} = \frac{206}{10000000} = 0.0000206$

(ii) $5 \times 10^{-7} = \frac{5}{10^7} = \frac{5}{10000000} = 0.0000005$

(iii) $6.82 \times 10^{-6} = \frac{682}{100} \times \frac{1}{10^6} = \frac{682}{10^2 \times 10^6} = \frac{682}{10^{(2+6)}} = \frac{682}{10^8} = \frac{682}{100000000} = 0.00000682$

(iv) $5.673 \times 10^{-4} = \frac{5673}{1000} \times \frac{1}{10^4} = \frac{5673}{10^3 \times 10^4} = \frac{5673}{10^{(3+4)}} = \frac{5673}{10^7} = \frac{5673}{10000000} = 0.0005673$

(v) $1.8 \times 10^{-2} = \frac{18}{10} \times \frac{1}{10^2} = \frac{18}{10 \times 10^2} = \frac{18}{10^{(1+2)}} = \frac{18}{10^3} = \frac{18}{1000} = 0.018$

(vi) $4.129 \times 10^{-3} = \frac{4129}{1000} \times \frac{1}{10^3} = \frac{4129}{10^3 \times 10^3} = \frac{4129}{10^{(3+3)}} = \frac{4129}{10^6} = \frac{4129}{1000000} = 0.004129$

Exponents Exercise 2C

Q1

Answer :

(c) $\frac{125}{8}$

$\left(\frac{2}{5}\right)^{-3} = \left(\frac{5}{2}\right)^{3} = \frac{5^3}{2^3} = \frac{125}{8}$

Q2

Answer :

(d) $\frac{1}{81}$

$(-3)^{-4} = \frac{1}{(-3)^4} = \frac{1}{(-1)^4 \times (3)^4} = \frac{1}{(3)^4} = \frac{1}{81}$

Q3

Answer :

(b) $\frac{-1}{32}$

$(-2)^{-5} = \frac{1}{(-2)^5} = \frac{1}{-32} = \frac{1 \times (-1)}{-32 \times (-1)} = \frac{-1}{32}$

Q4

Answer :

(d) $\frac{1}{8}$

$\left(2^{-5} \div 2^{-2}\right) = \left(\frac{1}{2^5} \div \frac{1}{2^2}\right) = \left(\frac{1}{32} \div \frac{1}{4}\right) = \left(\frac{1}{32} \times 4\right) = \frac{4}{32} = \frac{1}{8}$

Q5

Answer :

(b) $\frac{60}{7}$

$\left(3^{-1} + 4^{-1}\right)^{-1} \div 5^{-1} = \left(\frac{1}{3} + \frac{1}{4}\right)^{-1} \div \frac{1}{5} = \left(\frac{4+3}{12}\right)^{-1} \div \frac{1}{5} = \left(\frac{7}{12}\right)^{-1} \div \frac{1}{5} = \left(\frac{12}{7}\right) \div \frac{1}{5} = \frac{12}{7} \times 5 = \frac{60}{7}$

Q6

Answer :

(c) 29

$\left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2} = \left(\frac{2}{1}\right)^{2} + \left(\frac{3}{1}\right)^{2} + \left(\frac{4}{1}\right)^{2}$
$= 2^2 + 3^2 + 4^2$
$= 4 + 9 + 16$
$= 29$

Q7

Answer :

(a) $\frac{19}{64}$

$$\left\{\left(\frac{1}{3}\right)^{-3}-\left(\frac{1}{2}\right)^{-3}\right\}\div\left(\frac{1}{4}\right)^{-3}$$
$$=\left\{3^3-2^3\right\}\div 4^3$$
$$=\{27-8\}\div 64$$
$$=19\div 64$$
$$=\frac{19}{64}$$

Q8

Answer :

(a) $\frac{1}{16}$

$$\left[\left\{\left(-\frac{1}{2}\right)^2\right\}^{-2}\right]^{-1}$$
$$=\left[\left\{-\frac{1}{2}\right\}^{-4}\right]^{-1}$$
$$=\left(-\frac{1}{2}\right)^{(-4\times -1)}$$
$$=\left(-\frac{1}{2}\right)^4$$
$$=\frac{1}{16}$$

Q9

Answer :

(d) 3

$$\left(\frac{7}{12}\right)^{-4}\times\left(\frac{7}{12}\right)^{3x}=\left(\frac{7}{12}\right)^5$$
$$\Rightarrow\left(\frac{7}{12}\right)^{-4+3x}=\left(\frac{7}{12}\right)^5$$
$$\Rightarrow 3x-4=5$$
$$3x=9$$
$$\text{or } x=\frac{9}{3}=3$$

Q10

Answer :

(d) 2

$$\left(2^{3x-1}+10\right)\div 7=6$$
$$\Rightarrow\frac{\left(2^{3x-1}+10\right)}{7}=\frac{6}{1}$$
$$\text{On cross multiplying :}$$
$$\left(2^{3x-1}+10\right)\times 1=6\times 7=42$$

$\Rightarrow 2^{3x-1} = 42 - 10$

$\Rightarrow 2^{3x-1} = 32$

$\Rightarrow 2^{3x-1} = 2^5$

$\Rightarrow 3x-1 = 5$

$\Rightarrow 3x = 6$

Therefore, $x = 2$

Q11

Answer :

(c) 1

Using the law of exponents $\left(\frac{a}{b}\right)^0 = 1$:

$\therefore \left(\frac{2}{3}\right)^0 = 1$

Q12

Answer :

(c) $\frac{-3}{5}$

$\left(\frac{-5}{3}\right)^{-1} = \left(\frac{3}{-5}\right)^1 = \frac{3}{-5} = \frac{3\times(-1)}{-5\times(-1)} = \frac{-3}{5}$

Q13

Answer :

(d) $\frac{-1}{8}$

$\left(\frac{-1}{2}\right)^3 = \frac{-1^3}{2^3} = \frac{-1}{8}$

Q14

Answer :

(b) $\frac{9}{16}$

$\left(\frac{-3}{4}\right)^2 = \frac{(-3)^2}{(4)^2} = \frac{9}{16}$

Q15

Answer :

(c) 3.67×10^6

$3670000 = 367 \times 10^4 = 3.67 \times 100 \times 10^4 = 3.67 \times 10^2 \times 10^4 = 3.67 \times 10^{(2+4)} = 3.67 \times 10^6$

Q16

Answer :

(b) 4.63×10^{-5}

$0.0000463 = \frac{463}{10^7} = \frac{4.63\times10^2}{10^7} = 4.63 \times 10^{(2-7)} = 4.63 \times 10^{-5}$

Q17

Answer :

(a) 3.67

$0.000367 \times 10^4 = \frac{367}{10^6} \times 10^4 = 367 \times 10^{(4-6)} = 367 \times 10^{-2} = \frac{367}{10^2} = \frac{367}{100} = 3.67$