







Quadrilaterals and Parallelograms

Exercise 9A

Type	Properties
Parallelogram 	<ul style="list-style-type: none"> • Opposite sides are equal and parallel • Opposite angles are equal
Rectangle 	<ul style="list-style-type: none"> • Opposite sides are equal and parallel • All angles are right angles (90°)
Square 	<ul style="list-style-type: none"> • Opposite sides are parallel • All sides are equal • All angles are right angles (90°)
Rhombus 	<ul style="list-style-type: none"> • Opposite sides are parallel • All sides are equal • Opposite angles are equal • Diagonals bisect each other at right angles (90°)
Trapezoid 	<ul style="list-style-type: none"> • One pair of opposite sides is parallel
Kite 	<ul style="list-style-type: none"> • Two pairs of adjacent sides are equal • One pair of opposite sides are equal • One diagonal bisects the other • Diagonals intersect at right angle (90°)

Question 1:

Let the fourth angle be x .

We know, that sum of the angles of a quadrilateral is 360°

$$\text{Then, } 56^\circ + 115^\circ + 84^\circ + x = 360^\circ$$

$$\Rightarrow 255^\circ + x = 360^\circ$$

$$\Rightarrow x = 360^\circ - 255^\circ = 105^\circ$$

\therefore The fourth angle is 105° .

Question 2:

Let the angles of a quadrilateral be $2x$, $4x$, $5x$ and $7x$.

We know, that sum of the angles of a quadrilateral is 360°

$$\text{Then, } 2x + 4x + 5x + 7x = 360^\circ$$

$$\Rightarrow 18x = 360^\circ$$

$$\Rightarrow x = \frac{360}{18} = 20^\circ$$

\therefore the angles of the quadrilateral are:

$$2x = 2 \times 20 = 40^\circ$$

$$4x = 4 \times 20 = 80^\circ$$

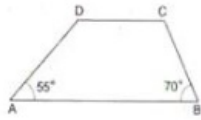
$$5x = 5 \times 20 = 100^\circ$$

$$7x = 7 \times 20 = 140^\circ$$

\therefore the required angles are 40° , 80° , 100° and 140° .

Question 3:

Since $AB \parallel DC$



Since $AB \parallel DC$, $\angle A$ and $\angle D$ are consecutive interior angles.
Consecutive interior angles sum upto 180° .

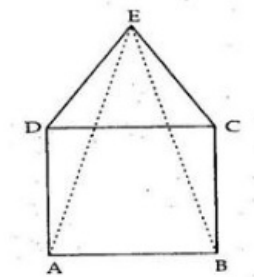
$$\begin{aligned}\text{So, } \angle A + \angle D &= 180^\circ \\ \Rightarrow 55^\circ + \angle D &= 180^\circ \\ \Rightarrow \angle D &= 180^\circ - 55^\circ = 125^\circ\end{aligned}$$

Also, we know that, sum of the angles of a quadrilateral is 360°

$$\begin{aligned}\Rightarrow \angle A + \angle B + \angle C + \angle D &= 360^\circ \\ \Rightarrow 55^\circ + 70^\circ + \angle C + 125^\circ &= 360^\circ \\ \Rightarrow 250^\circ + \angle C &= 360^\circ \\ \Rightarrow \angle C &= 360^\circ - 250^\circ = 110^\circ \\ \therefore \angle C = 110^\circ \text{ and } \angle D = 125^\circ\end{aligned}$$

Question 4:

Given: $\triangle EDC$ is an equilateral triangle and $ABCD$ is a square



To Prove: $AE = BE$
and $\angle DAE = 15^\circ$

(i) Proof: Since $\triangle EDC$ is an equilateral triangle,

$$\angle EDC = 60^\circ \text{ and } \angle ECD = 60^\circ$$

Since $ABCD$ is a square,

$$\angle CDA = 90^\circ \text{ and } \angle DCB = 90^\circ$$

In $\triangle EDA$

$$\begin{aligned}\angle EDA &= \angle EDC + \angle CDA \\ &= 60^\circ + 90^\circ \\ &= 150^\circ \quad \dots\dots(1)\end{aligned}$$

In $\triangle ECB$

$$\begin{aligned}\angle ECB &= \angle ECD + \angle DCB \\ &= 60^\circ + 90^\circ = 150^\circ\end{aligned}$$

$$\Rightarrow \angle EDA = \angle ECB \quad \dots\dots(2)$$

Thus, in $\triangle EDA$ and $\triangle ECB$

$$\begin{aligned}ED &= EC && [\text{sides of equilateral triangle } \triangle EDC] \\ \angle EDA &= \angle ECB && [\text{from (2)}] \\ DA &= CB && [\text{sides of square } \square ABCD]\end{aligned}$$

Thus, by Side-Angle-Side criterion of congruence, we have

$$\therefore \triangle EDA \cong \triangle ECB \quad [\text{By SAS}]$$

The corresponding parts of the congruent triangles are equal.

$$\therefore AE = BE \quad [\text{C.P.C.T}]$$

(ii) Now in $\triangle EDA$, we have

$$ED = DA$$

$$\Rightarrow \angle DEA = \angle DAE \quad [\text{base angles are equal}]$$

$$\text{But } \angle EDA = 150^\circ \quad [\text{from (1)}]$$

So, by angle sum property in $\triangle EDA$

$$\angle EDA + \angle DAE + \angle DEA = 180^\circ$$

$$\Rightarrow 150^\circ + \angle DAE + \angle DAE = 180^\circ$$

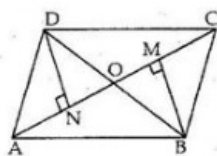
$$\Rightarrow 2 \angle DAE = 180^\circ - 150^\circ$$

$$\Rightarrow 2 \angle DAE = 30^\circ$$

$$\Rightarrow \angle DAE = \frac{30}{2} = 15^\circ$$

Question 5:

Given : $BM \perp AC$ and $DN \perp AC$ and $BM = DN$



To Prove : AC bisects BD.

We have,

$$\angle DON = \angle MOB \quad [\text{Vertically opposite angles}]$$

$$\angle DNO = \angle BMO = 90^\circ$$

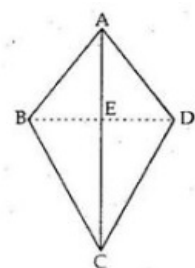
$$BM = DN \quad [\text{Given}]$$

$$\therefore \triangle DNO \cong \triangle BMO \quad [\text{By AAS}]$$

$$\therefore OD = OB \quad [\text{C.P.C.T}]$$

So, AC bisects BD.

Question 6:



Given : ABCD is quadrilateral in which $AB = AD$ and $BC = DC$

To Prove: (i) AC bisects $\angle A$ and $\angle C$

(ii) $BE = DE$

(iii) $\angle ABC = \angle ADC$

Proof: In $\triangle ABC$ and $\triangle ADC$, we have

$$AB = AD \quad [\text{Given}]$$

$$BC = DC \quad [\text{Given}]$$

$$AC = AC \quad [\text{Common}]$$

Thus by Side-Side-Side criterion of congruence,

$$\triangle ABC \cong \triangle ADC \quad \dots\dots(1)$$

The corresponding parts of the congruent triangles are equal.

$$\text{So, } \angle BAC = \angle DAC \quad [\text{C.P.C.T}]$$

$$\Rightarrow \angle BAE = \angle DAE$$

It means that AC bisects $\angle BAD$, that is $\angle A$

$$\text{Also, } \angle BCA = \angle DCA \quad [\text{C.P.C.T}]$$

$$\Rightarrow \angle BCE = \angle DCE$$

It means that AC bisects $\angle BCD$, that is $\angle C$

(ii) In $\triangle ABE$ and $\triangle ADE$, we have

$$AB = AD \quad [\text{given}]$$

$$\angle BAE = \angle DAE \quad [\text{from (i)}]$$

$$AE = AE \quad [\text{Common}]$$

Thus by Side-Angle-Side criterion of congruence, we have

$$\therefore \triangle ABE \cong \triangle ADE \quad [\therefore \text{By SAS}]$$

$$\text{So, } BE = DE \quad [\text{By c.p.c.t}]$$

(iii) Since from equation (1) in subpart (i), we have

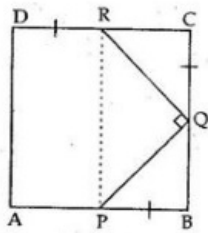
$$\triangle ABC \cong \triangle ADC,$$

Thus, by c.p.c.t, $\angle ABC = \angle ADC$

Question 7:

Given : A square ABCD in which $\angle PQR = 90^\circ$ and $PB = QC = DR$

- To Prove : (i) $QB = RC$
(ii) $PQ = QR$
(iii) $\angle QPR = 45^\circ$



Proof :

(i) Consider the line segment QB:

$$QB = BC - QC$$

$$= CD - DR \quad [\because ABCD \text{ is a square, so } BC = DC, QC = DR(\text{given})]$$

$$QB = RC \quad \dots\dots(1)$$

(ii) In $\triangle PBQ$ and $\triangle QCR$, we have

$$PB = QC \quad [\text{Given}]$$

$$\angle PBQ = \angle QCR = 90^\circ \quad [\because ABCD \text{ is a square}]$$

$$\text{and} \quad QB = RC \quad [\text{from (1)}]$$

Thus by Side-Angle-Side criterion of congruence, we have

$$\triangle PBQ \cong \triangle QCR \quad [\text{By SAS}]$$

$$\Rightarrow \quad PQ = QR \quad [\text{By cp.ct}]$$

(iii) Given that, $PQ = QR$

So, in $\triangle PQR$

$$\angle QPR = \angle QRP \quad [\text{isosceles triangle, so base angles are equal}]$$

By the angle sum property, in $\triangle PQR$

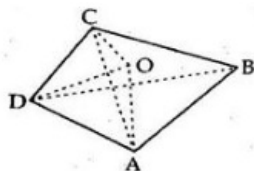
$$\angle QPR + \angle QRP + 90^\circ = 180^\circ$$

$$\Rightarrow \angle QPR + \angle QPR = 180^\circ - 90^\circ = 90^\circ$$

$$\therefore \quad \angle QPR = \frac{90}{2} = 45^\circ.$$

Question 8:

Given: O is a point within a quadrilateral ABCD



To Prove : $OA + OB + OC + OD > AC + BD$

Construction : Join AC and BD

Proof : In $\triangle ACO$,

$$OA + OC > AC \quad \dots(i)$$

$[\because \text{in a triangle, sum of any two sides is greater than the third side}]$

Similarly, In $\triangle BOD$,

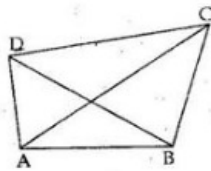
$$OB + OD > BD \quad \dots(ii)$$

Adding both sides of (i) and (ii), we get;

$$OA + OC + OB + OD > AC + BD \quad (\text{Proved})$$

Question 9:

Given: ABCD is a quadrilateral and AC is one of its diagonals.



To Prove :

(i) $AB + BC + CD + DA > 2AC$

(ii) $AB + BC + CD > DA$

(iii) $AB + BC + CD + DA > AC + BD$

Construction : Join BD.

Proof : (i) In $\triangle ABC$,

$$AB + BC > AC \quad \dots(1)$$

and, in $\triangle ACD$

$$AD + CD > AC \quad \dots(2)$$

Adding both sides of (1) and (2), we get :

$$AB + BC + CD + DA > 2AC \quad \dots(3)$$

(ii) In $\triangle ABC$,

$$AB + BC > AC$$

On adding CD to both sides of this inequality, we have,

$$AB + BC + CD > AC + CD \quad \dots(4)$$

Now, in $\triangle ACD$, we have,

$$AC + CD > DA \quad \dots(5)$$

From (4) and (5) we get

$$AB + BC + CD > DA \quad \dots(6)$$

(iii) In $\triangle ABD$ and $\triangle BDC$, we have

$$AB + DA > BD \quad \dots(7)$$

$$\text{and } BC + CD > BD \quad \dots(8)$$

On adding (7) and (8), we get

$$AB + BC + CD + DA > 2BD \quad \dots(9)$$

Adding (9) and (3), we have,

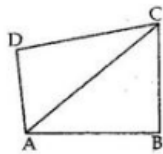
$$2(AB + BC + CD + DA) > 2BD + 2AC$$

$$\text{i.e. } AB + BC + CD + DA > BD + AC$$

[Dividing both sides by 2]

Question 10:

Given: ABCD is a quadrilateral.



To Prove : $\angle A + \angle B + \angle C + \angle D = 360^\circ$

Construction : Join AC

Proof : In $\triangle ABC$

$$\angle CAB + \angle B + \angle BCA = 180^\circ \quad \dots(i)$$

In $\triangle ACD$,

$$\angle DAC + \angle ACD + \angle D = 180^\circ \quad \dots(ii)$$

Adding both sides of (i) and (ii) we get

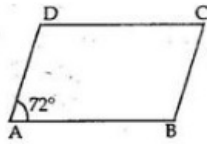
$$\angle CAB + \angle B + \angle BCA + \angle DAC + \angle ACD + \angle D = 180^\circ + 180^\circ$$

$$\Rightarrow \angle CAB + \angle DAC + \angle B + \angle BCA + \angle ACD + \angle D = 360^\circ$$

$$\Rightarrow \angle A + \angle B + \angle C + \angle D = 360^\circ$$

Exercise 9B

Question 1:



In a parallelogram, opposite angles are equal.

$$\therefore \angle A = \angle C = 72^\circ$$

The sum of all the four angles of a parallelogram is 360°

$$\text{So, } \angle A + \angle B + \angle C + \angle D = 360^\circ$$

$$\Rightarrow 72^\circ + \angle B + 72^\circ + \angle D = 360^\circ \quad [\because \angle A = \angle C]$$

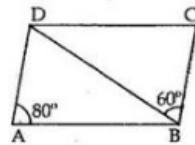
$$\Rightarrow 2\angle B + 144^\circ = 360^\circ \quad [\because \angle B = \angle D]$$

$$\Rightarrow 2\angle B = 360^\circ - 144^\circ = 216^\circ$$

$$\Rightarrow \angle B = \frac{216}{2} = 108^\circ$$

$$\therefore \angle B = 108^\circ, \angle C = 72^\circ \text{ and } \angle D = 108^\circ.$$

Question 2:



ABCD is a parallelogram,
so opposite angles are equal.

$$\therefore \angle C = \angle A = 80^\circ$$

As $AD \parallel BC$ and BD is a transversal.

$$\text{So, } \angle ADB = \angle DBC = 60^\circ$$

[Alternate angles]

In $\triangle ABD$

$$\angle A + \angle ADB + \angle ABD = 180^\circ$$

$$\Rightarrow 80^\circ + 60^\circ + \angle ABD = 180^\circ$$

$$\Rightarrow 140^\circ + \angle ABD = 180^\circ$$

$$\Rightarrow \angle ABD = 180^\circ - 140^\circ = 40^\circ$$

$$\therefore \begin{aligned} \angle ABC &= \angle ABD + \angle DBC \\ &= 40^\circ + 60^\circ = 100^\circ \end{aligned}$$

In a parallelogram, opposite angles are equal.

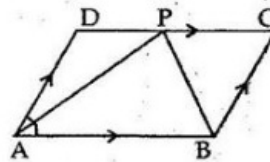
$$\text{So, } \angle ADC = \angle ABC = 100^\circ$$

$$\therefore \begin{aligned} \angle CDB &= \angle ADC - \angle ADB \\ &= 100^\circ - 60^\circ = 40^\circ \end{aligned}$$

$$\text{and } \angle ADB = 60^\circ.$$

Question 3:

ABCD is a parallelogram in which $\angle A = 60^\circ$ and bisectors of A and B meet DC at P.



(i) In a parallelogram, opposite angles are equal.

So, $\angle C = \angle A = 60^\circ$

In a parallelogram the sum of all the four angles is 360° .

$$\Rightarrow \angle A + \angle B + \angle C + \angle D = 360^\circ$$

$$\begin{aligned} \text{Now, } \angle B + \angle D &= 360^\circ - (\angle A + \angle C) \\ &= 360^\circ - (60^\circ + 60^\circ) = 240^\circ \end{aligned}$$

$$\therefore 2\angle B = 240^\circ \quad [\because \angle B = \angle D]$$

$$\text{So, } \angle B = \angle D = \frac{240^\circ}{2} = 120^\circ$$

Since $AB \parallel DC$ and AP is a transversal

$$\begin{aligned} \text{So, } \angle APD &= \angle PAB = \frac{60^\circ}{2} = 30^\circ \quad \dots(1) \\ &[\because \text{alternate angles}] \end{aligned}$$

Also, $AB \parallel DC$ and BP is a transversal.

$$\text{So, } \angle ABP = \angle CPB$$

$$\text{But, } \angle ABP = \frac{\angle B}{2} = \frac{120^\circ}{2} = 60^\circ$$

$$\therefore \angle CPB = 60^\circ \quad \dots(2)$$

$$\text{Now, } \angle APD + \angle APB + \angle CPB = 180^\circ$$

[As DPC is a straight line]

$$\begin{aligned} 30^\circ + \angle APB + 60^\circ &= 180^\circ \\ \Rightarrow \angle APB &= 180^\circ - 30^\circ - 60^\circ = 90^\circ \end{aligned}$$

$$(ii) \quad \text{Since } \angle APD = 30^\circ \quad [\text{from (1)}]$$

$$\text{and } \angle DAP = \frac{60^\circ}{2} = 30^\circ$$

$$\text{So, } \angle APD = \angle DAP$$

Now in $\triangle APD$,

$$\angle APD = \angle DAP \dots(3)$$

$$\therefore DP = AD \quad [\text{isosceles triangle, sides are equal}]$$

$$\text{As } \angle CPB = 60^\circ \quad [\text{from (2)}]$$

$$\text{and } \angle C = 60^\circ$$

$$\text{So, } \angle PBC = 180^\circ - 60^\circ - 60^\circ = 60^\circ$$

Since all angles in the $\triangle PCB$ are equal, it is an equilateral triangle.

$$\therefore PB = PC = BC \dots(4)$$

$$(iii) \quad \angle DPA = \angle PAD, \quad [\text{from (3)}]$$

$$\therefore DP = AD \quad [\text{isosceles triangle, sides are equal}]$$

$$= BC \quad [\text{opposite sides are equal}]$$

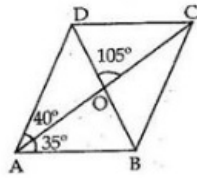
$$= PC \quad [\text{from (4)}]$$

$$= \frac{1}{2} DC \quad [\because DP = PC \Rightarrow P \text{ is the midpoint of } DC]$$

$$\therefore DC = 2AD.$$

Question 4:

ABCD is a parallelogram



(i) $\angle AOB = \angle COD = 105^\circ$
[vertical opposite angle]

Now in $\triangle AOB$, we have

$$\angle OAB + \angle AOB + \angle ABO = 180^\circ$$

$$\Rightarrow 35^\circ + 105^\circ + \angle ABO = 180^\circ$$

$$\Rightarrow 140^\circ + \angle ABO = 180^\circ$$

$$\Rightarrow \angle ABO = 180^\circ - 140^\circ = 40^\circ$$

(ii) Since $AB \parallel DC$ and BD is a transversal

$$\text{So, } \angle ABD = \angle CDB \quad [\text{alternate angles}]$$

$$\Rightarrow \angle CDO = \angle CDB = \angle ABD = \angle ABO = 40^\circ$$

$$\therefore \angle ODC = 40^\circ$$

(iii) As $AB \parallel CD$ and AC is a transversal

$$\text{So, } \angle ACB = \angle DAC = 40^\circ$$

[alternate opposite angles]

(iv) $\angle CBD = \angle B - \angle ABO$

$$\text{But, } \angle A + \angle B + \angle C + \angle D = 360^\circ$$

[\because ABCD is a parallelogram]

$$\Rightarrow 2\angle A + 2\angle B = 360^\circ$$

$$\Rightarrow 2 \times (40^\circ + 35^\circ) + 2\angle B = 360^\circ$$

$$\Rightarrow 150^\circ + 2\angle B = 360^\circ$$

$$\Rightarrow 2\angle B = 360^\circ - 150^\circ = 210^\circ$$

$$\Rightarrow \angle B = \frac{210^\circ}{2} = 105^\circ$$

$$\text{and } \angle CBD = \angle B - \angle ABO$$

$$= 105^\circ - 40^\circ = 65^\circ$$

$$\angle CBD = 65^\circ$$

Question 5:

In a parallelogram, the opposite angles are equal.

So, in the parallelogram ABCD,

$$\angle A = \angle C$$

and

$$\angle B = \angle D$$

Since

$$\angle A = (2x + 25)^\circ$$

\therefore

$$\angle C = (2x + 25)^\circ$$

and

$$\angle B = (3x - 5)^\circ$$

\therefore

$$\angle D = (3x - 5)^\circ$$

In a parallelogram, the sum of all the four angles is 360°

$$\therefore \angle A + \angle B + \angle C + \angle D = 360^\circ$$

$$\Rightarrow (2x + 25) + (3x - 5) + (2x + 25) + (3x - 5) = 360^\circ$$

$$\Rightarrow 10x + 40 = 360^\circ$$

$$\Rightarrow 10x = 360^\circ - 40^\circ = 320^\circ$$

$$\Rightarrow x = \frac{320}{10} = 32^\circ$$

$$\therefore \angle A = (2x + 25) = (2 \times 32 + 25) = 89^\circ$$

$$\angle B = (3x - 5) = (3 \times 32 - 5) = 91^\circ$$

$$\angle C = (2x + 25) = (2 \times 32 + 25) = 89^\circ$$

$$\angle D = (3x - 5) = (3 \times 32 - 5) = 91^\circ$$

$$\therefore \angle A = \angle C = 89^\circ \text{ and } \angle B = \angle D = 91^\circ$$

Question 6:

Lets ABCD be a parallelogram.

$$\text{Suppose, } \angle A = x^\circ$$

Then, $\angle B$, which is adjacent angle of A is $\frac{4}{5}x^\circ$.

In a parallelogram, the opposite angles are equal

$$\Rightarrow \angle A = \angle C = x^\circ \text{ and } \angle B = \angle D = \frac{4}{5}x^\circ$$

The sum of all the four angles of a parallelogram is 360° .

$$\Rightarrow \angle A + \angle B + \angle C + \angle D = 360^\circ$$

$$\Rightarrow x + \frac{4}{5}x + x + \frac{4}{5}x = 360^\circ$$

$$\Rightarrow 2x + \frac{8}{5}x = 360^\circ$$

$$\Rightarrow \frac{18}{5}x = 360^\circ$$

$$\Rightarrow x = \frac{360 \times 5}{18} = 100^\circ$$

$$\therefore \angle A = x = 100^\circ$$

$$\angle B = \frac{4}{5}x = \frac{4}{5} \times 100 = 80^\circ$$

$$\angle C = x = 100^\circ$$

$$\angle D = \frac{4}{5}x = \frac{4}{5} \times 100 = 80^\circ$$

$$\therefore \angle A = \angle C = 100^\circ \text{ and } \angle B = \angle D = 80^\circ.$$

Question 7:

Lets ABCD be the given parallelogram.

If $\angle A$ is smallest angle, then the greater angle

$$\Rightarrow \angle B = 2\angle A - 30^\circ$$

In a parallelogram, the opposite angles are equal

$$\Rightarrow \angle A = \angle C \text{ and } \angle B = \angle D = 2\angle A - 30^\circ$$

The sum of all the four angles of a parallelogram is 360° .

$$\Rightarrow \angle A + \angle B + \angle C + \angle D = 360^\circ$$

$$\Rightarrow \angle A + (2\angle A - 30^\circ) + \angle A + (2\angle A - 30^\circ) = 360^\circ$$

$$\Rightarrow \angle A + 2\angle A - 30^\circ + \angle A + 2\angle A - 30^\circ = 360^\circ$$

$$\Rightarrow 6\angle A - 60^\circ = 360^\circ$$

$$\Rightarrow 6\angle A = 360^\circ + 60^\circ = 420^\circ$$

$$\Rightarrow \angle A = \frac{420^\circ}{6} = 70^\circ$$

$$\therefore \angle A = 70^\circ \Rightarrow \angle C = 70^\circ$$

$$\angle B = (2\angle A - 30^\circ) = (2 \times 70^\circ - 30^\circ) = 110^\circ$$

$$\angle D = \angle B = 110^\circ$$

$$\therefore \angle A = \angle C = 70^\circ \text{ and } \angle B = \angle D = 110^\circ.$$

Question 8:

Perimeter of a parallelogram ABCD

$$= AB + BC + CD + DA$$

$$= 9.5 + BC + 9.5 + BC$$

[\because ABCD is a parallelogram and its opposite sides are equal

i.e. $AB = CD$ and $BC = DA$]

$$30 = 19 + 2BC$$

[Perimeter = 30 cm(given)]

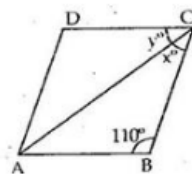
$$\Rightarrow 2BC = 30 - 19 = 11$$

$$\Rightarrow BC = \frac{11}{2} = 5.5 \text{ cm}$$

$$\therefore AB = 9.5 \text{ cm, } BC = 5.5 \text{ cm, } CD = 9.5 \text{ cm, } DA = 5.5 \text{ cm.}$$

Question 9:

(i) ABCD is a rhombus so its all sides are equal.



In $\triangle ABC$, we have

$$AB = BC$$

$$\Rightarrow \angle CAB = \angle ACB = x^\circ$$

$$\text{As, } \angle CAB + \angle ABC + \angle ACB = 180^\circ$$

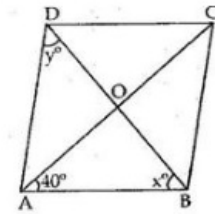
$$\Rightarrow x + 110^\circ + x = 180^\circ$$

$$\Rightarrow 2x = 180^\circ - 110^\circ = 70^\circ$$

$$\Rightarrow x = \frac{70^\circ}{2} = 35^\circ$$

$$\therefore x = 35^\circ \text{ and } y = 35^\circ$$

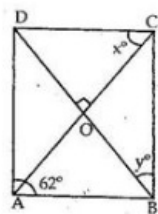
(ii) Since in a rhombus, all sides are equal



$$\begin{aligned}
 &\text{So in } \triangle ABD, \quad AB = AD \\
 &\Rightarrow \quad \angle ABD = \angle ADB \\
 &\Rightarrow \quad x = y \quad \dots\dots(1) \\
 &\text{Now in } \triangle ABC, \quad AB = BC \\
 &\Rightarrow \quad \angle CAB = \angle ACB \\
 &\Rightarrow \quad \angle ACB = 40^\circ \\
 &\quad \therefore \angle B = 180^\circ - \angle CAB - \angle ACB \\
 &\quad \quad \quad = 180^\circ - 40^\circ - 40^\circ = 100^\circ \\
 &\Rightarrow \quad \angle DBC = \angle B - x^\circ = 100 - x^\circ \\
 &\text{But} \quad \angle DBC = \angle ADB = y^\circ \quad [\text{alternate angle}] \\
 &\Rightarrow \quad 100 - x^\circ = y^\circ \\
 &\Rightarrow \quad 100^\circ - x^\circ = x^\circ \quad [\text{from (1)}] \\
 &\Rightarrow \quad 2x^\circ = 100 \\
 &\Rightarrow \quad x^\circ = \frac{100}{2} = 50^\circ
 \end{aligned}$$

So, $x = 50^\circ$ and $y = 50^\circ$.

(iii) Since ABCD is a rhombus



So, $\angle A = \angle C$, i.e. $\angle C = 62^\circ$

Now in $\triangle BCD$,

$$\begin{aligned}
 &\quad BC = DC \\
 &\Rightarrow \quad \angle CDB = \angle DBC = y^\circ \\
 &\text{As, } \angle BDC + \angle DBC + \angle BCD = 180^\circ \\
 &\Rightarrow \quad y + y + 62^\circ = 180^\circ \\
 &\Rightarrow \quad 2y = 180^\circ - 62^\circ = 118^\circ \\
 &\Rightarrow \quad y = \frac{118}{2} = 59^\circ
 \end{aligned}$$

As diagonals of a rhombus are perpendicular to each other,

$\triangle COD$ is a right triangle and $\angle DOC = 90^\circ$, $\angle ODC = y = 59^\circ$

$$\begin{aligned}
 &\Rightarrow \angle DCO = 90^\circ - \angle ODC \\
 &\quad \quad \quad = 90^\circ - 59^\circ = 31^\circ
 \end{aligned}$$

$$\therefore \angle DCO = x = 31^\circ$$

$$\therefore x = 31^\circ \text{ and } y = 59^\circ$$

Question 10:

ABCD is a rhombus in which diagonal AC = 24 cm and BD = 18 cm.

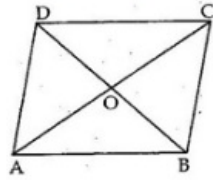
We know that in a rhombus, diagonals bisect each other at right angles.

So in $\triangle AOB$

$$\angle AOB = 90^\circ$$

$$AO = \frac{1}{2}AC = \frac{1}{2} \times 24 = 12 \text{ cm}$$

$$\text{and, } BO = \frac{1}{2}BD = \frac{1}{2} \times 18 = 9 \text{ cm}$$



Now, by Pythagoras Theorem, we have

$$AB^2 = AO^2 + OB^2$$

\Rightarrow

$$AB^2 = 12^2 + 9^2$$

$$= 144 + 81 = 225$$

\Rightarrow

$$AB = \sqrt{225} = 15 \text{ cm}$$

So the length of each side of the rhombus is 15 cm.

Question 11:

Since diagonals of a rhombus bisect each other at right angles.

$$\text{So, } AO = OC = \frac{1}{2}AC = \frac{1}{2} \times 16 = 8 \text{ cm.}$$

\therefore In right $\triangle AOB$,

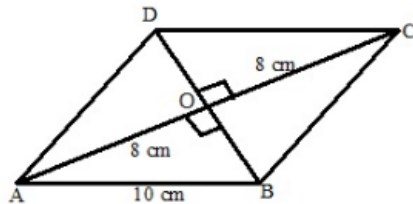
$$AB^2 = AO^2 + OB^2$$

$$\Rightarrow 10^2 = 8^2 + OB^2$$

$$\Rightarrow OB^2 = 100 - 64 = 36$$

$$\Rightarrow OB = \sqrt{36} = 6 \text{ cm.}$$

$$\therefore \text{Length of the other diagonal } BD = 2 \times OB \\ = 2 \times 6 = 12 \text{ cm.}$$

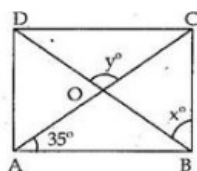


$$\begin{aligned} \text{Area of } \triangle ABC &= \frac{1}{2} \times AC \times OB \\ &= \frac{1}{2} \times 16 \times 6 = 48 \text{ cm}^2. \end{aligned}$$

$$\begin{aligned} \text{Area of } \triangle ACD &= \frac{1}{2} \times AC \times OD \\ &= \frac{1}{2} \times 16 \times 6 = 48 \text{ cm}^2. \end{aligned}$$

$$\begin{aligned} \therefore \text{Area of rhombus } ABCD &= (\text{Area of } \triangle ABC + \text{Area of } \triangle ACD) \\ &= (48 + 48) \text{ cm}^2 = 96 \text{ cm}^2. \end{aligned}$$

Question 12:



We know that diagonals of a rectangle are equal and bisect each other.

So, in $\triangle AOB$

$$AO = OB$$

$$\Rightarrow \angle OAB = \angle OBA \quad [\text{base angles are equal}]$$

$$\text{i.e. } \angle OBA = 35^\circ \quad [\because \angle OAB = 35^\circ, \text{ given}]$$

$$\angle AOB = 180^\circ - 35^\circ - 35^\circ = 110^\circ$$

$$\text{and, } \angle DOC = y^\circ = \angle AOB = 110^\circ$$

[Vertically opp. angles]

Consider the right triangle, $\triangle ABC$, right angled at B.

$$\text{So, } \angle ABC = 90^\circ \quad [\because ABCD \text{ is a rectangle}]$$

Now, consider the $\triangle OBC$

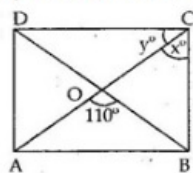
$$\text{So, } \angle OBC = x^\circ = \angle ABC - \angle OBA$$

$$= 90^\circ - 35^\circ$$

$$= 55^\circ$$

$$\therefore x = 55^\circ \text{ and } y = 110^\circ.$$

(ii) We know that diagonals of a rectangle are equal and bisect each other.



So, in $\triangle AOB$, $OA = OB$

$$\Rightarrow \angle OAB = \angle OBA$$

Again in $\triangle AOB$,

$$\angle AOB + \angle OAB + \angle OBA = 180^\circ$$

$$\Rightarrow 110^\circ + \angle OAB + \angle OBA = 180^\circ$$

$$\Rightarrow 2\angle OAB = 180^\circ - 110^\circ = 70^\circ$$

$$\Rightarrow \angle OAB + \angle OBA = \frac{70}{2} = 35^\circ$$

Since $AB \parallel CD$ and AC is a transversal, $\angle DCA$ and $\angle CAB$ are alternate angles, and thus they are equal.

$$\text{So, } \angle DCA = y^\circ = \angle CAB \text{ and } \angle CAB = 35^\circ \dots\dots(1)$$

$$\Rightarrow y^\circ = 35^\circ$$

Now consider the right triangle, $\triangle ABC$

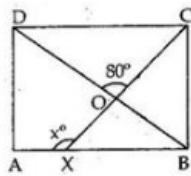
$$\angle ACB = x^\circ = 90^\circ - \angle CAB$$

$$= 90^\circ - 35^\circ \quad [\text{from (1)}]$$

$$= 55^\circ$$

$$\therefore x = 55^\circ \text{ and } y = 35^\circ.$$

Question 13:



Consider the triangle $\triangle ABD$

$$AB = AD$$

[\therefore ABCD is a square]

$$\text{So, } \angle ADB = \angle ABD$$

[base angles are equal]

$$\therefore \angle ADB + \angle ABD = 90^\circ$$

[$\therefore \angle A = 90^\circ$ as ABCD is a square]

$$2\angle ADB = 90^\circ$$

$$\Rightarrow \angle ADB = \frac{90}{2} = 45^\circ$$

Now in $\triangle OXB$,

$$\angle XOB = \angle DOC = 80^\circ \text{ [vertically opposite angle]}$$

$$\text{and } \angle ABD = 45^\circ \Rightarrow \angle XBD = 45^\circ \dots (1)$$

$$\text{So, exterior } \angle AXO = \angle XOB + \angle XBD$$

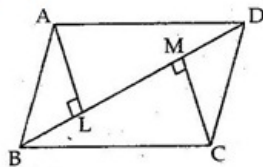
$$x^\circ = 80^\circ + 45^\circ \text{ [from (1)]}$$

$$= 125^\circ$$

$$\therefore x^\circ = 125^\circ$$

Question 14:

A parallelogram ABCD in which AL and CM are perpendiculars to its diagonal BD



To Prove : (i) $\triangle ALD \cong \triangle CMB$

(ii) $AL = CM$

Proof : (i) In $\triangle ALD$ and $\triangle CMB$, we have

$$\angle ALD = \angle CMB = 90^\circ \text{ [Given]}$$

$$\angle ADL = \angle CBM \text{ [AD \parallel BC, BD is a transversal, so alternate angles are equal]}$$

$$AD = BC \text{ [Opposite sides of a parallelogram]}$$

Thus by Angle-Angle-Side criterion of congruence, we have

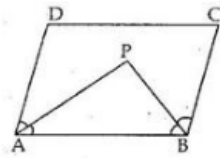
$$\therefore \triangle ALD \cong \triangle CMB \text{ [By AAS]}$$

(ii) Since $\triangle ALD \cong \triangle CMB$, the corresponding parts of the congruent triangles are equal.

$$\therefore AL = CM \text{ [C.P.C.T.]}$$

Question 15:

Given: A parallelogram ABCD in which angle bisectors of $\angle A$ and $\angle B$ intersect at P.



To Prove : $\angle APB = 90^\circ$

Proof : $\angle PAB = \frac{1}{2} \angle A$

and $\angle PBA = \frac{1}{2} \angle B$ [Given]

\therefore AD and BC are parallel and AB is a transversal.

So sum of consecutive angles is 180° .

$$\Rightarrow \angle A + \angle B = 180^\circ \quad \dots(1)$$

$$\begin{aligned} \angle PAB + \angle PBA &= \frac{1}{2} \angle A + \frac{1}{2} \angle B \\ &= \frac{1}{2} (\angle A + \angle B) \\ &= \frac{1}{2} \times 180^\circ \quad [\text{from (1)}] \end{aligned}$$

$$\angle PAB + \angle PBA = 90^\circ \quad \dots(2)$$

Now in $\triangle PAB$,

$$\angle PAB + \angle PBA + \angle APB = 180^\circ$$

$$\Rightarrow 90^\circ + \angle APB = 180^\circ \quad [\text{from (2)}]$$

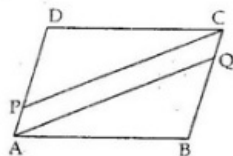
$$\Rightarrow \angle APB = 180^\circ - 90^\circ = 90^\circ$$

$$\therefore \angle APB = 90^\circ$$

Question 16:

Given: A parallelogram ABCD in which $AP = \frac{1}{3}AD$ and

$$CQ = \frac{1}{3}BC$$



To Prove: PAQC is a parallelogram.

Proof: In $\triangle ABQ$ and $\triangle CDP$

$$AB = CD$$

[\because opposite sides of a parallelogram]

$$\angle B = \angle D$$

$$\text{and } DP = AD - PA = \frac{2}{3}AD$$

$$\text{and, } BQ = BC - CQ = BC - \frac{1}{3}BC$$

$$= \frac{2}{3}BC = \frac{2}{3}AD \quad [\because AD = BC]$$

$$\therefore BQ = DP$$

Thus, by Side-Angle-Side criterion of congruence, we have,

$$\text{So, } \triangle ABQ \cong \triangle CDP \quad [\text{By SAS}]$$

The corresponding parts of the congruent triangles are equal.

$$AQ = CP \quad [\text{By cpct}]$$

$$\text{and } PA = \frac{1}{3}AD$$

$$\text{and } CQ = \frac{1}{3}BC = \frac{1}{3}AD$$

$$PA = CQ \quad [\because AD = BC]$$

Also, by c.p.c.t, $\angle QAB = \angle PCD \dots (1)$

Therefore,

$$\angle QAP = \angle A - \angle QAB$$

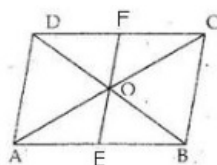
$$= \angle C - \angle PCD \quad [\text{since } \angle A = \angle C \text{ and from (1)}]$$

$$= \angle PCQ \quad [\text{alternate interior angles are equal}]$$

Therefore, AQ and CP are two parallel lines.

So, PAQC is a parallelogram.

Question 17:



Given: A parallelogram ABCD, in which diagonals intersect at O. E and F are the points on AB and CD

To Prove: $OE = OF$

Proof: In $\triangle AOE$ and $\triangle COF$, we have

$$\angle CAE = \angle DCA \quad [\text{Alternate angles}]$$

$$AO = CO \quad [\text{diagonals are equal and bisect each other}]$$

$$\text{and, } \angle AOE = \angle COF \quad [\text{Vertically opposite angles}]$$

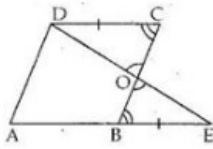
Thus by Angle-Side-Angle criterion of congruence, we have,

$$\therefore \triangle AOE \cong \triangle COF \quad [\text{By ASA}]$$

The corresponding parts of the congruent triangles are equal.

$$\therefore OE = OF \quad [\text{By cpct}]$$

Question 18:



Given : ABCD is a parallelogram in which AB is produced to E such that $BE = AB$. DE is joined which cuts BC at O.

To Prove : $OB = OC$

Proof : In $\triangle OCD$ and $\triangle OBE$, we have,

$$\angle DOC = \angle EOB \quad [\text{vertically opposite angles are equal}]$$

$$\angle OCD = \angle OBE \quad [AB \parallel CD, BC \text{ is a transversal} \\ \text{thus, alternate angles are equal}]$$

$$DC = BE \quad [AB = CD \text{ and } BE = AB]$$

Thus, by Angle-Angle-Side criterion of congruence, we have

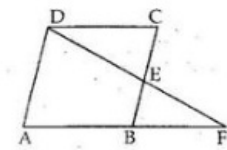
$$\therefore \triangle OCD \cong \triangle OBE \quad [\text{by AAS}]$$

The corresponding parts of the congruent triangles are equal.

$$\therefore OC = OB$$

Hence, ED bisects BC.

Question 19:



Given : A parallelogram ABCD in which E is the mid point of side BC. DE and AB when produced meet at F.

To Prove : $AF = 2AB$

Proof : In $\triangle DEC$ and $\triangle FEB$

$$\angle DEC = \angle FEB \quad [\text{Vertically opposite angles}]$$

$$\angle DCE = \angle FBE \quad [\text{alternate angles}]$$

$$CE = EB \quad [\text{Given}]$$

Thus by Angle-Angle-Side criterion of congruence, we have

$$\triangle DEC \cong \triangle FEB \quad [\text{By AAS}]$$

The corresponding parts of the congruent triangles are equal.

$$\therefore DC = FB \quad [\text{By cpct}]$$

$$\text{So, } AF = AB + BF$$

$$= AB + DC$$

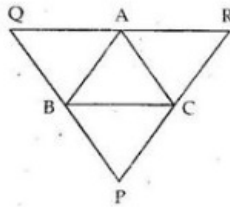
$$= AB + AB$$

$$= 2AB$$

$$\therefore AF = 2AB$$

Question 20:

Given : A $\triangle ABC$ in which through points A, B and C, lines QR, QP and RP are drawn parallel to BC, CA and AB.



To prove : $BC = \frac{1}{2} QR$

Proof : Since $AR \parallel BC$ and $AB \parallel RC$ [Given]

So, ABCR is a parallelogram. Therefore

$$AR = BC \quad \dots\dots(i)$$

Also, $AQ \parallel BC$ and $QB \parallel AC$

So, AQBC is a parallelogram. Therefore

$$QA = BC \quad \dots\dots(ii)$$

Adding both side of (i) and (ii), we get

$$AR + QA = BC + BC$$

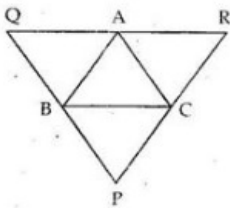
$$\Rightarrow QR = 2BC$$

$$\Rightarrow BC = \frac{QR}{2}$$

$$\therefore BC = \frac{1}{2} QR$$

Question 21:

Given : A $\triangle ABC$, in which through points A, B and C, lines QR, QP and RP have been drawn parallel to BC, AC and AB of $\triangle ABC$ respectively.



To Prove : Perimeter of $\triangle PQR = 2(\text{Perimeter of } \triangle ABC)$

Proof :

Since $AR \parallel BC$ and $AB \parallel RC$ [Given]

So, ABCR is a parallelogram. Therefore

$$AR = BC \quad \dots\dots(i)$$

Also, $AQ \parallel BC$ and $QB \parallel AC$

So, AQBC is a parallelogram. Therefore

$$QA = BC \quad \dots\dots(ii)$$

Adding both side of (i) and (ii), we get

$$AR + QA = BC + BC$$

$$\Rightarrow QR = 2BC$$

$$\Rightarrow BC = \frac{QR}{2}$$

$$\therefore BC = \frac{1}{2} QR$$

Similarly, we can prove $AB = \frac{1}{2} RP$ and $AC = \frac{1}{2} PQ$

So, Perimeter of $\triangle PQR = PQ + QR + RP$

$$= 2AC + 2BC + 2AB$$

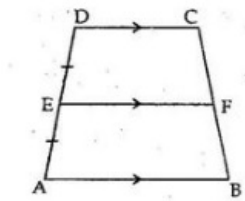
$$= 2(AC + BC + AB)$$

$$= 2(\text{Perimeter of } \triangle ABC)$$

Exercise 9C

Question 1:

Given : ABCD is trapezium in which $AB \parallel DC$ and through the mid – point E of AD a line drawn parallel to AB which cuts BC at F.



To prove : F is the mid – point of BC

Proof : Since $AB \parallel DC$ and $EF \parallel AB$

So, $AB \parallel EF \parallel DC$

Intercept Theorem: If there are three parallel lines and the intercepts made by them on one transversal are equal then the intercept on any other transversal are also equal.

Now AD is a transversal and therefore,

Let us apply Intercepts Theorem.

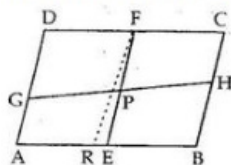
Thus, the intercepts made by AB, EF and DC on transversal BC are also equal

$\therefore CF = FB$

\therefore F is mid – Point of BC.

Question 2:

Given : A parallelogram ABCD in which E and F are the mid points of AB and CD. A line segment GH cuts EF at P.



To prove : $GP = PH$

Proof : AD, EF and BC are three line segments and DC and AB are two transversal.

The intercepts made by the line on transversal AB and CD are equal because,

$$AE = EB$$

and $DF = FC$

We need to prove that FE is parallel to AD.

Let us prove by the method of contradiction.

Let us assume that FE is not parallel to AD.

Now, draw FR parallel to AD.

Intercept Theorem: If there are three parallel lines and the intercepts made by them on one transversal are equal then the intercept on any other transversal are also equal.

Thus, by Intercept Theorem, $AR = RB$ because

$$DF = FC$$

But $AE = EB$ [Given]

There cannot be two mid points R and E of AB.

Hence our assumption is wrong.

So, $AD \parallel EF \parallel BC$

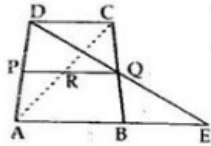
Now, again by Intercept Theorem, we have

$$GP = PH$$

because GH is transversal and intercept made by AD, EF and BC on GH are equal as $DF = FC$.

Question 3:

Given : ABCD is trapezium in which $AB \parallel DC$
P and Q are the mid – points of AD and BC. DQ is joined and produced and AB is also produced and so that they meet at E.
AC cuts PQ at R.



To prove :

(i) $DQ = QE$

(ii) $PQ \parallel AB$

(iii) $AR = RC$

Proof :

(i) Consider the triangles $\triangle QCD$ and $\triangle QBE$

$\angle DQC = \angle BQE$ [vertically opposite angles]

$CQ = BQ$ [\because Q is the midpoint of BC]

$\angle QDC = \angle QEB$ [$AE \parallel DC$, BC is a transversal,
and thus alternate angles are equal]

Thus, by Angle-Side-Angle criterion of congruence, we have

$\triangle QCD \cong \triangle QEB$ [by ASA]

The corresponding parts of the congruent triangles are equal.

Thus, $DQ = QE$ [by c.p.c.t]

(ii) Midpoint Theorem: The line segment joining the midpoints of any two sides of a triangle is parallel to the third side and equal to half of it.

Thus by the midpoint Theorem, $PQ \parallel AE$.

AB is a part of AE and hence, we have $PQ \parallel AB$

Since the intercepts made by the lines AB, PQ and DC on AD

Since $PQ \parallel AB \parallel DC$

So, PR which is part of PQ is also parallel to AB

$\therefore PR \parallel AB \parallel DC$

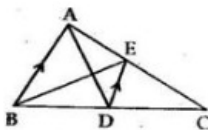
(iii) Intercept Theorem: If there are three parallel lines and the intercepts made by them on one transversal are equal then the intercept on any other transversal are also equal.

The three lines PR, AB and DC are cut by AC and AD.

So, by intercept Theorem, $AR = RC$

Question 4:

Given: $\triangle ABC$ in which AD is its median and $DE \parallel AB$



To Prove : BE is a median of $\triangle ABC$.

Proof : In $\triangle ABC$,

$DE \parallel AB$ [Given]

D is the mid – point of BC.

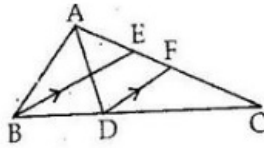
The line drawn through the midpoint of one side of a triangle, parallel to another side, intersects the third side at its midpoint.

So, by Mid point Theorem, E is the mid – point of AC.

\therefore BE is the median of $\triangle ABC$ drawn through B.

Question 5:

Given : $\triangle ABC$ in which AD and BE are the medians. DF is drawn parallel to BE.



To prove : $CF = \frac{1}{4}AC$

Proof : In $\triangle CBE$,

D is the mid point of BC and DF is parallel to BE.

The line drawn through the midpoint of one side of a triangle, parallel to another side, intersects the third side at its midpoint.

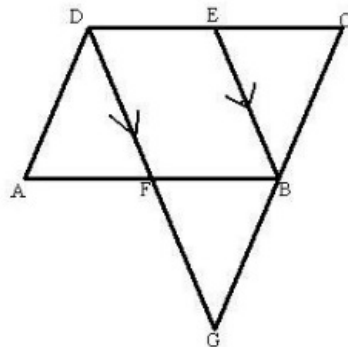
So, by Mid point Theorem F is the mid point of EC.

$$\begin{aligned}\therefore CF &= \frac{1}{2}EC \\ &= \frac{1}{2}\left(\frac{1}{2}AC\right) \quad [\text{BE is the median through B}] \\ &= \frac{1}{4}AC.\end{aligned}$$

Thus, $CF = \frac{1}{4}AC$.

Question 6:

Given : ABCD is a parallelogram in which E is the mid point of DC.



Through D, a line is drawn parallel to EB meeting AB at F and BC produced at G.

To Prove : (i) $AD = \frac{1}{2}GC$

(ii) $DG = 2EB$

Proof : (i) In $\triangle CDG$,

$EB \parallel DG$ and E is the mid - point of CD.

The line drawn through the midpoint of one side of a triangle, parallel to another side, intersects the third side at its midpoint.

So, by Mid - point Theorem, B is the mid - point of CG.

$$\therefore CB = BG$$

As, ABCD is a parallelogram,

$$\text{So, } AD = BC$$

$$\Rightarrow BG = CB$$

$$\Rightarrow AD = BG = \frac{1}{2}CG$$

(ii) Midpoint Theorem: The line segment joining the midpoints of any two sides of a triangle is parallel to the third side and equal to half of it.

Since E is the mid point of DC and B is the mid point of CG

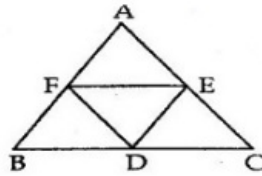
\therefore By Mid point Theorem, in $\triangle CDG$

$$EB = \frac{1}{2}DG$$

$$\Rightarrow DG = 2EB$$

Question 7:

Given : A $\triangle ABC$ in which D, E and F are the mid points of BC, AC and AB respectively.
DE, EF and FD are joined to get four triangles.



To Prove : Four triangle AFE, BFD, FDE and EDC are Congruent.

Proof : Since F, E are mid point of AB and AC

$$\text{So, } EF = \frac{1}{2} BC \quad [\text{By Mid point Theorem}]$$

$$\text{Similarly } FD = \frac{1}{2} AC$$

$$\text{and } ED = \frac{1}{2} AB$$

Now, in $\triangle AFE$ and $\triangle BFD$, we have

$$AF = FB$$

$$FE = \frac{1}{2} BC = BD$$

$$FD = \frac{1}{2} AC = AE$$

Thus by Side-Side-Side criterion of congruence, we have

$$\therefore \triangle AFE \cong \triangle BFD \quad [\text{By SSS}]$$

Again, in $\triangle BFD$ and $\triangle FED$, we have

$$FE \parallel BC$$

$$\text{i.e. } FE \parallel BD \text{ and } AB \parallel ED$$

$$\text{i.e. } FB \parallel ED, \text{ by Mid point Theorem.}$$

So, BDEF is a parallelogram.

\therefore FD being a diagonal divides the parallelogram into two congruent triangles

$$\therefore \triangle BFD \cong \triangle FDE$$

Similarly we can prove FECD is a parallelogram.

$$\text{So, } \triangle FED \cong \triangle EDC$$

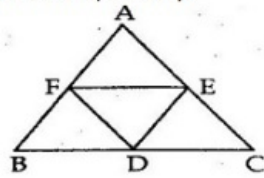
Thus, all the four triangles

$\triangle BFD, \triangle FDE, \triangle FED$ and $\triangle EDC$

are congruent to each other.

Question 8:

Given : A triangle ABC in which D,E and F are the mid points of BC,AC and AB respectively.



To prove : $\angle EDF = \angle A$

$\angle DEF = \angle B$

and $\angle DFE = \angle C$

Proof :

Midpoint Theorem: The line segment joining the midpoints of any two sides of a triangle is parallel to the third side and equal to half of it.

In $\triangle AFE$ and $\triangle DFE$

$$AF = \frac{1}{2} AB = ED \quad [\text{By Mid point Theorem}]$$

$$AE = \frac{1}{2} AC = FD \quad [\text{By Mid point Theorem}]$$

$$FE = EF \quad [\text{Common}]$$

Thus by Side-Side-Side criterion of congruence, we have

$$\triangle AFE \cong \triangle DFE \quad [\text{By SSS}]$$

The corresponding parts of the congruent triangles are equal.

$$\therefore \angle A = \angle FDE \quad [\text{C.P.C.T.}]$$

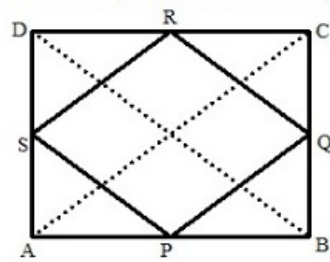
Similarly we can prove that

$$\angle B = \angle DEF$$

$$\text{and } \angle C = \angle DFE.$$

Question 9:

Given : ABCD is a rectangle and P, Q, R and S are the mid points of AB, BC, CD and DA respectively.



To prove : PQRS is a rhombus.

Construction : Join AC and BD

Proof : In $\triangle ABC$,

P and Q are the mid –points of AB and BC.

Midpoint Theorem: The line segment joining the midpoints of any two sides of a triangle is parallel to the third side and equal to half of it.

So by Mid – point Theorem,

$$PQ \parallel AC \text{ and } PQ = \frac{1}{2} AC$$

Similarly, from $\triangle ADC$,

$$RS \parallel AC \text{ and } RS = \frac{1}{2} AC$$

$$\Rightarrow PQ \parallel RS \text{ and } PQ = RS = \frac{1}{2} AC \dots\dots(1)$$

Now, in $\triangle BAD$,

P and S are the mid –point s of AB and AD.

So by Mid – point Theorem, we have

$$PS \parallel BD \text{ and } PS = \frac{1}{2} DB$$

Similarly, from $\triangle BCD$,

$$RQ \parallel BD \text{ and } RQ = \frac{1}{2} DB$$

$$\Rightarrow PS \parallel RQ \text{ and } PS = RQ = \frac{1}{2} DB \dots\dots(2)$$

The diagonals of a rectangle are equal

$$\therefore AC = BD \dots\dots(3)$$

From (1), (2) and (3) we have

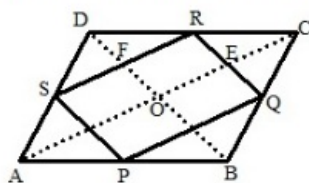
$$PQ \parallel RS \text{ and } PS \parallel RQ \text{ and}$$

$$\therefore PQ = QR = RS = SP$$

\therefore PQRS is a rhombus.

Question 10:

Given : ABCD is a rhombus in which P,Q,R and S are the mid – points of AB, BC, CD and DA respectively.



To Prove : PQRS is a rectangle.

Construction : Join AC and BD.

Proof :

Midpoint Theorem: The line segment joining the midpoints of any two sides of a triangle is parallel to the third side and equal to half of it.

In $\triangle ABC$

P and Q are the mid points of AB and BC.

So by Mid point Theorem,

$$PQ \parallel AC \text{ and } PQ = \frac{1}{2}AC$$

Similarly, from $\triangle ADC$,

$$RS \parallel AC \text{ and } RS = \frac{1}{2}AC$$

$$\Rightarrow PQ \parallel RS \text{ and } PQ = RS = \frac{1}{2}AC \dots\dots(1)$$

Now, in $\triangle BAD$,

P and S are the mid – points of AB and AD.

So by Mid – point Theorem, we have

$$PS \parallel BD \text{ and } PS = \frac{1}{2}DB$$

Similarly, from $\triangle BCD$,

$$RQ \parallel BD \text{ and } RQ = \frac{1}{2}DB$$

$$\Rightarrow PS \parallel RQ \text{ and } PS = RQ = \frac{1}{2}DB \dots\dots(2)$$

From (1) and (2), we have

PQRS is a parallelogram as its opposite sides are parallel.

We know ,that in a rhombus, diagonals intersect at right angles.

$$\therefore \angle EOF = 90^\circ$$

$$\text{Now, } RQ \parallel DB$$

$$\Rightarrow RE \parallel FO$$

$$\text{Also, } SR \parallel AC$$

$$\Rightarrow FR \parallel OE$$

$$\therefore OERF \text{ is a parallelogram}$$

In a parallelogram, opposite angles are equal.

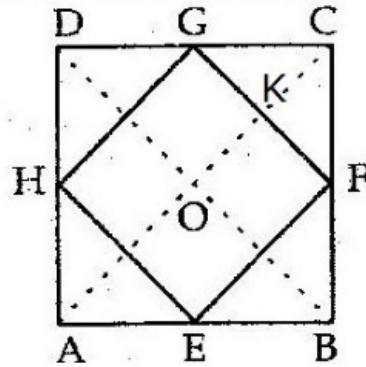
$$\text{So, } \angle FRE = \angle EOF = 90^\circ$$

Thus, PQRS is a parallelogram with $\angle R = 90^\circ$

Hence, PQRS is a rectangle.

Question 11:

Given : ABCD is a square in which E, F, G and H are the mid points of AB, BC, CD and AD, respectively.
The mid points are joined together.



To prove : EFGH is a square.

Construction : Join AC and BD

Proof :

Midpoint Theorem: The line segment joining the midpoints of any two sides of a triangle is parallel to the third side and equal to half of it.

In $\triangle ABC$

E and F are the mid – points and by the Mid points Theorem , we have

$$EF \parallel AC \text{ and } EF = \frac{1}{2} AC$$

Similarly, in $\triangle ADC$,

H and G are the midpoints and by the Mid points Theorem , we have

$$HG \parallel AC \text{ and } HG = \frac{1}{2} AC$$

Thus, we have,

$$EF \parallel HG \text{ and } EF = HG = \frac{1}{2} AC \dots (1)$$

In $\triangle BAD$,

H and E are the midpoints and by the Mid points Theorem , we have,

$$HE \parallel BD \text{ and } HE = \frac{1}{2} BD$$

In $\triangle BCD$,

G and F are the midpoints and by the Mid points Theorem , we have,

$$GF \parallel BD \text{ and } GF = \frac{1}{2} BD$$

Thus, we have,

$$HE \parallel GF \text{ and } HE = GF = \frac{1}{2} BD \dots (2)$$

The diagonals of a square are equal.

$$\Rightarrow AC = BD \dots (3)$$

From (1), (2) and (3), we have

$GF \parallel BD$ and $HE \parallel GF$.

Also, we have $EF = GF = GH = HE$

So, EFGH is a rhombus

Now, as diagonals of a square are equal and intersect at right angles.

$$\text{So, } \angle DOC = 90^\circ$$

In a parallelogram the sum of adjacent angles is 180° .

$$\text{So, } \angle DOC + \angle GKO = 180^\circ$$

$$\Rightarrow \angle GKO = 180^\circ - 90^\circ = 90^\circ$$

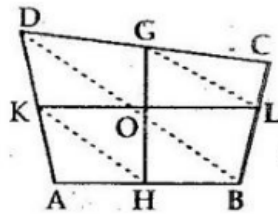
But $\angle GKO = \angle EFG$ [Corresponding angles]

$$\therefore \angle EFG = 90^\circ$$

\therefore EFGH is a square.

Given: A quadrilateral ABCD in which H, L, G and K are the midpoints of AB, BC, CD and AD.

Points G and H are joined and K and L are joined.



To prove: GH and KL bisect each other.

Construction: Join KH, BD and GL.

Proof: Since K and H are the midpoints of AD and AB.

So in $\triangle ABD$, by mid point theorem,

$$\Rightarrow KH = \frac{1}{2} BD$$

Similarly, in $\triangle CBD$,

$$GL = \frac{1}{2} BD$$

$$\Rightarrow KH = GL$$

Now in $\triangle KOH$ and $\triangle GOL$, we have

$$KH = GL$$

$$\angle OKH = \angle GLO \quad [\text{Alternate angles}]$$

$$\angle OHK = \angle OGL \quad [\text{Alternate angles}]$$

$$\therefore \triangle KOH = \triangle GOL \quad [\text{SAS}]$$

$$\Rightarrow OK = OL \text{ and } OG = OH \quad [\text{C.P.C.T.}]$$

$$\therefore GH \text{ and } KL \text{ bisect each other.}$$