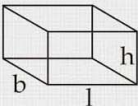
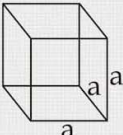
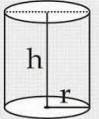
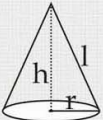
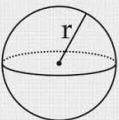
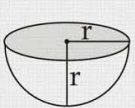


Volume and Surface Area

Exercise 13A

Name of the solid	Figure	Volume	Lateral/Curved Surface Area	Total Surface Area
Cuboid		lbh	$2lh + 2bh$ or $2h(l+b)$	$2lh+2bh+2lb$ or $2(lh+bh+lb)$
Cube		a^3	$4a^2$	$4a^2+2a^2$ or $6a^2$
Right circular cylinder		$\pi r^2 h$	$2\pi rh$	$2\pi rh + 2\pi r^2$ or $2\pi r(h+r)$
Right circular cone		$\frac{1}{3}\pi r^2 h$	πrl	$\pi rl + \pi r^2$ or $\pi r(l+r)$
Sphere		$\frac{4}{3}\pi r^3$	$4\pi r^2$	$4\pi r^2$
Hemisphere		$\frac{2}{3}\pi r^3$	$2\pi r^2$	$2\pi r^2 + \pi r^2$ or $3\pi r^2$

Question 1:

(i) length = 12 cm, breadth = 8 cm and height = 4.5 cm

\therefore Volume of cuboid = $l \times b \times h$

$$= (12 \times 8 \times 4.5) \text{ cm}^3 = 432 \text{ cm}^3$$

\therefore Lateral surface area of a cuboid = $2(l + b) \times h$

$$= [2(12 + 8) \times 4.5] \text{ cm}^2$$

$$= (2 \times 20 \times 4.5) \text{ cm}^2 = 180 \text{ cm}^2$$

\therefore Total surface area cuboid = $2(lb + bh + lh)$

$$= 2(12 \times 8 + 8 \times 4.5 + 12 \times 4.5) \text{ cm}^2$$

$$= 2(96 + 36 + 54) \text{ cm}^2$$

$$= (2 \times 186) \text{ cm}^2$$

$$= 372 \text{ cm}^2$$

(ii) Length = 26 m, breadth = 14 m and height = 6.5 m

\therefore Volume of a cuboid = $l \times b \times h$

$$= (26 \times 14 \times 6.5) \text{ m}^3$$

$$= 2366 \text{ m}^3$$

\therefore Lateral surface area of a cuboid = $2(l + b) \times h$

$$= [2(26 + 14) \times 6.5] \text{ m}^2$$

$$= (2 \times 40 \times 6.5) \text{ m}^2$$

$$= 520 \text{ m}^2$$

$$\therefore \text{Total surface area} = 2(lb + bh + lh)$$

$$= 2(26 \times 14 + 14 \times 6.5 + 26 \times 6.5)$$

$$= 2(364 + 91 + 169) \text{ m}^2$$

$$= (2 \times 624) \text{ m}^2 = 1248 \text{ m}^2.$$

(iii) Length = 15 m, breadth = 6 m and height = 5 dm = 0.5 m

$$\therefore \text{Volume of a cuboid} = l \times b \times h$$

$$= (15 \times 6 \times 0.5) \text{ m}^3 = 45 \text{ m}^3.$$

$$\therefore \text{Lateral surface area} = 2(l + b) \times h$$

$$= [2(15 + 6) \times 0.5] \text{ m}^2$$

$$= (2 \times 21 \times 0.5) \text{ m}^2 = 21 \text{ m}^2$$

$$\therefore \text{Total surface area} = 2(lb + bh + lh)$$

$$= 2(15 \times 6 + 6 \times 0.5 + 15 \times 0.5) \text{ m}^2$$

$$= 2(90 + 3 + 7.5) \text{ m}^2$$

$$= (2 \times 100.5) \text{ m}^2$$

$$= 201 \text{ m}^2$$

(iv) Length = 24 m, breadth = 25 cm = 0.25 m, height = 6 m.

$$\therefore \text{Volume of cuboid} = l \times b \times h$$

$$= (24 \times 0.25 \times 6) \text{ m}^3.$$

$$= 36 \text{ m}^3.$$

$$\therefore \text{Lateral surface area} = 2(l + b) \times h$$

$$= [2(24 + 0.25) \times 6] \text{ m}^2$$

$$= (2 \times 24.25 \times 6) \text{ m}^2$$

$$= 291 \text{ m}^2.$$

$$\therefore \text{Total surface area} = 2(lb + bh + lh)$$

$$= 2(24 \times 0.25 + 0.25 \times 6 + 24 \times 6) \text{ m}^2$$

$$= 2(6 + 1.5 + 144) \text{ m}^2$$

$$= (2 \times 151.5) \text{ m}^2$$

$$= 303 \text{ m}^2.$$

Question 2:

Length of Cistern = 8 m

Breadth of Cistern = 6 m

And Height (depth) of Cistern = 2.5 m

$$\therefore \text{Capacity of the Cistern} = \text{Volume of cistern}$$

$$\therefore \text{Volume of Cistern} = (l \times b \times h)$$

$$= (8 \times 6 \times 2.5) \text{ m}^3$$

$$= 120 \text{ m}^3$$

Area of the iron sheet required = Total surface area of the cistern.

$$\therefore \text{Total surface area} = 2(lb + bh + lh)$$

$$= 2(8 \times 6 + 6 \times 2.5 + 2.5 \times 8) \text{ m}^2$$

$$= 2(48 + 15 + 20) \text{ m}^2$$

$$= (2 \times 83) \text{ m}^2 = 166 \text{ m}^2$$

Question 3:

Length of a room = 9m,

Breadth of a room = 8m

And height of room = 6.5 m

∴ Area of 4 walls = Lateral surface area

$$= 2(l + b) \times h$$

$$= [2(9+8) \times 6.5] \text{ m}^2$$

$$= (2 \times 17 \times 6.5) \text{ m}^2$$

$$= 221 \text{ m}^2$$

∴ Area not to be whitewashed = (area of 1 door) + (area of 2 windows)

$$= (2 \times 1.5) \text{ m}^2 + (2 \times 1.5 \times 1) \text{ m}^2$$

$$= 3 \text{ m}^2 + 3 \text{ m}^2 = 6 \text{ m}^2$$

∴ Area to be whitewashed = $(221 - 6) \text{ m}^2 = 215 \text{ m}^2$

∴ Cost of whitewashing the walls at the rate of Rs.6.40 per

Square meter = Rs. $(6.40 \times 215) = \text{Rs. } 1376$

Question 4:

Length of plank = 5m = 500 cm

Breadth of plank = 25 m

Height of plank = 10 cm

∴ Volume of plank = $l \times b \times h$

$$= (500 \times 25 \times 10) \text{ cm}^3$$

Now,

Length of pit = 20 m = 2000 cm

Breadth of pit = 6m = 600cm

Height of pit = 80 cm

∴ Volume of one pit = $(2000 \times 600 \times 80) \text{ cm}^3$

$$\begin{aligned} \therefore \text{Number of planks that can be stored} &= \frac{\text{Volume of pit}}{\text{Volume of plank}} \\ &= \frac{(2000 \times 600 \times 80)}{(500 \times 25 \times 10)} = 768 \end{aligned}$$

Question 5:

Length of wall = 8m = 800cm

Breadth of wall = 6m = 600 cm

Height of wall = 22.5 cm

∴ Volume of wall = $l \times b \times h$

$$= (800 \times 600 \times 22.5) \text{ cm}^3$$

Length of brick = 25cm

Breadth of brick = 11.25cm

Height of brick = 6cm

∴ Volume of brick = $(25 \times 11.25 \times 6) \text{ cm}^3$

∴ Number of bricks required = $\frac{\text{Volume of the wall}}{\text{Volume of brick}}$

$$= \frac{(800 \times 600 \times 22.5)}{(25 \times 11.25 \times 6)} = 6400$$

Question 6:

$$\begin{aligned}\text{Length of wall} &= 15\text{m} \\ \text{Breadth of wall} &= 0.3\text{m} \\ \text{Height of wall} &= 4\text{m} \\ \therefore \text{Volume of the wall} &= (15 \times 0.3 \times 4) \text{ m}^3 = 18\text{m}^3 \\ \text{Volume of mortar} &= \left(\frac{1}{12} \times 18\right) = 1.5 \text{ m}^3 \\ \text{Volume of wall} &= (18 - 1.5)\text{m}^3 = 16.5 = \frac{33}{2} \text{ m}^3\end{aligned}$$

$$\begin{aligned}\text{Length of brick} &= 22 \text{ cm} \\ \text{Breadth of brick} &= 12.5 \text{ cm} \\ \text{Height of brick} &= 7.5 \text{ cm} \\ \therefore \text{Volume of 1 brick} &= \left(\frac{22}{100} \times \frac{12.5}{100} \times \frac{7.5}{100}\right) \text{ m}^3 \\ &= \left(\frac{33}{16000}\right) \text{ m}^3\end{aligned}$$

$$\begin{aligned}\therefore \text{Number of bricks} &= \frac{\text{Volume of bricks}}{\text{Volume of 1brick}} \\ &= \left(\frac{33}{2} \times \frac{16000}{33}\right) = 8000\end{aligned}$$

Question 7:

$$\begin{aligned}\text{External length of cistern} &= 1.35 \text{ m} = 135 \text{ cm} \\ \text{External breadth of cistern} &= 1.08 \text{ m} = 108 \text{ cm} \\ \text{External height of cistern} &= 90\text{cm} \\ \therefore \text{External volume of cistern} &= (135 \times 108 \times 90) \text{ cm}^3 \\ &= 1312200 \text{ cm}^3 \\ \text{Internal length of cistern} &= (135 - 2 \times 2.5) \text{ cm} \\ &= (135 - 5) \text{ cm} = 130 \text{ cm} \\ \text{Internal breadth of cistern} &= (108 - 2 \times 2.5) \text{ cm} \\ &= (108 - 5) \text{ cm} = 103 \text{ cm} \\ \therefore \text{Internal height of cistern} &= (90 - 2.5) \text{ cm} = 87.5 \text{ cm} \\ \therefore \text{Capacity of the cistern} &= \text{Internal volume of} \\ \text{cistern} &= (130 \times 103 \times 87.5) \text{ cm}^3 \\ &= 1171625 \text{ cm}^3 \\ \text{Volume of the iron used} &= \text{External volume of the} \\ \text{cistern} &\quad - \text{Internal volume of the} \\ \text{cistern} &= (1312200 - 1171625) \text{ cm}^3 \\ &= 140575 \text{ cm}^3\end{aligned}$$

Question 8:

$$\begin{aligned}
 \text{Depth of the river} &= 2 \text{ m} \\
 \text{Breadth of the river} &= 45 \text{ m} \\
 \text{Length of the river} &= 3 \text{ K M /h} = \left(\frac{3 \times 1000}{60} \right) \text{ m/min} \\
 &= 50 \text{ m /min.}
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{ Volume of water running into the sea per minute} &= (50 \times 45 \times 2) \text{ m}^3 \\
 &= 4500 \text{ m}^3
 \end{aligned}$$

Question 9:

$$\begin{aligned}
 \text{Total cost of sheet} &= \text{Rs. 1620} \\
 \text{Cost of metal sheet per square meter} &= \text{Rs.30}
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{ Area of the sheet required} &= \left(\frac{\text{Total cost}}{\text{rate /m}^2} \right) \text{ sq.m.} \\
 &= \left(\frac{1620}{30} \right) \text{ sq.m} = 54 \text{ sq.m.}
 \end{aligned}$$

$$\begin{aligned}
 \text{Length of box} &= 5 \text{ m} \\
 \text{Breadth of box} &= 3 \text{ m}
 \end{aligned}$$

Now, Let the height of the box be x meters.

$$\begin{aligned}
 \therefore \text{ Area of the sheet} &= \text{Total surface area of the box.} \\
 &= 2(lb + bh + lh) \\
 54 &= 2(5 \times 3 + 3 \times x + 5 \times x) \\
 54 &= 2(15 + 3x + 5x) \\
 54 &= 2(15 + 8x)
 \end{aligned}$$

$$\begin{aligned}
 \therefore 2(15 + 8x) &= 54 \\
 \Rightarrow 30 + 16x &= 54 \\
 \Rightarrow 16x &= 54 - 30 \\
 \Rightarrow x &= \frac{24}{16} = 1.5 \text{ m}
 \end{aligned}$$

$$\therefore \text{ The height of the box} = 1.5 \text{ m.}$$

Question 10:

$$\begin{aligned}
 \text{Length of room} &= 10 \text{ m} \\
 \text{Breadth of room} &= 10 \text{ m} \\
 \text{Height of room} &= 5 \text{ m}
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{ Length of the longest pole} &= \text{length of diagonal} \\
 &= \sqrt{l^2 + b^2 + h^2} \\
 &= \sqrt{10^2 + 10^2 + 5^2} \\
 &= \sqrt{100 + 100 + 25} = \sqrt{225} = 15 \text{ m} \\
 \therefore \text{ The length of the longest pole that can be put in a room with} \\
 \text{given} \\
 \text{Dimensions} &= 15 \text{ m.}
 \end{aligned}$$

Question 11:

$$\begin{aligned}
 \text{Length of hall} &= 20 \text{ m} \\
 \text{Breadth of hall} &= 16 \text{ m} \\
 \text{Height of hall} &= 4.5 \text{ m} \\
 \therefore \text{ Volume of hall} &= l \times b \times h \\
 &= (20 \times 16 \times 4.5) \text{ m}^3
 \end{aligned}$$

$$\text{Volume of air needed per person} = 5 \text{ m}^3$$

$$\begin{aligned}
 \therefore \text{ Number of persons} &= \left(\frac{\text{Volume of the hall}}{\text{Volume of air needed per person}} \right) \\
 &= \left(\frac{20 \times 16 \times 4.5}{5} \right) = 288.
 \end{aligned}$$

Question 12:

Length of classroom = 10m
 Breadth of classroom = 6.4 m
 Height of classroom = 5 m
 Each student is given 1.6 m² of the floor area.

$$\begin{aligned}\text{Number of students} &= \frac{(\text{area of the room})}{1.6} \\ &= \frac{(10 \times 6.4)}{1.6} = \frac{64}{1.6} = 40\end{aligned}$$

∴ Number of students = 40

$$\begin{aligned}\therefore \text{Air required by each student} &= \left(\frac{\text{Volume of the room}}{\text{number of students}} \right) \text{m}^3 \\ &= \left(\frac{10 \times 6.4 \times 5}{40} \right) \text{m}^3 = \left(\frac{320}{40} \right) \text{m}^3 \\ &= 8 \text{m}^3\end{aligned}$$

Question 13:

Volume of a cuboid = 1536 m³
 Length of the cuboid = 16 m
 Let the breadth and height of the cuboid be 3x and 2x.
 ∴ Volume of cuboid = l × b × h

$$\Rightarrow 1536 = (16 \times 3x \times 2x)$$

$$\Rightarrow 1536 = 96x^2$$

$$\Rightarrow x^2 = \frac{1536}{96} = 16$$

$$\therefore x = \sqrt{16} = 4 \text{ m.}$$

$$\therefore \text{Breadth of the cuboid} = 3x = 3 \times 4 = 12 \text{m}$$

$$\text{And height of the cuboid} = 2x = 2 \times 4 = 8 \text{ m}$$

Question 14:

Surface area of a cuboid = 758 cm²
 Length = 14 cm
 Breadth = 11 cm

Let the height of the cuboid = h cm

$$\therefore \text{Surface area of cuboid} = 2(lb + bh + lh)$$

$$\Rightarrow 758 = 2(14 \times 11 + 11 \times h + 14 \times h)$$

$$\Rightarrow 758 = 2(154 + 11h + 14h)$$

$$\Rightarrow 758 = 2(154 + 25h)$$

$$\Rightarrow 758 = 308 + 50h$$

$$\Rightarrow 50h = 758 - 308$$

$$\therefore h = \frac{450}{50} = 9 \text{cm.}$$

$$\therefore \text{The height of the cuboid} = 9 \text{ cm}$$

Question 15:

(a) Each edge of a cube = 9m
 \therefore Volume of a cube = a^3
 $= (9)^3 \text{ m}^3 = 729 \text{ m}^3$
 \therefore Lateral surface area of cube = $4a^2$
 $= 4 \times (9)^2$
 $= (4 \times 81) \text{ m}^2$
 $= 324 \text{ m}^2$
 \therefore Total surface area of a cube = $6a^2$
 $= 6 \times (9)^2$
 $= (6 \times 81) \text{ m}^2$
 $= 486 \text{ m}^2$
 \therefore Diagonal of cube = $\sqrt{3} a$
 $= \sqrt{3} \times 9$
 $= (1.73 \times 9) \text{ m} = 15.57 \text{ m}$

(b) \therefore Each edge of a cube = 6.5 cm
Volume of a cube = $a^3 = (6.5)^3 \text{ cm}^3$
 $= 274.625 \text{ cm}^3$
 \therefore Lateral surface area of a cube = $4a^2$
 $= 4 \times (6.5)^2 \text{ cm}^2$
 $= (4 \times 42.25) \text{ cm}^2$
 $= 169 \text{ cm}^2$
Total surface area of a cube = $6a^2$
 $= 6 \times (6.5)^2 \text{ cm}^2$
 $= (6 \times 42.25) \text{ cm}^2$
 $= 253.5 \text{ cm}^2$
 \therefore Diagonal of cube = $\sqrt{3} a$
 $= \sqrt{3} \times 6.5$
 $= (1.73 \times 6.5) \text{ cm}$
 $= 11.245 \text{ cm}.$

Question 16:

Let each side of the cube be a cm.
Then, the total surface area of the cube = $(6a^2) \text{ cm}^2$
 $\therefore 6a^2 = 1176$
 $\Rightarrow a^2 = \frac{1176}{6} = 196$
 $\Rightarrow a = \sqrt{196} = 14 \text{ cm}$
 \therefore Volume of the cube = a^3
 $= (14)^3 = (14 \times 14 \times 14) \text{ cm}^3$
 $= 2744 \text{ cm}^3.$

Question 17:

Let each side of the cube be a cm
Then, the lateral surface area of the cube = $(4a^2) \text{ cm}^2$
 $\therefore 4a^2 = 900$
 $\Rightarrow a^2 = \frac{900}{4} = 225$
 $\therefore a = \sqrt{225} = 15 \text{ cm}$
 \therefore Volume of the cube = a^3
 $= (15)^3 = (15 \times 15 \times 15) \text{ cm}^3$
 $= 3375 \text{ cm}^3.$

Question 18:

Volume of the cube = 512 cm^3 [Volume = a^3]
 \therefore Each edge of the cube = $\sqrt[3]{512} = 8 \text{ cm}.$
 \therefore Surface area of cube = $6a^2$
 $= 6 \times (8)^2 \text{ cm}^2$
 $= (6 \times 64) \text{ cm}^2$
 $= 384 \text{ cm}^2$

Question 19:

$$\begin{aligned}\text{Volume of the new cube} &= [(3)^3 + (4)^3 + (5)^3] \text{ cm} \\ &= (27 + 64 + 125) \text{ cm}^3 \\ &= 216 \text{ cm}^3\end{aligned}$$

$$\text{Now edge of this cube} = a \text{ cm}$$

$$\text{And, } a^3 = 216$$

$$\therefore a = 6 \text{ cm}$$

$$\begin{aligned}\text{Lateral surface area of the new cube} &= 4a^2 \text{ cm}^2. \\ &= 4 \times (6)^2 \text{ cm}^2 \\ &= (4 \times 36) \text{ cm}^2 \\ &= 144 \text{ cm}^2\end{aligned}$$

$$\therefore \text{The lateral surface area of the new cube formed} = 144 \text{ cm}^2.$$

Question 20:

$$1 \text{ hectare} = 10000 \text{ m}^2$$

$$\text{Area} = 2 \text{ hectares} = 2 \times 10000 \text{ m}^2$$

$$\text{Depth of the ground} = 5 \text{ cm} = \frac{5}{100} \text{ m}$$

$$\begin{aligned}\text{Volume of water} &= (\text{area} \times \text{depth}) \\ &= \left(2 \times 10000 \times \frac{5}{100} \right) \text{ m}^3 \\ &= 1000 \text{ m}^3\end{aligned}$$

$$\therefore \text{Volume of water that falls} = 1000 \text{ m}^3$$

Exercise 13B

Question 1:

$$\text{Here, } r = 5 \text{ cm and } h = 21 \text{ cm}$$

$$\begin{aligned}\therefore \text{Volume of the cylinder} &= (\pi r^2 h) \\ &= \left(\frac{22}{7} \times 5^2 \times 21 \right) \text{ cm}^3 \\ &= \left(\frac{22}{7} \times 25 \times 21 \right) \text{ cm}^3 \\ &= 1650 \text{ cm}^3.\end{aligned}$$

$$\begin{aligned}\therefore \text{Curved surface area of a cylinder} &= (2\pi rh) \\ &= 2 \times \left(\frac{22}{7} \times 5 \times 21 \right) \text{ cm}^2 \\ &= 660 \text{ cm}^2\end{aligned}$$

Question 2:

Here, diameter = 28 cm

$$\text{Radius} = \left(\frac{28}{2}\right) \text{ cm} = 14 \text{ cm and}$$

$$\text{height} = 40 \text{ cm}$$

$$\therefore \text{Curved surface area} = (2\pi rh)$$

$$= \left(2 \times \frac{22}{7} \times 14 \times 40\right) \text{ cm}^2$$

$$= 3520 \text{ cm}^2$$

$$\therefore \text{Total surface area} = (2\pi rh + 2\pi r^2)$$

$$= \left(2 \times \frac{22}{7} \times 14 \times 40 + 2 \times \frac{22}{7} \times 14^2\right)$$

$$= (3520 + 1232) = 4752 \text{ cm}^2$$

$$\therefore \text{Volume of the cylinder} = (\pi r^2 h)$$

$$= \left(\frac{22}{7} \times 14^2 \times 40\right) \text{ cm}^3$$

$$= \left(\frac{22}{7} \times 14 \times 14 \times 40\right) \text{ cm}^3$$

$$= 24640 \text{ cm}^3.$$

Question 3:

Here, radius (r) = 10.5 cm and height = 60 cm.

$$\therefore \text{Volume of the cylinder} = (\pi r^2 h)$$

$$= \left(\frac{22}{7} \times 10.5 \times 10.5 \times 60\right) \text{ cm}^3$$

$$= 20790 \text{ cm}^3$$

\therefore Weight of the solid cylinder if the material of the cylinder

$$\text{Weighs } 5 \text{ g per cm}^3 = (20790 \times 5) = 103950 \text{ g}$$

$$= \frac{103950}{1000} \quad [\because 1000 \text{ g} = 1 \text{ kg}]$$

$$= 103.95 \text{ kg}$$

Question 4:

Here, curved surface area = 1210 cm²

$$\text{Diameter} = 20 \text{ cm} \Rightarrow \text{radius} = \frac{20}{2} = 10 \text{ cm}$$

$$\therefore \text{Curved surface area of the cylinder} = 2\pi rh$$

$$\Rightarrow 1210 = 2 \times \frac{22}{7} \times 10 \times h$$

$$\Rightarrow h = \left(\frac{1210 \times 7}{2 \times 22 \times 10}\right) \text{ cm} = 19.25 \text{ cm}$$

$$\therefore \text{Height} = 19.25 \text{ cm}$$

$$\therefore \text{Volume of the cylinder} = (\pi r^2 h)$$

$$= \left(\frac{22}{7} \times 10^2 \times 19.25\right) \text{ cm}^3$$

$$= \left(\frac{22}{7} \times 10 \times 10 \times 19.25\right) \text{ cm}^3$$

$$= 6050 \text{ cm}^3$$

$$\therefore \text{Volume of the cylinder} = 6050 \text{ cm}^3.$$

Question 5:

Let base radius be r and height be h

$$\text{Then, } 2\pi rh = 4400 \text{ cm}^2$$

$$\text{And } 2\pi r = 110 \text{ cm}$$

$$\Rightarrow \frac{2\pi rh}{2\pi r} = \frac{4400}{110}$$

$$\Rightarrow h = 40 \text{ cm}$$

$$\therefore 2 \times \frac{22}{7} \times r \times h \times 40 = 4400 \text{ cm}.$$

$$\Rightarrow r = \left(\frac{4400 \times 7}{44 \times 40} \right) \text{ cm} = \frac{35}{2} \text{ cm}.$$

$$\begin{aligned} \therefore \text{Volume of the cylinder} &= \pi r^2 h \\ &= \left(\frac{22}{7} \times \frac{35}{2} \times \frac{35}{2} \times 40 \right) \text{ cm}^3 \\ &= 38500 \text{ cm}^3. \end{aligned}$$

Question 6:

Let the radius (r) = $2x$ cm and height (h) = $3x$ cm

Then, Volume of cylinder = $(\pi r^2 h)$

$$\text{Volume} = \left[\frac{22}{7} \times (2x)^2 \times 3x \right]$$

$$\text{Volume} = \left[\frac{22}{7} \times 4x^2 \times 3x \right]$$

$$\text{Volume} = \frac{22}{7} \times 12x^3$$

$$\Rightarrow 1617 = \frac{22}{7} \times 12x^3$$

[\because volume given = 1617 cm^3]

$$\Rightarrow 12x^3 = \frac{1617 \times 7}{22}$$

$$\Rightarrow x^3 = \frac{1617 \times 7}{22 \times 12} = \left(\frac{7}{2} \right)^3$$

$$\Rightarrow x = \frac{7}{2}$$

$$\therefore \text{radius} = 2x = 2 \times \frac{7}{2} = 7 \text{ cm}$$

$$\text{and height} = 3x = 3 \times \frac{7}{2} = \frac{21}{2} \text{ cm}$$

$$\text{Total surface area} = 2\pi r(h + r)$$

$$= 2 \times \frac{22}{7} \times 7 \left(\frac{21}{2} + 7 \right) \text{ cm}^2$$

$$= 44 \times \left(\frac{21 + 14}{2} \right) \text{ cm}^2$$

$$= (22 \times 35) \text{ cm}^2 = 770 \text{ cm}^2$$

Question 7:

$$\begin{aligned}
\text{Curved surface area} &= \frac{1}{3} \times (\text{total surface area}) \\
&= \left(\frac{1}{3} \times 462 \right) \text{cm}^2 = 154 \text{cm}^2 \\
(\text{Total surface area}) - (\text{Curved surface area}) \\
&= (462 - 154) \text{cm}^2 = 308 \text{cm}^2 \\
\Rightarrow 2\pi r^2 &= 308 \\
\Rightarrow 2 \times \frac{22}{7} \times r^2 &= 308 \\
\Rightarrow r^2 &= \frac{308 \times 7}{44} = 49 \\
\Rightarrow r &= \sqrt{49} = 7 \text{cm} \\
\text{Now, curved surface area} &= 2\pi rh = 154 \text{cm}^2 \\
&= 2 \times \frac{22}{7} \times 7 \times h = 154 \text{cm}^2 \\
&= h = \frac{154}{44} = 3.5 \text{cm} \\
\text{Now, } r &= 7 \text{ cm and } h = 3.5 \text{ cm} \\
\text{Volume of the cylinder} &= (\pi r^2 h) \\
&= \left(\frac{22}{7} \times 7 \times 7 \times 3.5 \right) \text{cm}^3 \\
&= 539 \text{cm}^3 \\
\therefore \text{ The volume of the cylinder} &= 539 \text{cm}^3.
\end{aligned}$$

Question 8:

$$\begin{aligned}
\text{Curved surface area} &= \frac{2}{3} \times (\text{total surface area}) \\
&= \left(\frac{2}{3} \times 231 \right) \text{cm}^2 = 154 \text{cm}^2 \\
(\text{Total surface area}) - (\text{Curved surface area}) \\
&= (231 - 154) \text{cm}^2 = 77 \text{cm}^2 \\
2\pi r^2 &= 77 \text{cm}^2 \\
\Rightarrow 2 \times \frac{22}{7} \times r^2 &= 77 \\
\Rightarrow r^2 &= \frac{77 \times 7}{44} = \frac{49}{4} \\
\Rightarrow r &= \sqrt{\frac{49}{4}} = \frac{7}{2} \text{cm} \\
\text{Now, } 2\pi rh &= 154 \text{cm}^2 \\
\Rightarrow 2 \times \frac{22}{7} \times \frac{7}{2} \times h &= 154 \text{cm}^2 \\
\Rightarrow h &= \frac{154}{22} = 7 \text{cm} \\
\text{Now, } r &= \frac{7}{2} \text{cm and } h = 7 \text{cm} \\
\text{Volume of the cylinder} &= \pi r^2 h \\
&= \left(\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 7 \right) \text{cm}^3 \\
&= 269.5 \text{cm}^3 \\
\text{Volume of the cylinder} &= 269.5 \text{cm}^3
\end{aligned}$$

Question 9:

Here, $(r + h) = 37 \text{ m}$ [\because given]

$$\text{And, total surface area} = 2\pi r(r + h) = 1628\text{m}^2$$

$$\Rightarrow 2\pi \times 37 = 1628\text{m}^2$$

$$\Rightarrow 2 \times \frac{22}{7} \times r \times 37 = 1628$$

$$\Rightarrow r = \frac{1628 \times 7}{44 \times 37} = 7\text{ m}$$

$$\text{And } (r + h) = 37\text{ m}$$

$$\Rightarrow (7 + h) = 37$$

$$\Rightarrow h = 37 - 7 = 30\text{ m}$$

$$\text{Volume} = \pi r^2 h$$

$$= \left(\frac{22}{7} \times 7 \times 7 \times 30 \right) \text{m}^3 = 4620\text{ m}^3.$$

Question 10:

$$\text{Curved surface area} = 2\pi rh$$

$$\text{Total surface area} = 2\pi r(h + r)$$

Since they are in the ratio of 1: 2

$$\therefore \frac{2\pi rh}{2\pi r(h + r)} = \frac{1}{2}$$

$$\Rightarrow \frac{h}{h + r} = \frac{1}{2}$$

$$\Rightarrow 2h = h + r$$

$$\Rightarrow 2h - h = r$$

$$\Rightarrow h = r$$

$$2\pi r(h + r) = 616\text{ cm}^2$$

$$\Rightarrow 4\pi r^2 = 616\text{ cm}^2 \quad [\text{Putting } h = r]$$

$$\Rightarrow 4 \times \frac{22}{7} \times r^2 = 616$$

$$\Rightarrow r^2 = \frac{616 \times 7}{88} = 49$$

$$\Rightarrow r = \sqrt{49} = 7\text{ cm}$$

Then, $r = 7\text{ cm}$ and $h = 7\text{ cm}$

$$\therefore \text{Volume} = (\pi r^2 h)$$

$$= \left(\frac{22}{7} \times 7 \times 7 \times 7 \right) \text{cm}^3 = 1078\text{ cm}^3$$

\therefore the volume of the cylinder = 1078 cm^3 .

Question 11:

$$1\text{ cm}^3 = 1\text{ cm} \times 1\text{ cm} \times 1\text{ cm} \text{ and } 1\text{ cm} = 0.01\text{ m}$$

Therefore,

Volume of the

$$\text{gold} = 0.01\text{ m} \times 0.01\text{ m} \times 0.01\text{ m} = 0.000001\text{ m}^3 \dots\dots(1)$$

$$\text{Diameter of the wire drawn} = 0.1\text{ mm}$$

$$\text{Radius of the wire drawn} = \frac{0.1}{2}\text{ mm} = 0.05\text{ mm}$$

$$r = 0.00005\text{ m} \dots\dots(2)$$

$$\text{Length of the wire} = h\text{ m} \dots\dots(3)$$

Volume of the wire drawn = Volume of the gold

$$\Rightarrow \pi r^2 h = 0.000001$$

$$\Rightarrow \pi \times 0.00005 \times 0.00005 \times h = 0.000001 \quad [\text{from equations (1), (2) and (3)}]$$

$$h = \frac{0.000001 \times 7}{0.00005 \times 0.00005 \times 22}$$

$$\therefore h = 127.27\text{ m}$$

\therefore the length of the wire is 127.27 m

Question 12:

Let the radii of the two cylinders be $2R$ and $3R$.

And their heights be $5H$ and $3H$.

$$\text{Then, } \frac{V_1}{V_2} = \frac{\pi \times (2R)^2 \times 5H}{\pi \times (3R)^2 \times 3H} = \frac{\pi \times 4R^2 \times 5H}{\pi \times 9R^2 \times 3H} = \frac{20}{27}$$

\therefore the ratio of their volumes = $20:27$

$$\text{Now, } \frac{S_1}{S_2} = \frac{2\pi(2R)(5H)}{2\pi(3R)(3H)} = \frac{10}{9}$$

\therefore the ratio of their curved surfaces = $10:9$

Question 13:

For the tin having square base,

side = 12 cm and height = 17.5 cm .

$$\therefore \text{Volume} = (12 \times 12 \times 17.5)\text{ cm}^3 = 2520\text{ cm}^3$$

Now, diameter of tin with cylindrical base = 12 cm

$$\therefore \text{radius} = \left(\frac{12}{2}\right)\text{ cm} = 6\text{ cm} \text{ and height} = 17.5\text{ cm}$$

$$\therefore \text{Volume} = \left(\frac{22}{7} \times 6 \times 6 \times 17.5\right)\text{ cm}^3 = 1980\text{ cm}^3$$

$$\begin{aligned} \text{Tin with square base has more capacity by } (2520 - 1980)\text{ cm}^3 \\ = 540\text{ cm}^3. \end{aligned}$$

Question 14:

Here, cylindrical bucket has diameter = 28 cm

$$\therefore \text{radius} = \left(\frac{28}{2}\right)\text{ cm} = 14\text{ cm} \text{ and height} = 72\text{ cm}$$

Length of the tank = 66 cm

Breadth of the tank = 28 cm

\therefore Volume of tank = Volume of cylindrical bucket

$$\Rightarrow l \times b \times h = \pi r^2 h$$

$$\Rightarrow 66 \times 28 \times h = \frac{22}{7} \times 14 \times 14 \times 72$$

$$\Rightarrow h = \left(\frac{22 \times 2 \times 14 \times 72}{66 \times 28}\right)\text{ cm}$$

$$\Rightarrow h = 24\text{ cm}$$

\therefore The height of the water level in the tank = 24 cm .

Question 15:

$$\text{Internal radius} = \left(\frac{3}{2}\right)\text{ cm} = 1.5\text{ cm}$$

$$\text{And, external radius} = (1.5 + 1)\text{ cm} = 2.5\text{ cm}$$

$$\text{Volume of cast iron} = \left[\pi \times (2.5)^2 \times 100 - \pi \times (1.5)^2 \times 100\right]\text{ cm}^3$$

$$= \pi \times 100 \times [(2.5)^2 - (1.5)^2]\text{ cm}^3$$

$$= \frac{22}{7} \times 100 \times [6.25 - 2.25]\text{ cm}^3$$

$$= \left(\frac{22}{7} \times 100 \times 4\right)\text{ cm}^3$$

$$\begin{aligned} \therefore \text{Weight} &= \left(\frac{22}{7} \times 100 \times 4 \times \frac{21}{1000}\right)\text{ kg} \\ &[\because 1\text{ kg} = 1000\text{ g}] \\ &= 26.4\text{ kg} \end{aligned}$$

\therefore the weight of the iron pipe = 26.4 kg .

Question 16:

Internal diameter of the tube = 10.4 cm

$$\text{internal radius} = \left(\frac{10.4}{2}\right) \text{cm} = 5.2 \text{ cm}$$

and length = 25 cm

and external radius = $(5.2 + 0.8) \text{ cm} = 6 \text{ cm}$

$$\begin{aligned}\text{Required volume} &= \left[\pi \times (6)^2 \times 25 - \pi \times (5.2)^2 \times 25 \right] \text{cm}^3 \\ &= \pi \times 25 \left[(6)^2 - (5.2)^2 \right] \text{cm}^3 \\ &= \frac{22}{7} \times 25 [36 - 27.04] \text{cm}^3 \\ &= \left(\frac{22}{7} \times 25 \times 8.96 \right) \text{cm}^3 \\ &= 704 \text{ cm}^3\end{aligned}$$

\therefore the volume of the metal = 704 cm³

Question 17:

Length = 7 cm = (height)

$$\begin{aligned}\text{Diameter} = 5 \text{ mm} \Rightarrow \text{radius} &= \left(\frac{5}{2}\right) \text{ mm} = 2.5 \text{ mm} \\ &= 0.25 \text{ cm}\end{aligned}$$

$$\begin{aligned}\therefore \text{Volume of the barrel} &= \pi r^2 h \\ &= \left(\frac{22}{7} \times 0.25 \times 0.25 \times 7 \right) \text{cm}^3 \\ &= \frac{11}{8} \text{ cm}^3\end{aligned}$$

$\frac{11}{8} \text{ cm}^3$ is used for writing 330 words.

So, $\left(\frac{1}{5} \times 1000\right) \text{ cm}^3$ will be used for writing

$$\begin{aligned}\left(330 \times \frac{8}{11} \times \frac{1}{5} \times 1000 \right) \text{ words} \\ = 48000 \text{ words}\end{aligned}$$

Question 18:

$$\begin{aligned}\text{Weight of the graphite} &= \left[\frac{22}{7} \times (0.05)^2 \times 10 \times 2.1 \right] \text{g} \\ &= \frac{33}{200} \text{ g}\end{aligned}$$

$$\begin{aligned}\text{Weight of wood} &= \left[\frac{22}{7} \times 10 \left\{ (0.35)^2 - (0.05)^2 \right\} \times 0.7 \right] \\ &= \left[\frac{22}{7} \times 10 (0.1225 - 0.0025) \times 0.7 \right] \\ &= \frac{66}{25} \text{ g}\end{aligned}$$

$$\begin{aligned}\therefore \text{Total weight of the pencil} &= \left(\frac{33}{200} + \frac{66}{25} \right) \text{g} \\ &= \left(\frac{33 + 528}{200} \right) \text{g} = \frac{561}{200} = 2.805 \text{ g}\end{aligned}$$

\therefore Weight of the whole pencil = 2.805 g

Exercise 13C

Question 1:

Here, $r = 35$ cm and $h = 84$ cm

$$\begin{aligned}\therefore \text{Volume of the cone} &= \frac{1}{3} \pi r^2 h \\ &= \left(\frac{1}{3} \times \frac{22}{7} \times 35 \times 35 \times 84 \right) \text{ cm}^3 \\ &= 107800 \text{ cm}^3 \\ \therefore \text{Curved surface area} &= \left(\pi r \sqrt{h^2 + r^2} \right) \quad [\because l = \sqrt{h^2 + r^2}] \\ &= \pi r \sqrt{84^2 + 35^2} \\ &= \pi r \sqrt{8281} \\ &= \frac{22}{7} \times 35 \times 91 \\ &= 10010 \text{ cm}^2\end{aligned}$$

$$\therefore \text{Total surface area} = \pi r(l + r)$$

$$\begin{aligned}\text{Now,} \quad l &= \sqrt{h^2 + r^2} \\ &= \sqrt{84^2 + 35^2} \\ &= \sqrt{7056 + 1225} = \sqrt{8281} = 91 \text{ cm}\end{aligned}$$

$$\begin{aligned}\therefore \text{Total surface area} &= \frac{22}{7} \times 35(91 + 35) \\ &= (22 \times 5 \times 126) \text{ cm}^2 = 13860 \text{ cm}^2\end{aligned}$$

Question 2:

Here, height (h) = 6 cm and slant height (ℓ) = 10 cm

$$\begin{aligned}\therefore \text{radius}(r) &= \sqrt{\ell^2 - h^2} \\ &= \sqrt{10^2 - 6^2} = \sqrt{100 - 36} \\ &= \sqrt{64} = 8 \text{ cm}\end{aligned}$$

$$\begin{aligned}\therefore \text{Volume of cone} &= \frac{1}{3} \pi r^2 h \\ &= \left(\frac{1}{3} \times 3.14 \times 8 \times 8 \times 6 \right) \text{ cm}^3 \\ &= 401.92 \text{ cm}^3\end{aligned}$$

$$\begin{aligned}\therefore \text{Curved surface area} &= \pi r \ell \\ &= (3.14 \times 8 \times 10) \text{ cm}^2 \\ &= 251.2 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\therefore \text{Total surface area} &= \pi r(\ell + r) \\ &= \pi r(10 + 8) \\ &= (3.14 \times 8 \times 18) \text{ cm}^2 \\ &= 452.16 \text{ cm}^2\end{aligned}$$

Question 3:

Here, Volume = (100π) cm³, height (h) = 12 cm

$$\begin{aligned}\text{Volume of the cone} &= \frac{1}{3} \pi r^2 h \\ \Rightarrow \quad 100 \pi &= \frac{1}{3} \pi \times r^2 \times 12 \\ \Rightarrow \quad r^2 &= \frac{100\pi \times 3}{\pi \times 12} \\ \Rightarrow \quad r^2 &= 25 \\ \Rightarrow \quad r &= \sqrt{25} = 5 \text{ cm.} \\ \text{Slant height}(\ell) &= \sqrt{h^2 + r^2} \\ &= \sqrt{12^2 + 5^2} \\ \ell &= \sqrt{144 + 25} = \sqrt{169} = 13 \text{ cm} \\ \therefore \text{Slant height, } \ell &= 13 \text{ cm} \\ \therefore \text{Curved surface area} &= \pi r \ell \\ &= \pi \times 5 \times 13 \text{ cm}^2 \\ &= 65\pi \text{ cm}^2\end{aligned}$$

Question 5:

Here, curved surface area = 550cm^2 and

slant height (ℓ) = 25 cm

\therefore Curved surface area = $\pi r \ell$

$$\Rightarrow 550 = \frac{22}{7} \times r \times 25$$

$$\Rightarrow r = \left(\frac{550 \times 7}{22 \times 25} \right) \text{cm} = 7\text{ cm}$$

$$\begin{aligned}\text{Now, height (h)} &= \sqrt{\ell^2 - r^2} \\ &= \sqrt{(25)^2 - (7)^2} \\ &= \sqrt{625 - 49} \\ &= \sqrt{576} = 24\text{ cm}\end{aligned}$$

$$\therefore \text{ height of the cone} = 24\text{ cm}$$

$$\text{Volume of the cone} = \frac{1}{3} \pi r^2 h$$

$$\begin{aligned}&= \left(\frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 24 \right) \text{cm}^3 \\ &= 1232\text{ cm}^3\end{aligned}$$

$$\therefore \text{ Volume of the cone} = 1232\text{ cm}^3$$

Question 6:

Here, radius, $r = 35\text{ cm}$ and slant height, $\ell = 37\text{ cm}$

$$\begin{aligned}\therefore h &= \sqrt{\ell^2 - r^2} \\ &= \sqrt{(37)^2 - (35)^2} \\ &= \sqrt{1369 - 1225} = \sqrt{144} = 12\text{ cm}\end{aligned}$$

$$\therefore \text{ height (h)} = 12\text{ cm}$$

$$\begin{aligned}\therefore \text{ Volume of the cone} &= \frac{1}{3} \pi r^2 h \\ &= \left(\frac{1}{3} \times \frac{22}{7} \times 35 \times 35 \times 12 \right) \text{cm}^3 \\ &= 15400\text{ cm}^3\end{aligned}$$

$$\therefore \text{ Volume of the cone} = 15400\text{ cm}^3$$

Question 7:

Here, curved surface area = 4070 cm^2

$$\text{Diameter} = 70\text{ cm} \Rightarrow \text{radius} = \left(\frac{70}{2} \right) \text{cm} = 35\text{ cm}$$

\therefore Curved surface area = $\pi r \ell$

$$\Rightarrow 4070 = \frac{22}{7} \times 35 \times \ell$$

$$\Rightarrow \ell = \left(\frac{4070}{110} \right) \text{cm} = 37\text{ cm}$$

$$\therefore \text{ slant height} = 37\text{ cm.}$$

Question 8:

Here, radius = 7 m and height(h) = 24 m

$$\begin{aligned}\therefore \text{ slant height } (\ell) &= \sqrt{h^2 + r^2} \\ &= \sqrt{(24)^2 + (7)^2} \\ \ell &= \sqrt{576 + 49} = \sqrt{625} = 25\text{ m}\end{aligned}$$

Now, area of cloth = $\pi r \ell$

$$= \left(\frac{22}{7} \times 7 \times 25 \right) \text{m}^2 = 550\text{ m}^2$$

$$\begin{aligned}\therefore \text{ length of cloth} &= \frac{\text{area of cloth}}{\text{width of cloth}} = \left(\frac{550}{2.5} \right) \text{m} \\ &= 220\text{ m}\end{aligned}$$

$$\therefore \text{ Length of cloth required to make a conical tent} = 220\text{ m}$$

Question 9:

Here, height of cone = 3.6 cm and radius = 1.6 cm

After melting, its radius = 1.2 cm

Volume of original cone = Volume of cone after melting

$$\therefore \frac{1}{3}\pi \times 1.6 \times 1.6 \times 3.6 = \frac{1}{3}\pi \times 1.2 \times 1.2 \times h$$

$$\Rightarrow h = \frac{\frac{1}{3}\pi \times 1.6 \times 1.6 \times 3.6}{\frac{1}{3}\pi \times 1.2 \times 1.2} = 6.4 \text{ cm}$$

\therefore height of new cone = 6.4 cm

Question 10:

Let their heights be h and $3h$

And, their radii be $3r$ and r .

$$\text{Then, } V_1 = \frac{1}{3}\pi(3r)^2 \times h$$

$$\text{and, } V_2 = \frac{1}{3}\pi r^2 \times 3h$$

$$\Rightarrow \frac{V_1}{V_2} = \frac{\frac{1}{3}\pi(3r)^2 \times h}{\frac{1}{3}\pi r^2 \times 3h} = \frac{3}{1}$$

$$\therefore V_1 : V_2 = 3 : 1$$

Question 11:

Radius of the cylinder, $R = \left(\frac{105}{2}\right)$ m and its height, $H = 3$ m

Slant height (ℓ) = 53 m

\therefore area of canvas = $(2\pi RH + \pi R\ell)$

$$\begin{aligned} &= \left[\left(2 \times \frac{22}{7} \times \frac{105}{2} \times 3 \right) + \left(\frac{22}{7} \times \frac{105}{2} \times 53 \right) \right] \text{ m}^2 \\ &= (990 + 8745) \text{ m}^2 \\ &= 9735 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \therefore \text{length of canvas} &= \left(\frac{\text{area of canvas}}{\text{width of canvas}} \right) \text{ m} \\ &= \left(\frac{9735}{5} \right) = 1947 \text{ m.} \end{aligned}$$

Question 12:

Let the radius be r metres and height be h metres.

Area of the base = $(11 \times 4) \text{ m}^2 = 44 \text{ m}^2$

$$\therefore \pi r^2 = 44$$

$$\Rightarrow r^2 = \left(44 \times \frac{7}{22} \right) = 14 \text{ m}$$

$$\Rightarrow r^2 = 14 \text{ m}$$

$$\text{Volume of the cone} = \frac{1}{3}\pi r^2 h$$

$$\therefore \text{Volume of the cone} = (11 \times 20) \text{ m}^3 = 220 \text{ m}^3$$

$$\Rightarrow 220 = \frac{1}{3} \times \frac{22}{7} \times 14 \times h$$

$$\Rightarrow h = \frac{220 \times 3}{22 \times 2} = 15 \text{ m}$$

\therefore the height of the cone = 15 m.

Question 13:

Here, height of the cylindrical bucket = 32 m and radius = 18 cm.

Now, let the radius of the heap be R cm and its slant height be ℓ cm

$$\begin{aligned}\text{Then, } \pi \times (18)^2 \times 32 &= \frac{1}{3} \pi \times R^2 \times 24 \\ \Rightarrow R^2 &= \frac{\pi \times 18 \times 18 \times 32 \times 3}{\pi \times 24} = 1296 \\ \Rightarrow R &= \sqrt{1296} = 36 \text{ cm.}\end{aligned}$$

\therefore Radius of the heap = 36 cm

$$\begin{aligned}\text{Slant height}(\ell) &= \sqrt{h^2 + R^2} \\ &= \sqrt{(24)^2 + (36)^2} \\ &= \sqrt{576 + 1296} \\ &= \sqrt{1872} = 43.27 \text{ cm}\end{aligned}$$

\therefore Slant height of the heap = 43.27 cm.

Question 14:

Let the curved surface areas of cylinder and cone be $8x$ and $5x$.

$$\text{Then, } 2\pi rh = 8x \dots\dots(i)$$

$$\text{and, } \pi r\sqrt{h^2 + r^2} = 5x \dots\dots(ii)$$

Squaring both sides of equation (i), we have

$$\begin{aligned}(2\pi rh)^2 &= (8x)^2 \\ 4\pi^2 r^2 h^2 &= 64x^2 \dots\dots(iii)\end{aligned}$$

From (ii) we have,

$$\pi r\sqrt{h^2 + r^2} = 5x$$

Squaring both sides,

$$\begin{aligned}\Rightarrow \pi^2 r^2 (h^2 + r^2) &= 25x^2 \dots\dots(iv) \\ \Rightarrow \frac{4\pi^2 r^2 h^2}{\pi^2 r^2 (h^2 + r^2)} &= \frac{64}{25} \quad [\text{Divide (iii) by (iv)}] \\ \Rightarrow \frac{h^2}{(h^2 + r^2)} &= \frac{16}{25} \\ \Rightarrow 9h^2 &= 16r^2 \\ \Rightarrow \frac{r^2}{h^2} &= \frac{9}{16} \\ \Rightarrow \frac{r}{h} &= \frac{3}{4}\end{aligned}$$

\therefore The ratio of radius and height = 3 : 4

Question 15:

Here, height (h) of cylinder = 2.8 m = 280 cm

and diameter = 20 cm

$$\Rightarrow \text{radius} = \left(\frac{20}{2}\right) = 10 \text{ cm}$$

height (H) of the cone = 42 cm

$$\begin{aligned}\therefore \text{Volume of the pillar} &= \left(\pi r^2 h + \frac{1}{3} \pi r^2 H\right) \text{ cm}^3 \\ &= \pi r^2 \left(h + \frac{1}{3} H\right) \text{ cm}^3 \\ &= \frac{22}{7} \times 10 \times 10 \left(280 + \frac{1}{3} \times 42\right) \text{ cm}^3 \\ &= \frac{2200}{7} \times [280 + 14] \\ &= 92400 \text{ cm}^3 \\ \therefore \text{Weight of pillar} &= \left(\frac{92400 \times 7.5}{1000}\right) \text{ kg} = 693 \text{ kg}\end{aligned}$$

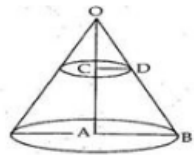
Question 16:

Let the smaller cone have radius = r cm and height = h cm
 And, let the radius of the given original cone be R cm
 Since the two triangles, $\triangle OCD$ and $\triangle OAB$
 are similar to each other, we have

$$\begin{aligned} \text{Then,} \quad \frac{r}{R} &= \frac{h}{30} \quad [\because \triangle OCD \sim \triangle OAB] \\ \Rightarrow \quad r &= \frac{Rh}{30} \dots\dots(1) \end{aligned}$$

Given that the volume of the small cone is
 $\frac{1}{27}$ of the volume of the given cone.

$$\begin{aligned} \therefore \quad \frac{1}{3} \pi r^2 h &= \frac{1}{27} \times \frac{1}{3} \pi R^2 \times 30 \quad [\text{given}] \\ \Rightarrow \quad \frac{1}{3} \pi \left(\frac{hR}{30} \right)^2 h &= \frac{1}{81} \pi R^2 \times 30 \quad [\text{from (1)}] \\ \Rightarrow \quad \frac{1}{3} \pi \frac{h^3 R^2}{900} &= \frac{1}{81} \pi R^2 \times 30 \\ \Rightarrow \quad h^3 &= \frac{1 \times 30 \times 900 \times 3}{81} \\ \Rightarrow \quad h^3 &= 1000 \text{ cm}^3 \\ \Rightarrow \quad h &= 10 \text{ cm} \end{aligned}$$



From the figure,
 $AC = (OA - OC)$
 $= (30 - 10) \text{ cm} = 20 \text{ cm}$
 \therefore the required height = 20 cm

Question 17:

Here, height(h) = 10 cm and radius = 6 cm

$$\begin{aligned} \therefore \text{Volume of the remaining solid} &= (\pi r^2 h) - \left(\frac{1}{3} \pi r^2 h \right) \\ &= (\pi \times 6 \times 6 \times 10) \text{ cm}^3 - \left(\frac{1}{3} \pi \times 6 \times 6 \times 10 \right) \text{ cm}^3 \\ &= \frac{2}{3} \pi \times 6 \times 6 \times 10 \text{ cm}^3 \\ &= \left(\frac{2}{3} \times 3.14 \times 360 \right) \text{ cm}^3 = 753.6 \text{ cm}^3 \\ \therefore \text{Volume of the remaining solid} &= 753.6 \text{ cm}^3 \end{aligned}$$

Question 18:

Diameter of the pipe = 5 mm = 0.5 cm

Radius of the pipe = $\frac{0.5}{2} = 0.25 \text{ cm}$

Length of the pipe = 10 metres = 1000 cm

Volume that flows in 1 min = $[\pi \times (0.25)^2 \times 1000] \text{ cm}^3$

\therefore Volume of the conical vessel = $\left[\frac{1}{3} \pi \times (20)^2 \times 24 \right] \text{ cm}^3$

$$\therefore \text{Required time} = \left[\frac{\frac{1}{3} \pi \times (20)^2 \times 24}{\pi \times (0.25)^2 \times 1000} \right] \text{ min}$$

$$\begin{aligned} &= \left[\frac{\frac{1}{3} \pi \times 400 \times 24}{\pi \times 0.0625 \times 1000} \right] \text{ min} \\ &= 51.2 \text{ min} \end{aligned}$$

= 51 min 12 sec

Exercise 13D

Question 1:

(i) Radius of sphere = 3.5 cm

$$\begin{aligned}\therefore \text{Volume of the sphere} &= \left(\frac{4}{3} \pi r^3\right) \\ &= \left(\frac{4}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 3.5\right) \text{ cm}^3 \\ &= 179.67 \text{ cm}^3\end{aligned}$$

$$\begin{aligned}\therefore \text{Surface area of the sphere} &= (4\pi r^2) \\ &= \left(4 \times \frac{22}{7} \times 3.5 \times 3.5\right) \text{ cm}^2 \\ &= 154 \text{ cm}^2\end{aligned}$$

(ii) Radius of the sphere = 4.2 cm

$$\begin{aligned}\therefore \text{Volume of the sphere} &= \left(\frac{4}{3} \pi r^3\right) \\ &= \left(\frac{4}{3} \times \frac{22}{7} \times 4.2 \times 4.2 \times 4.2\right) \text{ cm}^3 \\ &= 310.464 \text{ cm}^3\end{aligned}$$

$$\begin{aligned}\therefore \text{Surface area of the sphere} &= (4\pi r^2) \\ &= \left(4 \times \frac{22}{7} \times 4.2 \times 4.2\right) \text{ cm}^2 \\ &= 221.76 \text{ cm}^2\end{aligned}$$

(iii) Radius of sphere = 5 m

$$\begin{aligned}\therefore \text{Volume of the sphere} &= \left(\frac{4}{3} \pi r^3\right) \\ &= \left(\frac{4}{3} \times \frac{22}{7} \times 5 \times 5 \times 5\right) \text{ m}^3 \\ &= 523.81 \text{ m}^3\end{aligned}$$

$$\begin{aligned}\therefore \text{Surface area of the sphere} &= (4\pi r^2) \\ &= \left(4 \times \frac{22}{7} \times 5 \times 5\right) \text{ m}^2 \\ &= 314.28 \text{ m}^2\end{aligned}$$

Question 2:

$$\text{Volume of the sphere} = \left(\frac{4}{3} \pi r^3\right)$$

$$\Rightarrow 38808 = \frac{4}{3} \times \frac{22}{7} \times r^3 \quad [\because \text{Volume} = 38808 \text{ cm}^3]$$

$$\Rightarrow r^3 = \frac{38808 \times 3 \times 7}{88} = 9261$$

$$\Rightarrow r = 21 \text{ cm}$$

$$\therefore \text{Surface area of the sphere} = 4\pi r^2$$

$$= \left(4 \times \frac{22}{7} \times 21 \times 21\right) \text{ cm}^2$$

$$= 5544 \text{ cm}^2$$

Question 3:

$$\text{Volume of the sphere} = 606.375 \text{ m}^3 \quad \dots (1)$$

$$\text{Volume of the sphere} = \frac{4}{3} \pi r^3$$

$$\Rightarrow 606.375 = \frac{4}{3} \times \frac{22}{7} \times r^3 \quad [\text{from (1)}]$$

$$\Rightarrow r^3 = \frac{606.375 \times 3 \times 7}{4 \times 22}$$

$$= 144.703125$$

$$\Rightarrow r = 5.25 \text{ m}$$

$$\text{Surface area of the sphere} = 4\pi r^2$$

$$= 4 \times \frac{22}{7} \times 5.25 \times 5.25 \text{ m}^2$$

$$= 346.5 \text{ m}^2$$

Question 4:

Let the radius of the sphere be r m

$$\text{Then, its surface area} = (4\pi r^2)$$

$$\therefore (4\pi r^2) = 394.24$$

$$[\text{Surface area} = 394.24 \text{ m}^2]$$

$$4 \times \frac{22}{7} \times r^2 = 394.24$$

$$r^2 = \left(\frac{394.24 \times 7}{4 \times 22}\right) = 31.36$$

$$r = \sqrt{31.36} = 5.6 \text{ m}$$

$$\therefore \text{radius of the sphere} = 5.6 \text{ m}$$

$$\therefore \text{Volume of the sphere} = \left(\frac{4}{3} \pi r^3\right)$$

$$= \left(\frac{4}{3} \times \frac{22}{7} \times 5.6 \times 5.6 \times 5.6\right) \text{ m}^3$$

$$= 735.91 \text{ m}^3$$

$$\therefore \text{Volume of the sphere} = 735.91 \text{ m}^3$$

Question 5:

Surface area of sphere = $(4\pi r^2)$

$$\therefore (4\pi r^2) = (576\pi)$$

$$[\text{Surface area} = 576\pi \text{ cm}^2]$$

$$\Rightarrow r^2 = \frac{(576\pi)}{(4\pi)}$$

$$\Rightarrow r = \sqrt{144} = 12 \text{ cm}$$

$$\therefore \text{Volume of the sphere} = \left(\frac{4}{3}\pi r^3\right)$$

$$= \left(\frac{4}{3} \times \pi \times 12 \times 12 \times 12\right) \text{ cm}^3$$

$$= (2304\pi) \text{ cm}^3$$

$$\therefore \text{Volume of the sphere} = (2304\pi) \text{ cm}^3$$

Question 6:

Outer diameter of spherical shell = 12 cm

$$\text{radius} = 6 \text{ cm} \quad \left[\text{radius} = \frac{D}{2}\right]$$

Outer diameter of spherical shell = 8 cm

$$\text{radius} = 4 \text{ cm}$$

$$\text{Now, Volume of the outer shell} = \left(\frac{4}{3}\pi r^3\right)$$

$$= \left(\frac{4}{3} \times \frac{22}{7} \times 6 \times 6 \times 6\right) \text{ cm}^3$$

$$= 905.15 \text{ cm}^3$$

$$\therefore \text{Volume of the inner shell} = \left(\frac{4}{3}\pi r^3\right)$$

$$= \left(\frac{4}{3} \times \frac{22}{7} \times 4 \times 4 \times 4\right) \text{ cm}^3$$

$$= 268.20 \text{ cm}^3$$

$$\therefore \text{Volume of metal contained in the shell} = (\text{Volume of outer}) \\ - (\text{Volume of inner})$$

$$= (905.15 - 268.20) \text{ cm}^3$$

$$= 636.95 \text{ cm}^3$$

$$\therefore \text{Outer surface area} = 4\pi r^2$$

$$= \left(4 \times \frac{22}{7} \times 6 \times 6\right) \text{ cm}^2$$

$$= 452.57 \text{ cm}^2$$

Question 7:

Here, diameter of the lead shot = 3 mm

$$\therefore \text{radius} = \left(\frac{3}{2}\right) \text{ mm} = \left(\frac{0.3}{2}\right) \text{ cm}$$

$$[1 \text{ mm} = 0.1 \text{ cm}]$$

$$\text{Now, number of lead shots} = \frac{\text{Volume of the cuboid}}{\text{Volume of 1 lead shot}}$$

$$= \left\{ \frac{(12 \times 11 \times 9)}{\frac{4}{3} \times \frac{22}{7} \times \left(\frac{0.3}{2}\right)^3} \right\}$$

$$= \left\{ \frac{(12 \times 11 \times 9)}{\frac{4}{3} \times \frac{22}{7} \times \frac{0.027}{8}} \right\}$$

$$= \left\{ \frac{12 \times 11 \times 9 \times 3 \times 7 \times 8}{4 \times 22 \times 0.027} \right\} = 84000$$

$$\therefore \text{number of lead shots} = 84000.$$

Question 8:

Here, radius of 1 lead ball = 1 cm

and radius of sphere = 8 cm

$$\therefore \text{Number of lead balls} = \frac{\text{Volume of the sphere}}{\text{Volume of 1 lead ball}}$$

$$= \frac{\left(\frac{4}{3}\pi R^3\right) \text{ cm}^3}{\left(\frac{4}{3}\pi r^3\right) \text{ cm}^3}$$

$$= \left\{ \frac{\frac{4}{3} \times \frac{22}{7} \times 8^3}{\frac{4}{3} \times \frac{22}{7} \times 1^3} \right\}$$

$$= \left\{ \frac{\frac{4}{3} \times \frac{22}{7} \times 512}{\frac{4}{3} \times \frac{22}{7} \times 1} \right\} = 512$$

\therefore number of lead balls = 512.

Question 9:

Here, radius of sphere = 3 cm

$$\text{Diameter of spherical ball} = 0.6 \text{ cm} \quad \left[\because \text{radius} = \frac{D}{2} \right]$$

Radius of spherical ball = 0.3 cm

$$\therefore \text{Number of balls} = \frac{\text{Volume of the sphere}}{\text{Volume of 1 small ball}}$$

$$= \left\{ \frac{\frac{4}{3} \times \frac{22}{7} \times 3^3 \text{ cm}^3}{\frac{4}{3} \times \frac{22}{7} \times (0.3)^3 \text{ cm}^3} \right\}$$

$$= \left\{ \frac{\frac{4}{3} \times \frac{22}{7} \times 27}{\frac{4}{3} \times \frac{22}{7} \times 0.027} \right\} = 1000$$

\therefore number of small balls obtained = 1000.

Question 10:

Here, radius of sphere = $10.5 \text{ cm} = \left(\frac{21}{2}\right) \text{ cm}$

Radius of smaller cone = $3.5 \text{ cm} = \left(\frac{7}{2}\right) \text{ cm}$ and height = 3 cm

$$\text{Now number of cones} = \frac{\text{Volume of the sphere}}{\text{Volume of 1 small cone}}$$

$$= \frac{\left\{ \frac{4}{3}\pi \times \left(\frac{21}{2}\right)^3 \text{ cm}^3 \right\}}{\left\{ \frac{1}{3}\pi \times \left(\frac{7}{2}\right)^2 \times 3 \text{ cm}^3 \right\}}$$

$$= \frac{\left(\frac{4}{3} \times \frac{9261}{8} \right)}{\left(\frac{1}{3} \times \frac{49}{4} \times 3 \right)} = \frac{\frac{9261}{6}}{\frac{49}{4}}$$

$$= \frac{9261}{6} \times \frac{4}{49} = 126$$

\therefore Number of cones obtained = 126.

Question 11:

Diameter of a sphere = 12 cm

$$\text{radius} = \frac{\text{Diameter}}{2}$$

$$= \frac{12}{2}$$

$$= 6 \text{ cm}$$

$$\begin{aligned}\therefore \text{Volume of the sphere} &= \frac{4}{3} \pi r^3 \\ &= \frac{4}{3} \times \frac{22}{7} \times 6 \times 6 \times 6 \quad (\text{i})\end{aligned}$$

Diameter of cylinder = 8 cm

$$\text{Radius of cylinder} = \frac{\text{Diameter}}{2}$$

$$\text{Radius of cylinder} = \frac{8}{2}$$

$$\text{Radius of cylinder} = 4 \text{ cm}$$

Height of the cylinder = 90 cm

$$\begin{aligned}\therefore \text{Volume of the cylinder} &= \pi r^2 h \\ &= \frac{22}{7} \times 4 \times 4 \times 90 \quad (\text{ii})\end{aligned}$$

$$\text{Number of spheres} = \frac{\text{Volume of cylinder}}{\text{Volume of sphere}}$$

$$\begin{aligned}\text{Number of spheres} &= \frac{\frac{22}{7} \times 4 \times 4 \times 90 \text{ cm}^3}{\frac{4}{3} \times \frac{22}{7} \times 6 \times 6 \times 6 \text{ cm}^3} \quad [(\text{ii}) \div (\text{i})] \\ \text{Number of spheres} &= 5.\end{aligned}$$

Question 12:

Here, Diameter of a sphere = 6 cm

$$\therefore \text{radius (R)} = \left(\frac{6}{2}\right) \text{ cm} = 3 \text{ cm}$$

Diameter of wire = 2 mm

$$\therefore \text{radius (r)} = 1 \text{ mm} = 0.1 \text{ cm}$$

Let the required length of wire be h cm.

Then,

$$\begin{aligned}\Rightarrow \pi \times (r)^2 \times h &= \frac{4}{3} \times \pi \times (R)^3 \\ \Rightarrow \pi \times (0.1)^2 \times h &= \frac{4}{3} \times \pi \times (3)^3 \\ \Rightarrow h &= \frac{\frac{4}{3} \times \pi \times 27}{\pi \times (0.1)^2} \\ &= \left(\frac{4 \times 9}{0.01}\right) \text{ cm} = \frac{36}{0.01} \\ &= 3600 \text{ cm} = 36 \text{ m}\end{aligned}$$

\therefore the length of the wire = 36 m.

Question 13:

Here, diameter of sphere = 18 cm

$$\therefore \text{radius of sphere} = \left(\frac{18}{2}\right) \text{ cm} = 9 \text{ cm}$$

$$\text{Length of the wire} = 108 \text{ m} = 10800 \text{ cm}$$

Then,

$$\frac{4}{3} \pi \times (r)^3 = \pi \times r^2 \times 10800$$

$$\Rightarrow \frac{4}{3} \pi \times (9)^3 = \pi \times r^2 \times 10800$$

$$\Rightarrow r^2 = \frac{\frac{4}{3} \times \pi \times 729}{\pi \times 10800}$$
$$= \frac{4 \times 243}{10800} = \frac{972}{10800} = \frac{9}{100}$$

$$\Rightarrow r = \sqrt{\frac{9}{100}} = \frac{3}{10} = 0.3$$

$$\therefore r = 0.3 \text{ cm}$$

$$\text{So, Diameter} = (2 \times 0.3) \text{ cm} = 0.6 \text{ cm.}$$

Question 14:

Here, diameter of sphere = 15.6 cm

$$\therefore \text{Radius of sphere} = \left(\frac{15.6}{2}\right) \text{ cm} = 7.8 \text{ cm}$$

and, height of cone = 31.2 cm

Then,

$$\frac{4}{3} \pi \times R^3 = \frac{1}{3} \pi \times r^2 \times h$$

$$\Rightarrow \frac{4}{3} \pi \times (7.8)^3 = \frac{1}{3} \pi \times r^2 \times 31.2$$

$$\Rightarrow r^2 = \frac{\frac{4}{3} \times \pi \times (7.8)^3}{\frac{1}{3} \times \pi \times 31.2}$$

$$r^2 = \left(\frac{4 \times 474.552}{31.2}\right) = (60.84) = (7.8)^2$$

$$\Rightarrow r = 7.8 \text{ cm}$$

$$\therefore \text{Diameter of cone} = (2 \times 7.8) \text{ cm} = 15.6 \text{ cm.}$$

Question 15:

Here, diameter of sphere = 28 cm

$$\therefore \text{radius of sphere} = \left(\frac{28}{2}\right) \text{ cm} = 14 \text{ cm}$$

Diameter of cone = 35

$$\therefore \text{radius of cone} = \left(\frac{35}{2}\right) \text{ cm} = 17.5 \text{ cm}$$

$$\therefore \frac{4}{3} \times \pi \times R^3 = \frac{1}{3} \pi \times (r)^2 \times h$$

$$\Rightarrow h = \frac{\frac{4}{3} \times \pi \times (14)^3}{\frac{1}{3} \times \pi \times (17.5)^2}$$
$$= \left(\frac{4 \times 2744}{306.25}\right) \text{ cm}$$
$$= \left(\frac{10976}{306.25}\right) \text{ cm} = 35.84 \text{ cm}$$

$$\therefore \text{Height of the cone} = 35.84 \text{ cm}$$

Question 16:

Let the radius of the third ball be r cm

Then,

$$\begin{aligned}\frac{4}{3} \times \pi \times (3)^3 &= \frac{4}{3} \pi \left(\frac{3}{2}\right)^3 + \frac{4}{3} \times \pi (2)^3 + \frac{4}{3} \pi \times (r)^3 \\ \Rightarrow \frac{4}{3} \times \pi \times 27 &= \frac{4}{3} \pi \times \frac{27}{8} + \frac{4}{3} \times \pi \times 8 + \frac{4}{3} \pi \times (r)^3 \\ \Rightarrow 27 &= \frac{27}{8} + 8 + (r)^3 \\ \Rightarrow r^3 &= \left\{ 27 - \left(\frac{27}{8} + 8 \right) \right\} \\ \Rightarrow r^3 &= \left\{ 27 - \left(\frac{27 + 64}{8} \right) \right\} \\ \Rightarrow r^3 &= \left\{ 27 - \frac{91}{8} \right\} \\ \Rightarrow r^3 &= \left\{ \frac{216 - 91}{8} \right\} \\ \Rightarrow r^3 &= \frac{125}{8} \Rightarrow r^3 = \left(\frac{5}{2} \right)^3 \\ \Rightarrow r &= \frac{5}{2} = 2.5 \text{ cm}\end{aligned}$$

\therefore radius of the third ball = 2.5 cm

Question 17:

Let the radii of two spheres be x and $2x$ and their respective surface areas be S_1 and S_2 .

$$\begin{aligned}\text{Then, } \frac{S_1}{S_2} &= \frac{4\pi x^2}{4\pi (2x)^2} \\ &= \frac{x^2}{4x^2} = \frac{1}{4}\end{aligned}$$

\therefore the ratio of their surface areas = 1 : 4.

Question 18:

Let the radii of two spheres be r and R

Then,

$$\begin{aligned}\frac{4\pi r^2}{4\pi R^2} &= \frac{1}{4} \\ \Rightarrow \left(\frac{r}{R} \right)^2 &= \left(\frac{1}{2} \right)^2 \Rightarrow \frac{r}{R} = \frac{1}{2}\end{aligned}$$

Let V_1 and V_2 be the volumes of the respective spheres whose radii are r and R

$$\therefore \frac{V_1}{V_2} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} = \left(\frac{r}{R} \right)^3 = \left(\frac{1}{2} \right)^3 = \frac{1}{8}$$

\therefore the ratio of their volume = 1 : 8.

Question 19:

Let the radius of ball be r cm and R be the radius of the cylindrical tub.

Then,

$$\frac{4}{3} \times \pi \times (r)^3 = \pi \times R^2 \times h$$

$$\Rightarrow \frac{4}{3} \times \pi \times (r)^3 = \pi \times (12)^2 \times 6.75$$

$$\Rightarrow (r)^3 = \frac{\pi \times 144 \times 6.75}{\frac{4}{3} \times \pi} = \frac{144 \times 6.75}{\frac{4}{3}}$$

$$r^3 = \frac{972 \times 3}{4} = \frac{2916}{4} = 729$$

$$\Rightarrow r = 9 \text{ cm}$$

\therefore the radius of the ball = 9 cm.

Question 20:

Radius of the cylindrical bucket = 15 cm

Height of the cylindrical bucket = 20 cm

Volume of the water in the bucket = $\pi \times 15 \times 15 \times 20 \text{ cm}^3$

Radius of spherical ball = 9 cm

Volume of the spherical ball = $\frac{4}{3} \times \pi \times 9 \times 9 \times 9 \text{ cm}^3 \dots (1)$

Increase in the water level = h cm

Volume of the increased water level = $\pi \times 15 \times 15 \times h \text{ cm}^3 \dots (2)$

Equating (1) and (2),

we have

$$\pi \times 15 \times 15 \times h = \frac{4}{3} \times \pi \times 9 \times 9 \times 9$$

$$h = \frac{\frac{4}{3} \times \pi \times 9 \times 9 \times 9}{\pi \times 15 \times 15}$$

$$h = 4.32 \text{ cm}$$

Question 21:

Radius of hemisphere = 9 cm

Height of cone = 72 cm

Let the radius of the base of cone be r cm.

Then,

$$\frac{1}{3} \times \pi \times r^2 \times h = \frac{2}{3} \times \pi \times R^3$$

$$\Rightarrow \frac{1}{3} \times \pi \times r^2 \times 72 = \frac{2}{3} \times \pi \times (9)^3$$

$$\Rightarrow r^2 = \frac{\frac{2}{3} \times \pi \times 729}{\frac{1}{3} \times \pi \times 72} = \frac{2 \times 729}{72}$$

$$r^2 = \frac{1458}{72} = 20.25$$

$$\Rightarrow r = 4.5 \text{ cm}$$

\therefore the radius of the base of the cone = 4.5 cm.

Question 22:

Here, internal radius of hemisphere bowl (R) = 9 cm

Diameter of bottle = 3 cm

$$\Rightarrow \text{radius } (r) = \left(\frac{3}{2}\right) \text{ cm}$$

and, height of bottle = 4 cm

$$\begin{aligned} \therefore \text{Number of bottles} &= \frac{\text{Volume of the bowl}}{\text{Volume of each bottle}} \\ &= \left\{ \frac{\frac{2}{3} \pi \times R^3}{\pi \times (r)^2 \times h} \right\} \\ &= \left\{ \frac{\frac{2}{3} \pi \times (9)^3}{\pi \times \left(\frac{3}{2}\right)^2 \times 4} \right\} \\ &= \left\{ \frac{\frac{2}{3} \times 9 \times 9 \times 9}{\frac{9}{4} \times 4} \right\} \\ &= \frac{2 \times 3 \times 81}{9} = 54 \end{aligned}$$

\therefore the number of bottle required = 54.

Question 23:

Internal radius(r) = 8 cm

External radius(R) = 9 cm

Density of metal = 4.5g per cm^3

$$\begin{aligned} \therefore \text{weight of the shell} &= \left[\frac{4}{3} \pi \times \{(R)^3 - (r)^3\} \times \text{density} \right] \\ &= \left[\frac{4}{3} \times \frac{22}{7} \times \{(9)^3 - (8)^3\} \times \frac{4.5}{1000} \right] \text{ kg} \\ &= \left[\frac{4}{3} \times \frac{22}{7} \times \{729 - 512\} \times \frac{4.5}{1000} \right] \text{ kg} \\ &= \left[\frac{4}{3} \times \frac{22}{7} \times 217 \times \frac{4.5}{1000} \right] \text{ kg} \\ &= \left(\frac{85932}{21000} \right) \text{ kg} = 4.092 \text{ kg} \end{aligned}$$

\therefore weight of the shell = 4.092 kg.