Exercise – 2A

1. Find the zeros of the polynomial $f(x) = x^2 + 7x + 12$ and verify the relation between its zeroes and coefficients.

Sol:

$x^2 + 7x + 12 = 0$

$\Rightarrow x^2 + 4x + 3x + 12 = 0$

$\Rightarrow x(x+4) + 3(x+4) = 0$

$\Rightarrow (x+4)(x+3) = 0$

$\Rightarrow (x + 4) = 0$ or $(x + 3) = 0$

$\Rightarrow x = -4$ or $x = -3$

Sum of zeroes $= -4 + (-3) = \frac{-7}{1} = \frac{-(coefficient\ of\ x)}{(coefficient\ of\ x^2)}$

Product of zeroes $= (-4)(-3) = \frac{12}{1} = \frac{constant\ term}{(coefficient\ of\ x^2)}$

2. Find the zeroes of the polynomial $f(x) = x^2 - 2x - 8$ and verify the relation between its zeroes and coefficients.

Sol:

$x^2 - 2x - 8 = 0$

$\Rightarrow x^2 - 4x + 2x - 8 = 0$

$\Rightarrow x(x - 4) + 2(x - 4) = 0$

$\Rightarrow (x - 4)(x + 2) = 0$

$\Rightarrow (x - 4) = 0$ or $(x+2) = 0$

$\Rightarrow x = 4$ or $x = -2$

Sum of zeroes $= 4 + (-2) = 2 = \frac{2}{1} = \frac{-(coefficient\ of\ x)}{(coefficient\ of\ x^2)}$

Product of zeroes $= (4)(-2) = \frac{-8}{1} = \frac{constant\ term}{(coefficient\ of\ x^2)}$

3. Find the zeroes of the quadratic polynomial $f(x) = x^2 + 3x - 10$ and verify the relation between its zeroes and coefficients.

Sol:

We have:

$f(x) = x^2 + 3x - 10$

$= x^2 + 5x - 2x - 10$

$= x(x + 5) - 2(x + 5)$

$= (x - 2)(x + 5)$

$\therefore f(x) = 0 \Rightarrow (x - 2)(x + 5) = 0$

$\Rightarrow x - 2 = 0$ or $x + 5 = 0$

$\Rightarrow x = 2$ or $x = -5$.

So, the zeroes of f(x) are 2 and -5.

Sum of zeroes = 2 + (−5) = −3 = $\frac{-3}{1} = \frac{-(coefficient\ of\ x)}{(coefficient\ of\ x^2)}$

Product of zeroes = 2 × (−5) = −10 = $\frac{-10}{1} = \frac{constant\ term}{(coefficient\ of\ x^2)}$

4. Find the zeroes of the quadratic polynomial f(x) = $4x^2$ - 4x - 3 and verify the relation between its zeroes and coefficients.

Sol:

We have:

f(x) = $4x^2$ - 4x - 3

= $4x^2$ - (6x - 2x) - 3

= $4x^2$ - 6x + 2x - 3

= 2x (2x - 3) + 1(2x - 3)

= (2x + 1) (2x - 3)

∴ f(x) = 0 ⇒ (2x + 1) (2x - 3)= 0

⇒ 2x + 1= 0 or 2x - 3 = 0

⇒ x = $\frac{-1}{2}$ or x = $\frac{3}{2}$

So, the zeroes of f(x) are $\frac{-1}{2}$ and $\frac{3}{2}$.

Sum of zeroes = $\left(\frac{-1}{2}\right) + \left(\frac{3}{2}\right) = \frac{-1+3}{2} = \frac{2}{2} = 1 = \frac{-(coefficient\ of\ x)}{(coefficient\ of\ x^2)}$

Product of zeroes = $\left(\frac{-1}{2}\right) \times \left(\frac{3}{2}\right) = \frac{-3}{4} = \frac{constant\ term}{(coefficient\ of\ x^2)}$

5. Find the zeroes of the quadratic polynomial f(x) = $5x^2$ - 4 - 8x and verify the relationship between the zeroes and coefficients of the given polynomial.

Sol:

We have:

f(x) = $5x^2$ - 4 - 8x

= $5x^2$ - 8x - 4

= $5x^2$ - (10x - 2x) - 4

= $5x^2$ - 10x + 2x - 4

= 5x (x - 2) + 2(x - 2)

= (5x + 2) (x - 2)

∴ f(x) = 0 ⇒ (5x + 2) (x - 2) = 0

⇒ 5x + 2= 0 or x - 2 = 0

⇒ x = $\frac{-2}{5}$ or x = 2

So, the zeroes of f(x) are $\frac{-2}{5}$ and 2.

Sum of zeroes = $\left(\frac{-2}{5}\right) + 2 = \frac{-2+10}{5} = \frac{8}{5} = \frac{-(coefficient\ of\ x)}{(coefficient\ of\ x^2)}$

Product of zeroes = $\left(\frac{-2}{5}\right) \times 2 = \frac{-4}{5} = \frac{constant\ term}{(coefficient\ of\ x^2)}$

6. Find the zeroes of the polynomial f(x) = $2\sqrt{3}x^2 - 5x + \sqrt{3}$ and verify the relation between its zeroes and coefficients.

Sol:

$2\sqrt{3}x^2 - 5x + \sqrt{3}$

$\Rightarrow 2\sqrt{3}x^2 - 2x - 3x + \sqrt{3}$

$\Rightarrow 2x(\sqrt{3}x - 1) - \sqrt{3}(\sqrt{3}x - 1) = 0$

$\Rightarrow (\sqrt{3}x - 1)$ or $(2x - \sqrt{3}) = 0$

$\Rightarrow (\sqrt{3}x - 1) = 0$ or $(2x - \sqrt{3}) = 0$

$\Rightarrow x = \frac{1}{\sqrt{3}}$ or $x = \frac{\sqrt{3}}{2}$

$\Rightarrow x = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ or $x = \frac{\sqrt{3}}{2}$

Sum of zeroes = $\frac{\sqrt{3}}{3} + \frac{\sqrt{3}}{2} = \frac{5\sqrt{3}}{6} = \frac{-(coefficient\ of\ x)}{(coefficient\ of\ x^2)}$

Product of zeroes = $\frac{\sqrt{3}}{3} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{6} = \frac{constant\ term}{(coefficient\ of\ x^2)}$

7. Find the zeroes of the quadratic polynomial $2x^2 - 11x + 15$ and verify the relation between the zeroes and the coefficients.

Sol:

$f(x) = 2x^2 - 11x + 15$

$= 2x^2 - (6x + 5x) + 15$

$= 2x^2 - 6x - 5x + 15$

$= 2x(x - 3) - 5(x - 3)$

$= (2x - 5)(x - 3)$

$\therefore f(x) = 0 \Rightarrow (2x - 5)(x - 3) = 0$

$\Rightarrow 2x - 5 = 0$ or $x - 3 = 0$

$\Rightarrow x = \frac{5}{2}$ or $x = 3$

So, the zeroes of f(x) are $\frac{5}{2}$ and 3.

Sum of zeroes = $\frac{5}{2} + 3 = \frac{5+6}{2} = \frac{11}{2} = \frac{-(coefficient\ of\ x)}{(coefficient\ of\ x^2)}$

Product of zeroes = $\frac{5}{2} \times 3 = \frac{-15}{2} = \frac{constant\ term}{(coefficient\ of\ x^2)}$

8. Find the zeroes of the quadratic polynomial $4x^2 - 4x + 1$ and verify the relation between the zeroes and the coefficients.

Sol:

$4x^2 - 4x + 1 = 0$

$\Rightarrow (2x)^2 - 2(2x)(1) + (1)^2 = 0$

$\Rightarrow (2x - 1)^2 = 0$ $\quad [\because a^2 – 2ab + b^2 = (a–b)^2]$

$\Rightarrow (2x - 1)^2 = 0$

$\Rightarrow x = \frac{1}{2}$ or $x = \frac{1}{2}$

Sum of zeroes = $\frac{1}{2} + \frac{1}{2} = 1 = \frac{1}{1} = \frac{-(coefficient\ of\ x)}{(coefficient\ of\ x^2)}$

Product of zeroes = $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = \frac{constant\ term}{(coefficient\ of\ x^2)}$

9. Find the zeroes of the quadratic polynomial $(x^2 - 5)$ and verify the relation between the zeroes and the coefficients.

Sol:

We have:

$f(x) = x^2 - 5$

It can be written as $x^2 + 0x - 5$.

$= \left(x^2 - (\sqrt{5})^2\right)$

$= (x + \sqrt{5})(x - \sqrt{5})$

$\therefore f(x) = 0 \Rightarrow (x + \sqrt{5})(x - \sqrt{5}) = 0$

$\Rightarrow x + \sqrt{5} = 0$ or $x - \sqrt{5} = 0$

$\Rightarrow x = -\sqrt{5}$ or $x = \sqrt{5}$

So, the zeroes of f(x) are $-\sqrt{5}$ and $\sqrt{5}$.

Here, the coefficient of x is 0 and the coefficient of x^2 is 1.

Sum of zeroes = $-\sqrt{5} + \sqrt{5} = \frac{0}{1} = \frac{-(coefficient\ of\ x)}{(coefficient\ of\ x^2)}$

Product of zeroes = $-\sqrt{5} \times \sqrt{5} = \frac{-5}{1} = \frac{constant\ term}{(coefficient\ of\ x^2)}$

10. Find the zeroes of the quadratic polynomial $(8x^2 - 4)$ and verify the relation between the zeroes and the coefficients.

Sol:

We have:

$f(x) = 8x^2 - 4$

It can be written as $8x^2 + 0x - 4$

$= 4\{(\sqrt{2}x)^2 - (1)^2\}$

$= 4(\sqrt{2}x + 1)(\sqrt{2}x - 1)$

$\therefore f(x) = 0 \Rightarrow (\sqrt{2}x + 1)(\sqrt{2}x - 1) = 0$

$\Rightarrow (\sqrt{2}x + 1) = 0$ or $\sqrt{2}x - 1 = 0$

$\Rightarrow x = \frac{-1}{\sqrt{2}}$ or $x = \frac{1}{\sqrt{2}}$

So, the zeroes of f(x) are $\frac{-1}{\sqrt{2}}$ and $\frac{1}{\sqrt{2}}$

Here the coefficient of x is 0 and the coefficient of x^2 is $\sqrt{2}$

Sum of zeroes = $\frac{-1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{-1+1}{\sqrt{2}} = \frac{0}{\sqrt{2}} = \frac{-(coefficient\ of\ x)}{(coefficient\ of\ x^2)}$

Product of zeroes = $\frac{-1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{-1 \times 4}{2 \times 4} = \frac{-4}{8} = \frac{constant\ term}{(coefficient\ of\ x^2)}$

11. Find the zeroes of the quadratic polynomial ($5y^2 + 10y$) and verify the relation between the zeroes and the coefficients.

Sol:

We have,

f (u) = $5u^2 + 10u$

It can be written as 5u (u+2)

$\therefore$ f (u) = 0 $\Rightarrow$ 5u = 0 or u + 2 = 0

$\Rightarrow$ u = 0 or u = −2

So, the zeroes of f (u) are −2 and 0.

Sum of the zeroes = −2 + 0 =−2 = $\frac{-2 \times 5}{1 \times 5} = \frac{-10}{5} = \frac{-(coefficient\ of\ x)}{(coefficient\ of\ u^2)}$

Product of zeroes = $-2 \times 0 = 0 = \frac{0 \times 5}{1 \times 5} = \frac{-0}{5} = \frac{constant\ term}{(coefficient\ of\ u^2)}$

12. Find the zeroes of the quadratic polynomial ($3x^2 - x - 4$) and verify the relation between the zeroes and the coefficients.

Sol:

$3x^2 - x - 4 = 0$

$\Rightarrow 3x^2 - 4x + 3x - 4 = 0$

$\Rightarrow x(3x - 4) + 1(3x - 4) = 0$

$\Rightarrow (3x - 4)(x + 1) = 0$

$\Rightarrow (3x - 4)$ or $(x + 1) = 0$

$\Rightarrow x = \frac{4}{3}$ or $x = -1$

Sum of zeroes = $\frac{4}{3} + (-1) = \frac{1}{3} = \frac{-(coefficient\ of\ x)}{(coefficient\ of\ x^2)}$

Product of zeroes = $\frac{4}{3} \times (-1) = \frac{-4}{3} = \frac{constant\ term}{(coefficient\ of\ x^2)}$

13. Find the quadratic polynomial whose zeroes are 2 and -6. Verify the relation between the coefficients and the zeroes of the polynomial.

Sol:

Let α = 2 and β = -6

Sum of the zeroes, $(\alpha + \beta)$ = 2 + (-6) = -4

Product of the zeroes, $\alpha\beta$ = 2 $\times$ (-6) = -12

$\therefore$ Required polynomial = $x^2 - (\alpha + \beta)x + \alpha\beta = x^2 - (-4)x - 12$

$= x^2 + 4x - 12$

Sum of the zeroes = -4 = $\frac{-4}{1} = \frac{-(coefficient\ of\ x)}{(coefficient\ of\ x^2)}$

Product of zeroes = -12 = $\frac{-12}{1} = \frac{constant\ term}{(coefficient\ of\ x^2)}$

14. Find the quadratic polynomial whose zeroes are $\frac{2}{3}$ and $\frac{-1}{4}$. Verify the relation between the coefficients and the zeroes of the polynomial.

Sol:

Let $\alpha = \frac{2}{3}$ and $\beta = \frac{-1}{4}$.

Sum of the zeroes = $(\alpha + \beta) = \frac{2}{3} + \left(\frac{-1}{4}\right) = \frac{8-3}{12} = \frac{5}{12}$

1

Product of the zeroes, $\alpha\beta = \frac{2}{3} \times \left(\frac{-1}{4}\right) = \frac{-2}{12} = \frac{-1}{6}$

6

$\therefore$ Required polynomial = $x^2 - (\alpha + \beta)x + \alpha\beta = x^2 - \frac{5}{12}x + \left(\frac{-1}{6}\right)$

$= x^2 - \frac{5}{12}x - \frac{1}{6}$

Sum of the zeroes = $\frac{5}{12} = \frac{-(coefficient\ of\ x)}{(coefficient\ of\ x^2)}$

Product of zeroes = $\frac{-1}{6} = \frac{constant\ term}{(coefficient\ of\ x^2)}$

15. Find the quadratic polynomial, sum of whose zeroes is 8 and their product is 12. Hence, find the zeroes of the polynomial.

Sol:

Let α and β be the zeroes of the required polynomial f(x).

Then $(\alpha + \beta) = 8$ and $\alpha\beta = 12$

$\therefore f(x) = x^2 - (\alpha + \beta)x + \alpha\beta$

$\Rightarrow f(x) = x^2 - 8x + 12$

Hence, required polynomial $f(x) = x^2 - 8x + 12$

$\therefore f(x) = 0 \Rightarrow x^2 - 8x + 12 = 0$

$\Rightarrow x^2 - (6x + 2x) + 12 = 0$

$\Rightarrow x^2 - 6x - 2x + 12 = 0$

$\Rightarrow x(x - 6) - 2(x - 6) = 0$

$\Rightarrow (x - 2)(x - 6) = 0$

$\Rightarrow (x - 2) = 0$ or $(x - 6) = 0$

$\Rightarrow$ x = 2 or x = 6

So, the zeroes of f(x) are 2 and 6.

16. Find the quadratic polynomial, sum of whose zeroes is 0 and their product is -1. Hence, find the zeroes of the polynomial.

Sol:

Let α and β be the zeroes of the required polynomial f(x).

Then $(\alpha + \beta) = 0$ and $\alpha\beta = -1$

$\therefore f(x) = x^2 - (\alpha + \beta)x + \alpha\beta$

$\Rightarrow f(x) = x^2 - 0x + (-1)$

$\Rightarrow f(x) = x^2 - 1$

Hence, required polynomial $f(x) = x^2 - 1$.

$\therefore f(x) = 0 \Rightarrow x^2 - 1 = 0$

$\Rightarrow (x + 1)(x - 1) = 0$

$\Rightarrow (x + 1) = 0$ or $(x - 1) = 0$

$\Rightarrow x = -1$ or $x = 1$

So, the zeroes of f(x) are -1 and 1.

17. Find the quadratic polynomial, sum of whose zeroes is $\left(\frac{5}{2}\right)$ and their product is 1. Hence, find the zeroes of the polynomial.

Sol:

Let α and β be the zeroes of the required polynomial f(x).

Then $(\alpha + \beta) = \frac{5}{2}$ and $\alpha\beta = 1$

$\therefore f(x) = x^2 - (\alpha + \beta)x + \alpha\beta$

$\Rightarrow f(x) = x^2 - \frac{5}{2}x + 1$

$\Rightarrow f(x) = 2x^2 - 5x + 2$

Hence, the required polynomial is $f(x) = 2x^2 - 5x + 2$

$\therefore f(x) = 0 \Rightarrow 2x^2 - 5x + 2 = 0$

$\Rightarrow 2x^2 - (4x + x) + 2 = 0$

$\Rightarrow 2x^2 - 4x - x + 2 = 0$

$\Rightarrow 2x(x - 2) - 1(x - 2) = 0$

$\Rightarrow (2x - 1)(x - 2) = 0$

$\Rightarrow (2x - 1) = 0$ or $(x - 2) = 0$

$\Rightarrow x = \frac{1}{2}$ or $x = 2$

So, the zeros of f(x) are $\frac{1}{2}$ and 2.

18. Find the quadratic polynomial, sum of whose zeroes is $\sqrt{2}$ and their product is ($\frac{1}{3}$).

Sol:

We can find the quadratic equation if we know the sum of the roots and product of the roots by using the formula

x^2 – (Sum of the roots)x + Product of roots = 0

$\Rightarrow x^2 - \sqrt{2}x + \frac{1}{3} = 0$

$\Rightarrow 3x^2 - 3\sqrt{2}x + 1 = 0$

19. If $x = \dfrac{2}{3}$ and x = -3 are the roots of the quadratic equation $ax^2 + 2ax + 5x + 10$ then find the value of a and b.

Sol:

Given: $ax^2 + 7x + b = 0$

Since, $x = \frac{2}{3}$ is the root of the above quadratic equation

Hence, it will satisfy the above equation.

Therefore, we will get

$a\left(\frac{2}{3}\right)^2 + 7\left(\frac{2}{3}\right) + b = 0$

$\Rightarrow \frac{4}{9}a + \frac{14}{3} + b = 0$

$\Rightarrow 4a + 42 + 9b = 0$

$\Rightarrow 4a + 9b = -42$ …(1)

Since, x = –3 is the root of the above quadratic equation

Hence, It will satisfy the above equation.

Therefore, we will get

$a(-3)^2 + 7(-3) + b = 0$

$\Rightarrow 9a - 21 + b = 0$

$\Rightarrow 9a + b = 21$ …..(2)

From (1) and (2), we get

a = 3, b = –6

20. If $(x+a)$ is a factor of the polynomial $2x^2 + 2ax + 5x + 10$, find the value of a.

Sol:

Given: (x + a) is a factor of $2x^2 + 2ax + 5x + 10$

So, we have
$x + a = 0$

$\Rightarrow x = -a$

Now, it will satisfy the above polynomial.
Therefore, we will get
$2(-a)^2 + 2a(-a) + 5(-a) + 10 = 0$

$\Rightarrow 2a^2 - 2a^2 - 5a + 10 = 0$

$\Rightarrow -5a = -10$

$\Rightarrow a = 2$

21. One zero of the polynomial $3x^3 + 16x^2 + 15x - 18$ is $\frac{2}{3}$. Find the other zeros of the polynomial.

Sol:

Given: $x = \frac{2}{3}$ is one of the zero of $3x^3 + 16x^2 + 15x - 18$

Now, we have

$x = \frac{2}{3}$

$\Rightarrow x - \frac{2}{3} = 0$

Now, we divide $3x^3 + 16x^2 + 15x - 18$ by $x - \frac{2}{3}$ to find the quotient

$$
\begin{array}{r|l}
 & 3x^2 + 18x + 27 \\
\hline
x - \frac{2}{3} & 3x^3 + 16x^2 + 15x - 18 \\
 & 3x^3 - 2x^2 \\
 & - \quad + \\
\hline
 & 18x^2 + 15x \\
 & 18x^2 - 12x \\
 & - \quad + \\
\hline
 & 27x - 18 \\
 & 27x - 18 \\
 & - \quad + \\
\hline
 & \text{X} \\
\hline
\end{array}
$$

So, the quotient is $3x^2 + 18x + 27$

Now,

$3x^2 + 18x + 27 = 0$

$\Rightarrow 3x^2 + 9x + 9x + 27 = 0$

$\Rightarrow 3x(x + 3) + 9(x + 3) = 0$

$\Rightarrow$ (x + 3) (3x + 9) = 0
$\Rightarrow$ (x + 3) = 0 or (3x + 9) = 0
$\Rightarrow$ x = –3 or x = –3

Exercise – 2B

1. Verify that 3, -2, 1 are the zeros of the cubic polynomial p(x) = $(x^3 - 2x^2 - 5x + 6)$ and verify the relation between it zeros and coefficients.

Sol:

The given polynomial is p(x) = $(x^3 - 2x^2 - 5x + 6)$

$\therefore$ p(3) = $(3^3 - 2 \times 3^2 - 5 \times 3 + 6) = (27 - 18 - 15 + 6) = 0$

p(-2) = $[(-2^3) - 2 \times (-2)^2 - 5 \times (-2) + 6] = (-8 - 8 + 10 + 6) = 0$

p(1) = $(1^3 - 2 \times 1^2 - 5 \times 1 + 6) = (1 - 2 - 5 + 6) = 0$

$\therefore$ 3, –2 and 1are the zeroes of p(x),

Let $\alpha = 3$, $\beta = -2$ and $\gamma = 1$. Then we have:

$(\alpha + \beta + \gamma) = (3 - 2 + 1) = 2 = \frac{-(coefficient\ of\ x^2)}{(coefficient\ of\ x^3)}$

$(\alpha\beta + \beta\gamma + \gamma\alpha) = (-6 - 2 + 3) = \frac{-5}{1} = \frac{coefficient\ of\ x}{coefficient\ of\ x^3}$

$\alpha\beta\gamma = \{3 \times (-2) \times 1\} = \frac{-6}{1} = \frac{-(constant\ term)}{(coefficient\ of\ x^3)}$

2. Verify that 5, -2 and $\frac{1}{3}$ are the zeroes of the cubic polynomial p(x) = $(3x^3 - 10x^2 - 27x + 10)$ and verify the relation between its zeroes and coefficients.

Sol:

p(x) = $(3x^3 - 10x^2 - 27x + 10)$

p(5) = $(3 \times 5^3 - 10 \times 5^2 - 27 \times 5 + 10) = (375 - 250 - 135 + 10) = 0$

p(–2) = $[3 \times (-2^3) - 10 \times (-2^2) - 27 \times (-2) + 10] = (-24 - 40 + 54 + 10) = 0$

$p\left(\frac{1}{3}\right) = \left\{3 \times \left(\frac{1}{3}\right)^3 - 10 \times \left(\frac{1}{3}\right)^2 - 27 \times \frac{1}{3} + 10\right\} = \left(3 \times \frac{1}{27} - 10 \times \frac{1}{9} - 9 + 10\right)$

$= \left(\frac{1}{9} - \frac{10}{9} + 1\right) = \left(\frac{1-10-9}{9}\right) = \left(\frac{0}{9}\right) = 0$

$\therefore$ 5, –2 and $\frac{1}{3}$ are the zeroes of p(x).

Let $\alpha = 5$, $\beta = -2$ and $\gamma = \frac{1}{3}$. Then we have:

$(\alpha + \beta + \gamma) = \left(5 - 2 + \frac{1}{3}\right) = \frac{10}{3} = \frac{-(coefficient\ of\ x^2)}{(coefficient\ of\ x^3)}$

$(\alpha\beta + \beta\gamma + \gamma\alpha) = (-10 - \frac{2}{3} + \frac{5}{3}) = \frac{-27}{3} = \frac{coefficient\ of\ x}{coefficient\ of\ x^3}$

$\alpha\beta\gamma = \left\{5 \times (-2) \times \frac{1}{3}\right\} = \frac{-10}{3} = \frac{-(constant\ term)}{(coefficient\ of\ x^3)}$

3. Find a cubic polynomial whose zeroes are 2, -3and 4.

Sol:

If the zeroes of the cubic polynomial are a, b and c then the cubic polynomial can be found as

$x^3 – (a + b + c)x^2 + (ab + bc + ca)x – abc$ ……(1)

Let a = 2, b = –3 and c = 4

Substituting the values in 1, we get

$x^3 – (2 – 3 + 4)x^2 + (– 6 – 12 + 8)x – (–24)$

$\Rightarrow x^3 – 3x^2 – 10x + 24$

4. Find a cubic polynomial whose zeroes are $\frac{1}{2}$, 1 and –3.

Sol:

If the zeroes of the cubic polynomial are a, b and c then the cubic polynomial can be found as

$x^3 – (a + b + c)x^2 + (ab + bc + ca)x – abc$ ……(1)

Let a = $\frac{1}{2}$, b = 1 and c = –3

Substituting the values in (1), we get

$x^3 – \left(\frac{1}{2} + 1 - 3\right)x^2 + \left(\frac{1}{2} - 3 - \frac{3}{2}\right)x – \left(\frac{-3}{2}\right)$

$\Rightarrow x^3 – \left(\frac{-3}{2}\right)x^2 – 4x + \frac{3}{2}$

$\Rightarrow 2x^3 +3x^2 – 8x + 3$

5. Find a cubic polynomial with the sum of its zeroes, sum of the products of its zeroes taken two at a time and the product of its zeroes as 5, -2 and -24 respectively.

Sol:

We know the sum, sum of the product of the zeroes taken two at a time and the product of the zeroes of a cubic polynomial then the cubic polynomial can be found as

x^3 – (sum of the zeroes)x^2 + (sum of the product of the zeroes taking two at a time)x – product of zeroes

Therefore, the required polynomial is

$x^3 – 5x^2 – 2x + 24$

6. If f(x) = $x^3 - 3x + 5x - 3$ is divided by g(x) = $x^2 - 2$

Sol:

$$\begin{array}{r|l} & \quad x - 3 \\ \hline x - 2 & x^3 - 3x^2 + 5x - 3 \\ & x^3 \qquad - 2x \\ & - \qquad\quad + \\ \hline & -3x^2 + 7x - 3 \\ & -3x^2 \qquad + 6 \\ & + \qquad\quad - \\ \hline & \quad 7x - 9 \\ \hline \end{array}$$

Quotient q(x) = x – 3

Remainder r(x) = 7x – 9

7. If $f(x) = x^4 - 3x^2 + 4x + 5$ is divided by $g(x) = x^2 - x + 1$

Sol:

$$
\begin{array}{r|l}
 & x^2 + x - 3 \\
\hline
x^2 - x + 1 & x^4 + 0x^3 - 3x^2 + 4x + 5 \\
 & x^4 - x^3 + x^2 \\
 & - \quad + \quad - \\
\hline
 & x^3 - 4x^2 + 4x + 5 \\
 & x^3 - x^2 + x \\
 & - \quad + \quad - \\
\hline
 & -3x^2 + 3x + 5 \\
 & -3x^2 + 3x - 3 \\
 & + \quad - \quad + \\
\hline
 & 8 \\
\hline
\end{array}
$$

Quotient $q(x) = x^2 + x - 3$

Remainder $r(x) = 8$

8. If $f(x) = x^4 - 5x + 6$ is divided by $g(x) = 2 - x^2$.

Sol:

We can write

$f(x)$ as $x^4 + 0x^3 + 0x^2 - 5x + 6$ and $g(x)$ as $-x^2 + 2$

$$
\begin{array}{r|l}
 & -x^2 - 2 \\
\hline
-x^2 + 2 & x^4 + 0x^3 + 0x^2 - 5x + 6 \\
 & x^4 \quad -2x^2 \\
 & - \quad + \\
\hline
 & 2x^2 - 5x + 6 \\
 & 2x^2 \quad - 4 \\
 & - \quad + \\
\hline
 & -5x + 10 \\
\hline
\end{array}
$$

Quotient $q(x) = -x^2 - 2$

Remainder $r(x) = -5x + 10$

9. By actual division, show that $x^2 - 3$ is a factor of $2x^4 + 3x^3 - 2x^2 - 9x - 12$.

Sol:

Let $f(x) = 2x^4 + 3x^3 - 2x^2 - 9x - 12$ and $g(x)$ as $x^2 - 3$

$$
\begin{array}{r|l}
 & 2x^2 + 3x + 4 \\
\hline
x^2 - 3 & 2x^4 + 3x^3 - 2x^2 - 9x - 12 \\
 & 2x^4 \qquad - 6x^2 \\
 & - \qquad\quad + \\
\hline
 & 3x^3 + 4x^2 - 9x - 12 \\
 & 3x^3 \qquad - 9x \\
 & - \qquad\quad + \\
\hline
 & 4x^2 - 12 \\
 & 4x^2 - 12 \\
 & - \quad + \\
\hline
 & x \\
\hline
\end{array}
$$

Quotient $q(x) = 2x^2 + 3x + 4$

Remainder $r(x) = 0$

Since, the remainder is 0.

Hence, $x^2 - 3$ is a factor of $2x^4 + 3x^3 - 2x^2 - 9x - 12$

10. On dividing $3x^3 + x^2 + 2x + 5$ is divided by a polynomial $g(x)$, the quotient and remainder are $(3x - 5)$ and $(9x + 10)$ respectively. Find $g(x)$.

Sol:

By using division rule, we have

Dividend = Quotient $\times$ Divisor + Remainder

$\therefore 3x^3 + x^2 + 2x + 5 = (3x - 5)g(x) + 9x + 10$

$\Rightarrow 3x^3 + x^2 + 2x + 5 - 9x - 10 = (3x - 5)g(x)$

$\Rightarrow 3x^3 + x^2 - 7x - 5 = (3x - 5)g(x)$

$\Rightarrow g(x) = \frac{3x^3 + x^2 - 7x - 5}{3x - 5}$

$$
\begin{array}{r|l}
 & x^2 + 2x + 1 \\
\hline
3x - 5 & 3x^3 + x^2 - 7x - 5 \\
 & 3x^3 - 5x^2 \\
 & - \quad + \\
\hline
 & 6x^2 - 7x - 5 \\
 & 6x^2 - 10x \\
 & - \quad + \\
\hline
 & 3x - 5 \\
 & 3x - 5 \\
 & - \quad + \\
\hline
 & X \\
\hline
\end{array}
$$

$\therefore g(x) = x^2 + 2x + 1$

11. Verify division algorithm for the polynomial f(x)= $(8 + 20x + x^2 – 6x^3)$ by g(x) =($2 + 5x – 3x^2$).

Sol:

We can write f(x) as $–6x^3 + x^2 + 20x + 8$ and g(x) as $–3x^2 + 5x + 2$

$$x^2 + 2x + 1$$

$$-3x^2 + 5x + 2 \;\big)\; -6x^3 + x^2 + 20x + 8$$

$$-6x^3 + 10x^2 + 4x$$

$$+ \quad - \quad -$$

$$-9x^2 + 16x + 8$$

$$-9x^2 + 15x + 6$$

$$+ \quad - \quad -$$

$$x + 2$$

Quotient = $2x + 3$

Remainder = $x + 2$

By using division rule, we have

Dividend = Quotient × Divisor + Remainder

$\therefore –6x^3 + x^2 + 20x + 8 = (–3x^2 + 5x + 2) (2x + 3) + x + 2$

$\Rightarrow –6x^3 + x^2 + 20x + 8 = –6x^3 + 10x^2 + 4x –9x^2 + 15x + 6 + x + 2$

$\Rightarrow –6x^3 + x^2 + 20x + 8 = –6x^3 + x^2 + 20x + 8$

12. It is given that –1 is one of the zeroes of the polynomial $x^3 + 2x^2 – 11x – 12$. Find all the zeroes of the given polynomial.

Sol:

Let $f(x) = x^3 + 2x^2 – 11x – 12$

Since – 1 is a zero of f(x), (x+1) is a factor of f(x).

On dividing f(x) by (x+1), we get

$$x + 1 \;\big)\; x^3 + 2x^2 - 11x - 12 \;\big(\; x^2 + x + 12$$

$$x^3 + x^2$$

$$- \quad -$$

$$x^2 - 11x - 12$$

$$x^2 + x$$

$$- \quad -$$

$$-12x - 12$$

$$-12x - 12$$

$$+ \quad +$$

$$\text{X}$$

$f(x) = x^3 + 2x^2 – 11x – 12$
$= (x + 1)(x^2 + x – 12)$
$= (x + 1)\{x^2 + 4x – 3x – 12\}$
$= (x + 1)\{x(x+4) – 3(x+4)\}$
$= (x + 1)(x – 3)(x + 4)$

$\therefore f(x) = 0 \Rightarrow (x + 1)(x – 3)(x + 4) = 0$

$\Rightarrow (x + 1) = 0$ or $(x – 3) = 0$ or $(x + 4) = 0$

$\Rightarrow x = –1$ or $x = 3$ or $x = –4$

Thus, all the zeroes are $–1$, 3 and $–4$.

13. If 1 and –2 are two zeroes of the polynomial $(x^3 – 4x^2 – 7x + 10)$, find its third zero.

Sol:

Let $f(x) = x^3 – 4x^2 – 7x + 10$

Since 1 and –2 are the zeroes of f(x), it follows that each one of $(x–1)$ and $(x+2)$ is a factor of f(x).

Consequently, $(x–1)(x+2) = (x^2 + x – 2)$ is a factor of f(x).

On dividing f(x) by $(x^2 + x – 2)$, we get:

$$
\begin{array}{r|l|l}
x^2 + x - 2 & x^3 - 4x^2 - 7x + 10 & x - 5 \\
 & x^3 + x^2 - 2x & \\
 & - \quad - \quad + & \\
\hline
 & -5x^2 - 5x + 10 & \\
 & -5x^2 - 5x + 10 & \\
 & + \quad + \quad - & \\
\hline
 & \text{X} & \\
\hline
\end{array}
$$

$f(x) = 0 \Rightarrow (x^2 + x – 2)(x – 5) = 0$

$\Rightarrow (x – 1)(x + 2)(x – 5) = 0$

$\Rightarrow x = 1$ or $x = –2$ or $x = 5$

Hence, the third zero is 5.

14. If 3 and –3 are two zeroes of the polynomial $(x^4 + x^3 – 11x^2 – 9x + 18)$, find all the zeroes of the given polynomial.

Sol:

Let $x^4 + x^3 – 11x^2 – 9x + 18$

Since 3 and – 3 are the zeroes of f(x), it follows that each one of $(x + 3)$ and $(x – 3)$ is a factor of f(x).

Consequently, $(x – 3)(x + 3) = (x^2 – 9)$ is a factor of f(x).

On dividing f(x) by $(x^2 – 9)$, we get:

$$x^2 - 9 \overline{\big)\, x^4 + x^3 - 11x^2 - 9x + 18 \,\big(} \; x^2 + x - 2$$

$$\begin{array}{l}
x^4 \qquad - 9x^2 \\
- \qquad\quad + \\
\hline
x^3 - 2x^2 - 9x + 18 \\
x^3 \qquad - 9x \\
- \qquad\quad + \\
\hline
\quad -2x^2 + 18 \\
\quad -2x^2 + 18 \\
\quad + \qquad - \\
\hline
\qquad x \\
\hline
\end{array}$$

$f(x) = 0 \Rightarrow (x^2 + x - 2)(x^2 - 9) = 0$

$\Rightarrow (x^2 + 2x - x - 2)(x - 3)(x + 3)$

$\Rightarrow (x - 1)(x + 2)(x - 3)(x + 3) = 0$

$\Rightarrow x = 1$ or $x = -2$ or $x = 3$ or $x = -3$

Hence, all the zeroes are 1, -2, 3 and -3.

15. If 2 and -2 are two zeroes of the polynomial $(x^4 + x^3 - 34x^2 - 4x + 120)$, find all the zeroes of the given polynomial.

Sol:

Let $f(x) = x^4 + x^3 - 34x^2 - 4x + 120$

Since 2 and -2 are the zeroes of f(x), it follows that each one of $(x - 2)$ and $(x + 2)$ is a factor of f(x).

Consequently, $(x - 2)(x + 2) = (x^2 - 4)$ is a factor of f(x).

On dividing f(x) by $(x^2 - 4)$, we get:

$$x^2 - 4 \overline{\big)\, x^4 + x^3 - 34x^2 - 4x + 120 \,\big(} \; x^2 + x - 2$$

$$\begin{array}{l}
x^4 \qquad - 4x^2 \\
- \qquad + \\
\hline
x^3 - 30x^2 - 4x + 120 \\
\quad x^3 \qquad\quad -4x \\
\quad - \qquad\qquad + \\
\hline
-30x^2 + 120 \\
-30x^2 + 120 \\
+ \qquad - \\
\hline
\qquad x \\
\hline
\end{array}$$

$f(x) = 0$

$\Rightarrow (x^2 + x - 30)(x^2 - 4) = 0$

$\Rightarrow (x^2 + 6x - 5x - 30)(x - 2)(x + 2)$

$\Rightarrow [x(x + 6) - 5(x + 6)](x - 2)(x + 2)$

$\Rightarrow (x - 5)(x + 6)(x - 2)(x + 2) = 0$

$\Rightarrow x = 5$ or $x = -6$ or $x = 2$ or $x = -2$

Hence, all the zeroes are 2, -2, 5 and -6.

16. Find all the zeroes of $(x^4 + x^3 - 23x^2 - 3x + 60)$, if it is given that two of its zeroes are $\sqrt{3}$ and $-\sqrt{3}$.

Sol:

Let $f(x) = x^4 + x^3 - 23x^2 - 3x + 60$

Since $\sqrt{3}$ and $-\sqrt{3}$ are the zeroes of f(x), it follows that each one of $(x - \sqrt{3})$ and $(x + \sqrt{3})$ is a factor of f(x).

Consequently, $(x - \sqrt{3})(x + \sqrt{3}) = (x^2 - 3)$ is a factor of f(x).

On dividing f(x) by $(x^2 - 3)$, we get:

$$
\begin{array}{l|l|l}
x^2 - 3 & x^4 + x^3 - 23x^2 - 3x + 60 & x^2 + x - 20 \\
 & x^4 \qquad - 3x^2 & \\
 & - \qquad\quad + & \\
\hline
 & x^3 - 20x^2 - 3x + 60 & \\
 & x^3 \qquad\quad -3x & \\
 & - \qquad\quad\; + & \\
\hline
 & -20x^2 + 60 & \\
 & -20x^2 + 60 & \\
 & + \qquad - & \\
\hline
 & \qquad x & \\
\hline
\end{array}
$$

f(x) = 0

$\Rightarrow (x^2 + x - 20)(x^2 - 3) = 0$

$\Rightarrow (x^2 + 5x - 4x - 20)(x^2 - 3)$

$\Rightarrow [x(x + 5) - 4(x + 5)](x^2 - 3)$

$\Rightarrow (x - 4)(x + 5)(x - \sqrt{3})(x + \sqrt{3}) = 0$

$\Rightarrow x = 4$ or $x = -5$ or $x = \sqrt{3}$ or $x = -\sqrt{3}$

Hence, all the zeroes are $\sqrt{3}$, $-\sqrt{3}$, 4 and -5.

17. Find all the zeroes of $(2x^4 - 3x^3 - 5x^2 + 9x - 3)$, it is being given that two of its zeroes are $\sqrt{3}$ and $-\sqrt{3}$.

Sol:

The given polynomial is $f(x) = 2x^4 - 3x^3 - 5x^2 + 9x - 3$

Since $\sqrt{3}$ and $-\sqrt{3}$ are the zeroes of f(x), it follows that each one of $(x - \sqrt{3})$ and $(x + \sqrt{3})$ is a factor of f(x).

Consequently, $(x - \sqrt{3})(x + \sqrt{3}) = (x^2 - 3)$ is a factor of f(x).

On dividing f(x) by $(x^2 - 3)$, we get:

$$
\begin{array}{r|l|l}
x^2 - 3 & 2x^4 - 3x^3 - 5x^2 + 9x - 3 & 2x^2 - 3x + 1 \\
 & 2x^4 \qquad\quad - 6x^2 & \\
 & - \qquad\qquad + & \\
\hline
 & -3x^3 + x^2 + 9x - 3 & \\
 & -3x^3 \qquad + 9x & \\
 & + \qquad\qquad - & \\
\hline
 & \qquad x^2 - 3 & \\
 & \qquad x^2 - 3 & \\
 & \qquad - \quad + & \\
\hline
 & \qquad\quad x & \\
\hline
\end{array}
$$

$f(x) = 0$

$\Rightarrow 2x^4 - 3x^3 - 5x^2 + 9x - 3 = 0$

$\Rightarrow (x^2 - 3)(2x^2 - 3x + 1) = 0$

$\Rightarrow (x^2 - 3)(2x^2 - 2x - x + 1) = 0$

$\Rightarrow (x - \sqrt{3})(x + \sqrt{3})(2x - 1)(x - 1) = 0$

$\Rightarrow x = \sqrt{3}$ or $x = -\sqrt{3}$ or $x = \frac{1}{2}$ or $x = 1$

Hence, all the zeroes are $\sqrt{3}$, $-\sqrt{3}$, $\frac{1}{2}$ and 1.

18. Obtain all other zeroes of $(x^4 + 4x^3 - 2x^2 - 20x - 15)$ if two of its zeroes are $\sqrt{5}$ and $-\sqrt{5}$.

Sol:

The given polynomial is $f(x) = x^4 + 4x^3 - 2x^2 - 20x - 15$.

Since $(x - \sqrt{5})$ and $(x + \sqrt{5})$ are the zeroes of f(x) it follows that each one of $(x - \sqrt{5})$ and $(x + \sqrt{5})$ is a factor of f(x).

Consequently, $(x - \sqrt{5})(x + \sqrt{5}) = (x^2 - 5)$ is a factor of f(x).

On dividing f(x) by $(x^2 - 5)$, we get:

$$x^2 - 5 \,\overline{)\, x^4 + 4x^3 - 2x^2 - 20x - 15 \,}\, (\, 2x^2 - 3x + 1$$

$$\begin{array}{l}
x^4 \qquad - 5x^2 \\
- \qquad\quad + \\
\hline
4x^3 + 3x^2 - 20x - 15 \\
4x^3 \qquad - 20x \\
- \qquad\quad + \\
\hline
3x^2 - 15 \\
3x^2 - 15 \\
- \quad + \\
\hline
x \\
\hline
\end{array}$$

$f(x) = 0$

$\Rightarrow x^4 + 4x^3 - 7x^2 - 20x - 15 = 0$

$\Rightarrow (x^2 - 5)(x^2 + 4x + 3) = 0$

$\Rightarrow (x - \sqrt{5})(x + \sqrt{5})(x + 1)(x + 3) = 0$

$\Rightarrow x = \sqrt{5}$ or $x = -\sqrt{5}$ or $x = -1$ or $x = -3$

Hence, all the zeroes are $\sqrt{5}$, $-\sqrt{5}$, -1 and -3.

19. Find all the zeroes of polynomial $(2x^4 - 11x^3 + 7x^2 + 13x - 7)$, it being given that two of its zeroes are $(3 + \sqrt{2})$ and $(3 - \sqrt{2})$.

Sol:

The given polynomial is $f(x) = 2x^4 - 11x^3 + 7x^2 + 13x - 7$.

Since $(3 + \sqrt{2})$ and $(3 - \sqrt{2})$ are the zeroes of f(x) it follows that each one of $(x + 3 + \sqrt{2})$ and $(x + 3 - \sqrt{2})$ is a factor of f(x).

Consequently, $[(x - (3 + \sqrt{2})][(x - (3 - \sqrt{2})] = [(x - 3) - \sqrt{2}\,][(x - 3) + \sqrt{2}\,]$

$= [(x - 3)^2 - 2\,] = x^2 - 6x + 7$, which is a factor of f(x).

On dividing f(x) by $(x^2 - 6x + 7)$, we get:

$$x^2 - 6x + 7 \,\overline{)\, 2x^4 - 11x^3 + 7x^2 + 13x - 7 \,}\, (\, 2x^2 + x - 1$$

$$\begin{array}{l}
2x^4 - 12x^3 + 14x^2 \\
- \qquad + \qquad - \\
\hline
x^3 - 7x^2 + 13x - 7 \\
x^3 - 6x^2 + 7x \\
- \quad + \quad - \\
\hline
-x^2 + 6x - 7 \\
-x^2 + 6x - 7
\end{array}$$

$+ \quad - \quad +$

x

$f(x) = 0$

$\Rightarrow 2x^4 - 11x^3 + 7x^2 + 13x - 7 = 0$

$\Rightarrow (x^2 - 6x + 7)(2x^2 + x - 7) = 0$

$\Rightarrow (x + 3 + \sqrt{2})(x + 3 - \sqrt{2})(2x - 1)(x + 1) = 0$

$\Rightarrow x = -3 - \sqrt{2}$ or $x = -3 + \sqrt{2}$ or $x = \frac{1}{2}$ or $x = -1$

Hence, all the zeroes are $(-3 - \sqrt{2})$, $(-3 + \sqrt{2})$, $\frac{1}{2}$ and -1.

Exercise – 2C

1. If one zero of the polynomial $x^2 - 4x + 1$ Is $\left(2 + \sqrt{3}\right)$, write the other zero.

Sol:

Let the other zeroes of $x^2 - 4x + 1$ be a.

By using the relationship between the zeroes of the quadratic polynomial.

We have, sum of zeroes = $\frac{-(coefficient\ of\ x)}{coefficient\ of\ x^2}$

$\therefore 2 + \sqrt{3} + a = \frac{-(-4)}{1}$

$\Rightarrow a = 2 - \sqrt{3}$

Hence, the other zeroes of $x^2 - 4x + 1$ is $2 - \sqrt{3}$.

2. Find the zeroes of the polynomial $x^2 + x - p(p + 1)$

Sol:

$f(x) = x^2 + x - p(p + 1)$

By adding and subtracting px, we get

$f(x) = x^2 + px + x - px - p(p + 1)$

$= x^2 + (p + 1)x - px - p(p + 1)$

$= x[x + (p + 1)] - p[x + (p + 1)]$

$= [x + (p + 1)](x - p)$

$f(x) = 0$

$\Rightarrow [x + (p + 1)](x - p) = 0$

$\Rightarrow [x + (p + 1)] = 0$ or $(x - p) = 0$

$\Rightarrow x = -(p + 1)$ or $x = p$

So, the zeroes of f(x) are $-(p + 1)$ and p.

3. Find the zeroes of the polynomial $x^2 - 3x - m(m + 3)$

Sol:

$f(x) = x^2 - 3x - m (m + 3)$

By adding and subtracting mx, we get

$f(x) = x^2 - mx - 3x + mx - m (m + 3)$

$= x[x - (m + 3)] + m[x - (m + 3)]$

$= [x - (m + 3)] (x + m)$

$f(x) = 0 \Rightarrow [x - (m + 3)] (x + m) = 0$

$\Rightarrow [x - (m + 3)] = 0$ or $(x + m) = 0$

$\Rightarrow x = m + 3$ or $x = -m$

So, the zeroes of f(x) are –m and +3.

4. Find α , β are the zeros of polynomial $\alpha + \beta = 6$ and $\alpha\beta = 4$ then write the polynomial.

Sol:

If the zeroes of the quadratic polynomial are α and β then the quadratic polynomial can be found as $x^2 - (\alpha + \beta)x + \alpha\beta$ …..(1)

Substituting the values in (1), we get

$x^2 - 6x + 4$

5. If one zero of the quadratic polynomial $kx^2 + 3x + k$ is 2, then find the value of k.

Sol:

Given: x = 2 is one zero of the quadratic polynomial $kx^2 + 3x + k$

Therefore, it will satisfy the above polynomial.

Now, we have

$k(2)^2 + 3(2) + k = 0$

$\Rightarrow 4k + 6 + k = 0$

$\Rightarrow 5k + 6 = 0$

$\Rightarrow k = -\frac{6}{5}$

6. If 3 is a zero of the polynomial $2x^2 + x + k$, find the value of k.

Sol:

Given: x = 3 is one zero of the polynomial $2x^2 + x + k$

Therefore, it will satisfy the above polynomial.

Now, we have

$2(3)^2 + 3 + k = 0$

$\Rightarrow 21 + k = 0$

$\Rightarrow k = -21$

7. If -4 is a zero of the polynomial $x^2 – x – (2k + 2)$ is –4, then find the value of k.

Sol:

Given: x = –4 is one zero of the polynomial $x^2 – x – (2k + 2)$

Therefore, it will satisfy the above polynomial.

Now, we have

$(–4)^2 – (–4) – (2k + 2) = 0$

$\Rightarrow 16 + 4 – 2k – 2 = 0$

$\Rightarrow 2k = – 18$

$\Rightarrow k = 9$

8. If 1is a zero of the quadratic polynomial $ax^2 – 3(a – 1)x – 1$is 1, then find the value of a.

Sol:

Given: x = 1 is one zero of the polynomial $ax^2 – 3(a – 1) x – 1$

Therefore, it will satisfy the above polynomial.

Now, we have

$a(1)^2 – (a – 1)1 – 1 = 0$

$\Rightarrow a – 3a + 3 – 1 = 0$

$\Rightarrow –2a = – 2$

$\Rightarrow a = 1$

9. If -2 is a zero of the polynomial $3x^2 + 4x + 2k$ then find the value of k.

Sol:

Given: x = –2 is one zero of the polynomial $3x^2 + 4x + 2k$

Therefore, it will satisfy the above polynomial.

Now, we have

$3(–2)^2 + 4(–2)1 + 2k = 0$

$\Rightarrow 12 – 8 + 2k = 0$

$\Rightarrow k = – 2$

10. Write the zeros of the polynomial $f(x) = x^2 – x – 6$.

Sol:

$f(x) = x^2 – x – 6$

$= x^2 – 3x + 2x – 6$

$= x(x – 3) + 2(x – 3)$

$= (x – 3) (x + 2)$

$f(x) = 0 \Rightarrow (x – 3) (x + 2) = 0$

⇒ (x – 3) = 0 or (x + 2) = 0

⇒ x = 3 or x = –2

So, the zeroes of f(x) are 3 and –2.

11. If the sum of the zeros of the quadratic polynomial kx^2-3x + 5 is 1 write the value of k..

Sol:

By using the relationship between the zeroes of the quadratic polynomial.

We have

Sum of zeroes = $\frac{-(coefficient\ of\ x)}{coefficient\ of\ x^2}$

⇒ 1 = $\frac{-(-3)}{k}$

⇒ k = 3

12. If the product of the zero of the polynomial (x^2 -4x + k) is 3. Find the value of k.

Sol:

By using the relationship between the zeroes of he quadratic polynomial.

We have

Product of zeroes = $\frac{constant\ term}{coefficient\ of\ x^2}$

⇒ 3 = $\frac{k}{1}$

⇒ k = 3

13. If (x + a) is a factor of (2x2 + 2ax + 5x + 10), then find the value of a.

Sol:

Given: (x + a) is a factor of $2x^2$ + 2ax + 5x + 10

We have

x + a = 0

⇒ x = –a

Since, (x + a) is a factor of $2x^2$ + 2ax + 5x + 10

Hence, It will satisfy the above polynomial

∴ $2(-a)^2 + 2a(-a) + 5(-a) + 10 = 0$

⇒ –5a + 10 = 0

⇒ a = 2

14. If (a-b) , a and (a + b) are zeros of the polynomial 2x3-6x2+5x-7 write the value of a.

Sol:

By using the relationship between the zeroes of the quadratic polynomial.

We have

Sum of zeroes = $\frac{-(coefficient\ of\ x^2)}{coefficient\ of\ x^3}$

$\Rightarrow$ a – b + a + a + b = $\frac{-(-6)}{2}$

$\Rightarrow$ 3a = 3

$\Rightarrow$ a = 1

15. If $x^3 + x^2 - ax + b$ is divisible by $(x^2 - x)$,write the value of a and b.

Sol:

Equating $x^2 - x$ to 0 to find the zeroes, we will get

$x(x - 1) = 0$

$\Rightarrow x = 0$ or $x - 1 = 0$

$\Rightarrow x = 0$ or $x = 1$

Since, $x^3 + x^2 - ax + b$ is divisible by $x^2 - x$.

Hence, the zeroes of $x^2 - x$ will satisfy $x^3 + x^2 - ax + b$

$\therefore (0)^3 + 0^2 - a(0) + b = 0$

$\Rightarrow b = 0$

And

$(1)^3 + 1^2 - a(1) + 0 = 0 \quad [\because b = 0]$

$\Rightarrow a = 2$

16. If α and β be the zeroes of the polynomial $2x^2 - 7x + k$ write the value of $(\alpha + \beta + \alpha\beta$.

Sol:

By using the relationship between the zeroes of he quadratic polynomial.

We have

Sum of zeroes = $\frac{-(coefficient\ of\ x)}{coefficient\ of\ x^2}$ and Product of zeroes = $\frac{constant\ term}{coefficient\ of\ x^2}$

$\therefore \alpha + \beta = \frac{-7}{2}$ and $\alpha\beta = \frac{5}{2}$

Now, $\alpha + \beta + \alpha\beta = \frac{-7}{2} + \frac{5}{2} = -1$

17. State Division Algorithm for Polynomials.

Sol:

"If f(x) and g(x) are two polynomials such that degree of f(x) is greater than degree of g(x) where $g(x) \neq 0$, there exists unique polynomials q(x) and r(x) such that

$$f(x) = g(x) \times q(x) + r(x),$$

where r(x) = 0 or degree of r(x) < degree of g(x).

18. Find the sum of the zeros and the product of zeros of a quadratic polynomial, are $-\frac{1}{2}$ and \ -3 respectively. Write the polynomial.

Sol:

We can find the quadratic polynomial if we know the sum of the roots and product of the roots by using the formula

x^2 – (sum of the zeroes)x + product of zeroes

$\Rightarrow x^2 - \left(-\frac{1}{2}\right)x + (-3)$

$\Rightarrow x^2 + \frac{1}{2}x - 3$

Hence, the required polynomial is $x^2 + \frac{1}{2}x - 3$.

19. Find the zeroes of the quadratic polynomial $f(x) = 6x^2 - 3$.

Sol:

To find the zeroes of the quadratic polynomial we will equate f(x) to 0

$\therefore f(x) = 0$

$\Rightarrow 6x^2 - 3 = 0$

$\Rightarrow 3(2x^2 - 1) = 0$

$\Rightarrow 2x^2 - 1 = 0$

$\Rightarrow 2x^2 = 1$

$\Rightarrow x^2 = \frac{1}{2}$

$\Rightarrow x = \pm\frac{1}{\sqrt{2}}$

Hence, the zeroes of the quadratic polynomial $f(x) = 6x^2 - 3$ are $\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}$.

20. Find the zeroes of the quadratic polynomial $f(x) = 4\sqrt{3}x^2 + 5x - 2\sqrt{3}$.

Sol:

To find the zeroes of the quadratic polynomial we will equate f(x) to 0

$\therefore f(x) = 0$

$\Rightarrow 4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$

$\Rightarrow 4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3} = 0$

$\Rightarrow 4x(\sqrt{3}x + 2) - \sqrt{3}(\sqrt{3}x + 2) = 0$

$\Rightarrow (\sqrt{3}x + 2) = 0$ or $(4x - \sqrt{3}) = 0$

$\Rightarrow x = -\frac{2}{\sqrt{3}}$ or $x = \frac{\sqrt{3}}{4}$

Hence, the zeroes of the quadratic polynomial $f(x) = 4\sqrt{3}x^2 + 5x - 2\sqrt{3}$ are $-\frac{2}{\sqrt{3}}$ or $\frac{\sqrt{3}}{4}$

21. If α, β are the zeroes of the polynomial f(x) = $x^2 - 5x + k$ such that $\alpha - \beta = 1$, find the value of k = ?

Sol:

By using the relationship between the zeroes of the quadratic polynomial.

We have

Sum of zeroes = $\frac{-(coefficient\ of\ x)}{coefficient\ of\ x^2}$ and Product of zeroes = $\frac{constant\ term}{coefficient\ of\ x^2}$

$\therefore \alpha + \beta = \frac{-(-5)}{1}$ and $\alpha\beta = \frac{k}{1}$

$\Rightarrow \alpha + \beta = 5$ and $\alpha\beta = \frac{k}{1}$

Solving $\alpha - \beta = 1$ and $\alpha + \beta = 5$, we will get

$\alpha = 3$ and $\beta = 2$

Substituting these values in $\alpha\beta = \frac{k}{1}$, we will get

k = 6

22. If α and β are the zeros of the polynomial f(x) = $6x^2 + x - 2$ find the value of $\left(\frac{\alpha}{\beta} + \frac{\alpha}{\beta}\right)$

Sol:

By using the relationship between the zeroes of the quadratic polynomial.

We have

Sum of zeroes = $\frac{-(coefficient\ of\ x)}{coefficient\ of\ x^2}$ and Product of zeroes = $\frac{constant\ term}{coefficient\ of\ x^2}$

$\therefore \alpha + \beta = \frac{-1}{6}$ and $\alpha\beta = -\frac{1}{3}$

Now, $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$

$= \frac{\alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta}{\alpha\beta}$

$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$

$= \frac{\left(\frac{-1}{6}\right)^2 - 2\left(-\frac{1}{3}\right)}{-\frac{1}{3}}$

$= \frac{\frac{1}{36} + \frac{2}{3}}{-\frac{1}{3}}$

$= -\frac{25}{12}$

23. If α, β are the zeroes of the polynomial f(x) = $5x^2 - 7x + 1$, then $\frac{1}{\alpha} + \frac{1}{\beta}$ = ?

Sol:

By using the relationship between the zeroes of he quadratic polynomial.

We have

Sum of zeroes = $\frac{-(coefficient\ of\ x)}{coefficient\ of\ x^2}$ and Product of zeroes = $\frac{constant\ term}{coefficient\ of\ x^2}$

$\therefore\ \alpha + \beta = \frac{-(-7)}{5}$ and $\alpha\beta = \frac{1}{5}$

$\Rightarrow \alpha + \beta = \frac{7}{5}$ and $\alpha\beta = \frac{1}{5}$

Now, $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta}$

$= \frac{\frac{7}{5}}{\frac{1}{5}}$

$= 7$

24. If α, β are the zeroes of the polynomial f(x) = $x^2 + x - 2$, then $\left(\frac{\alpha}{\beta} - \frac{\alpha}{\beta}\right)$.

Sol:

By using the relationship between the zeroes of the quadratic polynomial.

We have

Sum of zeroes = $\frac{-(coefficient\ of\ x)}{coefficient\ of\ x^2}$ and Product of zeroes = $\frac{constant\ term}{coefficient\ of\ x^2}$

$\therefore\ \alpha + \beta = \frac{-1}{1}$ and $\alpha\beta = \frac{-2}{1}$

$\Rightarrow \alpha + \beta = -1$ and $\alpha\beta = -2$

Now, $\left(\frac{1}{\alpha} - \frac{1}{\beta}\right)^2 = \left(\frac{\beta - \alpha}{\alpha\beta}\right)^2$

$= \frac{(\alpha+\beta)^2 - 4\alpha\beta}{(\alpha\beta)^2}$ $\quad [\because (\beta - \alpha)^2 = (\alpha + \beta)^2 - 4\alpha\beta]$

$= \frac{(-1)^2 - 4(-2)}{(-2)^2}$ $\quad [\because \alpha + \beta = -1$ and $\alpha\beta = -2]$

$= \frac{(-1)^2 - 4(-2)}{4}$

$= \frac{9}{4}$

$\because \left(\frac{1}{\alpha} - \frac{1}{\beta}\right)^2 = \frac{9}{4}$

$\Rightarrow \frac{1}{\alpha} - \frac{1}{\beta} = \pm \frac{3}{2}$

25. If the zeroes of the polynomial f(x) = $x^3 - 3x^2 + x + 1$ are (a – b), a and (a + b), find the values of a and b.

Sol:

By using the relationship between the zeroes of he quadratic polynomial.

We have, Sum of zeroes = $\frac{-(coefficient\ of\ x^2)}{coefficient\ of\ x^3}$

$\therefore$ a – b + a + a + b = $\frac{-(-3)}{1}$

$\Rightarrow$ 3a = 3

$\Rightarrow$ a = 1

Now, Product of zeroes = $\frac{-(constant\ term)}{coefficient\ of\ x^3}$

$\therefore$ (a – b) (a) (a + b) = $\frac{-1}{1}$

$\Rightarrow$ (1 – b) (1) (1 + b) = –1 [$\because$a =1]

$\Rightarrow$ $1 - b^2 = -1$

$\Rightarrow$ $b^2 = 2$

$\Rightarrow$ b = $\pm\sqrt{2}$

Exercise – MCQ

1. Which of the following is a polynomial?

(a) $x^2 - 5x + 6\sqrt{x} + 3$ (b) $x^{3/2} - x + x^{1/2} + 1$

(c) $\sqrt{x} + \frac{1}{\sqrt{x}}$ (d) None of these

Sol:

(d) none of these

A polynomial in x of degree n is an expression of the form $p(x) = a_0 + a_1x + a_2x^2 + \ldots\ldots + a_nx^n$, where $a_n \neq 0$.

2. Which of the following is not a polynomial?

(a) $\sqrt{3}x^2 - 2\sqrt{3}x + 5$ (b) $9x^2 - 4x + \sqrt{2}$

(c) $\frac{3}{2}x^3 + 6x^2 - \frac{1}{\sqrt{2}}x - 8$ (d) $x + \frac{3}{x}$

Sol:

(d) $x + \frac{3}{x}$ is not a polynomial.

It is because in the second term, the degree of x is –1 and an expression with a negative degree is not a polynomial.

3. The Zeroes of the polynomial $x^2 - 2x - 3$ are

(a) -3, 1 (b) -3, -1 (c) 3, -1 (d) 3, 1

Sol:

(c) 3, –1

Let $f(x) = x^2 - 2x - 3= 0$

$= x^2 - 3x + x - 3= 0$

$= x(x - 3) + 1(x - 3) = 0$

$= (x - 3)(x + 1) = 0$

$\Rightarrow$ x = 3 or x = –1

4. The zeroes of the polynomial $x^2 - \sqrt{2}x - 12$ are

(a) $\sqrt{2}, -\sqrt{2}$ (b) $3\sqrt{2}, -2\sqrt{2}$ (c) $-3\sqrt{2}, 2\sqrt{2}$ (d) $3\sqrt{2}, 2\sqrt{2}$

Sol:

(b) $3\sqrt{2}, -2\sqrt{2}$

Let $f(x) = x^2 - \sqrt{2}x - 12 = 0$

$\Rightarrow x^2 - 3\sqrt{2}x + 2\sqrt{2}x - 12 = 0$

$\Rightarrow x(x - 3\sqrt{2}) + 2\sqrt{2}(x - 3\sqrt{2}) = 0$

$\Rightarrow (x - 3\sqrt{2})(x + 2\sqrt{2}) = 0$

$\Rightarrow x = 3\sqrt{2}$ or $x = -2\sqrt{2}$

5. The zeroes of the polynomial $4x^2 + 5\sqrt{2}x - 3$ are

(a) $-3\sqrt{2}, \sqrt{2}$ (b) $-3\sqrt{2}, \frac{\sqrt{2}}{2}$ (c) $\frac{-3}{\sqrt{2}}, \frac{\sqrt{2}}{4}$ (d) none of these

Sol:

(c) $-\frac{3}{\sqrt{2}}, \frac{\sqrt{2}}{4}$

Let $f(x) = 4x^2 + 5\sqrt{2}x - 3 = 0$

$\Rightarrow 4x^2 + 6\sqrt{2}x - \sqrt{2}x - 3 = 0$

$\Rightarrow 2\sqrt{2}x(\sqrt{2}x + 3) - 1(\sqrt{2}x + 3) = 0$

$\Rightarrow (\sqrt{2}x + 3)(2\sqrt{2}x - 1) = 0$

$\Rightarrow x = -\frac{3}{\sqrt{2}}$ or $x = \frac{1}{2\sqrt{2}}$

$\Rightarrow x = -\frac{3}{\sqrt{2}}$ or $x = \frac{1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{4}$

6. The zeros of the polynomial $x^2 + \frac{1}{6}x - 2$ are

(a) -3, 4 (b) $\frac{-3}{2}, \frac{4}{3}$ (c) $\frac{-4}{3}, \frac{3}{2}$ (d) none of these

Sol:

(b) $\frac{-3}{2}, \frac{4}{3}$

Let $f(x) = x^2 + \frac{1}{6}x - 2 = 0$

$\Rightarrow 6x^2 + x - 12 = 0$

$\Rightarrow 6x^2 + 9x - 8x - 12 = 0$

$\Rightarrow 3x(2x + 3) - 4(2x + 3) = 0$

$\Rightarrow (2x + 3)(3x - 4) = 0$

$\therefore x = \frac{-3}{2}$ or $x = \frac{4}{3}$

7. The zeros of the polynomial $7x^2 - \frac{11}{3}x - \frac{2}{3}$ are

(a) $\frac{2}{3}, \frac{-1}{7}$ (a) $\frac{2}{7}, \frac{-1}{3}$ (c) $\frac{-2}{3}, \frac{1}{7}$ (d) none of these

Sol:

(a) $\frac{2}{3}, \frac{-1}{7}$

Let $f(x) = 7x^2 - \frac{11}{3}x - \frac{2}{3} = 0$

$\Rightarrow 21x^2 - 11x - 2 = 0$

$\Rightarrow 21x^2 - 14x + 3x - 2 = 0$

$\Rightarrow 7x(3x - 2) + 1(3x - 2) = 0$

$\Rightarrow (3x - 2)(7x + 1) = 0$

$\Rightarrow x = \frac{2}{3}$ or $x = \frac{-1}{7}$

8. The sum and product of the zeroes of a quadratic polynomial are 3 and -10 respectively. The quadratic polynomial is

(a) $x^2 - 3x + 10$ (b) $x^2 + 3x - 10$ (c) $x^2 - 3x - 10$ (d) $x^2 + 3x + 10$

Sol:

(c) $x^2 - 3x - 10$

Given: Sum of zeroes, $\alpha + \beta = 3$

Also, product of zeroes, $\alpha\beta = -10$

$\therefore$ Required polynomial $= x^2 - (\alpha + \beta) + \alpha\beta = x^2 - 3x - 10$

9. A quadratic polynomial whose zeroes are 5 and -3, is

(a) $x^2 + 2x - 15$ (b) $x^2 - 2x + 15$ (c) $x^2 - 2x - 15$ (d) none of these

Sol:

(c) $x^2 - 2x - 15$

Here, the zeroes are 5 and –3.

Let $\alpha = 5$ and $\beta = -3$

So, sum of the zeroes, $\alpha + \beta = 5 + (-3) = 2$

Also, product of the zeroes, $\alpha\beta = 5 \times (-3) = -15$

The polynomial will be $x^2 - (\alpha + \beta)x + \alpha\beta$

$\therefore$ The required polynomial is $x^2 - 2x - 15$.

10. A quadratic polynomial whose zeroes are $\frac{3}{5}$ and $\frac{-1}{2}$, is

(a) $10x^2 + x + 3$ (b) $10x^2 + x - 3$ (c) $10x^2 - x + 3$ (d) $x^2 - \frac{1}{10}x - \frac{3}{10}$

Sol:

(d) $x^2 - \frac{1}{10}x - \frac{3}{10}$

Here, the zeroes are $\frac{3}{5}$ and $\frac{-1}{2}$

Let $\alpha = \frac{3}{5}$ and $\beta = \frac{-1}{2}$

So, sum of the zeroes, $\alpha + \beta = \frac{3}{5} + \left(\frac{-1}{2}\right) = \frac{1}{10}$

Also, product of the zeroes, $\alpha\beta = \frac{3}{5} \times \left(\frac{-1}{2}\right) = \frac{-3}{10}$

The polynomial will be $x^2 - (\alpha + \beta)x + \alpha\beta$.

$\therefore$ The required polynomial is $x^2 - \frac{1}{10}x - \frac{3}{10}$.

11. The zeroes of the quadratic polynomial $x^2 + 88x + 125$ are

(a) both positive (b) both negative

(c) one positive and one negative (d) both equal

Sol:

(b) both negative

Let α and β be the zeroes of $x^2 + 88x + 125$.

Then $\alpha + \beta = -88$ and $\alpha \times \beta = 125$

This can only happen when both the zeroes are negative.

12. If α and β are the zeros of $x^2 + 5x + 8$, then the value of $(\alpha + \beta)$ is

(a) 5 (b) -5 (c) 8 (d) -8

Sol:

(b) –5

Given: α and β be the zeroes of $x^2 + 5x + 8$.

If $\alpha + \beta$ is the sum of the roots and $\alpha\beta$ is the product, then the required polynomial will be $x^2 - (\alpha + \beta)x + \alpha\beta$.

$\therefore \alpha + \beta = -5$

13. If α and β are the zeroes of $2x^2 + 5x - 9$, then the value of $\alpha\beta$ is

(a) $\frac{-5}{2}$ (b) $\frac{5}{2}$ (c) $\frac{-9}{2}$ (d) $\frac{9}{2}$

Sol:

(c) $\frac{-9}{2}$

Given: α and β be the zeroes of $2x^2 + 5x - 9$.

If $\alpha + \beta$ are the zeroes, then $x^2 - (\alpha + \beta)x + \alpha\beta$ is the required polynomial.

The polynomial will be $x^2 - \frac{5}{2}x - \frac{9}{2}$.

$\therefore \alpha\beta = \frac{-9}{2}$

14. If one zero of the quadratic polynomial $kx^2 + 3x + k$ is 2, then the value of k is

(a) $\frac{5}{6}$ (b) $\frac{-5}{6}$ (c) $\frac{6}{5}$ (d) $\frac{-6}{5}$

Sol:

(d) $\frac{-6}{5}$

Since 2 is a zero of $kx^2 + 3x + k$, we have:

$k \times (2)^2 + 3(2) + k = 0$

$\Rightarrow 4k + k + 6 = 0$

$\Rightarrow 5k = -6$

$\Rightarrow k = \frac{-6}{5}$

15. If one zero of the quadratic polynomial $(k - 1)x^2 - kx + 1$ is -4, then the value of k is

(a) $\frac{-5}{4}$ (b) $\frac{5}{4}$ (c) $\frac{-4}{3}$ (d) $\frac{4}{3}$

Sol:

(b) $\frac{5}{4}$

Since –4 is a zero of $(k - 1)x^2 + kx + 1$, we have:

$(k - 1) \times (-4)^2 + k \times (-4) + 1 = 0$

$\Rightarrow 16k - 16 - 4k + 1 = 0$

$\Rightarrow 12k - 15 = 0$

$\Rightarrow k = \frac{\overset{5}{\cancel{15}}}{\underset{4}{\cancel{12}}}$

$\Rightarrow k = \frac{5}{4}$

16. If -2 and 3 are the zeroes of the quadratic polynomial $x^2 + (a + 1)x + b$, then

(a) $a = -2, b = 6$ (b) $a = 2, b = -6$

(c) $a = -2, b = -6$ (d) $a = 2, b = 6$

Sol:

(c) $a = -2, b = -6$

Given: –2 and 3 are the zeroes of $x^2 + (a + 1)x + b$.

Now, $(-2)^2 + (a + 1) \times (-2) + b = 0 \Rightarrow 4 - 2a - 2 + b = 0$

$\Rightarrow b - 2a = -2$ ….(1)

Also, $3^2 + (a + 1) \times 3 + b = 0 \Rightarrow 9 + 3a + 3 + b = 0$

$\Rightarrow b + 3a = -12$ ….(2)

On subtracting (1) from (2), we get a = –2

∴ b = –2 – 4 = –6 [From (1)]

17. If one zero of $3x^2 - 8x + k$ be the reciprocal of the other, then k = ?

(a) 3 (b) -3 (c) $\frac{1}{3}$ (d) $\frac{-1}{3}$

Sol:

(a) k = 3

Let α and $\frac{1}{\alpha}$ be the zeroes of $3x^2 - 8x + k$.

Then the product of zeroes = $\frac{k}{3}$

$\Rightarrow \alpha \times \frac{1}{\alpha} = \frac{k}{3}$

$\Rightarrow 1 = \frac{k}{3}$

$\Rightarrow k = 3$

18. If the sum of the zeroes of the quadratic polynomial $kx^2 + 2x + 3k$ is equal to the product of its zeroes, then k = ?

(a) $\frac{1}{3}$ (b) $\frac{-1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{-2}{3}$

Sol:

(d) $\frac{-2}{3}$

Let α and β be the zeroes of $kx^2 + 2x + 3k$.

Then $\alpha + \beta = \frac{-2}{k}$ and $\alpha\beta = 3$

$\Rightarrow \alpha + \beta = \alpha\beta$

$\Rightarrow \frac{-2}{k} = 3$

$\Rightarrow k = \frac{-2}{3}$

19. If α, β are the zeroes of the polynomial $x^2 + 6x + 2$, then $\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$ = ?

(a) 3 (b) -3 (c) 12 (d) -12

Sol:

(b) –3

Since α and β be the zeroes of $x^2 + 6x + 2$, we have:

$\alpha + \beta = -6$ and $\alpha\beta = 2$

$\therefore \left(\frac{1}{\alpha} + \frac{1}{\beta}\right) = \left(\frac{\alpha + \beta}{\alpha\beta}\right) = \frac{-6}{2} = -3$

20. If α, β, γ be the zeroes of the polynomial $x^3 - 6x^2 - x + 30$, then $(\alpha\beta + \beta\gamma + \gamma\alpha)$ = ?

(a) -1 (b) 1 (c) -5 (d) 30

Sol:

(a) -1

It is given that α, β and γ are the zeroes of $x^3 - 6x^2 - x + 30$.

$\therefore (\alpha\beta + \beta\gamma + \gamma\alpha) = \frac{co-efficient\ of\ x}{co-efficient\ of\ x^3} = \frac{-1}{1} = -1$

21. If α, β, γ be the zeroes of the polynomial $2x^3 + x^2 - 13x + 6$, then $\alpha\beta\gamma$ = ?

(a) -3 (b) 3 (c) $\frac{-1}{2}$ (d) $\frac{-13}{2}$

Sol:

(a) –3

Since, α, β and γ are the zeroes of $2x^3 + x^2 - 13x + 6$, we have:

$\alpha\beta\gamma = \frac{-(constant\ term)}{co-efficient\ of\ x^3} = \frac{-6}{2} = -3$

22. If α, β, γ be the zeroes of the polynomial p(x) such that $(\alpha + \beta + \gamma) = 3$, $(\alpha\beta + \beta\gamma + \gamma\alpha) = -10$ and $\alpha\beta\gamma = -24$, then p(x) = ?

(a) $x^3 + 3x^2 - 10x + 24$ (b) $x^3 + 3x^2 + 10x - 24$

(c) $x^3 - 3x^2 - 10x + 24$ (d) none of these

Sol:

(c) $x^3 - 3x^2 - 10x + 24$

Given: α, β and γ are the zeroes of polynomial p(x).

Also, $(\alpha + \beta + \gamma) = 3$, $(\alpha\beta + \beta\gamma + \gamma\alpha) = -10$ and $\alpha\beta\gamma = -24$

$\therefore p(x) = x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma$

$= x^3 - 3x^2 - 10x + 24$

23. If two of the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$ are 0, then the third zero is

(a) $\frac{-b}{a}$ (b) $\frac{b}{a}$ (c) $\frac{c}{a}$ (d) $\frac{-d}{a}$

Sol:

(a) $\frac{-b}{a}$

Let α, 0 and 0 be the zeroes of $ax^3 + bx^2 + cx + d = 0$

Then the sum of zeroes = $\frac{-b}{a}$

$\Rightarrow \alpha + 0 + 0 = \frac{-b}{a}$

$\Rightarrow \alpha = \frac{-b}{a}$

Hence, the third zero is $\frac{-b}{a}$.

24. If one of the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$ is 0, then the product of the other two zeroes is

(a) $\frac{-c}{a}$ (b) $\frac{c}{a}$ (b) 0 (b) $\frac{-b}{a}$

Sol:

(b) $\frac{c}{a}$

Let α, β and 0 be the zeroes of $ax^3 + bx^2 + cx + d$.

Then, sum of the products of zeroes taking two at a time is given by

$(\alpha\beta + \beta \times 0 + \alpha \times 0) = \frac{c}{a}$

$\Rightarrow \alpha\beta = \frac{c}{a}$

$\therefore$ The product of the other two zeroes is $\frac{c}{a}$.

25. If one of the zeroes of the cubic polynomial $x^3 + ax^2 + bx + c$ is -1, then the product of the other two zeroes is

(a) $a – b – 1$ (b) $b – a – 1$ (c) $1 – a + b$ (d) $1 + a – b$

Sol:

(c) $1 – a + b$

Since –1 is a zero of $x^3 + ax^2 + bx + c$, we have:

$(–1)^3 + a \times (–1)^2 + b \times (–1) + c = 0$

$\Rightarrow a – b + c + 1 = 0$

$\Rightarrow c = 1 – a + b$

Also, product of all zeroes is given by

$\alpha\beta \times (–1) = –c$

$\Rightarrow \alpha\beta = c$

$\Rightarrow \alpha\beta = 1 – a + b$

26. If α, β be the zeroes of the polynomial $2x^2 + 5x + k$ such that $(\alpha + \beta)^2 - \alpha\beta = \frac{21}{4}$, then k = ?

(a) 3 (b) -3 (c) -2 (d) 2

Sol:

(d) 2

Since α and β are the zeroes of $2x^2 + 5x + k$, we have:

$\alpha + \beta = \frac{-5}{2}$ and $\alpha\beta = \frac{k}{2}$

Also, it is given that $\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$.

$\Rightarrow (\alpha + \beta)^2 - \alpha\beta = \frac{21}{4}$

$\Rightarrow \left(\frac{-5}{2}\right)^2 - \frac{k}{2} = \frac{21}{4}$

$\Rightarrow \frac{25}{4} - \frac{k}{2} = \frac{21}{4}$

$\Rightarrow \frac{k}{2} = \frac{25}{4} - \frac{21}{4} = \frac{4}{4} = 1$

$\Rightarrow$ k = 2

27. On dividing a polynomial p(x) by a non-zero polynomial q(x), let g(x) be the quotient and r(x) be the remainder, then p(x) = q(x). g(x) + r(x), where

(a) r(x) = 0 always

(b) deg r(x) < deg g(x) always

(c) either r(x) = 0 or deg r(x) < deg g(x)

(d) r(x) = g(x)

Sol:

(c) either r(x) = 0 or deg r(x) < deg g(x)

By division algorithm on polynomials, either r(x) = 0 or deg r(x) < deg g(x).

28. Which of the following is a true statement?

(a) $x^2 + 5x - 3$ is a linear polynomial.

(b) $x^2 + 4x - 1$ is a binomial

(c) x + 1 is a monomial

(d) $5x^2$ is a monomial

Sol:

(d) $5x^2$ is a monomial.

$5x^2$ consists of one term only. So, it is a monomial.

Exercise – Formative Assesment

1. The zeroes of the polynomial $P(x) = x^2 - 2x - 3$ are

(a) -3, 1 (b) -3, -1 (c) 3, -1 (d) 3, 1

Sol:

(c) 3, -1

Here, $p(x) = x^2 - 2x - 3$

Let $x^2 - 2x - 3 = 0$

$\Rightarrow x^2 - (3 - 1)x - 3 = 0$

$\Rightarrow x^2 - 3x + x - 3 = 0$

$\Rightarrow x(x - 3) + 1(x - 3) = 0$

$\Rightarrow (x - 3)(x + 1) = 0$

$\Rightarrow x = 3, -1$

2. If $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{\gamma}$ be the zeroes of the polynomial $x^3 - 6x^2 - x + 3$, then the values of $(\boldsymbol{\alpha\beta} + \boldsymbol{\beta\gamma} + \boldsymbol{\gamma\alpha}) = ?$

(a) -1 (b) 1 (c) -5 (d) 3

Sol:

(a) -1

Here, $p(x) = x^3 - 6x^2 - x + 3$

Comparing the given polynomial with $x^3 - (\boldsymbol{\alpha} + \boldsymbol{\beta} + \boldsymbol{\gamma})x^2 + (\boldsymbol{\alpha\beta} + \boldsymbol{\beta\gamma} + \boldsymbol{\gamma\alpha})x - \boldsymbol{\alpha\beta\gamma}$, we get:$(\boldsymbol{\alpha\beta} + \boldsymbol{\beta\gamma} + \boldsymbol{\gamma\alpha}) = -1$

3. If α, β are the zeros of $kx^2 - 2x + 3k$ is equal $\alpha + \beta = \alpha\beta$ then k = ?

(a) $\frac{1}{3}$ (b) $\frac{-1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{-2}{3}$

Sol:

(c) $\frac{2}{3}$

Here, $p(x) = x^2 - 2x + 3k$

Comparing the given polynomial with $ax^2 + bx + c$, we get:

$a = 1$, $b = -2$ and $c = 3k$

It is given that $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are the roots of the polynomial.

$\therefore \boldsymbol{\alpha} + \boldsymbol{\beta} = \frac{-b}{a}$

$\Rightarrow \boldsymbol{\alpha} + \boldsymbol{\beta} = -\left(\frac{-2}{1}\right)$

$\Rightarrow \boldsymbol{\alpha} + \boldsymbol{\beta} = 2$ ….(i)

Also, $\boldsymbol{\alpha\beta} = \frac{c}{a}$

$\Rightarrow \boldsymbol{\alpha\beta} = \frac{3k}{1}$

$\Rightarrow \boldsymbol{\alpha\beta} = 3k$ ….(ii)

Now, $\boldsymbol{\alpha} + \boldsymbol{\beta} = \boldsymbol{\alpha\beta}$

$\Rightarrow 2 = 3k$ [Using (i) and (ii)]

$\Rightarrow k = \frac{2}{3}$

4. It is given that the difference between the zeroes of $4x^2 - 8kx + 9$ is 4 and $k > 0$. Then, k = ?

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) $\frac{5}{2}$ (d) $\frac{7}{2}$

Sol:

(c) $\frac{5}{2}$

Let the zeroes of the polynomial be $\boldsymbol{\alpha}$ and $\boldsymbol{\alpha} + 4$

Here, $p(x) = 4x^2 - 8kx + 9$

Comparing the given polynomial with $ax^2 + bx + c$, we get:

$a = 4$, $b = -8k$ and $c = 9$

Now, sum of the roots = $\frac{-b}{a}$

$\Rightarrow \boldsymbol{\alpha} + \boldsymbol{\alpha} + 4 = \frac{-(-8)}{4}$

$\Rightarrow 2\boldsymbol{\alpha} + 4 = 2k$

$\Rightarrow \boldsymbol{\alpha} + 2 = k$

$\Rightarrow \boldsymbol{\alpha} = (k - 2)$ ….(i)

Also, product of the roots, $\boldsymbol{\alpha\beta} = \frac{c}{a}$

$\Rightarrow \boldsymbol{\alpha}(\boldsymbol{\alpha} + 4) = \frac{9}{4}$

$\Rightarrow (k - 2)(k - 2 + 4) = \frac{9}{4}$

$\Rightarrow (k - 2)(k + 2) = \frac{9}{4}$

$\Rightarrow k^2 - 4 = \frac{9}{4}$

$\Rightarrow 4k^2 - 16 = 9$

$\Rightarrow 4k^2 = 25$

$\Rightarrow k^2 = \frac{25}{4}$

$\Rightarrow k = \frac{5}{2}$ $(\because k > 0)$

5. Find the zeroes of the polynomial $x^2 + 2x - 195$.

Sol:

Here, $p(x) = x^2 + 2x - 195$

Let $p(x) = 0$

$\Rightarrow x^2 + (15 - 13)x - 195 = 0$

$\Rightarrow x^2 + 15x - 13x - 195 = 0$

$\Rightarrow x(x + 15) - 13(x + 15) = 0$

$\Rightarrow (x + 15)(x - 13) = 0$

$\Rightarrow x = -15, 13$

Hence, the zeroes are –15 and 13.

6. If one zero of the polynomial $(a^2 + 9)x^2 - 13x + 6a$ is the reciprocal of the other, find the value of a.

Sol:

$(a + 9)x^2 - 13x + 6a = 0$

Here, $A = (a^2 + 9)$, $B = 13$ and $C = 6a$

Let $\boldsymbol{\alpha}$ and $\frac{1}{\alpha}$ be the two zeroes.

Then, product of the zeroes = $\frac{C}{A}$

$\Rightarrow \boldsymbol{\alpha}.\frac{1}{\alpha} = \frac{6a}{a^2+9}$

$\Rightarrow 1 = \frac{6a}{a^2+9}$

$\Rightarrow a^2 + 9 = 6a$
$\Rightarrow a^2 - 6a + 9 = 0$
$\Rightarrow a^2 - 2 \times a \times 3 + 3^2 = 0$
$\Rightarrow (a - 3)^2 = 0$
$\Rightarrow a - 3 = 0$
$\Rightarrow a = 3$

7. Find a quadratic polynomial whose zeroes are 2 and -5.
Sol:
It is given that the two roots of the polynomial are 2 and -5.
Let $\boldsymbol{\alpha} = 2$ and $\boldsymbol{\beta} = -5$
Now, the sum of the zeroes, $\boldsymbol{\alpha} + \boldsymbol{\beta} = 2 + (-5) = -3$
Product of the zeroes, $\boldsymbol{\alpha\beta} = 2 \times (-5) = -15$
$\therefore$ Required polynomial = $x^2 - (\boldsymbol{\alpha} + \boldsymbol{\beta})x + \boldsymbol{\alpha\beta}$
$= x^2 - (-3)x + 10$
$= x^2 + 3x - 10$

8. If the zeroes of the polynomial $x^3 - 3x^2 + x + 1$ are $(a - b)$, a and $(a + b)$, find the values of a and b.
Sol:
The given polynomial = $x^3 - 3x^2 + x + 1$ and its roots are $(a - b)$, a and $(a + b)$.
Comparing the given polynomial with $Ax^3 + Bx^2 + Cx + D$, we have:
A = 1, B = -3, C = 1 and D = 1
Now, $(a - b) + a + (a + b) = \frac{-B}{A}$
$\Rightarrow 3a = -\frac{-3}{1}$
$\Rightarrow a = 1$
Also, $(a - b) \times a \times (a + b) = \frac{-D}{A}$
$\Rightarrow a(a^2 - b^2) = \frac{-1}{1}$
$\Rightarrow 1(1^2 - b^2) = -1$
$\Rightarrow 1 - b^2 = -1$
$\Rightarrow b^2 = 2$
$\Rightarrow b = \pm\sqrt{2}$
$\therefore$ a = 1 and $b = \pm\sqrt{2}$

9. Verify that 2 is a zero of the polynomial $x^3 + 4x^2 - 3x - 18$.
Sol:
Let $p(x) = x^3 + 4x^2 - 3x - 18$
Now, $p(2) = 2^3 + 4 \times 2^2 - 3 \times 2 - 18 = 0$

∴ 2 is a zero of $p(x)$.

10. Find the quadratic polynomial, the sum of whose zeroes is -5 and their product is 6.
Sol:
Given:
Sum of the zeroes = -5
Product of the zeroes = 6
∴ Required polynomial = x^2 – (sum of the zeroes) x + product of the zeroes
$= x^2 - (-5)x + 6$
$= x^2 + 5x + 6$

11. Find a cubic polynomial whose zeroes are 3, 5 and -2.
Sol:
Let α, β and γ are the zeroes of the required polynomial.
Then we have:
$\alpha + \beta + \gamma = 3 + 5 + (-2) = 6$
$\alpha\beta + \beta\gamma + \gamma\alpha = 3 \times 5 + 5 \times (-2) + (-2) \times 3 = -1$
and $\alpha\beta\gamma = 3 \times 5 \times -2 = -30$
Now, $p(x) = x^3 - x^2(\alpha + \beta + \gamma) + x(\alpha\beta + \beta\gamma + \gamma\alpha) - \alpha\beta\gamma$
$= x^3 - x^2 \times 6 + x \times (-1) - (-30)$
$= x^3 - 6x^2 - x + 30$
So, the required polynomial is $p(x) = x^3 - 6x^2 - x + 30$.

12. Using remainder theorem, find the remainder when $p(x) = x^3 + 3x^2 - 5x + 4$ is divided by $(x - 2)$.
Sol:
Given: $p(x) = x^3 + 3x^2 - 5x + 4$
Now, $p(2) = 2^3 + 3(2^2) - 5(2) + 4$
$= 8 + 12 - 10 + 4$
$= 14$

13. Show that $(x + 2)$ is a factor of $f(x) = x^3 + 4x^2 + x - 6$.
Sol:
Given: $f(x) = x^3 + 4x^2 + x - 6$
Now, $f(-2) = (-2)^3 + 4(-2)^2 + (-2) - 6$
$= -8 + 16 - 2 - 6$
$= 0$
∴ $(x + 2)$ is a factor of $f(x) = x^3 + 4x^2 + x - 6$.

14. If **α**, **β**, **γ** are the zeroes of the polynomial p(x) = $6x^3 + 3x^2 - 5x + 1$, find the value of $\left(\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}\right)$.

Sol:

Given: p(x) = $6x^3 + 3x^2 - 5x + 1$

$= 6x^3 - (-3)\,x^2 + (-5)\,x - 1$

Comparing the polynomial with $x^3 - x^2(\alpha + \beta + \gamma) + x(\alpha\beta + \beta\gamma + \gamma\alpha) - \alpha\beta\gamma$, we get:

$\alpha\beta + \beta\gamma + \gamma\alpha = -5$

and $\alpha\beta\gamma = -1$

$\therefore \left(\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}\right)$

$= \left(\frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma}\right)$

$= \left(\frac{-5}{-1}\right)$

$= 5$

15. If **α**, **β** are the zeroes of the polynomial f(x) = $x^2 - 5x + k$ such that $\alpha - \beta = 1$, find the value of k.

Sol:

Given: $x^2 - 5x + k$

The co-efficients are a = 1, b = -5 and c = k.

$\therefore \alpha + \beta = \frac{-b}{a}$

$\Rightarrow \alpha + \beta = -\frac{(-5)}{1}$

$\Rightarrow \alpha + \beta = 5$ ….(1)

Also, $\alpha - \beta = 1$ …..(2)

From (1) and (2), we get:

$2\alpha = 6$

$\Rightarrow \alpha = 3$

Putting the value of **α** in (1), we get **β** = 2.

Now, $\alpha\beta = \frac{c}{a}$

$\Rightarrow 3 \times 2 = \frac{k}{1}$

$\therefore k = 6$

16. Show that the polynomial f(x) = $x^2 + 4x + 6$ has no zero.

Sol:

Let $t = x^2$

So, $f(t) = t^2 + 4t + 6$

Now, to find the zeroes, we will equate f(t) = 0

$\Rightarrow t^2 + 4t + 6 = 0$

Now, t = $\frac{\mathbf{-4 \pm \sqrt{16-24}}}{\mathbf{2}}$

= $\frac{\mathbf{-4 \pm \sqrt{-8}}}{\mathbf{2}}$

= -2 ± $\sqrt{-2}$

i.e., x^2= -2 ± $\sqrt{-2}$

⇒ x = $\sqrt{-2 \pm \sqrt{-2}}$, which is not a real number.

The zeroes of a polynomial should be real numbers.

∴The given f(x) has no zeroes.

17. If one zero of the polynomial p(x) = $x^3 - 6x^2 + 11x - 6$ is 3, find the other two zeroes.

Sol:

p(x) = $x^3 - 6x^2 + 11x - 6$ and its factor, x + 3

Let us divide p(x) by (x – 3).

Here, $x^3 - 6x^2 + 11x - 6 = (x - 3)(x^2 - 3x + 2)$

$= (x - 3)[(x^2 - (2 + 1)x + 2]$

$= (x - 3)(x^2 - 2x - x + 2)$

$= (x - 3)[x(x - 2) - 1(x - 2)]$

$= (x - 3)(x - 1)(x - 2)$

∴The other two zeroes are 1 and 2.

18. If two zeroes of the polynomial p(x) = $2x^4 - 3x^3 - 3x^2 + 6x - 2$ are $\sqrt{2}$ and $-\sqrt{2}$, find its other two zeroes.

Sol:

Given: p(x) = $2x^4 - 3x^3 - 3x^2 + 6x - 2$ and the two zeroes, $\sqrt{2}$ and $-\sqrt{2}$

So, the polynomial is $(x + \sqrt{2})(x - \sqrt{2}) = x^2 - 2$.

Let us divide p(x) by $(x^2 - 2)$

Here, $2x^4 - 3x^3 - 3x^2 + 6x - 2 = (x^2 - 2)(2x^2 - 3x + 1)$

$= (x^2 - 2)[(2x^2 - (2 + 1)x + 1]$

$= (x^2 - 2)(2x^2 - 2x - x + 1)$

$= (x^2 - 2)[(2x(x - 1) - 1(x - 1)]$

$= (x^2 - 2)(2x - 1)(x - 1)$

The other two zeroes are $\frac{\mathbf{1}}{\mathbf{2}}$ and 1.

19. Find the quotient when $p(x) = 3x^4 + 5x^3 - 7x^2 + 2x + 2$ is divided by $(x^2 + 3x + 1)$.

Sol:

Given: $p(x) = 3x^4 + 5x^3 - 7x^2 + 2x + 2$

Dividing $p(x)$ by $(x^2 + 3x + 1)$, we have:

$$
\begin{array}{r|l|l}
x^2 + 3x + 1 & 3x^4 + 5x^3 - 7x^2 + 2x + 2 & 3x^2 - 4x + 2 \\
 & 3x^4 + 9x^3 + 3x^2 & \\
 & - \quad - \quad - & \\
\hline
 & -4x^3 - 10x^2 + 2x + 2 & \\
 & -4x^3 - 12x^2 - 4x & \\
 & + \quad + \quad + & \\
\hline
 & 2x^2 + 6x + 2 & \\
 & 2x^2 + 6x + 2 & \\
 & - \quad - \quad - & \\
\hline
 & x & \\
\hline
\end{array}
$$

$\therefore$The quotient is $3x^2 - 4x + 2$

20. Use remainder theorem to find the value of k, it being given that when $x^3 + 2x^2 + kx + 3$ is divided by $(x - 3)$, then the remainder is 21.

Sol:

Let $p(x) = x^3 + 2x^2 + kx + 3$

Now, $p(3) = (3)^3 + 2(3)^2 + 3k + 3$

$= 27 + 18 + 3k + 3$

$= 48 + 3k$

It is given that the reminder is 21

$\therefore 3k + 48 = 21$

$\Rightarrow 3k = -27$

$\Rightarrow k = -9$