Exercise – 3A

1. Solve the system of equations graphically:
2x + 3y = 2,
x – 2y = 8

Sol:

On a graph paper, draw a horizontal line X'OX and a vertical line YOY' representing the x-axis and y-axis, respectively.

Graph of 2x + 3y = 2

$2x + 3y = 2$

$\Rightarrow 3y = (2 - 2x)$

$\Rightarrow 3y = 2(1 - x)$

$\Rightarrow y = \frac{2(1-x)}{3}$ …(i)

Putting x = 1, we get y = 0
Putting x = -2, we get y = 2
Putting x = 4, we get y = -2

Thus, we have the following table for the equation 2x + 3y = 2

x	1	-2	4
y	0	2	-2

Now, plot the points A(1, 0), B(-2, 2) and C(4, -2) on the graph paper.
Join AB and AC to get the graph line BC. Extend it on both ways.
Thus, the line BC is the graph of 2x + 3y = 2.

Graph of x - 2y = 8

$x - 2y = 8$

$\Rightarrow 2y = (x - 8)$

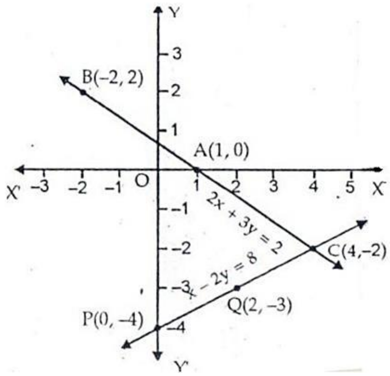
$\Rightarrow y = \frac{x-8}{2}$ …(ii)

Putting x = 2, we get y = -3
Putting x = 4, we get y = -2
Putting x = 0, we get y = -4

Thus, we have the following table for the equation x – 2y = 8.

x	2	4	0
y	-3	-2	-4

Now, plot the points P(0, -4) and Q(2, -3). The point C(4, -2) has already been plotted. Join PQ and QC and extend it on both ways.
Thus, line PC is the graph of x – 2y = 8.



The two graph lines intersect at C(4, -2).

∴ x = 4 and y = -2 are the solutions of the given system of equations.

2. Solve the system of equations graphically:
3x + 2y = 4,
2x – 3y = 7

Sol:

On a graph paper, draw a horizontal line X'OX and a vertical line YOY' representing the x-axis and y-axis, respectively.

Graph of 3x + 2y = 4

3x + 2y = 4

⇒2y = (4 – 3x)

$\Rightarrow y = \frac{4-3x)}{2}$ …(i)

Putting x = 0, we get y = 2
Putting x = 2, we get y = -1
Putting x = -2, we get y = 5
Thus, we have the following table for the equation 3x + 2y = 4

x	0	2	-2
y	2	-1	5

Now, plot the points A(0, 2), B(2, -1) and C(-2, 5) on the graph paper.
Join AB and AC to get the graph line BC. Extend it on both ways.
Thus, BC is the graph of 3x + 2y = 4.

Graph of 2x - 3y = 7

2x – 3y = 7

⇒ 3y = (2x – 7)

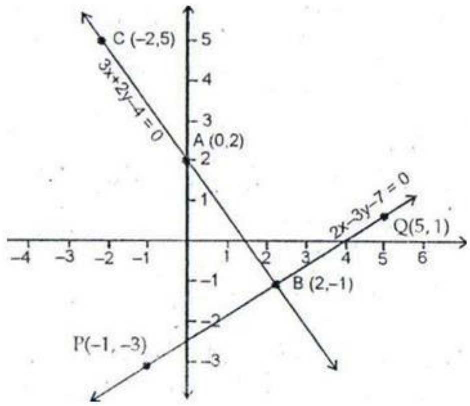
$\Rightarrow y = \frac{2x-7}{3}$ …(ii)

Putting x = 2, we get y = -1
Putting x = -1, we get y = -3
Putting x = 5, we get y = 1

Thus, we have the following table for the equation 2x – 3y = 7.

x	2	-1	5
y	-1	-3	1

Now, plot the points P(-1, -3) and Q(5, 1). The point C(2, -1) has already been plotted. Join PB and QB and extend it on both ways.
Thus, line PQ is the graph of 2x – 3y = 7.



The two graph lines intersect at B(2, -1).

∴x = 2 and y = -1 are the solutions of the given system of equations.

3. Solve the system of equations graphically:
2x + 3y = 8,
x – 2y + 3 = 0
Sol:
On a graph paper, draw a horizontal line X'OX and a vertical line YOY' as the x-axis and y-axis, respectively.

Graph of 2x + 3y = 8

2x + 3y = 8

⇒3y = (8 – 2x)

⇒$y = \frac{8-2x}{3}$ …(i)

Putting x = 1, we get y = 2.
Putting x = -5, we get y = 6.
Putting x = 7, we get y = -2.
Thus, we have the following table for the equation 2x + 3y = 8.

x	1	-5	7
y	2	6	-2

Now, plot the points A(1, 2), B(5, -6) and C(7, -2) on the graph paper.
Join AB and AC to get the graph line BC. Extend it on both ways.
Thus, BC is the graph of 2x + 3y = 8.

Graph of x - 2y + 3 = 0

$x - 2y + 3 = 0$

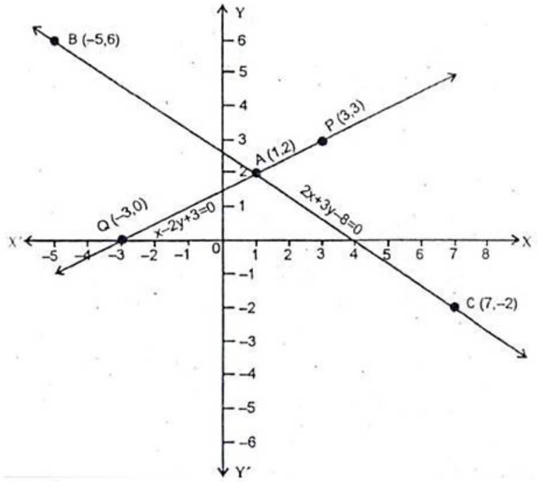
$\Rightarrow 2y = (x + 3)$

$\Rightarrow y = \frac{x+3}{2}$ ...(ii)

Putting x = 1, we get y = 2.
Putting x = 3, we get y = 3.
Putting x = -3, we get y = 0.
Thus, we have the following table for the equation $x - 2y + 3 = 0$.

x	1	3	-3
y	2	3	0

Now, plot the points P (3, 3) and Q (-3, 0). The point A (1, 2) has already been plotted. Join AP and QA and extend it on both ways.
Thus, PQ is the graph of $x - 2y + 3 = 0$.



The two graph lines intersect at A (1, 2).

$\therefore$ x = 1 and y = 2.

4. Solve the system of equations graphically:
$2x - 5y + 4 = 0$,
$2x + y - 8 = 0$
Sol:
On a graph paper, draw a horizontal line X'OX and a vertical line YOY' as the x-axis and y-axis, respectively.

Graph of 2x - 5y + 4 = 0

$2x - 5y + 4 = 0$

$\Rightarrow 5y = (2x + 4)$

$\Rightarrow y = \frac{2x+4}{5}$ ...(i)

Putting x = -2, we get y = 0.
Putting x = 3, we get y = 2.
Putting x = 8, we get y = 4.
Thus, we have the following table for the equation 2x - 5y + 4 = 0.

x	-2	3	8
y	0	2	4

Now, plot the points A (-2, 0), B (3, 2) and C(8, 4) on the graph paper.
Join AB and BC to get the graph line AC. Extend it on both ways.
Thus, AC is the graph of 2x - 5y + 4 = 0.

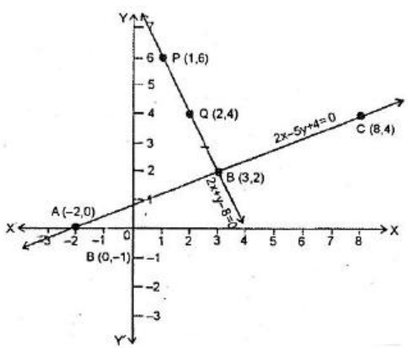
Graph of 2x + y - 8 = 0

2x + y - 8 = 0

$\Rightarrow$ y = (8 – 2x) …(ii)
Putting x = 1, we get y = 6.
Putting x = 3, we get y = 2.
Putting x = 2, we get y = 4.
Thus, we have the following table for the equation 2x + y - 8 = 0.

x	1	3	2
y	6	2	4

Now, plot the points P (1, 6) and Q (2, 4). The point B (3, 2) has already been plotted. Join PQ and QB and extend it on both ways.
Thus, PB is the graph of 2x + y - 8 = 0.



The two graph lines intersect at B (3, 2).

$\therefore$x = 3 and y = 2

5. Solve the system of equations graphically:
3x + 2y = 12,
5x – 2y = 4
Sol:
The given equations are:
3x + 2y = 12 …..(i)

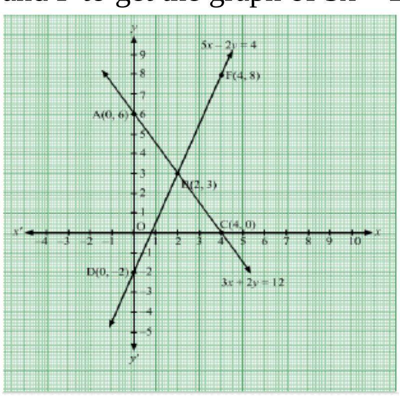
5x – 2y = 4 …..(ii)
From (i), write y in terms of x
y = $\frac{12 - 3x}{2}$ …..(iii)
Now, substitute different values of x in (iii) to get different values of y
For x = 0, y = $\frac{12 - 3x}{2} = \frac{12 - 0}{2}$ = 6
For x = 2, y = $\frac{12 - 3x}{2} = \frac{12 - 6}{2}$ = 3
For x = 4, y = $\frac{12 - 3x}{2} = \frac{12 - 12}{2}$ = 0
Thus, the table for the first equation (3x + 2y = 12) is

x	0	2	4
y	6	3	0

Now, plot the points A (0, 6), B(2, 3) and C(4, 0) on a graph paper and join A, B and C to get the graph of 3x + 2y = 12.
From (ii), write y in terms of x
y = $\frac{5x - 4}{2}$ ….(iv)
Now, substitute different values of x in (iv) to get different values of y
For x = 0, y = $\frac{5x - 4}{2} = \frac{0 - 4}{2}$ = -2
For x = 2, y = $\frac{5x - 4}{2} = \frac{10 - 4}{2}$ = 3
For x = 4, y = $\frac{5x - 4}{2} = \frac{20 - 4}{2}$ = 8
Thus, the table for the first equation (5x – 2y = 4) is

x	0	2	4
y	-2	3	8

Now, plot the points D (0, -2), E (2, 3) and F (4, 8) on the same graph paper and join D, E and F to get the graph of 5x – 2y = 4.



From the graph it is clear that, the given lines intersect at (2, 3).
Hence, the solution of the given system of equations is (2, 3).

6. Solve the system of equations graphically:
3x + y + 1 = 0,

2x - 3y + 8 = 0

Sol:

On a graph paper, draw a horizontal line X'OX and a vertical line YOY' as the x-axis and y-axis, respectively.

Graph of 3x + y + 1 = 0

3x + y + 1 = 0

$\Rightarrow$y = (-3x - 1) …(i)

Putting x = 0, we get y = -1.

Putting x = -1, we get y = 2.

Putting x = 1, we get y = -4.

Thus, we have the following table for the equation 3x + y + 1 = 0.

x	0	-1	1
y	-1	2	-4

Now, plot the points A(0, -1), B(-1, 2) and C(1, -4) on the graph paper.

Join AB and AC to get the graph line BC. Extend it on both ways.

Thus, BC is the graph of 3x + y + 1 = 0.

Graph of 2x - 3y + 8 = 0

2x - 3y + 8 = 0

$\Rightarrow$ 3y = (2x + 8)

$\therefore$ y = $\frac{2x+8}{3}$

Putting x = -1, we get y = 2.

Putting x = 2, we get y = 4.

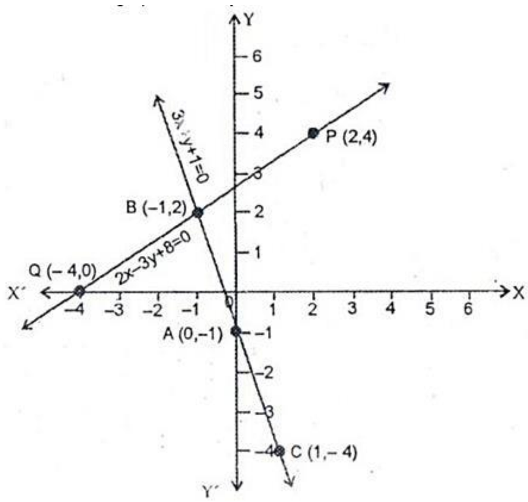
Putting x = -4, we get y = 0.

Thus, we have the following table for the equation 2x - 3y + 8 = 0.

x	-1	2	-4
y	2	4	0

Now, plot the points P(2, 4) and Q(-4, 0). The point B(-1, 2) has already been plotted. Join PB and BQ and extend it on both ways.

Thus, PQ is the graph of 2x + y - 8 = 0.



The two graph lines intersect at B (-1. 2).

∴x = -1 and y = 2

7. Solve the system of equations graphically:
2x + 3y + 5 = 0,
3x – 2y – 12 = 0

Sol:

From the first equation, write y in terms of x

$y = -\left(\frac{5 + 2x}{3}\right)$ …….(i)

Substitute different values of x in (i) to get different values of y

For x = -1, y = - $\frac{5-2}{3}$ = -1

For x = 2, y = - $\frac{5+4}{3}$ = -3

For x = 5, y = - $\frac{5+10}{3}$ = -5

Thus, the table for the first equation (2x + 3y + 5 = 0) is

x	-1	2	5
y	-1	-3	-5

Now, plot the points A (-1, -1), B (2, -3) and C (5, -5) on a graph paper and join them to get the graph of 2x + 3y + 5 = 0.

From the second equation, write y in terms of x

$y = \frac{3x - 12}{2}$ …….(ii)

Now, substitute different values of x in (ii) to get different values of y

For x = 0, y = $\frac{0-12}{2}$ = -6

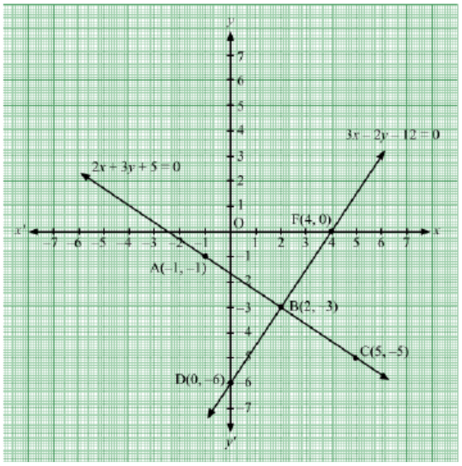
For x = 2, y = $\frac{6-12}{2}$ = -3

For x = 4, y = $\frac{12-12}{2}$ = 0

So, the table for the second equation (3x – 2y – 12 = 0) is

x	0	2	4
y	-6	-3	0

Now, plot the points D (0, -6), E (2, -3) and F (4, 0) on the same graph paper and join D, E and F to get the graph of 3x – 2y – 12 = 0.



From the graph it is clear that, the given lines intersect at (2, -3).
Hence, the solution of the given system of equation is (2, -3).

8. Solve the system of equations graphically:
2x – 3y + 13 = 0,
3x – 2y + 12 = 0

Sol:
From the first equation, write y in terms of x

y = $\frac{2x + 13}{3}$ …….(i)

Substitute different values of x in (i) to get different values of y

For x = -5, y = $\frac{-10 + 13}{3}$ = 1

For x = 1, y = $\frac{2 + 13}{3}$ = 5

For x = 4, y = $\frac{8 + 13}{3}$ = 7

Thus, the table for the first equation (2x - 3y + 13 = 0) is

x	-5	1	4
y	1	5	7

Now, plot the points A (-5, 1), B (1, 5) and C (4, 7) on a graph paper and join A, B and C to get the graph of 2x - 3y + 13 = 0.
From the second equation, write y in terms of x

y = $\frac{3x + 12}{2}$ …….(ii)

Now, substitute different values of x in (ii) to get different values of y

For x = -4, y = $\frac{-12+12}{2}$ = 0

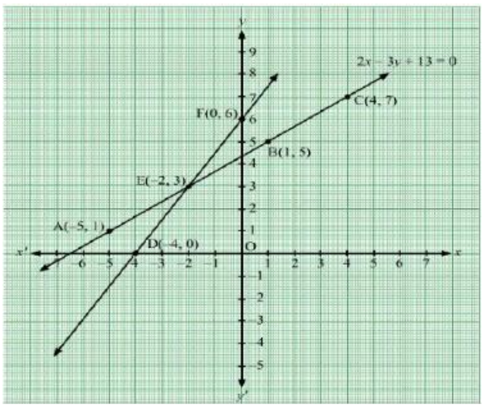
For x = -2, y = $\frac{-6+12}{2}$ = 3

For x = 0, y = $\frac{0+12}{2}$ = 6

So, the table for the second equation (3x – 2y + 12 = 0) is

x	-4	-2	0
y	0	3	6

Now, plot the points D (-4, 0), E (-2, 3) and F (0, 6) on the same graph paper and join D, E and F to get the graph of 3x – 2y + 12 = 0.



From the graph, it is clear that, the given lines intersect at (-2, 3).

Hence, the solution of the given system of equation is (-2, 3).

9. Solve the system of equations graphically:

2x + 3y = 4,

3x – y = -5

Sol:

On a graph paper, draw a horizontal line X'OX and a vertical line YOY' as the x-axis and y-axis, respectively.

Graph of 2x + 3y = 4

2x + 3y = 4

⇒3y = (4 – 2x)

∴ y = $\frac{4-2x}{3}$ …(i)

Putting x = -1, we get y = 2.

Putting x = 2, we get y = 0.

Putting x = 5, we get y = -2.

Thus, we have the following table for the equation 2x + 3y = 4.

x	-1	2	5
y	2	0	-2

Now, plot the points A (-1, 2), B (2, 0) and C (5, -2) on the graph paper.

Join AB and BC to get the graph line AC. Extend it on both ways.

Thus, AC is the graph of 2x + 3y = 4.

Graph of 3x – y = -5

3x – y = -5

⇒ y = (3x + 5) ……..(ii)

Putting x = -1, we get y = 2.

Putting x = 0, we get y = 5.

Putting x = -2, we get y = -1.

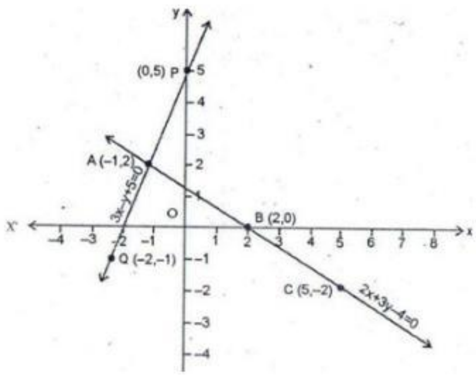
Thus, we have the following table for the equation 3x – y = -5.

x	-1	0	-2
y	2	5	-1

Now, plot the points P (0, 5) and Q (-2, -1). The point A (-1, 2) has already been plotted.

Join PA and QA and extend it on both ways.

Thus, PQ is the graph of 3x - y = - 5.



The two graph lines intersect at A (-1. 2).

∴x = -1 and y = 2 are the solutions of the given system of equations.

10. Solve the system of equations graphically:

x + 2y + 2 = 0

3x + 2y -2=0

Sol:

On a graph paper, draw a horizontal line X'OX and a vertical line YOY' as the x-axis and y-axis, respectively.

Graph of 2x + 3y = 4

x + 2y + 2 = 0

⇒2y = (–2 – x)

∴ y = $\frac{-2-x}{2}$ …(i)

Putting x = -2, we get y = 0.

Putting x = 0, we get y = -1.

Putting x = 2, we get y = -2.

Thus, we have the following table for the equation x + 2y + 2 = 0.

x	-2	0	2
y	0	-1	-2

Now, plot the points A (-2, 0), B (0, -1) and C (2, -2) on the graph paper.
Join AB and BC to get the graph line AC. Extend it on both ways.
Thus, AC is the graph of x + 2y + 2 = 0.

Graph of 3x + 2y - 2 = 0

3x + 2y – 2 = 0

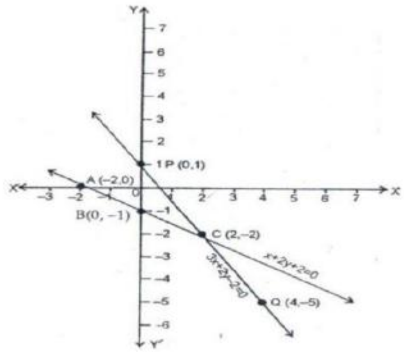
⇒ 2y = (2 – 3x)

∴ $y = \frac{2 - 3x}{2}$ ……..(ii)

Putting x = 0, we get y = 1.
Putting x = 2, we get y = -2.
Putting x = 4, we get y = -5.
Thus, we have the following table for the equation 3x + 2y – 2 = 0.

x	0	2	4
y	1	-2	-5

Now, plot the points P (0, 1) and Q(4, -5). The point C(2, -2) has already been plotted. Join PC and QC and extend it on both ways.
Thus, PQ is the graph of 3x + 2y – 2 = 0.



The two graph lines intersect at A(2, -2).

∴x = 2 and y = -2.

11. Solve graphically the system of equations
x – y – 3 = 0
2x – 3y – 4 = 0.
Find the coordinates of the vertices of the triangle formed by these two lines and the y-axis.
Sol:
From the first equation, write y in terms of x
y = x + 3 …….(i)
Substitute different values of x in (i) to get different values of y
For x = -3, y = -3 + 3 = 0

For x = -1, y = -1 + 3 = 2
For x = 1, y = 1 + 3 = 4
Thus, the table for the first equation (x - y + 3 = 0) is

x	-3	-1	1
y	0	2	4

Now, plot the points A(-3, 0), B(-1, 2) and C(1, 4) on a graph paper and join A, B and C to get the graph of x - y + 3 = 0.
From the second equation, write y in terms of x

$y = \frac{4-2x}{3}$ …….(ii)

Now, substitute different values of x in (ii) to get different values of y

For x = -4, $y = \frac{4+8}{3} = 4$

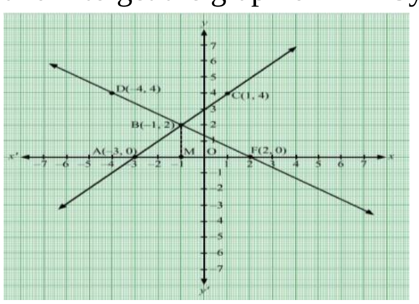
For x = -1, $y = \frac{4+12}{3} = 2$

For x = 2, $y = \frac{4-4}{3} = 0$

So, the table for the second equation (2x + 3y - 4 = 0) is

x	-4	-1	2
y	4	2	0

Now, plot the points D(-4, 4), E(-1, 2) and F(2, 0) on the same graph paper and join D, E and F to get the graph of 2x + 3y - 4 = 0.



From the graph, it is clear that, the given lines intersect at (-1, 2).
So, the solution of the given system of equation is (-1, 2).
The vertices of the triangle formed by the given lines and the x-axis are (-3, 0), (-1, 2) and (2, 0).
Now, draw a perpendicular from the intersection point E on the x-axis. So,

Area (ΔEAF) $= \frac{1}{2} \times AF \times EM$

$= \frac{1}{2} \times 5 \times 2$

= 5 sq. units

Hence, the vertices of the triangle formed by the given lines and the x-axis are (-3, 0), (-1, 2) and (2, 0) and its area is 5 sq. units.

12. Solve graphically the system of equations
2x – 3y + 4 = 0

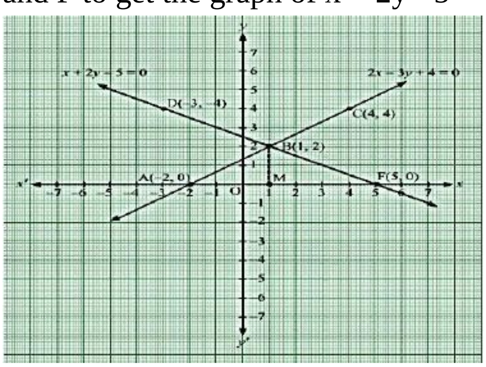
x + 2y – 5 = 0.
Find the coordinates of the vertices of the triangle formed by these two lines and the x-axis.
Sol:
From the first equation, write y in terms of x
y = $\frac{2x+4}{3}$ …….(i)
Substitute different values of x in (i) to get different values of y
For x = -2, y = $\frac{-4+4}{3}$ = 0
For x = 1, y = $\frac{2+4}{3}$ = 2
For x = 4, y = $\frac{8+4}{3}$ = 4
Thus, the table for the first equation (2x - 3y + 4 = 0) is

x	-2	1	4
y	0	2	4

Now, plot the points A(-2, 0), B(1, 2) and C(4, 4) on a graph paper and join A, B and C to get the graph of 2x - 3y + 4 = 0.
From the second equation, write y in terms of x
y = $\frac{5-x}{2}$ …….(ii)
Now, substitute different values of x in (ii) to get different values of y
For x = -3, y = $\frac{5+3}{2}$ = 4
For x = 1, y = $\frac{5-1}{2}$ = 2
For x = 5, y = $\frac{5-5}{2}$ = 0
So, the table for the second equation (x + 2y - 5 = 0) is

x	-3	1	5
y	4	2	0

Now, plot the points D(-3, 4), B(1, 2) and F(5, 0) on the same graph paper and join D, E and F to get the graph of x + 2y - 5 = 0.



From the graph, it is clear that, the given lines intersect at (1, 2).
So, the solution of the given system of equation is (1, 2).
From the graph, the vertices of the triangle formed by the given lines and the x-axis are (-2, 0), (1, 2) and (5, 0).

Now, draw a perpendicular from the intersection point B on the x-axis. So,

Area (ΔBAF) = $\frac{1}{2} \times$ AF $\times$ BM

= $\frac{1}{2} \times 7 \times 2$

= 7 sq. units

Hence, the vertices of the triangle formed by the given lines and the x-axis are (-2, 0), (1, 2) and (5, 0) and the area of the triangle is 7 sq. units.

13. Solve the following system of linear equations graphically

4x – 3y + 4 = 0, 4x + 3y – 20 = 0.

Find the area of the region bounded by these lines and the x-axis.

Sol:

On a graph paper, draw a horizontal line X'OX and a vertical line YOY' as the x-axis and y-axis, respectively.

Graph of 4x - 3y + 4 = 0

4x - 3y + 4 = 0

$\Rightarrow$3y = (4x + 4)

$\therefore$ y = $\frac{4x + 4}{3}$ …(i)

Putting x = -1, we get y = 0.

Putting x = 2, we get y = 4.

Putting x = 5, we get y = 8.

Thus, we have the following table for the equation 4x - 3y + 4 = 0.

x	-1	2	5
y	0	4	8

Now, plot the points A(-1, 0), B(2, 4) and C(5, 8) on the graph paper.

Join AB and BC to get the graph line AC. Extend it on both ways.

Thus, AC is the graph of 4x - 3y + 4 = 0.

Graph of 4x + 3y - 20 = 0

4x + 3y - 20 = 0

$\Rightarrow$ 3y = (–4x + 20)

$\therefore$ y = $\frac{-4x + 20}{3}$ ……..(ii)

Putting x = 2, we get y = 4.

Putting x = -1, we get y = 8.

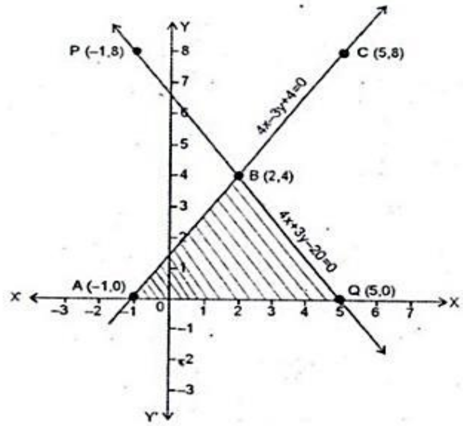
Putting x = 5, we get y = 0.

Thus, we have the following table for the equation 4x + 3y - 20 = 0.

x	2	-1	5
y	4	8	0

Now, plot the points P(1, -8) and Q(5, 0). The point B(2, 4) has already been plotted. Join PB and QB to get the graph line. Extend it on both ways.

Then, line PQ is the graph of the equation 4x + 3y - 20 = 0.



The two graph lines intersect at B(2, 4).

∴ The solution of the given system of equations is x = 2 and y = 4.

Clearly, the vertices of ΔABQ formed by these two lines and the x-axis are Q(5, 0), B(2, 4) and A(-1, 0).

Now, consider ΔABQ.

Here, height = 4 units and base (AQ) = 6 units

∴ Area ΔABQ = $\frac{1}{2} \times$ base $\times$ height sq. units

$= \frac{1}{2} \times 6 \times 4$

= 12 sq. units.

14. Solve the following system of linear equations graphically

x –y + 1 = 0, 3x + 2y – 12 = 0.

Calculate the area bounded by these lines and the x-axis.

Sol:

On a graph paper, draw a horizontal line X'OX and a vertical line YOY' as the x-axis and y-axis, respectively.

Graph of x - y + 1 = 0

x - y + 1 = 0

⇒y = x + 1 …(i)

Putting x = -1, we get y = 0.

Putting x = 1, we get y = 2.

Putting x = 2, we get y = 3.

Thus, we have the following table for the equation x - y + 1 = 0.

x	-1	1	2
y	0	2	3

Now, plot the points A(-1, 0), B(1, 2) and C(2, 3) on the graph paper.

Join AB and BC to get the graph line AC. Extend it on both ways.

Thus, AC is the graph of x - y + 1 = 0.

Graph of 3x + 2y - 12 = 0

3x + 2y - 12 = 0

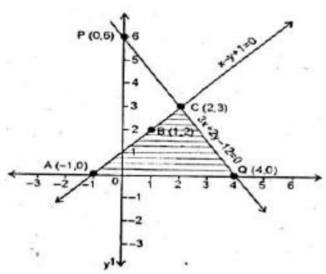
$\Rightarrow$ 2y = (–3x + 12)

$\therefore$ y = $\frac{-3x+12}{2}$ ……..(ii)

Putting x = 0, we get y = 6.
Putting x = 2, we get y = 3.
Putting x = 4, we get y = 0.
Thus, we have the following table for the equation 3x + 2y - 12 = 0.

x	0	2	4
y	6	3	0

Now, plot the points P(0, 6) and Q(4, 0). The point B(2, 3) has already been plotted. Join PC and CQ to get the graph line PQ. Extend it on both ways.
Then, PQ is the graph of the equation 3x + 2y - 12 = 0.



The two graph lines intersect at C(2, 3).

$\therefore$ The solution of the given system of equations is x = 2 and y = 3.
Clearly, the vertices of ΔACQ formed by these two lines and the x-axis are Q(4, 0), C(2, 3) and A(-1, 0).
Now, consider ΔACQ.
Here, height = 3 units and base (AQ) = 5 units

$\therefore$ Area ΔACQ = $\frac{1}{2} \times$ base $\times$ height sq. units

= $\frac{1}{2} \times 5 \times 3$

= 7.5 sq. units.

15. Solve graphically the system of equations
x – 2y + 2 = 0
2x + y – 6 = 0.
Find the coordinates of the vertices of the triangle formed by these two lines and the x-axis.
Sol:
From the first equation, write y in terms of x

y = $\frac{x+2}{2}$ …….(i)

Substitute different values of x in (i) to get different values of y

For x = -2, y = $\frac{-2+2}{2}$ = 0

For x = 2, y = $\frac{2+2}{2}$ = 2

For x = 4, y = $\frac{4+\ 2}{2}$ = 3

Thus, the table for the first equation (x - 2y + 2 = 0) is

x	-2	2	4
y	0	2	3

Now, plot the points A(-2, 0), B(2, 2) and C(4, 3) on a graph paper and join A, B and C to get the graph of x - 2y + 2 = 0.

From the second equation, write y in terms of x

y = 6 – 2x ……. (ii)

Now, substitute different values of x in (ii) to get different values of y

For x = 1, y = 6 – 2 = 4

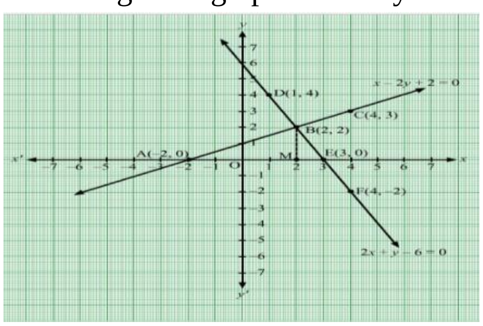
For x = 3, y = 0

For x = 4, y = 6 – 8 = -2

So, the table for the second equation (2x + y - 6 = 0) is

x	1	3	4
y	4	0	-2

Now, plot the points D(1, 4), E(3, 0) and F(4, -2) on the same graph paper and join D, E and F to get the graph of 2x + y - 6 = 0.



From the graph, it is clear that, the given lines intersect at (2, 2).

So, the solution of the given system of equation is (2, 2).

From the graph, the vertices of the triangle formed by the given lines and the x-axis are (-2, 0), (2, 2) and (3, 0).

Now, draw a perpendicular from the intersection point B on the x-axis. So,

Area (ΔBAE) = $\frac{1}{2} \times AE \times BM$

$= \frac{1}{2} \times 5 \times 2$

= 5 sq. units

Hence, the vertices of the triangle formed by the given lines and the x-axis are (-2, 0), (2, 2) and (3, 0) and the area of the triangle is 5 sq. units.

16. Solve graphically the system of equations

$2x - 3y + 6 = 0$

$2x + 3y - 18 = 0.$

Find the coordinates of the vertices of the triangle formed by these two lines and the y-axis.

Sol:

From the first equation, write y in terms of x

$y = \frac{2x+6}{3}$ …….(i)

Substitute different values of x in (i) to get different values of y

For $x = -3$, $y = \frac{-6+6}{3} = 0$

For $x = 0$, $y = \frac{0+6}{3} = 2$

For $x = 3$, $y = \frac{6+6}{3} = 4$

Thus, the table for the first equation ($2x - 3y + 6 = 0$) is

x	-3	0	3
y	0	2	4

Now, plot the points A(-3, 0), B(0, 2) and C(3, 4) on a graph paper and join A, B and C to get the graph of $2x - 3y + 6 = 0$.

From the second equation, write y in terms of x

$y = \frac{18-2x}{3}$ …….(ii)

Now, substitute different values of x in (ii) to get different values of y

For $x = 0$, $y = \frac{18-0}{3} = 6$

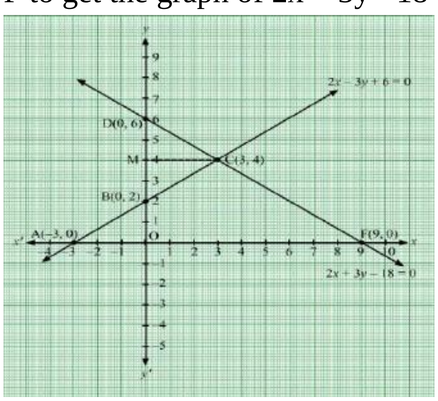
For $x = 3$, $y = \frac{18-6}{3} = 4$

For $x = 9$, $y = \frac{18-18}{3} = 0$

So, the table for the second equation ($2x + 3y - 18 = 0$) is

x	0	3	9
y	6	4	0

Now, plot the points D(0, 6), E(3, 4) and F(9, 0) on the same graph paper and join D, E and F to get the graph of $2x + 3y - 18 = 0$.



From the graph, it is clear that, the given lines intersect at (3, 4).

So, the solution of the given system of equation is (3, 4).
From the graph, the vertices of the triangle formed by the given lines and the y-axis are (0, 2), (0, 6) and (3, 4).
Now, draw a perpendicular from the intersection point E (or C) on the y-axis. So,
Area (ΔEDB) = $\frac{1}{2} \times BD \times EM$
$= \frac{1}{2} \times 4 \times 3$
=6 sq. units
Hence, the vertices of the triangle formed by the given lines and the y-axis are (0, 2), (0, 6) and (3, 4) and the area of the triangle is 6 sq. units.

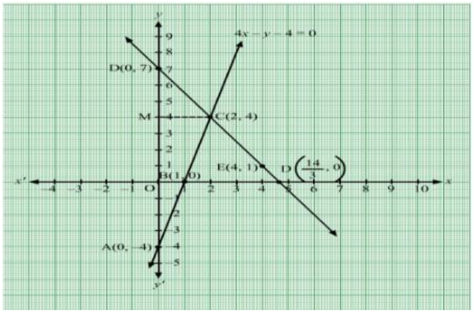
17. Solve graphically the system of equations
$4x - y - 4 = 0$
$3x + 2y - 14 = 0$.
Find the coordinates of the vertices of the triangle formed by these two lines and the y-axis.
Sol:
From the first equation, write y in terms of x
$y = 4x - 4$ …….(i)
Substitute different values of x in (i) to get different values of y
For x = 0, y = 0 – 4 = -4
For x = 1, y = 4 – 4 = 0
For x = 2, y = 8 – 4 = 4
Thus, the table for the first equation (4x - y – 4 = 0) is

x	0	1	2
y	-4	0	4

Now, plot the points A(0, -4), B(1, 0) and C(2, 4) on a graph paper and join A, B and C to get the graph of 4x – y – 4 = 0.
From the second equation, write y in terms of x
$y = \frac{14-3x}{2}$ …….(ii) 2y = 14 – 3x - 3x = 2y - 14
Now, substitute different values of x in (ii) to get different values of y
For x = 0, $y = \frac{14-0}{2} = 7$
For x = 4, $y = \frac{14-12}{2} = 1$
For $x = \frac{14}{3}$, $y = \frac{14-14}{2} = 0$
So, the table for the second equation (3x + 2y - 14 = 0) is

x	0	4	$\frac{14}{3}$
y	7	1	0

Now, plot the points D(0, 7), E(4, 1) and F($\frac{14}{3}$, 0) on the same graph paper and join D, E and F to get the graph of 3x + 2y - 14 = 0.



From the graph, it is clear that, the given lines intersect at (2, 4).
So, the solution of the given system of equation is (2, 4).
From the graph, the vertices of the triangle formed by the given lines and the y-axis are 0, 7), (0, -4) and (2, 4).
Now, draw a perpendicular from the intersection point C on the y-axis. So,

Area (ΔDAB) = $\frac{1}{2} \times$ DA $\times$ CM

$= \frac{1}{2} \times 11 \times 2$

= 11 sq. units

Hence, the vertices of the triangle formed by the given lines and the y-axis are (0, 7), (0, -4) and (2, 4) and the area of the triangle is 11 sq. units.

18. Solve graphically the system of equations
x – y – 5 = 0
3x + 5y - 15 = 0.
Find the coordinates of the vertices of the triangle formed by these two lines and the y-axis.
Sol:
From the first equation, write y in terms of x
y = x – 5 …….(i)
Substitute different values of x in (i) to get different values of y
For x = 0, y = 0 – 5 **=** -5
For x = 2, y = 2 – 5 **=** -3
For x = 5, y = 5 – 5 **=** 0
Thus, the table for the first equation (x – y – 5 = 0) is

x	0	2	5
y	-5	-3	0

Now, plot the points A(0, -5), B(2, -3) and C(5, 0) on a graph paper and join A, B and C to get the graph of x – y – 5 = 0.
From the second equation, write y in terms of x

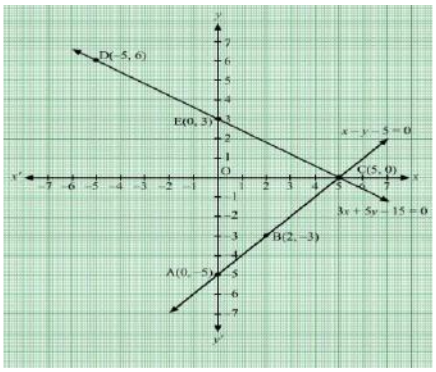
y = $\frac{15-3x}{5}$ …….(ii)

Now, substitute different values of x in (ii) to get different values of y

For x = -5, y = $\frac{\mathbf{15+15}}{\mathbf{5}}$ = 6
For x = 0, y = $\frac{\mathbf{15-0}}{\mathbf{5}}$ = 3
For x = 5, y = $\frac{\mathbf{15-15}}{\mathbf{5}}$ = 0
So, the table for the second equation (3x + 5y - 15 = 0) is

x	-5	0	5
y	6	3	0

Now, plot the points D(-5, 6), E(0, 3) and F(5, 0) on the same graph paper and join D, E and F to get the graph of 3x + 5y - 15 = 0.



From the graph, it is clear that, the given lines intersect at (5, 0).
So, the solution of the given system of equation is (5, 0).
From the graph, the vertices of the triangle formed by the given lines and the y-axis are 0, 3), (0, -5) and (5, 0).
Now, draw a perpendicular from the intersection point C on the y-axis. So,

Area (ΔCEA) = $\frac{\mathbf{1}}{\mathbf{2}} \times$ EA $\times$ OC
= $\frac{\mathbf{1}}{\mathbf{2}} \times 8 \times 5$
= 20 sq. units

Hence, the vertices of the triangle formed by the given lines and the y-axis are (0, 3), (0, -5) and (5, 0) and the area of the triangle is 20 sq. units.

19. Solve graphically the system of equations
2x – 5y + 4 = 0
2x + y - 8 = 0.
Find the coordinates of the vertices of the triangle formed by these two lines and the y-axis.
Sol:
From the first equation, write y in terms of x
y = $\frac{\mathbf{2x+4}}{\mathbf{5}}$ …….(i)
Substitute different values of x in (i) to get different values of y
For x = -2, y = $\frac{\mathbf{-4+4}}{\mathbf{5}}$ = 0

For x = 0, y = $\frac{\mathbf{0+4}}{\mathbf{5}} = \frac{4}{5}$

For x = 3, y = $\frac{\mathbf{6+4}}{\mathbf{5}} = 2$

Thus, the table for the first equation (2x – 5y + 4 = 0) is

x	-2	0	3
y	0	$\frac{4}{5}$	2

Now, plot the points A(-2, 0), B(0, $\frac{4}{5}$) and C(3, 2) on a graph paper and join A, B and C to get the graph of 2x – 5y + 4 = 0.

From the second equation, write y in terms of x

y = 8 – 2x …….(ii)

Now, substitute different values of x in (ii) to get different values of y

For x = 0, y = 8 – 0 = 8

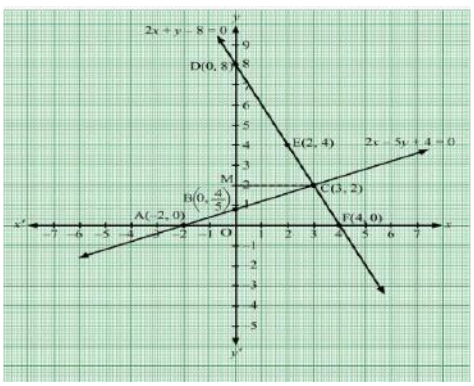
For x = 2, y = 8 – 4 = 3

For x = 4, y = 8 – 8 = 0

So, the table for the second equation (2x – 5y + 4 = 0) is

x	0	2	4
y	8	4	0

Now, plot the points D(0, 8), E(2, 4) and F(4, 0) on the same graph paper and join D, E and F to get the graph of 2x + y - 8 = 0.



From the graph, it is clear that, the given lines intersect at (3, 2).

So, the solution of the given system of equation is (3, 2).

The vertices of the triangle formed by the system of equations and y-axis are (0, 8), (0, $\frac{4}{5}$) and (3, 2).

Draw a perpendicular from point C on the y-axis. So,

Area (ΔDBC) = $\frac{1}{2} \times DB \times CM$

$= \frac{1}{2} \times \left(\mathbf{8} - \frac{4}{5}\right) \times 3$

$= \frac{54}{5}$ sq. units

Hence, the vertices of the triangle are (0, 8), (0, $\frac{4}{5}$) and (3, 2) and its area is $\frac{54}{5}$ sq. units.

20. Solve graphically the system of equations

5x - y = 7

x - y + 1 = 0.

Find the coordinates of the vertices of the triangle formed by these two lines and the y-axis.

Sol:

On a graph paper, draw a horizontal line X'OX and a vertical line YOY' as the x-axis and y-axis, respectively.

Graph of 5x - y = 7

5x - y = 7

$\Rightarrow y = (5x - 7)$ …(i)

Putting x = 0, we get y = -7.

Putting x = 1, we get y = -2.

Putting x = 2, we get y = 3.

Thus, we have the following table for the equation 5x - y = 7.

x	0	1	2
y	-7	-2	3

Now, plot the points A(0, -7), B(1, -2) and C(2, 3) on the graph paper.

Join AB and BC to get the graph line AC. Extend it on both ways.

Thus, AC is the graph of 5x - y = 7.

Graph of x - y + 1 = 0

x - y + 1 = 0

$\Rightarrow y = x + 1$ ……..(ii)

Putting x = 0, we get y = 1.

Putting x = 1, we get y = 2.

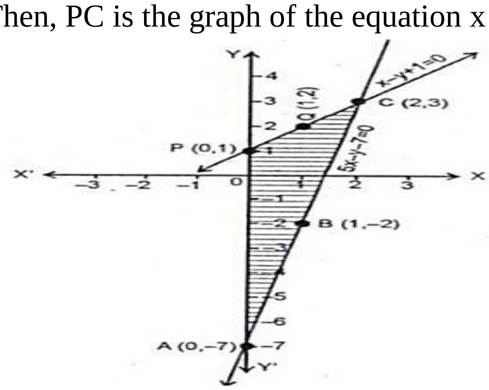
Putting x = 2, we get y = 3.

Thus, we have the following table for the equation x - y + 1 = 0.

x	0	1	2
y	1	2	3

Now, plot the points P(0, 1) and Q(1, 2). The point C(2, 3) has already been plotted. Join PQ and QC to get the graph line PC. Extend it on both ways.

Then, PC is the graph of the equation x - y + 1 = 0.



The two graph lines intersect at C(2, 3).

∴ The solution of the given system of equations is x = 2 and y = 3.
Clearly, the vertices of ΔAPC formed by these two lines and the y-axis are P(0, 1), C(2, 3) and A(0, -7).
Now, consider ΔAPC.
Here, height = 2 units and base (AP) = 8 units
∴ Area ΔAPC = $\frac{1}{2} \times$ base $\times$ height sq. units
$= \frac{1}{2} \times 8 \times 2$
= 8 sq. units.

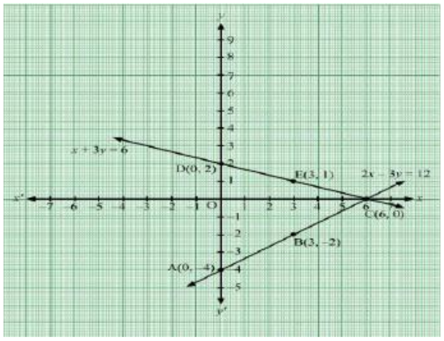
21. Solve graphically the system of equations
2x – 3y = 12
x + 3y = 6.
Find the coordinates of the vertices of the triangle formed by these two lines and the y-axis.
Sol:
From the first equation, write y in terms of x
$y = \frac{2x - 12}{3}$ …….(i)
Substitute different values of x in (i) to get different values of y
For x = 0, $y = \frac{0-12}{3} = -4$
For x = 3, $y = \frac{6 - 12}{3} = -2$
For x = 6, $y = \frac{12-12}{3} = 0$
Thus, the table for the first equation (2x – 3y = 12) is

x	0	3	6
y	-4	-2	0

Now, plot the points A(0, -4), B(3,-2) and C(6, 0) on a graph paper and join A, B and C to get the graph of 2x – 3y = 12.
From the second equation, write y in terms of x
$y = \frac{6 - x}{3}$ …….(ii)
Now, substitute different values of x in (ii) to get different values of y
For x = 0, $y = \frac{6 - 0}{3} = 2$
For x = 3, $y = \frac{6 - 3}{3} = 1$
For x = 6, $y = \frac{6 - 6}{3} = 0$
So, the table for the second equation (x + 3y = 6) is

x	0	3	6
y	2	1	0

Now, plot the points D(0, 2), E(3, 1) and F(6, 0) on the same graph paper and join D, E and F to get the graph of x + 3y = 6.



From the graph, it is clear that, the given lines intersect at (6, 0).
So, the solution of the given system of equation is (6, 0).
The vertices of the triangle formed by the system of equations and y-axis are (0, 2), (6, 0) and (0, -4).

Area (ΔDAC) = $\frac{1}{2} \times DA \times OC$

$= \frac{1}{2} \times 6 \times 6$

= 18 sq. units

Hence, the vertices of the triangle are (0, 2), (6, 0) and (0, -4) and its area is 18 sq. units.

22. Show graphically that the system of equations 2x + 3y = 6, 4x + 6y = 12 has infinitely many solutions.

Sol:

From the first equation, write y in terms of x

y = $\frac{6 - 2x}{3}$ …….(i)

Substitute different values of x in (i) to get different values of y

For x = -3, y = $\frac{6 + 6}{3}$ = 4

For x = 3, y = $\frac{6 - 6}{3}$= 0

For x = 6, y = $\frac{6 - 12}{3}$ = -2

Thus, the table for the first equation (2x + 3y = 6) is

x	-3	3	6
y	4	0	-2

Now, plot the points A(-3, 4), B(3, 0) and C(6, -2) on a graph paper and join A, B and C to get the graph of 2x + 3y = 6.

From the second equation, write y in terms of x

y = $\frac{12 - 4x}{6}$ …….(ii)

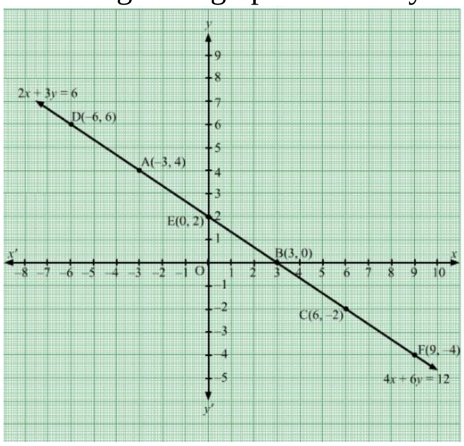
Now, substitute different values of x in (ii) to get different values of y

For x = -6, y = $\frac{12 + 24}{6}$ = 6

For x = 0, y = $\frac{\mathbf{12-0}}{\mathbf{6}}$ = 2
For x = 9, y = $\frac{\mathbf{12-36}}{\mathbf{6}}$ = -4
So, the table for the second equation (4x + 6y = 12) is

x	-6	0	9
y	6	2	-4

Now, plot the points D(-6, 6), E(0, 2) and F(9, -4) on the same graph paper and join D, E and F to get the graph of 4x + 6y = 12.



From the graph, it is clear that, the given lines coincidence with each other.
Hence, the solution of the given system of equations has infinitely many solutions.

23. Show graphically that the system of equations 3x - y = 5, 6x - 2y = 10 has infinitely many solutions.

Sol:

On a graph paper, draw a horizontal line X'OX and a vertical line YOY' representing the x-axis and y-axis, respectively.

Graph of 3x - y = 5

3x - y = 5

⇒y = 3x - 5 …(i)

Putting x = 1, we get y = -2
Putting x = 0, we get y = -5
Putting x = 2, we get y = 1
Thus, we have the following table for the equation 3x - y = 5

x	1	0	2
y	-2	-5	1

Now, plot the points A(1, -2), B(0, -5) and C(2, 1) on the graph paper.
Join AB and AC to get the graph line BC. Extend it on both ways.
Thus, the line BC is the graph of 3x - y = 5.

Graph of 6x - 2y = 10

6x - 2y = 10

⇒ 2y = (6x – 10)

⇒ y = $\frac{6x-10}{2}$ …(ii)

Putting x = 0, we get y = -5

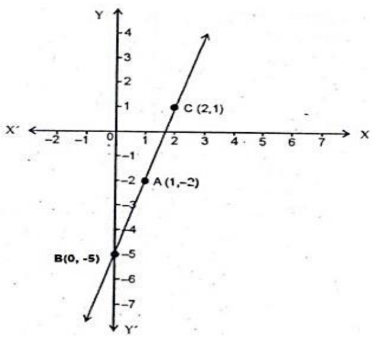
Putting x = 1, we get y = -2

Putting x = 2, we get y = 1

Thus, we have the following table for the equation 6x - 2y = 10.

x	0	1	2
y	-5	-2	1

These are the same points as obtained for the graph line of equation (i).



It is clear from the graph that these two lines coincide.

Hence, the given system of equations has infinitely many solutions.

24. Show graphically that the system of equations 2x + y = 6, 6x + 3y = 18 has infinitely many solutions.

Sol:

On a graph paper, draw a horizontal line X'OX and a vertical line YOY' representing the x-axis and y-axis, respectively.

Graph of 2x + y = 6

2x + y = 6

⇒y = (6 – 2x) …(i)

Putting x = 3, we get y = 0

Putting x = 1, we get y = 4

Putting x = 2, we get y = 2

Thus, we have the following table for the equation 2x + y = 6

x	3	1	2
y	0	4	2

Now, plot the points A(3, 0), B(1, 4) and C(2, 2) on the graph paper.

Join AC and CB to get the graph line AB. Extend it on both ways.

Thus, the line AB is the graph of 2x + y = 6.

Graph of 6x + 3y = 18

6x + 3y = 18

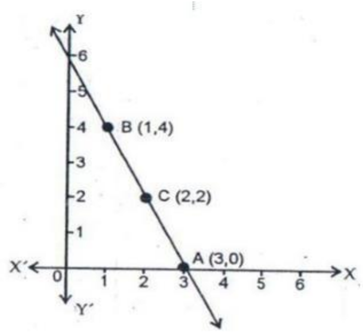
⇒ 3y = (18 - 6x)

⇒ y = $\frac{18 - 6x}{3}$ …(ii)

Putting x = 3, we get y = 0
Putting x = 1, we get y = 4
Putting x = 2, we get y = 2
Thus, we have the following table for the equation 6x + 3y = 18.

x	3	1	2
y	0	4	2

These are the same points as obtained for the graph line of equation (i).



It is clear from the graph that these two lines coincide.
Hence, the given system of equations has an infinite number of solutions.

25. Show graphically that the system of equations x - 2y = 5, 3x - 6y = 15 has infinitely many solutions.

Sol:

From the first equation, write y in terms of x

y = $\frac{x - 5}{2}$ …….(i)

Substitute different values of x in (i) to get different values of y

For x = -5, y = $\frac{-5-5}{2}$ = -5

For x = 1, y = $\frac{1 - 5}{2}$= -2

For x = 3, y = $\frac{3 - 5}{2}$ = -1

Thus, the table for the first equation (x - 2y = 5) is

x	-5	1	3
y	-5	-2	-1

Now, plot the points A(-5, -5), B(1, -2) and C(3, -1) on a graph paper and join A, B and C to get the graph of x - 2y = 5.

From the second equation, write y in terms of x

y = $\frac{3x - 15}{6}$ …….(ii)

Now, substitute different values of x in (ii) to get different values of y

For x = -3, y = $\frac{-9 - 15}{6}$ = -4

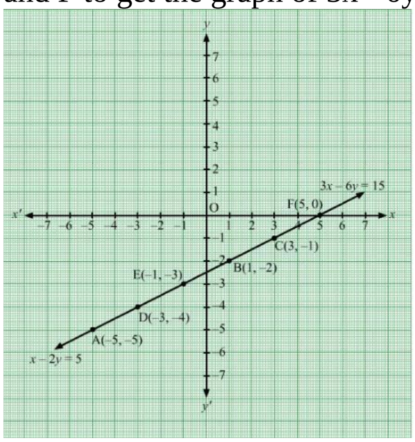
For x = -1, y = $\frac{-3 - 15}{6}$ = -3

For x = 5, y = $\frac{15 - 15}{6}$ = 0

So, the table for the second equation (3x - 6y = 15) is

x	-3	-1	5
y	-4	-3	0

Now, plot the points D(-3, -4), E(-1, -3) and F(5, 0) on the same graph paper and join D, E and F to get the graph of 3x - 6y = 15.



From the graph, it is clear that, the given lines coincide with each other.

Hence, the solution of the given system of equations has infinitely many solutions.

26. Show graphically that the system of equations x - 2y = 6, 3x - 6y = 0 is inconsistent.

Sol:

From the first equation, write y in terms of x

y = $\frac{x - 6}{2}$ …….(i)

Substitute different values of x in (i) to get different values of y

For x = -2, y = $\frac{-2-6}{2}$ = -4

For x = 0, y = $\frac{0 - 6}{2}$= -3

For x = 2, y = $\frac{2 - 6}{2}$ = -2

Thus, the table for the first equation (x - 2y = 5) is

x	-2	0	2
y	-4	-3	-2

Now, plot the points A(-2, -4), B(0, -3) and C(2, -2) on a graph paper and join A, B and C to get the graph of x - 2y = 6.

From the second equation, write y in terms of x

y = $\frac{1}{2}x$ …….(ii)

Now, substitute different values of x in (ii) to get different values of y

For x = -4, y = $\frac{-4}{2}$ = -2

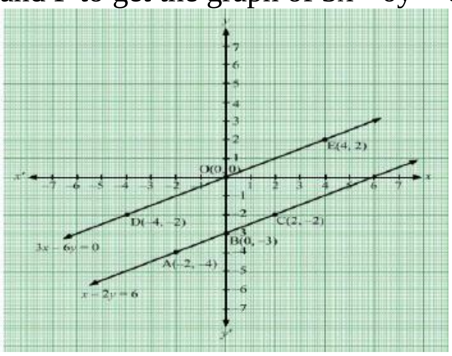
For x = 0, y = $\frac{0}{2}$ = 0

For x = 4, y = $\frac{4}{2}$ = 2

So, the table for the second equation (3x - 6y = 0) is

x	-4	0	4
y	-2	0	2

Now, plot the points D(-4, -2), O(0, 0) and E(4, 2) on the same graph paper and join D, E and F to get the graph of 3x - 6y = 0.



From the graph, it is clear that, the given lines do not intersect at all when produced.
Hence, the system of equations has no solution and therefore is inconsistent.

27. Show graphically that the system of equations 2x + 3y = 4, 4x + 6y = 12 is inconsistent.

Sol:

On a graph paper, draw a horizontal line X'OX and a vertical line YOY' as the x-axis and y-axis, respectively.

Graph of 2x +3y = 4

2x + 3y = 4

⇒ 3y = (–2x + 4) …(i)

Putting x = 2, we get y = 0

Putting x = -1, we get y = 2

Putting x = -4, we get y = 4

Thus, we have the following table for the equation 2x + 3y = 4.

x	2	-1	-4
y	0	2	4

Now, plot the points A(2, 0), B(-1, 2) and C(-4, 4) on the graph paper.
Join AB and BC to get the graph line AC. Extend it on both ways.
Thus, the line AC is the graph of 2x + 3y = 4.

Graph of 4x + 6y = 12

4x + 6y = 12

⇒ 6y = (-4x + 12)

⇒ y = $\frac{-4x+12}{6}$ …(ii)

Putting x = 3, we get y = 0

Putting x = 0, we get y = 2

Putting x = 6, we get y = -2

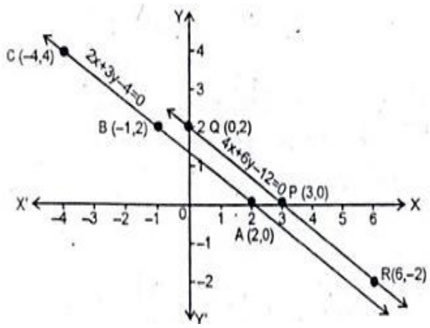
Thus, we have the following table for the equation 4x + 6y = 12.

x	3	0	6
y	0	2	-2

Now, on the same graph, plot the points A(3, 0), B(0, 2) and C(6, -2).

Join PQ and PR to get the graph line QR. Extend it on both ways.

Thus, QR is the graph of the equation 4x + 6y = 12.



It is clear from the graph that these two lines are parallel and do not intersect when produced.

Hence, the given system of equations is inconsistent.

28. Show graphically that the system of equations 2x + y = 6, 6x + 3y = 20 is inconsistent.

Sol:

From the first equation, write y in terms of x

y = 6 – 2x …….(i)

Substitute different values of x in (i) to get different values of y

For x = 0, y = 6 – 0 = 6

For x = 2, y = 6 – 4 = 2

For x = 4, y = 6 – 8 = -2

Thus, the table for the first equation (2x + y = 6) is

x	0	2	4
y	6	2	-2

Now, plot the points A(0, 6), B(2, 2) and C(4, -2) on a graph paper and join A, B and C to get the graph of 2x + y = 6.

From the second equation, write y in terms of x

y = $\frac{20-6x}{3}$ …….(ii)

Now, substitute different values of x in (ii) to get different values of y

For x = 0, y = $\frac{20-0}{3} = \frac{20}{3}$

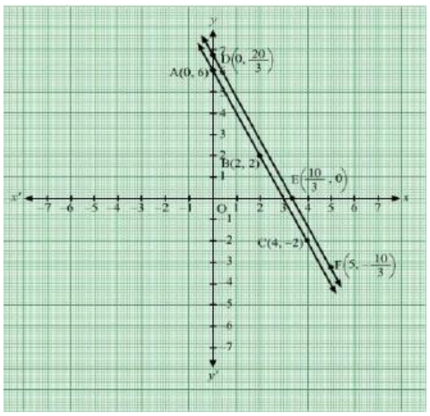
For x = $\frac{10}{3}$, y = $\frac{20-20}{3}$ = 0

For x = 5, y = $\frac{20-30}{3} = -\frac{10}{3}$

So, the table for the second equation (6x + 3y = 20) is

x	0	$\frac{10}{3}$	5
y	$\frac{20}{3}$	0	$-\frac{10}{3}$

Now, plot the points D(0, $\frac{20}{3}$), O($\frac{10}{3}$, 0) and E(5, $-\frac{10}{3}$) on the same graph paper and join D, E and F to get the graph of 6x + 3y = 20.



From the graph, it is clear that, the given lines do not intersect at all when produced.
Hence, the system of equations has no solution and therefore is inconsistent.

29. Draw the graphs for the following equations on the same graph paper:

2x + y = 2

2x + y = 6

Find the co-ordinates of the vertices of the trapezium formed by these lines. Also, find the area of the trapezium so formed.

Sol:

From the first equation, write y in terms of x

y = 2 – 2x …….(i)

Substitute different values of x in (i) to get different values of y

For x = 0, y = 2 – 0 = 2

For x = 1, y = 2 – 2 = 0

For x = 2, y = 2 – 4 = -2

Thus, the table for the first equation (2x + y = 2) is

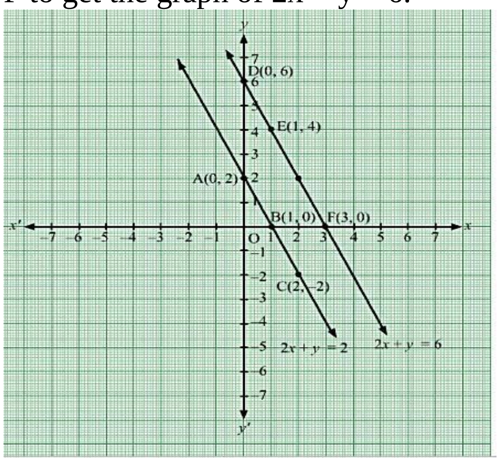
x	0	1	2
y	2	0	-2

Now, plot the points A(0, 2), B(1, 0) and C(2, -2) on a graph paper and join A, B and C to get the graph of 2x + y = 2.

From the second equation, write y in terms of x
$y = 6 - 2x$ …….(ii)
Now, substitute different values of x in (ii) to get different values of y
For $x = 0$, $y = 6 - 0 = 6$
For $x = 1$, $y = 6 - 2 = 4$
For $x = 3$, $y = 6 - 6 = 0$
So, the table for the second equation ($2x + y = 6$) is

x	0	1	3
y	6	4	0

Now, plot the points D(0,6), E(1, 4) and F(3,0) on the same graph paper and join D, E and F to get the graph of $2x + y = 6$.



From the graph, it is clear that, the given lines do not intersect at all when produced. So, these lines are parallel to each other and therefore, the quadrilateral DABF is a trapezium. The vertices of the required trapezium are D(0, 6), A (0, 2), B(1, 0) and F(3, 0).
Now,
Area(Trapezium DABF) = Area (ΔDOF) – Area(ΔAOB)
$= \frac{1}{2} \times 3 \times 6 - \frac{1}{2} \times 1 \times 2$
$= 9 - 1$
$= 8$ sq. units
Hence, the area of the required trapezium is 8 sq. units.

Exercise – 3B

1. Solve for x and y:
$x + y = 3$, $4x - 3y = 26$
Sol:
The given system of equation is:
$x + y = 3$…….(i)
$4x - 3y = 26$ ……(ii)
On multiplying (i) by 3, we get:

$3x + 3y = 9$ ….(iii)
On adding (ii) and (iii), we get:
$7x = 35$
$\Rightarrow x = 5$
On substituting the value of $x = 5$ in (i), we get:
$5 + y = 3$
$\Rightarrow y = (3 – 5) = -2$
Hence, the solution is $x = 5$ and $y = -2$

2. Solve for x and y:
$x – y = 3, \frac{x}{3} + \frac{y}{2} = 6$
Sol:
The given system of equations is
$x – y = 3$ …..(i)
$\frac{x}{3} + \frac{y}{2} = 6$ ……(ii)
From (i), write y in terms of x to get
$y = x – 3$
Substituting $y = x – 3$ in (ii), we get
$\frac{x}{3} + \frac{x-3}{2} = 6$
$\Rightarrow 2x + 3(x – 3) = 36$
$\Rightarrow 2x + 3x – 9 = 36$
$\Rightarrow x = \frac{45}{5} = 9$
Now, substituting $x = 9$ in (i), we have
$9 – y = 3$
$\Rightarrow y = 9 – 3 = 6$
Hence, $x = 9$ and $y = 6$.

3. Solve for x and y:
$2x + 3y = 0, 3x + 4y = 5$
Sol:
The given system of equation is:
$2x + 3y = 0$ ……(i)
$3x + 4y = 5$ ……(ii)
On multiplying (i) by 4 and (ii) by 3, we get:
$8x + 12y = 0$ ……(iii)
$9x + 12y = 15$ …….(iv)
On subtracting (iii) from (iv) we get:

$x = 15$
On substituting the value of $x = 15$ in (i), we get:
$30 + 3y = 0$

$\Rightarrow 3y = -30$

$\Rightarrow y = -10$
Hence, the solution is $x = 15$ and $y = -10$.

4. Solve for x and y:
$2x - 3y = 13$, $7x - 2y = 20$
Sol:
The given system of equation is:
$2x - 3y = 13$ ……(i)
$7x - 2y = 20$ ……(ii)
On multiplying (i) by 2 and (ii) by 3, we get:
$4x - 6y = 26$ ……(iii)
$21x - 6y = 60$ …….(iv)
On subtracting (iii) from (iv) we get:
$17x = (60 – 26) = 34$

$\Rightarrow x = 2$
On substituting the value of $x = 2$ in (i), we get:
$4 – 3y = 13$

$\Rightarrow 3y = (4 – 13) = -9$

$\Rightarrow y = -3$
Hence, the solution is $x = 2$ and $y = -3$.

5. Solve for x and y:
$3x - 5y - 19 = 0$, $-7x + 3y + 1 = 0$
Sol:
The given system of equation is:
$3x - 5y - 19 = 0$ ……(i)
$-7x + 3y + 1 = 0$ ……(ii)
On multiplying (i) by 3 and (ii) by 5, we get:
$9x - 15y = 57$ ……(iii)
$-35x + 15y = -5$ …….(iv)
On subtracting (iii) from (iv) we get:
$-26x = (57 – 5) = 52$

$\Rightarrow x = -2$
On substituting the value of $x = -2$ in (i), we get:

$-6 - 5y - 19 = 0$

$\Rightarrow 5y = (-6 - 19) = -25$

$\Rightarrow y = -5$

Hence, the solution is $x = -2$ and $y = -5$.

6. Solve for x and y:

$2x - y + 3 = 0, 3x - 7y + 10 = 0$

Sol:

The given system of equation is:

$2x - y + 3 = 0$…….(i)

$3x - 7y + 10 = 0$ ……(ii)

From (i), write y in terms of x to get

$y=2x + 3$

Substituting $y = 2x + 3$ in (ii), we get

$3x - 7(2x + 3) + 10 = 0$

$\Rightarrow 3x - 14x - 21 + 10 = 0$

$\Rightarrow -7x = 21 - 10 = 11$

$x = -\frac{11}{7}$

Now substituting $x = -\frac{11}{7}$ in (i), we have

$-\frac{22}{7} - y + 3 = 0$

$y = 3 - \frac{22}{7} = -\frac{1}{7}$

Hence, $x = -\frac{11}{7}$ and $y = -\frac{1}{7}$.

7. Solve for x and y:

$9x - 2y = 108, 3x + 7y = 105$

Sol:

The given system of equation can be written as:

$9x - 2y = 108$ ……(i)

$3x + 7y = 105$ ……(ii)

On multiplying (i) by 7 and (ii) by 2, we get:

$63x + 6x = 108 \times 7 + 105 \times 2$

$\Rightarrow 69x = 966$

$\Rightarrow x = \frac{966}{69} = 14$

Now, substituting $x = 14$ in (i), we get:

$9 \times 14 - 2y = 108$

$\Rightarrow 2y = 126 - 108$

$\Rightarrow y = \frac{18}{2} = 9$
Hence, x = 14 and y = 9.

8. Solve for x and y:
$\frac{x}{3} + \frac{y}{4} = 11, \frac{5x}{6} - \frac{y}{3} + 7 = 0$
Sol:
The given equations are:
$\frac{x}{3} + \frac{y}{4} = 11$
$\Rightarrow 4x + 3y = 132$ …….(i)
and $\frac{5x}{6} - \frac{y}{3} + 7 = 0$
$\Rightarrow 5x - 2y = -42$ ……..(ii)
On multiplying (i) by 2 and (ii) by 3, we get:
$8x + 6y = 264$ …...(iii)
$15x - 6y = -126$ ….(iv)
On adding (iii) and (iv), we get:
$23x = 138$
$\Rightarrow x = 6$
On substituting x = 6 in (i), we get:
$24 + 3y = 132$
$\Rightarrow 3y = (132 - 24) = 108$
$\Rightarrow y = 36$
Hence, the solution is x = 6 and y = 36.

9. Solve for x and y:
$4x - 3y = 8, 6x - y = \frac{29}{3}$
Sol:
The given system of equation is:
$4x - 3y = 8$ ……(i)
$6x - y = \frac{29}{3}$ ……(ii)
On multiplying (ii) by 3, we get:
$18x - 3y = 29$ ….(iii)
On subtracting (iii) from (i) we get:
$-14x = -21$
$x = \frac{21}{14} = \frac{3}{2}$
Now, substituting the value of $x = \frac{3}{2}$ in (i), we get:

$4 \times \frac{3}{2} – 3y = 8$

$\Rightarrow 6 – 3y = 8$

$\Rightarrow 3y = 6 – 8 = -2$

$y = \frac{-2}{3}$

Hence, the solution $x = \frac{3}{2}$ and $y = \frac{-2}{3}$.

10. Solve for x and y:

$2x - \frac{3y}{4} = 3,\ 5x = 2y + 7$

Sol:

The given equations are:

$2x - \frac{3y}{4} = 3$ …..(i)

$5x = 2y + 7$ ……..(ii)

On multiplying (i) by 2 and (ii) by $\frac{3}{4}$, we get:

$4x - \frac{3}{2}y = 6$ …...(iii)

$\frac{15}{4}x = \frac{3}{2}y + \frac{21}{4}$….(iv)

On subtracting (iii) and (iv), we get:

$-\frac{1}{4}x = -\frac{3}{4}$

$\Rightarrow x = 3$

On substituting x = 3 in (i), we get:

$2 \times 3 - \frac{3y}{4} = 3$

$\Rightarrow \frac{3y}{4} = (6 – 3) = 3$

$\Rightarrow y = \frac{3 \times 4}{3} = 4$

Hence, the solution is x = 3 and y = 4.

11. Solve for x and y:

$2x + 5y = \frac{8}{3},\ 3x – 2y = \frac{5}{6}$

Sol:

The given equations are:

$2x – 5y = \frac{8}{3}$ …..(i)

$3x – 2y = \frac{5}{6}$ ……..(ii)

On multiplying (i) by 2 and (ii) by 5, we get:

$4x - 10y = \frac{16}{3}$ …...(iii)

$15x - 10y = \frac{25}{6}$……(iv)
On adding (iii) and (iv), we get:
$19x = \frac{57}{6}$
$\Rightarrow x = \frac{57}{6 \times 19} = \frac{3}{6} = \frac{1}{2}$
On substituting $x = \frac{1}{2}$ in (i), we get:
$2 \times \frac{1}{2} + 5y = \frac{8}{3}$
$\Rightarrow 5y = \left(\frac{8}{3} - 1\right) = \frac{5}{3}$
$\Rightarrow y = \frac{5}{3 \times 5} = \frac{1}{3}$
Hence, the solution is $x = \frac{1}{2}$ and $y = \frac{1}{3}$.

12. Solve for x and y:
$2x + 3y + 1 = 0$
$\frac{7-4x}{3} = y$
Sol:
The given equations are:
$\frac{7-4x}{3} = y$
$\Rightarrow 4x + 3y = 7$ ……..(i)
and $2x + 3y + 1 = 0$
$\Rightarrow 2x + 3y = -1$ ……….(ii)
On subtracting (ii) from (i), we get:
$2x = 8$
$\Rightarrow x = 4$
On substituting x = 4 in (i), we get:
$16x + 3y = 7$
$\Rightarrow 3y = (7 - 16) = -9$
$\Rightarrow y = -3$
Hence, the solution is x = 4 and y = -3.

13. Solve for x and y:
$0.4x + 0.3y = 1.7$, $0.7x - 0.2y = 0.8$.
Sol:
The given system of equations is
$0.4x + 0.3y = 1.7$ …….(i)
$0.7x - 0.2y = 0.8$ …….(ii)

Multiplying (i) by 0.2 and (ii) by 0.3 and adding them, we get
$0.8x + 2.1x = 3.4 + 2.4$

$\Rightarrow 2.9x = 5.8$

$\Rightarrow x = \frac{5.8}{2.9} = 2$

Now, substituting x = 2 in (i), we have

$0.4 \times 2 + 0.3y = 1.7$

$\Rightarrow 0.3y = 1.7 – 0.8$

$\Rightarrow y = \frac{0.9}{0.3} = 3$

Hence, x = 2 and y = 3.

14. Solve for x and y:
$0.3x + 0.5y = 0.5, 0.5x + 0.7y = 0.74$
Sol:
The given system of equations is
$0.3x + 0.5y = 0.5$ …….(i)
$0.5x + 0.7y = 0.74$ …….(ii)
Multiplying (i) by 5 and (ii) by 3 and subtracting (ii) from (i), we get
$2.5y - 2.1y = 2.5 - 2.2$

$\Rightarrow 0.4y = 0.28$

$\Rightarrow y = \frac{0.28}{0.4} = 0.7$

Now, substituting y = 0.7 in (i), we have

$0.3x + 0.5 \times 0.7 = 0.5$

$\Rightarrow 0.3x = 0.50 – 0.35 = 0.15$

$\Rightarrow x = \frac{0.15}{0.3} = 0.5$

Hence, x = 0.5 and y = 0.7.

15. Solve for x and y:
$7(y + 3) – 2(x + 2) = 14, 4(y – 2) + 3(x – 3) = 2$
Sol:
The given equations are:
$7(y + 3) – 2(x + 2) = 14$

$\Rightarrow 7y + 21 – 2x – 4 = 14$

$\Rightarrow -2x + 7y = -3$ ……..(i)
and $4(y – 2) + 3(x – 3) = 2$

$\Rightarrow 4y – 8 + 3x – 9 = 2$

$\Rightarrow 3x + 4y = 19$ ……….(ii)

On multiplying (i) by 4 and (ii) by 7, we get:

$-8x + 28y = -12$ ……(iii)

$21x + 28y = 133$ ……(iv)

On subtracting (iii) from (iv), we get:

$29x = 145$

$\Rightarrow x = 5$

On substituting x = 5 in (i), we get:

$-10 + 7y = -3$

$\Rightarrow 7y = (-3 + 10) = 7$

$\Rightarrow y = 1$

Hence, the solution is x = 5 and y = 1.

16. Solve for x and y:

$6x + 5y = 7x + 3y + 1 = 2(x + 6y – 1)$

Sol:

The given equations are:

$6x + 5y = 7x + 3y + 1 = 2(x + 6y – 1)$

$\Rightarrow 6x + 5y = 2(x + 6y – 1)$

$\Rightarrow 6x + 5y = 2x + 12y – 2$

$\Rightarrow 6x – 2x + 5y – 12y = -2$

$\Rightarrow 4x – 7y = -2$ …….(i)

and $7x + 3y + 1 = 2(x + 6y – 1)$

$\Rightarrow 7x + 3y + 1 = 2x + 12y – 2$

$\Rightarrow 7x – 2x + 3y – 12y = -2 – 1$

$\Rightarrow 5x – 9y = -3$ …….(ii)

On multiplying (i) by 9 and (ii) by 7, we get:

$36x - 63y = -18$ ……(iii)

$35x - 63y = -21$ ……(iv)

On subtracting (iv) from (iii), we get:

$x = (-18 + 21) = 3$

On substituting x = 3 in (i), we get:

$12 - 7y = -2$

$\Rightarrow 7y = (2 + 12) = 14$

$\Rightarrow y = 2$

Hence, the solution is x = 3 and y = 2.

17. Solve for x and y:

$\frac{x+y-8}{2} = \frac{x+2y-14}{3} = \frac{3x+y-12}{11}$

Sol:

The given equations are:

$\frac{x+y-8}{2} = \frac{x+2y-14}{3} = \frac{3x+y-12}{11}$

i.e., $\frac{x+y-8}{2} = \frac{3x+y-12}{11}$

By cross multiplication, we get:

11x + 11y – 88 = 6x + 2y – 24

⇒11x – 6x + 11y – 2y = -24 + 88

⇒5x + 9y = 64 …….(i)

and $\frac{x+2y-14}{3} = \frac{3x+y-12}{11}$

⇒11x + 22y – 154 = 9x + 3y – 36

⇒11x – 9x + 22y – 3y = -36 + 154

⇒2x + 19y = 118 …….(ii)

On multiplying (i) by 19 and (ii) by 9, we get:

95x + 171y = 1216 ……(iii)

18x + 171y = 1062 ……(iv)

On subtracting (iv) from (iii), we get:

77x = 154

⇒x = 2

On substituting x = 2 in (i), we get:

10 + 9y = 64

⇒9y = (64 – 10) = 54

⇒y = 6

Hence, the solution is x = 2 and y = 6.

18. Solve for x and y:

$\frac{5}{x}$ + 6y = 13, $\frac{3}{x}$ + 4y = 7

Sol:

The given equations are:

$\frac{5}{x}$ + 6y = 13 ……..(i)

$\frac{3}{x}$ + 4y = 7 ……..(ii)

Putting $\frac{1}{x}$ = u, we get:

5u + 6y = 13 …….(iii)
3u + 4y = 7 ……(iv)
On multiplying (iii) by 4 and (iv) by 6, we get:
20u + 24y = 52 ……..(v)
18u + 24y = 42 ……..(vi)
On subtracting (vi) from (v), we get:

2u = 10 ⇒ u = 5
⇒ $\frac{1}{x}$ = 5 ⇒ x = $\frac{1}{5}$
On substituting x = $\frac{1}{5}$ in (i), we get:
$\frac{5}{1/3}$ + 6y = 13
25 + 6y = 13
6y = (13 – 25) = -12
y = -2
Hence, the required solution is x = $\frac{1}{5}$ and y = -2.

19. Solve for x and y:
x + $\frac{6}{y}$ = 6, 3x - $\frac{8}{y}$ = 5
Sol:
The given equations are:
x + $\frac{6}{y}$ = 6 ……..(i)
3x - $\frac{8}{y}$ = 5 ……..(ii)
Putting $\frac{1}{y}$ = v, we get:
x + 6v = 6 …….(iii)
3x – 8v = 5 ……(iv)
On multiplying (iii) by 4 and (iv) by 3, we get:
4x + 24v = 24 ……..(v)
9x – 24v = 15 ……..(vi)
On adding (v) from (vi), we get:

13x = 39 ⇒ x = 3
On substituting x = 3 in (i), we get:
3 + $\frac{6}{y}$ = 6
⇒$\frac{6}{y}$ = (6 – 3) = 3 ⇒ 3y = 6 ⇒ y = 2
Hence, the required solution is x = 3 and y = 2.

20. Solve for x and y:

$2x - \frac{3}{y} = 9$, $3x + \frac{7}{y} = 2$

Sol:

The given equations are:

$2x - \frac{3}{y} = 9$ ……..(i)

$3x + \frac{7}{y} = 2$ ……..(ii)

Putting $\frac{1}{y} = v$, we get:

$2x - 3v = 6$ …….(iii)

$3x + 7v = 2$ ……(iv)

On multiplying (iii) by 7 and (iv) by 3, we get:

$14x - 21v = 63$ ……..(v)

$9x + 21v = 6$ ……..(vi)

On adding (v) from (vi), we get:

$23x = 69 \Rightarrow x = 3$

On substituting x = 3 in (i), we get:

$2 \times 3 - \frac{3}{y} = 9$

$\Rightarrow 6 - \frac{3}{y} = 9 \Rightarrow \frac{3}{y} = -3 \Rightarrow y = -1$

Hence, the required solution is x = 3 and y = -1.

21. Solve for x and y:

$\frac{3}{x} - \frac{1}{y} + 9 = 0$, $\frac{2}{x} + \frac{3}{y} = 5$

Sol:

The given equations are:

$\frac{3}{x} - \frac{1}{y} + 9 = 0$,

$\Rightarrow \frac{3}{x} - \frac{1}{y} = -9$ ……..(i)

$\Rightarrow \frac{2}{x} - \frac{3}{y} = 5$……..(ii)

Putting $\frac{1}{x} = u$ and $\frac{1}{y} = v$, we get:

$3u – v = -9$ …….(iii)

$2u + 3v = 5$ ……(iv)

On multiplying (iii) by 3, we get:

$9u – 3v = -27$ ……..(v)

On adding (iv) and (v), we get:

$11u = -22 \Rightarrow u = -2$

$\Rightarrow \frac{1}{x} = -2 \Rightarrow x = \frac{-1}{2}$

On substituting $x = \frac{-1}{2}$ in (i), we get:

$\frac{3}{-1/2} - \frac{1}{y} = -9$

$\Rightarrow -6 - \frac{1}{y} = -9 \Rightarrow \frac{1}{y} = (-6 + 9) = 3$

$\Rightarrow y = \frac{1}{3}$

Hence, the required solution is $x = \frac{-1}{2}$ and $y = \frac{1}{3}$.

22. Solve for x and y:

$\frac{9}{x} - \frac{4}{y} = 8, \frac{13}{x} + \frac{7}{y} = 101$

Sol:

The given equations are:

$\frac{9}{x} - \frac{4}{y} = 8$ ……..(i)

$\frac{13}{x} + \frac{7}{y} = 101$ ……..(ii)

Putting $\frac{1}{x} = u$ and $\frac{1}{y} = v$, we get:

$9u - 4v = 8$ …….(iii)

$13u + 7v = 101$ ……(iv)

On multiplying (iii) by 7 and (iv) by 4, we get:

$63u - 28v = 56$ ……..(v)

$52u + 28v = 404$ ……..(vi)

On adding (v) from (vi), we get:

$115u = 460 \Rightarrow u = 4$

$\Rightarrow \frac{1}{x} = 4 \Rightarrow x = \frac{1}{4}$

On substituting $x = \frac{1}{4}$ in (i), we get:

$\frac{9}{1/4} - \frac{4}{y} = 8$

$\Rightarrow 36 - \frac{4}{y} = 8 \Rightarrow \frac{4}{y} = (36 – 8) = 28$

$y = \frac{4}{28} = \frac{1}{7}$

Hence, the required solution is $x = \frac{1}{4}$ and $y = \frac{1}{7}$.

23. Solve for x and y:

$\frac{5}{x} - \frac{3}{y} = 1, \frac{3}{2x} + \frac{2}{3y} = 5$

Sol:

The given equations are:

$\frac{5}{x} - \frac{3}{y} = 1$ ……..(i)

$\frac{3}{2x} + \frac{2}{3y} = 5$ ……..(ii)

Putting $\frac{1}{x}$ = u and $\frac{1}{y}$ = v, we get:
5u - 3v = 1 …….(iii)
$\Rightarrow\frac{3}{2}$ u + $\frac{2}{3}$ v = 5
$\Rightarrow\frac{9u+4v}{6}$ = 5
$\Rightarrow$9u + 4v = 30 …….(iv)
On multiplying (iii) by 4 and (iv) by 3, we get:
20u - 12v = 4 ……..(v)
27u + 12v = 90 ……..(vi)
On adding (iv) and (v), we get:
47u = 94 $\Rightarrow$ u = 2
$\Rightarrow\frac{1}{x}$ = 2 $\Rightarrow$ x = $\frac{1}{2}$
On substituting x = $\frac{1}{2}$ in (i), we get:
$\frac{5}{1/2}$ - $\frac{3}{y}$ = 1
$\Rightarrow$10 - $\frac{3}{y}$ = 1 $\Rightarrow$ $\frac{3}{y}$ = (10 – 1) = 9
y = $\frac{3}{9}$ = $\frac{1}{3}$
Hence, the required solution is x = $\frac{1}{2}$ and y = $\frac{1}{3}$.

24. Solve for x and y:
$\frac{3}{x}$ + $\frac{2}{y}$ = 12, $\frac{2}{x}$ + $\frac{3}{y}$ = 13
Sol:
The given equations are:
$\frac{3}{x}$ + $\frac{2}{y}$ = 12 ……..(i)
$\frac{2}{x}$ + $\frac{3}{y}$ = 13 ……..(ii)
Multiplying (i) by 3 and (ii) by 2 and subtracting (ii) from (i), we get:
$\frac{9}{x}$ - $\frac{4}{x}$ = 36 – 26
$\Rightarrow\frac{5}{x}$ = 10
$\Rightarrow$x = $\frac{5}{10}$ = $\frac{1}{2}$
Now, substituting x = $\frac{1}{2}$ in (i), we have
6 + $\frac{2}{y}$ = 12
$\Rightarrow\frac{2}{y}$ = 6
$\Rightarrow$y = $\frac{1}{3}$

Hence, $x = \frac{1}{2}$ and $y = \frac{1}{3}$.

25. Solve for x and y:
$4x + 6y = 3xy$, $8x + 9y = 5xy$
Sol:
The given equations are:
$4x + 6y = 3xy$ …..(i)
$8x + 9y = 5xy$ ……(ii)
From equation (i), we have:
$\frac{4x + 6y}{xy} = 3$
$\Rightarrow \frac{4}{y} + \frac{6}{x} = 3$ ……(iii)
For equation (ii), we have:
$\frac{8x + 9y}{xy} = 5$
$\Rightarrow \frac{8}{y} + \frac{9}{x} = 5$ ……(iv)
On substituting $\frac{1}{y} = v$ and $\frac{1}{x} = u$, we get:
$4v + 6u = 3$ ……(v)
$8v + 9u = 5$ …….(vi)
On multiplying (v) by 9 and (vi) by 6, we get:
$36v + 54u = 27$ ….(vii)
$48v + 54u = 30$ ….(viii)
On subtracting (vii) from (viii), we get:
$12v = 3 \Rightarrow v = \frac{3}{12} = \frac{1}{4}$
$\Rightarrow \frac{1}{y} = \frac{1}{4} \Rightarrow y = 4$
On substituting y =4 in (iii), we get:
$\frac{4}{4} + \frac{6}{x} = 3$
$\Rightarrow 1 + \frac{6}{x} = 3 \Rightarrow \frac{6}{x} = (3 – 1) = 2$
$\Rightarrow 2x = 6 \Rightarrow x = \frac{6}{2} = 3$
Hence, the required solution is x = 3 and y = 4.

26. Solve for x and y:
$x + y = 5xy$, $3x + 2y = 13xy$
Sol:
The given equations are:
$x + y = 5xy$ …..(i)
$3x + 2y = 13xy$ ……(ii)

From equation (i), we have:
$\frac{x + y}{xy} = 5$
$\Rightarrow \frac{1}{y} + \frac{1}{x} = 5$ ……(iii)
For equation (ii), we have:
$\frac{3x + 2y}{xy} = 13$
$\Rightarrow \frac{3}{y} + \frac{2}{x} = 13$ ……(iv)
On substituting $\frac{1}{y}$ = v and $\frac{1}{x}$ = u, we get:
v + u = 5 ……(v)
3v + 2u = 13 …….(vi)
On multiplying (v) by 2, we get:
2v + 2u = 10 ….(vii)
On subtracting (vii) from (vi), we get:
v = 3
$\Rightarrow \frac{1}{y} = 3 \Rightarrow y = \frac{1}{3}$
On substituting y = $\frac{1}{3}$ in (iii), we get:
$\frac{1}{1/3} + \frac{1}{x} = 5$
$\Rightarrow 3 + \frac{1}{x} = 5 \Rightarrow \frac{1}{x} = 2 \Rightarrow x = \frac{1}{2}$
Hence, the required solution is x = $\frac{1}{2}$ and y = $\frac{1}{3}$ or x= 0 and y = 0.

27. Solve for x and y:
$\frac{5}{x+y} - \frac{2}{x-y} = -1, \frac{15}{x+y} - \frac{7}{x-y} = 10$
Sol:
The given equations are
$\frac{5}{x+y} - \frac{2}{x-y} = -1$ ……(i)
$\frac{15}{x+y} - \frac{7}{x-y} = 10$ ……(ii)
Substituting $\frac{1}{x+y}$ = u and $\frac{1}{x-y}$ = v in (i) and (ii), we get
5u – 2v = -1 ……..(iii)
15u + 7v = 10 …….(iv)
Multiplying (iii) by 3 and subtracting it from (iv), we get
7v + 6v = 10 + 3
⇒13v = 13
⇒v = 1

$\Rightarrow$x – y = 1 $\left(\because \frac{1}{x-y} = v\right)$ …..(v)

Now, substituting v = 1 in (iii), we get

5u – 2 = -1

$\Rightarrow$5u = 1

$\Rightarrow$u = $\frac{1}{5}$

x + y = 5 …….(vi)

Adding (v) and (vi), we get

2x = 6 $\Rightarrow$ x = 3

Substituting x = 3 in (vi), we have

3 + y = 5 $\Rightarrow$ y = 5 – 3 = 2

Hence, x = 3 and y = 2.

28. Solve for x and y:

$\frac{3}{x+y} + \frac{2}{x-y}$ = 2, $\frac{3}{x+y} + \frac{2}{x-y}$ = 2

Sol:

The given equations are

$\frac{3}{x+y} + \frac{2}{x-y}$ = 2 ……(i)

$\frac{9}{x+y}$ - $\frac{4}{x-y}$ = 1 ……(ii)

Substituting $\frac{1}{x+y}$ = u and $\frac{1}{x-y}$ = v, we get:

3u + 2v = 2 ……..(iii)

9u - 4v = 1 …….(iv)

On multiplying (iii) by 2, we get:

6u + 4v = 4 …..(v)

On adding (iv) and (v), we get:

15u = 5

$\Rightarrow$u = $\frac{5}{15} = \frac{1}{3}$

$\Rightarrow$ $\frac{1}{x+y} = \frac{1}{3}$ $\Rightarrow$ x + y = 3 …….(vi)

On substituting u = $\frac{1}{3}$ in (iii), we get

1 + 2v = 2

$\Rightarrow$2v = 1

$\Rightarrow$v = $\frac{1}{2}$

$\Rightarrow$ $\frac{1}{x-y} = \frac{1}{2}$ $\Rightarrow$ x – y = 2 …….(vii)

On adding (vi) and (vii), we get

2x = 5

$\Rightarrow$ x = $\frac{5}{2}$

On substituting x = $\frac{5}{2}$ in (vi), we have

$\frac{5}{2}$ + y = 3

$\Rightarrow$ y = $\left(3 - \frac{5}{2}\right) = \frac{1}{2}$

Hence, the required solution is x = $\frac{5}{2}$ and y = $\frac{1}{2}$.

29. Solve for x and y:

$\frac{5}{x+1} + \frac{2}{y-1} = \frac{1}{2}$, $\frac{10}{x+1} - \frac{2}{y-1} = \frac{5}{2}$, where x ≠ 1, y ≠ 1.

Sol:

The given equations are

$\frac{5}{x+1} + \frac{2}{y-1} = \frac{1}{2}$ ……(i)

$\frac{10}{x+1} - \frac{2}{y-1} = \frac{5}{2}$ ……(ii)

Substituting $\frac{1}{x+1}$ = u and $\frac{1}{y-1}$ = v, we get:

5u - 2v = $\frac{1}{2}$ ……..(iii)

10u + 2v = $\frac{5}{2}$ …….(iv)

On adding (iii) and (iv), we get:

15u = 3

$\Rightarrow$u = $\frac{3}{15} = \frac{1}{5}$

$\Rightarrow$ $\frac{1}{x+1} = \frac{1}{5}$ $\Rightarrow$ x + 1 = 5 $\Rightarrow$ x = 4

On substituting u = $\frac{1}{5}$ in (iii), we get

5 $\times$ $\frac{1}{5}$ - 2v = $\frac{1}{2}$ $\Rightarrow$ 1 – 2v = $\frac{1}{2}$

$\Rightarrow$2v = $\frac{1}{2}$ $\Rightarrow$v = $\frac{1}{4}$

$\Rightarrow$ $\frac{1}{y-1} = \frac{1}{4}$ $\Rightarrow$ y – 1 = 4 $\Rightarrow$ y = 5

Hence, the required solution is x = 4 and y = 5.

30. Solve for x and y:

$\frac{44}{x+y} + \frac{30}{x-y}$ = 10, $\frac{55}{x+y} - \frac{40}{x-y}$ = 13.

Sol:

The given equations are

$\frac{44}{x+y} + \frac{30}{x-y}$ = 10 ……(i)

$\frac{55}{x+y} - \frac{40}{x-y}$ = 13 ……(ii)

Putting $\frac{1}{x+y}$ = u and $\frac{1}{x-y}$ = v, we get:
44u + 30v = 10 ……..(iii)
55u + 40v = 13 …….(iv)
On multiplying (iii) by 4 and (iv) by 3, we get:
176u + 120v = 40 …..(v)
165u + 120v = 39 …..(vi)
On subtracting (vi) and (v), we get:
11u = 1
$\Rightarrow$u = $\frac{1}{11}$
$\Rightarrow$ $\frac{1}{x+y}$ = $\frac{1}{11}$ $\Rightarrow$ x + y = 11 …….(vii)
On substituting u = $\frac{1}{11}$ in (iii), we get:
4 + 30v = 10
$\Rightarrow$30v = 6
$\Rightarrow$v = $\frac{6}{30}$ = $\frac{1}{5}$
$\Rightarrow$ $\frac{1}{x-y}$ = $\frac{1}{5}$ $\Rightarrow$ x – y = 5 …….(viii)
On adding (vii) and (viii), we get
2x = 16
$\Rightarrow$x = 8
On substituting x =8 in (vii), we get:
8 + y = 11
$\Rightarrow$ y = 11 – 8 = 3
Hence, the required solution is x = 8 and y =3.

31. Solve for x and y:
$\frac{10}{x+y}$ + $\frac{2}{x-y}$ = 4, $\frac{15}{x+y}$ - $\frac{9}{x-y}$ = -2, where x ≠ y, x ≠ -y.
Sol:
The given equations are
$\frac{10}{x+y}$ + $\frac{2}{x-y}$ = 4 ……(i)
$\frac{15}{x+y}$ - $\frac{9}{x-y}$ = -2 ……(ii)
Substituting $\frac{1}{x+y}$ = u and $\frac{1}{x-y}$ = v in (i) and (ii), we get:
10u + 2v = 4 ……..(iii)
15u - 9v = -2 …….(iv)
Multiplying (iii) by 9 and (iv) by 2 and adding, we get:
90u + 30u = 36 – 4

$\Rightarrow$120u = 32

$\Rightarrow$u = $\frac{32}{120} = \frac{4}{15}$

$\Rightarrow$x + y = $\frac{15}{4}$ $\left(\because \frac{1}{x+y} = u\right)$ …..(v)

On substituting u = $\frac{4}{15}$ in (iii), we get:

$10 \times \frac{4}{15} + 2v = 4$

$\frac{8}{3} + 2v = 4$

$\Rightarrow 2v = 4 - \frac{8}{3} = \frac{4}{3}$

$\Rightarrow v = \frac{2}{3}$

$\Rightarrow$ x – y = $\frac{3}{2}$ $\left(\because \frac{1}{x-y} = v\right)$ …..(vi)

Adding (v) and (vi), we get

$2x = \frac{15}{4} + \frac{3}{2} \Rightarrow 2x = \frac{21}{4} \Rightarrow x = \frac{21}{8}$

Substituting x = $\frac{21}{8}$ in (v), we have

$\frac{21}{8} + y = \frac{15}{4} \Rightarrow y = \frac{15}{4} - \frac{21}{8} = \frac{9}{8}$

Hence, x = $\frac{21}{8}$ and y =$\frac{9}{8}$.

32. Solve for x and y:

71x + 37y = 253, 37x + 71y = 287

Sol:

The given equations are:

71x + 37y = 253 …..(i)

37x + 71y = 287 ……(ii)

On adding (i) and (ii), we get:

108x + 108y = 540

$\Rightarrow$108(x + y) = 540

$\Rightarrow$(x + y) = 5 ……(iii)

On subtracting (ii) from (i), we get:

34x – 34y = -34

$\Rightarrow$34(x – y) = -34

$\Rightarrow$(x – y) = -1 ……(iv)

On adding (iii) from (i), we get:

2x = 5 – 1 = 4

$\Rightarrow$x = 2

On subtracting (iv) from (iii), we get:

$2y = 5 + 1 = 6$

$\Rightarrow y = 3$

Hence, the required solution is x = 2 and y = 3.

33. Solve for x and y:
$217x + 131y = 913$, $131x + 217y = 827$

Sol:

The given equations are:

$217x + 131y = 913$ …..(i)

$131x + 217y = 827$ ……(ii)

On adding (i) and (ii), we get:

$348x + 348y = 1740$

$\Rightarrow 348(x + y) = 1740$

$\Rightarrow x + y = 5$ ……(iii)

On subtracting (ii) from (i), we get:

$86x – 86y = 86$

$\Rightarrow 86(x – y) = 86$

$\Rightarrow x – y = 1$ ……(iv)

On adding (iii) from (i), we get:

$2x = 6$

$\Rightarrow x = 3$

On substituting x = 3 in (iii), we get:

$3 + y = 5$

$\Rightarrow y = 5 – 3 = 2$

Hence, the required solution is x = 3 and y = 2.

34. Solve for x and y:
$23x - 29y = 98$, $29x - 23y = 110$

Sol:

The given equations are:

$23x - 29y = 98$ …..(i)

$29x - 23y = 110$ ……(ii)

Adding (i) and (ii), we get:

$52x – 52y = 208$

$\Rightarrow x – y = 4$ ……(iii)

Subtracting (i) from (ii), we get:

$6x + 6y = 12$

$\Rightarrow x + y = 2$ ……(iv)

Now, adding equation (iii) and (iv), we get:

$2x = 6$

$\Rightarrow x = 3$

On substituting x = 3 in (iv), we have:

$3 + y = 2$

$\Rightarrow y = 2 – 3 = -1$

Hence, x = 3 and y = -1.

35. Solve for x and y:

$\frac{5}{x} + \frac{2}{y} = 6, \frac{-5}{x} + \frac{4}{y} = -3$

Sol:

The given equations can be written as

$\frac{5}{x} + \frac{2}{y} = 6$ …….(i)

$\frac{-5}{x} + \frac{4}{y} = -3$ ……(ii)

Adding (i) and (ii), we get

$\frac{6}{y} = 3 \Rightarrow y = 2$

Substituting y = 2 in (i), we have

$\frac{5}{x} + \frac{2}{2} = 6 \Rightarrow x = 1$

Hence, x = 1 and y = 2.

36. Solve for x and y:

$\frac{1}{3x+y} + \frac{1}{3x-y} = \frac{3}{4}, \frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = -\frac{1}{8}$

Sol:

The given equations are

$\frac{1}{3x+y} + \frac{1}{3x-y} = \frac{3}{4}$ ……(i)

$\frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = -\frac{1}{8}$

$\frac{1}{3x+y} - \frac{1}{3x-y} = -\frac{1}{4}$ (Multiplying by 2) ……(ii)

Substituting $\frac{1}{3x+y} = u$ and $\frac{1}{3x-y} = v$ in (i) and (ii), we get:

$u + v = \frac{3}{4}$ ……..(iii)

$u – v = -\frac{1}{4}$ …….(iv)

Adding (iii) and (iv), we get:

$2u = \frac{1}{2}$

$\Rightarrow$ u = $\frac{1}{4}$

$\Rightarrow$3x + y = 4 $\left(\because \frac{1}{3x+y} = u\right)$ …..(v)

Now, substituting u = $\frac{1}{4}$ in (iii), we get:

$\frac{1}{4}$ + v = $\frac{3}{4}$

v = $\frac{3}{4}$ - $\frac{1}{4}$

$\Rightarrow$v = $\frac{1}{2}$

$\Rightarrow$ 3x – y = 2 $\left(\because \frac{1}{3x-y} = v\right)$ …..(vi)

Adding (v) and (vi), we get

6x = 6 $\Rightarrow$x = 1

Substituting x = 1 in (v), we have

3 + y = 4 $\Rightarrow$y = 1

Hence, x = 1 and y = 1.

37. Solve for x and y:

$\frac{1}{2(x+2y)} + \frac{5}{3(3x-2y)} = -\frac{3}{2}, \frac{1}{4(x+2y)} - \frac{3}{5(3x-2y)} = \frac{61}{60}$ where x + 2y ≠ 0 and 3x – 2y ≠ 0.

Sol:

The given equations are

$\frac{1}{2(x+2y)} + \frac{5}{3(3x-2y)} = -\frac{3}{2}$ ……(i)

$\frac{1}{4(x+2y)} - \frac{3}{5(3x-2y)} = \frac{61}{60}$ ……(ii)

Putting $\frac{1}{x+2y}$ = u and $\frac{1}{3x-2y}$ = v, we get:

$\frac{1}{2}$u + $\frac{5}{3}$v = - $\frac{3}{2}$ ……..(iii)

$\frac{5}{4}$u – $\frac{3}{5}$v = $\frac{61}{60}$ …….(iv)

On multiplying (iii) by 6 and (iv) by 20, we get:

3u + 10v = -9 …..(v)

25u – 12v = $\frac{61}{3}$ …..(vi)

On multiplying (v) by 6 and (vi) by 5, we get

18u + 60v = -54 …….(vii)

125u – 60v = $\frac{305}{3}$ ……..(viii)

On adding(vii) and (viii), we get:

143u = $\frac{305}{3}$ – 54 = $\frac{305-162}{3}$ = $\frac{143}{3}$

$\Rightarrow$u = $\frac{1}{3}$ = $\frac{1}{x+2y}$

$\Rightarrow x + 2y = 3$ …….(ix)

On substituting $u = \frac{1}{3}$ in (v), we get:

$1 + 10v = -9$

$\Rightarrow 10v = -10$

$\Rightarrow v = -1$

$\Rightarrow \frac{1}{3x-2y} = -1 \Rightarrow 3x - 2y = -1$ …….(x)

On adding (ix) and (x), we get:

$4x = 2$

$\Rightarrow x = \frac{1}{2}$

On substituting $x = \frac{1}{2}$ in (x), we get:

$\frac{3}{2} - 2y = -1$

$2y = \left(\frac{3}{2} + 1\right) = \frac{5}{2}$

$y = \frac{5}{4}$

Hence, the required solution is $x = \frac{1}{2}$ and $y = \frac{5}{4}$.

38. Solve for x and y:

$\frac{2}{3x+2y} + \frac{3}{3x-2y} = \frac{17}{5}, \frac{5}{3x+2y} + \frac{1}{3x-2y} = 2$

Sol:

The given equations are

$\frac{2}{3x+2y} + \frac{3}{3x-2y} = \frac{17}{5}$ ……(i)

$\frac{5}{3x+2y} + \frac{1}{3x-2y} = 2$ ……(ii)

Substituting $\frac{1}{3x+2y} = u$ and $\frac{1}{3x-2y} = v$, in (i) and (ii), we get:

$2u + 3v = \frac{17}{5}$ ……..(iii)

$5u + v = 2$ …….(iv)

Multiplying (iv) by 3 and subtracting from (iii), we get:

$2u - 15u = \frac{17}{5} - 6$

$\Rightarrow -13u = \frac{-13}{5} \Rightarrow u = \frac{1}{5}$

$\Rightarrow 3x + 2y = 5 \quad \left(\because \frac{1}{3x+2y} = u\right)$ …..(v)

Now, substituting $u = \frac{1}{5}$ in (iv), we get

$1 + v = 2 \Rightarrow v = 1$

$\Rightarrow 3x - 2y = 1 \quad \left(\because \frac{1}{3x-2y} = v\right)$ …….(vi)

Adding(v) and (vi), we get:

$\Rightarrow 6x = 6 \Rightarrow x = 1$

Substituting x = 1 in (v), we get:

$3 + 2y = 5 \Rightarrow y = 1$

Hence, x = 1 and y =1.

39. Solve for x and y:

$\frac{3}{x} + \frac{6}{y} = 7, \frac{9}{x} + \frac{3}{y} = 11$

Sol:

The given equations can be written as

$\frac{3}{x} + \frac{6}{y} = 7$ …….(i)

$\frac{9}{x} + \frac{3}{y} = 11$ ……(ii)

Multiplying (i) by 3 and subtracting (ii) from it, we get

$\frac{18}{y} - \frac{3}{y} = 21 - 11$

$\Rightarrow \frac{15}{y} = 10$

$\Rightarrow y = \frac{15}{10} = \frac{3}{2}$

Substituting $y = \frac{3}{2}$ in (i), we have

$\frac{3}{x} + \frac{6\times2}{3} = 7$

$\Rightarrow \frac{3}{x} = 7 - 4 = 3$

Hence, x = 1 and $y = \frac{3}{2}$.

40. Solve for x and y:

$x + y = a + b, ax - by = a^2 - b^2$

Sol:

The given equations are

$x + y = a + b$ …….(i)

$ax - by = a^2 - b^2$ ……..(ii)

Multiplying (i) by b and adding it with (ii), we get

$bx + ax = ab + b^2 + a^2 - b^2$

$\Rightarrow x = \frac{ab + a^2}{a+b} = a$

Substituting x = a in (i), we have

$a + y = a + b$

$\Rightarrow y = b$

Hence, x = a and y = b.

41. Solve for x and y:

$\frac{x}{a} + \frac{y}{b} = 2$, $ax - by = (a^2 - b^2)$

Sol:

The given equations are:

$\frac{x}{a} + \frac{y}{b} = 2$

$\Rightarrow \frac{bx+ay}{ab} = 2$ [Taking LCM]

$\Rightarrow bx + ay = 2ab$ …….(i)

Again, $ax - by = (a^2 - b^2)$ …..(ii)

On multiplying (i) by b and (ii) by a, we get:

$b^2x + bay = 2ab^2$ ……..(iii)

$a^2x - bay = a(a^2 - b^2)$ …….(iv)

On adding (iii) from (iv), we get:

$(b^2 + a^2)x = 2a^2b + a(a^2 - b^2)$

$\Rightarrow (b^2 + a^2)x = 2ab^2 + a^3 - ab^2$

$\Rightarrow (b^2 + a^2)x = ab^2 + a^3$

$\Rightarrow (b^2 + a^2)x = a(b^2 + a^2)$

$\Rightarrow x = \frac{a(b^2 + a^2)}{(b^2 + a^2)} = a$

On substituting x = a in (i), we get:

$ba + ay = 2ab$

$\Rightarrow ay = ab$

$\Rightarrow y = b$

Hence, the solution is x = a and y = b.

42. Solve for x and y:

$px + qy = p - q$,

$qx - py = p + q$

Sol:

The given equations are

$px + qy = p - q$ ………(i)

$qx - py = p + q$ ………(ii)

Multiplying (i) by p and (ii) by q and adding them , we get

$p^2x + q^2x = p^2 - pq + pq + q^2$

$x = \frac{p^2 + q^2}{p^2 + q^2} = 1$

Substituting x = 1 in (i), we have

$p + qy = p - q$

$\Rightarrow qy = -p$

$\Rightarrow$y = -1
Hence, x = 1 and y = -1.

43. Solve for x and y:
$\frac{x}{a} - \frac{y}{b} = 0$, $ax + by = a^2 + b^2$
Sol:
The given equations can be written as
$\frac{x}{a} - \frac{y}{b} = 0$ …….(i)
$ax + by = a^2 + b^2$ ……(ii)
From (i),
$y = \frac{bx}{a}$
Substituting $y = \frac{bx}{a}$ in (ii), we get
$ax + \frac{b \times bx}{a} = a^2 + b^2$
$\Rightarrow x = \frac{(a^2+b^2) \times a}{a^2+b^2} = a$
Now, substitute x = a in (ii) to get
$a^2 + by = a^2 + b^2$
$\Rightarrow by = b^2$
$\Rightarrow$y = b
Hence, x = a and y = b.

44. Solve for x and y:
6(ax + by) = 3a + 2b,
6(bx – ay) = 3b – 2a
Sol:
The given equations are
6(ax + by) = 3a + 2b
$\Rightarrow$6ax + 6by = 3a + 2b ………(i)
and 6(bx – ay) = 3b – 2a
$\Rightarrow$6bx – 6ay = 3b – 2a ………(ii)
On multiplying (i) by a and (ii) by b, we get
$6a^2x + 6aby = 3a^2 + 2ab$ ……….(iii)
$6b^2x - 6aby = 3b^2 - 2ab$ ……….(iv)
On adding (iii) and (iv), we get
$6(a^2 + b^2)x = 3(a^2 + b^2)$
$x = \frac{3(a^2+b^2)}{6(a^2+b^2)} = \frac{1}{2}$

On substituting x = $\frac{1}{2}$ in (i), we get:

$6a \times \frac{1}{2} + 6by = 3a + 2b$

$6by = 2b$

$y = \frac{2b}{6b} = \frac{1}{3}$

Hence, the required solution is $x = \frac{1}{2}$ and $y = \frac{1}{3}$.

45. Solve for x and y:

$ax - by = a^2 + b^2$, $x + y = 2a$

Sol:

The given equations are

$ax - by = a^2 + b^2$ ………(i)

$x + y = 2a$ ………(ii)

From (ii)

$y = 2a - x$

Substituting $y = 2a - x$ in (i), we get

$ax - b(2a - x) = a^2 + b^2$

$\Rightarrow ax - 2ab + bx = a^2 + b^2$

$\Rightarrow x = \frac{a^2 + b^2 + 2ab}{a+b} = \frac{(a+b)^2}{a+b} = a + b$

Now, substitute $x = a + b$ in (ii) to get

$a + b + y = 2a$

$\Rightarrow y = a - b$

Hence, $x = a + b$ and $y = a - b$.

46. Solve for x and y:

$\frac{bx}{a} - \frac{ay}{b} + a + b = 0$, $bx - ay + 2ab = 0$

Sol:

The given equations are:

$\frac{bx}{a} - \frac{ay}{b} + a + b = 0$

By taking LCM, we get:

$b^2x - a^2y = -a^2b - b^2a$ …..(i)

and $bx - ay + 2ab = 0$

$bx - ay = -2ab$ …….(ii)

On multiplying (ii) by a, we get:

$abx - a^2y = -2a^2b$ ……(iii)

On subtracting (i) from (iii), we get:

$abx - b^2x = 2a^2b + a^2b + b^2a = -a^2b + b^2a$

$\Rightarrow x(ab - b^2) = -ab(a - b)$

$\Rightarrow x(a - b)b = -ab(a - b)$

$\therefore x = \frac{-ab(a-b)}{(a-b)b} = -a$

On substituting x = -a in (i), we get:

$b^2(-a) - a^2y = -a^2b - b^2a$

$\Rightarrow -b^2a - a^2y = -a^2b - b^2a$

$\Rightarrow -a^2y = -a^2b$

$\Rightarrow y = b$

Hence, the solution is x = -a and y = b.

47. Solve for x and y:

$\frac{bx}{a} + \frac{ay}{b} = a^2 + b^2$, $x + y = 2ab$

Sol:

The given equations are:

$\frac{bx}{a} + \frac{ay}{b} = a^2 + b^2$

By taking LCM, we get:

$\frac{b^2x + a^2y}{ab} = a^2 + b^2$

$\Rightarrow b^2x + a^2y = (ab)a^2 + b^2$

$\Rightarrow b^2x + a^2y = a^3b + ab^3$ …..(i)

Also, x + y = 2ab …….(ii)

On multiplying (ii) by a^2, we get:

$a^2x + a^2y = 2a^3b$ ……(iii)

On subtracting (iii) from (i), we get:

$(b^2 - a^2)x = a^3b + ab^3 - 2a^3b$

$\Rightarrow (b^2 - a^2)x = -a^3b + ab^3$

$\Rightarrow (b^2 - a^2)x = ab(b^2 - a^2)$

$\Rightarrow (b^2 - a^2)x = ab(b^2 - a^2)$

$\therefore x = \frac{ab(b^2 - a^2)}{(b^2 - a^2)} = ab$

On substituting x = ab in (i), we get:

$b^2(ab) + a^2y = a^3b + ab^3$

$\Rightarrow a^2y = a^3b$

$\Rightarrow \frac{a^3b}{a^2} = ab$

Hence, the solution is x = ab and y = ab.

48. Solve for x and y:
$x + y = a + b$, $ax - by = a^2 - b^2$
Sol:
The given equations are
$x + y = a + b$ ....(i)
$ax - by = a^2 - b^2$ ....(ii)
From (i)
$y = a + b - x$
Substituting $y = a + b - x$ in (ii), we get
$ax – b(a + b - x) = a^2 - b^2$
$\Rightarrow ax – ab - b^2 + bx = a^2 - b^2$
$\Rightarrow x = \frac{a^2 + ab}{a+b} = a$
Now, substitute x = a in (i) to get
$a + y = a + b$
$\Rightarrow y = b$
Hence, x = a and y = b.

49. Solve for x and y:
$a^2x + b^2y = c^2$, $b^2x + a^2y = d^2$
Sol:
The given equations are
$a^2x + b^2y = c^2$ ....(i)
$b^2x + a^2y = d^2$ ....(ii)
Multiplying (i) by a^2 and (ii) by b^2 and subtracting, we get
$a^4x – b^4x = a^2c^2 - b^2 d^2$
$\Rightarrow x = \frac{a^2c^2 - b^2d^2}{a^4 - b^4}$
Now, multiplying (i) by b^2 and (ii) by a^2 and subtracting, we get
$b^4y – a^4y = b^2c^2 - a^2 d^2$
$\Rightarrow y = \frac{b^2c^2 - a^2d^2}{b^4 - a^4}$
Hence, $x = \frac{a^2c^2 - b^2d^2}{a^4 - b^4}$ and $y = \frac{b^2c^2 - a^2d^2}{b^4 - a^4}$.

50. Solve for x and y:
$\frac{x}{a} + \frac{y}{b} = a + b$, $\frac{x}{a^2} + \frac{y}{b^2} = 2$
Sol:
The given equations are
$\frac{x}{a} + \frac{y}{b} = a + b$ ....(i)
$\frac{x}{a^2} + \frac{y}{b^2} = 2$ ......(ii)

Multiplying (i) by b and (ii) by b^2 and subtracting, we get

$\frac{bx}{a} - \frac{b^2x}{a^2} = ab + b^2 - 2b^2$

$\Rightarrow \frac{ab - b^2}{a^2} x = ab - b^2$

$\Rightarrow x = \frac{(ab - b^2)a^2}{ab - b^2} = a^2$

Now, substituting $x = a^2$ in (i) we get

$\frac{a^2}{a} + \frac{y}{b} = a + b$

$\Rightarrow \frac{y}{b} = a + b – a = b$

$\Rightarrow y = b^2$

Hence, $x = a^2$ and $y = b^2$.

Exercise – 3C

1. Solve the system of equations by using the method of cross multiplication:

x + 2y + 1 = 0,

2x – 3y – 12 = 0.

Sol:

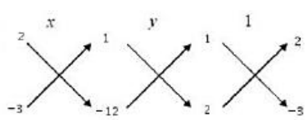
The given equations are:

x + 2y + 1 = 0 …….(i)

2x – 3y – 12 = 0 …….(ii)

Here $a_1 = 1$, $b_1 = 2$, $c_1 = 1$, $a_2 = 2$, $b_2 = -3$ and $c_2 = -12$

By cross multiplication, we have:



$\therefore \frac{x}{[2 \times (-12) - 1 \times (-3)]} = \frac{y}{[1 \times 2 - 1 \times (-12)]} = \frac{1}{[1 \times (-3) - 2 \times 2]}$

$\Rightarrow \frac{x}{(-24+3)} = \frac{y}{(2+12)} = \frac{1}{(-3-4)}$

$\Rightarrow \frac{x}{(-21)} = \frac{y}{(14)} = \frac{1}{(-7)}$

$\Rightarrow x = \frac{-21}{-7} = 3$, $y = \frac{14}{-7} = -2$

Hence, x = 3 and y = -2 is the required solution.

2. Solve the system of equations by using the method of cross multiplication:

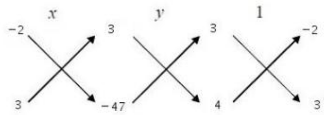
3x - 2y + 3 = 0,

4x + 3y – 47 = 0

Sol:

The given equations are:

3x - 2y + 3 = 0 …….(i)
4x + 3y – 47 = 0 …….(ii)
Here $a_1 = 3$, $b_1 = -2$, $c_1 = 3$, $a_2 = 4$, $b_2 = 3$ and $c_2 = -47$
By cross multiplication, we have:



$\therefore \frac{x}{[(-2)\times(-47)-3\times 3]} = \frac{y}{[3\times 4-(-47)\times 3]} = \frac{1}{[3\times 3-(-2)\times 4]}$

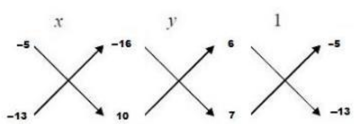
$\Rightarrow \frac{x}{(94-9)} = \frac{y}{(12+141)} = \frac{1}{(9+8)}$

$\Rightarrow \frac{x}{(85)} = \frac{y}{(153)} = \frac{1}{(17)}$

$\Rightarrow x = \frac{85}{17} = 5$, $y = \frac{153}{17} = 9$

Hence, x = 5 and y = 9 is the required solution.

3. Solve the system of equations by using the method of cross multiplication:
6x - 5y - 16 = 0,
7x - 13y + 10 = 0
Sol:
The given equations are:
6x - 5y - 16 = 0 …….(i)
7x - 13y + 10 = 0 …….(ii)
Here $a_1 = 6$, $b_1 = -5$, $c_1 = -16$, $a_2 = 7$, $b_2 = -13$ and $c_2 = 10$
By cross multiplication, we have:



$\therefore \frac{x}{[(-5)\times 10-(-16)\times(-13)]} = \frac{y}{[(-16)\times 7-10\times 6]} = \frac{1}{[6\times(-13)-(-5)\times 7]}$

$\Rightarrow \frac{x}{(-50-208)} = \frac{y}{(-112-60)} = \frac{1}{(-78+35)}$

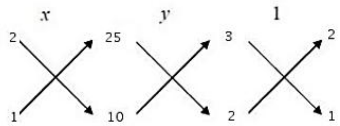
$\Rightarrow \frac{x}{(-258)} = \frac{y}{(-172)} = \frac{1}{(43)}$

$\Rightarrow x = \frac{-258}{-43} = 6$, $y = \frac{-172}{-43} = 4$

Hence, x = 6 and y = 4 is the required solution.

4. Solve the system of equations by using the method of cross multiplication:
3x + 2y + 25 = 0, 2x + y + 10 = 0
Sol:
The given equations are:
3x + 2y + 25 = 0 …….(i)

$2x + y + 10 = 0$ ....(ii)
Here $a_1 = 3$, $b_1 = 2$, $c_1 = 25$, $a_2 = 2$, $b_2 = 1$ and $c_2 = 10$
By cross multiplication, we have:



$\therefore \frac{x}{[2\times10-25\times1]} = \frac{y}{[25\times2-10\times3]} = \frac{1}{[3\times1-2\times2]}$

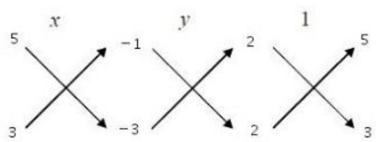
$\Rightarrow \frac{x}{(20-25)} = \frac{y}{(50-30)} = \frac{1}{(3-4)}$

$\Rightarrow \frac{x}{(-5)} = \frac{y}{20} = \frac{1}{(-1)}$

$\Rightarrow x = \frac{-5}{-1} = 5$, $y = \frac{20}{(-1)} = -20$

Hence, $x = 5$ and $y = -20$ is the required solution.

5. Solve the system of equations by using the method of cross multiplication:
$2x + 5y - 1 = 0$, $2x + 3y - 3 = 0$
Sol:
The given equations may be written as:
$2x + 5y - 1 = 0$ ....(i)
$2x + 3y - 3 = 0$ ....(ii)
Here $a_1 = 2$, $b_1 = 5$, $c_1 = -1$, $a_2 = 2$, $b_2 = 3$ and $c_2 = -3$
By cross multiplication, we have:



$\therefore \frac{x}{[5\times(-3)-3\times(-1)]} = \frac{y}{[(-1)\times2-(-3)\times2]} = \frac{1}{[2\times3-2\times5]}$

$\Rightarrow \frac{x}{(-15+3)} = \frac{y}{(-2+6)} = \frac{1}{(6-10)}$

$\Rightarrow \frac{x}{-12} = \frac{y}{4} = \frac{1}{-4}$

$\Rightarrow x = \frac{-12}{-4} = 3$, $y = \frac{4}{-4} = -1$

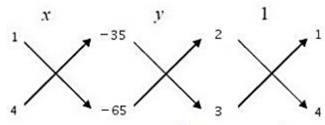
Hence, $x = 3$ and $y = -1$ is the required solution.

6. Solve the system of equations by using the method of cross multiplication:
$2x + y - 35 = 0$,
$3x + 4y - 65 = 0$
Sol:
The given equations may be written as:
$2x + y - 35 = 0$ ....(i)

$3x + 4y – 65 = 0$ …….(ii)

Here $a_1 = 2$, $b_1 = 1$, $c_1 = -35$, $a_2 = 3$, $b_2 = 4$ and $c_2 = -65$

By cross multiplication, we have:



$\therefore \frac{x}{[1\times(-65) - 4\times(-35)]} = \frac{y}{[(-35)\times 3 - (-65)\times 2]} = \frac{1}{[2\times 4 - 3\times 1]}$

$\Rightarrow \frac{x}{(-65+140)} = \frac{y}{(-105+130)} = \frac{1}{(8-3)}$

$\Rightarrow \frac{x}{75} = \frac{y}{25} = \frac{1}{5}$

$\Rightarrow x = \frac{75}{5} = 15, \; y = \frac{25}{5} = 5$

Hence, x = 15 and y = 5 is the required solution.

7. Solve the system of equations by using the method of cross multiplication:

$7x - 2y – 3 = 0,$

$11x - \frac{3}{2}y – 8 = 0.$

Sol:

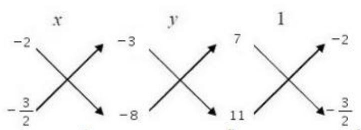
The given equations may be written as:

$7x - 2y – 3 = 0$ …….(i)

$11x - \frac{3}{2}y – 8 = 0$ …….(ii)

Here $a_1 = 7$, $b_1 = -2$, $c_1 = -3$, $a_2 = 11$, $b_2 = -\frac{3}{2}$ and $c_2 = -8$

By cross multiplication, we have:



$\therefore \frac{x}{\left[(-2)\times(-8) - \left(-\frac{3}{2}\right)\times(-3)\right]} = \frac{y}{[(-3)\times 11 - (-8)\times 7]} = \frac{1}{\left[7\times\left(-\frac{3}{2}\right) - 11\times(-2)\right]}$

$\Rightarrow \frac{x}{(16 - \frac{9}{2})} = \frac{y}{(-33+56)} = \frac{1}{(-\frac{21}{2}+22)}$

$\Rightarrow \frac{x}{\left(\frac{23}{2}\right)} = \frac{y}{23} = \frac{1}{\left(\frac{23}{2}\right)}$

$\Rightarrow x = \frac{\frac{23}{2}}{\frac{23}{2}} = 1, \; y = \frac{23}{\frac{23}{2}} = 2$

Hence, x = 1 and y = 2 is the required solution.

8. Solve the system of equations by using the method of cross multiplication:

$\frac{x}{6} + \frac{y}{15} – 4 = 0, \frac{x}{3} - \frac{y}{12} – \frac{19}{4} = 0$

Sol:

The given equations may be written as:

$\frac{x}{6} + \frac{y}{15} - 4 = 0$ …….(i)

$\frac{x}{3} - \frac{y}{12} - \frac{19}{4} = 0$ …….(ii)

Here $a_1 = \frac{1}{6}$, $b_1 = \frac{1}{15}$, $c_1 = -4$, $a_2 = \frac{1}{3}$, $b_2 = -\frac{1}{12}$ and $c_2 = -\frac{19}{4}$

By cross multiplication, we have:

$\therefore \frac{x}{\left[\frac{1}{15} \times \left(-\frac{19}{4}\right) - \left(-\frac{1}{12}\right) \times (-4)\right]} = \frac{y}{\left[(-4) \times \frac{1}{3} - \left(\frac{1}{6}\right) \times \left(-\frac{19}{4}\right)\right]} = \frac{1}{\left[\frac{1}{6} \times \left(-\frac{1}{12}\right) \times \frac{1}{3} \times \frac{1}{15}\right]}$

$\Rightarrow \frac{x}{\left(-\frac{19}{60} - \frac{1}{3}\right)} = \frac{y}{\left(-\frac{4}{3} + \frac{19}{34}\right)} = \frac{1}{\left(-\frac{1}{72} - \frac{1}{45}\right)}$

$\Rightarrow \frac{x}{\left(-\frac{39}{60}\right)} = \frac{y}{\left(-\frac{13}{24}\right)} = \frac{1}{\left(-\frac{13}{360}\right)}$

$\Rightarrow x = \left[\left(-\frac{39}{60}\right) \times \left(-\frac{360}{13}\right)\right] = 18$, $y = \left[\left(-\frac{13}{24}\right) \times \left(-\frac{360}{13}\right)\right] = 15$

Hence, x = 18 and y = 15 is the required solution.

9. Solve the system of equations by using the method of cross multiplication:

$\frac{1}{x} + \frac{1}{y} = 7$, $\frac{2}{x} + \frac{3}{y} = 17$

Sol:

Taking $\frac{1}{x} = u$ and $\frac{1}{y} = v$, the given equations become:

u + v = 7

2u + 3v = 17

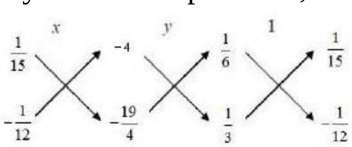
The given equations may be written as:

u + v – 7 = 0 …..(i)

2u + 3v – 17 = 0 ……(ii)

Here, $a_1 = 1$, $b_1 = 1$, $c_1 = -7$, $a_2 = 2$, $b_2 = 3$ and $c_2 = -17$

By cross multiplication, we have:



$\therefore \frac{u}{[1 \times (-17) - 3 \times (-7)]} = \frac{v}{[(-7) \times 2 - 1 \times (-17)]} = \frac{1}{[3-2]}$

$\Rightarrow \frac{u}{(-17+21)} = \frac{v}{(-14+17)} = \frac{1}{(1)}$

$\Rightarrow \frac{u}{4} = \frac{v}{3} = \frac{1}{1}$

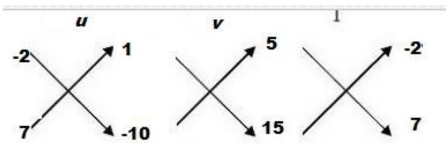
$\Rightarrow u = \frac{4}{1} = 4$, $v = \frac{3}{1} = 3$

$\Rightarrow \frac{1}{x} = 4$, $\frac{1}{y} = 3$

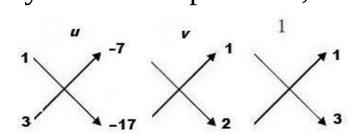
$\Rightarrow x = \frac{1}{4}$, $y = \frac{1}{3}$

Hence, x = $\frac{1}{4}$ and y = $\frac{1}{3}$ is the required solution.

10. Solve the system of equations by using the method of cross multiplication:
$\frac{5}{x+y} - \frac{2}{x-y} + 1 = 0$, $\frac{15}{x+y} + \frac{7}{x-y} - 10 = 0$
Sol:
Taking $\frac{1}{x+y}$ = u and $\frac{1}{x-y}$ = v, the given equations become:
5u - 2v + 1 = 0 …..(i)
15u + 7v – 10 = 0 ……(ii)
Here, a_1 = 5, b_1 = -2, c_1 = 1, a_2 = 15, b_2 = -7 and c_2 = -10
By cross multiplication, we have:



$\therefore \frac{u}{[-2\times(-10) - 1\times7]} = \frac{v}{[1\times15 - (-10)\times5]} = \frac{1}{[35+30]}$
$\Rightarrow \frac{u}{(20-7)} = \frac{v}{(15+50)} = \frac{1}{65}$
$\Rightarrow \frac{u}{13} = \frac{v}{65} = \frac{1}{65}$
$\Rightarrow$u = $\frac{13}{65} = \frac{1}{5}$, v = $\frac{65}{65}$ = 1
$\Rightarrow \frac{1}{x+y} = \frac{1}{5}, \frac{1}{x-y} = 1$
So, (x + y) = 5 …….(iii)
and (x – y) = 1 ……(iv)
Again, the above equations (ii) and (iv) may be written as:
x + y – 5 = 0 …..(i)
x – y – 1 = 0 ……(ii)
Here, a_1 = 1, b_1 = 1, c_1 = -5, a_2 = 1, b_2 = -1 and c_2 = -1
By cross multiplication, we have:



$\therefore \frac{x}{[1\times(-1) - (-5)\times(-1)]} = \frac{y}{[(-5)\times1 - (-1)\times1]} = \frac{1}{[1\times(-1) - 1\times1]}$
$\Rightarrow \frac{x}{(-1-5)} = \frac{y}{(-5+1)} = \frac{1}{(-1-1)}$
$\Rightarrow \frac{x}{-6} = \frac{v}{-4} = \frac{1}{-2}$
$\Rightarrow$x = $\frac{-6}{-2}$ = 3, y = $\frac{-4}{-2}$ = 2
Hence, x = 3 and y = 2 is the required solution.

11. Solve the system of equations by using the method of cross multiplication:

$\frac{ax}{b} - \frac{by}{a} - (a + b) = 0$, $ax - by - 2ab = 0$

Sol:

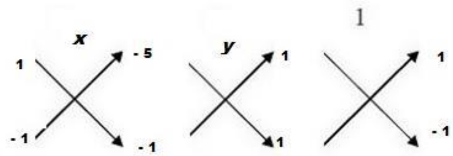
The given equations may be written as:

$\frac{ax}{b} - \frac{by}{a} - (a + b) = 0$ ……(i)

$ax - by - 2ab = 0$ ……(ii)

Here, $a_1 = \frac{a}{b}$, $b_1 = \frac{-b}{a}$, $c_1 = -(a + b)$, $a_2 = a$, $b_2 = -b$ and $c_2 = -2ab$

By cross multiplication, we have:



$\therefore \frac{x}{\left[\left(-\frac{b}{a}\right)\times(-2ab) - (-b)\times(-(a+b))\right]} = \frac{y}{\left[-(a+b)\times a - (-2ab)\times\frac{a}{b}\right]} = \frac{1}{\left[\frac{a}{b}\times(-b) - a\times\left(-\frac{b}{a}\right)\right]}$

$\Rightarrow \frac{x}{(2b^2 - b(a+b))} = \frac{y}{-a(a+b)+2a^2} = \frac{1}{-a+b}$

$\Rightarrow \frac{x}{2b^2 - ab - b^2} = \frac{y}{-a^2 - ab + 2a^2} = \frac{1}{-a+b}$

$\Rightarrow \frac{x}{b^2 - ab} = \frac{y}{a^2 - ab} = \frac{1}{-(a-b)}$

$\Rightarrow \frac{x}{-b(a-b)} = \frac{y}{a(a-b)} = \frac{1}{-(a-b)}$

$\Rightarrow x = \frac{-b(a-b)}{-(a-b)} = b$, $y = \frac{a(a-b)}{-(a-b)} = -a$

Hence, $x = b$ and $y = -a$ is the required solution.

12. Solve the system of equations by using the method of cross multiplication:

$2ax + 3by - (a + 2b) = 0$,

$3ax + 2by - (2a + b) = 0$

Sol:

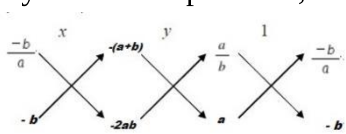
The given equations may be written as:

$2ax + 3by - (a + 2b) = 0$ ……(i)

$3ax + 2by - (2a + b) = 0$ ……(ii)

Here, $a_1 = 2a$, $b_1 = 3b$, $c_1 = -(a + 2b)$, $a_2 = 3a$, $b_2 = 2b$ and $c_2 = -(2a + b)$

By cross multiplication, we have:



$\therefore \frac{x}{[3b\times(-(2a+b)) - 2b\times(-(a+2b))]} = \frac{y}{[-(a+2b)\times 3a - 2a\times(-(2a+b))]} = \frac{1}{[2a\times 2b - 3a\times 3b]}$

$\Rightarrow \frac{x}{(-6ab - 3b^2 + 2ab + 4b^2)} = \frac{y}{(-3a^2 - 6ab + 4a^2 + 2ab)} = \frac{1}{4ab - 9ab}$

$\Rightarrow \frac{x}{b^2-4ab} = \frac{y}{a^2-4ab} = \frac{1}{-5ab}$

$\Rightarrow \frac{x}{-b(4a-b)} = \frac{y}{-a(4b-a)} = \frac{1}{-5ab}$

$\Rightarrow x = \frac{-b(4a-b)}{-5ab} = \frac{(4a-b)}{5a}$, $y = \frac{-a(4b-a)}{-5ab} = \frac{(4b-a)}{5b}$

Hence, $x = \frac{(4a-b)}{5a}$ and $y = \frac{(4b-a)}{5b}$ is the required solution.

13. Solve the system of equations by using the method of cross multiplication:

$\frac{a}{x} - \frac{b}{y} = 0$, $\frac{ab^2}{x} + \frac{a^2b}{y} = (a^2 + b^2)$, where $x \neq 0$ and $y \neq 0$.

Sol:

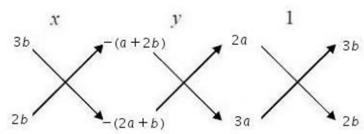
Substituting $\frac{1}{x} = u$ and $\frac{1}{y} = v$ in the given equations, we get

$au - bv + 0 = 0$ …….(i)

$ab^2u + a^2bv - (a^2 + b^2) = 0$ ……(ii)

Here, $a_1 = a$, $b_1 = -b$, $c_1 = 0$, $a_2 = ab^2$, $b_2 = a^2b$ and $c_2 = -(a^2 + b^2)$.

So, by cross-multiplication, we have



$$\frac{u}{b_1c_2 - b_2c_1} = \frac{v}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\Rightarrow \frac{u}{(-b)[-(a^2+b^2)]-(a^2b)(0)} = \frac{v}{(0)(ab^2)-(-a^2-b^2)(a)} = \frac{1}{(a)(a^2b)-(ab^2)(-b)}$$

$$\Rightarrow \frac{u}{b(a^2+b^2)} = \frac{v}{a(a^2+b^2)} = \frac{1}{ab(a^2+b^2)}$$

$$\Rightarrow u = \frac{b(a^2+b^2)}{ab(a^2+b^2)}, \ v = \frac{a(a^2+b^2)}{ab(a^2+b^2)}$$

$$\Rightarrow u = \frac{1}{a}, \ v = \frac{1}{b}$$

$$\Rightarrow \frac{1}{x} = \frac{1}{a}, \ \frac{1}{y} = \frac{1}{b}$$

$\Rightarrow x = a, y = b$

Hence, $x = a$ and $y = b$.

Exercise – 3D

1. Show that the following system of equations has a unique solution:

$3x + 5y = 12$,

$5x + 3y = 4$.

Also, find the solution of the given system of equations.

Sol:

The given system of equations is:

3x + 5y = 12
5x + 3y = 4
These equations are of the forms:
$a_1x+b_1y+c_1 = 0$ and $a_2x+b_2y+c_2 = 0$
where, $a_1 = 3$, $b_1= 5$, $c_1 = -12$ and $a_2 = 5$, $b_2 = 3$, $c_2 = -4$
For a unique solution, we must have:
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, i.e., $\frac{3}{5} \neq \frac{5}{3}$
Hence, the given system of equations has a unique solution.
Again, the given equations are:
3x + 5y = 12 …..(i)
5x + 3y = 4 …..(ii)
On multiplying (i) by 3 and (ii) by 5, we get:
9x + 15y = 36 …….(iii)
25x + 15y = 20 ……(iv)
On subtracting (iii) from (iv), we get:
16x = -16
$\Rightarrow$x = -1
On substituting x = -1 in (i), we get:
3(-1) + 5y = 12
$\Rightarrow$5y = (12 + 3) = 15
$\Rightarrow$y = 3
Hence, x = -1 and y = 3 is the required solution.

2. Show that the following system of equations has a unique solution:
2x - 3y = 17,
4x + y = 13.
Also, find the solution of the given system of equations.
Sol:
The given system of equations is:
2x - 3y - 17 = 0 ….(i)
4x + y - 13 = 0 …..(ii)
The given equations are of the form
$a_1x+b_1y+c_1 = 0$ and $a_2x+b_2y+c_2 = 0$
where, $a_1 = 2$, $b_1= -3$, $c_1 = -17$ and $a_2 = 4$, $b_2 = 1$, $c_2 = -13$
Now,
$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$ and $\frac{b_1}{b_2} = \frac{-3}{1} = -3$
Since, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, therefore the system of equations has unique solution.
Using cross multiplication method, we have

$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$

$\Rightarrow \frac{x}{-3(-13)-1\times(-17)} = \frac{y}{-17\times4-(-13)\times2} = \frac{1}{2\times1-4\times(-3)}$

$\Rightarrow \frac{x}{39+17} = \frac{y}{-68+26} = \frac{1}{2+12}$

$\Rightarrow \frac{x}{56} = \frac{y}{-42} = \frac{1}{14}$

$\Rightarrow$ x = $\frac{56}{14}$, y = $\frac{-42}{14}$

$\Rightarrow$ x = 4, y = -3

Hence, x = 4 and y = -3.

3. Show that the following system of equations has a unique solution:

$\frac{x}{3} + \frac{y}{2} = 3$, x – 2y = 2.

Also, find the solution of the given system of equations.

Sol:

The given system of equations is:

$\frac{x}{3} + \frac{y}{2} = 3$

$\Rightarrow \frac{2x+3y}{6} = 3$

2x + 3y = 18

$\Rightarrow$2x + 3y – 18 = 0 ….(i)

and

x – 2y = 2

x – 2y – 2 = 0 …..(ii)

These equations are of the forms:

$a_1x+b_1y+c_1 = 0$ and $a_2x+b_2y+c_2 = 0$

where, $a_1 = 2$, $b_1= 3$, $c_1 = -18$ and $a_2 = 1$, $b_2 = -2$, $c_2 = -2$

For a unique solution, we must have:

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, i.e., $\frac{2}{1} \neq \frac{3}{-2}$

Hence, the given system of equations has a unique solution.

Again, the given equations are:

2x + 3y – 18 = 0 …..(iii)

x – 2y – 2 = 0 …..(iv)

On multiplying (i) by 2 and (ii) by 3, we get:

4x + 6y – 36 = 0 …….(v)

3x - 6y – 6 = 0 ……(vi)

On adding (v) from (vi), we get:

7x = 42

$\Rightarrow$x = 6

On substituting x = 6 in (iii), we get:

$2(6) + 3y = 18$

$\Rightarrow 3y = (18 - 12) = 6$

$\Rightarrow y = 2$

Hence, x = 6 and y = 2 is the required solution.

4. Find the value of k for which the system of equations has a unique solution:
$2x + 3y = 5$,
$kx - 6y = 8$.

Sol:

The given system of equations are
$2x + 3y – 5 = 0$
$kx - 6y - 8 = 0$
This system is of the form:
$a_1x+b_1y+c_1 = 0$ and $a_2x+b_2y+c_2 = 0$
where, $a_1 = 2$, $b_1= 3$, $c_1 = -5$ and $a_2 = k$, $b_2 = -6$, $c_2 = -8$
Now, for the given system of equations to have a unique solution, we must have:
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
$\Rightarrow \frac{2}{k} \neq \frac{3}{-6}$
$\Rightarrow k \neq -4$
Hence, $k \neq -4$

5. Find the value of k for which the system of equations has a unique solution:
$x – ky = 2$,
$3x + 2y + 5=0$.

Sol:

The given system of equations are
$x - ky – 2 = 0$
$3x + 2y + 5 = 0$
This system of equations is of the form:
$a_1x+b_1y+c_1 = 0$ and $a_2x+b_2y+c_2 = 0$
where, $a_1 = 1$, $b_1= -k$, $c_1 = -2$ and $a_2 = 3$, $b_2 = 2$, $c_2 = 5$
Now, for the given system of equations to have a unique solution, we must have:
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
$\Rightarrow \frac{1}{3} \neq \frac{-k}{2}$
$\Rightarrow k \neq -\frac{2}{3}$
Hence, $k \neq -\frac{2}{3}$.

6. Find the value of k for which the system of equations has a unique solution:
$5x - 7y = 5$,
$2x + ky = 1$.
Sol:
The given system of equations are
$5x - 7y - 5 = 0$ ….(i)
$2x + ky - 1 = 0$ …(ii)
This system is of the form:
$a_1x+b_1y+c_1 = 0$
$a_2x+b_2y+c_2 = 0$
where, $a_1 = 5$, $b_1 = -7$, $c_1 = -5$ and $a_2 = 2$, $b_2 = k$, $c_2 = -1$
Now, for the given system of equations to have a unique solution, we must have:
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
$\Rightarrow \frac{5}{2} \neq \frac{-7}{k}$
$\Rightarrow k \neq -\frac{14}{5}$
Hence, $k \neq -\frac{14}{5}$.

7. Find the value of k for which the system of equations has a unique solution:
$4x + ky + 8=0$,
$x + y + 1 = 0$.
Sol:
The given system of equations are
$4x + ky + 8 = 0$
$x + y + 1 = 0$
This system is of the form:
$a_1x+b_1y+c_1 = 0$
$a_2x+b_2y+c_2 = 0$
where, $a_1 = 4$, $b_1 = k$, $c_1 = 8$ and $a_2 = 1$, $b_2 = 1$, $c_2 = 1$
For the given system of equations to have a unique solution, we must have:
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
$\Rightarrow \frac{4}{1} \neq \frac{k}{1}$
$\Rightarrow k \neq 4$
Hence, $k \neq 4$.

8. Find the value of k for which the system of equations has a unique solution:
$4x - 5y = k$,
$2x - 3y = 12$.
Sol:

The given system of equations are
$4x - 5y = k$
$\Rightarrow 4x - 5y - k = 0$ ….(i)
And, $2x - 3y = 12$
$\Rightarrow 2x - 3y - 12 = 0$ …(ii)
These equations are of the following form:
$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$
Here, $a_1 = 4$, $b_1= -5$, $c_1 = -k$ and $a_2 = 2$, $b_2 = -3$, $c_2 = -12$
For a unique solution, we must have:
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
i.e., $\frac{4}{2} \neq \frac{-5}{-3}$
$\Rightarrow 2 \neq \frac{5}{3} \Rightarrow 6 \neq 5$
Thus, for all real values of k, the given system of equations will have a unique solution.

9. Find the value of k for which the system of equations has a unique solution:
$kx + 3y = (k – 3)$,
$12x + ky = k$
Sol:
The given system of equations:
$kx + 3y = (k – 3)$
$\Rightarrow kx + 3y – (k - 3) = 0$ ….(i)
And, $12x + ky = k$
$\Rightarrow 12x + ky - k = 0$ …(ii)
These equations are of the following form:
$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$
Here, $a_1 = k$, $b_1= 3$, $c_1 = -(k – 3)$ and $a_2 = 12$, $b_2 = k$, $c_2 = -k$
For a unique solution, we must have:
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
i.e., $\frac{k}{12} \neq \frac{3}{k}$
$\Rightarrow k^2 \neq 36 \Rightarrow k \neq \pm 6$
Thus, for all real values of k, other than ± 6, the given system of equations will have a unique solution.

10. Show that the system equations
$2x - 3y = 5$,
$6x - 9y = 15$

has an infinite number of solutions

Sol:

The given system of equations:

2x - 3y = 5

$\Rightarrow$ 2x - 3y – 5 = 0 ….(i)

6x - 9y = 15

$\Rightarrow$6x - 9y - 15 = 0 …(ii)

These equations are of the following forms:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$

Here, $a_1 = 2$, $b_1= -3$, $c_1 = -5$ and $a_2 = 6$, $b_2 = -9$, $c_2 = -15$

$\therefore \frac{a_1}{a_2} = \frac{2}{6} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{-3}{-9} = \frac{1}{3}$ and $\frac{c_1}{c_2} = \frac{-5}{-15} = \frac{1}{3}$

Thus, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Hence, the given system of equations has an infinite number of solutions.

11. Show that the system of equations

6x + 5y = 11,

$9x + \frac{15}{2}y = 21$

has no solution.

Sol:

The given system of equations can be written as

6x + 5y – 11 = 0 ….(i)

$\Rightarrow 9x + \frac{15}{2}y - 21 = 0$ …(ii)

This system is of the form

$a_1x+b_1y+c_1 = 0$

$a_2x+b_2y+c_2 = 0$

Here, $a_1 = 6$, $b_1= 5$, $c_1 = -11$ and $a_2 = 9$, $b_2 = \frac{15}{2}$, $c_2 = -21$

Now,

$\frac{a_1}{a_2} = \frac{6}{9} = \frac{2}{3}$

$\frac{b_1}{b_2} = \frac{5}{\frac{15}{2}} = \frac{2}{3}$

$\frac{c_1}{c_2} = \frac{-11}{-21} = \frac{11}{21}$

Thus, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, therefore the given system has no solution.

12. For what value of k, the system of equations

kx + 2y = 5,

3x - 4y = 10

has (i) a unique solution, (ii) no solution?

Sol:

The given system of equations:

kx + 2y = 5

$\Rightarrow$ kx + 2y - 5 = 0 ….(i)

3x - 4y = 10

$\Rightarrow$3x - 4y - 10 = 0 …(ii)

These equations are of the forms:

$a_1x+b_1y+c_1 = 0$ and $a_2x+b_2y+c_2 = 0$

where, $a_1 = k$, $b_1= 2$, $c_1 = -5$ and $a_2 = 3$, $b_2 = -4$, $c_2 = -10$

(i) For a unique solution, we must have:

$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ i.e., $\frac{k}{3} \neq \frac{2}{-4} \Rightarrow k \neq \frac{-3}{2}$

Thus for all real values of k other than $\frac{-3}{2}$, the given system of equations will have a unique solution.

(ii) For the given system of equations to have no solutions, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \frac{k}{3} = \frac{2}{-4} \neq \frac{-5}{-10}$

$\Rightarrow \frac{k}{3} = \frac{2}{-4}$ and $\frac{k}{3} \neq \frac{1}{2}$

$\Rightarrow k = \frac{-3}{2}, k \neq \frac{3}{2}$

Hence, the required value of k is $\frac{-3}{2}$.

13. For what value of k, the system of equations

x + 2y = 5,

3x + ky + 15 = 0

has (i) a unique solution, (ii) no solution?

Sol:

The given system of equations:

x + 2y = 5

$\Rightarrow$ x + 2y - 5 = 0 ….(i)

3x + ky + 15 = 0 …(ii)

These equations are of the forms:

$a_1x+b_1y+c_1 = 0$ and $a_2x+b_2y+c_2 = 0$

where, $a_1 = 1$, $b_1= 2$, $c_1 = -5$ and $a_2 = 3$, $b_2 = k$, $c_2 = 15$

(i) For a unique solution, we must have:

$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ i.e., $\frac{1}{3} \neq \frac{2}{k} \Rightarrow k \neq 6$

Thus for all real values of k other than 6, the given system of equations will have a unique solution.
(ii) For the given system of equations to have no solutions, we must have:
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
$\Rightarrow \frac{1}{3} = \frac{2}{k} \neq \frac{-5}{15}$
$\Rightarrow \frac{1}{3} = \frac{2}{k}$ and $\frac{2}{k} \neq \frac{-5}{15}$
$\Rightarrow k = 6, k \neq -6$
Hence, the required value of k is 6.

14. For what value of k, the system of equations
x + 2y = 3,
5x + ky + 7 = 0
Have (i) a unique solution, (ii) no solution?
Also, show that there is no value of k for which the given system of equation has infinitely namely solutions
Sol:
The given system of equations:
x + 2y = 3
$\Rightarrow$ x + 2y - 3 = 0 ….(i)
And, 5x + ky + 7 = 0 …(ii)
These equations are of the following form:
$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$
where, $a_1 = 1$, $b_1= 2$, $c_1 = -3$ and $a_2 = 5$, $b_2 = k$, $c_2 = 7$
(i) For a unique solution, we must have:
$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ i.e., $\frac{1}{5} \neq \frac{2}{k} \Rightarrow k \neq 10$
Thus for all real values of k other than 10, the given system of equations will have a unique solution.
(ii) In order that the given system of equations has no solution, we must have:
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
$\Rightarrow \frac{1}{5} \neq \frac{2}{k} \neq \frac{-3}{7}$
$\Rightarrow \frac{1}{5} \neq \frac{2}{k}$ and $\frac{2}{k} \neq \frac{-3}{7}$
$\Rightarrow k = 10, k \neq \frac{14}{-3}$
Hence, the required value of k is 10.
There is no value of k for which the given system of equations has an infinite number of solutions.

15. Find the value of k for which the system of linear equations has an infinite number of solutions:
$2x + 3y = 7$,
$(k – 1)x + (k + 2)y = 3k$.
Sol:
The given system of equations:
$2x + 3y = 7$,
$\Rightarrow 2x + 3y - 7 = 0$ ….(i)
And, $(k – 1)x + (k + 2)y = 3k$
$\Rightarrow (k – 1)x + (k + 2)y - 3k = 0$ …(ii)
These equations are of the following form:
$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$
where, $a_1 = 2$, $b_1= 3$, $c_1 = -7$ and $a_2 = (k – 1)$, $b_2 = (k + 2)$, $c_2 = -3k$
For an infinite number of solutions, we must have:
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
$\frac{2}{(k-1)} = \frac{3}{(k+2)} = \frac{-7}{-3k}$
$\Rightarrow \frac{2}{(k-1)} = \frac{3}{(k+2)} = \frac{7}{3k}$
Now, we have the following three cases:
Case I:
$\frac{2}{(k-1)} = \frac{3}{k+2}$
$\Rightarrow 2(k + 2) = 3(k – 1) \Rightarrow 2k + 4 = 3k – 3 \Rightarrow k = 7$
Case II:
$\frac{3}{(k+2)} = \frac{7}{3k}$
$\Rightarrow 7(k + 2) = 9k \Rightarrow 7k + 14 = 9k \Rightarrow 2k = 14 \Rightarrow k = 7$
Case III:
$\frac{2}{(k-1)} = \frac{7}{3k}$
$\Rightarrow 7k – 7 = 6k \Rightarrow k = 7$
Hence, the given system of equations has an infinite number of solutions when k is equal to 7.

16. Find the value of k for which the system of linear equations has an infinite number of solutions:
$2x + (k – 2)y = k$,
$6x + (2k - 1)y = (2k + 5)$.
Sol:
The given system of equations:

$2x + (k - 2)y = k$

$\Rightarrow 2x + (k - 2)y - k = 0$ ….(i)

And, $6x + (2k - 1)y = (2k + 5)$

$\Rightarrow 6x + (2k - 1)y - (2k + 5) = 0$ …(ii)

These equations are of the following form:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$

where, $a_1 = 2$, $b_1= (k - 2)$, $c_1 = -k$ and $a_2 = 6$, $b_2 = (2k - 1)$, $c_2 = -(2k + 5)$

For an infinite number of solutions, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\frac{2}{6} = \frac{(k-2)}{(2k-1)} = \frac{-k}{-(2k+5)}$

$\Rightarrow \frac{1}{3} = \frac{(k-2)}{(2k-1)} = \frac{k}{(2k+5)}$

Now, we have the following three cases:

Case I:

$\frac{1}{3} = \frac{(k-2)}{(2k-1)}$

$\Rightarrow (2k - 1) = 3(k - 2)$

$\Rightarrow 2k - 1 = 3k - 6 \Rightarrow k = 5$

Case II:

$\frac{(k-2)}{(2k-1)} = \frac{k}{(2k+5)}$

$\Rightarrow (k - 2)(2k + 5) = k(2k - 1)$

$\Rightarrow 2k^2 + 5k - 4k - 10 = 2k^2 - k$

$\Rightarrow k + k = 10 \Rightarrow 2k = 10 \Rightarrow k = 5$

Case III:

$\frac{1}{3} = \frac{k}{(2k+5)}$

$\Rightarrow 2k + 5 = 3k \Rightarrow k = 5$

Hence, the given system of equations has an infinite number of solutions when k is equal to 5.

17. Find the value of k for which the system of linear equations has an infinite number of solutions:

$kx + 3y = (2k + 1)$,

$2(k + 1)x + 9y = (7k + 1)$.

Sol:

The given system of equations:

$kx + 3y = (2k + 1)$

$\Rightarrow kx + 3y - (2k + 1) = 0$ ….(i)

And, $2(k + 1)x + 9y = (7k + 1)$

$\Rightarrow 2(k + 1)x + 9y - (7k + 1) = 0$ …(ii)

These equations are of the following form:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$

where, $a_1 = k$, $b_1= 3$, $c_1 = -(2k + 1)$ and $a_2 = 2(k + 1)$, $b_2 = 9$, $c_2 = -(7k + 1)$

For an infinite number of solutions, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

i.e., $\frac{k}{2(k+1)} = \frac{3}{9} = \frac{-(2k+1)}{-(7k+1)}$

$\Rightarrow \frac{k}{2(k+1)} = \frac{1}{3} = \frac{(2k+1)}{(7k+1)}$

Now, we have the following three cases:

Case I:

$\frac{k}{2(k+1)} = \frac{1}{3}$

$\Rightarrow 2(k + 1) = 3k$

$\Rightarrow 2k + 2 = 3k$

$\Rightarrow k = 2$

Case II:

$\frac{1}{3} = \frac{(2k+1)}{(7k+1)}$

$\Rightarrow (7k + 1) = 6k + 3$

$\Rightarrow k = 2$

Case III:

$\frac{k}{2(k+1)} = \frac{(2k+1)}{(7k+1)}$

$\Rightarrow k(7k + 1) = (2k + 1) \times 2(k + 1)$

$\Rightarrow 7k^2 + k = (2k + 1)(2k + 2)$

$\Rightarrow 7k^2 + k = 4k^2 + 4k + 2k + 2$

$\Rightarrow 3k^2 – 5k – 2 = 0$

$\Rightarrow 3k^2 – 6k + k – 2 = 0$

$\Rightarrow 3k(k – 2) + 1(k – 2) = 0$

$\Rightarrow (3k + 1)(k – 2) = 0$

$\Rightarrow k = 2$ or $k = \frac{-1}{3}$

Hence, the given system of equations has an infinite number of solutions when k is equal to 2.

18. Find the value of k for which the system of linear equations has an infinite number of solutions:

5x + 2y = 2k,

2(k + 1)x + ky = (3k + 4).

Sol:

The given system of equations:

5x + 2y = 2k

⇒ 5x + 2y - 2k = 0 ….(i)

And, 2(k + 1)x + ky = (3k + 4)

⇒ 2(k + 1)x + ky - (3k + 4) = 0 …(ii)

These equations are of the following form:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$

where, $a_1 = 5$, $b_1= 2$, $c_1 = -2k$ and $a_2 = 2(k + 1)$, $b_2 = k$, $c_2 = -(3k + 4)$

For an infinite number of solutions, we must have:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{5}{2(k+1)} = \frac{2}{k} = \frac{-2k}{-(3k+4)}$$

$$\Rightarrow \frac{5}{2(k+1)} = \frac{2}{k} = \frac{2k}{(3k+4)}$$

Now, we have the following three cases:

Case I:

$$\frac{5}{2(k+1)} = \frac{2}{k}$$

⇒ 2 × 2(k + 1) = 5k

⇒ 4(k + 1) = 5k

⇒ 4k + 4 = 5k

⇒ k = 4

Case II:

$$\frac{2}{k} = \frac{2k}{(3k+4)}$$

⇒ $2k^2 = 2 \times (3k + 4)$

⇒ $2k^2 = 6k + 8$ ⇒ $2k^2 – 6k – 8 = 0$

⇒ $2(k^2 – 3k – 4) = 0$

⇒ $k^2 – 4k + k – 4 = 0$

⇒ k(k – 4) + 1(k – 4) = 0

⇒ (k + 1) (k – 4) = 0

⇒ (k + 1) = 0 or (k – 4) = 0

$\Rightarrow$ k = -1 or k = 4

Case III:

$\frac{5}{2(k+1)} = \frac{2k}{(3k+4)}$

$\Rightarrow$ 15k + 20 = $4k^2$ + 4k

$\Rightarrow$ $4k^2$ – 11k – 20 = 0

$\Rightarrow$ $4k^2$– 16k + 5k – 20 = 0

$\Rightarrow$ 4k(k – 4) + 5(k – 4) = 0

$\Rightarrow$ (k – 4) (4k + 5) = 0

$\Rightarrow$ k = 4 or k = $\frac{-5}{4}$

Hence, the given system of equations has an infinite number of solutions when k is equal to 4.

19. Find the value of k for which the system of linear equations has an infinite number of solutions:

(k – 1)x – y = 5,

(k + 1)x + (1 – k)y = (3k + 1).

Sol:

The given system of equations:

(k – 1)x – y = 5

$\Rightarrow$ (k – 1)x – y – 5 = 0 ….(i)

And, (k + 1)x + (1 – k)y = (3k + 1)

$\Rightarrow$ (k + 1)x + (1 – k)y – (3k + 1) = 0 …(ii)

These equations are of the following form:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$

where, $a_1 = (k – 1)$, $b_1= -1$, $c_1 = -5$ and $a_2 = (k + 1)$, $b_2 = (1 – k)$, $c_2 = -(3k + 1)$

For an infinite number of solutions, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

i.e., $\frac{(k-1)}{(k+1)} = \frac{-1}{-(k-1)} = \frac{-5}{-(3k+1)}$

$\Rightarrow \frac{(k-1)}{(k+1)} = \frac{1}{(k-1)} = \frac{5}{(3k+1)}$

Now, we have the following three cases:

Case I:

$\frac{(k-1)}{(k+1)} = \frac{1}{(k-1)}$

$\Rightarrow$ $(k - 1)^2 = (k + 1)$

$\Rightarrow$ $k^2 + 1 – 2k = k + 1$

$\Rightarrow k^2 - 3k = 0 \Rightarrow k(k-3) = 0$

$\Rightarrow k = 0$ or $k = 3$

Case II:

$\frac{1}{(k-1)} = \frac{5}{(3k+1)}$

$\Rightarrow 3k + 1 = 5k - 5$

$\Rightarrow 2k = 6 \Rightarrow k = 3$

Case III:

$\frac{(k-1)}{(k+1)} = \frac{5}{(3k+1)}$

$\Rightarrow (3k + 1)(k - 1) = 5(k + 1)$

$\Rightarrow 3k^2 + k - 3k - 1 = 5k + 5$

$\Rightarrow 3k^2 - 2k - 5k - 1 - 5 = 0$

$\Rightarrow 3k^2 - 7k - 6 = 0$

$\Rightarrow 3k^2 - 9k + 2k - 6 = 0$

$\Rightarrow 3k(k - 3) + 2(k - 3) = 0$

$\Rightarrow (k - 3)(3k + 2) = 0$

$\Rightarrow (k - 3) = 0$ or $(3k + 2) = 0$

$\Rightarrow k = 3$ or $k = \frac{-2}{3}$

Hence, the given system of equations has an infinite number of solutions when k is equal to 3.

20. Find the value of k for which the system of linear equations has a unique solution:
$(k - 3)x + 3y - k$, $kx + ky - 12 = 0$.

Sol:

The given system of equations can be written as

$(k - 3)x + 3y - k = 0$

$kx + ky - 12 = 0$

This system is of the form:

$a_1x+b_1y+c_1 = 0$

$a_2x+b_2y+c_2 = 0$

where, $a_1 = k$, $b_1= 3$, $c_1 = -k$ and $a_2 = k$, $b_2 = k$, $c_2 = -12$

For the given system of equations to have a unique solution, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{k-3}{k} = \frac{3}{k} = \frac{-k}{-12}$

$\Rightarrow k - 3 = 3$ and $k^2 = 36$

$\Rightarrow$ k = 6 and k = ± 6
$\Rightarrow$ k = 6
Hence, k = 6.

21. Find the values of a and b for which the system of linear equations has an infinite number of solutions:
(a – 1) x + 3y = 2, 6x + (1 – 2b)y = 6
Sol:
The given system of equations can be written as
(a – 1) x + 3y = 2
$\Rightarrow$(a – 1) x + 3y – 2 = 0 ….(i)
and 6x + (1 – 2b)y = 6
$\Rightarrow$6x + (1 – 2b)y – 6 = 0 ….(ii)
These equations are of the following form:
$a_1x+b_1y+c_1 = 0$
$a_2x+b_2y+c_2 = 0$
where, $a_1 = (a - 1)$, $b_1= 3$, $c_1 = -2$ and $a_2 = 6$, $b_2 = (1 - 2b)$, $c_2 = -6$
For an infinite number of solutions, we must have:
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
$\Rightarrow \frac{a-1}{6} = \frac{3}{(1-2b)} = \frac{-2}{-6}$
$\Rightarrow \frac{a-1}{6} = \frac{3}{(1-2b)} = \frac{1}{3}$
$\Rightarrow \frac{a-1}{6} = \frac{1}{3}$ and $\frac{3}{(1-2b)} = \frac{1}{3}$
$\Rightarrow$ 3a – 3 = 6 and 9 = 1 – 2b
$\Rightarrow$ 3a = 9 and 2b = -8
$\Rightarrow$ a = 3 and b = -4
$\therefore$ a = 3 and b = -4

22. Find the values of a and b for which the system of linear equations has an infinite number of solutions:
(2a – 1) x + 3y = 5, 3x + (b – 1)y = 2.
Sol:
The given system of equations can be written as
(2a – 1) x + 3y = 5
$\Rightarrow$(2a – 1) x + 3y – 5 = 0 ….(i)
and 3x + (b – 1)y = 2
$\Rightarrow$3x + (b – 1)y – 2 = 0 ….(ii)
These equations are of the following form:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$
where, $a_1 = (2a – 1)$, $b_1= 3$, $c_1 = -5$ and $a_2 = 3$, $b_2 = (b – 1)$, $c_2 = -2$
For an infinite number of solutions, we must have:
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
$\Rightarrow \frac{(2a-1)}{3} = \frac{3}{(b-1)} = \frac{-5}{-2}$
$\Rightarrow \frac{(2a-1)}{6} = \frac{3}{(b-1)} = \frac{5}{2}$
$\Rightarrow \frac{(2a-1)}{6} = \frac{5}{2}$ and $\frac{3}{(b-1)} = \frac{5}{2}$
$\Rightarrow 2(2a – 1) = 15$ and $6 = 5(b – 1)$
$\Rightarrow 4a – 2 = 15$ and $6 = 5b – 5$
$\Rightarrow 4a = 17$ and $5b = 11$
$\therefore a = \frac{17}{4}$ and $b = \frac{11}{5}$

23. Find the values of a and b for which the system of linear equations has an infinite number of solutions:
$2x - 3y = 7$, $(a + b)x - (a + b – 3)y = 4a + b$.
Sol:
The given system of equations can be written as
$2x - 3y = 7$
$\Rightarrow 2x - 3y - 7 = 0$ ….(i)
and $(a + b)x - (a + b – 3)y = 4a + b$
$\Rightarrow (a + b)x - (a + b – 3)y - 4a + b = 0$ ….(ii)
These equations are of the following form:
$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$
Here, $a_1 = 2$, $b_1= -3$, $c_1 = -7$ and $a_2 = (a + b)$, $b_2 = -(a + b - 3)$, $c_2 = -(4a + b)$
For an infinite number of solutions, we must have:
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
$\frac{2}{a+b} = \frac{-3}{-(a+b-3)} = \frac{-7}{-(4a+b)}$
$\Rightarrow \frac{2}{a+b} = \frac{3}{(a+b-3)} = \frac{7}{(4a+b)}$
$\Rightarrow \frac{2}{a+b} = \frac{7}{(4a+b)}$ and $\frac{3}{(a+b-3)} = \frac{7}{(4a+b)}$
$\Rightarrow 2(4a + b) = 7(a + b)$ and $3(4a + b) = 7(a + b - 3)$
$\Rightarrow 8a + 2b = 7a + 7b$ and $12a + 3b = 7a + 7b - 21$
$\Rightarrow 4a = 17$ and $5b = 11$
$\therefore a = 5b$ …….(iii)
and $5a = 4b – 21$ ……(iv)
On substituting $a = 5b$ in (iv), we get:

25b = 4b – 21
$\Rightarrow$21b = -21
$\Rightarrow$b = -1
On substituting b = -1 in (iii), we get:
a = 5(-1) = -5
$\therefore$a = -5 and b = -1.

24. Find the values of a and b for which the system of linear equations has an infinite number of solutions:
2x + 3y = 7, (a + b + 1)x - (a + 2b + 2)y = 4(a + b) + 1.
Sol:
The given system of equations can be written as
2x + 3y = 7
$\Rightarrow$2x + 3y - 7 = 0 ….(i)
and (a + b + 1)x - (a + 2b + 2)y = 4(a + b) + 1
(a + b + 1)x - (a + 2b + 2)y – [4(a + b) + 1] = 0 ….(ii)
These equations are of the following form:
$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$
where, $a_1 = 2$, $b_1= 3$, $c_1 = -7$ and $a_2 = (a + b + 1)$, $b_2 = (a + 2b + 2)$, $c_2 = -[4(a + b) + 1]$
For an infinite number of solutions, we must have:
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
$\frac{2}{(a+b+1)} = \frac{3}{(a+2b+2)} = \frac{-7}{-[4(a+b)+1]}$
$\Rightarrow \frac{2}{(a+b+1)} = \frac{3}{(a+2b+2)} = \frac{7}{[4(a+b)+1]}$
$\Rightarrow \frac{2}{(a+b+1)} = \frac{3}{(a+2b+2)}$ and $\frac{3}{(a+2b+2)} = \frac{7}{[4(a+b)+1]}$
$\Rightarrow$ 2(a + 2b + 2) = 3(a + b + 1) and 3[4(a + b) + 1] = 7(a + 2b + 2)
$\Rightarrow$ 2a + 4b + 4 = 3a + 3b + 3 and 3(4a + 4b + 1) = 7a + 14b + 14
$\Rightarrow$ a – b - 1=0 and 12a + 12b + 3 = 7a + 14b + 14
$\Rightarrow$ a - b = 1 and 5a – 2b = 11
a = (b + 1) …….(iii)
5a - 2b = 11 ……(iv)
On substituting a = (b + 1) in (iv), we get:
5(b + 1) – 2b = 11
$\Rightarrow$5b + 5 – 2b = 11
$\Rightarrow$ 3b = 6
$\Rightarrow$ b = 2
On substituting b = 2 in (iii), we get:
a = 3

$\therefore$a = 3 and b = 2.

25. Find the values of a and b for which the system of linear equations has an infinite number of solutions:
2x + 3y = 7, (a + b)x + (2a - b)y = 21.
Sol:
The given system of equations can be written as
2x + 3y - 7 = 0 ….(i)
(a + b)x + (2a - b)y – 21 = 0 ….(ii)
This system is of the form:
$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$
where, $a_1 = 2$, $b_1= 3$, $c_1 = -7$ and $a_2 = a + b$, $b_2 = 2a - b$, $c_2 = – 21$
For the given system of linear equations to have an infinite number of solutions, we must have:
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
$\Rightarrow \frac{2}{a+b} = \frac{3}{2a-b} = \frac{-7}{-21}$
$\Rightarrow \frac{2}{a+b} = \frac{-7}{-21} = \frac{1}{3}$ and $\frac{3}{2a-b} = \frac{-7}{-21} = \frac{1}{3}$
$\Rightarrow$ a + b = 6 and 2a –b = 9
Adding a + b = 6 and 2a – b = 9 ,we get
3a = 15 $\Rightarrow$ a = $\frac{15}{3}$ = 3
Now substituting a = 5 in a + b = 6, we have
5 + b = 6 $\Rightarrow$ b = 6 – 5 = 1
Hence, a = 5 and b = 1.

26. Find the values of a and b for which the system of linear equations has an infinite number of solutions:
2x + 3y = 7, 2ax + (a + b)y = 28.
Sol:
The given system of equations can be written as
2x + 3y - 7 = 0 ….(i)
2ax + (a + b)y – 28 = 0 ….(ii)
This system is of the form:
$a_1x+b_1y+c_1 = 0$
$a_2x+b_2y+c_2 = 0$
where, $a_1 = 2$, $b_1= 3$, $c_1 = -7$ and $a_2 = 2a$, $b_2 = a + b$, $c_2 = – 28$
For the given system of linear equations to have an infinite number of solutions, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{2}{2a} = \frac{3}{a+b} = \frac{-7}{-28}$

$\Rightarrow \frac{2}{2a} = \frac{-7}{-28} = \frac{1}{4}$ and $\frac{3}{a+b} = \frac{-7}{-28} = \frac{1}{4}$

$\Rightarrow$ a = 4 and a + b = 12

Substituting a = 4 in a + b = 12, we get

4 + b = 12 $\Rightarrow$ b = 12 – 4 = 8

Hence, a = 4 and b = 8.

27. Find the value of k for which the system of equations

8x + 5y = 9, kx + 10y = 15

has a non-zero solution.

Sol:

The given system of equations:

8x + 5y = 9

8x + 5y - 9 = 0 ….(i)

kx + 10y = 15

kx + 10y - 15 = 0 ….(ii)

These equations are of the following form:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$

where, $a_1 = 8$, $b_1= 5$, $c_1 = -9$ and $a_2 = k$, $b_2 = 10$, $c_2 = -15$

In order that the given system has no solution, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

i.e., $\frac{8}{k} = \frac{5}{10} \neq \frac{-9}{-15}$

i.e., $\frac{8}{k} = \frac{1}{2} \neq \frac{3}{5}$

$\frac{8}{k} = \frac{1}{2}$ and $\frac{8}{k} \neq \frac{3}{5}$

$\Rightarrow$ k = 16 and k $\neq \frac{40}{3}$

Hence, the given system of equations has no solutions when k is equal to 16.

28. Find the value of k for which the system of equations

kx + 3y = 3, 12x + ky = 6 has no solution.

Sol:

The given system of equations:

kx + 3y = 3

kx + 3y - 3 = 0 ….(i)

12x + ky = 6

12x + ky - 6 = 0 ….(ii)

These equations are of the following form:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$
where, $a_1 = k$, $b_1= 3$, $c_1 = -3$ and $a_2 = 12$, $b_2 = k$, $c_2 = -6$
In order that the given system has no solution, we must have:
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
i.e., $\frac{k}{12} = \frac{3}{k} \neq \frac{-3}{-6}$
$\frac{k}{12} = \frac{3}{k}$ and $\frac{3}{k} \neq \frac{1}{2}$
$\Rightarrow k^2 = 36$ and $k \neq 6$
$\Rightarrow k = \pm 6$ and $k \neq 6$
Hence, the given system of equations has no solution when k is equal to -6.

29. Find the value of k for which the system of equations
$3x - y = 5$, $6x - 2y = k$
has no solution.
Sol:
The given system of equations:
$3x - y - 5 = 0$ ….(i)
And, $6x - 2y + k = 0$ ….(ii)
These equations are of the following form:
$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$
where, $a_1 = 3$, $b_1= -1$, $c_1 = -5$ and $a_2 = 6$, $b_2 = -2$, $c_2 = k$
In order that the given system has no solution, we must have:
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
i.e., $\frac{3}{6} = \frac{-1}{-2} \neq \frac{-5}{k}$
$\Rightarrow \frac{-1}{-2} \neq \frac{-5}{k} \Rightarrow k \neq -10$
Hence, equations (i) and (ii) will have no solution if $k \neq -10$.

30. Find the value of k for which the system of equations
$kx + 3y + 3 - k = 0$, $12x + ky - k = 0$
has no solution.
Sol:
The given system of equations can be written as
$kx + 3y + 3 - k = 0$ ….(i)
$12x + ky - k = 0$ ….(ii)
This system of the form:
$a_1x+b_1y+c_1 = 0$
$a_2x+b_2y+c_2 = 0$
where, $a_1 = k$, $b_1= 3$, $c_1 = 3 - k$ and $a_2 = 12$, $b_2 = k$, $c_2 = -k$

For the given system of linear equations to have no solution, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \frac{k}{12} = \frac{3}{k} \neq \frac{3-k}{-k}$

$\Rightarrow \frac{k}{12} = \frac{3}{k}$ and $\frac{3}{k} \neq \frac{3-k}{-k}$

$\Rightarrow k^2 = 36$ and $-3 \neq 3 - k$

$\Rightarrow k = \pm 6$ and $k \neq 6$

$\Rightarrow k = -6$

Hence, k = -6.

31. Find the value of k for which the system of equations

5x - 3y = 0, 2x + ky = 0

has a non-zero solution.

Sol:

The given system of equations:

5x - 3y = 0 ….(i)

2x + ky = 0 ….(ii)

These equations are of the following form:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$

where, $a_1 = 5$, $b_1 = -3$, $c_1 = 0$ and $a_2 = 2$, $b_2 = k$, $c_2 = 0$

For a non-zero solution, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2}$

$\Rightarrow \frac{5}{2} = \frac{-3}{k}$

$\Rightarrow 5k = -6 \Rightarrow k = \frac{-6}{5}$

Hence, the required value of k is $\frac{-6}{5}$.

Linear equations in two variables – 3E

32. 5 chairs and 4 tables together cost ₹5600, while 4 chairs and 3 tables together cost ₹ 4340. Find the cost of each chair and that of each table.

Sol:

Let the cost of a chair be ₹ x and that of a table be ₹ y, then

5x + 4y = 5600 ……..(i)

4x + 3y = 4340 ……..(ii)

Multiplying (i) by 3 and (ii) by 4, we get

15x – 16x = 16800 – 17360

$\Rightarrow$ -x = -560

$\Rightarrow$ x = 560
Substituting x = 560 in (i), we have
$5 \times 560 + 4y = 5600$
$\Rightarrow 4y = 5600 – 2800$
$\Rightarrow y = \frac{2800}{4} = 700$
Hence, the cost of a chair and that a table are respectively ₹ 560 and ₹ 700.

33. 23 spoons and 17 forks cost Rs.1770, while 17 spoons and 23 forks cost Rs.1830. Find the cost of each spoon and that of a fork.
Sol:
Let the cost of a spoon be Rs.x and that of a fork be Rs.y. Then
$23x + 17y = 1770$ ……….(i)
$17x + 23y = 1830$ ………(ii)
Adding (i) and (ii), we get
$40x + 40y = 3600$
$\Rightarrow x + y = 90$ ………..(iii)
Now, subtracting (ii) from (i), we get
$6x – 6y = -60$
$\Rightarrow x – y = -10$ ………..(iv)
Adding (iii) and (iv), we get
$2x = 80 \Rightarrow x = 40$
Substituting x = 40 in (iii), we get
$40 + y = 90 \Rightarrow y = 50$
Hence, the cost of a spoon that of a fork is Rs.40 and Rs.50 respectively.

34. A lady has only 50-paisa coins and 25-paisa coins in her purse. If she has 50 coins in all totaling Rs.19.50, how many coins of each kind does she have?
Sol:
Let x and y be the number of 50-paisa and 25-paisa coins respectively. Then
$x + y = 50$ ……….(i)
$0.5x + 0.25y = 19.50$ ………(ii)
Multiplying (ii) by 2 and subtracting it from (i), we get
$0.5y = 50 - 39$
$\Rightarrow y = \frac{11}{0.5} = 22$
Subtracting y = 22 in (i), we get
$x + 22 = 50$
$\Rightarrow x = 50 – 22 = 28$
Hence, the number of 25-paisa and 50-paisa coins is 22 and 28 respectively.

35. The sum of two numbers is 137 and their differences are 43. Find the numbers.
Sol:
Let the larger number be x and the smaller number be y.
Then, we have:
$x + y = 137$ ……….(i)
$x - y = 43$ ………(ii)
On adding (i) and (ii), we get
$2x = 180 \Rightarrow x = 90$
On substituting x = 90 in (i), we get
$90 + y = 137$
$\Rightarrow y = (137 - 90) = 47$
Hence, the required numbers are 90 and 47.

36. Find two numbers such that the sum of twice the first and thrice the second is 92, and four times the first exceeds seven times the second by 2.
Sol:
Let the first number be x and the second number be y.
Then, we have:
$2x + 3y = 92$ ……….(i)
$4x - 7y = 2$ ………(ii)
On multiplying (i) by 7 and (ii) by 3, we get
$14x + 21y = 644$ ………..(iii)
$12x - 21y = 6$ ………..(iv)
On adding (iii) and (iv), we get
$26x = 650$
$\Rightarrow x = 25$
On substituting x = 25 in (i), we get
$2 \times 25 + 3y = 92$
$\Rightarrow 50 + 3y = 92$
$\Rightarrow 3y = (92 - 50) = 42$
$\Rightarrow y = 14$
Hence, the first number is 25 and the second number is 14.

37. Find the numbers such that the sum of thrice the first and the second is 142, and four times the first exceeds the second by 138.
Sol:
Let the first number be x and the second number be y.
Then, we have:

$3x + y = 142$ ……….(i)
$4x - y = 138$ ………(ii)
On adding (i) and (ii), we get
$7x = 280$

$\Rightarrow x = 40$
On substituting x = 40 in (i), we get:

$3 \times 40 + y = 142$

$\Rightarrow y = (142 - 120) = 22$

$\Rightarrow y = 22$
Hence, the first number is 40 and the second number is 22.

38. If 45 is subtracted from twice the greater of two numbers, it results in the other number. If 21 is subtracted from twice the smaller number, it results in the greater number. Find the numbers.
Sol:
Let the greater number be x and the smaller number be y.
Then, we have:
$25x - 45 = y$ or $2x - y = 45$ ……….(i)
$2y - 21 = x$ or $-x + 2y = 21$ ………(ii)
On multiplying (i) by 2, we get:
$4x - 2y = 90$ ………..(iii)
On adding (ii) and (iii), we get
$3x = (90 + 21) = 111$

$\Rightarrow x = 37$
On substituting x = 37 in (i), we get

$2 \times 37 - y = 45$

$\Rightarrow 74 - y = 45$

$\Rightarrow y = (74 - 45) = 29$
Hence, the greater number is 37 and the smaller number is 29.

39. If three times the larger of two numbers is divided by the smaller, we get 4 as the quotient and 8 as the remainder. If five times the smaller is divided by the larger, we get 3 as the quotient and 5 as the remainder. Find the numbers.
Sol:
We know:

Dividend = Divisor $\times$ Quotient + Remainder
Let the larger number be x and the smaller be y.

Then, we have:

$3x = y \times 4 + 8$ or $3x - 4y = 8$ ……….(i)

$5y = x \times 3 + 5$ or $-3x + 5y = 5$ ………(ii)

On adding (i) and (ii), we get:

$y = (8 + 5) = 13$

On substituting $y = 13$ in (i) we get

$3x - 4 \times 13 = 8$

$\Rightarrow 3x = (8 + 52) = 60$

$\Rightarrow x = 20$

Hence, the larger number is 20 and the smaller number is 13.

40. If 2 is added to each of two given numbers, their ratio becomes 1 : 2. However, if 4 is subtracted from each of the given numbers, the ratio becomes 5 : 11. Find the numbers.

Sol:

Let the required numbers be x and y.

Now, we have:

$\frac{x+2}{y+2} = \frac{1}{2}$

By cross multiplication, we get:

$2x + 4 = y + 2$

$\Rightarrow 2x - y = -2$ ……(i)

Again, we have:

$\frac{x-4}{y-4} = \frac{5}{11}$

By cross multiplication, we get:

$11x - 44 = 5y - 20$

$\Rightarrow 11x - 5y = 24$ ……(ii)

On multiplying (i) by 5, we get:

$10x - 5y = -10$

On subtracting (iii) from (ii), we get:

$x = (24 + 10) = 34$

On substituting $x = 34$ in (i), we get:

$2 \times 34 - y = -2$

$\Rightarrow 68 - y = -2$

$\Rightarrow y = (68 + 2) = 70$

Hence, the required numbers are 34 and 70.

41. The difference between two numbers is 14 and the difference between their squares is 448. Find the numbers.

Sol:

Let the larger number be x and the smaller number be y.

Then, we have:

$x - y = 14$ or $x = 14 + y$ ……….(i)

$x^2 - y^2 = 448$ ………(ii)

On substituting $x = 14 + y$ in (ii) we get

$(14 + y)^2 - y^2 = 448$

$\Rightarrow 196 + y^2 + 28y - y^2 = 448$

$\Rightarrow 196 + 28y = 448$

$\Rightarrow 28y = (448 - 196) = 252$

$\Rightarrow y = \frac{252}{28} = 9$

On substituting $y = 9$ in (i), we get:

$x = 14 + 9 = 23$

Hence, the required numbers are 23 and 9.

42. The sum of the digits of a two-digit number is 12. The number obtained by interchanging its digits exceeds the given number by 18. Find the number.

Sol:

Let the tens and the units digits of the required number be x and y, respectively.

Required number = $(10x + y)$

$x + y = 12$ ……….(i)

Number obtained on reversing its digits = $(10y + x)$

$\therefore (10y + x) - (10x + y) = 18$

$\Rightarrow 10y + x - 10x - y = 18$

$\Rightarrow 9y - 9x = 18$

$\Rightarrow y - x = 2$ ……..(ii)

On adding (i) and (ii), we get:

$2y = 14$

$\Rightarrow y = 7$

On substituting $y = 7$ in (i) we get

$x + 7 = 12$

$\Rightarrow x = (12 - 7) = 5$

Number = $(10x + y) = 10 \times 5 + 7 = 50 + 7 = 57$

Hence, the required number is 57.

43. A number consisting of two digits is seven times the sum of its digits. When 27 is subtracted from the number, the digits are reversed. Find the number.

Sol:

Let the tens and the units digits of the required number be x and y, respectively.

Required number = $(10x + y)$

$10x + y = 7(x + y)$

$10x + 7y = 7x + 7y$ or $3x – 6y = 0$ ……….(i)

Number obtained on reversing its digits = $(10y + x)$

$(10x + y) - 27 = (10y + x)$

$\Rightarrow 10x – x + y – 10y = 27$

$\Rightarrow 9x – 9y = 27$

$\Rightarrow 9(x – y) = 27$

$\Rightarrow x – y = 3$ ……..(ii)

On multiplying (ii) by 6, we get:

$6x – 6y = 18$ ………(iii)

On subtracting (i) from (ii), we get:

$3x = 18$

$\Rightarrow x = 6$

On substituting x = 6 in (i) we get

$3 \times 6 – 6y = 0$

$\Rightarrow 18 – 6y = 0$

$\Rightarrow 6y = 18$

$\Rightarrow y = 3$

Number = $(10x + y) = 10 \times 6 + 3 = 60 + 3 = 63$

Hence, the required number is 63.

44. The sum of the digits of a two-digit number is 15. The number obtained by interchanging the digits exceeds the given number by 9. Find the number.

Sol:

Let the tens and the units digits of the required number be x and y, respectively.

Required number = $(10x + y)$

$x + y = 15$ ……….(i)

Number obtained on reversing its digits = $(10y + x)$

$\therefore (10y + x) - (10x + y) = 9$

$\Rightarrow 10y + x – 10x – y = 9$

$\Rightarrow 9y – 9x = 9$

$\Rightarrow y - x = 1$ ……..(ii)
On adding (i) and (ii), we get:
$2y = 16$
$\Rightarrow y = 8$
On substituting $y = 8$ in (i) we get
$x + 8 = 15$
$\Rightarrow x = (15 - 8) = 7$
Number $= (10x + y) = 10 \times 7 + 8 = 70 + 8 = 78$
Hence, the required number is 78.

45. A two-digit number is 3 more than 4 times the sum of its digits. If 18 is added to the number, the digits are reversed. Find the number.
Sol:
Let the tens and the units digits of the required number be x and y, respectively.
Required number $= (10x + y)$
$10x + y = 4(x + y) + 3$
$\Rightarrow 10x + y = 4x + 4y + 3$
$\Rightarrow 6x - 3y = 3$
$\Rightarrow 2x - y = 1$ ……….(i)
Again, we have:
$10x + y + 18 = 10y + x$
$\Rightarrow 9x - 9y = -18$
$\Rightarrow x - y = -2$ ……..(ii)
On subtracting (ii) from (i), we get:
$x = 3$
On substituting $x = 3$ in (i) we get
$2 \times 3 - y = 1$
$\Rightarrow y = 6 - 1 = 5$
Required number $= (10x + y) = 10 \times 3 + 5 = 30 + 5 = 35$
Hence, the required number is 35.

46. A number consists of two digits. When it is divided by the sum of its digits, the quotient is 6 with no remainder. When the number is diminished by 9, the digits are reversed. Find the number.
Sol:

We know:

Dividend = Divisor $\times$ Quotient + Remainder

Let the tens and the units digits of the required number be x and y, respectively.
Required number = $(10x + y)$

$10x + y = (x + y) \times 6 + 0$

$\Rightarrow 10x - 6x + y - 6y = 0$

$\Rightarrow 4x - 5y = 0$ …….(i)

Number obtained on reversing its digits = $(10y + x)$

$\therefore 10x + y - 9 = 10y + x$

$\Rightarrow 9x - 9y = 9$

$\Rightarrow x - y = 1$ ……..(ii)

On multiplying (ii) by 5, we get:

$5x - 5y = 5$ ……..(iii)

On subtracting (i) from (iii), we get:

$x = 5$

On substituting $x = 5$ in (i) we get

$4 \times 5 - 5y = 0$

$\Rightarrow 20 - 5y = 0$

$\Rightarrow y = 4$

$\therefore$ The number = $(10x + y) = 10 \times 5 + 4 = 50 + 4 = 54$

Hence, the required number is 54.

47. A two-digit number is such that the product of its digits is 35. If 18 is added to the number, the digits interchange their places. Find the number.

Sol:

Let the tens and the units digits of the required number be x and y, respectively.

Then, we have:

$xy = 35$ …….(i)

Required number = $(10x + y)$

Number obtained on reversing its digits = $(10y + x)$

$\therefore (10x + y) + 18 = 10y + x$

$\Rightarrow 9x - 9y = -18$

$\Rightarrow 9(y - x) = 18$

$\Rightarrow y - x = 2$ ……..(ii)

We know:
$(y + x)^2 - (y - x)^2 = 4xy$
$\Rightarrow (y + x) = \pm \sqrt{(y-x)^2 + 4xy}$
$\Rightarrow (y + x) = \pm \sqrt{4 + 4 \times 35} = \pm \sqrt{144} = \pm 12$
$\Rightarrow$ y + x = 12 ……..(iii) ($\because$ x and y cannot be negative)
On adding (ii) and (iii), we get:
2y = 2 +12 = 14
$\Rightarrow$y = 7
On substituting y = 7in (ii) we get
7 – x = 2
$\Rightarrow$ x = (7 – 2) = 5
$\therefore$ The number = (10x + y) = 10 $\times$ 5 + 7 = 50 + 7 = 57
Hence, the required number is 57.

48. A two-digit number is such that the product of its digits is 18. When 63 is subtracted from the number, the digits interchange their places. Find the number.
Sol:
Let the tens and the units digits of the required number be x and y, respectively.
Then, we have:
xy = 18 …….(i)
Required number = (10x + y)
Number obtained on reversing its digits = (10y + x)
$\therefore$(10x + y) - 63 = 10y + x
$\Rightarrow$9x – 9y = 63
$\Rightarrow$ 9(x – y) = 63
$\Rightarrow$ x – y = 7 ……..(ii)
We know:
$(x + y)^2 - (x - y)^2 = 4xy$
$\Rightarrow (x + y) = \pm \sqrt{(x-y)^2 + 4xy}$
$\Rightarrow (x + y) = \pm \sqrt{49 + 4 \times 18}$
$= \pm \sqrt{49 + 72}$
$= \pm \sqrt{121} = \pm 11$
$\Rightarrow$ x + y = 11 ……..(iii) ($\because$ x and y cannot be negative)
On adding (ii) and (iii), we get:

2x = 7 +11 = 18

$\Rightarrow$x = 9

On substituting x = 9in (ii) we get

9 – y = 7

$\Rightarrow$ y = (9 – 7) = 2

$\therefore$ Number = (10x + y) = 10 $\times$ 9 + 2 = 90 + 2 = 92

Hence, the required number is 92.

49. The sum of a two-digit number and the number obtained by reversing the order of its digits is 121, and the two digits differ by 3. Find the number,

Sol:

Let x be the ones digit and y be the tens digit. Then

Two digit number before reversing = 10y + x

Two digit number after reversing = 10x + y

As per the question

(10y + x) + (10x + y) = 121

$\Rightarrow$11x + 11y = 121

$\Rightarrow$x + y = 11 …….(i)

Since the digits differ by 3, so

x – y = 3 ……….(ii)

Adding (i) and (ii), we get

2x = 14 $\Rightarrow$ x = 7

Putting x = 7 in (i), we get

7 + y = 11 $\Rightarrow$ y = 4

Changing the role of x and y, x = 4 and y = 7

Hence, the two-digit number is 74 or 47.

50. The sum of the numerator and denominator of a fraction is 8. If 3 is added to both of the numerator and the denominator, the fraction becomes $\frac{3}{4}$. Find the fraction.

Sol:

Let the required fraction be $\frac{x}{y}$.

Then, we have:

x + y = 8 ……(i)

And, $\frac{x+3}{y+3} = \frac{3}{4}$

$\Rightarrow$4(x + 3) = 3(y + 3)

$\Rightarrow$4x + 12 = 3y + 9

⇒ 4x – 3y = -3 ……(ii)
On multiplying (i) by 3, we get:
3x + 3y = 24
On adding (ii) and (iii), we get:
7x = 21
⇒ x = 3
On substituting x = 3 in (i), we get:
3 + y = 8
⇒ y = (8 – 3) = 5
∴ x = 3 and y = 5
Hence, the required fraction is $\frac{3}{5}$.

51. If 2 is added to the numerator of a fraction, it reduces to $\left(\frac{1}{2}\right)$ and if 1 is subtracted from the denominator, it reduces to $\left(\frac{1}{3}\right)$. Find the fraction.

Sol:
Let the required fraction be $\frac{x}{y}$.
Then, we have:
$\frac{x+2}{y} = \frac{1}{2}$
⇒ 2(x + 2) = y
⇒ 2x + 4 = y
⇒2x - y = -4 ……(i)
Again, $\frac{x}{y-1} = \frac{1}{3}$
⇒3x = 1(y – 1)
⇒3x – y = -1 ……(ii)
On subtracting (i) from (ii), we get:
x = (-1 + 4) = 3
On substituting x = 3 in (i), we get:
2 × 3 – y = -4
⇒ 6 – y = -4
⇒ y = (6 + 4) = 10
∴ x = 3 and y = 10
Hence, the required fraction is $\frac{3}{10}$.

52. The denominator of a fraction is greater than its numerator by 11. If 8 is added to both its numerator and denominator, it becomes $\frac{3}{4}$. Find the fraction.

Sol:

Let the required fraction be $\frac{x}{y}$.

Then, we have:

$y = x + 11$

$\Rightarrow y - x = 11$ ……(i)

Again, $\frac{x+8}{y+8} = \frac{3}{4}$

$\Rightarrow 4(x + 8) = 3(y + 8)$

$\Rightarrow 4x + 32 = 3y + 24$

$\Rightarrow 4x - 3y = -8$ ……(ii)

On multiplying (i) by 4, we get:

$4y - 4x = 44$

On adding (ii) and (iii), we get:

$y = (-8 + 44) = 36$

On substituting y = 36 in (i), we get:

$36 - x = 11$

$\Rightarrow x = (36 - 11) = 25$

$\therefore x = 25$ and $y = 36$

Hence, the required fraction is $\frac{25}{36}$.

53. Find a fraction which becomes $\left(\frac{1}{2}\right)$ when 1 is subtracted from the numerator and 2 is added to the denominator, and the fraction becomes $\left(\frac{1}{3}\right)$ when 7 is subtracted from the numerator and 2 is subtracted from the denominator.

Sol:

Let the required fraction be $\frac{x}{y}$.

Then, we have:

$\frac{x-1}{y+2} = \frac{1}{2}$

$\Rightarrow 2(x - 1) = 1(y + 2)$

$\Rightarrow 2x - 2 = y + 2$

$\Rightarrow 2x - y = 4$ ……(i)

Again, $\frac{x-7}{y-2} = \frac{1}{3}$

$\Rightarrow 3(x - 7) = 1(y - 2)$

$\Rightarrow 3x - 21 = y - 2$

$\Rightarrow 3x - y = 19$ ……(ii)

On subtracting (i) from (ii), we get:

$x = (19 - 4) = 15$

On substituting $x = 15$ in (i), we get:

$2 \times 15 - y = 4$

$\Rightarrow 30 - y = 4$

$\Rightarrow y = 26$

$\therefore x = 15$ and $y = 26$

Hence, the required fraction is $\frac{15}{26}$.

54. The sum of the numerator and denominator of a fraction is 4 more than twice the numerator. If the numerator and denominator are increased by 3. They are in the ratio of 2: 3. Determine the fraction.

Sol:

Let the required fraction be $\frac{x}{y}$.

As per the question

$x + y = 4 + 2x$

$\Rightarrow y - x = 4$ ……(i)

After changing the numerator and denominator

New numerator = $x + 3$

New denominator = $y + 3$

Therefore

$\frac{x+3}{y+3} = \frac{2}{3}$

$\Rightarrow 3(x + 3) = 2(y + 3)$

$\Rightarrow 3x + 9 = 2y + 6$

$\Rightarrow 2y - 3x = 3$ ……(ii)

Multiplying (i) by 3 and subtracting (ii), we get:

$3y - 2y = 12 - 3$

$\Rightarrow y = 9$

Now, putting $y = 9$ in (i), we get:

$9 - x = 4 \Rightarrow x = 9 - 4 = 5$

Hence, the required fraction is $\frac{5}{9}$.

55. The sum of two numbers is 16 and the sum of their reciprocals is $\frac{1}{3}$. Find the numbers.

Sol:

Let the larger number be x and the smaller number be y.

Then, we have:

$x + y = 16$ ……(i)

And, $\frac{1}{x} + \frac{1}{y} = \frac{1}{3}$ ……(ii)

$\Rightarrow 3(x + y) = xy$

$\Rightarrow 3 \times 16 = xy$ [Since from (i), we have: $x + y = 16$]

$\therefore xy = 48$ …….(iii)

We know:

$(x - y)^2 = (x + y)^2 - 4xy$

$(x - y)^2 = (16)^2 - 4 \times 48 = 256 - 192 = 64$

$\therefore (x - y) = \pm\sqrt{64} = \pm 8$

Since x is larger and y is smaller, we have:

$x - y = 8$ ………(iv)

On adding (i) and (iv), we get:

$2x = 24$

$\Rightarrow x = 12$

On substituting x = 12 in (i), we get:

$12 + y = 16 \Rightarrow y = (16 - 12) = 4$

Hence, the required numbers are 12 and 4.

56. There are two classrooms A and B. If 10 students are sent from A to B, the number of students in each room becomes the same. If 20 students are sent from B to A, the number of students in A becomes double the number of students in B. Find the number of students in each room.

Sol:

Let the number of students in classroom A be x

Let the number of students in classroom B be y.

If 10 students are transferred from A to B, then we have:

$x - 10 = y + 10$

$\Rightarrow x - y = 20$ …..(i)

If 20 students are transferred from B to A, then we have:

$2(y - 20) = x + 20$

$\Rightarrow 2y - 40 = x + 20$

$\Rightarrow -x + 2y = 60$ …..(ii)

On adding (i) and (ii), we get:
y = (20 + 60) = 80
On substituting y = 80 in (i), we get:
x – 80 = 20
$\Rightarrow$x = (20 + 80) = 100
Hence, the number of students in classroom A is 100 and the number of students in classroom B is 80.

57. Taxi charges in a city consist of fixed charges per day and the remaining depending upon the distance travelled in kilometers. If a person travels 80km, he pays Rs. 1330, and for travelling 90km, he pays Rs. 1490. Find the fixed charges per day and the rate per km.

Sol:
Let fixed charges be Rs.x and rate per km be Rs.y.
Then as per the question
x + 80y = 1330 ………(i)
x + 90y = 1490 ……..(ii)
Subtracting (i) from (ii), we get
$10y = 160 \Rightarrow y = \frac{160}{10} = 16$
Now, putting y = 16, we have
$x + 80 \times 16 = 1330$
$\Rightarrow x = 1330 - 1280 = 50$
Hence, the fixed charges be Rs.50 and the rate per km is Rs.16.

58. A part of monthly hostel charges in a college are fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 25days, he has to pay Rs. 4550 as hostel charges whereas a student B, who takes food for 30 days, pays Rs. 5200 as hostel charges. Find the fixed charges and the cost of the food per day.

Sol:
Let the fixed charges be Rs.x and the cost of food per day be Rs.y.
Then as per the question
x + 25y = 4500 ………(i)
x + 30y = 5200 ……..(ii)
Subtracting (i) from (ii), we get
$5y = 700 \Rightarrow y = \frac{700}{5} = 140$
Now, putting y = 140, we have
$x + 25 \times 140 = 4500$
$\Rightarrow x = 4500 - 3500 = 1000$
Hence, the fixed charges be Rs.1000 and the cost of the food per day is Rs.140.

59. A man invested an amount at 10% per annum simple interest and another amount at 10% per annum simple interest. He received an annual interest of Rs. 1350. But, if he had interchanged the amounts invested, he would have received Rs. 45 less. What amounts did he invest at different rates?

Sol:

Let the amounts invested at 10% and 8% be Rs.x and Rs.y respectively.

Then as per the question

$\frac{x \times 10 \times 1}{100} = \frac{y \times 8 \times 1}{100} = 1350$

$10x + 8y = 135000$ …………..(i)

After the amounts interchanged but the rate being the same, we have

$\frac{x \times 8 \times 1}{100} = \frac{y \times 10 \times 1}{100} = 1350 - 45$

$8x + 10y = 130500$ …………….(ii)

Adding (i) and (ii) and dividing by 9, we get

$2x + 2y = 29500$ ……………(iii)

Subtracting (ii) from (i), we get

$2x - 2y = 4500$

Now, adding (iii) and (iv), we have

$4x = 34000$

$x = \frac{34000}{4} = 8500$

Putting x = 8500 in (iii), we get

$2 \times 8500 + 2y = 29500$

$2y = 29500 - 17000 = 12500$

$y = \frac{12500}{2} = 6250$

Hence, the amounts invested are Rs. 8,500 at 10% and Rs. 6,250 at 8%.

60. The monthly incomes of A and B are in the ratio of 5 : 4 and their monthly expenditures are in the ratio of 7 : 5. If each saves Rs. 9000 per month, find the monthly income of each.

Sol:

Let the monthly income of A and B are Rs.x and Rs.y respectively.

Then as per the question

$\frac{x}{y} = \frac{5}{4}$

$\Rightarrow y = \frac{4x}{5}$

Since each save Rs.9,000, so

Expenditure of A = Rs.$(x - 9000)$

Expenditure of B = Rs.$(y - 9000)$

The ratio of expenditures of A and B are in the ratio 7:5.

$\therefore \frac{x-9000}{y-9000} = \frac{7}{5}$

$\Rightarrow$7y – 63000 = 5x – 45000

$\Rightarrow$7y – 5x = 18000

From (i), substitute y = $\frac{4x}{5}$ in (ii) to get

$7 \times \frac{4x}{5} - 5x = 18000$

$\Rightarrow$28x – 25x = 90000

$\Rightarrow$3x = 90000

$\Rightarrow$x = 30000

Now, putting x = 30000, we get

y = $\frac{4 \times 30000}{5} = 4 \times 6000 = 24000$

Hence, the monthly incomes of A and B are Rs. 30,000 and Rs.24,000.

61. A man sold a chair and a table together for Rs. 1520, thereby making a profit of 25% on chair and 10% on table. By selling them together for Rs. 1535, he would have made a profit of 10% on the chair and 25% on the table. Find the cost price of each.

Sol:

Let the cost price of the chair and table be Rs.x and Rs.y respectively.

Then as per the question

Selling price of chair + Selling price of table = 1520

$\frac{100+25}{100} \times x + \frac{100+10}{100} \times y = 1520$

$\Rightarrow \frac{125}{100} x + \frac{110}{100} y = 1520$

$\Rightarrow$25x + 22y – 30400 = 0 ……….(i)

When the profit on chair and table are 10% and 25% respectively, then

$\frac{100+10}{100} \times x + \frac{100+25}{100} \times y = 1535$

$\Rightarrow \frac{110}{100} x + \frac{125}{100} y = 1535$

$\Rightarrow$22x + 25y – 30700 = 0 ……….(ii)

Solving (i) and (ii) by cross multiplication, we get

$\frac{x}{(22)(-30700)-(25)(-30400)} = \frac{y}{(-30400)(22)-(-30700)(25)} = \frac{1}{(25)(25)-(22)(22)}$

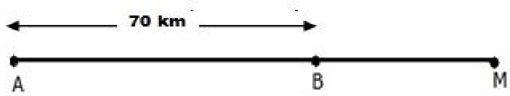
$\Rightarrow \frac{x}{7600-6754} = \frac{y}{7675-6688} = \frac{100}{3 \times 47}$

$\Rightarrow \frac{x}{846} = \frac{y}{987} = \frac{100}{3 \times 47}$

$\Rightarrow x = \frac{100 \times 846}{3 \times 47}, y = \frac{100 \times 987}{3 \times 47}$

$\Rightarrow$ x = 600, y = 700
Hence, the cost of chair and table are Rs.600 and Rs.700 respectively.

62. Points A and B are 70 km apart on a highway. A car starts from A and another car starts from B simultaneously. If they travel in the same direction, they meet in 7 hours. But, if they travel towards each other, they meet in 1 hour. Find the speed of each car.
Sol:
Let X and Y be the cars starting from points A and B, respectively and let their speeds be x km/h and y km/h, respectively.
Then, we have the following cases:
Case I: When the two cars move in the same direction
In this case, let the two cars meet at point M.



Distance covered by car X in 7 hours = 7x km
Distance covered by car Y in 7 hours = 7y km

$\therefore$ AM = (7x) km and BM = (7y) km

$\Rightarrow$(AM – BM) = AB

$\Rightarrow$(7x – 7y) = 70

$\Rightarrow$7(x – y) = 70

$\Rightarrow$(x – y) = 10 ……..(i)

Case II: When the two cars move in opposite directions
In this case, let the two cars meet at point N.
Distance covered by car X in 1 hour = x km
Distance covered by car Y in 1 hour = y km

$\therefore$ AN = x km and BN = y km

$\Rightarrow$ AN + BN = AB

$\Rightarrow$ x + y = 70 ………(ii)

On adding (i) and (ii), we get:
2x = 80

$\Rightarrow$x = 40

On substituting x = 40 in (i), we get:
40 – y = 10

$\Rightarrow$y = (40 – 10) = 30

Hence, the speed of car X is 40km/h and the speed of car Y is 30km/h.

63. A train covered a certain distance at a uniform speed. If the train had been 5 kmph faster, it would have taken 3 hours less than the scheduled time. And, if the train were slower by 4 kmph, it would have taken 3 hours more than the scheduled time. Find the length of the journey.

Sol:

Let the original speed be x kmph and let the time taken to complete the journey be y hours.

$\therefore$ Length of the whole journey = (xy) km

Case I:

When the speed is (x + 5) kmph and the time taken is (y – 3) hrs:

Total journey = (x + 5) (y – 3) km

$\Rightarrow (x + 5)(y - 3) = xy$

$\Rightarrow xy + 5y - 3x - 15 = xy$

$\Rightarrow 5y - 3x = 15$ ………(i)

Case II:

When the speed is (x – 4) kmph and the time taken is (y + 3) hrs:

Total journey = (x – 4) (y + 3) km

$\Rightarrow (x - 4)(y + 3) = xy$

$\Rightarrow xy - 4y + 3x - 12 = xy$

$\Rightarrow 3x - 4y = 12$ ………(ii)

On adding (i) and (ii), we get:

$y = 27$

On substituting y = 27 in (i), we get:

$5 \times 27 - 3x = 15$

$\Rightarrow 135 - 3x = 15$

$\Rightarrow 3x = 120$

$\Rightarrow x = 40$

$\therefore$ Length of the journey = (xy) km = (40×27) km = 1080 km

64. Abdul travelled 300 km by train and 200 km by taxi taking 5 hours and 30 minutes. But, if he travels 260km by train and 240km by taxi, he takes 6 minutes longer. Find the speed of the train and that of taxi.

Sol:

Let the speed of the train and taxi be x km/h and y km/h respectively.

Then as per the question

$\frac{3}{x} + \frac{2}{y} = \frac{11}{200}$ …………(i)

When the speeds of the train and taxi are 260 km and 240 km respectively, then

$\frac{260}{x} + \frac{240}{y} = \frac{11}{2} + \frac{6}{60}$

$\Rightarrow \frac{13}{x} + \frac{12}{y} = \frac{28}{100}$ …………..(ii)

Multiplying (i) by 6 and subtracting (ii) from it, we get

$\frac{18}{x} - \frac{13}{x} = \frac{66}{200} - \frac{28}{100}$

$\Rightarrow \frac{5}{x} = \frac{10}{200} \Rightarrow$ x = 100

Putting x = 100 in (i), we have

$\frac{3}{100} + \frac{2}{y} = \frac{11}{200}$

$\Rightarrow \frac{2}{y} = \frac{11}{200} - \frac{3}{100} = \frac{1}{40}$

$\Rightarrow$ y = 80

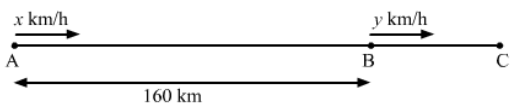
Hence, the speed of the train and that of the taxi are 100 km/h and 80 km/h respectively.

65. Places A and B are 160 km apart on a highway. A car starts from A and another car starts from B simultaneously. If they travel in the same direction, they meet in 8 hours. But, if they travel towards each other, they meet in 2 hours. Find the speed of each car.

Sol:

Let the speed of the car A and B be x km/h and y km/h respectively. Let x > y.

Case-1: When they travel in the same direction



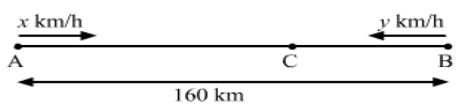
From the figure

AC – BC = 160

$\Rightarrow x \times 8 - y \times 8 = 160$

$\Rightarrow x - y = 20$

Case-2: When they travel in opposite direction



From the figure

AC + BC = 160

$\Rightarrow x \times 2 + y \times 2 = 160$

$\Rightarrow x + y = 80$

Adding (i) and (ii), we get

2x = 100 $\Rightarrow$ x = 50 km/h

Putting x = 50 in (ii), we have

$50 + y = 80 \Rightarrow y = 80 - 50 = 30$ km/h

Hence, the speeds of the cars are 50 km/h and 30 km/h.

66. A sailor goes 8 km downstream in 420 minutes and returns in 1 hour. Find the speed of the sailor in still water and the speed of the current .

Sol:

Let the speed of the sailor in still water be x km/h and that of the current y km/h.

Speed downstream = (x + y) km/h

Speed upstream = (x – y) km/h

As per the question

$(x + y) \times \frac{40}{60} = 8$

$\Rightarrow x + y = 12$ ………(i)

When the sailor goes upstream, then

$(x - y) \times 1 = 8$

$x - y = 8$ ………(ii)

Adding (i) and (ii), we get

$2x = 20 \Rightarrow x = 10$

Putting x = 10 in (i), we have

$10 + y = 12 \Rightarrow y = 2$

Hence, the speeds of the sailor in still water and the current are 10 km/h and 2 km/h respectively.

67. A boat goes 12 km upstream and 40 km downstream in 8 hours. It can go 16 km upstream and 32 km downstream in the same time. Find the speed of the boat in still water and the speed of the stream

Sol:

Let the speed of the boat in still water be x km/h and the speed of the stream be y km/h.

Then we have

Speed upstream = (x – y) km/hr

Speed downstream = (x + y) km/hr

Time taken to cover 12 km upstream = $\frac{12}{(x-y)}$ hrs

Time taken to cover 40 km downstream = $\frac{40}{(x+y)}$ hrs

Total time taken = 8 hrs

$\therefore \frac{12}{(x-y)} + \frac{40}{(x+y)} = 8$ ……….(i)

Again, we have:

Time taken to cover 16 km upstream = $\frac{16}{(x-y)}$ hrs

Time taken to cover 32 km downstream = $\frac{32}{(x+y)}$ hrs

Total time taken = 8 hrs

$\therefore \frac{16}{(x-y)} + \frac{32}{(x+y)} = 8$ ……….(ii)

Putting $\frac{1}{(x-y)}$ = u and $\frac{1}{(x+y)}$ = v in (i) and (ii), we get:

12u + 40v = 8

3u + 10v = 2 ………(a)

And, 16u + 32v = 8

⇒2u + 4v = 1 ……….(b)

On multiplying (a) by 4 and (b) by 10, we get:

12u + 40v = 8 …….(iii)

And, 20u + 40v = 10 …….(iv)

On subtracting (iii) from (iv), we get:

8u = 2

$\Rightarrow u = \frac{2}{8} = \frac{1}{4}$

On substituting u = $\frac{1}{4}$ in (iii), we get:

40v = 5

$\Rightarrow v = \frac{5}{40} = \frac{1}{8}$

Now, we have:

$u = \frac{1}{4}$

$\Rightarrow \frac{1}{(x-y)} = \frac{1}{4} \Rightarrow x - y = 4$ ……..(v)

$v = \frac{1}{8}$

$\Rightarrow \frac{1}{(x+y)} = \frac{1}{8} \Rightarrow x + y = 8$ ……..(vi)

On adding (v) and (vi), we get:

2x = 12

⇒ x = 6

On substituting x = 6 in (v), we get:

6 – y = 4

y = (6 – 4) = 2

∴ Speed of the boat in still water = 6km/h

And, speed of the stream = 2 km/h

68. 2 men and 5 boys can finish a piece of work in 4 days, while 3 men and 6 boys can finish it in 3 days. Find the time taken by one man alone to finish the work and that taken by one boy alone to finish the work.

Sol:

Let us suppose that one man alone can finish the work in x days and one boy alone can finish it in y days.

$\therefore$ One man's one day's work = $\frac{1}{x}$

And, one boy's one day's work = $\frac{1}{y}$

2 men and 5 boys can finish the work in 4 days.

$\therefore$ (2 men's one day's work) + (5 boys' one day's work) = $\frac{1}{4}$

$\Rightarrow \frac{2}{x} + \frac{5}{y} = \frac{1}{4}$

$\Rightarrow$ 2u + 5v = $\frac{1}{4}$ …….(i) Here, $\frac{1}{x}$ = u and $\frac{1}{y}$ = v

Again, 3 men and 6 boys can finish the work in 3days.

$\therefore$ (3 men's one day's work) + (6 boys' one day's work) = $\frac{1}{3}$

$\Rightarrow \frac{3}{x} + \frac{6}{y} = \frac{1}{3}$

$\Rightarrow$ 3u + 6v = $\frac{1}{3}$ …….(ii) Here, $\frac{1}{x}$ = u and $\frac{1}{y}$ = v

On multiplying (iii) from (iv), we get:

$3u = \left(\frac{5}{3} - \frac{6}{4}\right) = \frac{2}{12} = \frac{1}{6}$

$\Rightarrow u = \frac{1}{6 \times 3} = \frac{1}{18} \Rightarrow \frac{1}{x} = \frac{1}{18} \Rightarrow x = 18$

On substituting u = $\frac{1}{18}$ in (i), we get:

$2 \times \frac{1}{18} + 5v = \frac{1}{4} \Rightarrow 5v = \left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5}{36}$

$\Rightarrow v = \left(\frac{5}{36} \times \frac{1}{5}\right) = \frac{1}{36} \Rightarrow \frac{1}{y} = \frac{1}{36} \Rightarrow y = 36$

Hence, one man alone can finish the work is 18days and one boy alone can finish the work in 36 days.

69. The length of a room exceeds its breadth by 3 meters. If the length is increased by 3 meters and the breadth is decreased by 2 meters, the area remains the same. Find the length and the breadth of the room.

Sol:

Let the length of the room be x meters and he breadth of the room be y meters.

Then, we have:

Area of the room = xy

According to the question, we have:

x = y + 3

$\Rightarrow x - y = 3$ …….(i)

And, $(x + 3)(y - 2) = xy$

$\Rightarrow xy - 2x + 3y - 6 = xy$

$\Rightarrow 3y - 2x = 6$ ……..(ii)

On multiplying (i) by 2, we get:

$2x - 2y = 6$ ……….(iii)

On adding (ii) and (iii), we get:

$y = (6 + 6) = 12$

On substituting $y = 12$ in (i), we get:

$x - 12 = 3$

$\Rightarrow x = (3 + 12) = 15$

Hence, the length of the room is 15 meters and its breadth is 12 meters.

70. The area of a rectangle gets reduced by $8m^2$, when its length is reduced by 5m and its breadth is increased by 3m. If we increase the length by 3m and breadth by 2m, the area is increased by $74m^2$. Find the length and the breadth of the rectangle.

Sol:

Let the length and the breadth of the rectangle be x m and y m, respectively.

$\therefore$ Area of the rectangle = (xy) sq.m

Case 1:

When the length is reduced by 5m and the breadth is increased by 3 m:

New length = $(x - 5)$ m

New breadth = $(y + 3)$ m

$\therefore$ New area = $(x - 5)(y + 3)$ sq.m

$\therefore xy - (x - 5)(y + 3) = 8$

$\Rightarrow xy - [xy - 5y + 3x - 15] = 8$

$\Rightarrow xy - xy + 5y - 3x + 15 = 8$

$\Rightarrow 3x - 5y = 7$ ………(i)

Case 2:

When the length is increased by 3 m and the breadth is increased by 2 m:

New length = $(x + 3)$ m

New breadth = $(y + 2)$ m

$\therefore$ New area = $(x + 3)(y + 2)$ sq.m

$\Rightarrow (x + 3)(y + 2) - xy = 74$

$\Rightarrow [xy + 3y + 2x + 6] - xy = 74$

$\Rightarrow 2x + 3y = 68$ ………(ii)

On multiplying (i) by 3 and (ii) by 5, we get:
$9x - 15y = 21$ ……….(iii)
$10x + 15y = 340$ ………(iv)
On adding (iii) and (iv), we get:
$19x = 361$

$\Rightarrow x = 19$
On substituting x = 19 in (iii), we get:

$9 \times 19 - 15y = 21$

$\Rightarrow 171 - 15y = 21$

$\Rightarrow 15y = (171 - 21) = 150$

$\Rightarrow y = 10$
Hence, the length is 19m and the breadth is 10m.

71. The area of a rectangle gets reduced by 67 square meters, when its length is increased by 3m and the breadth is decreased by 4m. If the length is reduced by 1m and breadth is increased by 4m, the area is increased by 89 square meters, Find the dimension of the rectangle.
Sol:
Let the length and the breadth of the rectangle be x m and y m, respectively.
Case 1: When length is increased by 3m and the breadth is decreased by 4m:
$xy - (x + 3)(y - 4) = 67$

$\Rightarrow xy - xy + 4x - 3y + 12 = 67$

$\Rightarrow 4x - 3y = 55$ ………(i)
Case 2: When length is reduced by 1m and breadth is increased by 4m:
$(x - 1)(y + 4) - xy = 89$

$\Rightarrow xy + 4x - y - 4 - xy = 89$

$\Rightarrow 4x - y = 93$ ………(ii)
Subtracting (i) and (ii), we get:

$2y = 38 \Rightarrow y = 19$
On substituting y = 19 in (ii), we have
$4x - 19 = 93$

$\Rightarrow 4x = 93 + 19 = 112$

$\Rightarrow x = 28$
Hence, the length = 28m and breadth = 19m.

72. A railway half ticket costs half the full fare and the reservation charge is the some on half ticket as on full ticket. One reserved first class ticket from Mumbai to Delhi costs ₹4150 while one full and one half reserved first class ticket cost ₹ 6255. What is the basic first class full fare and what is the reservation charge?

Sol:

Let the basic first class full fare be Rs.x and the reservation charge be Rs.y.

Case 1: One reservation first class full ticket cost Rs.4, 150

$x + y = 4150$ ………(i)

Case 2: One full and one and half reserved first class tickets cost Rs.6,255

$(x + y) + \left(\frac{1}{2}x + y\right) = 6255$

$\Rightarrow 3x + 4y = 12510$ ………(ii)

Substituting $y = 4150 - x$ from (i) in (ii), we get

$3x + 4(4150 - x) = 12510$

$\Rightarrow 3x - 4x + 16600 = 12510$

$\Rightarrow x = 16600 - 12510 = 4090$

Now, putting $x = 4090$ in (i), we have

$4090 + y = 4150$

$\Rightarrow y = 4150 - 4090 = 60$

Hence, cost of basic first class full fare = Rs.4,090 and reservation charge = Rs.60.

73. Five years hence, a man's age will be three times the sum of the ages of his son. Five years ago, the man was seven times as old as his son. Find their present ages

Sol:

Let the present age of the man be x years and that of his son be y years.

After 5 years man's age = $x + 5$

After 5 years ago son's age = $y + 5$

As per the question

$x + 5 = 3(y + 5)$

$\Rightarrow x - 3y = 10$ ……………(i)

5 years ago man's age = $x - 5$

5 years ago son's age = $y - 5$

As per the question

$x - 5 = 7(y - 5)$

$\Rightarrow x - 7y = -30$ …….(ii)

Subtracting (ii) from (i), we have

$4y = 40 \Rightarrow y = 10$

Putting $y = 10$ in (i), we get

$x - 3 \times 10 = 10$

$\Rightarrow x = 10 + 30 = 40$

Hence, man's present age = 40 years and son's present age = 10 years.

74. The present age of a man is 2 years more than five times the age of his son. Two years hence, the man's age will be 8 years more than three times the age of his son. Find their present ages.

Sol:

Let the man's present age be x years.
Let his son's present age be y years.
According to the question, we have:
Two years ago:
Age of the man = Five times the age of the son

$\Rightarrow (x - 2) = 5(y - 2)$

$\Rightarrow x - 2 = 5y - 10$

$\Rightarrow x - 5y = -8$ …….(i)

Two years later:
Age of the man = Three times the age of the son + 8

$\Rightarrow (x + 2) = 3(y + 2) + 8$

$\Rightarrow x + 2 = 3y + 6 + 8$

$\Rightarrow x - 3y = 12$ …………(ii)

Subtracting (i) from (ii), we get:
$2y = 20$

$\Rightarrow y = 10$

On substituting y = 10 in (i), we get:

$x - 5 \times 10 = -8$

$\Rightarrow x - 50 = -8$

$\Rightarrow x = (-8 + 50) = 42$

Hence, the present age of the man is 42 years and the present age of the son is 10 years.

75. If twice the son's age in years is added to the mother's age, the sum is 70 years. But, if twice the mother's age is added to the son's age, the sum is 95 years. Find the age of the mother and that of the son.

Sol:

Let the mother's present age be x years.
Let her son's present age be y years.
Then, we have:
$x + 2y = 70$ …….(i)
And, $2x + y = 95$ ……(ii)

On multiplying (ii) by 2, we get:
$4x + 2y = 190$ ……..(iii)
On subtracting (i) from (iii), we get:
$3x = 120$
$\Rightarrow x = 40$
On substituting $x = 40$ in (i), we get:
$40 + 2y = 70$
$\Rightarrow 2y = (70 - 40) = 30$
$\Rightarrow y = 15$
Hence, the mother's present age is 40 years and her son's present age is 15 years.

76. The present age of a woman is 3 years more than three times the age of her daughter. Three years hence, the woman's age will be 10 years more than twice the age of her daughter. Find their present ages.
Sol:
Let the woman's present age be x years.
Let her daughter's present age be y years.
Then, we have:
$x = 3y + 3$
$\Rightarrow x - 3y = 3$ …….(i)
After three years, we have:
$(x + 3) = 2(y + 3) + 10$
$\Rightarrow x + 3 = 2y + 6 + 10$
$\Rightarrow x - 2y = 13$ ……(ii)
Subtracting (ii) from (i), we get:
$-y = (3 - 13) = -10$
$\Rightarrow y = 10$
On substituting $y = 10$ in (i), we get:
$x - 3 \times 10 = 3$
$\Rightarrow x - 30 = 3$
$\Rightarrow x = (3 + 30) = 33$
Hence, the woman's present age is 33 years and her daughter's present age is 10 years.

77. On selling a tea-set at 5% loss and a lemon-set at 15% gain, a shopkeeper gains Rs. 7. However, if he sells the tea-set at 5% gain and the lemon-set at 10% gain, he gains Rs. 14. Find the price of the tea-set and that of the lemon-set paid by the shopkeeper.
Sol:

Let the actual price of the tea and lemon set be Rs.x and Rs.y respectively.
When gain is Rs.7, then
$\frac{y}{100} \times 15 - \frac{x}{100} \times 5 = 7$
$\Rightarrow 3y - x = 140$ ……..(i)
When gain is Rs.14, then
$\frac{y}{100} \times 5 + \frac{x}{100} \times 10 = 14$
$\Rightarrow y + 2x = 280$ ……..(ii)
Multiplying (i) by 2 and adding with (ii), we have
$7y = 280 + 280$
$\Rightarrow y = \frac{560}{7} = 80$
Putting y = 80 in (ii), we get
$80 + 2x = 280$
$\Rightarrow x = \frac{200}{2} = 100$
Hence, actual price of the tea set and lemon set are Rs.100 and Rs.80 respectively.

78. A lending library has fixed charge for the first three days and an additional charge for each day thereafter. Mona paid ₹27 for a book kept for 7 days, while Tanvy paid ₹21 for the book she kept for 5 days find the fixed charge and the charge for each extra day.
Sol:
Let the fixed charge be Rs.x and the charge for each extra day be Rs.y.
In case of Mona, as per the question
$x + 4y = 27$ ………(i)
In case of Tanvy, as per the question
$x + 2y = 21$ ………(ii)
Subtracting (ii) from (i), we get
$2y = 6 \Rightarrow y = 3$
Now, putting y = 3 in (ii), we have
$x + 2 \times 3 = 21$
$\Rightarrow x = 21 - 6 = 15$
Hence, the fixed charge be Rs.15 and the charge for each extra day is Rs.3.

79. A chemist has one solution containing 50% acid and a second one containing 25% acid. How much of each should be used to make 10 litres of a 40% acid solution?
Sol:
Let x litres and y litres be the amount of acids from 50% and 25% acid solutions respectively.
As per the question
50% of x + 25% of y = 40% of 10

$\Rightarrow 0.50x + 0.25y = 4$

$\Rightarrow 2x + y = 16$ ………(i)

Since, the total volume is 10 liters, so

$x + y = 10$

Subtracting (ii) from (i), we get

$x = 6$

Now, putting $x = 6$ in (ii), we have

$6 + y = 10 \Rightarrow y = 4$

Hence, volume of 50% acid solution = 6litres and volume of 25% acid solution = 4litres.

80. A jeweler has bars of 18-carat gold and 12-carat gold. How much of each must be melted together to obtain a bar of 16-carat gold, weighing 120gm? (Given: Pure gold is 24-carat).

Sol:

Let x g and y g be the weight of 18-carat and 12- carat gold respectively.

As per the given condition

$\frac{18x}{24} + \frac{12y}{24} = \frac{120 \times 16}{24}$

$\Rightarrow 3x + 2y = 320$ …………(i)

And

$x + y = 120$ …………(ii)

Multiplying (ii) by 2 and subtracting from (i), we get

$x = 320 – 240 = 80$

Now, putting $x = 80$ in (ii), we have

$80 + y = 120 \Rightarrow y = 40$

Hence, the required weight of 18-carat and 12-carat gold bars are 80 g and 40 g respectively.

81. 90% and 97% pure acid solutions are mixed to obtain 21 litres of 95% pure acid solution. Find the quantity of each type of acid to be mixed to form the mixture.

Sol:

Let x litres and y litres be respectively the amount of 90% and 97% pure acid solutions.

As per the given condition

$0.90x + 0.97y = 21 \times 0.95$

$\Rightarrow 0.90x + 0.97y = 21 \times 0.95$ ……….(i)

And

$x + y = 21$

From (ii), substitute $y = 21 – x$ in (i) to get

$0.90x + 0.97(21 – x) = 21 \times 0.95$

$\Rightarrow 0.90x + 0.97\times 21 – 0.97x = 21 \times 0.95$

$\Rightarrow 0.07x = 0.97 \times 21 - 21 \times 0.95$

$\Rightarrow x = \frac{21 \times 0.02}{0.07} = 6$

Now, putting x = 6 in (ii), we have

$6 + y = 21 \Rightarrow y = 15$

Hence, the request quantities are 6 litres and 15 litres.

82. The larger of the two supplementary angles exceeds the smaller by 180^0. Find them.

Sol:

Let x and y be the supplementary angles, where $x > y$.

As per the given condition

$x + y = 180^0$ ....(i)

And

$x - y = 18^0$ ....(ii)

Adding (i) and (ii), we get

$2x = 198^0 \Rightarrow x = 99^0$

Now, substituting $x = 99^0$ in (ii), we have

$99^0 - y = 18^0 \Rightarrow x = 99^0 - 18^0 = 81^0$

Hence, the required angles are 99^0 and 81^0.

83. In a $\Delta ABC, \angle A = x°, \angle B = (3x-2)°, \angle C = y°$ *and* $\angle C - \angle B = 9°$. Find the there angles.

Sol:

$\because \angle C - \angle B = 9^0$

$\therefore y^0 - (3x - 2)^0 = 9^0$

$\Rightarrow y^0 - 3x^0 + 2^0 = 9^0$

$\Rightarrow y^0 - 3x^0 = 7^0$

The sum of all the angles of a triangle is 180^0, therefore

$\angle A + \angle B + \angle C = 180^0$

$\Rightarrow x^0 + (3x - 2)^0 + y^0 = 180^0$

$\Rightarrow 4x^0 + y^0 = 182^0$

Subtracting (i) from (ii), we have

$7x^0 = 182^0 - 7^0 = 175^0$

$\Rightarrow x^0 = 25^0$

Now, substituting $x^0 = 25^0$ in (i), we have

$y^0 = 3x^0 + 7^0 = 3 \times 25^0 + 7^0 = 82^0$

Thus

$\angle A = x^0 = 25^0$

$\angle B = (3x - 2)^0 = 75^0 - 2^0 = 73^0$
$\angle C = y^0 = 82^0$
Hence, the angles are 25^0, 73^0 and 82^0.

84. In a cyclic quadrilateral ABCD, it is given $\angle A = (2x + 4)^0$, $\angle B = (y + 3)^0$, $\angle C = (2y + 10)^0$ and $\angle D = (4x - 5)^0$. Find the four angles.
Sol:
The opposite angles of cyclic quadrilateral are supplementary, so
$\angle A + \angle C = 180^0$
$\Rightarrow (2x + 4)^0 + (2y + 10)^0 = 180^0$
$\Rightarrow x + y = 83^0$
And
$\angle B + \angle D = 180^0$
$\Rightarrow (y + 3)^0 + (4x - 5)^0 = 180^0$
$\Rightarrow 4x + y = 182^0$
Subtracting (i) from (ii), we have
$3x = 99 \Rightarrow x = 33^0$
Now, substituting $x = 33^0$ in (i), we have
$33^0 + y = 83^0 \Rightarrow y = 83^0 - 33^0 = 50^0$
Therefore
$\angle A = (2x + 4)^0 = (2 \times 33 + 4)^0 = 70^0$
$\angle B = (y + 3)^0 = (50 + 3)^0 = 53^0$
$\angle C = (2y + 10)^0 = (2 \times 50 + 10)^0 = 110^0$
$\angle D = (4x - 5)^0 = (4 \times 33 - 5)^0 = 132^0 - 5^0 = 127^0$
Hence, $\angle A = 70^0$, $\angle B = 53^0$, $\angle C = 110^0$ and $\angle D = 127^0$.

Exercise – 3F

1. Write the number of solutions of the following pair of linear equations:
$x + 2y - 8 = 0$,
$2x + 4y = 16$
Sol:
The given equations are
$x + 2y - 8 = 0$ ……(i)
$2x + 4y - 16 = 0$ ……(ii)
Which is of the form $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, where
$a_1 = 1$, $b_1 = 2$, $c_1 = -8$, $a_2 = 2$, $b_2 = 4$ and $c_2 = -18$

Now

$\frac{a_1}{a_2} = \frac{1}{2}$

$\frac{b_1}{b_2} = \frac{2}{4} = \frac{1}{2}$

$\frac{c_1}{c_2} = \frac{-8}{-16} = \frac{1}{2}$

$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{2}$

Thus, the pair of linear equations are coincident and therefore has infinitely many solutions.

2. Find the value of k for which the system of linear equations has an infinite number of solutions.

2x + 3y – 7 = 0,

(k – 1)x + (k + 2)y=3k

Sol:

The given equations are

2x + 3y – 7 = 0 ……(i)

(k – 1)x + (k + 2)y – 3k = 0 ……(ii)

Which is of the form $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, where

$a_1 = 2$, $b_1 = 3$, $c_1 = -7$, $a_2 = k – 1$, $b_2 = k + 2$ and $c_2 = -3k$

For the given pair of linear equations to have infinitely many solutions, we must have

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{2}{k-1} = \frac{3}{k+2} = \frac{-7}{-3k}$

$\Rightarrow \frac{2}{k-1} = \frac{3}{k+2}$, $\frac{3}{k+2} = \frac{-7}{-3k}$ and $\frac{2}{k-1} = \frac{-7}{-3k}$

⇒ 2(k + 2) = 3(k – 1), 9k = 7k + 14 and 6k = 7k – 7

⇒ k = 7, k = 7 and k = 7

Hence, k = 7.

3. Find the value of k for which the system of linear equations has an infinite number of solutions.

10x + 5y – (k – 5) = 0,

20x + 10y – k = 0.

Sol:

The given pair of linear equations are

10x + 5y – (k – 5) = 0 ……(i)

20x + 10y – k = 0 ……(ii)

Which is of the form $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, where

$a_1 = 10$, $b_1 = 5$, $c_1 = -(k – 5)$, $a_2 = 20$, $b_2 = 10$ and $c_2 = -k$

For the given pair of linear equations to have infinitely many solutions, we must have

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{10}{20} = \frac{5}{10} = \frac{-(k-5)}{-k}$

$\Rightarrow \frac{1}{2} = \frac{k-5}{k}$

$\Rightarrow$ 2k – 10 = k $\Rightarrow$ k = 10

Hence, k = 10.

4. Find the value of k for which the system of linear equations has an infinite number of solutions.

2x + 3y=9,

6x + (k – 2)y =(3k – 2

Sol:

The given pair of linear equations are

2x + 3y – 9 = 0 ……(i)

6x + (k – 2)y – (3k – 2) = 0 ……(ii)

Which is of the form $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, where

$a_1 = 2$, $b_1 = 3$, $c_1 = -9$, $a_2 = 6$, $b_2 = k - 2$ and $c_2 = -(3k - 2)$

For the given pair of linear equations to have infinitely many solutions, we must have

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \frac{2}{6} = \frac{3}{k-2} \neq \frac{-9}{-(3k-2)}$

$\Rightarrow \frac{2}{6} = \frac{3}{k-2}, \frac{3}{k-2} \neq \frac{-9}{-(3k-2)}$

$\Rightarrow$ k = 11, $\frac{3}{k-2} \neq \frac{9}{(3k-2)}$

$\Rightarrow$ k = 11, 3(3k – 2) ≠ 9(k – 2)

$\Rightarrow$ k = 11, 1 ≠ 3 (true)

Hence, k = 11.

5. Write the number of solutions of the following pair of linear equations:

x + 3y – 4 = 0, 2x + 6y – 7 = 0.

Sol:

The given pair of linear equations are

x + 3y – 4 = 0 ……(i)

2x + 6y – 7 = 0 ……(ii)

Which is of the form $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, where

$a_1 = 1$, $b_1 = 3$, $c_1 = -4$, $a_2 = 2$, $b_2 = 6$ and $c_2 = -7$

Now

$\frac{a_1}{a_2} = \frac{1}{2}$

$\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$

$\frac{c_1}{c_2} = \frac{-4}{-7} = \frac{4}{7}$

$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Thus, the pair of the given linear equations has no solution.

6. Find the values of k for which the system of equations 3x + ky = 0, 2x – y = 0 has a unique solution.

Sol:

The given pair of linear equations are

3x + ky = 0 ……(i)

2x – y = 0 ……(ii)

Which is of the form $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, where

$a_1 = 3$, $b_1 = k$, $c_1 = 0$, $a_2 = 2$, $b_2 = -1$ and $c_2 = 0$

For the system to have a unique solution, we must have

$\frac{a_1}{a_2} = \frac{b_1}{b_2}$

$\Rightarrow \frac{3}{2} \neq \frac{k}{-1}$

$\Rightarrow k \neq -\frac{3}{2}$

Hence, $k \neq -\frac{3}{2}$.

7. The difference of two numbers is 5 and the difference between their squares is 65. Find the numbers.

Sol:

Let the numbers be x and y, where x > y.

Then as per the question

x – y = 5 …….(i)

$x^2 – y^2 = 65$ …….(ii)

Dividing (ii) by (i), we get

$\frac{x^2 - y^2}{x-y} = \frac{65}{5}$

$\Rightarrow \frac{(x-y)(x+y)}{x-y} = 13$

$\Rightarrow$ x + y = 13 ……….(iii)

Now, adding (i) and (ii), we have

2x = 18 $\Rightarrow$ x = 9

Substituting x = 9 in (iii), we have

9 + y = 13 $\Rightarrow$ y = 4

Hence, the numbers are 9 and 4.

8. The cost of 5 pens and 8 pencils together cost Rs. 120 while 8 pens and 5 pencils together cost Rs. 153. Find the cost of a 1 pen and that of a 1pencil.

Sol:

Let the cost of 1 pen and 1 pencil are ₹x and ₹y respectively.

Then as per the question

$5x + 8y = 120$ …….(i)

$8x + 5y = 153$ …….(ii)

Adding (i) and (ii), we get

$13x + 13y = 273$

$\Rightarrow x + y = 21$ …….(iii)

Subtracting (i) from (ii), we get

$3x - 3y = 33$

$\Rightarrow x - y = 11$ ………(iv)

Now, adding (iii) and (iv), we get

$2x = 32 \Rightarrow x = 16$

Substituting x = 16 in (iii), we have

$16 + y = 21 \Rightarrow y = 5$

Hence, the cost of 1 pen and 1 pencil are respectively ₹16 and ₹5.

9. The sum of two numbers is 80. The larger number exceeds four times the smaller one by 5. Find the numbers.

Sol:

Let the larger number be x and the smaller number be y.

Then as per the question

$x + y = 80$ …….(i)

$x = 4y + 5$

$x - 4y = 5$ …….(ii)

Subtracting (ii) from (i), we get

$5y = 75 \Rightarrow y = 15$

Now, putting y = 15 in (i), we have

$x + 15 = 80 \Rightarrow x = 65$

Hence, the numbers are 65 and 15.

10. A number consists of two digits whose sum is 10. If 18 is subtracted form the number, its digits are reversed. Find the number.

Sol:

Let the ones digit and tens digit be x and y respectively.

Then as per the question

$x + y = 10$ …….(i)
$(10y + x) - 18 = 10x + y$
$x - y = -2$ …….(ii)
Adding (i) and (ii), we get
$2x = 8 \Rightarrow x = 4$
Now, putting $x = 4$ in (i), we have
$4 + y = 10 \Rightarrow y = 6$
Hence, the number is 64.

11. A man purchased 47 stamps of 20p and 25p for ₹10. Find the number of each type of stamps
Sol:
Let the number of stamps of 20p and 25p be x and y respectively.
Then as per the question
$x + y = 47$ …….(i)
$0.20x + 0.25y = 10$
$4x + 5y = 200$ …….(ii)
From (i), we get
$y = 47 - x$
Now, substituting $y = 47 - x$ in (ii), we have
$4x + 5(47 - x) = 200$
$\Rightarrow 4x - 5x + 235 = 200$
$\Rightarrow x = 235 - 200 = 35$
Putting $x = 35$ in (i), we get
$35 + y = 47$
$\Rightarrow y = 47 - 35 = 12$
Hence, the number of 20p stamps and 25p stamps are 35 and 12 respectively.

12. A man has some hens and cows. If the number of heads be 48 and number of feet by 140. How many cows are there.
Sol:
Let the number of hens and cow be x and y respectively.
As per the question
$x + y = 48$ …….(i)
$2x + 4y = 140$
$x + 2y = 70$ …….(ii)
Subtracting (i) from (ii), we have
$y = 22$

Hence, the number of cows is 22.

13. If $\frac{2}{x}+\frac{3}{y}=-\frac{9}{xy}$ and $\frac{4}{x}+\frac{9}{y}=\frac{21}{xy}$, find the values of x and y.

Sol:

The given pair of equation is

$\frac{2}{x}+\frac{3}{y}=\frac{9}{xy}$ ………(i)

$\frac{4}{x}+\frac{9}{y}=\frac{21}{xy}$ ………(ii)

Multiplying (i) and (ii) by xy, we have

$3x + 2y = 9$ ……….(iii)

$9x + 4y = 21$ ………(iv)

Now, multiplying (iii) by 2 and subtracting from (iv), we get

$9x – 6x = 21 – 18 \Rightarrow x = \frac{3}{3} = 1$

Putting x = 1 in (iii), we have

$3 \times 1 + 2y = 9 \Rightarrow y = \frac{9-3}{2} = 3$

Hence, x = 1 and y = 3.

14. If $\frac{x}{4}+\frac{y}{3}=-\frac{15}{12}$ and $\frac{x}{2}+y=1,$ then find the value of (x + y).

Sol:

The given pair of equations is

$\frac{x}{4}+\frac{y}{3}=\frac{5}{12}$ ………(i)

$\frac{x}{2}+ y = 1$ ………(ii)

Multiplying (i) by 12 and (ii) by 4, we have

$3x + 4y = 5$ ……….(iii)

$2x + 4y = 4$ ………(iv)

Now, subtracting (iv) from (iii), we get

$x = 1$

Putting x = 1 in (iv), we have

$2 + 4y =4$

$\Rightarrow 4y = 2$

$\Rightarrow y = \frac{1}{2}$

$\therefore x + y = 1 + \frac{1}{2} = \frac{3}{2}$

Hence, the value of x + y is $\frac{3}{2}$.

15. If $12x + 17y = 53$ and $17x + 12y = 63$ then find the value of $(x + y)$

Sol:

The given pair of equations is

$12x + 17y = 53$ ……..(i)

$17x + 12y = 63$ …….(ii)

Adding (i) and (ii), we get

$29x + 29y = 116$

$\Rightarrow x + y = 4$ (Dividing by 4)

Hence, the value of $x + y$ is 4.

16. Find the value of k for which the system of equations $3x + 5y = 0$ and $kx + 10y = 0$ has infinite nonzero solutions.

Sol:

The given system is

$3x + 5y = 0$ ……(i)

$kx + 10y = 0$ ……(ii)

This is a homogeneous system of linear differential equation, so it always has a zero solution i.e., $x = y = 0$.

But to have a non-zero solution, it must have infinitely many solutions.

For this, we have

$\frac{a_1}{a_2} = \frac{b_1}{b_2}$

$\Rightarrow \frac{3}{k} = \frac{5}{10} = \frac{1}{2}$

$\Rightarrow k = 6$

Hence, $k = 6$.

17. Find the value of k for which the system of equations $kx – y = 2$ and $6x – 2y = 3$ has a unique solution.

Sol:

The given system is

$kx - y - 2 = 0$ ……(i)

$6x - 2y - 3 = 0$ ……(ii)

Here, $a_1 = k$, $b_1 = -1$, $c_1 = -2$, $a_2 = 6$, $b_2 = -2$ and $c_2 = -3$

For the system, to have a unique solution, we must have

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\Rightarrow \frac{k}{6} \neq \frac{-1}{-2} = \frac{1}{2}$

$\Rightarrow k \neq 3$

Hence, $k \neq 3$.

18. Find the value of k for which the system of equations 2x + 3y -5 = 0 and 4x + ky – 10 = 0 has infinite number of solutions.

Sol:

The given system is

2x + 3y – 5 = 0 ……(i)

4x + ky – 10 = 0 ……(ii)

Here, $a_1 = 2$, $b_1 = 3$, $c_1 = -5$, $a_2 = 4$, $b_2 = k$ and $c_2 = -10$

For the system, to have an infinite number of solutions, we must have

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{2}{4} = \frac{3}{k} = \frac{-5}{-10}$

$\Rightarrow \frac{1}{2} = \frac{3}{k} = \frac{1}{2}$

⇒ k = 6

Hence, k = 6.

19. Show that the system 2x + 3y -1= 0 and 4x + 6y - 4 = 0 has no solution.

Sol:

The given system is

2x + 3y – 1 = 0 ……(i)

4x + 6y – 4 = 0 ……(ii)

Here, $a_1 = 2$, $b_1 = 3$, $c_1 = -1$, $a_2 = 4$, $b_2 = 6$ and $c_2 = -4$

Now,

$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$

$\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$

$\frac{c_1}{c_2} = \frac{-1}{-4} = \frac{1}{4}$

Thus, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ and therefore the given system has no solution.

20. Find the value of k for which the system of equations x + 2y – 3 = 0 and 5x + ky + 7 = 0 is inconsistent.

Sol:

The given system is

x + 2y – 3 = 0 ……(i)

5x + ky + 7 = 0 ……(ii)

Here, $a_1 = 1$, $b_1 = 2$, $c_1 = -3$, $a_2 = 5$, $b_2 = k$ and $c_2 = 7$.

For the system, to be consistent, we must have

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \frac{1}{5} = \frac{2}{k} \neq \frac{-3}{7}$

$\Rightarrow \frac{1}{5} = \frac{2}{k}$
$\Rightarrow$ k = 10
Hence, k = 10.

21. Solve for x and y: $\frac{3}{x+y} + \frac{2}{x-y} = 2, \frac{9}{x+y} - \frac{4}{x-y} = 1$

Sol:
The given system of equations is
$\frac{3}{x+y} + \frac{2}{x-y} = 2$ ………..(i)
$\frac{9}{x+y} - \frac{4}{x-y} = 1$ ………..(ii)
Substituting $\frac{1}{x+y}$ = u and $\frac{1}{x-y}$ = v in (i) and (ii), the given equations are changed to
3u + 2v = 2 ………(iii)
9u – 4v = 1 ………(iv)
Multiplying (i) by 2 and adding it with (ii), we get
15u = 4 + 1 $\Rightarrow$ u = $\frac{1}{3}$
Multiplying (i) by 3 and subtracting (ii) from it, we get
6u + 4v = 6 – 1 $\Rightarrow$ u = $\frac{5}{10} = \frac{1}{2}$
Therefore
x + y = 3 ………….(v)
x – y = 2 …………(vi)
Now, adding (v) and (vi) we have
2x = 5 $\Rightarrow$ x = $\frac{5}{2}$
Substituting x = $\frac{5}{2}$ in (v), we have
$\frac{5}{2}$ + y = 3 $\Rightarrow$ y = 3 – $\frac{5}{2} = \frac{1}{2}$
Hence, x = $\frac{5}{2}$ and y = $\frac{1}{2}$.

Exercise – MCQ

1. If 2x+ 3y =12 and 3x - 2y =5 then
(a) x = 2, y = 3 (b) x = 2, y = -3 (c) x = 3, y = 2 (d) x = 3, y = -2
Answer: (c) x = 3, y = 2
Sol:
The given system of equations is
2x + 3y = 12 ……..(i)
3x – 2y = 5 ……..(ii)
Multiplying (i) by 2 and (ii) by 3 and then adding, we get

4x + 9x = 24 + 15

⇒ x = $\frac{39}{13}$ = 3

Now, putting x = 3 in (i), we have

2 × 3 + 3y = 12 ⇒ y = $\frac{12-6}{3}$ = 2

Thus, x = 3 and y = 2.

2. If x - y = 2 and $\frac{2}{x+y}=\frac{1}{5}$ then

(a) x = 4, y = 2 (b) x = 5, y = 3 (c) x = 6, y = 4 (d) x = 7, y = 5

Answer: (c) x = 6, y = 4

Sol:

The given system of equations is

x – y = 2 ……..(i)

x + y = 10 ……..(ii)

Adding (i) and (ii), we get

2x = 12 ⇒ x = 6

Now, putting x = 6 in (ii), we have

6 + y = 10 ⇒ y = 10 – 6 = 4

Thus, x = 6 and y = 4.

3. If $\frac{2x}{3}-\frac{y}{2}+\frac{1}{6}=0$ and $\frac{x}{2}+\frac{2y}{3}=3$ then

(a) x = 2, y = 3 (b) x = - 2, y = 3 (c) x = 2, y = - 3 (d) x = - 2, y = - 3

Answer: (a) x = 2, y = 3

Sol:

The given system of equations is

$\frac{2x}{3}-\frac{y}{2}=-\frac{1}{6}$ ……..(i)

$\frac{x}{2}+\frac{2y}{3}$ = 3 ……..(ii)

Multiplying (i) and (ii) by 6, we get

4x – 3y = –1 ……..(iii)

3x + 4y = 18 ………(iv)

Multiplying (iii) by 4 and (iv) by 3 and adding, we get

16x + 9x = –4 + 54

⇒ x = $\frac{50}{25}$ = 2

Now, putting x = 2 in (iv), we have

3 × 2 + 4y = 18 ⇒ y = $\frac{18-6}{4}$ = 3

Thus, x = 2 and y = 3.

4. If $\frac{1}{x}+\frac{2}{y}=4$ and $\frac{3}{y}-\frac{1}{x}=11$ then

(a) x = 2, y = 3 (b) x = - 2, y = 3 (c) x = $\frac{-1}{2}$, y = 3 (d) x = $\frac{-1}{2}$, y = $\frac{1}{3}$

Answer: (d) x = $\frac{-1}{2}$, y = $\frac{1}{3}$

Sol:

The given system of equations is

$\frac{1}{x}+\frac{2}{y}=4$ ……..(i)

$\frac{3}{y}-\frac{1}{x}=11$ ……..(ii)

Adding (i) and (ii), we get

$\frac{2}{y}+\frac{3}{y}=15$

$\Rightarrow \frac{5}{y}=15 \Rightarrow y=\frac{5}{15}=\frac{1}{3}$

Now, putting y = $\frac{1}{3}$ in (i), we have

$\frac{1}{x}+2\times 3=4 \Rightarrow \frac{1}{x}=4-6 \Rightarrow x=-\frac{1}{2}$

Thus, $x=-\frac{1}{2}$ and $y=\frac{1}{3}$.

5. If $\frac{2x+y+2}{5}=\frac{3x-y+1}{3}=\frac{3x+2y+1}{6}$ then

(a) x = 1, y = 1 (b) x = -1, y = -1 (c) x = 1, y = 2 (d) x = 2, y = 1

Answer: (a) x = 1, y = 1

Sol:

Consider $\frac{2x+y+2}{5}=\frac{3x-y+1}{3}$ and $\frac{3x-y+1}{3}=\frac{3x+2y+1}{3}$. Now, simplifying these equations, we get

$3(2x+y+2)=5(3x-y+1)$

$\Rightarrow 6x+3y+6=15x-5y+5$

$\Rightarrow 9x-8y=1$ ……….(i)

And

$6(3x-y+1)=3(3x+2y+1)$

$\Rightarrow 18x-6y+6=9x+6y+3$

$\Rightarrow 3x-4y=-1$ ……………(ii)

Multiplying (ii) by 2 and subtracting it from (i)

$9x-6x=1+2 \Rightarrow x=1$

Now, putting x = 1 in (ii), we have

$3\times 1-4y=-1 \Rightarrow y=\frac{3+1}{4}=1$

Thus, x = 1, y = 1.

6. If $\frac{3}{x+y}+\frac{2}{x-y}=2$ and $\frac{9}{x+y}-\frac{4}{x-y}=1$ then

(a) $x=\frac{1}{2}, y=\frac{3}{2}$ (b) $x=\frac{5}{2}, y=\frac{1}{2}$ (c) $x=\frac{3}{2}, y=\frac{1}{2}$ (d) $x=\frac{1}{2}, y=\frac{5}{2}$

Answer: (b) $x=\frac{5}{2}, y=\frac{1}{2}$

Sol:

The given equations are

$\frac{3}{x+y}+\frac{2}{x-y}=2$ ……..(i)

$\frac{9}{x+y}-\frac{4}{x-y}=1$ ……..(ii)

Substituting $\frac{1}{x+y}$ = u and $\frac{1}{x-y}$ = v in (i) and (ii), the new system becomes

3u + 2v = 2 ……..(iii)

9u – 4v = 1 …….(iv)

Now, multiplying (iii) by 2 and adding it with (iv), we get

6u + 9u = 4 + 1 ⇒ u = $\frac{5}{15}=\frac{1}{3}$

Again, multiplying (iii) by 2 and subtracting (iv) from , we get

6v + 4v = 6 – 1 ⇒ v = $\frac{5}{10}=\frac{1}{2}$

Therefore

x + y = 3 ……….(v)

x – y = 2 ………(vi)

Adding (v) and (vi), we get

2x = 3 + 2 ⇒ x = $\frac{5}{2}$

Substituting x = $\frac{5}{2}$, in (v), we have

$\frac{5}{2}$ + y = 3 ⇒ y = 3 – $\frac{5}{2}=\frac{1}{2}$.

Thus, x = $\frac{5}{2}$ and y = $\frac{1}{2}$.

7. If 4x+6y=3xy and 8x +9y=5xy then

(a) x = 2, y = 3 (b) x = 1, y = 2 (c) x = 3, y = 4 (d) x = 1, y = - 1

Answer: (c) x = 3, y = 4

Sol:

The given equations are

4x + 6y = 3xy ……..(i)

8x + 9y = 5xy ……..(ii)

Dividing (i) and (ii) by xy, we get

$\frac{6}{x}+\frac{4}{y}=3$ ………(iii)

$\frac{9}{x} + \frac{8}{y} = 5$ ………(iv)

Multiplying (iii) by 2 and subtracting (iv) from it, we get

$\frac{12}{x} - \frac{9}{x} = 6 – 5 \Rightarrow \frac{3}{x} = 1 \Rightarrow x = 3$

Substituting x = 3 in (iii), we get

$\frac{6}{3} + \frac{4}{y} = 3 \Rightarrow \frac{4}{y} = 1 \Rightarrow y = 4$

Thus, x = 3 and y = 4.

8. If 29x + 37y = 103 and 37x + 29y = 95 then

(a) x = 1, y = 2 (b) x = 2, y = 1 (c) x = 3, y = 2 (d) x = 2, y = 3

Answer: (a) x = 1, y = 2

Sol:

The given system of equations is

29x + 37y = 103 ……..(i)

37x + 29y = 95 ……..(ii)

Adding (i) and (ii), we get

66x + 66y = 198

⇒ x + y = 3 ………(iii)

Subtracting (i) from (ii), we get

8x – 8y = –8

⇒ x – y = –1

Adding (iii) and (iv), we get

2x = 2 ⇒ x = 1

Substituting x = 1 in (iii), we have

1 + y = 3 ⇒ y = 2

Thus, x = 1 and y = 2.

9. If $2^{x+y} = 2^{x-y} = \sqrt{8}$ then the value of y is

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) 0 (d) none of these

Answer: (c) 0

Sol:

$\because 2^{x+y} = 2^{x-y} = \sqrt{8}$

$\therefore$ x + y = x – y

⇒ y = 0

10. If $\frac{2}{x}+\frac{3}{y}=6$ and $\frac{1}{x}+\frac{1}{2y}=2$ then

(a) x = 1, y = $\frac{2}{3}$ (b) x = $\frac{2}{3}$, y = 1 (c) x = 1, y = $\frac{3}{2}$ (d) x= $\frac{3}{2}$, y = 1

Answer: (b) x = $\frac{2}{3}$, y = 1

Sol:

The given equations are

$\frac{2}{x}+\frac{3}{y}=6$ ……..(i)

$\frac{1}{x}+\frac{1}{2y}=2$ ……..(ii)

Multiplying (ii) by 2 and subtracting it from (ii), we get

$\frac{3}{y}-\frac{1}{y}=6-4$

$\Rightarrow \frac{2}{y}=2 \Rightarrow y=1$

Substituting y = 1 in (ii), we get

$\frac{1}{x}+\frac{1}{2}=2$

$\Rightarrow \frac{1}{x}=2-\frac{1}{2} \Rightarrow \frac{3}{2}$

$\Rightarrow x=\frac{2}{3}$.

11. The system of kx – y = 2 and 6x – 2y = 3 has a unique solution only when

(a) k = 0 (b) $k \neq 0$ (c) k = 3 (d) $k \neq 3$

Answer: (d) $k \neq 3$

Sol:

The given equations are

kx – y – 2 = 0 ………(i)

6x – 2y – 3 = 0 ……..(ii)

Here, $a_1 = k$, $b_1 = -1$, $c_1 = -2$, $a_2 = 6$, $b_2 = -2$ and $c_2 = -3$.

For the given system to have a unique solution, we must have

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\Rightarrow \frac{k}{6} \neq \frac{-1}{-2}$

$\Rightarrow k \neq 3$

12. The system x – 2y = 3 and 3x + ky = 1 have a unique solution only when ?

(a) k = -6 (b) k ≠ -6 (c) k = 0 (d) k ≠ 0

Answer: (b) k ≠ -6

Sol:

The correct option is (b).

The given system of equations can be written as follows:

x – 2y – 3 = 0 and 3x + ky – 1 = 0
The given equations are of the following form:
$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$
Here, $a_1 = 1$, $b_1 = -2$, $c_1 = -3$, $a_2 = 3$, $b_2 = k$ and $c_2 = -1$.
$\therefore \frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{-2}{k}$ and $\frac{c_1}{c_2} = \frac{-3}{-1} = 3$
These graph lines will intersect at a unique point when we have:
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow \frac{1}{3} \neq \frac{-2}{k} \Rightarrow k \neq -6$
Hence, k has all real values other than –6.

13. The system x + 2y = 3 and 5x + ky + 7 = 0 have no solution when?
(a) k = 10 (b) k ≠ 10 (c) $k = \frac{-7}{3}$ (d) k = -21
Answer: (a) k = 10
Sol:
The correct option is (a).
The given system of equations can be written as follows:
x + 2y – 3 = 0 and 5x + ky + 7 = 0
The given equations are of the following form:
$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$
Here, $a_1 = 1$, $b_1 = 2$, $c_1 = -3$, $a_2 = 5$, $b_2 = k$ and $c_2 = 7$.
$\therefore \frac{a_1}{a_2} = \frac{1}{5}, \frac{b_1}{b_2} = \frac{2}{k}$ and $\frac{c_1}{c_2} = \frac{-3}{7}$
For the system of equations to have no solution, we must have:
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
$\therefore \frac{1}{5} = \frac{2}{k} \neq \frac{-3}{7} \Rightarrow k = 10$

14. If the lines given by 3x + 2ky = 2 and 2x + 5y + 1 = 0 are parallel, then the value of k is
(a) $\frac{-5}{4}$ (b) $\frac{2}{5}$ (c) $\frac{3}{2}$ (d) $\frac{15}{4}$
Answer: (d) $\frac{15}{4}$
Sol:
The given system of equations can be written as follows:
3x + 2ky – 2 = 0 and 2x + 5y + 1 = 0
The given equations are of the following form:
$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$
Here, $a_1 = 3$, $b_1 = 2k$, $c_1 = -2$, $a_2 = 2$, $b_2 = 5$ and $c_2 = 1$
$\therefore \frac{a_1}{a_2} = \frac{3}{2}, \frac{b_1}{b_2} = \frac{2k}{5}$ and $\frac{c_1}{c_2} = \frac{-2}{1}$
For parallel lines, we have:
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\therefore \frac{3}{2} = \frac{2k}{5} \neq \frac{-2}{1}$
$\Rightarrow$ k = $\frac{15}{4}$

15. For what value of k do the equations kx – 2y = 3 and 3x + y = 5 represent two lines intersecting at a unique point?
(a) k = 3 (b) k = -3 (c) k = 6 (d) all real values except -6
Answer: (d) all real values except -6
Sol:
The given system of equations can be written as follows:
kx – 2y – 3 = 0 and 3x + y – 5 = 0
The given equations are of the following form:
$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$
Here, $a_1 = k$, $b_1 = -2$, $c_1 = -3$ and a2 = 3, $b_2 = 1$ and $c_2 = -5$
$\therefore \frac{a_1}{a_2} = \frac{k}{3}, \frac{b_1}{b_2} = \frac{-2}{1}$ and $\frac{c_1}{c_2} = \frac{-3}{-5} = \frac{3}{5}$
Thus, for these graph lines to intersect at a unique point, we must have:
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
$\Rightarrow \frac{k}{3} \neq \frac{-2}{1} \Rightarrow k \neq -6$
Hence, the graph lines will intersect at all real values of k except –6.

16. The pair of equations x + 2y + 5 = 0 and–3x – 6y + 1 = 0 has
(a) a unique solution (b) exactly two solutions
(c) infinitely many solutions (d) no solution
Answer: (d) no solution
Sol:
The given system of equations can be written as:
x + 2y + 5 = 0 and -3x - 6y + 1 = 0
The given equations are of the following form:
$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$
Here, $a_1 = 1$, $b_1 = 2$, $c_1 = 5$, $a_2 = -3$, $b_2 = -6$ and $c_2 = 1$
$\therefore \frac{a_1}{a_2} = \frac{1}{-3}, \frac{b_1}{b_2} = \frac{2}{6} = \frac{1}{-3}$ and $\frac{c_1}{c_2} = \frac{5}{1}$
$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
Hence, the given system has no solution.

17. The pair of equations 2x + 3y = 5 and 4x + 6y = 15 has
(a) a unique solution (b) exactly two solutions
(c) infinitely many solutions (d) no solution
Answer: (d) no solution

Sol:
The given system of equations can be written as:
$2x + 3y - 5 = 0$ and $4x + 6y - 15 = 0$
The given equations are of the following form:
$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$
Here, $a_1 = 2$, $b_1 = 3$, $c_1 = -5$, $a_2 = 4$, $b_2 = 6$ and $c_2 = -15$
$\therefore \frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$ and $\frac{c_1}{c_2} = \frac{-5}{-15} = \frac{1}{3}$
$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
Hence, the given system has no solution.

18. If a pair of linear equations is consistent, then their graph lines will be
(a) parallel (b) always coincident
(c) always intersecting (d) intersecting or coincident
Answer: (d) intersecting or coincident
Sol:
If a pair of linear equations is consistent, then the two graph lines either intersect at a point or coincidence.

19. If a pair of linear equations is inconsistent, then their graph lines will be
(a) parallel (b) always coincident
(c) always intersecting (d) intersecting or coincident
Answer: (a) parallel
Sol:
If a pair of linear equations in two variables is inconsistent, then no solution exists as they have no common point. And, since there is no common solution, their graph lines do not intersect. Hence, they are parallel.

20. In a ΔABC, $\angle C = 3\angle B = 2(\angle A + \angle B)$, then $\angle B$ = ?
(a) 20^0 (b) 40^0 (c) 60^0 (d) 80^0
Answer: (b) 40^0
Sol:
Let $\angle A = x^0$ and $\angle B = y^0$
$\therefore \angle A = 3\angle B = (3y)^0$
Now, $\angle A + \angle B + \angle C = 180^0$
$\Rightarrow x + y + 3y = 180$
$\Rightarrow x + 4y = 180$ ……(i)
Also, $\angle C = 2(\angle A + \angle B)$
$\Rightarrow 3y = 2(x + y)$
$\Rightarrow 2x – y = 0$ ……..(ii)

On multiplying (ii) by 4, we get:
$8x - 4y = 0$ …….(iii)
On adding (i) and (iii) we get:
$9x = 180 \Rightarrow x = 20$
On substituting x = 20 in (i), we get:
$20 + 4y = 180 \Rightarrow 4y = (180 - 20) = 160 \Rightarrow y = 40$
$\therefore x = 20$ and $y = 40$
$\therefore \angle B = y^0 = 40^0$

21. In a cyclic quadrilateral ABCD, it is being given that $\angle A = (x + y + 10)^0$, $\angle B = (y + 20)^0$, $\angle C = (x + y - 30)^0$ and $\angle D = (x + y)^0$. Then, $\angle B$ = ?
(a) 70^0 (b) 80^0 (c) 100^0 (d) 110^0
Answer: (b) 80^0
Sol:
The correct option is (b).
In a cyclic quadrilateral ABCD:
$\angle A = (x + y + 10)^0$
$\angle B = (y + 20)^0$
$\angle C = (x + y - 30)^0$
$\angle D = (x + y)^0$
We have:
$\angle A + \angle C = 180^0$ and $\angle B + \angle D = 180^0$ [Since ABCD is a cyclic quadrilateral]
Now, $\angle A + \angle C = (x + y + 10)^0 + (x + y - 30)^0 = 180^0$
$\Rightarrow 2x + 2y - 20 = 180$
$\Rightarrow x + y - 10 = 90$
$\Rightarrow x + y = 160$ ……(i)
Also, $\angle B + \angle D = (y + 20)^0 + (x + y)^0 = 180^0$
$\Rightarrow x + 2y + 20 = 180$
$\Rightarrow x + 2y = 160$
On subtracting (i) from (ii), we get:
$y = (160 - 100) = 60$
On substituting y = 60 in (i), we get:
$x + 60 = 100 \Rightarrow x = (100 - 60) = 40$
$\therefore \angle B = (y + 20)^0 = (60 + 20)^0 = 80^0$

22. 5 years hence, the age of a man shall be 3 times the age of his son while 5 years earlier the age of the man was 7 times the age of his son. The present age of the man is
(a) 45 years (b) 50 years (c) 47 years (d) 40 years
Answer: (d) 40 years
Sol:

Let the man's present age be x years.
Let his son's present age be y years.
Five years later:
$(x + 5) = 3(y + 5)$
$\Rightarrow x + 5 = 3y + 15$
$\Rightarrow x - 3y = 10$ ………..(i)
Five years ago:
$(x - 5) = 7(y - 5)$
$\Rightarrow x - 5 = 7y - 35$
$\Rightarrow x - 7y = -30$ ………..(ii)
On subtracting (i) from (ii), we get:
$-4y = -40 \Rightarrow y = 10$
On substituting y = 10 in (i), we get:
$x - 3 \times 10 = 10 \Rightarrow x - 30 = 10 \Rightarrow x = (10 + 30) = 40$ years
Hence, the man's present age is 40 years.

23.

Assertion (A)	Reason (R)
The system of equations $x + y - 8 = 0$ and $x - y - 2 = 0$ has a unique solutions.	The system of equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ has a unique solution when $\frac{a_1}{a_2} = \frac{b_1}{b_2}$.

The correct answer is: (a) / (b)/ (c)/ (d).
Answer: (c)
Sol:
Option (c) is the correct answer.
Clearly, Reason (R) is false.
On solving $x + y = 8$ and $x - y = 2$, we get:
$x = 5$ and $y = 3$
Thus, the given system has a unique solution. So, assertion (A) is true.
∴ Assertion (A) is true and Reason (R) is false.

24. The graphs of the equations $6x - 2y + 9 = 0$ and $3x - y + 12 = 0$ are two lines which are
(a) coincident
(b) parallel
(c) intersecting exactly at one point
(d) perpendicular to each other
Answer: (b) parallel
Sol:

The given equations are as follows:
6x - 2y + 9 = 0 and 3x - y + 12 = 0
They are of the following form:
$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$
Here, $a_1 = 6$, $b_1 = -2$, $c_1 = 9$ and $a_2 = 3$, $b_2 = -1$ and $c_2 = 12$
$\therefore \frac{a_1}{a_2} = \frac{6}{3} = \frac{2}{1}, \frac{b_1}{b_2} = \frac{-2}{-1} = \frac{2}{1}$ and $\frac{c_1}{c_2} = \frac{9}{12} = \frac{3}{4}$
$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
The given system has no solution.
Hence, the lines are parallel.

25. The graphs of the equations 2x + 3y – 2 = 0 and x – 2y – 8 = 0 are two lines which are
(a) coincident
(b) parallel
(c) intersecting exactly at one point
(d) perpendicular to each other
Answer:
Sol:
The given equations are as follows:
2x + 3y – 2 = 0 and x – 2y – 8 = 0
They are of the following form:
$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$
Here, $a_1 = 2$, $b_1 = 3$, $c_1 = -2$ and $a_2 = 1$, $b_2 = -2$ and $c_2 = -8$
$\therefore \frac{a_1}{a_2} = \frac{2}{1}, \frac{b_1}{b_2} = \frac{3}{-2}$ and $\frac{c_1}{c_2} = \frac{-2}{-8} = \frac{1}{4}$
$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
The given system has a unique solution.
Hence, the lines intersect exactly at one point.

26. The graphs of the equations 5x – 15y = 8 and 3x – 9y = $\frac{24}{5}$ are two lines which are
(a) coincident
(b) parallel
(c) intersecting exactly at one point
(d) perpendicular to each other
Answer: (a) coincident
Sol:
The correct option is (a).
The given system of equations can be written as follows:
5x - 15y - 8 = 0 and 3x - 9y - $\frac{24}{5}$ = 0
The given equations are of the following form:

$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

Here, $a_1 = 5$, $b_1 = -15$, $c_1 = -8$ and $a_2 = 3$, $b_2 = -9$ and $c_2 = -\frac{24}{5}$

$\therefore \frac{a_1}{a_2} = \frac{5}{3}, \frac{b_1}{b_2} = \frac{-15}{-9} = \frac{5}{3}$ and $\frac{c_1}{c_2} = -8 \times \frac{5}{-24} = \frac{5}{3}$

$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

The given system of equations will have an infinite number of solutions.

Hence, the lines are coincident.

27. The sum of the digits of a two digit number is 15. The number obtained by interchanging the digits exceeds the given number by 9. The number is

(a) 96 (b) 69 (c) 87 (d) 78

Answer: (a) 96

Sol:

Let the tens and the units digits of the required number be x and y, respectively.

Required number = (10x + y)

According to the question, we have:

$x + y = 15$ ……..(i)

Number obtained on reversing its digits = (10y + x)

$\therefore (10y + x) = (10x + y) + 9$

$\Rightarrow 10y + x – 10x – y = 9$

$\Rightarrow 9y – 9x = 9$

$\Rightarrow y – x = 1$ …….(ii)

On adding (i) and (ii), we get:

$2y = 16 \Rightarrow y = 8$

On substituting y = 8 in (i), we get:

$x + 8 = 15 \Rightarrow x = (15 – 8) = 7$

Number = $(10x + y) = 10 \times 7 + 8 = 70 + 8 = 78$

Hence, the required number is 78.

Exercise – Formative Assessment

1. The graphic representation of the equations $x + 2y = 3$ and $2x + 4y + 7 = 0$ gives a pair of

(a) parallel lines (b) intersecting lines

(c) coincident lines (d) none of these

Answer: (a) parallel lines

Sol:

The given system of equations can be written as follows:

$x + 2y – 3 = 0$ and $2x + 4y + 7 = 0$

The given equations are of the following form:

$a_1x + b_1x + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

Here, $a_1 = 1$, $b_1 = 2$, $c_1 = -3$ and $a_2 = 2$, $b_2 = 4$ and $c_2 = 7$

$\therefore \frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{2}{4} = \frac{1}{2}$ and $\frac{c_1}{c_2} = \frac{-3}{7}$

$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

So, the given system has no solution.

Hence, the lines are parallel.

2. If 2x – 3y = 7 and (a + b)x – (a + b – 3)y = (4a + b) have an infinite number of solutions, then

(a) a = 5, b = 1 (b) a = -5, b = 1

(c) a = 5, b = -1 (d) a = -5, b = -1

Answer: (d) a = -5, b = -1

Sol:

The given system of equations can be written as follows:

2x – 3y – 7 = 0 and (a + b)x – (a + b – 3)y – (4a + b) = 0

The given equations are of the following form:

$a_1x + b_1x + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

Here, $a_1 = 2$, $b_1 = -3$, $c_1 = -7$ and $a_2 = (a + b)$, $b_2 = -(a + b - 3)$ and $c_2 = -(4a + b)$

$\therefore \frac{a_1}{a_2} = \frac{2}{(a+b)}, \frac{b_1}{b_2} = \frac{-3}{-(a+b-3)} = \frac{3}{(a+b-3)}$ and $\frac{c_1}{c_2} = \frac{-7}{-(4a+b)} = \frac{7}{(4a+b)}$

For an infinite number of solutions, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\therefore \frac{2}{(a+b)} = \frac{3}{(a+b-3)} = \frac{7}{(4a+b)}$

Now, we have:

$\frac{2}{(a+b)} = \frac{3}{(a+b-3)} \Rightarrow 2a + 2b - 6 = 3a + 3b$

$\Rightarrow a + b + 6 = 0$ ……(i)

Again, we have:

$\frac{3}{(a+b-3)} = \frac{7}{(4a+b)} \Rightarrow 12a + 3b = 7a + 7b - 21$

$\Rightarrow 5a - 4b + 21 = 0$ …….(ii)

On multiplying (i) by 4, we get:

$4a + 4b + 24 = 0$ ……(iii)

On adding (ii) and (iii), we get:

$9a = -45 \Rightarrow a = -5$

On substituting a = -5 in (i), we get:

$-5 + b + 6 = 0 \Rightarrow b = -1$

$\therefore a = -5$ and $b = -1$.

3. The pair of equations $2x + y = 5$, $3x + 2y = 8$ has
(a) a unique solution (b) two solutions
(c) no solution (d) infinitely many solutions
Answer: (a) a unique solution
Sol:
The given system of equations can be written as follows:
$2x + y – 5 = 0$ and $3x + 2y – 8 = 0$
The given equations are of the following form:
$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$
Here, $a_1 = 2$, $b_1 = 1$, $c_1 = -5$ and $a_2 = 3$, $b_2 = 2$ and $c_2 = –8$
$\therefore \frac{a_1}{a_2} = \frac{2}{3}, \frac{b_1}{b_2} = \frac{1}{2}$ and $\frac{c_1}{c_2} = \frac{-5}{-8} = \frac{5}{8}$
$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
The given system has a unique solution.
Hence, the lines intersect at one point.

4. If $x = -y$ and $y > 0$, which of the following is wrong?
(a) $x^2y > 0$ (b) $x + y = 0$ (c) $xy < 0$ (d) $\frac{1}{x} - \frac{1}{y} = 0$
Answer: (d) $\frac{1}{x} - \frac{1}{y} = 0$
Sol:
Given:
$x = -y$ and $y > 0$
Now, we have:
(i) x^2y
On substituting $x = -y$, we get:
$(-y)^2y = y^3 > 0$ ($\because y > 0$)
This is true.

(ii) $x + y$
On substituting $x = -y$, we get:
$(-y) + y = 0$
This is also true.

(iii) xy
On substituting $x = -y$, we get:
$(-y)y = -y^2$ ($\because y > 0$)
This is again true.
(iv) $\frac{1}{x} - \frac{1}{y} = 0$

$\Rightarrow \frac{y-x}{xy} = 0$

On substituting x = -y, we get:

$\frac{y-(-y)}{(-y)y} = 0 \Rightarrow \frac{2y}{-y^2} = 0 \Rightarrow 2y = 0 \Rightarrow y = 0.$

5. Show that the system of equations $-x + 2y + 2 = 0$ and $\frac{1}{2}x - \frac{1}{4}y - 1 = 0$ has a unique solution.

Sol:

The given system of equations:

$-x + 2y + 2 = 0$ and $\frac{1}{2}x - \frac{1}{4}y - 1 = 0$

The given equations are of the following form:

$a_1x + b_1x + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

Here, $a_1 = -1$, $b_1 = 2$, $c_1 = 2$ and $a_2 = \frac{1}{2}$, $b_2 = -\frac{1}{4}$ and $c_2 = -1$

$\therefore \frac{a_1}{a_2} = \frac{-1}{(1/2)} = -2$, $\frac{b_1}{b_2} = \frac{2}{(-1/4)} = -8$ and $\frac{c_1}{c_2} = \frac{2}{-1} = -2$

$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

The given system has a unique solution.

Hence, the lines intersect at one point.

6. For what values of k is the system of equations $kx + 3y = (k - 2)$, $12x + ky = k$ inconsistent?

Sol:

The given system of equations can be written as follows:

$kx + 3y - (k - 2) = 0$ and $12x + ky - k = 0$

The given equations are of the following form:

$a_1x + b_1x + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

Here, $a_1 = k$, $b_1 = 3$, $c_1 = -(k - 2)$ and $a_2 = 12$, $b_2 = k$ and $c_2 = -k$

$\therefore \frac{a_1}{a_2} = \frac{k}{12}, \frac{b_1}{b_2} = \frac{3}{k}$ and $\frac{c_1}{c_2} = \frac{-(k-2)}{-k} = \frac{(k-2)}{k}$

For inconsistency, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \frac{k}{12} = \frac{3}{k} \neq \frac{(k-2)}{k} \Rightarrow k^2 = (3 \times 12) = 36$

$\Rightarrow k = \sqrt{36} = \pm 6$

Hence, the pair of equations is inconsistent if $k = \pm 6$.

7. Show that the equations $9x - 10y = 21$, $\frac{3x}{2} - \frac{5y}{3} = \frac{7}{2}$ have infinitely many solutions.

Sol:

The given system of equations can be written as follows:

$9x - 10y - 21 = 0$ and $\frac{3x}{2}x - \frac{5y}{3} - \frac{7}{2} = 0$

The given equations are of the following form:

$a_1x + b_1x + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

Here, $a_1 = 9$, $b_1 = -10$, $c_1 = -21$ and $a_2 = \frac{3}{2}$, $b_2 = \frac{-5}{3}$ and $c_2 = \frac{-7}{2}$

$\therefore \frac{a_1}{a_2} = \frac{9}{3/2} = 6$, $\frac{b_1}{b_2} = \frac{-10}{(-5/3)} = 6$ and $\frac{c_1}{c_2} = -21 \times \frac{2}{-7} = 6$

$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

This shows that the given system of equations has an infinite number of solutions.

8. Solve the system of equations: $x - 2y = 0$, $3x + 4y = 20$.

Sol:

The given equations are as follows:

$x - 2y = 0$ ....(i)

$3x + 4y = 20$ ....(ii)

On multiplying (i) by 2, we get:

$2x - 4y = 0$ ....(iii)

On adding (ii) and (iii), we get:

$5x = 20 \Rightarrow x = 4$

On substituting $x = 4$ in (i), we get:

$4 - 2y = 0 \Rightarrow 4 = 2y \Rightarrow y = 2$

Hence, the required solution is $x = 4$ and $y = 2$.

9. Show that the paths represented by the equations $x - 3y = 2$ and $-2x + 6y = 5$ are parallel.

Sol:

The given system of equations can be written as follows:

$x - 3y - 2 = 0$ and $-2x + 6y - 5 = 0$

The given equations are of the following form:

$a_1x + b_{1y} + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

Here, $a_1 = 1$, $b_1 = -3$, $c_1 = -2$ and $a_2 = -2$, $b_2 = 6$ and $c_2 = -5$

$\therefore \frac{a_1}{a_2} = \frac{1}{-2} = \frac{-1}{2}$, $\frac{b_1}{b_2} = \frac{-3}{6} = \frac{-1}{2}$ and $\frac{c_1}{c_2} = \frac{-2}{-5} = \frac{2}{5}$

$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Thus, the given system of equations has no solution.

Hence, the paths represented by the equations are parallel.

10. The difference between two numbers is 26 and one number is three times the other. Find the numbers.

Sol:

Let the larger number be x and the smaller number be y.

Then, we have:

$x - y = 26$ ………….(i)

$x = 3y$ …………(ii)

On substituting $x = 3y$ in (i), we get:

$3y - y = 26 \Rightarrow 2y = 26 \Rightarrow y = 13$

On substituting $y = 13$ in (i), we get:

$x - 13 = 26 \Rightarrow x = 26 + 13 = 39$

Hence, the required numbers are 39 and 13.

11. Solve: $23x + 29y = 98$, $29x + 23y = 110$.

Sol:

The given equations are as follows:

$23x + 29y = 98$ ………….(i)

$29x + 23y = 110$ …………(ii)

On adding (i) and (ii), we get:

$52x + 52y = 208$

$\Rightarrow x + y = 4$ …………(iii)

On subtracting (i) from (ii), we get:

$6x - 6y = 12$

$\Rightarrow x - y = 2$ …………(iv)

On adding (iii) and (iv), we get:

$2x = 6 \Rightarrow x = 3$

On substituting $x = 3$ in (iii), we get:

$3 + y = 4$

$\Rightarrow y = 4 - 3 = 1$

Hence, the required solution is $x = 3$ and $y = 1$.

12. Solve: $6x + 3y = 7xy$ and $3x + 9y = 11xy$

Sol:

The given equations are as follows:

$6x + 3y = 7xy$ ………….(i)

$3x + 9y = 11xy$ …………(ii)

For equation (i), we have:

$\frac{6x+3y}{xy} = 7$

$\Rightarrow \frac{6x}{xy} + \frac{3y}{xy} = 7 \Rightarrow \frac{6}{y} + \frac{3}{x} = 7$ …….(iii)

For equation (ii), we have:

$\frac{3x+9y}{xy} = 11$

$\Rightarrow \frac{3x}{xy} + \frac{9y}{xy} = 11 \Rightarrow \frac{3}{y} + \frac{9}{x} = 11$ …….(iii)

On substituting $\frac{3}{y} = v$ and $\frac{1}{x} = u$ in (iii) and (iv), we get:

$6v + 3u = 7$ ………..(v)

$3v + 9u = 11$ ……….(vi)

On multiplying (v) by 3, we get:

$18v + 9u = 21$ ………(vii)

On substituting $y = \frac{3}{2}$ in (iii), we get:

$\frac{6}{(3/2)} + \frac{3}{x} = 7$

$\Rightarrow 4 + \frac{3}{x} = 7 \Rightarrow \frac{3}{x} = 3 \Rightarrow 3x = 3$

$\Rightarrow x = 1$

Hence, the required solution is $x = 1$ and $y = \frac{3}{2}$.

13. Find the value of k for which the system of equations $3x + y = 1$ and $kx + 2y = 5$ has (i) a unique solution, (ii) no solution.

Sol:

The given system of equations can be written as follows:

$3x + y = 1$

$\Rightarrow 3x + y - 1 = 0$ ……….(i)

$kx + 2y = 5$

$\Rightarrow kx + 2y - 5 = 0$ ………(ii)

These equations are of the following form:

$a_1x + b_1x + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

Here, $a_1 = 3$, $b_1 = 1$, $c_1 = -1$ and $a_2 = k$, $b_2 = 2$ and $c_2 = -5$

(i) For a unique solution, we must have:

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ i.e. $\frac{3}{k} \neq \frac{1}{2} \Rightarrow k \neq 6$

Thus, for all real values of k other than 6, the given system of equations will have a unique solution.

(ii) In order that the given equations have no solution, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\frac{3}{k} = \frac{1}{2} \neq \frac{-1}{-5}$

$\Rightarrow \frac{3}{k} = \frac{1}{2}$ and $\frac{3}{k} = \frac{-1}{-5}$

$\Rightarrow k = 6, k \neq 15$
Thus, for k = 6, the given system of equations will have no solution.

14. In a ΔABC, $\angle C = 3\angle B = 2(\angle A + \angle B)$, find the measure of each one of $\angle A$, $\angle B$ and $\angle C$.
Sol:
Let $\angle A = x^0$ and $\angle B = y^0$
Then, $\angle C = 3\angle B = 3y^0$
Now, we have:
$\angle A + \angle B + \angle C = 180^0$
$\Rightarrow x + y + 3y = 180$
$\Rightarrow x + 4y = 180$ ……..(i)
Also, $\angle C = 2(\angle A + \angle B)$
$\Rightarrow 3y = 2(x + y)$
$\Rightarrow 2x - y = 0$ ……..(ii)
On multiplying (ii) by 4, we get:
$8x - 4y = 0$ ………(iii)
On adding (i) and (iii), we get:
$9x = 180 \Rightarrow x = 20$
On substituting x = 20 in (i), we get:
$20 + 4y = 180 \Rightarrow 4y = (180 - 20) = 160 \Rightarrow y = 40$
$\therefore x = 20$ and $y = 40$
$\therefore \angle A = 20^0, \angle B = 40^0, \angle C = (3 \times 40^0) = 120^0.$

15. 5 pencils and 7 pens together cost Rs 195 while 7 pencils and 5 pens together cost Rs 153. Find the cost of each one of the pencil and pen.
Sol:
Let the cost of each pencil be Rs. x and that of each pen be Rs. y.
Then, we have:
$5x + 7y = 195$ ………(i)
$7x + 5y = 153$ ………(ii)
Adding (i) and (ii), we get:
$12x + 12y = 348$
$\Rightarrow 12(x + y) = 348$
$\Rightarrow x + y = 29$ ………(iii)
Subtracting (i) from (ii), we get:
$2x - 2y = -42$

$\Rightarrow 2(x – y) = –42$

$\Rightarrow x – y = –21$ ………(iv)

On adding (iii) and (iv), we get:

$4 + y = 29 \Rightarrow y = (29 – 4) = 25$

Hence, the cost of each pencil is Rs. 4 and the cost of each pen is Rs. 25.

16. Solve the following system of equations graphically:

$$2x – 3y = 1, 4x – 3y + 1 = 0$$

Sol:

On a graph paper, draw a horizontal line X'OX and a vertical line YOY' as the x-axis and the y-axis, respectively.

Graph of $2x – 3y = 1$

$2x – 3y = 1$

$\Rightarrow 3y = (2x – 1)$

$\therefore y = \frac{2x-1}{3}$ ……..(i)

Putting x = -1, we get:

y = -1

Putting x = 2, we get:

y = 1

Putting x = 5, we get:

y = 3

Thus, we have the following table for the equation $2x – 3y = 1$.

x	-1	2	5
y	-1	1	3

Now, plots the points A(-1, -1), B(2, 1) and C(5, 3) on the graph paper.

Join AB and BC to get the graph line AC. Extend it on both the sides.

Thus, the line AC is the graph of $2x – 3y = 1$.

Graph of $4x – 3y + 1 = 0$

$4x – 3y + 1 = 0$

$\Rightarrow 3y = (4x + 1)$

$\therefore y = \frac{4x+1}{3}$ ……..(ii)

Putting x = -1, we get:

y = -1

Putting x = 2, we get:

y = 3

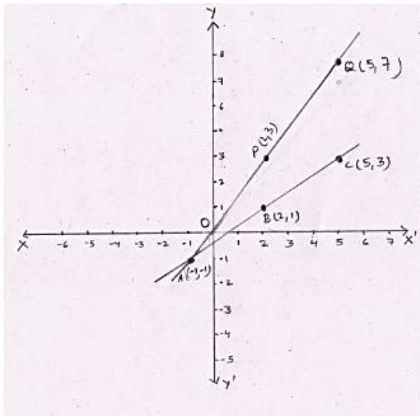
Putting x = 5, we get:

y = 7

Thus, we have the following table for the equation $4x – 3y + 1 = 0$.

x	-1	2	5
y	-1	3	7

Now, Plot the points P(2, 3) and Q(5, 7). The point A(-1, -1) has already been plotted. Join PA and QP to get the graph line AQ. Extend it on both sides.
Thus, the line AQ is the graph of the equation $4x - 3y + 1 = 0$.



The two lines intersect at A(-1. -1).
Thus, $x = -1$ and $y = -1$ is the solution of the given system of equations.

17. Find the angles of a cyclic quadrilateral ABCD in which $\angle A = (4x + 20)^0$, $\angle B = (3x - 5)^0$, $\angle C = 4y^0$ and $\angle D = (7y + 5)^0$.

Sol:
Given:
In a cyclic quadrilateral ABCD, we have:
$\angle A = (4x + 20)^0$
$\angle B = (3x - 5)^0$
$\angle C = 4y^0$
$\angle D = (7y + 5)^0$
$\angle A + \angle C = 180^0$ and $\angle B + \angle D = 180^0$ [Since ABCD is a cyclic quadrilateral]
Now, $\angle A + \angle C = (4x + 20)^0 + (4y^0) = 180^0$

$\Rightarrow 4x + 4y + 20 = 180$

$\Rightarrow 4x + 4y = 180 - 20 = 160$

$\Rightarrow x + y = 40$ …….(i)
Also, $\angle B + \angle D = (3x - 5)^0 + (7y + 5)^0 = 180^0$

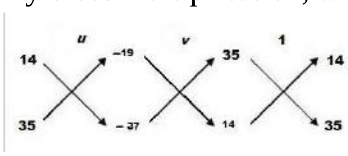
$\Rightarrow 3x + 7y = 180$ …….(ii)
On multiplying (i) by 3, we get:
$3x + 3y = 120$ ……(iii)
On subtracting (iii) from (ii), we get:

$4y = 60 \Rightarrow y = 15$
On substituting $y = 15$ in (i), we get:

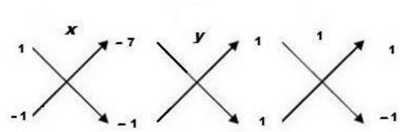
$x + 15 = 40 \Rightarrow x = (40 - 15) = 25$
Therefore, we have:
$\angle A = (4x + 20)^0 = (4 \times 25 + 20)^0 = 120°$
$\angle B = (3x - 5)^0 = (3 \times 25 - 5)^0 = 70°$
$\angle C = 4y^0 = (4 \times 15)^0 = 60^0$
$\angle D = (7y + 5)^0 = (7 \times 15 + 5)^0 = (105 + 5)^0 = 110^0$.

18. Solve for x and y: $\frac{35}{x+y} + \frac{14}{x-y} = 19, \frac{14}{x+y} + \frac{35}{x-y} = 37$

Sol:
We have:
$\frac{35}{x+y} + \frac{14}{x-y} = 19$ and $\frac{14}{x+y} + \frac{35}{x-y} = 37$
Taking $\frac{1}{x+y} = u$ and $\frac{1}{x-y} = v$.
$35u + 14v - 19 = 0$ ……….(i)
$14u + 35v - 37 = 0$ ……….(ii)
Here, $a_1 = 35, b_1 = 14, c_1 = -19$ and $a_2 = 14, b_2 = 35$ and $c_2 = -37$
By cross multiplication, we have:



$\therefore \frac{u}{[14 \times (-37) - 35 \times (-19)]} = \frac{v}{[(-19) \times 14 - (-37) \times (35)]} = \frac{1}{[35 \times 35 - 14 \times 14]}$
$\Rightarrow \frac{u}{-518+665} = \frac{v}{-266+1295} = \frac{1}{1225-196}$
$\Rightarrow \frac{u}{147} = \frac{v}{1029} = \frac{1}{1029}$
$\Rightarrow u = \frac{147}{1029} = \frac{1}{7}, v = \frac{1029}{1029} = 1$
$\Rightarrow \frac{1}{x+y} = \frac{1}{7}, \frac{1}{x-y} = 1$
$\therefore (x + y) = 7$ ………(iii)
And, $(x - y) = 1$ ……...(iv)
Again, the equations (iii) and (iv) can be written as follows:
$x + y - 7 = 0$ ……….(v)
$x - y - 1 = 0$ ………(vi)
Here, $a_1 = 1, b_1 = 1, c_1 = -7$ and $a_2 = 1, b_2 = -1$ and $c_2 = -1$
By cross multiplication, we have:



$\therefore \frac{x}{[1\times(-1)-(-1)\times(-7)]} = \frac{y}{[(-7)\times 1 \ -(-1)\times 1]} = \frac{1}{[1\times(-1) - 1\times 1]}$

$\Rightarrow \frac{x}{-1-7} = \frac{y}{-7+1} = \frac{1}{-1-1}$

$\Rightarrow \frac{x}{-8} = \frac{y}{-6} = \frac{1}{-2}$

$\Rightarrow x = \frac{-8}{-2} = 4$, $y = \frac{-6}{-2} = 3$

Hence, x = 4 and y = 3 is the required solution.

19. If 1 is added to both of the numerator and denominator of a fraction, it becomes $\frac{4}{5}$. If however, 5 is subtracted from both numerator and denominator, the fraction becomes $\frac{1}{2}$. Find the fraction.

Sol:

Let the required fraction be $\frac{x}{y}$.

Then, we have:

$\frac{x+1}{y+1} = \frac{4}{5}$

$\Rightarrow 5(x + 1) = 4(y + 1)$

$\Rightarrow 5x + 5 = 4y + 4$

$\Rightarrow 5x - 4y = -1$ ………(i)

Again, we have:

$\frac{x-5}{y-5} = \frac{1}{2}$

$\Rightarrow 2(x - 5) = 1(y - 5)$

$\Rightarrow 2x - 10 = y - 5$

$\Rightarrow 2x - y = 5$ ………(ii)

On multiplying (ii) by 4, we get:

$8x - 4y = 20$ ……..(iii)

On subtracting (i) from (iii), we get:

$3x = (20 - (-1)) = 20 + 1 = 21$

$\Rightarrow 3x = 21$

$\Rightarrow x = 7$

On substituting x = 7 in (i), we get

$5 \times 7 - 4y = -1$

$\Rightarrow 35 - 4y = -1$

$\Rightarrow 4y = 36$

$\Rightarrow y = 9$

$\therefore x = 7$ and $y = 9$

Hence, the required fraction is $\frac{7}{9}$.

20. Solve: $\frac{ax}{b} - \frac{by}{a} = (a + b)$, $ax - by = 2ab$.

Sol:

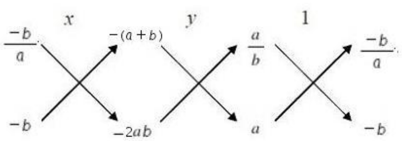
The given equations may be written as follows:

$\frac{ax}{b} - \frac{by}{a} - (a + b) = 0$ ………(i)

$ax - by - 2ab = 0$ ………(ii)

Here, $a_1 = \frac{a}{b}$, $b_1 = \frac{-b}{a}$, $c_1 = -(a + b)$ and $a_2 = a$, $b_2 = -b$ and $c_2 = -2ab$

By cross multiplication, we have:



$$\therefore \frac{x}{\left(-\frac{b}{a}\right)\times(-2ab)-(-b)\times(-(a+b))} = \frac{y}{-(a+b)\times a-(-2ab)\times\frac{a}{b}} = \frac{1}{\frac{a}{b}\times(-b)-a\times\left(-\frac{b}{a}\right)}$$

$$\Rightarrow \frac{x}{2b^2-b(a+b)} = \frac{y}{-a(a+b)+2a^2} = \frac{1}{-a+b}$$

$$\Rightarrow \frac{x}{2b^2-ab-b^2} = \frac{y}{-a^2-ab+2a^2} = \frac{1}{-a+b}$$

$$\Rightarrow \frac{x}{b^2-ab} = \frac{y}{a^2-ab} = \frac{1}{-(a-b)}$$

$$\Rightarrow \frac{x}{-b(a-b)} = \frac{y}{a(a-b)} = \frac{1}{-(a-b)}$$

$$\Rightarrow x = \frac{-b(a-b)}{-(a-b)} = b,\ y = \frac{a(a-b)}{-(a-b)} = -a$$

Hence, $x = b$ and $y = -a$ is the required solution.