Chapter 19 - Volume and Surface Areas of Solids

Excercise 19A

Solution 1

Let the length of each side of each cube = s cm.
Now,
Volume of each cuboid = 27 cm^3
$\Rightarrow s^3 = 27$
$\Rightarrow s = 3$ cm
When two cubes of each side, 3 cm is joined end to end, then a cuboid is formed.
Now, length of cubiod (l) = 6 cm,
breadth of cubiod (b) = 3 cm and
height of cubiod (h) = 3 cm

$$\begin{aligned}\therefore \text{ Total surface area} &= 2(lb + bh + lh)\\ &= 2[(6\times3)+(3\times3)+(3\times6)]\\ &= 2[18+9+18]\\ &= 2\times45\\ &= 90 \text{ cm}^2\end{aligned}$$

Solution 2

Let r be the radius of the hemisphere.

Now, Volume of hemisphere = $2425\frac{1}{2}$ cm^3

$$\Rightarrow \frac{2}{3}\pi r^3 = \frac{4851}{2}$$

$$\Rightarrow \frac{2}{3} \times \frac{22}{7} \times r^3 = \frac{4851}{2}$$

$$\Rightarrow \frac{44}{21} \times r^3 = \frac{4851}{2}$$

$$\Rightarrow r^3 = \frac{4851 \times 21}{88}$$

$$\Rightarrow r^3 = \frac{4851 \times 21}{88}$$

$$\Rightarrow r = \sqrt[3]{1157.625}$$

$$\Rightarrow r = 10.5 \text{ cm}$$

Now,

Curved surface area $= 2\pi r^2$

$$= 2 \times \frac{22}{7} \times (10.5)^2$$

$$= \frac{44}{7} \times 110.25$$

$$= 693 \text{ cm}^2$$

Solution 3

Total surface area of solid hemisphere = $3\pi r^2$

$\Rightarrow 462 = 3\pi r^2$

$\Rightarrow 462 = 3 \times \frac{22}{7} \times r^2$

$\Rightarrow 462 = \frac{66}{7} \times r^2$

$\Rightarrow r^2 = \frac{3234}{66}$

$\Rightarrow r^2 = 49$

$\Rightarrow r = 7$ cm

Now,

Volume of a solid hemisphere $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times 7 \times 7 \times 7$

$= \frac{2156}{3}$

$= 718.67\ cm^3$

Solution 4

Let the length of the cloth be l m

Given radius of cone = 7 m and heigth = 24 m

Now,

Slant height = $\sqrt{h^2 + r^2}$

$\Rightarrow l = \sqrt{h^2 + r^2}$

$\Rightarrow l = \sqrt{(24)^2 + (7)^2}$

$\Rightarrow l = \sqrt{576 + 49}$

$\Rightarrow l = \sqrt{625}$

$\Rightarrow l = 25$ m

Area of the doth = Curved surface area of cone

$= \pi rl$

$= \frac{22}{7} \times 7 \times 25$

$= 22 \times 25$

$= 550\ m^2$

Now,

Area of the doth = length $\times$ breadth

$\Rightarrow 550 = \text{length} \times 5$

$\Rightarrow \text{length} = \frac{550}{5}$

$\Rightarrow \text{length} = 110$ m

Given Cost of 1 m doth = Rs. 25

$\Rightarrow$ Cost of 110 m cloth = 110×25

= Rs. 2750

Solution 5

Given ratio of the volumes = 1 : 4 and

the ratio of the diameters = 4 : 5

$$\therefore \frac{d_1}{d_2} = \frac{2r_1}{2r_2}$$

$$\therefore \frac{4}{5} = \frac{2r_1}{2r_2}$$

$$\therefore \frac{r_1}{r_2} = \frac{4}{5} \quad \ldots(i)$$

Now,

$$\frac{V_1}{V_2} = \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2}$$

$$\Rightarrow \frac{1}{4} = \left(\frac{r_1}{r_2}\right)^2 \times \frac{h_1}{h_2}$$

$$\Rightarrow \frac{1}{4} = \left(\frac{4}{5}\right)^2 \times \frac{h_1}{h_2} \quad \ldots(\text{From (i)})$$

$$\Rightarrow \frac{1}{4} = \frac{16}{25} \times \frac{h_1}{h_2}$$

$$\Rightarrow \frac{h_1}{h_2} = \frac{25}{16 \times 4}$$

$$\Rightarrow \frac{h_1}{h_2} = \frac{25}{64}$$

Thus, the ratio of their heights is 25 : 64.

Solution 6

Let the radius of the base of the conical mountain be r km.

$\Rightarrow$ Area of the base of the conical mountain = πr^2

$$\Rightarrow 1.54 = \frac{22}{7} \times r^2$$

$$\Rightarrow r^2 = \frac{10.78}{22}$$

$$\Rightarrow r^2 = 0.49$$

$$\Rightarrow r = 0.7 \text{ km}$$

Slant height of the conical mountain (l) = 2.5 km

Let the height of the mountain be h km.

Now,

$$\Rightarrow l^2 = h^2 + r^2$$

$$\Rightarrow (2.5)^2 = h^2 + (0.7)^2$$

$$\Rightarrow h = \sqrt{(2.5)^2 - (0.7)^2}$$

$$\Rightarrow h = \sqrt{6.25 - 0.49}$$

$$\Rightarrow h = \sqrt{5.76}$$

$$\Rightarrow h = 2.4 \text{ km}$$

Thus, the height of the moutain is 2.4 km.

Solution 7

Let r and h be the radius and height of the solid cylinder respectively.
Given $r + h = 37$ m
Now,
Total surface area of the cylinder $= 2\pi r(r+h)$

$\Rightarrow 1628 = 2 \times \frac{22}{7} \times r \times 37$

$\Rightarrow 1628 = \frac{148}{7} \times r$

$\Rightarrow r = \frac{11396}{1628}$

$\Rightarrow r = 7$ m

$\Rightarrow r + h = 37$

$\Rightarrow 7 + h = 37$

$\Rightarrow h = 30$ m

$\Rightarrow$ Volume of the cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times 7 \times 7 \times 30$$
$$= 22 \times 7 \times 30$$
$$= 4620 \text{ m}^3$$

Solution 8

Let the original radius be r.

$\Rightarrow$ Original surface area $= 4\pi r^2 = 2464 \text{ cm}^2$ ....(i)

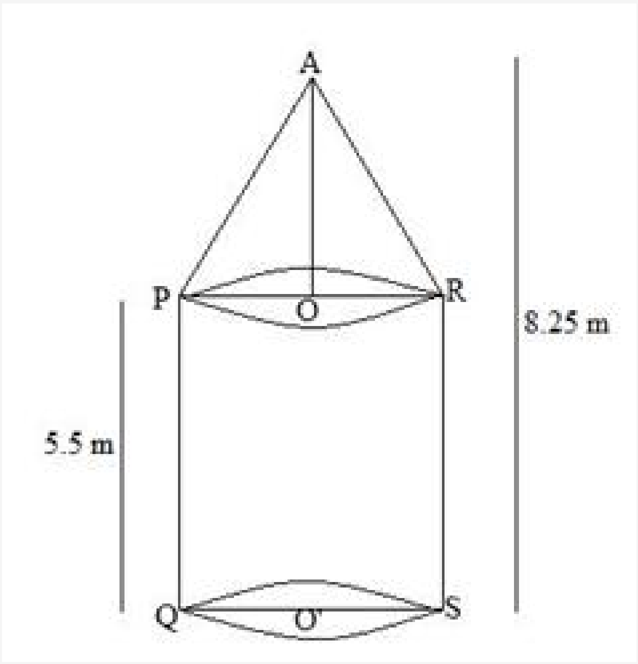
Given new radius $= 2r$

$\Rightarrow$ New surface area $= 4\pi(2r)^2$

$$= 4 \times 4\pi r^2$$
$$= 4 \times 2464 \quad \text{....(From (i))}$$
$$= 9856 \text{ cm}^2$$

Solution 9

s



Given Height of the tent (h) = 8.25 m,

Diameter of cylindrical base = 30 m

$\Rightarrow$ Radius of cylindrical base (r) = Radius of the cone = 15 m,

Height of cylinder = 5.5 m

Height of cone (h) = Height og the tent − Height of the cylinder

$= 8.25 - 5.5$

2.75 m

Slant height $= \sqrt{r^2 + h^2}$

$= \sqrt{(15)^2 + (2.75)^2}$

$= \sqrt{225 + 7.5625}$

$= \sqrt{232.5625}$

$= 15.25$ m

Total surface area of the tent = Curved surface area of cone + Curved surface area of cylinder

$= \pi rl + 2\pi rh$

$= \pi r(l + 2h)$

$= \frac{22}{7} \times 15(15.25 + 2 \times 5.5)$

$= \frac{22}{7} \times 15 \times 26.25$

$= 1237.5\ m^2$

Now, breadth of the canvas (b) = 1.5 m

Let the length of the canvas = x m

Area of the triangular canvas = Total surface area of the tent

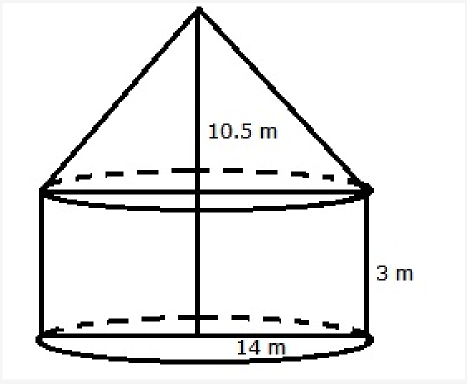
$\Rightarrow l \times b = 1237.5$

$\Rightarrow l \times 1.5 = 1237.5$

$\Rightarrow l = 825$ m

Thus, length of the canvas is 825 m.

Solution 10



Radius of the cylinder = 14 m

And its height = 3 m

Radius of cone = 14 m

And its height = 10.5 m

Let l be the slant height

$$\therefore \quad l^2 = (14)^2 + (10.5)^2$$
$$l^2 = (196 + 110.25)\, m^2$$
$$l^2 = 306.25\ m^2$$
$$l = \sqrt{306.25}\, m$$
$$= 17.5\ m$$

Curved surface area of tent

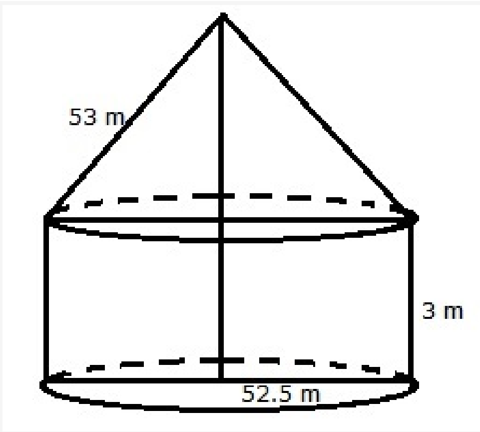
= (curved area of cylinder + curved surface area of cone)

$$= 2\pi rh + \pi rl$$
$$= \left[\left(2 \times \frac{22}{7} \times 14 \times 3\right) + \left(\frac{22}{7} \times 14 \times 17.5\right)\right] m^2$$
$$= (264 + 770) m^2 = 1034\ m^2$$

Hence, the curved surface area of the tent = 1034 m^2

Cost of canvas = Rs.(1034 × 80) = Rs. 82720

Solution 11



For the cylindrical portion, we have radius = 52.5 m and height = 3 m

For the conical portion, we have radius = 52.5 m

And slant height = 53 m

Area of canvas = 2 πrh + πrl = πr(2h + l)

$$= \left[\frac{22}{7} \times 52.5 \times (2 \times 3 + 53)\right] m^2$$
$$= \left(22 \times \frac{15}{2} \times 59\right) m^2 = 9735 m^2$$

Solution 12

Radius o f cylinder = 2.5 m

Height of cylinder = 21 m

Slant height of cone = 8 m

Radius of cone = 2.5 m

Total surface area of the rocket = (curved surface area of cone

+ curved surface area of cylinder + area of base)

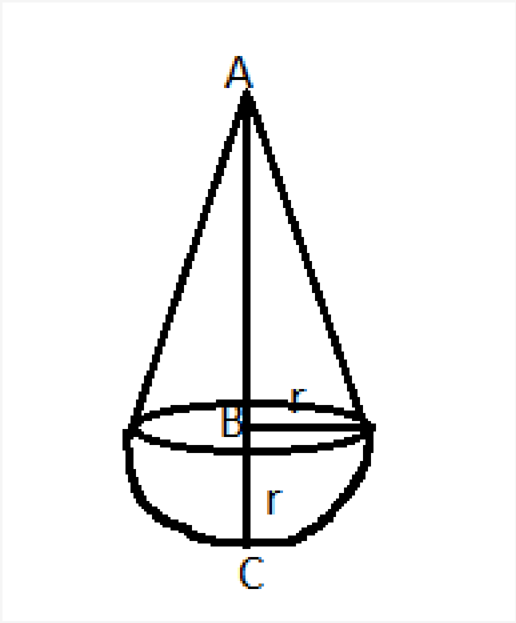
$$= \left(\pi rl + 2\pi rh + \pi r^2\right)$$

$$\text{where } l = 8\,m,\ h = 21\,m,\ r = 2.5\,m$$

$$= \left(\frac{22}{7} \times 2.5 \times 8 + 2 \times \frac{22}{7} \times 2.5 \times 21 + \frac{22}{7} \times 2.5 \times 2.5\right) m^2$$

$$= (62.85 + 330 + 19.64)\, m^2 = 412.5\, m^2$$

Solution 13



From the given figure,

Height (AB) of the cone = AC − BC (Radius of the hemisphere)

Thus, height of the cone = Total height − Radius of the hemisphere

$= 9.5 - 3.5$

$= 6\text{ cm}$

Volume of the solid = Volume of the cone + Volume of the hemisphere

$$= \left(\frac{1}{3}\pi r^2 h\right) + \left(\frac{2}{3}\pi r^3\right)$$

$$= \frac{1}{3}\pi r^2 (h + 2r)$$

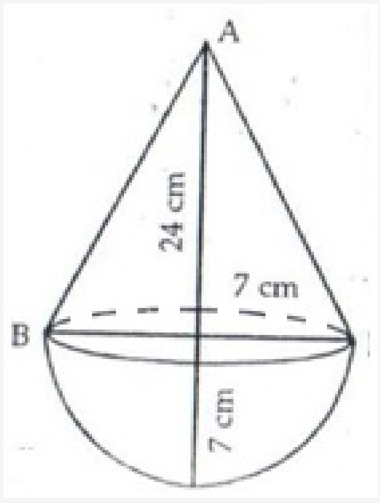
$$= \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5 (6 + 2 \times 3.5)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 13$$

$$= 166.83\text{ cm}^3$$

Thus, total volume of the solid is 166.83 cm^3.

Solution 14



Height of cone = h = 24 cm

Its radius = 7 cm

$$\therefore \text{Slant height} = \sqrt{(24)^2 + 7^2}$$
$$= \sqrt{576 + 49}$$
$$= \sqrt{625} = 25 \text{ cm}$$

Total surface area of toy

$$= \left(\pi r l + 2\pi r^2\right)$$
$$= \pi r\left(l + 2r\right)$$
$$= \frac{22}{7} \times 7 \times \left(25 + 14\right)$$
$$= 22 \times 39 = 858 \text{ cm}^2$$

Solution 15

Given the base radius of the cone and the hemisphere are equal.
Diameter of hemisphere = 7 cm
$\Rightarrow$ Radius of hemisphere = 3.5 cm
Radius of the cone = Radius of the hemisphere = 3.5 cm
Let H be the total height of the top.
H = h + r
H = h + 3.5 ...(where h is the height of the cone.) ...(i)

Now,
Volume of toy = Volume of cone + Volume of hemisphere

$$\Rightarrow 231 = \left(\frac{1}{3}\pi r^2 h\right) + \left(\frac{2}{3}\pi r^3\right)$$

$$\Rightarrow 231 = \frac{1}{3}\pi r^2 (h + 2r)$$

$$\Rightarrow 231 = \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5 (h + 2 \times 3.5)$$

$$\Rightarrow 231 = \frac{269.5}{21}(h + 7)$$

$$\Rightarrow h + 7 = \frac{4851}{269.5}$$

$$\Rightarrow h + 7 = 18$$

$$\Rightarrow h = 11 \text{ cm}$$

$$\Rightarrow \text{Height of the toy} = h + r$$

$$= 11 + 3.5 \quad \ldots(\text{From (i)})$$

$$= 14.5 \text{ cm}$$

Solution 16

Let R and H be the radius and height of the cylindrical container respectively.
Given R = 6 cm and H = 15 cm
Now,
Volume of the ice cream in the cylindrical container = $\pi R^2 H$

$$= \pi \times 6 \times 6 \times 15$$
$$= 540\pi \text{ cm}^3$$

Let the radius of the cone be r cm.
Height of the cone (h) = $2(2r) = 4r$ ...(Given)
Radius of the hemispherical portion = r cm
Volume of ice cream in the cone = Volume of the cone + Volume of hemisphere

$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$
$$= \frac{1}{3}\pi r^2 (h + 2r)$$
$$= \frac{1}{3}\pi r^2 (4r + 2r)$$
$$= \frac{1}{3} \times \pi \times r^2 \times 6r$$
$$= \frac{1}{3} \times \pi \times 6r^3$$
$$= 2\pi r^3$$

Number of ice cream cones distributed to the children = 10 ...(Given)
$\Rightarrow$ 10 × Volume of ice cream in the cone = Volume of ice cream in the cylindrical container
$\Rightarrow 10 \times 2\pi r^3 = 540\pi$
$\Rightarrow 20r^3 = 540$
$\Rightarrow r^3 = 27$
$\Rightarrow r = 3$ cm
Thus, the radius of the ice cream cone is 3 cm.

Solution 17

Radius of hemisphere = 10.5 cm

Height of cylinder = (14.5 10.5) cm = 4 cm

Radius of cylinder = 10.5 cm

Capacity = Volume of cylinder + Volume of hemisphere

$$= \left(\pi r^2 h + \frac{2}{3}\pi r^3\right) cm^3 = \pi r^2\left(h + \frac{2}{3}r\right) cm^3$$

$$= \left[\frac{22}{7} \times 10.5 \times 10.5 \times \left(4 + \frac{2}{3} \times 10.5\right)\right] cm^3$$

$$= (346.5 \times 11) cm^2 = 3811.5 cm^2$$

Solution 18

Given Diameter of hemispherical ends = 42 cm

$\Rightarrow$ Radius of hemispherical ends = 21 cm

Radius of the cylinder = Radius of hemispherical ends = 21 cm

Height of the cylinder (h) $= 90 - (21 + 21)$

$= 90 - 42$

$= 48$ cm

Total surface area of the solid = Surface area of cylinder + 2 (Surface area of hemispherical ends)

$= 2\pi rh + 2(2\pi r^2)$

$= 2 \times \frac{22}{7} \times 21 \times 48 + 2\left(2 \times \frac{22}{7} \times 21 \times 21\right)$

$= 6336 + 2 \times 2772$

$= 6336 + 5544$

$= 11880$ cm^2

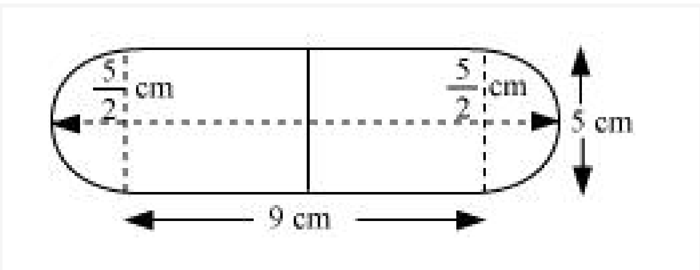
Thus,

Cost of painting the solid = 70 paise per sq cm = Rs. 0.70 per sq cm

Total cost $= 11880 \times 0.70$

= Rs. 8316

Solution 19



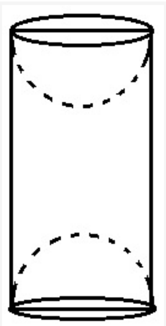
From the given figure,
Radius (r) of cylindrical part = Radius (r) of hemispherical part
$\Rightarrow$ Radius (r) of cylindrical part = $\frac{\text{Diameter of the capsule}}{2}$
$\Rightarrow$ Radius (r) of cylindrical part = $\frac{5}{2}$
$\Rightarrow 2r = 5$

Length of the cylindrical part (h) = Length of the entire capsule – 2r
= 14 – 5
= 9 mm

Surface area of capsule = 2 x Curved surface area of hemispherical part + Curved surface area of cylindrical part

$$= 2 \times 2\pi r^2 + 2\pi rh$$
$$= 4\pi\left(\frac{5}{2}\right)^2 + 2\pi\left(\frac{5}{2}\right) \times 9$$
$$= 4\pi \times \frac{25}{4} + 2\pi\left(\frac{5}{2}\right) \times 9$$
$$= 25\pi + 45\pi$$
$$= 70\pi$$
$$= 70 \times \frac{22}{7}$$
$$= 220 \text{ mm}^2$$

Solution 20



Height of cylinder = 20 cm

And diameter = 7 cm and then radius = 3.5 cm

Total surface area of article

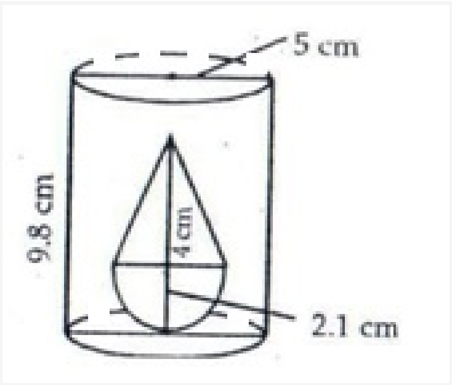
= (lateral surface of cylinder with r = 3.5 cm and h = 20 cm)

$$= \left[2\pi rh + 2 \times \left(2\pi r^2\right)\right] \text{sq.units}$$

$$= \left[\left(2 \times \frac{22}{7} \times \frac{7}{2} \times 20\right) + \left(4 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\right)\right] cm^2$$

$$= (440 + 154)\, cm^2 = 594\, cm^2$$

Solution 21



Radius of cylinder $r_1 = 5$ cm

And height of cylinder $h_1 = 9.8 cm$

Radius of cone r = 2.1 cm

And height of cone $h_2 = 4cm$

Volume of water left in tub

= (volume of cylindrical tub – volume of solid)

$$= \left(\pi r_1^2 h_1 - \frac{2}{3}\pi r^3 - \frac{1}{3}\pi r^2 h_2\right)$$

$$= \left(\frac{22}{7} \times 5 \times 5 \times 9.8 - \frac{2}{3} \times \frac{22}{7} \times 2.1 \times 2.1 \times 2.1 - \frac{1}{3} \times \frac{22}{7} \times 2.1 \times 2.1 \times 4\right)$$

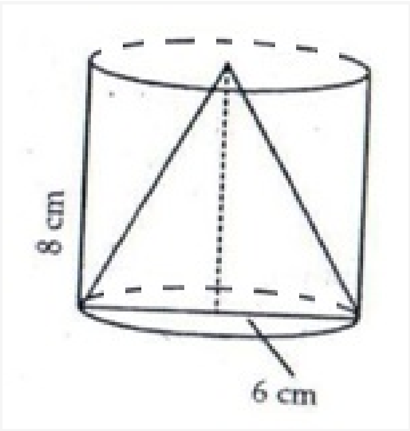
$$= \left[(770 - 19.404) - 18.48\right] cm^3$$

$$= 732.116 cm^3$$

Solution 22

(i)Radius of cylinder = 6 cm

Height of cylinder = 8 cm



Volume of cylinder

$$= \pi r^2 h \text{ cu. units}$$

$$= \pi \times 6 \times 6 \times 8 cm^3$$

$$= 288\pi\ cm^3$$

Volume of cone removed

$$= \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \pi \times 6 \times 6 \times 8 \text{ cm}^3$$

$$= 96\ \pi \text{ cm}^3$$

(ii)Surface area of cylinder = 2π = $2\pi \times 6 \times 8$ $\text{cm}^2 = 96\ \pi \text{ cm}^2$

Slant height of cone = $\sqrt{6^2 + 8^2} = \sqrt{36 + 64}$ cm

$= \sqrt{100}$ cm = 10 cm

Curved surface area of cone = $\pi rl = \pi \times 6 \times 10 = 60\ \pi$

Area of base of cylinder = $\pi r^2 = \pi \times 6 \times 6 = 36\ \pi$

Total surface area of remaining solid

$$= (96\pi + 60\pi + 36\pi) \text{cm}^2$$

$$= 192\pi \text{ cm}^2 = 602.88 \text{cm}^2$$

Solution 23

Given height (h) of the conical part = Height (h) of the cylindrical part = 2.8 cm
Diameter of the cylindrical part = 4.2 cm
$\Rightarrow$ Radius of the cylindrical part = 2.1 cm
Curved surface area of the cylindrical part = $2\pi rh$

$$= 2 \times \frac{22}{7} \times 2.1 \times 2.8$$
$$= 44 \times 0.3 \times 2.8$$
$$= 36.96 \text{ cm}^2$$

Now,
Slant height (l) = $\sqrt{h^2 + r^2}$
$\Rightarrow l = \sqrt{(2.8)^2 + (2.1)^2}$
$\Rightarrow l = \sqrt{7.84 + 4.41}$
$\Rightarrow l = \sqrt{12.25}$
$\Rightarrow l = 3.5$
$\Rightarrow$ Curved surface area of the concial part = πrl

$$= \frac{22}{7} \times 2.1 \times 3.5$$
$$= 22 \times 0.5 \times 2.1$$
$$= 23.1 \text{ cm}^2$$

Area of cylindrical base = πr^2

$$= \frac{22}{7} \times 2.1 \times 2.1$$
$$= 22 \times 0.3 \times 2.1$$
$$= 13.86 \text{ cm}^2$$

Total surface area of the remaining solid
= Curved surface area of the cylindrical part + Curved surface area of the concial part + Area of cylindrical base
= 36.96 + 23.1 + 13.86
= 73.92 cm^2
Thus, the total surface area of the remaining solid is 73.92 cm^2.

Solution 24

Height of the cylinder (h) = 14 cm,
Base diameter = 7 cm
$\Rightarrow$ Radius of the base of the cylinder (r) = 3.5 cm
Volume of the cylinder = $\pi r^2 h$

$$= \frac{22}{7} \times 3.5 \times 3.5 \times 14$$
$$= 22 \times 0.5 \times 3.5 \times 14$$
$$= 539 \text{ cm}^3$$

Radius of the conical holes (r_1) = 2.1 cm,
Height of the conical holes (h_1) = 4 cm
Volume of the conical hole = $\frac{1}{3}\pi r_1^2 h_1$

$$= \frac{1}{3} \times \frac{22}{7} \times 2.1 \times 2.1 \times 4$$
$$= 18.48 \text{ cm}^3$$

Volume of the two conical hole = 2×18.48

$$= 36.96 \text{ cm}^3$$

Volume of the remaining solid = Volume of the cylinder – Volume of two conical hole

$$= 539 - 36.96$$
$$= 502.04 \text{ cm}^3$$

Solution 25

Given Radius of the base of the metal cylinder (R) = 3 cm,
Height of the metal cylinder (H) = 5 cm
Now,
Volume of the metal cylinder = $\pi R^2 h$

$$= \pi \times (3)^2 \times 5$$
$$= 45\pi \text{ cm}^3$$

Given A conical hole is drilled in this metal cylinder.

Radius of the base of the cone (r) = $\frac{3}{2}$ cm

Height of the cylinder (h) = $\frac{8}{9}$ cm

Volume of the cone = $\frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \pi \times \left(\frac{3}{2}\right)^2 \times \frac{8}{9}$$
$$= \pi \times \frac{9}{4} \times \frac{8}{9}$$
$$= \frac{2}{3}\pi \text{ cm}^3$$

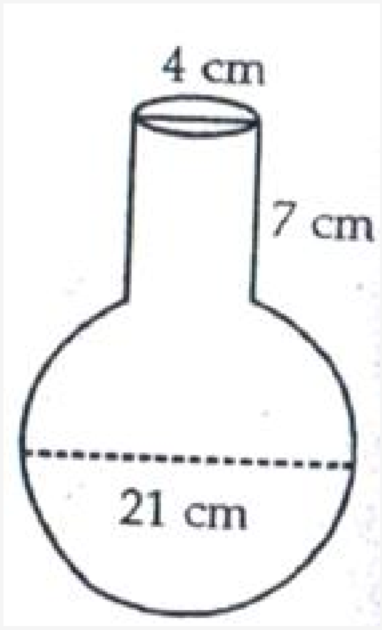
When the conical hole is drilled out, then
Volume of the metal left in the solid = Volume of the metal cylinder – Volume of the cone

$$= 45\pi - \frac{2}{3}\pi$$
$$= \frac{133\pi}{3} \text{ cm}^3$$

Now,

$$\frac{\text{Volume of the metal left in the solid}}{\text{Volume of the cone}} = \frac{\frac{133\pi}{3}}{\frac{2}{3}\pi} = \frac{133}{2}$$

Solution 26



Diameter of spherical part of vessel = 21 cm

$$\text{Its radius} = \frac{21}{2}\text{ cm}$$

$$\text{Its volume} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3}\times\frac{22}{7}\times\frac{21}{2}\times\frac{21}{2}\times\frac{21}{2}$$

$$= 11\times 21\times 21\text{ cm}^3 = 4851\text{ cm}^3$$

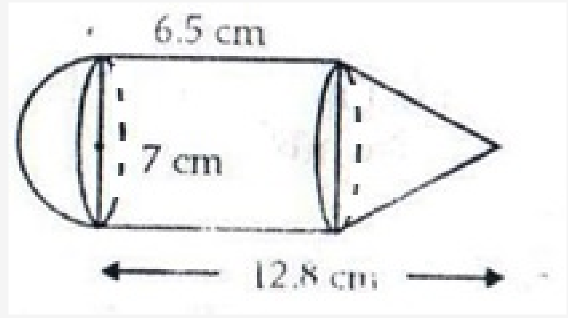
Volume of cylindrical part of vessel

$$= \pi r^2 h = \frac{22}{7}\times 2\times 2\times 7\text{ cm}^3$$

$$= 88\text{ cm}^3$$

$$\therefore \text{Volume of whole vessel} = (4851 + 88)\text{cm}^3 = 4939\text{ cm}^3$$

Solution 27



Height of cylinder = 6.5 cm

Height of cone = $h_2 = (12.8 - 6.5)\text{cm} = 6.3\text{ cm}$

Radius of cylinder = radius of cone

= radius of hemisphere

= $\left(\frac{7}{2}\right)\text{cm}$

Volume of solid = Volume of cylinder + Volume of cone

+ Volume of hemisphere

$$= \pi r^2 h_1 + \frac{1}{3}\pi r^2 h_2 + \frac{2}{3}\pi r^3 = \pi r^2\left(h_1 + \frac{1}{3}h_2 + \frac{2}{3}r\right)$$

$$= \left[\frac{22}{7} \times 3.5 \times 3.5 \times \left(6.5 + 6.3 \times \frac{1}{3} + \frac{2}{3} \times 3.5\right)\right]$$

$$= \left[(38.5) \times (6.5 + 2.1 + 2.33)\right]\text{cm}^3$$

$$= (38.5 \times 10.93)\text{cm}^3 = 420.80\text{ cm}^3$$

Solution 28

Given side of a cube = 21 cm

Diameter of the hemisphere is equal to the side of the cubical piece (d) = 21 cm

$\Rightarrow$ Radius of the hemisphere = 10.5 cm

$$\text{Volume of cube} = \text{Side}^3 = (21)^3 = 9261\ \text{cm}^3$$

$$\text{Volume of the hemisphere} = \frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times 10.5 \times 10.5 \times 10.5 = 44 \times 0.5 \times 10.5 \times 10.5 = 2425.5\ \text{cm}^3$$

$$\Rightarrow \text{Volume of the remaining piece} = 9261 - 2425.5 = 6835.5\ \text{cm}^3$$

Now,

Surface area of the cube (without the side carved)

$= 5(\text{side})^2$

$= 5 \times 21 \times 21$

$= 2205\ \text{cm}^2$

$$\text{Surface area of hemisphere} = 2\pi r^2 = 2 \times \frac{22}{7} \times 10.5 \times 10.5 = 693\ \text{cm}^2$$

$$\Rightarrow \text{Surface area of remaining piece} = 2205 + 693 = 2898\ \text{cm}^2$$

Solution 29

The greatest diameter that the hemisphere can have is 10 cm.
So, the radius = 5 cm
TSA of the solid = Surface area of the cube + CSA of the hemisphere – Area of the base of the hemisphere

$$= 6a^2 + 2\pi r^2 - \pi r^2$$
$$= 6(10)^2 + 2 \times 3.14 \times 5 \times 5 - 3.14 \times 5 \times 5$$
$$= 600 + 157 - 78.5$$
$$= 678.5\ cm^2$$

$\Rightarrow$ Cost of painting the block at the rate of Rs. 5 per 100 sq cm

So, the cost of painting 1 sq cm = Rs. $\frac{5}{100}$.

Cost of painting 678.5 cm² = $\frac{678.5}{100} \times 5 = 33.93$

Thus, cost of painting the block at the rate of Rs. 5 per 100 sq cm is Rs. 33.93.

Note : The answer in the textbook is incorrect.

Solution 30

Total surface area of the toy = CSA of the cylinder + CSA of hemisphere + CSA of cone

$$= 2\pi rh + 2\pi r^2 + \pi rh$$
$$= \left(2 \times \frac{22}{7} \times 5 \times 13\right) + \left(2 \times \frac{22}{7} \times 5 \times 5\right) + \left(\frac{22}{7} \times 5 \times 13\right)$$
$$= \frac{22}{7}(130 + 50 + 65)$$
$$= \frac{22}{7} \times 245$$
$$= 770\ cm^2$$

Solution 31

Given Diameter of the cylinder = 7 cm

$\Rightarrow$ Radius of the cylinder = 3.5 cm,

Height of the cylinder = 16 cm

Apparent volume of the glass = $\pi r^2 h$

$$= \frac{22}{7} \times (3.5)^2 \times 16$$

$$= \frac{4312}{7}$$

$$= 616 \text{ cm}^3$$

$\Rightarrow$ Apparent capacity of the glass = 616 cm^3

Actual volume of the glass = $\pi r^2 h - \frac{2}{3}\pi r^3$

$$= 616 - \frac{2}{3} \times \frac{22}{7} \times (3.5)^3$$

$$= 616 - \frac{1886.5}{3}$$

$$= 616 - 89.833$$

$$= 526.167 \text{ cm}^3$$

$\Rightarrow$ Actual capacity of the glass = 526.17 cm^3

Solution 32

Given diameter of the cone = 5 cm

$\Rightarrow$ Radius of cone (r) = 2.5 cm,

Height of the cone (h) = 6 cm,

Diameter of the cylinder = 4 cm

$\Rightarrow$ Radius of cylinder (R) = 2 cm,

Height of the cylinder (H) = 26 = 6 = 20 cm

Now,

Slant height (l) of cone = $\sqrt{h^2 + r^2}$

$\Rightarrow l = \sqrt{h^2 + r^2}$

$\Rightarrow l = \sqrt{(6)^2 + (2.5)^2}$

$\Rightarrow l = \sqrt{36 + 6.25}$

$\Rightarrow l = \sqrt{42.25}$

$\Rightarrow l = 6.5$ cm

Here, the conical portion has its circular base resting on the base of the cylinder, but the base of the cone is larger than the base of the cylinder, so a part of the base of the cone is to be painted.

Area of the top of the cylinder = πr^2

$= \frac{22}{7} \times 2^2$

$= \frac{88}{7}\ \text{cm}^2$

Area of the base of the cone = πr^2

$= \frac{22}{7} \times 2.5^2$

$= \frac{137.5}{7}\ \text{cm}^2$

Area of the painted region on the base of the cone

= Area of the base of the cone – Area of the top of the cylinder

$= \frac{137.5}{7}\ \text{cm}^2 - \frac{88}{7}\ \text{cm}^2$

$= \frac{49.5}{7}\ \text{cm}^2$

Curved surface of the cone = $\pi r l$

$= \frac{22}{7} \times 2.5 \times 6.5$

$= 51.07\ \text{cm}^2$

Curved surface of the cylinder = $2\pi rh$

$= 2 \times \frac{22}{7} \times 2.5 \times 20$

$= \frac{2200}{7}\ \text{cm}^2$

So, the area painted red = $51.07\ \text{cm}^2 + \frac{49.5}{7}\ \text{cm}^2$

$= 58.143\ \text{cm}^2$

So, the area painted white = $51.07\ \text{cm}^2 + \frac{49.5}{7}\ \text{cm}^2$

$= 58.143\ \text{cm}^2$

R S Aggarwal and V Aggarwal Solution for Class 10 Mathematics Chapter 19 - Volume and Surface Areas of Solids Page/Excercise 19B

Solution 1

Given The dimensions of the cuboid are 100 cm , 80 cm and 64 cm respectively.

Now,

$$\text{Volume of cuboid} = 100 \times 80 \times 64 = 512000 \text{ cm}^3$$

Let the side of the cuve be a.

Given

Volume of cube = Volume of cuboid

$\Rightarrow a^3 = 512000$

$\Rightarrow a = \sqrt[3]{512000}$

$\Rightarrow a = 80$ cm

Now,

$$\text{Total surface area of cube} = 6a^2 = 6 \times (80)^2 = 6 \times 6400 = 38400 \text{ cm}^2$$

Solution 2

Given height of the cone = 20 cm and
Radius of the cone = 5 cm
Let the radius of the sphere be R cm.
Given
Volume of cone = Volume of sphere

$\Rightarrow \frac{1}{3}\pi r^2 h = \frac{4}{3}\pi R^3$

$\Rightarrow r^2 h = 4R^3$

$\Rightarrow R^3 = \frac{5 \times 5 \times 20}{4}$

$\Rightarrow R^3 = 125$

$\Rightarrow R = 5$ cm

$\Rightarrow$ Radius of the sphere is 5 cm

$\Rightarrow$ The diameter of the sphere = $5 \times 2 = 10$ cm

Solution 3

Given Radius of the 1st sphere $(r_1) = 6$ cm,
Radius of the 2nd sphere $(r_2) = 8$ cm and
Radius of the 3rd sphere $(r_3) = 10$ cm
Let the radius of the resulting sphere be r cm.
The object formed by recasting these spheres will be same in volume as the sum of the volumes of these spheres.
Now,
Volume of 3 spheres = Volume of resulting sphere

$$\Rightarrow \frac{4}{3}\pi\left(r_1^3 + r_2^3 + r_3^3\right) = \frac{4}{3}\pi r^3$$

$$\Rightarrow \left(6^3 + 8^3 + 10^3\right) = r^3$$

$$\Rightarrow 216 + 512 + 1000 = r^3$$

$$\Rightarrow r^3 = 1728$$

$$\Rightarrow r = 12 \text{ cm}$$

Thus, the radius of the resulting sphere is 12 cm.

Solution 4

Radius of the cone = 12 cm and its height = 24 cm

Volume of cone = $\frac{1}{3}\pi r^2 h = \left(\frac{1}{3} \times \pi \times 12 \times 12 \times 24\right) \text{cm}^3$

$$= (48 \times 24)\pi \text{ cm}^3$$

$$\text{Volume of each ball} = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi \times 3 \times 3 \times 3 = (36\pi)\,cm^3$$

$$\text{Number of balls formed} = \frac{\text{Volume of solid cone}}{\text{Volume of each ball}}$$

$$= \frac{(48 \times 24\pi)}{36\pi} = 32$$

Solution 5

Let R_1 and R_2 be the internal and external radii of the hollow spherical shell respectively.

$R_1 = 3$ cm $R_2 = 5$ cm

$$\text{Volume of the hollow spherical shell} = \frac{4}{3}\pi\left(R_2^3 - R_1^3\right)$$

$$= \frac{4}{3}\pi\left(5^3 - 3^3\right)$$

$$= \frac{4}{3}\pi(125 - 27)$$

$$= \frac{392}{3}\pi$$

Let r and h be the radius and the height of the cylinder respectively.

$\Rightarrow r = \frac{14}{2} = 7$ cm

$$\text{Volume of the cylinder} = \pi r^2 h$$

$$= \pi \times (7)^2 \times h$$

$$= 49\pi h$$

If the hollow spherical shell is melted to form a solid cylinder, then

Volume of the cylinder = Volume of the hollow spherical shell

$\Rightarrow 49\pi h = \frac{64}{3}\pi$

$\Rightarrow h = \frac{392}{3 \times 49}$

$\Rightarrow h = \frac{8}{3}$ cm

Hence, the height of the cylinder is $\frac{8}{3}$ cm.

Solution 6

Internal radius = 3 cm and external radius = 5 cm

Volume of material in the shell $= \frac{2}{3}\pi \times \left[(5)^3 - (3)^3\right] \text{cm}^2$

$$= \frac{2}{3} \times \frac{22}{7} \times 98 = \frac{616}{3} \text{ cm}^3$$

Radius of the cone = 7 cm

Let height of cone be h cm

Volume of cone $= \left(\frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times h\right) \text{cm}^3 = \frac{154h}{3} \text{ cm}^3$

$\therefore \frac{154h}{3} = \frac{616}{3}$

$\Rightarrow h = \frac{616}{154} = 4 \text{ cm}$

Hence, height of the cone = 4 cm

Solution 7

We know that,

1 m = 1000 mm and 1 cm = 10 mm

Given Diameter of silver rod = 2 cm = 20 mm

$\Rightarrow$ Radius of silver rod (r_1) = 1 cm = 10 mm

Length of the silver rod (h_1) = 10 cm = 100 mm

Length of the wire (h_2) = 10 m = 10000 mm

Let the radius of the wire be r_2 cm.

As the wire is made from the rod

$\Rightarrow$ Volume of the rod = Volume of the wire

$\Rightarrow \pi r_1^2 h_1 = \pi r_2^2 h_2$

$\Rightarrow \pi \times (10)^2 \times 100 = \pi \times r_2^2 \times 10000$

$\Rightarrow 10000 = r_2^2 \times 10000$

$\Rightarrow r_2^2 = 1$

$\Rightarrow r_2 = 1$ cm

Diameter of the wire $(d_2) = 2 \times 1$

$= 2$ mm

Diameter of the wire (d_2) = Thickness of the wire = 2 mm

Solution 8

Inner radius of the bowl = 15 cm

Volume of liquid in it = $\frac{2}{3}\pi r^3 = \left(\frac{2}{3}\pi \times (15)^3\right) cm^3$

Radius of each cylindrical bottle = 2.5 cm and its height = 6 cm

Volume of each cylindrical bottle

$$= \pi r^2 h = \left(\pi \times \left(\frac{5}{2}\right)^2 \times 6\right) cm^2$$

$$= \left(\frac{25}{4} \times 6\pi\right) = \left(\frac{75\pi}{2}\right) cm^3$$

Required number of bottles = $\dfrac{\text{Volume of liquid}}{\text{Volume of each cylindrical bottle}}$

$$= \frac{\frac{2}{3} \times \pi \times 15 \times 15 \times 15}{\frac{75}{2} \times \pi} = 60$$

Hence, bottles required = 60

Solution 9

Radius of the sphere= $\dfrac{21}{2}$ cm

Volume of the sphere = $\left(\frac{4}{3}\pi r^3\right) = \left[\frac{4}{3}\pi \times \left(\frac{21}{2}\right)^3\right] cm^3$

Radius of cone = $\frac{7}{4}$ cm and height 3 cm

Volume of cone = $\frac{1}{3}\pi r^2 h = \left(\frac{1}{3} \times \pi \times \left(\frac{7}{4}\right)^2 \times 3\right) cm^3$

Let the number of cones formed be n, then

$$n \times \frac{1}{3}\pi \times \left(\frac{7}{4}\right)^2 \times 3 = \frac{4}{3}\pi \times \left(\frac{21}{2}\right)^3$$

$$n = \frac{4}{3}\pi \times \frac{21}{2} \times \frac{21}{2} \times \frac{21}{2} \times \frac{3}{\pi} \times \frac{4}{7} \times \frac{4}{7} \times \frac{1}{3}$$

$$n = 504$$

Hence, number of cones formed = 504

Solution 10

Radius of the cannon ball = 14 cm

Volume of cannon ball = $\frac{4}{3}\pi r^3 = \left[\frac{4}{3}\pi \times (14)^3\right] cm^3$

Radius of the cone = $\frac{35}{2}$ cm,

Let the height of cone be h cm

$\therefore$ Volume of cone = $\left[\frac{1}{3}\pi \times \left(\frac{35}{2}\right)^2 \times h\right] cm^3$

$$\therefore \frac{4}{3}\pi \times (14)^3 = \frac{1}{3}\pi \times \left(\frac{35}{2}\right)^2 \times h$$

$$h = \frac{4}{3}\pi \times 14 \times 14 \times 14 \times \frac{3}{\pi} \times \frac{2}{35} \times \frac{2}{35}$$

$$= 35.84 \text{ cm}$$

Hence, height of the cone = 35.84 cm

Solution 11

Let the radius of the third ball be r cm, then,

Volume of third ball = Volume of spherical ball volume of 2 small balls

$$\text{Volume of third ball} = \left[\frac{4}{3}\pi(3)^3 - \left\{\frac{4}{3}\pi\left(\frac{3}{2}\right)^3 + \frac{4}{3}\pi(2)^3\right\}\right]$$

$$= \left[36\pi - \left(\frac{9\pi}{2} + \frac{32\pi}{3}\right)\right]cm^3 = \frac{125\pi}{6}cm^3$$

$$\therefore \quad \frac{4}{3}\pi r^3 = \frac{125\pi}{6}$$

$$r^3 = \frac{125\pi \times 3}{6 \times 4 \times \pi} = \frac{125}{8}$$

$$r = \left(\frac{5}{2}\right)cm = 2.5 \ cm$$

Solution 12

External radius of shell = 12 cm and internal radius = 9 cm

Volume of lead in the shell = $\frac{4}{3}\pi\left[(12)^3 - (9)^3\right]cm^3$

Let the radius of the cylinder be r cm

Its height = 37 cm

Volume of cylinder = $\pi r^2 h = \left(\pi r^2 \times 37\right)$

$$\therefore \frac{4}{3}\pi\left[(12)^3 - (9)^3\right] = \pi r^2 \times 37$$

$$\frac{4}{3} \times \pi \times 999 = \pi r^2 \times 37$$

$$r^2 = \frac{4}{3} \times \pi \times 999 \times \frac{1}{37\pi} = 36\ cm^2$$

$$r = \sqrt{36}\ cm^2 = 6 cm$$

Hence diameter of the base of the cylinder = 12 cm

Solution 13

Volume of hemisphere of radius 9 cm

$$= \left(\frac{2}{3} \times \pi \times 9 \times 9 \times 9\right) cm^3$$

Volume of circular cone (height = 72 cm)

$$= \frac{1}{3}\left(\pi \times r^2 \times 72\right) cm$$

Volume of cone = Volume of hemisphere

$$\therefore \frac{1}{3} \times \pi r^2 \times 72 = \frac{2}{3}\pi \times 9 \times 9 \times 9$$

$$r^2 = \frac{2\pi}{3} \times 9 \times 9 \times 9 \times \frac{1}{24\pi} = 20.25$$

$$r = \sqrt{20.25} = 4.5\ cm$$

Hence radius of the base of the cone = 4.5 cm

Solution 14

Diameter of sphere = 21 cm

Hence, radius of sphere = $\left(\frac{21}{2}\right)cm$

Volume of sphere = $\frac{4}{3}\pi r^3$ = $\left(\frac{4}{3} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times \frac{21}{2}\right)$

Volume of cube = a3 = (1 1 1) $cm^3 = 1 cm^3$

Let number of cubes formed be n

$\therefore$ Volume of sphere = n Volume of cube

$$\therefore \frac{4}{3} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times \frac{21}{2} = n \times 1$$
$$= (441 \times 11) = n$$
$$4851 = n$$

Hence, number of cubes is 4851.

Solution 15

Volume of sphere (when r = 1 cm) = $\frac{4}{3}\pi r^3$ = $\left(\frac{4}{3} \times 1 \times 1 \times 1\right)\pi\ cm^3$

Volume of sphere (when r = 8 cm) = $\frac{4}{3}\pi r^3$ = $\left(\frac{4}{3} \times 8 \times 8 \times 8\right)\pi\ cm^3$

Let the number of balls = n

$$n \times \left(\frac{4}{3} \times 1 \times 1 \times 1\right)\pi = \left(\frac{4}{3} \times 8 \times 8 \times 8\right)\pi$$

$$n = \frac{4 \times 8 \times 8 \times 8 \times 3}{3 \times 4} = 512$$

Solution 16

Radius of sphere = 3 cm

Volume of sphere = $\left(\frac{4}{3} \times \pi \times 3 \times 3 \times 3\right) cm^3 = 36\pi\ cm^3$

Radius of small sphere = $\frac{0.6}{2} cm = 0.3\ cm$

Volume of small sphere = $\left(\frac{4}{3} \times \pi \times 0.3 \times 0.3 \times 0.3\right) cm^3$

$$= \left(\frac{4}{3} \times \pi \times \frac{3}{10} \times \frac{3}{10} \times \frac{3}{10}\right) cm^3$$

$$= \left(\frac{4\pi}{3} \times \frac{3}{10} \times \frac{3}{10} \times \frac{3}{10}\right) cm^2$$

Let number of small balls be n

$$n \times \left(\frac{4\pi}{3} \times \frac{3}{10} \times \frac{3}{10} \times \frac{3}{10}\right) = \frac{4}{3}\pi \times 3 \times 3 \times 3$$

$$n = 1000$$

Hence, the number of small balls = 1000.

Solution 17

Diameter of sphere = 42 cm

Radius of sphere = $\left(\frac{42}{2}\right)$cm = 21 cm

Volume of sphere = $\frac{4}{3}\pi r^3 = \left(\frac{4}{3} \times \pi \times 21 \times 21 \times 21\right)\text{cm}^3$

Diameter of cylindrical wire = 2.8 cm

Radius of cylindrical wire = $\left(\frac{2.8}{2}\right)$cm = 1.4 cm

Volume of cylindrical wire = $\pi r^2 h = (\pi \times 1.4 \times 1.4 \times h)\text{cm}^3$

$= (1.96\pi h)\text{cm}^3$

Volume of cylindrical wire = volume of sphere

$$\therefore\ 1.96\pi h = \frac{4}{3} \times \pi \times 21 \times 21 \times 21$$

$$h = \left(\frac{4}{3} \times \pi \times 21 \times 21 \times 21 \times \frac{1}{1.96} \times \frac{1}{\pi}\right)\text{cm}$$

$$h = 6300$$

$$h\left(\frac{6300}{100}\right)\text{m} = 63\ \text{m}$$

Hence length of the wire 63 m.

Solution 18

Diameter of sphere = 18 cm

Radius of copper sphere = $\left(\frac{3600}{100}\right)m = 36m$

$$\text{Volume of sphere} = \left(\frac{4}{3} \times \pi \times r^3\right)cm^3$$
$$= \left(\frac{4}{3}\pi \times 9 \times 9 \times 9\right)cm^3 = 972\,\pi\,cm^3$$

Length of wire = 108 m = 10800 cm

Let the radius of wire be r cm

$$= \left(\pi r^2 l\right)cm^3 = \left(\pi r^2 \times 10800\right)cm$$

But the volume of wire = Volume of sphere

$$\Rightarrow \pi r^2 \times 10800 = 972\pi$$
$$r^2 = \frac{972\pi}{10800\pi} = 0.09\ cm^2$$
$$r = \sqrt{0.09}\ cm = 0.3$$

Hence the diameter = 2r = (0.3 2) cm = 0.6 cm

Solution 19

Let the height of water in the cylindrical vessel be h cm.
Given Internal radius of hemispherical bowl $(r_1) = 9$ cm,
Internal radius of the cylindrical bowl $(r_2) = 6$ cm
As the content of hemispherical bowl has been put into the cylindrical vessel.
$\Rightarrow$ Volume of hemispherical bowl = Volume of cylindrical bowl

$$\Rightarrow \frac{2}{3}\pi r_1^3 = \pi r_2^2 h$$

$$\Rightarrow h = \frac{2}{3} \times \left(\frac{r_1^3}{r_2^2}\right)$$

$$\Rightarrow h = \frac{2}{3} \times \left(\frac{9^3}{6^2}\right)$$

$$\Rightarrow h = \frac{2}{3} \times \frac{729}{36}$$

$\Rightarrow h = 13.5$ cm
Thus, the height of the water in the cylindrical vessel is 13.5 cm.

Solution 20

Given diameter of the tank = 3 m

$\Rightarrow$ Radius of the tank = 1.5 m

$$\text{Volume of the tank} = \frac{2}{3}\pi r^3$$

$$= \frac{2}{3}\pi(1.5)^3$$

$$= \frac{6.75}{3} \times \frac{22}{7}$$

$$= \frac{99}{14}\ \text{m}^3$$

Since, $1\ \text{m}^3 = 1000$ litre

$$\frac{99}{14}\ \text{m}^3 = \frac{99000}{14}\ \text{litre}$$

$$\text{Half of the tank} = \frac{99000}{28}\ \text{litre}$$

Required time

$$= \frac{25}{7} \times \frac{99000}{28}$$

= 990 secs

= 16 mins 30 seconds

Solution 21

Length of the roof (l) = 44 m,

Breadth of the roof (b) = 20 m

Let the height of the water on the roof be h m.

Volume of water falling on the roof = $l \times b \times h$

$= 44 \times 20 \times h$

$= 880h$

Radius of the cylindrical vessel (R) = $\frac{4}{2} = 2$ m

Height of the water in the cylindrical vessel (H) = 3.5 m

Volume of the water in the cylindrical vessel = $\pi R^2 H$

$= \frac{22}{7} \times 2 \times 2 \times 3.5$

$= \frac{308}{7}$

$= 44$

Volume of water falling on the roof = Volume of the water in the cylindrical vessel

$\Rightarrow 880h = 44$

$\Rightarrow h = \frac{44}{880}$

$\Rightarrow h = \frac{44}{880} \times 100$

$\Rightarrow h = 5$ cm

$\Rightarrow$ Height of the water on the roof is 5 cm.

Solution 22

Given Length of the roof (l) = 22 m,
Breadth of the roof (b) = 20 m
Let the height of the rainfall be h m.
Now,
Volume of cuboidal water column = $l \times b \times h$

$$= 22 \times 20 \times h$$

$$= 440h \text{ m}^3$$

Now, the rainwater drains into the cylindrical vessel.
Diameter of the cylindrical vessel (R) = 2 m
Radius of the cylindrical vessel (r) = 1 m
Height of the vessel (h) = 3.5 m
$\Rightarrow$ Capacity of the vessel = Volume of the water in the cylindrical vessel

$$= \pi r^2 h$$

$$= \frac{22}{7} \times 1 \times 1 \times 3.5$$

$$= 11 \text{ m}^3$$

Given that the vessel is filled upto $\frac{4}{5}$th of its volume by the rainwater.

$\Rightarrow$ Volume of the water inside the vessel = $\frac{4}{5} \times$ volume by the rainwater

$$= \frac{4}{5} \times 11$$

$$= 8.8 \text{ m}^3$$

Now,
Volume of the cuboidal water column = Volume of the water inside the vessel

$$\Rightarrow 440h = 8.8$$

$$\Rightarrow h = \frac{8.8}{440}$$

$$\Rightarrow h = \frac{8.8}{440} \times 100$$

$$\Rightarrow h = 2 \text{ cm}$$

Thus, height of the rainfall is 2 cm.

Solution 23

We know that,
1 cm = 0.01 m
Given Radius of the cone (r) = 30 cm = 0.3 m,
Height of the cone (h) = 60 cm = 0.6 m,
Radius of the cylinder (R) = 60 cm = 0.6 m,
Height of the cylinder (H) = 180 cm = 1.8 m
Given that the cone is inserted in the cylinder touches its bottom.
⇒ Volume of the water left in the cylinder = Volume of the cylinder − Volume of the cone

$$= \pi R^2 H - \frac{1}{3}\pi r^2 h$$
$$= \frac{22}{7} \times 0.6 \times 0.6 \times 1.8 - \frac{1}{3} \times \frac{22}{7} \times 0.3 \times 0.3 \times 0.6$$
$$= \frac{14.256}{7} - \frac{1.188}{21}$$
$$= \frac{42.768 - 1.188}{21}$$
$$= \frac{41.58}{21}$$
$$= 1.98 \text{ m}^3$$

Solution 24

Given Internal Diameter of cylindrical pipe = 2 cm

$\Rightarrow$ Radius of the cylindrical pipe $(r) = 1$ cm

Now,

Area of cross-section of the pipe = πr^2

$$= \pi(1)^2$$
$$= \pi \text{ cm}^2$$

Speed of water = 0.4 m/sec

$\Rightarrow$ Volume of water flown out in half and hour = $40 \times 30 \times 60$

$$= 72000\pi \text{ cm}^2$$

Radius of cylindrical tank $(R) = 40$ cm

Let the level of the water rise to the height be h cm.

$\Rightarrow$ Volume of cylindrical tank = $\pi R^2 h$

$$= \pi(40)^2 h$$
$$= 1600\pi h \text{ cm}^2$$

Now,

Volume of cylindrical tank = Volume of water flown out in half and hour

$\Rightarrow 1600\pi h = 72000\pi$

$\Rightarrow 16h = 720$

$\Rightarrow h = 45$ cm

Thus, level of the water rise to the height of 45 cm.

Note: The answer in the textbook is incorrect.

Solution 25

Given that r = 7 cm = 0.07 m

h = 6 km/hr = 100 m/min

Volume of the pipe = $\pi r^2 h$

$$= \frac{22}{7} \times 0.07^2 \times 100$$

$$= 1.54$$

Volume of the rectangle = $60 \times 22 \times 0.07 = 92.4 \text{ m}^3$

$$\text{Time taken} = \frac{\text{Volume of the rectangle}}{\text{Volume of the pipe}}$$

$$= \frac{92.4}{1.54}$$

= 60 mins = 1 hour

Hence, the time in which the level of water in the tank will rise is 1 hour.

Solution 26

Width of the canal = 6 m
Depth of the canal = 1.5 m
It is given that the water is flowing at a speed of 4 km/hr = 4000 m/hr

Thus, the length of the water column formed in 10 minutes that is, $\frac{1}{6}$ hour

$= \frac{1}{6} \times 4000 = 666.67$ m

Hence, the volume of the water flowing in $\frac{1}{6}$ hour

= Volume of the cuboid of length 666.67 m, width 6m and depth 1.5 m.

$\Rightarrow$ Volume of the water flowing in $\frac{1}{6}$ hour

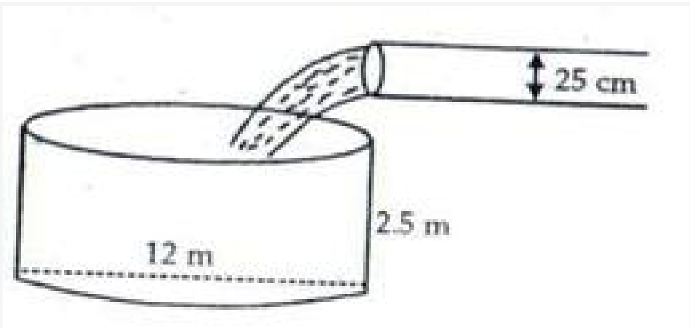
$= 666.67 \times 6 \times 1.5$
= 6000.03 approximately

Let x be the area irrigated in $\frac{1}{6}$ hour.

Then, $x \times \frac{8}{100} = 6000$

$\Rightarrow x = \frac{60000}{8} = 75000 \text{ m}^2$

Solution 27



Height of cylindrical tank = 2.5 m

Its diameter = 12 m, Radius = 6 m

Volume of tank = $\pi r^2 h = \frac{22}{7} \times 6 \times 6 \times 2.5 \text{ m}^3 = \frac{1980}{7} \text{ m}^3$

Water is flowing at the rate of 3.6 km/ hr = 3600 m/hr

Diameter of pipe = 25 cm, radius = 0.125 m

Volume of water flowing per hour

$$= \frac{22}{7} \times 0.125 \times 0.125 \times 3600 \text{ m}^3$$

$$= \frac{22 \times 3600}{7 \times 8 \times 8} \text{m}^3 = \frac{2475}{14} \text{m}^3$$

$$\text{Time taken to fill the tank} = \frac{1980}{7} \div \frac{2475}{14} \text{ hr}$$

$$= \frac{1980}{7} \times \frac{14}{2475} \text{hr} = \frac{792}{495} \text{hr}$$

$$= 1.36 \text{ hr} = 1 \text{ hr } 36 \text{ min.}$$

$$\text{Water charges} = \text{Rs.} \frac{1980}{7} \times 0.07 = \text{Rs.} 19.80$$

Solution 28

$$\text{Flow rate} = \frac{\text{Volumetric flow rate}}{\text{Area}}$$

$\text{Area} = \pi r^2$

$$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = 38.5\ \text{cm}^2$$

Volumetric flow rate = 192.5 l / min.

Since 1 l = 1 dm^3 = 1000 cm^3

So, volumetric flow rate = $192.5 \times 1000\ \text{cm}^3/\text{min}$

So, flow rate per 38.5 cm^2 $= \dfrac{192.5 \times 1000}{38.5}\ \text{cm}^3/\text{min}/\text{cm}^2$

$\Rightarrow$ flow rate = 5000 cm / min

$$= \frac{5000 \times 0.00001\ \text{km}}{\left(\frac{1}{60}\right)\ \text{h}}$$

$$= 3\ \text{km}/\text{h}$$

Solution 29

Diameter of marble = 14 cm
Radius of marble = 7 cm
Number of marbles = 150
Diameter of cylinder = 7 cm
Radius of cylinder = 3.5 cm
Let the height of the water raised when 150 spheres are dropped in the vessel.
Volume of 150 marbles = Volume of water raised by height 'h' inside the vessel.

$\Rightarrow 150 \times \frac{4}{3} \times \pi \times 7 \times 7 \times 7 = \pi \times 3.5 \times 3.5 \times h$

$\ldots..\left(\text{Since volume of a sphere} = \frac{4}{3}\pi r^2 h \text{ and volume of a cylinder} = \pi r^2 h\right)$

$\Rightarrow 200 \times 343 = 12.25 \times h$

$\Rightarrow h = \frac{68600}{12.25} = 5600\ \text{cm} = 56\ \text{m}$

Hence, the rise in the level of water in the vessel is 56 m.

Solution 30

$$\text{Radius of marbles} = \frac{\text{Diameter}}{2} = \left(\frac{1.4}{2}\right)\text{cm}$$

$$\text{Volume of marbles} = \frac{4}{3}\pi r^3$$

$$= \left[\frac{4}{3} \times \pi \times \left(\frac{1.4}{2}\right) \times \left(\frac{1.4}{2}\right) \times \left(\frac{1.4}{2}\right)\right]\text{cm}^3$$

$$\text{Radius of beaker} = \left(\frac{7}{2}\right)\text{cm}$$

Volume of rising water in beaker

$$= \pi r^2 h = \left(\pi \times \left(\frac{7}{2}\right)^2 \times \left(\frac{56}{10}\right)\right)\text{cm}^3$$

Let the number of marbles be n

$\therefore$ n volume of marble = volume of rising water in beaker

$$n \times \left(\frac{4}{3}\pi \times \frac{1.4}{2} \times \frac{1.4}{2} \times \frac{1.4}{2}\right) = \pi \times \frac{7}{2} \times \frac{7}{2} \times \frac{56}{10}$$

$$n = 150$$

Hence the number of marbles is 150

Solution 31

Given the diameter = 10 m

So, the radius of the well = 5 m

Height of the well = 14 m

Width of the embankment = 5 m

Therefore, radius of the embankment = 5 + 5 = 10 m

Let h be the height of the embankment.

Hence the volume of the embankment = volume of the well

That is, $\pi(R-r)^2h = \pi r^2 h$

$\Rightarrow (10^2 - 5^2) \times h = 5^2 \times 14$

$\Rightarrow (100-25) \times h = 25 \times 14$

$\Rightarrow h = \dfrac{25 \times 14}{75} = \dfrac{14}{3}$

Therefore, h = 4.67 cm approximately.

The value reflected by the villagers is that

we must work hard and make maximum use of the available resources.

Note : The answer given in the textbook is incorrect.

Solution 32

Given Diameter of the well = 14 m

$\Rightarrow$ Radius of the well (r) = 7 m and

Height of the well (h) = 8 m

Volume of the earth dug out of the well = $\pi r^2 h$

$$= \frac{22}{7} \times (7)^2 \times 8$$

$$= 22 \times 56$$

$$= 1232 \text{ m}^3$$

Area of the field on which the earth is spread = $l \times b - \pi r^2$

$$= (35 \times 22) - \frac{22}{7} \times (7)^2$$

$$= 770 - 154$$

$$= 616 \text{ m}^2$$

$\Rightarrow$ Level of eath raised in the field = $\dfrac{\text{Volume of the earth dug out of the well}}{\text{Area of the field on which the earth is spread}}$

$$= \frac{1232}{616}$$

$$= 2 \text{ m}$$

Thus, the level of earth raised in the field is 2 m.

Solution 33

Diameter of the copper wire = 6 mm

So, the radius = 3 mm

Diameter of the cylinder = 49 cm = 490 mm

Radius of the cylinder = 245 mm

Length of the cylinder = 18 cm = 180 mm

Number of turns of the copper wire = $\frac{180}{6} = 30$

Length of one turn = $2\pi(245)$

$= 2 \times \frac{22}{7} \times 245$

$= 1540$ mm

So, the total length of the copper wire

$= 30 \times 1540$

$= 46200$ mm

$= 46.2$ m

Thus, the volume of the copper wire

$= \pi \times (3)^2 \times 46200$

$= 1306800 \text{ mm}^3$

$= 1306.8 \text{ cm}^3$

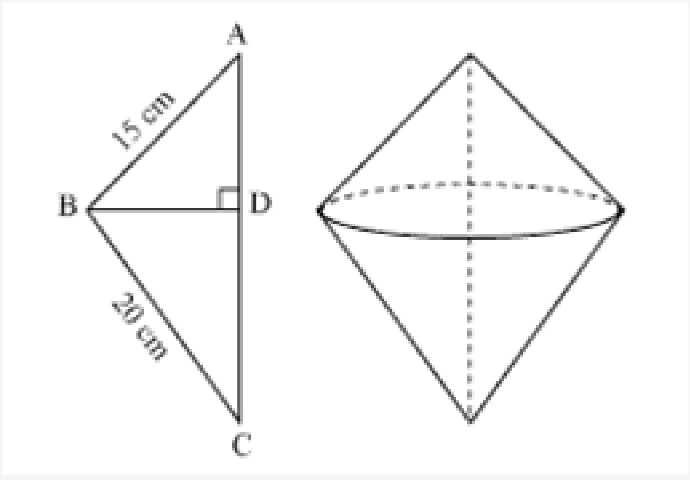
Given that the density = 8.8 g per cu. cm.

So, weight of the wire = 8.8×1306.8

$= 11499.84$ g

$= 11.5$ kg

Solution 34


A
15 cm
B
D
20 cm
C

Consider the following right angled triangle ABC is rotated
through its hypotenuse AC.
BD $\perp$ AC. In this case BD is the radius of the double cone generated.
Using Pythagoras theorem for ΔABC,

$AC^2 = AB^2 + BC^2$

$\Rightarrow AC^2 = 15^2 + 20^2$

$\Rightarrow AC^2 = 225 + 400$

$\Rightarrow AC^2 = 625$

$\Rightarrow AC = 25$ cm

Let $AD = x$ cm ...(i)

$\Rightarrow DC = 25 - x$ ...(ii)

Using Pythagoras theorem in ΔCBD

$AB^2 = AD^2 + BD^2$ and $BC^2 = BD^2 + CD^2$

$\Rightarrow 15^2 = x^2 + BD^2$ and $20^2 = BD^2 + (25 - x)^2$

$\Rightarrow BD^2 = 15^2 - x^2$ and $BD^2 = 20^2 - (25 - x)^2$

$\Rightarrow 20^2 - (25 - x)^2 = 15^2 - x^2$

$\Rightarrow 400 - (625 - 50x + x^2) = 225 - x^2$

$\Rightarrow 50x = 450$

$\Rightarrow x = 9$

$\Rightarrow BD^2 = 15^2 - 9^2$

$\Rightarrow BD^2 = 144$

$\Rightarrow BD = 12$ cm

Radius of the generated double cone = 12 cm

From (i)

$DC = 25 - 9 = 16$ cm

From (ii)

$AD + DC = 9 + 16 = 25$ cm

Now,

Volume of the cone generated = Volume of the upper cone + Volume of the lower cone

$$= \frac{1}{3} \times \pi \times BD^2 \times AD + \frac{1}{3} \times \pi \times BD^2 \times DC$$

$$= \frac{1}{3} \times \pi \times BD^2 \times (AD + DC)$$

$$= \frac{1}{3} \times \pi \times 12^2 \times 25$$

$$= 1200 \times \frac{22}{7}$$

$$= 3771 \text{ cm}^3$$

Surface area of the double cone formed = L.S.A of upper cone + L.S.A of the lower cone

$$= \pi \times BD \times AD + \pi \times BD \times DC$$

$$= \pi \times 12 \times 15 + \pi \times 12 \times 20$$

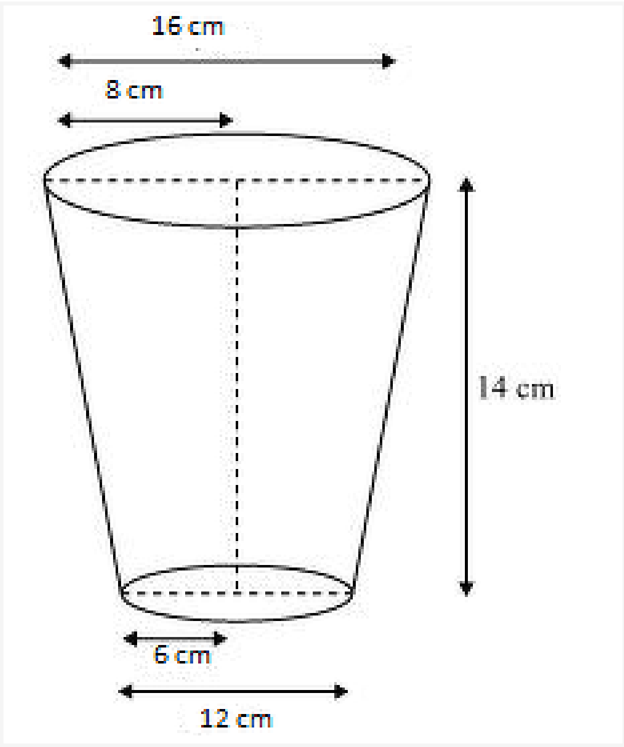
$$= 180\pi + 240\pi$$

$$= 420 \times \frac{22}{7}$$

$$= 1320 \text{ cm}^2$$

R S Aggarwal and V Aggarwal Solution for Class 10 Mathematics Chapter 19 - Volume and Surface Areas of Solids Page/Excercise 19C

Solution 1



Given Diameter of the upper end of the glass = 16 cm

$\Rightarrow$ Radius of the upper end of the glass (R) = 8 cm,

Diameter of the lower end of the glass = 12 cm

$\Rightarrow$ Radius of the lower end of the glass (R) = 6 cm

Now, height of the glass (h) = 14 cm

$$\text{Capacity of the glass} = \frac{1}{3} \times \pi \times h\left(R^2 + r^2 + R \times r\right)$$
$$= \frac{1}{3} \times \frac{22}{7} \times 14\left(8^2 + 6^2 + 8 \times 6\right)$$
$$= \frac{44}{3}(64 + 36 + 48)$$
$$= \frac{44}{3} \times 148$$
$$= 2170.67 \text{ cm}^3$$

Solution 2

$$\text{Slant height of the frustum} = \sqrt{h^2 + (R - r)^2}$$
$$= \sqrt{8^2 + (18 - 12)^2}$$
$$= \sqrt{64 + 36}$$
$$= \sqrt{100}$$
$$= 10 \text{ cm}$$

$$\text{Total surface area} = \pi(R + r)l + \pi r^2 + \pi R^2$$
$$= \pi\left[(18 + 12)10 + 12^2 + 18^2\right]$$
$$= \pi\left[300 + 144 + 324\right]$$
$$= 3.14 \times 768$$
$$= 2411.52 \text{ cm}^2$$

Solution 3

Slant height of frustum, $l = \sqrt{(R-r)^2 + h^2}$

$= \sqrt{(14-7)^2 + 24^2}$

$= \sqrt{7^2 + 24^2}$

$= \sqrt{49 + 576}$

$= \sqrt{625}$

$= 25$ cm

Volume of frustum of cone $= \frac{1}{3}\pi h\left(R^2 + Rr + r^2\right)$

$= \frac{1}{3} \times \frac{22}{7} \times 24 \times \left(14^2 + 14 \times 7 + 7^2\right)$

$= \frac{1}{3} \times \frac{22}{7} \times 24 \times (196 + 98 + 49)$

$= \frac{1}{3} \times \frac{22}{7} \times 24 \times (343)$

$= 22 \times 8 \times 343$

$= 8624\ cm^3$

Curved surface area $= \pi l(R + r)$

$= \frac{22}{7} \times 25 \times (14 + 7)$

$= \frac{22}{7} \times 25 \times 21$

$= \frac{11550}{7}$

$= 1650\ cm^2$

Area of the base (lower end) of the bucket

$= \pi r^2$

$= \frac{22}{7} \times 7 \times 7$

$= 154\ cm^2$

Area of the metal sheet used to make the bucket

= Curved surface area + Area of the base (lower end)

$= 1650 + 154$

$= 1804\ cm^2$

Solution 4

Given radius of the upper end of container $(r_1) = 20$ cm

Radius of the lower end of container $(r_2) = 8$ cm

Height of the container $(h) = 24$ cm

Slant height of frustum $(l) = \sqrt{(r_1 - r_2)^2 + h^2}$

$$= \sqrt{(20-8)^2 + 24^2}$$

$$= \sqrt{12^2 + 24^2}$$

$$= \sqrt{144 + 576}$$

$$= 26.83 \text{ cm}$$

Capacity of the container = Volume of frustum

$$= \frac{1}{3}\pi h\left(r_1^{\,2} + r_2^{\,2} + r_1 r_2\right)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 24 \times \left(20^2 + 8^2 + 20 \times 8\right)$$

$$= \frac{528}{21} \times (400 + 64 + 160)$$

$$= 15689.14 \text{ cm}^3$$

$$= 15.68914 \text{ litres}$$

Cost of 1 litre milk = Rs. 21

Cost of 15.68914 litre milk = 15.68914×21 = Rs. 329.47

Solution 5

Given height of the frustum of cone = 16 cm,

Diameter of lower end = 16 cm

$\Rightarrow$ Radius of lower end (r) = 8 cm,

Diameter of upper end = 40 cm

$\Rightarrow$ Radius of upper end (R) = 20 cm

$$\begin{aligned}
\text{Slant height of the frustum } (l) &= \sqrt{h^2 + (R - r)^2} \\
&= \sqrt{16^2 + (20 - 8)^2} \\
&= \sqrt{16^2 + 12^2} \\
&= \sqrt{256 + 144} \\
&= \sqrt{400} \\
&= 20 \text{ cm}
\end{aligned}$$

$$\begin{aligned}
\text{Surface area of the frustum of the cone} &= \pi(R + r)l + \pi r^2 \\
&= \pi\left[(20 + 8)20 + 8^2\right] \\
&= \pi[560 + 64] \\
&= 624 \times \frac{22}{7} \\
&= 1961.14 \text{ cm}^2
\end{aligned}$$

Cost of metal sheet per 100 cm^2 = Rs. 10

$$\begin{aligned}
\text{Cost of metal for Rs. } 1961.14 \text{ cm}^2 &= \frac{1961.14 \times 10}{100} \\
&= \text{Rs. } 196.114
\end{aligned}$$

Note : The answer in the book is incorrect.

Solution 6

Here R = 33 cm, r = 27 cm and l = 10 cm

$$\therefore\ h = \sqrt{l^2 - \left(R^2 - r^2\right)}\ cm = \sqrt{(10)^2 - (33 - 27)^2}\ cm$$

$$= \sqrt{(10)^2 - (6)^2} = \sqrt{64}\, cm = 8\, cm$$

Capacity of the frustum $= \frac{1}{3}\pi h\left(R^2 + r^2 + Rr\right)cm^3$

$$= \frac{1}{3} \times \frac{22}{7} \times 8\left[(33)^2 + (27)^2 + 33 \times 27\right]cm^3$$

$$= (8.38 \times 2709)\, cm^3 = 22701.4\ cm^3$$

Total surface area = $\left[\pi R^2 + \pi r^2 + \pi l\,(R + r)\right]cm^2$

$$= \pi\left[R^2 + r^2 + l(R + r)\right]cm^2$$

$$= \frac{22}{7}\left[(33)^2 + (27)^2 + 10 \times (33 + 27)\right]cm^2$$

$$= \left(\frac{22}{7} \times 2418\right)cm^2 = 7599.43\, cm^2$$

Solution 7

Height = 15 cm, R = $\frac{56}{2} = 28\, cm$ and $r = \frac{42}{2} = 21\, cm$

Capacity of the bucket = $\frac{1}{3}\pi h\left(R^2 + r^2 + Rr\right)cm^3$

$$= \frac{1}{3} \times \frac{22}{7} \times 15\left[(28)^2 + (21)^2 + 28 \times 21\right] cm^3$$
$$= (15.71 \times 1831)\, cm^3$$
$$= (28482.23)\, cm^3$$

Quantity of water in bucket = 28.49 litres

Solution 8

R = 20 cm, r = 8 cm and h = 16 cm

$$\therefore\ l = \sqrt{h^2 + (R - r)^2} = \sqrt{(16)^2 + (20 - 8)^2}$$
$$= \sqrt{256 + 144}\, cm = 20\, cm$$

Total surface area of container = $\pi l(R + r) + \pi r^2$

$$= \left[3.14 \times 20 \times (20 + 8) + 3.14 \times 8 \times 8\right] cm^2$$
$$= (3.14 \times 20 \times 28 + 3.14 \times 8 \times 8)\, cm^2$$
$$= (1758.4 + 200.96)\, cm^2$$
$$= 1959.36\ cm^2$$

Cost of metal sheet used = $Rs\left(1959.36 \times \frac{15}{100}\right) = Rs.\,293.90$

Solution 9

R = 15 cm, r = 5 cm and h = 24 cm

$$\therefore \; l = \sqrt{h^2 + (R-r)^2} = \sqrt{(24)^2 + (10)^2}\,\text{cm}$$
$$= \sqrt{576 + 100}\,\text{cm} = \sqrt{676}\;\text{cm} = 26\,\text{cm}$$

(i)Volume of bucket = $\frac{1}{3}\pi h\left(R^2 + r^2 + Rr\right)$

$$= \frac{1}{3} \times 3.14 \times 24 \times \left[(15)^2 + (5)^2 + 15 \times 5\right]$$
$$= (25.12 \times 325)\,\text{cm}^3$$
$$= 8164\ \text{cm}^3 = 8.164\ \text{litres}$$

Cost of milk = Rs. (8.164 20) = Rs. 163.28

(ii)Total surface area of the bucket

$$= \pi l(R + r) + \pi r^2$$
$$= (3.14 \times 26 \times 20 \times 3.14 \times 5 \times 5)\,\text{cm}^2$$
$$= 1711.3\ \text{cm}^2$$

Cost of sheet = $\text{Rs}\left(\frac{1711.3 \times 10}{100}\right) = \text{Rs.}\,171.13$

Solution 10

Given Diameter of the upper face = 35 cm

$\Rightarrow$ Radius of the upper face (R) = 17.5 cm,

Diameter of the lower face = 30 cm

$\Rightarrow$ Radius of the upper face (r) = 15 cm and

Vertial height of the frustum (h) = 14 cm

$$\text{Volume of the frustum} = \frac{\pi h}{3}\left(R^2 + r^2 + Rr\right)$$

$$= \frac{22}{7} \times \frac{14}{3}\left(17.5^2 + 15^2 + 17.5 \times 15\right)$$

$$= \frac{44}{3}\left(306.25 + 225 + 262.5\right)$$

$$= \frac{44}{3} \times 793.75$$

$$= 11641.667 \text{ cm}^3$$

Volume of the oil in the container = 11641.667 cm^3

1 cubic cm of oil = 1.2 g

$\Rightarrow$ Total mass of the oil in the container = 11641.667×1.2

= 13970 kg

Total cost of the oil in the container = 13970×40

= Rs. 558800

Solution 11

Given R = 28 cm and r = 21 cm.

We know that,

$1\ l = 1000\ cm^3$

$\Rightarrow 28.49\ l = (28.49 \times 1000)\ cm^3 = 28490\ cm^3$

Now,

$$\text{Volume of frustum} = \pi \frac{h}{3}\left(Rr + R^2 + r^2\right)$$

$$\Rightarrow 28490 = \frac{22}{7} \times \frac{h}{3}\left[(28)(21) + 28^2 + 21^2\right]$$

$$\Rightarrow 28490 = \frac{22}{7} \times \frac{h}{3}\left[588 + 784 + 441\right]$$

$$\Rightarrow 28490 = \frac{22}{7} \times \frac{h}{3}\left[1813\right]$$

$$\Rightarrow h = \frac{28490 \times 21}{22 \times 1813}$$

$\Rightarrow h = 15\ cm$

Hence, the height of the bucket is 15 cm.

Solution 12

Given Height of the bucket (h) = 15 cm

r = 14 cm

R = ?

Now,

$$\text{Volume of the bucket} = \pi \times \frac{1}{3} \times \left(r^2 + R^2 + rR\right) \times h$$

$$\Rightarrow 5390 = \frac{22}{7} \times \frac{1}{3} \times \left(14^2 + R^2 + 14R\right) \times 15$$

$$\Rightarrow 5390 = \frac{110}{7} \times \left(196 + R^2 + 14R\right)$$

$$\Rightarrow \frac{539 \times 7}{11} = 196 + R^2 + 14R$$

$$\Rightarrow 343 = 196 + R^2 + 14R$$

$$\Rightarrow R^2 + 14R = 147$$

$$\Rightarrow R^2 + 14R - 147 = 0$$

$$\Rightarrow R^2 + 21R - 7R - 147 = 0$$

$$\Rightarrow R\left(R + 21\right) - 7\left(R + 21\right) = 0$$

$$\Rightarrow \left(R - 7\right)\left(R + 21\right) = 0$$

$$\Rightarrow R = -21 \text{ or } R = 7$$

$$\Rightarrow R = 7 \text{ cm} \qquad \ldots.(\because R \text{ cannot be negative})$$

Solution 13

Given Radius of the circular ends of the solid frustum of a cone are 33 cm and 27 cm.

$\Rightarrow R = 33$ cm and $r = 27$ cm and slant height = 10 cm

$$\text{Total surface area of frustum} = \pi\left[R^2 + r^2 + l\left(R + r\right)\right]$$

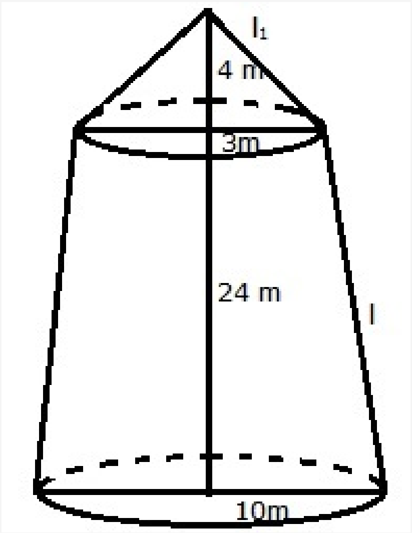
$$= 3.14 \times \left[33^2 + 27^2 + 10\left(33 + 27\right)\right]$$

$$= 3.14 \times \left[1089 + 729 + 600\right]$$

$$= 3.14 \times 2418$$

$$= 7592.52 \text{ cm}^2$$

Solution 14



R = 10cm, r = 3 m and h = 24 m

Let l be the slant height of the frustum, then

$$l = \sqrt{h^2 + (R - r)^2}$$
$$= \sqrt{(24)^2 + (10 - 3)^2}$$
$$= \sqrt{(24)^2 + (7)^2}$$
$$= \sqrt{576 + 49}$$
$$= \sqrt{625}\ \text{m} = 25\ \text{m}$$

Let l_1 be the slant height of conical part

$r = 3$ m

and $h = 4$m

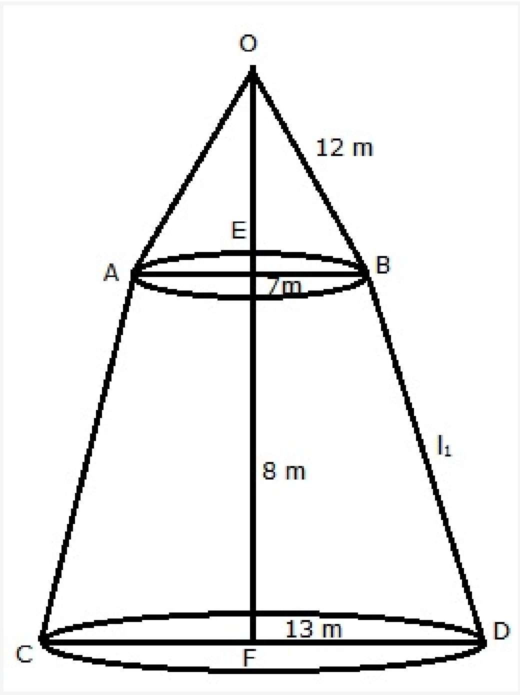
$$\therefore \quad l_1 = \sqrt{3^2 + 4^2}\ \text{m}$$
$$= \sqrt{25}\ \text{m} = 5\ \text{m}$$

Quantity of canvas = (Lateral surface area of the frustum)

+ (lateral surface area of the cone)

$$= \left[\pi l(R+r) + \pi r l_1\right] m^2$$
$$= \pi[25 \times (10+3) + (3 \times 5)]m^2$$
$$= \frac{22}{7} \times \left[(25 \times 13) + (3 \times 5)\right] m^2$$
$$= 1068.57\ m^2$$

Solution 15



ABCD is the frustum in which upper and lower radii are EB = 7 m and FD = 13 m

Height of frustum= 8 m

Slant height l_1 of frustum

$$= \sqrt{h^2 + (R - r)^2}$$
$$= \sqrt{8^2 + (13 - 7)^2}$$
$$= \sqrt{64 + 36}$$
$$= \sqrt{100} = 10\,m$$

Radius of the cone = EB = 7 m

Slant height l_2 of cone = 12 m

$\therefore$ Surface area of canvas required

$$= \pi(R + r)l_1 + \pi r l_2$$
$$= \pi[(13 + 7) \times 10 + 7 \times 12]$$
$$= \frac{22}{7} \times [200 + 84] = \frac{22}{7} \times 284\ m^2$$
$$= 892.6\ m^2$$

Solution 16

Let R be the radius of the bigger end and r be the radius of the smaller end of the frustum of a cone.

Given Perimeter of bigger end = 48 cm

$\Rightarrow 2\pi R = 48$

$\Rightarrow 2 \times \frac{22}{7} \times R = 48$

$\Rightarrow \frac{44}{7} \times R = 48$

$\Rightarrow \frac{11}{7} \times R = 12$

$\Rightarrow R = \frac{84}{11}$

Given Perimeter of smaller end = 36 cm

$\Rightarrow 2\pi r = 36$

$\Rightarrow 2 \times \frac{22}{7} \times r = 36$

$\Rightarrow \frac{44}{7} \times r = 36$

$\Rightarrow \frac{22}{7} \times r = 18$

$\Rightarrow r = \frac{63}{11}$

Given Height (h) = 11 cm

$$\text{Volume} = \frac{1}{3}\pi h\left(R^2 + r^2 + Rr\right)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 11\left(\left(\frac{84}{11}\right)^2 + \left(\frac{63}{11}\right)^2 + \frac{84 \times 63}{(11)^2}\right)$$

$$= \frac{1}{3} \times \frac{22}{7} \times \frac{11}{(11)^2}(7056 + 3969 + 5292)$$

$$= \frac{1}{3} \times \frac{22}{7} \times \frac{1}{11} \times 16317$$

$$= 1554 \text{ cm}^3$$

Slant height of the frustum,

$l = \sqrt{h^2 + (R - r)^2}$

$\Rightarrow l = \sqrt{11^2 + \left(\frac{84}{11} - \frac{63}{11}\right)^2}$

$\Rightarrow l = \sqrt{11^2 + \left(\frac{21}{11}\right)^2}$

$\Rightarrow l = \sqrt{\left(\frac{11^4 + 21^2}{11^2}\right)}$

$\Rightarrow l = \sqrt{\frac{14662}{121}}$

$\Rightarrow l = \sqrt{124.64}$

$\Rightarrow l = 11.164$ cm

Curved surface area of frustum $= \pi l\left(r + R\right)$

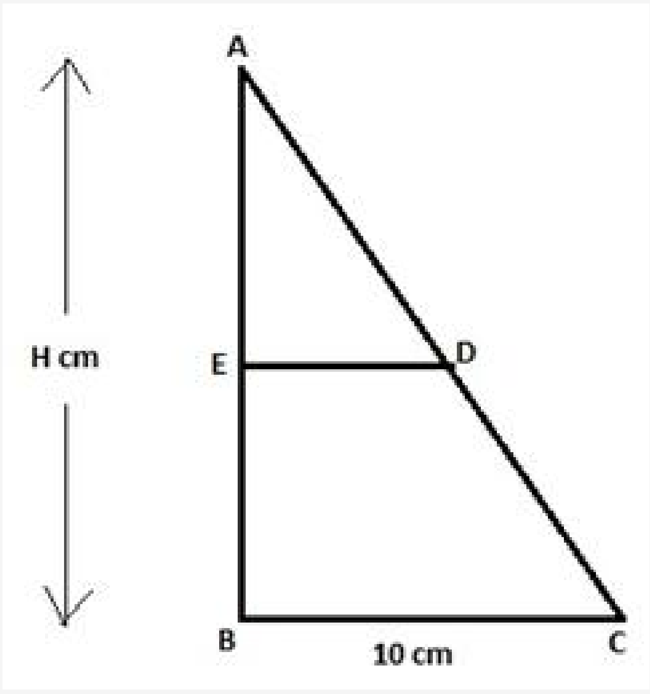
$$= \frac{22}{7} \times 11\left(\frac{84}{11} + \frac{63}{11}\right)$$

$$= \frac{22}{7} \times 11.164(7.636 + 5.727)$$

$$= \frac{22}{7} \times 11.164 \times 13.36$$

$$= 468.89 \text{ cm}^2$$

Solution 17


A
H cm
E
D
B
10 cm
C

Given that AE = EB = $\frac{H}{2}$, where H is the height of the cone ABC.

Consider the ΔAED and ΔABC.

$\angle AED = \angle ABC = 90°$

$\angle EAD = \angle BAC$ .....(common angle)

So, $\Delta AED \sim \Delta ABC$ ....(AA criterion for similarity)

$\Rightarrow \frac{AE}{AB} = \frac{ED}{BC} = \frac{r}{10}$, where r is the radius of the cone,

when a plane parallel to its base cuts the height of its mid-point.

$\Rightarrow \frac{\frac{H}{2}}{H} = \frac{r}{10}$, where H is the height of the cone

$\Rightarrow \frac{1}{2} = \frac{r}{10}$

$\Rightarrow r = 5$ cm

Volume of the frustum of the cone

= Volume of the cone ABC – Volume of the upper part of the cone AED

$= \frac{1}{3}\pi R^2 H - \frac{1}{3}\pi\left(\frac{R}{4}\right)^2\left(\frac{H}{2}\right)$

Here, R = 10 cm and $\frac{R}{2} = 5$ cm

$\Rightarrow$ Volume of the frustum of the cone

$= \frac{1}{3}\pi(100)H - \frac{1}{3}\pi(25)\frac{H}{2}$

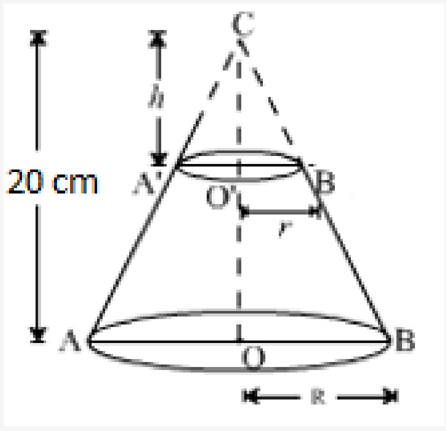
$= \frac{175\pi H}{6}$

Volume of the cone AED = $\frac{1}{3}\pi(25)\frac{H}{2} = \frac{25\pi H}{6}$

Ratio of their volumes = $\frac{\frac{25\pi H}{6}}{\frac{175\pi H}{6}} = \frac{1}{7}$

Hence, the ratio is 1:7.

Solution 18


C
h
20 cm
A'
O'
B
r
A
O
B
R

H = 20 cm = height of the right circular cone.

R = radius of the base of the cone

V = Volume of the cone = $\frac{1}{3}\pi r^2 H$

Let the radius of the base of the small cone = r cm

h = height of the small cone

v = volume of small cone = $\frac{1}{3}\pi r^2 h$

From the similar triangles principles,

$\frac{r}{h} = \frac{R}{H}$

$r = \frac{Rh}{H}$

Given $v = \frac{1}{8}V$

$\Rightarrow V = 8v$

$\Rightarrow \frac{1}{3}\pi R^2 H = 8 \times \frac{1}{3}\pi r^2 h$

$\Rightarrow R^2 H = 8 \times r^2 h$

$\Rightarrow R^2 H = 8 \times \left(\frac{R^2 h^2}{H^2}\right) \times h$

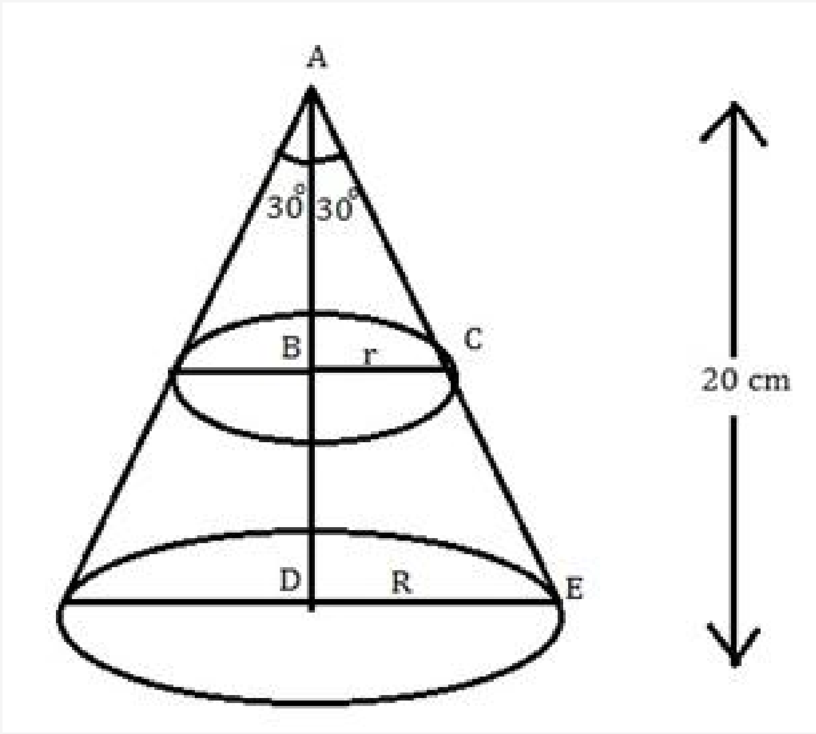
$\Rightarrow H^3 = 8 h^3$

$\Rightarrow h = \frac{H}{2}$

$\Rightarrow h = \frac{20}{2}$ cm = 10 cm

Hence, the height above the base is 10 cm.

Solution 19


A
30°
30°
B
r
C
20 cm
D
R
E

ΔABC and ΔADE are similar triangles and hence the corresponding sides are proportional.
B is the mid-point of AD.

Thus, $\frac{AB}{AD} = \frac{BC}{DE}$

Let r and R be the radii of both the ends of the frustum.

So, $\frac{10}{20} = \frac{r}{R}$

$\Rightarrow R = 2r$

Also consider the ΔADE.

$\tan 30^\circ = \frac{\text{opposite side}}{\text{adjacent side}}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{R}{20}$

$\Rightarrow R = \frac{20}{\sqrt{3}}$ cm

$\Rightarrow r = \frac{10}{\sqrt{3}}$ cm

Volume of the frustum $= \frac{\pi}{3}\left[R^2 + Rr + r^2\right]h$

$\Rightarrow \pi\left(\frac{\frac{1}{12}}{2}\right)^2 h = \frac{\pi}{3}\left[\frac{400}{3} + \frac{200}{3} + \frac{100}{3}\right] \times 10$

$\Rightarrow \frac{1}{576} \times h = \frac{1}{3}\left[\frac{700}{3}\right] \times 10$

$\Rightarrow h = \frac{1}{3}\left[\frac{700}{3}\right] \times 10 \times 576$

$\Rightarrow h = 100 \times 10 \times 576$

$\Rightarrow h = 448000$ cm

$\Rightarrow h = 4480$ m

Hence, the length of the wire is 4480 m.

Solution 20

Area of the material used for making the fez

= Curved surface area of the frustum + Area of the upper circular end

$= \pi(r_1 + r_2)l + \pi r_2^2$

$= \frac{22}{7} \times (10 + 4) \times 15 + \frac{22}{7} \times 4 \times 4$

$= 710.28\ cm^2$

Solution 21

Let r_1 and r_2 be the radii of the ends of the frustum of a cone.

Let l, h and H be the slant height of the frustum, height of the frustum and the height of the cylinder respectively.

Diameter of the top of the frustum, $2r_1 = 18$ cm

$\therefore\ r_1 = 9$ cm

Diameter of the bottom of the frustum, $2r_2 = 8$ cm

$\therefore\ r_1 = 4$ cm

Height of the frustum = Total height of the funner – Height of the cylinder

= 22 cm – 10 cm

= 12 cm

Slant height of the frustum,

$l = \sqrt{h^2 + (r_1 - r_2)^2}$

$\Rightarrow l = \sqrt{12^2 + (9 - 4)^2}$

$\Rightarrow l = \sqrt{144 + 25}$

$\Rightarrow l = \sqrt{169}$

$\Rightarrow l = 13$ cm

Curved surface area of the oil funnel

$= \pi(r_1 + r_2)l + 2\pi r_2 H$

$= \frac{22}{7} \times (9 + 14) \times 13 + 2 \times \frac{22}{7} \times 4 \times 10$

$= 782.6\ m^2$

R S Aggarwal and V Aggarwal Solution for Class 10 Mathematics Chapter 19 - Volume and Surface Areas of Solids Page/Excercise 19D

Solution 1

First change the rate of flow of water of the river in metre/min.

Since 1 km = 1000 m and 1 hour = 60 minutes, we have

$$3.5 \text{ km per hour} = \frac{3.5 \text{ km}}{1 \text{ hour}}$$

$$= \frac{3.5 \times 1000 \text{ m}}{1 \times 60 \text{ min}}$$

$$= \frac{350}{6} \text{ m / min}$$

This means that in 1 min, the river travels $\frac{350}{6}$ m

Futher, it is given that the river is 1.5 m deep and 36 m wide.

So, the amount of water that runs into the sea per minute wil be

$$\frac{350}{6} \times 1.5 \times 36 = 350 \times 1.5 \times 6$$

$$= 3150$$

Hence, the required amount of water is $3150 \text{ m}^3 / \text{min}$.

Solution 2

Let the side of the cube be a cm.

Given Volume of cube = 729 cm^3

$\Rightarrow a^3 = 729$

$\Rightarrow a = \sqrt[3]{729}$

$\Rightarrow a = \sqrt[3]{3 \times 3 \times 3 \times 3 \times 3 \times 3}$

$\Rightarrow a = 3 \times 3$

$\Rightarrow a = 9$ cm

Now, Surface area of cube = $6a^2$

$= 6 \times 9 \times 9$

$= 486\ cm^2$

Solution 3

Given Edge = 1 m = 100 cm

Volume = $side^3$

$= (100)^3$

$= 1000000\ cm^3$

Volumes of cubes of 10 cm edge = $10^3 = 1000\ cm^3$

Number of cubes = $\frac{1000000}{1000}$

$= 1000$ cubes

Solution 4

Three cubes are recast into a bigger cube.

The edges of the cube are 6 cm, 8 cm and 10 cm.

Let the edge of the new cube be a cm.

$\Rightarrow$ Volume of 3 cubes = Volume of the new cube

$\Rightarrow 6^3 + 8^3 + 10^3 = a^3$

$\Rightarrow 216 + 512 + 1000 = a^3$

$\Rightarrow a^3 = 1728$

$\Rightarrow a = \sqrt[3]{1728}$

$\Rightarrow a = 12$ cm

Thus, the edge of the new cube is 12 cm.

Solution 5

Since the 5 identical cubes are places adjacent to each other, the length of the cuboid formed

$= 5 + 5 + 5 + 5 + 5$

$= 25$ cm

Volume of the resulting cuboid = lbh

$= 25 \times 5 \times 5$

$= 625 \text{ cm}^3$

Solution 6

Let the volumes of the two cubes be 8x and 27x.

Since volume of a cube = $(\text{side})^3$

So, $8x = (\text{side of the first cube})^3$

$\Rightarrow 2\sqrt[3]{x}$ = side of the first cube

Similarly, $27x = (\text{side of the second cube})^3$

$\Rightarrow 3\sqrt[3]{x}$ = side of the second cube

Surface area of a cube = $6(\text{side})^2$

So, ratio of the surface areas of the cubes

$$= \frac{6\left(2\sqrt[3]{x}\right)^2}{6\left(3\sqrt[3]{x}\right)^2} = \frac{4}{9}$$

Hence, the ratio is 4:9.

Solution 7

Given that the height of the cylinder = radius of the cylinder = r(say)

Volume of the cylinder = $\pi r^2 h$

$\Rightarrow 25\frac{1}{7} = \frac{22}{7} \times r^2(r)$

$\Rightarrow \frac{176}{7} = \frac{22}{7} \times r^2(r)$

$\Rightarrow r^3 = \frac{176}{22}$

$\Rightarrow r^3 = 8$

$\Rightarrow r = 2$ cm

Solution 8

Let the radius and height of the cylinder be 2x and 3x.

Volume of the cylinder = $\pi r^2 h$

$\Rightarrow 12936 = \frac{22}{7} \times (2x)^2 \times (3x)$

$\Rightarrow \frac{12936 \times 7}{22 \times 12} = x^3$

$\Rightarrow x^3 = 343$

$\Rightarrow x = 7$ cm

So, the radius of the base is 2(7) = 14 cm.

Solution 9

Let the radii of the cylinders be 2r and 3r and their heights be 5h and 3h respectively.

$$\text{Ratio of their volumes} = \frac{\pi(2r)^2(5h)}{\pi(3r)^2(3h)}$$

$$= \frac{4 \times 5}{9 \times 3}$$

$$= \frac{20}{27}$$

So, the ratio of their volumes is 20:27.

Solution 10

Let the length of the wire be h.

Diameter of the wire = 1 mm = $\frac{1}{2}$ mm = $\frac{1}{20}$ cm

Since the silver wire is drawn into a wire,
the volume of the silver drawn = volume of the wire

$\Rightarrow 66 = \pi r^2 h$

$\Rightarrow 66 = \frac{22}{7} \times \left(\frac{1}{20}\right)^2 h$

$\Rightarrow h = \frac{66 \times 7 \times 400}{22}$

$\Rightarrow h = 8400$ cm

$\Rightarrow h = 84$ m

Hence, the length of the wire is 84 m.

Solution 11

Area of the base = 3850

$\Rightarrow \pi r^2 = 3850$

$\Rightarrow \frac{22}{7} \times r^2 = 3850$

$\Rightarrow r^2 = \frac{3850 \times 7}{22}$

$\Rightarrow r^2 = 1225$

$\Rightarrow r = 35$ cm

Slant height, $l = \sqrt{h^2 + r^2}$

$= \sqrt{84^2 + 35^2}$

$= \sqrt{7045 + 1225}$

$= \sqrt{8281}$

$= 91$ cm

Solution 12

Let the radius of the cone be r cm

$$\Rightarrow \frac{1}{3}\pi \times r^2 \times 6 = \pi \times 8 \times 8 \times 2$$

$$\Rightarrow r^2 = 64$$

$$\Rightarrow r = 8 \text{ cm}$$

Solution 13

Let n be the number of cones that will be needed to store the water,
and R and H be the radius and height of the cylindrical vessel and cone.
Volume of the cylindrical vessel = n x Volume of each cone

$$\Rightarrow \pi R^2 H = n \times \frac{1}{3}\pi R^2 H$$

$$\Rightarrow 1 = n \times \frac{1}{3}$$

$$\Rightarrow n = 3$$

Solution 14

Volume of the sphere = 4851

$\Rightarrow \frac{4}{3}\pi r^3 = 4851$

$\Rightarrow 4 \times \frac{22}{7} \times r^3 = 4851$

$\Rightarrow r^3 = \frac{4851 \times 7}{88}$

$\Rightarrow r^3 = \sqrt[3]{\frac{4851 \times 7}{88}}$

$\Rightarrow r^3 = \sqrt[3]{385.875}$

$\Rightarrow r = 7.28 \text{ cm}^3$

Curved surface area of a sphere $= 4\pi r^2$

$$= 4 \times \frac{22}{7} \times (7.280)^2$$

$$= \frac{4663.8592}{7}$$

$$= 666.2656 \text{ cm}^2$$

Note: The answer in the textbook is incorrect.

Solution 15

Curved surface area of a sphere = 5544

$\Rightarrow 4\pi r^2 = 5544$

$\Rightarrow 4 \times \frac{22}{7} \times r^2 = 5544$

$\Rightarrow r^2 = \frac{5544 \times 7}{4 \times 22}$

$\Rightarrow r^2 = 441$

$\Rightarrow r = 21 \text{ cm}$

So, volume of the sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times (21)^3$

$= 38808 \text{ cm}^3$

Solution 16

Surface areas of two spheres = $\frac{4}{25}$

$$\Rightarrow \frac{4R^2}{4r^2} = \frac{4}{25}$$

$$\Rightarrow \frac{R^2}{r^2} = \frac{4}{25}$$

$$\Rightarrow \frac{R}{r} = \frac{2}{5}$$

$$\text{Ratio of their volumes} = \frac{\frac{4}{3}\pi R^3}{\frac{4}{3}\pi r^3}$$

$$= \left(\frac{R}{r}\right)^3$$

$$= \left(\frac{2}{5}\right)^3$$

$$= \frac{8}{125}$$

Hence, the ratio of their volumes is 8:125.

Solution 17

Let the radius of the metallic sphere be R and the radius of each spherical ball be r.

$$\text{Number of spherical balls} = \frac{\text{Volume of the sphere}}{\text{Volume of each spherical ball}}$$

$$= \frac{\frac{4}{3}\pi R^3}{\frac{4}{3}\pi r^3}$$

$$= \frac{(8)^3}{(2)^3}$$

$$= 64$$

Solution 18

The diameter of each lead shot = 3 mm

So, the radius of each lead shot = 1.5 mm = 0.15 cm

$$\text{Number of lead shots} = \frac{\text{Volume of the cuboid}}{\text{Volume of each lead shot}}$$

$$= \frac{9 \times 11 \times 12}{\frac{4}{3}\pi R^3}$$

$$= \frac{9 \times 11 \times 12}{\frac{4}{3} \times \frac{22}{7} \times (0.15)^3}$$

$$= \frac{24948}{0.297}$$

$$= 84000$$

Solution 19

$$\text{Number of spheres} = \frac{\text{Volume of the cone}}{\text{Volume of each sphere}}$$

$$= \frac{\frac{1}{3}\pi r^2 h}{\frac{4}{3}\pi R^3}$$

$$= \frac{(12)^2(24)}{4 \times (2)^3}$$

$$= 108$$

Solution 20

Let the radius of the base of the cone be R and the radius of the hemisphere be r.

Volume of the hemisphere = Volume of the cone

$$\Rightarrow \frac{2}{3}\pi r^3 = \frac{1}{3}\pi R^2 h$$

$$\Rightarrow 2(6)^3 = R^2(75)$$

$$\Rightarrow R^2 = \frac{432}{75}$$

$$\Rightarrow R^2 = 5.76$$

$$\Rightarrow R = 2.4 \text{ cm}$$

Hence, the radius of the cone is 2.4 cm.

Solution 21

The diameter of the copper sphere = 18 cm

So, the radius of the sphere = 9 cm

Diameter of the wire = 4 mm = 0.4 cm

So, the radius of the wire = 0.2 cm

Let the length of the wire be h.

Volume of the sphere = Volume of the wire

$$\Rightarrow \frac{4}{3}\pi r^3 = \pi R^2 h$$

$$\Rightarrow \frac{4}{3} \times (9)^3 = (0.2)^2 h$$

$$\Rightarrow h = \frac{4 \times 729}{3 \times 0.04}$$

$\Rightarrow h = 24300$ cm = 243 m

So, the length of the wire is 243 m.

Solution 22

$$\text{Slant height, } l = \sqrt{h^2 + (R - r)^2}$$

$$= \sqrt{6^2 + (14 - 6)^2}$$

$$= \sqrt{36 + 64}$$

$$= \sqrt{100}$$

$$= 10 \text{ cm}$$

Hence, the slant height of the frustum is 10 cm.

Solution 23

Since the sphere fits inside the cube, the diameter of the sphere is equal to the side of the cube.
Let the diameter of the sphere be 2r which is the same as the edge of the cube.
So, the radius of the cube = r
Volume of the cube : Volume of a sphere

$= (2r)^3 : \frac{4}{3}\pi r^3$

$= 8r^3 : \frac{4}{3}\pi r^3$

$= 24 : 4\pi$

$= 6 : \pi$

Solution 24

Let the diameter of the cylinder, cone and sphere be 2r.
So, their radius will be r.
Height of the cylinder and the cone = 2r
Required ratio
= Volume of the cylinder:Volume of the cone:Volume of the sphere

$= \pi r^2 h : \frac{1}{3}\pi r^2 h : \frac{4}{3}\pi r^3$

$= \pi r^2(2r) : \frac{1}{3}\pi r^2(2r) : \frac{4}{3}\pi r^3$

$= 2 : \frac{2}{3} : \frac{4}{3}$

$= 6 : 2 : 4$

$= 3 : 1 : 2$

Solution 25

Volume of the cube = $125\ cm^3$

$\Rightarrow$ Side of each side = 5 cm

Total length = $(5+5)$ cm = 10 cm

Breadth = 5 cm

Height = 5 cm

$$\begin{aligned}\text{Total surface area} &= 2(lb+bh+lh)\\ &= 2(10\times 5+5\times 5+10\times 5)\\ &= 2(50+25+50)\\ &= 2(125)\\ &= 250\ cm^2\end{aligned}$$

Solution 26

$$\begin{aligned}\text{Total volume of 3 cubes} &= \left(3^3+4^3+5^3\right)\ cm^3\\ &= (27+64+125)\ cm^3\\ &= 216\ cm^3\end{aligned}$$

Now volume of new cube = $216\ cm^3$

$$\begin{aligned}\text{So the edge of new cube} &= \sqrt[3]{216}\ cm^3\\ &= 6\ cm\end{aligned}$$

Solution 27

The diameter of the sphere = 8 cm

Radius of the sphere = 4 cm

Length of the wire = 12 m = 1200 cm

Volume of the sphere = Volume of the cylindrical wire

$$\frac{4}{3}\pi r^3 = \pi R^2 h$$

$$\Rightarrow \frac{4}{3}(4)^3 = R^2(1200)$$

$$\Rightarrow R^2 = \frac{4 \times 4 \times 4 \times 4}{3 \times 1200}$$

$$\Rightarrow R^2 = \frac{4 \times 4 \times 4 \times 4}{3 \times 3 \times 400}$$

$$\Rightarrow R = \frac{16}{3 \times 20}$$

$$\Rightarrow R = \frac{4}{15} \text{ cm}$$

The width of the wire = the diameter of the base of the wire

$$= 2\left(\frac{4}{15}\right) = \frac{8}{15} \text{ cm}$$

Solution 28

Height of the cone = 24 m

Area of the cloth = Curved surface area of the cone

$l^2 = h^2 + r^2$

$= 576 + 49$

$= 625$

$\therefore l = 25$ m

Area of the cloth = πrl

$= \frac{22}{7} \times 7 \times 25$

$= 550\ m^2$

Now, length x width = 550 m^2

So, length = $\frac{550}{5} = 110$ m

Cost of 1 m cloth = Rs. 25

Cost of 110 m doth = Rs. (110×25) = Rs. 2750

Solution 29

Volume of the wood in the toy

= Volume of the cylinder – Volume of the 2 hemispheres

$= \pi r^2 h - 2\left(\frac{2}{3}\pi r^3\right)$

$= \pi r^2\left[h - \frac{4}{3}r\right]$

$= \frac{22}{7} \times 3.5^2\left[10 - \frac{4}{3}(3.5)\right]$

$= 205.33\ cm^3$

Solution 30

Let the edges of the three cubes be 3x, 4x and 5x.

Volume = $(3x)^3 (4x)^3 (5x)^3 = 27x^3 \cdot 64x^3 \cdot 125x^3$

We know that,

the length of the diagonal of a single cube of side a = $a\sqrt{3}$

The length of the diagonal of a cube is given to be $12\sqrt{3}$.

So, the edge of a single cube is 12 cm.

Volume of the single cube = $(12)^3 = 1728$ cm^3

Since the three cubes are melted and converted into a single cube,

sum of the volumes of the three cubes = volume of a single cube

$\Rightarrow 27x^3 + 64x^3 + 125x^3 = 1728$

$\Rightarrow 216x^3 = 1728$

$\Rightarrow x^3 = 8$

$\Rightarrow x = 2$ cm

So, the edges of the cubes are 6 cm, 8 cm and 10 cm.

Solution 31

The internal diameter of the hollow sphere = 4 cm
So, the internal radius = 2 cm
The external diameter of the hollow sphere = 8 cm
So, the external radius = 4 cm

$$\text{Volume of the metal} = \frac{4}{3}\pi\left(R^3 - r^3\right)$$
$$= \frac{4}{3}\pi\left(4^3 - 2^3\right)$$
$$= \frac{4}{3}\pi(64 - 8)$$
$$= \frac{4}{3}\pi(56)$$

Diameter of the cone = 8 cm
So, radius = 4 cm
Let the height of the cone be h cm.
Volume of the cone = Volume of the metal

$$\frac{1}{3}\pi r_c^2 h = \frac{4}{3}\pi(56)$$
$$\Rightarrow r_c^2 h = 4 \times 56$$
$$\Rightarrow (4)^2 h = 4 \times 56$$
$$\Rightarrow h = \frac{4 \times 56}{16}$$
$$\Rightarrow h = 14 \text{ cm}$$

Hence, the height of the cone is 14 cm.

Solution 32

Since the circular ends of the diameter are 28 cm and 42 cm, the radii of are 14 cm and 21 cm respectively.

$$\text{Capacity of the bucket} = \frac{1}{3}\pi h\left(R^2 + r^2 + Rr\right)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 24\left[14^2 + 21^2 + 14 \times 21\right]$$

$$= \frac{22}{7} \times 8\left[196 + 441 + 294\right]$$

$$= \frac{22}{7} \times 8 \times \left[196 + 441 + 294\right]$$

$$= 23408 \text{ cm}^3$$

$$= 23.408 \text{ l} \quad .....(\text{Since } 1000 \text{ cm}^3 = 1 \text{ l})$$

Cost of milk at the rate of Rs. 30 per litre

$= 23.408 \times 30$

$=$ Rs. 702.24

Solution 33

Given that height of the cone = 2.8 m
Diameter of cylinder = 4.2 m
radius of the cylinder = 2.1 m
radius of the cylinder = radius of the cone = 2.1 m

Slant height of the cone $l = \sqrt{(2.8)^2 + (2.1)^2}$

$= \sqrt{7.84 + 4.41}$

$= \sqrt{(2.8)^2 + (2.1)^2}$

$= 3.5$ m

Outer surface area of the building
= Curved surface area of cylinder + Curved surface area of cone

$= 2\pi rh + \pi rl$

$= 2 \times \left(\frac{22}{7}\right) \times 2.1 \times 4 + \left(\frac{22}{7}\right) \times 2.1 \times 3.5$

$= 44 \times 0.3 \times 4 + 22 \times 0.3 \times 3.5$

$= 44 \times 1.2 + 6.6 \times 3.5$

$= 52.8 + 23.1$

$= 75.90 \text{ cm}^2$

Solution 34

Given that the metallic cone is melted and recast into a solid sphere.
Let the radius of the sphere be r.
So, volume of the cone = volume of the sphere

$\Rightarrow \frac{1}{3}\pi(21)^2(84) = \frac{4}{3}\pi r^3$

$\Rightarrow r^3 = 9261$

$\Rightarrow r = 21$ cm

So, the diameter of the sphere = 42 cm

Solution 35

Total height of the cone = 15.5 cm

So, the height of the cone = 15.5 cm – radius of the hemisphere

= 15.5 cm – 3.5 cm

= 12 cm

Slant height of the cone, l

$= \sqrt{h^2 + r^2}$

$= \sqrt{12^2 + 3.5^2}$

$= \sqrt{144 + 12.25}$

$= \sqrt{156.25}$

= 12.5 cm

Curved surface area of the cone

$= \pi rl$

$= \frac{22}{7} \times 3.5 \times 12.5$

$= 137.5 \text{ cm}^2$

Curved surface area of the sphere

$= 2\pi r^2$

$= 2 \times \frac{22}{7} \times 3.5 \times 3.5$

$= 77 \text{ cm}^2$

So, the total surface area of the toy

$= 137.5 \text{ cm}^2 + 77 \text{ cm}^2$

$= 214.5 \text{ cm}^2$

Note: The answer in the text is incorrect.

Solution 36

$$\text{Capacity of the bucket} = \frac{1}{3}\pi h\left(R^2 + r^2 + Rr\right)$$
$$= \frac{1}{3} \times \frac{22}{7} \times 28\left[28^2 + 7^2 + 28 \times 7\right]$$
$$= \frac{1}{3} \times 22 \times 4\left[784 + 49 + 196\right]$$
$$= \frac{1}{3} \times 22 \times 4 \times 1029$$
$$= 30184 \text{ cm}^3$$

$$\text{Slant height, } l = \sqrt{h^2 + (R - r)^2}$$
$$= \sqrt{28^2 + (28 - 7)^2}$$
$$= \sqrt{784 + 441}$$
$$= \sqrt{1225}$$
$$= 35 \text{ cm}$$

Total surface area of the bucket

$$= \pi(R + r)l + \pi R^2 + \pi r^2$$
$$= \pi\left[(R + r)l + R^2 + r^2\right]$$
$$= \frac{22}{7}\left[(28 + 7)35 + 28^2 + 7^2\right]$$
$$= \frac{22}{7}\left[1225 + 784 + 49\right]$$
$$= \frac{22}{7}\left[2058\right]$$
$$= 6468 \text{ cm}^2$$

Solution 37

$$\text{Volume of a frustum} = \frac{1}{3}\pi h\left(R^2 + r^2 + Rr\right)$$

$$\Rightarrow 12308.8 = \frac{1}{3} \times 3.14 \times h\left[20^2 + 12^2 + 20 \times 12\right]$$

$$\Rightarrow 12308.8 = \frac{1}{3} \times 3.14 \times h\left[400 + 144 + 240\right]$$

$$\Rightarrow h = \frac{12308.8 \times 3}{3.14 \times 784}$$

$$\Rightarrow h = 15 \text{ cm}$$

Hence, the height is 15 cm.

Solution 38

Volume of a frustum = volume of milk container

$\frac{1}{3}\pi h\left(R^2 + r^2 + Rr\right) = 10459\frac{3}{7}$

$\frac{1}{3} \times \frac{22}{7} \times h\left[\left(20^2\right) + 8^2 + 20 \times 8\right] = \frac{73216}{7}$

$\frac{1}{3} \times \frac{22}{7} \times h\left(400 + 64 + 160\right) = \frac{73216}{7}$

$\frac{1}{3} \times \frac{22}{7} \times h \times 624 = \frac{73216}{7}$

$h = \frac{73216 \times 3}{22 \times 624} = 16 \text{ cm}$

$h = 16 \text{ cm}$

$$\begin{aligned}
\text{Slant height}(l) &= \sqrt{h^2 + (R - r)^2} \\
&= \sqrt{16^2 + (20 - 8)^2} \\
&= \sqrt{256 + 144} \\
&= \sqrt{400} \\
&= 20 \text{ cm}
\end{aligned}$$

Surface area of the cone

$= \pi(R + r)l + \pi r^2 + \pi R^2$

$= \pi\left[(R + r)l + r^2 + R^2\right]$

$= \frac{22}{7}\left[(20 + 8)20 + 8^2 + 20^2\right]$

$= \frac{22}{7}\left[560 + 64 + 400\right]$

$= \frac{22}{7}\left[1024\right]$

$= 3218.285714 \text{ cm}^2$

Cost of a metal sheet used in a container at 1.40 per cm^2

$= 3218.285714 \times 1.40$

= Rs.4505.6

Hence, the cost of the metal sheet used in making the container is Rs.4505.6.

Note : The answer in the text is incorrect.

Solution 39

Diameter of the metallic sphere = 28 cm

Radius of the metallic sphere, R = 14 cm

Diameter of the cone = $4\frac{2}{3}$ cm = $\frac{14}{3}$ cm

$\therefore$ Radius of each cone, r = $\frac{7}{3}$ cm

$$\text{Number of cones formed} = \frac{\text{Volume of the sphere}}{\text{Volume of the cone}}$$

$$= \frac{\frac{4}{3}\pi R^3}{\frac{1}{3}\pi r^2 h}$$

$$= \frac{4R^3}{r^2 h}$$

$$= \frac{4 \times 14 \times 14 \times 14}{\frac{7}{3} \times \frac{7}{3} \times 3}$$

$$= \frac{4 \times 14 \times 14 \times 14 \times 3}{7 \times 7}$$

$$= 672$$

Note : The answer given in the text is incorrect.

Solution 40

The internal diameter of the cylinder = 10 cm
So, the internal radius of the cylinder = 5 cm
Height of the cylinder = 10.5 cm
Diameter of the cone = 7 cm
Radius of the cone = 3.5 cm
Height of the cone = 6 cm
(i) Volume of water displayed out of the cylindrical vessel
= Volume of the cone

$= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 3.5^2 \times 10.5$

$= 77\ cm^2$

(ii) Volume of water left in the cylindrical vessel
= Volume of the cylinder – Volume of the cone

$= \pi r^2 H - 77$

$= \frac{22}{7} \times 5 \times 5 \times 10.5 - 77$

$= 825 - 77$

$= 748\ cm^3$

Excercise MCQ

Solution 1

Correct option: (a)

A cylindrical pencil sharpened at one edge is the combination of a cylinder and a cone. Observe the figure, the lower portion is a cylinder and the upper tapering portion is a cone.

Solution 2

Correct option: (b)

A shuttlecock used for playing badminton is the combination of a frustum of a cone and a hemisphere, the lower portion being the hemisphere and the portion above that being the frustum of the cone.

Solution 3

Correct option: (c)

A funnel is the combination of a cylinder and frustum of a cone. The lower portion is cylindrical and the upper portion is a frustum of a cone.

Solution 4

Correct option: (a)

A surahi is a combination of a sphere and a cylinder, the lower portion is the sphere and the upper portion is the cylinder.

Solution 5

Correct option: (b)

The shape of a glass (tumbler) is usually in the form of a frustum of a cone.

Solution 6

Correct option: (c)

The shape of a gill in the gilli-danda game is a combination of two cones and a cylinder. The cones at either ends with the cylinder in the middle.

Solution 7

Correct option: (a)

A plumbline (sahul) is the combination of a hemisphere and a cone, the hemisphere being on top and the lower portion being the cone.

Solution 8

Correct option: (d)

A cone is cut by a plane parallel to its base and the upper part is removed. The part that is left over is called the frustum of a cone.

Solution 9

Correct option: (c)

During conversion of a solid from one shape to another, the volume of the new shape will remain altered.

Solution 10

Correct option: (c)

In a right circular cone, the cross section made by a plane parallel to the base is a circle.

Solution 11

Correct option: (b)

Since the cuboid is moulded to form a solid sphere,

the volume of sphere = volume of the cuboid

$$\Rightarrow \frac{4}{3}\pi r^3 = 49 \times 33 \times 24$$

$$\Rightarrow \frac{4}{3} \times \frac{22}{7} \times r^3 = 49 \times 33 \times 24$$

$$\Rightarrow r^3 = \frac{49 \times 33 \times 24 \times 3 \times 7}{22 \times 4}$$

$$\Rightarrow r^3 = 7 \times 7 \times 7 \times 3 \times 3 \times 3$$

$$\Rightarrow r = 7 \times 3$$

$$\Rightarrow r = 21 \text{ cm}$$

Solution 12

Correct option: (a)
The diameter of such a cone is equal to the edge of the cube.
So, the diameter = 4.2 cm.
Hence, the radius = 2.1 cm.

Solution 13

Correct option: (a)
The metallic solid sphere is melted to form a solid cylinder.
Let the height of the cylinder be h.
So, volume of the sphere = volume of the cylinder

$\Rightarrow \frac{4}{3}\pi r_1^3 = \pi r^2 h$

$\Rightarrow \frac{4}{3} r_1 = h$

$\Rightarrow h = \frac{4}{3} \times 9 = 12$ cm

Solution 14

Correct option: (a)
Since the height of the cylinder is given to be 40 cm,
the sheet of paper when converted to a cylinder,
has its circumference to be 22 cm.
So, circumference = 22 cm

$\Rightarrow 2\pi r = 22$

$\Rightarrow 2 \times \frac{22}{7} \times r = 22$

$\Rightarrow r = 3.5$ cm

Hence, the radius of the cylinder is 3.5 cm.

Solution 15

Correct option: (c)

Let the number of solid spheres be n.

Since the solid metal cylinder is melted and recast into n solid spheres,

volume of n solid sphere = volume of the solid metal cylinder

$\Rightarrow n \times \frac{4}{3}\pi r^3 = \pi R^2 h$

$\Rightarrow n = \frac{3R^2 h}{4r^3}$

$\Rightarrow n = \frac{3 \times 2^2 \times 45}{4 \times 3^3}$

.....(Since diameter of the cylinder = 4 cm and diameter of each sphere = 6 cm)

$\Rightarrow n = 5$

Hence, 5 solid spheres can be formed.

Solution 16

Correct option: (a)

Given that the surface areas of the two spheres are in the ratio 16:9.

So, $\frac{4\pi r^2}{4\pi R^2} = \frac{16}{9}$

$\Rightarrow \frac{r^2}{R^2} = \frac{16}{9}$

$\Rightarrow \frac{r}{R} = \frac{4}{3}$

Let the volumes of the sphere with radius r and R be V_1 and V_2 respectively.

$$\frac{V_1}{V_2} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3}$$

$$\Rightarrow \frac{V_1}{V_2} = \left(\frac{r}{R}\right)^3$$

$$\Rightarrow \frac{V_1}{V_2} = \left(\frac{4}{3}\right)^3 = \frac{64}{27}$$

Hence, the ratio of their volumes is 64:27.

Solution 17

Correct option: (b)

Surface area of a sphere = 616 cm^2

$\Rightarrow 4\pi r^2 = 616$

$\Rightarrow 4 \times \frac{22}{7} \times r^2 = 616$

$\Rightarrow r^2 = \frac{616 \times 7}{22 \times 4}$

$\Rightarrow r = 7$ cm

So, the diameter = $2r = 2 \times 7 = 14$ cm.

Solution 18

Correct option: (d)

Let the radius of the sphere be r.

So, the volume of the sphere = $\frac{4}{3}\pi r^3$

If the radius becomes 3r,

the volume = $\frac{4}{3}\pi(3r)^3 = 27 \times \frac{4}{3}\pi r^3 = 27$ times the original sphere

Solution 19

Correct option: (a)

Height of the frustum, h = 16 cm

Radii of the circular ends, R and r are:

$R = \frac{40}{2} = 20$ cm and $r = \frac{16}{2} = 8$ cm

The slant height of the frustum,

$$l = \sqrt{(R-r)^2 + h^2}$$

$$\Rightarrow l = \sqrt{(20-8)^2 + 16^2}$$

$$\Rightarrow l = \sqrt{12^2 + 16^2}$$

$$\Rightarrow l = \sqrt{144 + 256}$$

$$\Rightarrow l = \sqrt{400}$$

$$\Rightarrow l = 20 \text{ cm}$$

Solution 20

Correct option: (a)

Let the rise in the water level be h.

Radii of the sphere and cylindrical vessel are:

$r = \frac{18}{2} = 9$ cm and $R = \frac{36}{2} = 18$ cm

Volume of the water level risen = volume of the sphere

$$\Rightarrow \pi R^2 h = \frac{4}{3}\pi r^3$$

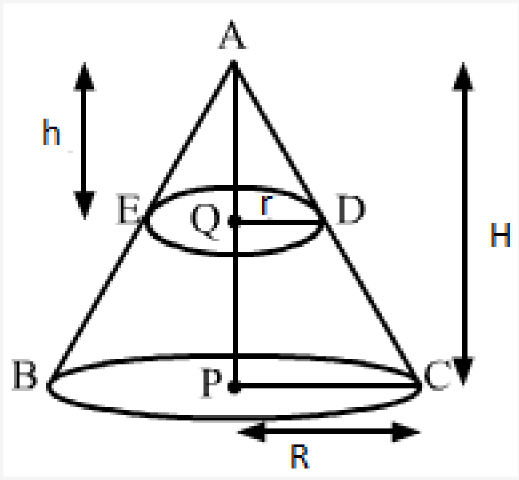
$$\Rightarrow h = \frac{4 \times 9 \times 9 \times 9}{3 \times 18 \times 18}$$

$$\Rightarrow h = 3 \text{ cm}$$

Hence, the water level rises by 3 cm.

Solution 21

Correct option: (d)



Let the heights of the smaller and larger cone be h and H respectively.
Let the radii of the smaller and larger cone be r and R respectively.
Given that the plane cuts the larger cone at the middle of its height.
So, $H = 2h$ ....(i)
Consider, ΔAQD and ΔAPC,
$\angle QAD = \angle PAC$ ....(Common angle)
$\angle AQD = \angle APC$ ....(90° angle)
$\therefore \Delta AQD \sim \Delta APC$ ....(AA criterion for Similarity)

$$\Rightarrow \frac{AQ}{AP} = \frac{QD}{PC}$$

$$\Rightarrow \frac{h}{H} = \frac{r}{R}$$

$$\Rightarrow \frac{h}{2h} = \frac{r}{R}$$

$$\Rightarrow \frac{r}{R} = \frac{1}{2}$$

that is, $R = 2r$ ....(ii)

$$\frac{\text{Volume of the smaller cone}}{\text{Volume of the larger cone}}$$

$$= \frac{\frac{1}{3}\pi r^2 h}{\frac{1}{3}\pi R^2 H}$$

$$= \frac{r^2 h}{(2r)^2(2h)}$$

$$= \frac{r^2 h}{8r^2 h}$$

$$= \frac{1}{8}$$

Hence, the ratio of the volume of the smaller cone to the larger cone is 1:8.

Solution 22

Correct option: (a)

Slant height of the bucket

$= \sqrt{(R - r)^2 + h^2}$

$= \sqrt{(24 - 15)^2 + 40^2}$

$= \sqrt{9^2 + 40^2}$

$= \sqrt{81 + 1600}$

$= \sqrt{1681}$

$= 41$ cm

Solution 23

Correct option: (a)

Given that the radius of the hemisphere and the cone are equal.

Since the surface of the two parts are given to be equal,

$2\pi r^2 = \pi r l$

$\Rightarrow 2r = l$

$\Rightarrow \frac{r}{l} = \frac{1}{2}$

So, the ratio is 1:2.

Solution 24

Correct option: (d)

Let the radius and height of the cylinder be r and h respectively.

Since the radius is halved keeping the height the same,

the new radius is $\frac{r}{2}$.

$$\frac{\text{Volume of the new cylinder}}{\text{Volume of the original cylinder}}$$

$$= \frac{\pi r^2 h}{\pi\left(\frac{r}{2}\right)^2 h}$$

$$= \frac{4}{1}$$

So, the ratio is 4:1.

Solution 25

Correct option: (c)

Given that the edge of the cubical ice-cream brick is 22 cm.

Volume of the cubical ice-cream brick

$= 22^3$

Volume of each ice-cream cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times (2)^2 \times 7$$

So, the number of ice-cream cones

$$= \frac{\text{Volume of the cubical ice-cream brick}}{\text{Volume of each ice-cream cone}}$$

$$= \frac{22 \times 22 \times 22}{\frac{1}{3} \times \frac{22}{7} \times (2)^2 \times 7}$$

$$= \frac{22 \times 22 \times 22 \times 7 \times 3}{22 \times 4 \times 7}$$

$= 363$

Hence, the number of ice-cream cones is 363.

Solution 26

Correct option: (c)

Dimensions of the wall are given to be 270 cm x 300 cm x 350 cm.

So, the volume of the wall = 270 cm x 300 cm x 350 cm.

$\frac{1}{8}$th of the wall is covered with mortar.

Volume of the wall filled with bricks

$= \left(\frac{7}{8} \times 270 \times 300 \times 350\right)\ cm^3$

Volume of each brick = $(22.5 \times 11.25 \times 8.75)\ cm^3$

Number of bricks used to construct the wall

$= \frac{\text{Volume of the wall filled with bricks}}{\text{Volume of each brick}}$

$= \frac{\frac{7}{8} \times 270 \times 300 \times 350}{22.5 \times 11.25 \times 8.75}$

$= \frac{\frac{7}{8} \times 270 \times 300 \times 350 \times 100000}{225 \times 1125 \times 875}$

$= 11200$

Solution 27

Correct option: (a)

Radius of the cylinder = $\frac{2}{2}$ = 1 cm

h = 16 cm

Since twelve solid spheres of the same size are made by melting a solid metallic cylinder of base diameter 2 cm and height 16 cm,

$$12 \times \frac{4}{3}\pi r^3 = \pi R^2 h$$

$$\Rightarrow 12 \times \frac{4}{3} r^3 = R^2 h$$

$$\Rightarrow 12 \times \frac{4}{3} \times \left(\frac{d}{2}\right)^3 = (1)^2 \times 16$$

$$\Rightarrow 16 \times \left(\frac{d}{2}\right)^3 = (1)^2 \times 16$$

$$\Rightarrow \left(\frac{d}{2}\right)^3 = 1$$

$$\Rightarrow \frac{d^3}{8} = 1$$

$$\Rightarrow d^3 = 8$$

$$\Rightarrow d = \pm 2$$

Since the diameter cannot be negative, d = 2 cm.

Solution 28

Correct option: (b)
Since the diameter of the two circular ends of the bucket are 44 cm and 24 cm, the radii of the circular ends are 22 cm and 12 cm.
Capacity of the bucket = Volume of the bucket

$$= \frac{1}{3}\pi h\left[R^2 + r^2 + Rr\right]$$

$$= \frac{1}{3} \times \frac{22}{7} \times 35 \times \left[22^2 + 12^2 + (22 \times 12)\right]$$

$$= 32.7 \text{ litres}$$

Hence, the capacity of the bucket is 32.7 litres.

Solution 29

Correct option: (d)
The curved surface area of the bucket

$$= \pi l(R + r)$$

$$= \frac{22}{7} \times 45 \times (28 + 7)$$

$$= 4950 \text{ cm}^2$$

Hence, the curved surface area of the bucket is 4950 cm^2.

Solution 30

Correct option: (b)

Let the radii of the two spheres be r and R.

The volume of the two spheres are in the ratio 64:27.

$$\Rightarrow \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} = \frac{64}{27}$$

$$\Rightarrow \frac{r^3}{R^3} = \frac{64}{27}$$

$$\Rightarrow \frac{r}{R} = \frac{4}{3}$$

Ratio of the surface area of the spheres

$$= \frac{4\pi r^2}{4\pi R^2}$$

$$= \left(\frac{r}{R}\right)$$

$$= \left(\frac{4}{3}\right)^2$$

$$= \frac{16}{9}$$

Hence, the ratio is 16:9.

Solution 31

Correct option: (a)

Volume of the cube with edge 22 cm = $(22)^3$

Given that $\frac{1}{8}$ of the cube remains unfilled.

So, $\frac{7}{8}$ of the volume of the cube is filled.

Let the number of marbles required be n.

Thus, $\frac{7}{8} \times (22)^3 = n \times \frac{4}{3}\pi(0.25)^3$ .....(Since diameter = 0.5 cm)

$\Rightarrow \frac{7}{8} \times (22)^3 = n \times \frac{4}{3} \times \frac{22}{7} \times (0.25)^3$

$\Rightarrow n = \frac{7 \times (22)^3 \times 3 \times 7}{8 \times 4 \times 22 \times (0.25)^3}$

$\Rightarrow n = 142296$

Hence, the number of marbles required is 142296.

Solution 32

Correct option: (b)

The radii of the spherical shell is 2 cm and 4 cm.

$$\text{Volume of the spherical shell} = \frac{4}{3}\pi\left(R^3 - r^3\right)$$
$$= \frac{4}{3}\pi\left(4^3 - 2^3\right)$$
$$= \frac{4}{3}\pi(56)$$

Radius of the cone = 4 cm

$$\text{Volume of the cone} = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3}\pi(4)^2 h$$

$$\therefore \frac{1}{3}\pi(4)^2 h = \frac{4}{3}\pi(56)$$
$$\Rightarrow 16h = 4(56)$$
$$\Rightarrow h = 14 \text{ cm}$$

Solution 33

Correct option: (d)

Radius of the capsule = 0.25 cm

Let the length of the cylindrical part of the capsule be x cm.

So, $0.25 + x + 0.25 = 2$

$\Rightarrow 0.5 + x = 2$

$\Rightarrow x = 1.5$

Capacity of the capsule

= 2 x (Volume of the hemisphere) + (Volume of the cylinder)

$= 2 \times \left(\frac{2}{3}\pi r^3\right) + \left(\pi r^2 h\right)$

$= 2 \times \left(\frac{2}{3} \times \frac{22}{7} \times 0.25^3\right) + \left(\frac{22}{7} \times 0.25^2 \times 1.5\right)$

$= 0.36\ cm^3$

Solution 34

Correct option: (d)

Length of the longest pole that can be kept in a room

= length of the diagonal of the room

$= \sqrt{l^2 + b^2 + h^2}$

$= \sqrt{12^2 + 9^2 + 8^2}$

$= \sqrt{289}$

$= 17\ m$

Solution 35

Correct option: (b)

Let the edge of the cube be x cm.

So, length of the diagonal of the cube = $\sqrt{3}x$

$\sqrt{3}x = 6\sqrt{3}$

$\Rightarrow x = 6$ cm

Thus, the total surface area of the cube = $6x^2$

$= 6(6)^2$

$= 216 \text{ cm}^2$

Solution 36

Correct option: (b)

Let the edge of the cube be x cm.

Volume of a cube = x^3

$\Rightarrow 2744 = x^3$

$\Rightarrow x = 14$ cm

So, the surface area of the cube = $6x^2$

$= 6(14)^2$

$= 1176 \text{ cm}^2$

Solution 37

Correct option: (c)

Let the edge of the cube be x cm.

Total surface area of a cube = $6x^2$

$\Rightarrow 6x^2 = 864$

$\Rightarrow x^2 = 144$

$\Rightarrow x = 12$ cm

So, the volume of the cube $= x^3$

$= (12)^3$

$= 1728 \text{ cm}^3$

Solution 38

Correct option: (b)

$$\text{Number of bricks} = \frac{\text{Volume of the wall}}{\text{Volume of each brick}}$$

$$= \frac{(800 \times 600 \times 22.5)}{(25 \times 11.25 \times 6)}$$

$$= 6400$$

Solution 39

Correct option: (b)
Area of the base of the rectangular tank
$= 6500\ cm^2$
$= \frac{(6500)}{(100^2)}\ m^2$
Let the depth of the water be h metres.
So, $\frac{(6500)}{(100^2)} \times h = 2.6$
$\Rightarrow h = 4\ m$
Hence, the depth of the water is 4 m.

Solution 40

Correct option: (b)
Let the breadth of the wall be x cm.
So, its height = 5x cm
Length of the wall $= 8 \times 5x = 40x$ cm
Given that the volume of the wall $= 12.8\ m^3 = 12800000\ cm^3$
Thus, the volume of the wall = length $\times$ breadth $\times$ height
$\Rightarrow 12800000 = 40x \times x \times 5x$
$\Rightarrow \frac{12800000}{200} = x^3$
$\Rightarrow x^3 = 64000$
$\Rightarrow x = 40$ cm

Solution 41

Correct option: (c)

Given that the areas of the three adjacent faces of a cuboid are x, y and z.

This means,

$lb = x, \ bh = y, \ lh = z$

$\therefore lb \times bh \times lh = xyz$

$\therefore l^2b^2h^2 = xyz$

$\therefore (lbh)^2 = xyz$

$\therefore (\text{Volume of the cuboid})^2 = xyz$

$\therefore \text{Volume of the cuboid} = \sqrt{xyz}$

Solution 42

Correct option: (c)

Given that $l + b + h = 19$

$\Rightarrow (l + b + h)^2 = 19^2$

$\Rightarrow l^2 + b^2 + h^2 + 2lb + 2bh + 2lh = 361$

$\Rightarrow l^2 + b^2 + h^2 + 2(lb + bh + lh) = 361$

We know that, the diagonal of a cuboid $= l^2 + b^2 + h^2$

that is, $(5\sqrt{5})^2 = l^2 + b^2 + h^2$

So, from (i), we get

$(5\sqrt{5})^2 + 2(lb + bh + lh) = 361$

$\Rightarrow 125 + 2(lb + bh + lh) = 361$

$\Rightarrow 2(lb + bh + lh) = 236$

$\Rightarrow \text{Surface area} = 236 \text{ cm}^2$

Hence, the surface area of the cuboid is 236 cm^2.

Solution 43

Correct option: (d)

Let the edge of the cube be x.

So, the surface area of the cube = $6x^2$

Since the edge of the cube is increased by 50%,

the new edge $= x + \frac{x}{2} = \frac{3x}{2}$

So, the new surface area $= 6\left(\frac{3x}{2}\right)^2$

$$= 6\left(\frac{9x^2}{4}\right)$$

$$= \frac{27x^2}{2}$$

Increase in the surface area $= \frac{27x^2}{2} - 6x^2 = \frac{15x^2}{2}$

Percentage increase $= \frac{\frac{15x^2}{2}}{6x^2} \times 100$

$$= \frac{15}{12} \times 100$$

$$= 125\%$$

Solution 44

Correct option: (d)

Volume of the cuboidal granary = $(8\ m \times 6\ m \times 3\ m)$

Volume of each bag = $0.64\ m^3$

Number of bags that can be stored in the cuboidal granary

$= \dfrac{\text{Volume of the cuboidal granary}}{\text{Volume of each bag}}$

$= \dfrac{8 \times 6 \times 3}{0.64}$

$= 225$

Solution 45

Correct option: (d)

Volume of the cube = $(6\ cm \times 6\ cm \times 6\ cm)$

Volume of each small cube = $(2\ cm \times 2\ cm \times 2\ cm)$

Number of cubes formed

$= \dfrac{\text{Volume of the cube}}{\text{Volume of each small cube}}$

$= \dfrac{6 \times 6 \times 6}{2 \times 2 \times 2}$

$= 27$

Solution 46

Correct option: (c)

Volume of the water that falls on 2 hectares of land

= (Amount of rainfall x Area of the ground)

$= \left(\dfrac{5}{100} \times 2 \times 1000\right) \quad \ldots.. \left(\text{Since } 5\text{ cm} = \dfrac{5}{100}\text{ m and 2 hectares} = 2000\ m^2\right)$

$= 1000\ m^3$

Solution 47

Correct option: (c)

The ratio of the volumes of the two cube is 1:27.

Let the sides of the two cubes be a and b.

So, $\frac{a^3}{b^3} = \frac{1}{27}$

$\Rightarrow \frac{a}{b} = \frac{1}{3}$

$\Rightarrow \left(\frac{a}{b}\right)^2 = \left(\frac{1}{3}\right)^2$

$\Rightarrow \frac{a^2}{b^2} = \frac{1}{9}$

$\Rightarrow \frac{6a^2}{6b^2} = \frac{1}{9}$

So, the ratio of the surface areas of the two cubes is 1:9.

Solution 48

Correct option: (a)

Volume of the cylinder $= \pi r^2 h$

$= \frac{22}{7} \times 2 \times 2 \times 14$ ....(Since the diameter = 4 cm)

$= 176\ \text{cm}^3$

Solution 49

Correct option: (b)

Diameter = 28 cm $\Rightarrow$ radius = 14 cm

The total surface area of the cylinder

$= 2\pi r(h + r)$

$= 2 \times \frac{22}{7} \times 14(20 + 14)$

$= 2992\ cm^2$

Solution 50

Correct option: (b)

The curved surface are of the cylinder = $2\pi rh$

$\Rightarrow 264 = 2 \times \frac{22}{7} \times r \times 14$

$\Rightarrow r = 3$ cm

Volume of the cylinder = $\pi r^2 h$

$= \frac{22}{7} \times 3 \times 3 \times 14$

$= 396\ cm^3$

Solution 51

Correct option: (c)

The curved surface area of the cylinder = $2\pi rh$

$$= 2 \times \frac{22}{7} \times 14 \times h$$

$\Rightarrow 2 \times \frac{22}{7} \times 14 \times h = 1760$

$\Rightarrow h = \frac{1760}{88} = 20$ cm

Hence, the height of the cylinder is 20 cm.

Solution 52

Correct option: (d)

The ratio of the total surface area to the lateral surface area

$= \frac{\text{Total surface area}}{\text{Lateral surface area}}$

$= \frac{2\pi r(h+r)}{2\pi rh}$

$= \frac{h+r}{h}$

$= \frac{20+80}{20}$

$= \frac{5}{1}$

So, the required ratio is 5:1.

Solution 53

Correct option: (c)

The curved surface area of the cylinder = $2\pi rh$

$\Rightarrow 264 = 2\pi rh$

Volume of the cylinder = $\pi r^2 h$

$\Rightarrow 924 = \pi r^2 h$

So, $\frac{264}{924} = \frac{2\pi rh}{\pi r^2 h}$

$\Rightarrow \frac{264}{924} = \frac{2}{r}$

$\Rightarrow r = \frac{924 \times 2}{264}$

$\Rightarrow r = 7 \text{ m}$

So, $2\pi rh = 264$

$\Rightarrow 2 \times \frac{22}{7} \times 7 \times h = 264$

$\Rightarrow h = \frac{264}{44}$

$\Rightarrow h = 6 \text{ m}$

Hence, the height of the pillar is 6 m.

Solution 54

Correct option: (d)

Let the radius of the cylinder be 2x and 3x.

Volume of the cylinder = $\pi r^2 h$

$\Rightarrow 1617 = \pi r^2 h$

$\Rightarrow 1617 = \pi(2x)^2(3x)$

$\Rightarrow 1617 = \frac{22}{7} \times \left(12x^3\right)$

$\Rightarrow \frac{343}{8} = x^3$

$\Rightarrow x = \frac{7}{2}$

So, radius $= 2\left(\frac{7}{2}\right) = 7$ cm and height $= 3\left(\frac{7}{2}\right) = \frac{21}{2}$ cm

Hence, the total surface area of the cylinder

$= 2\pi rh + 2\pi r^2$

$= 2\pi r(h + r)$

$= 2 \times \frac{22}{7} \times 7\left(\frac{21}{2} + 7\right)$

$= 770 \text{ cm}^2$

Solution 55

Correct option: (b)

Let the radii of the two cylinders be 2x and 3x,

and the heights of the two cylinders be 5y and 3y respectively.

Ratio of the volume of the cylinders $= \frac{\pi(2x)^2(5y)}{\pi(3x)^2(3y)}$

$= \frac{20}{27}$

That is, the ratio of their volumes is 20:27.

Solution 56

Correct option: (b)

Let the heights of the two cylinders be h and 2h,
and the radii of the cylinders be r_1 and r_2 respectively.

Since the volume of the cylinders are equal,

$\pi(r_1)^2 h = \pi(r_2)^2 (2h)$

$\Rightarrow \frac{r_1^2}{r_2^2} = \frac{2}{1}$

$\Rightarrow \frac{r_1}{r_2} = \frac{\sqrt{2}}{1}$

Hence, the ratio of their radii is $\sqrt{2} : 1$.

Solution 57

Correct option: (b)

Slant height, $l = \sqrt{r^2 + h^2}$

$\Rightarrow l = \sqrt{5^2 + 12^2}$

$\Rightarrow l = \sqrt{25 + 144}$

$\Rightarrow l = \sqrt{169}$

$\Rightarrow l = 13$ cm

Curved surface area of the cone $= \pi rl$

$= \pi \times 5 \times 13$

$= 65\pi$ cm^2

Solution 58

Correct option: (a)

Diameter = 42

So, radius = $\frac{42}{2}$ = 21 cm

Volume of the cone = $\frac{1}{3}\pi r^2 h$

$\Rightarrow 12936 = \frac{1}{3} \times \frac{22}{7} \times (21)^2 h$

$\Rightarrow h = \frac{12936}{22 \times 21} = 28$ cm

Hence, the height of the cone is 28 cm.

Solution 59

Correct option: (a)

Area of the base of the cone = 154

$\Rightarrow \pi r^2 = 154$

$\Rightarrow \frac{22}{7} \times r^2 = 154$

$\Rightarrow r^2 = 49$

$\Rightarrow r = 7$ cm

$l = \sqrt{r^2 + h^2}$

$\Rightarrow l = \sqrt{7^2 + 14^2}$

$\Rightarrow l = \sqrt{49 + 196}$

$\Rightarrow l = \sqrt{245}$

$\Rightarrow l = 7\sqrt{5}$ cm

Curve surface area of the cone = πrl

$= \frac{22}{7} \times 7 \times 7\sqrt{5}$

$= 154\sqrt{5}$ cm^2

Solution 60

Correct option: (d)

Let the radius and height of the cone be r and h respectively.

Original volume = $\frac{1}{3}\pi r^2 h$

On increasing each by 20%, the new radius and height become $r + \frac{1}{5}r = \frac{6}{5}r$ and $h + \frac{1}{5}h = \frac{6}{5}h$.

$$\text{New volume} = \frac{1}{3}\pi\left(\frac{6}{5}r\right)^2\left(\frac{6}{5}h\right)$$
$$= \frac{1}{3}\pi\left(\frac{36}{25}r^2\right)\left(\frac{6}{5}h\right)$$
$$= \frac{216}{125}\left(\frac{1}{3}\pi r^2 h\right)$$
$$= \frac{216}{125}(\text{Original volume})$$

So, change in the volume

$$= \frac{216}{125}(\text{Original volume}) - (\text{Original volume})$$
$$= \frac{91}{125}(\text{Original volume})$$

$$\text{Increase percentage} = \frac{\frac{91}{125}(\text{Original volume})}{\text{Original volume}} \times 100$$
$$= 72.8\%$$

Solution 61

Correct option: (a)

Let the radii of the base of the cylinder and the cone be 3r and 4r and their heights be 2h and 3h respectively.

$$\text{Ratio of their volumes} = \frac{\pi(3r)^2(2h)}{\frac{1}{3}\pi(4r)^2(3h)}$$

$$= \frac{\pi \times 9r^2 \times 2h \times 3}{\pi \times 16r^2 \times 3h}$$

$$= \frac{9}{8}$$

Hence, the ratio is 9:8.

Solution 62

Correct option: (d)

Volume of the cylinder = volume of the cone

$$\Rightarrow \pi(8)^2(2) = \frac{1}{3} \times \pi(r)^2(6)$$

$$\Rightarrow r^2 = 64$$

$$\Rightarrow r = 8 \text{ cm}$$

Hence, the radius of the base of the cone is 8 cm.

Solution 63

Correct option: (b)

Area of the floor of a conical tent = $\pi(r)^2$

$\Rightarrow \pi r^2 = 346.5$

$\Rightarrow \frac{22}{7} \times r^2 = 346.5$

$\Rightarrow r^2 = \left(\frac{3465}{10} \times \frac{7}{22}\right)$

$\Rightarrow r^2 = \frac{441}{4}$

$\Rightarrow r = \frac{21}{2}$ cm

Slant height of the cone, $l = \sqrt{r^2 + h^2}$

$\Rightarrow l = \sqrt{\left(\frac{21}{2}\right)^2 + 14^2}$

$\Rightarrow l = \sqrt{\frac{1225}{4}}$

$\Rightarrow l = \frac{35}{2}$ m

Area of the canvas = curved surface area of the conical tent

$\Rightarrow$ Area of the canvas = πrl

$\Rightarrow$ Area of the canvas = $\frac{22}{7} \times \frac{21}{2} \times \frac{35}{2} = 577.5$ m^2

$$\text{Length of the canvas} = \frac{\text{Area of the canvas}}{\text{Width of the canvas}}$$

$$= \frac{577.5}{1.1}$$

$$= 525 \text{ m}$$

Solution 64

Correct option: (c)

Diameter = 14 cm

So, the radius = 7 cm

$$\text{Volume of the sphere} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times (7)^3$$

$$= 1437\frac{1}{3}\ \text{cm}^3$$

Solution 65

Correct option: (d)

Let the radii of the spheres be R and r.

Ratio of their volumes = $\dfrac{\frac{4}{3}\pi R^3}{\frac{4}{3}\pi r^3}$

$$\Rightarrow \frac{\frac{4}{3}\pi R^3}{\frac{4}{3}\pi r^3} = \frac{8}{27}$$

$$\Rightarrow \left(\frac{R}{r}\right)^3 = \left(\frac{2}{3}\right)^3$$

$$\Rightarrow \frac{R}{r} = \frac{2}{3}$$

Ratio between their surface areas

$$= \frac{4\pi R^2}{4\pi r^2}$$

$$= \left(\frac{R}{r}\right)^2$$

$$= \left(\frac{2}{3}\right)^2$$

$$= \frac{4}{9}$$

Solution 66

Correct option: (b)

The radii of the spherical shell is 2 cm and 4 cm.

$$\text{Volume of the spherical shell} = \frac{4}{3}\pi\left(R^3 - r^3\right)$$
$$= \frac{4}{3}\pi\left(4^3 - 2^3\right)$$
$$= \frac{4}{3}\pi(56)$$

Radius of the cone = 4 cm

$$\text{Volume of the cone} = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3}\pi(4)^2 h$$

$$\therefore \frac{1}{3}\pi(4)^2 h = \frac{4}{3}\pi(56)$$
$$\Rightarrow 16h = 4(56)$$
$$\Rightarrow h = 14 \text{ cm}$$

Solution 67

Solution 68

Correct option: (a)

Volume of hemisphere = $\frac{2}{3}\pi r^3$

$\frac{2}{3}\pi r^3 = 19404$

$\frac{2}{3} \times \frac{22}{7} r^3 = 19404$

$r^3 = 19404 \times \frac{3 \times 7}{2 \times 22}$

$r^3 = 9261$

$r^3 = 21^3$

$r = 21$ cm

Surface area of hemisphere = $3\pi r^2$

$= 3 \times \frac{22}{7} \times 21^2$

$= 4158\ cm^2$

Solution 69

Correct option: (a)

Surface area of sphere = 154 cm^2

$4\pi r^2 = 154$

$4 \times \frac{22}{7} r^2 = 154$

$r^2 = 154 \times \frac{7}{4 \times 22}$

$r^2 = \frac{49}{4}$

$r = \frac{7}{2}$ cm

Volume of sphere = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^3$

$= 179\frac{2}{3}$ cm^3

Solution 70

Correct option: (c)

The total surface area of a hemisphere

$= 3\pi r^2$

$= 3 \times \pi \times 7^2$

$= 147\pi$ cm^2

Solution 71

Correct option: (b)

Volume of the bucket = Volume of the frustum of the cone

$$= \frac{1}{3}\pi h\left[R^2 + r^2 + Rr\right]$$

$$= \frac{1}{3} \times \frac{22}{7} \times 40\left[35^2 + 14^2 + (35 \times 14)\right]$$

$$= \frac{880}{21} \times 1911$$

$$= 80080 \text{ cm}^3$$

Hence, the volume of the bucket is 80080 cm^3.

Solution 72

Correct option: (b)

$$l = \sqrt{h^2 + (R - r)^2}$$

$$\Rightarrow l = \sqrt{24^2 + (15 - 5)^2}$$

$$\Rightarrow l = \sqrt{576 + 100}$$

$$\Rightarrow l = \sqrt{676}$$

$$\Rightarrow l = 26 \text{ cm}$$

Surface area of the bucket

$$= \pi\left[r^2 + l(R + r)\right]$$

$$= 3.14 \times \left[5^2 + 26(15 + 5)\right]$$

$$= 3.14 \times [545]$$

$$= 1711.3 \text{ cm}^2$$

Solution 73

Correct option: (d)

Total area of the canvas required

= Curved surface area of the cylinder + Curved surface area of the cone

$= 2\pi rh + \pi rl$

$= \left(2 \times \frac{22}{7} \times \frac{105}{2} \times 4\right) + \left(\frac{22}{7} \times \frac{105}{2} \times 40\right)$

$= (1320) + (6600)$

$= 7920\ m^2$

Solution 74

Number of cones formed = $\frac{\text{Volume of the sphere}}{\text{Volume of each cone}}$

$$= \frac{\frac{4}{3}\pi r^3}{\frac{1}{3}\pi r^2 h}$$

$$= \frac{4r}{h}$$

$$= \frac{4 \times 8}{4}$$

$$= 8$$

(a) - (q)

Volume of the earth dug out = Volume of the cylinder

$$= \pi r^2 h$$

$$= \frac{22}{7} \times 7^2 \times 20$$

Let the height of the platform be h.

Volume of the platform = Volume of the cuboid

$$= 44 \times 14 \times h$$

$\Rightarrow \frac{22}{7} \times 7^2 \times 20 = 44 \times 14 \times h$

$\Rightarrow 3080 = 616 \times h$

$\Rightarrow h = \frac{3080}{616}$

$\Rightarrow h = 5$ m

(b) - (s)

Volume of the sphere = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi(6)^3$

Let h be the height of the cylinder.

Volume of the cylinder = $\pi r^2 h$

$$= \pi(4)^2 h$$

$\Rightarrow \frac{4}{3}\pi(6)^3 = \pi(4)^2 h$

$\Rightarrow \frac{1}{3}(6)^3 = (4)^2 h$

$\Rightarrow h = \frac{228}{16} = 18$ cm

(c) - (p)

Let the radii of the sphere be R and r.

Ratio of their volumes = $\frac{\frac{4}{3}\pi R^3}{\frac{4}{3}\pi r^3}$

$\Rightarrow \frac{\frac{4}{3}\pi R^3}{\frac{4}{3}\pi r^3} = \frac{64}{27}$

$\Rightarrow \left(\frac{R}{r}\right)^3 = \left(\frac{4}{3}\right)^3$

$\Rightarrow \frac{R}{r} = \frac{4}{3}$

Ratio of their surface areas = $\frac{4\pi R^2}{4\pi r^2}$

$$= \left(\frac{R}{r}\right)^2$$

$$= \left(\frac{4}{3}\right)^2$$

$$= \frac{16}{9}$$

(d) - (r)

Solution 75

Let R and r be the top and base of the bucket and h be the height.
Capacity of the bucket = Volume of the frustum of the cone

$$= \frac{\pi h}{3}\left(R^2 + r^2 + Rr\right)$$

$$= \frac{22}{7} \times \frac{1}{3} \times 30 \times \left(20^2 + 10^2 + 20 \times 10\right)$$

$$= \frac{220}{7} \times 700$$

$$= 22000 \text{ cm}^3$$

(a) - (q)

$$\text{Slant height, } l = \sqrt{h^2 + (R - r)^2}$$

$$= \sqrt{15^2 + (20 - 12)^2}$$

$$= \sqrt{225 + 64}$$

$$= \sqrt{289}$$

$$= 17 \text{ cm}$$

(b) - (s)

$$\text{Total surface area of the bucket} = \pi\left[R^2 + r^2 + l(R + r)\right]$$

$$= \pi\left[33^2 + 27^2 + 10(33 + 27)\right]$$

$$= \pi\left[1089 = 729 + 600\right]$$

$$= 2418\pi \text{ cm}^2$$

(c) - (p)

Let the diameter be d.

So, the radius $= \frac{d}{2}$

$$\text{Volume of the sphere} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3$$

$$\Rightarrow \frac{4}{3}\pi\left(\frac{d}{2}\right)^3 = \frac{4}{3}\pi(3)^3 + \frac{4}{3}\pi(4)^3 + \frac{4}{3}\pi(5)^3$$

$$\Rightarrow \left(\frac{d}{2}\right)^3 = (3)^3 + (4)^3 + (5)^3$$

$$\Rightarrow \frac{d^3}{8} = 27 + 64 + 125$$

$$\Rightarrow \frac{d^3}{8} = 216$$

$$\Rightarrow d^3 = 1728$$

$$\Rightarrow d = 12 \text{ cm}$$

(d) - (r)

Solution 76

Correct option: (d)

$$\text{Slant height} = \sqrt{h^2 + (R - r)^2}$$
$$= \sqrt{24^2 + (15 - 5)^2}$$
$$= \sqrt{576 + 100}$$
$$= \sqrt{676}$$
$$= 26 \text{ cm}$$

$$\text{Surface area of the bucket} = \pi\left[R^2 + r^2 + l(R + r)\right]$$
$$= \pi\left[15^2 + 5^2 + 26(15 + 5)\right]$$
$$= \pi\left[225 + 25 + 520\right]$$
$$= 770\pi \text{ cm}^2$$

The Assertion (A) and the Reason (R) are incorrect.

Note: The answer given in the text is incorrect.

Solution 77

Correct option: (d)

The total surface area of the hemisphere

$$= \pi r^2 + 2\pi r^2$$
$$= 3\pi r^2$$
$$= 3 \times \frac{22}{7} \times 7 \times 7$$
$$= 462 \text{ cm}^2$$

Cost of painting at Rs. 5 per cm^2 = Rs. (462×5) = Rs. 2310

So, the Assertion (A) is false.

The Reason (R) is true.

Solution 78

Correct option: (a)

Volume of the cuboid = $(10 \times 5.5 \times 3.5)\ cm^3$

Volume of each coin = $\pi r^2 h$

$$= \frac{22}{7} \times \frac{1.75}{2} \times \frac{1.75}{2} \times \frac{1}{5}$$

$$\text{Number of coins} = \frac{\text{Volume of the cuboid}}{\text{Volume of each coin}}$$

$$= \frac{10 \times 5.5 \times 3.5}{\frac{22}{7} \times \frac{1.75}{2} \times \frac{1.75}{2} \times \frac{1}{5}}$$

$$= 400$$

So, the Assertion (A) is true.

The Reason (R) is also true and is the correct explanation for the Assertion (A).

Solution 79

Correct option: (d)

Let r and R be the radii of the two spheres.

Ratio of their volumes = $\frac{27}{8}$

$$\Rightarrow \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} = \frac{27}{8}$$

$$\Rightarrow \frac{r^3}{R^3} = \frac{27}{8}$$

$$\Rightarrow \frac{r}{R} = \frac{3}{2}$$

Ratio of their surface areas $= \frac{4\pi r^2}{4\pi R^2}$

$$= \left(\frac{r}{R}\right)^2$$

$$= \left(\frac{3}{2}\right)^2$$

$$= \frac{9}{4}$$

So, the Assertion (A) is false.

The Reason (R)s true.

Solution 80

Correct option: (c)

$l = \sqrt{r^2 + h^2}$

$\Rightarrow l = \sqrt{3^2 + 4^2}$

$\Rightarrow l = \sqrt{9 + 16}$

$\Rightarrow l = \sqrt{25}$

$\Rightarrow l = 5$ cm

Curved surface area of a cone

$= \pi rl$

$= \pi(3)(5)$

$= 15\pi\ cm^2$

So, the Assertion (A) is true.

Volume of a cone $= \frac{1}{3}\pi r^2 h$

So, the Reason (R) is false.

Excercise FA

Solution 1

Let the number of solid spheres be n.
Given Diameter of sphere = 6 cm $\Rightarrow$ radius = 3 cm,
Diameter of cylinder = 4 cm $\Rightarrow$ radius = 2 cm and height of the cylinder = 45 cm
Now,
Volume of the cylinder = Volume of the sphere $\times$ n

$$\Rightarrow \pi r^2 h = \frac{4}{3}\pi r^3 \times n$$

$$\Rightarrow \pi \times 2 \times 2 \times 45 = \frac{4}{3} \times \pi \times 3 \times 3 \times 3 \times n$$

$$\Rightarrow 45 = 9 \times n$$

$$\Rightarrow n = \frac{45}{9}$$

$$\Rightarrow n = 5$$

Hence, number of solid spheres is 5.

Solution 2

Let r_1 and h_1 be the radius and height of the first cylinder and r_2 and h_2 be the radius and height of the second cylinder.
Given height of first cylinder : height of the second cylinder = 1 : 4

$$\Rightarrow h_1 : h_2 = 1 : 4$$

$$\Rightarrow \frac{h_1}{h_2} = \frac{1}{4} \qquad \ldots(i)$$

Volume of the first cylinder = Volume of the second cylinder

$$\Rightarrow \pi r_1^2 h_1 = \pi r_2^2 h_2$$

$$\Rightarrow \frac{r_1^2}{r_2^2} = \frac{h_2}{h_1}$$

$$\Rightarrow \frac{r_1^2}{r_2^2} = \frac{4}{1} \qquad \ldots\text{(From (i))}$$

$$\Rightarrow \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{2}{1}\right)^2$$

$$\Rightarrow \frac{r_1}{r_2} = \frac{2}{1}$$

$$\Rightarrow r_1 : r_2 = 2 : 1$$

Hence, the ratio of their radii is 2 : 1.

Solution 3

Given diameter of cylindrical portion = 105 m $\Rightarrow$ radius = 52.5 m and
height of cylindrical portion (h) = 4 m
$\Rightarrow$ Curved Surface area of cylindrical portion = $2\pi rh$

$$= 2 \times \frac{22}{7} \times 52.5 \times 4$$
$$= \frac{9240}{7}$$
$$= 1320 \text{ m}^2$$

Given slant height of conical ortion (l) = 40 m and
Radius of conical portion = Radius of cylindrical portion = 52.5 m
$\Rightarrow$ Curved Surface area of conical portion = πrl

$$= \frac{22}{7} \times 52.5 \times 40$$
$$= \frac{46200}{7}$$
$$= 6600 \text{ m}^2$$

Now,
Total surfae area = Curved Surface area of cylindrical portion + Curved Surface area of conical portion

$$= 1320 + 6600$$
$$= 7920 \text{ m}^2$$

Solution 4

Given radius of the top of the bucket (R) = 28 cm,
radius of the bottom of the bucket (r) = 7 cm and
slant height of the bucket (l) = 45 cm
$\because$ The bucket will be in the form of a frustum
$\therefore$ Curved surface area of the bucket = $\pi(r + R)l$

$$= \frac{22}{7} \times (28 + 7) \times 45$$
$$= 22 \times 5 \times 45$$
$$= 4950 \text{ cm}^2$$

Thus, Curved surface area of the bucket is 4950 cm^2.

Solution 5

Given radius of cone = 12 cm, height of cone = 24 cm and diameter of sphere = 6 cm $\Rightarrow$ radius = 3 cm

As the right circular cone has been melted to number of spheres.

Thus, the volume of cone = volume of all such spheres

Let the number of spheres be n

Volume of cone = Volume of sphere

$$\Rightarrow \frac{1}{3}\pi r^2 h = n \times \frac{4}{3}\pi R^3$$

$$\Rightarrow r^2 h = n \times 4 \times R^3$$

$$\Rightarrow 12^2 \times 24 = n \times 4 \times 3^3$$

$$\Rightarrow n = \frac{12^2 \times 24}{4 \times 3^3}$$

$$\Rightarrow n = \frac{864}{27}$$

$$\Rightarrow n = 32$$

Thus, the number of spheres is 32.

Solution 6

Let the required bottles = x

Given Internal diameter of hemispherical sphere = 30 cm

Internal radius of hemispherical sphere = 15 cm

$$\text{Volume of hemispherical sphere} = \frac{2}{3}\pi r^3$$

$$= \frac{2}{3} \times \pi \times 15 \times 15 \times 15$$

$$= 2250\pi \text{ cm}^3$$

Also,

Given Diameter of the cylindrical bottle = 5 cm $\Rightarrow$ radius = 2.5 cm and

Height of the cylindrical bottle = 6 cm

$$\text{Volume of 1 cylindrical bottle} = \pi r^2 h$$

$$= \pi (2.5)^2 \times 6$$

$$= 37.5\pi \text{ cm}^3$$

Now,

Amount of water in x bottle = Amount of water in bowl

$\Rightarrow 37.5\pi \times x = 2250\pi$

$\Rightarrow x = \dfrac{2250}{37.5}$

$\Rightarrow x = 60$

Thus, 60 bottles are required.

Solution 7

Given Diameter of sphere = 21 cm
$\Rightarrow$ Radius of sphere (R) = 10.5 cm
Now,
Diameter of small cone = 3.5 cm
$\Rightarrow$ Radius of the cone (r) = 1.75 cm and
Height of small cone (h) = 3 cm
Let the number of small cones formed by melting the metallic sphere = n.
$\Rightarrow$ n x Volume of small cone = Volume of the sphere

$$\Rightarrow n \times \frac{1}{3}\pi r^2 h = \frac{4}{3}\pi R^3$$

$$\Rightarrow n \times r^2 h = 4R^3$$

$$\Rightarrow n \times (1.75)^2 h = 4(10.5)^3$$

$$\Rightarrow n \times 9.1875 = 4630.5$$

$$\Rightarrow n = \frac{4630.5}{9.1875}$$

$$\Rightarrow n = 504$$

Hence, 504 small cones can be formed by melting the given metallic sphere.

Solution 8

Given Diameter of sphere = 42 cm

$\Rightarrow$ Radius of sphere (R) = 21 cm

Volume of the sphere $= \frac{4}{3}\pi R^3$

$= \frac{4}{3} \times \frac{22}{7} \times 21 \times 21 \times 21$

$= 88 \times 21 \times 21$

$= 38808 \text{ cm}^3$

Given Diameter of cylindrical wire = 2.8 cm

$\Rightarrow$ Radius of cylindrical wire (r) = 1.4 cm

Let the length of the wire be h cm.

Volume of the cylindrical wire $= \pi r^2 h$

$= \frac{22}{7} \times 1.4 \times 1.4 \times 1.4 \times h$

$= \frac{60.368}{7} \times h$

$= 8.624 \times h \text{ cm}^3$

Since volume of the cylindrical wire = Volume of the sphere

$\Rightarrow 8.624 \times h = 38808$

$\Rightarrow h = \frac{38808}{8.624}$

$\Rightarrow h = 4500 \text{ cm}$

$\Rightarrow h = 45 \text{ m}$

Thus, the length of the wire is 45 m.

Solution 9

Given Diameter of circular ends are 6 cm and 4 cm

$\Rightarrow$ Radius if one circular end (r_1) = 3 cm and radius of other circular end (r_2) = 2 cm

Height of frustum of the cone (h) = 21 cm

Now,

Capacity of the glass = Volume of the frustum of the cone

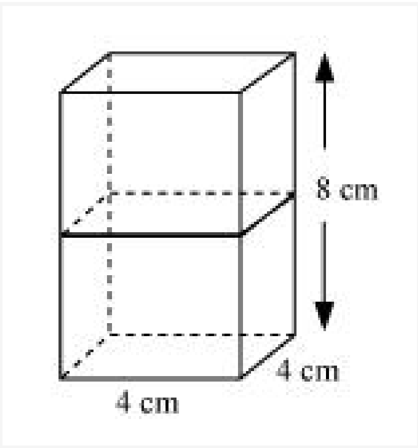
$$= \frac{1}{3} \times \pi \times \left(r_1^2 + r_2^2 + r_1 r_2\right) \times h$$

$$= \frac{1}{3} \times \frac{22}{7} \times \left(3^2 + 2^2 + 3 \times 2\right) \times 21$$

$$= 22 \times (9 + 4 + 6)$$

$$= 418 \text{ cm}^3$$

Solution 10



Given that,

Volume of cube = 64 cm^3

$\therefore (side)^3 = 64$

$\therefore$ side = 4 cm

If cubes are joined end to end, the dimensions of the resulting cuboid will be 4 cm, 4 cm and 8 cm.

$$
\begin{aligned}
\text{Surface area of cuboid} &= 2(lb + bh + lh) \\
&= 2((4 \times 4) + (4 \times 8) + (4 \times 8)) \\
&= 2(16 + 32 + 32) \\
&= 160 \text{ cm}^2
\end{aligned}
$$

Solution 11

Let r and h be the radius and height of the cylinder respectively.
Given r : h = 2 : 3
Let the common multiple be x.
$\therefore$ r = 2x and h = 3x
Now,
Volume of the cylinder = 1617 cm^3 ...(Given)

$\therefore \pi r^2 h = 1617$

$\therefore \frac{22}{7} \times (2x)^2 \times (3x) = 1617$

$\therefore \frac{22}{7} \times 12x^3 = 1617$

$\therefore x^3 = \frac{1617 \times 7}{22 \times 12}$

$\therefore x^3 = 42.875$

$\therefore x = \sqrt[3]{42.875}$

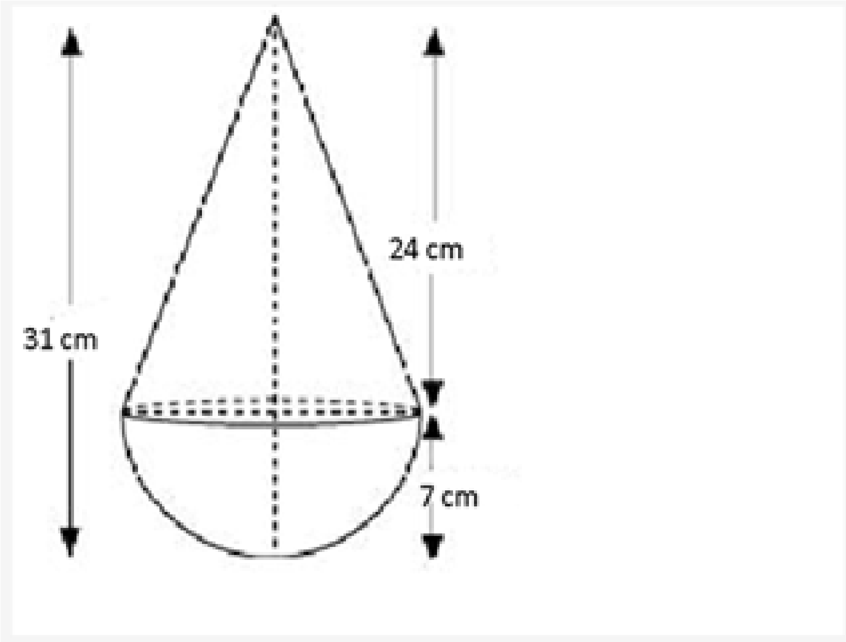
$\therefore$ x = 3.5 cm (Approx)

$\therefore$ Radius of the cylinder (r) = 2x = 2 × 3.5 = 7 cm and
Height of the cylinder (h) = 3x = 3 × 3.5 = 10.5 cm

$$\begin{aligned}\text{Total surface area of cylinder} &= 2\pi r(r+h)\\ &= 2 \times \frac{22}{7} \times 7(7+10.5)\\ &= 44 \times 17.5\\ &= 770\ cm^2\end{aligned}$$

Hence, total surface area of the cylinder is 770 cm^2 (Approx).

Solution 12



Given Radius of the conical part = Radius of the hemispherical part = 7 cm

Height of the hemispherical part = Radius (r) = 7 cm

Height of the conical part (h) = 31 − 7 = 24 cm

Slant height (l) of the conical part $= \sqrt{r^2 + h^2}$

$$= \sqrt{(7)^2 + (24)^2}$$

$$= \sqrt{49 + 576}$$

$$= \sqrt{625}$$

$$= 25 \text{ cm}$$

Total surface area of top = CSA of conical part + CSA of hemispherical part

$$= \pi r l + 2\pi r^2$$

$$= \frac{22}{7} \times 7 \times 25 + 2 \times \frac{22}{7} \times 7 \times 7$$

$$= 550 + 308$$

$$= 858 \text{ cm}^2$$

Solution 13

Given Internal radius of the hemispherical bowl (R) = 9 cm

Amount of the liquid in the bowl = Capacity of the bowl

$$= \frac{2}{3}\pi R^3$$

$$= \frac{2}{3}\pi (9)^3$$

$$= 486\pi \text{ cm}^3$$

Now, liquid from the bowl is to be emptied into cylindrical bottles.

Diameter of each cylindrical bottle (d) = 3 cm

$\Rightarrow$ Radius of each cylindrical bottle (r) = $\frac{3}{2}$ cm

Height of each cylindrical bottle (h) = 4 cm

$\therefore$ Capacity of each cylindrical bottle = $\pi r^2 h$

$$= \pi\left(\frac{3}{2}\right)^2 \times 4$$

$$= 9\pi \text{ cm}^3$$

Number of cylindrical bottles filled = $\dfrac{\text{Capacity of the bowl}}{\text{Capacity of each cylindrical bottle}}$

$$= \frac{486\pi}{9\pi}$$

$$= 54$$

Thus, 54 cylindrical bottles can be filled with the liquid available in the bowl.

Solution 14

We know that,

Surface area of sphere = $4\pi r^2$ and

Surface area of a cube = $6a^2$

Given Surface area of sphere = Surface area of cube

$\Rightarrow 4\pi r^2 = 6a^2$

$\Rightarrow \frac{r^2}{a^2} = \frac{6}{4\pi}$

$\Rightarrow \left(\frac{r}{a}\right)^2 = \frac{6}{4\pi}$

$\Rightarrow \frac{r}{a} = \sqrt{\frac{6}{4\pi}}$ ...(i)

Now,

$$\frac{\text{Volume of sphere}}{\text{Volume of cube}} = \frac{\frac{4}{3}\pi r^3}{a^3}$$

$$= \frac{4}{3}\pi\left(\frac{r}{a}\right)^3$$

$$= \frac{4}{3}\pi\left(\sqrt{\frac{6}{4\pi}}\right)^3 \quad \text{....(From (i))}$$

$$= \frac{4}{3} \times \pi \times \sqrt{\frac{6}{4\pi}} \times \frac{6}{4\pi}$$

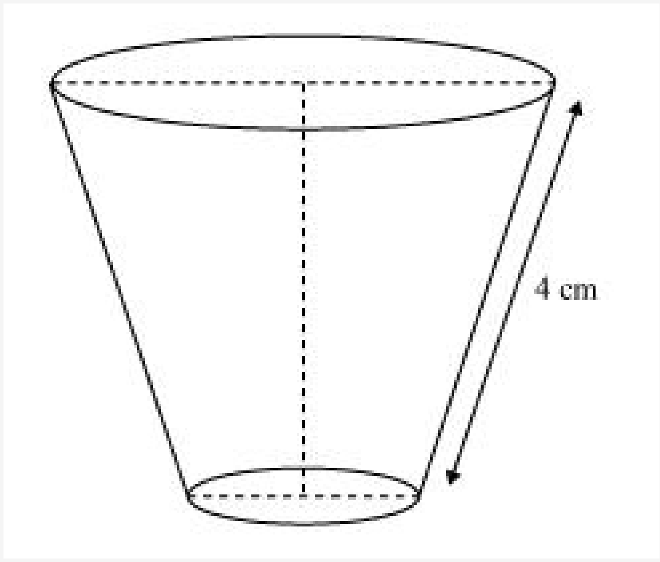
$$= 2 \times \sqrt{\frac{6}{4\pi}}$$

$$= \sqrt{\frac{6}{\frac{22}{7}}}$$

$$= \sqrt{\frac{6 \times 7}{22}}$$

$$= \sqrt{\frac{21}{11}}$$

Solution 15



Perimeter of upper circular end of frustum = 18 cm

$\Rightarrow 2\pi r_1 = 18$

$\Rightarrow r_1 = \dfrac{9}{\pi}$

Perimeter of lower end frustum = 6 cm

$\Rightarrow 2\pi r_2 = 6$

$\Rightarrow r_2 = \dfrac{3}{\pi}$

Slant height (l) of fristum = 4 cm

Curved Surface Area of frustum $= \pi\left(r_1 + r_2\right)l$

$$= \pi\left(\frac{9}{\pi} + \frac{3}{\pi}\right) \times 4$$

$$= \pi\left(\frac{12}{\pi}\right) \times 4$$

$$= 12 \times 4$$

$$= 48\ cm^2$$

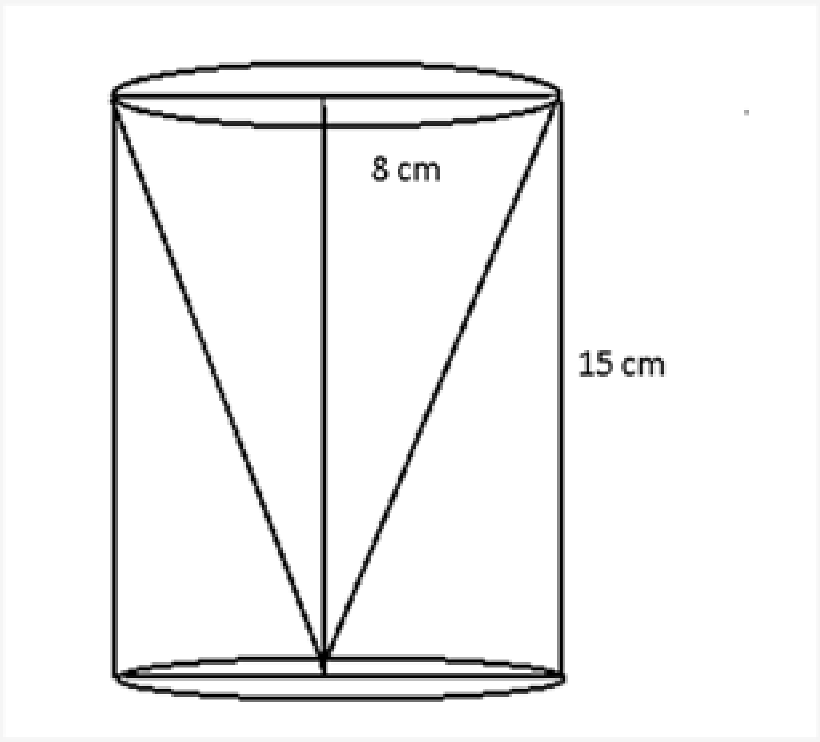
Thus, the Curved Surface Area of frustum is 48 cm^2.

Solution 16

Given Radius of hemispherical end = 7 cm,
Radius of the cylinder = Radius of hemispherical end = 7 cm and
Height of cylinder (h) = 104 cm
Total surface area of solid = Surface area of cylinder + 2(Surface area of hemispherical ends)

$$= 2\pi rh + 2 \times \left(2\pi r^2\right)$$
$$= \left[2 \times \frac{22}{7} \times 7 \times 104\right] + \left[2 \times \left(2 \times \frac{22}{7} \times (7)^2\right)\right]$$
$$= 4576 + 616$$
$$= 5192 \text{ cm}^2$$

Solution 17



Given that,
Height of the conical part (h) = Height of the cylindrical part (h) = 15 cm
Diameter of the cylindrical part = 16 cm
$\Rightarrow$ Radius of the cylindrical part (r) = 8 cm
Now,
Total Surface area of the remaining solid = CSA of the cylindrical part + CSA of the conical part + Area of cylindrical base

$$= 2\pi rh + \pi r\sqrt{r^2 + h^2} + \pi r^2$$
$$= \left[2 \times \frac{22}{7} \times 8 \times 15\right] + \left[\frac{22}{7} \times 8 \times \sqrt{8^2 + 15^2}\right] + \left[\frac{22}{7} \times (8)^2\right]$$
$$= 754.286 + 427.4 + 201.1$$
$$= 1382.786 \text{ cm}^2$$

Solution 18

Given Length of the cuboid = 4.4 m

Breadth of the cuboid = 2.6 m

Height of the cuboid = 1 m

Internal radius (r) = 30 cm

$$= \frac{30}{100} \text{ m}$$

$$= 0.3 \text{ m}$$

Thickness = 5 cm = 0.05 m

Outer radius (R) = inner radius + thickness

$$= 0.3 + 0.05$$

$$= 0.35 \text{ m}$$

Let h be the length of the pipe.

Volume of the cuboid = Volume of the cylindrical pipe

$$\Rightarrow \text{length} \times \text{breadth} \times \text{height} = \pi h\left(R^2 - r^2\right)$$

$$\Rightarrow 4.4 \times 2.6 \times 1 = \frac{22}{7} \times h \times \left((0.35)^2 - (0.3)^2\right)$$

$$\Rightarrow 11.44 = \frac{22}{7} \times h \times 0.0325$$

$$\Rightarrow h = 112 \text{ m}$$

Hence, the length of the pipe is 112 m.

Solution 19

The total height = 40 cm which includes the height of the base.

So, the height of the frustum of the cone = 40 – 6 = 34 cm

$\therefore$ Slant height of frustum (l) = $\sqrt{h^2 + (r_1 - r_2)^2}$

$= \sqrt{34^2 + \left(\frac{45}{2} - \frac{25}{2}\right)^2}$

$= \sqrt{34^2 + (22.5 + -12.5)^2}$

$= \sqrt{34^2 + (10)^2}$

$= \sqrt{1256}$

= 35.44 cm

Area of the metallic sheet used = Curved surface area of frustum of cone + Area of circular base + Curved surface area of cylinder

$= \pi \times 35.44 \times (22.5 + 12.5) + \pi \times (12.5)^2 + 2\pi \times 12.5 \times 6$

$= \frac{22}{7} \times 35.44 \times 35 + \frac{22}{7} \times 156.25 + 2 \times \frac{22}{7} \times 12.5 \times 6$

$= \frac{27288.8}{7} + \frac{3437.5}{7} + \frac{3300}{7}$

$= \frac{27288.8 + 3437.5 + 3300}{7}$

$= \frac{34026.3}{7}$

= 4860.9 cm²705925

Now,

Volume of the water that the bucket can hold = $\frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$

$= \frac{1}{3} \times \frac{22}{7} \times 34\left((22.5)^2 + (12.5)^2 + 22.5 \times 12.5\right)$

$= \frac{748}{21} \times 943.75$

= 33615.12

= 33.62 litres (approx.) .....(Since 1 litres = 1000 cm³)

Solution 20

Let x hours be the time taken for the pipe to fill the tank.

$\because$ The water is flowing at the rate of 4 km/hr,

$\therefore$ Length of the water column in x hours is 4x km = 4000x m.

$\therefore$ The length of the pipe is 4000x m

The diameter of the pipe = 20 cm

$\Rightarrow$ radius = 10 cm

$= \frac{10}{100}$ m

$= 0.1$ m

$\therefore$ Volume of the water flowing through the pipe in x hours = V_1

$= \pi r^2 h$

$= \pi \times (0.1)^2 \times 4000x \quad \ldots(i)$

Given Diameter of the cylindrical tank = 10 m

$\Rightarrow$ radius = 5 cm and

Volume of the water that falls into the tank in x hours = V_1

$= \pi r^2 h$

$= \pi \times (5)^2 \times 2 \quad \ldots(ii)$

$\therefore$ Volume of the water flowing through the pipe in x hours = Volume of the water that falls into the tank in x hours

$\Rightarrow \pi \times (0.1)^2 \times 4000x = \pi \times (5)^2 \times 2$

$\Rightarrow 40x = 50$

$\Rightarrow x = \frac{50}{40}$ hour

$\Rightarrow x = \frac{50}{40} \times 60$ minutes

$\Rightarrow x = 75$ minutes = 1 hour 15 mins

Thus, the water in the tank will fill in 1 hour 15 minutes.