

# Polynomials

If ' $x$ ' is a variable, ' $n$ ' is a positive integer and  $a_0, a_1, a_2, \dots, a_n$  are constants, then a polynomial in variable  $x$  is  $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$

$$f(x) = \underbrace{a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0}_{\text{Terms}}$$

Coefficients

**Degree of a Polynomial:** The power of the highest degree term

**Zero of a Polynomial:** A real number  $\alpha$  is a zero of a polynomial  $f(x)$ , iff  $f(\alpha) = 0$ .  
Finding the zero of a polynomial  $f(x)$  means solving the polynomial equation  $f(x) = 0$

## Polynomial Classification

Degree	Name
0	Constant
1	Linear
2	Quadratic
3	Cubic
4	Quartic
5	Quintic

Number of Terms	Name
1	Monomial
2	Binomial
3	Trinomial
4	Polynomial of 4 Terms

## Exercise 2A

### Question 1:

- (i) It is a polynomial, Degree = 5.
- (ii) It is polynomial, Degree = 3.
- (iii) It is polynomial, Degree = 2.
- (iv) It is not a polynomial.
- (v) It is not a polynomial.
- (vi) It is polynomial, Degree = 108.
- (vii) It is not a polynomial.
- (viii) It is a polynomial, Degree = 2.
- (ix) It is not a polynomial.
- (x) It is a polynomial, Degree = 0.
- (xi) It is a polynomial, Degree = 0.
- (xii) It is a polynomial, Degree = 2.

### Question 2:

The degree of a polynomial in one variable is the highest power of the variable.

- (i) Degree of  $2x - \sqrt{5}$  is 1.
- (ii) Degree of  $3 - x + x^2 - 6x^3$  is 3.
- (iii) Degree of 9 is 0.
- (iv) Degree of  $8x^4 - 36x + 5x^7$  is 7.
- (v) Degree of  $x^9 - x^5 + 3x^{10} + 8$  is 10.

(vi) Degree of  $2 - 3x^2$  is 2.

**Question 3:**

(i) Coefficient of  $x^3$  in  $2x + x^2 - 5x^3 + x^4$  is -5

(ii) Coefficient of  $x$  in  $\sqrt{3} - 2\sqrt{2}x + 4x^2$  is  $-2\sqrt{2}$

(iii) Coefficient of  $x^2$  in  $\frac{\pi}{3}x^2 + 7x - 3$  is  $\frac{\pi}{3}$

(iv) Coefficient of  $x^2$  in  $3x - 5$  is 0.

**Question 4:**

(i)  $x^{27} - 36$

(ii)  $y^{16}$

(iii)  $5x^3 - 8x + 7$

**Question 5:**

(i) It is a quadratic polynomial.

(ii) It is a cubic polynomial.

(iii) It is a quadratic polynomial.

(iv) It is a linear polynomial.

(v) It is a linear polynomial.

(vi) It is a cubic polynomial.

## Exercise 2B

**Question 1:**

$$p(x) = 5 - 4x + 2x^2$$

$$(i) p(0) = 5 - 4(0) + 2(0)^2 = 5$$

$$(ii) p(3) = 5 - 4(3) + 2(3)^2$$

$$= 5 - 12 + 18$$

$$= 23 - 12 = 11$$

$$(iii) p(-2) = 5 - 4(-2) + 2(-2)^2$$

$$= 5 + 8 + 8 = 21$$

**Question 2:**

$$p(y) = 4 + 3y - y^2 + 5y^3$$

$$(i) p(0) = 4 + 3(0) - 0^2 + 5(0)^3$$

$$= 4 + 0 - 0 + 0 = 4$$

$$(ii) p(2) = 4 + 3(2) - 2^2 + 5(2)^3$$

$$= 4 + 6 - 4 + 40$$

$$= 10 - 4 + 40 = 46$$

$$(iii) p(-1) = 4 + 3(-1) - (-1)^2 + 5(-1)^3$$

$$= 4 - 3 - 1 - 5 = -5$$

**Question 3:**

$$f(t) = 4t^2 - 3t + 6$$

$$(i) f(0) = 4(0)^2 - 3(0) + 6$$

$$= 0 - 0 + 6 = 6$$

$$(ii) f(4) = 4(4)^2 - 3(4) + 6$$

$$= 64 - 12 + 6 = 58$$

$$\begin{aligned} \text{(iii) } f(-5) &= 4(-5)^2 - 3(-5) + 6 \\ &= 100 + 15 + 6 = 121 \end{aligned}$$

#### Question 4:

$$\text{(i) } p(x) = 0$$

$$\Rightarrow x - 5 = 0$$

$$\Rightarrow x = 5$$

$\Rightarrow 5$  is the zero of the polynomial  $p(x)$ .

$$\text{(ii) } q(x) = 0$$

$$\Rightarrow x + 4 = 0$$

$$\Rightarrow x = -4$$

$\Rightarrow -4$  is the zero of the polynomial  $q(x)$ .

$$\text{(iii) } p(t) = 0$$

$$\Rightarrow 2t - 3 = 0$$

$$\Rightarrow 2t = 3$$

$$\Rightarrow t = \frac{3}{2}$$

$\Rightarrow t = \frac{3}{2}$  is the zero of the polynomial  $p(t)$ .

$$\text{(iv) } f(x) = 0$$

$$\Rightarrow 3x + 1 = 0$$

$$\Rightarrow 3x = -1$$

$$\Rightarrow x = \frac{-1}{3}$$

$\Rightarrow x = \frac{-1}{3}$  is the zero of the polynomial  $f(x)$ .

$$\text{(v) } g(x) = 0$$

$$\Rightarrow 5 - 4x = 0$$

$$\Rightarrow -4x = -5$$

$$\Rightarrow x = \frac{5}{4}$$

$\Rightarrow x = \frac{5}{4}$  is the zero of the polynomial  $g(x)$ .

$$\text{(vi) } h(x) = 0$$

$$\Rightarrow 6x - 1 = 0$$

$$\Rightarrow 6x = 1$$

$$\Rightarrow x = \frac{1}{6}$$

$\Rightarrow x = \frac{1}{6}$  is the zero of the polynomial  $h(x)$ .

$$\text{(vii) } p(x) = 0$$

$$\Rightarrow ax + b = 0$$

$$\Rightarrow ax = -b$$

$$\Rightarrow x = \frac{-b}{a}$$

$\Rightarrow x = \frac{-b}{a}$  is the zero of the polynomial  $p(x)$

$$\text{(viii) } q(x) = 0$$

$$\Rightarrow 4x = 0$$

$$\Rightarrow x = 0$$

$\Rightarrow 0$  is the zero of the polynomial  $q(x)$ .

$$(ix) p(x) = 0$$

$$\Rightarrow ax = 0$$

$$\Rightarrow x = 0$$

$\Rightarrow 0$  is the zero of the polynomial  $p(x)$ .

#### Question 5:

$$(i) p(x) = x - 4$$

$$\text{Then, } p(4) = 4 - 4 = 0$$

$\Rightarrow 4$  is a zero of the polynomial  $p(x)$ .

$$(ii) p(x) = x - 3$$

$$\text{Then, } p(-3) = -3 - 3 = -6$$

$\Rightarrow -3$  is not a zero of the polynomial  $p(x)$ .

$$(iii) p(y) = 2y + 1$$

$$p\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right) + 1 = 0$$

Then,

$\Rightarrow -\frac{1}{2}$  is a zero of the polynomial  $p(y)$ .

$$(iv) p(x) = 2 - 5x$$

$$p\left(\frac{2}{5}\right) = 2 - 5\left(\frac{2}{5}\right) = 2 - 2 = 0$$

Then,

$\Rightarrow \frac{2}{5}$  is a zero of the polynomial  $p(x)$ .

$$(v) p(x) = (x - 1)(x - 2)$$

$$\text{Then, } p(1) = (1 - 1)(1 - 2) = 0 \cdot 1 = 0$$

$\Rightarrow 1$  is a zero of the polynomial  $p(x)$ .

$$\text{Also, } p(2) = (2 - 1)(2 - 2) = 1 \cdot 0 = 0$$

$\Rightarrow 2$  is a zero of the polynomial  $p(x)$ .

Hence, 1 and 2 are the zeroes of the polynomial  $p(x)$ .

$$(vi) p(x) = x^2 - 3x.$$

$$\text{Then, } p(0) = 0^2 - 3(0) = 0$$

$$p(3) = (3^2) - 3(3) = 9 - 9 = 0$$

$\Rightarrow 0$  and 3 are the zeroes of the polynomial  $p(x)$ .

$$(vii) p(x) = x^2 + x - 6$$

$$\text{Then, } p(2) = 2^2 + 2 - 6$$

$$= 4 + 2 - 6$$

$$= 6 - 6 = 0$$

$\Rightarrow 2$  is a zero of the polynomial  $p(x)$ .

$$\text{Also, } p(-3) = (-3)^2 - 3 - 6$$

$$= 9 - 3 - 6 = 0$$

$\Rightarrow -3$  is a zero of the polynomial  $p(x)$ .

Hence, 2 and -3 are the zeroes of the polynomial  $p(x)$ .

## Exercise 2C

## Remainder Theorem

Let  $p(x)$  be a polynomial of degree greater than or equal to one and ' $a$ ' be a real number. If  $p(x)$  is divided by  $(x - a)$ , then the remainder is equal to  $p(a)$ .  
 $p(x) = (x - a) q(x) + r(x)$

Proof:

Divide  $p(x)$  by  $(x - a)$ , and let  $q(x)$  be the quotient and  $r(x)$  be the remainder, where  $r(x) = 0$  or degree of  $r(x) <$  degree of  $(x - a)$ .

But degree of  $(x - a)$  is 1,

$\therefore$  degree of  $r(x) = 0$

Let  $r(x) = r$ , then  $p(x) = (x - a) q(x) + r$

Substituting  $x = a$ , we have

$$p(a) = (a - a) q(a) + r$$

$$\Rightarrow p(a) = 0 \times q(a) + r$$

$$\Rightarrow p(a) = 0 + r$$

$$\Rightarrow p(a) = r$$

Thus remainder is  $p(a)$  when  $p(x)$  is divided by  $(x - a)$

To divide:  $(8x^2 + 4x - 2) \div (4x - 2)$

$$\begin{array}{r} 2x + 2 \\ 4x - 2 \overline{) 8x^2 + 4x + 2} \\ \underline{8x^2 - 4x} \phantom{+ 2} \\ 8x - 2 \\ \underline{8x - 4} \\ 2 \end{array}$$

Using the remainder theorem

$$p(x) = (x - a) q(x) + r(x),$$

$$p(x) = 8x^2 + 4x - 2 \text{ and } x = \frac{2}{4} = \frac{1}{2}$$

$$p\left(\frac{1}{2}\right) = 8 \times \left(\frac{1}{2}\right)^2 + 4 \times \frac{1}{2} - 2$$

$$= 8 \times \frac{1}{4} + 2 - 2$$

$$= 2 + 0 = 2$$

### Question 1:

$$f(x) = x^3 - 6x^2 + 9x + 3$$

$$\text{Now, } x - 1 = 0 \Rightarrow x = 1$$

By the remainder theorem, we know that when  $f(x)$  is divided by  $(x - 1)$  the remainder is  $f(1)$ .

$$\text{Now, } f(1) = 1^3 - 6 \times 1^2 + 9 \times 1 + 3$$

$$= 1 - 6 + 9 + 3$$

$$= 13 - 6 = 7$$

$\therefore$  The required remainder is 7.

### Question 2:

$$f(x) = (2x^3 - 5x^2 + 9x - 8)$$

$$\text{Now, } x - 3 = 0 \Rightarrow x = 3$$

By the remainder theorem, we know that when  $f(x)$  is divided by  $(x - 3)$  the remainder is  $f(3)$ .

$$\text{Now, } f(3) = 2 \times 3^3 - 5 \times 3^2 + 9 \times 3 - 8$$

$$= 54 - 45 + 27 - 8$$

$$= 81 - 53 = 28$$

$\therefore$  The required remainder is 28.

### Question 3:

$$f(x) = (3x^4 - 6x^2 - 8x + 2)$$

$$\text{Now, } x - 2 = 0 \Rightarrow x = 2$$

By the remainder theorem, we know that when  $f(x)$  is divided by  $(x - 2)$  the remainder is  $f(2)$ .

$$\text{Now, } f(2) = 3 \times 2^4 - 6 \times 2^2 - 8 \times 2 + 2$$

$$= 48 - 24 - 16 + 2$$

$$= 50 - 40 = 10$$

$\therefore$  The required remainder is 10.

### Question 4:

$$f(x) = x^3 - 7x^2 + 6x + 4$$

$$\text{Now, } x - 6 = 0 \Rightarrow x = 6$$

By the remainder theorem, we know that when  $f(x)$  is divided by  $(x - 6)$  the remainder is  $f(6)$

$$\text{Now, } f(6) = 6^3 - 7 \times 6^2 + 6 \times 6 + 4$$

$$= 216 - 252 + 36 + 4$$

$$= 256 - 252 = 4$$

∴ The required remainder is 4.

#### Question 5:

$$f(x) = (x^3 - 6x^2 + 13x + 60)$$

$$\text{Now, } x + 2 = 0 \Rightarrow x = -2$$

By the remainder theorem, we know that when  $f(x)$  is divided by  $(x + 2)$  the remainder is  $f(-2)$ .

$$\text{Now, } f(-2) = (-2)^3 - 6(-2)^2 + 13(-2) + 60$$

$$= -8 - 24 - 26 + 60$$

$$= -58 + 60 = 2$$

∴ The required remainder is 2.

#### Question 6:

$$f(x) = (2x^4 + 6x^3 + 2x^2 + x - 8)$$

$$\text{Now, } x + 3 = 0 \Rightarrow x = -3$$

By the remainder theorem, we know that when  $f(x)$  is divided by  $(x + 3)$  the remainder is  $f(-3)$ .

$$f(-3) = 2(-3)^4 + 6(-3)^3 + 2(-3)^2 - 3 - 8$$

$$= 162 - 162 + 18 - 3 - 8$$

$$= 18 - 11 = 7$$

∴ The required remainder is 7.

#### Question 7:

$$f(x) = (4x^3 - 12x^2 + 11x - 5)$$

$$\text{Now, } 2x - 1 = 0 \Rightarrow x = \frac{1}{2}$$

By the remainder theorem, we know that when  $f(x)$  is divided by  $(2x - 1)$  the remainder is  $f\left(\frac{1}{2}\right)$

$$\text{Now, } f\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^3 - 12\left(\frac{1}{2}\right)^2 + 11\left(\frac{1}{2}\right) - 5$$

$$= 4 \times \frac{1}{8} - 12 \times \frac{1}{4} + \frac{11}{2} - 5$$

$$= \frac{1}{2} - 3 + \frac{11}{2} - 5$$

$$= \frac{1 - 6 + 11 - 10}{2}$$

$$= \frac{-4}{2}$$

$$= -2$$

∴ The required remainder is -2.

#### Question 8:

$$f(x) = (81x^4 + 54x^3 - 9x^2 - 3x + 2)$$

$$\text{Now, } 3x + 2 = 0 \Rightarrow x = -\frac{2}{3}$$

By the remainder theorem, we know that when  $f(x)$  is divided by  $(3x + 2)$  the remainder is

$$f\left(-\frac{2}{3}\right)$$

$$\text{Now, } f\left(-\frac{2}{3}\right) = 81\left(-\frac{2}{3}\right)^4 + 54\left(-\frac{2}{3}\right)^3 - 9\left(-\frac{2}{3}\right)^2 - 3\left(-\frac{2}{3}\right) + 2$$

$$= 81 \times \frac{16}{81} + 54\left(-\frac{8}{27}\right) - 9\left(\frac{4}{9}\right) + 2 + 2$$

$$= 16 - 16 - 4 + 4 = 0$$

∴ The required remainder is 0.

**Question 9:**

$$f(x) = (x^3 - ax^2 + 2x - a)$$

$$\text{Now, } x - a = 0 \Rightarrow x = a$$

By the remainder theorem, we know that when  $f(x)$  is divided by  $(x - a)$  the remainder is  $f(a)$

$$\text{Now, } f(a) = a^3 - a^2 + 2a - a$$

$$= a^3 - a^2 + 2a - a$$

$$= a$$

∴ The required remainder is  $a$ .

**Question 10:**

$$\text{Let } f(x) = ax^3 + 3x^2 - 3$$

$$\text{and } g(x) = 2x^3 - 5x + a$$

$$\therefore f(4) = a \times 4^3 + 3 \times 4^2 - 3$$

$$= 64a + 48 - 3$$

$$= 64a + 45$$

$$g(4) = 2 \times 4^3 - 5 \times 4 + a$$

$$= 128 - 20 + a$$

$$= 108 + a$$

It is given that:

$$f(4) = g(4)$$

$$\Rightarrow 64a + 45 = 108 + a$$

$$\Rightarrow 64a - a = 108 - 45$$

$$\Rightarrow 63a = 63$$

$$\Rightarrow a = \frac{63}{63} = 1$$

∴ The value of  $a$  is 1.

**Question 11:**

$$\text{Let } f(x) = (x^4 - 2x^3 + 3x^2 - ax + b)$$

∴ From the given information,

$$f(1) = 1^4 - 2(1)^3 + 3(1)^2 - a(1) + b = 5$$

$$\Rightarrow 1 - 2 + 3 - a + b = 5$$

$$\Rightarrow 2 - a + b = 5 \dots (i)$$

And,

$$f(-1) = (-1)^4 - 2(-1)^3 + 3(-1)^2 - a(-1) + b = 19$$

$$\Rightarrow 1 + 2 + 3 + a + b = 19$$

$$\Rightarrow 6 + a + b = 19 \dots (ii)$$

Adding (i) and (ii), we get

$$\Rightarrow 8 + 2b = 24$$

$$\Rightarrow 2b = 24 - 8 = 16$$

$$\Rightarrow b = \frac{16}{2}$$

Substituting the value of  $b = 8$  in (i), we get

$$2 - a + 8 = 5$$

$$\Rightarrow -a + 10 = 5$$

$$\Rightarrow -a = -10 + 5$$

$$\Rightarrow -a = -5$$

$$\Rightarrow a = 5$$

∴  $a = 5$  and  $b = 8$

$$f(x) = x^4 - 2x^3 + 3x^2 - ax + b$$

$$= x^4 - 2x^3 + 3x^2 - 5x + 8$$



$$\begin{aligned}
 \therefore f(2) &= (2)^4 - 2(2)^3 + 3(2)^2 - 5(2) + 8 \\
 &= 16 - 16 + 12 - 10 + 8 \\
 &= 20 - 10 = 10 \\
 \therefore \text{The required remainder is } 10.
 \end{aligned}$$

## Exercise 2D

### Factor Theorem

Let  $p(x)$  be a polynomial of degree greater than or equal to one and 'a' be a real number such that  $p(a) = 0$ , then  $(x - a)$  is a factor of  $p(x)$ .  
i.e.  $(x - a)$  is a factor of  $p(x)$ , if  $p(a) = 0$

**Proof:**

$p(x)$  is a polynomial of degree greater than or equal to one and 'a' is a real number such that  $p(a) = 0$ .  
To prove :  $(x - a)$  is a factor of  $p(x)$   
Divide  $p(x)$  by  $(x - a)$ , and let  $q(x)$  be the quotient.  
By Remainder theorem,  $p(x)$  when divided by  $(x - a)$  gives remainder  $p(a)$ .  
 $\therefore p(x) = (x - a) q(x) + p(a)$   
 $\Rightarrow p(x) = (x - a) q(x) [\because p(a) = 0]$   
 $\Rightarrow (x - a)$  is a factor of  $p(x)$

Find if  $(x + 1)$  and  $(2x - 4)$  are factors of  $2x^3 - 9x^2 + x + 12 = p(x)$

**Using Factor theorem:**

$$(i) p(-1) = 2(-1)^3 - 9(-1)^2 + 1(-1) + 12 = -2 - 9 - 1 + 12 = 0$$

$$(ii) p\left(\frac{4}{2}\right) = 2(2)^3 - 9(2)^2 + 1(2) + 12 = 16 - 36 + 2 + 12 = -6$$

Since (i) = 0,  $(x + 1)$  is a factor and (ii)  $\neq 0$ ,  $(2x - 4)$  is not a factor of  $2x^3 - 9x^2 + x + 12$ .

#### Question 1:

$$f(x) = (x^3 - 8)$$

By the Factor Theorem,  $(x - 2)$  will be a factor of  $f(x)$  if  $f(2) = 0$ .

$$\text{Here, } f(2) = (2)^3 - 8$$

$$= 8 - 8 = 0$$

$\therefore (x - 2)$  is a factor of  $(x^3 - 8)$ .

#### Question 2:

$$f(x) = (2x^3 + 7x^2 - 24x - 45)$$

By the Factor Theorem,  $(x - 3)$  will be a factor of  $f(x)$  if  $f(3) = 0$ .

$$\text{Here, } f(3) = 2 \times 3^3 + 7 \times 3^2 - 24 \times 3 - 45$$

$$= 54 + 63 - 72 - 45$$

$$= 117 - 117 = 0$$

$\therefore (x - 3)$  is a factor of  $(2x^3 + 7x^2 - 24x - 45)$ .

#### Question 3:

$$f(x) = (2x^4 + 9x^3 + 6x^2 - 11x - 6)$$

By the Factor Theorem,  $(x - 1)$  will be a factor of  $f(x)$  if  $f(1) = 0$ .

$$\text{Here, } f(1) = 2 \times 1^4 + 9 \times 1^3 + 6 \times 1^2 - 11 \times 1 - 6$$

$$= 2 + 9 + 6 - 11 - 6$$

$$= 17 - 17 = 0$$

$\therefore (x - 1)$  is factor of  $(2x^4 + 9x^3 + 6x^2 - 11x - 6)$ .

#### Question 4:

$$f(x) = (x^4 - x^2 - 12)$$

By the Factor Theorem,  $(x + 2)$  will be a factor of  $f(x)$  if  $f(-2) = 0$ .

$$\text{Here, } f(-2) = (-2)^4 - (-2)^2 - 12$$

$$= 16 - 4 - 12$$

$$= 16 - 16 = 0$$

$\therefore (x + 2)$  is a factor of  $(x^4 - x^2 - 12)$ .

**Question 5:**

$$f(x) = 2x^3 + 9x^2 - 11x - 30$$

By the Factor Theorem,  $(x + 5)$  will be a factor of  $f(x)$  if  $f(-5) = 0$ .

$$\text{Here, } f(-5) = 2(-5)^3 + 9(-5)^2 - 11(-5) - 30$$

$$= -250 + 225 + 55 - 30$$

$$= -280 + 280 = 0$$

$\therefore (x + 5)$  is a factor of  $(2x^3 + 9x^2 - 11x - 30)$ .

**Question 6:**

$$f(x) = (2x^4 + x^3 - 8x^2 - x + 6)$$

By the Factor Theorem,  $(x - a)$  will be a factor of  $f(x)$  if  $f(a) = 0$ .

$$\text{Here, } 2x - 3 = 0 \Rightarrow x = \frac{3}{2}$$

$$\therefore f\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}\right)^4 + \left(\frac{3}{2}\right)^3 - 8\left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right) + 6$$

$$= 2 \times \frac{81}{16} + \frac{27}{8} - 8 \times \frac{9}{4} - \frac{3}{2} + 6$$

$$= \frac{81}{8} + \frac{27}{8} - 18 - \frac{3}{2} + 6$$

$$= \frac{81 + 27 - 144 - 12 + 48}{8}$$

$$= \frac{156 - 156}{8} = 0$$

$\therefore (2x - 3)$  is a factor of  $(2x^4 + x^3 - 8x^2 - x + 6)$ .

**Question 7:**

$$f(x) = (7x^2 - 4\sqrt{2}x - 6 = 0)$$

By the Factor Theorem,  $(x - a)$  will be a factor of  $f(x)$  if  $f(a) = 0$ .

$$\text{Here, } f(\sqrt{2}) = 7(\sqrt{2})^2 - 4\sqrt{2} \times \sqrt{2} - 6$$

$$= 14 - 8 - 6$$

$$= 14 - 14 = 0$$

$\therefore (x - \sqrt{2})$  is a factor of  $(7 - 4\sqrt{2}x - 6 = 0)$ .

**Question 8:**

$$f(x) = (4\sqrt{2}x^2 + 5x + \sqrt{2} = 0)$$

By the Factor Theorem,  $(x - a)$  will be a factor of  $f(x)$  if  $f(a) = 0$ .

$$f(-\sqrt{2}) = 2\sqrt{2}(-\sqrt{2})^2 + 5(-\sqrt{2}) + \sqrt{2}$$

$$= 2\sqrt{2} \times 2 - 5\sqrt{2} + \sqrt{2}$$

$$= 4\sqrt{2} - 5\sqrt{2} + \sqrt{2}$$

$$\text{Here, } = 5\sqrt{2} - 5\sqrt{2} = 0.$$

$\therefore (x + \sqrt{2})$  is a factor of  $(4\sqrt{2}x^2 + 5x + \sqrt{2} = 0)$ .

**Question 9:**

$$f(x) = (2x^3 + 9x^2 + x + k)$$

$$x - 1 = 0 \Rightarrow x = 1$$

$$\therefore f(1) = 2 \times 1^3 + 9 \times 1^2 + 1 + k$$

$$= 2 + 9 + 1 + k$$

$$= 12 + k$$

Given that  $(x - 1)$  is a factor of  $f(x)$ .

By the Factor Theorem,  $(x - a)$  will be a factor of  $f(x)$  if  $f(a) = 0$  and therefore  $f(1) = 0$ .

$$\Rightarrow f(1) = 12 + k = 0$$

$$\Rightarrow k = -12.$$

**Question 10:**

$$f(x) = (2x^3 - 3x^2 - 18x + a)$$

$$x - 4 = 0 \Rightarrow x = 4$$

$$\therefore f(4) = 2(4)^3 - 3(4)^2 - 18 \times 4 + a$$

$$= 128 - 48 - 72 + a$$

$$= 128 - 120 + a$$

$$= 8 + a$$

Given that  $(x - 4)$  is a factor of  $f(x)$ .

By the Factor Theorem,  $(x - a)$  will be a factor of  $f(x)$  if  $f(a) = 0$  and therefore  $f(4) = 0$ .

$$\Rightarrow f(4) = 8 + a = 0$$

$$\Rightarrow a = -8$$

**Question 11:**

$$f(x) = x^4 - x^3 - 11x^2 - x + a$$

$$x + 3 = 0 \Rightarrow x = -3$$

$$\therefore f(-3) = (-3)^4 - (-3)^3 - 11(-3)^2 - (-3) + a$$

$$= 81 + 27 - 11 \times 9 + 3 + a$$

$$= 81 + 27 - 99 + 3 + a$$

$$= 111 - 99 + a$$

$$= 12 + a$$

Given that  $f(x)$  is divisible by  $(x + 3)$ , that is  $(x+3)$  is a factor of  $f(x)$ .

By the Factor Theorem,  $(x - a)$  will be a factor of  $f(x)$  if  $f(a) = 0$  and therefore  $f(-3) = 0$ .

$$\Rightarrow f(-3) = 12 + a = 0$$

$$\Rightarrow a = -12.$$

**Question 12:**

$$f(x) = (2x^3 + ax^2 + 11x + a + 3)$$

$$2x - 1 = 0 \Rightarrow x = \frac{1}{2}$$

Given that  $f(x)$  is exactly divisible by  $(2x - 1)$ , that is  $(2x - 1)$  is a factor of  $f(x)$ .

By the Factor Theorem,  $(x - a)$  will be a factor of  $f(x)$  if  $f(a) = 0$

and therefore  $f\left(\frac{1}{2}\right) \neq 0$ .

Therefore, we have

$$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + a\left(\frac{1}{2}\right)^2 + 11 \times \frac{1}{2} + a + 3 = 0$$

$$\Rightarrow 2 \times \frac{1}{8} + a \times \frac{1}{4} + \frac{11}{2} + a + 3 = 0$$

$$\Rightarrow \frac{1}{4} + \frac{1}{4}a + \frac{11}{2} + a + 3 = 0$$

$$\Rightarrow \frac{1 + a + 22 + 4a + 12}{4} = 0$$

$$\Rightarrow \frac{5a + 35}{4} = 0$$

$$\Rightarrow 5a + 35 = 0$$

$$\Rightarrow 5a = -35$$

$$\Rightarrow a = \frac{-35}{5} = -7$$

$\therefore$  The value of  $a = -7$ .

**Question 13:**

Let  $f(x) = (x^3 - 10x^2 + ax + b)$ , then by factor theorem

$(x - 1)$  and  $(x - 2)$  will be factors of  $f(x)$  if  $f(1) = 0$  and  $f(2) = 0$ .

$$f(1) = 1^3 - 10 \times 1^2 + a \times 1 + b = 0$$

$$\Rightarrow 1 - 10 + a + b = 0$$

$$\Rightarrow a + b = 9 \dots (i)$$

$$\text{And } f(2) = 2^3 - 10 \square 2^2 + a \square 2 + b = 0$$

$$\Rightarrow 8 - 40 + 2a + b = 0$$

$$\Rightarrow 2a + b = 32 \dots (ii)$$

Subtracting (i) from (ii), we get

$$a = 23$$

Substituting the value of  $a = 23$  in (i), we get

$$\Rightarrow 23 + b = 9$$

$$\Rightarrow b = 9 - 23$$

$$\Rightarrow b = -14$$

$$\therefore a = 23 \text{ and } b = -14.$$

#### Question 14:

$$\text{Let } f(x) = (x^4 + ax^3 - 7x^2 - 8x + b)$$

$$\text{Now, } x + 2 = 0 \Rightarrow x = -2 \text{ and } x + 3 = 0 \Rightarrow x = -3$$

By factor theorem,  $(x + 2)$  and  $(x + 3)$  will be factors of  $f(x)$  if  $f(-2) = 0$  and  $f(-3) = 0$

$$\therefore f(-2) = (-2)^4 + a(-2)^3 - 7(-2)^2 - 8(-2) + b = 0$$

$$\Rightarrow 16 - 8a - 28 + 16 + b = 0$$

$$\Rightarrow -8a + b = -4$$

$$\Rightarrow 8a - b = 4 \dots (i)$$

$$\text{And, } f(-3) = (-3)^4 + a(-3)^3 - 7(-3)^2 - 8(-3) + b = 0$$

$$\Rightarrow 81 - 27a - 63 + 24 + b = 0$$

$$\Rightarrow -27a + b = -42$$

$$\Rightarrow 27a - b = 42 \dots (ii)$$

Subtracting (i) from (ii), we get,

$$19a = 38$$

$$\text{So, } a = 2$$

Substituting the value of  $a = 2$  in (i), we get

$$8(2) - b = 4$$

$$\Rightarrow 16 - b = 4$$

$$\Rightarrow -b = -16 + 4$$

$$\Rightarrow -b = -12$$

$$\Rightarrow b = 12$$

$$\therefore a = 2 \text{ and } b = 12.$$

#### Question 15:

$$\text{Let } f(x) = x^3 - 3x^2 - 13x + 15$$

$$\text{Now, } x^2 + 2x - 3 = x^2 + 3x - x - 3$$

$$= x(x + 3) - 1(x + 3)$$

$$= (x + 3)(x - 1)$$

Thus,  $f(x)$  will be exactly divisible by  $x^2 + 2x - 3 = (x + 3)(x - 1)$  if  $(x + 3)$  and  $(x - 1)$  are both factors of  $f(x)$ , so by factor theorem, we should have  $f(-3) = 0$  and  $f(1) = 0$ .

$$\text{Now, } f(-3) = (-3)^3 - 3(-3)^2 - 13(-3) + 15$$

$$= -27 - 3 \times 9 + 39 + 15$$

$$= -27 - 27 + 39 + 15$$

$$= -54 + 54 = 0$$

$$\text{And, } f(1) = 1^3 - 3 \times 1^2 - 13 \times 1 + 15$$

$$= 1 - 3 - 13 + 15$$

$$= 16 - 16 = 0$$

$$\therefore f(-3) = 0 \text{ and } f(1) = 0$$

So,  $x^2 + 2x - 3$  divides  $f(x)$  exactly.

#### Question 16:

$$\text{Let } f(x) = (x^3 + ax^2 + bx + 6)$$

Now, by remainder theorem,  $f(x)$  when divided by  $(x - 3)$  will leave a remainder as  $f(3)$ .

$$\text{So, } f(3) = 3^3 + a \times 3^2 + b \times 3 + 6 = 3$$

$$\Rightarrow 27 + 9a + 3b + 6 = 3$$

$$\Rightarrow 9a + 3b + 33 = 3$$

$$\Rightarrow 9a + 3b = 3 - 33$$

$$\Rightarrow 9a + 3b = -30$$

$$\Rightarrow 3a + b = -10 \dots (i)$$

Given that  $(x - 2)$  is a factor of  $f(x)$ .

By the Factor Theorem,  $(x - a)$  will be a factor of  $f(x)$  if  $f(a) = 0$  and therefore  $f(2) = 0$ .

$$f(2) = 2^3 + a \times 2^2 + b \times 2 + 6 = 0$$

$$\Rightarrow 8 + 4a + 2b + 6 = 0$$

$$\Rightarrow 4a + 2b = -14$$

$$\Rightarrow 2a + b = -7 \dots (ii)$$

Subtracting (ii) from (i), we get,

$$\Rightarrow a = -3$$

Substituting the value of  $a = -3$  in (i), we get,

$$\Rightarrow 3(-3) + b = -10$$

$$\Rightarrow -9 + b = -10$$

$$\Rightarrow b = -10 + 9$$

$$\Rightarrow b = -1$$

$$\therefore a = -3 \text{ and } b = -1.$$

## Exercise 2E

1.  $(a + b)^2 = a^2 + 2ab + b^2 = (-a - b)^2$
2.  $(a - b)^2 = a^2 - 2ab + b^2$
3.  $(a - b)(a + b) = a^2 - b^2$
4.  $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
5.  $(a + b - c)^2 = a^2 + b^2 + c^2 + 2ab - 2bc - 2ca$
6.  $(a - b + c)^2 = a^2 + b^2 + c^2 - 2ab - 2bc + 2ca$
7.  $(-a + b + c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$
8.  $(a - b - c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$
9.  $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
10.  $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
11.  $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$   
 $= (a + b)(a^2 - ab + b^2)$
12.  $a^3 - b^3 = (a - b)^3 + 3ab(a - b)$   
 $= (a - b)(a^2 + ab + b^2)$
13.  $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$   
 if  $a + b + c = 0$  then  $a^3 + b^3 + c^3 = 3abc$

### Question 1:

$$9x^2 + 12xy = 3x(3x + 4y)$$

### Question 2:

$$18x^2y - 24xyz = 6xy(3x - 4z)$$

### Question 3:

$$27a^3b^3 - 45a^4b^2 = 9a^3b^2(3b - 5a)$$

### Question 4:

$$2a(x + y) - 3b(x + y) = (x + y)(2a - 3b)$$

**Question 5:**

$$\begin{aligned} & 2x(p^2 + q^2) + 4y(p^2 + q^2) \\ &= (2x + 4y)(p^2 + q^2) \\ &= 2(x + 2y)(p^2 + q^2) \end{aligned}$$

**Question 6:**

$$\begin{aligned} & x(a - 5) + y(5 - a) \\ &= x(a - 5) + y(-1)(a - 5) \\ &= (x - y)(a - 5) \end{aligned}$$

**Question 7:**

$$\begin{aligned} & 4(a + b) - 6(a + b)^2 \\ &= (a + b)[4 - 6(a + b)] \\ &= 2(a + b)(2 - 3a - 3b) \\ &= 2(a + b)(2 - 3a - 3b) \end{aligned}$$

**Question 8:**

$$\begin{aligned} & 8(3a - 2b)^2 - 10(3a - 2b) \\ &= (3a - 2b)[8(3a - 2b) - 10] \\ &= (3a - 2b)2[4(3a - 2b) - 5] \\ &= 2(3a - 2b)(12a - 8b - 5) \end{aligned}$$

**Question 9:**

$$\begin{aligned} & x(x + y)^3 - 3x^2y(x + y) \\ &= x(x + y)[(x + y)^2 - 3xy] \\ &= x(x + y)(x^2 + y^2 + 2xy - 3xy) \\ &= x(x + y)(x^2 + y^2 - xy) \end{aligned}$$

**Question 10:**

$$\begin{aligned} & x^3 + 2x^2 + 5x + 10 \\ &= x^2(x + 2) + 5(x + 2) \\ &= (x^2 + 5)(x + 2) \end{aligned}$$

**Question 11:**

$$\begin{aligned} & x^2 + xy - 2xz - 2yz \\ &= x(x + y) - 2z(x + y) \\ &= (x + y)(x - 2z) \end{aligned}$$

**Question 12:**

$$\begin{aligned} & a^3b - a^2b + 5ab - 5b \\ &= a^2b(a - 1) + 5b(a - 1) \\ &= (a - 1)(a^2b + 5b) \\ &= (a - 1)b(a^2 + 5) \\ &= b(a - 1)(a^2 + 5) \end{aligned}$$

**Question 13:**

$$\begin{aligned} & 8 - 4a - 2a^3 + a^4 \\ &= 4(2 - a) - a^3(2 - a) \\ &= (2 - a)(4 - a^3) \end{aligned}$$

**Question 14:**

$$\begin{aligned}
 & x^3 - 2x^2y + 3xy^2 - 6y^3 \\
 &= x^2(x - 2y) + 3y^2(x - 2y) \\
 &= (x - 2y)(x^2 + 3y^2)
 \end{aligned}$$

**Question 15:**

$$\begin{aligned}
 & px + pq - 5q - 5x \\
 &= p(x + q) - 5(q + x) \\
 &= (x + q)(p - 5)
 \end{aligned}$$

**Question 16:**

$$\begin{aligned}
 & x^2 - xy + y - x \\
 &= x(x - y) - 1(x - y) \\
 &= (x - y)(x - 1)
 \end{aligned}$$

**Question 17:**

$$\begin{aligned}
 & (3a - 1)^2 - 6a + 2 \\
 &= (3a - 1)^2 - 2(3a - 1) \\
 &= (3a - 1)[(3a - 1) - 2] \\
 &= (3a - 1)(3a - 3) \\
 &= 3(3a - 1)(a - 1)
 \end{aligned}$$

**Question 18:**

$$\begin{aligned}
 & (2x - 3)^2 - 8x + 12 \\
 &= (2x - 3)^2 - 4(2x - 3) \\
 &= (2x - 3)(2x - 3 - 4) \\
 &= (2x - 3)(2x - 7)
 \end{aligned}$$

**Question 19:**

$$\begin{aligned}
 & a^3 + a - 3a^2 - 3 \\
 &= a(a^2 + 1) - 3(a^2 + 1) \\
 &= (a - 3)(a^2 + 1)
 \end{aligned}$$

**Question 20:**

$$\begin{aligned}
 & 3ax - 6ay - 8by + 4bx \\
 &= 3a(x - 2y) + 4b(x - 2y) \\
 &= (x - 2y)(3a + 4b)
 \end{aligned}$$

**Question 21:**

$$\begin{aligned}
 & abx^2 + a^2x + b^2x + ab \\
 &= ax(bx + a) + b(bx + a) \\
 &= (bx + a)(ax + b)
 \end{aligned}$$

**Question 22:**

$$\begin{aligned}
 & x^3 - x^2 + ax + x - a - 1 \\
 &= x^3 - x^2 + ax - a + x - 1 \\
 &= x^2(x - 1) + a(x - 1) + 1(x - 1) \\
 &= (x - 1)(x^2 + a + 1)
 \end{aligned}$$

**Question 23:**

$$\begin{aligned}
 & 2x + 4y - 8xy - 1 \\
 &= 2x - 1 - 8xy + 4y
 \end{aligned}$$

$$= (2x - 1) - 4y (2x - 1)$$

$$= (2x - 1) (1 - 4y)$$

**Question 24:**

$$ab (x^2 + y^2) - xy (a^2 + b^2)$$

$$= abx^2 + aby^2 - a^2xy - b^2xy$$

$$= abx^2 - a^2xy + aby^2 - b^2xy$$

$$= ax (bx - ay) + by (ay - bx)$$

$$= (bx - ay) (ax - by)$$

**Question 25:**

$$a^2 + ab (b + 1) + b^3$$

$$= a^2 + ab^2 + ab + b^3$$

$$= a^2 + ab + ab^2 + b^3$$

$$= a (a + b) + b^2 (a + b)$$

$$= (a + b) (a + b^2)$$

**Question 26:**

$$a^3 + ab (1 - 2a) - 2b^2$$

$$= a^3 + ab - 2a^2b - 2b^2$$

$$= a (a^2 + b) - 2b (a^2 + b)$$

$$= (a^2 + b) (a - 2b)$$

**Question 27:**

$$2a^2 + bc - 2ab - ac$$

$$= 2a^2 - 2ab - ac + bc$$

$$= 2a (a - b) - c (a - b)$$

$$= (a - b) (2a - c)$$

**Question 28:**

$$(ax + by)^2 + (bx - ay)^2$$

$$= a^2x^2 + b^2y^2 + 2abxy + b^2x^2 + a^2y^2 - 2abxy$$

$$= a^2x^2 + b^2y^2 + b^2x^2 + a^2y^2$$

$$= a^2x^2 + b^2x^2 + b^2y^2 + a^2y^2$$

$$= x^2 (a^2 + b^2) + y^2 (a^2 + b^2)$$

$$= (a^2 + b^2) (x^2 + y^2)$$

**Question 29:**

$$a (a + b - c) - bc$$

$$= a^2 + ab - ac - bc$$

$$= a(a + b) - c (a + b)$$

$$= (a - c) (a + b)$$

**Question 30:**

$$a(a - 2b - c) + 2bc$$

$$= a^2 - 2ab - ac + 2bc$$

$$= a (a - 2b) - c (a - 2b)$$

$$= (a - 2b) (a - c)$$

**Question 31:**

$$a^2x^2 + (ax^2 + 1)x + a$$

$$= a^2x^2 + ax^3 + x + a$$



$$= ax^2(a+x) + 1(x+a)$$

$$= (ax^2 + 1)(a+x)$$

**Question 32:**

$$ab(x^2 + 1) + x(a^2 + b^2)$$

$$= abx^2 + ab + a^2x + b^2x$$

$$= abx^2 + a^2x + ab + b^2x$$

$$= ax(bx + a) + b(bx + a)$$

$$= (bx + a)(ax + b)$$

**Question 33:**

$$x^2 - (a+b)x + ab$$

$$= x^2 - ax - bx + ab$$

$$= x(x-a) - b(x-a)$$

$$= (x-a)(x-b)$$

**Question 34:**

$$x^2 + \frac{1}{x^2} - 2 - 3x + \frac{3}{x}$$

$$= \left(x - \frac{1}{x}\right)^2 - 3\left(x - \frac{1}{x}\right)$$

$$= \left(x - \frac{1}{x}\right)\left(x - \frac{1}{x} - 3\right)$$

## Exercise 2F

**Question 1:**

$$25x^2 - 64y^2$$

$$= (5x)^2 - (8y)^2$$

$$= (5x + 8y)(5x - 8y)$$

$$\left[\because a^2 - b^2 = (a + b)(a - b)\right]$$

**Question 2:**

$$100 - 9x^2$$

$$= (10)^2 - (3x)^2$$

$$= (10 + 3x)(10 - 3x)$$

$$\left[\because a^2 - b^2 = (a + b)(a - b)\right]$$

**Question 3:**

$$5x^2 - 7y^2$$

$$= (\sqrt{5}x)^2 - (\sqrt{7}y)^2$$

$$= (\sqrt{5}x + \sqrt{7}y)(\sqrt{5}x - \sqrt{7}y) \quad \left[\because a^2 - b^2 = (a + b)(a - b)\right]$$

**Question 4:**

$$(3x + 5y)^2 - 4z^2$$

$$= (3x + 5y)^2 - (2z)^2$$

$$= (3x + 5y + 2z)(3x + 5y - 2z)$$

$$\left[\because a^2 - b^2 = (a + b)(a - b)\right]$$

**Question 5:**

$$150 - 6x^2$$

$$\begin{aligned}
 &= 6(25 - x^2) \\
 &= 6(5^2 - x^2) \\
 &= 6(5 + x)(5 - x) \\
 &\left[ \because a^2 - b^2 = (a + b)(a - b) \right]
 \end{aligned}$$

**Question 6:**

$$\begin{aligned}
 &20x^2 - 45 \\
 &= 5(4x^2 - 9) \\
 &= 5[(2x)^2 - (3)^2] \\
 &= 5(2x + 3)(2x - 3) \\
 &\left[ \because a^2 - b^2 = (a + b)(a - b) \right]
 \end{aligned}$$

**Question 7:**

$$\begin{aligned}
 &3x^3 - 48x \\
 &= 3x(x^2 - 16) \\
 &= 3x[(x)^2 - (4)^2] \\
 &= 3x(x + 4)(x - 4) \\
 &\left[ \because a^2 - b^2 = (a + b)(a - b) \right]
 \end{aligned}$$

**Question 8:**

$$\begin{aligned}
 &2 - 50x^2 \\
 &= 2(1 - 25x^2) \\
 &= 2[(1)^2 - (5x)^2] \\
 &= 2(1 + 5x)(1 - 5x) \\
 &\left[ \because a^2 - b^2 = (a + b)(a - b) \right]
 \end{aligned}$$

**Question 9:**

$$\begin{aligned}
 &27a^2 - 48b^2 \\
 &= 3(9a^2 - 16b^2) \\
 &= 3[(3a)^2 - (4b)^2] \\
 &= 3(3a + 4b)(3a - 4b) \\
 &\left[ \because a^2 - b^2 = (a + b)(a - b) \right]
 \end{aligned}$$

**Question 10:**

$$\begin{aligned}
 &x - 64x^3 \\
 &= x(1 - 64x^2) \\
 &= x[(1)^2 - (8x)^2] \\
 &= x(1 + 8x)(1 - 8x) \\
 &\left[ \because a^2 - b^2 = (a + b)(a - b) \right]
 \end{aligned}$$

**Question 11:**

$$\begin{aligned}
 &8ab^2 - 18a^3 \\
 &= 2a(4b^2 - 9a^2) \\
 &= 2a[(2b)^2 - (3a)^2] \\
 &= 2a(2b + 3a)(2b - 3a) \\
 &\left[ \because a^2 - b^2 = (a + b)(a - b) \right]
 \end{aligned}$$

**Question 12:**

$$\begin{aligned}
& 3a^3b - 243ab^3 \\
&= 3ab(a^2 - 81b^2) \\
&= 3ab[(a)^2 - (9b)^2] \\
&= 3ab(a + 9b)(a - 9b) \\
&\left[ \because a^2 - b^2 = (a + b)(a - b) \right]
\end{aligned}$$

**Question 13:**

$$\begin{aligned}
& (a + b)^3 - a - b \\
&= (a + b)^3 - (a + b) \\
&= (a + b)[(a + b)^2 - 1^2] \\
&= (a + b)(a + b + 1)(a + b - 1) \\
&\left[ \because a^2 - b^2 = (a + b)(a - b) \right]
\end{aligned}$$

**Question 14:**

$$\begin{aligned}
& 108a^2 - 3(b - c)^2 \\
&= 3[(36a^2 - (b - c)^2)] \\
&= 3[(6a)^2 - (b - c)^2] \\
&= 3(6a + b - c)(6a - b + c) \\
&\left[ \because a^2 - b^2 = (a + b)(a - b) \right]
\end{aligned}$$

**Question 15:**

$$\begin{aligned}
& x^3 - 5x^2 - x + 5 \\
&= x^2(x - 5) - 1(x - 5) \\
&= (x - 5)(x^2 - 1) \\
&= (x - 5)(x + 1)(x - 1) \\
&\left[ \because a^2 - b^2 = (a + b)(a - b) \right]
\end{aligned}$$

**Question 16:**

$$\begin{aligned}
& a^2 + 2ab + b^2 - 9c^2 \\
&= (a + b)^2 - (3c)^2 \\
&= (a + b + 3c)(a + b - 3c) \\
&\left[ \because a^2 - b^2 = (a + b)(a - b) \right]
\end{aligned}$$

**Question 17:**

$$\begin{aligned}
& 9 - a^2 + 2ab - b^2 \\
&= 9 - (a^2 - 2ab + b^2) \\
&= 3^2 - (a - b)^2 \\
&= (3 + a - b)(3 - a + b) \\
&\left[ \because a^2 - b^2 = (a + b)(a - b) \right]
\end{aligned}$$

**Question 18:**

$$\begin{aligned}
& a^2 - 4ac + 4c^2 - b^2 \\
&= a^2 - 4ac + 4c^2 - b^2 \\
&= a^2 - 2 \cdot a \cdot 2c + (2c)^2 - b^2 \\
&= (a - 2c)^2 - b^2 \\
&= (a - 2c + b)(a - 2c - b) \\
&\left[ \because a^2 - b^2 = (a + b)(a - b) \right]
\end{aligned}$$

**Question 19:**

$$\begin{aligned}
& 9a^2 + 3a - 8b - 64b^2 \\
&= 9a^2 - 64b^2 + 3a - 8b \\
&= (3a)^2 - (8b)^2 + (3a - 8b) \\
&= (3a + 8b)(3a - 8b) + (3a - 8b) \\
&\left[ \because a^2 - b^2 = (a + b)(a - b) \right] \\
&= (3a - 8b)(3a + 8b + 1)
\end{aligned}$$

**Question 20:**

$$\begin{aligned}
& x^2 - y^2 + 6y - 9 \\
&= x^2 - (y^2 - 6y + 9) \\
&= x^2 - (y^2 - 2 \cdot y \cdot 3 + 3^2) \\
&= x^2 - (y - 3)^2 \\
&= [x + (y - 3)][x - (y - 3)] \\
&\left[ \because a^2 - b^2 = (a + b)(a - b) \right] \\
&= (x + y - 3)(x - y + 3)
\end{aligned}$$

**Question 21:**

$$\begin{aligned}
& 4x^2 - 9y^2 - 2x - 3y \\
&= (2x)^2 - (3y)^2 - (2x + 3y) \\
&= (2x + 3y)(2x - 3y) - (2x + 3y) \\
&\left[ \because a^2 - b^2 = (a + b)(a - b) \right] \\
&= (2x + 3y)(2x - 3y - 1)
\end{aligned}$$

**Question 22:**

$$\begin{aligned}
& x^4 - 1 \\
&= (x^2)^2 - 1^2 \\
&= (x^2 + 1)(x^2 - 1) \left[ \because a^2 - b^2 = (a + b)(a - b) \right] \\
&= (x^2 + 1)(x + 1)(x - 1) \\
&\left[ \because a^2 - b^2 = (a + b)(a - b) \right]
\end{aligned}$$

**Question 23:**

$$\begin{aligned}
& a - b - a^2 + b^2 \\
&= (a - b) - (a^2 - b^2) \\
&= (a - b) - (a - b)(a + b) \\
&\left[ \because a^2 - b^2 = (a + b)(a - b) \right] \\
&= (a - b)(1 - a - b)
\end{aligned}$$

**Question 24:**

$$\begin{aligned}
& x^4 - 625 \\
&= (x^2)^2 - (25)^2 \\
&= (x^2 + 25)(x^2 - 25) \\
&\left[ \because a^2 - b^2 = (a + b)(a - b) \right] \\
&= (x^2 + 25)(x^2 - 5^2) \\
&= (x^2 + 25)(x + 5)(x - 5) \\
&\left[ \because a^2 - b^2 = (a + b)(a - b) \right]
\end{aligned}$$

## Exercise 2G

### Question 1:

$$\begin{aligned}x^2 + 11x + 30 \\&= x^2 + 6x + 5x + 30 \\&= x(x + 6) + 5(x + 6) \\&= (x + 6)(x + 5).\end{aligned}$$

### Question 2:

$$\begin{aligned}x^2 + 18x + 32 \\&= x^2 + 16x + 2x + 32 \\&= x(x + 16) + 2(x + 16) \\&= (x + 16)(x + 2).\end{aligned}$$

### Question 3:

$$\begin{aligned}x^2 + 7x - 18 \\&= x^2 + 9x - 2x - 18 \\&= x(x + 9) - 2(x + 9) \\&= (x + 9)(x - 2).\end{aligned}$$

### Question 4:

$$\begin{aligned}x^2 + 5x - 6 \\&= x^2 + 6x - x - 6 \\&= x(x + 6) - 1(x + 6) \\&= (x + 6)(x - 1).\end{aligned}$$

### Question 5:

$$\begin{aligned}y^2 - 4y + 3 \\&= y^2 - 3y - y + 3 \\&= y(y - 3) - 1(y - 3) \\&= (y - 3)(y - 1).\end{aligned}$$

### Question 6:

$$\begin{aligned}x^2 - 21x + 108 \\&= x^2 - 12x - 9x + 108 \\&= x(x - 12) - 9(x - 12) \\&= (x - 12)(x - 9).\end{aligned}$$

### Question 7:

$$\begin{aligned}x^2 - 11x - 80 \\&= x^2 - 16x + 5x - 80 \\&= x(x - 16) + 5(x - 16) \\&= (x - 16)(x + 5).\end{aligned}$$

### Question 8:

$$\begin{aligned}x^2 - x - 156 \\&= x^2 - 13x + 12x - 156 \\&= x(x - 13) + 12(x - 13) \\&= (x - 13)(x + 12).\end{aligned}$$

### Question 9:

$$z^2 - 32z - 105$$

$$\begin{aligned}
 &= z^2 - 35z + 3z - 105 \\
 &= z(z - 35) + 3(z - 35) \\
 &= (z - 35)(z + 3)
 \end{aligned}$$

**Question 10:**

$$\begin{aligned}
 &40 + 3x - x^2 \\
 &= 40 + 8x - 5x - x^2 \\
 &= 8(5 + x) - x(5 + x) \\
 &= (5 + x)(8 - x).
 \end{aligned}$$

**Question 11:**

$$\begin{aligned}
 &6 - x - x^2 \\
 &= 6 + 2x - 3x - x^2 \\
 &= 2(3 + x) - x(3 + x) \\
 &= (3 + x)(2 - x).
 \end{aligned}$$

**Question 12:**

$$\begin{aligned}
 &7x^2 + 49x + 84 \\
 &= 7(x^2 + 7x + 12) \\
 &= 7[x^2 + 4x + 3x + 12] \\
 &= 7[x(x + 4) + 3(x + 4)] \\
 &= 7(x + 4)(x + 3).
 \end{aligned}$$

**Question 13:**

$$\begin{aligned}
 &m^2 + 17mn - 84n^2 \\
 &= m^2 + 21mn - 4mn - 84n^2 \\
 &= m(m + 21n) - 4n(m + 21n) \\
 &= (m + 21n)(m - 4n).
 \end{aligned}$$

**Question 14:**

$$\begin{aligned}
 &5x^2 + 16x + 3 \\
 &= 5x^2 + 15x + x + 3 \\
 &= 5x(x + 3) + 1(x + 3) \\
 &= (5x + 1)(x + 3).
 \end{aligned}$$

**Question 15:**

$$\begin{aligned}
 &6x^2 + 17x + 12 \\
 &= 6x^2 + 9x + 8x + 12 \\
 &= 3x(2x + 3) + 4(2x + 3) \\
 &= (2x + 3)(3x + 4).
 \end{aligned}$$

**Question 16:**

$$\begin{aligned}
 &9x^2 + 18x + 8 \\
 &= 9x^2 + 12x + 6x + 8 \\
 &= 3x(3x + 4) + 2(3x + 4) \\
 &= (3x + 4)(3x + 2).
 \end{aligned}$$

**Question 17:**

$$\begin{aligned}
 &14x^2 + 9x + 1 \\
 &= 14x^2 + 7x + 2x + 1 \\
 &= 7x(2x + 1) + (2x + 1) \\
 &= (7x + 1)(2x + 1).
 \end{aligned}$$

**Question 18:**

$$\begin{aligned} & 2x^2 + 3x - 90 \\ &= 2x^2 - 12x + 15x - 90 \\ &= 2x(x - 6) + 15(x - 6) \\ &= (x - 6)(2x + 15). \end{aligned}$$

**Question 19:**

$$\begin{aligned} & 2x^2 + 11x - 21 \\ &= 2x^2 + 14x - 3x - 21 \\ &= 2x(x + 7) - 3(x + 7) \\ &= (x + 7)(2x - 3). \end{aligned}$$

**Question 20:**

$$\begin{aligned} & 3x^2 - 14x + 8 \\ &= 3x^2 - 12x - 2x + 8 \\ &= 3x(x - 4) - 2(x - 4) \\ &= (x - 4)(3x - 2). \end{aligned}$$

**Question 21:**

$$\begin{aligned} & 18x^2 + 3x - 10 \\ &= 18x^2 - 12x + 15x - 10 \\ &= 6x(3x - 2) + 5(3x - 2) \\ &= (6x + 5)(3x - 2). \end{aligned}$$

**Question 22:**

$$\begin{aligned} & 15x^2 + 2x - 8 \\ &= 15x^2 - 10x + 12x - 8 \\ &= 5x(3x - 2) + 4(3x - 2) \\ &= (3x - 2)(5x + 4). \end{aligned}$$

**Question 23:**

$$\begin{aligned} & 6x^2 + 11x - 10 \\ &= 6x^2 + 15x - 4x - 10 \\ &= 3x(2x + 5) - 2(2x + 5) \\ &= (2x + 5)(3x - 2). \end{aligned}$$

**Question 24:**

$$\begin{aligned} & 30x^2 + 7x - 15 \\ &= 30x^2 - 18x + 25x - 15 \\ &= 6x(5x - 3) + 5(5x - 3) \\ &= (5x - 3)(6x + 5). \end{aligned}$$

**Question 25:**

$$\begin{aligned} & 24x^2 - 41x + 12 \\ &= 24x^2 - 32x - 9x + 12 \\ &= 8x(3x - 4) - 3(3x - 4) \\ &= (3x - 4)(8x - 3). \end{aligned}$$

**Question 26:**

$$\begin{aligned} & 2x^2 - 7x - 15 \\ &= 2x^2 - 10x + 3x - 15 \\ &= 2x(x - 5) + 3(x - 5) \end{aligned}$$

$$= (x - 5) (2x + 3).$$

**Question 27:**

$$\begin{aligned} & 6x^2 - 5x - 21 \\ &= 6x^2 + 9x - 14x - 21 \\ &= 3x (2x + 3) - 7 (2x + 3) \\ &= (3x - 7) (2x + 3). \end{aligned}$$

**Question 28:**

$$\begin{aligned} & 10x^2 - 9x - 7 \\ &= 10x^2 + 5x - 14x - 7 \\ &= 5x (2x + 1) - 7 (2x + 1) \\ &= (2x + 1) (5x - 7). \end{aligned}$$

**Question 29:**

$$\begin{aligned} & 5x^2 - 16x - 21 \\ &= 5x^2 + 5x - 21x - 21 \\ &= 5x (x + 1) - 21 (x + 1) \\ &= (x + 1) (5x - 21). \end{aligned}$$

**Question 30:**

$$\begin{aligned} & 2x^2 - x - 21 \\ &= 2x^2 + 6x - 7x - 21 \\ &= 2x (x + 3) - 7 (x + 3) \\ &= (x + 3) (2x - 7). \end{aligned}$$

**Question 31:**

$$\begin{aligned} & 15x^2 - x - 28 \\ &= 15x^2 + 20x - 21x - 28 \\ &= 5x (3x + 4) - 7 (3x + 4) \\ &= (3x + 4) (5x - 7). \end{aligned}$$

**Question 32:**

$$\begin{aligned} & 8a^2 - 27ab + 9b^2 \\ &= 8a^2 - 24ab - 3ab + 9b^2 \\ &= 8a (a - 3b) - 3b (a - 3b) \\ &= (a - 3b) (8a - 3b). \end{aligned}$$

**Question 33:**

$$\begin{aligned} & 5x^2 + 33xy - 14y^2 \\ &= 5x^2 + 35xy - 2xy - 14y^2 \\ &= 5x (x + 7y) - 2y (x + 7y) \\ &= (x + 7y) (5x - 2y). \end{aligned}$$

**Question 34:**

$$\begin{aligned} & 3x^3 - x^2 - 10x \\ &= x (3x^2 - x - 10) \\ &= x [3x^2 - 6x + 5x - 10] \\ &= x [3x (x - 2) + 5 (x - 2)] \\ &= x (x - 2) (3x + 5). \end{aligned}$$

**Question 35:**



$$\begin{aligned}
& \frac{1}{3}x^2 - 2x - 9 \\
&= \frac{1}{3}x^2 - 3x + x - 9 \\
&= x\left(\frac{x}{3} - 3\right) + (x - 9) \\
&= \frac{x}{3}(x - 9) + (x - 9) \\
&= (x - 9)\left(\frac{x}{3} + 1\right) \\
&= (x - 9)\frac{(x + 3)}{3} = \frac{1}{3}(x - 9)(x + 3).
\end{aligned}$$

**Question 36:**

$$\begin{aligned}
& x^2 - 2x + \frac{7}{16} \\
&= \frac{1}{16}(16x^2 - 32x + 7) \\
&= \frac{1}{16}(16x^2 - 4x - 28x + 7) \\
&= \frac{1}{16}[4x(4x - 1) - 7(4x - 1)] \\
&= \frac{1}{16}(4x - 1)(4x - 7).
\end{aligned}$$

**Question 37:**

$$\begin{aligned}
& \sqrt{2}x^2 + 3x + \sqrt{2} \\
&= \sqrt{2}x^2 + x + 2x + \sqrt{2} \\
&= x(\sqrt{2}x + 1) + \sqrt{2}(\sqrt{2}x + 1) \\
&= (\sqrt{2}x + 1)(x + \sqrt{2}).
\end{aligned}$$

**Question 38:**

$$\begin{aligned}
& \sqrt{5}x^2 + 2x - 3\sqrt{5} \\
&= \sqrt{5}x^2 + 5x - 3x - 3\sqrt{5} \\
&= \sqrt{5}x(x + \sqrt{5}) - 3(x + \sqrt{5}) \\
&= (\sqrt{5}x - 3)(x + \sqrt{5}).
\end{aligned}$$

**Question 39:**

$$\begin{aligned}
& 2x^2 + 3\sqrt{3}x + 3 \\
&= 2x^2 + 2\sqrt{3}x + \sqrt{3}x + 3 \\
&= 2x(x + \sqrt{3}) + \sqrt{3}(x + \sqrt{3}) \\
&= (x + \sqrt{3})(2x + \sqrt{3}).
\end{aligned}$$

**Question 40:**

$$\begin{aligned}
& 2\sqrt{3}x^2 + x - 5\sqrt{3} \\
&= 2\sqrt{3}x^2 + 6x - 5x - 5\sqrt{3} \\
&= 2\sqrt{3}x(x + \sqrt{3}) - 5(x + \sqrt{3}) \\
&= (x + \sqrt{3})(2\sqrt{3}x - 5).
\end{aligned}$$

**Question 41:**

$$\begin{aligned}
& 5\sqrt{5}x^2 + 20x + 3\sqrt{5} \\
&= 5\sqrt{5}x^2 + 15x + 5x + 3\sqrt{5} \\
&= 5x(\sqrt{5}x + 3) + \sqrt{5}(\sqrt{5}x + 3) \\
&= (\sqrt{5}x + 3)(5x + \sqrt{5}).
\end{aligned}$$

**Question 42:**

$$\begin{aligned}
& 7\sqrt{2}x^2 - 10x - 4\sqrt{2} \\
&= 7\sqrt{2}x^2 - 14x + 4x - 4\sqrt{2} \\
&= 7\sqrt{2}x(x - \sqrt{2}) + 4(x - \sqrt{2}) \\
&= (x - \sqrt{2})(7\sqrt{2}x + 4).
\end{aligned}$$

**Question 43:**

$$\begin{aligned}
& 6\sqrt{3}x^2 - 47x + 5\sqrt{3} \\
&= 6\sqrt{3}x^2 - 45x - 2x + 5\sqrt{3} \\
&= 3\sqrt{3}x(2x - 5\sqrt{3}) - 1(2x - 5\sqrt{3}) \\
&= (2x - 5\sqrt{3})(3\sqrt{3}x - 1).
\end{aligned}$$

**Question 44:**

$$\begin{aligned}
& 7x^2 + 2\sqrt{14}x + 2 \\
&= 7x^2 + \sqrt{2}\sqrt{7}x + \sqrt{2}\sqrt{7}x + 2 \\
&= \sqrt{7}x(\sqrt{7}x + \sqrt{2}) + \sqrt{2}(\sqrt{7}x + \sqrt{2}) \\
&= (\sqrt{7}x + \sqrt{2})(\sqrt{7}x + \sqrt{2}) = (\sqrt{7}x + \sqrt{2})^2.
\end{aligned}$$

**Question 45:**

Let  $x + y = z$

Then,  $2(x + y)^2 - 9(x + y) - 5$

$$\begin{aligned}
&= 2z^2 - 9z - 5 \\
&= 2z^2 - 10z + z - 5 \\
&= 2z(z - 5) + 1(z - 5) \\
&= (z - 5)(2z + 1)
\end{aligned}$$

Now, replacing  $z$  by  $(x + y)$ , we get

$$\begin{aligned}
& 2(x + y)^2 - 9(x + y) - 5 \\
&= [(x + y) - 5][2(x + y) + 1] \\
&= (x + y - 5)(2x + 2y + 1).
\end{aligned}$$

**Question 46:**

Let  $2a - b = c$

Then,  $9(2a - b)^2 - 4(2a - b) - 13$

$$\begin{aligned}
&= 9c^2 - 4c - 13 \\
&= 9c^2 - 13c + 9c - 13 \\
&= c(9c - 13) + 1(9c - 13) \\
&= (c + 1)(9c - 13)
\end{aligned}$$

Now, replacing  $c$  by  $(2a - b)$ , we get

$$\begin{aligned}
& 9(2a - b)^2 - 4(2a - b) - 13 \\
&= (2a - b + 1)[9(2a - b) - 13] \\
&= (2a - b + 1)(18a - 9b - 13)
\end{aligned}$$

**Question 47:**

Let  $x - 2y = z$

Then,  $7(x - 2y)^2 - 25(x - 2y) + 12$

$$\begin{aligned}
&= 7z^2 - 25z + 12 \\
&= 7z^2 - 21z - 4z + 12 \\
&= 7z(z - 3) - 4(z - 3) \\
&= (z - 3)(7z - 4)
\end{aligned}$$

Now replace  $z$  by  $(x - 2y)$ , we get

$$7(x - 2y)^2 - 25(x - 2y) + 12$$

$$= (x - 2y - 3) [7(x - 2y) - 4]$$

$$= (x - 2y - 3) (7x - 14y - 4).$$

#### Question 48:

Let  $x^2 = y$

Then,  $4x^4 + 7x^2 - 2$

$$= 4y^2 + 7y - 2$$

$$= 4y^2 + 8y - y - 2$$

$$= 4y(y + 2) - 1(y + 2)$$

$$= (y + 2)(4y - 1)$$

Now replacing  $y$  by  $x^2$ , we get

$$4x^4 + 7x^2 - 2$$

$$= (x^2 + 2)(4x^2 - 1) \quad \left[ \because a^2 - b^2 = (a - b)(a + b) \right]$$

$$= (x^2 + 2)(2x + 1)(2x - 1).$$

### Exercise 2H

1.  $(a + b)^2 = a^2 + 2ab + b^2 = (-a - b)^2$
2.  $(a - b)^2 = a^2 - 2ab + b^2$
3.  $(a - b)(a + b) = a^2 - b^2$
4.  $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
5.  $(a + b - c)^2 = a^2 + b^2 + c^2 + 2ab - 2bc - 2ca$
6.  $(a - b + c)^2 = a^2 + b^2 + c^2 - 2ab - 2bc + 2ca$
7.  $(-a + b + c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$
8.  $(a - b - c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$
9.  $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
10.  $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
11.  $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$   
 $= (a + b)(a^2 - ab + b^2)$
12.  $a^3 - b^3 = (a - b)^3 + 3ab(a - b)$   
 $= (a - b)(a^2 + ab + b^2)$
13.  $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$   
 if  $a + b + c = 0$  then  $a^3 + b^3 + c^3 = 3abc$

#### Question 1:

We know:

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$$(i) (a + 2b + 5c)^2$$

$$= (a)^2 + (2b)^2 + (5c)^2 + 2(a)(2b) + 2(2b)(5c) + 2(5c)(a)$$

$$= a^2 + 4b^2 + 25c^2 + 4ab + 20bc + 10ac$$

$$(ii) (2a - b + c)^2$$

$$= (2a)^2 + (-b)^2 + (c)^2 + 2(2a)(-b) + 2(-b)(c) + 2(c)(2a)$$

$$= 4a^2 + b^2 + c^2 - 4ab - 2bc + 4ac.$$

$$(iii) (a - 2b - 3c)^2$$

$$= (a)^2 + (-2b)^2 + (-3c)^2 + 2(a)(-2b) + 2(-2b)(-3c) + 2(-3c)(a)$$

$$= a^2 + 4b^2 + 9c^2 - 4ab + 12bc - 6ac.$$

#### Question 2:

We know:

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$$(i) (2a - 5b - 7c)^2$$

$$= (2a)^2 + (-5b)^2 + (-7c)^2 + 2(2a)(-5b) + 2(-5b)(-7c) + 2(-7c)(2a)$$

$$= 4a^2 + 25b^2 + 49c^2 - 20ab + 70bc - 28ac.$$

$$(ii) (-3a + 4b - 5c)^2$$

$$= (-3a)^2 + (4b)^2 + (-5c)^2 + 2(-3a)(4b) + 2(4b)(-5c) + 2(-5c)(-3a)$$

$$= 9a^2 + 16b^2 + 25c^2 - 24ab - 40bc + 30ac.$$

$$(iii) \left( \frac{1}{2}a - \frac{1}{4}b + 2 \right)^2$$

$$= \left( \frac{1}{2}a \right)^2 + \left( -\frac{1}{4}b \right)^2 + (2)^2 + 2\left( \frac{1}{2}a \right) \left( -\frac{1}{4}b \right) + 2\left( -\frac{1}{4}b \right) (2) + 2(2) \left( \frac{1}{2}a \right) = \frac{a^2}{4} + \frac{b^2}{16} + 4 - \frac{ab}{4} - b + 2a$$

### Question 3:

$$4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$$

$$= (2x)^2 + (3y)^2 + (-4z)^2 + 2(2x)(3y) + 2(3y)(-4z) + 2(-4z)(2x)$$

$$= (2x + 3y - 4z)^2$$

### Question 4:

$$9x^2 + 16y^2 + 4z^2 - 24xy + 16yz - 12xz$$

$$= (-3x)^2 + (4y)^2 + (2z)^2 + 2(-3x)(4y) + 2(4y)(2z) + 2(2z)(-3x)$$

$$= (-3x + 4y + 2z)^2.$$

### Question 5:

$$25x^2 + 4y^2 + 9z^2 - 20xy - 12yz + 30xz$$

$$= (5x)^2 + (-2y)^2 + (3z)^2 + 2(5x)(-2y) + 2(-2y)(3z) + 2(3z)(5x)$$

$$= (5x - 2y + 3z)^2$$

### Question 6:

$$(i) (99)^2$$

$$= (100 - 1)^2$$

$$\left[ \because (a - b)^2 = a^2 - 2ab + b^2 \right]$$

$$= (100)^2 - 2(100)(1) + (1)^2$$

$$= 10000 - 200 + 1$$

$$= 9801.$$

$$(ii) (998)^2$$

$$= (1000 - 2)^2$$

$$= (1000)^2 - 2(1000)(2) + (2)^2$$

$$= 1000000 - 4000 + 4$$

$$= 996004.$$

## Exercise 21

1.  $(a + b)^2 = a^2 + 2ab + b^2 = (-a - b)^2$
2.  $(a - b)^2 = a^2 - 2ab + b^2$
3.  $(a - b)(a + b) = a^2 - b^2$
4.  $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
5.  $(a + b - c)^2 = a^2 + b^2 + c^2 + 2ab - 2bc - 2ca$
6.  $(a - b + c)^2 = a^2 + b^2 + c^2 - 2ab - 2bc + 2ca$
7.  $(-a + b + c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$
8.  $(a - b - c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$
9.  $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
10.  $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
11.  $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$   
 $= (a + b)(a^2 - ab + b^2)$
12.  $a^3 - b^3 = (a - b)^3 + 3ab(a - b)$   
 $= (a - b)(a^2 + ab + b^2)$
13.  $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$   
 if  $a + b + c = 0$  then  $a^3 + b^3 + c^3 = 3abc$

**Question 1:**

$$\begin{aligned} \text{(i)} \quad & (3x + 2)^3 \\ &= (3x)^3 + (2)^3 + 3 \times 3x \times 2(3x + 2) \\ & \left[ \because (a + b)^3 = a^3 + b^3 + 3ab(a + b) \right] \end{aligned}$$

$$\begin{aligned} &= 27x^3 + 8 + 18x(3x + 2) \\ &= 27x^3 + 8 + 54x^2 + 36x. \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad & (3a - 2b)^3 \\ &= (3a)^3 - (2b)^3 - 3 \times 3a \times 2b(3a - 2b) \\ & \left[ \because (a - b)^3 = a^3 - b^3 - 3ab(a - b) \right] \end{aligned}$$

$$\begin{aligned} &= 27a^3 - 8b^3 - 18ab(3a - 2b) \\ &= 27a^3 - 8b^3 - 54a^2b + 36ab^2. \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad & \left( \frac{2}{3}x + 1 \right)^3 \\ &= \left( \frac{2}{3}x \right)^3 + (1)^3 + 3 \times \frac{2}{3}x \times 1 \left( \frac{2}{3}x + 1 \right) \\ & \left[ \because (a + b)^3 = a^3 + b^3 + 3ab(a + b) \right] \\ &= \frac{8}{27}x^3 + 1 + 2x \left( \frac{2}{3}x + 1 \right) \\ &= \frac{8}{27}x^3 + 1 + \frac{4}{3}x^2 + 2x. \end{aligned}$$

**Question 2:**

(i)

$$\begin{aligned} & \left(2x - \frac{2}{x}\right)^3 \\ &= (2x)^3 - \left(\frac{2}{x}\right)^3 - 3 \times 2x \times \frac{2}{x} \left(2x - \frac{2}{x}\right) \\ & \quad \left[\because (a-b)^3 = a^3 - b^3 - 3ab(a-b)\right] \\ &= 8x^3 - \frac{8}{x^3} - 12 \left(2x - \frac{2}{x}\right) \\ &= 8x^3 - \frac{8}{x^3} - 24x + \frac{24}{x}. \end{aligned}$$

(ii)

$$\begin{aligned} & \left(3a + \frac{1}{4b}\right)^3 \\ &= (3a)^3 + \left(\frac{1}{4b}\right)^3 + 3 \times 3a \times \frac{1}{4b} \left(3a + \frac{1}{4b}\right) \\ & \quad \left[\because (a+b)^3 = a^3 + b^3 + 3ab(a+b)\right] \\ &= 27a^3 + \frac{1}{64b^3} + \frac{9a}{4b} \left(3a + \frac{1}{4b}\right) \\ &= 27a^3 + \frac{1}{64b^3} + \frac{27a^2}{4b} + \frac{9a}{16b^2}. \end{aligned}$$

(iii)

$$\begin{aligned} & \left(\frac{4}{5}x - 2\right)^3 \\ &= \left(\frac{4}{5}x\right)^3 - (2)^3 - 3 \times \frac{4}{5}x \times 2 \left(\frac{4}{5}x - 2\right) \\ & \quad \left[\because (a-b)^3 = a^3 - b^3 - 3ab(a-b)\right] \\ &= \frac{64}{125}x^3 - 8 - \frac{24}{5}x \left(\frac{4}{5}x - 2\right) \\ &= \frac{64}{125}x^3 - 8 - \frac{96}{25}x^2 + \frac{48}{5}x. \end{aligned}$$

### Question 3:

$$\begin{aligned} & \text{(i) } (95)^3 \\ &= (100 - 5)^3 \\ &= (100)^3 - (5)^3 - 3 \times 100 \times 5 (100 - 5) \\ &= 1000000 - 125 - (1500 \times 95) \\ &= 857375. \\ & \text{(ii) } (999)^3 \\ &= (1000 - 1)^3 \\ &= (1000)^3 - (1)^3 - 3 \times 1000 \times 1 (1000 - 1) \\ &= 1000000000 - 1 - 3000 (1000 - 1) \\ &= 1000000000 - 1 - (3000 \times 999) \\ &= 997002999. \end{aligned}$$

## Exercise 2J

1.  $(a + b)^2 = a^2 + 2ab + b^2 = (-a - b)^2$
2.  $(a - b)^2 = a^2 - 2ab + b^2$
3.  $(a - b)(a + b) = a^2 - b^2$
4.  $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
5.  $(a + b - c)^2 = a^2 + b^2 + c^2 + 2ab - 2bc - 2ca$
6.  $(a - b + c)^2 = a^2 + b^2 + c^2 - 2ab - 2bc + 2ca$
7.  $(-a + b + c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$
8.  $(a - b - c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$
9.  $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
10.  $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
11.  $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$   
 $= (a + b)(a^2 - ab + b^2)$
12.  $a^3 - b^3 = (a - b)^3 + 3ab(a - b)$   
 $= (a - b)(a^2 + ab + b^2)$
13.  $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$   
 if  $a + b + c = 0$  then  $a^3 + b^3 + c^3 = 3abc$

**Question 1:**

$$\begin{aligned}
 & x^3 + 27 \\
 &= x^3 + 3^3 \\
 &= (x + 3)(x^2 - 3x + 9) \\
 &\text{Since } a^3 + b^3 = (a + b)(a^2 - a \times b + b^2)
 \end{aligned}$$

**Question 2:**

$$\begin{aligned}
 & 8x^3 + 27y^3 \\
 &= (2x)^3 + (3y)^3 \\
 &= (2x + 3y)[(2x)^2 - (2x)(3y) + (3y)^2] \\
 &\text{Since } a^3 + b^3 = (a + b)(a^2 - a \times b + b^2) \\
 &= (2x + 3y)(4x^2 - 6xy + 9y^2).
 \end{aligned}$$

**Question 3:**

$$\begin{aligned}
 & 343 + 125b^3 \\
 &= (7)^3 + (5b)^3 \\
 &= (7 + 5b)[(7)^2 - (7)(5b) + (5b)^2] \\
 &\text{Since } a^3 + b^3 = (a + b)(a^2 - a \times b + b^2) \\
 &= (7 + 5b)(49 - 35b + 25b^2)
 \end{aligned}$$

**Question 4:**

$$\begin{aligned}
 & 1 + 64x^3 \\
 &= (1)^3 + (4x)^3 \\
 &= (1 + 4x)[(1)^2 - 1(4x) + (4x)^2] \\
 &\text{Since } a^3 + b^3 = (a + b)(a^2 - a \times b + b^2) \\
 &= (1 + 4x)(1 - 4x + 16x^2).
 \end{aligned}$$

**Question 5:**

$$\begin{aligned}
 & 125a^3 + \frac{1}{8} \\
 &\text{We know that} \\
 &a^3 + b^3 = (a + b)(a^2 - a \times b + b^2)
 \end{aligned}$$

Let us rewrite

$$\begin{aligned}
& 125a^3 + \frac{1}{8} \\
&= (5a)^3 + \left(\frac{1}{2}\right)^3 \\
&= \left(5a + \frac{1}{2}\right) \left[(5a)^2 - 5a \times \frac{1}{2} + \left(\frac{1}{2}\right)^2\right] \\
&= \left(5a + \frac{1}{2}\right) \left(25a^2 - \frac{5a}{2} + \frac{1}{4}\right).
\end{aligned}$$

**Question 6:**

$$216x^3 + \frac{1}{125}$$

We know that

$$a^3 + b^3 = (a+b)(a^2 - a \times b + b^2)$$

Let us rewrite

$$\begin{aligned}
& 216x^3 + \frac{1}{125} \\
&= (6x)^3 + \left(\frac{1}{5}\right)^3 \\
&= \left(6x + \frac{1}{5}\right) \left[(6x)^2 - 6x \times \frac{1}{5} + \left(\frac{1}{5}\right)^2\right] \\
&= \left(6x + \frac{1}{5}\right) \left(36x^2 - \frac{6x}{5} + \frac{1}{25}\right).
\end{aligned}$$

**Question 7:**

$$\begin{aligned}
& 16x^4 + 54x \\
&= 2x(8x^3 + 27) \\
&= 2x[(2x)^3 + (3)^3] \\
&= 2x(2x+3)[(2x)^2 - 2x(3) + 3^2] \\
&\text{Since } a^3 + b^3 = (a+b)(a^2 - a \times b + b^2) \\
&= 2x(2x+3)(4x^2 - 6x + 9)
\end{aligned}$$

**Question 8:**

$$\begin{aligned}
& 7a^3 + 56b^3 \\
&= 7(a^3 + 8b^3) \\
&= 7[(a)^3 + (2b)^3] \\
&= 7(a+2b)[a^2 - a \times 2b + (2b)^2] \\
&\text{Since } a^3 + b^3 = (a+b)(a^2 - a \times b + b^2) \\
&= 7(a+2b)(a^2 - 2ab + 4b^2).
\end{aligned}$$

**Question 9:**

$$\begin{aligned}
& x^5 + x^2 \\
&= x^2(x^3 + 1) \\
&= x^2(x+1)[(x)^2 - x(1) + (1)^2] \\
&\text{Since } a^3 + b^3 = (a+b)(a^2 - a \times b + b^2) \\
&= x^2(x+1)(x^2 - x + 1).
\end{aligned}$$

**Question 10:**

$$\begin{aligned}
& a^3 + 0.008 \\
&= (a)^3 + (0.2)^3 \\
&= (a+0.2)[(a)^2 - a(0.2) + (0.2)^2] \\
&\text{Since } a^3 + b^3 = (a+b)(a^2 - a \times b + b^2) \\
&= (a+0.2)(a^2 - 0.2a + 0.04).
\end{aligned}$$



**Question 11:**

$$\begin{aligned}
& x^6 + y^6 \\
&= (x^2)^3 + (y^2)^3 \\
&= (x^2 + y^2) [(x^2)^2 - x^2 (y^2) + (y^2)^2] \\
&\text{Since } a^3 + b^3 = (a + b)(a^2 - a \times b + b^2) \\
&= (x^2 + y^2) (x^4 - x^2 y^2 + y^4).
\end{aligned}$$

**Question 12:**

$$\begin{aligned}
& 2a^3 + 16b^3 - 5a - 10b \\
&= 2(a^3 + 8b^3) - 5(a + 2b) \\
&= 2[(a)^3 + (2b)^3] - 5(a + 2b) \\
&= 2(a + 2b) [(a)^2 - a(2b) + (2b)^2] - 5(a + 2b) \\
&\text{Since } a^3 + b^3 = (a + b)(a^2 - a \times b + b^2) \\
&= (a + 2b) [2(a^2 - 2ab + 4b^2) - 5]
\end{aligned}$$

**Question 13:**

$$\begin{aligned}
& x^3 - 512 \\
&= (x)^3 - (8)^3 \\
&= (x - 8) [(x)^2 + x(8) + (8)^2] \\
&\text{Since } a^3 - b^3 = (a - b)(a^2 + a \times b + b^2) \\
&= (x - 8) (x^2 + 8x + 64).
\end{aligned}$$

**Question 14:**

$$\begin{aligned}
& 64x^3 - 343 \\
&= (4x)^3 - (7)^3 \\
&= (4x - 7) [(4x)^2 + 4x(7) + (7)^2] \\
&\text{Since } a^3 - b^3 = (a - b)(a^2 + a \times b + b^2) \\
&= (4x - 7) (16x^2 + 28x + 49).
\end{aligned}$$

**Question 15:**

$$\begin{aligned}
& 1 - 27x^3 \\
&= (1)^3 - (3x)^3 \\
&= (1 - 3x) [(1)^2 + 1(3x) + (3x)^2] \\
&\text{Since } a^3 - b^3 = (a - b)(a^2 + a \times b + b^2) \\
&= (1 - 3x) (1 + 3x + 9x^2).
\end{aligned}$$

**Question 16:**

$$\begin{aligned}
& 1 - 27x^3 \\
&= (1)^3 - (3x)^3 \\
&= (1 - 3x) [(1)^2 + 1(3x) + (3x)^2] \\
&\text{Since } a^3 - b^3 = (a - b)(a^2 + a \times b + b^2) \\
&= (1 - 3x) (1 + 3x + 9x^2).
\end{aligned}$$

**Question 17:**

We know that

$$a^3 - b^3 = (a - b)(a^2 + a \times b + b^2)$$

Let us rewrite

$$\begin{aligned}
& 8x^3 - \frac{1}{27y^3} \\
&= (2x)^3 - \left(\frac{1}{3y}\right)^3 \\
&= \left(2x - \frac{1}{3y}\right) \left[ (2x)^2 + 2x \times \frac{1}{3y} + \left(\frac{1}{3y}\right)^2 \right] \\
&= \left(2x - \frac{1}{3y}\right) \left( 4x^2 + \frac{2x}{3y} + \frac{1}{9y^2} \right).
\end{aligned}$$

**Question 18:**

$$\begin{aligned}
& a^3 - 0.064 \\
&= (a)^3 - (0.4)^3 \\
&= (a - 0.4) [(a)^2 + a(0.4) + (0.4)^2] \\
&\text{Since } a^3 - b^3 = (a - b)(a^2 + a \times b + b^2) \\
&= (a - 0.4)(a^2 + 0.4a + 0.16).
\end{aligned}$$

**Question 19:**

$$\begin{aligned}
& (a + b)^3 - 8 \\
&= (a + b)^3 - (2)^3 \\
&= (a + b - 2) [(a + b)^2 + (a + b)2 + (2)^2] \\
&\text{Since } a^3 - b^3 = (a - b)(a^2 + a \times b + b^2) \\
&= (a + b - 2)(a^2 + b^2 + 2ab + 2(a + b) + 4).
\end{aligned}$$

**Question 20:**

$$\begin{aligned}
& x^6 - 729 \\
&= (x^2)^3 - (9)^3 \\
&= (x^2 - 9) [(x^2)^2 + x^2 \cdot 9 + (9)^2] \\
&\text{Since } a^3 - b^3 = (a - b)(a^2 + a \times b + b^2) \\
&= (x^2 - 9)(x^4 + 9x^2 + 81) \\
&= (x + 3)(x - 3) [(x^2 + 9)^2 - (3x)^2] \\
&= (x + 3)(x - 3)(x^2 + 3x + 9)(x^2 - 3x + 9).
\end{aligned}$$

**Question 21:**

We know that,  
 $a^3 - b^3 = (a - b)(a^2 + a \times b + b^2)$

$$\begin{aligned}
& \text{Therefore,} \\
& (a + b)^3 - (a - b)^3 \\
&= [a + b - (a - b)] [(a + b)^2 + (a + b)(a - b) + (a - b)^2] \\
&= (a + b - a + b) [a^2 + b^2 + 2ab + a^2 - b^2 + a^2 + b^2 - 2ab] \\
&= 2b(3a^2 + b^2).
\end{aligned}$$

**Question 22:**

$$\begin{aligned}
& x - 8xy^3 \\
&= x(1 - 8y^3) \\
&= x[(1)^3 - (2y)^3] \\
&= x(1 - 2y) [(1)^2 + 1(2y) + (2y)^2] \\
&\text{Since } a^3 - b^3 = (a - b)(a^2 + a \times b + b^2) \\
&= x(1 - 2y)(1 + 2y + 4y^2).
\end{aligned}$$

**Question 23:**

$$32x^4 - 500x$$

$$\begin{aligned}
&= 4x(8x^3 - 125) \\
&= 4x[(2x)^3 - (5)^3] \\
&= 4x[(2x - 5)[(2x)^2 + 2x(5) + (5)^2]] \\
&\text{Since } a^3 - b^3 = (a - b)(a^2 + a \times b + b^2) \\
&= 4x(2x - 5)(4x^2 + 10x + 25).
\end{aligned}$$

**Question 24:**

$$\begin{aligned}
&3a^7b - 81a^4b^4 \\
&= 3a^4b(a^3 - 27b^3) \\
&= 3a^4b[(a)^3 - (3b)^3] \\
&= 3a^4b(a - 3b)[(a)^2 + a(3b) + (3b)^2] \\
&\text{Since } a^3 - b^3 = (a - b)(a^2 + a \times b + b^2) \\
&= 3a^4b(a - 3b)(a^2 + 3ab + 9b^2).
\end{aligned}$$

**Question 25:**

We know that

$$a^3 - b^3 = (a - b)(a^2 + a \times b + b^2)$$

$$\begin{aligned}
&a^3 - \frac{1}{a^3} - 2a + \frac{2}{a} \\
&= a^3 - \frac{1}{a^3} - 2\left(a - \frac{1}{a}\right) \\
&= \left(a - \frac{1}{a}\right)\left[a^2 + a \times \frac{1}{a} + \frac{1}{a^2}\right] - 2\left(a - \frac{1}{a}\right) \\
&= \left(a - \frac{1}{a}\right)\left[a^2 + 1 + \frac{1}{a^2} - 2\right] \\
&= \left(a - \frac{1}{a}\right)\left[a^2 + \frac{1}{a^2} - 1\right].
\end{aligned}$$

**Question 26:**

$$\begin{aligned}
&8a^3 - b^3 - 4ax + 2bx \\
&= 8a^3 - b^3 - 2x(2a - b) \\
&= (2a)^3 - (b)^3 - 2x(2a - b) \\
&= (2a - b)[(2a)^2 + 2a(b) + (b)^2] - 2x(2a - b) \\
&\text{Since } a^3 - b^3 = (a - b)(a^2 + a \times b + b^2) \\
&= (2a - b)(4a^2 + 2ab + b^2) - 2x(2a - b) \\
&= (2a - b)(4a^2 + 2ab + b^2 - 2x).
\end{aligned}$$

**Question 27:**

$$\begin{aligned}
&8a^3 - b^3 - 4ax + 2bx \\
&= 8a^3 - b^3 - 2x(2a - b) \\
&= (2a)^3 - (b)^3 - 2x(2a - b) \\
&= (2a - b)[(2a)^2 + 2a(b) + (b)^2] - 2x(2a - b) \\
&\text{Since } a^3 - b^3 = (a - b)(a^2 + a \times b + b^2) \\
&= (2a - b)(4a^2 + 2ab + b^2) - 2x(2a - b) \\
&= (2a - b)(4a^2 + 2ab + b^2 - 2x).
\end{aligned}$$

## Exercise 2K

1.  $(a + b)^2 = a^2 + 2ab + b^2 = (-a - b)^2$
2.  $(a - b)^2 = a^2 - 2ab + b^2$
3.  $(a - b)(a + b) = a^2 - b^2$
4.  $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
5.  $(a + b - c)^2 = a^2 + b^2 + c^2 + 2ab - 2bc - 2ca$
6.  $(a - b + c)^2 = a^2 + b^2 + c^2 - 2ab - 2bc + 2ca$
7.  $(-a + b + c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$
8.  $(a - b - c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$
9.  $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
10.  $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
11.  $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$   
 $= (a + b)(a^2 - ab + b^2)$
12.  $a^3 - b^3 = (a - b)^3 + 3ab(a - b)$   
 $= (a - b)(a^2 + ab + b^2)$
13.  $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$   
 if  $a + b + c = 0$  then  $a^3 + b^3 + c^3 = 3abc$

**Question 1:**

$$\begin{aligned}
 &125a^3 + b^3 + 64c^3 - 60abc \\
 &= (5a)^3 + (b)^3 + (4c)^3 - 3(5a)(b)(4c) \\
 &= (5a + b + 4c)[(5a)^2 + b^2 + (4c)^2 - (5a)(b) - (b)(4c) - (5a)(4c)] \\
 &[\because a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)] \\
 &= (5a + b + 4c)(25a^2 + b^2 + 16c^2 - 5ab - 4bc - 20ac).
 \end{aligned}$$

**Question 2:**

$$\begin{aligned}
 &a^3 + 8b^3 + 64c^3 - 24abc \\
 &= (a)^3 + (2b)^3 + (4c)^3 - 3a(2b)(4c) \\
 &= (a + 2b + 4c)[a^2 + 4b^2 + 16c^2 - 2ab - 8bc - 4ca].
 \end{aligned}$$

**Question 3:**

$$\begin{aligned}
 &1 + b^3 + 8c^3 - 6bc \\
 &= 1 + (b)^3 + (2c)^3 - 3(b)(2c) \\
 &= (1 + b + 2c)[1 + b^2 + (2c)^2 - b - b(2c) - 2c] \\
 &= (1 + b + 2c)(1 + b^2 + 4c^2 - b - 2bc - 2c).
 \end{aligned}$$

**Question 4:**

$$\begin{aligned}
 &216 + 27b^3 + 8c^3 - 108bc \\
 &= (6)^3 + (3b)^3 + (2c)^3 - 3(6)(3b)(2c) \\
 &= (6 + 3b + 2c)[(6)^2 + (3b)^2 + (2c)^2 - 6(3b) - 3(6)(2c) - 2c(6)] \\
 &= (6 + 3b + 2c)(36 + 9b^2 + 4c^2 - 18b - 6bc - 12c).
 \end{aligned}$$

**Question 5:**

$$\begin{aligned}
 &27a^3 - b^3 + 8c^3 + 18abc \\
 &= (3a)^3 + (-b)^3 + (2c)^3 + 3(3a)(-b)(2c) \\
 &= [3a + (-b) + 2c][(3a)^2 + (-b)^2 + (2c)^2 - 3a(-b) - (-b)(2c) - (2c)(3a)] \\
 &= (3a - b + 2c)(9a^2 + b^2 + 4c^2 + 3ab + 2bc - 6ca).
 \end{aligned}$$

**Question 6:**

$$\begin{aligned}
 &8a^3 + 125b^3 - 64c^3 + 120abc \\
 &= (2a)^3 + (5b)^3 + (-4c)^3 - 3(2a)(5b)(-4c) \\
 &= (2a + 5b - 4c)[(2a)^2 + (5b)^2 + (-4c)^2 - (2a)(5b) - (5b)(-4c) - (-4c)(2a)]
 \end{aligned}$$

$$= (2a + 5b - 4c)(4a^2 + 25b^2 + 16c^2 - 10ab + 20bc + 8ca).$$

**Question 7:**

$$\begin{aligned} & 8 - 27b^3 - 343c^3 - 126bc \\ &= (2)^3 + (-3b)^3 + (-7c)^3 - 3(2)(-3b)(-7c) \\ &= (2 - 3b - 7c)[(2)^2 + (-3b)^2 + (-7c)^2 - (2)(-3b) - (-3b)(-7c) - (-7c)(2)] \\ &= (2 - 3b - 7c)(4 + 9b^2 + 49c^2 + 6b - 21bc + 14c). \end{aligned}$$

**Question 8:**

$$\begin{aligned} & 125 - 8x^3 - 27y^3 - 90xy \\ &= (5)^3 + (-2x)^3 + (-3y)^3 - 3(5)(-2x)(-3y) \\ &= (5 - 2x - 3y)[(5)^2 + (-2x)^2 + (-3y)^2 - (5)(-2x) - (-2x)(-3y) - (-3y)(5)] \\ &= (5 - 2x - 3y)(25 + 4x^2 + 9y^2 + 10x - 6xy + 15y). \end{aligned}$$

**Question 9:**

$$\begin{aligned} & 2\sqrt{2}a^3 + 16\sqrt{2}b^3 + c^3 - 12abc \\ &= (\sqrt{2}a)^3 + (2\sqrt{2}b)^3 + (c)^3 - 3(\sqrt{2}a)(2\sqrt{2}b)(c) \\ &= (\sqrt{2}a + 2\sqrt{2}b + c) \left[ (\sqrt{2}a)^2 + (2\sqrt{2}b)^2 + c^2 - (\sqrt{2}a)(2\sqrt{2}b) - (2\sqrt{2}b)(c) - (c)(\sqrt{2}a) \right] \\ &= (\sqrt{2}a + 2\sqrt{2}b + c)(2a^2 + 8b^2 + c^2 - 4ab - 2\sqrt{2}bc - \sqrt{2}ac). \end{aligned}$$

**Question 10:**

$$\begin{aligned} & x^3 + y^3 - 12xy + 64 \\ &= x^3 + y^3 + 64 - 12xy \\ &= (x)^3 + (y)^3 + (4)^3 - 3(x)(y)(4) \\ &= (x + y + 4)[(x)^2 + (y)^2 + (4)^2 - x \times y - y \times 4 - 4 \times x] \\ &= (x + y + 4)(x^2 + y^2 + 16 - xy - 4y - 4x). \end{aligned}$$

**Question 11:**

Putting  $(a - b) = x$ ,  $(b - c) = y$  and  $(c - a) = z$ , we get,

$$\begin{aligned} & (a - b)^3 + (b - c)^3 + (c - a)^3 \\ &= x^3 + y^3 + z^3, \text{ where } (x + y + z) = (a - b) + (b - c) + (c - a) = 0 \\ &= 3xyz \text{ [}\because (x + y + z) = 0 \Rightarrow (x^3 + y^3 + z^3) = 3xyz\text{]} \\ &= 3(a - b)(b - c)(c - a). \end{aligned}$$

**Question 12:**

We have:

$$\begin{aligned} & (3a - 2b) + (2b - 5c) + (5c - 3a) = 0 \\ & \text{So, } (3a - 2b)^3 + (2b - 5c)^3 + (5c - 3a)^3 \\ &= 3(3a - 2b)(2b - 5c)(5c - 3a). \end{aligned}$$

**Question 13:**

$$\begin{aligned} & a^3(b - c)^3 + b^3(c - a)^3 + c^3(a - b)^3 \\ &= [a(b - c)]^3 + [b(c - a)]^3 + [c(a - b)]^3 \\ & \text{Now, since, } a(b - c) + b(c - a) + c(a - b) \\ &= ab - ac + bc - ba + ca - bc = 0 \\ & \text{So, } a^3(b - c)^3 + b^3(c - a)^3 + c^3(a - b)^3 \\ &= 3a(b - c)b(c - a)c(a - b) \\ &= 3abc(a - b)(b - c)(c - a). \end{aligned}$$

**Question 14:**

$$(5a - 7b)^3 + (9c - 5a)^3 + (7b - 9c)^3$$

$$\begin{aligned}
&\text{Since, } (5a - 7b) + (9c - 5a) + (7b - 9c) \\
&= 5a - 7b + 9c - 5a + 7b - 9c = 0 \\
&\text{So, } (5a - 7b)^3 + (9c - 5a)^3 + (7b - 9c)^3 \\
&= 3(5a - 7b)(9c - 5a)(7b - 9c).
\end{aligned}$$

**Question 15:**

$$\begin{aligned}
&(x + y - z)(x^2 + y^2 + z^2 - xy + yz + zx) \\
&= [x + y + (-z)][(x)^2 + (y)^2 + (-z)^2 - (x)(y) - (y)(-z) - (-z)(x)] \\
&= x^3 + y^3 - z^3 + 3xyz.
\end{aligned}$$

**Question 16:**

$$\begin{aligned}
&(x - 2y + 3)(x^2 + 4y^2 + 2xy - 3x + 6y + 9) \\
&= [x + (-2y) + 3][(x)^2 + (-2y)^2 + (3)^2 - (x)(-2y) - (-2y)(3) - (3)(x)] \\
&= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) \\
&= a^3 + b^3 + c^3 - 3abc \\
&\text{Where, } x = a, (-2y) = b \text{ and } 3 = c \\
&(x - 2y + 3)(x^2 + 4y^2 + 2xy - 3x + 6y + 9) \\
&= (x)^3 + (-2y)^3 + (3)^3 - 3(x)(-2y)(3) \\
&= x^3 - 8y^3 + 27 + 18xy.
\end{aligned}$$

**Question 17:**

$$\begin{aligned}
&(x - 2y - z)(x^2 + 4y^2 + z^2 + 2xy + zx - 2yz) \\
&= [x + (-2y) + (-z)][(x)^2 + (-2y)^2 + (-z)^2 - (x)(-2y) - (-2y)(-z) - (-z)(x)] \\
&= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) \\
&= a^3 + b^3 + c^3 - 3abc \\
&\text{Where } x = a, (-2y) = b \text{ and } (-z) = c \\
&(x - 2y - z)(x^2 + 4y^2 + z^2 + 2xy + zx - 2yz) \\
&= (x)^3 + (-2y)^3 + (-z)^3 - 3(x)(-2y)(-z) \\
&= x^3 - 8y^3 - z^3 - 6xyz.
\end{aligned}$$

**Question 18:**

$$\begin{aligned}
&\text{Given, } x + y + 4 = 0 \\
&\text{We have } (x^3 + y^3 - 12xy + 64) \\
&= (x)^3 + (y)^3 + (4)^3 - 3(x)(y)(4) \\
&= 0. \\
&\text{Since, we know } a + b + c = 0 \Rightarrow (a^3 + b^3 + c^3) = 3abc
\end{aligned}$$

**Question 19:**

$$\begin{aligned}
&\text{Given } x = 2y + 6 \\
&\text{Or, } x - 2y - 6 = 0 \\
&\text{We have, } (x^3 - 8y^3 - 36xy - 216) \\
&= (x^3 - 8y^3 - 216 - 36xy) \\
&= (x)^3 + (-2y)^3 + (-6)^3 - 3(x)(-2y)(-6) \\
&= (x - 2y - 6)[(x)^2 + (-2y)^2 + (-6)^2 - (x)(-2y) - (-2y)(-6) - (-6)(x)] \\
&= (x - 2y - 6)(x^2 + 4y^2 + 36 + 2xy - 12y + 6x) \\
&= 0(x^2 + 4y^2 + 36 + 2xy - 12y + 6x) \\
&= 0.
\end{aligned}$$