Cubes and Cube Roots
Ex 4A

Q1

Answer :

(i) $(8)^3 = (8 \times 8 \times 8) = 512$.
Thus, the cube of 8 is 512.
(ii) $(15)^3 = (15 \times 15 \times 15) = 3375$.
Thus, the cube of 15 is 3375.
(iii) $(21)^3 = (21 \times 21 \times 21) = 9261$.
Thus, the cube of 21 is 9261.
(iv) $(60)^3 = (60 \times 60 \times 60) = 216000$.
Thus, the cube of 60 is 216000.

Q2

Answer :

(i) $(1.2)^3 = (1.2 \times 1.2 \times 1.2) = 1.728$
Thus, the cube of 1.2 is 1.728.
(ii) $(3.5)^3 = (3.5 \times 3.5 \times 3.5) = 42.875$
Thus, the cube of 3.5 is 42.875.
(iii) $(0.8)^3 = (0.8 \times 0.8 \times 0.8) = 0.512$
Thus, the cube of 0.8 is 0.512.
(iv) $(0.05)^3 = (0.05 \times 0.05 \times 0.05) = 0.000125$
Thus, the cube of 0.05 is 0.000125.

Q3

Answer :

(i) $\left(\frac{4}{7}\right)^3 = \left(\frac{4}{7} \times \frac{4}{7} \times \frac{4}{7}\right) = \left(\frac{64}{343}\right)$
Thus, the cube of $\left(\frac{4}{7}\right)$ is $\left(\frac{64}{343}\right)$.

(ii) $\left(\frac{10}{11}\right)^3 = \left(\frac{10}{11} \times \frac{10}{11} \times \frac{10}{11}\right) = \left(\frac{1000}{1331}\right)$
Thus, the cube of $\left(\frac{10}{11}\right)$ is $\left(\frac{1000}{1331}\right)$.
(iii) $\left(\frac{1}{15}\right)^3 = \left(\frac{1}{15} \times \frac{1}{15} \times \frac{1}{15}\right) = \left(\frac{1}{3375}\right)$
Thus, the cube of $\left(\frac{1}{15}\right)$ is $\left(\frac{1}{3375}\right)$ $\left(1\frac{3}{10}\right)^3 = \left(\frac{13}{10}\right)^3 = \left(\frac{13}{10} \times \frac{13}{10} \times \frac{13}{10}\right) = \left(\frac{2197}{1000}\right)$
Thus, the cube of $\left(1\frac{3}{10}\right)$ is $\left(\frac{2197}{1000}\right)$.

Q4

Answer :

(i) 125
Resolving 125 into prime factors:
125 = 5×5×5
Here, one triplet is formed, which is 5^3. Hence, 125 can be expressed as the product of the triplets of 5.
Therefore, 125 is a perfect cube.

(ii) 243 is not a perfect cube.

(iii) 343
Resolving 125 into prime factors:
343 = 7×7×7
Here, one triplet is formed, which is 7^3. Hence, 343 can be expressed as the product of the triplets of 7.
Therefore, 343 is a perfect cube.

(iv) 256 is not a perfect cube.

(v) 8000
Resolving 8000 into prime factors:
8000 = 2×2×2×2×2×2×5×5×5
Here, three triplets are formed, which are 2^3, 2^3 and 5^3. Hence, 8000 can be expressed as the product of the triplets of 2, 2 and 5, i.e. $2^3 \times 2^3 \times 5^3 = 20^3$.
Therefore, 8000 is a perfect cube.

(vi) 9261
Resolving 9261 into prime factors:
9261 = 3×3×3×7×7×7
Here, two triplets are formed, which are 3^3 and 7^3. Hence, 9261 can be expressed as the product of the triplets of 3 and 7, i.e. $3^3 \times 7^3 = 21^3$.
Therefore, 9261 is a perfect cube.

(vii) 5324 is not a perfect cube.

(viii) 3375 .
Resolving 3375 into prime factors:
3375 = 3×3×3×5×5×5.
Here, two triplets are formed, which are 3^3 and 5^3. Hence, 3375 can be expressed as the product of the triplets of 3 and 5, i.e. $3^3 \times 5^3 = 15^3$.
Therefore, 3375 is a perfect cube.

Q5

Answer :

The cubes of even numbers are always even. Therefore, 216, 512 and 1000 are the cubes of even numbers.

216 = 2×2×2×3×3×3 = $2^3 \times 3^3 = 6^3$
512 = $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^3 \times 2^3 \times 2^3 = 8^3$
1000 = $2 \times 2 \times 2 \times 5 \times 5 \times 5 = 2^3 \times 5^3 = 10^3$

Q6

Answer :

The cube of an odd number is an odd number. Therefore, 125, 343 and 9261 are the cubes of odd numbers.

125 = 5×5×5 = 5^3

343 = 7×7×7 = 7^3

9261 = 3×3×3×7×7×7 = $3^3 \times 7^3 = 21^3$

Q7

Answer :

1323

3	1323
3	441
3	147
7	49
7	7
	1

1323 = $3 \times 3 \times 3 \times 7 \times 7$.
To make it a perfect cube, it has to be multiplied by 7.

Q8

Answer :

2560

2560 can be expressed as the product of prime factors in the following manner:

2	2560
2	1280
2	640
2	320
2	160
2	80
2	40
2	20
2	10
5	5
	1

2560 = $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5$

To make this a perfect square, we have to multiply it by 5×5.
Therefore, 2560 should be multiplied by 25 so that the product is a perfect cube.

Q9

Answer :

1600

1600 can be expressed as the product of prime factors in the following manner:

2	1600
2	800
2	400
2	200
2	100
2	50
5	25
5	5
	1

1600 = $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5$

Therefore, to make the quotient a perfect cube, we have to divide 1600 by:
$5 \times 5 = 25$

Q10

Answer :

2	8788
2	4394
13	2197
13	169
13	13
	1

8788
8788 can be expressed as the product of prime factors as $2 \times 2 \times 13 \times 13 \times 13$.
Therefore, 8788 should be divided by 4, i.e. (2×2), so that the quotient is a perfect cube.

Cubes and Cube Roots
Ex 4B

Q1

Answer :

$(25)^3$

Here, $a = 2$ and $b = 5$

Using the formula $a^3 + 3a^2b + 3ab^2 + b^3$:

4 × 2	4 × 15	25 × 6	25 ×5
8 +7	60 + 16	150 + 12	125
15	76	162	

$\therefore (25)^3$ = 15625

Q2

Answer :

$(47)^3$

Here, $a = 4$ and $b = 7$

Using the formula $a^3 + 3a^2b + 3ab^2 + b^3$:

16 × 4	16 × 21	49 × 12	49 ×7
64 +39	336 + 62	588 + 34	343
103	398	622	

$\therefore (47)^3$ = 103823

Q3

Answer :

$(68)^3$

Here, $a = 6$ and $b = 8$

Using the formula $a^3 + 3a^2b + 3ab^2 + b^3$:

36 × 6	36 × 24	64 × 18	64 ×8
216 + 98	864 + 120	1152 + 51	512
314	984	1203	

$\therefore (68)^3$ = 314432

Q4

Answer :

$(84)^3$

Here, $a = 8$ and $b = 4$

Using the formula $a^3 + 3a^2b + 3ab^2 + b^3$:

64 × 8	64 × 12	16 × 24	16 × 4
512 + 80	768 + 39	384 + 6	64
592	807	390	

$\therefore (84)^3$ = 592704

Cubes and Cube Roots
Ex 4C

Q1

Answer :

$\sqrt[3]{64}$

By prime factorisation:

$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$= (2 \times 2 \times 2) \times (2 \times 2 \times 2)$

$\therefore \sqrt[3]{64} = \sqrt[3]{(2)^3 \times (2)^3} = (2 \times 2) = 4$

Q2

Answer :

$\sqrt[3]{343}$

By prime factorisation:

$343 = 7 \times 7 \times 7$

$= (7 \times 7 \times 7)$

$\therefore \sqrt[3]{343} = \sqrt[3]{7^3} = 7$

Q3

Answer :

$\sqrt[3]{729}$

By prime factorisation:

3	729
3	243
3	81
3	27
3	9
3	3
	1

$729 = 3 \times 3 \times 3 \times 3 \times 3 \times 3$

$= (3 \times 3 \times 3) \times (3 \times 3 \times 3)$

$\therefore \sqrt[3]{729} = (3 \times 3) = 9$

Q4

Answer :

$\sqrt[3]{1728}$

By prime factorisation:

2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

1728 = $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$

$= (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (3 \times 3 \times 3) = 2^3 \times 2^3 \times 3^3$

$\therefore \sqrt[3]{1728} = (2 \times 2 \times 3) = 12$

Q5

Answer :

$\sqrt[3]{9261}$

By prime factorisation:

3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

9261 = $3 \times 3 \times 3 \times 7 \times 7 \times 7$

$= (3 \times 3 \times 3) \times (7 \times 7 \times 7) = 3^3 \times 7^3$

$\therefore \sqrt[3]{9261} = (3 \times 7) = 21$

Q6

Answer :

$\sqrt[3]{4096}$

By prime factorisation:

2	4096
2	2048
2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

4096 = $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$= (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2)$

$= 2^3 \times 2^3 \times 2^3 \times 2^3$

$\therefore \sqrt[3]{4096} = (2 \times 2 \times 2 \times 2) = 16$

Q7

Answer :

$\sqrt[3]{8000}$

By prime factorisation:

2	8000
2	4000
2	2000
2	1000
2	500
2	250
5	125
5	25
5	5
	1

8000 = $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5$

$= (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (5 \times 5 \times 5)$

$\therefore \sqrt[3]{8000} = (2 \times 2 \times 5) = 20$

Q8

Answer :

$\sqrt[3]{3375}$

By prime factorisation:

5	3375
5	675
5	135
3	27
3	9
3	3
	1

3375 = $3 \times 3 \times 3 \times 5 \times 5 \times 5$

$= (3 \times 3 \times 3) \times (5 \times 5 \times 5)$

$\therefore \sqrt[3]{3375} = (3 \times 5) = 15$

Q9

Answer :

$\sqrt[3]{-216}$

By prime factorisation:

2	216
2	108
2	54
3	27
3	9
3	3
	1

216 = $2 \times 2 \times 2 \times 3 \times 3 \times 3$

$= (2 \times 2 \times 2) \times (3 \times 3 \times 3)$

$\sqrt[3]{-216} = -(2 \times 3) = -6$

$\therefore \sqrt[3]{-216} = -\left(\sqrt[3]{216}\right) = -6$

Q10

Answer :

$\sqrt[3]{-512}$

By prime factorisation:

2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

$\sqrt[3]{512} = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$= (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2)$

$\sqrt[3]{-512} = -\sqrt[3]{(2 \times 2 \times 2)} = -8$

$\therefore \sqrt[3]{-512} = -\left(\sqrt[3]{512}\right) = -8$

Q11

Answer :

$\sqrt[3]{-1331}$

By prime factorisation:

$\sqrt[3]{1331} = \sqrt[3]{11 \times 11 \times 11}$

11	1331
11	121
11	11
	1

$\sqrt[3]{-1331} = -(11 \times 11 \times 11)^{\frac{1}{3}} = -11$

$\therefore \sqrt[3]{-1331} = -\left(\sqrt[3]{1331}\right) = -11$

Q12

Answer :

$\sqrt[3]{\frac{27}{64}}$

By prime factorisation:

3	27
3	9
3	3
	1

2	64
2	32
2	16
2	8
2	4
2	2
	1

$\sqrt[3]{\frac{27}{64}} = \frac{\sqrt[3]{27}}{\sqrt[3]{64}} = \frac{\sqrt[3]{(3\times3\times3)}}{\sqrt[3]{(2\times2\times2)\times(2\times2\times2)}} = \frac{\sqrt[3]{(3\times3\times3)}}{\sqrt[3]{(4\times4\times4)}} = \frac{3}{4}$

$\therefore \sqrt[3]{\frac{27}{64}} = \frac{3}{4}$

Q13

Answer :

$\sqrt[3]{\frac{125}{216}}$

By prime factorisation:

5	125
5	25
5	5
	1

2	216
2	108
2	54
3	27
3	9
3	3
	1

$\sqrt[3]{\frac{125}{216}} = \frac{\sqrt[3]{5\times5\times5}}{\sqrt[3]{(2\times2\times2)\times(3\times3\times3)}} = \frac{\sqrt[3]{5\times5\times5}}{\sqrt[3]{(6\times6\times6)}} = \frac{5}{6}$

$\therefore \sqrt[3]{\frac{125}{216}} = \frac{5}{6}$

Q14

Answer :

$\sqrt[3]{\frac{-27}{125}}$

3	27
3	9
3	3
	1

5	125
5	25
5	5
	1

By factorisation:

$\sqrt[3]{\frac{27}{125}} = \sqrt[3]{\frac{3\times3\times3}{5\times5\times5}}$

$\therefore \sqrt[3]{\frac{-27}{125}} = \frac{-3}{5}$

Q15

Answer :

$\sqrt[3]{\frac{-64}{343}}$

On factorisation:

2	64
2	32
2	16
2	8
2	4
2	2
	1

7	343
7	49
7	7
	1

$\sqrt[3]{\frac{64}{343}} = \sqrt[3]{\frac{2\times2\times2\times2\times2\times2}{7\times7\times7}}$

$\therefore \sqrt[3]{\frac{-64}{343}} = \frac{-4}{7}$

Q16

Answer :

$\sqrt[3]{64 \times 729}$

$\sqrt[3]{64 \times 729} = \sqrt[3]{64} \times \sqrt[3]{729}$

$= \sqrt[3]{4 \times 4 \times 4} \times \sqrt[3]{(3 \times 3 \times 3) \times (3 \times 3 \times 3)}$

$= \sqrt[3]{4 \times 4 \times 4} \times \sqrt[3]{(9 \times 9 \times 9)}$

$\sqrt[3]{64 \times 729} = (4) \times (9) = 36$

Q17

Answer :

$\sqrt[3]{\frac{729}{1000}}$

3	729
3	243
3	81
3	27
3	9
3	3
	1

2	1000
2	500
2	250
5	125
5	25
5	5
	1

On factorisation:

$\sqrt[3]{\frac{729}{1000}} = \frac{\sqrt[3]{(3\times3\times3)\times(3\times3\times3)}}{\sqrt[3]{(2\times2\times2)\times(5\times5\times5)}} = \frac{\sqrt[3]{9\times9\times9}}{\sqrt[3]{10\times10\times10}}$

$\sqrt[3]{\frac{729}{1000}} = \frac{9}{10}$

Q18

Answer :

$\sqrt[3]{\frac{-512}{343}}$

By factorisation:

2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

7	343
7	49
7	7
	1

$\sqrt[3]{\frac{512}{343}} = \frac{\sqrt[3]{8\times8\times8}}{\sqrt[3]{7\times7\times7}}$

$\sqrt[3]{\frac{-512}{343}} = \frac{-8}{7}$

Cubes and Cube Roots
Ex 4D

Q1

Answer :

(a)
141 is not a perfect cube.

(b)
294 is not a perfect cube.

(c) (✓)
216 is a perfect cube.
216 = $(2 \times 2 \times 2) \times (3 \times 3 \times 3) = \left(2^3\right) \times \left(3^3\right) = 6^3$

(d)
496 is not a perfect cube.

Q2

Answer :

(a)
1152 = $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 = (2)^3 \times (2)^3 \times (2 \times 3 \times 3)$.
Hence, 1152 is not a perfect cube.

(b) (✓)
1331 = $11 \times 11 \times 11 = (11)^3$
Hence, 1331 is a perfect cube.

(c)
2016 = $2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 7 = (2)^3 \times 2 \times 2 \times 3 \times 3 \times 7$
Hence, 2016 is not a perfect cube.

(d)
739 is not a perfect cube.

Q3

Answer :

(c) 8

$\sqrt[3]{512} = \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2} = \sqrt[3]{(2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2)}$

$\sqrt[3]{512} = \sqrt[3]{(2)^3 \times (2)^3 \times (2)^3} = 8$

Hence, the cube root of 512 is 8.

Q4

Answer :

(c) 20

$\sqrt[3]{125 \times 64} = \sqrt[3]{125} \times \sqrt[3]{64} = \sqrt[3]{5 \times 5 \times 5} \times \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2}$

$\sqrt[3]{125 \times 64} = \sqrt[3]{(5)^3} \times \sqrt[3]{(2)^3 \times (2)^3} = \sqrt[3]{(5)^3} \times \sqrt[3]{(4)^3}$

$\sqrt[3]{125 \times 64} = 5 \times 4 = 20$

Hence, the cube root of $\sqrt[3]{125 \times 64}$ is 20.

Q5

Answer :

(b) $\frac{4}{7}$

$\sqrt[3]{\frac{64}{343}} = \frac{\sqrt[3]{64}}{\sqrt[3]{343}} = \frac{\sqrt[3]{4 \times 4 \times 4}}{\sqrt[3]{7 \times 7 \times 7}} = \frac{\sqrt[3]{(4)^3}}{\sqrt[3]{(7)^3}}$

$\sqrt[3]{\frac{64}{343}} = \frac{4}{7}$

$\therefore \sqrt[3]{\frac{64}{343}} = \frac{4}{7}$

Q6

Answer :

(b) $\frac{-8}{9}$

$\sqrt[3]{\frac{-512}{729}} = \frac{\sqrt[3]{-512}}{\sqrt[3]{729}} = \frac{\sqrt[3]{(-8) \times (-8) \times (-8)}}{\sqrt[3]{9 \times 9 \times 9}} = \frac{\sqrt[3]{(-8)^3}}{\sqrt[3]{(9)^3}}$

$\sqrt[3]{\frac{-512}{729}} = \frac{-8}{9}$

$\therefore \sqrt[3]{\frac{-512}{729}} = \frac{-8}{9}$

Q7

Answer :

(c) 9

2	648
2	324
2	162
3	81
3	27
3	9
3	3
	1

$648 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 = (2)^3 \times (3)^3 \times 3$

Therefore, to get a perfect cube, we need to multiply 648 by 9, i.e. (3×3).

Q8

Answer :

(a) 3

2	1536
2	768
2	384
2	192
2	96
2	48
2	24
2	12
2	6
3	1

$1536 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 = (2)^3 \times (2)^3 \times (2)^3 \times 3$

Therefore, to get a perfect cube, we need to divide 1536 by 3.

Q9

Answer :

(c) $2\frac{197}{1000}$

$\left(1\frac{3}{10}\right)^3 = \left(\frac{13}{10}\right)^3 = \frac{(13)^3}{(10)^3} = \frac{(13\times13\times13)}{(10\times10\times10)}$

$\left(1\frac{3}{10}\right)^3 = \frac{2197}{1000} = 2\frac{197}{1000}$

$\therefore \left(1\frac{3}{10}\right)^3 = 2\frac{197}{1000}$

Q10

Answer :

(c) 0.512

$(0.8)^3 = (0.8) \times (0.8) \times (0.8) = 0.512$

$\therefore (0.8)^3 = 0.512$