# Chapter 18 - Areas of Circle, Sector and Segment

## Excercise 18A

Solution 1

Let r be the radius of a circle.

Then, circumference of a circle = $2\pi r$

We have,

Circumference − Radius = 37 cm

$\Rightarrow 2\pi r - r = 37$

$\Rightarrow r(2\pi - 1) = 37$

$\Rightarrow r\left(2 \times \frac{22}{7} - 1\right) = 37$

$\Rightarrow r\left(\frac{44}{7} - 1\right) = 37$

$\Rightarrow r\left(\frac{37}{7}\right) = 37$

$\Rightarrow r = \frac{37 \times 7}{37}$

$\Rightarrow r = 7$ cm

$\therefore$ Circumference of the circle = $2\pi r = \left(2 \times \frac{22}{7} \times 7\right)$cm = 44 cm

Solution 2

Let r be the radius of a cirde

Circumference of a circle = 22 cm

$\Rightarrow 2\pi r = 22$

$\Rightarrow 2 \times \frac{22}{7} \times r = 22$

$\Rightarrow r = \frac{7}{2}$ cm

Area of a quadrant of a cirde $= \frac{1}{4} \times \pi r^2 = \left(\frac{1}{4} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\right) cm^2 = \frac{77}{8} cm^2$

Solution 3

Let the radius of the large circle be R.

Then, we have

Area of large cirde of radius R

= Area of a cirde of radius 5 cm + Area of a cirde of radius 12 cm

$\Rightarrow \pi R^2 = \left(\pi \times 5^2 + \pi \times 12^2\right)$

$\Rightarrow \pi R^2 = (25\pi + 144\pi)$

$\Rightarrow \pi R^2 = 169\pi$

$\Rightarrow R^2 = 169$

$\Rightarrow R = 13$ cm

$\Rightarrow$ Diameter $= 2R = 26$ cm

Solution 4

Let r be the radius of the circle.

It is given that,

Area of a circle = 2 x Circumference of a circle

$\Rightarrow \pi r^2 = 2 \times 2\pi r$

$\Rightarrow r^2 = 4r$

$\Rightarrow r = 4$

$\Rightarrow$ Diameter = 2r = 2 x 4 = 8 cm

## Solution 5

Since square circumscribes a circle of radius a cm, we have

Side of the square = 2 × radius of circle = 2a cm

Then, Perimeter of the square = (4 × 2a) = 8a cm

## Solution 6

Diameter of a circle = 42 cm

$\Rightarrow$ Radius of a circle = $r = \frac{42}{2} = 21$ cm

Central angle = $\theta = 60^\circ$

$\therefore$ Length of the arc = $\frac{2\pi r\theta}{360}$

$$= \left(\frac{2 \times \frac{22}{7} \times 21 \times 60^\circ}{360^\circ}\right) \text{cm}$$

$$= 22 \text{ cm}$$

## Solution 7

Let the radius of the large circle be R.
Then, we have
Area of large circle of radius R
= Area of a circle of radius 4 cm + Area of a circle of radius 3 cm

$\Rightarrow \pi R^2 = (\pi \times 4^2 + \pi \times 3^2)$

$\Rightarrow \pi R^2 = (16\pi + 9\pi)$

$\Rightarrow \pi R^2 = 25\pi$

$\Rightarrow R^2 = 25$

$\Rightarrow R = 5$ cm

$\Rightarrow$ Diameter $= 2R = 10$ cm

Solution 8

Let r be the radius of the circle.
Circumference of a circle $= 8\pi$

$\Rightarrow 2\pi r = 8\pi$

$\Rightarrow r = \frac{8}{2}$

$\Rightarrow r = 4$

$\therefore$ Area of a circle $= \pi r^2 = \pi \times 4 \times 4 = 16\pi$

Solution 9

Diameter of a semicircular protractor = 14 cm

$\Rightarrow$ Radius of a semicircular protractor = r = 7 cm

$\therefore$ Perimeter of a semicircular protractor $= (\pi r + 2r)$

$$= r(\pi + 2)$$

$$= 7\left(\frac{22}{7} + 2\right) \text{cm}$$

$$= 7\left(\frac{22 + 14}{7}\right) \text{ cm}$$

$$= 7 \times \frac{36}{7}$$

$$= 36 \text{ cm}$$

Solution 10

Let r be the radius of a circle.

Then, area of a circle $= \pi r^2$

Perimeter of a circle $= 2\pi r$

It is given that,

Area of a circle = Perimeter of a circle

$\Rightarrow \pi r^2 = 2\pi r$

$\Rightarrow r = 2$ units

Solution 11

Let R be the radius of the circle.

Then, we have

Circumference of a circle of radius R = Circumference of a circle of radius 19 cm + Circumference of a circle of radius 9 cm

$\Rightarrow 2\pi R = 2\pi \times 19 + 2\pi \times 9$

$\Rightarrow R = 19 + 9$

$\Rightarrow R = 28$ cm

Solution 12

Let R be the radius of the circle.

Then, we have

Area of a circle of radius R = Area of a circle of radius 8 cm + Area of a circle of radius 6 cm

$\Rightarrow \pi R^2 = \pi \times (8)^2 + \pi \times (6)^2$

$\Rightarrow R^2 = 64 + 36$

$\Rightarrow R^2 = 100$

$\Rightarrow R = 10$ cm

Solution 13

Radius of a circle = r = 6 cm

Central angle = $\theta = 30°$

$$\therefore \text{ Area of the sector } = \frac{\pi r^2 \theta}{360} = \left(\frac{3.14 \times 6 \times 6 \times 30°}{360°}\right) \text{cm}^2 = 9.42 \text{ cm}^2$$

Solution 14

Radius of a circle = r = 21 cm

Central angle = $\theta = 60°$

$$\therefore \text{ Length of the arc } = \frac{2\pi r\theta}{360} = \left(2 \times \frac{22}{7} \times 21 \times \frac{60}{360}\right) \text{cm} = 22 \text{cm}$$

Solution 15

Let $R_1$ and $R_2$ be the radii of two circles respectively.
Then, we have

$$\frac{2\pi R_1}{2\pi R_2} = \frac{2}{3}$$

$$\Rightarrow \frac{R_1}{R_2} = \frac{2}{3}$$

Now, $\frac{\pi R_1^{\ 2}}{\pi R_2^{\ 2}} = \frac{R_1^{\ 2}}{R_2^{\ 2}} = \left(\frac{R_1}{R_2}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$

$\Rightarrow$ Ratio of their areas is $4 : 9$.

Solution 16

Let $R_1$ and $R_2$ be the radii of two circles respectively.
Then, we have

$$\frac{\pi R_1^{\ 2}}{\pi R_2^{\ 2}} = \frac{4}{9}$$

$$\Rightarrow \frac{R_1^{\ 2}}{R_2^{\ 2}} = \frac{4}{9}$$

$$\Rightarrow \left(\frac{R_1}{R_2}\right)^2 = \left(\frac{2}{3}\right)^2$$

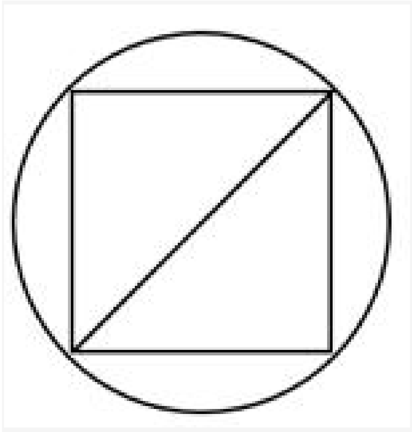
$$\Rightarrow \frac{R_1}{R_2} = \frac{2}{3}$$

Now,

$$\frac{2\pi R_1}{2\pi R_2} = \frac{R_1}{R_2} = \frac{2}{3}$$

$\therefore$ Ratio of their circumferences is $2 : 3$.

Solution 17



Let the radius of the circle be r cm.

Then, diagonal of the square = diameter of the circle = 2r cm

Area of the circle = $\pi r^2$ sq. units

Area of the square = $\frac{1}{2} \times (\text{diagonal})^2 = \frac{1}{2} \times (2r)^2 = \frac{1}{2} \times 4r^2 = 2r^2$ sq. units

Now,

$$\frac{\text{Area of the circle}}{\text{Area of the square}} = \frac{\pi r^2}{2r^2} = \frac{\pi}{2}$$

$\therefore$ Required ratio is $\pi : 2$.

Solution 18

Let r be the radius of a circle.

Circumference of a circle = 8 cm

Central angle = $\theta = 72°$

Now,

Cirumference of a circle = $2\pi r$

$\Rightarrow 88 = 2\pi r$

$\Rightarrow 88 = 2 \times \frac{22}{7} \times r$

$\Rightarrow r = \frac{88 \times 7}{2 \times 22}$

$\Rightarrow r = 14$ cm

$\therefore$ Area of the sector $= \frac{\pi r^2 \theta}{360} = \left(\frac{22}{7} \times 14 \times 14 \times \frac{72}{360}\right) cm^2 = 123.2\ cm^2$

## Solution 19

Length of the pendulum = radius of sector = r cm

$$\text{Arc length} = 8.8 \Rightarrow 2 \times \frac{22}{7} \times r \times \frac{30}{360} = 8.8$$

$$\Rightarrow r = \frac{8.8 \times 7 \times 360}{2 \times 22 \times 30} = 16.8 \text{ cm}$$

## Solution 20

Angle described by the minute hand in 60 minutes = 360°

Angle described by minute hand in 20 minutes

$$= \left(\frac{360}{60} \times 20\right) = 120°$$

Required area swept by the minute hand in 20 minutes

=Area of the sector(with r = 15 cm and $\theta$ = 120°)

$$= \left(\frac{\pi r^2\theta}{360°}\right)cm^2 = \left(3.14 \times 15 \times 15 \times \frac{120°}{360°}\right)$$
$$= 235.5\,cm^2$$

## Solution 21

$\theta$ = 56o and let radius is r cm

Area of sector = $\frac{\pi r^2\theta}{360°} = 17.6\ cm^2$

$$\Rightarrow \frac{22}{7} \times r^2 \times \frac{56°}{360°} = 17.6$$
$$r^2 = \left(\frac{17.6 \times 360 \times 7}{22 \times 56}\right)cm^2$$
$$r^2 = 36\ cm^2 \Rightarrow r = \sqrt{36}\ cm = 6\ cm$$

Hence radius= 6cm

## Solution 22

Area of the sector of circle = $\frac{\pi r^2\theta}{360} = 69.3$

Radius = 10.5 cm

$$\Rightarrow \frac{\pi \times (10.5)^2 \times \theta}{360} = 69.3$$

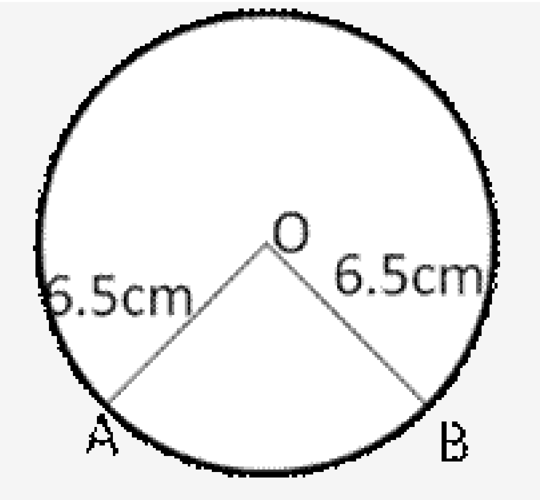
$$\Rightarrow \quad \theta = \frac{69.3 \times 360 \times 7}{10.5 \times 10.5 \times 22} = 72^\circ$$

## Solution 23

Let sector of circle is OAB

Perimeter of a sector of circle =31 cm

OA + OB + length of arc AB = 31 cm




6.5 + 6.5 + arc AB = 31 cm

arc AB = 31 - 13

= 18 cm

$$\text{Area of circle} = \frac{1}{2} lr$$

$$= \frac{1}{2} \times 18 \times 6.5 = 58.5 \text{ cm}^2$$

## Solution 24

Length of arc of circle = 44 cm

Radius of circle = 17.5 cm

Area of sector = $\frac{1}{2}lr = \left(\frac{1}{2} \times 44 \times 17.5\right) cm^2$

$= (22 \times 17.5) cm^2 = 385\ cm^2$

## Solution 25

Since the dimensions of a rectangular cardboard are $14\ cm \times 7\ cm$, the diameter of each circle is 7 cm.

Now,

Area of the rectangular cardboard $= 14 \times 7 = 98\ cm^2$

Area of two circles $= 2 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = 77\ cm^2$

$\therefore$ Area of the remaining cardboard

= Area of the rectangular cardboard – Area of two circles

$= (98 - 77)\ cm^2$

$= 21\ cm^2$

## Solution 26

Side of a square = 4 cm

$\Rightarrow$ Area of a square = $(4)^2 = 16\ cm^2$

Radius of a circle = r = 1 cm

$\Rightarrow$ Area of 4 quadrants of circle = $4 \times \frac{1}{4} \times 3.14 \times 1 \times 1 = 3.14\ cm^2$

Area of a circle of diameter 2 cm = $3.14 \times 1 \times 1 = 3.14\ cm^2$

$\therefore$ Area of the shaded region = Area of a square – Area of 4 quadrants of circle – Area of a circle of diameter 2 cm

$= (16 - 3.14 - 3.14)\ cm^2$

$= 9.72\ cm^2$

Solution 27

Length of a rectangular sheet of paper = AB = 40 cm

Breadth of a rectangular sheet of paper = AD = 28 cm

$\Rightarrow$ Area of a rectangular sheet of paper = AB x AD = $40 \times 28 = 1120\ cm^2$

Diameter of a Semicircular portion = AD = 28 cm

$\Rightarrow$ Radius = 14 cm

$\Rightarrow$ Area of a Semicircular portion = $\frac{1}{2} \times \frac{22}{7} \times 14 \times 14 = 308\ cm^2$

$\therefore$ Area of the remaining paper = Area of a rectangular sheet of paper – Area of a Semicircular portion

$= (1120 - 308)\ cm^2$

$= 812\ cm^2$

Solution 28

Side of a square = 7 cm

$\Rightarrow$ Area of a square = $(7)^2 = 49 \text{ cm}^2$

Now, radius of a circle, r = side of a square = 7 cm

$\Rightarrow$ Area of a quadrant of a circle = $\frac{1}{4} \times \frac{22}{7} \times 7 \times 7 = \frac{77}{2} = 38.5 \text{ cm}^2$

$\therefore$ Area of the shaded region = Area of a square − Area of a quadrants of circle

$= (49 - 38.5) \text{ cm}^2$

$= 10.5 \text{ cm}^2$

Solution 29

Radius of a circle = r = 7 cm

$$\text{Area of a sector} = \frac{\pi r^2 \theta}{360}$$

$$\therefore \text{ Area of the shaded region} = \frac{\pi r^2 \times 60^\circ}{360} + \frac{\pi r^2 \times 40^\circ}{360} + \frac{\pi r^2 \times 80^\circ}{360}$$

$$= \pi r^2 \left( \frac{60^\circ + 40^\circ + 80^\circ}{360^\circ} \right)$$

$$= \frac{22}{7} \times 7 \times 7 \times \frac{180^\circ}{360^\circ}$$

$$= 77 \text{ cm}^2$$

Solution 30

Area of a shaded region = Area of sector OPQ – Area of sector OAB

$$= \left[\left(\frac{\frac{22}{7} \times 7^2 \times 30}{360}\right) - \left(\frac{\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 30}{360}\right)\right] cm^2$$

$$= \left(\frac{77}{6} - \frac{77}{24}\right) cm^2$$

$$= 77\left(\frac{1}{6} - \frac{1}{24}\right) cm^2$$

$$= 77\left(\frac{4-1}{24}\right) cm^2$$

$$= 77 \times \frac{3}{24}\ cm^2$$

$$= \frac{77}{8}\ cm^2$$

Solution 31

Side of a square = 14 cm

$\Rightarrow$ Diameter of a semicircle = 14 cm

$\Rightarrow$ Radius of a semicircle = 7 cm

Now,

Area of a shaded region = Area of a square – Area of two semicircles

$$= \left[(14 \times 14) - \left(\frac{22}{7} \times 7 \times 7\right)\right] cm^2$$

$$= (196 - 154)\ cm^2$$

$$= 42\ cm^2$$

Solution 32

$\angle AOB = 90°$

$AO = OB = 42$ cm

$\Rightarrow$ Radius of a circle = 42 cm

$\therefore$ Required perimeter = Circumference of a circle − Length of arc AB + (AO + OB)

$$= \left\{\left(2 \times \frac{22}{7} \times 42\right) - \left(2 \times \frac{22}{7} \times 42 \times \frac{90}{360}\right) + (42 + 42)\right\} \text{cm}$$

$$= \left\{\left(2 \times \frac{22}{7} \times 42\right) - \left(2 \times \frac{22}{7} \times 42 \times \frac{90}{360}\right) + (42 + 42)\right\} \text{cm}$$

$$= (264 - 66 + 84) \text{cm}$$

$$= 282 \text{ cm}$$

## Solution 33

Area of the shaded region

= Area of quadrant DPBA + Area of quadrant DQBC − Area of a square ABCD

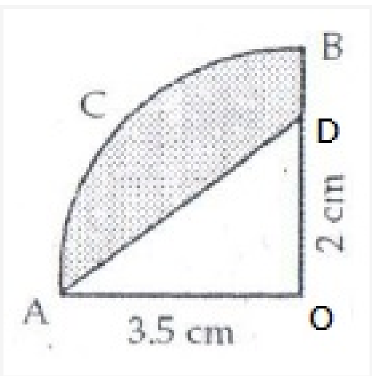
$$= \left\{\left(\frac{1}{4} \times \frac{22}{7} \times 7 \times 7\right) + \left(\frac{1}{4} \times \frac{22}{7} \times 7 \times 7\right) - (7 \times 7)\right\} \text{cm}^2$$

$$= \left(\frac{77}{2} + \frac{77}{2} - 49\right) \text{cm}^2$$

$$= (77 - 49) \text{ cm}^2$$

$$= 28 \text{ cm}^2$$

## Solution 34




Shaded area = (area of quadrant) - (area of DAOD)

$$= \left[\frac{1}{4}\pi r^2 - \frac{1}{2} \times h \times b\right]$$

$$= \left[\frac{1}{4} \times \frac{22}{7} \times 3.5 \times 3.5 - \frac{1}{2} \times 2 \times 3.5\right] cm^2$$

$$= (9.625 - 3.5) cm^2 = 6.125 cm^2$$

Solution 35

Side of a square = 14 cm

$\Rightarrow$ Diameter of a semicircle = 14 cm

$\Rightarrow$ Radius of a semicircle = 7 cm

$\therefore$ Perimeter of the shaded region

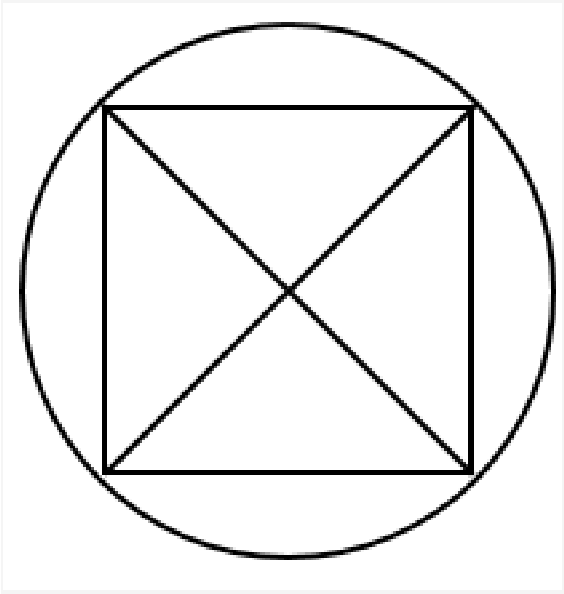
= Arc of semicircle DPC + Arc of semicircle APB + AD + BC

$$= \left\{\left(\frac{22}{7} \times 7\right) + \left(\frac{22}{7} \times 7\right) + 14 + 14\right\} cm$$

= (22 + 22 + 28) cm

= 72 cm

Solution 36



Radius of a circle = 7 cm

$\Rightarrow$ Diagonal of the square = $2 \times 7 = 14$ cm

Now,

$$\text{Area of the square} = \frac{1}{2} \times (\text{diagonal})^2 = \left(\frac{1}{2} \times 14 \times 14\right) \text{cm}^2 = 98\ \text{cm}^2$$

$$\text{Area of the circle} = \left(\frac{22}{7} \times 7 \times 7\right) \text{cm}^2 = 154\ \text{cm}^2$$

$$\begin{aligned} \therefore \text{Required area} &= \text{Area of the circle} - \text{Area of the square} \\ &= (154 - 98)\ \text{cm}^2 \\ &= 56\ \text{cm}^2 \end{aligned}$$

Solution 37

(i) Perimeter of the shaded region

$= \text{Perimeter of semicircles } (ARC + BSD) + \text{Perimeter of semicircles} (APB + CQD)$

$= \left\{2\left(\frac{22}{7} \times 7\right) + 2\left(\frac{22}{7} \times \frac{7}{2}\right)\right\} \text{cm}$

$= (44 + 22) \text{ cm}$

$= 66 \text{ cm}$

(ii) Area of the shaded region

$= \text{Area of semicircles } (ARC + BSD) - \text{Area of semicircles} (APB + CQD)$

$= \left\{2\left(\frac{1}{2} \times \frac{22}{7} \times 7 \times 7\right) - 2\left(\frac{1}{2} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\right)\right\} \text{cm}^2$

$= \left(2 \times 77 - 2 \times \frac{77}{4}\right) \text{ cm}^2$

$= (154 - 38.5) \text{ cm}^2$

$= 115.5 \text{ cm}^2$

## Solution 38

Perimeter of shaded region

$= \text{Perimeter of semicircle } PSR + \text{Perimeter of semicircle } RTQ$
$\quad + \text{Perimeter of semicircle } PAQ$

$\Rightarrow \text{Perimeter} = (5\pi + 1.5\pi + 3.5\pi) \text{ cm}$

$= 10\pi \text{ cm}$

$= 10 \times 3.14 \text{ cm}$

$= 31.4 \text{ cm}$

## Solution 39

Side of a square = 20 cm.

$\therefore$ Area of the square $= (20 \times 20)\text{ cm}^2 = 400\text{ cm}^2$

Diagonal of square $= \sqrt{(20)^2 + (20)^2} = \sqrt{800} = 20\sqrt{2}$ cm

$\Rightarrow$ Radius of the quadrant $= 20\sqrt{2}$ cm

$\therefore$ Area of a quadrant $= \frac{1}{4} \times 3.14 \times \left(20\sqrt{2}\right)^2 = 628\text{ cm}^2$

Thus, area of the shaded region = Area of a quadrant – Area of the square

$= (628 - 400)\text{ cm}^2$

$= 228\text{ cm}^2$

## Solution 40

Let AO = OB = r

Perimeter of the given figure = Perimeter of arc APB + OB + Perimeter of arc OQA

$$\Rightarrow 40 = \pi r + r + \frac{\pi r}{2}$$

$$= 40 = \left(\frac{3\pi}{2} + 1\right)r$$

$$= 40 = \left(\frac{3 \times 22}{2 \times 7} + 1\right)r$$

$$= 40 = \left(\frac{33}{7} + 1\right)r$$

$$\Rightarrow 40 = \frac{40}{7}r$$

$$\Rightarrow r = 7 \text{ cm}$$

Now, Area of the shaded region

Area of semicircle APB + Area of semicircle AQO

$$= \left[\left(\frac{1}{2} \times \frac{22}{7} \times 7 \times 7\right) + \left(\frac{1}{2} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\right)\right] \text{cm}^2$$

$$= \left(77 + \frac{77}{4}\right) \text{cm}^2$$

$$= \frac{385}{4} \text{cm}^2$$

$$= 96.25 \text{ cm}^2$$

Solution 41

Let r be the radius of the circle.

Then, circumference of a circle = $2\pi r$

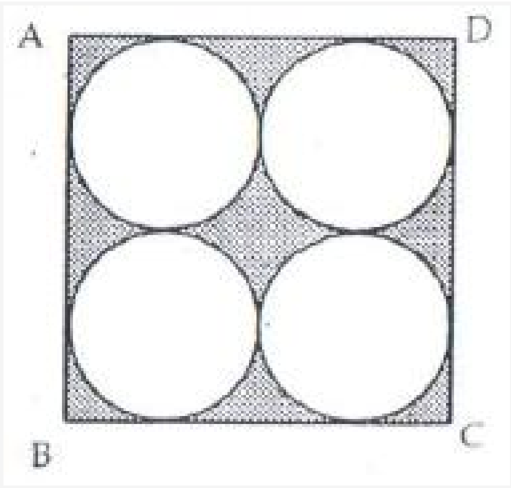
$\Rightarrow 2\pi r = 44$

$\Rightarrow 2 \times \frac{22}{7} \times r = 44$

$\Rightarrow r = 7$ cm

$\therefore$ Area of the quadrant = $\left(\frac{1}{4} \times \frac{22}{7} \times 7 \times 7\right) cm^2 = \frac{77}{2} cm^2 = 38.5\ cm^2$

## Solution 42




Side of the square ABCD = 14 cm

Area of square ABCD = 14 14 = 196 $cm^2$

Radius of each circle = $\frac{14}{4} = 3.5$ cm

Area of the circles = 4 area of one circle

$= 4 \times \pi(3.5)^2$

$= 4 \times \frac{22}{7} \times 3.5 \times 3.5$

$= 154\ cm^2$

Area of shaded region = Area of square - area of 4 circles

= 196 - 154 = 42 $cm^2$

## Solution 43

Length of a rectangle = 8 cm
Breadth of a rectangle = 6 cm
$\therefore$ Area of rectangle ABCD = $8 \times 6 = 48\ cm^2$
Now,
$AC^2 = AB^2 + BC^2 = 8^2 + 6^2 = 64 + 36 = 100$
$\Rightarrow AC = \sqrt{100} = 10$ cm
$\Rightarrow$ Diameter of a circle = 10 cm
$\Rightarrow$ Radius of a circle = 5 cm
$\therefore$ Area of a circle = $\frac{22}{7} \times 5 \times 5 = 78.57\ cm^2$
Thus, area of shaded region = Area of a circle − Area of rectangle ABCD

$= (78.57 - 48)\ cm^2$

$= 30.57\ cm^2$

## Solution 44

Area of a square formed = 484 $m^2$

$\Rightarrow (\text{Side})^2 = 484$

$\Rightarrow \text{Side} = \sqrt{484}\text{ m} = 22 \text{ m}$

$\therefore$ Perimeter of a square = 4 x side = 4 x 22 = 88 m

Let r be the radius of the circle formed.

Now,

Circumference of a circle = Perimeter of square

$\Rightarrow 2 \times \frac{22}{7} \times r = 88$

$\Rightarrow r = \left(88 \times \frac{7}{44}\right) = 14 \text{ m}$

$\therefore$ Area of the circle $= \pi r^2 = \left(\frac{22}{7} \times 14 \times 14\right) m^2 = 616 \text{ } m^2$

Solution 45

Since square ABCD is inscribed in a circle of radius r,

diagonal of a square = AC = 2r

$\therefore$ Area of a square ABCD $= \frac{1}{2} \times (\text{diagonal})^2 = \frac{1}{2} \times (2r)^2 = 2r^2$ sq. units

Solution 46

Cost of fencing a circular field = Rs. 5500

Rate of fencing per metre = Rs. 25

$\therefore$ Perimeter of a circular field = $\dfrac{\text{Cost of fencing}}{\text{Rate per metre}} = \left(\dfrac{5500}{25}\right)$ m = 220 m

Let r be the radius of the circular field.

Then, $2\pi r = 220$

$\Rightarrow 2 \times \dfrac{22}{7} \times r = 220$

$\Rightarrow r = 220 \times \dfrac{7}{44}$

$\Rightarrow r = 35$ m

$\therefore$ Area of the circular field = $\pi r^2 = \left(\dfrac{22}{7} \times 35 \times 35\right) = 3850\ m^2$

Cost of ploughing per $m^2$ = 50 paise

$\therefore$ Cost of ploughing 3850 $m^2$ = Rs. $\dfrac{50}{100} \times 3850$ = Rs. 1925

## Solution 47

Area of rectangle = (120 × 90) $m^2$

= 10800 $m^2$

Area of circular lawn = [Area of rectangle - Area of park excluding circular lawn]

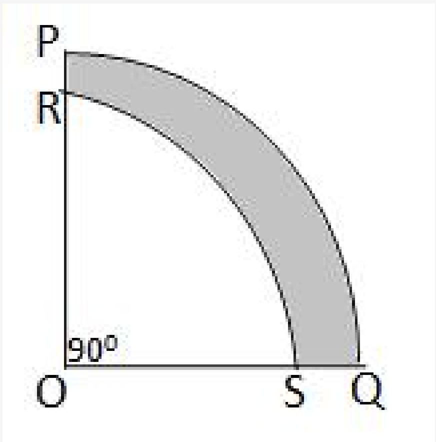
= [10800 - 2950] $m^2$ = 7850 $m^2$

Area of circular lawn = 7850 $\Rightarrow \pi r^2 = 7850\ m^2$

$$3.14 \times r^2 = 7850 \text{ m}^2$$
$$r^2 = \left(\frac{7850}{3.14}\right) \text{m}^2$$
$$= 2500 \text{ m}^2$$
$$r = \sqrt{2500} \text{ m}$$
$$\text{or} \quad r = 50 \text{ m}$$

Hence, radius of the circular lawn = 50 m

## Solution 48

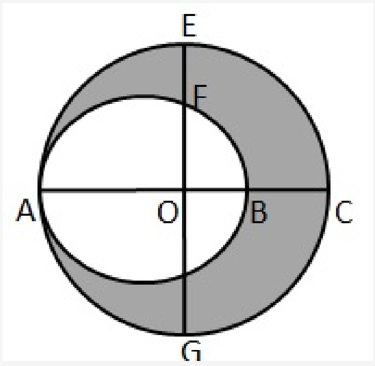



Area of flower bed = (area of quadrant OPQ)

-(area of the quadrant ORS)

$$= \left[\frac{1}{4}\pi r_1^2 - \frac{1}{4}\pi r_2^2\right]$$
$$= \left[\frac{1}{4} \times \frac{22}{7} \times 21 \times 21 - \frac{1}{4} \times \frac{22}{7} \times 14 \times 14\right] \text{m}^2$$
$$= [346.5 - 154] \text{m}^2 = 192.5 \text{ m}^2$$

## Solution 49




Diameter of bigger circle = AC = 54 cm

Radius of bigger circle = $\frac{AC}{2}$

$$= \left(\frac{54}{2}\right) cm$$
$$= 27 \text{ cm}$$

Diameter AB of smaller circle

$$= AC - BC$$
$$= (54 - 10) cm = 44 cm$$

Radius of smaller circle = $\frac{44}{2} cm = 22 cm$

Area of bigger circle = $\pi R^2 = \left(\frac{22}{7} \times 27 \times 27\right) cm^2$

$= 2291.14\ \text{cm}^2$

Area of smaller circle = $\pi r^2 = \left(\frac{22}{7} \times 22 \times 22\right) \text{cm}^2$

$= 1521.11\ \text{cm}^2$

Area of shaded region = area of bigger circle - area of smaller circle

$$= (2291.14 - 1521.14)\ \text{cm}^2$$
$$= 770\ \text{cm}^2$$

## Solution 50

Clearly, $AB = BC = CE = 3.5$ cm and $DE = 2$ cm

$\Rightarrow CD = DE + EC = 2 + 3.5 = 5.5$ cm

$\therefore$ Area of the shaded part = Area of trapezium ABCD − Area of quadrant BCE

$$= \left[\left\{\frac{1}{2}(AB + CD) \times BC\right\} - \left\{\frac{1}{4} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\right\}\right] \text{cm}^2$$
$$= \left[\left\{\frac{1}{2}(3.5 + 5.5) \times 3.5\right\} - \frac{77}{8}\right] \text{cm}^2$$
$$= \left[\left\{\frac{1}{2} \times 9 \times 3.5\right\} - \frac{77}{8}\right] \text{cm}^2$$
$$= [15.75 - 9.625]\ \text{cm}^2$$
$$= 6.125\ \text{cm}^2$$

## Solution 51

Area of the minor segment ACBA = Area of sector OACBO − Area of $\Delta OAB$

$$= \left[\left(\frac{22}{7} \times 35 \times 35 \times \frac{90}{360}\right) - \left(\frac{1}{2} \times 35 \times 35\right)\right] cm^2$$
$$= [962.50 - 612.50]\ cm^2$$
$$= 350\ cm^2$$

Area of a circle $= \frac{22}{7} \times 35 \times 35 = 3850\ cm^2$

$\therefore$ Area of the major segment = Area of a circle − Area of a minor segment

$$= (3850 - 350)\ cm^2$$
$$= 3500\ cm^2$$

# R S Aggarwal and V Aggarwal Solution for Class 10 Mathematics Chapter 18 - Areas of Circle, Sector and Segment Page/Excercise 18B

Solution 1

Circumference of circle = 2 $\pi$ r = 39.6 cm

$$\Rightarrow 2 \times \frac{22}{7} \times r = 39.6$$
$$r = \left(39.6 \times \frac{7}{44}\right) cm = 6.3$$
$$r = 6.3\ cm$$

Area of circle $= \pi r^2 = \left(\frac{22}{7} \times 6.3 \times 6.3\right) cm^2$

$$= 124.74\ cm^2$$

Solution 2

Let r be the radius of the circle.

Area of the circle = 98.56 $cm^2$

$\Rightarrow \pi r^2 = 98.56$

$\Rightarrow \frac{22}{7} \times r^2 = 98.56$

$\Rightarrow r^2 = \frac{98.56 \times 7}{22}$

$\Rightarrow r^2 = 31.36$

$\Rightarrow r = 5.6$

$\therefore$ Circumference of a circle $= 2 \times \frac{22}{7} \times 5.6 = 35.2$ cm

Solution 3

Let r be the radius of the circle.

$\Rightarrow$ Diameter of a circle = 2r

And, circumference of a circle = $2\pi r$

It is given that,

Circumference of a circle – Diameter of a circle = 45 cm

$\Rightarrow 2\pi r - 2r = 45$

$\Rightarrow 2r(\pi - 1) = 45$

$$\Rightarrow 2r\left(\frac{22}{7} - 1\right) = 45$$

$$\Rightarrow r\left(\frac{22-7}{7}\right) = \frac{45}{2}$$

$$\Rightarrow r \times \frac{15}{7} = \frac{45}{2}$$

$$\Rightarrow r = \frac{45 \times 7}{15 \times 2}$$

$\Rightarrow r = 10.5$ cm

$$\therefore \text{Circumference of a circle} = 2 \times \frac{22}{7} \times 10.5 = 66 \text{ cm}$$

## Solution 4

Area of square = $(\text{side})^2 = 484 \text{ cm}^2$

$\Rightarrow \text{side} = \sqrt{484} \text{ cm} = 22\text{cm}$

Perimeter of square = 4 side = 4 22 = 88 cm

Circumference of circle = Perimeter of square

$$2\pi r = 88\text{cm} \Rightarrow r = \frac{88 \times 7}{2 \times 22} = 14 \text{ cm}$$

$$\text{Area of circle} = \pi r^2 = \left(\frac{22}{7} \times 14 \times 14\right)\text{cm}^2 = 616\text{cm}^2$$

## Solution 5

Area of equilateral = $\frac{\sqrt{3}a^2}{4} = 121\sqrt{3}$

$$a^2 = 121 \times \frac{\sqrt{3}}{\sqrt{3}} \times 4$$

$$a^2 = 484 \Rightarrow a = \sqrt{484}$$

$$a = 22 \text{ cm}$$

Perimeter of equilateral triangle = 3a = (3 22) cm

= 66 cm

Circumference of circle = Perimeter of circle

$2\pi r = 66 \Rightarrow r = 66 \times \frac{7}{22 \times 2} = 10.5\text{cm}$

Area of circle = $\pi r^2 = \left(\frac{22}{7} \times 10.5 \times 10.5\right)\text{cm}^2$

$= 346.5 \text{ cm}^2$

## Solution 6

The length of a chain used as the boundary of a semicircular park is 108 m. Find the area of the park.

Let r be the radius of the semicircular park.
Now, perimeter of a semicircular park = 108 m

$\Rightarrow \pi r + 2r = 108$

$\Rightarrow \left(\frac{22}{7} + 2\right)r = 108$

$\Rightarrow \left(\frac{22+14}{7}\right)r = 108$

$\Rightarrow \frac{36}{7}r = 108$

$\Rightarrow r = \frac{108 \times 7}{36} = 21 \text{ cm}$

$\therefore$ Area of the park $= \frac{1}{2} \times \frac{22}{7} \times 21 \times 21 = 693 \text{ m}^2$

## Solution 7

Let the radii of circles be x cm and (7 - x) cm

Then,

$2\pi x - [2\pi(7 - x)] = 8$

$2\pi x - [14\pi - 2\pi x] = 8$

$2\pi x - 14\pi + 2\pi x = 8$

$4\pi x - 14\pi = 8$

$2\pi x = 4 + 7\pi$

$2\pi x = 4 + 22$

$2\pi x = 26$

Substitute the value of $2\pi x$ in $2\pi(7 - x)$

$= 14\pi - 2\pi x = 14 \times \frac{22}{7} - 26$

$= 44 - 26 = 18 \text{ cm}$

Circumference of the circles are 26 cm and 18 cm

## Solution 8

Area of outer circle = $\pi r_1^2 = \left(\frac{22}{7} \times 23 \times 23\right) cm^2$

= 1662.5 $cm^2$

$$\text{Area of inner circle} = \pi r_2^2 = \left(\frac{22}{7} \times 12 \times 12\right) cm^2$$
$$= 452.2\ cm^2$$

Area of ring = Outer area - inner area

= (1662.5 - 452.5) $cm^2 = 1210\ cm^2$

## Solution 9

Inner radius of the circular park = 17 m

Width of the path = 8 m

Outer radius of the circular park = (17 + 8)m = 25 m

Area of path = $\pi\left[(25)^2 - (17)^2\right] cm^2$

$$= \pi(25 + 17)(25 - 17) m^2$$
$$= \left[\frac{22}{7} \times 42 \times 8\right] m^2$$

$\therefore$ Area = 1056 $m^2$

## Solution 10

Let r m and R m be the radii of inner circle and outer boundaries respectively.

Then, 2 $\pi$ r = 352 and 2 $\pi$ R = 396

$$r = \frac{352}{2\pi}, \quad R = \frac{396}{2\pi}$$

Width of the track = (R - r) m

$$= \left(\frac{396}{2\pi} - \frac{352}{2\pi}\right)m = \left(\frac{44}{2\pi}\right)m$$
$$= \left(\frac{44}{2} \times \frac{7}{22}\right)m = 7 \text{ m}$$

Area the track = $\pi\left(R^2 - r^2\right) = \pi(R + r)(R - r)$

$$= \left[\pi\left(\frac{352}{2\pi} + \frac{396}{2\pi}\right) \times 7\right]m^2$$
$$= \left[\left(\pi \times \frac{748}{2\pi}\right) \times 7\right]m^2 = (374 \times 7)m^2$$
$$= 2618 \text{ m}^2$$

## Solution 11

Length of the arc $= \frac{2\pi r\theta}{360}$, r = 21 cm, $\theta = 150°$

$$= \left(\frac{2\pi \times 21 \times 150}{360}\right)\text{cm} = (17.5\,\pi)\text{cm}$$

Length of arc = $\left(17.5 \times \frac{22}{7}\right)\text{cm} = 55\,\text{cm}$

Area of the sector = $\frac{\pi r^2\theta}{360} = \left(\frac{\pi \times 21 \times 21 \times 150}{360}\right)\text{cm}^2$

$$= \left(\frac{22}{7} \times 183.75\right)\text{cm}^2 = 577.5\ \text{cm}^2$$

Solution 12

Radius of a circle = r = 10.5 cm

Area of a sector = 69.3 $\text{cm}^2$

Now, area of the sector = $\frac{\pi r^2\theta}{360}$

$$\Rightarrow 69.3 = \frac{\frac{22}{7} \times 10.5 \times 10.5 \times \theta}{360^\circ}$$

$$\Rightarrow 69.3 = \frac{11 \times 1.5 \times 10.5 \times \theta}{180}$$

$$\Rightarrow \theta = \frac{69.3 \times 180}{11 \times 1.5 \times 10.5}$$

$$\Rightarrow \theta = 72^\circ$$

Solution 13

Length of arc = $\dfrac{2\pi r\theta}{360} = 16.5 \text{ cm}$

$$2 \times \frac{22}{7} \times r \times \frac{54^\circ}{360^\circ} = 16.5$$
$$r = \frac{16.5 \times 7 \times 360}{2 \times 22 \times 54} = 17.5 \text{cm}$$

Circumference of circle = 2 $\pi$ r

$$\left(2 \times \frac{22}{7} \times 17.5\right) = 110 \text{ cm}$$

Area of circle = $\pi r^2 = \left(\dfrac{22}{7} \times 17.5 \times 17.5\right) \text{cm}^2$

$$= 962.5 \text{ cm}^2$$

## Solution 14

Radius of the circle = r = 7 cm

Central angle = $\theta$ = 90°

$$\text{Area of the minor segment} = \frac{\pi r^2 \theta}{360} - \frac{1}{2} r^2 \sin\theta$$

$$= \frac{22}{7} \times 7 \times 7 \times \frac{90}{360} - \frac{1}{2} \times 7 \times 7 \times \sin 90°$$

$$= \frac{77}{2} - \frac{49}{2}$$

$$= \frac{77 - 49}{2}$$

$$= \frac{28}{2}$$

$$= 14\,\text{cm}^2$$

$$\text{Area of a circle} = \pi r^2 = \frac{22}{7} \times 7 \times 7 = 154\ \text{cm}^2$$

Area of the major segment = Area of a circle – Area of the minor segment

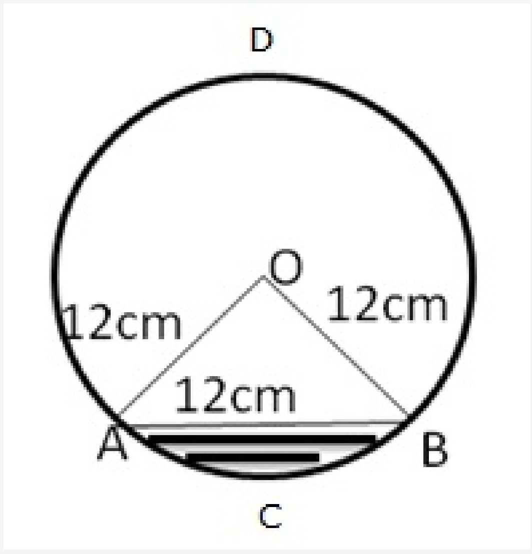
$$= (154 - 14)\ \text{cm}^2$$

$$= 140\ \text{cm}^2$$

## Solution 15

$\triangle$ OAB is equilateral.

So, $\angle$ AOB = 60




$$\begin{aligned} arcACB &= \left(2\pi \times 12 \times \frac{60}{360}\right) cm \\ &= 4\pi \ cm \\ &= (4 \times 3.14) cm \\ &= 12.56 \ cm \end{aligned}$$

Length of arc BDA = (2 $\pi$ 12 - arc ACB) cm

= (24 $\pi$ - 4 $\pi$ ) cm = (20 $\pi$ ) cm

= (20 3.14) cm = 62.8 cm

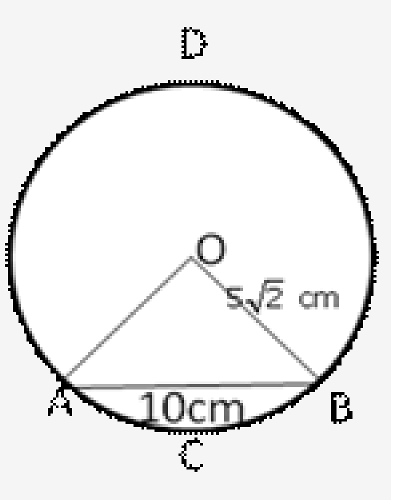
Area of the minor segment ACBA

$$\begin{aligned} &= \left[\pi \times (12)^2 \times \frac{60}{360} - \frac{\sqrt{3}}{4} \times (12)^2\right] cm^2 \\ &= \left(3.14 \times 12 \times 12 \times \frac{60}{360} - \frac{1.73}{4} \times 12 \times 12\right) cm^2 \\ &= (75.36 - 62.28) cm^2 = 13.08 \ cm^2 \end{aligned}$$

Solution 16

Let OA = $5\sqrt{2}$ cm , OB = $5\sqrt{2}$ cm

And AB = 10 cm




Then, $OA^2 + OB^2 = AB^2$

$\Rightarrow \angle AOB = 90^\circ$

Area of the sector OACBO

$$= \frac{\pi r^2 \theta}{360} \text{ cm}^2$$

$$= \left(3.14 \times (5\sqrt{2}) \times (5\sqrt{2}) \times \frac{90}{360}\right) \text{cm}^2$$

$$= 39.25 \text{ cm}^2$$

Area of $\Delta$AOB = $\frac{1}{2} r^2 \sin\theta = \left(\frac{1}{2} \times 5\sqrt{2} \times 5\sqrt{2} \times \sin 90^\circ\right)$

$$= 25 \text{ cm}^2$$

Area of minor segment = (area of sector OACBO) - (area of $\Delta$ OAB)

$$= (39.25 - 25)\ cm^2 = 14.25\ cm^2$$

Area of the major segment BDAB

= area of circle – area of minor segment

$$= \left(\frac{22}{7} \times 5\sqrt{2} \times 5\sqrt{2} - 14.25\right) cm^2$$

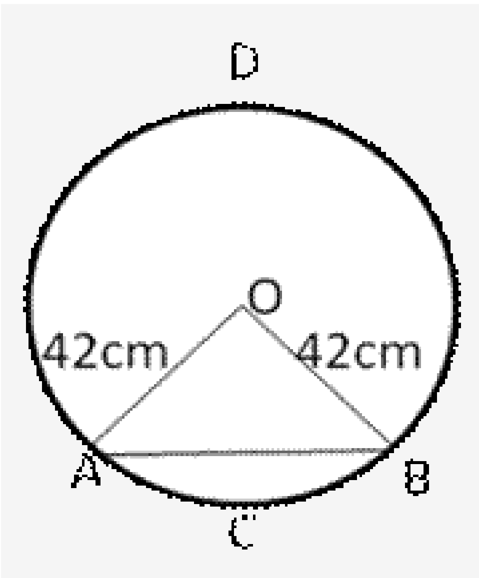
$$= \left(\frac{1100}{7} - 14.25\right) cm^2 = (157 - 14.25)\ cm^2$$

$$= 142.75\ cm^2$$

## Solution 17

Area of sector OACBO

$$= \frac{\pi r^2 \theta}{360} cm^2 = \left(\frac{22}{7} \times 42 \times 42 \times \frac{120}{360}\right) cm^2 = 1848\ cm^2$$



$$\text{Area of } \Delta OAB = \frac{1}{2} r^2 \sin\theta$$
$$= \left(\frac{1}{2} \times 42 \times 42 \times \sin 120°\right)$$
$$= \left(21 \times 42 \times \frac{\sqrt{3}}{2}\right) cm^2$$
$$= (21 \times 21 \times 1.73)\, cm^2 = 762.93\, cm^2$$

Area of minor segment ACBA

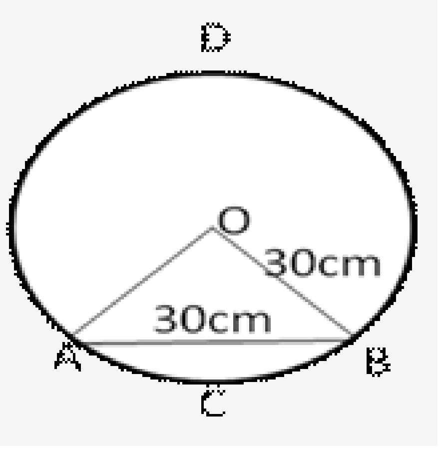
$$= (\text{area of sector OACBO}) - (\text{area of the } \Delta OAB)$$
$$= (1848 - 762.93)\, cm^2 = 1085.07\, cm^2$$

Area of major segment BADB

$$= (\text{area of the circle}) - (\text{area of minor segment})$$
$$= \frac{22}{7} \times 42 \times 42 - 1085.07$$
$$= (5544 - 1085.07)\, cm^2 = 4458.93\, cm^2$$

## Solution 18

Let AB be the chord of circle of centre O and radius = 30 cm such that $\angle$ AOB = 60°



Area of the sector OACBO

$$= \frac{\pi r^2 \theta}{360} \text{cm}^2$$
$$= \left(3.14 \times 30 \times 30 \times \frac{60}{360}\right) \text{cm}^2$$
$$= 471 \text{ cm}^2$$

Area of $\Delta$OAB = $\frac{1}{2} r^2 \sin\theta = \left(\frac{1}{2} \times 30 \times 30 \times \sin 60^\circ\right) \text{cm}^2$

$$= \left(\frac{1}{2} \times 30 \times 30 \times \frac{\sqrt{3}}{2}\right) \text{cm}^2 = \left(225\sqrt{3}\right) \text{cm}^2$$
$$= (225 \times 1.73) \text{cm}^2 = 389.25 \text{ cm}^2$$

Area of the minor segment ACBA

= (area of the sector OACBO) - (area of the $\Delta$OAB)

=(471 - 389.25) $\text{cm}^2$ = 81.75 $\text{cm}^2$

Area of the major segment BADB

= (area of circle) - (area of the minor segment)

= [(3.14 × 30 × 30) - 81.75)] $\text{cm}^2$ = 2744.25 $\text{cm}^2$

## Solution 19

Let the major arc be x cm long

Then, length of the minor arc = $\frac{1}{5}x$ cm

Circumference = $\left(x + \frac{1}{5}x\right)\text{cm} = \frac{6x}{5}$ cm

$$\frac{6x}{5} = 2 \times \frac{22}{7} \times \frac{21}{2} \Rightarrow x = 55 \text{ cm}$$

$$\text{Required area} = \left(\frac{1}{2} \times 55 \times \frac{21}{2}\right)\text{cm}^2$$

$$\left[\text{Area} = \frac{1}{2}rl\right]$$

$$= 288.75 \text{ cm}^2$$

## Solution 20

In 2 days, the short hand will complete 4 rounds

$\therefore$ Distance travelled by its tip in 2 days

=4(circumference of the circle with r = 4 cm)

= $(4 \times 2\pi \times 4)$ cm = $32\pi$ cm

In 2 days, the long hand will complete 48 rounds

$\therefore$ length moved by its tip

= 48(circumference of the circle with r = 6cm)

= $(48 \times 2\pi \times 6)$ cm = $576\pi$ cm

$\therefore$ Sum of the lengths moved

$= (32\pi + 576\pi) = 608\pi$ cm

$= (608 \times 3.14)$ cm $= 1909.12$ cm

## Solution 21

Let r be the radius of a circle.

Circumference of a circle = 88 cm

$\Rightarrow 2\pi r = 88$

$\Rightarrow 2 \times \frac{22}{7} \times r = 88$

$\Rightarrow r = \frac{88 \times 7}{2 \times 22}$

$\Rightarrow r = 14 \text{cm}$

$\therefore$ Area of a quadrant $= \frac{1}{4}\pi r^2 = \frac{1}{4} \times \frac{22}{7} \times 14 \times 14 = 154 \text{ cm}^2$

## Solution 22

Area of plot which cow can graze when r = 16 m is $\pi r^2$

$$= \left(\frac{22}{7} \times 16 \times 16\right) m^2$$

= 804.5 m2

Area of plot which cow can graze when radius is increased to 23 m

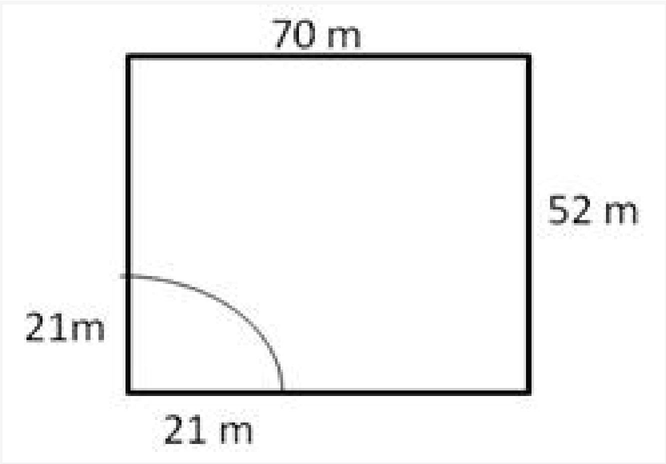
$$= \left(\frac{22}{7} \times 23 \times 23\right) m^2$$

$$= 1662.57\ m^2$$

Additional ground = Area covered by increased rope - old area

= (1662.57 - 804.5) $m^2$ = 858 $m^2$

## Solution 23




Area which the horse can graze = Area of the quadrant of radius 21 m

$$= \left(\frac{1}{4} \times \frac{22}{7} \times 21 \times 21\right) m^2$$

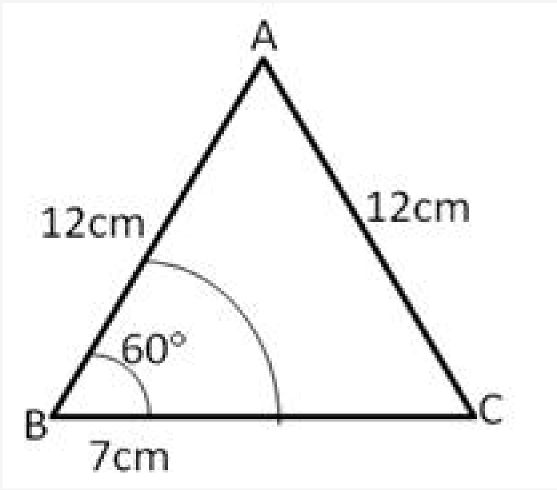
$$= 346.5\, m^2$$

Area ungrazed = $\left[(70 \times 52) - 346.5\right] m^2$

$$= 3293.5\, m^2$$

## Solution 24

Each angle of equilateral triangle is 60




Area which cannot be grazed =(area of equilateral $\Delta ABC$)

- (area of the sector with $r = 7m, \theta = 60°$)

$$= \left[\frac{\sqrt{3}}{4}\times(12)^2 - \frac{22}{7}\times(7)^2\times\frac{60}{360}\right]m^2$$

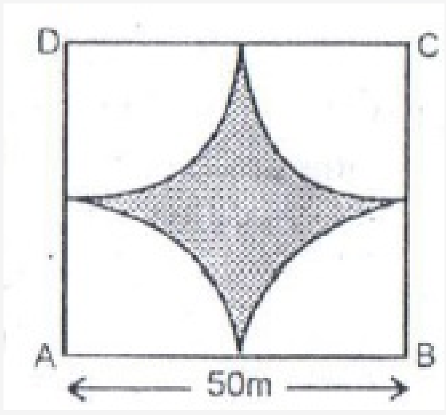
$$= \left[(\sqrt{3}\times 12\times 3) - \frac{(22\times 7)}{6}\right]$$

$$= 62.35 - 25.66\ m^2$$

$$= 36.68\ m^2$$

Area that the horse cannot graze is 36.68 m2

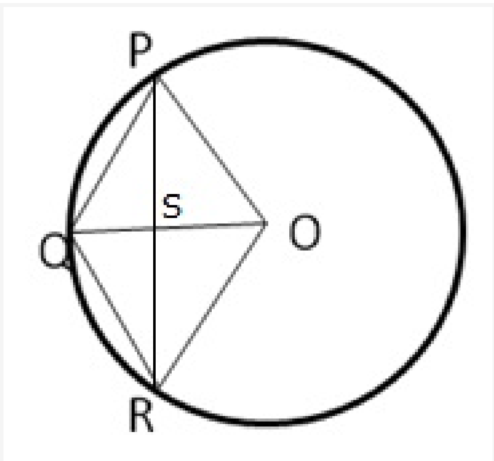
## Solution 25




Ungrazed area

$$
\begin{aligned}
&= \text{shaded area}\\
&= \left[(50 \times 50) - \frac{4 \times \pi \times (25)^2 \times 90}{360}\right] m^2\\
&= [2500 - 3.14 \times 25 \times 25] m^2\\
&= [2500 - 1962.5] m^2\\
&= 537.5\ m^2
\end{aligned}
$$

## Solution 26



OP = OR = OQ = r

Let OQ and PR intersect at S

We know the diagonals of a rhombus bisect each other at right angle.

Therefore we have

$$OS = \frac{1}{2}r \text{ and } \angle OSR = 90^\circ$$

$$\therefore SR = \sqrt{OR^2 - OS^2}$$

$$= \sqrt{r^2 - \frac{r^2}{4}} = \frac{\sqrt{3}r}{2}$$

$$\therefore PR = 2 \times SR = \sqrt{3}r$$

$$\text{Area of rhombus} = \frac{1}{2} \times OQ \times PR$$

$$= \frac{1}{2} \times r \times \sqrt{3}r = \frac{\sqrt{3}r^2}{2}$$

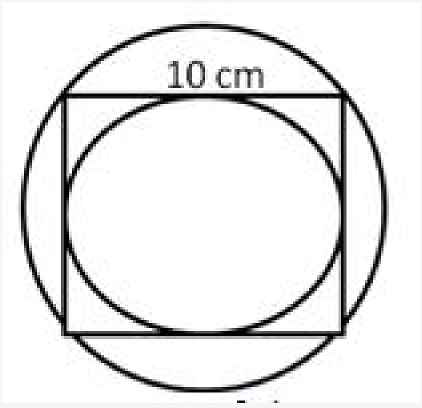
$$\therefore \frac{\sqrt{3}r^2}{2} = 32\sqrt{3} \Rightarrow r^2 = \frac{32\sqrt{3}}{\sqrt{3}} \times 2 = 64cm$$

$$r = 8 \text{ cm}$$

## Solution 27

Diameter of the inscribed circle = Side of the square = 10 cm

$\therefore$ Radius of the inscribed circle = 5 cm




Diameter of the circumscribed circle

= Diagonal of the square

$= \left(\sqrt{2} \times 10\right) cm$

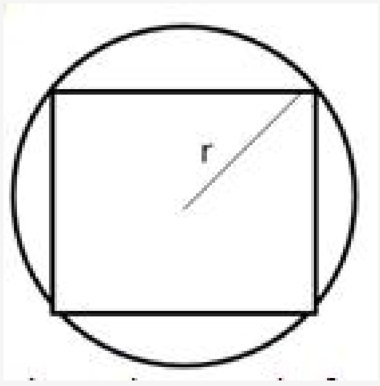
Radius of circumscribed circle = $5\sqrt{2}$ cm

(i)Area of inscribed circle = $\left(\frac{22}{7} \times 5 \times 5\right) = 78.57\ cm^2$

(ii)Area of the circumscribed circle $= \left(\frac{22}{7} \times 5\sqrt{2} \times 5\sqrt{2}\right) = 157.14\,cm^2$

## Solution 28

Let the radius of circle be r cm



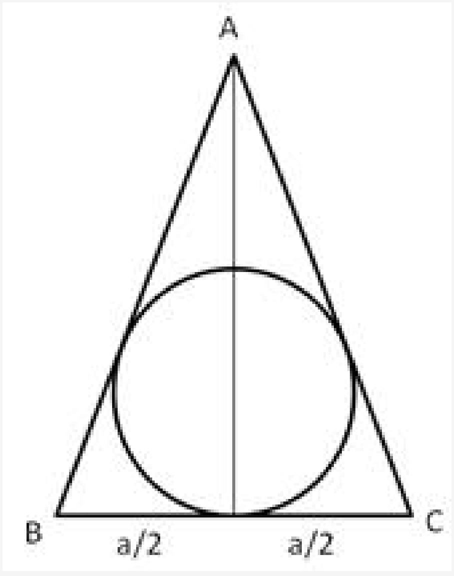

Then diagonal of square = diameter of circle = 2r cm

Area of the circle = $\left(\pi r^2\right) = cm^2$

$$\text{Area of square} = \frac{1}{2} \times (\text{diagonal})^2$$
$$= \frac{1}{2} \times 4r^2 = 2r^2 \text{ cm}$$
$$\text{Ratio} = \frac{\text{Area of circle}}{\text{Area of square}} = \frac{\pi r^2}{2r^2} = \frac{\pi}{2} = (\pi : 2)$$

## Solution 29

Let the radius of circle be r cm




$$\text{Then, } \pi r^2 = 154$$
$$\Rightarrow \quad r^2 = \left(154 \times \frac{7}{22}\right)$$
$$\Rightarrow \quad r = 7 \text{ cm}$$

Let each side of the triangle be a cm

And height be h cm

$$\text{Then, } \quad r = \frac{h}{3}$$
$$\Rightarrow \quad h = 3r = 21 \text{ cm}$$
$$h = \sqrt{a^2 - \frac{a^2}{4}} = \frac{\sqrt{3a^2}}{2} = \frac{\sqrt{3}a}{2} = 21$$
$$a = \frac{42}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 14\sqrt{3} \text{ cm}$$
$$\text{Perimeter} = 3a = \left(3 \times 14 \times \sqrt{3}\right) = (42 \times 1.73) \text{ cm}$$
$$= 72.66 \text{ cm}$$

## Solution 30

Radius of the wheel = 42 cm

Circumference of wheel =2 $\pi$ r = $\left(2 \times \frac{22}{7} \times 42\right)$cm = 264 cm

Distance travelled = 19.8 km = 1980000 cm

Number of revolutions = $\left(\frac{1980000}{264}\right) = 7500$

## Solution 31

Radius of wheel = 2.1 m

Circumference of wheel = $(2\pi r)$m = $\left(2 \times \frac{22}{7} \times 2.1\right)$m = 13.2 m

Distance covered in one revolution = 13.2 m

Distance covered in 75 revolutions = (13.2 75) m = 990 m

= $\frac{990}{1000}$ km

Distance a covered in 1 minute = $\frac{99}{100}$km

Distance covered in 1 hour = $\left(\frac{99}{100} \times 60\right)$km = 59.4 km

## Solution 32

Distance covered by the wheel in 1 revolution

$$= \left(\frac{4.95 \times 1000 \times 100}{2500}\right) \text{cm} = 198 \text{ cm}$$

$\therefore$ The circumference of the wheel = 198 cm

Let the diameter of the wheel be d cm

$$\text{Then,} \quad \pi d = 198 \Rightarrow \frac{22}{7} \times d = 198$$

$$\Rightarrow \quad d = \frac{198 \times 7}{22} = 63 \text{ cm}$$

Hence diameter of the wheel is 63 cm

## Solution 33

Radius of the wheel $= r = \dfrac{60}{2} = 30 \text{ cm}$

Circumference of the wheel = 2 $\pi$ r = $\left(2 \times \dfrac{22}{7} \times 30\right) \text{cm}$

$$= \frac{1320}{7} \text{cm}$$

Distance covered in 140 revolution

$$= \left(\frac{1320}{7} \times 140\right) cm = (1320 \times 20) cm$$

$$= 26400 \, cm = \frac{26400}{100} m = 264 \, m = \frac{264}{1000} km$$

Distance covered in one hour = $\left(\frac{264}{1000} \times 60\right) km = 15.84 km$

## Solution 34

Distance covered by a wheel in 1minute

$$= \left(\frac{72.6 \times 1000 \times 100}{60}\right) cm = 121000 \ cm$$

Circumference of a wheel = $\left(2 \times \frac{22}{7} \times 70\right) cm = 440 cm$

Number of revolution in 1 min = $\left(\frac{121000}{440}\right) = 275$

## Solution 35

Radius of the front wheel = 40 cm = $\frac{2}{5} m$

Circumference of the front wheel= $\left(2\pi \times \frac{2}{5}\right) m = \frac{4\pi}{5} m$

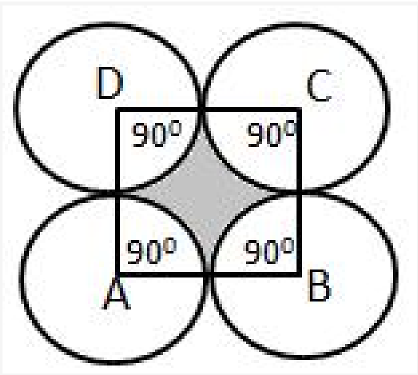
Distance moved by it in 800 revolution

$$= \left(\frac{4\pi}{5} \times 800\right) m = (640\pi) m$$

Circumference of rear wheel = (2 $\pi$ 1)m = (2 $\pi$) m

Required number of revolutions = $\left(\frac{640\pi}{2\pi}\right) = 320$

## Solution 36




Each side of the square is 14 cm

Then, area of square = (14 × 14) $cm^2$

= 196 $cm^2$

Thus, radius of each circle 7 cm

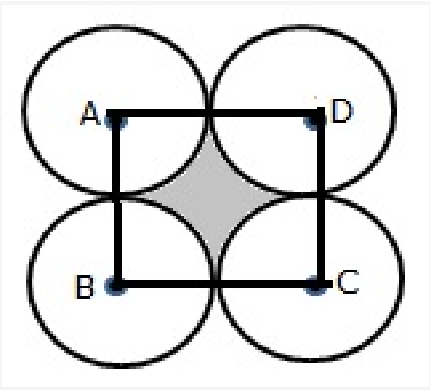
Required area = area of square ABCD

-4 (area of sector with r = 7 cm, $\theta$ = 90°)

$$= \left[196 - 4 \times \frac{22}{7} \times 7 \times 7 \times \frac{90}{360}\right] cm^2$$
$$= [196 - 154] cm^2$$
$$= 42\ cm^2$$

Area of the shaded region = 42 $cm^2$

## Solution 37




Let A, B, C, D be the centres of these circles

Join AB, BC, CD and DA

Side of square = 10 cm

Area of square ABCD

$$= (10 \times 10) cm^2$$
$$= 100\ cm^2$$

Area of each sector = $\left(\pi r^2 \times \frac{\theta}{360}\right) = 3.14 \times 5 \times 5 \times \frac{90}{360}$

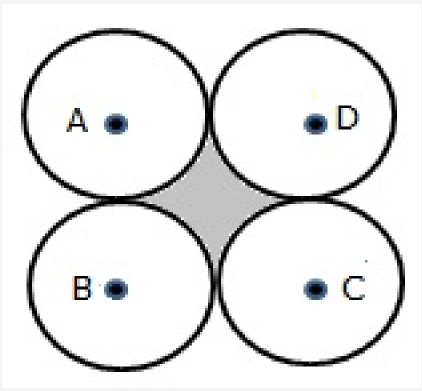
$= 19.625\ cm^2$

Required area = [area of sq. ABCD - 4(area of each sector)]

$= (100 - 4\ 19.625)\ cm^2$

$= (100 - 78.5)\ cm^2 = 21.5\ cm^2$

## Solution 38




Required area = [area of square - areas of quadrants of circles]

Let the side = 2a unit and radius = a units

Area of square = (side side) = (2a 2a) sq. units

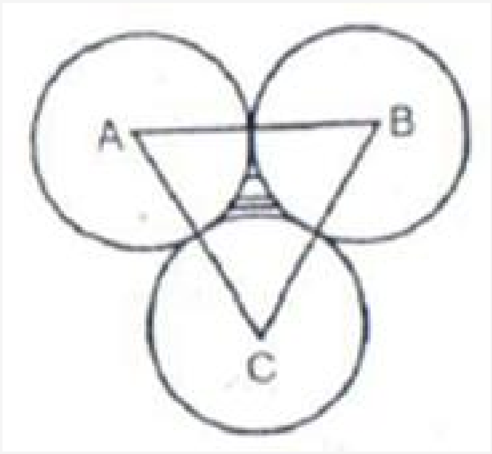
$= 4a^2$ sq. units

$$\text{Area of quadrant} = \frac{1}{4}\pi r^2$$

$$\text{Area of 4 quadrants} = 4 \times \frac{1}{4}\pi r^2 = \pi r^2 = \frac{22}{7} \times a \times a = \frac{22}{7}a^2 \text{ sq.unit}$$

$$\text{Required area} = \left(4a^2 - \frac{22}{7}a^2\right)\text{sq.unit} = \frac{6a^2}{7}$$

## Solution 39




Let A, B, C be the centres of these circles. Joint AB, BC, CA

Required area=(area of $\Delta$ ABC with each side a = 12 cm)

-3(area of sector with r = 6, $\theta$ = 60°)

$$= \left[\frac{\sqrt{3}}{4} \times (12)^2 - 3 \times \left(3.14 \times (6)^2 \times \frac{60}{360}\right)\right]$$

$$= \left[\frac{\sqrt{3}}{4} \times 12 \times 12 - 3 \times 3.14 \times 6\right] cm$$

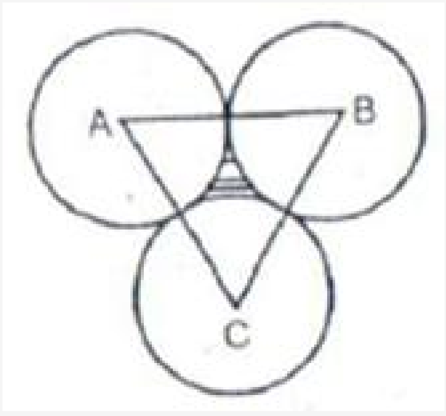
$$= (36 \times 1.73 - 56.52)\, cm^2$$

$$= (62.28 - 56.52)\, cm^2$$

$$= 5.76\ cm^2$$

The area enclosed = 5.76 cm2

## Solution 40




Let A, B, C be the centers of these circles. Join AB, BC, CA

Required area= (area of $\Delta$ ABC with each side 2)

-3[area of sector with r = a cm, $\theta$ = 60°]

$$= \left[\frac{\sqrt{3}}{4} \times (2a)^2 - \frac{3\pi a^2 \times 60}{360}\right]$$
$$= \left(1.73a^2 - 1.57\ a^2\right)$$
$$= 0.16\ a^2$$
$$= \frac{16}{100} a^2$$
$$= \left(\frac{4}{25} a^2\right) \text{sq unit}$$

## Solution 41

Area of the trapezium ABCD = $\frac{1}{2}(AD + BC) \times AB$

$\Rightarrow 24.5 = \frac{1}{2} \times (10 + 4) \times AB$

$\Rightarrow 24.5 = \frac{1}{2} \times 14 \times AB$

$\Rightarrow 24.5 = 7AB$

$\Rightarrow AB = \frac{24.5}{7}$

$\Rightarrow AB = 3.5$ cm

$\Rightarrow$ Radius of a quadrant ABE = 3.5 cm

$\therefore$ Area of a quadrant ABE = $\frac{1}{4} \times \frac{22}{7} \times 3.5 \times 3.5 = 9.625\ cm^2$

Now, Area of the shaded region

= Area of the trapezium ABCD – Area of a quadrant ABE

= 24.5 – 9.625

= 14.875 $cm^2$

Solution 42

i. Total area of 4 sectors $= \left\{\frac{22}{7} \times (14)^2 \times \left(\frac{90}{360} + \frac{90}{360} + \frac{120}{360} + \frac{60}{360}\right)\right\} m^2$

$= \left\{22 \times 2 \times 14 \times \left(\frac{1}{4} + \frac{1}{4} + \frac{1}{3} + \frac{1}{6}\right)\right\} m^2$

$= \left\{616 \times \frac{3+3+4+2}{12}\right\} m^2$

$= \left\{616 \times \frac{12}{12}\right\} m^2$

$= 616\ m^2$

ii. Area of trapezium ABCD $= \frac{1}{2} \times (AD + BC) \times AB$

$= \frac{1}{2} \times (55 + 45) \times 30$

$= 100 \times 15$

$= 1500\ m^2$

Now, area of the remaining portion

= Ar(trapezium ABCD) − Total area of 4 sectors

$= (1500 - 616)\ m^2$

$= 884\ m^2$

## Solution 43

Since $\Delta OAB$ is an equilateral triangle,

$\angle O = \angle A = \angle B = 60°$ and $OA = OB = AB = 12$ cm

$\Rightarrow$ Radius of the circle = r = 6 cm

Area of the shaded circular part

$$= \pi r^2 - \frac{\pi r^2 \times 60}{360}$$

$$= \pi r^2 \left(1 - \frac{1}{6}\right)$$

$$= 3.14 \times 6 \times 6 \times \frac{5}{6}$$

$$= 94.2 \text{ cm}^2$$

Area of shaded triangular region

$$= \frac{\sqrt{3}}{4} \times (\text{side})^2 - \frac{\pi r^2 \times 60}{360}$$

$$= \frac{\sqrt{3}}{4} \times 12 \times 12 - \frac{\frac{22}{7} \times 6 \times 6 \times 60}{360}$$

$$= 36\sqrt{3} - 18.86$$

$$= 36 \times 1.73 - 18.86$$

$$= 62.28 - 18.86$$

$$= 43.42 \text{ cm}^2$$

$\therefore$ Area of the shaded region

= Area of the shaded circular part + Area of shaded triangular region

$$= 94.2 \text{ cm}^2 + 43.42 \text{ cm}^2$$

$$= 137.62 \text{ cm}^2$$

## Solution 44

Length of rectangle ABCD = AB = 80 cm
Breadth of rectangle ABCD = BC = 70 cm
$\therefore$ Area of rectangle ABCD = AB x BC = 80 x 70 = 5600 $cm^2$

In right-angled $\Delta$AED,
$AE^2 = (AD^2 - DE^2) = (70^2 - 42^2) = (70 + 42)(70 - 42) = 112 \times 28 = 4 \times 28 \times 28$
$\Rightarrow AE = 2 \times 28 = 56$ cm
$\therefore$ Area of $\Delta AED = \frac{1}{2} \times DE \times AE = \frac{1}{2} \times 42 \times 56 = 1176\ cm^2$

Area of semi-circle $= \frac{1}{2}\pi \times \left(\frac{70}{2}\right)^2 = \left\{\frac{1}{2} \times \frac{22}{7} \times 35 \times 35\right\} cm^2 = 1925\ cm^2$

Thus, Area of the shaded region
= Area of rectangle ABCD – (Area of $\Delta$AED + Area of semi-circle)
= 5600 – (1176 + 1925)
= 5600 – 3101
= 2499 $cm^2$

## Solution 45

In right-angled $\Delta AED$,

$AD^2 = DE^2 + AE^2 = 12^2 + 9^2 = 144 + 81 = 225$

$\Rightarrow AD = \sqrt{225} = 15\,cm$

Now, Area of $\Delta AED = \frac{1}{2} \times DE \times AE = \frac{1}{2} \times 12 \times 9 = 54\ cm^2$

Length of rectangle ABCD = AB = 20 cm

Breadth of rectangle ABCD = AD = 15 cm

$\therefore$ Area of rectangle ABCD = AB x BC = 20 x 15 = 300 $cm^2$

Area of semi-circle $= \frac{1}{2}\pi \times \left(\frac{15}{2}\right)^2 = \left\{\frac{1}{2} \times 3.14 \times 7.5 \times 7.5\right\} cm^2 = 88.3125\ cm^2$

Thus, Area of the shaded region

= Area of rectangle ABCD + Area of semi-circle − Area of $\Delta AED$

$= 300 + 88.31 - 54$

$= 334.31\ cm^2$

## Solution 46

In right-angled $\Delta BAC$,

$CB^2 = AC^2 + AB^2 = 24^2 + 7^2 = 576 + 49 = 625$

$\Rightarrow CB = \sqrt{625} = 25$ cm

$\Rightarrow OC = \frac{1}{2} CB = \frac{25}{2}$ cm $= 12.5$ cm = radius of the circle

Now, area of $\Delta BAC = \frac{1}{2} \times AC \times AB = \frac{1}{2} \times 24 \times 7 = 84\ cm^2$

Area of the circle $= 3.14 \times 12.5 \times 12.5 = 490.625\ cm^2$

Area of quadrant COD $= \frac{1}{4} \times 3.14 \times 12.5 \times 12.5 = 122.66\ cm^2$

Now, area of the shaded region

= Area of the circle – Area of $\Delta BAC$ – Area of quadrant COD

$= (490.625 - 84 - 122.66)\ cm^2$

$= 283.96\ cm^2$

Solution 47

Since AD ⊥ BC, D is the midpoint of BC.

$\therefore$ $DC = \frac{1}{2} \times 12 = 6$ cm

In right-angled $\Delta ADC$,

$AD^2 = AC^2 - DC^2 = 12^2 - 6^2 = 144 - 36 = 108$

$\Rightarrow AD = \sqrt{108} = 6\sqrt{3}$ cm

$\Rightarrow OD = \frac{6\sqrt{3}}{3} \Rightarrow 2\sqrt{3}$ cm

So, the radius of inscribed circle is $2\sqrt{3}$ cm.

Now, area of the shaded region

= Area of equilateral $\Delta ABC$ – Area of circle

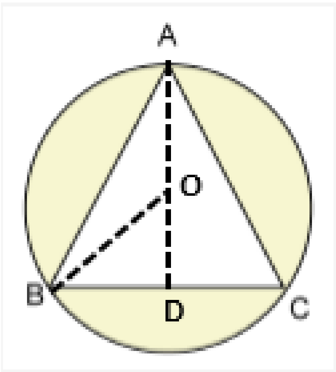
$= \frac{\sqrt{3}}{4} \times 12 \times 12 - 3.14 \times \left(2\sqrt{3}\right)^2$

$= 36\sqrt{3} - 37.68$

$= 62.28 - 37.68$

$= 24.6$ cm$^2$

Solution 48



Let O be the centre of the circumcircle.

Join OB and draw $AD \perp BC$.

Then, OB = 42 cm and $\angle OBD = 30°$

In $\Delta OBD$,

$$\sin 30° = \frac{OD}{OB}$$

$$\Rightarrow \frac{1}{2} = \frac{OD}{42}$$

$$\Rightarrow OD = 21 \text{ cm}$$

Now, $BD^2 = OB^2 - OD^2 = 42^2 - 21^2 = (42+21)(42-21) = 63 \times 21$

$$\Rightarrow BD = \sqrt{63 \times 21} = \sqrt{3 \times 21 \times 21} = 21\sqrt{3} \text{ cm}$$

$$\Rightarrow BC = 2 \times 21\sqrt{3} = 42\sqrt{3} \text{ cm}$$

Now, area of the shaded region

= Area of the circle − Area of an equilateral $\Delta ABC$

$$= \frac{22}{7} \times 42 \times 42 - \frac{\sqrt{3}}{4} \times 42\sqrt{3} \times 42\sqrt{3}$$

$$= (5544 - 2288.79) \text{ cm}^2$$

$$= 3255.21 \text{ cm}^2$$

Solution 49

Let r be the radius of the quadrant.

$$\text{Perimeter of the quadrant} = \left(2r + \frac{2\pi r \times 90}{360}\right)\text{cm}$$

$$= \left(2r + \frac{\pi r}{2}\right)\text{cm}$$

$$= \left(2r + \frac{22r}{7 \times 2}\right)\text{cm}$$

$$= \left(2r + \frac{11r}{7}\right)\text{cm}$$

$$= \left(\frac{14r + 11r}{7}\right)\text{cm}$$

$$= \frac{25r}{7}\text{cm}$$

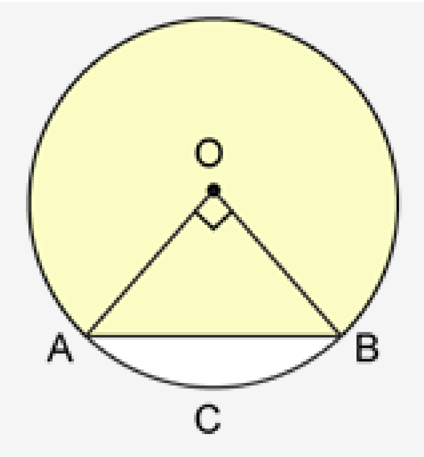
Given, perimeter of a quadrant = 25 cm

$$\Rightarrow \frac{25r}{7} = 25$$

$$\Rightarrow r = 7 \text{ cm}$$

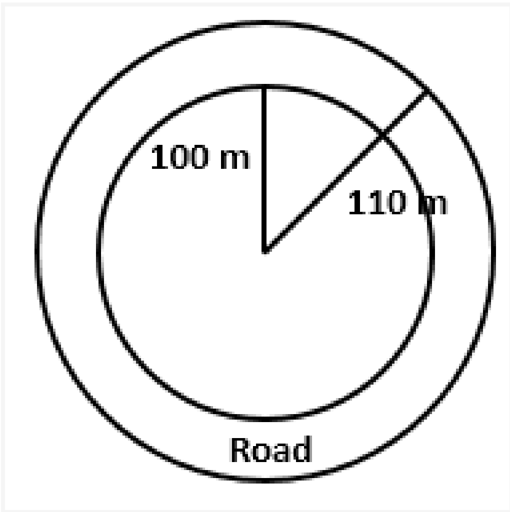
$$\therefore \text{ Area of the quadrant} = \frac{1}{4}\pi r^2 = \left(\frac{1}{4} \times \frac{22}{7} \times 7 \times 7\right)\text{cm}^2 = \frac{77}{2}\text{ cm}^2 = 38.5 \text{ cm}^2$$

## Solution 50



Let O be the center of the circle and AB be the chord.
Now, Area of the minor segment
$= \text{Area of sector OACBO} - \text{Area of } \Delta OAB$
$= 3.14 \times 10 \times 10 \times \frac{90}{360} - \frac{1}{2} \times 10 \times 10$
$= 78.5 - 50$
$= 28.5 \text{ cm}^2$

Solution 51



Area of the road

$= \text{Ar(circular region of radius 110 m)} - \text{Ar(circular region of radius 100 m)}$

$= \pi(110)^2 - \pi(100)^2$

$= \pi(110^2 - 100^2)$

$= 3.14 \times (110 + 100)(110 - 100)$

$= 3.14 \times 210 \times 10$

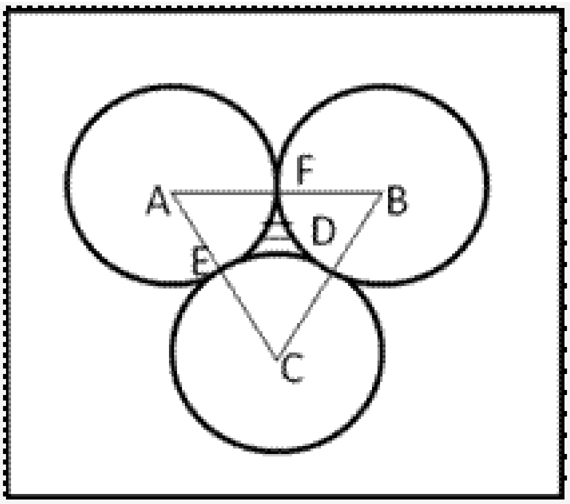
$= 6954 \text{ m}^2$

Cost of levelling per $\text{m}^2$ = Rs. 20

$\therefore$ Cost of levelling $6954 \text{ m}^2$ = Rs. $20 \times 6594$ = Rs. 131880

## Solution 52

Area of equilateral triangle ABC = 49 $\sqrt{3}\ \text{cm}^2$




Let a be its side

$\therefore \quad \frac{\sqrt{3}}{4} a^2 = 49\sqrt{3}$

or $a^2 = 49 \times 4$

$\therefore \quad a = 7 \times 2$

$\Rightarrow a = 14 \text{ cm}$

Area of sector BDF = $\pi r^2 \times \frac{\theta}{360^\circ}$

$$= \frac{22}{7} \times 7 \times 7 \times \frac{60}{360} \text{ cm}$$
$$= \frac{11 \times 7}{3} \text{ cm}^2 = \frac{77}{3} \text{ cm}^2$$

Area of sector BDF = Area of sector CDE = Area of sector AEF

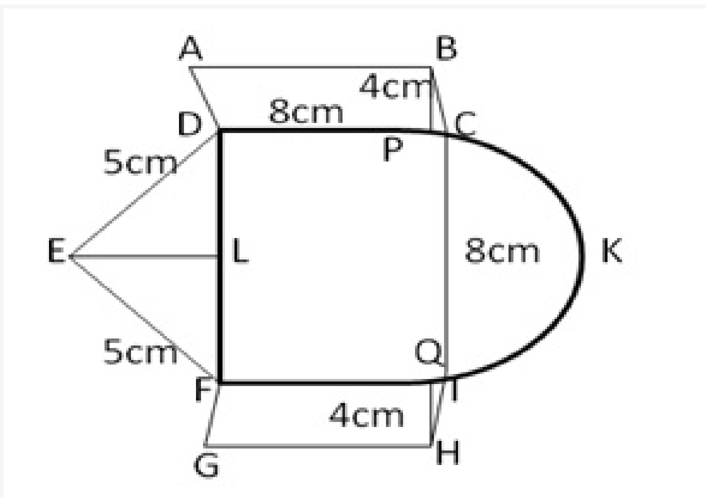
Sum of area of all the sectors

$$= \frac{77}{3} \times 3 \text{ cm}^2 = 77 \text{ cm}^2$$

$\therefore$ Shaded area = Area of $\Delta$ABC - sum of area of all sectors

$$= 49\sqrt{3} - 77 \text{ cm}^2 = (84.77 - 77.00) \text{ cm}^2$$
$$= 7.77 \text{ cm}^2$$

## Solution 53



Given BP $\perp$ CD, HQ $\perp$ FI and EL $\perp$DF,

DC=8 cm, BP = HQ = 4 cm and DE = EF = 5 cm

Area of parallelogram ABCD = BP x DC

$$= 4 \times 8 = 32 \text{ cm}^2$$

Area of parallelogram FGHI = FI x HQ

$$= 8 \times 4 = 32 \text{ cm}^2$$

Area of semicircle CKI = $\frac{1}{2}\pi r^2$

$$= \frac{1}{2} \times 3.14 \times (4)^2 = 25.12 \text{ cm}^2$$

Area of isosceles $\Delta$DEF = $\frac{1}{4} b\sqrt{4a^2 - b^2}$

$$= \frac{1}{4}(8)\sqrt{4(5)^2 - (8)^2} = 2\sqrt{100 - 64}$$

$$= 2\sqrt{36} = 12 \text{ cm}^2$$

Area of square CDFI = $(\text{side})^2 = (8)^2 = 64 \text{ cm}^2$

Area of whole figure = area of $||^{gm}$ ABCD + area of $||^{gm}$ FGHI

+ area of semi-circle CKI + area of $\Delta$DEF

+ area of square CDFI

$$= (32+32+25.12+12+64) \text{ cm}^2$$

$$= 165.12 \text{ cm}^2$$

Solution 54

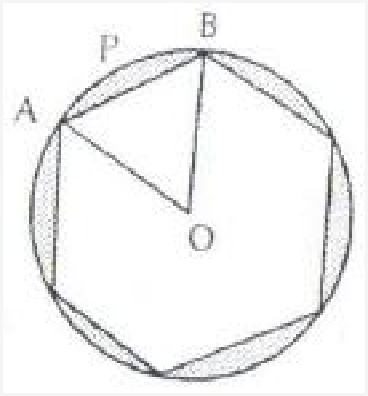
$$\frac{\text{Area of sector with } \theta = 150^\circ}{\text{Area of the circle}} = \frac{\pi \times (6)^2 \times \frac{150}{360}}{\pi \times (6)^2}$$

$$= \frac{150}{360} = \frac{5}{12}$$

$$\text{Required ratio} = \left(36\pi \times \frac{90}{360}\right) : \left(36\pi \times \frac{120}{360}\right) : \left(36\pi \times \frac{150}{360}\right)$$

$$= \frac{1}{4} : \frac{1}{3} : \frac{5}{12} = 3:4:5$$

## Solution 55




ABCDEF is a hexagon

$\therefore \angle AOB = 60$, Radius = 35 cm

Area of sector AOB

$$= \pi r^2 \times \frac{60^\circ}{360^\circ} = \frac{\pi \times 35 \times 35}{6} \text{ cm}^2$$

$$= \frac{3.14 \times 35 \times 35}{6} \text{ cm}^2$$

$$= 641.083 \text{ cm}^2$$

Area of $\Delta$AOB = $\frac{\sqrt{3}}{4} \times r^2 = \frac{\sqrt{3}}{4} \times 35 \times 35 \text{ cm}^2$

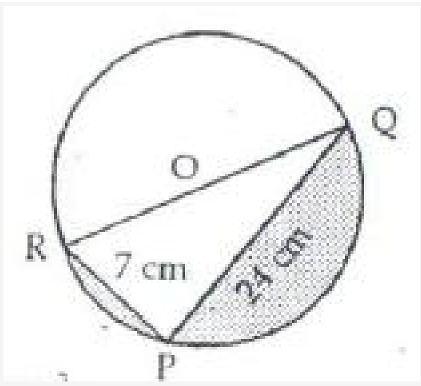
$= 530.425\ cm^2$

Area of segment APB = (641.083 = 530.425) $cm^2$ = 110.658 $cm^2$

Area of design (shaded area) = 6 110.658 $cm^2$ = 663.948 $cm^2$

= 663.95 $cm^2$

## Solution 56




In $\Delta$ PQR, $\angle$ P = 90, PQ = 24 cm, PR = 7 cm

$$\therefore\ QR^2 = RP^2 + PQ^2 = 7^2 + 24^2$$
$$= 49 + 576 = 625$$
$$\therefore\ QR = 25\,cm$$

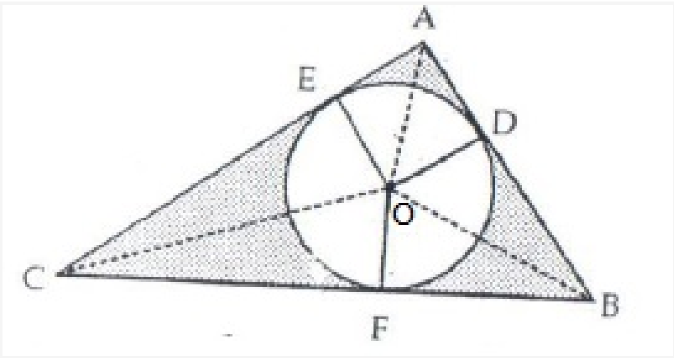
Area of semicircle

$$= \frac{1}{2} \times \pi \times \left(\frac{25}{2}\right)^2$$
$$= \frac{1}{2} \times 3.14 \times \frac{25 \times 25}{4} \text{ cm}^2$$
$$= \frac{625 \times 3.14}{8} = 245.31 \text{ cm}^2$$

Area of $\Delta$ PQR = $\frac{1}{2} \times 7 \times 24 \text{ cm}^2 = 84 \text{ cm}^2$

$\therefore$ Shaded area = 245.31 - 84 = 161.31 $\text{cm}^2$

## Solution 57




In $\Delta$ ABC, $\angle$ A = 90°, AB = 6cm, BC = 10 cm

$$BC^2 = AC^2 + AB^2$$
$$\therefore \quad AC^2 = BC^2 - AB^2 = 10^2 - 6^2 = 100 - 36 = 64$$
$$\therefore \quad AC = 8 \text{ cm}$$

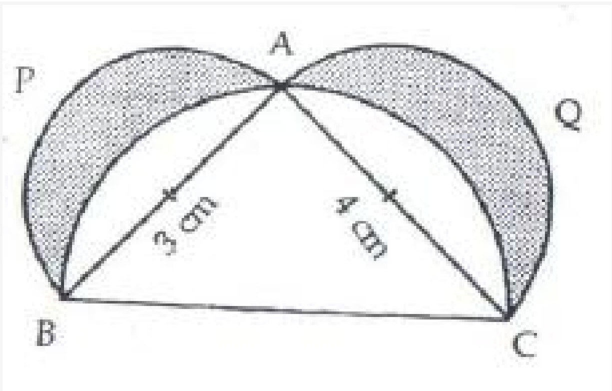
Area of ABC = $\frac{1}{2} \times AC \times AB = \frac{1}{2} \times 8 \times 6 \text{cm}^3 = 24 \text{ cm}^2$

Let r be the radius of circle of centre O

$$\text{Area of } \Delta OCB = \frac{1}{2} \times 10 \times r \text{ cm}^2 = 5r \text{ cm}^2$$
$$\text{Area of } \Delta OAB = \frac{1}{2} \times 6 \times r \text{ cm}^2 = 3r \text{ cm}^2$$
$$\text{Area of } \Delta OCA = \frac{1}{2} \times 8 \times r \text{ cm}^2 = 4r \text{ cm}^2$$
$$\text{Area of } (\Delta OCB + \Delta OAB + \Delta OCA) = \text{Area of } \Delta ABC$$
$$\therefore 5r + 3r + 4r = 24$$
$$\text{or } 12r = 24 \quad \therefore r = 2 \text{ cm}$$

$$\therefore \text{Area of incircle} = \pi r^2 = 3.14 \times 2 \times 2 \text{ cm}^2$$
$$= 12.56 \text{ cm}^2$$
$$\Rightarrow \text{Shaded area} = \text{Area of } \Delta ABC - \text{Area of incircle}$$
$$= (24 - 12.56) \text{cm}^2 = 11.44 \text{ cm}^2$$

## Solution 58




Area of shaded region = Area of $\Delta$ ABC + Area of semi-circle APB

+ Area of semi circle AQC - Area of semicircle BAC

$$\text{Now, Area of a } \Delta ABC = \frac{1}{2} \times 3 \times 4 = 6\ cm^2 \ --(1)$$

$$\text{Area of semi} - \text{cirde APB} = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi \times \left(\frac{3}{2}\right)^2 = \frac{9}{8}\pi --(2)$$

$$\text{Area of semi} - \text{cirde AQC} = \frac{1}{2}\pi r_2^2$$

$$= \frac{1}{2}\pi\left(\frac{4}{2}\right)^2 = 2\pi\ cm^2 ----(3)$$

Further in $\Delta$ ABC, $\angle$ A = 90

$$\therefore BC^2 = AB^2 + AC^2 = 9 + 16 = 25$$

$$\therefore BC = 5$$

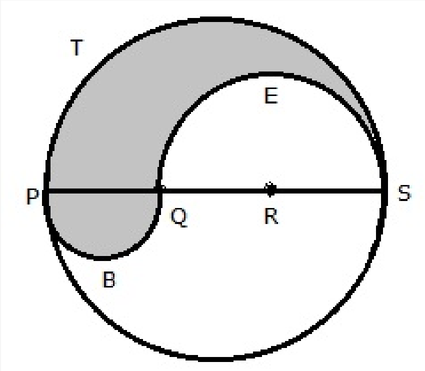
$$\text{Area of semi} - \text{cirdeBAC} = \frac{1}{2}\pi\left(\frac{5}{2}\right)^2 = \frac{25}{8}\pi --(4)$$

Adding (1), (2), (3) and subtracting (4)

$$\therefore \text{ Area of shaded region} = 6 + \frac{9}{8}\pi + 2\pi - \frac{25}{8}\pi$$

$$= 6 + \frac{25}{8}\pi - \frac{25}{8}\pi = 6\ cm^2$$

## Solution 59




PS = 12 cm

PQ = QR = RS = 4 cm, QS = 8 cm

Perimeter = arc PTS + arc PBQ + arc QES

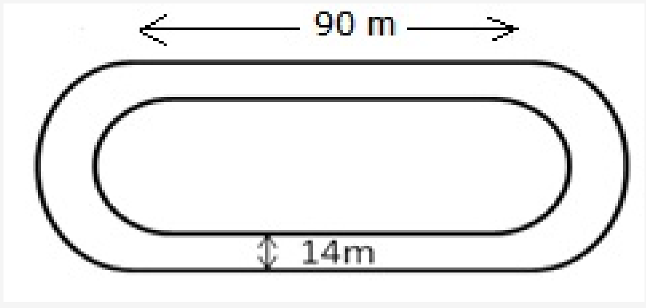
$$= (\pi \times 6 + \pi \times 2 + \pi \times 4)\,cm$$
$$= 12\pi\ cm$$
$$= 12\pi = 12 \times 3.14\ cm$$
$$= 37.68\ cm$$

Area of shaded region = (area of the semicircle PBQ)

+ (area of semicircle PTS)-(Area of semicircle QES)

$$= \left[\frac{1}{2}\pi \times (2)^2 + \frac{1}{2} \times \pi \times (6)^2 - \frac{1}{2} \times \pi \times (4)^2\right] cm^2$$
$$= [2\pi + 18\pi - 8\pi] = 12\pi\ cm^2 = (12 \times 3.14)\,cm^2$$
$$= 37.68\ cm^2$$

## Solution 60




Length of the inner curved portion

= (400 - 2 90) m

= 220 m

Let the radius of each inner curved part be r

$$\text{Then, } \frac{22}{7} \times r = 110 \text{ m}$$

$$r = \left(110 \times \frac{7}{22}\right) \text{m} = 35 \text{ m}$$

Inner radius = 35 m, outer radius = (35 + 14) = 49 m

$\therefore$ Area of the track = (area of 2 rectangles each 90 m 14 m)

+ (area of circular ring with R = 49 m, r = 35 m

$$= \left[2 \times 90 \times 14 + \frac{22}{7}\left\{(49)^2 - (35)^2\right\}\right] \text{m}^2$$

$$= \left[2520 + \frac{22}{7}(49+35)(49-35)\right] \text{m}^2$$

$$= [2520 + 3696] \text{m}^2 = 6216 \text{ m}^2$$

Length of outer boundary of the track

$$= \left[2 \times 90 + 2 \times \frac{22}{7} \times 49\right] m = 488\ m$$

## R S Aggarwal and V Aggarwal Solution for Class 10 Mathematics Chapter 18 - Areas of Circle, Sector and Segment Page/Excercise MCQ

Solution 1

Correct option: (d)

Let r be the radius of the circle.

Area of the circle = 38.5

$\Rightarrow \pi r^2 = 38.5$

$\Rightarrow \frac{22}{7} \times r^2 = 38.5$

$\Rightarrow r^2 = \frac{38.5 \times 7}{22} = 1.75 \times 7 = 12.25$

$\Rightarrow r = 3.5$ cm

$\therefore$ Circumference $= 2\pi r = 2 \times \frac{22}{7} \times 3.5 = 22$ cm

Solution 2

Correct option: (b)

Let r be the radius of the circle.

Area of the circle = $49\pi$ cm$^2$

$\Rightarrow \pi r^2 = 49\pi$

$\Rightarrow r^2 = 49$

$\Rightarrow r = 7$ cm

$\therefore$ Circumference $= 2\pi r = 2 \times \pi \times 7 = 14\pi$ cm

Solution 3

Correct option: (c)

Let r be the radius of the circle.

Circumference of the circle – Radius of the circle = 37 cm

$\Rightarrow 2\pi r - r = 37$

$$\Rightarrow r\left(2 \times \frac{22}{7} - 1\right) = 37$$

$$\Rightarrow r\left(\frac{37}{7}\right) = 37$$

$$\Rightarrow r = \frac{37 \times 7}{37} = 7 \text{ cm}$$

$$\therefore \text{Area of the circle} = \pi r^2 = \left(\frac{22}{7} \times 7 \times 7\right) \text{cm}^2 = 154 \text{cm}^2$$

Solution 4

Correct option: (c)

Let r be the radius of the circular field.

Perimeter of the circular field = 242 m

$\Rightarrow 2\pi r = 242$

$\Rightarrow 2 \times \frac{22}{7} \times r = 242$

$\Rightarrow r = \frac{242 \times 7}{2 \times 22}$

$\Rightarrow r = \frac{77}{2}$ m

$\therefore$ Area of the circle = $\pi r^2 = \left(\frac{22}{7} \times \frac{77}{2} \times \frac{77}{2}\right) cm^2 = 4658.5\ m^2$

Solution 5

Correct option: (c)

Let original diameter of a circle be 100 units.

$\Rightarrow$ Original radius = 50 units

$\therefore$ Original area of the circle $= \pi \times (50)^2 = 2500\pi$ sq. units

Now, new diameter = 140 units

$\Rightarrow$New radius = 70 units

$\therefore$ New area $= \pi \times (70)^2 = 4900\pi$ sq. units

$$\therefore \text{ Increase \%} = \left(\frac{\text{New area} - \text{Original area}}{\text{Original area}} \times 100\right)\%$$

$$= \left(\frac{4900\pi - 2500\pi}{2500} \times 100\right)\%$$

$$= \left(\frac{2400}{2500} \times 100\right)\%$$

$$= 96\%$$

Solution 6

Correct option: (d)

Let original radius of the circle = 100 units

Then, original area of the circle = $\pi \times (100)^2 = 10000\pi$ sq. units

Now, new radius = 70 units

$\therefore$ New area of the circle = $\pi \times (70)^2 = 4900\pi$ sq. units

$$\therefore \text{Decrease\%} = \left(\frac{\text{Original area} - \text{New area}}{\text{Original area}} \times 100\right)\%$$

$$= \left(\frac{10000\pi - 4900\pi}{10000} \times 100\right)\%$$

$$= \left(\frac{5100}{10000} \times 100\right)\%$$

$$= 51\%$$

Solution 7

Correct option: (d)

Let s be the side of the square and r be the radius of the circle.

Then, $s^2 = \pi r^2$

$$\Rightarrow \frac{r^2}{s^2} = \frac{1}{\pi}$$

$$\Rightarrow \frac{r}{s} = \frac{1}{\sqrt{\pi}}$$

Now,

Ratio of their perimeters $= \frac{2\pi r}{4s} = \frac{\pi}{2} \times \left(\frac{r}{s}\right) = \frac{\pi}{2} \times \frac{1}{\sqrt{\pi}} = \frac{\sqrt{\pi}}{2} = \sqrt{\pi} : 2$

Solution 8

Correct option: (b)
Let r be the radius of the big circle.

Circumference of the big circle = $\left(2\pi \times \frac{36}{2} + 2\pi \times \frac{20}{2}\right)$

$\Rightarrow 2\pi r = 2\pi \times (18 + 10)$

$\Rightarrow 2\pi r = 2\pi \times 28$

$\Rightarrow r = 28$ cm

Solution 9

Correct option: (c)
Let r be the radius of the big circle.

Area of the big circle = $\left(\pi \times 24^2 + \pi \times 7^2\right)$

$\Rightarrow \pi r^2 = \pi \times (576 + 49)$

$\Rightarrow \pi r^2 = \pi \times 625$

$\Rightarrow r^2 = 625$

$\Rightarrow r = 25$ cm

$\Rightarrow$ Diameter $= 2r = 2 \times 25 = 50$ cm

Solution 10

Correct option: (b)
Let s be the side of the square and r be the radius of the circle.

Then, $4s = 2\pi r$

$\Rightarrow \frac{s}{r} = \frac{\pi}{2}$

$$\frac{\text{Area of square}}{\text{Area of circle}} = \frac{s^2}{\pi r^2} = \frac{1}{\pi} \times \left(\frac{s}{r}\right)^2 = \frac{1}{\pi} \times \left(\frac{\pi}{2}\right)^2 = \frac{\pi^2}{4\pi} = \frac{\pi}{4} = \pi : 4$$

Solution 11

Correct option: (d)

$\pi R_1^2 + \pi R_2^2 = \pi R^2$

$\Rightarrow R_1^2 + R_2^2 = R^2$

Solution 12

Correct option: (a)

$2\pi R_1 + 2\pi R_2 = 2\pi R$

$\Rightarrow R_1 + R_2 = R$

Solution 13

Correct option: (b)

Let s be the side of the square and r be the radius of the circle.

Then, $2\pi r = 4s$

$\Rightarrow \frac{r}{s} = \frac{2}{\pi}$

$\Rightarrow \frac{r^2}{s^2} = \frac{4}{\pi^2}$

$\Rightarrow \frac{\pi r^2}{s^2} = \frac{4}{\pi} > 1 \quad [\because \pi = 3.14 < 4]$

$\Rightarrow \pi r^2 > s^2$

$\Rightarrow$ Area of the circle > Area of the square

Solution 14

Correct option: (b)

Area of the ring enclosed by two concentric circles

$= \pi\{(19)^2 - (16)^2\}$

$= \pi\{(19+16)(19-16)\}$

$= \frac{22}{7} \times 35 \times 3$

$= 330 \text{cm}^2$

Solution 15

Correct option: (b)

Area of a circle of radius R = 1386 $\text{cm}^2$

$\Rightarrow \pi R^2 = 1386$

$\Rightarrow \frac{22}{7} \times R^2 = 1386$

$\Rightarrow R^2 = 1386 \times \frac{7}{22}$

$\Rightarrow R^2 = 441 \Rightarrow R = 21 \text{ cm}$

Area of a circle of radius r = 962.5 $\text{cm}^2$

$\Rightarrow \pi r^2 = 962.5$

$\Rightarrow r^2 = \frac{9625}{10} \times \frac{7}{22} = \frac{1375 \times 7}{10} \times \frac{7}{22} = \frac{25 \times 49}{4}$

$\Rightarrow r = \frac{5 \times 7}{2} = \frac{35}{2} = 17.5 \text{ cm}$

$\therefore$ Width of the ring = R − r = 21 − 17.5 = 3.5 cm

Solution 16

Correct option: (c)

$$\frac{2\pi R_1}{2\pi R_2} = \frac{3}{4}$$

$$\Rightarrow \frac{R_1}{R_2} = \frac{3}{4}$$

$$\Rightarrow \frac{{R_1}^2}{{R_2}^2} = \frac{9}{16}$$

$$\Rightarrow \frac{\pi {R_1}^2}{\pi {R_2}^2} = \frac{9}{16}$$

Solution 17

Correct option: (a)

$$\frac{\pi {R_1}^2}{\pi {R_2}^2} = \frac{9}{4}$$

$$\Rightarrow \frac{{R_1}^2}{{R_2}^2} = \frac{3^2}{2^2}$$

$$\Rightarrow \frac{R_1}{R_2} = \frac{3}{2}$$

$$\Rightarrow \frac{2\pi R_1}{2\pi R_2} = \frac{3}{2}$$

Solution 18

Correct option: (d)

Radius of the whel = r = 0.25 m

Distance covered in 1 revolution = $2\pi r = \left(2 \times \frac{22}{7} \times \frac{25}{100}\right) m = \frac{11}{7}$ m

Total distance covered = 11 km = 11000 m

$\therefore$ Number of revolutions = $\left(11000 \times \frac{7}{11}\right) = 7000$

Solution 19

Correct option: (a)

Diameter of a wheel = d = 40 cm = $\frac{40}{100}$ m

Distance covered in 1 revolution = $\pi d = \left(\frac{22}{7} \times \frac{40}{100}\right) m = \frac{44}{35}$ m

Total distance covered = 176 m

$\therefore$ Number of revolutions = $\left(176 \times \frac{35}{44}\right) = 140$

Solution 20

Correct option: (c)

Distance covered in 1000 revolutions = 88 km = 88000 m

$\therefore$ Distance covered in 1 revolution = $\frac{88000}{1000}$ m = 88 m

$\Rightarrow \pi d = 88$

$\Rightarrow \frac{22}{7} \times d = 88$

$\Rightarrow d = \frac{88 \times 7}{22} = 28$ m

## Solution 21

Correct option: (d)

Area of a sector of angle $\theta°$ of a circle with radius R = $\frac{\pi R^2\theta}{360}$

## Solution 22

Correct option: (b)

Length of an arc of a sector of angle $\theta°$ of a circle with radius R = $\frac{2\pi R\theta}{360}$

## Solution 23

Correct option: (a)

Area swept by minute hand in 60 minutes = $\pi r^2$

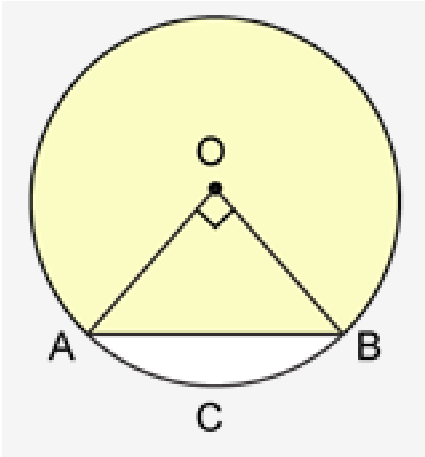
$\therefore$ Area swept by it in 10 minutes = $\left(\frac{\pi r^2}{60} \times 10\right)$ cm$^2$

$$= \left(\frac{22}{7} \times 21 \times 21 \times \frac{10}{60}\right) \text{cm}^2$$

$$= (11 \times 21)\ \text{cm}^2$$

$$= 231\ \text{cm}^2$$

## Solution 24




Correct option: (c)

Let O be the center of the circle and AB be the chord.

Now, Area of the minor segment

= Area of sector OACBO – Area of $\Delta OAB$

$$= 3.14 \times 10 \times 10 \times \frac{90}{360} - \frac{1}{2} \times 10 \times 10$$

$= 78.5 - 50$

$= 28.5 \text{ cm}^2$

Solution 25

Correct option: (b)

$$\text{Length of an arc} = \frac{2\pi r\theta}{360}$$

$$= \left(2 \times \frac{22}{7} \times 21 \times \frac{60}{360}\right) \text{cm}$$

$= 22$ cm

Solution 26

Correct option: (a)

$$\text{Area of the segment} = \frac{\pi r^2\theta}{360} - r^2 \sin\frac{\theta}{2}\cos\frac{\theta}{2}$$

$$= \left(\frac{22}{7}\times 14\times 14\times\frac{120}{360}\right) - (14\times 14\times \sin 60^\circ \cos 60^\circ)$$

$$= \frac{616}{3} - \frac{\sqrt{3}}{2}\times\frac{1}{2}\times 14\times 14$$

$$= 205.33 - 49\sqrt{3}$$

$$= 205.33 - 84.77$$

$$= 120.56\ \text{cm}^2$$

# R S Aggarwal and V Aggarwal Solution for Class 10 Mathematics Chapter 18 - Areas of Circle, Sector and Segment Page/Excercise FA

Solution 1

Correct option: (b)

Side of a square = 20 cm.

$\therefore$ Area of the square $= (20\times 20)\ \text{cm}^2 = 400\ \text{cm}^2$

Diagonal of square $= \sqrt{(20)^2 + (20)^2} = \sqrt{800} = 20\sqrt{2}$ cm

$\Rightarrow$ Radius of the quadrant $= 20\sqrt{2}$ cm

$\therefore$ Area of a quadrant $= \frac{1}{4}\times 3.14\times\left(20\sqrt{2}\right)^2 = 628\ \text{cm}^2$

Thus, area of the shaded region = Area of a quadrant − Area of the square

$$= (628 - 400)\ \text{cm}^2$$

$$= 228\ \text{cm}^2$$

Solution 2

Correct option: (c)

Diameter of a wheel = d = 84 cm = $\frac{84}{100}$ m

Distance covered in 1 revolution = $\pi d = \left(\frac{22}{7} \times \frac{84}{100}\right)$ m = $\frac{66}{25}$ m

Total distance covered = 792 m

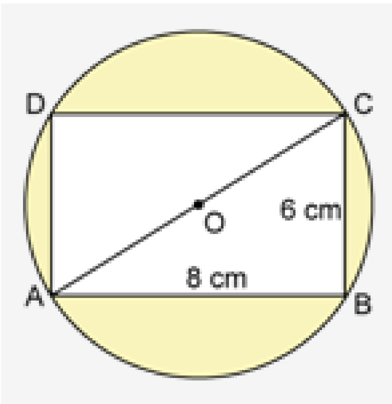
$\therefore$ Number of revolutions = $792 \times \frac{25}{66} = 12 \times 25 = 300$

Solution 3

Correct option: (d)

Area of a sector of a circle with radius r and making an angle of $x°$ at the centre = $\frac{x}{360} \times \pi r^2$

Solution 4



Note: Given options not matching with the answer.

Length of a rectangle = 8 cm
Breadth of a rectangle = 6 cm
$\therefore$ Area of rectangle ABCD = $8 \times 6 = 48$ cm$^2$
Now,
$AC^2 = AB^2 + BC^2 = 8^2 + 6^2 = 64 + 36 = 100$
$\Rightarrow AC = \sqrt{100} = 10$ cm
$\Rightarrow$ Diameter of a circle = 10 cm
$\Rightarrow$ Radius of a circle = 5 cm
$\therefore$ Area of a circle = $3.14 \times 5 \times 5 = 78.5$ cm$^2$
Thus, area of shaded region = Area of a circle − Area of rectangle ABCD
$= (78.5 - 48)$ cm$^2$
$= 30.5$ cm$^2$

Solution 5

Let r be the radius of the given circle.
Circumference of the circle = 22 cm

$$\Rightarrow 2\pi r = 22$$

$$\Rightarrow 2 \times \frac{22}{7} \times r = 22$$

$$\Rightarrow r = \frac{22 \times 7}{2 \times 22}$$

$$\Rightarrow r = \frac{7}{2} \text{ cm}$$

$$\therefore \text{Area of the circle} = \pi r^2 = \left(\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\right) \text{cm}^2 = 38.5 \text{ cm}^2$$

Solution 6

Radius of a circle = r = 21 cm

Central angle = $\theta = 60^\circ$

$\therefore$ Length of the arc $= \frac{2\pi r\theta}{360} = \left(2 \times \frac{22}{7} \times 21 \times \frac{60}{360}\right) cm = 22 cm$

Solution 7

Angle described by the minute hand in 60 minutes = $360^\circ$

$\therefore$ Angle described by the minute hand in 35 minutes $= \left(\frac{360}{60} \times 35\right)^\circ = 210^\circ$

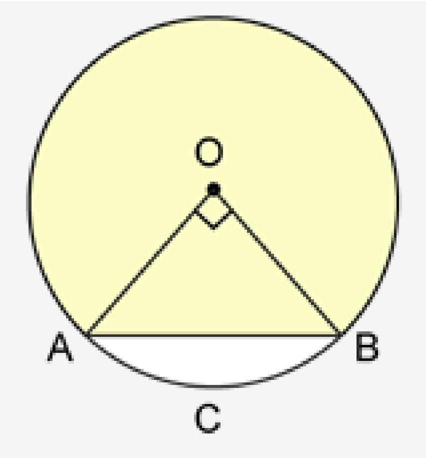
$\therefore$ $\theta = 210^\circ$ and r = 12 cm

$\therefore$ Area swept by the minute hand in 35 minutes $= \left(\frac{\pi r^2 \theta}{360}\right)$

$= \left(\frac{22}{7} \times 12 \times 12 \times \frac{210}{360}\right) cm^2$

$= 264 cm^2$

Solution 8



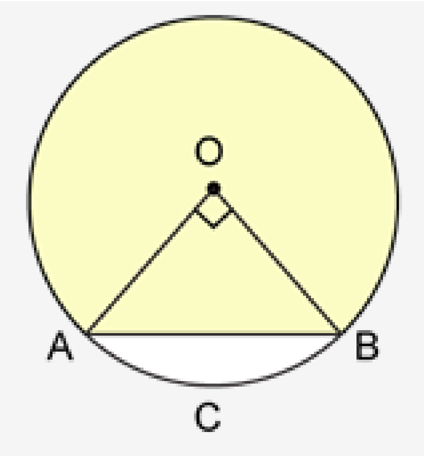
Let O be the centre of a circle of radius 5.6 cm.
Let OACBO be the sector whose perimeter is 27.2 cm.
Then, OA + OB + arc ACB = 27.2 cm

$\Rightarrow$ 5.6 cm + 5.6 cm + arc ACB = 27.2 cm

$\Rightarrow$ arc ACB $= (27.2 - 11.2)$ cm = 16 cm

$$\therefore \text{ Area of sector OACBO} = \frac{1}{2} \times \text{radius} \times \text{arc length}$$
$$= \frac{1}{2} \times 5.6 \times 16$$
$$= 44.8 \text{ cm}^2$$

Solution 9




Let AB be the chord of a circle of centre O and radius = 14 cm such that $\angle AOB = 90°$

$$\therefore \text{ Area of the sector OACBO} = \frac{\pi r^2 \theta}{360}$$
$$= \left(\frac{22}{7} \times 14 \times 14 \times \frac{90}{360}\right) \text{cm}^2$$
$$= 154 \text{ cm}^2$$

Solution 10

Area of a shaded region = Area of sector OAB – Area of sector OCD

$$= \left[\left(\frac{\frac{22}{7} \times 7^2 \times 30}{360}\right) - \left(\frac{\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 30}{360}\right)\right] cm^2$$

$$= \left(\frac{77}{6} - \frac{77}{24}\right) cm^2$$

$$= 77\left(\frac{1}{6} - \frac{1}{24}\right) cm^2$$

$$= 77\left(\frac{4-1}{24}\right) cm^2$$

$$= 77 \times \frac{3}{24}\ cm^2$$

$$= \frac{77}{8}\ cm^2$$

$$= 9.625\ cm^2$$

Solution 11

Area of an equilateral triangle formed = $121\sqrt{3}$ cm$^2$

$\Rightarrow \frac{\sqrt{3}}{4} \times (\text{Side})^2 = 121\sqrt{3}$

$\Rightarrow (\text{Side})^2 = \frac{121\sqrt{3} \times 4}{\sqrt{3}} = 484$

$\Rightarrow \text{Side} = \sqrt{484}\,\text{m} = 22\text{ cm}$

$\therefore$ Perimeter of an equilateral triangle = $3 \times$ side = $3 \times 22 = 66$ cm

Let r be the radius of the circle formed.

Now,

Circumference of a circle = Perimeter of an equilateral triangle

$\Rightarrow 2 \times \frac{22}{7} \times r = 66$

$\Rightarrow r = \frac{66 \times 7}{2 \times 22} = \frac{21}{2}$ cm

$\therefore$ Area of the circle = $\pi r^2 = \left(\frac{22}{7} \times \frac{21}{2} \times \frac{21}{2}\right)$ cm$^2$ = 346.5 cm$^2$

Solution 12

Diameter of a wheel = 84 cm

$\Rightarrow$ Circumference of the wheel = $\pi d = \frac{22}{7} \times 84 = 264$ cm

$\therefore$ Distance covered in 1 revolution = 264 cm

Distance covered in 1 second = $5 \times 264 = 1320$ cm

$\Rightarrow$ Distance covered in 1hour (3600 seconds) = $1320 \times 3600$ cm

= 4752000 cm

= 47.52 km

Hence, speed of the cart is 47.52 km/hr.

Solution 13

(i) Area of quadrant OACB = $\left(\frac{1}{4} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\right)$ cm$^2$

$= \frac{77}{8}$ cm$^2$

$= 9.625$ cm$^2$

(ii) Area of $\Delta AOD = \frac{1}{2} \times AO \times OD = \frac{1}{2} \times 3.5 \times 2 = 3.5$ cm$^2$

$\therefore$ Area of the shaded region = Area of quadrant OACB – Area of $\Delta AOD$

$= (9.625 - 3.5)$ cm$^2$

$= 6.125$ cm$^2$

Solution 14

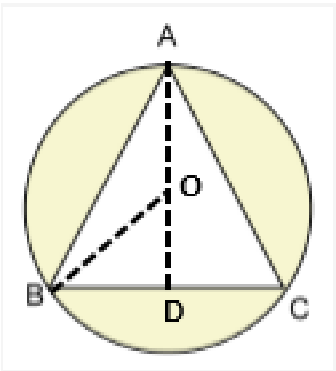
Required area = Area of a square ABCD – 4 x Area of 1 quadrant

$= 28 \times 28 - 4 \times \frac{1}{4} \times \frac{22}{7} \times 14 \times 14$

$= 784 - 616$

$= 168$ cm$^2$

Solution 15



Let O be the centre of the circumcircle.

Join OB and draw AD $\perp$ BC.

Then, OB = 4cm and $\angle OBD = 30°$

In $\Delta OBD$,

$$\sin 30° = \frac{OD}{OB}$$

$$\Rightarrow \frac{1}{2} = \frac{OD}{4}$$

$\Rightarrow$ OD = 2 cm

Now, $BD^2 = OB^2 - OD^2 = 4^2 - 2^2 = 16 - 4 = 12$

$\Rightarrow BD = 2\sqrt{3}$ cm

$\Rightarrow BC = 2 \times 2\sqrt{3} = 4\sqrt{3}$ cm

Now, area of the shaded region

= Area of the circle – Area of an equilateral $\Delta ABC$

$$= 3.14 \times 4 \times 4 - \frac{\sqrt{3}}{4} \times 4\sqrt{3} \times 4\sqrt{3}$$

$= 50.24 - 12 \times \sqrt{3}$

$= 50.24 - 20.76$

$= 29.48\ cm^2$

Solution 16

Angle described by the minute hand in 60 minutes = 360°

$\therefore$ Angle described by the minute hand in 56 minutes = $\left(\frac{360}{60} \times 56\right)^\circ = 336°$

$\therefore$ $\theta = 336°$ and r = 7.5 cm

$\therefore$ Area swept by the minute hand in 56 minutes = $\left(\frac{\pi r^2 \theta}{360}\right)$

$$= \left(3.14 \times 7.5 \times 7.5 \times \frac{336}{360}\right) cm^2$$

$$= 165 cm^2$$

Solution 17

Inner circumference of a racetrack = 352 m

$\Rightarrow 2\pi r = 352$

$\Rightarrow 2 \times \frac{22}{7} \times r = 352$

$\Rightarrow r = \frac{352 \times 7}{2 \times 22} = 56 \text{ m}$

Outer circumference of a racetrack = 396 m

$\Rightarrow 2\pi R = 396$

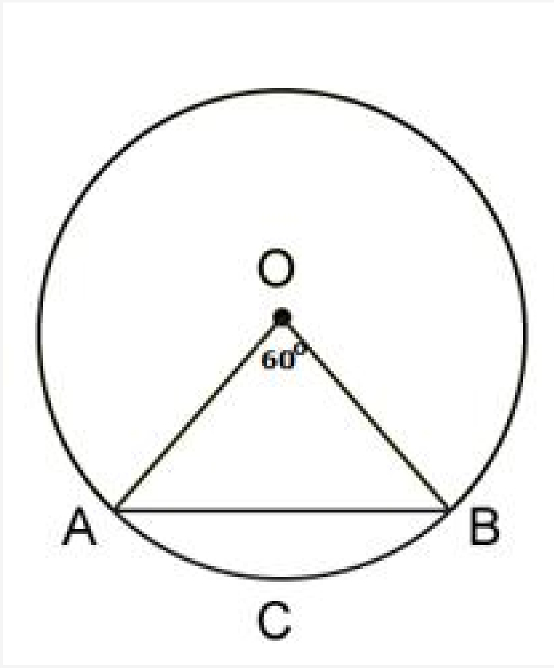
$\Rightarrow 2 \times \frac{22}{7} \times R = 396$

$\Rightarrow R = \frac{396 \times 7}{2 \times 22} = 63 \text{ m}$

$\therefore$ Width of the track = $R - r = 63 - 56 = 7$ m

$$\text{Area of the track} = \pi(R^2 - r^2)$$
$$= \frac{22}{7}(63^2 - 56^2)$$
$$= \frac{22}{7}(63 + 56)(63 - 56)$$
$$= \frac{22}{7} \times 119 \times 7$$
$$= 2618 \text{ m}^2$$

Solution 18




Area of minor segment ACBA = Ar (sector OACBO) − Ar (ΔOAB)

$$= \frac{\pi r^2 \theta}{360} - \frac{1}{2} r^2 \sin\theta$$

$$= \frac{3.14 \times 30 \times 30 \times 60}{360} - \frac{1}{2} \times 30 \times 30 \times \sin 60^\circ$$

$$= \frac{3.14 \times 30 \times 30 \times 60}{360} - \frac{1}{2} \times 30 \times 30 \times \frac{\sqrt{3}}{2}$$

$$= 471 - 389.7$$

$$= 81.3 \text{ cm}^2$$

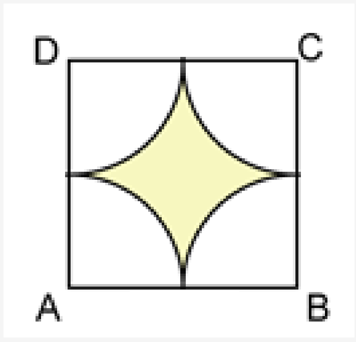
Area of major segment BDAB = Area of a circle − Area of minor segment

$$= 3.14 \times 30 \times 30 - 81.3$$

$$= 2826 - 81.3$$

$$= 2744.7 \text{ cm}^2$$

Solution 19




Ungrazed area = Shaded area

$$= \left[(50 \times 50) - 4 \times \frac{3.14 \times (25)^2 \times 90}{360}\right] m^2$$

$$= \left[2500 - \frac{3.14\pi \times 25 \times 25 \times}{}\right] m^2$$

$$= [2500 - 1962.5] m^2$$

$$= 537.5 \ m^2$$

Solution 20

Area of a square tank = 1600 $m^2$

$\Rightarrow (\text{Side})^2 = 1600$

$\Rightarrow$ Side = 40 m

Now, side of a square tank = Diameter of a semicircular plot = 40 m

$\Rightarrow$ Radius of a semicircular plot = 20 m

$$\text{Area of 4 semicircular plots} = 4\left(\frac{1}{2} \times 3.14 \times 20 \times 20\right)$$

$$= 4(3.14 \times 10 \times 20)$$

$$= 4 \times 628$$

$$= 2512 \ m^2$$

Cost of turfing the plot = Rs. 12.50 per $m^2$

$\therefore$ Cost of turfing 2512 $m^2$ plot = Rs. 12.50 $\times$ 2512 = Rs. 31400