Exercise 15.1

1. Find the circumference and area of circle of radius 4.2 cm

Sol:

Radius (r) = 4.2 cm

Circumference = 2 × r

$= 2 \times \frac{22}{7} \times 4.2$

$= \left(\frac{44}{10} \times 6\right) = \frac{264}{10}$

$= 26.4\ cm$

Area = $\pi r^2 = \frac{22}{7} \times 4.2 \times 4.2$

$= \frac{22 \times 6 \times 42}{10 \times 10} = \frac{5544}{100} = 55.44 cm^2$

2. Find the circumference of a circle whose area is 301.84 cm^2.

Sol:

Area of circle = 301.84 cm^2.

Let radius = r cm

Area of circle = πr^2

$\pi r^2 = 301.84$

$\frac{22}{7} \times r^2 = 301.84$

$r^2 = \frac{301.84 \times 7}{22} = \left(\sqrt{7 \times 7}\right)^{\frac{1}{2}} \times 13.75$

$r = \sqrt{13.72\ \times 7} = \sqrt{7 \times 7 \times 1.96} = 7 \times 1.4 = 9.8\ cm$

Radius = r = 9.8 cm

Circumference = 2 × r = $2 \times \frac{22}{7} \times 9.8$

$= 44 \times 1.4$

$= 61.6\ cm$

3. Find the area of circle whose circumference is 44 cm.

Sol:

Circumference = 44 cm

Let radius = r cm

Circumference = 2 × r = 44 cm

$2 \times \frac{22}{7} \times r = 44$

r = $\frac{44 \times 7}{2 \times 22} = 7cm$

radius = 7 cm

Area of circle = πr^2

$= \frac{22}{7} \times 7 \times 7 = (22 \times 7) = 154 cm^2$

4. The circumference of a circle exceeds diameter by 16.8 cm. Find the circumference of circle.

Sol:

Let radius of circle = r cms

Diameter(d) = $2 \times radius = 2r$

Circumference (c) = $2\pi r$

Given circumference exceeds diameter by 16.8cm

C = d + 16.8

$\Rightarrow 2\pi r = 2r + 16.8$

$\Rightarrow 2r(\pi - 1) = 16.8$

$\Rightarrow 2r \times \left(\frac{22}{7} - 1\right) = 16.8$

$\Rightarrow 2r \times \frac{15}{7} = 16.8$

$\Rightarrow r = \frac{16.8 \times 7}{30} = 5.6 \times 0.7$

$\Rightarrow r = 3.92$ cms

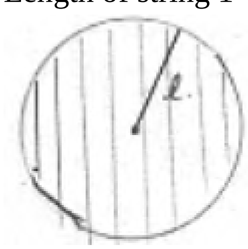
Circumference $= 2\pi r = 2 \times \frac{22}{7} \times 3.92$

$= \frac{2464}{100} = 24.64\ cms$

5. A horse is tied to a pole with 28m long string. Find the area where the horse can graze.

Sol:

Length of string 1 = 28m



Area it can graze is area of circle with radius equal to length of string

Area = πl^2

$= \frac{22}{7} \times 28 \times 28$

$= 88 \times 28$

$= 2464\ cm^2$

$\therefore$ area grazed by horse $= 2464 cm^2$.

6. A steel wire when bent is the form of square encloses an area of 12 cm^2. If the same wire is bent in form of circle. Find the area of circle.

Sol:

Let side of square = s and length of wire be l. As wire is bent into square

l = perimeter of square = 4s.

Area of square = $121cm^2 = s^2$.

S = $\sqrt{121} = 11cm$

∴ length of wire l = 4(11) = 44cm

As wire is bent into circle (let radius be r)

Length of wire = circumference

44 = 2πr

$\frac{22}{7} \times 2 \times r = 44 \Rightarrow r = \frac{44\times7}{2\times22} = 7cm$

$Area\ of\ circle = \pi r^2$

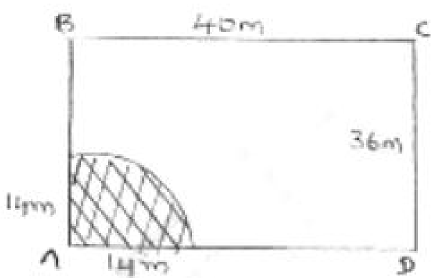
$= \frac{22}{7} \times 7 \times 7$

$= 22 \times 7$

$= 154cm^2$

7. A horse is placed for grazing inside a rectangular field 40m by 36m and is tethered to one corner by a rope 14m long. Over how much area can it graze.

Sol:



The fig shows rectangular field ABCD at corner A, a horse is tied with rope length = 14m. The area it can graze is represented A as shaded region= area of quadrant with (radius = length) of string

Area = $\frac{1}{4} \times (area\ of\ circle) = \pi r^2$

= $\frac{1}{4} \times \frac{22}{7} \times 14 \times 14$

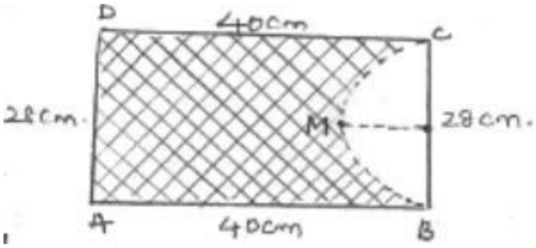
= (22 × 7)

= 154 m^2.

Area it can graze = $154m^2$.

8. A sheet of paper is in the form of rectangle ABCD in which AB = 40cm and AD = 28 cm. A semicircular portion with BC as diameter is cut off. Find the area of remaining paper.

Sol:



Given sheet of paper ABCD

AB = 40 cm, AD = 28 cm

⇒ CD = 40 cm, BC = 28 cm [since ABCD is rectangle]

Semicircle be represented as BMC with BC as diameter

Radius = $\frac{1}{2} \times BC = \frac{1}{2} \times 28 = 14cms$

Area of remaining (shaded region) = (area of rectangle) – (area of semicircle)

= $(AB \times BC) - \left(\frac{1}{2}\pi r^2\right)$

= $(40 \times 28) - \left(\frac{1}{7} \times \frac{22}{7} \times 14 \times 14\right)$

= $1120 - 308$

= 812 cm^2.

9. The circumference of two circles are in ratio 2:3. Find the ratio of their areas

Sol:

Let radius of two circles be r_1 and r_2 then their circumferences will be $2\pi r_1$: $2\pi r_2$

= $r_1 : r_2$

But circumference ratio is given as 2 : 3

$r_1 : r_2 = 2 : 3$

Ratio of areas = $\pi r_2^2 : \pi r_2^2$

$= \left(\frac{r_1}{r_2}\right)^2$

$= \left(\frac{2}{3}\right)^2$

$= \frac{4}{9}$

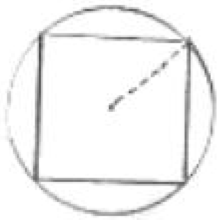
$= 4 : 9$

$\therefore ratio\ of\ areas = 4 : 9$

10. The side of a square is 10 cm. find the area of circumscribed and inscribed circles.

Sol:

Circumscribed circle



Radius = $\frac{1}{2}(diagonal\ of\ square)$

= $\frac{1}{2} \times \sqrt{2}$ side

= $\frac{1}{2} \times \sqrt{2} \times 10$

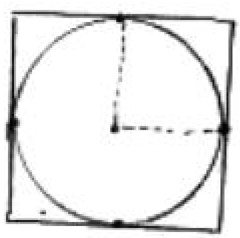
= $5\sqrt{2}$ cm

Area = πr^2

= $\frac{22}{7} \times 25 \times 2$

= $\frac{1100}{7} cm^2$

Inscribed circle



Radius = $\frac{1}{2}$ (sides)

= $\frac{1}{2} \times 10$

= 5 cm

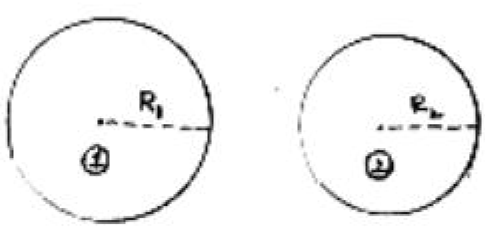
Area = πr^2

= $\frac{22}{7} \times 5 \times 5$

= $\frac{550}{7} cm^2$

11. The sum of the radii of two circles is 140 cm and the difference of their circumferences in 88 cm. Find the diameters of the circles.

Sol:



Let radius of circles be $r_1 and\ r_2$

Given sum of radius = 140cm

$r_1 + r_2 = 140$ …..(i)

$difference\ in\ circumgerences = 88\ cm$

$2 \times r_1 - 2\pi r_2 = 88$

$2 \times \frac{22}{7}(r_1 - r_2)\ \ 88$

$r_1 - r_2 = \frac{88\times7}{2\times22} = 14$

$r_1 = r_2 + 14$ ….(ii)

$(ii) in\ (i) \Rightarrow\ r_2 + r_2 + 14 = 140$

$\Rightarrow 2r_2 = 126$

$\Rightarrow r_2 = \frac{126}{2} = 63cms$

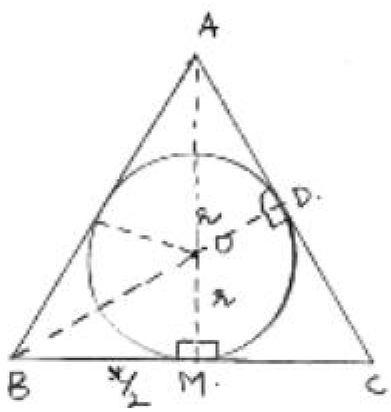
$r_2 = 63cms\ in\ (ii)\ r_1 = 63 + 14 = 77\ cms$

Diameter of circle (i) = $2r_1 = 2 \times 77 = 154\ cms$

Diameter of circle (ii) = $2r_2 = 2 \times 63 = 126\ cms$

12. The area of circle, inscribed in equilateral triangle is 154 cms^2. Find the perimeter of triangle.

Sol:



Let circle inscribed in equilateral triangle

Be with centre O and radius 'r'

Area of circle = πr^2

But given that area = 154 cm2.

$\pi r^2 = 154$

$\frac{22}{7} \times r^2 = 154$

$r^2 = 7 \times 7$

r = 7cms

Radius of circle = 7cms

From fig. at point M, BC side is tangent at point M, BM ⊥ OM. In equilateral triangle, the perpendicular from vertex divides the side into two halves

BM = $\frac{1}{2}BC = \frac{1}{2}(side = x) = \frac{x}{2}$

ΔBMO is right triangle, by Pythagoras theorem

$OB^2 = BM^2 + MO^2$

$OB = \sqrt{r^2 + \frac{x^2}{4}} = \sqrt{49 + \frac{x^2}{4}} \qquad OD = r$

Altitude BD = $\frac{\sqrt{3}}{2}\ (side) = \frac{\sqrt{3}}{2}x = OB + OD$

BD – OD = OB

$\Rightarrow \frac{\sqrt{3}}{2}x - r = \sqrt{49 + \frac{x^2}{4}}$

$\Rightarrow \frac{\sqrt{3}}{2}x - 7 = \sqrt{49 + \frac{x^2}{4}}$

$\Rightarrow \left(\frac{\sqrt{3}}{2}x - 7\right)^2 = \left(\sqrt{\frac{x^2}{4} + 49}\right)^2$

$\Rightarrow \frac{3}{4}x^2 - 7\sqrt{3}x + 49 = \frac{x^2}{4} + 49$

$\Rightarrow \frac{x}{2} = 7\sqrt{3} \Rightarrow x = 14\sqrt{3}\ cm$

Perimeter = 3x = 3 × $14\sqrt{3}$

= $42\sqrt{3}$ cms

13. A field is in the form of circle. A fence is to be erected around the field. The cost of fencing would to Rs. 2640 at rate of Rs.12 per metre. Then the field is to be thoroughs ploughed at cost of Rs. 0.50 per m^2. What is amount required to plough the field?

Sol:

Given

Total cost of fencing the circular field = Rs. 2640

Cost per metre fencing = Rs 12

Total cost of fencing = circumference × cost per fencing

⇒ 2640 = circumference × 12

⇒ circumference = $\frac{2640}{12} = 220m$

Let radius of field be r m

Circumference = 2 πr m

$2\pi r = 220$

$2 \times \frac{22}{7} \times r = 220$

r = $\frac{70}{2} = 35m$

Area of field = πr^2

= $\frac{22}{7} \times 35 \times 35$

= 3850 m^2.

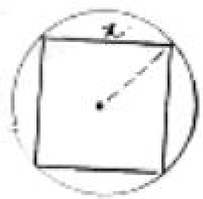
Cost of ploughing per m^2 land = Rs. 0.50

Cost of ploughing 3850 m^2 land $= \frac{1}{2} \times 3850$

= Rs. 1925.

14. If a square is inscribed in a circle, find the ratio of areas of the circle and the square.

Sol:



Let side of square be x cms inscribed in a circle.

Radius of circle (r) = $\frac{1}{2}(diagonal\ of\ square)$

$= \frac{1}{2}\left(\sqrt{2}x\right)$

$= \frac{x}{\sqrt{2}}$

Area of square = $(\text{side})^2 = x^2$

Area of circle = πr^2

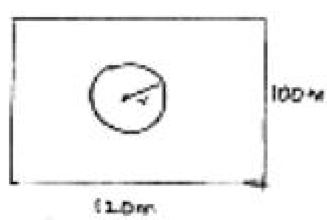
$= \pi\left(\frac{x}{\sqrt{2}}\right)^2$

$= \frac{\pi x^2}{2}$

$\frac{area\ of\ circle}{area\ of\ square} = \frac{\frac{\pi}{2}x^2}{x^2} = \frac{\pi}{2} = \pi : 2$

15. A park is in the form of rectangle 120m × 100m. At the centre of park there is a circular lawn. The area of park excluding lawn is 8700m^2. Find the radius of circular lawn.

Sol:



Dimensions of rectangular park length = 120m

Breadth = 100m

Area of park = l × b

$= 120 \times 100 = 12000m^2$.

$Let\ radius\ of\ circular\ lawn\ be\ r$

Area of circular lawn $= \pi r^2$

Area of remaining park excluding lawn = (area of park) – (area of circular lawn)

$\Rightarrow 8700 = 12000 - \pi r^2$

$\Rightarrow \pi r^2 = 12000 - 8700 = 3300$

$\Rightarrow \frac{22}{7} \times r^2 = 3300$
$\Rightarrow r^2 = 150 \times 7 = 1050$
$\Rightarrow \text{r} = \sqrt{1050} = 5\sqrt{42}\ metres$
$\therefore$ radius of circular lawn = $5\sqrt{42}$ metres.

16. The radii of two circles are 8 cm and 6 cm respectively. Find the radius of the circle having its are equal to the sum of the areas of two circles.

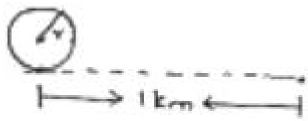
Sol:
Radius of circles are 8cm and 6 cm
Area of circle with radius 8 cm = $\pi(8)^2 = 64\pi cm^2$
Area of circle with radius 6cm = $\pi(6)^2 = 36\pi\ cm^2$
Areas sum = $64\pi + 36\pi = 100\pi$ cm^2
Radius of circle be x cm
Area = πx^2
$\pi x^2 = 100\pi$
$x^2 = 100 \Rightarrow x = \sqrt{100} = 10cm$

17. The radii of two circles are 19cm and 9 cm respectively. Find the radius and area of the circle which has circumferences is equal to sum of circumference of two circles.

Sol:
Radius of 1st circle = 19cm
Radius of 2nd circle = 9 cm
Circumference of 1st circle = 2(19) = 38π cm
Circumference of 2nd circle = 2π (9) = 18π cm
Let radius of required circle = R cm
Circumference of required circle = $2\pi\text{R} = c_1 + c_2$
$2\pi\text{R} = 38\pi + 18\pi$
$2\pi\text{R} = 56\pi$
R = 28 cms
Area of required circle = πr^2
$= \frac{22}{7} \times 28 \times 28$
$= 2464\ cm^2$

18. A car travels 1 km distance in which each wheel makes 450 complete revolutions. Find the radius of wheel.

Sol:
Let radius of wheel = ‘r’ m



Circumference of wheel = $(2\pi r)$m.
No. of revolutions = 450
Distance for 450 revolutions = $450 \times 2\pi r = 900\pi r\ m$
But distance travelled = 1000 m.
900πr = 1000
r = 10000 $9\pi \times 100$
$= \frac{10}{9\pi} m$
$= \frac{1000}{9\pi} cms$
$radius\ (r) = \frac{1000}{9\pi}\ cms$

19. The area enclosed between the concentric circles is 770cm^2. If the radius of inner circle.
Sol:
Radius of outer circle = $21cm$

Radius of inner circle = R_2
Area between concentric circles = area of outer circle – area of inner circle
$\Rightarrow 770 = \frac{22}{7}(21^2 - {R_2}^2)$
$\Rightarrow 21^2 - {R_2}^2 = 35 \times 7 = 245$
$\Rightarrow 441 - 245 = {R_2}^2$
$\Rightarrow R_2 = \sqrt{196} = 14\ cm$
Radius of inner circle = 14cm.

Exercise 15.2

1. Find in terms of x the length of the arc that subtends an angle of 30°, at the centre of circle of radius 4 cm.

Sol:

Length of arc = $\frac{\theta}{360°} \times 2\pi r$

Radius = r = 4 cm

θ = angle subtended at centre = 30°

Arc length = $\frac{30°}{360°} \times 2 \times (7)$

= $\frac{2\pi}{3}$ cm

2. Find the angle subtended at the centre of circle of radius 5cm by an arc of length $\left(\frac{5\pi}{3}\right) cm$

Sol:

Radius (r) = 5 cm

θ = angle subtended at centre (degrees)

Length of Arc = $\frac{\theta}{360°} \times 2\pi$r cm

But arc length = $\frac{5\pi}{3} cm$

$\frac{\theta}{360°} \times 2\pi \times 5 = \frac{5\pi}{3}$

θ= $\frac{360° \times \pi}{3 \times 2\pi} = 60°$

∴ Angle subtended at centre = 60°

3. An arc of length 20π cm subtends an angle of 144° at centre of circle. Find the radius of the circle.

Sol:

Length of arc = 20πcm

Let radius = 'r' cm

O = angle subtended at centre = 144°

Length of arc = $\frac{\theta}{360°} \times 2\pi r$

= $\frac{144}{360} \times 2\pi r = \frac{4\pi}{5} r$

$= \frac{4\pi}{5} r = 20\pi$

$r = \frac{20\pi \times 5}{4\pi} = 25\ cms$

4. An arc of length 15 cm subtends an angle of 45° at the centre of a circle. Find in terms of π, radius of the circle.

Sol:

Length of arc = 15cm

θ = angle subtended at centre = 45°

Let radius = r cm

arc length = $\frac{\theta}{360°} \times 2\pi r$

= $\frac{45°}{360°} \times 2\pi r$

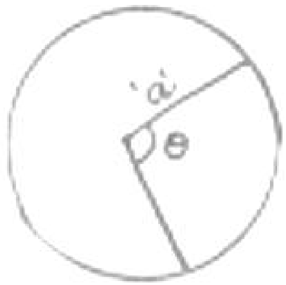
$\frac{45}{360} \times 2\pi r = 15$

$r = \frac{15 \times 360}{45 \times 2\pi} = \frac{60}{\pi}$ cms

$Radius = \frac{60}{\pi} cms$

5. Find the angle subtended at the centre of circle of radius 'a' cm by an arc of length $\frac{a\pi}{4}\ cm$

Sol:



Length of arc = $\frac{a\pi}{4}\ cm$

Radius r = 'a' cm

θ = angle subtended at centre

arc length = $\frac{\theta}{360°} \times 2\pi r$

= $\frac{\theta}{360°} \times 2\pi a$

$\therefore \frac{\theta}{360°} \times 2\pi a = \frac{a\pi}{4}$

$\Rightarrow \theta = \frac{9\pi \times 360°}{4 \times 2\pi a} = 45°$

6. A sector of circle of radius 4cm contains an angle of 30°. Find the area of sector

Sol:

Radius = 4 cm = r

Angle subtended at centre = θ = 30°

Area of sector (shaded region)

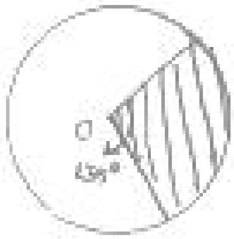
$= \frac{\theta}{360°} \times \pi r^2$

$= \frac{30}{360} \times \frac{22}{7} \times 4 \times 4$

$= \frac{88}{21} cm^2$

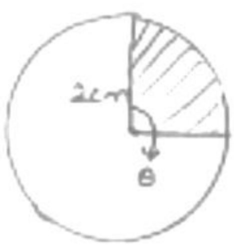
$\therefore area\ of\ required\ sector = \frac{88}{21} cm^2$

7. A sector of a circle of radius 8cm contains the angle of 135°. Find the area of sector.
Sol:
Radius (r) = 8cm



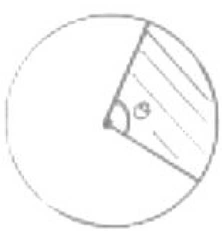
θ = angle subtended at centre = 135°
Area of sector = $\frac{x}{360°} \times \pi r^2$
$= \frac{135}{360} \times \frac{22}{7} \times 8 \times 8$
= $\frac{528}{7} cm^2$

8. The area of sector of circle of radius 2cm is πcm^2. Find the angle contained by the sector.
Sol:



Area of sector = $\pi\ cm^2$
Radius of circle = $2cm$
Let θ = angle subtended by arc at centre
Area of sector = $\frac{\theta}{360°} \times \pi r^2$
= $\frac{\theta}{360°} \times \pi \times 2 \times 2$
= $\frac{\pi\theta}{90°}$
$\frac{\pi\theta}{90°} = \pi \Rightarrow\ \theta = 90°$

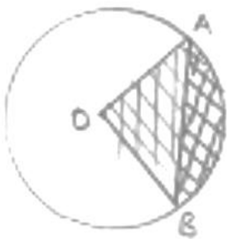
9. The area of sector of circle of radius 5cm is 5π cm^2. Find the angle contained by the sector.
Sol:



Area of sector = 5π cm^2.
Radius (r) = 5cm
Let θ = angle subtended at centre area of sector = $\frac{\theta}{360°} \times \pi r^2$
$= \frac{\theta}{360} \times \pi \times 5 \times 5 = \frac{5\pi\theta}{72°}$
$= \frac{5\pi\theta}{72°} = 5\pi$
$\Rightarrow \theta = 72°$

10. AB is a chord of circle with centre O and radius 4cm. AB is length of 4cm. Find the area of sector of the circle formed by chord AB

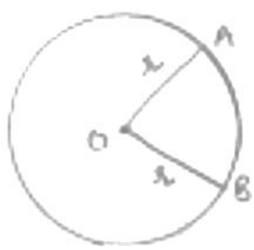
Sol:



AB is chord AB = 4cm
OA = OB = 4cm
OAB is equilateral triangle ∠AOB = 60°
Area of sector (formed by chord [shaded region]) = (area of sector)
= $\frac{\theta}{360°} \times \pi r^2 = \frac{60}{360} \times \pi \times 4 \times 4 = \frac{8\pi}{3} cm^2$

11. In a circle of radius 35 cm, an arc subtends an angle of 72° at the centre. Find the length of arc and area of sector

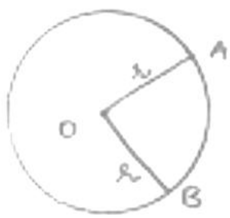
Sol:



Radius (r) = 35 cm
θ = angle subtended at centre = 72°
Length of arc = $\frac{\theta}{360°} \times 2\pi r$
= $\frac{72}{360} \times 2 \times \frac{22}{7} \times 35$
= $2 \times 22 = 44 cms$
Area of sector = $\frac{\theta}{360°} \times \pi r^2$

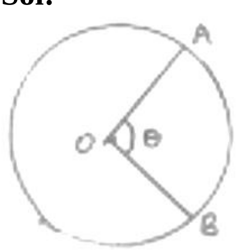
$= \frac{72}{360} \times \frac{22}{7} \times 35 \times 35$
$= (35 \times 22) = 770\ cm^2$

12. The perimeter of a sector of circle of radius 5.7m is 27.2 m. Find the area of sector.
Sol:



Radius = OA = OB (From fig) = r
= 5.7 m
Perimeter = 27.2 m
Let angle subtended at centre = θ
Perimeter = $\left(\frac{\theta}{360°} \times 2\pi r\right) + OA + OB$
$= \frac{\theta}{360°} \times 2(5.7) \times \pi + 2(5.7)$
$= \frac{2\pi(5.7)\theta}{360°} + 11.4$
$= \frac{\pi(5.7)\theta}{180°} + 11.4 = 27.2$
$= \frac{\pi(5.7)\theta}{180°} = 15.8$
$Area\ of\ sector = \frac{\theta}{360°} \times \pi r^2$
$= \frac{158.8}{360} \times \frac{22}{7} \times 5.7 \times 5.7$
$= 45.048\ cm^2$

13. The perimeter of certain sector of circle of radius 5.6 m is 27.2 m. Find the area of sector.
Sol:



θ = angle subtended at centre
Radius (r) = 5.6m = OA $\pm$ OB
Perimeter of sector = 27.2 m
(AB arc length) + OA + OB = 27.2
$\Rightarrow \left(\frac{\theta}{360°} \times 2\pi r\right) + 5.6 + 5.6 \pm 27.2$

$\Rightarrow \frac{5.6\,\pi\,\theta}{180°} + 11.2 = 27.2$

$\Rightarrow 5.6 \times \frac{22}{7} \times \theta = 16 \times 180$

$\Rightarrow \theta = \frac{16\times180}{0.8\times22} = 163.64°$

Area of sector = $\frac{\theta}{360°} \times \pi r^2 = \frac{163.64°}{360°} \times \frac{22}{7} \times 5.6 \times 5.6$

= $\frac{163.64}{180} \times 11 \times 0.8 \times 5.6$

= $44.8\ cm^2$

14. A sector is cut-off from a circle of radius 21 cm the angle of sector is 120°. Find the length of its arc and its area.

Sol:

Radius of circle (r) = 21 cm

θ = angle subtended at centre = 120°

Length of its arc = $\frac{\theta}{360°} \times 2\pi r$

= $\frac{120}{360} \times 2 \times \frac{22}{7} \times 21$

= 44 cms

Area of sector = $\frac{\theta}{360°} \times \pi r^2$

= $\frac{120}{360} \times \frac{22}{7} \times 21 \times 21$

$= (22 \times 21)$

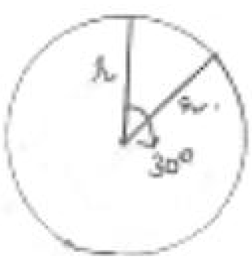
$= 462\ cm^2$

$Length\ of\ arc = 44\ cm$

Area of sector = 462 cm^2

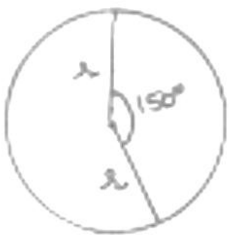
15. The minute hand of a clock is $\sqrt{21}\ cm$ long. Find area described by the minute hand on the face of clock between 7 am and 7:05 am

Sol:



Radius of minute hand (r) = $\sqrt{21}\ cm$
For 1hr = 60 min, minute hand completes one revolution = 360°
60 min = 360°
1 min = 6°
From 7 am to 7:05 am it is 5 min angle subtended = $5 \times 6° = 30° = \theta$
Area described = $\frac{\theta}{360°} \times \pi r^2$
$= \frac{30}{360} \times \frac{22}{7} \times 21$
$= \frac{22}{4} = 5.5 cm^2$

16. The minute hand of clock is10cm long. Find the area of the face of the clock described by the minute hand between 8am and 8:25 am
Sol:



Radius of minute hand (r) = 10 cm
For 1 hr = 60 min, minute hand completes one revolution = 360°
60 min = 360°
1 min = 6°
From 8 am to 8:25 am it is 25 min angle subtended = 6° × 25 = 150° = θ
Area described = $\frac{\theta}{360°} \times \pi r^2$
$= \frac{150}{360} \times \frac{22}{7} \times 10 \times 10$
$= \frac{250 \times 11}{3}$
$= \frac{2750}{3}\ cm^2$

17. A sector of 56° cut out from a circle contains area of 4.4 cm^2. Find the radius of the circle
Sol:
Angle subtended by sector at centre θ = 56°
Let radius be ‘x’ cm
Area of sector = $\frac{\theta}{360°} \times \pi r^2$
= $\frac{56}{360} \times \frac{22}{7} \times r^2$
= $\frac{22}{45} r^2$
But area of sector = 4.4cm^2 = $\frac{44}{10} cm^2$

$\frac{22}{45}\ r^2 = \frac{44}{10}$
$\Rightarrow r^2 = \frac{45\times44}{22\times10} = 9$
$\Rightarrow r = \sqrt{9}$
$= 3\ cm$
∴ radius (r) = 3cm

18. In circle of radius 6cm, chord of length 10 cm makes an angle of 110° at the centre of circle find
(i) Circumference of the circle
(ii) Area of the circle
(iii) Length of arc
(iv) The area of sector

Sol:

(i) Radius of circle (r) = 6 cm
Angle subtended at the centre = 110°
Circumference of the circle = 2πr
= $2 \times \frac{22}{7} \times 6$
= $\frac{264}{7}\ cm$

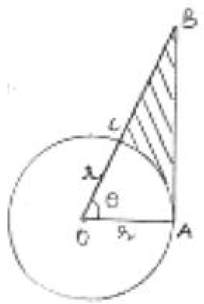
(ii) Area of circle = πr^2 = $\frac{22}{7} \times 6 \times 6$
= $\frac{792}{7}\ cm^2$

(iii) Length of arc = $\frac{\theta}{360°}\ \times 2\pi r$
$= \frac{110}{360} \times 2 \times \frac{22}{7} \times 6$
$= \frac{232}{21}\ cm$

(iv) Area of sector = $\frac{\theta}{360°} \times \pi r^2$
= $\frac{110}{360} \times \frac{22}{7} \times 6 \times 6$
= $\frac{232}{7}\ cm^2$

19. Below fig shows a sector of a circle, centre O. containing an angle θ°. Prove that
(i) Perimeter of shaded region is $r\left(\tan\ \theta + \sec\theta + \frac{\pi\theta}{180} - 1\right)$
(ii) Area of shaded region is $\frac{r^2}{2}\left(\tan\theta - \frac{\pi\theta}{180}\right)$

Sol:



Given angle subtended at centre of circle = θ

∠OAB = 90° [At joint of contact, tangent is perpendicular to radius]

OAB is right angle triangle

Cos $\theta = \frac{adj.side}{hypotenuse} = \frac{r}{OB} \Rightarrow OB = r\sec\theta \ldots\ldots (i)$

$\tan\theta = \frac{opp.side}{adj.\ side} = \frac{AB}{r} \Rightarrow AB = r\tan\theta \ldots\ldots.(ii)$

Perimeter of shaded region = AB + BC + (CA arc)

$= r\tan\theta + (OB - OC) + \frac{\theta}{360°} \times 2\pi r$

$= r\tan\theta + r\sec\theta - r + \frac{\pi\,\theta\, r}{180°}$

$= r\left(\tan\theta + \sec\theta + \frac{\pi\theta}{180°} - 1\right)$

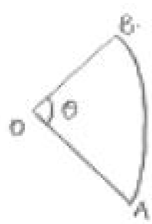
Area of shaded region = (area of triangle) – (area of sector)

$= \left(\frac{1}{2} \times OA \times AB\right) - \frac{\theta}{360°} \times \pi r^2$

$= \frac{1}{2} \times r \times r\tan\theta - \frac{r^2}{2}\left[\frac{\theta}{180°} \times \pi\right]$

$= \frac{r^2}{2}\left[\tan\theta - \frac{\pi\theta}{180}\right]$

20. The diagram shows a sector of circle of radius ‘r’ can containing an angle θ. The area of sector is A cm^2 and perimeter of sector is 50 cm. Prove that



(i) $\theta = \frac{360}{\pi}\left(\frac{25}{r} - 1\right)$

(ii) A = 25r – r^2

Sol:

(i) Radius of circle = ‘r’ cm

Angle subtended at centre = θ

Perimeter = OA + OB + (AB arc)

$= r + r + \frac{\theta}{360°} \times 2\pi r = 2r + 2r\left[\frac{\pi\theta}{360°}\right]$

But perimeter given as 50

$50 = 2r\left[1 + \frac{\pi\theta}{360°}\right]$

$\Rightarrow \frac{\pi\theta}{360°} = \frac{50}{2r} - 1$

$\Rightarrow \theta = \frac{360°}{\pi}\left[\frac{25}{r} - 1\right]$ …..(i)

(ii) Area of sector = $\frac{\theta}{360°} \times \pi r^2$

$= \frac{\frac{360°}{\pi}\left(\frac{25}{r} - 1\right)}{360°} \times \pi r^2$

$= \frac{25}{r} \times r^2 - r^2$

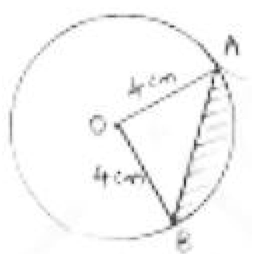
$= 25r - r^2$

$\Rightarrow A = 25r - r^2$ …..(ii)

Exercise 15.3

1. AB is a chord of a circle with centre O and radius 4cm. AB is length 4cm and divides circle into two segments. Find the area of minor segment

Sol:



Radius of circle r = 4cm = OA = OB

Length of chord AB = 4cm

OAB is equilateral triangle ∠AOB = 60° ⟶ θ

Angle subtended at centre θ = 60°

Area of segment (shaded region) = (area of sector) – (area of ΔAOB)

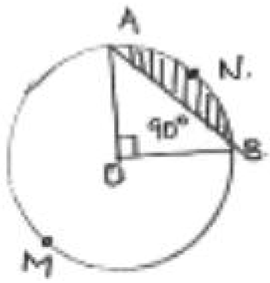
$= \frac{\theta}{360°} \times \pi r^2 = \frac{\sqrt{3}}{4}(side)^2$

$= \frac{60}{360} \times \frac{22}{7} \times 4 \times 4 = \frac{\sqrt{2}}{4} \times 4 \times 4$

$= \frac{176}{3} - 4\sqrt{3} = 58.67 - 6.92 = 51.75\ cm^2$

2. A chord of circle of radius 14cm makes a right angle at the centre. Find the areas of minor and major segments of the circle.

Sol:



Radius (r) = 14cm

θ = 90°

= OA = OB

Area of minor segment (ANB)

= $(area\ of\ ANB\ sector) - (area\ of\ \Delta AOB)$

= $\frac{\theta}{360°} \times \pi r^2 - \frac{1}{2} \times OA \times OB$

= $\frac{90}{360} \times \frac{22}{7} \times 14 \times 14 - \frac{1}{2} \times 14 \times 14$

= $154 - 98 = 56cm^2$

Area of major segment (other than shaded)

= area of circle – area of segment ANB

= $\pi r^2 - 56$

= $\frac{22}{7} \times 14 \times 14 - 56$

= 616 – 56

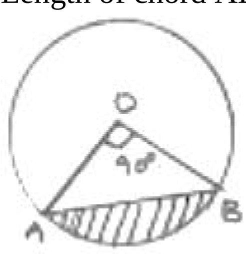
= 560 cm^2.

3. A chord 10 cm long is drawn in a circle whose radius is $5\sqrt{2}$ cm. Find the area of both segments

Sol:

Given radius = r = $5\sqrt{2}$ cm = OA = OB

Length of chord AB = 10cm



In ΔOAB, OA = OB = $5\sqrt{2}\ cm$ AB = 10cm

$OA^2 + OB^2 = \left(5\sqrt{2}\right)^2 + \left(5\sqrt{2}\right)^2 = 50 + 50 = 100 = (AB)^2$

Pythagoras theorem is satisfied OAB is right triangle

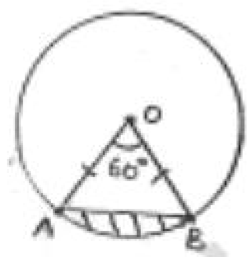
θ = angle subtended by chord = ∠AOB = 90°

Area of segment (minor) = shaded region

= $area\ of\ sector - area\ of\ \Delta OAB$

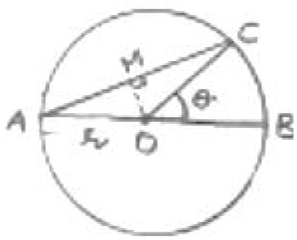
$= \frac{\theta}{360} \times \pi r^2 - \frac{1}{2} \times OA \times OB$
$= \frac{90}{360} \times \frac{22}{7} \left(5\sqrt{2}\right)^2 - \frac{1}{2} \times 5\sqrt{2} \times 5\sqrt{2}$
$= \frac{275}{7} - 25 - \frac{100}{7}\ cm^2$
Area of major segment = (area of circle) – (area of minor segment)
$= \pi r^2 2 - \frac{100}{7}$
$= \frac{22}{7} \times \left(5\sqrt{2}\right)^2 - \frac{100}{7}$
$= \frac{1100}{7} - \frac{100}{7} = \frac{1000}{7}\ cm^2$

4. A chord AB of circle, of radius 14cm makes an angle of 60° at the centre. Find the area of minor segment of circle.
Sol:



Given radius (r) = 14cm = OA = OB
θ = angle at centre = 60°
In ΔAOB, ∠A = ∠B [angles opposite to equal sides OA and OB] = x
By angle sum property ∠A + ∠B + ∠O = 180°
x + x + 60° = 180° ⇒ 2x = 120° ⇒ x = 60°
All angles are 60°, OAB is equilateral OA = OB = AB
Area of segment = area of sector – area Δle OAB
$= \frac{\theta}{360°} \times \pi r^2 - \frac{\sqrt{3}}{4} \times (-AB)^2$
$= \frac{60}{360} \times \frac{22}{7} \times 14 \times 14 - \frac{\sqrt{3}}{4} \times 14 \times 14$
$= \frac{308}{3} - 49\sqrt{3} = \frac{308-147\sqrt{3}}{3}\ cm^2$

5. AB is the diameter of a circle, centre O. C is a point on the circumference such that ∠COB $= \theta$. The area of the minor segment cutoff by AC is equal to twice the area of sector BOC. Prove that $\sin\frac{\theta}{2} . \cos\frac{\theta}{2} = \pi\left(\frac{1}{2} - \frac{\theta}{120°}\right)$
Sol:



Given AB is diameter of circle with centre O

∠COB = θ

Area of sector BOC = $\frac{\theta}{360°} \times \pi r^2$

Area of segment cut off, by AC = (area of sector) – (area of ΔAOC)

∠AOC = 180 – θ [∠AOC and ∠BOC form linear pair]

Area of sector $= \frac{(180-\theta)}{360°} \times \pi r^2 = \frac{\pi r^2}{2} - \frac{\pi\theta r^2}{360°}$

In ΔAOC, drop a perpendicular AM, this bisects ∠AOC and side AC.

Now, In ΔAMO, sin∠AOM = $\frac{AM}{DA} \Rightarrow \sin\left(\frac{180-\theta}{2}\right) = \frac{AM}{R}$

$\Rightarrow$ AM = R $\sin\left(90 - \frac{\theta}{2}\right) = R.\cos\frac{\theta}{2}$

cos ∠ADM = $\frac{OM}{OA} \Rightarrow \cos\left(90 - \frac{\theta}{2}\right) = \frac{OM}{Y} \Rightarrow OM = R.Sin\frac{\theta}{2}$

Area of segment = $\frac{\pi r^2}{2} - \frac{\pi\theta r^2}{360°} - \frac{1}{2}(AC \times OM)$ $[AC = 2\ AM]$

$= \frac{\pi r^2}{2} - \frac{\pi\theta r^2}{360°} - \frac{1}{2} \times \left(2\ R\cos\frac{\theta}{2}\ R\sin\frac{\theta}{2}\right)$

$= r^2\left[\frac{\pi}{2} - \frac{\pi\theta}{360°} - \cos\frac{\theta}{2}\sin\frac{\theta}{2}\right]$

Area of segment by AC = 2 (Area of sector BDC)

$r^2\left[\frac{\pi}{2} - \frac{\pi\theta}{360°} - \cos\frac{\theta}{2}\ .\sin\frac{\theta}{2}\right] = 2r^2\left[\frac{\pi\theta}{360°}\right]$

$\cos\frac{\theta}{2}\ .\sin\frac{\theta}{2} = \frac{\pi}{2} - \frac{\pi\theta}{360} - \frac{2\pi\theta}{360°}$

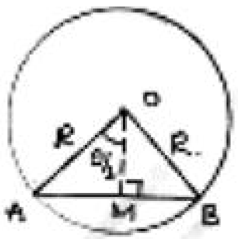
$= \frac{\pi}{2} - \frac{\pi\theta}{360°}[1+2]$

$= \frac{\pi}{2} - \frac{\pi\theta}{360°} = \pi\left(\frac{1}{2} - \frac{\theta}{120°}\right)$

$\cos\frac{\theta}{2}\ .\sin\frac{\theta}{2} = \pi\ \left(\frac{1}{2} - \frac{\theta}{120°}\right)$

6. A chord of a circle subtends an angle θ at the centre of circle. The area of the minor segment cut off by the chord is one eighth of the area of circle. Prove that 8 sin $\frac{\theta}{2}$. cos $\frac{\theta}{2}$ + $\pi = \frac{\pi\theta}{45}$

Sol:



Let radius of circle = r

Area of circle = πr^2

AB is a chord, OA, OB are joined drop OM ⊥ AB. This OM bisects AB as well as ∠AOB.

$\angle AOM = \angle MOB = \frac{1}{2}(0) = \frac{\theta}{2}$ $\qquad$ $AB = 2AM$

In ΔAOM, $\angle AMO = 90°$

$\text{Sin}\,\frac{\theta}{2} = \frac{AM}{AD} \Rightarrow AM = R.\sin\frac{\theta}{2}$ $\qquad$ $AB = 2R\sin\frac{\theta}{2}$

$\text{Cos}\,\frac{\theta}{2} = \frac{OM}{AD} \Rightarrow OM = R\cos\frac{\theta}{2}$

Area of segment cut off by AB = (area of sector) – (area of triangles)

$= \frac{\theta}{360} \times \pi r^2 - \frac{1}{2} \times AB\ \times OM$

$= r^2\left[\frac{\pi\theta}{360°} - \frac{1}{2}.2rsin\frac{\theta}{2}\ .R\cos\frac{\theta}{2}\right]$

$= R^2\left[\frac{\pi\theta}{360°} - \sin\frac{\theta}{2}.\cos\frac{\theta}{2}\right]$

Area of segment $= \frac{1}{2}(area\ of\ circle)$

$r^2\left[\frac{\pi\theta}{360} - \sin\frac{\theta}{2}\ .\cos\frac{\theta}{2}\right] = \frac{1}{8}\pi r^2$

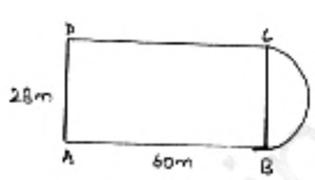
$\frac{8\pi\theta}{360°} - 8\sin\frac{\theta}{2}\ .\cos\frac{\theta}{2} =\ \pi$

$8\sin\frac{\theta}{2}\ .\cos\frac{\theta}{2} + \pi = \frac{\pi\theta}{45}$

Exercise 15.4

1. A plot is in the form of rectangle ABCD having semi-circle on BC. If AB = 60m and BC = 28m, find the area of plot.

Sol:



Given AB = 60m = DC [length]

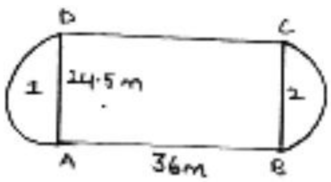
BC = 28m = AD [breadth]

Radius of semicircle r = $\frac{1}{2} \times BC = 14m$

Area of semicircle r = $\frac{1}{2} \times BC = 14m$
Area of plot = (Area of rectangle ABCD) + (area of semicircle)
= $(length \times breadth) + \frac{1}{2}\pi r^2$
= $(60 \times 28) + \left[\frac{1}{2} \times \frac{22}{7} \times 14 \times 14\right]$
= $1680 + 308 = 1988m^2$

2. A playground has the shape of rectangle, with two semicircles on its smaller sides as diameters, added to its outside. If the sides of rectangle are 36m and 24.5m. find the area of playground.

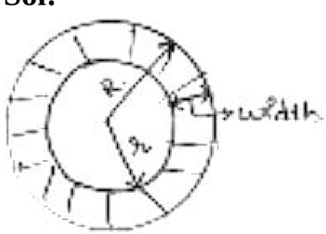
Sol:



Let rectangular play area be ABCD
AB = CD = 36m [length]
AD = BC = 24.5 m [breadth]
Radius of the semicircle = $\frac{1}{2}(BC) = R$
= $\frac{1}{2} \times (24.5) = 12.25cm$
Area of playground = (Area of rectangle) + 2(Area of semicircle)
= $(AB \times BC) + \left(\frac{1}{2}\pi r^2\right)2$
$= (36 \times 24.5) + \left(\frac{1}{2} \times \frac{22}{7} \times 12.25 \times 12.25\right)2$
$= 882 + 471.625$
$= 1353.625\ m^2$

3. The outer circumference of a circular race track is 528m. The track is everywhere 14m wide. Calculate the cost of leveling the track at rate of 50 paise per square metre.

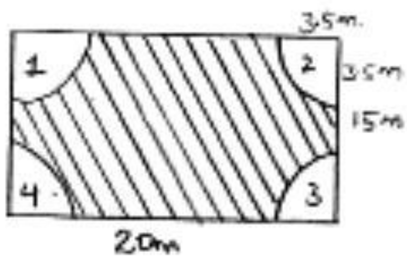
Sol:



Let inner radius = r width(d) = 14m
Outer radius = R
Outer circumference of track = 2 πr
$\therefore 2\ \pi r = 528$

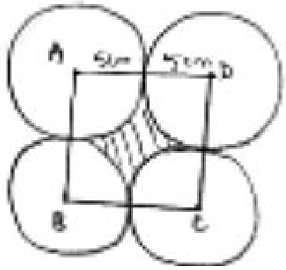
$2 \times \frac{22}{7} \times R = 528 \Rightarrow R = \frac{528 \times 7}{2 \times 22} = 84\ m$
Inner radius r = R – d = 84 – 14 = 70m
Area of track = (area of outer circle) – (area of inner circles)
= $\pi R^2 - \pi r^2$
= $\pi(R^2 - r^2) = \frac{22}{7}(84^2 - 70^2)$
= $\frac{22}{7}(84 + 70)(84 - 70) = \frac{22}{7} \times 154 \times 14$
= $6776\ m^2$
Cost of leveling $m^2 = Rs. 0.50$
Total cost of leveling track = $6776 \times \frac{1}{2} = Rs. 3388$

4. A rectangular piece is 20m long and 15m wide from its four corners, quadrants of 3.5m radius have been cut. Find the area of remaining part.
Sol:



Length of rectangular piece l = 20m
Breadth of rectangular piece b = 15m
Radius of each quadrant r = 3.5m
Area of rectangular piece = (length × breadth) = 20 × 15 = 300m^2.
Area of quadrant each = $\frac{1}{4}(area\ of\ circle\ with\ radius\ 3.5m)$
= $\frac{1}{4} \times \pi r^2$
= $\frac{1}{4} \times \frac{22}{7} \times 3.5 \times 3.5 = \frac{38.5}{4} m^2$
Area of remaining part = [area of rectangular piece] – 4[area of each quadrant]
= 300 – 4 [$\frac{385}{4}$] = 300 – 38.5
= 261.5m^2

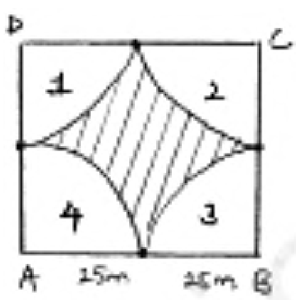
5. Four equal circles, each of radius 5 cm touch each other as shown in fig. Find the area included between them.
Sol:



Area required shaded = (area of square ABCD) – (Area of 4 quadrant)
Side of square = 5cm + 5cm
= 10cm
Area of square = side × side
= $10cm \times 10cm = 100cm^2$
Area of quadrant = $\frac{1}{4}(area\ of\ circle\ with\ radius\ 5\ cm)$
= $\frac{1}{4} \times \pi r^2$
= $\frac{1}{4} \times \frac{22}{7} \times 5 \times 5 = (25 \times 3.14)\frac{1}{4}\ cm^2$
Area included between circles = (area of square) – 4(area of quadrant)
= 100 – $\left(\frac{1}{4} \times 25 \times 2.14\right)$
= 100 – 78.5
= 21.5cm^2

6. Four cows are tethered at four corners of a square plot of side 50m, so that' they just cant reach one another. What area will be left ungrazed.

Sol:



Side of square plot (s) = 50m
Area grazed by four cows is area of sectors represented by 1, 2, 3 and 4.
Radius of each quadrant = 25m = r.
Area of square plot = $s^2 = 50^2 = 2500m^2$
Area of each quadrant = $\frac{1}{4}\pi r^2 = \frac{1}{4} \times \frac{22}{7} \times 25 \times 25 = (625 \times 3.14) \times \frac{1}{4}$
Area of ungrazed land = (area of square plot) – 4(area of quadrant)
= $2500 - 4\left(\frac{1}{4} \times 3.14 \times 625\right)$
= 2500 – 1962.5 = 537.5 m^2

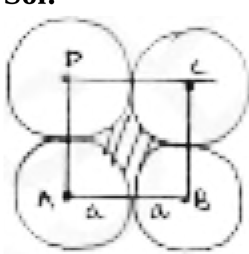
7. A road which is 7m wide surrounds a circular park whose circumference is 352m. Find the area of road.

Sol:

Outer radius of road = R
Inner radius of road = r
Width of park road = d
R = 2 + d
Circumference of road (outer) = 2πR
2πR = 352 [from problem given]
$2 \times \frac{22}{7} \times R = 352$
R = $\frac{352\times7}{2\times22} = 56m.$
Inner radius = R – d = 56 – 7 = 49 m
Area of road = (area of circle with radius 56m) – (area of circle with radius 49m)
= $\pi R^2 - \pi r^2$
= $\frac{22}{7}(56^2 - 49^2) = \frac{22}{7}(56 - 49)\ (56 + 49)$
= $\frac{22}{7} \times 7 \times 105 = 2310m^2$

8. Four equal circles each of radius a, touch each other. Show that area between them is $\frac{6}{7}a^2$

Sol:



Let circles be with centres A, B, C, D
Join A, B, C and D then ABCD is square formed with side = (a + a) = 2a
Radius = a
Area between circles = area of square – 4(area of quadrant)
(shaded region)
= $(2a)^2 - 4\left(\frac{1}{4}\ area\ of\ circle\ with\ radius\ 'a'\right)$

$= 4a^2 - 4\left(\frac{1}{4}\right) \times a^2$

$= a^2(4 - \pi)$

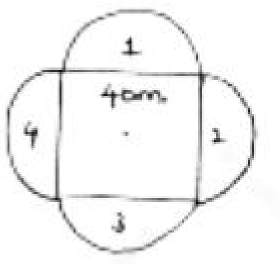
$= a^2\left(4 - \frac{22}{7}\right)$

$= \left(\frac{28-22}{7}\right)a^2 = \frac{6}{7}\ a^2$

$\therefore$ Area between circles $= \frac{6}{7}\ a^2$.

9. A square water tank has its side equal to 40m, there are 4 semicircular flower beds grassy plots all around it. Find the cost of turfing the plot at Rs 1.25/sq.m

Sol:



Side of water tank = 40m

Grassy plot is semicircular with radius $= \frac{side}{2} = \frac{40}{2} = 20m = r$

Area of grassy plot = 4(area od semicircular grassy plot with radius 20m)

$= 4\left[\frac{1}{2}\ (area\ of\ circle\ with\ radius)\right]$

$= 4 \times \frac{1}{2} \times \pi(20)^2$

$= 2 \times 20 \times 20 \times \pi = 800\pi\ m^2$.

Cost of turfing 1m^2 = Rs. 1.25

Total cost of turfing the grassy plot around tank

$= 800\pi\ \times 1.25$

$= 1000\pi$

$= 1000 \times 3.14$

= Rs. 3140.