1. Evaluate:

$\sin 60^0 \cos 30^0 + \cos 60^0 \sin 30^0$

Sol:

On substituting the values of various T-ratios, we get:

$\sin 60^0 \cos 30^0 + \cos 60^0 \sin 30^0$

$= \left(\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2}\right) = \left(\frac{3}{4} + \frac{1}{4}\right) = \frac{4}{4} = 1$

2. Evaluate:

$\cos 60^0 \cos 30^0 - \sin 60^0 \sin 30^0$

Sol:

On substituting the values of various T-ratios, we get:

$\cos 60^0 \cos 30^0 - \sin 60^0 \sin 30^0$

$= \left(\frac{1}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \times \frac{1}{2}\right) = \left(\frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}\right) = 0$

3. Evaluate:

$\cos 45^0 \cos 30^0 + \sin 45^0 \sin 30^0$

Sol:

On substituting the values of various T-ratios, we get:

$\cos 45^0 \cos 30^0 + \sin 45^0 \sin 30^0$

$= \left(\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}\right) = \left(\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}\right) = \left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)$

4. Evaluate:

$\frac{\sin 30^0}{\cos 45^0} + \frac{\cot 45^0}{\sec 60^0} - \frac{\sin 60^0}{\tan 45^0} + \frac{\cos 30^0}{\sin 90^0}$

Sol:

$\frac{\sin 30^0}{\cos 45^0} + \frac{\cot 45^0}{\sec 60^0} - \frac{\sin 60^0}{\tan 45^0} + \frac{\cos 30^0}{\sin 90^0}$

$= \frac{\left(\frac{1}{2}\right)}{\left(\frac{1}{\sqrt{2}}\right)} + \frac{1}{2} - \frac{\left(\frac{\sqrt{3}}{2}\right)}{1} + \frac{\left(\frac{\sqrt{3}}{2}\right)}{1}$

$= \frac{\sqrt{2}}{2} + \frac{1}{2} - \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{2}+1}{2}$

5. Evaluate:

$\frac{5\cos^2 60^0 + 4\sec^2 30^0 - \tan^2 45^0}{\sin^2 30^0 + \cos^2 30^0}$

Sol:

$\frac{5\cos^2 60^0 + 4\sec^2 30^0 - \tan^2 45^0}{\sin^2 30^0 + \cos^2 30^0}$

$= \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$

$= \frac{\left(\frac{5}{4} + \frac{4 \times 4}{3} - 1\right)}{\left(\frac{1}{4} + \frac{3}{4}\right)}$

$= \frac{\left(\frac{5}{4} + \frac{16}{3} - \frac{1}{1}\right)}{\left(\frac{4}{4}\right)}$

$= \frac{\left(\frac{15+64-12}{12}\right)}{\left(\frac{4}{4}\right)}$

$= \frac{\left(\frac{67}{12}\right)}{(1)}$

$= \frac{67}{12}$

6. Evaluate:

$2\cos^2 60^0 + 3\sin^2 45^0 - 3\sin^2 30^0 + 2\cos^2 90^0$

Sol:

On substituting the values of various T-ratios, we get:

$2\cos^2 60^0 + 3\sin^2 45^0 - 3\sin^2 30^0 + 2\cos^2 90^0$

$= 2 \times \left(\frac{1}{2}\right)^2 + 3 \times \left(\frac{1}{\sqrt{2}}\right)^2 - 3 \times \left(\frac{1}{2}\right)^2 + 2 \times (0)^2$

$= 2 \times \frac{1}{4} + 3 \times \frac{1}{2} - 3 \times \frac{1}{4} + 0$

$= \left(\frac{1}{2} + \frac{3}{2} - \frac{3}{4}\right) = \left(\frac{2+6-3}{4}\right) = \frac{5}{4}$

7. Evaluate:

$\cot^2 30^0 - 2\cos^2 30^0 - \frac{3}{4}\sec^2 45^0 + \frac{1}{4}\operatorname{cosec}^2 30^0$

Sol:

On substituting the values of various T-ratios, we get:

$\cot^2 30^0 - 2\cos^2 30^0 - \frac{3}{4}\sec^2 45^0 + \frac{1}{4}\operatorname{cosec}^2 30^0$

$= \left(\sqrt{3}\right)^2 - 2 \times \left(\frac{\sqrt{3}}{2}\right)^2 - \frac{3}{4} \times \left(\sqrt{2}\right)^2 + \frac{1}{4} \times (2)^2$

$= 3 - 2 \times \frac{3}{4} - \frac{3}{4} \times 2 + \frac{1}{4} \times 4$

$= 3 - \frac{3}{2} - \frac{3}{2} + 1$

$= 4 - \left(\frac{3}{2} + \frac{3}{2}\right)$

$= 4 - 3 = 1$

8. Evaluate:

$(\sin^2 30^0 + 4\cot^2 45^0 - \sec^2 60^0)(\text{cosec}^2 45^0 \sec^2 30^0)$

Sol:

On substituting the values of various T-ratios, we get:

$(\sin^2 30^0 + 4\cot^2 45^0 - \sec^2 60^0)(\text{cosec}^2 45^0 \sec^2 30^0)$

$= \left[\left(\frac{1}{2}\right)^2 + 4\times(1)^2 - (2)^2\right]\left[\left(\sqrt{2}\right)^2 \left(\frac{2}{\sqrt{3}}\right)^2\right]$

$= \left(\frac{1}{4} + 4 - 4\right)\left(2 \times \frac{4}{3}\right)$

$= \frac{1}{4} \times \frac{8}{3} = \frac{2}{3}$

9. Evaluate:

$\frac{4}{cot^2 30^0} + \frac{1}{sin^2 30^0} - 2\cos^2 45^0 - \sin^2 0^0$

Sol:

On substituting the values of various T-ratios, we get:

$\frac{4}{cot^2 30^0} + \frac{1}{sin^2 30^0} - 2\cos^2 45^0 - \sin^2 0^0$

$= \frac{4}{\left(\sqrt{3}\right)^2} + \frac{1}{\left(\frac{1}{2}\right)^2} - 2 \times \left(\frac{1}{\sqrt{2}}\right)^2 - (0)^2$

$= \frac{4}{3} + \frac{1}{\frac{1}{4}} - 2 \times \frac{1}{2} - 0$

$= \frac{4}{3} + 4 - 1$

$= \frac{4}{3} + 3 = \frac{4+9}{3} = \frac{13}{3}$

10. Show that:

(i) $\frac{1-\sin 60^0}{\cos 60^0} = \frac{\tan 60^0 - 1}{\tan 60^0 + 1}$

(ii) $\frac{\cos 30^0 + \sin 60^0}{1 + \sin 30^0 + \cos 60^0} = \cos 30^0$

Sol:

(i)

LHS $= \frac{1-\sin 60^0}{\cos 60^0} = \frac{1-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\left(\frac{2-\sqrt{3}}{2}\right)}{\frac{1}{2}} = \left(\frac{2-\sqrt{3}}{2}\right) \times 2 = 2 - \sqrt{3}$

RHS $= \frac{\tan 60^0 - 1}{\tan 60^0 + 1} = \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{\left(\sqrt{3}-1\right)^2}{\left(\sqrt{3}\right)^2 - 1^2} = \frac{3+1-2\sqrt{3}}{3-1} = \frac{4-2\sqrt{3}}{2} = 2 - \sqrt{3}$

Hence, LHS = RHS

$\therefore \frac{1-\sin 60^0}{\cos 60^0} = \frac{\tan 60^0 - 1}{\tan 60^0 + 1}$

(ii)

LHS $= \frac{\cos 30^0 + \sin 60^0}{1 + \sin 30^0 + \cos 60^0} = \frac{\left(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right)}{1 + \frac{1}{2} + \frac{1}{2}} = \frac{\frac{\sqrt{3}+\sqrt{3}}{2}}{\frac{2+1+1}{2}} = \frac{\sqrt{3}}{2}$

Also, RHS = $\cos 30^0 = \frac{\sqrt{3}}{2}$

Hence, LHS = RHS

$\therefore \frac{\cos 30^0 + \sin 60^0}{1 + \sin 30^0 + \cos 60^0} = \cos 30^0$

11. Verify each of the following:

(i) $\sin 60^0 \cos 30^0 - \cos 60^0 \sin 30^0$

(ii) $\cos 60^0 \cos 30^0 + \sin 60^0 \sin 30^0$

(iii) $2 \sin 30^0 \cos 30^0$

(iv) $2 \sin 45^0 \cos 45^0$

Sol:

(i) $\sin 60^0 \cos 30^0 - \cos 60^0 \sin 30^0$

$= \left(\frac{\sqrt{3}}{2}\right) \times \left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) = \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$

Also, $\sin 30^0 = \frac{1}{2}$

$\therefore \sin 60^0 \cos 30^0 - \cos 60^0 \sin 30^0 = \sin 30^0$

(ii) $\cos 60^0 \cos 30^0 + \sin 60^0 \sin 30^0$

$= \left(\frac{1}{2}\right) \times \left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right) \times \left(\frac{1}{2}\right) = \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$

Also, $\cos 30^0 = \frac{\sqrt{3}}{2}$

$\therefore \cos 60^0 \cos 30^0 + \sin 60^0 \sin 30^0 = \cos 30^0$

(iii) $2 \sin 30^0 \cos 30^0$

$= 2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$

Also, $\sin 60^0 = \frac{\sqrt{3}}{2}$

$\therefore 2 \sin 30^0 \cos 30^0 = \sin 60^0$

(iv) $2 \sin 45^0 \cos 45^0$

$= 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 1$

Also, $\sin 90^0 = 1$

$\therefore 2 \sin 45^0 \cos 45^0 = \sin 90^0$

12. If $A = 45^0$, verify that:

(i) $\sin 2A = 2 \sin A \cos A$

(ii) $\cos 2A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$

Sol:

$A = 45^0$

$\Rightarrow 2A = 2 \times 45^0 = 90^0$

(i) $\sin 2A = \sin 90^0 = 1$

$2 \sin A \cos A = 2 \sin 45^0 \cos 45^0 = 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 1$

$\therefore \sin 2A = 2 \sin A \cos A$

(ii) $\cos 2A = \cos 90^0 = 0$

$2 \cos^2 A - 1 = 2 \cos^2 45^0 - 1 = 2 \times \left(\frac{1}{\sqrt{2}}\right)^2 - 1 = 2 \times \frac{1}{2} - 1 = 1 - 1 = 0$

Now, $1 - 2 \sin^2 A = 1 - 2 \times \left(\frac{1}{\sqrt{2}}\right)^2 - 1 = 1 - 2 \times \frac{1}{2} = 1 - 1 = 0$

$\therefore \cos 2A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$

13. If $A = 30^0$, verify that:

(i) $\sin 2A = \frac{2 \tan A}{1 + \tan^2 A}$

(ii) $\cos 2A = \frac{1 - \tan^2 A}{1 + \tan^2 A}$

(iii) $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$

Sol:

$A = 30^0$

$\Rightarrow 2A = 2 \times 30^0 = 60^0$

(i) $\sin 2A = \sin 60^0 = \frac{\sqrt{3}}{2}$

$$\frac{2 \tan A}{1 + \tan^2 A} = \frac{2 \tan 30^0}{1 + \tan^2 30^0} = \frac{2 \times \left(\frac{1}{\sqrt{3}}\right)}{1 + \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\left(\frac{2}{\sqrt{3}}\right)}{1 + \frac{1}{3}} = \frac{\left(\frac{2}{\sqrt{3}}\right)}{\frac{4}{3}} = \left(\frac{2}{\sqrt{3}}\right) \times \frac{3}{4} = \frac{\sqrt{3}}{2}$$

$\therefore \sin 2A = \frac{2 \tan A}{1 + \tan^2 A}$

(ii) $\cos 2A = \cos 60^0 = \frac{1}{2}$

$$\frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{1 - \tan^2 30^0}{1 + \tan^2 30^0} = \frac{1 - \left(\frac{1}{\sqrt{3}}\right)^2}{1 + \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\left(1 - \frac{1}{3}\right)}{1 + \frac{1}{3}} = \frac{\left(\frac{2}{3}\right)}{\frac{4}{3}} = \left(\frac{2}{3}\right) \times \frac{3}{4} = \frac{1}{2}$$

$\therefore \cos 2A = \frac{1 - \tan^2 A}{1 + \tan^2 A}$

(iii) $\tan 2A = \tan 60^0 = \sqrt{3}$

$$\frac{2 \tan A}{1 - \tan^2 A} = \frac{2 \tan 30^0}{1 - \tan^2 30^0} = \frac{2 \times \left(\frac{1}{\sqrt{3}}\right)}{1 - \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\left(\frac{2}{\sqrt{3}}\right)}{1 - \frac{1}{3}} = \frac{\left(\frac{2}{\sqrt{3}}\right)}{\frac{2}{3}} = \left(\frac{2}{\sqrt{3}}\right) \times \frac{3}{2} = \sqrt{3}$$

$\therefore \tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$

14. If A = 60^0 and B = 30^0, verify that:

(i) sin (A + B) = sin A cos B + cos A sin B

(ii) cos (A + B) = cos A cos B - sin A sin B

Sol:

A = 60^0 and B = 30^0

Now, A + B = $60^0 + 30^0 = 90^0$

Also, A – B = $60^0 – 30^0 = 30^0$

(i) sin (A + B) = sin 90^0 = 1

sin A cos B + cos A sin B = sin 60^0 cos 30^0 + cos 60^0 sin 30^0

$= \left(\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2}\right) = \left(\frac{3}{4} + \frac{1}{4}\right) = 1$

$\therefore$ sin (A + B) = sin A cos B + cos A sin B

(ii) cos (A + B) = cos 90^0 = 0

cos A cos B - sin A sin B = cos 60^0 cos 30^0 - sin 60^0 sin 30^0

$= \left(\frac{1}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \times \frac{1}{2}\right) = \left(\frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}\right) = 0$

$\therefore$ cos (A + B) = cos A cos B - sin A sin B

15. If A = 60^0 and B = 30^0, verify that:

(i) sin (A – B) = sin A cos B – cos A sin B

(ii) cos (A – B) = cos A cos B + sin A sin B

(iii) tan (A – B) = $\frac{\tan A - \tan B}{1 + tan\,A \tan B}$

Sol:

(i) sin (A – B) = sin 30^0 = $\frac{1}{2}$

sin A cos B – cos A sin B = sin 60^0 cos 30^0 – cos 60^0 sin 30^0

$= \left(\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2}\right) = \left(\frac{3}{4} - \frac{1}{4}\right) = \frac{2}{4} = \frac{1}{2}$

$\therefore$ sin (A – B) = sin A cos B – cos A sin B

(ii) cos (A – B) = cos 30^0 = $\frac{\sqrt{3}}{2}$

cos A cos B + sin A sin B = cos 60^0 cos 30^0 + sin 60^0 sin 30^0

$= \left(\frac{1}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \times \frac{1}{2}\right) = \left(\frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}\right) = 2 \times \frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$

$\therefore$ cos (A – B) = cos A cos B + sin A sin B

(iii) tan (A – B) = tan 60^0 = $\frac{1}{\sqrt{3}}$

$\frac{\tan A - \tan B}{1 + tan\,A \tan B} = \frac{\tan 60^0 - \tan 30^0}{1 + tan\,60^0\,tan 30^0} = \frac{\sqrt{3} - \left(\frac{1}{\sqrt{3}}\right)}{1 + \left(\sqrt{3} \times \frac{1}{\sqrt{3}}\right)} = \frac{1}{2} \times \frac{3-1}{\sqrt{3}} = \frac{3}{\sqrt{3}}$

$\therefore$ tan (A – B) = $\frac{\tan A - \tan B}{1 + tan\,A \tan B}$

16. If A and B are acute angles such that tan A = $\frac{1}{3}$, tan B = $\frac{1}{2}$ and tan (A + B) = $\frac{\tan A+\tan B}{1-\tan A \tan B}$, show that A + B = 45^0.

Sol:

Given:

tan A = $\frac{1}{3}$ and tan B = $\frac{1}{2}$

tan (A + B) = $\frac{\tan A+\tan B}{1-\tan A \tan B}$

On substituting these values in RHS of the expression, we get:

$\frac{\tan A+\tan B}{1-\tan A \tan B} = \frac{\left(\frac{1}{3}+\frac{1}{2}\right)}{\left(1-\frac{1}{3}\times\frac{1}{3}\right)} = \frac{\left(\frac{5}{6}\right)}{\left(1-\frac{1}{6}\right)} = \frac{\left(\frac{5}{6}\right)}{\left(\frac{5}{6}\right)} = 1$

$\Rightarrow$ tan (A + B) = 1 = tan 45^0 [$\because$ tan 450 = 1]

$\therefore$ A + B = 45^0

17. Using the formula, tan 2A = $\frac{2\tan A}{1-tan^2A}$, find the value of tan 60^0, it being given that tan 30^0 = $\frac{1}{\sqrt{3}}$.

Sol:

A = 30^0

$\Rightarrow$ 2A = 2 $\times$ 30^0 = 60^0

By substituting the value of the given T-ratio, we get:

tan 2A = $\frac{2\tan A}{1-tan^2A}$

$\Rightarrow$ tan 60^0 = $\frac{2\tan 30^0}{1-tan^2 30^0} = \frac{2\times\left(\frac{1}{\sqrt{3}}\right)}{1-\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\left(\frac{2}{\sqrt{3}}\right)}{1-\frac{1}{3}} = \frac{\left(\frac{2}{\sqrt{3}}\right)}{\frac{2}{3}} = \left(\frac{2}{\sqrt{3}}\right)\times\frac{3}{2} = \sqrt{3}$

$\therefore$ tan 60^0 = $\sqrt{3}$

18. Using the formula, cos A = $\sqrt{\frac{1+\cos 2A}{2}}$, find the value of cos 30^0, it being given that cos 60^0 = $\frac{1}{2}$.

Sol:

A = 30^0

$\Rightarrow$ 2A = 2 $\times$ 30^0 = 60^0

By substituting the value of the given T-ratio, we get:

Cos A = $\sqrt{\frac{1+\cos 2A}{2}}$

Cos 30^0 = $\sqrt{\frac{1+\cos 60^0}{2}} = \sqrt{\frac{1+\frac{1}{2}}{2}} = \sqrt{\frac{\frac{3}{2}}{2}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

$\therefore$ cos A = $\frac{\sqrt{3}}{2}$

19. Using the formula, sin A = $\sqrt{\frac{1-\cos 2A}{2}}$, find the value of sin 30^0, it being given that cos 60^0 = $\frac{1}{2}$.

Sol:

A = 30^0

$\Rightarrow$ 2A = 2 × 30^0 = 60^0

By substituting the value of the given T-ratio, we get:

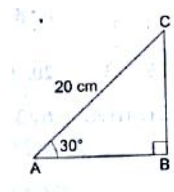
sin A = $\sqrt{\frac{1-\cos 2A}{2}}$

sin 30^0 = $\sqrt{\frac{1-\cos 60^0}{2}} = \sqrt{\frac{1-\frac{1}{2}}{2}} = \sqrt{\frac{\frac{1}{2}}{2}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

$\therefore$ sin 30^0 = $\frac{1}{2}$

20. In the adjoining figure, ΔABC is a right-angled triangle in which ∠B = 90^0, ∠30^0 and AC = 20cm.

Find (i) BC, (ii) AB.



Sol:

From the given right-angled triangle, we have:

$\frac{BC}{AC}$ = sin 30^0

$\Rightarrow \frac{BC}{20} = \frac{1}{2}$

$\Rightarrow$ BC = $\frac{20}{2}$ = 10cm

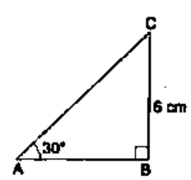
Also, $\frac{AB}{AC}$ = cos 30^0

$\Rightarrow \frac{AB}{20} = \frac{\sqrt{3}}{2}$

$\Rightarrow$ AB = $\left(20 \times \frac{\sqrt{3}}{2}\right)$ = $10\sqrt{3}$ cm

$\therefore$ BC = 10cm and AB = $10\sqrt{3}$ cm

21. In the adjoining figure, ΔABC is right-angled at B and ∠A = 30^0. If BC = 6cm, find (i) AB, (ii) AC.



Sol:

From the given right-angled triangle, we have:

$\frac{BC}{AB}$ = tan 30^0

$\Rightarrow \frac{6}{AB} = \frac{1}{\sqrt{3}}$

$\Rightarrow$ AB = $6\sqrt{3}$cm

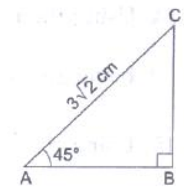
Also, $\frac{BC}{AC}$ = sin 30^0

$\Rightarrow \frac{6}{AC} = \frac{1}{2}$

$\Rightarrow$ AC = (2 × 6) = 12 cm

$\therefore$ AB = $6\sqrt{3}$ cm and AC = 12 cm

22. In the adjoining figure, ΔABC is right-angled at B and $\angle A = 45^0$. If AC = $3\sqrt{2}$cm, find (i) BC, (ii) AB.



Sol:

From the right-angled ΔABC, we have:

$\frac{BC}{AC} = \sin 45^0$

$\Rightarrow \frac{BC}{3\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow$ BC = 3cm

Also, $\frac{AB}{AC} = \cos 45^0$

$\Rightarrow \frac{AB}{3\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow$ AB = 3 cm

$\therefore$ BC = 3 cm and AB = 3 cm

23. If sin (A + B) = 1 and cos (A – B) = 1, $0^0 \le (A + B) \le 90^0$ and A > B, then find A and B.

Sol:

Here, sin (A + B) = 1

$\Rightarrow \sin (A + B) = 90^0$ [$\because \sin 90^0 = 1$]

$\Rightarrow (A + B) = 90^0$ …….(i)

Also, cos (A – B) = 1

$\Rightarrow \cos (A - B) = 0^0$ [$\because \cos 0^0 = 1$]

$\Rightarrow A - B = 0^0$ ….(ii)

Solving (i) and (ii), we get:

$A = 45^0$ and $B = 45^0$

24. If sin (A – B) = $\frac{1}{2}$ and cos (A + B) = $\frac{1}{2}$, $0^0 \le (A + B) \le 90^0$ and A > B, then find A and B.

Sol:

Here, sin (A – B) = $\frac{1}{2}$

$\Rightarrow \sin (A - B) = 30^0$ [$\because \sin 30^0 = \frac{1}{2}$]

$\Rightarrow (A - B) = 30^0$ …….(i)

Also, cos (A + B) = $\frac{1}{2}$

$\Rightarrow \cos (A + B) = \cos 60^0$ [$\because \cos 60^0 = \frac{1}{2}$]

$\Rightarrow A + B = 60^0$ ….(ii)

Solving (i) and (ii), we get:

$A = 45^0$ and $B = 15^0$

25. If tan (A – B) = $\frac{1}{\sqrt{3}}$ and tan (A + B) = $\sqrt{3}$, $0^0 \le (A + B) \le 90^0$ and A > B, then find A and B.

Sol:

Here, tan (A – B) = $\frac{1}{\sqrt{3}}$

$\Rightarrow \tan (A - B) = \tan 30^0$ [$\because \tan 30^0 = \frac{1}{\sqrt{3}}$]

$\Rightarrow (A - B) = 30^0$ …….(i)

Also, tan (A + B) = $\sqrt{3}$

$\Rightarrow$ tan (A + B) = tan 60^0 [$\because$ tan $60^0 = \sqrt{3}$]

$\Rightarrow$ A + B = 60^0 …….(ii)

Solving (i) and (ii), we get:

A = 45^0 and B = 15^0

26. If 3x = cosec θ = and $\frac{3}{x}$ = cot θ find the value of $3\left(x^2 - \frac{1}{x^2}\right)$

Sol:

$3\left(x^2 - \frac{1}{x^2}\right)$

$= \frac{9}{3}\left(x^2 - \frac{1}{x^2}\right)$

$= \frac{1}{3}\left(9x^2 - \frac{9}{x^2}\right)$

$= \frac{1}{3}\left[(3x^2) - \left(\frac{3}{x}\right)^2\right]$

$= \frac{1}{3}\left[(cosec\ \theta)^2 - (cot\ \theta)^2\right]$

$= \frac{1}{3}(\text{cosec}^2\theta - \cot^2\theta)$

$= \frac{1}{3}(1) = \frac{1}{3}$

27. If sin (A+B) = sin A cos B + cos A sin B and cos (A-B) = cos A cos B + sin A sin B

(i) sin (75^0)

(ii) cos (15^0)

Sol:

Let A = 45^0 and B = 30^0

(i) As, sin(A + B) = sin A cos B + cos A sin B

$\Rightarrow$ sin ($45^0 + 30^0$) = sin 45^0 cos 30^0 + cos 45^0 sin 30^0

$\Rightarrow$ sin (75^0) = $\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$\Rightarrow$ sin (75^0) = $\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$

$\therefore$ sin (75^0) = $\frac{\sqrt{3}+1}{2\sqrt{2}}$

(ii) As, cos (A – B) = cos A cos B + sin A sin B

$\Rightarrow$ cos ($45^0 - 30^0$) = cos 45^0 cos 30^0 + sin 45^0 sin 30^0

$\Rightarrow$ cos (15^0) = $\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$\Rightarrow$ cos (15^0) = $\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$

$\therefore$ cos (15^0) = $\frac{\sqrt{3}+1}{2\sqrt{2}}$

Disclaimer: cos 15^0 can also be written by taking A = 60^0 and B = 45^0.