

## Exercise – 4A

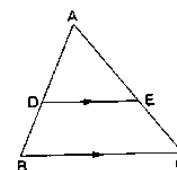
1. D and E are points on the sides AB and AC respectively of a  $\triangle ABC$  such that  $DE \parallel BC$ .

(i) If  $AD = 3.6\text{cm}$ ,  $AB = 10\text{cm}$  and  $AE = 4.5\text{cm}$ , find  $EC$  and  $AC$ .

(ii) If  $AB = 13.3\text{cm}$ ,  $AC = 11.9\text{cm}$  and  $EC = 5.1\text{cm}$ , find  $AD$ .

(iii) If  $\frac{AD}{DB} = \frac{4}{7}$  and  $AC = 6.6\text{cm}$ , find  $AE$ .

(iv) If  $\frac{AD}{AB} = \frac{8}{15}$  and  $EC = 3.5\text{cm}$ , find  $AE$ .



**Sol:**

- (i) In  $\triangle ABC$ , it is given that  $DE \parallel BC$ .

Applying Thales' theorem, we get:

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\because AD = 3.6 \text{ cm}, AB = 10 \text{ cm}, AE = 4.5 \text{ cm}$$

$$\therefore DB = 10 - 3.6 = 6.4 \text{ cm}$$

$$\text{Or, } \frac{3.6}{6.4} = \frac{4.5}{EC}$$

$$\text{Or, } EC = \frac{6.4 \times 4.5}{3.6}$$

$$\text{Or, } EC = 8 \text{ cm}$$

$$\text{Thus, } AC = AE + EC$$

$$= 4.5 + 8 = 12.5 \text{ cm}$$

- (ii) In  $\triangle ABC$ , it is given that  $DE \parallel BC$ .

Applying Thales' Theorem, we get :

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Adding 1 to both sides, we get :

$$\frac{AD}{DB} + 1 = \frac{AE}{EC} + 1$$

$$\Rightarrow \frac{AB}{DB} = \frac{AC}{EC}$$

$$\Rightarrow \frac{13.3}{DB} = \frac{11.9}{5.1}$$

$$\Rightarrow DB = \frac{13.3 \times 5.1}{11.9} = 5.7 \text{ cm}$$

$$\text{Therefore, } AD = AB - DB = 13.3 - 5.7 = 7.6 \text{ cm}$$

- (iii) In  $\triangle ABC$ , it is given that  $DE \parallel BC$ .

Applying Thales' theorem, we get :

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{4}{7} = \frac{AE}{EC}$$

Adding 1 to both the sides, we get :

$$\frac{11}{7} = \frac{AC}{EC}$$

$$\Rightarrow EC = \frac{6.6 \times 7}{11} = 4.2 \text{ cm}$$

Therefore,

$$AE = AC - EC = 6.6 - 4.2 = 2.4 \text{ cm}$$

(iv) In  $\triangle ABC$ , it is given that  $DE \parallel BC$ .

Applying Thales' theorem, we get:

$$\begin{aligned}\frac{AD}{AB} &= \frac{AE}{AC} \\ \Rightarrow \frac{8}{15} &= \frac{AE}{AE + EC} \\ \Rightarrow \frac{8}{15} &= \frac{AE}{AE + 3.5} \\ \Rightarrow 8AE + 28 &= 15AE \\ \Rightarrow 7AE &= 28 \\ \Rightarrow AE &= 4 \text{ cm}\end{aligned}$$

2. D and E are points on the sides AB and AC respectively of a  $\triangle ABC$  such that  $DE \parallel BC$ . Find the value of x, when

- (i)  $AD = x \text{ cm}$ ,  $DB = (x - 2) \text{ cm}$ ,  $AE = (x + 2) \text{ cm}$  and  $EC = (x - 1) \text{ cm}$ .  
 (ii)  $AD = 4 \text{ cm}$ ,  $DB = (x - 4) \text{ cm}$ ,  $AE = 8 \text{ cm}$  and  $EC = (3x - 19) \text{ cm}$ .  
 (iii)  $AD = (7x - 4) \text{ cm}$ ,  $AE = (5x - 2) \text{ cm}$ ,  $DB = (3x + 4) \text{ cm}$  and  $EC = 3x \text{ cm}$ .

**Sol:**

- (i) In  $\triangle ABC$ , it is given that  $DE \parallel BC$ .

Applying Thales' theorem, we have :

$$\begin{aligned}\frac{AD}{DB} &= \frac{AE}{EC} \\ \Rightarrow \frac{x}{x-2} &= \frac{x+2}{x-1} \\ \Rightarrow x(x-1) &= (x-2)(x+2) \\ \Rightarrow x^2 - x &= x^2 - 4 \\ \Rightarrow x &= 4 \text{ cm}\end{aligned}$$

- (ii) In  $\triangle ABC$ , it is given that  $DE \parallel BC$ .

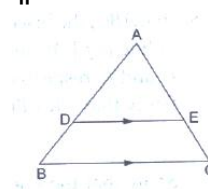
Applying Thales' theorem, we have :

$$\begin{aligned}\frac{AD}{DB} &= \frac{AE}{EC} \\ \Rightarrow \frac{4}{x-4} &= \frac{8}{3x-19} \\ \Rightarrow 4(3x-19) &= 8(x-4) \\ \Rightarrow 12x - 76 &= 8x - 32 \\ \Rightarrow 4x &= 44 \\ \Rightarrow x &= 11 \text{ cm}\end{aligned}$$

- (iii) In  $\triangle ABC$ , it is given that  $DE \parallel BC$ .

Applying Thales' theorem, we have :

$$\begin{aligned}\frac{AD}{DB} &= \frac{AE}{EC} \\ \Rightarrow \frac{7x-4}{3x+4} &= \frac{5x-2}{3x} \\ \Rightarrow 3x(7x-4) &= (5x-2)(3x+4)\end{aligned}$$



$$\Rightarrow 21x^2 - 12x = 15x^2 + 14x - 8$$

$$\Rightarrow 6x^2 - 26x + 8 = 0$$

$$\Rightarrow (x-4)(6x-2) = 0$$

$$\Rightarrow x = 4, \frac{1}{3}$$

$$\because x \neq \frac{1}{3} \text{ (as if } x = \frac{1}{3} \text{ then } AE \text{ will become negative)}$$

$$\therefore x = 4 \text{ cm}$$

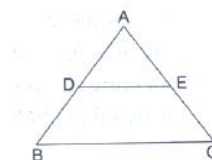
3. D and E are points on the sides AB and AC respectively of a  $\triangle ABC$ . In each of the following cases, determine whether  $DE \parallel BC$  or not.

(i)  $AD = 5.7\text{cm}$ ,  $DB = 9.5\text{cm}$ ,  $AE = 4.8\text{cm}$  and  $EC = 8\text{cm}$ .

(ii)  $AB = 11.7\text{cm}$ ,  $AC = 11.2\text{cm}$ ,  $BD = 6.5\text{cm}$  and  $AE = 4.2\text{cm}$ .

(iii)  $AB = 10.8\text{cm}$ ,  $AD = 6.3\text{cm}$ ,  $AC = 9.6\text{cm}$  and  $EC = 4\text{cm}$ .

(iv)  $AD = 7.2\text{cm}$ ,  $AE = 6.4\text{cm}$ ,  $AB = 12\text{cm}$  and  $AC = 10\text{cm}$ .



**Sol:**

(i) We have:

$$\frac{AD}{DB} = \frac{5.7}{9.5} = 0.6 \text{ cm}$$

$$\frac{AE}{EC} = \frac{4.8}{8} = 0.6 \text{ cm}$$

$$\text{Hence, } \frac{AD}{DB} = \frac{AE}{EC}$$

Applying the converse of Thales' theorem,

We conclude that  $DE \parallel BC$ .

(ii) We have:

$$AB = 11.7 \text{ cm, } DB = 6.5 \text{ cm}$$

Therefore,

$$AD = 11.7 - 6.5 = 5.2 \text{ cm}$$

Similarly,

$$AC = 11.2 \text{ cm, } AE = 4.2 \text{ cm}$$

Therefore,

$$EC = 11.2 - 4.2 = 7 \text{ cm}$$

Now,

$$\frac{AD}{DB} = \frac{5.2}{6.5} = \frac{4}{5}$$

$$\frac{AE}{EC} = \frac{4.2}{7}$$

$$\text{Thus, } \frac{AD}{DB} \neq \frac{AE}{EC}$$

Applying the converse of Thales' theorem,

We conclude that  $DE$  is not parallel to  $BC$ .

(iii) We have:

$$AB = 10.8 \text{ cm, } AD = 6.3 \text{ cm}$$

Therefore,

$$DB = 10.8 - 6.3 = 4.5 \text{ cm}$$

Similarly,

$$AC = 9.6 \text{ cm}, EC = 4 \text{ cm}$$

Therefore,

$$AE = 9.6 - 4 = 5.6 \text{ cm}$$

Now,

$$\frac{AD}{DB} = \frac{6.3}{4.5} = \frac{7}{5}$$

$$\frac{AE}{EC} = \frac{5.6}{4} = \frac{7}{5}$$

$$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$$

Applying the converse of Thales' theorem,

We conclude that  $DE \parallel BC$ .

(iv) We have :

$$AD = 7.2 \text{ cm}, AB = 12 \text{ cm}$$

Therefore,

$$DB = 12 - 7.2 = 4.8 \text{ cm}$$

Similarly,

$$AE = 6.4 \text{ cm}, AC = 10 \text{ cm}$$

Therefore,

$$EC = 10 - 6.4 = 3.6 \text{ cm}$$

Now,

$$\frac{AD}{DB} = \frac{7.2}{4.8} = \frac{3}{2}$$

$$\frac{AE}{EC} = \frac{6.4}{3.6} = \frac{16}{9}$$

$$\text{This, } \frac{AD}{DB} \neq \frac{AE}{EC}$$

Applying the converse of Thales' theorem,

We conclude that  $DE$  is not parallel to  $BC$ .

4. In a  $\triangle ABC$ ,  $AD$  is the bisector of  $\angle A$ .

(i) If  $AB = 6.4 \text{ cm}$ ,  $AC = 8 \text{ cm}$  and  $BD = 5.6 \text{ cm}$ , find  $DC$ .

(ii) If  $AB = 10 \text{ cm}$ ,  $AC = 14 \text{ cm}$  and  $BC = 6 \text{ cm}$ , find  $BD$  and  $DC$ .

(iii) If  $AB = 5.6 \text{ cm}$ ,  $BD = 3.2 \text{ cm}$  and  $BC = 6 \text{ cm}$ , find  $AC$ .

(iv) If  $AB = 5.6 \text{ cm}$ ,  $AC = 4 \text{ cm}$  and  $DC = 3 \text{ cm}$ , find  $BC$ .

**Sol:**

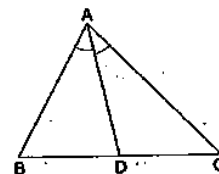
(i) It is given that  $AD$  bisects  $\angle A$ .

Applying angle – bisector theorem in  $\triangle ABC$ , we get:

$$\frac{BD}{DC} = \frac{AB}{AC}$$

$$\Rightarrow \frac{5.6}{DC} = \frac{6.4}{8}$$

$$\Rightarrow DC = \frac{8 \times 5.6}{6.4} = 7 \text{ cm}$$



- (ii) It is given that AD bisects  $\angle A$ .

Applying angle – bisector theorem in  $\triangle ABC$ , we get:

$$\frac{BD}{DC} = \frac{AB}{AC}$$

Let BD be x cm.

Therefore, DC = (6- x) cm

$$\Rightarrow \frac{x}{6-x} = \frac{10}{14}$$

$$\Rightarrow 14x = 60 - 10x$$

$$\Rightarrow 24x = 60$$

$$\Rightarrow x = 2.5 \text{ cm}$$

Thus, BD = 2.5 cm

$$DC = 6 - 2.5 = 3.5 \text{ cm}$$

- (iii) It is given that AD bisector  $\angle A$ .

Applying angle – bisector theorem in  $\triangle ABC$ , we get:

$$\frac{BD}{DC} = \frac{AB}{AC}$$

BD = 3.2 cm, BC = 6 cm

Therefore, DC = 6 - 3.2 = 2.8 cm

$$\Rightarrow \frac{3.2}{2.8} = \frac{5.6}{AC}$$

$$\Rightarrow AC = \frac{5.6 \times 2.8}{3.2} = 4.9 \text{ cm}$$

- (iv) It is given that AD bisects  $\angle A$ .

Applying angle – bisector theorem in  $\triangle ABC$ , we get:

$$\frac{BD}{DC} = \frac{AB}{AC}$$

$$\Rightarrow \frac{BD}{3} = \frac{5.6}{4}$$

$$\Rightarrow BD = \frac{5.6 \times 3}{4}$$

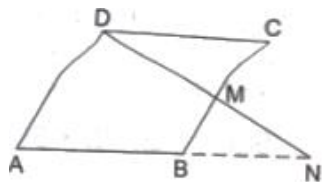
$$\Rightarrow BD = 4.2 \text{ cm}$$

$$\text{Hence, } BC = 3 + 4.2 = 7.2 \text{ cm}$$

5. M is a point on the side BC of a parallelogram ABCD. DM when produced meets AB produced at N. Prove that

(i)  $\frac{DM}{MN} = \frac{DC}{BN}$

(ii)  $\frac{DN}{DM} = \frac{AN}{DC}$



**Sol:**

- (i) Given: ABCD is a parallelogram

To prove:

$$(i) \quad \frac{DM}{MN} = \frac{DC}{BN}$$

$$(ii) \quad \frac{DN}{DM} = \frac{AN}{DC}$$

Proof: In  $\triangle DMC$  and  $\triangle NMB$

$\angle DMC = \angle NMB$  (Vertically opposite angle)

$\angle DCM = \angle NBM$  (Alternate angles)

By AAA- Similarity

$\triangle DMC \sim \triangle NMB$

$$\therefore \frac{DM}{MN} = \frac{DC}{BN}$$

$$\text{NOW, } \frac{MN}{DM} = \frac{BN}{DC}$$

Adding 1 to both sides, we get

$$\frac{MN}{DM} + 1 = \frac{BN}{DC} + 1$$

$$\Rightarrow \frac{MN+DM}{DM} = \frac{BN+DC}{DC}$$

$$\Rightarrow \frac{MN+DM}{DM} = \frac{BN+AB}{DC} \quad [\because ABCD \text{ is a parallelogram}]$$

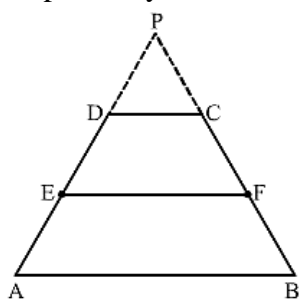
$$\Rightarrow \frac{DN}{DM} = \frac{AN}{DC}$$

6. Show that the line segment which joins the midpoints of the oblique sides of a trapezium is parallel sides

**Sol:**

(i)

Let the trapezium be ABCD with E and F as the mid Points of AD and BC, Respectively Produce AD and BC to Meet at P.



In  $\triangle PAB$ ,  $DC \parallel AB$ .

Applying Thales' theorem, we get

$$\frac{PD}{DA} = \frac{PC}{CB}$$

Now, E and F are the midpoints of AD and BC, respectively.

$$\Rightarrow \frac{PD}{2DE} = \frac{PC}{2CF}$$

$$\Rightarrow \frac{PD}{DE} = \frac{PC}{CF}$$

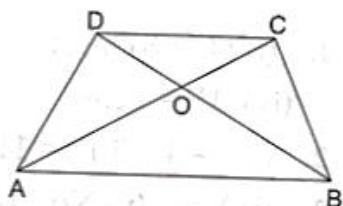
Applying the converse of Thales' theorem in  $\triangle PEF$ , we get that DC

Hence,  $EF \parallel AB$ .

Thus, EF is parallel to both AB and DC.

This completes the proof.

7. In the given figure, ABCD is a trapezium in which  $AB \parallel DC$  and its diagonals intersect at O. If  $AO = (5x - 7)$ ,  $OC = (2x + 1)$ ,  $BO = (7x - 5)$  and  $OD = (7x + 1)$ , find the value of x.



**Sol:**

In trapezium ABCD,  $AB \parallel CD$  and the diagonals AC and BD intersect at O.

Therefore,

$$\frac{AO}{OC} = \frac{BO}{OD}$$

$$\Rightarrow \frac{5x-7}{2x+1} = \frac{7x-5}{7x+1}$$

$$\Rightarrow (5x-7)(7x+1) = (7x-5)(2x+1)$$

$$\Rightarrow 35x^2 + 5x - 49x - 7 = 14x^2 - 10x + 7x - 5$$

$$\Rightarrow 21x^2 - 41x - 2 = 0$$

$$\Rightarrow 21x^2 - 42x + x - 2 = 0$$

$$\Rightarrow 21x(x-2) + 1(x-2) = 0$$

$$\Rightarrow (x-2)(21x+1) = 0$$

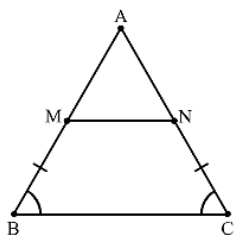
$$\Rightarrow x = 2, -\frac{1}{21}$$

$$\because x \neq -\frac{1}{21}$$

$$\therefore x = 2$$

8. In  $\triangle ABC$ , M and N are points on the sides AB and AC respectively such that  $BM = CN$ . If  $\angle B = \angle C$  then show that  $MN \parallel BC$

**Sol:**



In  $\triangle ABC$ ,  $\angle B = \angle C$

$\therefore AB = AC$  (Sides opposite to equal angle are equal)

Subtracting BM from both sides, we get

$$AB - BM = AC - BM$$

$$\Rightarrow AB - BM = AC - CN \quad (\because BM = CN)$$

$$\Rightarrow AM = AN$$

$\therefore \angle AMN = \angle ANM$  (Angles opposite to equal sides are equal)

Now, in  $\triangle ABC$ ,

$$\angle A + \angle B + \angle C = 180^\circ \quad \text{----(1)}$$

(Angle Sum Property of triangle)

Again In  $\triangle AMN$ ,

$$\angle A + \angle AMN + \angle ANM = 180^\circ \quad \text{----(2)}$$

(Angle Sum Property of triangle)

From (1) and (2), we get

$$\angle B + \angle C = \angle AMN + \angle ANM$$

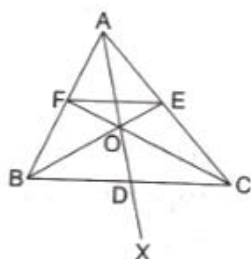
$$\Rightarrow 2\angle B = 2\angle AMN$$

$$\Rightarrow \angle B = \angle AMN$$

Since,  $\angle B$  and  $\angle AMN$  are corresponding angles.

$\therefore MN \parallel BC$ .

9.  $\triangle ABC$  and  $\triangle DBC$  lie on the same side of  $BC$ , as shown in the figure. From a point  $P$  on  $BC$ ,  $PQ \parallel AB$  and  $PR \parallel BD$  are drawn, meeting  $AC$  at  $Q$  and  $CD$  at  $R$  respectively. Prove that  $QR \parallel AD$ .



**Sol:**

In  $\triangle CAB$ ,  $PQ \parallel AB$ .

Applying Thales' theorem, we get:

$$\frac{CP}{PB} = \frac{CQ}{QA} \quad \dots(1)$$

Similarly, applying Thales theorem in  $\triangle BDC$ , Where  $PR \parallel DM$  we get:

$$\frac{CP}{PB} = \frac{CR}{RD} \quad \dots(2)$$

Hence, from (1) and (2), we have :

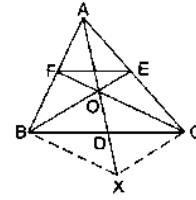
$$\frac{CQ}{QA} = \frac{CR}{RD}$$

Applying the converse of Thales' theorem, we conclude that  $QR \parallel AD$  in  $\triangle ADC$ .

This completes the proof.



10. In the given figure, side BC of a  $\triangle ABC$  is bisected at D and O is any point on AD. BO and CO produced meet AC and AB at E and F respectively, and AD is produced to X so that D is the midpoint of OX. Prove that  $AO : AX = AF : AB$  and show that  $EF \parallel BC$ .



**Sol:**

It is given that BC is bisected at D.

$$\therefore BD = DC$$

It is also given that  $OD = OX$

The diagonals OX and BC of quadrilateral BOCX bisect each other.

Therefore, BOCX is a parallelogram.

$$\therefore BO \parallel CX \text{ and } BX \parallel CO$$

$$BX \parallel CF \text{ and } CX \parallel BE$$

$$BX \parallel OF \text{ and } CX \parallel OE$$

Applying Thales' theorem in  $\triangle ABX$ , we get:

$$\frac{AO}{AX} = \frac{AF}{AB} \quad \dots(1)$$

Also, in  $\triangle ACX$ ,  $CX \parallel OE$ .

Therefore by Thales' theorem, we get:

$$\frac{AO}{AX} = \frac{AE}{AC} \quad \dots(2)$$

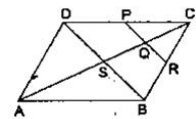
From (1) and (2), we have:

$$\frac{AO}{AX} = \frac{AE}{AC}$$

Applying the converse of Theorem in  $\triangle ABC$ ,  $EF \parallel CB$ .

This completes the proof.

11. ABCD is a parallelogram in which P is the midpoint of DC and Q is a point on AC such that  $CQ = \frac{1}{4} AC$ . If PQ produced meets BC at R, prove that R is the midpoint of BC.



**Sol:**

We know that the diagonals of a parallelogram bisect each other.

Therefore,

$$CS = \frac{1}{2} AC \quad \dots(i)$$

$$\text{Also, it is given that } CQ = \frac{1}{4} AC \quad \dots(ii)$$

Dividing equation (ii) by (i), we get:

$$\frac{CQ}{CS} = \frac{\frac{1}{4} AC}{\frac{1}{2} AC}$$

$$\text{Or, } CQ = \frac{1}{2} CS$$

Hence, Q is the midpoint of CS.

Therefore, according to midpoint theorem in  $\triangle CSD$

$PQ \parallel DS$

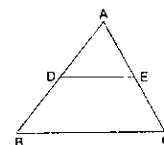
If  $PQ \parallel DS$ , we can say that  $QR \parallel SB$

In  $\triangle CSB$ , Q is midpoint of CS and  $QR \parallel SB$ .

Applying converse of midpoint theorem, we conclude that R is the midpoint of CB.

This completes the proof.

12. In the adjoining figure, ABC is a triangle in which  $AB = AC$ . IF D and E are points on AB and AC respectively such that  $AD = AE$ , show that the points B, C, E and D are concyclic.



**Sol:**

Given:

$$AD = AE \quad \dots(i)$$

$$AB = AC \quad \dots(ii)$$

Subtracting AD from both sides, we get:

$$\Rightarrow AB - AD = AC - AD$$

$$\Rightarrow AB - AD = AC - AE \text{ (Since, } AD = AE)$$

$$\Rightarrow BD = EC \quad \dots(iii)$$

Dividing equation (i) by equation (iii), we get:

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Applying the converse of Thales' theorem,  $DE \parallel BC$

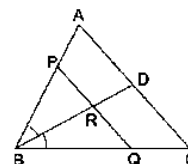
$$\Rightarrow \angle DEC + \angle ECB = 180^\circ \text{ (Sum of interior angles on the same side of a Transversal Line is } 180^\circ \text{.)}$$

$$\Rightarrow \angle DEC + \angle CBD = 180^\circ \text{ (Since, } AB = AC \Rightarrow \angle B = \angle C)$$

Hence, quadrilateral BCED is cyclic.

Therefore, B, C, E and D are concyclic points.

13. In  $\triangle ABC$ , the bisector of  $\angle B$  meets AC at D. A line  $OQ \parallel AC$  meets AB, BC and BD at O, Q and R respectively. Show that  $BP \times QR = BQ \times PR$



**Sol:**

In triangle BQO, BR bisects angle B.

Applying angle bisector theorem, we get:

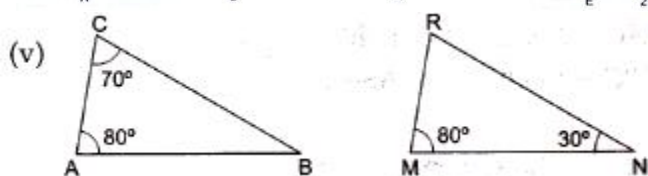
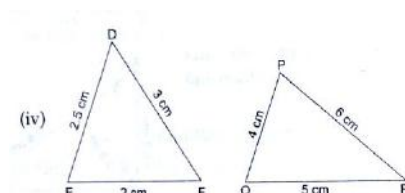
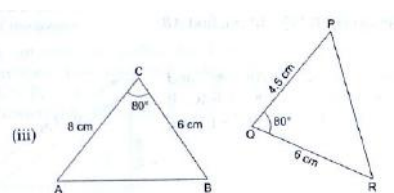
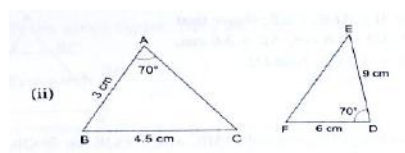
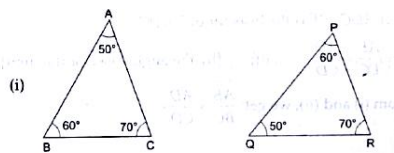
$$\frac{QR}{PR} = \frac{BQ}{BP}$$

$$\Rightarrow BP \times QR = BQ \times PR$$

This completes the proof.

## Exercise – 4B

1. In each of the given pairs of triangles, find which pair of triangles are similar. State the similarity criterion and write the similarity relation in symbolic form:



**Sol:**

(i)

We have:

$$\angle BAC = \angle PQR = 50^\circ$$

$$\angle ABC = \angle QPR = 60^\circ$$

$$\angle ACB = \angle PRQ = 70^\circ$$

Therefore, by AAA similarity theorem,  $\Delta ABC \sim \Delta PQR$

(ii)

We have:

$$\frac{AB}{DF} = \frac{3}{6} = \frac{1}{2} \text{ and } \frac{BC}{DE} = \frac{4.5}{9} = \frac{1}{2}$$

But,  $\angle ABC \neq \angle EDF$  (Included angles are not equal)

Thus, these triangles are not similar.

(iii)

We have:

$$\frac{CA}{QR} = \frac{8}{6} = \frac{4}{3} \text{ and } \frac{CB}{PQ} = \frac{6}{4.5} = \frac{4}{3}$$

$$\Rightarrow \frac{CA}{QR} = \frac{CB}{PQ}$$

Also,  $\angle ACB = \angle PQR = 80^\circ$

Therefore, by SAS similarity theorem,  $\Delta ACB \sim \Delta RQP$ .

(iv)

We have

$$\frac{DE}{QR} = \frac{2.5}{5} = \frac{1}{2}$$

$$\frac{EF}{PQ} = \frac{2}{4} = \frac{1}{2}$$

$$\frac{DF}{PR} = \frac{3}{6} = \frac{1}{2}$$

$$\Rightarrow \frac{DE}{QR} = \frac{EF}{PQ} = \frac{DF}{PR}$$

Therefore, by SSS similarity theorem,  $\Delta FED \sim \Delta PQR$

(v)

In  $\Delta ABC$

$$\angle A + \angle B + \angle C = 180^\circ \text{ (Angle Sum Property)}$$

$$\Rightarrow 80^\circ + \angle B + 70^\circ = 180^\circ$$

$$\Rightarrow \angle B = 30^\circ$$

$$\angle A = \angle M \text{ and } \angle B = \angle N$$

Therefore, by AA similarity,  $\Delta ABC \sim \Delta MNR$

2. In the given figure,  $\Delta ODC \sim \Delta OBA$ ,  $\angle BOC = 115^\circ$  and  $\angle CDO = 70^\circ$ .

Find (i)  $\angle DCO$  (ii)  $\angle DCO$  (iii)  $\angle OAB$  (iv)  $\angle OBA$ .

**Sol:**

(i)

It is given that DB is a straight line.

Therefore,

$$\angle DOC + \angle COB = 180^\circ$$

$$\angle DOC = 180^\circ - 115^\circ = 65^\circ$$

(ii)

In  $\Delta DOC$ , we have:

$$\angle ODC + \angle DCO + \angle DOC = 180^\circ$$

Therefore,

$$70^\circ + \angle DCO + 65^\circ = 180^\circ$$

$$\Rightarrow \angle DCO = 180 - 70 - 65 = 45^\circ$$

(iii)

It is given that  $\Delta ODC \sim \Delta OBA$

Therefore,

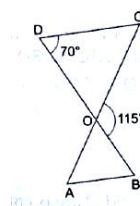
$$\angle OAB = \angle OCD = 45^\circ$$

(iv)

Again,  $\Delta ODC \sim \Delta OBA$

Therefore,

$$\angle OBA = \angle ODC = 70^\circ$$



3. In the given figure,  $\triangle OAB \sim \triangle OCD$ . If  $AB = 8\text{cm}$ ,  $BO = 6.4\text{cm}$ ,  $OC = 3.5\text{cm}$  and  $CD = 5\text{cm}$ , find (i)  $OA$  (ii)  $DO$ .

**Sol:**

- (i) Let  $OA$  be  $X$  cm.

$$\because \triangle OAB \sim \triangle OCD$$

$$\therefore \frac{OA}{OC} = \frac{AB}{CD}$$

$$\Rightarrow \frac{x}{3.5} = \frac{8}{5}$$

$$\Rightarrow x = \frac{8 \times 3.5}{5} = 5.6$$

Hence,  $OA = 5.6$  cm

- (ii) Let  $OD$  be  $Y$  cm

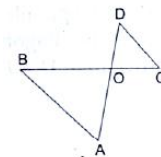
$$\because \triangle OAB \sim \triangle OCD$$

$$\therefore \frac{AB}{CD} = \frac{OB}{OD}$$

$$\Rightarrow \frac{8}{5} = \frac{6.4}{y}$$

$$\Rightarrow y = \frac{6.4 \times 5}{8} = 4$$

Hence,  $DO = 4$  cm



4. In the given figure, if  $\angle ADE = \angle B$ , show that  $\triangle ADE \sim \triangle ABC$ . If  $AD = 3.8\text{cm}$ ,  $AE = 3.6\text{cm}$ ,  $BE = 2.1\text{cm}$  and  $BC = 4.2\text{cm}$ , find  $DE$ .

**Sol:**

Given :

$$\angle ADE = \angle ABC \text{ and } \angle A = \angle A$$

Let  $DE$  be  $X$  cm

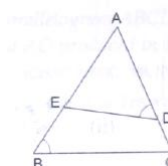
Therefore, by AA similarity theorem,  $\triangle ADE \sim \triangle ABC$

$$\Rightarrow \frac{AD}{AB} = \frac{DE}{BC}$$

$$\Rightarrow \frac{3.8}{3.6+2.1} = \frac{x}{4.2}$$

$$\Rightarrow x = \frac{3.8 \times 4.2}{5.7} = 2.8$$

Hence,  $DE = 2.8$  cm



5. The perimeter of two similar triangles  $ABC$  and  $PQR$  are  $32\text{cm}$  and  $24\text{cm}$  respectively. If  $PQ = 12\text{cm}$ , find  $AB$ .

**Sol:**

It is given that triangles  $ABC$  and  $PQR$  are similar.

Therefore,

$$\frac{\text{Perimeter}(\triangle ABC)}{\text{Perimeter}(\triangle PQR)} = \frac{AB}{PQ}$$

$$\Rightarrow \frac{32}{24} = \frac{AB}{12}$$

$$\Rightarrow AB = \frac{32 \times 12}{24} = 16 \text{ cm}$$

6. The corresponding sides of two similar triangles ABC and DEF are  $BC = 9.1\text{cm}$  and  $EF = 6.5\text{cm}$ . If the perimeter of  $\triangle DEF$  is  $25\text{cm}$ , find the perimeter of  $\triangle ABC$ .

**Sol:**

It is given that  $\triangle ABC \sim \triangle DEF$ .

Therefore, their corresponding sides will be proportional.

Also, the ratio of the perimeters of similar triangles is same as the ratio of their corresponding sides.

$$\Rightarrow \frac{\text{Perimeter of } \triangle ABC}{\text{Perimeter of } \triangle DEF} = \frac{BC}{EF}$$

Let the perimeter of  $\triangle ABC$  be  $X \text{ cm}$

Therefore,

$$\frac{x}{25} = \frac{9.1}{6.5}$$

$$\Rightarrow x = \frac{9.1 \times 25}{6.5} = 35$$

Thus, the perimeter of  $\triangle ABC$  is  $35 \text{ cm}$ .

7. In the given figure,  $\angle CAB = 90^\circ$  and  $AD \perp BC$ . Show that  $\triangle BDA \sim \triangle BAC$ . If  $AC = 75\text{cm}$ ,  $AB = 1\text{m}$  and  $BC = 1.25\text{m}$ , find  $AD$ .

**Sol:**

In  $\triangle BDA$  and  $\triangle BAC$ , we have :

$$\angle BDA = \angle BAC = 90^\circ$$

$$\angle DBA = \angle CBA \quad (\text{Common})$$

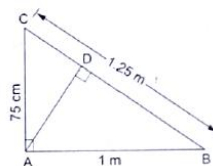
Therefore, by AA similarity theorem,  $\triangle BDA \sim \triangle BAC$

$$\Rightarrow \frac{AD}{AC} = \frac{AB}{BC}$$

$$\Rightarrow \frac{AD}{0.75} = \frac{1}{1.25}$$

$$\Rightarrow AD = \frac{0.75}{1.25}$$

$$= 0.6 \text{ m or } 60 \text{ cm}$$



8. In the given figure,  $\angle ABC = 90^\circ$  and  $BD \perp AC$ . If  $AB = 5.7\text{cm}$ ,  $BD = 3.8\text{cm}$  and  $CD = 5.4\text{cm}$ , find  $BC$ .

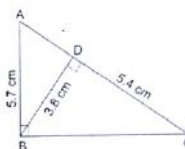
**Sol:**

It is given that  $\triangle ABC$  is a right angled triangle and  $BD$  is the altitude drawn from the right angle to the hypotenuse.

In  $\triangle BDC$  and  $\triangle ABC$ , we have :

$$\angle ABC = \angle BDC = 90^\circ \quad (\text{given})$$

$$\angle C = \angle C \quad (\text{common})$$



By AA similarity theorem, we get :

$$\triangle BDC \sim \triangle ABC$$

$$\frac{AB}{BD} = \frac{BC}{DC}$$

$$\Rightarrow \frac{5.7}{3.8} = \frac{BC}{5.4}$$

$$\Rightarrow BC = \frac{5.7}{3.8} \times 5.4$$

$$= 8.1$$

Hence,  $BC = 8.1$  cm

9. In the given figure,  $\angle ABC = 90^\circ$  and  $BD \perp AC$ .  
If  $BD = 8$  cm,  $AD = 4$  cm, find  $CD$ .

**Sol:**

It is given that  $ABC$  is a right angled triangle  
and  $BD$  is the altitude drawn from the right angle to the hypotenuse.

In  $\triangle DBA$  and  $\triangle DCB$ , we have :

$$\angle BDA = \angle CDB$$

$$\angle DBA = \angle DCB = 90^\circ$$

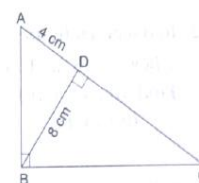
Therefore, by AA similarity theorem, we get :

$$\triangle DBA \sim \triangle DCB$$

$$\Rightarrow \frac{BD}{CD} = \frac{AD}{BD}$$

$$\Rightarrow CD = \frac{BD^2}{AD}$$

$$CD = \frac{8 \times 8}{4} = 16 \text{ cm}$$



10. P and Q are points on the sides AB and AC respectively of a  $\triangle ABC$ . If  $AP = 2$  cm,  $PB = 4$  cm,  $AQ = 3$  cm and  $QC = 6$  cm, show that  $BC = 3PQ$ .

**Sol:**

We have :

$$\frac{AP}{AB} = \frac{2}{6} = \frac{1}{3} \text{ and } \frac{AQ}{AC} = \frac{3}{9} = \frac{1}{3}$$

$$\Rightarrow \frac{AP}{AB} = \frac{AQ}{AC}$$

In  $\triangle APQ$  and  $\triangle ABC$ , we have:

$$\frac{AP}{AB} = \frac{AQ}{AC}$$

$$\angle A = \angle A$$

Therefore, by AA similarity theorem, we get:

$$\triangle APQ \sim \triangle ABC$$

$$\text{Hence, } \frac{PQ}{BC} = \frac{AQ}{AC} = \frac{1}{3}$$

$$\Rightarrow \frac{PQ}{BC} = \frac{1}{3}$$

$$\Rightarrow BC = 3PQ$$

This completes the proof.

11. ABCD is parallelogram and E is a point on BC. If the diagonal BD intersects AE at F, prove that

$$AF \times FB = EF \times FD.$$

**Sol:**

We have:

$$\angle AFD = \angle EFB \quad (\text{Vertically Opposite angles})$$

$$\because DA \parallel BC$$

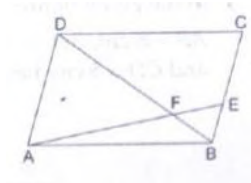
$$\therefore \angle DAF = \angle BEF \quad (\text{Alternate angles})$$

$$\Delta DAF \sim \Delta BEF \quad (\text{AA similarity theorem})$$

$$\Rightarrow \frac{AF}{EF} = \frac{FD}{FB}$$

$$\text{Or, } AF \times FB = FD \times EF$$

This completes the proof.



12. In the given figure,  $DB \perp BC$ ,  $DE \perp AB$  and  $AC \perp BC$ .

$$\text{Prove that } \frac{BE}{DE} = \frac{AC}{BC}.$$

**Sol:**

In  $\Delta BED$  and  $\Delta ACB$ , we have:

$$\angle BED = \angle ACB = 90^\circ$$

$$\because \angle B + \angle C = 180^\circ$$

$$\therefore BD \parallel AC$$

$$\angle EBD = \angle CAB \quad (\text{Alternate angles})$$

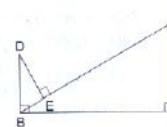
Therefore, by AA similarity theorem, we get :

$$\Delta BED \sim \Delta ACB$$

$$\Rightarrow \frac{BE}{AC} = \frac{DE}{BC}$$

$$\Rightarrow \frac{BE}{DE} = \frac{AC}{BC}$$

This completes the proof.



13. A vertical pole of length 7.5m casts a shadow 5m long on the ground and at the same time a tower casts a shadow 24m long. Find the height of the tower.

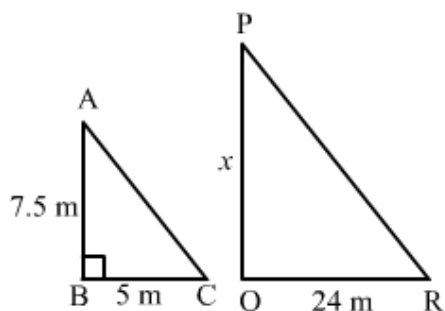
**Sol:**

Let AB be the vertical stick and BC be its shadow.

Given:

$$AB = 7.5 \text{ m, } BC = 5 \text{ m}$$





Let PQ be the tower and QR be its shadow.

Given:

$$QR = 24 \text{ m}$$

Let the length of PQ be  $x$  m.

In  $\triangle ABC$  and  $\triangle PQR$ , we have:

$$\angle ABC = \angle PQR = 90^\circ$$

$$\angle ACB = \angle PRQ \text{ (Angular elevation of the Sun at the same time)}$$

Therefore, by AA similarity theorem, we get :

$$\triangle ABC \sim \triangle PQR$$

$$\Rightarrow \frac{AB}{BC} = \frac{PQ}{QR}$$

$$\Rightarrow \frac{7.5}{5} = \frac{x}{24}$$

$$x = \frac{7.5}{5} \times 24 = 36 \text{ m}$$

Therefore,  $PQ = 36 \text{ m}$

Hence, the height of the tower is 36 m.

14. In an isosceles  $\triangle ABC$ , the base AB is produced both ways in P and Q such that

$$AP \times BQ = AC^2.$$

Prove that  $\triangle ACP \sim \triangle BCQ$ .

**Sol:**

Disclaimer: It should be  $\triangle APC \sim \triangle BCQ$

$\triangle BCQ$

It is given that  $\triangle ABC$  is an isosceles triangle.

Therefore,

$$CA = CB$$

$$\Rightarrow \angle CAB = \angle CBA$$

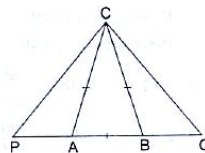
$$\Rightarrow 180^\circ - \angle CAB = 180^\circ - \angle CBA$$

$$\Rightarrow \angle CAP = \angle CBQ$$

Also,

$$AP \times BQ = AC^2$$

$$\Rightarrow \frac{AP}{AC} = \frac{AC}{BQ}$$



instead of  $\triangle ACP \sim$

$$\Rightarrow \frac{AP}{AC} = \frac{BQ}{BC} (\because AC = BC)$$

Thus, by SAS similarity theorem, we get

$$\triangle APC \sim \triangle BCQ$$

This completes the proof.

15. In the given figure,  $\angle 1 = \angle 2$  and  $\frac{AC}{BD} = \frac{CB}{CE}$ .

Prove that  $\triangle ACB \sim \triangle DCE$ .

**Sol:**

We have :

$$\frac{AC}{BD} = \frac{CB}{CE}$$

$$\Rightarrow \frac{AC}{CB} = \frac{BD}{CE}$$

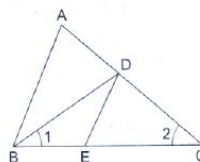
$$\Rightarrow \frac{AC}{CB} = \frac{CD}{CE} \text{ (Since, } BD = DC \text{ as } \angle 1 = \angle 2 \text{)}$$

Also,  $\angle 1 = \angle 2$

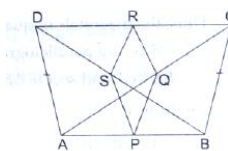
i.e,  $\angle DBC = \angle ACB$

Therefore, by SAS similarity theorem, we get :

$$\triangle ACB \sim \triangle DCE$$



16. ABCD is a quadrilateral in which  $AD = BC$ .  
If P, Q, R, S be the midpoints of AB, AC, CD and BD respectively, show that PQRS is a rhombus.



**Sol:**

In  $\triangle ABC$ , P and Q are mid points of AB and AC respectively.

$$\text{So, } PQ \parallel BC, \text{ and } PQ = \frac{1}{2} BC \quad \dots(1)$$

Similarly, in  $\triangle ADC$ ,  $\dots(2)$

$$\text{Now, in } \triangle BCD, SR = \frac{1}{2} BC \quad \dots(3)$$

$$\text{Similarly, in } \triangle ABD, PS = \frac{1}{2} AD = \frac{1}{2} BC \quad \dots(4)$$

Using (1), (2), (3), and (4).

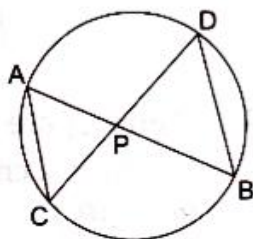
$$PQ = QR = SR = PS$$

Since, all sides are equal

Hence, PQRS is a rhombus.

17. In a circle, two chords AB and CD intersect at a point P inside the circle. Prove that

- (a)  $\Delta PAC \sim \Delta PDB$       (b)  $PA \cdot PB = PC \cdot PD$



**Sol:**

Given : AB and CD are two chords

To Prove:

- (a)  $\Delta PAC \sim \Delta PDB$

- (b)  $PA \cdot PB = PC \cdot PD$

Proof: In  $\Delta PAC$  and  $\Delta PDB$

$$\angle APC = \angle DPB \text{ (Vertically Opposite angles)}$$

$$\angle CAP = \angle BDP \text{ (Angles in the same segment are equal)}$$

by AA similarity criterion  $\Delta PAC \sim \Delta PDB$

When two triangles are similar, then the ratios of lengths of their corresponding sides are proportional.

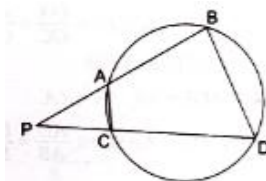
$$\therefore \frac{PA}{PD} = \frac{PC}{PB}$$

$$\Rightarrow PA \cdot PB = PC \cdot PD$$

18. Two chords AB and CD of a circle intersect at a point P outside the circle.

Prove that: (i)  $\Delta PAC \sim \Delta PDB$

- (ii)  $PA \cdot PB = PC \cdot PD$



**Sol:**

Given : AB and CD are two chords

To Prove:

- (a)  $\Delta PAC \sim \Delta PDB$

- (b)  $PA \cdot PB = PC \cdot PD$

Proof:  $\angle ABD + \angle ACD = 180^\circ \dots(1)$  (Opposite angles of a cyclic quadrilateral are supplementary)

$$\angle PCA + \angle ACD = 180^\circ \dots(2) \quad \text{(Linear Pair Angles)}$$

Using (1) and (2), we get

$$\angle ABD = \angle PCA$$

$$\angle A = \angle A$$

(Common)

By AA similarity-criterion  $\Delta PAC \sim \Delta PDB$

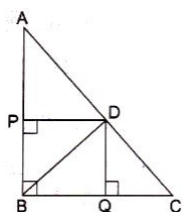
When two triangles are similar, then the ratios of the lengths of their corresponding sides are proportional.

$$\therefore \frac{PA}{PD} = \frac{PC}{PB}$$

$$\Rightarrow PA \cdot PB = PC \cdot PD$$

19. In a right triangle ABC, right angled at B, D is a point on hypotenuse such that  $BD \perp AC$ , if  $DP \perp AB$  and  $DQ \perp BC$  then prove that

(a)  $DQ^2 = DP \cdot QC$       (b)  $DP^2 = DQ \cdot AP$



**Sol:**

We know that if a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse then the triangles on the both sides of the perpendicular are similar to the whole triangle and also to each other.

- (a) Now using the same property in  $\Delta BDC$ , we get

$$\Delta CQD \sim \Delta DQB$$

$$\frac{CQ}{DQ} = \frac{DQ}{QB}$$

$$\Rightarrow DQ^2 = QB \cdot CQ$$

Now, Since all the angles in quadrilateral BQDP are right angles.

Hence, BQDP is a rectangle.

So,  $QB = DP$  and  $DQ = PB$

$$\therefore DQ^2 = DP \cdot CQ$$

(b)

Similarly,  $\Delta APD \sim \Delta DPB$

$$\frac{AP}{DP} = \frac{PD}{PB}$$

$$\Rightarrow DP^2 = AP \cdot PB$$

$$\Rightarrow DP^2 = AP \cdot DQ \quad [\because DQ = PB]$$

## Exercise – 4C

1.  $\triangle ABC \sim \triangle DEF$  and their areas are respectively  $64 \text{ cm}^2$  and  $121 \text{ cm}^2$ . If  $EF = 15.4 \text{ cm}$ , find  $BC$ .

**Sol:**

It is given that  $\triangle ABC \sim \triangle DEF$ .

Therefore, ratio of the areas of these triangles will be equal to the ratio of squares of their corresponding sides.

$$\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \frac{BC^2}{EF^2}$$

Let  $BC$  be  $x \text{ cm}$ .

$$\Rightarrow \frac{64}{121} = \frac{x^2}{(15.4)^2}$$

$$\Rightarrow x^2 = \frac{64 \times 15.4 \times 15.4}{121}$$

$$\Rightarrow x = \sqrt{\frac{64 \times 15.4 \times 15.4}{121}}$$

$$= \frac{8 \times 15.4}{11}$$

$$= 11.2$$

Hence,  $BC = 11.2 \text{ cm}$

2. The areas of two similar triangles  $ABC$  and  $PQR$  are in the ratio  $9:16$ . If  $BC = 4.5 \text{ cm}$ , find the length of  $QR$ .

**Sol:**

It is given that  $\triangle ABC \sim \triangle PQR$

Therefore, the ratio of the areas of triangles will be equal to the ratio of squares of their corresponding sides.

$$\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle PQR)} = \frac{BC^2}{QR^2}$$

$$\Rightarrow \frac{9}{16} = \frac{4.5^2}{QR^2}$$

$$\Rightarrow QR^2 = \frac{4.5 \times 4.5 \times 16}{9}$$

$$\Rightarrow QR = \sqrt{\frac{4.5 \times 4.5 \times 16}{9}}$$

$$= \frac{4.5 \times 4}{3}$$

$$= 6 \text{ cm}$$

Hence,  $QR = 6 \text{ cm}$

3.  $\triangle ABC \sim \triangle PQR$  and  $\text{ar}(\triangle ABC) = 4$ ,  $\text{ar}(\triangle PQR)$ . If  $BC = 12 \text{ cm}$ , find  $QR$ .

**Sol:**

Given :  $\text{ar}(\triangle ABC) = 4 \text{ar}(\triangle PQR)$

$$\frac{ar(\triangle ABC)}{ar(\triangle PQR)} = \frac{4}{1}$$

$$\therefore \triangle ABC \sim \triangle PQR$$

$$\therefore \frac{ar(\triangle ABC)}{ar(\triangle PQR)} = \frac{BC^2}{QR^2}$$

$$\therefore \frac{BC^2}{QR^2} = \frac{4}{1}$$

$$\Rightarrow QR^2 = \frac{12^2}{4}$$

$$\Rightarrow QR^2 = 36$$

$$\Rightarrow QR = 6 \text{ cm}$$

Hence, QR = 6 cm

4. The areas of two similar triangles are  $169\text{cm}^2$  and  $121\text{cm}^2$  respectively. If the longest side of the larger triangle is 26cm, find the longest side of the smaller triangle.

**Sol:**

It is given that the triangles are similar.

Therefore, the ratio of the areas of these triangles will be equal to the ratio of squares of their corresponding sides.

Let the longest side of smaller triangle be X cm.

$$\frac{ar(\text{Larger triangle})}{ar(\text{Smaller triangle})} = \frac{(\text{Longest side of larger triangle})^2}{(\text{Longest side of smaller triangle})^2}$$

$$\Rightarrow \frac{169}{121} = \frac{26^2}{x^2}$$

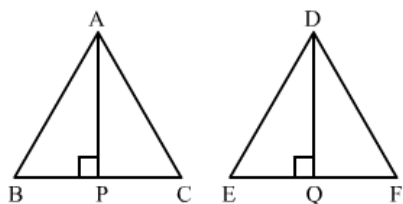
$$\Rightarrow x = \sqrt{\frac{26 \times 26 \times 121}{169}}$$

$$= 22$$

Hence, the longest side of the smaller triangle is 22 cm.

5.  $\triangle ABC \sim \triangle DEF$  and their areas are respectively  $100\text{cm}^2$  and  $49\text{cm}^2$ . If the altitude of  $\triangle ABC$  is 5cm, find the corresponding altitude of  $\triangle DEF$ .

**Sol:**



It is given that  $\triangle ABC \sim \triangle DEF$ .

Therefore, the ratio of the areas of these triangles will be equal to the ratio of squares of their corresponding sides.

Also, the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding altitudes.

Let the altitude of  $\triangle ABC$  be  $AP$ , drawn from  $A$  to  $BC$  to meet  $BC$  at  $P$  and the altitude of  $\triangle DEF$  be  $DQ$ , drawn from  $D$  to meet  $EF$  at  $Q$ .

Then,

$$\frac{ar(\triangle ABC)}{ar(\triangle DEF)} = \frac{AP^2}{DQ^2}$$

$$\Rightarrow \frac{100}{49} = \frac{5^2}{DQ^2}$$

$$\Rightarrow \frac{100}{49} = \frac{25}{DQ^2}$$

$$\Rightarrow DQ^2 = \frac{49 \times 25}{100}$$

$$\Rightarrow DQ = \sqrt{\frac{49 \times 25}{100}}$$

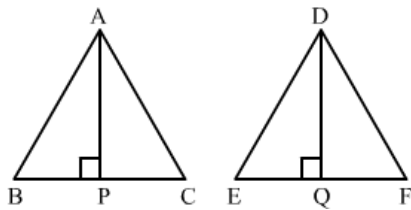
$$\Rightarrow DQ = 3.5 \text{ cm}$$

Hence, the altitude of  $\triangle DEF$  is 3.5 cm

6. The corresponding altitudes of two similar triangles are 6cm and 9cm respectively. Find the ratio of their areas.

**Sol:**

Let the two triangles be  $ABC$  and  $DEF$  with altitudes  $AP$  and  $DQ$ , respectively.



It is given that  $\triangle ABC \sim \triangle DEF$ .

We know that the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding altitudes.

$$\therefore \frac{ar(\triangle ABC)}{ar(\triangle DEF)} = \frac{(AP)^2}{(DQ)^2}$$

$$\Rightarrow \frac{ar(\triangle ABC)}{ar(\triangle DEF)} = \frac{6^2}{9^2}$$

$$= \frac{36}{81}$$

$$= \frac{4}{9}$$

Hence, the ratio of their areas is 4 : 9

7. The areas of two similar triangles are  $81\text{cm}^2$  and  $49\text{cm}^2$  respectively. If the altitude of the first triangle is 6.3cm, find the corresponding altitude of the other.

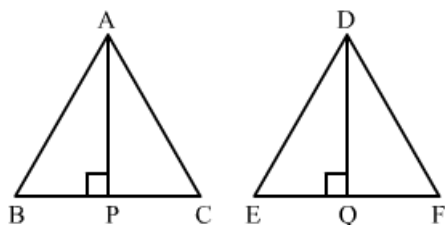
**Sol:**

It is given that the triangles are similar.

Therefore, the areas of these triangles will be equal to the ratio of squares of their corresponding sides.

Also, the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding altitudes.

Let the two triangles be ABC and DEF with altitudes AP and DQ, respectively.



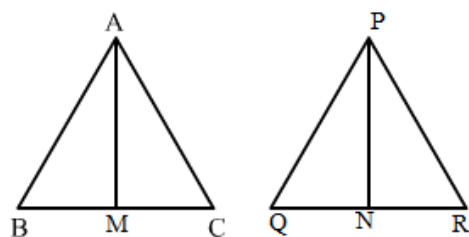
$$\begin{aligned}\frac{ar(\triangle ABC)}{ar(\triangle PQR)} &= \frac{AP^2}{DQ^2} \\ \Rightarrow \frac{81}{49} &= \frac{6.3^2}{DQ^2} \\ \Rightarrow DQ^2 &= \frac{49}{81} \times 6.3^2 \\ \Rightarrow DQ^2 &= \sqrt{\frac{49}{81} \times 6.3 \times 6.3}\end{aligned}$$

Hence, the altitude of the other triangle is 4.9 cm.

8. The areas of two similar triangles are  $64\text{cm}^2$  and  $100\text{cm}^2$  respectively. If a median of the smaller triangle is 5.6cm, find the corresponding median of the other.

**Sol:**

Let the two triangles be ABC and PQR with medians AM and PN, respectively.



Therefore, the ratio of areas of two similar triangles will be equal to the ratio of squares of their corresponding medians.

$$\begin{aligned}\therefore \frac{ar(\triangle ABC)}{ar(\triangle PQR)} &= \frac{AM^2}{PN^2} \\ \Rightarrow \frac{64}{100} &= \frac{5.6^2}{PN^2} \\ \Rightarrow PN^2 &= \frac{64}{100} \times 5.6^2 \\ \Rightarrow PN^2 &= \sqrt{\frac{100}{64} \times 5.6 \times 5.6} \\ &= 7 \text{ cm}\end{aligned}$$

Hence, the median of the larger triangle is 7 cm.



9. In the given figure, ABC is a triangle and PQ is a straight line meeting AB in P and AC in Q. If AP = 1 cm, PB = 3 cm, AQ = 1.5 cm, QC = 4.5 cm, prove that area of  $\triangle APQ$  is  $\frac{1}{16}$  of the area of  $\triangle ABC$ .

**Sol:**

We have :

$$\frac{AP}{AB} = \frac{1}{1+3} = \frac{1}{4} \text{ and } \frac{AQ}{AC} = \frac{1.5}{1.5+4.5} = \frac{1.5}{6} = \frac{1}{4}$$

$$\Rightarrow \frac{AP}{AB} = \frac{AQ}{AC}$$

Also,  $\angle A = \angle A$

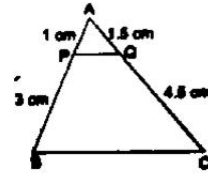
By SAS similarity, we can conclude that  $\triangle APQ \sim \triangle ABC$ .

$$\frac{ar(\triangle APQ)}{ar(\triangle ABC)} = \frac{AP^2}{AB^2} = \frac{1^2}{4^2} = \frac{1}{16}$$

$$\Rightarrow \frac{ar(\triangle APQ)}{ar(\triangle ABC)} = \frac{1}{16}$$

$$\Rightarrow ar(\triangle APQ) = \frac{1}{16} \times ar(\triangle ABC)$$

Hence proved.



10. In the given figure,  $DE \parallel BC$ . If  $DE = 3$  cm,  $BC = 6$  cm and  $ar(\triangle ADE) = 15 \text{ cm}^2$ , find the area of  $\triangle ABC$ .

**Sol:**

It is given that  $DE \parallel BC$

$\therefore \angle ADE = \angle ABC$  (Corresponding angles)

$\angle AED = \angle ACB$  (Corresponding angles)

By AA similarity, we can conclude that  $\triangle ADE \sim \triangle ABC$

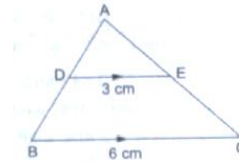
$$\therefore \frac{ar(\triangle ADE)}{ar(\triangle ABC)} = \frac{DE^2}{BC^2}$$

$$\Rightarrow \frac{15}{ar(\triangle ABC)} = \frac{3^2}{6^2}$$

$$\Rightarrow ar(\triangle ABC) = \frac{15 \times 36}{9}$$

$$= 60 \text{ cm}^2$$

Hence, area of triangle ABC is  $60 \text{ cm}^2$



11.  $\triangle ABC$  is right angled at A and  $AD \perp BC$ . If  $BC = 13$  cm and  $AC = 5$  cm, find the ratio of the areas of  $\triangle ABC$  and  $\triangle ADC$ .

**Sol:**

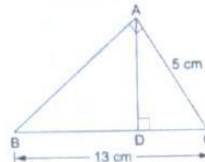
In  $\triangle ABC$  and  $\triangle ADC$ , we have:

$\angle BAC = \angle ADC = 90^\circ$

$\angle ACB = \angle ACD$  (common)

By AA similarity, we can conclude that  $\triangle BAC \sim \triangle ADC$ .

Hence, the ratio of the areas of these triangles is equal to the ratio of squares of their corresponding sides.



$$\begin{aligned}\therefore \frac{ar(\triangle BAC)}{ar(\triangle ADC)} &= \frac{BC^2}{AC^2} \\ \Rightarrow \frac{ar(\triangle BAC)}{ar(\triangle ADC)} &= \frac{13^2}{5^2} \\ &= \frac{169}{25}\end{aligned}$$

Hence, the ratio of areas of both the triangles is 169:25

12. In the given figure,  $DE \parallel BC$  and  $DE:BC = 3:5$ . Calculate the ratio of the areas of  $\triangle ADE$  and the trapezium  $BCED$ .

**Sol:**

It is given that  $DE \parallel BC$ .

$$\therefore \angle ADE = \angle ABC \text{ (Corresponding angles)}$$

$$\angle AED = \angle ACB \text{ (Corresponding angles)}$$

Applying AA similarity theorem, we can conclude that  $\triangle ADE \sim \triangle ABC$ .

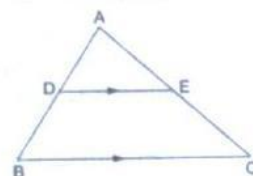
$$\therefore \frac{ar(\triangle ABC)}{ar(\triangle ADE)} = \frac{BC^2}{DE^2}$$

Subtracting 1 from both sides, we get:

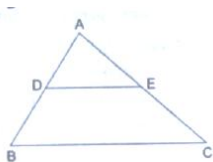
$$\begin{aligned}\frac{ar(\triangle ABC)}{ar(\triangle ADE)} - 1 &= \frac{5^2}{3^2} - 1 \\ \Rightarrow \frac{ar(\triangle ABC) - ar(\triangle ADE)}{ar(\triangle ADE)} &= \frac{25-9}{9}\end{aligned}$$

$$\Rightarrow \frac{ar(BCED)}{ar(\triangle ADE)} = \frac{16}{9}$$

$$\text{Or, } \frac{ar(\triangle ADE)}{ar(BCED)} = \frac{9}{16}$$



13. In  $\triangle ABC$ , D and E are the midpoints of AB and AC respectively. Find the ratio of the areas of  $\triangle ADE$  and  $\triangle ABC$ .



**Sol:**

It is given that D and E are midpoints of AB and AC.

Applying midpoint theorem, we can conclude that  $DE \parallel BC$ .

Hence, by B.P.T., we get :

$$\frac{AD}{AB} = \frac{AE}{AC}$$

$$\text{Also, } \angle A = \angle A$$

Applying SAS similarity theorem, we can conclude that  $\triangle ADE \sim \triangle ABC$ .

Therefore, the ratio of areas of these triangles will be equal to the ratio of squares of their corresponding sides.

$$\therefore \frac{ar(\triangle ADE)}{ar(\triangle ABC)} = \frac{DE^2}{BC^2}$$

$$\begin{aligned} &= \frac{\left(\frac{1}{2}BC\right)^2}{BC^2} \\ &= \frac{1}{4} \end{aligned}$$

**Exercise – 4D**

1. The sides of certain triangles are given below. Determine which of them right triangles are.

(i) 9cm, 16cm, 18cm

(ii) 7cm, 24cm, 25cm

(iii) 1.4cm, 4.8cm, 5cm

(iv) 1.6cm, 3.8cm, 4cm

(v)  $(a - 1)$  cm,  $2\sqrt{a}$  cm,  $(a + 1)$  cm

**Sol:**

For the given triangle to be right-angled, the sum of the two sides must be equal to the square of the third side.

Here, let the three sides of the triangle be a, b and c.

(i)

$a = 9$  cm,  $b = 16$  cm and  $c = 18$  cm

Then,

$$a^2 + b^2 = 9^2 + 16^2$$

$$= 81 + 256$$

$$= 337$$

$$c^2 = 18^2$$

$$= 361$$

$$a^2 + b^2 \neq c^2$$

Thus, the given triangle is not right-angled.

(ii)

$a = 7$  cm,  $b = 24$  cm and  $c = 25$  cm

Then,

$$a^2 + b^2 = 7^2 + 24^2$$

$$= 49 + 576$$

$$= 625$$

$$c^2 = 25^2$$

$$= 625$$

$$a^2 + b^2 = c^2$$

Thus, the given triangle is a right-angled.

(iii)

$a = 1.4$  cm,  $b = 4.8$  cm and  $c = 5$  cm

Then,

$$a^2 + b^2 = (1.4)^2 + (4.8)^2$$

$$= 1.96 + 23.04$$

$$= 25$$

$$c^2 = 5^2$$

$$= 25$$

$$a^2 + b^2 = c^2$$

Thus, the given triangle is right-angled.

(iv)  $A = 1.6$  cm,  $b = 3.8$  cm and  $c = 4$  cm

Then

$$a^2 + b^2 = (1.6)^2 + (3.8)^2$$

$$= 2.56 + 14.44$$

$$= 16$$

$$a^2 + b^2 \neq c^2$$

Thus, the given triangle is not right-angled.

(v)

$P = (a-1)$  cm,  $q = 2\sqrt{a}$  cm and  $r = (a+1)$  cm

Then,

$$p^2 + q^2 = (a-1)^2 + (2\sqrt{a})^2$$

$$= a^2 + 1 - 2a + 4a$$

$$= a^2 + 1 + 2a$$

$$= (a+1)^2$$

$$r^2 = (a+1)^2$$

$$p^2 + q^2 = r^2$$

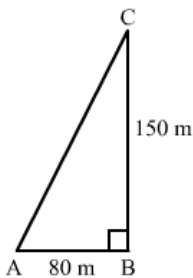
Thus, the given triangle is right-angled.

2. A man goes 80m due east and then 150m due north. How far is he from the starting point?

**Sol:**

Let the man starts from point A and goes 80 m due east to B.

Then, from B, he goes 150 m due north to C.



We need to find AC.

In right-angled triangle ABC, we have:

$$AC^2 = AB^2 + BC^2$$

$$AC = \sqrt{80^2 + 150^2}$$

$$= \sqrt{6400 + 22500}$$

$$= \sqrt{28900}$$

$$= 170 \text{ m}$$

Hence, the man is 170 m away from the starting point.

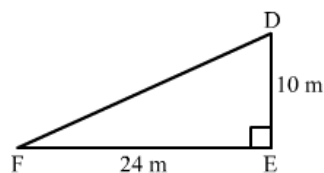
3. A man goes 10m due south and then 24m due west. How far is he from the starting point?

**Sol:**

Let the man starts from point D and goes 10 m due south at E. He then goes 24 m due west at F.

In right  $\triangle DEF$ , we have:

DE = 10 m, EF = 24 m



$$DF^2 = EF^2 + DE^2$$

$$DF = \sqrt{10^2 + 24^2}$$

$$= \sqrt{100 + 576}$$

$$= \sqrt{676}$$

$$= 26 \text{ m}$$

Hence, the man is 26 m away from the starting point.

4. A 13m long ladder reaches a window of a building 12m above the ground. Determine the distance of the foot of the ladder from the building.

**Sol:**

Let AB and AC be the ladder and height of the building.

It is given that :

AB = 13 m and AC = 12 m

We need to find distance of the foot of the ladder from the building, i.e, BC.

In right-angled triangle ABC, we have:



$$AB^2 = AC^2 + BC^2$$

$$\Rightarrow BC = \sqrt{13^2 - 12^2}$$

$$= \sqrt{169 - 144}$$

$$= \sqrt{25}$$

$$= 5 \text{ m}$$

Hence, the distance of the foot ladder from the building is 5 m

5. A ladder is placed in such a way that its foot is at a distance of 15m from a wall and its top reaches a window 20m above the ground. Find the length of the ladder.

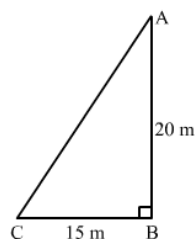
**Sol:**

Let the height of the window from the ground and the distance of the foot of the ladder from the wall be AB and BC, respectively.

We have :

AB = 20 m and BC = 15 m

Applying Pythagoras theorem in right-angled ABC, we get:



$$\begin{aligned}AC^2 &= AB^2 + BC^2 \\ \Rightarrow AC &= \sqrt{20^2 + 15^2} \\ &= \sqrt{400 + 225} \\ &= \sqrt{625} \\ &= 25 \text{ m}\end{aligned}$$

Hence, the length of the ladder is 25 m.

6. Two vertical poles of height 9m and 14m stand on a plane ground. If the distance between their feet is 12m, find the distance between their tops.

**Sol:**

Let the two poles be DE and AB and the distance between their bases be BE.

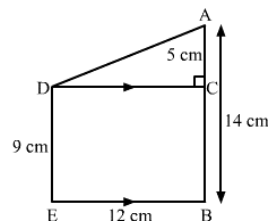
We have:

DE = 9 m, AB = 14 m and BE = 12 m

Draw a line parallel to BE from D, meeting AB at C.

Then, DC = 12 m and AC = 5 m

We need to find AD, the distance between their tops.



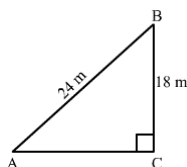
Applying Pythagoras theorem in right-angled ACD, we have:

$$\begin{aligned}AD^2 &= AC^2 + DC^2 \\ AD^2 &= 5^2 + 12^2 = 25 + 144 = 169 \\ AD &= \sqrt{169} = 13 \text{ m}\end{aligned}$$

Hence, the distance between the tops to the two poles is 13 m.

7. A guy wire attached to a vertical pole of height 18 m is 24m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?

**Sol:**



Let AB be a guy wire attached to a pole BC of height 18 m. Now, to keep the wire taut let it to be fixed at A.

Now, In right triangle ABC

By using Pythagoras theorem, we have

$$AB^2 = BC^2 + CA^2$$

$$\Rightarrow 24^2 = 18^2 + CA^2$$

$$\Rightarrow CA^2 = 576 - 324$$

$$\Rightarrow CA^2 = 252$$

$$\Rightarrow CA = 6\sqrt{7} \text{ m}$$

Hence, the stake should be driven  $6\sqrt{7} \text{ m}$  far from the base of the pole.

8. In the given figure, O is a point inside a  $\triangle PQR$  such that  $\angle POR = 90^\circ$ ,  $OP = 6 \text{ cm}$  and  $OR = 8 \text{ cm}$ . If  $PQ = 24 \text{ cm}$  and  $QR = 26 \text{ cm}$ , prove that  $\triangle PQR$  is right-angled.

**Sol:**

Applying Pythagoras theorem in right-angled triangle POR, we have:

$$PR^2 = PO^2 + OR^2$$

$$\Rightarrow PR^2 = 6^2 + 8^2 = 36 + 64 = 100$$

$$\Rightarrow PR = \sqrt{100} = 10 \text{ cm}$$

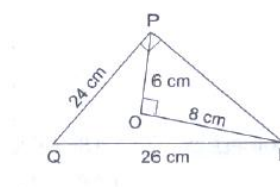
In  $\triangle PQR$ ,

$$PQ^2 + PR^2 = 24^2 + 10^2 = 576 + 100 = 676$$

$$\text{And } QR^2 = 26^2 = 676$$

$$\therefore PQ^2 + PR^2 = QR^2$$

Therefore, by applying Pythagoras theorem, we can say that  $\triangle PQR$  is right-angled at P.



9.  $\triangle ABC$  is an isosceles triangle with  $AB = AC = 13 \text{ cm}$ . The length of altitude from A on BC is 5 cm. Find BC.

**Sol:**

It is given that  $\triangle ABC$  is an isosceles triangle.

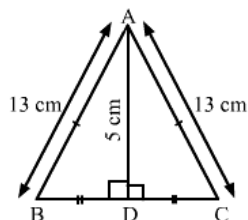
Also,  $AB = AC = 13 \text{ cm}$

Suppose the altitude from A on BC meets BC at D. Therefore, D is the midpoint of BC.

$AD = 5$  cm

$\triangle ADB$  and  $\triangle ADC$  are right-angled triangles.

Applying Pythagoras theorem, we have;



$$AB^2 = AD^2 + BD^2$$

$$BD^2 = AB^2 - AD^2 = 13^2 - 5^2$$

$$BD^2 = 169 - 25 = 144$$

$$BD = \sqrt{144} = 12$$

Hence,

$$BC = 2(BD) = 2 \times 12 = 24 \text{ cm}$$

10. Find the length of altitude AD of an isosceles  $\triangle ABC$  in which  $AB = AC = 2a$  units and  $BC = a$  units.

**Sol:**

In isosceles  $\triangle ABC$ , we have:

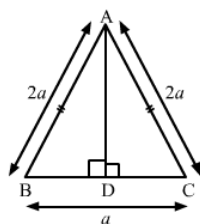
$AB = AC = 2a$  units and  $BC = a$  units

Let AD be the altitude drawn from A that meets BC at D.

Then, D is the midpoint of BC.

$$BD = DC = \frac{a}{2} \text{ units}$$

Applying Pythagoras theorem in right-angled  $\triangle ABD$ , we have:



$$AB^2 = AD^2 + BD^2$$

$$AD^2 = AB^2 - BD^2 = (2a)^2 - \left(\frac{a}{2}\right)^2$$

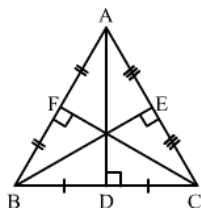
$$AD^2 = 4a^2 - \frac{a^2}{4} = \frac{15a^2}{4}$$

$$AD = \sqrt{\frac{15a^2}{4}} = \frac{a\sqrt{15}}{2} \text{ units.}$$

11.  $\triangle ABC$  is an equilateral triangle of side  $2a$  units. Find each of its altitudes.

**Sol:**





Let AD, BE and CF be the altitudes of  $\triangle ABC$  meeting BC, AC and AB at D, E and F, respectively.

Then, D, E and F are the midpoint of BC, AC and AB, respectively.

In right-angled  $\triangle ABD$ , we have:

$$AB = 2a \text{ and } BD = a$$

Applying Pythagoras theorem, we get:

$$AB^2 = AD^2 + BD^2$$

$$AD^2 = AB^2 - BD^2 = (2a)^2 - a^2$$

$$AD^2 = 4a^2 - a^2 = 3a^2$$

$$AD = \sqrt{3}a \text{ units}$$

Similarly,

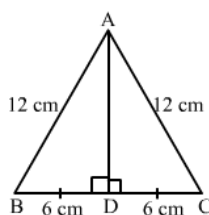
$$BE = a\sqrt{3} \text{ units and } CF = a\sqrt{3} \text{ units}$$

12. Find the height of an equilateral triangle of side 12cm.

**Sol:**

Let ABC be the equilateral triangle with AD as an altitude from A meeting BC at D. Then, D will be the midpoint of BC.

Applying Pythagoras theorem in right-angled triangle ABD, we get:



$$AB^2 = AD^2 + BD^2$$

$$\Rightarrow AD^2 = 12^2 - 6^2 \quad (\because BD = \frac{1}{2} BC = 6)$$

$$\Rightarrow AD^2 = 144 - 36 = 108$$

$$\Rightarrow AD = \sqrt{108} = 6\sqrt{3} \text{ cm.}$$

Hence, the height of the given triangle is  $6\sqrt{3} \text{ cm}$ .

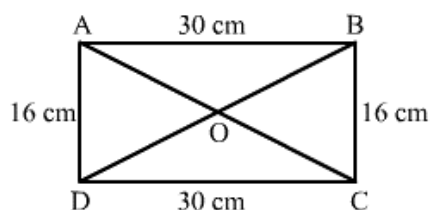
13. Find the length of a diagonal of a rectangle whose adjacent sides are 30cm and 16cm.

**Sol:**

Let ABCD be the rectangle with diagonals AC and BD meeting at O.

According to the question:

$AB = CD = 30$  cm and  $BC = AD = 16$  cm



Applying Pythagoras theorem in right-angled triangle ABC, we get:

$$AC^2 = AB^2 + BC^2 = 30^2 + 16^2 = 900 + 256 = 1156$$

$$AC = \sqrt{1156} = 34 \text{ cm}$$

Diagonals of a rectangle are equal.

Therefore,  $AC = BD = 34$  cm

14. Find the length of each side of a rhombus whose diagonals are 24cm and 10cm long.

**Sol:**

Let ABCD be the rhombus with diagonals ( $AC = 24$  cm and  $BD = 10$  cm) meeting at O.

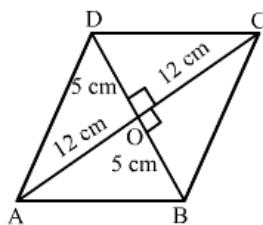
We know that the diagonals of a rhombus bisect each other at angles.

Applying Pythagoras theorem in right-angled AOB, we get:

$$AB^2 = AO^2 + BO^2 = 12^2 + 5^2$$

$$AB^2 = 144 + 25 = 169$$

$$AB = \sqrt{169} = 13 \text{ cm}$$



Hence, the length of each side of the rhombus is 13 cm.

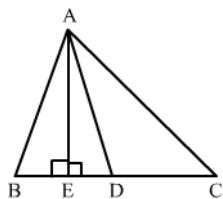
15. In  $\triangle ABC$ , D is the midpoint of BC and  $AE \perp BC$ . If  $AC > AB$ , show that  $AB^2 = AD^2 + \frac{1}{4} BC^2 - BC \cdot DE$

**Sol:**

In right-angled triangle AED, applying Pythagoras theorem, we have:

$$AB^2 = AE^2 + ED^2 \dots (i)$$

In right-angled triangle AED, applying Pythagoras theorem, we have:



$$AD^2 = AE^2 + ED^2$$

$$\Rightarrow AE^2 = AD^2 - ED^2 \dots (ii)$$

Therefore,

$$AB^2 = AD^2 - ED^2 + EB^2 \text{ (from (i) and (ii))}$$

$$AB^2 = AD^2 - ED^2 + (BD - DE)^2$$

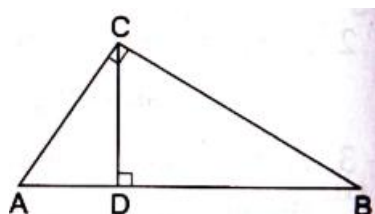
$$= AD^2 - ED^2 + \left(\frac{1}{2} BC - DE\right)^2$$

$$= AD^2 - DE^2 + \frac{1}{4} BC^2 + DE^2 - BC \cdot DE$$

$$= AD^2 + \frac{1}{4} BC^2 - BC \cdot DE$$

This completes the proof.

16. In the given figure,  $\angle ACB = 90^\circ$   $CD \perp AB$  Prove that  $\frac{BC^2}{AC^2} = \frac{BD}{AD}$



**Sol:**

Given:  $\angle ACB = 90^\circ$  and  $CD \perp AB$

To Prove;  $\frac{BC^2}{AC^2} = \frac{BD}{AD}$

Proof: In  $\triangle ACB$  and  $\triangle CDB$

$$\angle ACB = \angle CDB = 90^\circ \text{ (Given)}$$

$$\angle ABC = \angle CBD \text{ (Common)}$$

By AA similarity-criterion  $\triangle ACB \sim \triangle CDB$

When two triangles are similar, then the ratios of the lengths of their corresponding sides are proportional.

$$\therefore \frac{BC}{BD} = \frac{AB}{BC}$$

$$\Rightarrow BC^2 = BD \cdot AB \dots (1)$$

In  $\triangle ACB$  and  $\triangle ADC$

$$\angle ACB = \angle ADC = 90^\circ \text{ (Given)}$$

$$\angle CAB = \angle DAC \text{ (Common)}$$

By AA similarity-criterion  $\triangle ACB \sim \triangle ADC$

When two triangles are similar, then the ratios of their corresponding sides are proportional.

$$\therefore \frac{AC}{AD} = \frac{AB}{AC}$$

$$\Rightarrow AC^2 = AD \cdot AB \quad \dots(2)$$

Dividing (2) by (1), we get

$$\frac{BC^2}{AC^2} = \frac{BD}{AD}$$

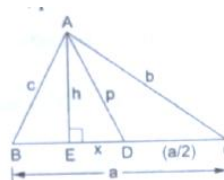
17. In the given figure, D is the midpoint of side BC and  $AE \perp BC$ . If  $BC = a$ ,  $AC = b$ ,  $AB = c$ ,  $AD = p$  and  $AE = h$ , prove that

$$(i) \quad b^2 = p^2 + ax + \frac{a^2}{x}$$

$$(ii) \quad c^2 = p^2 - ax + \frac{a^2}{x}$$

$$(iii) \quad b^2 + c^2 = 2p^2 + \frac{a^2}{2}$$

$$(iv) \quad b^2 - c^2 = 2ax$$



**Sol:**

(i)

In right-angled triangle AEC, applying Pythagoras theorem, we have:

$$AC^2 = AE^2 + EC^2$$

$$\Rightarrow b^2 = h^2 + \left(x + \frac{a}{2}\right)^2 = h^2 + x^2 + \frac{a^2}{4} + ax \dots (i)$$

In right-angled triangle AED, we have:

$$AD^2 = AE^2 + ED^2$$

$$\Rightarrow p^2 = h^2 + x^2 \dots (ii)$$

Therefore,

from (i) and (ii),

$$b^2 = p^2 + ax + \frac{a^2}{x}$$

(ii)

In right-angled triangle AEB, applying Pythagoras, we have:

$$AB^2 = AE^2 + EB^2$$

$$\Rightarrow c^2 = h^2 + \left(\frac{a}{2} - x\right)^2 \quad (\because BD = \frac{a}{2} \text{ and } BE = BD - x)$$

$$\Rightarrow c^2 = h^2 + x^2 - \frac{a^2}{4} \quad (\because h^2 + x^2 = p^2)$$

$$\Rightarrow c^2 = p^2 - ax + \frac{a^2}{x}$$

(iii)

Adding (i) and (ii), we get:

$$\begin{aligned} \Rightarrow b^2 + c^2 &= p^2 + ax + \frac{a^2}{x} + p^2 - ax + \frac{a^2}{x} \\ &= 2p^2 + ax - ax + \frac{a^2 + a^2}{x} \end{aligned}$$

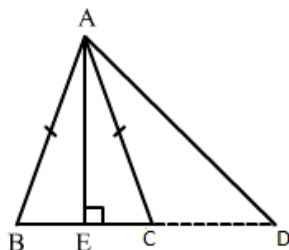
$$= 2p^2 + \frac{a^2}{2}$$

(iv)

Subtracting (ii) from (i), we get:

$$\begin{aligned} b^2 - c^2 &= p^2 + ax + \frac{a^2}{4} - (p^2 - ax + \frac{a^2}{4}) \\ &= p^2 - p^2 + ax + ax + \frac{a^2}{4} - \frac{a^2}{4} \\ &= 2ax \end{aligned}$$

18. In  $\triangle ABC$ ,  $AB = AC$ . Side  $BC$  is produced to  $D$ . Prove that  $AD^2 - AC^2 = BD \cdot CD$

**Sol:**Draw  $AE \perp BC$ , meeting  $BC$  at  $D$ .Applying Pythagoras theorem in right-angled triangle  $AED$ , we get:

Since,  $\triangle ABC$  is an isosceles triangle and  $AE$  is the altitude and we know that the altitude is also the median of the isosceles triangle.

So,  $BE = CE$ And  $DE + CE = DE + BE = BD$ 

$$AD^2 = AE^2 + DE^2$$

$$\Rightarrow AE^2 = AD^2 - DE^2 \dots (i)$$

In  $\triangle ACE$ ,

$$AC^2 = AE^2 + EC^2$$

$$\Rightarrow AE^2 = AC^2 - EC^2 \dots (ii)$$

Using (i) and (ii),

$$\Rightarrow AD^2 - DE^2 = AC^2 - EC^2$$

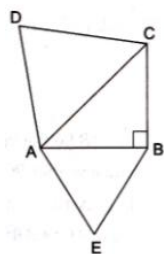
$$\Rightarrow AD^2 - AC^2 = DE^2 - EC^2$$

$$= (DE + EC)(DE - EC)$$

$$= (DE + BE) CD$$

$$= BD \cdot CD$$

19.  $\triangle ABC$  is an isosceles triangle, right-angled at  $B$ . Similar triangles  $\triangle ACD$  and  $\triangle ABE$  are constructed on sides  $AC$  and  $AB$ . Find the ratio between the areas of  $\triangle ABE$  and  $\triangle ACD$ .

**Sol:**

We have, ABC as an isosceles triangle, right angled at B.

Now,  $AB = BC$

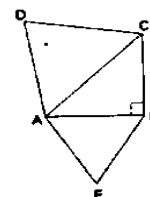
Applying Pythagoras theorem in right-angled triangle ABC, we get:

$$AC^2 = AB^2 + BC^2 = 2AB^2 (\because AB = BC) \dots (i)$$

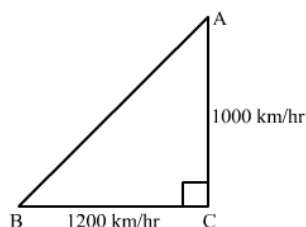
$$\therefore \triangle ACD \sim \triangle ABE$$

We know that ratio of areas of 2 similar triangles is equal to squares of the ratio of their corresponding sides.

$$\begin{aligned} \therefore \frac{ar(\triangle ABE)}{ar(\triangle ACD)} &= \frac{AB^2}{AC^2} = \frac{AB^2}{2AB^2} [from (i)] \\ &= \frac{1}{2} = 1 : 2 \end{aligned}$$



20. An aeroplane leaves an airport and flies due north at a speed of 1000km per hour. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1200 km per hour. How far apart will be the two planes after  $1\frac{1}{2}$  hours?

**Sol:**

Let A be the first aeroplane flew due north at a speed of 1000 km/hr and B be the second aeroplane flew due west at a speed of 1200 km/hr

$$\text{Distance covered by plane A in } 1\frac{1}{2} \text{ hours} = 1000 \times \frac{3}{2} = 1500 \text{ km}$$

$$\text{Distance covered by plane B in } 1\frac{1}{2} \text{ hours} = 1200 \times \frac{3}{2} = 1800 \text{ km}$$

Now, In right triangle ABC

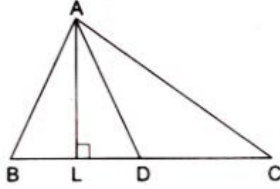
By using Pythagoras theorem, we have

$$\begin{aligned} AB^2 &= BC^2 + CA^2 \\ &= (1800)^2 + (1500)^2 \\ &= 3240000 + 2250000 \\ &= 5490000 \\ \therefore AB^2 &= 5490000 \end{aligned}$$

$$\Rightarrow AB = 300 \sqrt{61} m$$

Hence, the distance between two planes after  $1\frac{1}{2}$  hours is  $300 \sqrt{61} m$

21. In a  $\triangle ABC$ ,  $AD$  is a median and  $AL \perp BC$ .



Prove that

$$(a) AC^2 = AD^2 + BC \cdot DL + \left(\frac{BC}{2}\right)^2$$

$$(b) AB^2 = AD^2 - BC \cdot DL + \left(\frac{BC}{2}\right)^2$$

$$(c) AC^2 + AB^2 = 2AD^2 + \frac{1}{2}BC^2$$

**Sol:**

(a) In right triangle ALD

Using Pythagoras theorem, we have

$$\begin{aligned} AC^2 &= AL^2 + LC^2 \\ &= AD^2 - DL^2 + (DL + DC)^2 \quad [\text{Using (1)}] \\ &= AD^2 - DL^2 + \left(DL + \frac{BC}{2}\right)^2 \quad [\because AD \text{ is a median}] \\ &= AD^2 - DL^2 + DL^2 + \left(\frac{BC}{2}\right)^2 + BC \cdot DL \\ \therefore AC^2 &= AD^2 + BC \cdot DL + \left(\frac{BC}{2}\right)^2 \quad \dots (2) \end{aligned}$$

(b) In right triangle ALD

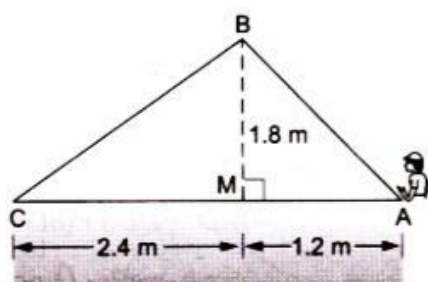
Using Pythagoras theorem, we have

$$\begin{aligned} AL^2 &= AD^2 - DL^2 \quad \dots (3) \\ \text{Again, In right triangle ABL} \\ \text{Using Pythagoras theorem, we have} \\ AB^2 &= AL^2 + LB^2 \\ &= AD^2 - DL^2 + LB^2 \quad [\text{Using (3)}] \\ &= AD^2 - DL^2 + (BD - DL)^2 \\ &= AD^2 - DL^2 + \left(\frac{1}{2}BC - DL\right)^2 \\ &= AD^2 - DL^2 + \left(\frac{BC}{2}\right)^2 - BC \cdot DL + DL^2 \\ \therefore AB^2 &= AD^2 - BC \cdot DL + \left(\frac{BC}{2}\right)^2 \quad \dots (4) \end{aligned}$$

(c) Adding (2) and (4), we get,

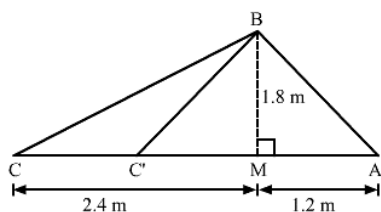
$$\begin{aligned}
 &= AC^2 + AB^2 = AD^2 + BC \cdot DL + \left(\frac{BC}{2}\right)^2 + AD^2 - BC \cdot DL + \left(\frac{BC}{2}\right)^2 \\
 &= 2AD^2 + \frac{BC^2}{4} + \frac{BC^2}{4} \\
 &= 2AD^2 + \frac{1}{2}BC^2
 \end{aligned}$$

22. Naman is doing fly-fishing in a stream. The trip fishing rod is 1.8m above the surface of the water and the fly at the end of the string rests on the water 3.6m away from him and 2.4m from the point directly under the tip of the rod. Assuming that the string( from the tip of his rod to the fly) is taut, how much string does he have out (see the adjoining figure) if he pulls in the string at the rate of 5cm per second, what will be the horizontal distance of the fly from him after 12 seconds?



23.

Sol:



Naman pulls in the string at the rate of 5 cm per second.

Hence, after 12 seconds the length of the string he will pulled is given by

$$12 \times 5 = 60 \text{ cm or } 0.6 \text{ m}$$

Now, in  $\triangle BMC$

By using Pythagoras theorem, we have

$$BC^2 = CM^2 + MB^2$$

$$= (2.4)^2 + (1.8)^2$$

$$= 9$$

$$\therefore BC = 3 \text{ m}$$

$$\text{Now, } BC' = BC - 0.6$$

$$= 3 - 0.6$$

$$= 2.4 \text{ m}$$

Now, In  $\triangle BC'M$

By using Pythagoras theorem, we have

$$C'M^2 = BC'^2 - MB^2$$



$$= (2.4)^2 - (1.8)^2$$

$$= 2.52$$

$$\therefore C'M = 1.6 \text{ m}$$

The horizontal distance of the fly from him after 12 seconds is given by

$$C'A = C'M + MA$$

$$= 1.6 + 1.2$$

$$= 2.8 \text{ m}$$

### Exercise – 4E

1. State the two properties which are necessary for given two triangles to be similar.

**Sol:**

The two triangles are similar if and only if

1. The corresponding sides are in proportion.
2. The corresponding angles are equal.

2. State the basic proportionality theorem.

**Sol:**

If a line is drawn parallel to one side of a triangle intersect the other two sides, then it divides the other two sides in the same ratio.

3. State and converse of Thale's theorem.

**Sol:**

If a line divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side.

4. State the midpoint theorem

**Sol:**

The line segment connecting the midpoints of two sides of a triangle is parallel to the third side and is equal to one half of the third side.

5. State the AAA-similarity criterion

**Sol:**

If the corresponding angles of two triangles are equal, then their corresponding sides are proportional and hence the triangles are similar.

6. State the AA-similarity criterion

**Sol:**

If two angles are correspondingly equal to the two angles of another triangle, then the two triangles are similar.

7. State the SSS-similarity criterion for similarity of triangles

**Sol:**

If the corresponding sides of two triangles are proportional then their corresponding angles are equal, and hence the two triangles are similar.

8. State the SAS-similarity criterion

**Sol:**

If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional then the two triangles are similar.

9. State Pythagoras theorem

**Sol:**

The square of the hypotenuse is equal to the sum of the squares of the other two sides. Here, the hypotenuse is the longest side and it's always opposite the right angle.

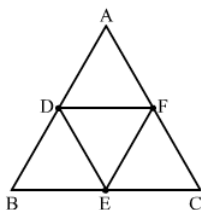
10. State the converse of Pythagoras theorem.

**Sol:**

If the square of one side of a triangle is equal to the sum of the squares of the other two sides, then the triangle is a right triangle.

11. If D, E, F are the respectively the midpoints of sides BC, CA and AB of  $\triangle ABC$ . Find the ratio of the areas of  $\triangle DEF$  and  $\triangle ABC$ .

**Sol:**



By using mid theorem i.e., the segment joining two sides of a triangle at the midpoints of those sides is parallel to the third side and is half the length of the third side.

$\therefore DF \parallel BC$

And  $DF = \frac{1}{2} BC$

$\Rightarrow DF = BE$

Since, the opposite sides of the quadrilateral are parallel and equal.

Hence, BDFE is a parallelogram

Similarly, DFCE is a parallelogram.

Now, in  $\triangle ABC$  and  $\triangle EFD$

$\angle ABC = \angle EFD$  (Opposite angles of a parallelogram)

$\angle BCA = \angle EDF$  (Opposite angles of a parallelogram)

By AA similarity criterion,  $\triangle ABC \sim \triangle EFD$

If two triangles are similar, then the ratio of their areas is equal to the squares of their corresponding sides.

$$\therefore \frac{\text{area}(\triangle DEF)}{\text{area}(\triangle ABC)} = \left(\frac{DF}{BC}\right)^2 = \left(\frac{DF}{2DF}\right)^2 = \frac{1}{4}$$

Hence, the ratio of the areas of  $\triangle DEF$  and  $\triangle ABC$  is 1 : 4.

12. Two triangles ABC and PQR are such that  $AB = 3$  cm,  $AC = 6$  cm,  $\angle A = 70^\circ$ ,  $PR = 9$  cm,  $\angle P = 70^\circ$  and  $PQ = 4.5$  cm. Show that  $\triangle ABC \sim \triangle PQR$  and state that similarity criterion.

**Sol:**

Now, In  $\triangle ABC$  and  $\triangle PQR$

$$\angle A = \angle P = 70^\circ \quad (\text{Given})$$

$$\frac{AB}{PQ} = \frac{AC}{PR} \left[ \because \frac{3}{4.5} = \frac{6}{9} \Rightarrow \frac{1}{1.5} = \frac{1}{1.5} \right]$$

By SAS similarity criterion,  $\triangle ABC \sim \triangle PQR$

13. In  $\triangle ABC \sim \triangle DEF$  such that  $2AB = DE$  and  $BC = 6$  cm, find  $EF$ .

**Sol:**

When two triangles are similar, then the ratios of the lengths of their corresponding sides are equal.

Here,  $\triangle ABC \sim \triangle DEF$

$$\therefore \frac{AB}{DE} = \frac{BC}{EF}$$

$$\Rightarrow \frac{AB}{2AB} = \frac{6}{EF}$$

$$\Rightarrow EF = 12 \text{ cm}$$

14. In the given figure,  $DE \parallel BC$  such that  $AD = x$  cm,  $DB = (3x + 4)$  cm,  $AE = (x + 3)$  cm and  $EC = (3x + 19)$  cm. Find the value of  $x$ .

**Sol:**

In  $\triangle ADE$  and  $\triangle ABC$

$$\angle ADE = \angle ABC \quad (\text{Corresponding angles in } DE \parallel BC)$$

$$\angle AED = \angle ACB \quad (\text{Corresponding angles in } DE \parallel BC)$$

By AA similarity criterion,  $\triangle ADE \sim \triangle ABC$

If two triangles are similar, then the ratio of their corresponding sides are proportional

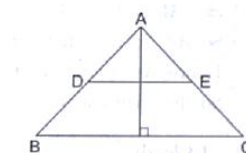
$$\therefore \frac{AD}{AB} = \frac{AE}{AC}$$

$$\Rightarrow \frac{AD}{AD+DB} = \frac{AE}{AE+EC}$$

$$\Rightarrow \frac{x}{x+3x+4} = \frac{x+3}{x+3+3x+19}$$

$$\Rightarrow \frac{x}{4x+4} = \frac{x+3}{x+3+3x+19}$$

$$\Rightarrow \frac{x}{2x+2} = \frac{x+3}{2x+11}$$



$$\Rightarrow 2x^2 + 11x = 2x^2 + 2x + 6x + 6$$

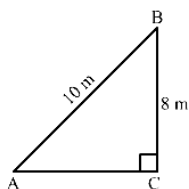
$$\Rightarrow 3x = 6$$

$$\Rightarrow x = 2$$

Hence, the value of  $x$  is 2.

15. A ladder 10m long reaches the window of a house 8m above the ground. Find the distance of the foot of the ladder from the base of the wall.

**Sol:**



Let AB be A ladder and B is the window at 8 m above the ground C.

Now, In right triangle ABC

By using Pythagoras theorem, we have

$$AB^2 = BC^2 + CA^2$$

$$\Rightarrow 10^2 = 8^2 + CA^2$$

$$\Rightarrow CA^2 = 100 - 64$$

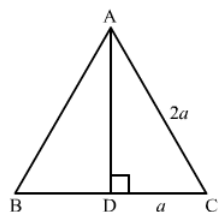
$$\Rightarrow CA^2 = 36$$

$$\Rightarrow CA = 6\text{m}$$

Hence, the distance of the foot of the ladder from the base of the wall is 6 m.

16. Find the length of the altitude of an equilateral triangle of side  $2a$  cm.

**Sol:**



We know that the altitude of an equilateral triangle bisects the side on which it stands and forms right angled triangles with the remaining sides.

Suppose ABC is an equilateral triangle having  $AB = BC = CA = 2a$ .

Suppose AD is the altitude drawn from the vertex A to the side BC.

So, it will bisects the side BC

$$\therefore DC = a$$

Now, In right triangle ADC

By using Pythagoras theorem, we have

$$AC^2 = CD^2 + DA^2$$

$$\Rightarrow (2a)^2 = a^2 + DA^2$$

$$\Rightarrow DA^2 = 4a^2 - a^2$$

$$\Rightarrow DA^2 = 3a^2$$

$$\Rightarrow DA = \sqrt{3}a$$

Hence, the length of the altitude of an equilateral triangle of side  $2a$  cm is  $\sqrt{3}a$  cm

17.  $\triangle ABC \sim \triangle DEF$  such that  $\text{ar}(\triangle ABC) = 64 \text{ cm}^2$  and  $\text{ar}(\triangle DEF) = 169 \text{ cm}^2$ . If  $BC = 4$  cm, find  $EF$ .

**Sol:**

We have  $\triangle ABC \sim \triangle DEF$

If two triangles are similar then the ratio of their areas is equal to the ratio of the squares of their corresponding sides.

$$\therefore \frac{\text{area}(\triangle ABC)}{\text{area}(\triangle DEF)} = \left(\frac{BC}{EF}\right)^2$$

$$\Rightarrow \frac{64}{169} = \left(\frac{BC}{EF}\right)^2$$

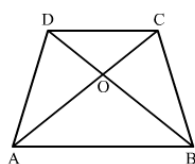
$$\Rightarrow \left(\frac{8}{13}\right)^2 = \left(\frac{4}{EF}\right)^2$$

$$\Rightarrow \frac{8}{13} = \frac{4}{EF}$$

$$\Rightarrow EF = 6.5 \text{ cm}$$

18. In a trapezium  $ABCD$ , it is given that  $AB \parallel CD$  and  $AB = 2CD$ . Its diagonals  $AC$  and  $BD$  intersect at the point  $O$  such that  $\text{ar}(\triangle AOB) = 84 \text{ cm}^2$ . Find  $\text{ar}(\triangle COD)$ .

**Sol:**



In  $\triangle AOB$  and  $\triangle COD$

$$\angle ABO = \angle CDO \quad (\text{Alternate angles in } AB \parallel CD)$$

$$\angle AOB = \angle COD \quad (\text{Vertically opposite angles})$$

By AA similarity criterion,  $\triangle AOB \sim \triangle COD$

If two triangles are similar, then the ratio of their areas is equal to the ratio of the squares of their corresponding sides.

$$\therefore \frac{\text{area}(\triangle AOB)}{\text{area}(\triangle COD)} = \left(\frac{AB}{CD}\right)^2$$

$$\Rightarrow \frac{84}{\text{area}(\triangle COD)} = \left(\frac{2CD}{CD}\right)^2$$

$$\Rightarrow \text{area}(\triangle COD) = 12 \text{ cm}^2$$

19. The corresponding sides of two similar triangles are in the ratio 2: 3. If the area of the smaller triangle is  $48 \text{ cm}^2$ , find the area of the larger triangle.

**Sol:**

If two triangles are similar, then the ratio of their areas is equal to the squares of their corresponding sides.

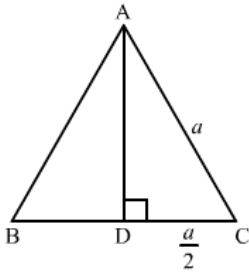
$$\therefore \frac{\text{area of smaller triangle}}{\text{area of larger triangle}} = \left( \frac{\text{Side of smaller triangle}}{\text{Side of larger triangle}} \right)^2$$

$$\Rightarrow \frac{48}{\text{area of larger triangle}} = \left( \frac{2}{3} \right)^2$$

$$\Rightarrow \text{area of larger triangle} = 108 \text{ cm}^2$$

20. In an equilateral triangle with side  $a$ , prove that  $\text{area} = \frac{\sqrt{3}}{4} a^2$ .

**Sol:**



We know that the altitude of an equilateral triangle bisects the side on which it stands and forms right angled triangles with the remaining sides.

Suppose ABC is an equilateral triangle having  $AB = BC = CA = a$ .

Suppose AD is the altitude drawn from the vertex A to the side BC.

So, It will bisect the side BC

$$\therefore DC = \frac{1}{2} a$$

Now, In right triangle ADC

By using Pythagoras theorem, we have

$$AC^2 = CD^2 + DA^2$$

$$\Rightarrow a^2 - \left( \frac{1}{2} a \right)^2 + DA^2$$

$$\Rightarrow DA^2 = a^2 - \frac{1}{4} a^2$$

$$\Rightarrow DA^2 = \frac{3}{4} a^2$$

$$\Rightarrow DA = \frac{\sqrt{3}}{2} a$$

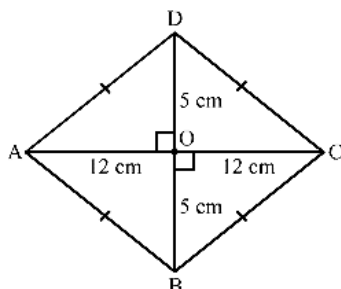
$$\text{Now, area } (\triangle ABC) = \frac{1}{2} \times BC \times AD$$

$$= \frac{1}{2} \times a \times \frac{\sqrt{3}}{2} a$$

$$= \frac{\sqrt{3}}{4} a^2$$

21. Find the length of each side of a rhombus whose diagonals are 24cm and 10cm long.

**Sol:**



Suppose ABCD is a rhombus.

We know that the diagonals of a rhombus perpendicularly bisect each other.

$\therefore \angle AOB = 90^\circ$ ,  $AO = 12 \text{ cm}$  and  $BO = 5 \text{ cm}$

Now, In right triangle AOB

By using Pythagoras theorem we have

$$AB^2 = AO^2 + BO^2$$

$$= 12^2 + 5^2$$

$$= 144 + 25$$

$$= 169$$

$$\therefore AB^2 = 169$$

$$\Rightarrow AB = 13 \text{ cm}$$

Since, all the sides of a rhombus are equal.

Hence,  $AB = BC = CD = DA = 13 \text{ cm}$

22. Two triangles DEF and GHK are such that  $\angle D = 48^\circ$  and  $\angle H = 57^\circ$ . If  $\triangle DEF \sim \triangle GHK$  then find the measures of  $\angle F$

**Sol:**

If two triangles are similar then the corresponding angles of the two triangles are equal.

Here,  $\triangle DEF \sim \triangle GHK$

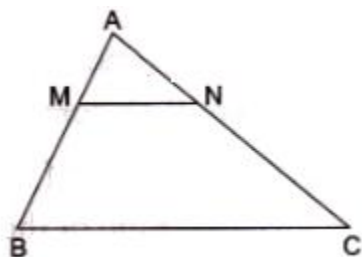
$$\therefore \angle E = \angle H = 57^\circ$$

Now, In  $\triangle DEF$

$$\angle D + \angle E + \angle F = 180^\circ \quad (\text{Angle sum property of triangle})$$

$$\Rightarrow \angle F = 180^\circ - 48^\circ - 57^\circ = 75^\circ$$

23. In the given figure  $MN \parallel BC$  and  $AM:MB = 1:2$



Find  $\frac{\text{area}(\triangle AMN)}{\text{area}(\triangle ABC)}$

**Sol:**

We have

$$AM : MB = 1 : 2$$

$$\Rightarrow \frac{MB}{AM} = \frac{2}{1}$$

Adding 1 to both sides, we get

$$\Rightarrow \frac{MB}{AM} + 1 = \frac{2}{1} + 1$$

$$\Rightarrow \frac{MB+AM}{AM} = \frac{2+1}{1}$$

$$\Rightarrow \frac{AB}{AM} = \frac{3}{1}$$

Now, In  $\triangle AMN$  and  $\triangle ABC$

$$\angle AMN = \angle ABC \quad (\text{Corresponding angles in } MN \parallel BC)$$

$$\angle ANM = \angle ACB \quad (\text{Corresponding angles in } MN \parallel BC)$$

By AA similarity criterion,  $\triangle AMN \sim \triangle ABC$

If two triangles are similar, then the ratio of their areas is equal to the ratio of the squares of their corresponding sides.

$$\therefore \frac{\text{area}(\triangle AMN)}{\text{area}(\triangle ABC)} = \left(\frac{AM}{AB}\right)^2 = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

- 24.** In triangle BMP and CNR it is given that PB = 5 cm, MP = 6 cm, BM = 9 cm and NR = 9 cm. If  $\triangle BMP \sim \triangle CNR$  then find the perimeter of  $\triangle CNR$

**Sol:**

When two triangles are similar, then the ratios of the lengths of their corresponding sides are proportional.

Here,  $\triangle BMP \sim \triangle CNR$

$$\therefore \frac{BM}{CN} = \frac{BP}{CR} = \frac{MP}{NR} \quad \dots (1)$$

$$\text{Now, } \frac{BM}{CN} = \frac{MP}{NR} \quad [\text{Using (1)}]$$

$$\Rightarrow CN = \frac{BM \times NR}{MP} = \frac{9 \times 9}{6} = 13.5 \text{ cm}$$

$$\text{Again, } \frac{BM}{CN} = \frac{BP}{CR} = [\text{Using (1)}]$$

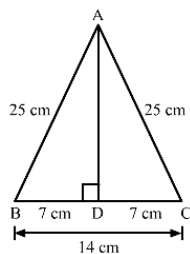
$$\Rightarrow CR = \frac{BP \times CN}{BM} = \frac{5 \times 13.5}{9} = 7.5 \text{ cm}$$

$$\text{Perimeter of } \triangle CNR = CN + NR + CR = 13.5 + 9 + 7.5 = 30 \text{ cm}$$

- 25.** Each of the equal sides of an isosceles triangle is 25 cm. Find the length of its altitude if the base is 14 cm.

**Sol:**





We know that the altitude drawn from the vertex opposite to the non-equal side bisects the non-equal side.

Suppose ABC is an isosceles triangle having equal sides AB and AC.

So, the altitude drawn from the vertex will bisect the opposite side.

Now, In right triangle ABD

By using Pythagoras theorem, we have

$$AB^2 = BD^2 + DA^2$$

$$\Rightarrow 25^2 = 7^2 + DA^2$$

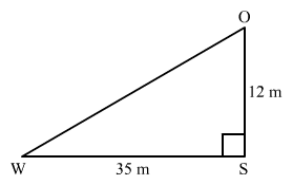
$$\Rightarrow DA^2 = 625 - 49$$

$$\Rightarrow DA^2 = 576$$

$$\Rightarrow DA = 24 \text{ cm}$$

26. A man goes 12m due south and then 35m due west. How far is he from the starting point.

**Sol:**



In right triangle OSW

By using Pythagoras theorem, we have

$$OW^2 = WS^2 + OS^2$$

$$= 35^2 + 12^2$$

$$= 1225 + 144$$

$$= 1369$$

$$\therefore OW^2 = 1369$$

$$\Rightarrow OW = 37 \text{ m}$$

Hence, the man is 37 m away from the starting point.

27. If the lengths of the sides BC, CA and AB of a  $\triangle ABC$  are a, b and c respectively and AD is the bisector  $\angle A$  then find the lengths of BD and DC

**Sol:**

Let DC = X

$$\therefore BD = a - X$$

By using angle bisector there in  $\triangle ABC$ , we have

$$\frac{AB}{AC} = \frac{BD}{DC}$$

$$\Rightarrow \frac{c}{b} = \frac{a-x}{x}$$

$$\Rightarrow cx = ab - bx$$

$$\Rightarrow x(b+c) = ab$$

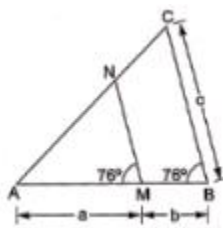
$$\Rightarrow x = \frac{ab}{(b+c)}$$

$$\text{Now, } a - x = a - \frac{ab}{b+c}$$

$$= \frac{ab+ac-ab}{b+c}$$

$$= \frac{ac}{a+b}$$

28. In the given figure,  $\angle AMN = \angle MBC = 76^\circ$ . If p, q and r are the lengths of AM, MB and BC respectively then express the length of MN of terms of P, q and r.



**Sol:**

In  $\triangle AMN$  and  $\triangle ABC$

$$\angle AMN = \angle ABC = 76^\circ \text{ (Given)}$$

$$\angle A = \angle A \text{ (Common)}$$

By AA similarity criterion,  $\triangle AMN \sim \triangle ABC$

If two triangles are similar, then the ratio of their corresponding sides are proportional

$$\therefore \frac{AM}{AB} = \frac{MN}{BC}$$

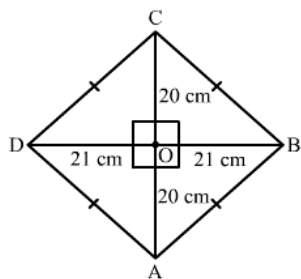
$$\Rightarrow \frac{AM}{AM+MB} = \frac{MN}{BC}$$

$$\Rightarrow \frac{a}{a+b} = \frac{MN}{c}$$

$$\Rightarrow MN = \frac{ac}{a+b}$$

29. Find the length of each side of a rhombus are 40 cm and 42 cm. find the length of each side of the rhombus.

**Sol:**



Suppose ABCD is a rhombus.

We know that the diagonals of a rhombus perpendicularly bisect each other.

$\therefore \angle AOB = 90^\circ, AO = 20 \text{ cm and } BO = 21 \text{ cm}$

Now, In right triangle AOB

By using Pythagoras theorem we have

$$AB^2 = AO^2 + OB^2$$

$$= 20^2 + 21^2$$

$$= 400 + 441$$

$$= 841$$

$$\therefore AB^2 = 841$$

$$\Rightarrow AB = 29 \text{ cm}$$

Since, all the sides of a rhombus are equal.

Hence,  $AB = BC = CD = DA = 29 \text{ cm}$

**30.** For each of the following statements state whether true(T) or false (F)

(i) Two circles with different radii are similar.

(ii) any two rectangles are similar

(iii) if two triangles are similar then their corresponding angles are equal and their corresponding sides are equal

(iv) The length of the line segment joining the midpoints of any two sides of a triangles is equal to half the length of the third side.

(v) In a  $\triangle ABC$ ,  $AB = 6 \text{ cm}$ ,  $\angle A = 45^\circ$  and  $AC = 8 \text{ cm}$  and in a  $\triangle DEF$ ,  $DF = 9 \text{ cm}$ ,  $\angle D = 45^\circ$  and  $DE = 12 \text{ cm}$  then  $\triangle ABC \sim \triangle DEF$ .

(vi) the polygon formed by joining the midpoints of the sides of a quadrilateral is a rhombus.

(vii) the ratio of the perimeter of two similar triangles is the same as the ratio of the their corresponding medians.

(ix) if O is any point inside a rectangle ABCD then  $OA^2 + OC^2 = OB^2 + OD^2$

(x) The sum of the squares on the sides of a rhombus is equal to the sum of the squares on its diagonals.

**Sol:**

(i)

Two rectangles are similar if their corresponding sides are proportional.

(ii) True

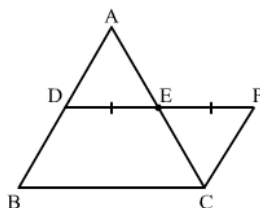
Two circles of any radii are similar to each other.

(iii) false

If two triangles are similar, their corresponding angles are equal and their corresponding sides are proportional.

(iv) True

Suppose ABC is a triangle and M, N are



Construction: DE is expanded to F such that  $EF = DE$

To prove  $DE = \frac{1}{2}BC$

Proof: In  $\triangle ADE$  and  $\triangle CEF$

$AE = EC$  (E is the mid point of AC)

$DE = EF$  (By construction)

$\angle AED = \angle CEF$  (Vertically Opposite angle)

By SAS criterion,  $\triangle ADE \cong \triangle CEF$

$CF = AD$  (CPCT)

$\Rightarrow BD = CF$

$\angle ADE = \angle EFC$  (CPCT)

Since,  $\angle ADE$  and  $\angle EFC$  are alternate angle

Hence,  $AD \parallel CF$  and  $BD \parallel CF$

When two sides of a quadrilateral are parallel, then it is a parallelogram

$\therefore DF = BC$  and  $BD \parallel CF$

$\therefore BDFC$  is a parallelogram

Hence,  $DF = BC$

$\Rightarrow DE + EF = BC$

$\Rightarrow DE = \frac{1}{2}BC$

(v) False

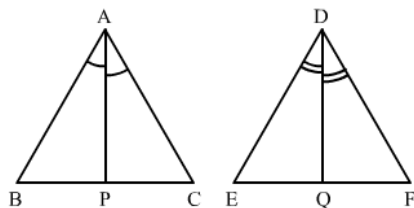
In  $\triangle ABC$ ,  $AB = 6$  cm,  $\angle A = 45^\circ$  and  $AC = 8$  cm and in  $\triangle DEF$ ,  $DF = 9$  cm,  $\angle D = 45^\circ$  and  $DE = 12$  cm, then  $\triangle ABC \sim \triangle DEF$ .

In  $\triangle ABC$  and  $\triangle DEF$

(vi) False

The polygon formed by joining the mid points of the sides of a quadrilateral is a parallelogram.

(vii) True



Given:  $\Delta ABC \sim \Delta DEF$

To prove =  $\frac{Ar(\Delta ABC)}{Ar(\Delta DEF)} = \left(\frac{AP}{DQ}\right)^2$

Proof: in  $\Delta ABP$  and  $\Delta DEQ$

$\angle BAP = \angle EDQ$  (As  $\angle A = \angle D$ , so their Half is also equal)

$\angle B = \angle E$  ( $\Delta ABC \sim \Delta DEF$ )

By AA criterion,  $\Delta ABP$  and  $\Delta DEQ$

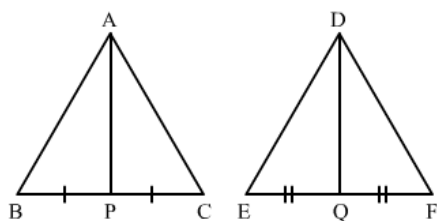
$$\frac{AB}{DE} = \frac{AP}{DQ} \quad \dots(1)$$

Since,  $\Delta ABC \sim \Delta DEF$

$$\therefore \frac{Ar(\Delta ABC)}{Ar(\Delta DEF)} = \left(\frac{AB}{DE}\right)^2$$

$$\Rightarrow \frac{Ar(\Delta ABC)}{Ar(\Delta DEF)} = \left(\frac{AP}{DQ}\right)^2 \quad [Using (1)]$$

(viii)



Given:  $\Delta ABC \sim \Delta DEF$

To Prove =  $\frac{Perimeter(\Delta ABC)}{Perimeter(\Delta DEF)} = \frac{AP}{DQ}$

Proof: In  $\Delta ABP$  and  $\Delta DEQ$

$\angle B = \angle E$  ( $\because \Delta ABC \sim \Delta DEF$ )

$\therefore \Delta ABC \sim \Delta DEF$

$$\therefore \frac{AB}{DE} = \frac{BC}{EF}$$

$$\Rightarrow \frac{AB}{DE} = \frac{2 BP}{2 EQ}$$

$$\Rightarrow \frac{AB}{DE} = \frac{BP}{EQ}$$

By SAS criterion,  $\Delta ABP \sim \Delta DEQ$

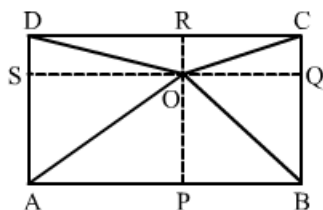
$$\frac{AB}{DE} = \frac{AP}{DQ} \quad \dots(1)$$

Since,  $\Delta ABC \sim \Delta DEF$

$$\therefore \frac{\text{Perimeter}(\triangle ABC)}{\text{Perimeter}(\triangle DEF)} = \frac{AB}{DE}$$

$$\Rightarrow \frac{\text{Perimeter}(\triangle ABC)}{\text{Perimeter}(\triangle DEF)} = \frac{AP}{DQ} \quad [\text{Using (1)}]$$

(ix) True



Suppose ABCD is a rectangle with O is any point inside it.

Construction:  $OA^2 + OC^2 = OB^2 + OD^2$

Proof:

$$OA^2 + OC^2 = (AS^2 + OS^2) + (OQ^2 + QC^2) \quad [\text{Using Pythagoras theorem in right triangle AOP and COQ}]$$

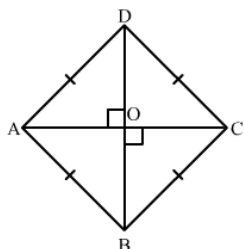
$$= (BQ^2 + OS^2) + (OQ^2 + DS^2)$$

$$= (BQ^2 + OQ^2) + (OS^2 + DS^2) \quad [\text{Using Pythagoras theorem in right triangle BOQ and DOS}]$$

$$= OB^2 + OD^2$$

Hence, LHS = RHS

(x) True



Suppose ABCD is a rhombus having AC and BD its diagonals.

Since, the diagonals of a rhombus perpendicular bisect each other.

Hence, AOC is a right angle triangle

In right triangle AOC

By using Pythagoras theorem, we have

$$AB^2 = \left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2$$

[ $\therefore$  Diagonals of a rhombus perpendicularly bisect each other]

$$\Rightarrow AB^2 = \frac{AC^2}{4} + \frac{BD^2}{4}$$

$$\Rightarrow 4AB^2 = AC^2 + BD^2$$

$$\Rightarrow AB^2 + AB^2 + AB^2 + AB^2 = AC^2 + BD^2$$

$$\Rightarrow AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2 \quad [\because \text{All sides of a rhombus are equal}]$$

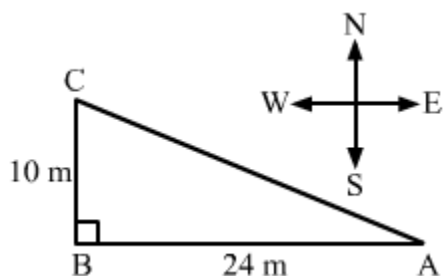
## Exercise – MCQ

1. A man goes 24m due west and then 10m due north. How far is he from the starting point?

(a) 34m      (b) 17m      (c) 26m      (d) 28m

**Sol:**

(c) 26 m



Suppose, the man starts from point A and goes 24 m due west to point B. From here, he goes 10 m due north and stops at C.

In right triangle ABC, we have:

$$AB = 24 \text{ m}, BC = 10 \text{ m}$$

Applying Pythagoras theorem, we get:

$$AC^2 = AB^2 + BC^2 = 24^2 + 10^2$$

$$AC^2 = 576 + 100 = 676$$

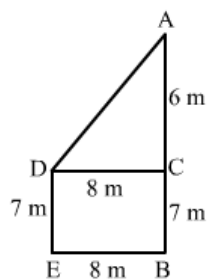
$$AC = \sqrt{676} = 26$$

2. Two poles of height 13m and 7m respectively stand vertically on a plane ground at a distance of 8m from each other. The distance between their tops is

(a) 9m      (b) 10m      (c) 11m      (d) 12m

**Sol:**

(b) 10 m



Let AB and DE be the two poles.

According to the question:

$$AB = 13 \text{ m}$$

$$DE = 7 \text{ m}$$

$$\text{Distance between their bottoms} = BE = 8 \text{ m}$$

Draw a perpendicular DC to AB from D, meeting AB at C. We get:

$$DC = 8\text{m}, AC = 6 \text{ m}$$

Applying, Pythagoras theorem in right-angled triangle ACD, we have

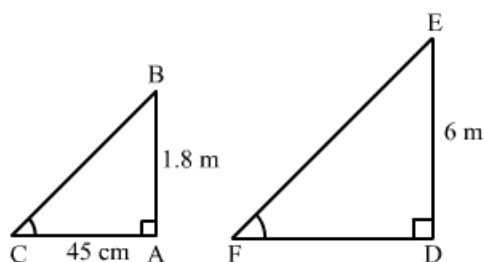
$$\begin{aligned} AD^2 &= DC^2 + AC^2 \\ &= 8^2 + 6^2 = 64 + 36 = 100 \end{aligned}$$

$$AD = \sqrt{100} = 10 \text{ m}$$

3. A vertical stick 1.8m long casts a shadow 45cm long on the ground. At the same time, what is the length of the shadow of a pole 6m high?

(a) 2.4m      (b) 1.35m      (c) 1.5m      (d) 13.5m

**Sol:**



Let AB and AC be the vertical stick and its shadow, respectively.

According to the question:

$$AB = 1.8 \text{ m}$$

$$AC = 45 \text{ cm} = 0.45 \text{ m}$$

Again, let DE and DF be the pole and its shadow, respectively.

According to the question:

$$DE = 6 \text{ m}$$

$$DF = ?$$

Now, in right-angled triangles ABC and DEF, we have:

$$\angle BAC = \angle EDF = 90^\circ$$

$$\angle ACB = \angle DFE \quad (\text{Angular elevation of the Sun at the same time})$$

Therefore, by AA similarity theorem, we get:

$$\triangle ABC \sim \triangle DFE$$

$$\Rightarrow \frac{AB}{AC} = \frac{DE}{DF}$$

$$\Rightarrow \frac{1.8}{0.45} = \frac{6}{DF}$$

$$\Rightarrow DF = \frac{6 \times 0.45}{1.8} = 1.5 \text{ m}$$

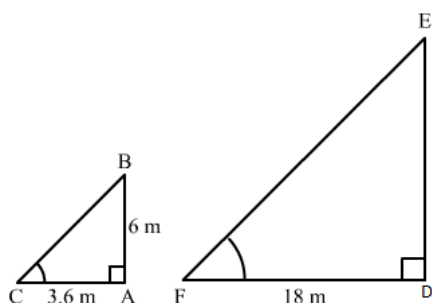
4. A vertical pole 6m long casts a shadow of length 3.6m on the ground. What is the height of a tower which casts a shadow of length 18m at the same time?

(a) 10.8m      (b) 28.8m      (c) 32.4m      (d) 30m

**Sol:**

(d)





Let AB and AC be the vertical pole and its shadow, respectively.

According to the question:

$$AB = 6 \text{ m}$$

$$AC = 3.6 \text{ m}$$

Again, let DE and DF be the tower and its shadow.

According to the question:

$$DF = 18 \text{ m}$$

$$DE = ?$$

Now, in right -angled triangles ABC and DEF, we have:

$$\angle BAC = \angle EDF = 90^\circ$$

$$\angle ABC = \angle DFE = \quad (\text{Angular elevation of the sun at the same time})$$

Therefore, by AA similarity theorem, we get:

$$\triangle ABC \sim \triangle DEF$$

$$\Rightarrow \frac{AB}{AC} = \frac{DE}{DF}$$

$$\Rightarrow \frac{6}{3.6} = \frac{DE}{18}$$

$$\Rightarrow DE = \frac{6 \times 18}{3.6} = 30 \text{ m}$$

5. The shadow of a 5m long stick is 2m long. At the same time the length of the shadow of a 12.5m high tree(in m) is

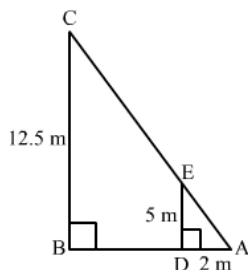
(a) 3.0

(b) 3.5

(c) 4.5

(d) 5.0

**Sol:**



Suppose DE is a 5 m long stick and BC is a 12.5 m high tree.

Suppose DA and BA are the shadows of DE and BC respectively.

Now, In  $\triangle ABC$  and  $\triangle ADE$

$$\angle ABC = \angle ADE = 90^\circ$$

$$\angle A = \angle A \text{ (Common)}$$

By AA- similarity criterion

$$\triangle ABC \sim \triangle ADE$$

If two triangles are similar, then the ratio of their corresponding sides are equal.

$$\therefore \frac{AB}{AD} = \frac{BC}{DE}$$

$$\Rightarrow \frac{AB}{2} = \frac{12.5}{5}$$

$$\Rightarrow AB = 5 \text{ cm}$$

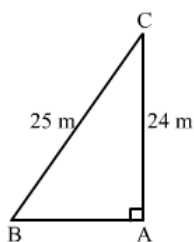
Hence, the correct answer is option (d).

6. A ladder 25m long just reaches the top of a building 24m high from the ground. What is the distance of the foot of the ladder from the building?

(a) 7m      (b) 14m      (c) 21m      (d) 24.5m

**Sol:**

(a) 7 m



Let the ladder BC reaches the building at C.

Let the height of building where the ladder reaches be AC.

According to the question:

$$BC = 25 \text{ m}$$

$$AC = 24 \text{ m}$$

In right-angled triangle CAB, we apply Pythagoras theorem to find the value of AB.

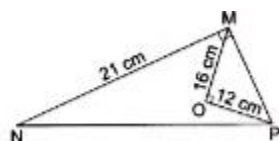
$$BC^2 = AC^2 + AB^2$$

$$\Rightarrow AB^2 = BC^2 - AC^2 = 25^2 - 24^2$$

$$\Rightarrow AB^2 = 625 - 576 = 49$$

$$\Rightarrow AB = \sqrt{49} = 7 \text{ m}$$

7. In the given figure, O is the point inside a  $\triangle MNP$  such that  $\angle MOP = 90^\circ$  OM = 16 cm and OP = 12 cm if MN = 21cm and  $\angle NMP = 90^\circ$  then NP = ?



**Sol:**

Now, In right triangle MOP

By using Pythagoras theorem, we have

$$MP^2 = PO^2 + OM^2$$

$$= 12^2 + 16^2$$

$$= 144 + 256$$

$$= 400$$

$$\therefore MP^2 = 400$$

$$\Rightarrow MO = 20 \text{ cm}$$

Now, In right triangle MPN

By using Pythagoras theorem, we have

$$PN^2 = NM^2 + MP^2$$

$$= 21^2 + 20^2$$

$$= 441 + 400$$

$$= 841$$

$$\therefore MP^2 = 841$$

$$\Rightarrow MP = 29 \text{ cm}$$

Hence, the correct answer is option (b).

8. The hypotenuse of a right triangle is 25cm. The other two sides are such that one is 5cm longer than the other. The lengths of these sides are

(a) 10cm, 15cm

(b) 15cm, 20cm

(c) 12cm, 17cm

(d) 13cm, 18cm

**Sol:**

(b) 15 cm, 20 cm

It is given that length of hypotenuse is 25 cm.

Let the other two sides be x cm and (x–5) cm.

Applying Pythagoras theorem, we get:

$$25^2 = x^2 + (x - 5)^2$$

$$\Rightarrow 625 = x^2 + x^2 + 25 - 10x$$

$$\Rightarrow 2x^2 - 10x - 600 = 0$$

$$\Rightarrow x^2 - 5x - 300 = 0$$

$$\Rightarrow x^2 - 20x + 15x - 300 = 0$$

$$\Rightarrow x(x - 20) + 15(x - 20) = 0$$

$$\Rightarrow (x - 20)(x + 15) = 0$$

$$\Rightarrow x - 20 = 0 \text{ or } x + 15 = 0$$

$$\Rightarrow x = 20 \text{ or } x = -15$$

Side of a triangle cannot be negative.

Therefore, x = 20 cm

Now,

$$x - 5 = 20 - 5 = 15 \text{ cm}$$

9. The height of an equilateral triangle having each side 12cm, is

(a)  $6\sqrt{2}$  cm

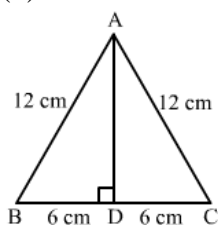
(b)  $6\sqrt{3}$  m

(c)  $3\sqrt{6}$  m

(d)  $6\sqrt{6}$  m

**Sol:**

(b)  $6\sqrt{3}$  cm



Let ABC be the equilateral triangle with AD as its altitude from A.

In right-angled triangle ABD, we have

$$AB^2 = AD^2 + BD^2$$

$$AD^2 = AB^2 - BD^2$$

$$= 12^2 - 6^2$$

$$= 144 - 36 = 108$$

$$AD = \sqrt{108} = 6\sqrt{3} \text{ cm}$$

- 10.**  $\triangle ABC$  is an isosceles triangle with  $AB = AC = 13$  cm and the length of altitude from A on BC is 5 cm. Then, BC = ?

(a) 12 cm

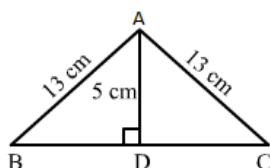
(b) 16 cm

(c) 18 cm

(d) 24 cm

**Sol:**

(d) 24 cm



In triangle ABC, let the altitude from A on BC meets BC at D.

We have:

$AD = 5$  cm,  $AB = 13$  cm and D is the midpoint of BC.

Applying Pythagoras theorem in right-angled triangle ABD, we get:

$$AB^2 = AD^2 + BD^2$$

$$\Rightarrow BD^2 = AB^2 - AD^2$$

$$\Rightarrow BD^2 = 13^2 - 5^2$$

$$\Rightarrow BD^2 = 169 - 25$$

$$\Rightarrow BD^2 = 144$$

$$\Rightarrow BD = \sqrt{144} = 12 \text{ cm}$$

Therefore,  $BC = 2BD = 24$  cm

11. In a  $\triangle ABC$ , it is given that  $AB = 6\text{cm}$ ,  $AC = 8\text{cm}$  and  $AD$  is the bisector of  $\angle A$ . Then,  $BD : DC = ?$

(a)  $3 : 4$       (b)  $9 : 16$       (c)  $4 : 3$       (d)  $\sqrt{3} : 2$

**Sol:**

(a)  $3 : 4$

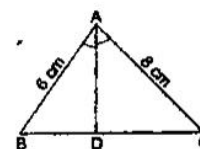
In  $\triangle ABD$  and  $\triangle ACD$ , we have:

$$\angle BAD = \angle CAD$$

Now,

$$\frac{BD}{DC} = \frac{AB}{AC} = \frac{6}{8} = \frac{3}{4}$$

$$BD : DC = 3 : 4$$



12. In  $\triangle ABC$ , it is given that  $AD$  is the internal bisector of  $\angle A$ . If  $BD = 4\text{cm}$ ,  $DC = 5\text{cm}$  and  $AB = 6\text{cm}$ , then  $AC = ?$

(a)  $4.5\text{cm}$       (b)  $8\text{cm}$       (c)  $9\text{cm}$       (d)  $7.5\text{cm}$

**Sol:**

(d)  $7.5\text{ cm}$

It is given that  $AD$  bisects angle  $A$

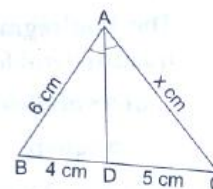
Therefore, applying angle bisector theorem, we get:

$$\frac{BD}{DC} = \frac{AB}{AC}$$

$$\Rightarrow \frac{4}{5} = \frac{6}{x}$$

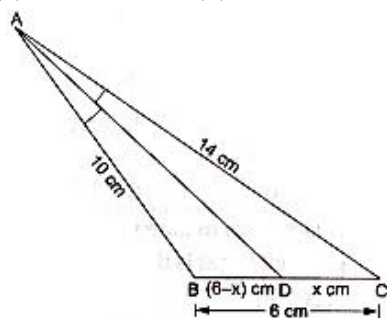
$$\Rightarrow x = \frac{5 \times 6}{4} = 7.5$$

Hence,  $AC = 7.5\text{ cm}$



13. In a  $\triangle ABC$ , it is given that  $AD$  is the internal bisector of  $\angle A$ . If  $AB = 10\text{cm}$ ,  $AC = 14\text{cm}$  and  $BC = 6\text{cm}$ , then  $CD = ?$

(a)  $4.8\text{cm}$       (b)  $3.5\text{cm}$       (c)  $7\text{cm}$       (d)  $10.5\text{cm}$



**Sol:**

By using angle bisector in  $\triangle ABC$ , we have

$$\frac{AB}{AC} = \frac{BD}{DC}$$

$$\Rightarrow \frac{10}{14} = \frac{6-x}{x}$$

$$\Rightarrow 10x = 84 - 14x$$

$$\Rightarrow 24x = 84$$

$$\Rightarrow x = 3.5$$

Hence, the correct answer is option (b).

14. In a triangle, the perpendicular from the vertex to the base bisects the base. The triangle is  
 (a) right-angled (b) isosceles  
 (c) scalene (d) obtuse-angled

**Sol:**

(b) Isosceles

In an isosceles triangle, the perpendicular from the vertex to the base bisects the base.

15. In an equilateral triangle ABC, if  $AD \perp BC$ , then which of the following is true?  
 (a)  $2AB^2 = 3AD^2$  (b)  $4AB^2 = 3AD^2$   
 (c)  $3AB^2 = 4AD^2$  (d)  $3AB^2 = 2AD^2$

**Sol:**

(c)  $3AB^2 = 4AD^2$

Applying Pythagoras theorem in right-angled triangles ABD and ADC, we get:

$$AB^2 = AD^2 + BD^2$$

$$\Rightarrow AB^2 = \left(\frac{1}{2} AB\right)^2 + AD^2 \quad \left(\because \triangle ABC \text{ is equilateral and } AD = \frac{1}{2} AB\right)$$

$$\Rightarrow AB^2 = \frac{1}{4} AB^2 + AD^2$$

$$\Rightarrow AB^2 - \frac{1}{4} AB^2 = AD^2$$

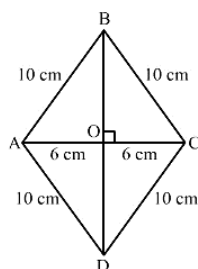
$$\Rightarrow \frac{3}{4} AB^2 = AD^2$$

$$\Rightarrow 3AB^2 = 4AD^2$$

16. In a rhombus of side 10cm, one of the diagonals is 12cm long. The length of the second diagonal is  
 (a) 20cm (b) 18cm (c) 16cm (d) 22cm

**Sol:**

(c) 16 cm



Let ABCD be the rhombus with diagonals AC and BD intersecting each other at O.  
 Also, diagonals of a rhombus bisect each other at right angles.

If  $AC = 12$  cm,  $AO = 6$  cm

Applying Pythagoras theorem in right-angled triangle AOB. We get:

$$AB^2 = AO^2 + BO^2$$

$$\Rightarrow BO^2 = AB^2 - AO^2$$

$$\Rightarrow BO^2 = 10^2 - 6^2 = 100 - 36 = 64$$

$$\Rightarrow BO = \sqrt{64} = 8$$

$$\Rightarrow BD = 2 \times BO = 2 \times 8 = 16 \text{ cm}$$

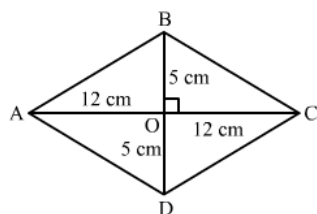
Hence, the length of the second diagonal BD is 16 cm.

17. The lengths of the diagonals of a rhombus are 24cm and 10cm. The length of each side of the rhombus is

(a) 12cm      (b) 13cm      (c) 14cm      (d) 17cm

**Sol:**

(b) 13 cm



Let ABCD be the rhombus with diagonals AC and BD intersecting each other at O.

We have:

$AC = 24$  cm and  $BD = 10$  cm

We know that diagonals of a rhombus bisect each other at right angles.

Therefore applying Pythagoras theorem in right-angled triangle AOB, we get:

$$AB^2 = AO^2 + BO^2 = 12^2 + 5^2$$

$$= 144 + 25 = 169$$

$$AB = \sqrt{169} = 13$$

Hence, the length of each side of the rhombus is 13 cm.

18. If the diagonals of a quadrilateral divide each other proportionally, then it is a

(a) parallelogram      (b) trapezium  
(c) rectangle      (d) square

**Sol:**

(b) trapezium

Diagonals of a trapezium divide each other proportionally.

19. The line segments joining the midpoints of the adjacent sides of a quadrilateral form

(a) parallelogram      (b) trapezium  
(c) rectangle      (d) square

**Sol:**

- (a) A parallelogram

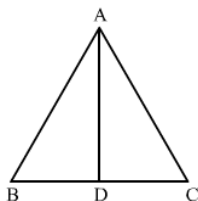
The line segments joining the midpoints of the adjacent sides of a quadrilateral form a parallelogram.

20. If the bisector of an angle of a triangle bisects the opposite side, then the triangle is

- (a) scalene (b) equilateral  
(c) isosceles (d) right-angled

**Sol:**

- (c) isosceles



Let AD be the angle bisector of angle A in triangle ABC.

Applying angle bisector theorem, we get:

$$\frac{AB}{AC} = \frac{BD}{DC}$$

It is given that AD bisects BC.

Therefore,  $BD = DC$

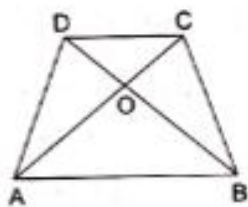
$$\Rightarrow \frac{AB}{AC} = 1$$

$$\Rightarrow AB = AC$$

Therefore, the triangle is isosceles.

21. In the given figure, ABCD is a trapezium whose diagonals AC and BD intersect at O such that  $OA = (3x - 1)$  cm,  $OB = (2x + 1)$  cm,  $OC = (5x - 3)$  cm and  $OD = (6x - 5)$  cm. Then,  $x = ?$

- (a) 2 (b) 3 (c) 2.5 (d) 4



**Sol:**

- (a) 2

We know that the diagonals of a trapezium are proportional.

$$\text{Therefore } \frac{OA}{OC} = \frac{OB}{OD}$$

$$\Rightarrow \frac{3x-1}{5x-3} = \frac{2x+1}{6x-5}$$

$$\Rightarrow (3X - 1) (6X - 5) = (2X + 1) (5X - 3)$$



$$\Rightarrow 18X^2 - 15X - 6X + 5 = 10X^2 - 6X + 5X - 3$$

$$\Rightarrow 18X^2 - 21X + 5 = 10X^2 - X - 3$$

$$\Rightarrow 18X^2 - 21X + 5 - 10X^2 + X + 3 = 0$$

$$\Rightarrow 8X^2 - 20X + 8 = 0$$

$$\Rightarrow 4(2X^2 - 5X + 2) = 0$$

$$\Rightarrow 2X^2 - 5X + 2 = 0$$

$$\Rightarrow 2X^2 - 4X - X + 2 = 0$$

$$\Rightarrow 2X(X - 2) - 1(X - 2) = 0$$

$$\Rightarrow (X - 2)(2X - 1) = 0$$

$$\Rightarrow \text{Either } x - 2 = 0 \text{ or } 2x - 1 = 0$$

$$\Rightarrow \text{Either } x = 2 \text{ or } x = \frac{1}{2}$$

When  $x = \frac{1}{2}$ ,  $6x - 5 = -2 < 0$ , which is not possible.

Therefore,  $x = 2$

22. In  $\triangle ABC$ , it is given that  $\frac{AB}{AC} = \frac{BD}{DC}$ . If  $\angle B = 70^\circ$  and  $\angle C = 50^\circ$ , then  $\angle BAD = ?$   
 (a)  $30^\circ$       (b)  $40^\circ$       (c)  $45^\circ$       (d)  $50^\circ$

**Sol:**

(a)  $30^\circ$

We have:

$$\frac{AB}{AC} = \frac{BD}{DC}$$

Applying angle bisector theorem, we can conclude that AD bisects  $\angle A$ .

In  $\triangle ABC$ ,

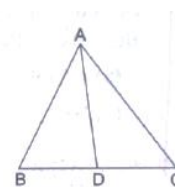
$$\angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow \angle A = 180 - \angle B - \angle C$$

$$\Rightarrow \angle A = 180 - 70 - 50 = 60^\circ$$

$$\therefore \angle BAD = \angle CAD = \frac{1}{2} \angle BAC$$

$$\therefore \angle BAD = \frac{1}{2} \times 60 = 30^\circ$$



23. In  $\triangle ABC$ ,  $DE \parallel BC$  so that  $AD = 2.4\text{cm}$ ,  $AE = 3.2\text{cm}$  and  $EC = 4.8\text{cm}$ . Then,  $AB = ?$   
 (a)  $3.6\text{cm}$       (b)  $6\text{cm}$       (c)  $6.4\text{cm}$       (d)  $7.2\text{cm}$

**Sol:**

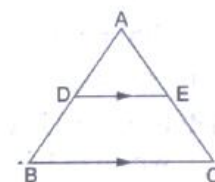
(b)  $6\text{ cm}$

It is given that  $DE \parallel BC$ .

Applying basic proportionality theorem, we have:

$$\frac{AD}{BD} = \frac{AE}{EC}$$

$$\Rightarrow \frac{2.4}{BD} = \frac{3.2}{4.8}$$



$$\Rightarrow BD = \frac{2.4 \times 4.8}{3.2} = 3.6 \text{ cm}$$

Therefore,  $AB = AD + BD = 2.4 + 3.6 = 6 \text{ cm}$

24. In a  $\triangle ABC$ , if  $DE$  is drawn parallel to  $BC$ , cutting  $AB$  and  $AC$  at  $D$  and  $E$  respectively such that  $AB = 7.2 \text{ cm}$ ,  $AC = 6.4 \text{ cm}$  and  $AD = 4.5 \text{ cm}$ . Then,  $AE = ?$

(a) 5.4 cm      (b) 4 cm      (c) 3.6 cm      (d) 3.2 cm

**Sol:**

(b) 4 cm

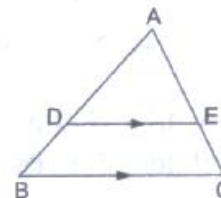
It is given that  $DE \parallel BC$ .

Applying basic proportionality theorem, we get:

$$\frac{AD}{AB} = \frac{AE}{AC}$$

$$\Rightarrow \frac{4.5}{7.2} = \frac{AE}{6.4}$$

$$\Rightarrow AE = \frac{4.5 \times 6.4}{7.2} = 4 \text{ cm}$$



25. In  $\triangle ABC$ ,  $DE \parallel BC$  so that  $AD = (7x - 4) \text{ cm}$ ,  $AE = (5x - 2) \text{ cm}$ ,  $DB = (3x + 4) \text{ cm}$  and  $EC = 3x \text{ cm}$ . Then, we have:

(a)  $x = 3$       (b)  $x = 5$       (c)  $x = 4$       (d)  $x = 2.5$

**Sol:**

(c)  $x = 4$

It is given  $DE \parallel BC$ .

Applying Thales' theorem. We get:

$$\frac{AD}{BD} = \frac{AE}{EC}$$

$$\Rightarrow \frac{7x-4}{3x+4} = \frac{5x-2}{3x}$$

$$\Rightarrow 3x(7x - 4) = (5x - 2)(3x + 4)$$

$$\Rightarrow 21x^2 - 12x = 15x^2 + 20x - 6x - 8$$

$$\Rightarrow 21x^2 - 12x = 15x^2 + 14x - 8$$

$$\Rightarrow 6x^2 - 26x + 8 = 0$$

$$\Rightarrow 2(3x^2 - 13x + 4) = 0$$

$$\Rightarrow 3x^2 - 13x + 4 = 0$$

$$\Rightarrow 3x^2 - 12x - x + 4 = 0$$

$$\Rightarrow 3x(x - 4) - 1(x - 4) = 0$$

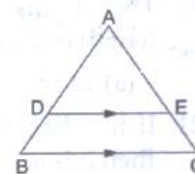
$$\Rightarrow (x - 4)(3x - 1) = 0$$

$$\Rightarrow x - 4 = 0 \text{ or } 3x - 1 = 0$$

$$\Rightarrow x - 4 \text{ or } x = \frac{1}{3}$$

If  $x = \frac{1}{3}$ ,  $7x - 4 = -\frac{5}{3} < 0$ ; it is not possible.

Therefore,  $x = 4$



26. In  $\triangle ABC$ ,  $DE \parallel BC$  such that  $\frac{AD}{DB} = \frac{3}{5}$ . If  $AC = 5.6\text{cm}$ , then  $AE = ?$

(a) 4.2cm      (b) 3.1cm      (c) 2.8cm      (d) 2.1cm

**Sol:**

(d) 2.1 cm

It is given that  $DE \parallel BC$ .

Applying Thales' theorem, we get:

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Let  $AE$  be  $x$  cm.

Therefore,  $EC = (5.6 - x)$  cm

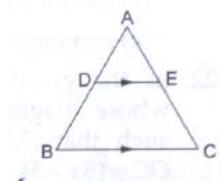
$$\Rightarrow \frac{3}{5} = \frac{x}{5.6 - x}$$

$$\Rightarrow 3(5.6 - x) = 5x$$

$$\Rightarrow 16.8 - 3x = 5x$$

$$\Rightarrow 8x = 16.8$$

$$\Rightarrow x = 2.1 \text{ cm}$$



27.  $\triangle ABC \sim \triangle DEF$  and the perimeters of  $\triangle ABC$  and  $\triangle DEF$  are 30cm and 18cm respectively. If  $BC = 9\text{cm}$ , then  $EF = ?$

(a) 6.3cm      (b) 5.4cm      (c) 7.2cm      (d) 4.5cm

**Sol:**

(b) 5.4 cm

$\triangle ABC \sim \triangle DEF$

Therefore,

$$\frac{\text{Perimeter}(\triangle ABC)}{\text{Perimeter}(\triangle DEF)} = \frac{BC}{EF}$$

$$\Rightarrow \frac{30}{18} = \frac{9}{EF}$$

$$\Rightarrow EF = \frac{9 \times 18}{30} = 5.4 \text{ cm}$$

28.  $\triangle ABC \sim \triangle DEF$  such that  $AB = 9.1\text{cm}$  and  $DE = 6.5\text{cm}$ . If the perimeter of  $\triangle DEF$  is 25cm, what is the perimeter of  $\triangle ABC$ ?

(a) 35cm      (b) 28cm      (c) 42cm      (d) 40cm

**Sol:**

(a) 35 cm

$\therefore \triangle ABC \sim \triangle DEF$

$$\therefore \frac{\text{Perimeter}(\triangle ABC)}{\text{Perimeter}(\triangle DEF)} = \frac{AB}{DE}$$

$$\Rightarrow \frac{\text{Perimeter}(\triangle ABC)}{25} = \frac{9.1}{6.5}$$

$$\Rightarrow \text{Perimeter}(\triangle ABC) = \frac{9.1 \times 25}{6.5} = 35 \text{ cm}$$

29. In  $\triangle ABC$ , it is given that  $AB = 9\text{cm}$ ,  $BC = 6\text{cm}$  and  $CA = 7.5\text{cm}$ . Also,  $\triangle DEF$  is given such that  $EF = 8\text{cm}$  and  $\triangle DEF \sim \triangle ABC$ . Then, perimeter of  $\triangle DEF$  is  
 (a) 22.5cm      (b) 25cm      (c) 27cm      (d) 30cm

**Sol:**

(d) 30 cm

Perimeter of  $\triangle ABC = AB + BC + CA = 9 + 6 + 7.5 = 22.5\text{ cm}$

$\therefore \triangle DEF \sim \triangle ABC$

$$\therefore \frac{\text{Perimeter}(\triangle ABC)}{\text{Perimeter}(\triangle DEF)} = \frac{BC}{EF}$$

$$\Rightarrow \frac{22.5}{\text{Perimeter}(\triangle DEF)} = \frac{6}{8}$$

$$\text{Perimeter}(\triangle DEF) = \frac{22.5 \times 8}{6} = 30\text{ cm}$$

30.  $ABC$  and  $BDE$  are two equilateral triangles such that  $D$  is the midpoint of  $BC$ . Ratio of these area of triangles  $ABC$  and  $BDE$  is  
 (a) 2 : 1      (b) 1 : 4      (c) 1 : 2      (d) 4 : 1

**Sol:**

Given:  $ABC$  and  $BDE$  are two equilateral triangles

Since,  $D$  is the midpoint of  $BC$  and  $BDE$  is also an equilateral triangle.

Hence,  $E$  is also the midpoint of  $AB$ .

Now,  $D$  and  $E$  are the midpoint of  $BC$  and  $AB$ .

In a triangle, the line segment that joins midpoint of the two sides of a triangle is parallel to the third side and is half of it.

$$DE \parallel CA \text{ and } DE = \frac{1}{2} CA$$

Now, in  $\triangle ABC$  and  $\triangle EBD$

$$\angle BED = \angle BAC \quad (\text{Corresponding angles})$$

$$\angle B = \angle B \quad (\text{Common})$$

By AA-similarity criterion

$$\triangle ABC \sim \triangle EBD$$

If two triangles are similar, then the ratio of their areas is equal to the ratio of the squares of their corresponding sides.

$$\therefore \frac{\text{area}(\triangle ABC)}{\text{area}(\triangle DBE)} = \left(\frac{AC}{ED}\right)^2 = \left(\frac{2ED}{ED}\right)^2 = \frac{4}{1}$$

Hence, the correct answer is option (d).

31. It is given that  $\triangle ABC \sim \triangle DEF$ . If  $\angle A = 30^\circ$ ,  $\angle C = 50^\circ$ ,  $AB = 5\text{cm}$ ,  $AC = 8\text{cm}$  and  $DF = 7.5\text{cm}$ , then which of the following is true?  
 (a)  $DE = 12\text{cm}$ ,  $\angle F = 50^\circ$       (b)  $DE = 12\text{cm}$ ,  $\angle F = 100^\circ$   
 (c)  $DE = 12\text{cm}$ ,  $\angle D = 100^\circ$       (d)  $EF = 12\text{cm}$ ,  $\angle D = 30^\circ$

**Sol:**

(b)  $DE = 12\text{ cm}$ ,  $\angle F = 100^\circ$

Disclaimer: In the question, it should be  $\triangle ABC \sim \triangle DFE$  instead of  $\triangle ABC \sim \triangle DEF$ .

In triangle ABC,

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\therefore \angle B = 180 - 30 - 50 = 100^\circ$$

$$\therefore \triangle ABC \sim \triangle DFE$$

$$\therefore \angle D = \angle A = 30^\circ$$

$$\angle F = \angle B = 100^\circ$$

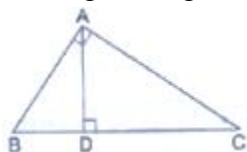
$$\text{And } \angle E = \angle C = 50^\circ$$

Also,

$$\frac{AB}{DF} = \frac{AC}{DE} \Rightarrow \frac{5}{7.5} = \frac{8}{DE}$$

$$\Rightarrow DE = \frac{8 \times 7.5}{5} = 12 \text{ cm}$$

32. In the given figure,  $\angle BAC = 90^\circ$  and  $AD \perp BC$ . Then,



(a)  $BC \cdot CD = BC^2$

(b)  $AB \cdot AC = BC^2$

(c)  $BD \cdot CD = AD^2$

(d)  $AB \cdot AC = AD^2$

**Sol:**

(c)  $BD \cdot CD = AD^2$

In  $\triangle BDA$  and  $\triangle ADC$ , we have:

$$\angle BDA = \angle ADC = 90^\circ$$

$$\angle ABD = 90^\circ - \angle DAB$$

$$= 90^\circ - (90^\circ - \angle DAC)$$

$$= 90^\circ - 90^\circ + \angle DAC$$

$$= \angle DAC$$

Applying AA similarity, we conclude that  $\triangle BDA \sim \triangle ADC$ .

$$\Rightarrow \frac{BD}{AD} = \frac{AD}{CD}$$

$$\Rightarrow AD^2 = BD \cdot CD$$

33. In  $\triangle ABC$ ,  $AB = 6\sqrt{3}$  cm,  $AC = 12$  cm and  $BC = 6$  cm. Then  $\angle B$  is

**Sol:**

$$AB = 6\sqrt{3} \text{ cm}$$

$$\Rightarrow AB^2 = 108 \text{ cm}^2$$

$$AC = 12 \text{ cm}$$

$$\Rightarrow AC^2 = 144 \text{ cm}^2$$

$$BC = 6 \text{ cm}$$

$$\Rightarrow BC^2 = 36 \text{ cm}$$

$$\therefore AC^2 = AB^2 + BC^2$$

Since, the square of the longest side is equal to the sum of two sides, so  $\triangle ABC$  is a right angled triangle.

$\therefore$  The angle opposite to  $\angle 90^\circ$

Hence, the correct answer is option (c)

34. In  $\triangle ABC$  and  $\triangle DEF$ , it is given that  $\frac{AB}{DE} = \frac{BC}{FD}$ , then

- (a)  $\angle B = \angle E$       (b)  $\angle A = \angle D$       (c)  $\angle B = \angle D$       (d)  $\angle A = \angle F$

**Sol:**

$$(c) \angle B = \angle D$$

Disclaimer: In the question, the ratio should be  $\frac{AB}{DE} = \frac{BC}{FD} = \frac{AC}{EF}$ .

We can write it as:

$$\frac{AB}{ED} = \frac{BC}{DF} = \frac{AC}{FE}$$

Therefore,  $\triangle ABC \sim \triangle EDF$

Hence, the corresponding angles, i.e.,  $\angle B$  and  $\angle D$ , will be equal.

$$i.e., \angle B = \angle D$$

35. In  $\triangle DEF$  and  $\triangle PQR$ , it is given that  $\angle D = \angle Q$  and  $\angle R = \angle E$ , then which of the following is not true?

- (a)  $\frac{EF}{PR} = \frac{DF}{PQ}$       (b)  $\frac{DE}{PQ} = \frac{EF}{RP}$       (c)  $\frac{DE}{QR} = \frac{DF}{PQ}$       (d)  $\frac{EF}{RP} = \frac{DE}{QR}$

**Sol:**

$$(b) \frac{DE}{PQ} = \frac{EF}{RP}$$

In  $\triangle DEF$  and  $\triangle PQR$ , we have:

$$\angle D = \angle Q \text{ and } \angle R = \angle E$$

Applying AA similarity theorem, we conclude that  $\triangle DEF \sim \triangle QRP$ .

$$\text{Hence, } \frac{DE}{QR} = \frac{DF}{QP} = \frac{EF}{PR}$$

36. If  $\triangle ABC \sim \triangle EDF$  and  $\triangle ABC$  is not similar to  $\triangle DEF$ , then which of the following is not true?

- (a)  $BC \cdot EF = AC \cdot FD$   
 (b)  $AB \cdot EF = AC \cdot DE$   
 (c)  $BC \cdot DE = AB \cdot EF$   
 (d)  $BC \cdot DE = AB \cdot FD$

**Sol:**

$$(c) BC \cdot DE = AB \cdot EF$$

$$\triangle ABC \sim \triangle EDF$$

Therefore,

$$\frac{AB}{DE} = \frac{AC}{EF} = \frac{BC}{DF}$$

$$\Rightarrow BC \cdot DE \neq AB \cdot EF$$

37. In  $\triangle ABC$  and  $\triangle DEF$ , it is given that  $\angle B = \angle E$ ,  $\angle F = \angle C$  and  $AB = 3DE$ , then the two triangles are

- (a) congruent but not similar (b) similar but not congruent  
(c) neither congruent nor similar (d) similar as well as congruent

**Sol:**

(b) similar but not congruent

In  $\triangle ABC$  and  $\triangle DEF$ , we have:

$$\angle B = \angle E \text{ and } \angle F = \angle C$$

Applying AA similarity theorem, we conclude that  $\triangle ABC \sim \triangle DEF$ .

Also,

$$AB = 3DE$$

$$\Rightarrow AB \neq DE$$

Therefore,  $\triangle ABC$  and  $\triangle DEF$  are not congruent.

38. If in  $\triangle ABC$  and  $\triangle PQR$ , we have:  $\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$ , then

- (a)  $\triangle PQR \sim \triangle CAB$  (b)  $\triangle PQR \sim \triangle ABC$   
(c)  $\triangle CBA \sim \triangle PQR$  (d)  $\triangle BCA \sim \triangle PQR$

**Sol:**

(a)  $\triangle PQR \sim \triangle CAB$

In  $\triangle ABC$  and  $\triangle PQR$ , we have:

$$\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$$

$$\Rightarrow \triangle ABC \sim \triangle QRP$$

We can also write it as  $\triangle PQR \sim \triangle CAB$ .

39. In the given figure, two line segment AC and BD intersect each other at the point P such that  $PA = 6\text{cm}$ ,  $PB = 3\text{cm}$ ,  $PC = 2.5\text{cm}$ ,  $PD = 5\text{cm}$ ,  $\angle APB = 50^\circ$  and  $\angle CDP = 30^\circ$ , then  $\angle PBA = ?$

- (a)  $50^\circ$  (b)  $30^\circ$   
(c)  $60^\circ$  (d)  $100^\circ$

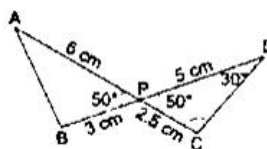
**Sol:**

(d)  $100^\circ$

In  $\triangle APB$  and  $\triangle DPC$ , we have:

$$\angle APB = \angle DPC = 50^\circ$$

$$\frac{AP}{BP} = \frac{6}{3} = 2$$



$$\frac{DP}{CP} = \frac{5}{2.5} = 2$$

$$\text{Hence, } \frac{AP}{BP} = \frac{DP}{CP}$$

Applying SAS theorem, we conclude that  $\triangle APB \sim \triangle DPC$ .

$$\therefore \angle PBA = \angle PCD$$

In  $\triangle DPC$ , we have:

$$\angle CDP + \angle CPD + \angle PCD = 180^\circ$$

$$\Rightarrow \angle PCD = 180^\circ - \angle CDP - \angle CPD$$

$$\Rightarrow \angle PCD = 180^\circ - 30^\circ - 50^\circ$$

$$\Rightarrow \angle PCD = 100^\circ$$

$$\text{Therefore, } \angle PBA = 100^\circ$$

40. Corresponding sides of two similar triangles are in the ratio 4:9 Areas of these triangles are in the ration

- (a) 2:3                      (b) 4:9                      (c) 9:4                      (d) 16:81

**Sol:**

If two triangles are similar, then the ratio of their areas is equal to the ratio of the squares of their corresponding sides.

$$\therefore \frac{\text{area of first triangle}}{\text{area of second triangle}} = \left( \frac{\text{Side of first triangle}}{\text{Side of second triangle}} \right)^2 = \left( \frac{4}{9} \right)^2 = \frac{16}{81}$$

Hence, the correct answer is option (d).

41. It is given that  $\triangle ABC \sim \triangle PQR$  and  $\frac{BC}{QR} = \frac{2}{3}$ , then  $\frac{\text{ar}(\triangle PQR)}{\text{ar}(\triangle ABC)} = ?$

- (a)  $\frac{2}{3}$                       (b)  $\frac{3}{2}$                       (c)  $\frac{4}{9}$                       (d)  $\frac{9}{4}$

**Sol:**

$$(d) 9:4$$

$$\text{It is given that } \triangle ABC \sim \triangle PQR \text{ and } \frac{BC}{QR} = \frac{2}{3}$$

Therefore,

$$\frac{\text{ar}(\triangle PQR)}{\text{ar}(\triangle ABC)} = \frac{QR^2}{BC^2} = \left( \frac{3}{2} \right)^2 = \frac{9}{4}$$

42. In an equilateral  $\triangle ABC$ , D is the midpoint of AB and E is the midpoint of AC. Then,  $\text{ar}(\triangle ABC) : \text{ar}(\triangle ADE) = ?$

- (a) 2 : 1                      (b) 4 : 1                      (c) 1 : 2                      (d) 1 : 4

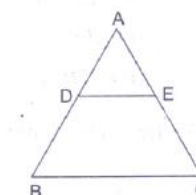
**Sol:**

$$(b) 4:1$$

In  $\triangle ABC$ , D is the midpoint of AB and E is the midpoint of AC.

Therefore, by midpoint theorem,

Also, by Basic Proportionality Theorem,





$$\frac{AD}{DB} = \frac{AE}{EC}$$

Also,

$AB = AC = BC$  ( $\because \triangle ABC$  is an equilateral triangle)

$$\text{So, } \frac{AD}{DB} = \frac{AE}{EC} = 1$$

In  $\triangle ABC$  and  $\triangle ADE$ , we have:

$$\angle A = \angle A$$

$$\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{2}$$

$\therefore \triangle ABC \sim \triangle ADE$  (SAS criterion)

$$\therefore ar(\triangle ABC) : ar(\triangle ADE) = (AB)^2 : (AD)^2$$

$$\Rightarrow ar(\triangle ABC) : ar(\triangle ADE) = 2^2 : 1^2$$

$$\Rightarrow ar(\triangle ABC) : ar(\triangle ADE) = 4 : 1$$

43. In  $\triangle ABC$  and  $\triangle DEF$ , we have:  $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{5}{7}$ , then  $ar(\triangle ABC) : ar(\triangle DEF) = ?$

(a) 5 : 7                      (b) 25 : 49                      (c) 49 : 25                      (d) 125 : 343

**Sol:**

(b) 25 : 49

In  $\triangle ABC$  and  $\triangle DEF$ , we have :

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{5}{7}$$

Therefore, by SSS criterion, we conclude that  $\triangle ABC \sim \triangle DEF$ .

$$\Rightarrow \frac{ar(\triangle ABC)}{ar(\triangle DEF)} = \frac{AB^2}{DE^2} = \left(\frac{5}{7}\right)^2 = \frac{25}{49} = 25 : 49$$

44.  $\triangle ABC \sim \triangle DEF$  such that  $ar(\triangle ABC) = 36\text{cm}^2$  and  $ar(\triangle DEF) = 49\text{cm}^2$ . Then, the ratio of their corresponding sides is

(a) 36 : 49                      (b) 6 : 7                      (c) 7 : 6                      (d)  $\sqrt{6} : \sqrt{7}$

**Sol:**

(b) 6 : 7

$\because \triangle ABC \sim \triangle DEF$

$$\therefore \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} \dots (i)$$

Also,

$$\frac{ar(\triangle ABC)}{ar(\triangle DEF)} = \frac{AB^2}{DE^2}$$

$$\Rightarrow \frac{36}{49} = \frac{AB^2}{DE^2}$$

$$\Rightarrow \frac{6}{7} = \frac{AB}{DE}$$

$$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{6}{7} \text{ (from (i))}$$

Thus, the ratio of corresponding sides is 6 : 7.



$$\therefore \frac{AB}{QR} = \frac{BC}{PR}$$

Now,

$$\frac{ar(\triangle ABC)}{ar(\triangle QRP)} = \frac{9}{4}$$

$$\Rightarrow \left(\frac{AB}{QR}\right)^2 = \frac{9}{4}$$

$$\Rightarrow \frac{AB}{QR} = \frac{BC}{PR} = \frac{3}{2}$$

Hence,  $3PR = 2BC = 2 \times 15 = 30$

$PR = 10$  cm

48. In the given figure, O is the point of intersection of two chords AB and CD such that  $OB = OD$  and  $\angle AOC = 45^\circ$ . Then,  $\triangle OAC$  and  $\triangle ODB$  are

- (a) equilateral and similar
- (b) equilateral but not similar
- (c) isosceles and similar
- (d) isosceles but not similar

**Sol:**

- (c) isosceles and similar

In  $\triangle AOC$  and  $\triangle ODB$ , we have:

$\angle AOC = \angle DOB$  (Vertically opposite angles)

and  $\angle OAC = \angle ODB$  (Angles in the same segment)

Therefore, by AA similarity theorem, we conclude that  $\triangle AOC \sim \triangle DOB$ .

$$\Rightarrow \frac{OC}{OB} = \frac{OA}{OD} = \frac{AC}{BD}$$

Now,  $OB = OD$

$$\Rightarrow \frac{OC}{OA} = \frac{OB}{OD} = 1$$

$$\Rightarrow OC = OA$$

Hence,  $\triangle OAC$  and  $\triangle ODB$  are isosceles and similar.

49. In an isosceles  $\triangle ABC$ , if  $AC = BC$  and  $AB^2 = 2AC^2$ , then  $\angle C = ?$

- (a)  $30^\circ$
- (b)  $45^\circ$
- (c)  $60^\circ$
- (d)  $90^\circ$

**Sol:**

- (d)  $90^\circ$

Given:

$$AC = BC$$

$$AB^2 = 2AC^2 = AC^2 + AC^2 = AC^2 + BC^2$$

Applying Pythagoras theorem, we conclude that  $\triangle ABC$  is right angled at C.

Or,  $\angle C = 90^\circ$

50. In  $\triangle ABC$ , if  $AB = 16\text{cm}$ ,  $BC = 12\text{cm}$  and  $AC = 20\text{cm}$ , then  $\triangle ABC$  is

- (a) acute-angled                      (b) right-angled                      (c) obtuse-angled

**Sol:**

(b) right-angled

We have:

$$AB^2 + BC^2 = 16^2 + 12^2 = 256 + 144 = 400$$

$$\text{and, } AC^2 = 20^2 = 400$$

$$\therefore AB^2 + BC^2 = AC^2$$

Hence,  $\triangle ABC$  is a right-angled triangle.

51. Which of the following is a true statement?

- (a) Two similar triangles are always congruent  
(b) Two figures are similar if they have the same shape and size.  
(c) Two triangles are similar if their corresponding sides are proportional.  
(d) Two polygons are similar if their corresponding sides are proportional.

**Sol:**

(c) Two triangles are similar if their corresponding sides are proportional.

According to the statement:

$$\triangle ABC \sim \triangle DEF$$

$$\text{if } \frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$$

52. Which of the following is a false statement?

- (a) If the areas of two similar triangles are equal, then the triangles are congruent.  
(b) The ratio of the areas of two similar triangles is equal to the ratio of their corresponding sides.  
(c) The ratio of the areas of two similar triangles is equal to the ratio of squares of their corresponding.  
(d) The ratio of the areas of two similar triangles is equal to the ratio of squares of their corresponding altitudes.

**Sol:**

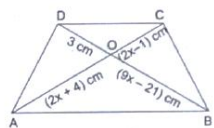
(b) The ratio of the areas of two similar triangles is equal to the ratio of their corresponding sides.

Because the ratio of the areas of two similar triangles is equal to the ratio of squares of their corresponding sides.

---

53. Match the following columns:

Column I	Column II
(a) In a given $\triangle ABC$ , $DE \parallel BC$ and $\frac{AD}{DB} = \frac{3}{5}$ . If $AC = 5.6\text{cm}$ , then $AE = \dots\dots\dots\text{cm}$ .	(p) 6
(b) If $\triangle ABC \sim \triangle DEF$ such that $2AB = 3DE$ and $BC = 6\text{cm}$ , then $EF = \dots\dots\dots\text{cm}$ .	(q) 4
(c) If $\triangle ABC \sim \triangle PQR$ such that $\text{ar}(\triangle ABC) : \text{ar}(\triangle PQR) = 9 : 16$ and $BC = 4.5\text{cm}$ , then $QR = \dots\dots\dots\text{cm}$ .	(r) 3
(d) In the given figure, $AB \parallel CD$ and $OA = (2x + 4)\text{cm}$ , $OB = (9x - 21)\text{cm}$ , $OC = (2x - 1)\text{cm}$ and $OD = 3\text{cm}$ . Then $x = ?$	(s) 2.1



The correct answer is:

(a) - ....., (b)-....., (c)-....., (d)-.....,

**Sol:**

(a) –(s)

Let  $AE$  be  $X$ .

Therefore,  $EC = 5.6 - X$

It is given that  $DE \parallel BC$ .

Therefore, by B.P.T., we get:

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{3}{5} = \frac{x}{5.6 - x}$$

$$\Rightarrow 3(5.6 - x) = 5x$$

$$\Rightarrow 16.8 - 3x = 5x$$

$$\Rightarrow 8x = 16.8$$

$$\Rightarrow x = 2.1 \text{ cm}$$

(b) –(q)

$\therefore \triangle ABC \sim \triangle DEF$

$$\therefore \frac{AB}{DE} = \frac{BC}{EF}$$

$$\Rightarrow \frac{3}{2} = \frac{6}{EF}$$

$$EF = \frac{6 \times 2}{3} = 4 \text{ cm}$$

(c) –(p)

$$\therefore \triangle ABC \sim \triangle PQR$$

$$\therefore \frac{ar(\triangle ABC)}{ar(\triangle PQR)} = \frac{BC^2}{QR^2}$$

$$\Rightarrow \frac{9}{16} = \frac{4.5^2}{QR^2} \Rightarrow QR = \sqrt{\frac{4.5 \times 4.5 \times 16}{9}} = \frac{4.5 \times 4}{3} = 6 \text{ cm}$$

(d) –(r)

$$\therefore AB \parallel CD$$

$$\therefore \frac{OA}{OB} = \frac{OC}{OD} \text{ (Thales' theorem)}$$

$$\Rightarrow \frac{2x+4}{9x-21} = \frac{2x-1}{3}$$

$$3(2x+4) = (2x-1)(9x-21)$$

$$\Rightarrow 6x + 12 = 18x^2 - 42x - 9x + 21$$

$$\Rightarrow 18x^2 - 57x + 9 = 0$$

$$\Rightarrow 6x^2 - 19x + 3 = 0$$

$$\Rightarrow 6x^2 - 18x - x + 3 = 0$$

$$\Rightarrow (6x - 1)(x - 3) = 0$$

$$\Rightarrow x = 3 \text{ or } x = -\frac{1}{6}$$

But  $x = -\frac{1}{6}$  makes  $(2x - 1) < 0$ , which is not possible.

Therefore,  $x = 3$

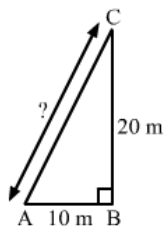
54. Match the following columns:

Column I	Column II
(a) A man goes 10m due east and then 20m due north. His distance from the starting point is .....m.	(p) $25\sqrt{3}$
(b) In an equilateral triangle with each side 10cm, the altitude is .....cm.	(q) $5\sqrt{3}$
(c) The area of an equilateral triangle having each side 10cm is .....cm <sup>2</sup> .	(r) $10\sqrt{5}$
(d) The length of a diagonal of a rectangle having length 8m and breadth 6m is .....m.	(s) 10

The correct answer is:

(a) - ....., (b)-....., (c)-....., (d)-.....,

**Sol:**



(a) – (r)

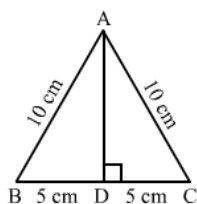
Let the man starts from A and goes 10 m due east at B and then 20 m due north at C.

Then, in right-angled triangle ABC, we have:

$$AB^2 + BC^2 = AC^2$$

$$\Rightarrow AC = \sqrt{10^2 + 20^2} = \sqrt{100 + 200} = 10\sqrt{3}$$

Hence, the man is  $10\sqrt{3}m$  away from the starting point.



(b) – (q)

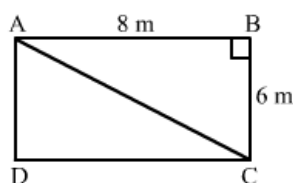
Let the triangle be ABC with altitude AD.

In right-angled triangle ABC, we have:

$$AB^2 = AD^2 + BD^2$$

$$\Rightarrow AD^2 = 10^2 - 5^2 \left( \because BD = \frac{1}{2} BC \right)$$

$$\Rightarrow AD = \sqrt{100 - 25} = \sqrt{75} = 5\sqrt{3} \text{ cm}$$



(c) – (p)

$$\begin{aligned} \text{Area of an equilateral triangle with side } a &= \frac{\sqrt{3}}{4} a^2 = \frac{\sqrt{3}}{4} \times 10^2 = \sqrt{3} \times 5 \times 5 \\ &= 25\sqrt{3} \text{ cm}^2 \end{aligned}$$

(d) – (s)

Let the rectangle be ABCD with diagonals AC and BD.

In right-angled triangle ABC, we have:

$$AC^2 = AB^2 + BC^2 = 8^2 + 6^2 = 64 + 36$$

$$\Rightarrow AC = \sqrt{100} = 10 \text{ m}$$

### Exercise – Formative Assessment

1.  $\triangle ABC \sim \triangle DEF$  and the perimeters of  $\triangle ABC$  and  $\triangle DEF$  are 32cm and 24cm respectively. If  $AB = 10$ cm, then  $DE = ?$

(a) 8cm                      (b) 7.5cm                      (c) 15cm                      (d)  $5\sqrt{3}$ cm

**Sol:**

(b) 7.5 cm

$\therefore \triangle ABC \sim \triangle DEF$

$$\therefore \frac{\text{Perimeter}(\triangle ABC)}{\text{Perimeter}(\triangle DEF)} = \frac{AB}{DE}$$

$$\Rightarrow \frac{32}{24} = \frac{10}{DE}$$

$$\Rightarrow DE = \frac{10 \times 24}{32} = 7.5 \text{ cm}$$

2. In  $\triangle ABC$ ,  $DE \parallel BC$ . If  $DE = 5$ cm,  $BC = 8$ cm and  $AD = 3.5$ cm, then  $AB = ?$

(a) 5.6cm                      (b) 4.8cm                      (c) 5.2cm                      (d) 6.4cm

**Sol:**

(a) 5.6 cm

$\therefore DE \parallel BC$

$$\therefore \frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC} \quad (\text{Thales' theorem})$$

$$\Rightarrow \frac{3.5}{AB} = \frac{5}{8}$$

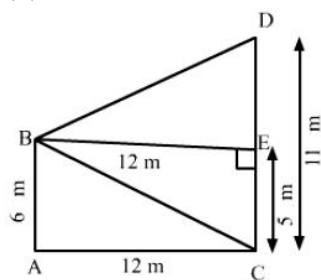
$$\Rightarrow AB = \frac{3.5 \times 8}{5} = 5.6 \text{ cm}$$

3. Two poles of heights 6m and 11m stand vertically on a plane ground. If the distance between their feet is 12m, find the distance between their tops.

(a) 12m                      (b) 13m                      (c) 14m                      (d) 15m

**Sol:**

(b) 13 m



Let the poles be AB and CD

It is given that:

$AB = 6$  m and  $CD = 11$  m

Let  $AC$  be 12 m

Draw a perpendicular from B to CD, meeting CD at E



Then,

$$BE = 12 \text{ m}$$

We have to find BD.

Applying Pythagoras theorem in right-angled triangle BED, we have:

$$BD^2 = BE^2 + ED^2$$

$$= 12^2 + 5^2 \quad (\because ED = CD - CE = 11 - 6)$$

$$= 144 + 25 = 169$$

$$BD = 13 \text{ m}$$

4. The areas of two similar triangles are  $25\text{cm}^2$  and  $36\text{cm}^2$  respectively. If the altitude of the first triangle is 3.5cm, then the corresponding altitude of the other triangle.

(a) 5.6cm              (b) 6.3cm              (c) 4.2cm              (d) 7cm

**Sol:**

(c)

We know that the ratio of areas of similar triangles is equal to the ratio of squares of their corresponding altitudes.

Let  $h$  be the altitude of the other triangle.

Therefore,

$$\frac{25}{36} = \frac{(3.5)^2}{h^2}$$

$$\Rightarrow h^2 = \frac{(3.5)^2 \times 36}{25}$$

$$\Rightarrow h^2 = 17.64$$

$$\Rightarrow h = 4.2 \text{ cm}$$

5. If  $\triangle ABC \sim \triangle DEF$  such that  $2AB = DE$  and  $BC = 6\text{cm}$ , find  $EF$ .

**Sol:**

$$\because \triangle ABC \sim \triangle DEF$$

$$\therefore \frac{AB}{DE} = \frac{BC}{EF}$$

$$\Rightarrow \frac{1}{2} = \frac{6}{EF}$$

$$\Rightarrow EF = 12 \text{ cm}$$

6. In the given figure,  $DE \parallel BC$  such that  $AD = x \text{ cm}$ ,  $DB = (3x + 4) \text{ cm}$ ,  $AE = (x + 3) \text{ cm}$  and  $EC = (3x + 19) \text{ cm}$ . Find the value of  $x$ .

**Sol:**

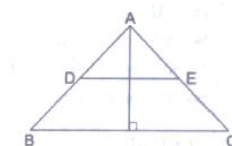
$$\because DE \parallel BC$$

$$\therefore \frac{AD}{DB} = \frac{AE}{EC} \quad (\text{Basic proportionality theorem})$$

$$\frac{x}{3x+4} = \frac{x+3}{3x+19}$$

$$\Rightarrow x(3x + 19) = (x + 3)(3x + 4)$$

$$\Rightarrow 3x^2 + 19x = 3x^2 + 4x + 9x + 12$$



$$\Rightarrow 19x - 13x = 12$$

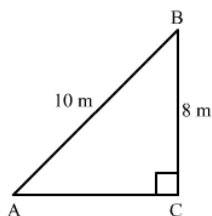
$$\Rightarrow 6x = 12$$

$$\Rightarrow x = 2$$

7. A ladder 10m long reaches the window of a house 8m above the ground. Find the distance of the foot of the ladder from the base of the wall.

**Sol:**

Let the ladder be AB and BC be the height of the window from the ground.



We have:

AB 10 m and BC = 8 m

Applying theorem in right-angled triangle ACB, we have:

$$AB^2 = AC^2 + BC^2$$

$$\Rightarrow AC^2 = AB^2 - BC^2 = 10^2 - 8^2 = 100 - 64 = 36$$

$$\Rightarrow AC = 6 \text{ m}$$

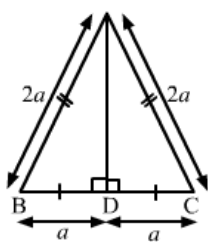
Hence, the ladder is 6 m away from the base of the wall.

8. Find the length of the altitude of an equilateral triangle of side 2a cm.

**Sol:**

Let the triangle be ABC with AD as its altitude. Then, D is the midpoint of BC.

In right-angled triangle ABD, we have:



$$AB^2 = AD^2 + DB^2$$

$$\Rightarrow AD^2 = AB^2 - DB^2 = 4a^2 - a^2 \quad \left( \because BD = \frac{1}{2} BC \right)$$

$$= 3a^2$$

$$AD = \sqrt{3}a$$

Hence, the length of the altitude of an equilateral triangle of side 2a cm is  $\sqrt{3}a$  cm.

9.  $\triangle ABC \sim \triangle DEF$  such that  $\text{ar}(\triangle ABC) = 64\text{cm}^2$  and  $\text{ar}(\triangle DEF) = 169\text{cm}^2$ . If  $BC = 4\text{cm}$ , find EF.

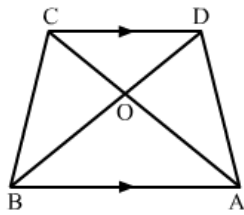
**Sol:**

$$\begin{aligned}
 &\because \triangle ABC \sim \triangle DEF \\
 &\therefore \frac{ar(\triangle ABC)}{ar(\triangle DEF)} = \frac{BC^2}{EF^2} \\
 &\Rightarrow \frac{64}{169} = \frac{4^2}{EF^2} \\
 &\Rightarrow EF^2 = \frac{16 \times 169}{64} \\
 &\Rightarrow EF = \frac{4 \times 13}{8} = 6.5 \text{ cm}
 \end{aligned}$$

10. In a trapezium ABCD, it is given that  $AB \parallel CD$  and  $AB = 2CD$ . Its diagonals AC and BD intersect at the point O such that  $ar(\triangle AOB) = 84 \text{ cm}^2$ . Find  $ar(\triangle COD)$ .

**Sol:**

In  $\triangle AOB$  and  $\triangle COD$ , we have:



$\angle AOB = \angle COD$  (Vertically opposite angles)  
 $\angle OAB = \angle OCD$  (Alternate angles as  $AB \parallel CD$ )

Applying AA similarity criterion, we get :

$\triangle AOB \sim \triangle COD$

$$\begin{aligned}
 &\therefore \frac{ar(\triangle AOB)}{ar(\triangle COD)} = \frac{AB^2}{CD^2} \\
 &\Rightarrow \frac{84}{ar(\triangle COD)} = \left(\frac{AB}{CD}\right)^2 \\
 &\Rightarrow \frac{84}{ar(\triangle COD)} = \left(\frac{2CD}{CD}\right)^2 \\
 &\Rightarrow ar(\triangle COD) = \frac{84}{4} = 21 \text{ cm}^2
 \end{aligned}$$

11. The corresponding sides of two similar triangles are in the ratio 2: 3. If the area of the smaller triangle is  $48 \text{ cm}^2$ , find the area of the larger triangle.

**Sol:**

It is given that the triangles are similar.

Therefore, the ratio of areas of similar triangles will be equal to the ratio of squares of their corresponding sides.

$$\begin{aligned}
 &\therefore \frac{48}{\text{Area of larger triangle}} = \frac{2^2}{3^2} \\
 &\Rightarrow \frac{48}{\text{Area of larger triangle}} = \frac{4}{9} \\
 &\Rightarrow \text{Area of larger triangle} = \frac{48 \times 9}{4} = 108 \text{ cm}^2
 \end{aligned}$$

12. In the given figure,  $LM \parallel CB$  and  $LN \parallel CD$ . Prove that  $\frac{AM}{AB} = \frac{AN}{AD}$ .

**Sol:**

$LM \parallel CB$  and  $LN \parallel CD$

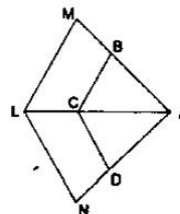
Therefore, applying Thales' theorem, we have:

$$\frac{AB}{AM} = \frac{AC}{AL} \text{ and } \frac{AD}{AN} = \frac{AC}{AL}$$

$$\Rightarrow \frac{AB}{AM} = \frac{AD}{AN}$$

$$\therefore \frac{AM}{AB} = \frac{AN}{AD}$$

This completes the proof.



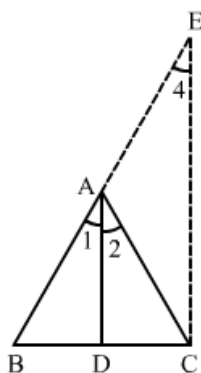
13. Prove that the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

**Sol:**

Let the triangle be  $ABC$  with  $AD$  as the bisector of  $\angle A$  which meets  $BC$  at  $D$ .

We have to prove:

$$\frac{BD}{DC} = \frac{AB}{AC}$$



Draw  $CE \parallel DA$ , meeting  $BA$  produced at  $E$ .

$CE \parallel DA$

Therefore,

$$\angle 2 = \angle 3 \quad (\text{Alternate angles})$$

$$\text{and } \angle 1 = \angle 4 \quad (\text{Corresponding angles})$$

But,

$$\angle 1 = \angle 2$$

Therefore,

$$\angle 3 = \angle 4$$

$$\Rightarrow AE = AC$$

In  $\triangle BCE$ ,  $DA \parallel CE$ .

Applying Thales' theorem, we gave:

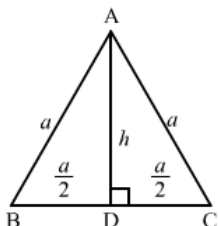
$$\frac{BD}{DC} = \frac{AB}{AE}$$

$$\Rightarrow \frac{BD}{DC} = \frac{AB}{AC}$$

This completes the proof.

14. In an equilateral triangle with side  $a$ , prove that  $\text{area} = \frac{\sqrt{3}}{4} a^2$ .

**Sol:**



Let ABC be the equilateral triangle with each side equal to  $a$ .

Let AD be the altitude from A, meeting BC at D.

Therefore, D is the midpoint of BC.

Let AD be  $h$ .

Applying Pythagoras theorem in right-angled ABD, we have:

$$AB^2 = AD^2 + BD^2$$

$$\Rightarrow a^2 = h^2 + \left(\frac{a}{2}\right)^2$$

$$\Rightarrow h^2 = a^2 - \frac{a^2}{4} = \frac{3}{4} a^2$$

$$\Rightarrow h = \frac{\sqrt{3}}{2} a$$

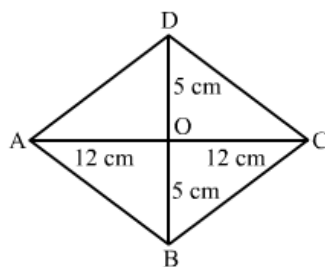
Therefore,

$$\text{Area of triangle ABC} = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times a \times \frac{\sqrt{3}}{2} a$$

This completes the proof.

15. Find the length of each side of a rhombus whose diagonals are 24cm and 10cm long.

**Sol:**



Let ABCD be the rhombus with diagonals AC and BD intersecting each other at O.

We know that the diagonals of a rhombus bisect each other at right angles.

$\therefore$  If AC = 24 cm and BD = 10 cm, AO = 12 cm and BO = 5 cm

Applying Pythagoras theorem in right-angled triangle AOB, we get:

$$AB^2 = AO^2 + BO^2 = 12^2 + 5^2 = 144 + 25 = 169$$

$$AB = 13 \text{ cm}$$

Hence, the length of each side of the given rhombus is 13 cm.

16. Prove that the ratio of the perimeters of two similar triangles is the same as the ratio of their corresponding sides.

**Sol:**

Let the two triangles be ABC and PQR.

We have:

$$\triangle ABC \sim \triangle PQR,$$

Here,

$$BC = a, AC = b \text{ and } AB = c$$

$$PQ = r, PR = q \text{ and } QR = p$$

We have to prove:

$$\frac{a}{p} = \frac{b}{q} = \frac{c}{r} = \frac{a+b+c}{p+q+r}$$

$\triangle ABC \sim \triangle PQR$ ; therefore, their corresponding sides will be proportional.

$$\Rightarrow \frac{a}{p} = \frac{b}{q} = \frac{c}{r} = k \quad (\text{say}) \dots (i)$$

$$\Rightarrow a = kp, b = kq \text{ and } c = kr$$

$$\therefore \frac{\text{Perimeter of } \triangle ABC}{\text{Perimeter of } \triangle PQR} = \frac{a+b+c}{p+q+r} = \frac{kp+kq+kr}{p+q+r} = k \dots (ii)$$

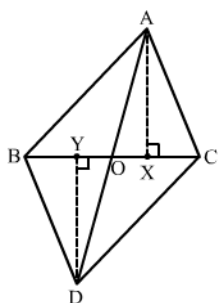
From (i) and (ii), we get:

$$\frac{a}{p} = \frac{b}{q} = \frac{c}{r} = \frac{a+b+c}{p+q+r} = \frac{\text{Perimeter of } \triangle ABC}{\text{Perimeter of } \triangle PQR}$$

This completes the proof.

17. In the given figure,  $\triangle ABC$  and  $\triangle DBC$  have the same base BC. If AD and BC intersect at O, prove that  $\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DBC)} = \frac{AO}{DO}$ .

**Sol:**



**Construction :** Draw  $AX \perp CO$  and  $DY \perp BO$ .

As,

$$\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DBC)} = \frac{\frac{1}{2} \times AX \times BC}{\frac{1}{2} \times DY \times BC}$$

$$\Rightarrow \frac{ar(\triangle ABC)}{ar(\triangle DBC)} = \frac{AX}{DY} \dots (i)$$

In  $\triangle ABC$  and  $\triangle DBC$ ,  $\angle AXY = \angle DYO = 90^\circ$  (BY constructin)  $\angle AOX = \angle DOY$  (Vertically opposite angles)  $\therefore \triangle AXO \sim \triangle DYO$  (BY AA criterion)  $\therefore \frac{AX}{DY}$

$$= \frac{AO}{DO} \text{ (Thales's theorem) } \dots (ii) \text{ From (i) and (ii), we have : } \frac{ar(\triangle ABC)}{ar(\triangle DBC)}$$

$$= \frac{AX}{DY} = \frac{AO}{DO} \text{ or, } \frac{ar(\triangle ABC)}{ar(\triangle DBC)} = \frac{AO}{DO}$$

This completes the proof.

18. In the given figure,  $XY \parallel AC$  and  $XY$  divides  $\triangle ABC$  into two regions, equal in area. Show that  $\frac{AX}{AB} = \frac{(2-\sqrt{2})}{2}$ .

**Sol:**

In  $\triangle ABC$  and  $\triangle BXY$ , we have:

$$\angle B = \angle B$$

$$\angle BXY = \angle BAC \text{ (Corresponding angles)}$$

Thus,  $\triangle ABC \sim \triangle BXY$  (AA criterion)

$$\therefore \frac{ar(\triangle ABC)}{ar(\triangle BXY)} = \frac{AB^2}{BX^2} = \frac{AB^2}{(AB-AX)^2} \dots (i)$$

$$\text{Also, } \frac{ar(\triangle ABC)}{ar(\triangle BXY)} = \frac{2}{1} \{ \because ar(\triangle BXY) = ar(\text{trapezium } AXYV) \} \dots (ii)$$

From (i) and (ii), we have:

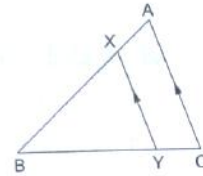
$$\frac{AB^2}{(AB-AX)^2} = \frac{2}{1}$$

$$\Rightarrow \frac{AB}{(AB-AX)} = \sqrt{2}$$

$$\Rightarrow \frac{(AB-AX)}{AB} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow 1 - \frac{AX}{AB} = \frac{1}{\sqrt{2}}$$

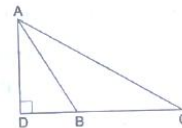
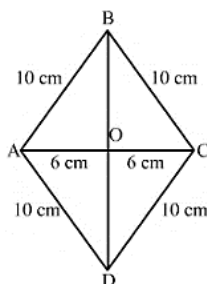
$$\Rightarrow \frac{AX}{AB} = 1 - \frac{1}{\sqrt{2}} = \frac{\sqrt{2}-1}{\sqrt{2}} = \frac{(2-\sqrt{2})}{2}$$



19. In the given figure,  $\triangle ABC$  is an obtuse triangle, obtuse-angled at B. If  $AD \perp CB$ , prove that  $AC^2 = AB^2 + BC^2 + 2BC \cdot BD$ .

**Sol:**

Applying Pythagoras theorem in right-angled triangle ADC, we get:



$$\begin{aligned}
 AC^2 &= AD^2 + DC^2 \\
 \Rightarrow AC^2 - DC^2 &= AD^2 \\
 \Rightarrow AD^2 &= AC^2 - DB^2 \quad \dots(1)
 \end{aligned}$$

Applying Pythagoras theorem in right-angled triangle ADB, we get:

$$\begin{aligned}
 AB^2 &= AD^2 + DB^2 \\
 \Rightarrow AB^2 - DB^2 &= AD^2 \\
 \Rightarrow AD^2 &= AB^2 - DB^2 \quad \dots(2)
 \end{aligned}$$

From equation (1) and (2), we have:

$$\begin{aligned}
 AC^2 - DC^2 &= AB^2 - DB^2 \\
 \Rightarrow AC^2 &= AB^2 + DC^2 - DB^2 \\
 \Rightarrow AC^2 &= AB^2 + (DB + BC)^2 - DB^2 \quad (\because DB + BC = DC) \\
 \Rightarrow AC^2 &= AB^2 + DB^2 + BC^2 + 2DB \cdot BC - DB^2 \\
 \Rightarrow AC^2 &= AB^2 + BC^2 + 2BC \cdot BD
 \end{aligned}$$

This completes the proof.

20. In the given figure, each one of PA, QB and RC is perpendicular to AC. If  $AP = x$ ,  $QB = z$ ,  $RC = y$ ,  $AB = a$  and  $BC = b$ , show that  $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$ .

**Sol:**

In  $\triangle PAC$  and  $\triangle QBC$ , we have:

$$\begin{aligned}
 \angle A &= \angle B \quad (\text{Both angles are } 90^\circ) \\
 \angle P &= \angle Q \quad (\text{Corresponding angles})
 \end{aligned}$$

And

$$\angle C = \angle C \quad (\text{common angles})$$

Therefore,  $\triangle PAC \sim \triangle QBC$

$$\begin{aligned}
 \frac{AP}{BQ} &= \frac{AC}{BC} \\
 \Rightarrow \frac{x}{z} &= \frac{a+b}{b}
 \end{aligned}$$

$$\Rightarrow a + b = \frac{ay}{z} \quad \dots (1)$$

In  $\triangle RCA$  and  $\triangle QBA$ , we have:

$$\begin{aligned}
 \angle C &= \angle B \quad (\text{Both angles are } 90^\circ) \\
 \angle R &= \angle Q \quad (\text{Corresponding angles})
 \end{aligned}$$

And

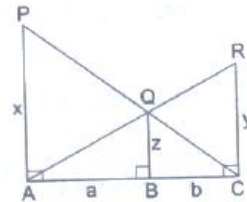
$$\angle A = \angle A \quad (\text{common angles})$$

Therefore,  $\triangle RCA \sim \triangle QBA$

$$\begin{aligned}
 \frac{RC}{BQ} &= \frac{AC}{AB} \\
 \Rightarrow \frac{y}{z} &= \frac{a+b}{a}
 \end{aligned}$$

$$\Rightarrow a + b = \frac{ay}{z} \quad \dots (2)$$

From equation (1) and (2), we have:





$$\frac{bx}{z} = \frac{ay}{z}$$

$$\Rightarrow bx = ay$$

$$\Rightarrow \frac{a}{b} = \frac{x}{y} \quad \dots (3)$$

Also,

$$\frac{x}{z} = \frac{a+b}{b}$$

$$\Rightarrow \frac{x}{z} = \frac{a}{b} + 1$$

Using the value of  $\frac{a}{b}$  from equation (3), we have:

$$\Rightarrow \frac{x}{z} = \frac{x}{y} + 1$$

Dividing both sides by  $x$ , we get:

$$\frac{1}{z} = \frac{1}{y} + \frac{1}{x}$$

$$\therefore \frac{1}{x} + \frac{1}{y} = \frac{1}{z}$$

This completes the proof.