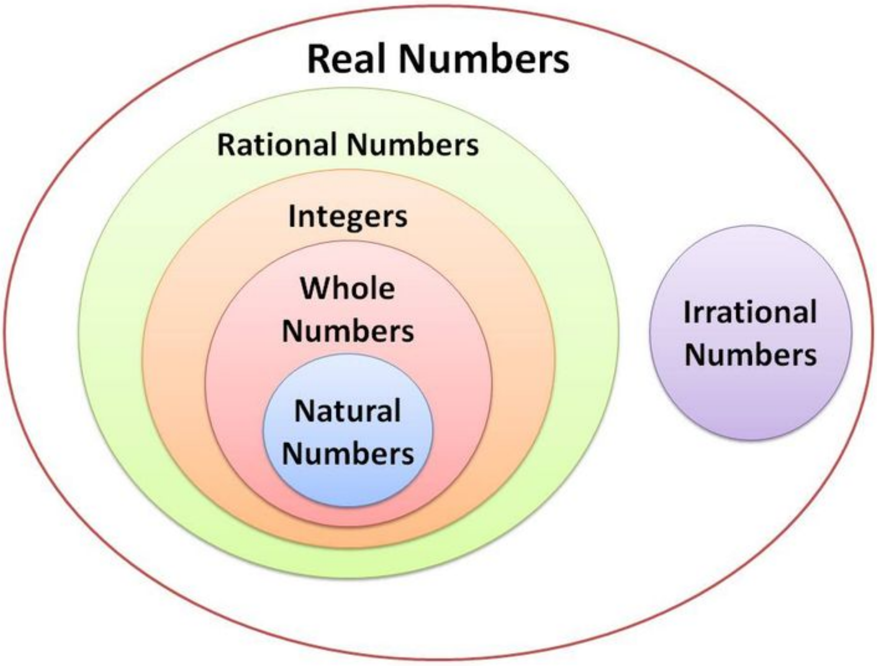
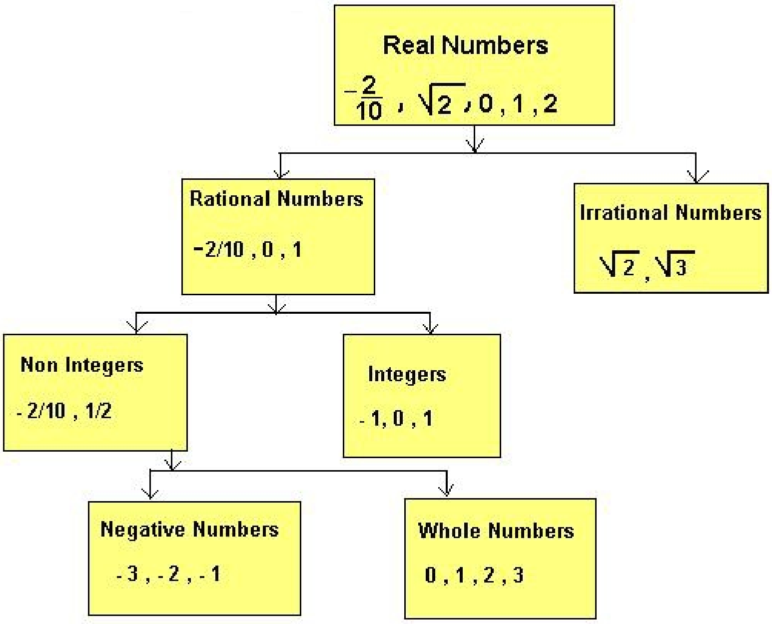
Real Numbers





Exercise 1A

Question 1:

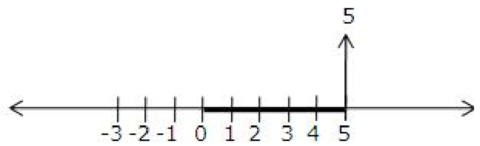
The numbers of the form $\frac{p}{q}$, where p and q are integers and q ≠ 0 are known as rational numbers.

Ten examples of rational numbers are:

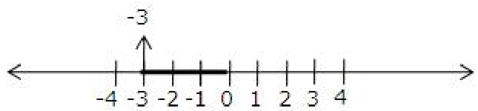
$\frac{2}{3}, \frac{4}{5}, \frac{7}{9}, \frac{8}{11}, \frac{15}{23}, \frac{23}{27}, \frac{25}{31}, \frac{26}{32}, 1, \frac{12}{5}$

Question 2:

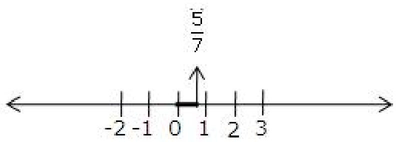
(i) 5



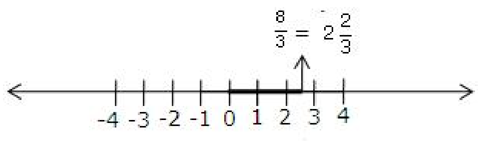
(ii) -3



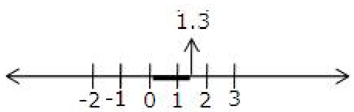
(iii) $\frac{5}{7}$



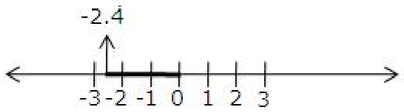
(iv) $\frac{8}{3} = 2\frac{2}{3}$



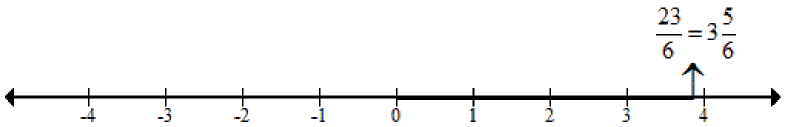
(v) 1.3



(vi) -2.4



(vii) $\frac{23}{6} = 3\frac{5}{6}$



Question 3:

(i) $\frac{1}{4}$ and $\frac{1}{3}$

Let x = $\frac{1}{4}$ and y = $\frac{1}{3}$

Then, x < y because $\frac{1}{4} < \frac{1}{3}$

$\therefore$ Rational number lying between x and y

$$= \frac{1}{2}(x + y)$$
$$= \frac{1}{2}\left(\frac{1}{4} + \frac{1}{3}\right)$$
$$= \frac{1}{2}\left(\frac{3 + 4}{12}\right)$$
$$= \frac{1}{2} \times \frac{7}{12} = \frac{7}{24}$$

Hence, $\frac{7}{24}$ is a rational number lying between $\frac{1}{4}$ and $\frac{1}{3}$.

(ii) $\frac{3}{8}$ and $\frac{2}{5}$

Let x = $\frac{3}{8}$ and y = $\frac{2}{5}$

Then, x < y because $\frac{3}{8} < \frac{2}{5}$

$\therefore$ Rational number lying between x and y

$$= \frac{1}{2}(x + y)$$
$$= \frac{1}{2}\left(\frac{3}{8} + \frac{2}{5}\right)$$
$$= \frac{1}{2}\left(\frac{15 + 16}{40}\right)$$
$$= \frac{1}{2} \times \frac{31}{40} = \frac{31}{80}$$

Hence, $\frac{31}{80}$ is a rational number lying between $\frac{3}{8}$ and $\frac{2}{5}$.

$$\frac{\frac{1}{5}+\frac{9}{40}}{2}=\frac{\frac{17}{40}}{2}=\frac{17}{80}$$

A rational number lying between $\frac{9}{40}$ and $\frac{1}{4}$ is

$$\frac{\frac{9}{40}+\frac{1}{4}}{2}=\frac{\frac{19}{40}}{2}=\frac{19}{80}$$

$$\frac{\frac{1}{5}+\frac{1}{4}}{2}=\frac{\frac{9}{20}}{2}=\frac{9}{40}$$

Therefore, we have $\frac{1}{5} < \frac{17}{80} < \frac{9}{40} < \frac{19}{80} < \frac{1}{4}$

Or we can say that, $\frac{1}{5} < \frac{17}{80} < \frac{9\times2}{40\times2} < \frac{19}{80} < \frac{1}{5}$

That is, $\frac{1}{5} < \frac{17}{80} < \frac{18}{80} < \frac{19}{80} < \frac{1}{5}$

Therefore, three rational numbers between $\frac{1}{5}$ and $\frac{1}{4}$ are

$\frac{17}{80}, \frac{18}{80}$ and $\frac{19}{80}$

Question 5:

Let $x = \frac{2}{5}$ and $y = \frac{3}{4}$

Then, x < y because $\frac{2}{5} < \frac{3}{4}$

Or we can say that, $\frac{2\times4}{5\times4} = \frac{3\times5}{4\times5}$

That is, $\frac{8}{20} < \frac{15}{20}$.

We know that, 8 < 9 < 10 < 11 < 12 < 13 < 14 < 15.

Therefore, we have, $\frac{8}{20} < \frac{9}{20} < \frac{10}{20} < \frac{11}{20} < \frac{12}{20} < \frac{13}{20} < \frac{14}{20} < \frac{15}{20}$

Thus, 5 rational numbers between, $\frac{8}{20} < \frac{15}{20}$ are:

$\frac{9}{20}, \frac{10}{20}, \frac{11}{20}, \frac{12}{20}$ and $\frac{13}{20}$

Question 6:

Let x = 3 and y = 4

Then, x < y, because 3 < 4

We can say that, $\frac{21}{7} < \frac{28}{7}$.

We know that, 21 < 22 < 23 < 24 < 25 < 26 < 27 < 28.

Therefore, we have, $\frac{21}{7} < \frac{22}{7} < \frac{23}{7} < \frac{24}{7} < \frac{25}{7} < \frac{26}{7} < \frac{27}{7} < \frac{28}{7}$

Therefore, 6 rational numbers between 3 and 4 are:

$\frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}$ and $\frac{26}{7}$

Question 7:

Let x = 2.1 and y = 2.2

Then, x < y because 2.1 < 2.2

Or we can say that, $\frac{21}{10} < \frac{22}{10}$

Or, $\frac{21\times100}{10\times100} = \frac{22\times100}{10\times100}$

That is, we have, $\frac{2100}{1000} < \frac{2200}{1000}$

We know that, 2100 < 2105 < 2110 < 2115 < 2120 < 2125 < 2130 < 2135 < 2140 < 2145 < 2150 < 2155 < 2160 < 2165 < 2170 < 2175 < 2180 < 2185 < 2190 < 2195 < 2200

Therefore, we can have,

$$\frac{2100}{1000}<\frac{2105}{1000}<\frac{2110}{1000}<\frac{2115}{1000}<\frac{2120}{1000}<\frac{2125}{1000}<\frac{2130}{1000}<\frac{2135}{1000}<\frac{2140}{1000}<\frac{2145}{1000}<\frac{2150}{1000}$$
$$<\frac{2155}{1000}<\frac{2160}{1000}<\frac{2165}{1000}<\frac{2170}{1000}<\frac{2175}{1000}<\frac{2180}{1000}<\frac{2185}{1000}<\frac{2190}{1000}<\frac{2195}{1000}<\frac{2200}{1000}$$

Therefore, 16 rational numbers between, 2.1 and 2.2 are:

$$\frac{2105}{1000},\frac{2110}{1000},\frac{2115}{1000},\frac{2120}{1000},\frac{2125}{1000},\frac{2130}{1000},\frac{2135}{1000},\frac{2140}{1000},\frac{2145}{1000},$$
$$\frac{2150}{1000},\frac{2155}{1000},\frac{2160}{1000},\frac{2165}{1000},\frac{2170}{1000},\frac{2175}{1000},\frac{2180}{1000}$$

So, 16 rational numbers between 2.1 and 2.2 are:
2.105, 2.11, 2.115, 2.12, 2.125, 2.13, 2.135, 2.14, 2.145, 2.15, 2.155, 2.16, 2.165, 2.17, 2.175, 2.18

Exercise 1B

Question 1:

(i) $\frac{13}{80}$

$$\frac{13}{80} = \frac{13}{2\times2\times2\times2\times5} = \frac{13}{2^4\times5}$$

If the prime factors of the denominator are 2 and/or 5 then the rational number is a terminating decimal.
Since, 80 has prime factors 2 and 5, $\frac{13}{80}$ is a terminating decimal.

(ii) $\frac{7}{24}$

$$\frac{7}{24} = \frac{7}{2\times2\times2\times3} = \frac{7}{2^3\times3}$$

If the prime factors of the denominators of the fraction are other than 2 and 5, then the rational number is not a terminating decimal.
Since, 24 has prime factors 2 and 3 and 3 is different from 2 and 5,
$\frac{7}{24}$ is not a terminating decimal.

(iii) $\frac{5}{12}$

$$\frac{5}{12} = \frac{5}{2\times2\times3} = \frac{5}{2^2\times3}$$

If the prime factors of the denominators of the fraction are other than 2 and 5, then the rational number is not a terminating decimal.
Since 12 has prime factors 2 and 3 and 3 is different from 2 and 5,
$\frac{5}{12}$ is not a terminating decimal.

(iv) $\frac{8}{35}$

$$\frac{8}{35} = \frac{8}{5\times7}$$

If the prime factors of the denominators of the fraction are other than 2 and 5, then the rational number is not a terminating decimal.
Since 35 has prime factors 5 and 7, and 7 is different from 2 and 5,
$\frac{8}{35}$ is not a terminating decimal.

(v) $\frac{16}{125}$

$$\frac{16}{125} = \frac{16}{5\times5\times5} = \frac{16}{5^3}$$

If the prime factors of the denominator are 2 and/or 5 then the rational number is a terminating decimal.
Since 125 has prime factors 5 only
$\frac{16}{125}$ is a terminating decimal.

Question 2:

(i) $\frac{5}{8}$

$$\begin{array}{r|l} & 0.625 \\ 8 & 5.000 \\ & 48 \\ \hline & \;\;20 \\ & \;\;16 \\ \hline & \;\;\;40 \\ & \;\;\;40 \\ \hline & \;\;\;\;0 \end{array}$$

$\frac{5}{8}$ = 0.625

(ii) $\frac{9}{16}$

$$\begin{array}{r}
0.5625 \\
16\overline{)\,9.0000} \\
80 \\
\hline
100 \\
96 \\
\hline
40 \\
32 \\
\hline
80 \\
80 \\
\hline
0 \\
\hline
\end{array}$$

$\frac{9}{16}$ = 0.5625

(iii) $\frac{7}{25}$

$$\begin{array}{r}
0.28 \\
25\overline{)\,7.00} \\
50 \\
\hline
200 \\
200 \\
\hline
0 \\
\hline
\end{array}$$

$\frac{7}{25}$ = 0.28

(iv)$\frac{11}{24}$

$$\begin{array}{r}
0.45833 \\
24\overline{)\,11.00000} \\
96 \\
\hline
140 \\
120 \\
\hline
200 \\
192 \\
\hline
80 \\
72 \\
\hline
80 \\
72 \\
\hline
8 \\
\hline
\end{array}$$

$\frac{11}{24}$ = 0.458$\bar{3}$

(v) $2\frac{5}{12} = \frac{29}{12}$

$$\begin{array}{r}
2.4166 \\
12\overline{)\,29.0000} \\
24 \\
\hline
50 \\
48 \\
\hline
20 \\
12 \\
\hline
80 \\
72 \\
\hline
80 \\
72 \\
\hline
8
\end{array}$$

$2\frac{5}{12}$ = 2.41$\bar{6}$

Question 3:

(i) Let x = 0.$\bar{3}$

i.e x = 0.333 …. (i)

⇒ 10x = 3.333 …. (ii)

Subtracting (i) from (ii), we get

9x = 3

⇒ x = $\frac{3}{9}$ = $\frac{1}{3}$

Hence, 0.$\bar{3}$ = $\frac{1}{3}$

(ii) Let x = 1.$\bar{3}$

i.e x = 1.333 …. (i)

⇒10x = 13.333 …. (ii)

Subtracting (i) from (ii) we get;

9x = 12

⇒ x = $\frac{12}{9}$ = $\frac{4}{3}$

Hence, 1.$\bar{3}$ = $\frac{4}{3}$

(iii) Let x = 0.$\bar{34}$

i.e x = 0.3434 .... (i)

⇒ 100x = 34.3434 .... (ii)

Subtracting (i) from (ii), we get

99x = 34

⇒ x = $\frac{34}{99}$

Hence, 0.$\bar{34}$ = $\frac{33}{99}$

(iv) Let x = 3.$\bar{14}$

i.e x = 3.1414 .... (i)

⇒ 100x = 314.1414 .... (ii)

Subtracting (i) from (ii), we get

99x = 311

⇒ x = $\frac{311}{99}$

Hence, 3.$\bar{14}$ = $\frac{311}{99}$

(v) Let x = 0.$\bar{324}$

i.e. x = 0.324324 ....(i)

⇒ 1000x = 324.324324....(ii)

Subtracting (i) from (ii), we get

999x = 324

⇒ x = $\frac{324}{999}$ = $\frac{12}{37}$

Hence, 0.$\bar{324}$ = $\frac{12}{37}$

(vi) Let x = 0.1$\bar{7}$

i.e. x = 0.177 .... (i)

⇒ 10x = 1.777 .... (ii)

and 100x = 17.777.... (iii)

Subtracting (ii) from (iii), we get

90x = 16

⇒ x = $\frac{16}{90}$ = $\frac{8}{45}$

Hence, 0.1$\bar{7}$ = $\frac{8}{45}$

(vii) Let x = 0.5$\bar{4}$

i.e. x = 0.544 .... (i)

⇒ 10 x = 5.44 .... (ii)

and 100x = 54.44 ....(iii)

Subtracting (ii) from (iii), we get

90x = 49

⇒ x = $\frac{49}{90}$

Hence, 0.5$\bar{4}$ = $\frac{49}{90}$

(vii) Let x = Let x = 0.1$\bar{63}$

i.e. x = 0.16363 .... (i)

⇒ 10x = 1.6363 .... (ii)

and 1000 x = 163.6363 .... (iii)

Subtracting (ii) from (iii), we get

990x = 162

⇒ x = $\frac{162}{990}$ = $\frac{9}{55}$

Hence, $0.1\bar{6}\bar{3}$ = $\frac{9}{55}$

Question 4:

(i) True. Since the collection of natural number is a sub collection of whole numbers, and every element of natural numbers is an element of whole numbers

(ii) False. Since 0 is whole number but it is not a natural number.

(iii) True. Every integer can be represented in a fraction form with denominator 1.

(iv) False. Since division of whole numbers is not closed under division, the value of $\frac{p}{q}$, p and q are integers and q ≠ 0, may not be a whole number.

(v) True. The prime factors of the denominator of the fraction form of terminating decimal contains 2 and/or 5, which are integers and are not equal to zero.

(vi) True. The prime factors of the denominator of the fraction form of repeating decimal contains integers, which are not equal to zero.

(vii) True. 0 can considered as a fraction $\frac{0}{1}$, which is a rational number.

Exercise 1C

Question 1:

Irrational number: A number which cannot be expressed either as a terminating decimal or a repeating decimal is known as irrational number. Rather irrational numbers cannot be expressed in the fraction form, $\frac{p}{q}$, p and q are integers and q ≠ 0

For example, 0.101001000100001 is neither a terminating nor a repeating decimal and so is an irrational number.

Also, $\sqrt{2}$, $\sqrt{5}$, $\sqrt{3}$, $\sqrt{6}$, $\sqrt{7}$ etc are examples of irrational numbers.

Question 2:

(i) $\sqrt{4}$

We know that, if n is a perfect square, then $\sqrt{n}$ is a rational number.

Here, 4 is a perfect square and hence, $\sqrt{4}$ = 2 is a rational number.

So, $\sqrt{4}$ is a rational number.

(ii) $\sqrt{196}$

We know that, if n is a perfect square, then $\sqrt{n}$ is a rational number.

Here, 196 is a perfect square and hence $\sqrt{196}$ is a rational number.

So, $\sqrt{196}$ is rational.

(iii) $\sqrt{21}$

We know that, if n is a not a perfect square, then $\sqrt{n}$ is an irrational number.
Here, 21 is a not a perfect square number and hence, $\sqrt{21}$ is an irrational number.
So, $\sqrt{21}$ is irrational.

(iv) $\sqrt{43}$
We know that, if n is a not a perfect square, then $\sqrt{n}$ is an irrational number.
Here, 43 is not a perfect square number and hence, $\sqrt{43}$ is an irrational number.
So, $\sqrt{43}$ is irrational.

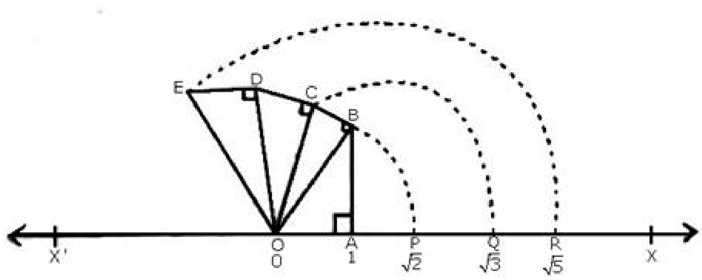
(v) $3+\sqrt{3}$
$3+\sqrt{3}$, is the sum of a rational number 3 and $\sqrt{3}$ irrational number .
Theorem: The sum of a rational number and an irrational number is an irrational number.
So by the above theorem, the sum, $3+\sqrt{3}$, is an irrational number.

(vi) $\sqrt{7}-2$
$\sqrt{7}-2$ = $\sqrt{7}$ + (-2) is the sum of a rational number and an irrational number.
Theorem: The sum of a rational number and an irrational number is an irrational number.
So by the above theorem, the sum, $\sqrt{7}$ + (-2) , is an irrational number.
So, $\sqrt{7}-2$ is irrational.

(vii) $\frac{2}{3}\sqrt{6}$
$\frac{2}{3}\sqrt{6}$ = $\frac{2}{3}$ × $\sqrt{6}$ is the product of a rational number and an irrational number .
Theorem: The product of a non-zero rational number and an irrational number is an irrational number.
Thus, by the above theorem, $\frac{2}{3}$ × $\sqrt{6}$ is an irrational number.
So, $\frac{2}{3}\sqrt{6}$ is an irrational number.

(viii) $0.\bar{6}$
Every rational number can be expressed either in the terminating form or in the non-terminating, recurring decimal form.
Therefore, $0.\bar{6}$ = 0.6666

Question 3:



Let X'OX be a horizontal line, taken as the x-axis and let O be the origin. Let O represent 0.
Take OA = 1 unit and draw BA ⊥ OA such that AB = 1 unit, join OB. Then,

$$OB = \sqrt{OA^2 + AB^2} = \sqrt{1^2 + 1^2} = \sqrt{2} \text{ units}$$

With O as centre and OB as radius, drawn an arc, meeting OX at P.
Then, OP = OB = $\sqrt{2}$ units
Thus the point P represents $\sqrt{2}$ on the real line.

Now draw BC ⊥ OB such that BC = 1 units

Join OC. Then,

$$OC = \sqrt{OB^2 + BC^2}$$
$$= \sqrt{(\sqrt{2})^2 + 1^2} = \sqrt{3} \text{ units}$$

With O as centre and OC as radius, draw an arc, meeting OX at Q. The,

OQ = OC = $\sqrt{3}$ units

Thus, the point Q represents $\sqrt{3}$ on the real line.

Now draw CD ⊥ OC such that CD = 1 units

Join OD. Then,

$$OD = \sqrt{OC^2 + CD^2}$$

$$= \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{4} = 2 \text{ units}$$

Now draw DE ⊥ OD such that DE = 1 units

Join OE. Then,

$$OE = \sqrt{OD^2 + DE^2}$$

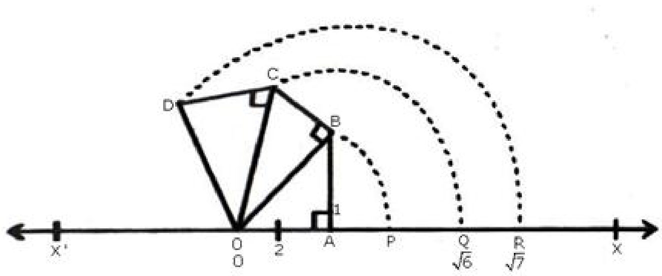
$$= \sqrt{2^2 + 1^2} = \sqrt{5} \text{ units}$$

With O as centre and OE as radius draw an arc, meeting OX at R.

Then, OR = OE = $\sqrt{5}$ units

Thus, the point R represents $\sqrt{5}$ on the real line.

Question 4:



Draw horizontal line X'OX taken as the x-axis

Take O as the origin to represent 0.

Let OA = 2 units and let AB ⊥ OA such that AB = 1 units

Join OB. Then,

$$OB = \sqrt{OA^2 + AB^2}$$
$$= \sqrt{2^2 + 1^2} = \sqrt{5}$$

With O as centre and OB as radius draw an arc meeting OX at P.

Then, OP = OB = $\sqrt{5}$

Now draw BC ⊥ OB and set off BC = 1 unit

Join OC. Then,

$$OC = \sqrt{OB^2 + BC^2}$$

$$= \sqrt{(\sqrt{5})^2 + 1^2} = \sqrt{6}$$

With O as centre and OC as radius, draw an arc, meeting OX at Q.

Then, OQ = OC = $\sqrt{6}$

Thus, Q represents $\sqrt{6}$ on the real line.

Now, draw CD ⊥ OC as set off CD = 1 units

Join OD. Then,

$$OD = \sqrt{OC^2 + CD^2}$$
$$= \sqrt{(\sqrt{6})^2 + 1^2} = \sqrt{7}$$

With O as centre and OD as radius, draw an arc, meeting OX at R. Then

OR = OD = $\sqrt{7}$

Thus, R represents $\sqrt{7}$ on the real line.

Question 5:

(i) $4 + \sqrt{5}$

Since 4 is a rational number and $\sqrt{5}$ is an irrational number.

So, $4 + \sqrt{5}$ is irrational because sum of a rational number and irrational number is always an irrational number.

(ii) $(-3 + \sqrt{6})$

Since – 3 is a rational number and $\sqrt{6}$ is irrational.

So, $(-3 + \sqrt{6})$ is irrational because sum of a rational number and irrational number is always an irrational number.

(iii) $5\sqrt{7}$

Since 5 is a rational number and $\sqrt{7}$ is an irrational number.

So, $5\sqrt{7}$ is irrational because product of a rational number and an irrational number is always irrational.

(iv) $-3\sqrt{8}$

Since -3 is a rational number and $\sqrt{8}$ is an irrational number.

So, $-3\sqrt{8}$ is irrational because product of a rational number and an irrational number is always irrational.

(v) $\frac{2}{\sqrt{5}}$

$$\frac{2}{\sqrt{5}} = \frac{2 \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}} = \frac{2\sqrt{5}}{5} = \frac{2}{5} \times \sqrt{5}$$

$\frac{2}{\sqrt{5}}$ is irrational because it is the product of a rational number and the irrational number $\sqrt{5}$.

(vi) $\frac{4}{\sqrt{3}}$

$$\frac{4}{\sqrt{3}} = \frac{4 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{4\sqrt{3}}{3} = \frac{4}{3} \times \sqrt{3}$$

$\frac{4}{\sqrt{3}}$ is an irrational number because it is the product of rational number and irrational number $\sqrt{3}$.

Question 6:

(i) True

(ii) False

(iii) True

(iv) False

(v) True

(vi) False

(vii) False

(viii) True

(ix) True

Exercise 1D

Question 1:

(i)

$\left(2\sqrt{3} - 5\sqrt{2}\right)$ and $\left(\sqrt{3} + 2\sqrt{2}\right)$

We have:

$$= \left(2\sqrt{3} - 5\sqrt{2}\right) + \left(\sqrt{3} + 2\sqrt{2}\right)$$
$$= \left(2\sqrt{3} + \sqrt{3}\right) + \left(-5\sqrt{2} + 2\sqrt{2}\right)$$
$$= (2 + 1)\sqrt{3} + (-5 + 2)\sqrt{2}$$
$$= 3\sqrt{3} - 3\sqrt{2}$$

(ii)

$\left(2\sqrt{2} + 5\sqrt{3} - 7\sqrt{5}\right)$ and $\left(3\sqrt{3} - \sqrt{2} + \sqrt{5}\right)$

We have:

$$\left(2\sqrt{2} + 5\sqrt{3} - 7\sqrt{5}\right) + \left(3\sqrt{3} - \sqrt{2} + \sqrt{5}\right)$$
$$= \left(2\sqrt{2} - \sqrt{2}\right) + \left(5\sqrt{3} + 3\sqrt{3}\right) + \left(-7\sqrt{5} + \sqrt{5}\right)$$
$$= (2 - 1)\sqrt{2} + (5 + 3)\sqrt{3} + (-7 + 1)\sqrt{5}$$
$$= \sqrt{2} + 8\sqrt{3} - 6\sqrt{5}.$$

(iii) $\left(\frac{2}{3}\sqrt{7} - \frac{1}{2}\sqrt{2} + 6\sqrt{11}\right)$ and $\left(\frac{1}{3}\sqrt{7} + \frac{3}{2}\sqrt{2} - \sqrt{11}\right)$

We have:

$$\left(\frac{2}{3}\sqrt{7} - \frac{1}{2}\sqrt{2} + 6\sqrt{11}\right) + \left(\frac{1}{3}\sqrt{7} + \frac{3}{2}\sqrt{2} - \sqrt{11}\right)$$
$$= \left(\frac{2}{3}\sqrt{7} + \frac{1}{3}\sqrt{7}\right) + \left(-\frac{1}{2}\sqrt{2} + \frac{3}{2}\sqrt{2}\right) + \left(6\sqrt{11} - \sqrt{11}\right)$$
$$= \left(\frac{2}{3} + \frac{1}{3}\right)\sqrt{7} + \left(-\frac{1}{2} + \frac{3}{2}\right)\sqrt{2} + (6 - 1)\sqrt{11}$$
$$= \sqrt{7} + \sqrt{2} + 5\sqrt{11}.$$

Question 2:

(i) $3\sqrt{5}$ by $2\sqrt{5}$

$$3\sqrt{5} \times 2\sqrt{5} = 3 \times 2 \times \sqrt{5} \times \sqrt{5}$$
$$= (3 \times 2 \times 5) = 30.$$

(ii) $6\sqrt{15}$ by $4\sqrt{3}$

$$6\sqrt{15} \times 4\sqrt{3} = 6 \times 4 \times \sqrt{15} \times \sqrt{3}$$
$$= 24 \times \sqrt{15 \times 3}$$
$$= 24 \times \sqrt{3 \times 5 \times 3}$$
$$= 24 \times 3\sqrt{5} = 72\sqrt{5}.$$

(iii) $2\sqrt{6}$ by $3\sqrt{3}$

$$2\sqrt{6} \times 3\sqrt{3} = 2 \times 3 \times \sqrt{6} \times \sqrt{3}$$
$$= 6 \times \sqrt{6 \times 3}$$
$$= 6 \times \sqrt{2 \times 3 \times 3}$$
$$= 6 \times 3\sqrt{2} = 18\sqrt{2}$$

(iv) $3\sqrt{8}$ by $3\sqrt{2}$

$$3\sqrt{8} \times 3\sqrt{2} = 3 \times 3 \times \sqrt{8} \times \sqrt{2}$$
$$= 9 \times \sqrt{8 \times 2}$$
$$= 9 \times \sqrt{2 \times 2 \times 2 \times 2}$$
$$= (9 \times 2 \times 2) = 36.$$

(v) $\sqrt{10}$ by $\sqrt{40}$

$$\begin{aligned}\sqrt{10}\times\sqrt{40} &= \sqrt{10\times 40}\\ &= \sqrt{2\times 5\times 2\times 2\times 2\times 5}\\ &= (2\times 2\times 5) = 20\end{aligned}$$

(vi) $3\sqrt{28}$ by $2\sqrt{7}$

$$\begin{aligned}3\sqrt{28}\times 2\sqrt{7} &= 3\times 2\times\sqrt{28}\times\sqrt{7}\\ &= 6\times\sqrt{28\times 7}\\ &= 6\times\sqrt{2\times 2\times 7\times 7}\\ &= (6\times 2\times 7) = 84.\end{aligned}$$

Question 3:

(i) $16\sqrt{6}$ by $4\sqrt{2}$

$$\begin{aligned}16\sqrt{6}\div 4\sqrt{2} &= \frac{16\sqrt{6}}{4\sqrt{2}} = \frac{4\sqrt{6}}{\sqrt{2}} = \frac{4\sqrt{6}\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}}\\ &= \frac{4\sqrt{6\times 2}}{2} = \frac{4\sqrt{2\times 3\times 2}}{2}\\ &= \frac{4\times 2\sqrt{3}}{2} = 4\sqrt{3}\end{aligned}$$

(ii) $12\sqrt{15}$ by $4\sqrt{3}$

$$\begin{aligned}12\sqrt{15}\div 4\sqrt{3} &= \frac{12\sqrt{15}}{4\sqrt{3}} = \frac{3\sqrt{15}}{\sqrt{3}} = \frac{3\sqrt{15}\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}}\\ &= \frac{3\sqrt{15\times 3}}{3} = \sqrt{3\times 5\times 3} = 3\sqrt{5}\end{aligned}$$

(iii) $18\sqrt{21}$ by $6\sqrt{7}$

$$\begin{aligned}18\sqrt{21}\div 6\sqrt{7} &= \frac{18\sqrt{21}}{6\sqrt{7}} = \frac{3\sqrt{21}}{\sqrt{7}} = \frac{3\sqrt{21}\times\sqrt{7}}{\sqrt{7}\times\sqrt{7}}\\ &= \frac{3\sqrt{3\times 7\times 7}}{7} = \frac{3\times 7\sqrt{3}}{7} = 3\sqrt{3}\end{aligned}$$

Question 4:

(i) $\left(4+\sqrt{2}\right)\left(4-\sqrt{2}\right)$

$$\begin{aligned}&= (4)^2 - \left(\sqrt{2}\right)^2 \qquad \left[\because a^2 - b^2 = (a-b)(a+b)\right]\\ &= 16 - 2 = 14\end{aligned}$$

(ii) $\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}-\sqrt{3}\right)$

$$\begin{aligned}&\left(\sqrt{5}\right)^2 - \left(\sqrt{3}\right)^2 \qquad \left[\because a^2 - b^2 = (a-b)(a+b)\right]\\ &= 5 - 3 = 2.\end{aligned}$$

(iii) $\left(6-\sqrt{6}\right)+\left(6+\sqrt{6}\right)$

$$\begin{aligned}&= (6)^2 - \left(\sqrt{6}\right)^2 \qquad \left[\because a^2 - b^2 = (a-b)(a+b)\right]\\ &= 36 - 6 = 30.\end{aligned}$$

(iv) $\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{2}-\sqrt{3}\right)$

$$\begin{aligned}&= \sqrt{5}\left(\sqrt{2}-\sqrt{3}\right) - \sqrt{2}\left(\sqrt{2}-\sqrt{3}\right)\\ &= \left(\sqrt{10}-\sqrt{15}-2+\sqrt{6}\right).\end{aligned}$$

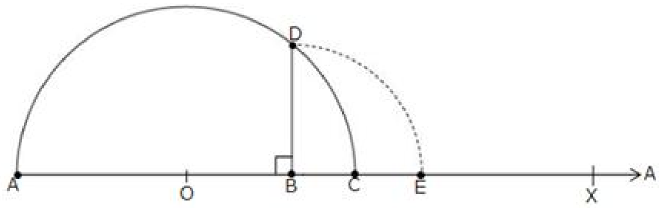
(v) $\left(\sqrt{5}-\sqrt{3}\right)^2$

$$\begin{aligned}&= (\sqrt{5})^2 + (\sqrt{3})^2 - 2\sqrt{5}.\sqrt{3}\\ &= 5+3-2\sqrt{15}\\ &= 8-2\sqrt{15}\end{aligned}$$

(vi) $\left(3-\sqrt{3}\right)^2$

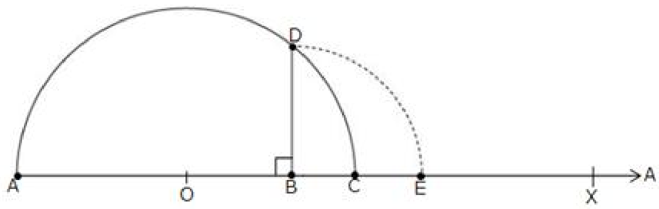
$$\begin{aligned}&= (3)^2 + (\sqrt{3})^2 - 2.3.\sqrt{3}\\ &= 9+3-6\sqrt{3}\\ &= 12-6\sqrt{3}\end{aligned}$$

Question 5:



Draw a line segment AB = 3.2 units and extend it to C such that BC = 1 units.
Find the midpoint O of AC.
With O as centre and OA as radius, draw a semicircle.
Now, draw BD AC, intersecting the semicircle at D.
Then, BD = $\sqrt{3.2}$ units.
With B as centre and BD as radius, draw an arc meeting AC produced at E.
Then, BE = BD = $\sqrt{3.2}$ units.

Question 6:



Draw a line segment AB = 7.28 units and extend it to C such that BC = 1 unit.
Find the midpoint O of AC.
With O as centre and OA as radius, draw a semicircle.

Now, draw BD AC, intersecting the semicircle at D.

Then, BD = $\sqrt{7.28}$ units.

With D as centre and BD as radius, draw an arc, meeting AC produced at E.

Then, BE = BD = $\sqrt{7.28}$ units.

Question 7:

Closure Property: The sum of two real numbers is always a real number.

Associative Law: (a + b) + c = a + (b + c) for al real numbers a, b, c.

Commutative Law: a + b = b + a, for all real numbers a and b.

Existence of identity: 0 is a real number such that 0 + a = a + 0, for every real number a.

Existence of inverse of addition: For each real number a, there exists a real number (-a) such that

a + (-a) = (-a) + a= 0

a and (-a) are called the additive inverse of each other.

Existence of inverse of multiplication:

For each non zero real number a, there exists a real number $\frac{1}{a}$ such that

$$a \times \frac{1}{a} = \frac{1}{a} \times a = 1$$

a and $\frac{1}{a}$ are called the multiplicative inverse of each other.

Exercise 1E

Question 1:

On multiplying the numerator and denominator of the given number by $\sqrt{7}$, we get

$$\frac{1}{\sqrt{7}} = \frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{7}}{7}.$$

Question 2:

On multiplying the numerator and denominator of the given number by $\sqrt{3}$, we get

$$\frac{\sqrt{5}}{2\sqrt{3}} = \frac{\sqrt{5}}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{15}}{2 \times 3} = \frac{\sqrt{15}}{6}.$$

Question 3:

If a and b are integers, then

$\left(a+\sqrt{b}\right)$ and $\left(a-\sqrt{b}\right)$ are rationalising factor of each other,

as $\left(a+\sqrt{b}\right)\left(a-\sqrt{b}\right) = \left(a^2 - b\right)$, which is rational.

Therefore, we have,

$$\frac{1}{\left(2+\sqrt{3}\right)} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2-\sqrt{3}}{(2)^2 - \left(\sqrt{3}\right)^2} = \frac{2-\sqrt{3}}{4-3}$$

$$= \frac{2-\sqrt{3}}{1} = 2-\sqrt{3}.$$

Question 4:

If a and b are integers, then

$\left(a+\sqrt{b}\right)$ and $\left(a-\sqrt{b}\right)$ are rationalising factor of each other,

as $\left(a+\sqrt{b}\right)\left(a-\sqrt{b}\right) = \left(a^2 - b\right)$, which is rational.

Therefore, we have,

$$\frac{1}{\left(\sqrt{5}-2\right)} = \frac{1}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2} = \frac{\sqrt{5}+2}{\left(\sqrt{5}\right)^2 - (2)^2} = \frac{\sqrt{5}+2}{5-4}$$

$$= \frac{\sqrt{5}+2}{1} = \sqrt{5}+2.$$

Question 5:

If a and b are integers and x is a natural number, then $\left(a+b\sqrt{x}\right)$ and $\left(a-b\sqrt{x}\right)$ are rationalising factor of each other, as $\left(a+b\sqrt{x}\right)\left(a-b\sqrt{x}\right)=\left(a^2-b^2x\right)$, which is rational.
Therefore, we have,

$$\frac{1}{\left(5+3\sqrt{2}\right)}=\frac{1}{5+3\sqrt{2}}\times\frac{5-3\sqrt{2}}{5-3\sqrt{2}}$$
$$=\frac{5-3\sqrt{2}}{(5)^2-\left(3\sqrt{2}\right)^2}=\frac{5-3\sqrt{2}}{25-18}=\left(\frac{5-3\sqrt{2}}{7}\right)$$

Question 6:

If a and b are integers, then $\left(\sqrt{a}+\sqrt{b}\right)$ and $\left(\sqrt{a}-\sqrt{b}\right)$ are rationalising factor of each other, as $\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}-\sqrt{b}\right)=(a-b)$, which is rational.
Therefore, we have,

$$\frac{1}{\left(\sqrt{6}-\sqrt{5}\right)}=\frac{1}{\sqrt{6}-\sqrt{5}}\times\frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}}=\frac{\sqrt{6}+\sqrt{5}}{\left(\sqrt{6^2}\right)-\left(\sqrt{5}\right)^2}=\frac{\sqrt{6}+\sqrt{5}}{6-5}$$
$$=\frac{\sqrt{6}+\sqrt{5}}{1}=\left(\sqrt{6}+\sqrt{5}\right).$$

Question 7:

If a and b are integers, then $\left(\sqrt{a}+\sqrt{b}\right)$ and $\left(\sqrt{a}-\sqrt{b}\right)$ are rationalising factor of each other, as $\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}-\sqrt{b}\right)=(a-b)$, which is rational.
Therefore, we have,

$$\frac{4}{\left(\sqrt{7}+\sqrt{3}\right)}=\frac{4}{\sqrt{7}+\sqrt{3}}\times\frac{\sqrt{7}-\sqrt{3}}{\sqrt{7}-\sqrt{3}}=\frac{4\left(\sqrt{7}-\sqrt{3}\right)}{\left(\sqrt{7}\right)^2-\left(\sqrt{3}\right)^2}$$
$$=\frac{4\left(\sqrt{7}-\sqrt{3}\right)}{7-3}=\frac{4\left(\sqrt{7}-\sqrt{3}\right)}{4}$$
$$=\left(\sqrt{7}-\sqrt{3}\right).$$

Question 8:

For rationalising the denominator of a number, we multiply its numerator and denominator by its rationalising factor.

If a and b are integers, then $\left(a+\sqrt{b}\right)$ and $\left(a-\sqrt{b}\right)$ are rationalising factor of each other, as $\left(a+\sqrt{b}\right)\left(a-\sqrt{b}\right)=\left(a^2-b\right)$, which is rational.

$$\frac{\sqrt{3}-1}{\sqrt{3}+1}=\frac{\sqrt{3}-1}{\sqrt{3}+1}\times\frac{\sqrt{3}-1}{\sqrt{3}-1}$$
$$=\frac{\left(\sqrt{3}-1\right)^2}{\left(\sqrt{3}\right)^2-(1)^2}$$
$$=\frac{\left(\sqrt{3}\right)^2-2\left(\sqrt{3}\right)(1)+1^2}{3-1}$$
$$=\frac{3-2\sqrt{3}+1}{2}=\frac{4-2\sqrt{3}}{2}$$
$$=\frac{2\left(2-\sqrt{3}\right)}{2}=\left(2-\sqrt{3}\right).$$

Question 9:

For rationalising the denominator of a number, we multiply its numerator and denominator by its rationalising factor.

If a and b are integers and x is a natural number, then $\left(a+b\sqrt{x}\right)$ and $\left(a-b\sqrt{x}\right)$ are rationalising factor of each other, as $\left(a+b\sqrt{x}\right)\left(a-b\sqrt{x}\right)=\left(a^2-b^2x\right)$, which is rational.

Therefore, we have,

$$\frac{3-2\sqrt{2}}{3+2\sqrt{2}}=\frac{3-2\sqrt{2}}{3+2\sqrt{2}}\times\frac{3-2\sqrt{2}}{3-2\sqrt{2}}$$
$$=\frac{\left(3-2\sqrt{2}\right)^2}{\left(3+2\sqrt{2}\right)\left(3-2\sqrt{2}\right)}$$
$$=\frac{9-12\sqrt{2}+8}{(3)^2-\left(2\sqrt{2}\right)^2}=\frac{17-12\sqrt{2}}{9-8}$$
$$=\frac{17-12\sqrt{2}}{1}=17-12\sqrt{2}.$$

Question 10:

Consider the given equation

$$\frac{\sqrt{3}+1}{\sqrt{3}-1}=a+b\sqrt{3}$$

For rationalising the denominator of a number, we multiply its numerator and denominator by its rationalising factor.

If a and b are integers, then $\left(a+\sqrt{b}\right)$ and $\left(a-\sqrt{b}\right)$ are rationalising factor of each other, as $\left(a+\sqrt{b}\right)\left(a-\sqrt{b}\right)=\left(a^2-b\right)$, which is rational.

Let us rationalise the denominator of the Left hand side.

$$\Rightarrow \frac{\sqrt{3}+1}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}=a+b\sqrt{3}$$
$$\Rightarrow \frac{\left(\sqrt{3}\right)^2+2\left(\sqrt{3}\right)(1)+(1)^2}{\left(\sqrt{3}\right)^2-(1)^2}=a+b\sqrt{3}$$
$$\Rightarrow \frac{3+2\sqrt{3}+1}{3-1}=a+b\sqrt{3}$$
$$\Rightarrow \frac{2\left(2+\sqrt{3}\right)}{2}=a+b\sqrt{3}$$
$$\Rightarrow 2+\sqrt{3}=a+b\sqrt{3}$$

$\therefore$ a = 2 and b = 1.

Question 11:

Consider the given equation

$$\frac{3+\sqrt{2}}{3-\sqrt{2}}=a+b\sqrt{2}$$

For rationalising the denominator of a number, we multiply its numerator and denominator by its rationalising factor.

If a and b are integers, then $\left(a+\sqrt{b}\right)$ and $\left(a-\sqrt{b}\right)$ are rationalising factor of each other, as $\left(a+\sqrt{b}\right)\left(a-\sqrt{b}\right)=\left(a^2-b\right)$, which is rational.

Let us rationalise the denominator of the Left hand side.

$$\Rightarrow \frac{3+\sqrt{2}}{3-\sqrt{2}}\times\frac{3+\sqrt{2}}{3+\sqrt{2}}=a+b\sqrt{2}$$
$$\Rightarrow \frac{\left(3+\sqrt{2}\right)^2}{(3)^2-\left(\sqrt{2}\right)^2}=a+b\sqrt{2}$$
$$\Rightarrow \frac{(3)^2+2(3)\left(\sqrt{2}\right)+\left(\sqrt{2}\right)^2}{9-2}=a+b\sqrt{2}$$
$$\Rightarrow \frac{11+6\sqrt{2}}{7}=a+b\sqrt{2}$$
$$\Rightarrow \frac{11}{7}+\frac{6\sqrt{2}}{7}=a+b\sqrt{2}$$

$\therefore a=\frac{11}{7}$ and $b=\frac{6}{7}$.

Question 12:

Consider the given equation

$$\frac{5-\sqrt{6}}{5+\sqrt{6}} = a - b\sqrt{6}$$

For rationalising the denominator of a number, we multiply its numerator and denominator by its rationalising factor.

If a and b are integers, then $\left(a+\sqrt{b}\right)$ and $\left(a-\sqrt{b}\right)$ are rationalising factor of each other, as $\left(a+\sqrt{b}\right)\left(a-\sqrt{b}\right)=\left(a^2-b\right)$, which is rational.

Let us rationalise the denominator of the Left hand side.

$$\Rightarrow \frac{5-\sqrt{6}}{5+\sqrt{6}} \times \frac{5-\sqrt{6}}{5-\sqrt{6}} = a - b\sqrt{6}$$

$$\Rightarrow \frac{\left(5-\sqrt{6}\right)^2}{(5)^2-\left(\sqrt{6}\right)^2} = a - b\sqrt{6}$$

$$\Rightarrow \frac{(5)^2-2(5)\left(\sqrt{6}\right)+\left(\sqrt{6}\right)^2}{25-6} = a - b\sqrt{6}$$

$$\Rightarrow \frac{25-10\sqrt{6}+6}{19} = a - b\sqrt{6}$$

$$\Rightarrow \frac{31-10\sqrt{6}}{19} = a - b\sqrt{6}$$

$$\Rightarrow \frac{31}{19} - \frac{10\sqrt{6}}{19} = a - b\sqrt{6}$$

$$\therefore a = \frac{31}{19} \text{ and } b = \frac{10}{19}.$$

Question 13:

Consider the given equation

$$\frac{5+2\sqrt{3}}{7+4\sqrt{3}} = a - b\sqrt{3}$$

For rationalising the denominator of a number, we multiply its numerator and denominator by its rationalising factor.

If a and b are integers and x is a natural number, then $\left(a+b\sqrt{x}\right)$ and $\left(a-b\sqrt{x}\right)$ are rationalising factor of each other, as $\left(a+b\sqrt{x}\right)\left(a-b\sqrt{x}\right)=\left(a^2-b^2x\right)$, which is rational.

Let us rationalise the denominator of the Left hand side.

$$\Rightarrow \frac{5+2\sqrt{3}}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}} = a - b\sqrt{3}$$

$$\Rightarrow \frac{\left(5+2\sqrt{3}\right)\left(7-4\sqrt{3}\right)}{(7)^2-\left(4\sqrt{3}\right)^2} = a - b\sqrt{3}$$

$$\Rightarrow \frac{5\left(7-4\sqrt{3}\right)+2\sqrt{3}\left(7-4\sqrt{3}\right)}{49-48} = a - b\sqrt{3}$$

$$\Rightarrow 35-20\sqrt{3}+14\sqrt{3}-24 = a - b\sqrt{3}$$

$$\Rightarrow 11-6\sqrt{3} = a - b\sqrt{3}$$

$$\therefore a = 11 \text{ and } b = 6.$$

Question 14:

For rationalising the denominator of a number, we multiply its numerator and denominator by its rationalising factor.

If a and b are integers, then
$\left(a+\sqrt{b}\right)$ and $\left(a-\sqrt{b}\right)$ are rationalising factor of each other,
as $\left(a+\sqrt{b}\right)\left(a-\sqrt{b}\right)=\left(a^2-b\right)$, which is rational.

Let us rationalise the denominator of the first term on the Left hand side.

We have,

$$\frac{\sqrt{5}-1}{\sqrt{5}+1}=\frac{\sqrt{5}-1}{\sqrt{5}+1}\times\frac{\sqrt{5}-1}{\sqrt{5}-1}$$
$$=\frac{\left(\sqrt{5}-1\right)^2}{\left(\sqrt{5}\right)^2-(1)^2}$$
$$=\frac{\left(\sqrt{5}\right)^2-2\left(\sqrt{5}\right)(1)+1}{5-1}$$
$$=\frac{5-2\sqrt{5}+1}{4}=\frac{6-2\sqrt{5}}{4}\,.....(1)$$

Now consider the denominator of the second

term on the left hand side:

$$\frac{\sqrt{5}+1}{\sqrt{5}-1}=\frac{\sqrt{5}+1}{\sqrt{5}-1}\times\frac{\sqrt{5}+1}{\sqrt{5}+1}$$
$$=\frac{\left(\sqrt{5}+1\right)^2}{\left(\sqrt{5}^2\right)-(1)^2}$$
$$=\frac{\left(\sqrt{5}\right)^2+2\left(\sqrt{5}\right)(1)+(1)^2}{5-1}$$
$$=\frac{5+2\sqrt{5}+1}{4}=\frac{6+2\sqrt{5}}{4}\,.......(2)$$

Adding equations (1) and (2), we have

$$\therefore\frac{\sqrt{5}-1}{\sqrt{5}+1}+\frac{\sqrt{5}+1}{\sqrt{5}-1}=\frac{6-2\sqrt{5}}{4}+\frac{6+2\sqrt{5}}{4}$$
$$=\frac{6-2\sqrt{5}+6+2\sqrt{5}}{4}=\frac{12}{4}=3.$$

Question 15:

For rationalising the denominator of a number, we multiply its numerator and denominator by its rationalising factor.

If a and b are integers, then
$\left(a+\sqrt{b}\right)$ and $\left(a-\sqrt{b}\right)$ are rationalising factor of each other,
as $\left(a+\sqrt{b}\right)\left(a-\sqrt{b}\right)=\left(a^2-b\right)$, which is rational.

Let us rationalise the denominator of the first term on the Left hand side.

We have,

$$\frac{4+\sqrt{5}}{4-\sqrt{5}}=\frac{4+\sqrt{5}}{4-\sqrt{5}}\times\frac{4+\sqrt{5}}{4+\sqrt{5}}$$
$$=\frac{\left(4+\sqrt{5}\right)^2}{(4)^2-\left(\sqrt{5}\right)^2}$$
$$=\frac{(4)^2+2(4)\left(\sqrt{5}\right)+\left(\sqrt{5}\right)^2}{16-5}$$
$$\frac{4+\sqrt{5}}{4-\sqrt{5}}=\frac{16+8\sqrt{5}+5}{11}=\frac{21+8\sqrt{5}}{11}\text{.....(1)}$$

Now consider the denominator of the second term on the left hand side:

$$\frac{4-\sqrt{5}}{4+\sqrt{5}}=\frac{4-\sqrt{5}}{4+\sqrt{5}}\times\frac{4-\sqrt{5}}{4-\sqrt{5}}$$
$$=\frac{\left(4-\sqrt{5}\right)^2}{(4)^2-\left(\sqrt{5}\right)^2}$$
$$=\frac{(4)^2-2(4)\left(\sqrt{5}\right)+\left(\sqrt{5}\right)^2}{16-5}$$
$$\frac{4-\sqrt{5}}{4+\sqrt{5}}=\frac{16-8\sqrt{5}+5}{11}=\frac{21-8\sqrt{5}}{11}\text{......(2)}$$

Adding equations (1) and (2), we have,

$$\therefore\frac{4+\sqrt{5}}{4-\sqrt{5}}+\frac{4-\sqrt{5}}{4+\sqrt{5}}=\frac{21+8\sqrt{5}}{11}+\frac{21-8\sqrt{5}}{11}$$
$$=\frac{21+8\sqrt{5}+21-8\sqrt{5}}{11}=\frac{42}{11}.$$

Question 16:

Given, $x=\left(4-\sqrt{15}\right)$

Then,

$$\left(x+\frac{1}{x}\right)=\left(4-\sqrt{15}+\frac{1}{4-\sqrt{15}}\right)$$
$$=\left(4-\sqrt{15}+\frac{1}{4-\sqrt{15}}\times\frac{4+\sqrt{15}}{4+\sqrt{15}}\right)\quad\text{[rationalisation]}$$
$$=\left(4-\sqrt{15}+\frac{4+\sqrt{15}}{(4)^2-\left(\sqrt{15}\right)^2}\right)$$
$$=\left(4-\sqrt{15}+\frac{4+\sqrt{15}}{16-15}\right)$$
$$=\left(4-\sqrt{15}+\frac{4+\sqrt{15}}{1}\right)$$
$$=4-\sqrt{15}+4+\sqrt{15}=8.$$

Question 17:

Given, $x = (2 + \sqrt{3})$

$$\therefore \frac{1}{x} = \frac{1}{2+\sqrt{3}} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} \text{ [rationalising the denominator]}$$
$$= \frac{2-\sqrt{3}}{(2)^2 - (\sqrt{3})^2}$$
$$= \frac{2-\sqrt{3}}{4-3} = \frac{(2-\sqrt{3})}{1} = (2-\sqrt{3})$$
$$\therefore \left(x + \frac{1}{x}\right) = (2+\sqrt{3}) + (2-\sqrt{3}) = 4$$
$$\Rightarrow \left(x + \frac{1}{x}\right)^2 = 4^2 = 16$$
$$\Rightarrow x^2 + \frac{1}{x^2} + 2 \times x \times \frac{1}{x} = 16$$
$$\Rightarrow x^2 + \frac{1}{x^2} = (16 - 2) = 14$$
$$\therefore \left(x^2 + \frac{1}{x^2}\right) = 14.$$

Question 18:

L.H.S =
$$\frac{1}{(3-\sqrt{8})} - \frac{1}{(\sqrt{8}-\sqrt{7})} + \frac{1}{(\sqrt{7}-\sqrt{6})} - \frac{1}{(\sqrt{6}-\sqrt{5})} + \frac{1}{(\sqrt{5}-2)}$$

$$= \frac{3+\sqrt{8}}{(3-\sqrt{8})(3+\sqrt{8})} - \frac{\sqrt{8}+\sqrt{7}}{(\sqrt{8}-\sqrt{7})(\sqrt{8}+\sqrt{7})} + \frac{\sqrt{7}+\sqrt{6}}{(\sqrt{7}-\sqrt{6})(\sqrt{7}+\sqrt{6})}$$
$$- \frac{\sqrt{6}+\sqrt{5}}{(\sqrt{6}-\sqrt{5})(\sqrt{6}+\sqrt{5})} + \frac{\sqrt{5}+2}{(\sqrt{5}-2)(\sqrt{5}+2)}$$

$$= \frac{3+\sqrt{8}}{9-8} - \frac{\sqrt{8}+\sqrt{7}}{8-7} + \frac{\sqrt{7}+\sqrt{6}}{7-6} - \frac{\sqrt{6}+\sqrt{5}}{6-5} + \frac{\sqrt{5}+2}{5-4}$$
$$= 3 + \sqrt{8} - \sqrt{8} - \sqrt{7} + \sqrt{7} + \sqrt{6} - \sqrt{6} - \sqrt{5} + \sqrt{5} + 2$$
$$= 3 + 2 = 5 = \text{R.H.S}$$
$$\therefore \quad \text{L.H.S} = \text{R.H.S}$$

Exercise 1F

Question 1:

(i)
$$\left(6^{\frac{2}{5}} \times 6^{\frac{3}{5}}\right) = 6^{\left(\frac{2}{5}+\frac{3}{5}\right)} = 6^1 = 6.$$

(ii)
$$\left(3^{\frac{1}{2}} \times 3^{\frac{1}{3}}\right) = 3^{\left(\frac{1}{2}+\frac{1}{3}\right)} = 3^{\left(\frac{3+2}{6}\right)} = 3^{\frac{5}{6}}.$$

(iii)
$$\left(7^{\frac{5}{6}} \times 7^{\frac{2}{3}}\right) = 7^{\left(\frac{5}{6}+\frac{2}{3}\right)} = 7^{\left(\frac{5+4}{6}\right)}$$
$$= 7^{\frac{9}{6}} = 7^{\frac{3}{2}}.$$

Question 2:

(i)
$$\frac{6^{\frac{1}{4}}}{6^{\frac{1}{5}}} = 6^{\left(\frac{1}{4}-\frac{1}{5}\right)}$$
$$= 6^{\left(\frac{5-4}{20}\right)} = 6^{\frac{1}{20}}.$$

(ii)

$$\frac{8^{\frac{1}{2}}}{8^{\frac{2}{3}}} = 8^{\left(\frac{1}{2} - \frac{2}{3}\right)} = 8^{\left(\frac{3-4}{6}\right)} = 8^{\frac{-1}{6}}.$$

(iii)

$$\frac{5^{\frac{6}{7}}}{5^{\frac{2}{3}}} = 5^{\left(\frac{6}{7} - \frac{2}{3}\right)} = 5^{\left(\frac{18-14}{21}\right)} = 5^{\frac{4}{21}}.$$

Question 3:

(i)

$$3^{\frac{1}{4}} \times 5^{\frac{1}{4}} = (3 \times 5)^{\frac{1}{4}} = (15)^{\frac{1}{4}}.$$

(ii)

$$2^{\frac{5}{8}} \times 3^{\frac{5}{8}} = (2 \times 3)^{\frac{5}{8}} = (6)^{\frac{5}{8}}.$$

(iii)

$$6^{\frac{1}{2}} \times 7^{\frac{1}{2}} = (6 \times 7)^{\frac{1}{2}} = (42)^{\frac{1}{2}}.$$

Question 4:

(i)

$$\left(3^4\right)^{\frac{1}{4}} = 3^{\left(4 \times \frac{1}{4}\right)} = (3)^1 = 3.$$

(ii)

$$\left(3^{\frac{1}{3}}\right)^4 = 3^{\left(\frac{1}{3} \times 4\right)} = 3^{\frac{4}{3}}$$

(iii)

$$\left[\frac{1}{3^4}\right]^{\frac{1}{2}} = \left[3^{-4}\right]^{\frac{1}{2}} = 3^{\left(-4 \times \frac{1}{2}\right)} = 3^{-2}.$$

Question 5:

(i)

$$(49)^{\frac{1}{2}} = \left(7^2\right)^{\frac{1}{2}} = 7^{\left(2 \times \frac{1}{2}\right)} = 7^1 = 7.$$

(ii)

$$(125)^{\frac{1}{3}} = \left(5^3\right)^{\frac{1}{3}} = 5^{3 \times \frac{1}{3}} = 5^1 = 5.$$

(iii)

$$(64)^{\frac{1}{6}} = \left(2^6\right)^{\frac{1}{6}} = 2^{\left(6 \times \frac{1}{6}\right)} = 2^1 = 2.$$

Question 6:

(i)

$$(25)^{\frac{3}{2}} = \left(5^2\right)^{\frac{3}{2}} = 5^{\left(2 \times \frac{3}{2}\right)}$$
$$= 5^3 = 125.$$

(ii)

$$(32)^{\frac{2}{5}} = \left(2^5\right)^{\frac{2}{5}} = (2)^{5 \times \frac{2}{5}} = 2^2 = 4.$$

(iii)

$$(81)^{\frac{3}{4}} = \left(3^4\right)^{\frac{3}{4}} = 3^{\left(4 \times \frac{3}{4}\right)} = 3^3 = 27.$$

Question 7:

(i)

$$(64)^{-\frac{1}{2}} = \frac{1}{(64)^{\frac{1}{2}}} = \frac{1}{\left(8^2\right)^{\frac{1}{2}}} = \frac{1}{(8)^{2 \times \frac{1}{2}}}$$

$$= \frac{1}{8^1} = \frac{1}{8}.$$

(ii)

$$(8)^{-\frac{1}{3}} = \frac{1}{(8)^{\frac{1}{3}}} = \frac{1}{\left(2^3\right)^{\frac{1}{3}}} = \frac{1}{2^{\left(3 \times \frac{1}{3}\right)}}$$

$$= \frac{1}{2^1} = \frac{1}{2}.$$

(iii)

$$(81)^{-\frac{1}{4}} = \frac{1}{(81)^{\frac{1}{4}}} = \frac{1}{\left(3^4\right)^{\frac{1}{4}}} = \frac{1}{3^{\left(4 \times \frac{1}{4}\right)}}$$

$$= \frac{1}{3^1} = \frac{1}{3}.$$