

Exercise – 8A

1. (i) $(1 - \cos^2 \theta) \operatorname{cosec}^2 \theta = 1$

(ii) $(1 + \cot^2 \theta) \sin^2 \theta = 1$

Sol:

$$\begin{aligned} \text{(i) } LHS &= (1 - \cos^2 \theta) \operatorname{cosec}^2 \theta \\ &= \sin^2 \theta \operatorname{cosec}^2 \theta \quad (\because \cos^2 \theta + \sin^2 \theta = 1) \\ &= \frac{1}{\operatorname{cosec}^2 \theta} \times \operatorname{cosec}^2 \theta \\ &= 1 \end{aligned}$$

Hence, LHS = RHS

$$\begin{aligned} \text{(ii) } LHS &= (1 + \cot^2 \theta) \sin^2 \theta \\ &= \operatorname{cosec}^2 \theta \sin^2 \theta \quad (\because \operatorname{cosec}^2 \theta - \cot^2 \theta = 1) \\ &= \frac{1}{\sin^2 \theta} \times \sin^2 \theta \\ &= 1 \end{aligned}$$

Hence, LHS = RHS

2. (i) $(\sec^2 \theta - 1) \cot^2 \theta = 1$

(ii) $(\sec^2 \theta - 1)(\operatorname{cosec}^2 \theta - 1) = 1$

(iii) $(1 - \cos^2 \theta) \sec^2 \theta = \tan^2 \theta$

Sol:

$$\begin{aligned} \text{(i) } LHS &= (\sec^2 \theta - 1) \cot^2 \theta \\ &= \tan^2 \theta \times \cot^2 \theta \quad (\because \sec^2 \theta - \tan^2 \theta = 1) \\ &= \frac{1}{\cot^2 \theta} \times \cot^2 \theta \\ &= 1 \\ &= \text{RHS} \end{aligned}$$

$$\begin{aligned} \text{(ii) } LHS &= (\sec^2 \theta - 1)(\operatorname{cosec}^2 \theta - 1) \\ &= \tan^2 \theta \times \cot^2 \theta \quad (\because \sec^2 \theta - \tan^2 \theta = 1 \text{ and } \operatorname{cosec}^2 \theta - \cot^2 \theta = 1) \\ &= \tan^2 \theta \times \frac{1}{\tan^2 \theta} \\ &= 1 \\ &= \text{RHS} \end{aligned}$$

$$\begin{aligned} \text{(iii) } LHS &= (1 - \cos^2 \theta) \sec^2 \theta \\ &= \sin^2 \theta \times \sec^2 \theta \quad (\because \sin^2 \theta + \cos^2 \theta = 1) \\ &= \sin^2 \theta \times \frac{1}{\cos^2 \theta} \\ &= \frac{\sin^2 \theta}{\cos^2 \theta} \\ &= \tan^2 \theta \\ &= \text{RHS} \end{aligned}$$

3. (i) $\sin^2 \theta + \frac{1}{(1 + \tan^2 \theta)} = 1$

(ii) $\frac{1}{(1 + \tan^2 \theta)} + \frac{1}{(1 + \cot^2 \theta)} = 1$

Sol:

$$\begin{aligned} \text{(i) } LHS &= \sin^2 \theta + \frac{1}{(1 + \tan^2 \theta)} \\ &= \sin^2 \theta + \frac{1}{\sec^2 \theta} \quad (\because \sec^2 \theta - \tan^2 \theta = 1) \\ &= \sin^2 \theta + \cos^2 \theta \\ &= 1 \\ &= \text{RHS} \end{aligned}$$

$$\begin{aligned} \text{(ii) } LHS &= \frac{1}{(1 + \tan^2 \theta)} + \frac{1}{(1 + \cot^2 \theta)} \\ &= \frac{1}{\sec^2 \theta} + \frac{1}{\operatorname{cosec}^2 \theta} \\ &= \cos^2 \theta + \sin^2 \theta \\ &= 1 \\ &= \text{RHS} \end{aligned}$$

4. (i) $(1 + \cos \theta)(1 - \cos \theta)(1 + \cos^2 \theta) = 1$

(ii) $\operatorname{cosec} \theta (1 + \cos \theta)(\operatorname{cosec} \theta - \cot \theta) = 1$

Sol:

$$\begin{aligned} \text{(i) } LHS &= (1 + \cos \theta)(1 - \cos \theta)(1 + \cos^2 \theta) \\ &= (1 - \cos^2 \theta) \operatorname{cosec}^2 \theta \\ &= \sin^2 \theta \times \operatorname{cosec}^2 \theta \\ &= \sin^2 \theta \times \frac{1}{\sin^2 \theta} \\ &= 1 \\ &= \text{RHS} \end{aligned}$$

$$\begin{aligned} \text{(ii) } LHS &= \operatorname{cosec} \theta (1 + \cos \theta)(\operatorname{cosec} \theta - \cot \theta) \\ &= (\operatorname{cosec} \theta + \operatorname{cosec} \theta \times \cos \theta)(\operatorname{cosec} \theta - \cot \theta) \\ &= \left(\operatorname{cosec} \theta + \frac{1}{\sin \theta} \times \cos \theta \right)(\operatorname{cosec} \theta - \cot \theta) \\ &= (\operatorname{cosec} \theta + \cot \theta)(\operatorname{cosec} \theta - \cot \theta) \\ &= \operatorname{cosec}^2 \theta - \cot^2 \theta \quad (\because \operatorname{cosec}^2 \theta - \cot^2 \theta = 1) \\ &= 1 \\ &= \text{RHS} \end{aligned}$$

5. (i) $\cot^2 \theta - \frac{1}{\sin^2 \theta} = -1$
 (ii) $\tan^2 \theta - \frac{1}{\cos^2 \theta} = -1$
 (iii) $\cos^2 \theta + \frac{1}{(1 + \cot^2 \theta)} = 1$

Sol:

$$\begin{aligned} \text{(i) } LHS &= \cot^2 \theta - \frac{1}{\sin^2 \theta} \\ &= \frac{\cos^2 \theta}{\sin^2 \theta} - \frac{1}{\sin^2 \theta} \\ &= \frac{\cos^2 \theta - 1}{\sin^2 \theta} \\ &= \frac{-\sin^2 \theta}{\sin^2 \theta} \\ &= -1 \\ &= \text{RHS} \end{aligned}$$

$$\begin{aligned} \text{(ii) } LHS &= \tan^2 \theta - \frac{1}{\cos^2 \theta} \\ &= \frac{\sin^2 \theta}{\cos^2 \theta} - \frac{1}{\cos^2 \theta} \\ &= \frac{\sin^2 \theta - 1}{\cos^2 \theta} \\ &= \frac{-\cos^2 \theta}{\cos^2 \theta} \\ &= -1 \\ &= \text{RHS} \end{aligned}$$

$$\begin{aligned} \text{(iii) } LHS &= \cos^2 \theta + \frac{1}{(1 + \cot^2 \theta)} \\ &= \cos^2 \theta + \frac{1}{\operatorname{cosec}^2 \theta} \\ &= \cos^2 \theta + \sin^2 \theta \\ &= 1 \\ &= \text{RHS} \end{aligned}$$

6. $\frac{1}{(1 + \sin \theta)} + \frac{1}{(1 - \sin \theta)} = 2 \sec^2 \theta$

Sol:

$$\begin{aligned} LHS &= \frac{1}{(1 + \sin \theta)} + \frac{1}{(1 - \sin \theta)} \\ &= \frac{(1 - \sin \theta) + (1 + \sin \theta)}{(1 + \sin \theta)(1 - \sin \theta)} \\ &= \frac{2}{1 - \sin^2 \theta} \\ &= \frac{2}{\cos^2 \theta} \\ &= 2 \sec^2 \theta \\ &= \text{RHS} \end{aligned}$$

7. (i) $\sec \theta (1 - \sin \theta) (\sec \theta + \tan \theta) = 1$
 (ii) $\sin \theta (1 + \tan \theta) + \cos \theta (1 + \cot \theta) = (\sec \theta + \operatorname{cosec} \theta)$

Sol:

$$\begin{aligned}
 \text{(i) } LHS &= \sec \theta (1 - \sin \theta) (\sec \theta + \tan \theta) \\
 &= (\sec \theta - \sec \theta \sin \theta) (\sec \theta + \tan \theta) \\
 &= \left(\sec \theta - \frac{1}{\cos \theta} \times \sin \theta \right) (\sec \theta + \tan \theta) \\
 &= (\sec \theta - \tan \theta) (\sec \theta + \tan \theta) \\
 &= \sec^2 \theta - \tan^2 \theta \\
 &= 1 \\
 &= RHS
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii) } LHS &= \sin \theta (1 + \tan \theta) + \cos \theta (1 + \cot \theta) \\
 &= \sin \theta + \sin \theta \times \frac{\sin \theta}{\cos \theta} + \cos \theta + \cos \theta \times \frac{\cos \theta}{\sin \theta} \\
 &= \frac{\cos \theta \sin^2 \theta + \sin^3 \theta + \cos^2 \theta \sin \theta + \cos^3 \theta}{\cos \theta \sin \theta} \\
 &= \frac{(\sin^3 \theta + \cos^3 \theta) + (\cos \theta \sin^2 \theta + \cos^2 \theta \sin \theta)}{\cos \theta \sin \theta} \\
 &= \frac{(\sin \theta + \cos \theta)(\sin^2 \theta - \sin \theta \cos \theta + \cos^2 \theta) + \sin \theta \cos \theta (\sin \theta + \cos \theta)}{\cos \theta \sin \theta} \\
 &= \frac{(\sin \theta + \cos \theta)(\sin^2 \theta + \cos^2 \theta - \sin \theta \cos \theta + \sin \theta \cos \theta)}{\cos \theta \sin \theta} \\
 &= \frac{(\sin \theta + \cos \theta) (1)}{\cos \theta \sin \theta} \\
 &= \frac{\sin \theta}{\cos \theta \sin \theta} + \frac{\cos \theta}{\cos \theta \sin \theta} \\
 &= \frac{1}{\cos \theta} + \frac{1}{\sin \theta} \\
 &= \sec \theta + \operatorname{cosec} \theta \\
 &= RHS
 \end{aligned}$$

8. (i) $1 + \frac{\cot^2 \theta}{(1 + \operatorname{cosec} \theta)} = \operatorname{cosec} \theta$

(ii) $1 + \frac{\tan^2 \theta}{(1 + \sec \theta)} = \sec \theta$

Sol:

$$\begin{aligned}
 \text{(i) } LHS &= 1 + \frac{\cot^2 \theta}{(1 + \operatorname{cosec} \theta)} \\
 &= 1 + \frac{(\operatorname{cosec}^2 \theta - 1)}{(\operatorname{cosec} \theta + 1)} \quad (\because \operatorname{cosec}^2 \theta - \cot^2 \theta = 1) \\
 &= 1 + \frac{(\operatorname{cosec} \theta + 1)(\operatorname{cosec} \theta - 1)}{(\operatorname{cosec} \theta + 1)} \\
 &= 1 + (\operatorname{cosec} \theta - 1) \\
 &= \operatorname{cosec} \theta
 \end{aligned}$$

= RHS

$$\begin{aligned}
 \text{(ii) } LHS &= 1 + \frac{\tan^2 \theta}{(1+\sec \theta)} \\
 &= 1 + \frac{(\sec^2 \theta - 1)}{(\sec \theta + 1)} \\
 &= 1 + \frac{(\sec \theta + 1)(\sec \theta - 1)}{(\sec \theta + 1)} \\
 &= 1 + (\sec \theta - 1) \\
 &= \sec \theta \\
 &= \text{RHS}
 \end{aligned}$$

9. $1 + \frac{(\tan^2 \theta) \cot \theta}{\operatorname{cosec}^2 \theta} = \tan \theta$

Sol:

$$\begin{aligned}
 LHS &= \frac{(1+\tan^2 \theta) \cot \theta}{\operatorname{cosec}^2 \theta} \\
 &= \frac{\sec^2 \theta \cot \theta}{\operatorname{cosec}^2 \theta} \\
 &= \frac{\frac{1}{\cos^2 \theta} \times \frac{\cos \theta}{\sin \theta}}{\frac{1}{\sin^2 \theta}} \\
 &= \frac{1}{\cos \theta \sin \theta} \times \sin^2 \theta \\
 &= \frac{\sin \theta}{\cos \theta} \\
 &= \tan \theta \\
 &= \text{RHS}
 \end{aligned}$$

Hence, LHS = RHS

10. $\frac{\tan^2 \theta}{(1+\tan^2 \theta)} + \frac{\cot^2 \theta}{(1+\cot^2 \theta)} = 1$

Sol:

$$\begin{aligned}
 LHS &= \frac{\tan^2 \theta}{(1+\tan^2 \theta)} + \frac{\cot^2 \theta}{(1+\cot^2 \theta)} \\
 &= \frac{\tan^2 \theta}{\sec^2 \theta} + \frac{\cot^2 \theta}{\operatorname{cosec}^2 \theta} \quad (\because \sec^2 \theta - \tan^2 \theta = 1 \text{ and } \operatorname{cosec}^2 \theta - \cot^2 \theta = 1) \\
 &= \frac{\frac{\sin^2 \theta}{\cos^2 \theta}}{\frac{1}{\cos^2 \theta}} + \frac{\frac{\cos^2 \theta}{\sin^2 \theta}}{\frac{1}{\sin^2 \theta}} \\
 &= \sin^2 \theta + \cos^2 \theta \\
 &= 1 \\
 &= \text{RHS}
 \end{aligned}$$

Hence, LHS = RHS

$$11. \frac{\sin \theta}{(1 + \cos \theta)} + \frac{(1 + \cos \theta)}{\sin \theta} = 2 \operatorname{cosec} \theta$$

Sol:

$$\begin{aligned} \text{LHS} &= \frac{\sin \theta}{(1 + \cos \theta)} + \frac{(1 + \cos \theta)}{\sin \theta} \\ &= \frac{\sin^2 \theta + (1 + \cos \theta)^2}{(1 + \cos \theta) \sin \theta} \\ &= \frac{\sin^2 \theta + 1 + \cos^2 \theta + 2 \cos \theta}{(1 + \cos \theta) \sin \theta} \\ &= \frac{1 + 1 + 2 \cos \theta}{(1 + \cos \theta) \sin \theta} \\ &= \frac{2 + 2 \cos \theta}{(1 + \cos \theta) \sin \theta} \\ &= \frac{2(1 + \cos \theta)}{(1 + \cos \theta) \sin \theta} \\ &= \frac{2}{\sin \theta} \\ &= 2 \operatorname{cosec} \theta \\ &= \text{RHS} \end{aligned}$$

Hence, L.H.S = R.H.S.

$$12. \frac{\tan \theta}{(1 - \cot \theta)} + \frac{\cot \theta}{(1 - \tan \theta)} = (1 + \sec \theta \operatorname{cosec} \theta)$$

Sol:

$$\begin{aligned} \text{LHS} &= \frac{\tan \theta}{(1 - \cot \theta)} + \frac{\cot \theta}{(1 - \tan \theta)} \\ &= \frac{\tan \theta}{(1 - \frac{\cos \theta}{\sin \theta})} + \frac{\cot \theta}{(1 - \frac{\sin \theta}{\cos \theta})} \\ &= \frac{\sin \theta \tan \theta}{(\sin \theta - \cos \theta)} + \frac{\cos \theta \cot \theta}{(\cos \theta - \sin \theta)} \\ &= \frac{\sin \theta \times \frac{\sin \theta}{\cos \theta} - \cos \theta \times \frac{\cos \theta}{\sin \theta}}{(\sin \theta - \cos \theta)} \\ &= \frac{\frac{\sin^2 \theta}{\cos \theta} - \frac{\cos^2 \theta}{\sin \theta}}{(\sin \theta - \cos \theta)} \\ &= \frac{\frac{\sin^3 \theta - \cos^3 \theta}{\cos \theta \sin \theta}}{(\sin \theta - \cos \theta)} \\ &= \frac{(\sin \theta - \cos \theta)(\sin^2 \theta + \sin \theta \cos \theta + \cos^2 \theta)}{\cos \theta \sin \theta (\sin \theta - \cos \theta)} \\ &= \frac{1 + \sin \theta \cos \theta}{\cos \theta \sin \theta} \\ &= \frac{1}{\cos \theta \sin \theta} + \frac{\sin \theta \cos \theta}{\cos \theta \sin \theta} \\ &= \frac{1}{\cos \theta \sin \theta} + \frac{\sin \theta \cos \theta}{\cos \theta \sin \theta} \\ &= \sec \theta \operatorname{cosec} \theta + 1 \\ &= 1 + \sec \theta \operatorname{cosec} \theta \\ &= \text{RHS} \end{aligned}$$

$$13. \frac{\cos^2 \theta}{(1 - \tan \theta)} + \frac{\sin^3 \theta}{(\sin \theta - \cos \theta)} = (1 + \sin \theta \cos \theta)$$

Sol:

$$\frac{\cos^2 \theta}{(1 - \tan \theta)} + \frac{\sin^3 \theta}{(\sin \theta - \cos \theta)} = (1 + \sin \theta \cos \theta)$$

$$\begin{aligned} LHS &= \frac{\cos^2 \theta}{(1 - \tan \theta)} + \frac{\sin^3 \theta}{(\sin \theta - \cos \theta)} \\ &= \frac{\cos^2 \theta}{\left(1 - \frac{\sin \theta}{\cos \theta}\right)} + \frac{\sin^3 \theta}{(\sin \theta - \cos \theta)} \\ &= \frac{\cos^3 \theta}{(\cos \theta - \sin \theta)} + \frac{\sin^3 \theta}{(\sin \theta - \cos \theta)} \\ &= \frac{\cos^3 \theta - \sin^3 \theta}{(\cos \theta - \sin \theta)} \\ &= \frac{(\cos \theta - \sin \theta)(\cos^2 \theta + \cos \theta \sin \theta + \sin^2 \theta)}{(\cos \theta - \sin \theta)} \\ &= (\sin^2 \theta + \cos^2 \theta + \cos \theta \sin \theta) \\ &= (1 + \sin \theta \cos \theta) \\ &= RHS \end{aligned}$$

Hence, L.H.S = R.H.S.

$$14. \frac{\cos \theta}{(1 - \tan \theta)} + \frac{\sin^2 \theta}{(\cos \theta - \sin \theta)} = (\cos \theta + \sin \theta)$$

Sol:

$$\begin{aligned} LHS &= \frac{\cos \theta}{(1 - \tan \theta)} - \frac{\sin^2 \theta}{(\cos \theta - \sin \theta)} \\ &= \frac{\cos \theta}{\left(1 - \frac{\sin \theta}{\cos \theta}\right)} - \frac{\sin^2 \theta}{(\cos \theta - \sin \theta)} \\ &= \frac{\cos^2 \theta}{(\cos \theta - \sin \theta)} - \frac{\sin^2 \theta}{(\cos \theta - \sin \theta)} \\ &= \frac{\cos^2 \theta - \sin^2 \theta}{(\cos \theta - \sin \theta)} \\ &= \frac{(\cos \theta + \sin \theta)(\cos \theta - \sin \theta)}{(\cos \theta - \sin \theta)} \\ &= (\cos \theta + \sin \theta) \\ &= RHS \end{aligned}$$

Hence, LHS = RHS

$$15. (1 + \tan^2 \theta)(1 + \cot^2 \theta) = \frac{1}{(\sin^2 \theta - \sin^4 \theta)}$$

Sol:

$$\begin{aligned} LHS &= (1 + \tan^2 \theta)(1 + \cot^2 \theta) \\ &= \sec^2 \theta \cdot \operatorname{cosec}^2 \theta \quad (\because \sec^2 \theta - \tan^2 \theta = 1 \text{ and } \operatorname{cosec}^2 \theta - \cot^2 \theta = 1) \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{\cos^2 \theta \cdot \sin^2 \theta} \\
 &= \frac{1}{(1 - \sin^2 \theta) \sin^2 \theta} \\
 &= \frac{1}{\sin^2 \theta - \sin^4 \theta} \\
 &= \text{RHS}
 \end{aligned}$$

Hence, LHS = RHS

16.
$$\frac{\tan \theta}{(1 + \tan^2 \theta)^2} + \frac{\cot \theta}{(1 + \cot^2 \theta)^2} = \sin \theta \cos \theta$$

Sol:

$$\begin{aligned}
 LHS &= \frac{\tan \theta}{(1 + \tan^2 \theta)^2} + \frac{\cot \theta}{(1 + \cot^2 \theta)^2} \\
 &= \frac{\tan \theta}{(\sec^2 \theta)^2} + \frac{\cot \theta}{(\operatorname{cosec}^2 \theta)^2} \\
 &= \frac{\tan \theta}{\sec^4 \theta} + \frac{\cot \theta}{\operatorname{cosec}^4 \theta} \\
 &= \frac{\sin \theta}{\cos \theta} \times \cos^4 \theta + \frac{\cos \theta}{\sin \theta} \times \sin^4 \theta \\
 &= \sin \theta \cos^3 \theta + \cos \theta \sin^3 \theta \\
 &= \sin \theta \cos \theta (\cos^2 \theta + \sin^2 \theta) \\
 &= \sin \theta \cos \theta \\
 &= \text{RHS}
 \end{aligned}$$

17. (i) $\sin^6 \theta + \cos^6 \theta = 1 - 3 \sin^2 \theta \cos^2 \theta$
 (ii) $\sin^2 \theta + \cos^4 \theta = \cos^2 \theta + \sin^4 \theta$
 (iii) $\operatorname{cosec}^4 \theta + \operatorname{cosec}^2 \theta = \cot^4 \theta + \cot^2 \theta$

Sol:

$$\begin{aligned}
 \text{(i) } LHS &= \sin^6 \theta + \cos^6 \theta \\
 &= (\sin^2 \theta)^3 + (\cos^2 \theta)^3 \\
 &= (\sin^2 \theta + \cos^2 \theta) (\sin^4 \theta - \sin^2 \theta \cos^2 \theta + \cos^4 \theta) \\
 &= 1 \times \{(\sin^2 \theta)^2 + 2 \sin^2 \theta \cos^2 \theta + (\cos^2 \theta)^2 - 3 \sin^2 \theta \cos^2 \theta\} \\
 &= (\sin^2 \theta + \cos^2 \theta)^2 - 3 \sin^2 \theta \cos^2 \theta \\
 &= (1)^2 - 3 \sin^2 \theta \cos^2 \theta \\
 &= 1 - 3 \sin^2 \theta \cos^2 \theta \\
 &= \text{RHS}
 \end{aligned}$$

Hence, LHS = RHS

$$\begin{aligned}
 \text{(ii) } LHS &= \sin^2 \theta + \cos^4 \theta \\
 &= \sin^2 \theta + (\cos^2 \theta)^2 \\
 &= \sin^2 \theta + (1 - \sin^2 \theta)^2 \\
 &= \sin^2 \theta + 1 - 2 \sin^2 \theta + \sin^4 \theta \\
 &= 1 - \sin^2 \theta + \sin^4 \theta
 \end{aligned}$$

$$= \cos^2 \theta + \sin^4 \theta$$

$$= \text{RHS}$$

Hence, LHS = RHS

$$\begin{aligned} \text{(iii) } LHS &= \operatorname{cosec}^4 \theta - \operatorname{cosec}^2 \theta \\ &= \operatorname{cosec}^2 \theta (\operatorname{cosec}^2 \theta - 1) \\ &= \operatorname{cosec}^2 \theta \times \cot^2 \theta \quad (\because \operatorname{cosec}^2 \theta - \cot^2 \theta = 1) \\ &= (1 + \cot^2 \theta) \times \cot^2 \theta \\ &= \cot^2 \theta + \cot^4 \theta \\ &= \text{RHS} \end{aligned}$$

Hence, LHS = RHS

$$18. \text{ (i) } \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} = (\cos^2 \theta - \sin^2 \theta)$$

$$\text{(ii) } \frac{1 - \tan^2 \theta}{\cot^2 - 1} = \tan^2 \theta$$

Sol:

$$\begin{aligned} \text{(i) } LHS &= \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} \\ &= \frac{1 - \frac{\sin^2 \theta}{\cos^2 \theta}}{1 + \frac{\sin^2 \theta}{\cos^2 \theta}} \\ &= \frac{\cos^2 \theta - \sin^2 \theta}{\cos^2 \theta + \sin^2 \theta} \\ &= \frac{\cos^2 \theta - \sin^2 \theta}{1} \\ &= \cos^2 \theta - \sin^2 \theta \\ &= \text{RHS} \end{aligned}$$

$$\begin{aligned} \text{(ii) } LHS &= \frac{1 - \tan^2 \theta}{\cot^2 \theta - 1} \\ &= \frac{1 - \frac{\sin^2 \theta}{\cos^2 \theta}}{\frac{\cos^2 \theta}{\sin^2 \theta} - 1} \\ &= \frac{\frac{\cos^2 \theta - \sin^2 \theta}{\cos^2 \theta}}{\frac{\cos^2 \theta - \sin^2 \theta}{\sin^2 \theta}} \\ &= \frac{\sin^2 \theta}{\cos^2 \theta} \\ &= \tan^2 \theta \\ &= \text{RHS} \end{aligned}$$

19. (i) $\frac{\tan \theta}{(\sec \theta - 1)} + \frac{\tan \theta}{(\sec \theta + 1)} = 2 \operatorname{cosec} \theta$

(ii) $\frac{\cot \theta}{(\operatorname{cosec} \theta + 1)} + \frac{(\operatorname{cosec} \theta + 1)}{\cot \theta} = 2 \sec \theta$

Sol:

$$\begin{aligned}
 \text{(i) } LHS &= \frac{\tan \theta}{(\sec \theta - 1)} + \frac{\tan \theta}{(\sec \theta + 1)} \\
 &= \tan \theta \left\{ \frac{\sec \theta + 1 + \sec \theta - 1}{(\sec \theta - 1)(\sec \theta + 1)} \right\} \\
 &= \tan \theta \left\{ \frac{2 \sec \theta}{(\sec^2 \theta - 1)} \right\} \\
 &= \tan \theta \times \frac{2 \sec \theta}{\tan^2 \theta} \\
 &= 2 \frac{\sec \theta}{\tan \theta} \\
 &= 2 \frac{\frac{1}{\cos \theta}}{\frac{\sin \theta}{\cos \theta}} \\
 &= 2 \frac{1}{\sin \theta} \\
 &= 2 \operatorname{cosec} \theta \\
 &= \text{RHS}
 \end{aligned}$$

Hence, LHS = RHS

$$\begin{aligned}
 \text{(ii) } LHS &= \frac{\cot \theta}{(\operatorname{cosec} \theta + 1)} + \frac{(\operatorname{cosec} \theta + 1)}{\cot \theta} \\
 &= \frac{\cot^2 \theta + (\operatorname{cosec} \theta + 1)^2}{(\operatorname{cosec} \theta + 1) \cot \theta} \\
 &= \frac{\cot^2 \theta + \operatorname{cosec}^2 \theta + 2 \operatorname{cosec} \theta + 1}{(\operatorname{cosec} \theta + 1) \cot \theta} \\
 &= \frac{\cot^2 \theta + \operatorname{cosec}^2 \theta + 2 \operatorname{cosec} \theta + \operatorname{cosec}^2 \theta - \cot^2 \theta}{(\operatorname{cosec} \theta + 1) \cot \theta} \\
 &= \frac{2 \operatorname{cosec}^2 \theta + 2 \operatorname{cosec} \theta}{(\operatorname{cosec} \theta + 1) \cot \theta} \\
 &= \frac{2 \operatorname{cosec} \theta (\operatorname{cosec} \theta + 1)}{(\operatorname{cosec} \theta + 1) \cot \theta} \\
 &= \frac{2 \operatorname{cosec} \theta}{\cot \theta} \\
 &= 2 \times \frac{1}{\sin \theta} \times \frac{\sin \theta}{\cos \theta} \\
 &= 2 \sec \theta \\
 &= \text{RHS}
 \end{aligned}$$

Hence, LHS = RHS

20. (i) $\frac{\sec \theta - 1}{\sec \theta + 1} = \frac{\sin^2 \theta}{(1 + \cos \theta)^2}$

(ii) $\frac{\sec \theta - \tan \theta}{\sec \theta + \tan \theta} = \frac{\cos^2 \theta}{(1 + \sin \theta)^2}$

Sol:

$$\begin{aligned} \text{(i) } LHS &= \frac{\sec \theta - 1}{\sec \theta + 1} \\ &= \frac{\frac{1}{\cos \theta} - 1}{\frac{1}{\cos \theta} + 1} \\ &= \frac{\frac{1 - \cos \theta}{\cos \theta}}{\frac{1 + \cos \theta}{\cos \theta}} \\ &= \frac{1 - \cos \theta}{1 + \cos \theta} \end{aligned}$$

$$= \frac{(1 - \cos \theta)(1 + \cos \theta)}{(1 + \cos \theta)(1 + \cos \theta)}$$

$\left\{ \begin{array}{l} \text{Dividing the numerator and} \\ \text{denominator by } (1 + \cos \theta) \end{array} \right\}$

$$\begin{aligned} &= \frac{1 - \cos^2 \theta}{(1 + \cos \theta)^2} \\ &= \frac{\sin^2 \theta}{(1 + \cos \theta)^2} \end{aligned}$$

= RHS

$$\begin{aligned} \text{(ii) } LHS &= \frac{\sec \theta - \tan \theta}{\sec \theta + \tan \theta} \\ &= \frac{\frac{1}{\cos \theta} - \frac{\sin \theta}{\cos \theta}}{\frac{1}{\cos \theta} + \frac{\sin \theta}{\cos \theta}} \\ &= \frac{\frac{1 - \sin \theta}{\cos \theta}}{\frac{1 + \sin \theta}{\cos \theta}} \\ &= \frac{1 - \sin \theta}{1 + \sin \theta} \end{aligned}$$

$$= \frac{(1 - \sin \theta)(1 + \sin \theta)}{(1 + \sin \theta)(1 + \sin \theta)}$$

$\left\{ \begin{array}{l} \text{Dividing the numerator and} \\ \text{denominator by } (1 + \sin \theta) \end{array} \right\}$

$$\begin{aligned} &= \frac{(1 - \sin^2 \theta)}{(1 + \sin \theta)^2} \\ &= \frac{\cos^2 \theta}{(1 + \sin \theta)^2} \end{aligned}$$

= RHS

21. (i) $\sqrt{\frac{1 + \sin \theta}{1 - \sin \theta}} = (\sec \theta + \tan \theta)$

(ii) $\sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} = (\sec \theta - \cot \theta)$

(iii) $\sqrt{\frac{1 + \cos \theta}{1 - \cos \theta}} + \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} = 2 \sec \theta$

Sol:

$$\begin{aligned}
 \text{(i) } LHS &= \sqrt{\frac{1+\sin \theta}{1-\sin \theta}} \\
 &= \sqrt{\frac{(1+\sin \theta)}{(1-\sin \theta)} \times \frac{(1+\sin \theta)}{(1+\sin \theta)}} \\
 &= \sqrt{\frac{(1+\sin \theta)^2}{1-\sin^2 \theta}} \\
 &= \sqrt{\frac{(1+\sin \theta)^2}{\cos^2 \theta}} \\
 &= \frac{1+\sin \theta}{\cos \theta} \\
 &= \frac{1}{\cos \theta} + \frac{\sin \theta}{\cos \theta} \\
 &= (\sec \theta + \tan \theta) \\
 &= \text{RHS}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii) } LHS &= \sqrt{\frac{1-\cos \theta}{1+\cos \theta}} \\
 &= \sqrt{\frac{(1-\cos \theta)}{(1+\cos \theta)} \times \frac{(1-\cos \theta)}{(1-\cos \theta)}} \\
 &= \sqrt{\frac{(1-\cos \theta)^2}{1-\cos^2 \theta}} \\
 &= \sqrt{\frac{(1-\cos \theta)^2}{\sin^2 \theta}} \\
 &= \frac{1-\cos \theta}{\sin \theta} \\
 &= \frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta} \\
 &= (\operatorname{cosec} \theta - \cot \theta) \\
 &= \text{RHS}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii) } LHS &= \sqrt{\frac{1+\cos \theta}{1-\cos \theta}} + \sqrt{\frac{1-\cos \theta}{1+\cos \theta}} \\
 &= \sqrt{\frac{(1+\cos \theta)^2}{(1-\cos \theta)(1+\cos \theta)}} + \sqrt{\frac{(1-\cos \theta)^2}{(1+\cos \theta)(1-\cos \theta)}} \\
 &= \sqrt{\frac{(1+\cos \theta)^2}{(1-\cos^2 \theta)}} + \sqrt{\frac{(1-\cos \theta)^2}{(1-\cos^2 \theta)}} \\
 &= \sqrt{\frac{(1+\cos \theta)^2}{\sin^2 \theta}} + \sqrt{\frac{(1-\cos \theta)^2}{\sin^2 \theta}} \\
 &= \frac{(1+\cos \theta)}{\sin \theta} + \frac{(1-\cos \theta)}{\sin \theta} \\
 &= \frac{1+\cos \theta+1-\cos \theta}{\sin \theta} \\
 &= \frac{2}{\sin \theta} \\
 &= 2 \operatorname{cosec} \theta \\
 &= \text{RHS}
 \end{aligned}$$

$$22. \frac{\cos^3 \theta + \sin^3 \theta}{\cos \theta + \sin \theta} + \frac{\cos^3 \theta - \sin^3 \theta}{\cos \theta - \sin \theta} = 2$$

Sol:

$$\begin{aligned} LHS &= \frac{\cos^3 \theta + \sin^3 \theta}{\cos \theta + \sin \theta} + \frac{\cos^3 \theta - \sin^3 \theta}{\cos \theta - \sin \theta} \\ &= \frac{(\cos \theta + \sin \theta)(\cos^2 \theta - \cos \theta \sin \theta + \sin^2 \theta)}{(\cos \theta + \sin \theta)} + \frac{(\cos \theta - \sin \theta)(\cos^2 \theta + \cos \theta \sin \theta + \sin^2 \theta)}{(\cos \theta - \sin \theta)} \\ &= (\cos^2 \theta + \sin^2 \theta - \cos \theta \sin \theta) + (\cos^2 \theta + \sin^2 \theta + \cos \theta \sin \theta) \\ &= (1 - \cos \theta \sin \theta) + (1 + \cos \theta \sin \theta) \\ &= 2 \\ &= \text{RHS} \end{aligned}$$

Hence, LHS = RHS

$$23. \frac{\sin \theta}{(\cot \theta + \operatorname{cosec} \theta)} - \frac{\sin \theta}{(\cot \theta - \operatorname{cosec} \theta)} = 2$$

Sol:

$$\begin{aligned} LHS &= \frac{\sin \theta}{(\cot \theta + \operatorname{cosec} \theta)} - \frac{\sin \theta}{(\cot \theta - \operatorname{cosec} \theta)} \\ &= \sin \theta \left\{ \frac{(\cot \theta - \operatorname{cosec} \theta) - (\cot \theta + \operatorname{cosec} \theta)}{(\cot \theta + \operatorname{cosec} \theta)(\cot \theta - \operatorname{cosec} \theta)} \right\} \\ &= \sin \theta \left\{ \frac{-2 \operatorname{cosec} \theta}{-1} \right\} \quad (\because \operatorname{cosec}^2 \theta - \cot^2 \theta = 1) \\ &= \sin \theta \cdot 2 \operatorname{cosec} \theta \\ &= \sin \theta \times 2 \times \frac{1}{\sin \theta} \\ &= 2 \\ &= \text{RHS} \end{aligned}$$

$$24. \text{ (i) } \frac{\sin \theta - \cos \theta}{\sin \theta + \cos \theta} + \frac{\sin \theta + \cos \theta}{\sin \theta - \cos \theta} = \frac{2}{(2 \sin^2 \theta - 1)}$$

$$\text{ (ii) } \frac{\sin \theta + \cos \theta}{\sin \theta - \cos \theta} + \frac{\sin \theta - \cos \theta}{\sin \theta + \cos \theta} = \frac{2}{(1 - 2 \cos^2 \theta)}$$

Sol:

$$\begin{aligned} \text{ (i) } LHS &= \frac{\sin \theta - \cos \theta}{\sin \theta + \cos \theta} + \frac{\sin \theta + \cos \theta}{\sin \theta - \cos \theta} \\ &= \frac{(\sin \theta - \cos \theta)^2 + (\sin \theta + \cos \theta)^2}{(\sin \theta + \cos \theta)(\sin \theta - \cos \theta)} \\ &= \frac{\sin^2 \theta + \cos^2 \theta - 2 \sin \theta \cos \theta + \sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta}{\sin^2 \theta - \cos^2 \theta} \\ &= \frac{1+1}{\sin^2 \theta - (1 - \sin^2 \theta)} \quad (\because \sin^2 \theta + \cos^2 \theta = 1) \\ &= \frac{2}{\sin^2 \theta - 1 + \sin^2 \theta} \\ &= \frac{2}{\sin^2 \theta - 1} \end{aligned}$$

= RHS

$$\begin{aligned}
 \text{(ii) } LHS &= \frac{\sin \theta + \cos \theta}{\sin \theta - \cos \theta} + \frac{\sin \theta - \cos \theta}{\sin \theta + \cos \theta} \\
 &= \frac{(\sin \theta + \cos \theta)^2 + (\sin \theta - \cos \theta)^2}{(\sin \theta - \cos \theta)(\sin \theta + \cos \theta)} \\
 &= \frac{\sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta + \sin^2 \theta + \cos^2 \theta - 2 \sin \theta \cos \theta}{(\sin^2 \theta - \cos^2 \theta)} \\
 &= \frac{1+1}{(1-\cos^2 \theta)-\cos^2 \theta} \quad (\because \sin^2 \theta + \cos^2 \theta = 1) \\
 &= \frac{2}{1-2 \cos^2 \theta} \\
 &= RHS
 \end{aligned}$$

25. $\frac{1 + \cos \theta - \sin^2 \theta}{\sin \theta (1 + \cos \theta)} = \cot \theta$

Sol:

$$\begin{aligned}
 LHS &= \frac{1 + \cos \theta - \sin^2 \theta}{\sin \theta (1 + \cos \theta)} \\
 &= \frac{(1 + \cos \theta) - (1 - \cos^2 \theta)}{\sin \theta (1 + \cos \theta)} \\
 &= \frac{\cos \theta + \cos^2 \theta}{\sin \theta (1 + \cos \theta)} \\
 &= \frac{\cos \theta (1 + \cos \theta)}{\sin \theta (1 + \cos \theta)} \\
 &= \frac{\cos \theta}{\sin \theta} \\
 &= \cot \theta \\
 &= RHS
 \end{aligned}$$

Hence, L.H.S. = R.H.S.

26. (i) $\frac{\operatorname{cosec} \theta + \cot \theta}{\operatorname{cosec} \theta - \cot \theta} = (\operatorname{cosec} \theta + \cot \theta)^2 = 1 + 2 \cot^2 \theta + 2 \operatorname{cosec} \theta \cot \theta$

(ii) $\frac{\sec \theta + \tan \theta}{\sec \theta - \tan \theta} = (\sec \theta + \tan \theta)^2 = 1 + 2 \tan^2 \theta + 2 \sec \theta \tan \theta$

Sol:

$$\begin{aligned}
 \text{(i) Here, } &\frac{\operatorname{cosec} \theta + \cot \theta}{\operatorname{cosec} \theta - \cot \theta} \\
 &= \frac{(\operatorname{cosec} \theta + \cot \theta)(\operatorname{cosec} \theta + \cot \theta)}{(\operatorname{cosec} \theta - \cot \theta)(\operatorname{cosec} \theta + \cot \theta)} \\
 &= \frac{(\operatorname{cosec} \theta + \cot \theta)^2}{(\operatorname{cosec}^2 \theta - \cot^2 \theta)} \\
 &= \frac{(\operatorname{cosec} \theta + \cot \theta)^2}{1} \\
 &= (\operatorname{cosec} \theta + \cot \theta)^2
 \end{aligned}$$

$$\begin{aligned}
 \text{Again, } &(\operatorname{cosec} \theta + \cot \theta)^2 \\
 &= \operatorname{cosec}^2 \theta + \cot^2 \theta + 2 \operatorname{cosec} \theta \cot \theta
 \end{aligned}$$

$$= 1 + \cot^2 \theta + \cot^2 \theta + 2 \operatorname{cosec} \theta \cot \theta \quad (\because \operatorname{cosec}^2 \theta - \cot^2 \theta = 1)$$

$$= 1 + 2 \cot^2 \theta + 2 \operatorname{cosec} \theta \cot \theta$$

(ii) Here, $\frac{\sec \theta + \tan \theta}{\sec \theta - \tan \theta}$

$$= \frac{(\sec \theta + \tan \theta)(\sec \theta + \tan \theta)}{(\sec \theta - \tan \theta)(\sec \theta + \tan \theta)}$$

$$= \frac{(\sec \theta + \tan \theta)^2}{\sec^2 \theta - \tan^2 \theta}$$

$$= \frac{(\sec \theta + \tan \theta)^2}{1}$$

$$= (\sec \theta + \tan \theta)^2$$

Again, $(\sec \theta + \tan \theta)^2$

$$= \sec^2 \theta + \tan^2 \theta + 2 \sec \theta \tan \theta$$

$$= 1 + \tan^2 \theta + \tan^2 \theta + 2 \sec \theta \tan \theta$$

$$= 1 + 2 \tan^2 \theta + 2 \sec \theta \tan \theta$$

27. (i) $\frac{1 + \cos \theta + \sin \theta}{1 + \cos \theta - \sin \theta} = \frac{1 + \sin \theta}{\cos \theta}$

(ii) $\frac{\sin \theta + 1 - \cos \theta}{\cos \theta - 1 + \sin \theta} = \frac{1 + \sin \theta}{\cos \theta}$

Sol:

(i) LHS = $\frac{1 + \cos \theta + \sin \theta}{1 + \cos \theta - \sin \theta}$

$$= \frac{\{(1 + \cos \theta) + \sin \theta\} \{(1 + \cos \theta) + \sin \theta\}}{\{(1 + \cos \theta) - \sin \theta\} \{(1 + \cos \theta) + \sin \theta\}}$$

$$= \frac{\{(1 + \cos \theta) + \sin \theta\}^2}{\{(1 + \cos \theta)^2 - \sin^2 \theta\}}$$

$$= \frac{1 + \cos^2 \theta + 2 \cos \theta + \sin^2 \theta + 2 \sin \theta (1 + \cos \theta)}{1 + \cos^2 \theta + 2 \cos \theta - \sin^2 \theta}$$

$$= \frac{2 + 2 \cos \theta + 2 \sin \theta (1 + \cos \theta)}{1 + \cos^2 \theta + 2 \cos \theta - (1 - \cos^2 \theta)}$$

$$= \frac{2(1 + \cos \theta) + 2 \sin \theta (1 + \cos \theta)}{2 \cos^2 \theta + 2 \cos \theta}$$

$$= \frac{2(1 + \cos \theta)(1 + \sin \theta)}{2 \cos \theta (1 + \cos \theta)}$$

$$= \frac{1 + \sin \theta}{\cos \theta}$$

$$= \text{RHS}$$

$\left\{ \begin{array}{l} \text{Multiplying the numerator and} \\ \text{denominator by } (1 + \cos \theta + \sin \theta) \end{array} \right\}$

(ii) LHS = $\frac{\sin \theta + 1 - \cos \theta}{\cos \theta - 1 + \sin \theta}$

$$= \frac{(\sin \theta + 1 - \cos \theta)(\sin \theta + \cos \theta + 1)}{(\cos \theta - 1 + \sin \theta)(\sin \theta + \cos \theta + 1)}$$

$$= \frac{(\sin \theta + 1)^2 - \cos^2 \theta}{(\sin \theta + \cos \theta)^2 - 1^2}$$

$$= \frac{\sin^2 \theta + 1 + 2 \sin \theta - \cos^2 \theta}{\sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta - 1}$$

$\left\{ \begin{array}{l} \text{Multiplying the numerator and} \\ \text{denominator by } (1 + \cos \theta + \sin \theta) \end{array} \right\}$

$$\begin{aligned}
 &= \frac{\sin^2 \theta + \sin^2 \theta + \cos^2 \theta + 2 \sin \theta - \cos^2 \theta}{2 \sin \theta \cos \theta} \\
 &= \frac{2 \sin^2 \theta + 2 \sin \theta}{2 \sin \theta \cos \theta} \\
 &= \frac{2 \sin \theta (1 + \sin \theta)}{2 \sin \theta \cos \theta} \\
 &= \frac{1 + \sin \theta}{\cos \theta} \\
 &= \text{RHS}
 \end{aligned}$$

$$28. \frac{\sin \theta}{(\sec \theta + \tan \theta - 1)} + \frac{\cos \theta}{(\operatorname{cosec} \theta + \cot \theta - 1)} = 1$$

Sol:

$$\begin{aligned}
 LHS &= \frac{\sin \theta}{(\sec \theta + \tan \theta - 1)} + \frac{\cos \theta}{(\operatorname{cosec} \theta + \cot \theta - 1)} \\
 &= \frac{\sin \theta \cos \theta}{1 + \sin \theta - \cos \theta} + \frac{\cos \theta \sin \theta}{1 + \cos \theta - \sin \theta} \\
 &= \sin \theta \cos \theta \left[\frac{1}{1 + (\sin \theta - \cos \theta)} + \frac{1}{1 - (\sin \theta - \cos \theta)} \right] \\
 &= \sin \theta \cos \theta \left[\frac{1 - (\sin \theta - \cos \theta) + 1 + (\sin \theta - \cos \theta)}{\{1 + (\sin \theta - \cos \theta)\}\{1 - (\sin \theta - \cos \theta)\}} \right] \\
 &= \sin \theta \cos \theta \left[\frac{1 - \sin \theta + \cos \theta + 1 + \sin \theta - \cos \theta}{1 - (\sin \theta - \cos \theta)^2} \right] \\
 &= \frac{2 \sin \theta \cos \theta}{1 - (\sin^2 \theta + \cos^2 \theta - 2 \sin \theta \cos \theta)} \\
 &= \frac{2 \sin \theta \cos \theta}{2 \sin \theta \cos \theta} \\
 &= 1 \\
 &= \text{RHS}
 \end{aligned}$$

Hence, LHS = RHS

$$29. \frac{\sin \theta + \cos \theta}{\sin \theta - \cos \theta} + \frac{\sin \theta - \cos \theta}{\sin \theta + \cos \theta} = \frac{2}{(\sin^2 \theta - \cos^2 \theta)} = \frac{2}{(2 \sin^2 \theta - 1)}$$

Sol:

$$\begin{aligned}
 \text{We have } &\frac{\sin \theta + \cos \theta}{\sin \theta - \cos \theta} + \frac{\sin \theta - \cos \theta}{\sin \theta + \cos \theta} \\
 &= \frac{(\sin \theta + \cos \theta)^2 + (\sin \theta - \cos \theta)^2}{(\sin \theta - \cos \theta)(\sin \theta + \cos \theta)} \\
 &= \frac{\sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta + \sin^2 \theta + \cos^2 \theta - 2 \sin \theta \cos \theta}{\sin^2 \theta - \cos^2 \theta} \\
 &= \frac{1 + 1}{\sin^2 \theta - \cos^2 \theta} \\
 &= \frac{2}{\sin^2 \theta - \cos^2 \theta} \\
 \text{Again, } &\frac{2}{\sin^2 \theta - \cos^2 \theta} \\
 &= \frac{2}{\sin^2 \theta - (1 - \sin^2 \theta)} \\
 &= \frac{2}{2 \sin^2 \theta - 1}
 \end{aligned}$$

$$30. \frac{\cos \theta \operatorname{cosec} \theta - \sin \theta \sec \theta}{\cos \theta + \sin \theta} = \operatorname{cosec} \theta - \sec \theta$$

Sol:

$$\begin{aligned} LHS &= \frac{\cos \theta \operatorname{cosec} \theta - \sin \theta \sec \theta}{\cos \theta + \sin \theta} \\ &= \frac{\cos \theta \cdot \frac{1}{\sin \theta} - \sin \theta \cdot \frac{1}{\cos \theta}}{\cos \theta + \sin \theta} \\ &= \frac{\frac{\cos \theta}{\sin \theta} - \frac{\sin \theta}{\cos \theta}}{\cos \theta + \sin \theta} \\ &= \frac{\frac{\cos^2 \theta - \sin^2 \theta}{\sin \theta \cos \theta}}{\cos \theta + \sin \theta} \\ &= \frac{(\cos \theta + \sin \theta)(\cos \theta - \sin \theta)}{\cos \theta \sin \theta (\cos \theta + \sin \theta)} \\ &= \frac{(\cos \theta - \sin \theta)}{\cos \theta \sin \theta} \\ &= \frac{1}{\sin \theta} - \frac{1}{\cos \theta} \\ &= \operatorname{cosec} \theta - \sec \theta \\ &= \text{RHS} \end{aligned}$$

Hence, LHS = RHS

$$31. (1 + \tan \theta + \cot \theta)(\sin \theta - \cos \theta) = \left(\frac{\sec \theta}{\cos^2 \theta} - \frac{\operatorname{cosec} \theta}{\sec^2 \theta} \right)$$

Sol:

$$\begin{aligned} LHS &= (1 + \tan \theta + \cot \theta)(\sin \theta - \cos \theta) \\ &= \sin \theta + \tan \theta \sin \theta + \cot \theta \sin \theta - \cos \theta - \tan \theta \cos \theta - \cot \theta \cos \theta \\ &= \sin \theta + \tan \theta \sin \theta + \frac{\cos \theta}{\sin \theta} \times \sin \theta - \cos \theta - \frac{\sin \theta}{\cos \theta} \times \cos \theta - \cot \theta \cos \theta \\ &= \sin \theta + \tan \theta \sin \theta + \cos \theta - \cos \theta - \sin \theta - \cot \theta \cos \theta \\ &= \tan \theta \sin \theta - \cot \theta \cos \theta \\ &= \frac{\sin \theta}{\cos \theta} \times \frac{1}{\operatorname{cosec} \theta} - \frac{\cos \theta}{\sin \theta} \times \frac{1}{\sec \theta} \\ &= \frac{1}{\operatorname{cosec} \theta} \times \frac{1}{\operatorname{cosec} \theta} \times \sec \theta - \frac{1}{\sec \theta} \times \frac{1}{\sec \theta} \times \operatorname{cosec} \theta \\ &= \frac{\sec \theta}{\operatorname{cosec}^2 \theta} - \frac{\operatorname{cosec} \theta}{\sec^2 \theta} \\ &= \text{RHS} \end{aligned}$$

Hence, LHS = RHS

$$32. \frac{\cot^2 \theta (\sec \theta - 1)}{(1 + \sin \theta)} + \frac{\sec^2 \theta (\sin \theta - 1)}{(1 + \sec \theta)} = 0$$

Sol:

$$\begin{aligned} LHS &= \frac{\cot^2 \theta (\sec \theta - 1)}{(1 + \sin \theta)} + \frac{\sec^2 \theta (\sin \theta - 1)}{(1 + \sec \theta)} \\ &= \frac{\frac{\cos^2 \theta}{\sin^2 \theta} \left(\frac{1}{\cos \theta} - 1 \right)}{(1 + \sin \theta)} + \frac{\frac{1}{\cos^2 \theta} (\sin \theta - 1)}{\left(1 + \frac{1}{\cos \theta} \right)} \end{aligned}$$

$$\begin{aligned}
&= \frac{\frac{\cos^2 \theta (1-\cos \theta)}{\sin^2 \theta (\frac{1-\cos \theta}{\cos \theta})}}{(1+\sin \theta)} + \frac{\frac{(\sin \theta - 1)}{\cos^2 \theta (\frac{\cos \theta + 1}{\cos \theta})}}{(\cos \theta + 1)} \\
&= \frac{\cos^2 \theta (1-\cos \theta)}{\sin^2 \theta \cos \theta (1+\sin \theta)} + \frac{(\sin \theta - 1) \cos \theta}{(\cos \theta + 1) \cos^2 \theta} \\
&= \frac{\cos \theta (1-\cos \theta)}{(1-\cos^2 \theta)(1+\sin \theta)} + \frac{(\sin \theta - 1) \cos \theta}{(\cos \theta + 1)(1-\sin^2 \theta)} \\
&= \frac{\cos \theta (1-\cos \theta)}{(1-\cos \theta)(1+\cos \theta)(1+\sin \theta)} + \frac{-(1-\sin \theta) \cos \theta}{(\cos \theta + 1)(1-\sin \theta)(1+\sin \theta)} \\
&= \frac{\cos \theta}{(1+\cos \theta)(1+\sin \theta)} - \frac{\cos \theta}{(\cos \theta + 1)(1+\sin \theta)} \\
&= \theta \\
&= \text{RHS}
\end{aligned}$$

$$33. \left\{ \frac{1}{(\sec^2 \theta - \cos^2 \theta)} + \frac{1}{(\operatorname{cosec}^2 \theta - \sin^2 \theta)} \right\} (\sin^2 \theta \cos^2 \theta) = \frac{1 - \sin^2 \theta \cos^2 \theta}{2 + \sin^2 \theta \cos^2 \theta}$$

Sol:

$$\begin{aligned}
LHS &= \left\{ \frac{1}{\sec^2 \theta - \cos^2 \theta} + \frac{1}{\operatorname{cosec}^2 \theta - \sin^2 \theta} \right\} (\sin^2 \theta \cos^2 \theta) \\
&= \left\{ \frac{\cos^2 \theta}{1 - \cos^4 \theta} + \frac{\sin^2 \theta}{1 - \sin^4 \theta} \right\} (\sin^2 \theta \cos^2 \theta) \\
&= \left\{ \frac{\cos^2 \theta}{(1 - \cos^2 \theta)(1 + \cos^2 \theta)} + \frac{\sin^2 \theta}{(1 - \sin^2 \theta)(1 + \sin^2 \theta)} \right\} (\sin^2 \theta \cos^2 \theta) \\
&= \left[\frac{\cot^2 \theta}{1 + \cos^2 \theta} + \frac{\tan^2 \theta}{1 + \sin^2 \theta} \right] \sin^2 \theta \cos^2 \theta \\
&= \frac{\cos^4 \theta}{1 + \cos^2 \theta} + \frac{\sin^4 \theta}{1 + \sin^2 \theta} \\
&= \frac{(\cos^2 \theta)^2}{1 + \cos^2 \theta} + \frac{(\sin^2 \theta)^2}{1 + \sin^2 \theta} \\
&= \frac{(1 - \sin^2 \theta)}{1 + \cos^2 \theta} + \frac{(1 - \cos^2 \theta)^2}{1 + \sin^2 \theta} \\
&= \frac{(1 - \sin^2 \theta)^2 (1 + \sin^2 \theta) + (1 - \cos^2 \theta)^2 (1 + \cos^2 \theta)}{(1 + \sin^2 \theta)(1 + \cos^2 \theta)} \\
&= \frac{\cos^4 \theta (1 + \sin^2 \theta) + \sin^4 \theta (1 + \cos^2 \theta)}{(1 + \sin^2 \theta + \cos^2 \theta + \sin^2 \theta \cos^2 \theta) + \sin^4 \theta + \sin^4 \theta \cos^2 \theta} \\
&= \frac{1 + \sin^2 \theta + \cos^2 \theta + \sin^2 \theta \cos^2 \theta}{\cos^4 \theta \cos^4 \theta \sin^2 \theta + \sin^4 \theta + \sin^4 \theta \cos^2 \theta} \\
&= \frac{1 + 1 \sin^2 \theta \cos^2 \theta}{\cos^4 \theta + \sin^4 \theta + \sin^2 \theta \cos^2 \theta (\sin^2 \theta + \cos^2 \theta)} \\
&= \frac{2 + \sin^2 \theta \cos^2 \theta}{(\cos^2 \theta)^2 + (\sin^2 \theta)^2 + \sin^2 \theta \cos^2 \theta (1)} \\
&= \frac{2 + \sin^2 \theta \cos^2 \theta}{(\cos^2 \theta + \sin^2 \theta)^2 - 2 \sin^2 \theta \cos^2 \theta + \sin^2 \theta \cos^2 \theta (1)} \\
&= \frac{2 + \sin^2 \theta \cos^2 \theta}{1^2 + \cos^2 \theta \sin^2 \theta - 2 \cos^2 \theta \sin^2 \theta} \\
&= \frac{2 + \sin^2 \theta \cos^2 \theta}{1 - \cos^2 \theta \sin^2 \theta} \\
&= \frac{2 + \sin^2 \theta \cos^2 \theta}{2 + \sin^2 \theta \cos^2 \theta} \\
&= \text{RHS}
\end{aligned}$$

$$34. \frac{(\sin A - \sin B)}{(\cos A + \cos B)} + \frac{(\cos A - \cos B)}{(\sin A + \sin B)} = 0$$

Sol:

$$\begin{aligned} LHS &= \frac{(\sin A - \sin B)}{(\cos A + \cos B)} + \frac{(\cos A - \cos B)}{(\sin A + \sin B)} \\ &= \frac{(\sin A - \sin B)(\sin A + \sin B) + (\cos A - \cos B)(\cos A - \cos B)}{(\cos A + \cos B)(\sin A + \sin B)} \\ &= \frac{\sin^2 A - \sin^2 B + \cos^2 A - \cos^2 B}{(\cos A + \cos B)(\sin A + \sin B)} \\ &= \frac{0}{(\cos A + \cos B)(\sin A + \sin B)} \\ &= 0 \\ &= RHS \end{aligned}$$

$$35. \frac{\tan A + \tan B}{\cot A + \cot B} = \tan A \tan B$$

Sol:

$$\begin{aligned} LHS &= \frac{\tan A + \tan B}{\cot A + \cot B} \\ &= \frac{\tan A + \tan B}{\frac{1}{\tan A} + \frac{1}{\tan B}} \\ &= \frac{\tan A + \tan B}{\frac{\tan A + \tan B}{\tan A \tan B}} \\ &= \frac{\tan A \tan B (\tan A + \tan B)}{(\tan A + \tan B)} \\ &= \tan A \tan B \\ &= RHS \end{aligned}$$

Hence, LHS = RHS

36. Show that none of the following is an identity:

(i) $\cos^2 \theta + \cos \theta = 1$

(ii) $\sin^2 \theta + \sin \theta = 2$

(iii) $\tan^2 \theta + \sin \theta = \cos^2 \theta$

Sol:

(i) $\cos^2 \theta + \cos \theta = 1$

$$\begin{aligned} LHS &= \cos^2 \theta + \cos \theta \\ &= 1 - \sin^2 \theta + \cos \theta \\ &= 1 - (\sin^2 \theta - \cos \theta) \end{aligned}$$

Since LHS \neq RHS, this not an identity.

(ii) $\sin^2 \theta + \sin \theta = 1$

$$\begin{aligned} LHS &= \sin^2 \theta + \sin \theta \\ &= 1 - \cos^2 \theta + \sin \theta \end{aligned}$$

$$= 1 - (\cos^2 \theta - \sin \theta)$$

Since LHS \neq RHS, this is not an identity.

$$(iii) \tan^2 \theta + \sin \theta = \cos^2 \theta$$

$$LHS = \tan^2 \theta + \sin \theta$$

$$= \frac{\sin^2 \theta}{\cos^2 \theta} + \sin \theta$$

$$= \frac{1 - \cos^2 \theta}{\cos^2 \theta} + \sin \theta$$

$$= \sec^2 \theta - 1 + \sin \theta$$

Since LHS \neq RHS, this is not an identity.

$$37. \quad \text{Prove that } (\sin \theta - 2 \sin^3 \theta) = (2 \cos^3 \theta - \cos \theta) \tan \theta$$

Sol:

$$\begin{aligned} RHS &= (2 \cos^3 \theta - \cos \theta) \tan \theta \\ &= (2 \cos^2 \theta - 1) \cos \theta \times \frac{\sin \theta}{\cos \theta} \\ &= [2(1 - \sin^2 \theta) - 1] \sin \theta \\ &= (2 - 2 \sin^2 \theta - 1) \sin \theta \\ &= (1 - 2 \sin^2 \theta) \sin \theta \\ &= (\sin \theta - 2 \sin^3 \theta) \\ &= LHS \end{aligned}$$

Exercise – 8B

$$1. \quad \text{If } a \cos \theta + b \sin \theta = m \text{ and } a \sin \theta - b \cos \theta = n, \text{ prove that, } (m^2 + n^2) = (a^2 + b^2)$$

Sol:

$$\begin{aligned} \text{We have } m^2 + n^2 &= [(a \cos \theta + b \sin \theta)^2 + (a \sin \theta - b \cos \theta)^2] \\ &= (a^2 \cos^2 \theta + b^2 \sin^2 \theta + 2ab \cos \theta \sin \theta) \\ &\quad + (a^2 \sin^2 \theta + b^2 \cos^2 \theta - 2ab \cos \theta \sin \theta) \\ &= a^2 \cos^2 \theta + b^2 \sin^2 \theta + a^2 \sin^2 \theta + b^2 \cos^2 \theta \\ &= (a^2 \cos^2 \theta + b^2 \sin^2 \theta) + (b^2 \cos^2 \theta + a^2 \sin^2 \theta) \\ &= a^2 (\cos^2 \theta + \sin^2 \theta) + b^2 (\cos^2 \theta + \sin^2 \theta) \\ &= a^2 + b^2 \quad [\because \sin^2 + \cos^2 = 1] \end{aligned}$$

$$\text{Hence, } m^2 + n^2 = a^2 + b^2$$

$$2. \quad \text{If } x = a \sec \theta + b \tan \theta \text{ and } y = a \tan \theta + b \sec \theta, \text{ prove that } (x^2 - y^2) = (a^2 - b^2).$$

Sol:

$$\begin{aligned} \text{We have } x^2 - y^2 &= [(a \sec \theta + b \tan \theta)^2 - (a \tan \theta + b \sec \theta)^2] \\ &= (a^2 \sec^2 \theta + b^2 \tan^2 \theta + 2ab \sec \theta \tan \theta) \end{aligned}$$

$$\begin{aligned}
 & -(a^2 \tan^2 \theta + b^2 \sec^2 \theta + 2ab \tan \theta \sec \theta) \\
 & = a^2 \sec^2 \theta + b^2 \tan^2 \theta - a^2 \tan^2 \theta - b^2 \sec^2 \theta \\
 & = (a^2 \sec^2 \theta - a^2 \tan^2 \theta) - (b^2 \sec^2 \theta - b^2 \tan^2 \theta) \\
 & = a^2 (\sec^2 \theta - \tan^2 \theta) - b^2 (\sec^2 \theta - \tan^2 \theta) \\
 & = a^2 - b^2 \quad [\because \sec^2 \theta - \tan^2 \theta = 1]
 \end{aligned}$$

Hence, $x^2 - y^2 = a^2 - b^2$

3. If $\left(\frac{x}{a} \sin \theta - \frac{y}{b} \cos \theta\right) = 1$ and $\left(\frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta\right) = 1$, prove that $\left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right) = 2$.

Sol:

We have $\left(\frac{x}{a} \sin \theta - \frac{y}{b} \cos \theta\right) = 1$

Squaring both side, we have:

$$\left(\frac{x}{a} \sin \theta - \frac{y}{b} \cos \theta\right)^2 = (1)^2$$

$$\Rightarrow \left(\frac{x^2}{a^2} \sin^2 \theta + \frac{y^2}{b^2} \cos^2 \theta - 2 \frac{x}{a} \times \frac{y}{b} \sin \theta \cos \theta\right) = 1 \quad \dots (i)$$

Again, $\left(\frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta\right) = 1$

Squaring both side, we get:

$$\left(\frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta\right)^2 = (1)^2$$

$$\Rightarrow \left(\frac{x^2}{a^2} \cos^2 \theta + \frac{y^2}{b^2} \sin^2 \theta + 2 \frac{x}{a} \times \frac{y}{b} \sin \theta \cos \theta\right) = \dots (ii)$$

Now, adding (i) and (ii), we get:

$$\left(\frac{x^2}{a^2} \sin^2 \theta + \frac{y^2}{b^2} \cos^2 \theta - 2 \frac{x}{a} \times \frac{y}{b} \sin \theta \cos \theta\right) + \left(\frac{x^2}{a^2} \cos^2 \theta + \frac{y^2}{b^2} \sin^2 \theta + 2 \frac{x}{a} \times \frac{y}{b} \sin \theta \cos \theta\right)$$

$$\Rightarrow \frac{x^2}{a^2} \sin^2 \theta + \frac{y^2}{b^2} \cos^2 \theta + \frac{x^2}{a^2} \cos^2 \theta + \frac{y^2}{b^2} \sin^2 \theta = 2$$

$$\Rightarrow \left(\frac{x^2}{a^2} \sin^2 \theta + \frac{x^2}{a^2} \cos^2 \theta\right) + \left(\frac{y^2}{b^2} \cos^2 \theta + \frac{y^2}{b^2} \sin^2 \theta\right) = 2$$

$$\Rightarrow \frac{x^2}{a^2} (\sin^2 \theta + \cos^2 \theta) + \frac{y^2}{b^2} (\cos^2 \theta + \sin^2 \theta) = 2$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 2 \quad [\because \sin^2 \theta + \cos^2 \theta = 1]$$

$$\therefore \frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$$

4. If $(\sec \theta + \tan \theta) = m$ and $(\sec \theta - \tan \theta) = n$, show that $mn = 1$.

Sol:

We have $(\sec \theta + \tan \theta) = m \quad \dots (i)$

Again, $(\sec \theta - \tan \theta) = n \quad \dots (ii)$

Now, multiplying (i) and (ii), we get:

$$(\sec \theta + \tan \theta) \times (\sec \theta - \tan \theta) = mn$$

$$\begin{aligned}
 &\Rightarrow \sec^2 \theta - \tan^2 \theta = mn \\
 &\Rightarrow 1 = mn \quad [\because \sec^2 \theta - \tan^2 \theta = 1] \\
 &\therefore mn = 1
 \end{aligned}$$

5. If $(\operatorname{cosec} \theta + \cot \theta) = m$ and $(\operatorname{cosec} \theta - \cot \theta) = n$, show that $mn = 1$.

Sol:

$$\text{We have } (\operatorname{cosec} \theta + \cot \theta) = m \quad \dots (i)$$

$$\text{Again, } (\operatorname{cosec} \theta - \cot \theta) = n \quad \dots (ii)$$

Now, multiplying (i) and (ii), we get:

$$(\operatorname{cosec} \theta + \cot \theta) \times (\operatorname{cosec} \theta - \cot \theta) = mn$$

$$\Rightarrow \operatorname{cosec}^2 \theta - \cot^2 \theta = mn$$

$$\Rightarrow 1 = mn \quad [\because \operatorname{cosec}^2 \theta - \cot^2 \theta = 1]$$

$$\therefore mn = 1$$

6. If $x = a \cos^3 \theta$ and $y = b \sin^3 \theta$, prove that $\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}} = 1$.

Sol:

$$\text{We have } x = a \cos^3 \theta$$

$$\Rightarrow \frac{x}{a} = \cos^3 \theta \quad \dots (i)$$

$$\text{Again, } y = b \sin^3 \theta$$

$$\Rightarrow \frac{y}{b} = \sin^3 \theta \quad \dots (ii)$$

$$\text{Now, LHS} = \left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}}$$

$$= (\cos^3 \theta)^{\frac{2}{3}} + (\sin^3 \theta)^{\frac{2}{3}} \quad [\text{From (i) and (ii)}]$$

$$= \cos^2 \theta + \sin^2 \theta$$

$$= 1$$

$$\text{Hence, LHS} = \text{RHS}$$

7. If $(\tan \theta + \sin \theta) = m$ and $(\tan \theta - \sin \theta) = n$, prove that $(m^2 - n^2)^2 = 16 mn$.

Sol:

$$\text{We have } (\tan \theta + \sin \theta) = m \text{ and } (\tan \theta - \sin \theta) = n$$

$$\text{Now, LHS} = (m^2 - n^2)^2$$

$$= [(\tan \theta + \sin \theta)^2 - (\tan \theta - \sin \theta)^2]^2$$

$$= [(\tan^2 \theta + \sin^2 \theta + 2 \tan \theta \sin \theta) - (\tan^2 \theta + \sin^2 \theta - 2 \tan \theta \sin \theta)]^2$$

$$= [(\tan^2 \theta + \sin^2 \theta + 2 \tan \theta \sin \theta - \tan^2 \theta - \sin^2 \theta + 2 \tan \theta \sin \theta)]^2$$

$$= (4 \tan \theta \sin \theta)^2$$

$$= 16 \tan^2 \theta \sin^2 \theta$$

$$\begin{aligned}
&= 16 \frac{\sin^2 \theta}{\cos^2 \theta} \sin^2 \theta \\
&= 16 \frac{(1 - \cos^2 \theta) \sin^2 \theta}{\cos^2 \theta} \\
&= 16[\tan^2 \theta(1 - \cos^2 \theta)] \\
&= 16(\tan^2 \theta - \tan^2 \theta \cos^2 \theta) \\
&= 16(\tan^2 \theta - \frac{\sin^2 \theta}{\cos^2 \theta} \times \cos^2 \theta) \\
&= 16(\tan^2 \theta - \sin^2 \theta) \\
&= 16(\tan \theta + \sin \theta)(\tan \theta - \sin \theta) \\
&= 16mn \quad [(\tan \theta + \sin \theta)(\tan \theta - \sin \theta) = mn] \\
\therefore (m^2 - n^2)(m^2 - n^2)^2 &= 16mn
\end{aligned}$$

8. If $(\cot \theta + \tan \theta) = m$ and $(\sec \theta - \cos \theta) = n$ prove that $(m^2 n)^{\frac{2}{3}} - (mn^2)^{\frac{2}{3}} = 1$

Sol:

We have $(\cot \theta + \tan \theta) = m$ and $(\sec \theta - \cos \theta) = n$

Now, $m^2 n = [(\cot \theta + \tan \theta)^2 (\sec \theta - \cos \theta)]$

$$\begin{aligned}
&= \left[\left(\frac{1}{\tan \theta} + \tan \theta \right)^2 \left(\frac{1}{\cos \theta} - \cos \theta \right) \right] \\
&= \frac{(1 + \tan^2 \theta)^2}{\tan^2 \theta} \times \frac{(1 - \cos^2 \theta)}{\cos \theta} \\
&= \frac{\sec^4 \theta}{\tan^2 \theta} \times \frac{\sin^2 \theta}{\cos \theta} \\
&= \frac{\sec^4 \theta}{\frac{\sin^2 \theta}{\cos^2 \theta}} \times \frac{\sin^2 \theta}{\cos \theta} \\
&= \frac{\cos^2 \theta \times \sec^4 \theta}{\cos \theta} \\
&= \cos \theta \sec^4 \theta \\
&= \frac{1}{\sec \theta} \times \sec^4 \theta = \sec^3 \theta
\end{aligned}$$

$$\therefore (m^2 n)^{\frac{2}{3}} = (\sec^3 \theta)^{\frac{2}{3}} = \sec^2 \theta$$

Again, $mn^2 = [(\cot \theta + \tan \theta)(\sec \theta - \cos \theta)^2]$

$$\begin{aligned}
&= \left[\left(\frac{1}{\tan \theta} + \tan \theta \right) \cdot \left(\frac{1}{\cos \theta} - \cos \theta \right)^2 \right] \\
&= \frac{(1 + \tan^2 \theta)}{\tan \theta} \times \frac{(1 - \cos^2 \theta)^2}{\cos^2 \theta} \\
&= \frac{\sec^2 \theta}{\tan \theta} \times \frac{\sin^4 \theta}{\cos^2 \theta} \\
&= \frac{\sec^2 \theta}{\frac{\sin \theta}{\cos \theta}} \times \frac{\sin^4 \theta}{\cos^2 \theta} \\
&= \frac{\sec^2 \theta \times \sin^3 \theta}{\cos \theta} \\
&= \frac{1}{\cos^2 \theta} \times \frac{\sec^3 \theta}{\cos \theta} = \tan^3 \theta
\end{aligned}$$

$$\therefore (mn^2)^{\frac{2}{3}} = (\tan^3 \theta)^{\frac{2}{3}} = \tan^2 \theta$$

$$\begin{aligned}
 \text{Now, } (m^2n)^{\frac{2}{3}} - (mn^2)^{\frac{2}{3}} \\
 = \sec^2 \theta - \tan^2 \theta = 1 \\
 = \text{RHS}
 \end{aligned}$$

Hence proved.

9. If $(\operatorname{cosec} \theta - \sin \theta) = a^3$ and $(\sec \theta - \cos \theta) = b^3$, prove that $a^2 b^2 (a^2 + b^2) = 1$

Sol:

$$\text{We have } (\operatorname{cosec} \theta - \sin \theta) = a^3$$

$$\Rightarrow a^3 = \left(\frac{1}{\sin \theta} - \sin \theta \right)$$

$$\Rightarrow a^3 = \frac{(1 - \sin^2 \theta)}{\sin \theta} = \frac{\cos^2 \theta}{\sin \theta}$$

$$\therefore a = \frac{\cos^{\frac{2}{3}} \theta}{\sin^{\frac{1}{3}} \theta}$$

$$\text{Again, } (\sec \theta - \cos \theta) = b^3$$

$$\Rightarrow b^3 = \left(\frac{1}{\cos \theta} - \cos \theta \right)$$

$$= \frac{(1 - \cos^2 \theta)}{\cos \theta}$$

$$= \frac{\sin^2 \theta}{\cos \theta}$$

$$\therefore b = \frac{\sin^{\frac{2}{3}} \theta}{\cos^{\frac{1}{3}} \theta}$$

$$\text{Now, LHS} = a^2 b^2 (a^2 + b^2)$$

$$= a^4 b^2 + a^2 b^4$$

$$= a^3 (ab^2) + (a^2 b^2) b^3$$

$$= \frac{\cos^2 \theta}{\sin \theta} \times \left[\frac{\cos^{\frac{2}{3}} \theta}{\sin^{\frac{1}{3}} \theta} \times \frac{\sin^{\frac{4}{3}} \theta}{\cos^{\frac{2}{3}} \theta} \right] + \left[\frac{\cos^{\frac{4}{3}} \theta}{\sin^{\frac{2}{3}} \theta} \times \frac{\sin^{\frac{2}{3}} \theta}{\cos^{\frac{1}{3}} \theta} \right] \times \frac{\sin^2 \theta}{\cos \theta}$$

$$= \frac{\cos^2 \theta}{\sin \theta} \times \sin \theta + \cos \theta \times \frac{\sin^2 \theta}{\cos \theta}$$

$$= \cos^2 \theta + \sin^2 \theta = 1$$

$$= \text{RHS}$$

Hence, proved.

10. If $(2 \sin \theta + 3 \cos \theta) = 2$, prove that $(3 \sin \theta - 2 \cos \theta) = \pm 3$.

Sol:

$$\text{Given, } (2 \sin \theta + 3 \cos \theta) = 2 \quad \dots (i)$$

$$\text{We have } (2 \sin \theta + 3 \cos \theta)^2 + (3 \sin \theta - 2 \cos \theta)^2$$

$$= 4 \sin^2 \theta + 9 \cos^2 \theta + 12 \sin \theta \cos \theta + 9 \sin^2 \theta + 4 \cos^2 \theta - 12 \sin \theta \cos \theta$$

$$= 4(\sin^2 \theta + \cos^2 \theta) + 9(\sin^2 \theta + \cos^2 \theta)$$

$$= 4 + 9$$

$$= 13$$

$$\begin{aligned}
 \text{i.e., } (2 \sin \theta + 3 \cos \theta)^2 + (3 \sin \theta - 2 \cos \theta)^2 &= 13 \\
 &\Rightarrow 2^2 + (3 \sin \theta - 2 \cos \theta)^2 = 13 \\
 &\Rightarrow (3 \sin \theta - 2 \cos \theta)^2 = 13 - 4 \\
 &\Rightarrow (3 \sin \theta - 2 \cos \theta)^2 = 9 \\
 &\Rightarrow (3 \sin \theta - 2 \cos \theta) = \pm 3
 \end{aligned}$$

11. If $(\sin \theta + \cos \theta) = \sqrt{2}$, prove that $\cot \theta = (\sqrt{2} + 1)$.

Sol:

We have, $(\sin \theta + \cos \theta) = \sqrt{2} \cos \theta$

Dividing both sides by $\sin \theta$, we get

$$\frac{\sin \theta}{\sin \theta} + \frac{\cos \theta}{\sin \theta} = \frac{\sqrt{2} \cos \theta}{\sin \theta}$$

$$\Rightarrow 1 + \cot \theta = \sqrt{2} \cot \theta$$

$$\Rightarrow \sqrt{2} \cot \theta - \cot \theta = 1$$

$$\Rightarrow (\sqrt{2} - 1) \cot \theta = 1$$

$$\Rightarrow \cot \theta = \frac{1}{(\sqrt{2}-1)}$$

$$\Rightarrow \cot \theta = \frac{1}{(\sqrt{2}-1)} \times \frac{(\sqrt{2}+1)}{(\sqrt{2}+1)}$$

$$\Rightarrow \cot \theta = \frac{(\sqrt{2}+1)}{2-1}$$

$$\Rightarrow \cot \theta = \frac{(\sqrt{2}+1)}{1}$$

$$\therefore \cot \theta = (\sqrt{2} + 1)$$

12. If $(\cos \theta + \sin \theta) = \sqrt{2} \sin \theta$, prove that $(\sin \theta - \cos \theta) = \sqrt{2} \cos \theta$.

Sol:

Given: $\cos \theta + \sin \theta = \sqrt{2} \sin \theta$

We have $(\sin \theta + \cos \theta)^2 + (\sin \theta - \cos \theta)^2 = 2(\sin^2 \theta + \cos^2 \theta)$

$$\Rightarrow (\sqrt{2} \sin \theta)^2 + (\sin \theta - \cos \theta)^2 = 2$$

$$\Rightarrow 2 \sin^2 \theta + (\sin \theta - \cos \theta)^2 = 2$$

$$\Rightarrow (\sin \theta - \cos \theta)^2 = 2 - 2 \sin^2 \theta$$

$$\Rightarrow (\sin \theta - \cos \theta)^2 = 2(1 - \sin^2 \theta)$$

$$\Rightarrow (\sin \theta - \cos \theta)^2 = 2 \cos^2 \theta$$

$$\Rightarrow (\sin \theta - \cos \theta) = \sqrt{2} \cos \theta$$

Hence proved.

13. If $\sec \theta + \tan \theta = p$, prove that

$$(i) \sec \theta = \frac{1}{2} \left(p + \frac{1}{p} \right) \quad (ii) \tan \theta = \frac{1}{2} \left(p - \frac{1}{p} \right) \quad (iii) \sin \theta = \frac{p^2 - 1}{p^2 + 1}$$

Sol:(i) We have, $\sec \theta + \tan \theta = p$ (1)

$$\Rightarrow \frac{\sec \theta + \tan \theta}{1} \times \frac{\sec \theta - \tan \theta}{\sec \theta - \tan \theta} = p$$

$$\Rightarrow \frac{\sec^2 \theta - \tan^2 \theta}{\sec \theta - \tan \theta} = p$$

$$\Rightarrow \frac{1}{\sec \theta - \tan \theta} = p$$

$$\Rightarrow \sec \theta - \tan \theta = \frac{1}{p} \quad \text{.....(2)}$$

Adding (1) and (2), we get

$$2 \sec \theta = p + \frac{1}{p}$$

$$\Rightarrow \sec \theta = \frac{1}{2} \left(p + \frac{1}{p} \right)$$

(ii) Subtracting (2) from (1), we get

$$2 \tan \theta = \left(p - \frac{1}{p} \right)$$

$$\Rightarrow \tan \theta = \frac{1}{2} \left(p - \frac{1}{p} \right)$$

(iii) Using (i) and (ii), we get

$$\begin{aligned} \sin \theta &= \frac{\tan \theta}{\sec \theta} \\ &= \frac{\frac{1}{2} \left(p - \frac{1}{p} \right)}{\frac{1}{2} \left(p + \frac{1}{p} \right)} \\ &= \frac{\left(\frac{p^2 - 1}{p} \right)}{\left(\frac{p^2 + 1}{p} \right)} \end{aligned}$$

$$\therefore \sin \theta = \frac{p^2 - 1}{p^2 + 1}$$

14. If $\tan A = n \tan B$ and $\sin A = m \sin B$, prove that $\cos^2 A = \frac{(m^2 - 1)}{(n^2 - 1)}$.

Sol:We have $\tan A = n \tan B$

$$\Rightarrow \cot B = \frac{n}{\tan A} \dots\dots(i)$$

Again, $\sin A = m \sin B$

$$\Rightarrow \operatorname{cosec} B = \frac{m}{\sin A} \dots\dots(ii)$$

Squaring (i) and (ii) and subtracting (ii) from (i), we get

$$\Rightarrow \frac{m^2}{\sin^2 A} - \frac{n^2}{\tan^2 A} = \operatorname{cosec}^2 B - \cot^2 B$$

$$\Rightarrow \frac{m^2}{\sin^2 A} - \frac{n^2 \cos A}{\sin^2 A} = 1$$

$$\Rightarrow m^2 - n^2 \cos^2 A = \sin^2 A$$

$$\Rightarrow m^2 - n^2 \cos^2 A = 1 - \cos^2 A$$

$$\Rightarrow n^2 \cos^2 A - \cos^2 A = m^2 - 1$$

$$\Rightarrow \cos^2 A (n^2 - 1) = (m^2 - 1)$$

$$\Rightarrow \cos^2 A = \frac{(m^2 - 1)}{(n^2 - 1)}$$

$$\therefore \cos^2 A = \frac{(m^2 - 1)}{(n^2 - 1)}$$

15. 15. if $m = (\cos \theta - \sin \theta)$ and $n = (\cos \theta + \sin \theta)$ then show that $\sqrt{\frac{m}{n}} + \sqrt{\frac{n}{m}} = \frac{2}{\sqrt{1 - \tan^2 \theta}}$.

Sol:

$$\begin{aligned} LHS &= \sqrt{\frac{m}{n}} + \sqrt{\frac{n}{m}} \\ &= \frac{\sqrt{m}}{\sqrt{n}} + \frac{\sqrt{n}}{\sqrt{m}} \\ &= \frac{m+n}{\sqrt{mn}} \\ &= \frac{(\cos \theta - \sin \theta) + (\cos \theta + \sin \theta)}{\sqrt{(\cos \theta - \sin \theta)(\cos \theta + \sin \theta)}} \\ &= \frac{2 \cos \theta}{\sqrt{\cos^2 \theta - \sin^2 \theta}} \\ &= \frac{2 \cos \theta}{\sqrt{\cos^2 \theta - \sin^2 \theta}} \end{aligned}$$

$$\begin{aligned}
 &= \frac{\left(\frac{2 \cos \theta}{\cos \theta} \right)}{\left(\frac{\sqrt{\cos^2 \theta - \sin^2 \theta}}{\cos \theta} \right)} \\
 &= \frac{2}{\frac{\sqrt{\cos^2 \theta - \sin^2 \theta}}{\cos \theta}} \\
 &= \frac{2}{\sqrt{1 - \tan^2 \theta}} \\
 &= RHS
 \end{aligned}$$

Exercise – 8C

1. Write the value of $(1 - \sin^2 \theta) \sec^2 \theta$.

Sol:

$$\begin{aligned}
 &(1 - \sin^2 \theta) \sec^2 \theta \\
 &= \cos^2 \theta \times \frac{1}{\cos^2 \theta} \\
 &= 1
 \end{aligned}$$

2. Write the value of $(1 - \cos^2 \theta) \operatorname{cosec}^2 \theta$.

Sol:

$$\begin{aligned}
 &(1 - \cos^2 \theta) \operatorname{cosec}^2 \theta \\
 &= \sin^2 \theta \times \frac{1}{\sin^2 \theta} \\
 &= 1
 \end{aligned}$$

3. Write the value of $(1 + \tan^2 \theta) \cos^2 \theta$.

Sol:

$$\begin{aligned}
 &(1 + \tan^2 \theta) \cos^2 \theta \\
 &= \sec^2 \theta \times \frac{1}{\sec^2 \theta} \\
 &= 1
 \end{aligned}$$

4. Write the value of $(1 + \cot^2 \theta) \sin^2 \theta$.

Sol:

$$\begin{aligned}
 &= (1 + \cot^2 \theta) \sin^2 \theta \\
 &= \operatorname{cosec}^2 \theta \times \frac{1}{\operatorname{cosec}^2 \theta} \\
 &= 1
 \end{aligned}$$

5. Write the value of $\left(\sin^2 \theta + \frac{1}{1 + \tan^2 \theta}\right)$.

Sol:

$$\begin{aligned} & \left(\sin^2 \theta + \frac{1}{1 + \tan^2 \theta}\right) \\ &= \left(\sin^2 \theta + \frac{1}{\sec^2 \theta}\right) \\ &= (\sin^2 \theta + \cos^2 \theta) \\ &= 1 \end{aligned}$$

6. Write the value of $\left(\cot^2 \theta - \frac{1}{\sin^2 \theta}\right)$.

Sol:

$$\begin{aligned} & \left(\cot^2 \theta - \frac{1}{\sin^2 \theta}\right) \\ &= (\cot^2 \theta - \operatorname{cosec}^2 \theta) \\ &= -1 \end{aligned}$$

7. Write the value of $\sin \theta \cos(90^\circ - \theta) + \cos \theta \sin(90^\circ - \theta)$.

Sol:

$$\begin{aligned} & \sin \theta \cos(90^\circ - \theta) + \cos \theta \sin(90^\circ - \theta) \\ &= \sin \theta \sin \theta + \cos \theta \cos \theta \\ &= \sin^2 \theta + \cos^2 \theta \\ &= 1 \end{aligned}$$

8. Write the value of $\operatorname{cosec}^2(90^\circ - \theta) - \tan^2 \theta$.

Sol:

$$\begin{aligned} & \operatorname{cosec}^2(90^\circ - \theta) - \tan^2 \theta \\ &= \sec^2 \theta - \tan^2 \theta \\ &= 1 \end{aligned}$$

9. Write the value of $\sec^2 \theta (1 + \sin \theta)(1 - \sin \theta)$.

Sol:

$$\begin{aligned} & \sec^2 \theta (1 + \sin \theta)(1 - \sin \theta) \\ &= \sec^2 \theta (1 - \sin^2 \theta) \\ &= \frac{1}{\cos^2 \theta} \times \cos^2 \theta \\ &= 1 \end{aligned}$$

10. Write the value of $\operatorname{cosec}^2 \theta (1 + \cos \theta)(1 - \cos \theta)$.

Sol:

$$\begin{aligned}\operatorname{cosec}^2 \theta (1 + \cos \theta)(1 - \cos \theta) \\&= \operatorname{cosec}^2 \theta (1 - \cos^2 \theta) \\&= \frac{1}{\sin^2 \theta} \times \sin^2 \theta \\&= 1\end{aligned}$$

11. Write the value of $\sin^2 \theta \cos^2 \theta (1 + \tan^2 \theta)(1 + \cot^2 \theta)$.

Sol:

$$\begin{aligned}\sin^2 \theta \cos^2 \theta (1 + \tan^2 \theta)(1 + \cot^2 \theta) \\&= \sin^2 \theta \cos^2 \theta \sec^2 \theta \operatorname{cosec}^2 \theta \\&= \sin^2 \theta \times \cos^2 \theta \times \frac{1}{\cos^2 \theta} \times \frac{1}{\sin^2 \theta} \\&= 1\end{aligned}$$

12. Write the value of $(1 + \tan^2 \theta)(1 + \sin \theta)(1 - \sin \theta)$.

Sol:

$$\begin{aligned}(1 + \tan^2 \theta)(1 + \sin \theta)(1 - \sin \theta) \\&= \sec^2 \theta (1 - \sin^2 \theta) \\&= \frac{1}{\cos^2 \theta} \times \cos^2 \theta \\&= 1\end{aligned}$$

13. Write the value of $3 \cot^2 \theta - 3 \operatorname{cosec}^2 \theta$.

Sol:

$$\begin{aligned}3 \cot^2 \theta - 3 \operatorname{cosec}^2 \theta \\&= 3(\cot^2 \theta - \operatorname{cosec}^2 \theta) \\&= 3(-1) \\&= -3\end{aligned}$$

14. Write the value of $4 \tan^2 \theta - \frac{4}{\cos^2 \theta}$.

Sol:

$$\begin{aligned}4 \tan^2 \theta - \frac{4}{\cos^2 \theta} \\&= 4 \tan^2 \theta - 4 \sec^2 \theta \\&= 4(\tan^2 \theta - \sec^2 \theta) \\&= 4(-1) \\&= -4\end{aligned}$$

15. Write the value of $\frac{\tan^2 \theta - \sec^2 \theta}{\cot^2 \theta - \operatorname{cosec}^2 \theta}$.

Sol:

$$\begin{aligned} & \frac{\tan^2 \theta - \sec^2 \theta}{\cot^2 \theta - \operatorname{cosec}^2 \theta} \\ &= \frac{-1}{-1} \\ &= 1 \end{aligned}$$

16. If $\sin \theta = \frac{1}{2}$, write the value of $(3\cot^2 \theta + 3)$.

Sol:

$$\text{As, } \sin \theta = \frac{1}{2}$$

$$\text{So, } \operatorname{cosec} \theta = \frac{1}{\sin \theta} = 2 \quad \dots (i)$$

Now,

$$\begin{aligned} 3\cot^2 \theta + 3 &= 3(\cot^2 \theta + 1) \\ &= 3\operatorname{cosec}^2 \theta \\ &= 3(2)^2 \quad [\text{Using (i)}] \\ &= 3(4) \\ &= 12 \end{aligned}$$

17. If $\cos \theta = \frac{2}{3}$, write the value of $(4 + 4\tan^2 \theta)$.

Sol:

$$\begin{aligned} 4 + 4\tan^2 \theta &= 4(1 + \tan^2 \theta) \\ &= 4\sec^2 \theta \\ &= \frac{4}{\cos^2 \theta} \\ &= \frac{4}{\left(\frac{2}{3}\right)^2} \\ &= \frac{4}{\left(\frac{4}{9}\right)} \\ &= \frac{4 \times 9}{4} \\ &= 9 \end{aligned}$$

18. If $\cos \theta = \frac{7}{25}$, write the value of $(\tan \theta + \cot \theta)$.

Sol:

$$\text{As } \sin^2 \theta = 1 - \cos^2 \theta$$

$$\begin{aligned}
 &= 1 - \left(\frac{7}{25}\right)^2 \\
 &= 1 - \frac{49}{625} \\
 &= \frac{625-49}{625} \\
 &\Rightarrow \sin^2 \theta = \frac{576}{625} \\
 &\Rightarrow \sin \theta = \sqrt{\frac{576}{625}} \\
 &\Rightarrow \sin \theta = \frac{24}{25}
 \end{aligned}$$

Now,

$$\begin{aligned}
 &\tan \theta + \cot \theta \\
 &= \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \\
 &= \frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta} \\
 &= \frac{1}{\left(\frac{7}{25} \times \frac{24}{25}\right)} \\
 &= \frac{1}{\left(\frac{168}{625}\right)} \\
 &= \frac{625}{168}
 \end{aligned}$$

19. If $\cos \theta = \frac{2}{3}$, write the value of $\frac{(\sec \theta - 1)}{(\sec \theta + 1)}$.

Sol:

$$\begin{aligned}
 &\frac{\sec \theta - 1}{\sec \theta + 1} \\
 &= \frac{\left(\frac{1}{\cos \theta} - 1\right)}{\left(\frac{1}{\cos \theta} + 1\right)} \\
 &= \frac{\left(\frac{1 - \cos \theta}{\cos \theta}\right)}{\left(\frac{1 + \cos \theta}{\cos \theta}\right)} \\
 &= \frac{1 - \cos \theta}{1 + \cos \theta} \\
 &= \frac{\left(\frac{1}{1} - \frac{2}{3}\right)}{\left(\frac{1}{1} + \frac{2}{3}\right)} \\
 &= \frac{\left(\frac{1}{3}\right)}{\left(\frac{5}{3}\right)} \\
 &= \frac{1}{5}
 \end{aligned}$$

20. If $5 \tan \theta = 4$, write the value of $\frac{(\cos \theta - \sin \theta)}{(\cos \theta + \sin \theta)}$.

Sol:

We have,

$$5 \tan \theta = 4$$

$$\Rightarrow \tan \theta = \frac{4}{5}$$

Now,

$$\begin{aligned} & \frac{(\cos \theta - \sin \theta)}{(\cos \theta + \sin \theta)} \\ &= \frac{\left(\frac{\cos \theta}{\cos \theta} - \frac{\sin \theta}{\cos \theta}\right)}{\left(\frac{\cos \theta}{\cos \theta} + \frac{\sin \theta}{\cos \theta}\right)} \quad (\text{Dividing numerator and denominator by } \cos \theta) \\ &= \frac{(1 - \tan \theta)}{(1 + \tan \theta)} \\ &= \frac{\left(1 - \frac{4}{5}\right)}{\left(1 + \frac{4}{5}\right)} \\ &= \frac{\left(\frac{1}{5}\right)}{\left(\frac{9}{5}\right)} \\ &= \frac{1}{9} \end{aligned}$$

21. If $3 \cot \theta = 4$, write the value of $\frac{(2 \cos \theta - \sin \theta)}{(4 \cos \theta - \sin \theta)}$.

Sol:

We have,

$$3 \cot \theta = 4$$

$$\Rightarrow \cot \theta = \frac{4}{3}$$

Now,

$$\begin{aligned} & \frac{(2 \cos \theta + \sin \theta)}{(4 \cos \theta - \sin \theta)} \\ &= \frac{\left(\frac{2 \cos \theta}{\sin \theta} + \frac{\sin \theta}{\sin \theta}\right)}{\left(\frac{4 \cos \theta}{\sin \theta} - \frac{\sin \theta}{\sin \theta}\right)} \quad (\text{Dividing numerator and denominator by } \sin \theta) \\ &= \frac{(2 \cot \theta + 1)}{(4 \cot \theta - 1)} \\ &= \frac{\left(2 \times \frac{4}{3} + 1\right)}{\left(4 \times \frac{4}{3} - 1\right)} \\ &= \frac{\left(\frac{8}{3} + 1\right)}{\left(\frac{16}{3} - 1\right)} \\ &= \frac{\left(\frac{8+3}{3}\right)}{\left(\frac{16-3}{3}\right)} \\ &= \frac{\left(\frac{11}{3}\right)}{\left(\frac{13}{3}\right)} \end{aligned}$$

$$= \frac{11}{13}$$

22. If $\cot \theta = \frac{1}{\sqrt{3}}$, write the value of $\frac{(1 - \cos^2 \theta)}{(2 - \sin^2 \theta)}$.

Sol:

We have,

$$\cot \theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \cot \theta = \cot \left(\frac{\pi}{3} \right)$$

$$\Rightarrow \theta = \frac{\pi}{3}$$

Now,

$$\begin{aligned} & \frac{(1 - \cos^2 \theta)}{(2 - \sin^2 \theta)} \\ &= \frac{1 - \cos^2 \left(\frac{\pi}{3} \right)}{2 - \sin^2 \left(\frac{\pi}{3} \right)} \\ &= \frac{1 - \left(\frac{1}{2} \right)^2}{2 - \left(\frac{\sqrt{3}}{2} \right)^2} \\ &= \frac{\left(\frac{1}{4} \right)}{\left(\frac{1}{4} \right)} \\ &= \frac{\left(\frac{3}{4} \right)}{\left(\frac{5}{4} \right)} \\ &= \frac{3}{5} \end{aligned}$$

23. If $\tan \theta = \frac{1}{\sqrt{5}}$, write the value of $\frac{(\cos \theta \sec^2 \theta - \sec^2 \theta)}{(\cos \theta \csc^2 \theta - \sec^2 \theta)}$.

Sol:

$$\begin{aligned} & \frac{(\cos \theta \sec^2 \theta - \sec^2 \theta)}{(\cos \theta \csc^2 \theta + \sec^2 \theta)} \\ &= \frac{(1 + \cot^2 \theta) - (1 + \tan^2 \theta)}{(1 + \cot^2 \theta) + (1 + \tan^2 \theta)} \\ &= \frac{\left(1 + \frac{1}{\tan^2 \theta} \right) - (1 + \tan^2 \theta)}{\left(1 + \frac{1}{\tan^2 \theta} \right) + (1 + \tan^2 \theta)} \\ &= \frac{\left(1 + \frac{1}{\tan^2 \theta} - 1 - \tan^2 \theta \right)}{\left(1 + \frac{1}{\tan^2 \theta} + 1 + \tan^2 \theta \right)} \\ &= \frac{\left(\frac{1}{\tan^2 \theta} - \tan^2 \theta \right)}{\left(\frac{1}{\tan^2 \theta} + \tan^2 \theta + 2 \right)} \end{aligned}$$

$$\begin{aligned}
 &= \frac{\left(\frac{\sqrt{5}}{1}\right)^2 - \left(\frac{1}{\sqrt{5}}\right)^2}{\left(\frac{\sqrt{5}}{1}\right)^2 + \left(\frac{1}{\sqrt{5}}\right)^2 + 2} \\
 &= \frac{\left(\frac{5}{1} - \frac{1}{5}\right)}{\left(\frac{5}{1} + \frac{1}{5} + \frac{2}{1}\right)} \\
 &= \frac{\left(\frac{24}{5}\right)}{\left(\frac{36}{5}\right)} \\
 &= \frac{24}{36} \\
 &= \frac{2}{3}
 \end{aligned}$$

24. If $\cot A = \frac{4}{3}$ and $(A + B) = 90^\circ$, what is the value of $\tan B$?

Sol:

We have,

$$\cot A = \frac{4}{3}$$

$$\Rightarrow \cot(90^\circ - B) = \frac{4}{3} \quad (\text{As, } A + B = 90^\circ)$$

$$\therefore \tan B = \frac{4}{3}$$

25. If $\cos B = \frac{3}{5}$ and $(A + B) = 90^\circ$, find the value of $\sin A$.

Sol:

We have,

$$\cos B = \frac{3}{5}$$

$$\Rightarrow \cos(90^\circ - A) = \frac{3}{5} \quad (\text{As, } A + B = 90^\circ)$$

$$\therefore \sin A = \frac{3}{5}$$

26. If $\sqrt{3} \sin \theta = \cos \theta$ and θ is an acute angle, find the value of θ .

Sol:

We have,

$$\sqrt{3} \sin \theta = \cos \theta$$

$$\Rightarrow \frac{\sin \theta}{\cos \theta} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \tan \theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \tan \theta = \tan 30^\circ$$

$$\therefore \theta = 30^\circ$$

27. Write the value of $\tan 10^\circ \tan 20^\circ \tan 70^\circ \tan 80^\circ$.

Sol:

$$\begin{aligned} & \tan 10^\circ \tan 20^\circ \tan 70^\circ \tan 80^\circ \\ &= \cot(90^\circ - 10^\circ) \cot(90^\circ - 20^\circ) \tan 70^\circ \tan 80^\circ \\ &= \cot 80^\circ \cot 70^\circ \tan 70^\circ \tan 80^\circ \\ &= \frac{1}{\tan 80^\circ} \times \frac{1}{\tan 70^\circ} \times \tan 70^\circ \times \tan 80^\circ \\ &= 1 \end{aligned}$$

28. Write the value of $\tan 1^\circ \tan 2^\circ \dots \tan 89^\circ$.

Sol:

$$\begin{aligned} & \tan 1^\circ \tan 2^\circ \dots \tan 89^\circ \\ &= \tan 1^\circ \tan 2^\circ \tan 3^\circ \dots \tan 45^\circ \dots \tan 87^\circ \tan 88^\circ \tan 89^\circ \\ &= \tan 1^\circ \tan 2^\circ \tan 3^\circ \dots \tan 45^\circ \dots \cot(90^\circ - 87^\circ) \cot(90^\circ - 88^\circ) \cot(90^\circ - 89^\circ) \\ &= \tan 1^\circ \tan 2^\circ \tan 3^\circ \dots \tan 45^\circ \dots \cot 3^\circ \cot 2^\circ \cot 1^\circ \\ &= \tan 1^\circ \times \tan 2^\circ \times \tan 3^\circ \times \dots \times 1 \times \dots \times \frac{1}{\tan 3^\circ} \times \frac{1}{\tan 2^\circ} \times \frac{1}{\tan 1^\circ} \\ &= 1 \end{aligned}$$

29. Write the value of $\cos 1^\circ \cos 2^\circ \dots \cos 180^\circ$.

Sol:

$$\begin{aligned} & \cos 1^\circ \cos 2^\circ \dots \cos 180^\circ \\ &= \cos 1^\circ \cos 2^\circ \dots \cos 90^\circ \dots \cos 180^\circ \\ &= \cos 1^\circ \cos 2^\circ \dots 0 \dots \cos 180^\circ \\ &= 0 \end{aligned}$$

30. If $\tan A = \frac{5}{12}$, find the value of $(\sin A + \cos A) \sec A$.

Sol:

$$\begin{aligned} & (\sin A + \cos A) \sec A \\ &= (\sin A + \cos A) \frac{1}{\cos A} \\ &= \frac{\sin A}{\cos A} + \frac{\cos A}{\cos A} \\ &= \tan A + 1 \\ &= \frac{5}{12} + \frac{1}{1} \\ &= \frac{5+12}{12} \\ &= \frac{17}{12} \end{aligned}$$

31. If $\sin \theta = \cos(\theta - 45^\circ)$, where θ is acute, find the value of θ .

Sol:

We have,

$$\begin{aligned} \sin \theta &= \cos(\theta - 45^\circ) \\ \Rightarrow \cos(90^\circ - \theta) &= \cos(\theta - 45^\circ) \end{aligned}$$

Comparing both sides, we get

$$\begin{aligned}
 90^\circ - \theta &= \theta - 45^\circ \\
 \Rightarrow \theta + \theta &= 90^\circ + 45^\circ \\
 \Rightarrow 2\theta &= 135^\circ \\
 \Rightarrow \theta &= \left(\frac{135}{2}\right)^\circ \\
 \therefore \theta &= 67.5^\circ
 \end{aligned}$$

32. Find the value of $\frac{\sin 50^\circ}{\cos 40^\circ} + \frac{\operatorname{cosec} 40^\circ}{\sec 50^\circ} - 4 \cos 50^\circ \operatorname{cosec} 40^\circ$.

Sol:

$$\begin{aligned}
 &\frac{\sin 50^\circ}{\cos 40^\circ} + \frac{\operatorname{cosec} 40^\circ}{\sec 50^\circ} - 4 \cos 50^\circ \operatorname{cosec} 40^\circ \\
 &= \frac{\cos(90^\circ - 50^\circ)}{\cos 40^\circ} + \frac{\sec(90^\circ - 40^\circ)}{\sec 50^\circ} - 4 \sin(90^\circ - 50^\circ) \operatorname{cosec} 40^\circ \\
 &= \frac{\cos 40^\circ}{\cos 40^\circ} + \frac{\sec 50^\circ}{\sec 50^\circ} - 4 \sin 40^\circ \times \frac{1}{\sin 40^\circ} \\
 &= 1 + 1 - 4 \\
 &= -2
 \end{aligned}$$

33. Find the value of $\sin 48^\circ \sec 42^\circ + \cos 48^\circ \operatorname{cosec} 42^\circ$.

Sol:

$$\begin{aligned}
 &\sin 48^\circ \sec 42^\circ + \cos 48^\circ \operatorname{cosec} 42^\circ \\
 &= \sin 48^\circ \operatorname{cosec}(90^\circ - 42^\circ) + \cos 48^\circ \sec(90^\circ - 42^\circ) \\
 &= \sin 48^\circ \operatorname{cosec} 48^\circ + \cos 48^\circ \sec 48^\circ \\
 &= \sin 48^\circ \times \frac{1}{\sin 48^\circ} + \cos 48^\circ \times \frac{1}{\cos 48^\circ} \\
 &= 1 + 1 \\
 &= 2
 \end{aligned}$$

34. If $x = a \sin \theta$ and $y = b \cos \theta$, write the value of $(b^2 x^2 + a^2 y^2)$.

Sol:

$$\begin{aligned}
 &(b^2 x^2 + a^2 y^2) \\
 &= b^2 (a \sin \theta)^2 + a^2 (b \cos \theta)^2 \\
 &= b^2 a^2 \sin^2 \theta + a^2 b^2 \cos^2 \theta \\
 &= a^2 b^2 (\sin^2 \theta + \cos^2 \theta) \\
 &= a^2 b^2 (1) \\
 &= a^2 b^2
 \end{aligned}$$

35. If $5x = \sec \theta$ and $\frac{5}{x} = \tan \theta$, find the value of $5\left(x^2 - \frac{1}{x^2}\right)$.

Sol:

$$\begin{aligned}
 &5\left(x^2 - \frac{1}{x^2}\right) \\
 &= \frac{25}{5}\left(x^2 - \frac{1}{x^2}\right)
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{5} \left(25x^2 - \frac{25}{x^2} \right) \\
 &= \frac{1}{5} \left[(5x)^2 - \left(\frac{5}{x} \right)^2 \right] \\
 &= \frac{1}{5} [(\sec \theta)^2 - (\tan \theta)^2] \\
 &= \frac{1}{5} (\sec^2 \theta - \tan^2 \theta) \\
 &= \frac{1}{5} (1) \\
 &= \frac{1}{5}
 \end{aligned}$$

36. If $\operatorname{cosec} \theta = 2x$ and $\cot \theta = \frac{2}{x}$, find the value of $2 \left(x^2 - \frac{1}{x^2} \right)$.

Sol:

$$\begin{aligned}
 &2 \left(x^2 - \frac{1}{x^2} \right) \\
 &= \frac{4}{2} \left(x^2 - \frac{1}{x^2} \right) \\
 &= \frac{1}{2} \left(4x^2 - \frac{4}{x^2} \right) \\
 &= \frac{1}{2} \left[(2x)^2 - \left(\frac{2}{x} \right)^2 \right] \\
 &= \frac{1}{2} [(\operatorname{cosec} \theta)^2 - (\sec \theta)^2] \\
 &= \frac{1}{2} (\operatorname{cosec}^2 \theta - \sec^2 \theta) \\
 &= \frac{1}{2} (1) \\
 &= \frac{1}{2}
 \end{aligned}$$

37. If $\sec \theta + \tan \theta = x$, find the value of $\sec \theta$.

Sol:

We have,

$$\sec \theta + \tan \theta = x \quad \dots \dots (i)$$

$$\Rightarrow \frac{\sec \theta + \tan \theta}{1} \times \frac{\sec \theta - \tan \theta}{\sec \theta - \tan \theta} = x$$

$$\Rightarrow \frac{\sec^2 \theta - \tan^2 \theta}{\sec \theta - \tan \theta} = x$$

$$\Rightarrow \frac{1}{\sec \theta - \tan \theta} = \frac{x}{1}$$

$$\Rightarrow \sec \theta - \tan \theta = \frac{1}{x} \quad \dots \dots (ii)$$

Adding (i) and (ii), we get

$$2 \sec \theta = x + \frac{1}{x}$$

$$\Rightarrow 2 \sec \theta = \frac{x^2 + 1}{x}$$

$$\therefore \sec \theta = \frac{x^2 + 1}{2x}$$

38. Find the value of $\frac{\cos 38^\circ \operatorname{cosec} 52^\circ}{\tan 18^\circ \tan 35^\circ \tan 60^\circ \tan 72^\circ \tan 55^\circ}$.

Sol:

$$\begin{aligned} & \frac{\cos 38^\circ \operatorname{cosec} 52^\circ}{\tan 18^\circ \tan 35^\circ \tan 60^\circ \tan 72^\circ \tan 55^\circ} \\ &= \frac{\cos 38^\circ \sec(90^\circ - 52^\circ)}{\cot(90^\circ - 18^\circ) \cot(90^\circ - 35^\circ) \tan 60^\circ \tan 72^\circ \tan 55^\circ} \\ &= \frac{\cos 38^\circ \sec 38^\circ}{\cot 72^\circ \cot 55^\circ \tan 60^\circ \tan 72^\circ \tan 55^\circ} \\ &= \frac{\cos 38^\circ \times \frac{1}{\cos 38^\circ}}{\frac{1}{\tan 72^\circ} \times \frac{1}{\tan 55^\circ} \times \sqrt{3} \times \tan 72^\circ \times \tan 55^\circ} \\ &= \frac{1}{\sqrt{3}} \end{aligned}$$

39. If $\sin \theta = x$, write the value of $\cot \theta$.

Sol:

$$\begin{aligned} \cot \theta &= \frac{\cos \theta}{\sin \theta} \\ &= \frac{\sqrt{1 - \sin^2 \theta}}{\sin \theta} \\ &= \frac{\sqrt{1 - x^2}}{x} \end{aligned}$$

40. If $\sec \theta = x$, write the value of $\tan \theta$.

Sol:

$$\text{As, } \tan^2 \theta = \sec^2 \theta - 1$$

$$\text{So, } \tan \theta = \sqrt{\sec^2 \theta - 1} = \sqrt{x^2 - 1}$$

Formative Assessment

1. $\frac{\cos^2 56^\circ + \cos^2 34^\circ}{\sin^2 56^\circ + \sin^2 34^\circ} + 3 \tan^2 56^\circ \tan^2 34^\circ = ?$

- (a) $3\frac{1}{2}$ (b) 4
(c) 6 (d) 5

Answer: (b) 4

Sol:

$$\begin{aligned} & \frac{\cos^2 56^\circ + \cos^2 34^\circ}{\sin^2 56^\circ + \sin^2 34^\circ} + 3 \tan^2 56^\circ \tan^2 34^\circ \\ &= \frac{\{\cos(90^\circ - 34^\circ)\}^2 + \cos^2 34^\circ}{\{\sin(90^\circ - 34^\circ)\}^2 + \sin^2 34^\circ} + 3\{\tan(90^\circ - 34^\circ)\}^2 \tan^2 34^\circ \\ &= \frac{\sin^2 34^\circ + \cos^2 34^\circ}{\cos^2 34^\circ + \sin^2 34^\circ} + 3 \cot^2 34^\circ \tan^2 34^\circ \quad \left[\begin{array}{l} \because \cos(90^\circ - \theta) = \sin \theta, \sin(90^\circ - \theta) \\ = \cos \theta \text{ and } \tan(90^\circ - \theta) = \cot \theta \end{array} \right] \end{aligned}$$

$$= \frac{1}{1} + 3 \times 1 \quad \left[\because \cot \theta = \frac{1}{\tan \theta} \text{ and } \sin^2 \theta + \cos^2 \theta = 1 \right]$$

$$= 4$$

2. The value of $\left(\sin^2 30^\circ \cos^2 45^\circ + 4 \tan^2 30^\circ + \frac{1}{2} \sin^2 90^\circ + \frac{1}{8} \cot^2 60^\circ \right) = ?$

- (a) $\frac{3}{8}$ (b) $\frac{5}{8}$
 (c) 6 (d) 2

Answer: (d) 2

Sol:

$$(\sin^2 30^\circ \cos^2 45^\circ) + 4 \tan^2 30^\circ + \frac{1}{2} \sin^2 90^\circ + \frac{1}{8} \cot^2 60^\circ$$

$$= \frac{1}{2^2} \times \frac{1}{(\sqrt{2})^2} + 4 \times \frac{1}{(\sqrt{3})^2} + \frac{1}{2} \times 1^2 + \frac{1}{8} \times \frac{1}{(\sqrt{3})^2} \quad \left[\begin{array}{l} \because \sin 30^\circ = \frac{1}{2} \text{ and } \cos 45^\circ = \frac{1}{\sqrt{2}} \\ \text{and } \tan 30^\circ = \frac{1}{2} \text{ and } \cot 60^\circ = \frac{1}{\sqrt{3}} \end{array} \right]$$

$$= \frac{1}{4} \times \frac{1}{2} + 4 \times \frac{1}{3} + \frac{1}{2} + \frac{1}{24}$$

$$= \frac{1}{8} + \frac{4}{3} + \frac{1}{2} + \frac{1}{24}$$

$$= \frac{3+32+12+1}{24}$$

$$= \frac{48}{24}$$

$$= 2$$

3. If $\cos A + \cos^2 A = 1$ then $(\sin^2 A + \sin^4 A) = ?$

- (a) $\frac{1}{2}$ (b) 2
 (c) 1 (d) 4

Answer: (c) 1

Sol:

$$\cos^2 A + A = 1$$

$$\Rightarrow \cos A = \sin^2 A \quad \dots (i)$$

Squaring both sides of (i), we get:

$$\cos^2 A = \sin^4 A \quad \dots (ii)$$

Adding (i) and (ii), we get:

$$\sin^2 A + \sin^4 A = \cos A + \cos^2 A$$

$$\Rightarrow \sin^2 A + \sin^4 A = 1 \quad [\because \cos A + \cos^2 A = 1]$$

4. If $\sin \theta = \frac{\sqrt{3}}{2}$ then $(\operatorname{cosec} \theta + \cot \theta) = ?$

(a) $(2 + \sqrt{3})$ (b) $2\sqrt{3}$

(c) $\sqrt{2}$ (d) $\sqrt{3}$

Answer: (d) $\sqrt{3}$

Sol:

Given: $\sin \theta = \frac{\sqrt{3}}{2}$ and $\operatorname{cosec} \theta = \frac{2}{\sqrt{3}}$

$$\operatorname{cosec}^2 \theta - \cot^2 \theta = 1$$

$$\Rightarrow \cot^2 \theta = \operatorname{cosec}^2 \theta - 1$$

$$\Rightarrow \cot^2 \theta = \frac{4}{3} - 1 \quad [\text{Given}]$$

$$\Rightarrow \cot \theta = \frac{1}{\sqrt{3}}$$

$$\begin{aligned} \therefore \operatorname{cosec} \theta + \cot \theta &= \frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}} \\ &= \frac{3}{\sqrt{3}} \\ &= \frac{\sqrt{3} \times \sqrt{3}}{\sqrt{3}} \\ &= \sqrt{3} \end{aligned}$$

5. If $\cot A = \frac{4}{5}$, prove that $\frac{(\sin A + \cos A)}{(\sin A - \cos A)} = 9$.

Sol:

Given : $\cot A = \frac{4}{5}$

Writing $\cot A = \frac{\cos A}{\sin A}$ and squaring the equation, we get :

$$\frac{\cos^2 A}{\sin^2 A} = \frac{16}{25}$$

$$\Rightarrow 25 \cos^2 A = 16 \sin^2 A$$

$$\Rightarrow 25 \cos^2 A = 16 - 16 \cos^2 A$$

$$\Rightarrow \cos^2 A = \frac{16}{41}$$

$$\Rightarrow \cos A = \frac{4}{\sqrt{41}}$$

$$\therefore \sin^2 A = 1 - \cos^2 A$$

$$= 1 - \frac{16}{41}$$

$$\text{Now, } \sin A = \sqrt{\frac{25}{41}}$$

$$\Rightarrow \sin A = \frac{5}{\sqrt{41}}$$

$$\therefore LHS = \frac{\sin A + \cos A}{\sin A - \cos A}$$

$$\begin{aligned}
 &= \frac{\frac{5}{\sqrt{41}} + \frac{4}{\sqrt{41}}}{\frac{5}{\sqrt{41}} - \frac{4}{\sqrt{41}}} \\
 &= \frac{9}{1} \\
 &= 9 = RHS
 \end{aligned}$$

6. If $2x = \sec A$ and $\frac{2}{x} = \tan A$, prove that $\left(x^2 - \frac{1}{x^2}\right) = \frac{1}{4}$.

Sol:

Given: $2x = \sec A$

$$\Rightarrow x = \frac{\sec A}{2} \quad \dots (i)$$

and $\frac{2}{x} = \tan A$

$$\Rightarrow \frac{1}{x} = \tan A \quad \dots (ii)$$

$$\therefore x + \frac{1}{x} = \frac{\sec A}{2} + \frac{\tan A}{2} \quad [\because \text{From (i) and (ii)}]$$

$$\text{Also, } x - \frac{1}{x} = \frac{\sec A}{2} - \frac{\tan A}{2}$$

$$\therefore \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right) = \left(\frac{\sec A}{2} + \frac{\tan A}{2}\right)\left(\frac{\sec A}{2} - \frac{\tan A}{2}\right)$$

$$\Rightarrow x^2 - \frac{1}{x^2} = \frac{1}{4} (\sec^2 A - \tan^2 A)$$

$$\begin{aligned}
 \therefore x^2 - \frac{1}{x^2} &= \frac{1}{4} \times 1 \quad (\because \sec^2 A - \tan^2 A = 1) \\
 &= \frac{1}{4}
 \end{aligned}$$

Hence proved.

7. If $\sqrt{3} \tan \theta = 3 \sin \theta$, prove that $(\sin^2 \theta - \cos^2 \theta) = \frac{1}{3}$.

Sol:

Given: $\sqrt{3} \tan \theta = 3 \sin \theta$

$$\Rightarrow \frac{\sqrt{3}}{\cos \theta} = 3 \quad \left[\because \tan \theta = \frac{\sin \theta}{\cos \theta}\right]$$

$$\Rightarrow \cos \theta = \frac{\sqrt{3}}{3}$$

$$\Rightarrow \cos^2 \theta = \frac{3}{9}$$

$$\therefore \sin^2 \theta = 1 - \frac{3}{9}$$

$$\Rightarrow \sin^2 \theta = \frac{6}{9}$$

$$\therefore LHS = \sin^2 \theta - \cos^2 \theta$$

$$= \frac{6}{9} - \frac{3}{9} \quad \left[\because \sin^2 \theta = \frac{6}{9}, \cos^2 \theta = \frac{3}{9}\right]$$

$$= \frac{3}{9}$$

$$= \frac{1}{3}$$

$$=RHS$$

Hence Proved.

8. Prove that $\frac{(\sin^2 73^\circ + \sin^2 17^\circ)}{(\cos^2 28^\circ + \cos^2 62^\circ)} = 1$.

Sol:

$$\frac{(\sin^2 73^\circ + \sin^2 17^\circ)}{(\cos^2 28^\circ + \cos^2 62^\circ)} = 1.$$

$$\begin{aligned} LHS &= \frac{\sin^2 73^\circ + \sin^2 17^\circ}{\cos^2 28^\circ + \cos^2 62^\circ} \\ &= \frac{[\sin(90^\circ - 17^\circ)]^2 + \sin^2 17^\circ}{[\cos(90^\circ - 62^\circ)]^2 + \cos^2 62^\circ} \\ &= \frac{\cos^2 17^\circ + \sin^2 17^\circ}{\sin^2 62^\circ + \cos^2 62^\circ} \\ &= \frac{1}{1} \quad [\because \sin^2 \theta + \cos^2 \theta = 1] \\ &= 1 = RHS \end{aligned}$$

9. If $2 \sin 2\theta = \sqrt{3}$, prove that $\theta = 30^\circ$.

Sol:

$$\begin{aligned} 2 \sin(2\theta) &= \sqrt{3} \\ \Rightarrow \sin(2\theta) &= \frac{\sqrt{3}}{2} \\ \Rightarrow \sin(2\theta) &= \sin(60^\circ) \\ \Rightarrow 2\theta &= 60^\circ \\ \Rightarrow \theta &= \frac{60^\circ}{2} \\ \Rightarrow \theta &= 30^\circ \end{aligned}$$

10. Prove that $\sqrt{\frac{1+\cos A}{1-\cos A}} = (\operatorname{cosec} A + \cot A)$.

Sol:

$$\sqrt{\frac{1+\cos A}{1-\cos A}} = (\operatorname{cosec} A + \cot A).$$

$$LHS = \sqrt{\frac{1+\cos A}{1-\cos A}}$$

Multiplying the numerator and denominator by $(1 + \cos A)$, we have:

$$\sqrt{\frac{(1+\cos A)^2}{(1-\cos A)(1+\cos A)}}$$

$$= \sqrt{\frac{(1+\cos A)^2}{1 - \cos^2 A}}$$

$$= \frac{1+\cos A}{\sqrt{\sin^2 A}}$$

$$= \frac{1+\cos A}{\sin A}$$

$$\begin{aligned}
 &= \frac{1}{\sin A} + \frac{\cos A}{\sin A} \\
 &= \operatorname{cosec} A + \cot A = RHS \\
 &\text{Hence proved.}
 \end{aligned}$$

11. If $\operatorname{cosec} \theta + \cot \theta = p$, prove that $\cos \theta = \frac{(p^2 - 1)}{(p^2 + 1)}$.

Sol:

$$\begin{aligned}
 \operatorname{cosec} \theta + \cot \theta &= p \\
 \Rightarrow \frac{1}{\sin \theta} + \frac{\cos \theta}{\sin \theta} &= p \\
 \Rightarrow \frac{1 + \cos \theta}{\sin \theta} &= p \\
 \text{Squaring both sides, we get:} \\
 \left(\frac{1 + \cos \theta}{\sin \theta} \right)^2 &= p^2 \\
 \Rightarrow \frac{(1 + \cos \theta)^2}{\sin^2 \theta} &= p^2 \\
 \Rightarrow \frac{(1 + \cos \theta)^2}{1 - \cos^2 \theta} &= p^2 \\
 \Rightarrow \frac{(1 + \cos \theta)^2}{(1 + \cos \theta)(1 - \cos \theta)} &= p^2 \\
 \Rightarrow \frac{(1 + \cos \theta)}{(1 - \cos \theta)} &= p^2 \\
 \Rightarrow 1 + \cos \theta &= p^2(1 - \cos \theta) \\
 = 1 + \cos \theta &= p^2 - p^2 \cos \theta \\
 \Rightarrow \cos \theta(1 + p^2) &= p^2 - 1 \\
 \Rightarrow \cos \theta &= \frac{p^2 - 1}{p^2 + 1}
 \end{aligned}$$

Hence proved.

12. Prove that $(\operatorname{cosec} A - \cot A)^2 = \frac{(1 - \cos A)}{(1 + \cos A)}$.

Sol:

$$\begin{aligned}
 (\operatorname{cosec} A - \cot A)^2 &= \frac{(1 - \cos A)}{(1 + \cos A)} \\
 LHS &= (\operatorname{cosec} A - \cot A)^2 \\
 &= \left(\frac{1}{\sin A} - \frac{\cos A}{\sin A} \right)^2 \\
 &= \left(\frac{1 - \cos A}{\sin A} \right)^2 \\
 &= \frac{(1 - \cos A)^2}{\sin^2 A} \\
 &= \frac{(1 - \cos A)^2}{1 - \cos^2 A} \quad [\because \sin^2 \theta + \cos^2 \theta = 1] \\
 &= \frac{(1 - \cos A)(1 - \cos A)}{(1 - \cos A)(1 + \cos A)} \\
 &= \frac{(1 - \cos A)}{(1 + \cos A)} = RHS
 \end{aligned}$$

Hence proved.

13. If $5 \cot \theta = 3$, show that the value of $\left(\frac{5 \sin \theta - 3 \cos \theta}{4 \sin \theta + 3 \cos \theta} \right)$ is $\frac{16}{29}$.

Sol:

$$\text{Given: } 5 \cot \theta = 3$$

$$\Rightarrow \frac{5 \cos \theta}{\sin \theta} = 3 \quad \left[\because \cot \theta = \frac{\cos \theta}{\sin \theta} \right]$$

$$\Rightarrow 5 \cos \theta = 3 \sin \theta$$

Squaring both sides, we get:

$$25 \cos^2 \theta = 9 \sin^2 \theta$$

$$\Rightarrow 25 \cos^2 \theta = 9 - 9 \cos^2 \theta \quad [\because \sin^2 \theta + \cos^2 \theta = 1]$$

$$\Rightarrow 34 \cos^2 \theta = 9$$

$$\Rightarrow \cos \theta = \sqrt{\frac{9}{34}}$$

$$\Rightarrow \cos \theta = \frac{3}{\sqrt{34}}$$

$$\text{Again, } \sin^2 \theta = 1 - \cos^2 \theta$$

$$\Rightarrow \sin^2 \theta = \frac{34-9}{34} = \frac{25}{34}$$

$$\Rightarrow \sin \theta = \frac{5}{\sqrt{34}}$$

$$\therefore LHS = \left(\frac{5 \sin \theta - 3 \cos \theta}{4 \sin \theta + 3 \cos \theta} \right)$$

$$= \frac{5 \times \frac{5}{\sqrt{34}} - 3 \times \frac{3}{\sqrt{34}}}{4 \times \frac{5}{\sqrt{34}} + 3 \times \frac{3}{\sqrt{34}}} \quad \left[\because \cos \theta = \frac{3}{\sqrt{34}}, \sin \theta = \frac{5}{\sqrt{34}} \right]$$

$$= \frac{25-9}{20+9}$$

$$= \frac{16}{29}$$

14. Prove that $(\sin 32^\circ \cos 58^\circ + \cos 32^\circ \sin 58^\circ) = 1$.

Sol:

$$(\sin 32^\circ \cos 58^\circ + \cos 32^\circ \sin 58^\circ) = 1$$

$$LHS = \sin 32^\circ \cos 58^\circ + \cos 32^\circ \sin 58^\circ$$

$$= \sin(90^\circ - 58^\circ) \cos 58^\circ + \cos(90^\circ - 58^\circ) \sin 58^\circ$$

$$= \cos 58^\circ \times \cos 58^\circ + \sin 58^\circ \times \sin 58^\circ \quad \left[\begin{array}{l} \because \sin(90^\circ - \theta) = \cos \theta, \\ \cos(90^\circ - \theta) = \sin \theta \end{array} \right]$$

$$= \cos^2 58^\circ + \sin^2 58^\circ$$

$$= 1 \quad [\because \sin^2 \theta + \cos^2 \theta = 1]$$

$$= RHS$$

15. If $x = a \sin \theta + b \cos \theta$ and $y = a \cos \theta - b \sin \theta$, prove that $x^2 + y^2 = a^2 + b^2$.

Sol:

Given: $x = a \sin \theta + b \cos \theta$

Squaring both sides, we get:

$$x^2 = a^2 \sin^2 \theta + 2ab \sin \theta \cos \theta + b^2 \cos^2 \theta \quad \dots (i)$$

Also, $y = a \cos \theta - b \sin \theta$

Squaring both sides, we get:

$$y^2 = a^2 \cos^2 \theta - 2ab \sin \theta \cos \theta + b^2 \sin^2 \theta \quad \dots (ii)$$

$$\begin{aligned} \therefore LHS &= x^2 + y^2 \\ &= a^2 \sin^2 \theta + 2ab \sin \theta \cos \theta + b^2 \cos^2 \theta + a^2 \cos^2 \theta - 2ab \sin \theta \cos \theta + b^2 \sin^2 \theta \\ &\quad \quad \quad [using (i) and (ii)] \\ &= a^2 (\sin^2 \theta + \cos^2 \theta) + b^2 (\sin^2 \theta + \cos^2 \theta) \\ &= a^2 + b^2 \quad [\because \sin^2 \theta + \cos^2 \theta = 1] \\ &= RHS \end{aligned}$$

Hence proved.

16. Prove that $\left(\frac{1 + \sin \theta}{1 - \sin \theta} \right) = (\sec \theta + \tan \theta)^2$.

Sol:

$$\frac{(1 + \sin \theta)}{(1 - \sin \theta)} = (\sec \theta + \tan \theta)^2$$

$$LHS = \frac{(1 + \sin \theta)}{(1 - \sin \theta)}$$

Multiplying the numerator and denominator by $(1 + \sin \theta)$, we get:

$$\begin{aligned} &\frac{(1 + \sin \theta)^2}{1 - \sin^2 \theta} \\ &= \frac{1 + 2 \sin \theta + \sin^2 \theta}{\cos^2 \theta} \quad [\because \sin^2 \theta + \cos^2 \theta = 1] \\ &= \sec^2 \theta + 2 \times \frac{\sin \theta}{\cos \theta} \times \sec \theta + \tan^2 \theta \\ &= \sec^2 \theta + 2 \times \tan \theta \times \sec \theta + \tan^2 \theta \\ &= (\sec \theta + \tan \theta)^2 \\ &= RHS \end{aligned}$$

Hence proved.

17. Prove that $\frac{1}{(\sec \theta - \tan \theta)} - \frac{1}{\cos \theta} = \frac{1}{\cos \theta} - \frac{1}{(\sec \theta + \tan \theta)}$.

Sol:

$$\begin{aligned} &\frac{1}{(\sec \theta - \tan \theta)} - \frac{1}{\cos \theta} = \frac{1}{\cos \theta} - \frac{1}{(\sec \theta + \tan \theta)} \\ LHS &= \frac{1}{\sec \theta - \tan \theta} - \frac{1}{\cos \theta} \end{aligned}$$

$$\begin{aligned}
&= \frac{\sec \theta + \tan \theta}{\sec^2 \theta - \tan^2 \theta} - \sec \theta \\
&= \sec \theta + \tan \theta - \sec \theta \quad [\because \sec^2 \theta - \tan^2 \theta = 1] \\
&= \tan \theta \\
RHS &= \frac{1}{\cos \theta} - \frac{1}{\sec \theta + \tan \theta} \\
&= \sec \theta - \frac{(\sec \theta - \tan \theta)}{\sec^2 \theta - \tan^2 \theta} \quad (\text{Multiplying the numerator and denominator by } (\sec \theta - \tan \theta)) \\
&= \sec \theta + \tan \theta - \sec \theta \quad [\because \sec^2 \theta - \tan^2 \theta = 1] \\
&= \tan \theta \\
\therefore LHS &= RHS \\
&\text{Hence Proved}
\end{aligned}$$

18. Prove that $\frac{(\sin A - 2 \sin^3 A)}{(2 \cos^3 A - \cos A)} = \tan A$.

Sol:

$$\begin{aligned}
LHS &= \frac{(\sin A - 2 \sin^3 A)}{(2 \cos^3 A - \cos A)} \\
&= \frac{\sin A(1 - 2 \sin^2 A)}{\cos A(2 \cos^2 A - 1)} \\
&= \tan A \left\{ \frac{(\sin^2 A + \cos^2 A - 2 \sin^2 A)}{2 \cos^2 A - \sin^2 A - \cos^2 A} \right\} \quad [\because \sin^2 A + \cos^2 A = 1] \\
&= \tan A \left\{ \frac{(\cos^2 A - \sin^2 A)}{(\cos^2 A - \sin^2 A)} \right\} \\
&= \tan A \\
&= RHS
\end{aligned}$$

19. Prove that $\frac{\tan A}{(1 - \cot A)} + \frac{\cot A}{(1 - \tan A)} = (1 + \tan A + \cot A)$.

Sol:

$$\begin{aligned}
LHS &= \frac{\tan A}{(1 - \cot A)} + \frac{\cot A}{(1 - \tan A)} \\
&= \frac{\tan A}{(1 - \cot A)} + \frac{\cot^2 A}{(\cot A - 1)} \quad \left[\because \tan A = \frac{1}{\cot A} \right] \\
&= \frac{\tan A}{(1 - \cot A)} - \frac{\cot^2 A}{(1 - \cot A)} \\
&= \frac{\tan A - \cot^2 A}{(1 - \cot A)} \\
&= \frac{\left(\frac{1}{\cot A}\right) - \cot^2 A}{(1 - \cot A)} \\
&= \frac{1 - \cot^3 A}{\cot A(1 - \cot A)} \\
&= \frac{(1 - \cot A)(1 + \cot A + \cot^2 A)}{\cot A(1 - \cot A)} \quad [\because a^3 - b^3 = (a - b)(a^2 + ab + b^2)]
\end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{\cot A} + \frac{\cot^2 A}{\cot A} + \frac{\cot A}{\cot A} \\
 &= 1 + \tan A + \cot A \\
 &= RHS
 \end{aligned}$$

Hence proved

20. If $\sec 5A = \operatorname{cosec}(A - 36^\circ)$ and $5A$ is an acute angle, show that $A = 21^\circ$.

Sol:

$$\begin{aligned}
 \text{Given: } \sec 5A &= \operatorname{cosec}(A - 36^\circ) \\
 \Rightarrow \operatorname{cosec}(90^\circ - 5A) &= \operatorname{cosec}(A - 36^\circ) \quad [\because \operatorname{cosec}(90^\circ - \theta) = \sec \theta] \\
 \Rightarrow 90^\circ - 5A &= A - 36^\circ \\
 \Rightarrow 6A &= 90^\circ + 36^\circ \\
 \Rightarrow 6A &= 126^\circ \\
 \Rightarrow A &= 21^\circ
 \end{aligned}$$

Multiple Choice Question

1. $\frac{\sec 30^\circ}{\operatorname{cosec} 60^\circ} = ?$

- (a) $\frac{2}{\sqrt{3}}$ (b) $\frac{\sqrt{3}}{2}$
 (c) $\sqrt{3}$ (d) 1

Answer: (d) 1

Sol:

$$\frac{\sec 30^\circ}{\operatorname{cosec} 60^\circ} = \frac{\sec 30^\circ}{\sec(90^\circ - 60^\circ)} = \frac{\sec 30^\circ}{\sec 30^\circ} = 1$$

2. $\frac{\tan 35^\circ}{\cot 55^\circ} + \frac{\cot 78^\circ}{\tan 12^\circ} = ?$

- (a) 0 (b) 1
 (c) 2 (d) none of these

Answer: (c) 2

Sol:

We have,

$$\begin{aligned}
 &\frac{\tan 35^\circ}{\cot 55^\circ} + \frac{\cot 78^\circ}{\tan 12^\circ} \\
 &= \frac{\tan 35^\circ}{\cot(90^\circ - 35^\circ)} + \frac{\cot(90^\circ - 12^\circ)}{\tan 12^\circ} \\
 &= \frac{\tan 35^\circ}{\tan 35^\circ} + \frac{\tan 12^\circ}{\tan 12^\circ} \quad [\because \cot(90^\circ - \theta) = \tan \theta] \\
 &= 1 + 1 = 2
 \end{aligned}$$

3. $\tan 10^\circ \tan 15^\circ \tan 75^\circ \tan 80^\circ = ?$

- (a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$
 (c) -1 (d) 1

Answer: (d) 1

Sol:

We have,

$$\begin{aligned} & \tan 10^\circ \tan 15^\circ \tan 75^\circ \tan 80^\circ \\ &= \tan 10^\circ \times \tan 15^\circ \times \tan(90^\circ - 15^\circ) \times \tan(90^\circ - 10^\circ) \\ &= \tan 10^\circ \times \tan 15^\circ \times \cot 15^\circ \times \cot 10^\circ \quad [\because \tan(90^\circ - \theta) = \cot \theta] \\ &= 1 \end{aligned}$$

4. $\tan 5^\circ \tan 25^\circ \tan 30^\circ \tan 65^\circ \tan 85^\circ = ?$

- (a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$
 (c) 1 (d) none of these

Answer: (b) $\frac{1}{\sqrt{3}}$

Sol:

We have:

$$\begin{aligned} & \tan 5^\circ \tan 25^\circ \tan 30^\circ \tan 65^\circ \tan 85^\circ \\ &= \tan 5^\circ \tan 25^\circ \tan 30^\circ \tan(90^\circ - 25^\circ) \tan(90^\circ - 5^\circ) \\ &= \tan 5^\circ \tan 25^\circ \times \frac{1}{\sqrt{3}} \times \cot 25^\circ \cot 5^\circ \quad \left[\because \tan(90^\circ - \theta) = \cot \theta \text{ and } \tan 30^\circ = \frac{1}{\sqrt{3}} \right] \\ &= \frac{1}{\sqrt{3}} \end{aligned}$$

5. $\cos 1^\circ \cos 2^\circ \cos 3^\circ \dots \cos 180^\circ = ?$

- (a) -1 (b) 1
 (c) 0 (d) $\frac{1}{2}$

Answer: (c) 0

Sol:

$$\begin{aligned} & \cos 1^\circ \cos 2^\circ \cos 3^\circ \dots \cos 180^\circ \\ &= \cos 1^\circ \cos 2^\circ \cos 3^\circ \dots \cos 90^\circ \dots \cos(180^\circ) \\ &= 0 \quad [\because \cos 90^\circ = 0] \end{aligned}$$

6. $\frac{2\sin^2 63^\circ + 1 + 2\sin^2 27^\circ}{3\cos^2 17^\circ - 2 + 3\cos^2 73^\circ} = ?$

(a) $\frac{3}{2}$

(b) $\frac{2}{3}$

(c) 2

(d) 3

Answer: (d) 3**Sol:**

$$\begin{aligned}
 \text{Given: } & \frac{2 \sin^2 63^\circ + 1 + 2 \sin^2 27^\circ}{3 \cos^2 17^\circ - 2 + 3 \cos^2 73^\circ} \\
 &= \frac{2(\sin^2 63^\circ + \sin^2 27^\circ) + 1}{3(\cos^2 17^\circ + \cos^2 73^\circ) - 2} \\
 &= \frac{2[\sin^2 63^\circ + \sin^2(90^\circ - 63^\circ)] + 1}{3[\cos^2 17^\circ + \cos^2(90^\circ - 17^\circ)] - 2} \\
 &= \frac{2(\sin^2 63^\circ + \cos^2 63^\circ) + 1}{3(\cos^2 17^\circ + \sin^2 17^\circ) - 2} \quad [\because \sin(90^\circ - \theta) = \cos \theta \text{ and } \cos(90^\circ - \theta) = \sin \theta] \\
 &= \frac{2 \times 1 + 1}{3 \times 1 - 2} \quad [\because \sin^2 \theta + \cos^2 \theta = 1] \\
 &= \frac{2+1}{3-2} \\
 &= \frac{3}{1} = 3
 \end{aligned}$$

7. $\sin 47^\circ \cos 43^\circ + \cos 47^\circ \sin 43^\circ = ?$

(a) $\sin 4^\circ$

(b) $\cos 4^\circ$

(c) 1

(d) 0

Answer: (c) 1**Sol:**

We have:

$$\begin{aligned}
 & (\sin 43^\circ \cos 47^\circ + \cos 43^\circ \sin 47^\circ) \\
 &= \sin 43^\circ \cos(90^\circ - 43^\circ) + \cos 43^\circ \sin(90^\circ - 43^\circ) \\
 &= \sin 43^\circ \sin 43^\circ \\
 &+ \cos 43^\circ \cos 43^\circ \quad [\because \cos(90^\circ - \theta) = \sin \theta \text{ and } \sin(90^\circ - \theta) = \cos \theta] \\
 &= \sin^2 43^\circ + \cos^2 43^\circ \\
 &= 1
 \end{aligned}$$

8. $\sec 70^\circ \sin 20^\circ + \cos 20^\circ \operatorname{cosec} 70^\circ = ?$

(a) 0

(b) 1

(c) -1

(d) 2

Answer: (d) 2**Sol:**

We have:

$$\begin{aligned}
 & \sec 70^\circ \sin 20^\circ + \cos 20^\circ \operatorname{cosec} 70^\circ \\
 &= \frac{\sin 20^\circ}{\cos 70^\circ} + \frac{\cos 20^\circ}{\sin 70^\circ} \\
 &= \frac{\sin 20^\circ}{\cos(90^\circ - 20^\circ)} + \frac{\cos 20^\circ}{\sin(90^\circ - 20^\circ)} \\
 &= \frac{\sin 20^\circ}{\sin 20^\circ} + \frac{\cos 20^\circ}{\cos 20^\circ} \quad [\because \cos(90^\circ - \theta) = \sin \theta \text{ and } \sin(90^\circ - \theta) = \cos \theta]
 \end{aligned}$$

$$= 1 + 1$$

$$= 2$$

OR

$$\sec 70^\circ \sin 20^\circ + \cos 20^\circ \operatorname{cosec} 70^\circ$$

$$= \operatorname{cosec}(90^\circ - 70^\circ) \sin 20^\circ + \cos 20^\circ \sec(90^\circ - 70^\circ)$$

$$= \operatorname{cosec} 20^\circ \sin 20^\circ + \cos 20^\circ \sec 20^\circ$$

$$= \frac{1}{\sin 20^\circ} \times \sin 20^\circ + \cos 20^\circ \times \frac{1}{\cos 20^\circ}$$

$$= 1 + 1$$

$$= 2$$

9. If $\sin 3A = \cos(A - 10^\circ)$ and $3A$ is acute then $\angle A = ?$

(a) 35° (b) 25°

(c) 20° (d) 45°

Answer: (b) 25°

Sol:

We have:

$$[\sin 3A = \cos(A - 10^\circ)]$$

$$=> \cos(90^\circ - 3A) = \cos(A - 10^\circ) \quad [\because \sin \theta = \cos(90^\circ - \theta)]$$

$$=> 90^\circ - 3A = A - 10^\circ$$

$$=> -4A = -100$$

$$=> A = \frac{100}{4}$$

$$=> A = 25^\circ$$

10. If $\sec 4A = \operatorname{cosec}(A - 10^\circ)$ and $4A$ is acute then $\angle A = ?$

(a) 20° (b) 30°

(c) 30° (d) 50°

Answer: (a) 20°

Sol:

We have,

$$\sec 4A = \operatorname{cosec}(A - 10^\circ)$$

$$\Rightarrow \operatorname{cosec}(90^\circ - 4A) = \operatorname{cosec}(A - 10^\circ)$$

Comparing both sides, we get

$$90^\circ - 4A = A - 10^\circ$$

$$\Rightarrow 4A + A = 90^\circ + 10^\circ$$

$$\Rightarrow 5A = 100^\circ$$

$$\Rightarrow A = \frac{100^\circ}{5}$$

$$\therefore A = 20^\circ$$

11. If A and B are acute angles such that $\sin A = \cos B$ then $(A + B) = ?$

- (a) 45° (b) 60°
(c) 90° (d) 180°

Answer: (c) 90°

12. If $\cos(\alpha + \beta) = 0$ then $\sin(\alpha - \beta) = ?$

- (a) $\sin \alpha$ (b) $\cos \beta$
(c) $\sin 2\alpha$ (d) $\cos 2\beta$

Answer: (d) $\cos 2\beta$

Sol:

We have:

$$\cos(\alpha + \beta) = 0$$

$$\Rightarrow \cos(\alpha + \beta) = \cos 90^\circ$$

$$\Rightarrow \alpha + \beta = 90^\circ$$

$$\Rightarrow \alpha = 90^\circ - \beta \quad \dots (i)$$

$$\text{Now, } \sin(\alpha - \beta)$$

$$= \sin[(90^\circ - \beta) - \beta] \quad [\text{Using (i)}]$$

$$= \sin(90^\circ - 2\beta)$$

$$= \cos 2\beta \quad [\because \sin(90^\circ - \theta) = \cos \theta]$$

13. $\sin(45^\circ + \theta) - \cos(45^\circ - \theta) = ?$

- (a) $2 \sin \theta$ (b) $2 \cos \theta$
(c) 0 (d) 1

Answer: (c) 0

Sol:

We have:

$$[\sin(45^\circ + \theta) - \cos(45^\circ - \theta)]$$

$$= [\sin\{90^\circ - (45^\circ - \theta)\} - \cos(45^\circ - \theta)]$$

$$= [\cos(45^\circ - \theta) - \cos(45^\circ - \theta)] \quad [\because \sin(90^\circ - \theta) = \cos \theta]$$

$$= 0$$

14. $\sec^2 10^\circ - \cot^2 80^\circ = ?$

- (a) 1 (b) 0
(c) $\frac{3}{2}$ (d) $\frac{1}{2}$

Answer: (a) 1

Sol:

$$\text{We have: } (\sin 79^\circ \cos 11^\circ + \cos 79^\circ \sin 11^\circ)$$

$$= \sin 79^\circ \cos(90^\circ - 79^\circ) + \cos 79^\circ \sin(90^\circ - 79^\circ)$$

$$\begin{aligned}
 &= \sin 79^\circ \sin 79^\circ + \cos 79^\circ \cos 79^\circ [\because \cos(90^\circ - \theta) = \sin \theta \text{ and } \sin(90^\circ - \theta) = \cos \theta] \\
 &= \sin^2 79^\circ + \cos^2 79^\circ \\
 &= 1
 \end{aligned}$$

15. $\operatorname{cosec}^2 57^\circ - \tan^2 33^\circ = ?$

- (a) 0 (b) 1
(c) -1 (d) 2

Answer: (b) 1

Sol:

We have:

$$\begin{aligned}
 &(\operatorname{cosec}^2 57^\circ - \tan^2 33^\circ) \\
 &= [\operatorname{cosec}^2(90^\circ - 33^\circ) - \tan^2 33^\circ] \\
 &= (\sec^2 33^\circ - \tan^2 33^\circ) [\because \operatorname{cosec}(90^\circ - \theta) = \sec \theta] \\
 &= 1 [\because \sec^2 \theta - \tan^2 \theta = 1]
 \end{aligned}$$

16. $\frac{2 \tan^2 30^\circ \sec^2 52^\circ \sin^2 38^\circ}{\operatorname{cosec}^2 70^\circ - \tan^2 20^\circ} = ?$

- (a) 2 (b) $\frac{1}{2}$
(c) $\frac{2}{3}$ (d) $\frac{3}{2}$

Answer: (c) $\frac{2}{3}$

Sol:

We have:

$$\begin{aligned}
 &\left[\frac{2 \tan^2 30^\circ \sec^2 52^\circ \sin^2 38^\circ}{\operatorname{cosec}^2 70^\circ - \tan^2 20^\circ} \right] \\
 &= \left[\frac{2 \times \left(\frac{1}{\sqrt{3}}\right)^2 \sec^2 52^\circ \{\sin^2(90^\circ - 52^\circ)\}}{\{\operatorname{cosec}^2(90^\circ - 20^\circ)\} - \tan^2 20^\circ} \right] \\
 &= \left[\frac{2}{3} \times \frac{\sec^2 52^\circ \cdot \cos^2 52^\circ}{\sec^2 20^\circ - \tan^2 20^\circ} \right] [\because \sin(90^\circ - \theta) = \cos \theta \text{ and } \operatorname{cosec}(90^\circ - \theta) = \sec \theta] \\
 &= \frac{2}{3} \times \frac{1}{1} [\because \sec^2 \theta - \tan^2 \theta = 1] \\
 &= \frac{2}{3}
 \end{aligned}$$

17. $\left\{ \frac{(\sin^2 22^\circ + \sin^2 68^\circ)}{(\cos^2 22^\circ + \cos^2 68^\circ)} + \sin^2 63^\circ + \cos 63^\circ \sin 27^\circ \right\} = ?$

- (a) 0 (b) 1
(c) 2 (d) 3

Answer: (c) 2

Sol:

We have:

$$\begin{aligned}
 & \left[\frac{\sin^2 22^\circ + \sin^2 68^\circ}{\cos^2 22^\circ + \cos^2 68^\circ} + \sin^2 63^\circ + \cos 63^\circ \sin 27^\circ \right] \\
 &= \left[\frac{\sin^2 22^\circ + \sin^2 (90^\circ - 22^\circ)}{\cos^2 (90^\circ - 68^\circ) + \cos^2 68^\circ} + \sin^2 63^\circ + \cos 63^\circ \{\sin(90^\circ - 63^\circ)\} \right] \\
 &= \left[\frac{\sin^2 22^\circ + \cos^2 22^\circ}{\sin^2 68^\circ + \cos^2 68^\circ} + \sin^2 63^\circ + \cos 63^\circ \cos 63^\circ \right] \quad [\because \sin(90^\circ - \theta) = \cos \theta \text{ and } \cos(90^\circ - \theta) = \sin \theta] \\
 &= \left[\frac{1}{1} + \sin^2 63^\circ + \cos^2 63^\circ \right] \quad [\because \sin^2 \theta + \cos^2 \theta = 1] \\
 &= 1 + 1 = 2
 \end{aligned}$$

18. $\frac{\cot(90^\circ - \theta) \cdot \sin(90^\circ - \theta)}{\sin \theta} + \frac{\cot 40^\circ}{\tan 50^\circ} - (\cos^2 20^\circ + \cos^2 70^\circ) = ?$

- (a) 0 (b) 1
(c) -1 (d) none of these

Answer: (b) 1

Sol:

We have:

$$\begin{aligned}
 & \left[\frac{\cot(90^\circ - \theta) \cdot \sin(90^\circ - \theta)}{\sin \theta} + \frac{\cot 40^\circ}{\tan 50^\circ} - (\cos^2 20^\circ + \cos^2 70^\circ) \right] \\
 &= \left[\frac{\tan \theta \cdot \cos \theta}{\sin \theta} + \frac{\cot(90^\circ - 50^\circ)}{\tan 50^\circ} - \{\cos^2(90^\circ - 70^\circ) + \cos^2 70^\circ\} \right] \quad [\because \cot(90^\circ - \theta) = \tan \theta \text{ and } \sin(90^\circ - \theta) = \cos \theta] \\
 &= \left[\frac{\sin \theta}{\sin \theta} + \frac{\tan 50^\circ}{\tan 50^\circ} - (\sin^2 70^\circ + \cos^2 70^\circ) \right] \quad [\because \cos(90^\circ - \theta) = \sin \theta] \\
 &= \left(\frac{\sin \theta}{\sin \theta} + 1 - 1 \right) \\
 &= 1 + 1 - 1 = 1
 \end{aligned}$$

19. $\frac{\cos 38^\circ \operatorname{cosec} 52^\circ}{\tan 18^\circ \tan 35^\circ \tan 60^\circ \tan 72^\circ \tan 55^\circ} = ?$

- (a) $\sqrt{3}$ (b) $\frac{1}{3}$
(c) $\frac{1}{\sqrt{3}}$ (d) $\frac{2}{\sqrt{3}}$

Answer: (c) $\frac{1}{\sqrt{3}}$

Sol:

We have:

$$\begin{aligned}
& \left[\frac{\cos 38^\circ \operatorname{cosec} 52^\circ}{\tan 18^\circ \tan 35^\circ \tan 60^\circ \tan 72^\circ \tan 55^\circ} \right] \\
&= \left[\frac{\cos 38^\circ \operatorname{cosec}(90^\circ - 38^\circ)}{\tan 18^\circ \tan 35^\circ \times \sqrt{3} \times \tan(90^\circ - 18^\circ) \tan(90^\circ - 35^\circ)} \right] \quad [\because \operatorname{cosec}(90^\circ - \theta) = \sec \theta \text{ and } \tan(90^\circ - \theta) = \cot \theta] \\
&= \left[\frac{\cos 38^\circ \sec 38^\circ}{\tan 18^\circ \tan 35^\circ \times \sqrt{3} \times \cot 18^\circ \cot 35^\circ} \right] \\
&= \left[\frac{\frac{1}{\sec 38^\circ} \times \sec 38^\circ}{\frac{1}{\cot 18^\circ \cot 35^\circ} \times \sqrt{3} \cot 18^\circ \cot 35^\circ} \right] \\
&= \frac{1}{\sqrt{3}}
\end{aligned}$$

20. If $2 \sin 2\theta = \sqrt{3}$ then $\theta = ?$

- (a) 30° (b) 45°
(c) 60° (d) 90°

Answer: (a) 30° **Sol:**

$$\begin{aligned}
2 \sin 2\theta &= \sqrt{3} \\
\Rightarrow \sin 2\theta &= \frac{\sqrt{3}}{2} = \sin 60^\circ \\
\Rightarrow \sin 2\theta &= \sin 60^\circ \\
\Rightarrow 2\theta &= 60^\circ \\
\Rightarrow \theta &= 30^\circ
\end{aligned}$$

21. If $2 \cos 3\theta = 1$ then $\theta = ?$

- (a) 10° (b) 15°
(c) 20° (d) 30°

Answer: (c) 20° **Sol:**

$$\begin{aligned}
2 \cos 3\theta &= 1 \\
\Rightarrow \cos 3\theta &= \frac{1}{2} \\
\Rightarrow \cos 3\theta &= \cos 60^\circ \quad \left[\because \cos 60^\circ = \frac{1}{2} \right] \\
\Rightarrow 3\theta &= 60^\circ \\
\Rightarrow \theta &= \frac{60^\circ}{3} = 20^\circ
\end{aligned}$$

22. If $\sqrt{3} \tan 2\theta - 3 = 0$ then $\theta = ?$

- (a) 15° (b) 30°

- (c) 45° (d) 60°

Answer: (b) 30°

Sol:

$$\begin{aligned}\sqrt{3} \tan 2\theta - 3 &= 0 \\ \Rightarrow \sqrt{3} \tan 2\theta &= 3 \\ \Rightarrow \tan 2\theta &= \frac{3}{\sqrt{3}} \\ \Rightarrow \tan 2\theta &= \sqrt{3} \quad [\because \tan 60^\circ = \sqrt{3}] \\ \Rightarrow \tan 2\theta &= \tan 60^\circ \\ \Rightarrow 2\theta &= 60^\circ \\ \Rightarrow \theta &= 30^\circ\end{aligned}$$

23. If $\tan x = 3 \cot x$ then $x = ?$

- (a) 45° (b) 60°
(c) 30° (d) 15°

Answer: (b) 60°

Sol:

$$\begin{aligned}\tan x &= 3 \cot x \\ \Rightarrow \frac{\tan x}{\cot x} &= 3 \\ \Rightarrow \tan^2 x &= 3 \quad \left[\because \cot x = \frac{1}{\tan x} \right] \\ \Rightarrow \tan x &= \sqrt{3} = \tan 60^\circ \\ \Rightarrow x &= 60^\circ\end{aligned}$$

24. If $x \tan 45^\circ \cos 60^\circ = \sin 60^\circ$ then $x = ?$

- (a) 1 (b) $\frac{1}{2}$
(c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{3}$

Answer: (a) 1

Sol:

$$\begin{aligned}x \tan 45^\circ \cos 60^\circ &= \sin 60^\circ \cot 60^\circ \\ \Rightarrow x (1) \left(\frac{1}{2}\right) &= \left(\frac{\sqrt{3}}{2}\right) \left(\frac{1}{\sqrt{3}}\right) \\ \Rightarrow x \left(\frac{1}{2}\right) &= \left(\frac{1}{2}\right) \\ \Rightarrow x &= 1\end{aligned}$$

25. If $t \tan^2 45^\circ - \cos^2 30^\circ = x \sin 45^\circ \cos 45^\circ$ then $x = ?$

- (a) 2 (b) -2

(c) $\frac{1}{2}$ (d) $\frac{-1}{2}$

Answer: (c) $\frac{1}{2}$

Sol:

$$\begin{aligned}
 (\tan^2 45^\circ - \cos^2 30^\circ) &= x \sin 45^\circ \cos 45^\circ \\
 \Rightarrow x &= \frac{(\tan^2 45^\circ - \cos^2 30^\circ)}{\sin 45^\circ \cos 45^\circ} \\
 &= \frac{\left[(1)^2 - \left(\frac{\sqrt{3}}{2} \right)^2 \right]}{\left(\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \right)} \\
 &= \frac{\left(1 - \frac{3}{4} \right)}{\left(\frac{1}{2} \right)} \\
 &= \frac{\left(\frac{1}{4} \right)}{\left(\frac{1}{2} \right)} \\
 &= \frac{1}{4} \times 2 = \frac{1}{2}
 \end{aligned}$$

26. $\sec^2 60^\circ - 1 = ?$

(a) 2 (b) 3
(c) 4 (d) 0

Answer: (b) 3

Sol:

$$\begin{aligned}
 \sec^2 60^\circ - 1 &= \\
 &= (2)^2 - 1 \\
 &= 4 - 1 \\
 &= 3
 \end{aligned}$$

27. $(\cos 0^\circ + \sin 30^\circ + \sin 45^\circ)(\sin 90^\circ + \cos 60^\circ - \cos 45^\circ) = ?$

(a) $\frac{5}{6}$ (b) $\frac{5}{8}$
(c) $\frac{3}{5}$ (d) $\frac{7}{4}$

Answer: (d) $\frac{7}{4}$

Sol:

$$\begin{aligned}
 &(\cos 0^\circ + \sin 30^\circ + \sin 45^\circ)(\sin 90^\circ + \cos 60^\circ - \cos 45^\circ) \\
 &= \left(1 + \frac{1}{2} + \frac{1}{\sqrt{2}} \right) \left(1 + \frac{1}{2} - \frac{1}{\sqrt{2}} \right) \\
 &= \left(\frac{3}{2} + \frac{1}{\sqrt{2}} \right) \left(\frac{3}{2} - \frac{1}{\sqrt{2}} \right) \\
 &= \left[\left(\frac{3}{2} \right)^2 - \left(\frac{1}{\sqrt{2}} \right)^2 \right] = \left(\frac{9}{4} \right) - \left(\frac{1}{2} \right) = \left(\frac{9-2}{4} \right) = \frac{7}{4}
 \end{aligned}$$

28. $\sin^2 30^\circ + 4 \cot^2 45^\circ - \sec^2 60^\circ = ?$

- (a) 0 (b) $\frac{1}{4}$
(c) 4 (d) 1

Answer: (b) $\frac{1}{4}$

Sol:

$$\begin{aligned} & (\sin^2 30^\circ + 4 \cot^2 45^\circ - \sec^2 60^\circ) \\ &= \left[\left(\frac{1}{2}\right)^2 + 4 \times (1)^2 - (2)^2 \right] \\ &= \left(\frac{1}{4} + 4 - 4 \right) = \frac{1}{4} \end{aligned}$$

29. $3 \cos^2 60^\circ + 2 \cot^2 30^\circ - 5 \sin^2 45^\circ = ?$

- (a) $\frac{13}{6}$ (b) $\frac{17}{4}$
(c) 1 (d) 4

Answer: (b) $\frac{17}{4}$

Sol:

$$\begin{aligned} & (3 \cos^2 60^\circ + 2 \cot^2 30^\circ - 5 \sin^2 45^\circ) \\ &= \left[3 \times \left(\frac{1}{2}\right)^2 + 2 \times (\sqrt{3})^2 - 5 \times \left(\frac{1}{\sqrt{2}}\right)^2 \right] \\ &= \left[\frac{3}{4} + 6 - \frac{5}{2} \right] \\ &= \frac{3+24-10}{4} = \frac{17}{4} \end{aligned}$$

30. $\cos^2 30^\circ \cos^2 45^\circ + 4 \sec^2 60^\circ + \frac{1}{2} \cos^2 90^\circ - 2 \tan^2 60^\circ = ?$

- (a) $\frac{73}{8}$ (b) $\frac{75}{8}$
(c) $\frac{81}{8}$ (d) $\frac{83}{8}$

Answer: (d) $\frac{83}{8}$

Sol:

$$\begin{aligned} & (\cos^2 30^\circ \cos^2 45^\circ + 4 \sec^2 60^\circ + \frac{1}{2} \cos^2 90^\circ - 2 \tan^2 60^\circ) \\ &= \left[\left(\frac{\sqrt{3}}{2}\right)^2 \times \left(\frac{1}{\sqrt{2}}\right)^2 + 4 \times (2)^2 + \frac{1}{2} \times (0)^2 - 2 \times (\sqrt{3})^2 \right] \end{aligned}$$

$$\begin{aligned}
 &= \left[\left(\frac{3}{4} \times \frac{1}{2} \right) + 16 - 6 \right] \\
 &= \left[\frac{3}{8} + 10 \right] \\
 &= \frac{3+80}{8} = \frac{83}{8}
 \end{aligned}$$

31. If $\operatorname{cosec} \theta = \sqrt{10}$ then $\sec \theta = ?$

- (a) $\frac{3}{\sqrt{10}}$ (b) $\frac{\sqrt{10}}{3}$
 (c) $\frac{1}{\sqrt{10}}$ (d) $\frac{2}{\sqrt{10}}$

Answer: (b) $\frac{\sqrt{10}}{3}$

Sol:

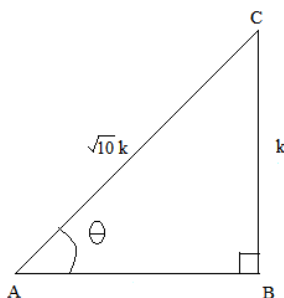
Let us first draw a right $\triangle ABC$ right angled at B and $\angle A = \theta$.

Given: $\operatorname{cosec} \theta = \sqrt{10}$, but $\sin \theta = \frac{1}{\operatorname{cosec} \theta} = \frac{1}{\sqrt{10}}$

Also, $\sin \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AC}$

So, $\frac{BC}{AC} = \frac{1}{\sqrt{10}}$

Thus, $BC = k$ and $AC = \sqrt{10}k$



Using Pythagoras theorem in triangle ABC, we have:

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow AB^2 = AC^2 - BC^2$$

$$\Rightarrow AB^2 = (\sqrt{10}k)^2 - (k)^2$$

$$\Rightarrow AB^2 = 9k^2$$

$$\Rightarrow AB = 3k$$

$$\therefore \sec \theta = \frac{AC}{AB} = \frac{\sqrt{10}k}{3k} = \frac{\sqrt{10}}{3}$$

32. If $\tan \theta = \frac{8}{15}$ then $\operatorname{cosec} \theta = ?$

- (a) $\frac{17}{8}$ (b) $\frac{8}{17}$
 (c) $\frac{17}{15}$ (d) $\frac{15}{17}$

Answer: (a) $\frac{17}{8}$

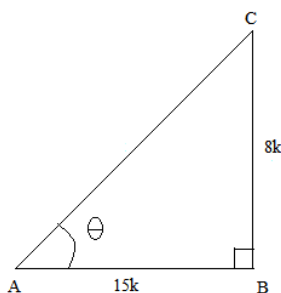
Sol:

Let us first draw a right $\triangle ABC$ right angled at B and $\angle A = \theta$.

Give: $\tan \theta = \frac{8}{5}$, but $\tan \theta = \frac{BC}{AB}$

So, $\frac{BC}{AB} = \frac{8}{15}$

Thus, $BC = 8k$ and $AB = 15k$



Using Pythagoras theorem in triangle ABC, we have:

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow AC^2 = (15k)^2 + (8k)^2$$

$$\Rightarrow AC^2 = 289k^2$$

$$\Rightarrow AC = 17k$$

$$\therefore \operatorname{cosec} \theta = \frac{AC}{BC} = \frac{17k}{8k} = \frac{17}{8}$$

33. If $\sin \theta = \frac{b}{a}$ then $\cos \theta = ?$

- (a) $\frac{b}{\sqrt{b^2 - a^2}}$ (b) $\frac{\sqrt{b^2 - a^2}}{b}$
 (c) $\frac{a}{\sqrt{b^2 - a^2}}$ (d) $\frac{b}{a}$

Answer: (b) $\frac{\sqrt{b^2 - a^2}}{b}$

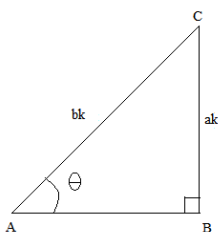
Sol:

Let us first draw a right $\triangle ABC$ right angled at B and $\angle A = \theta$.

Given: $\sin \theta = \frac{a}{b}$, but $\sin \theta = \frac{BC}{AC}$

So, $\frac{BC}{AC} = \frac{a}{b}$

Thus, $BC = ak$ and $AC = bk$



Using Pythagoras theorem in triangle ABC, we have:

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow AB^2 = AC^2 - BC^2$$

$$\Rightarrow AB^2 = (bk)^2 - (ak)^2$$

$$\Rightarrow AB^2 = (b^2 - a^2)k^2$$

$$\Rightarrow AB = (\sqrt{b^2 - a^2})k$$

$$\therefore \cos \theta = \frac{AB}{AC} = \frac{\sqrt{b^2 - a^2}k}{bk} = \frac{\sqrt{b^2 - a^2}}{b}$$

34. If $\tan \theta = \sqrt{3}$ then $\sec \theta = ?$

(a) $\frac{2}{\sqrt{3}}$

(b) $\frac{\sqrt{3}}{2}$

(c) $\frac{1}{2}$

(d) 2

Answer: (d) 2

Sol:

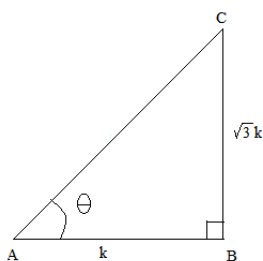
Let us first draw a right $\triangle ABC$ right angled at B and $\angle A = \theta$.

Given: $\tan \theta = \sqrt{3}$

But $\tan \theta = \frac{BC}{AB}$

So, $\frac{BC}{AB} = \frac{\sqrt{3}}{1}$

Thus, $BC = \sqrt{3}k$ and $AB = k$



Using Pythagoras theorem, we get:

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow AC^2 = (\sqrt{3}k)^2 + (k)^2$$

$$\Rightarrow AC^2 = 4k^2$$

$$\Rightarrow AC = 2k$$

$$\therefore \sec \theta = \frac{AC}{AB} = \frac{2k}{k} = \frac{2}{1}$$

35. If $\sec \theta = \frac{25}{7}$ then $\sin \theta = ?$

(a) $\frac{7}{24}$

(b) $\frac{24}{7}$

(c) $\frac{24}{25}$

(d) none of these

Answer: (c) $\frac{24}{25}$

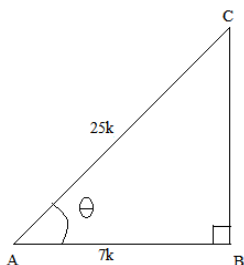
Sol:

Let us first draw a right $\triangle ABC$ right angled at B and $\angle A = \theta$.

Given $\sec \theta = \frac{25}{7}$

But $\cos \theta = \frac{1}{\sec \theta} = \frac{AB}{AC} = \frac{7}{25}$

Thus, $AC = 25k$ and $AB = 7k$



Using Pythagoras theorem, we get:

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow BC^2 = AC^2 - AB^2$$

$$\Rightarrow BC^2 = (25k)^2 - (7k)^2$$

$$\Rightarrow BC^2 = 576k^2$$

$$\Rightarrow BC = 24k$$

$$\therefore \sin \theta = \frac{BC}{AC} = \frac{24k}{25k} = \frac{24}{25}$$

36. If $\sin \theta = \frac{1}{2}$ then $\cot \theta = ?$

(a) $\frac{1}{\sqrt{3}}$ (b) $\sqrt{3}$

(c) $\frac{\sqrt{3}}{2}$ (d) 1

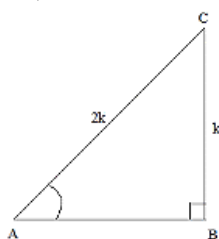
Answer: (b) $\sqrt{3}$

Sol:

Given: $\sin \theta = \frac{1}{2}$, but $\sin \theta = \frac{BC}{AC}$

So, $\frac{BC}{AC} = \frac{1}{2}$

Thus, $BC = k$ and $AC = 2k$



Using Pythagoras theorem in triangle ABC, we have:

$$AC^2 = AB^2 + BC^2$$

$$AB^2 = AC^2 - BC^2$$

$$AB^2 = (2k)^2 - (k)^2$$

$$AB^2 = 3k^2$$

$$AB = \sqrt{3}k$$

$$\text{So, } \tan \theta = \frac{BC}{AB} = \frac{k}{\sqrt{3}k} = \frac{1}{\sqrt{3}}$$

$$\therefore \cot \theta = \frac{1}{\tan \theta} = \sqrt{3}$$

37. If $\cos \theta = \frac{4}{5}$ then $\tan \theta = ?$

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$

(c) $\frac{3}{5}$ (d) $\frac{5}{3}$

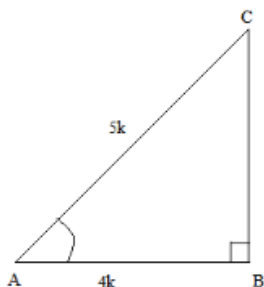
Answer: (a) $\frac{3}{4}$

Sol:

Since $\cos \theta = \frac{4}{5}$ but $\cos \theta = \frac{AB}{AC}$

$$\text{So, } \frac{AB}{AC} = \frac{4}{5}$$

Thus, $AB = 4k$ and $AC = 5k$



Using Pythagoras theorem in triangle ABC, we have:

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow BC^2 = AC^2 - AB^2$$

$$\Rightarrow BC^2 = (5k)^2 - (4K)^2$$

$$\Rightarrow BC^2 = 9k^2$$

$$\Rightarrow BC = 3k$$

$$\therefore \tan \theta = \frac{BC}{AB} = \frac{3}{4}$$

38. If $3x = \operatorname{cosec} \theta$ and $\frac{3}{x} = \cot \theta$ then $\left(x^2 - \frac{1}{x^2}\right) = ?$

(a) $\frac{1}{27}$

(b) $\frac{1}{81}$

(c) $\frac{1}{3}$

(d) $\frac{1}{9}$

Answer: (c) $\frac{1}{3}$

Sol:

Given: $3x = \operatorname{cosec} \theta$ and $\frac{3}{x} = \cot \theta$

Also, we can deduce that $x = \frac{\operatorname{cosec} \theta}{3}$ and $\frac{1}{x} = \frac{\cot \theta}{3}$

So, substituting the values of x and $\frac{1}{x}$ in the given expression, we get:

$$3 \left(x^2 - \frac{1}{x^2}\right) = 3 \left(\left(\frac{\operatorname{cosec} \theta}{3}\right)^2 - \left(\frac{\cot \theta}{3}\right)^2\right)$$

$$= 3 \left(\left(\frac{\operatorname{cosec}^2 \theta}{9}\right) - \left(\frac{\cot^2 \theta}{9}\right)\right)$$

$$= \frac{3}{9} (\operatorname{cosec}^2 \theta - \cot^2 \theta)$$

$$= \frac{1}{3} \quad [\text{By using the identity: } (\sec^2 \theta - \cot^2 \theta = 1)]$$

39. If $2x = \sec A$ and $\frac{2}{x} = \tan A$ then $2\left(x^2 - \frac{1}{x^2}\right) = ?$

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{1}{8}$ (d) $\frac{1}{16}$

Answer: (a) $\frac{1}{2}$

Sol:

Given: $2x = \sec A$ and $\frac{2}{x} = \tan A$

Also, we can deduce that $x = \frac{\sec A}{2}$ and $\frac{1}{x} = \frac{\tan A}{2}$

So, substituting the values of x and $\frac{1}{x}$ in the given expression, we get:

$$\begin{aligned} 2\left(x^2 - \frac{1}{x^2}\right) &= 2\left(\left(\frac{\sec A}{2}\right)^2 - \left(\frac{\tan A}{2}\right)^2\right) \\ &= 2\left(\frac{\sec^2 A}{4} - \frac{\tan^2 A}{4}\right) \\ &= \frac{2}{4}(\sec^2 A - \tan^2 A) \\ &= \frac{1}{2} \quad [\text{By using the identity: } (\sec^2 \theta - \tan^2 \theta = 1)] \end{aligned}$$

40. If $\tan \theta = \frac{4}{3}$ then $(\sin \theta + \cos \theta) = ?$

(a) $\frac{7}{3}$ (b) $\frac{7}{4}$

(c) $\frac{7}{5}$ (d) $\frac{5}{7}$

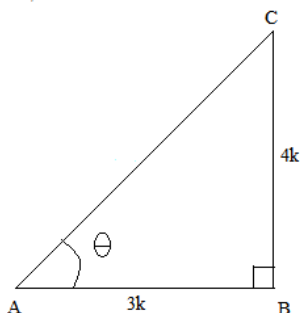
Answer: (c) $\frac{7}{5}$

Sol:

Let us first draw a right $\triangle ABC$ right angled at B and $\angle A = \theta$.

$$\tan \theta = \frac{4}{3} = \frac{BC}{AB}$$

So, $AB = 3k$ and $BC = 4k$



Using Pythagoras theorem, we get:

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow AC^2 = (3k)^2 + (4k)^2$$

$$\Rightarrow AC^2 = 25k^2$$

$$\Rightarrow AC = 5k$$

$$\text{Thus, } \sin \theta = \frac{BC}{AC} = \frac{4}{5}$$

$$\text{And } \cos \theta = \frac{AB}{AC} = \frac{3}{5}$$

$$\therefore (\sin \theta + \cos \theta) = \left(\frac{4}{5} + \frac{3}{5}\right) = \frac{7}{5}$$

41. If $(\tan \theta + \cot \theta) = 5$ then $(\tan^2 \theta + \cot^2 \theta) = ?$

- (a) 27 (b) 25
(c) 24 (d) 23

Answer: (d) 23

Sol:

We have $(\tan \theta + \cot \theta) = 5$

Squaring both sides, we get:

$$(\tan \theta + \cot \theta)^2 = 5^2$$

$$\Rightarrow \tan^2 \theta + \cot^2 \theta + 2 \tan \theta \cot \theta = 25$$

$$\Rightarrow \tan^2 \theta + \cot^2 \theta + 2 = 25 \quad \left[\because \tan \theta = \frac{1}{\cot \theta} \right]$$

$$\Rightarrow \tan^2 \theta + \cot^2 \theta = 25 - 2 = 23$$

42. If $(\cos \theta + \sec \theta) = \frac{5}{2}$ then $(\cos^2 \theta + \sec^2 \theta) = ?$

- (a) $\frac{21}{4}$ (b) $\frac{17}{4}$
(c) $\frac{29}{4}$ (d) $\frac{33}{4}$

Answer: (b) $\frac{17}{4}$

Sol:

We have $(\cos \theta + \sec \theta) = \frac{5}{2}$

Squaring both sides, we get:

$$(\cos \theta + \sec \theta)^2 = \left(\frac{5}{2}\right)^2$$

$$\Rightarrow \cos^2 \theta + \sec^2 \theta + 2\theta = \frac{25}{4}$$

$$\Rightarrow \cos^2 \theta + \sec^2 \theta + 2 = \frac{25}{4} \quad \left[\because \sec \theta = \frac{1}{\cos \theta} \right]$$

$$\Rightarrow \cos^2 \theta + \sec^2 \theta = \frac{25}{4} - 2 = \frac{17}{4}$$

43. If $\tan \theta = \frac{1}{\sqrt{7}}$ then $\frac{(\operatorname{cosec}^2 \theta + \sec^2 \theta)}{(\cos^2 \theta + \sec^2 \theta)} = ?$

(a) $\frac{-2}{3}$

(b) $\frac{-3}{4}$

(c) $\frac{2}{3}$

(d) $\frac{3}{4}$

Answer: (d) $\frac{3}{4}$

Sol:

$$= \frac{\operatorname{cosec}^2 \theta - \sec^2 \theta}{\operatorname{cosec}^2 \theta + \sec^2 \theta}$$

$$= \frac{\sin^2 \theta \left(\frac{1}{\sin^2 \theta} - \frac{1}{\cos^2 \theta} \right)}{\sin^2 \theta \left(\frac{1}{\sin^2 \theta} + \frac{1}{\cos^2 \theta} \right)}$$

[Multiplying the numerator and denominator by $\sin^2 \theta$]

$$= \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$$

$$= \frac{1 - \frac{1}{7}}{1 + \frac{1}{7}} = \frac{\frac{6}{7}}{\frac{8}{7}} = \frac{6}{8} = \frac{3}{4}$$

44. If $7 \tan \theta = 4$ then $\frac{(7 \sin \theta - 3 \cos \theta)}{(7 \sin \theta + 3 \cos \theta)} = ?$

(a) $\frac{1}{7}$

(b) $\frac{5}{7}$

(c) $\frac{3}{7}$

(d) $\frac{5}{14}$

Answer: (a) $\frac{1}{7}$

Sol:

$$7 \tan \theta = 4$$

Now, dividing the numerator and denominator of the given expression by $\cos \theta$,

We get:

$$\begin{aligned} & \frac{\frac{1}{\cos \theta}(7 \sin \theta - 3 \cos \theta)}{\frac{1}{\cos \theta}(7 \sin \theta + 3 \cos \theta)} \\ &= \frac{7 \tan \theta - 3}{7 \tan \theta + 3} \\ &= \frac{4 - 3}{4 + 3} \quad [\because 7 \tan \theta = 4] \\ &= \frac{1}{7} \end{aligned}$$

45. If $3 \cot \theta = 4$ then $\frac{(5 \sin \theta + 3 \cos \theta)}{(5 \sin \theta - 3 \cos \theta)} = ?$

(a) $\frac{1}{3}$ (b) 3

(c) $\frac{1}{9}$ (d) 9

Answer: (d) 9

Sol:

We have $\frac{(5 \sin \theta + 3 \cos \theta)}{(5 \sin \theta - 3 \cos \theta)}$

Dividing the numerator and denominator of the given expression by $\sin \theta$, we get:

$$\begin{aligned} & \frac{\frac{1}{\sin \theta}(5 \sin \theta + 3 \cos \theta)}{\frac{1}{\sin \theta}(5 \sin \theta - 3 \cos \theta)} \\ &= \frac{5 + 3 \cot \theta}{5 - 3 \cot \theta} \\ &= \frac{5 + 4}{5 - 4} = 9 \quad [\because 3 \cot \theta = 4] \end{aligned}$$

46. If $\tan \theta = \frac{a}{b}$ then $\frac{(a \sin \theta - b \cos \theta)}{(a \sin \theta + b \cos \theta)} = ?$

(a) $\frac{(a^2 + b^2)}{(a^2 - b^2)}$ (b) $\frac{(a^2 - b^2)}{(a^2 + b^2)}$

(c) $\frac{a^2}{(a^2 + b^2)}$ (d) $\frac{a^2}{(a^2 - b^2)}$

Answer: (b) $\frac{(a^2 - b^2)}{(a^2 + b^2)}$

Sol:

We have $\tan \theta = \frac{a}{b}$

Now, dividing the numerator and denominator of the given expression by $\cos \theta$

We get:

$$\frac{a \sin \theta - b \cos \theta}{a \sin \theta + b \cos \theta} = \frac{\frac{1}{\cos \theta}(a \sin \theta - b \cos \theta)}{\frac{1}{\cos \theta}(a \sin \theta + b \cos \theta)} = \frac{a \tan \theta - b}{a \tan \theta + b} = \frac{\frac{a^2}{b} - b}{\frac{a^2}{b} + b} = \frac{a^2 - b^2}{a^2 + b^2}$$

47. If $\sin A + \sin^2 A = 1$ then $\cos^2 A + \cos^4 A = ?$

- (a) $\frac{1}{2}$ (b) 1
(c) 2 (d) 3

Answer: (b) 1

Sol:

$$\begin{aligned} \sin A + \sin^2 A &= 1 \\ \Rightarrow \sin A &= 1 - \sin^2 A \\ \Rightarrow \sin A &= \cos^2 A \quad (\because 1 - \sin^2 A) \\ \Rightarrow \sin^2 A &= \cos^4 A \quad (\text{Squaring both sides}) \\ \Rightarrow 1 - \cos^2 A &= \cos^4 A \\ \Rightarrow \cos^4 A + \cos^2 A &= 1 \end{aligned}$$

48. If $\cos A + \cos^2 A = 1$ then $\sin^2 A + \sin^4 A = ?$

- (a) 1 (b) 2
(c) 4 (d) 3

Answer: (a) 1

Sol:

$$\begin{aligned} \cos A + \cos^2 A &= 1 \\ \Rightarrow \cos A &= 1 - \cos^2 A \\ \Rightarrow \cos A &= \sin^2 A \quad (\because 1 - \cos^2 A = \sin^2 A) \\ \Rightarrow \cos^2 A &= \sin^4 A \quad (\text{Squaring both sides}) \\ \Rightarrow 1 - \sin^2 A &= \sin^4 A \\ \Rightarrow \sin^4 A + \sin^2 A &= 1 \end{aligned}$$

49. $\frac{\sqrt{1 - \sin A}}{\sqrt{1 + \sin A}} = ?$

- (a) $\sec A + \tan A$ (b) $\sec A - \tan A$
(c) $\sec A \tan A$ (d) none of these

Answer: (b) $\sec A - \tan A$

Sol:

$$\begin{aligned}
 & \sqrt{\frac{1-\sin A}{1+\sin A}} \\
 &= \sqrt{\frac{(1-\sin A)}{(1+\sin A)} \times \frac{(1-\sin A)}{(1-\sin A)}} \quad [\text{Multiplying the denominator and numerator by } (1 - \sin A)] \\
 &= \frac{(1-\sin A)}{\sqrt{1-\sin^2 A}} \\
 &= \frac{(1-\sin A)}{\sqrt{\cos^2 A}} \\
 &= \frac{(1-\sin A)}{\cos A} \\
 &= \frac{1}{\cos A} - \frac{\sin A}{\cos A} \\
 &= \sec A - \tan A
 \end{aligned}$$

50. $\frac{\sqrt{1+\cos A}}{\sqrt{1-\cos A}} = ?$

- (a) $\cos \sec A - \cot A$ (b) $\cos \sec A + \cot A$
 (c) $\operatorname{cosec} A \cot A$ (d) none of these

Answer: (b) $\cos \sec A + \cot A$

Sol:

$$\begin{aligned}
 & \sqrt{\frac{1-\cos A}{1+\cos A}} \\
 &= \sqrt{\frac{(1-\cos A)}{(1+\cos A)} \times \frac{(1-\cos A)}{(1-\cos A)}} \quad [\text{Multiplying the numerator and denominator by } (1 - \cos A)] \\
 &= \sqrt{\frac{(1-\cos A)(1-\cos A)}{1-\cos^2 A}} \\
 &= \frac{1-\cos A}{\sqrt{\sin^2 A}} \\
 &= \frac{\sin A}{1-\cos A} \\
 &= \frac{1}{\sin A} - \frac{\cos A}{\sin A} \\
 &= \operatorname{cosec} A - \cot A
 \end{aligned}$$

51. If $\tan \theta = \frac{a}{b}$ then $\frac{(\cos \theta + \sin \theta)}{(\cos \theta - \sin \theta)} = ?$

- (a) $\frac{a+b}{a-b}$ (b) $\frac{a+b}{a-b}$
 (c) $\frac{b+a}{b-a}$ (d) $\frac{b-a}{b+a}$

Answer: (c) $\frac{b+a}{b-a}$

Sol:

Given: $\tan \theta = \frac{a}{b}$

$$\begin{aligned}
 &\text{Now, } \frac{(\cos \theta + \sin \theta)}{(\cos \theta - \sin \theta)} \\
 &= \frac{(1 + \tan \theta)}{(1 - \tan \theta)} \quad [\text{Dividing the numerator and denominator by } \cos \theta] \\
 &= \frac{\left(1 + \frac{a}{b}\right)}{\left(1 - \frac{a}{b}\right)} \\
 &= \frac{\left(\frac{b+a}{b}\right)}{\left(\frac{b-a}{b}\right)} \\
 &= \frac{(b+a)}{(b-a)}
 \end{aligned}$$

52. $(\operatorname{cosec} \theta - \cot \theta)^2 = ?$

- (a) $\frac{1 + \cos \theta}{1 - \cos \theta}$ (b) $\frac{1 - \cos \theta}{1 + \cos \theta}$
 (c) $\frac{1 + \sin \theta}{1 - \sin \theta}$ (d) none of these

Answer: (b) $\frac{1 - \cos \theta}{1 + \cos \theta}$

Sol:

$$\begin{aligned}
 &(\operatorname{cosec} \theta - \cot \theta)^2 \\
 &= \left(\frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta} \right)^2 \\
 &= \left(\frac{1 - \cos \theta}{\sin \theta} \right)^2 \\
 &= \frac{(1 - \cos \theta)^2}{(1 - \cos^2 \theta)} \\
 &= \frac{(1 - \cos \theta)^2}{(1 + \cos \theta)(1 - \cos \theta)} \\
 &= \frac{(1 - \cos \theta)}{(1 + \cos \theta)}
 \end{aligned}$$

53. $(\sec A + \tan A)(1 - \sin A) = ?$

- (a) $\sin A$ (b) $\cos A$
 (c) $\sec A$ (d) $\operatorname{cosec} A$

Answer: (b) $\cos A$

Sol:

$$\begin{aligned}
 &(\sec A + \tan A)(1 - \sin A) \\
 &= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A} \right) (1 - \sin A) \\
 &= \left(\frac{1 + \sin A}{\cos A} \right) (1 - \sin A) \\
 &= \left(\frac{1 - \sin^2 A}{\cos A} \right) \\
 &= \left(\frac{\cos^2 A}{\cos A} \right) \\
 &= \cos A
 \end{aligned}$$