Probability

1. **Experiment:**
   An operation which can produce some well-defined outcomes is called an experiment.

2. **Random Experiment:**
   An experiment in which all possible outcomes are know and the exact output cannot be predicted in advance, is called a random experiment.
   Examples:

   i. Rolling an unbiased dice.

   ii. Tossing a fair coin.

   iii. Drawing a card from a pack of well-shuffled cards.

   iv. Picking up a ball of certain colour from a bag containing balls of different colours.

   Details:

   i. When we throw a coin, then either a Head (H) or a Tail (T) appears.

   ii. A dice is a solid cube, having 6 faces, marked 1, 2, 3, 4, 5, 6 respectively. When we throw a die, the outcome is the number that appears on its upper face.

   iii. A pack of cards has 52 cards.
   It has 13 cards of each suit, name **Spades, Clubs, Hearts and Diamonds**.
   Cards of spades and clubs are **black cards**.
   Cards of hearts and diamonds are **red cards**.
   There are 4 honours of each unit.
   There are **Kings, Queens and Jacks**. These are all called face cards.

3. **Sample Space:**

When we perform an experiment, then the set S of all possible outcomes is called the **sample space**.

Examples:

1. In tossing a coin, S = {H, T}

2. If two coins are tossed, the S = {HH, HT, TH, TT}.

3. In rolling a dice, we have, S = {1, 2, 3, 4, 5, 6}.

4. **Event:**

Any subset of a sample space is called an **event**.

5. **Probability of Occurrence of an Event:**

Let S be the sample and let E be an event.

Then, $E \subseteq S$.

$\therefore$ $P(E) = \dfrac{n(E)}{n(S)}$.

6. **Results on Probability:**

  i. $P(S) = 1$

  ii. $0 \leq P(E) \leq 1$

  iii. $P(\phi) = 0$

  iv. For any events A and B we have : $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

  v. If $\overline{A}$ denotes (not-A), then $P(\overline{A}) = 1 - P(A)$.

Exercise 15A

Question 1:

Total numbers of trials = 500
Numbers of heads = 285
Numbers of tails = 215

(i) Let E be the event of getting a head

$\therefore$ P(getting a head)=P (E) $= \dfrac{\text{numbers of heads coming up}}{\text{total number of trials}}$

$$= \dfrac{285}{500} = 0.57$$

(ii) Let F be the event of getting a tail

$\therefore$ P (getting a tail)=P(F)= $\dfrac{\text{numbers of tails coming up}}{\text{total number of trials}}$

$$= \dfrac{215}{500}$$

$$= 0.43$$

Question 2:

Total numbers of trials = 400
Numbers of times 2 head appears=112
Number of times 1 head appears = 160
Number of times 0 head appears= 128

In a random toss of two coins , Let E_1, E_2, E_3, be the events of P(getting 2 heads)

=P(E_1)= $\dfrac{\text{numbers of times 2 heads appear}}{\text{total number of trials}} = \dfrac{112}{400} = 0.28$

P(getting 1 head)=P(E_2)=

$\dfrac{\text{numbers of times 1 head appears}}{\text{total number of trials}} = \dfrac{160}{400} = 0.4$

P(getting 0 head)=P(E_3)=

$\dfrac{\text{numbers of times 0 head appears}}{\text{total number of trials}} = \dfrac{128}{400} = 0.32$

Question 3:

Total number of trials=200

Number of times 3 heads appeared=39

Number of times 2 heads appeared = 58

Number of times 1 head appeared =67

Number of times 0 head appeared=36

The random toss of 3 coins , Let E1, E2, E3 and E4 be the events of getting 3 heads , 1 head and 0 head and 2 heads respectively . Then;

(i) P(getting 3 heads)=$P(E_1)$=

$$\frac{\text{numbers of times 3 head appeared}}{\text{total number of trials}}$$

$$=\frac{39}{200}$$

$$= 0.195$$

(ii) P (getting 1 head)= $P(E_2)$=
$$\frac{\text{numbers of times 1 head appeared}}{\text{total number of trials}}$$

$$=\frac{67}{200}$$

$$=0.335$$

(iii) P(getting 0head)= $P(E_3)$=
$$\frac{\text{numbers of times 0 head appeared}}{\text{total number of trials}}$$

$$=\frac{36}{200}$$

$$=0.18$$

(iii)   P(getting 2heads)= $P(E_4)$=
$$\frac{\text{numbers of times 2head appeared}}{\text{total number of trials}}$$

$$=\frac{58}{200}$$

$$=0.29$$

Question 4:

Total number of trials = 300

In a random throw of a die let E_1, E_2, E_3, and E_4 be the events of 3,6,5, and 1 respectively .Then;

(i) P(getting 3)= P(E_1)=

$$\frac{\text{numbers of times 3 appeared}}{\text{total number of trials}}$$

$$= \frac{54}{300}$$

$$= 0.18$$

(ii) P(getting 6)= P(E_2)=

$$\frac{\text{numbers of times 6 appeared}}{\text{total numberof trials}}$$

$$= \frac{33}{300}$$

$$= 0.11$$

(iii)   P(getting 5)= P(E_3)=

$$\frac{\text{numbers of times 5 appeared}}{\text{total number of trials}}$$

$$= \frac{39}{300}$$

$$= 0.13$$

(iv)   P(getting 1)= P(E_4)=

$$\frac{\text{numbers of times 2 head appeared}}{\text{total number of trials}}$$

$$= \frac{60}{300}$$

$$= 0.2$$

Question 5:

The number of ladies =200

Number of ladies who like coffee =142

Number of ladies who do not like coffee=58

Let E_1= event that the selected lady likes coffee.

$$\therefore P(E1) = \frac{\text{numbers of ladies who like coffee}}{\text{total number of trials}} = \frac{142}{200} = 0.71$$

Let ($E2$)= event that the selected lady dislikes coffee. Then

$$\therefore P(E2)= \frac{\text{numbers of ladies who dislike coffee}}{\text{total number of trials}} = \frac{58}{200} = 0.29$$

Question 6:

Number of tests in which he gets more than 60% marks =2

Total numbers of tests =6

$\therefore$ Required probability

$$= \frac{\text{numbers of tests in which he gets more than 60\% marks}}{\text{total number of trials}}$$

$$= \frac{2}{6} = \frac{1}{3}$$

Question 7:

Total numbers of vehicles = 240

Numbers of two wheelers = 84

∴ Required probability

$$= \frac{\text{numbers of two wheelers}}{\text{total number of vehicles}}$$

$$= \frac{84}{240}$$

$$= 0.35$$

Question 8:

Total phone numbers = 200

Numbers of phone numbers with unit digit 5 = 24

∴ Required probability

$$= \frac{\text{numbers of phone numbers with units digits 5}}{\text{total number of numbers}}$$

$$= \frac{24}{200}$$
$$= 0.12$$

Numbers of phone numbers with units digit 8 = 16

∴ Required probability

$$= \frac{\text{numbers of phone numbers with units digits 8}}{\text{total number of phone numbers}}$$

$$= \frac{16}{200}$$
$$= 0.08$$

Question 9:

Total number of students=40

(i) Numbers of students having blood group O = 14

∴ Required probability

$$= \frac{\text{numbers of students having blood group O}}{\text{total number of students}} = \frac{14}{40} = 0.35$$

(ii) Numbers of students having blood group AB = 6

∴ Required probability

$$= \frac{\text{numbers of students having blood group AB}}{\text{total number of students}} = \frac{6}{40} = 0.15$$

Question 10:

Total numbers of students =30

Numbers of students who lie in the interval 21-30=6

∴ Required probability

$$= \frac{\text{numbers of students in the interval}}{\text{total number of students}} = \frac{6}{30} = 0.2$$

Question 11:

Total number of patients=360

(i)P (getting a patient of age 30 years or more but less than 40 years)$= \dfrac{60}{360} = \dfrac{1}{6}$

(ii)P (getting a patient of age 50 years or more but less than 70 years)

$= \left(\dfrac{50 + 30}{360} \right) = \dfrac{80}{360} = \dfrac{2}{9}$

(iii) P (getting a patient of age less than 10 years) $= \dfrac{0}{360} = 0$

(iv) P (getting a patient of age 10 years or more) $= \dfrac{360}{360} = 1$