Ratio and Proportion
Exercise 8A

Ratio and Proportion

- A ratio is a comparison of two values expressed as a quotient
  - Example: A class has 12 girls and 18 boys. The ratio of girls to boys is $\frac{12}{18}$
  - This ratio can also be expressed as an equivalent fraction $\frac{2}{3}$
- A proportion is an equation stating that two ratios are equal.
  - Example: $\frac{12}{18} = \frac{2}{3}$

1. **Ratio:**

   The ratio of two quantities a and b in the same units, is the fraction $\frac{a}{b}$ and we write it as $a : b$.

   In the ratio $a : b$, we call a as the first term or **antecedent** and b, the second term or **consequent**.

   Eg. The ratio 5 : 9 represents $\frac{5}{9}$ with antecedent = 5, consequent = 9.

   Rule: The multiplication or division of each term of a ratio by the same non-zero number does not affect the ratio.

   Eg. 4 : 5 = 8 : 10 = 12 : 15. Also, 4 : 6 = 2 : 3.

2. **Proportion:**

   The equality of two ratios is called proportion.

   If $a : b = c : d$, we write $a : b :: c : d$ and we say that a, b, c, d are in proportion.

   Here a and d are called **extremes**, while b and c are called **mean terms**.

   Product of means = Product of extremes.

   Thus, $a : b :: c : d \Leftrightarrow (b \times c) = (a \times d)$.

3. **Fourth Proportional:**

   If $a : b = c : d$, then d is called the fourth proportional to a, b, c.

   Third Proportional:

   $a : b = c : d$, then c is called the third proportion to a and b.

   Mean Proportional:

   Mean proportional between a and b is $\sqrt{ab}$.

4. **Comparison of Ratios:**

   We say that $(a : b) > (c : d) \Leftrightarrow \frac{a}{b} > \frac{c}{d}$.

   Compounded Ratio:

   The compounded ratio of the ratios: $(a : b)$, $(c : d)$, $(e : f)$ is $(ace : bdf)$.

5. **Duplicate Ratios:**

   Duplicate ratio of $(a : b)$ is $(a^2 : b^2)$.

   Sub-duplicate ratio of $(a : b)$ is $(\sqrt{a} : \sqrt{b})$.

   Triplicate ratio of $(a : b)$ is $(a^3 : b^3)$.

   Sub-triplicate ratio of $(a : b)$ is $(a^{1/3} : b^{1/3})$.

   If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$. [componendo and dividendo]

6. **Variations:**

   We say that x is directly proportional to y, if $x = ky$ for some constant k and we write, $x \propto y$.

   We say that x is inversely proportional to y, if $xy = k$ for some constant k *and* we write, $x \propto \frac{1}{y}$.

Properties of proportions:
Convertendo: If a : b :: c : d ,then a : (a - b) :: c : (c – d) .

Invertendo: If $\frac{a}{b}=\frac{c}{d}\Rightarrow\frac{b}{a}=\frac{d}{c}$.

Alternendo: If $\frac{a}{b}=\frac{c}{d}\Rightarrow\frac{a}{c}=\frac{b}{d}$.

Componendo: If $\frac{a}{b}=\frac{c}{d}\Rightarrow\frac{a+b}{b}=\frac{c+d}{d}$.

Dividendo: $\frac{a}{b}=\frac{c}{d}\Rightarrow\frac{a-b}{b}=\frac{c-d}{d}$

Componendo and Dividendo: If $\frac{a}{b}=\frac{c}{d}\Rightarrow\frac{a+b}{a-b}=\frac{c+d}{c-d}$.

Q1

Answer :

(i) HCF of 24 and 40 is 8.
∴ 24 : 40 = $\frac{24}{40}=\frac{24\div 8}{40\div 8}=\frac{3}{5}$ = 3 : 5

Hence, 24 : 40 in its simplest form is 3 : 5.

(ii) HCF of 13.5 and 15 is 1.5.

$\frac{13.5}{15}=\frac{135}{150}$
The HCF of 135 and 150 is 15.
$=\frac{135\div 15}{150\div 15}=\frac{9}{10}$

Hence, 13.5 : 15 in its simplest form is 9 : 10.

(iii) $\frac{20}{3}:\frac{15}{2}=40:45$
The HCF of 40 and 45 is 5.

∴ 40 : 45 = $\frac{40}{45}=\frac{40\div 5}{45\div 5}=\frac{8}{9}$ = 8 : 9

Hence, $6\frac{2}{3}:7\frac{1}{2}$ in its simplest form is 8 : 9

(iv) 9 : 6
The HCF of 9 and 6 is 3.
∴ 9 : 6 = $\frac{9}{6}=\frac{9\div 3}{6\div 3}$ = 3 : 2
Hence, $\frac{1}{6}:\frac{1}{9}$ in its simplest form is 3 : 2.

(v) LCM of the denominators is 2.
∴ 4 : 5 : $\frac{9}{2}$ = 8 : 10 : 9
The HCF of these 3 numbers is 1.

∴ 8 : 10 : 9 is the simplest form.

(vi) 2.5 : 6.5 : 8 = 25 : 65 : 80

The HCF of 25, 65 and 80 is 5.
∴ 25 : 65 : 80 = $\frac{25}{\frac{65}{80}}=\frac{25\div 5}{\frac{65\div 5}{80\div 5}}=\frac{5}{\frac{13}{16}}$ = 5 : 13 : 16

Q2

Answer :

(i) Converting both the quantities into the same unit, we have:
75 paise : (3 × 100) paise = 75 : 300

$= \frac{75}{300} = \frac{75 \div 75}{300 \div 75} = \frac{1}{4}$ (∵ HCF of 75 and 300 = 75)
= 1 paise : 4 paise

(ii) Converting both the quantities into the same unit, we have:
105 cm : 63 cm = $\frac{105}{63} = \frac{105 \div 21}{63 \div 21} = \frac{5}{3}$ (∵ HCF of 105 and 63 = 21)
= 5 cm : 3 cm

(iii) Converting both the quantities into the same unit
65 min : 45 min = $\frac{65}{45} = \frac{65 \div 5}{45 \div 5} = \frac{13}{9}$ (∵ HCF of 65 and 45 = 5)
= 13 min : 9 min

(iv) Converting both the quantities into the same unit, we get:
8 months : 12 months = $\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}$ (∵ HCF of 8 and 12 = 4)
= 2 months : 3 months

(v) Converting both the quantities into the same unit, we get:

2250g : 3000 g = $\frac{2250}{3000} = \frac{2250 \div 750}{3000 \div 750} = \frac{3}{4}$ (∵ HCF of 2250 and 3000 = 750)

= 3 g : 4 g

(vi) Converting both the quantities into the same unit, we get:
1000 m : 750 m = $\frac{1000}{750} = \frac{1000 \div 250}{750 \div 250} = \frac{4}{3}$ (∵ HCF of 1000 and 750 = 250)
= 4 m : 3 m

Q3

Answer :

$\frac{A}{B} = \frac{7}{5}$ and $\frac{B}{C} = \frac{9}{14}$

Therefore, we have:

$\frac{A}{B} \times \frac{B}{C} = \frac{7}{5} \times \frac{9}{14}$
$\frac{A}{C} = \frac{9}{10}$

∴ A : C = 9 : 10

Q4

Answer :

$\frac{A}{B} = \frac{5}{8}$ and $\frac{B}{C} = \frac{16}{25}$
Now, we have : $\frac{A}{B} \times \frac{B}{C} = \frac{5}{8} \times \frac{16}{25} \Rightarrow \frac{A}{C} = \frac{2}{5}$

∴ A : C = 2 : 5

Q5

Answer :

$A : B = 3 : 5$

$B : C = 10 : 13 = \frac{10 : 2}{13 : 2} = 5 : \frac{13}{2}$

Now, $A : B : C = 3 : 5 : \frac{13}{2}$

$\therefore A : B : C = 6 : 10 : 13$

Q6

Answer :

We have the following:

$A : B = 5 : 6$

$B : C = 4 : 7 = \frac{4}{7} = \frac{4 \times \frac{6}{4}}{7 \times \frac{6}{4}} = 6 : \frac{21}{2}$

$\therefore A : B : C = 5 : 6 : \frac{21}{2} = 10 : 12 : 21$

Q7

Answer :

Sum of the ratio terms = 7 + 8 = 15

Now, we have the following:

Kunal's share = Rs 360 $\times \frac{7}{15} = 24 \times 7$ = Rs 168

Mohit's share = Rs 360 $\times \frac{8}{15} = 24 \times 8$ = Rs 192

Q8

Answer :

Sum of the ratio terms = $\frac{1}{5} + \frac{1}{6} = \frac{11}{30}$

Now, we have the following:

Rajan's share = Rs 880 $\times \frac{\frac{1}{5}}{\frac{11}{30}} = \text{Rs } 880 \times \frac{6}{11} = \text{Rs } 80 \times 6$ = Rs 480

Kamal's share = Rs 880 $\times \frac{\frac{1}{6}}{\frac{11}{30}} = \text{Rs } 880 \times \frac{5}{11} = \text{Rs } 80 \times 5$ = Rs 400

Q9

Answer :

Sum of the ratio terms is (1 + 3 + 4) = 8

We have the following:

A's share = Rs 5600 × $\frac{1}{8}$ = $\mathbf{Rs}\ \frac{5600}{8} = \mathbf{Rs}\ 700$

B's share = Rs 5600 × $\frac{3}{8} = \mathbf{Rs}\ 700 \times 3$ = Rs 2100

C's share = Rs 5600 × $\frac{4}{8} = \mathbf{Rs}\ 700 \times 4$ = Rs 2800

Q10

Answer :

Let *x* be the required number.
Then, (9 + *x*) : (16 + *x*) = 2 : 3

$\Rightarrow \frac{9+x}{16+x} = \frac{2}{3}$
$\Rightarrow 27 + 3x = 32 + 2x \Rightarrow x = 5$

Hence, 5 must be added to each term of the ratio 9 : 16 to make it 2 : 3.

Q11

Answer :

Suppose that *x* is the number that must be subtracted.
Then, (17 − *x*) : (33 − *x*) = 7 : 15

$\Rightarrow \frac{17-x}{33-x} = \frac{7}{15}$
$\Rightarrow 255 - 15x = 231 - 7x \Rightarrow 8x = 255 - 231 = 24 \Rightarrow x = 3$

Hence, 3 must be subtracted from each term of ratio 17 : 33 so that it becomes 7 : 15.

Q12

Answer :

Suppose that the numbers are 7*x* and 11*x*.

Then, (7*x* + 7) : (11*x* + 7) = 2 : 3
$\Rightarrow \frac{7x+7}{11x+7} = \frac{2}{3}$

⇒ 21*x* + 21 = 22*x* + 14

⇒ x = 7

Hence, the numbers are (7 × 7 =) 49 and (11 × 7 =) 77.

Q13

Answer :

Suppose that the numbers are 5*x* and 9*x*.
Then, (5*x* − 3) : (9*x* − 3) = 1 : 2

$\Rightarrow \frac{5x-3}{9x-3} = \frac{1}{2}$

⇒ 10*x* − 6 = 9*x* − 3
⇒ *x* = 3

Hence, the numbers are (5 × 3 =) 15 and (9 × 3 =) 27.

Q14

Answer :

Let the numbers be $3x$ and $4x$.
Their LCM is $12x$.
Then, $12x = 180$
$\Rightarrow x = 15$

$\therefore$ The numbers are $(3 \times 15 =) 45$ and $(4 \times 15 =) 60$.

Q15

Answer :

Suppose that the present ages of A and B are $8x$ yrs and $3x$ yrs.
Then, $(8x + 6) : (3x + 6) = 9 : 4$
$\Rightarrow \frac{8x+6}{3x+6} = \frac{9}{4}$
$\Rightarrow 32x + 24 = 27x + 54$
$\Rightarrow 5x = 30$
$\Rightarrow x = 6$

Now, present age of A $= 8 \times 6$ yrs $= 48$ yrs
Present age of B $= 3 \times 6$ yrs $= 18$ yrs

Q16

Answer :

Suppose that the weight of zinc is x g.

Then, $48.6 : x = 9 : 5$

$\Rightarrow x = \frac{48.6 \times 5}{9} = \frac{243}{9} = 27$

Hence, the weight of zinc in the alloy is 27 g.

Q17

Answer :

Suppose that the number of boys is x.
Then, $x : 375 = 8 : 3$

$\Rightarrow x = \frac{8 \times 375}{3} = 8 \times 125 = 1000$

Hence, the number of girls in the school is 1000.

Q18

Answer :

Suppose that the monthly income of the family is Rs x.

Then, $x : 2500 = 11 : 2$

$\Rightarrow x = \frac{11 \times 2500}{2} = 11 \times 1250$
$\Rightarrow x =$ Rs 13750

Hence, the income is Rs 13,750.
$\therefore$ Expenditure = (monthly income − savings)
=Rs (13750 − 2500)
= Rs 11250

Q19

Answer :

Let the numbers one rupee, fifty paise and twenty-five paise coins be 5*x*, 8*x* and 4*x*, respectively.

Total value of these coins = $(5x \times \frac{100}{100} + 8x \times \frac{50}{100} + 4x \times \frac{25}{100})$

$\Rightarrow 5x + \frac{8x}{2} + \frac{4x}{4}$

$= \frac{20x + 16x + 4x}{4} = \frac{40x}{4} = 10x$

However, the total value is Rs 750.

$\therefore 750 = 10x$

$\Rightarrow x = 75$

Hence, number of one rupee coins = 5 × 75 = 375

Number of fifty paise coins = 8 × 75 = 600

Number of twenty-five paise coins = 4 × 75 = 300

Q20

Answer :

(4*x* + 5) : (3*x* + 11) = 13 : 17

$\Rightarrow \frac{4x+5}{3x+11} = \frac{13}{17}$

$\Rightarrow 68x + 85 = 39x + 143 \Rightarrow 29x = 58 \Rightarrow x = 2$

Q21

Answer :

$\frac{x}{y} = \frac{3}{4}$

$\Rightarrow x = \frac{3y}{4}$

Now, we have (3*x* + 4*y*) : (5*x* + 6*y*)

$= \frac{3x+4y}{5x+6y} = \frac{3\times\frac{3y}{4}+4y}{5\times\frac{3y}{4}+6y}$

$= \frac{9y+16y}{15y+24y} = \frac{25y}{39y} = \frac{25}{39}$

= 25 : 39

Q22

Answer :

$\frac{x}{y} = \frac{6}{11}$

$\Rightarrow x = \frac{6y}{11}$

Now, we have:

$\frac{8x-3y}{3x+2y}$

$= \frac{8\times\frac{6y}{11}-3y}{3\times\frac{6y}{11}+2y}$

$= \frac{48y-33y}{18y+22y}$

$= \frac{15y}{40y} = \frac{3}{8}$

$\therefore$ (8*x* − 3*y*) : (3*x* + 2*y*) = 3 : 8

Q23

Answer :

Suppose that the numbers are 5*x* and 7*x*.
The sum of the numbers is 720.
i.e., 5*x* + 7*x* = 720
⇒ 12*x* = 720
⇒ *x* = 60

Hence, the numbers are (5 × 60 =) 300 and (7 × 60 =) 420.

Q24

Answer :

(i) The LCM of 6 and 9 is 18.

$\frac{5}{6} = \frac{5\times3}{6\times3} = \frac{15}{18}$
$\frac{7}{9} = \frac{7\times2}{9\times2} = \frac{14}{18}$ **Clearly,** $\frac{14}{18} < \frac{15}{18}$

∴ (7 : 9) < (5 : 6)

(ii) The LCM of 3 and 7 is 21.

$\frac{2}{3} = \frac{2\times7}{3\times7} = \frac{14}{21}$
$\frac{4}{7} = \frac{4\times3}{7\times3} = \frac{12}{21}$
Clearly, $\frac{12}{21} < \frac{14}{21}$

∴ (4 : 7) < (2 : 3)

(iii) The LCM of 2 and 7 is 14.

$\frac{1\times7}{2\times7} = \frac{7}{14}$
$\frac{4\times2}{7\times2} = \frac{8}{14}$

Clearly, $\frac{7}{14} < \frac{8}{14}$

∴ (1 : 2) < (4 : 7)

(iv) The LCM of 5 and 13 is 65.

$\frac{3}{5} = \frac{3\times13}{5\times13} = \frac{39}{65}$
$\frac{8}{13} = \frac{8\times5}{13\times5} = \frac{40}{65}$
Clearly, $\frac{39}{65} < \frac{40}{65}$

∴ (3 : 5) < (8 : 13)

Q25

Answer :

(i) We have $\frac{5}{6}$, $\frac{8}{9}$ **and** $\frac{11}{18}$.

$2 \mid 6, 9, 18$
$3 \mid 3, 9, 9$
$3 \mid 1, 3, 3$
$\mid 1, 1, 1$

The LCM of 6, 9 and 18 is 18. Therefore, we have:

$\frac{5}{6} = \frac{5\times3}{6\times3} = \frac{15}{18}$
$\frac{8}{9} = \frac{8\times2}{9\times2} = \frac{16}{18}$ $\frac{11}{18} = \frac{11}{18}$ **Clearly,** $\frac{11}{18} < \frac{15}{18} < \frac{16}{18}$

Hence, (11 : 18) < (5 : 6) < (8 : 9)

(ii) We have $\frac{11}{14}$, $\frac{17}{21}$, $\frac{5}{7}$ and $\frac{2}{3}$.

$2 \mid 14, 21, 7, 3$

$7 \mid 7, 21, 7, 3$,

$3 \mid 1, 3, 1, 3$

$\mid 1, 1, 1, 1$

The LCM of 14, 21, 7 and 3 is 42.

$\frac{11}{14} = \frac{11 \times 3}{14 \times 3} = \frac{33}{28}$

$\frac{17}{21} = \frac{17 \times 2}{21 \times 2} = \frac{34}{42}$

$\frac{5}{7} = \frac{5 \times 6}{7 \times 6} = \frac{30}{42}$

$\frac{2}{3} = \frac{2 \times 14}{3 \times 14} = \frac{28}{42}$

Clearly, $\frac{28}{42} < \frac{30}{42} < \frac{33}{28} < \frac{34}{42}$

Hence, $\left(2 : 3\right) < \left(5 : 7\right) < \left(11 : 14\right) < \left(17 : 21\right)$

Ratio and Proportion
Exercise 8B

Q1

Answer :

We have:

Product of the extremes = $30 \times 60 = 1800$
Product of the means = $40 \times 45 = 1800$
Product of extremes = Product of means

Hence, 30 : 40 :: 45 : 60

Q2

Answer :

We have:
Product of the extremes = $36 \times 7 = 252$
Product of the means = $49 \times 6 = 294$
Product of the extremes $\neq$ Product of the means

Hence, 36, 49, 6 and 7 are not in proportion.

Q3

Answer :

Product of the extremes = $2 \times 27 = 54$
Product of the means $= 9 \times x = 9x$

Since $2 : 9 :: x : 27$, we have:
Product of the extremes = Product of the means
$\Rightarrow 54 = 9x$
$\Rightarrow x = 6$

Q4

Answer :

Product of the extremes = 8 × 35 = 280
Product of the means = 16 × x = 16x

Since 8 : x :: 16 : 35, we have:
Product of the extremes = Product of the means
⇒ 280 = 16x
⇒ x = 17.5

Q5

Answer :

Product of the extremes = x × 60 = 60x
Product of the means = 35 × 48 = 1680

Since x : 35 :: 48 : 60, we have:
Product of the extremes = Product of the means
⇒ 60x= 1680
⇒ x = 28

Q6

Answer :

(i) Let the fourth proportional be x.
Then, 8 : 36 :: 6 : x

$8 \times x = 36 \times 6$ [Product of extremes = Product of means]
⇒ 8x = 216
⇒ x = 27
Hence, the fourth proportional is 27.

(ii) Let the fourth proportional be x.
Then, 5 : 7 :: 30 : x
$\Rightarrow 5 \times x = 7 \times 30$ [Product of extremes = Product of means]
⇒ 8x = 216
⇒ 5x = 210
⇒ x = 42

Hence, the fourth proportional is 42.

(iii) Let the fourth proportional be x.
Then, $2.8 \times x = 14 \times 3.5$ [Product of extremes = Product of means]
⇒ 8x = 216
⇒ 2.8x = 49
⇒ x = 17.5
Hence, the fourth proportional is 17.5.

Q7

Answer :

36, 54 and x are in continued proportion.
Then, 36 : 54 :: 54 : x
$\Rightarrow 36 \times x = 54 \times 54$ [Product of extremes = Product of means]

⇒ 36x = 2916
⇒ x = 81

Q8

Answer :

27, 36 and x are in continued proportion.
Then, 27 : 36 :: 36 : x
$\Rightarrow 27 \times x = 36 \times 36$ [Product of extremes = Product of means]
$\Rightarrow 27x = 1296$
$\Rightarrow x = 48$

Hence, the value of x is 48.

Q9

Answer :

(i) Suppose that x is the third proportional to 8 and 12.
Then, 8 :12 :: 12 : x
$\Rightarrow 8 \times x = 12 \times 12$ (Product of extremes = Product of means)
$\Rightarrow 8x = 144$
$\Rightarrow x = 18$

Hence, the required third proportional is 18.

(ii) Suppose that x is the third proportional to 12 and 18.
Then, 12 : 18 :: 18 : x
$\Rightarrow 12 \times x = 18 \times 18$ (Product of extremes = Product of means)
$\Rightarrow 12x = 324$
$\Rightarrow x = 27$

Hence, the third proportional is 27.

(iii) Suppose that x is the third proportional to 4.5 and 6.
Then, 4.5 : 6:: 6 : x
$\Rightarrow 4.5 \times x = 6 \times 6$ (Product of extremes = Product of means)
$\Rightarrow 4.5x = 36$
$\Rightarrow x = 8$

Hence, the third proportional is 8.

Q10

Answer :

The third proportional to 7 and x is 28.
Then, 7 : x :: x : 28
$\Rightarrow 7 \times 28 = x^2$ (Product of extremes = Product of means)
$\Rightarrow x = 14$

Q11

Answer :

(i) Suppose that x is the mean proportional.

Then, 6 : x :: x : 24

$\Rightarrow 6 \times 24 = x \times x$ (Product of extremes = Product of means)
$\Rightarrow x^2 = 144$
$\Rightarrow x = 12$

Hence, the mean proportional to 6 and 24 is 12.

(ii) Suppose that x is the mean proportional.

Then, 3 : x :: x : 27
$\Rightarrow 3 \times 27 = x \times x$ (Product of extremes =Product of means)
$\Rightarrow x^2 = 81$
$\Rightarrow x = 9$

Hence, the mean proportional to 3 and 27 is 9.

(iii) Suppose that x is the mean proportional.

Then, 0.4 : x :: x : 0.9

$\Rightarrow 0.4 \times 0.9 = x \times x$ (Product of extremes =Product of means)
$\Rightarrow x^2 = 0.36$
$\Rightarrow x = 0.6$

Hence, the mean proportional to 0.4 and 0.9 is 0.6.

Q12

Answer :

Suppose that the number is x.

Then, $(5 + x) : (9 + x) :: (7 + x) : (12 + x)$

$\Rightarrow (5 + x) \times (12 + x) = (9 + x) \times (7 + x)$
$(\text{Product of extremes} = \text{Product of means})$
$\Rightarrow 60 + 5x + 12x + x^2 = 63 + 9x + 7x + x^2$
$\Rightarrow 60 + 17x = 63 + 16x$
$\Rightarrow x = 3$

Hence, 3 must be added to each of the numbers: 5, 9, 7 and 12, to get the numbers which are in proportion.

Q13

Answer :

Suppose that x is the number that is to be subtracted.

Then, $(10 - x) : (12 - x) :: (19 - x) : (24 - x)$

$\Rightarrow (10 - x) \times (24 - x) = (12 - x) \times (19 - x)$
$(\text{Product of extremes} = \text{Product of means})$
$\Rightarrow 240 - 10x - 24x + x^2 = 228 - 12x - 19x + x^2$
$\Rightarrow 240 - 34x = 228 - 31x$
$\Rightarrow 3x = 12$
$\Rightarrow x = 4$

Hence, 4 must be subtracted from each of the numbers: 10, 12, 19 and 24, to get the numbers which are in proportion.

Q14

Answer :

Distance represented by 1 cm on the map = 5000000 cm = 50 km

Distance represented by 3 cm on the map = 50 × 4 km = 200 km

∴ The actual distance is 200 km.

Q15

Answer :

(Height of tree) : (height of its shadow) = (height of the pole) : (height of its shadow)

Suppose that the height of pole is x cm.

Then, 6 : 8 = x : 20

$\Rightarrow x = \frac{6 \times 20}{8} = 15$
∴ Height of the pole = 15 cm

Ratio and Proportion
Exercise 8C

Q1

Answer :

The correct option is (d).

$$\frac{a}{c} = \frac{a}{b} \times \frac{b}{c} = \frac{3}{4} \times \frac{8}{9}$$
$$= \frac{2}{3}$$

Hence, $a : c = 2 : 3$

Q2

Answer :

(a) 15 : 8

$$\frac{A}{B} = \frac{2}{3}$$
$$\frac{B}{C} = \frac{4}{5}$$
$$\text{Then, } \frac{A}{B} \times \frac{B}{C} = \frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$$
$$\text{Hence, } C : A = 15 : 8$$

Q3

Answer :

The correct option is (d).

$$A = \frac{3B}{2}$$
$$C = \frac{4B}{5}$$
$$\therefore A : C = \frac{A}{C} = \frac{\frac{3B}{2}}{\frac{4B}{5}} = \frac{15}{8}$$

Hence, $A : C = 15 : 8$

Q4

Answer :

The correct option is (b).

$\frac{15}{100}A = \frac{20}{100}B$
$\Rightarrow \frac{A}{B} = \frac{4}{3}$

Hence, $A : B = 4 : 3$

Q5

Answer :

(a) 1 : 3 : 6

$A = \frac{1}{3}B$
$C = 2B$
$\therefore A : B : C = \frac{1}{3}B : B : 2B = 1 : 3 : 6$

Q6

Answer :

(b) 30 : 42 : 77

$\frac{A}{B} = \frac{5}{7}$
$\Rightarrow A = \frac{5B}{7}\ \frac{B}{C} = \frac{6}{11} \Rightarrow C = \frac{11B}{6}$
$\therefore A : B : C = \frac{5B}{7} : B : \frac{11B}{6} = 30 : 42 : 77$

Q7

Answer :

(c) 6 : 4 : 3

$2A = 3B = 4C$
Then, $A = \frac{3B}{2}$ and $C = \frac{3B}{4}$
$\therefore A : B : C = \frac{3B}{2} : B : \frac{3B}{4} = 6 : 4 : 3$

Q8

Answer :

(a) 3 : 4 : 5

$A = \frac{3B}{4}$
$C = \frac{5B}{4}$
$\therefore A : B : C = \frac{3B}{4} : B : \frac{5B}{4}$

= 3 : 4 : 5

Q9

Answer :

(b) 15 : 10 : 6

$\frac{1}{x} : \frac{1}{y} = 2 : 3$
Then, $y : x = 2 : 3$ and $y = \frac{2}{3}x$
$\frac{1}{y} : \frac{1}{z} = 3 : 5$
Then, $z : y = 3 : 5$ and $z = \frac{3}{5}y$
$\therefore x : y : z = x : \frac{2}{3}x : \frac{3}{5}y = x : \frac{2}{3}x : \frac{3}{5} \times \frac{2}{3}x$
$= x : \frac{2}{3}x : \frac{2}{5}x = 15 : 10 : 6$

Q10

Answer :

$\frac{x}{y} = \frac{3}{4}$

$x = \frac{3y}{4}$

$\therefore \frac{7x+3y}{7x-3y} = \frac{7\frac{3y}{4}+3y}{7\frac{3y}{4}-3y}$

$= \frac{21y+12y}{21y-12y} = \frac{33y}{9y} = \frac{11}{3}$

Hence, $(7x + 3y) : (7x - 3y) = 11 : 3$

The correct option is (c).

Q11

Answer :

(c) 5 : 2

$\frac{3a+5b}{3a-5b} = \frac{5}{1}$

$3a + 5b = 15a - 25b$

$12a = 30b$

$\frac{a}{b} = \frac{30}{12} = \frac{5}{2}$

$\therefore a : b = 5 : 2$

Q12

Answer :

(c) 9

$7 \times 45 = x \times 35 \quad \text{(Product of extremes = Product of means)}$

$\Rightarrow 35x = 315$

$\Rightarrow x = 9$

Q13

Answer :

(b) 7

Suppose that x is the number that is to be added.

Then, $(3 + x) : (5 + x) = 5 : 6$

$\Rightarrow \frac{3+x}{5+x} = \frac{5}{6}$

$\Rightarrow 18 + 6x = 25 + 5x$

$\Rightarrow x = 7$

Q14

Answer :

(d) 40

Suppose that the numbers are x and y.

Then, $x : y = 3 : 5$ and $(x + 10) : (y + 10) = 5 : 7$

$\frac{x}{y} = \frac{3}{5}$

$x = \frac{3y}{5}$

$=> \frac{x+10}{y+10} = \frac{5}{7} => 7x + 70 = 5y + 50 => 7 \times \frac{3y}{5} + 70 = 5y + 50 => 5y - \frac{21y}{5} = 20 => \frac{4y}{5} = 20 => y = 25 \text{ Therefore, } x = \frac{3 \times 25}{5} = 15$

Hence, sum of numbers = 15 + 25 = 40

Q15

Answer :

(a) 3

Suppose that *x* is the number that is to be subtracted.

Then, (15 – x) : (19 – x) = 3 : 4

$\Rightarrow \frac{15-x}{19-x} = \frac{3}{4}$

Cross multiplying, we get :

$60 - 4x = 57 - 3x$

$\Rightarrow x = 3$

Q16

Answer :

(a) Rs 180

A's share = $\frac{3}{7} \times 420 = 180$

Q17

Answer :

(d) 416

Let *x* be the number of boys.

Then, 8 : 5 = *x* : 160

$\Rightarrow \frac{8}{5} = \frac{x}{160}$

$\Rightarrow x = \frac{8 \times 160}{5} = 256$

$\therefore$ Total strength of the school $= 256 + 160 = 416$

Q18

Answer :

(a) (2 :3)

LCM of 3 and 7 = $7 \times 3 = 21$

$\frac{2 \times 7}{3 \times 7} = \frac{14}{21}$ and $\frac{4 \times 3}{7 \times 3} = \frac{12}{21}$

Clearly, $\frac{12}{21} < \frac{14}{21}$

Hence, $\left(4 : 7\right) < \left(2 : 3\right)$

Q19

Answer :

(c) 16

Suppose that the third proportional is *x*.

Then, 9 : 12 :: 12 : x

$\Rightarrow 9 \times x = 12 \times 12$ (Product of extremes = Product of means)

$\Rightarrow 9x = 144$

$\Rightarrow x = 16$

Q20

Answer :

(b) 12

Suppose that the mean proportional is *x*.

Then, 9 : x :: x : 16

$9 \times 16 = x \times x \qquad (\text{Product of extremes} = \text{Product of means})$
$\Rightarrow x^2 = 144$
$\Rightarrow x = 12$

Q21

Answer :

(a) 18 years

Suppose that the present ages of A and B are 3*x* yrs and 8*x* yrs, respectively.
After six years, the age of A will be (3*x*+6) yrs and that of B will be (8*x*+6) yrs.
Then, (3*x* +6) : (8*x* + 6) = 4 : 9

$\Rightarrow \frac{3x+6}{8x+6} = \frac{4}{9}$
$\Rightarrow 27x + 54 = 32x + 24$
$\Rightarrow 5x = 30$
$\Rightarrow x = 6$
Hence, the present ages of A and B are 18 yrs and 48 yrs, respectively.