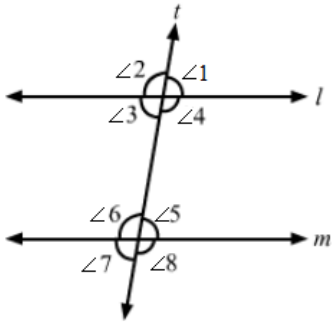
Properties of Parallel Lines

Q1

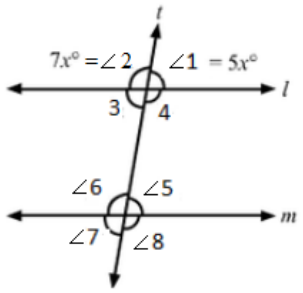
Answer :

Given : l ∥ m
t is a transversal.
$\angle 5 = 70^\circ$
$\angle 5 = \angle 3 = 70^\circ$ (alternate interior angles)
$\angle 5 + \angle 8 = 180^\circ$ (linear pair)
or $70^\circ + \angle 8 = 180^\circ$
$\angle 8 = 110^\circ$
$\angle 1 = \angle 3 = 70^\circ$ (vertically opposite angles)
$\angle 3 + \angle 4 = 180^\circ$ (linear pair)
or $\angle 4 = 180 - \angle 3 = 180 - 70 = 110^\circ$



Q2

Answer :



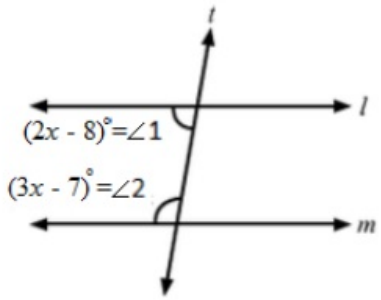
Given : $l \parallel m$
t is a transversal.
$\angle 1 : \angle 2 = 5 : 7$
Let the angles measure 5x and 7x.
$\angle 1 + \angle 2 = 180^\circ$ (linear pair)
$\therefore$ 5x + 7x = 180
or 12x = 180
or x = 15
$\therefore \angle 1 = 5x = 5(15) = 75^\circ$
and $\angle 2 = 7x = 7(15) = 105^\circ$

$\angle 2 + \angle 3 = 180^\circ$ (linear pair)
$\angle 3 = 180 - 105 = 75^\circ$
$\angle 3 + \angle 6 = 180$ (interior angles on the same side of *the* transversal are supplementary)
$\angle 6 = 180 - \angle 3 = 105^\circ$
and $\angle 6 = \angle 8 = 105^\circ$ (vertically opposite angles)
$\therefore \angle 1 = 75^\circ$
$\angle 2 = 105^\circ$
$\angle 3 = 75^\circ$
$\angle 8 = 105^\circ$

Q3

Answer :

Given : l $\parallel$ m
t is a transversal.
Let :
$\angle 1 = (2x - 8)^\circ$
$\angle 2 = (3x - 7)^\circ$
We know that the consecutive interior angles are supplementary.
$\therefore \angle 1 + \angle 2 = 180^\circ$
or $(2x - 8) + (3x - 7) = 180$
or $5x - 15 = 180$
or $5x = 195$
or x = 39
$\angle 1 = (2x - 8) = (2 \times 39 - 8) = 70^\circ$
$\angle 2 = (3x - 7) = (3 \times 39 - 7) = 110^\circ$

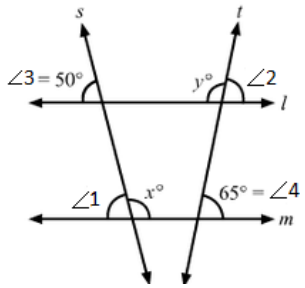


Q4

Answer :

From the given figure:

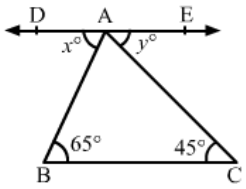
$\angle 1 = \angle 3 = 50^\circ$ (corresponding angles)
and $\angle 1 + x^\circ = 180^\circ$ (linear pair)
or $x^\circ = 180^\circ - 50^\circ = 130^\circ$
or $x = 130$
$\angle 2 = \angle 4 = 65^\circ$ (corresponding angles)
and $\angle 2 + y^\circ = 180^\circ$ (linear pair)
or $y^\circ = 180^\circ - 65^\circ = 115^\circ$
or $y = 115$



Q5

Answer :

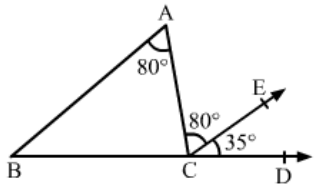
Given :
$\angle B = 65^\circ$
$\angle C = 45^\circ$
DAE $\parallel$ BC
The given lines are parallel.
$\therefore x^\circ = \angle B = 65^\circ$ (alternate angles when AB is taken as the transversal)
$y^\circ = \angle C = 45^\circ$ (alternate angles when AC is taken as the transversal)
$\therefore x = 65$
$y = 45$



Q6

Answer :

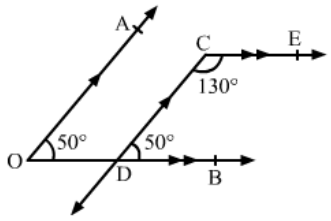
Given : CE $\parallel$ BA
$\angle BAC = 80^\circ, \angle ECD = 35^\circ$
(i) $\angle BAC = \angle ACE = 80^\circ$ (alternate angles with AC as a transversal)
(ii) $\angle ACB + \angle ACD = 180^\circ$ (linear pair)
or $\angle ACB + \angle ACE + \angle ECD = 180^\circ$
or $\angle ACB + 80^\circ + 35^\circ = 180^\circ$
or $\angle ACB = 65^\circ$
(iii) In $\triangle ABC$:
$\angle BAC + \angle ACB + \angle ABC = 180^\circ$ (angle sum property)
$80^\circ + 65^\circ + \angle ABC = 180^\circ$
$\angle ABC = 35^\circ$



Q7

Answer :

Given : AO $\parallel$ CD
OB $\parallel$ CE
$\angle AOB = 50^\circ$
$\angle AOD = \angle CDB = 50^\circ$ (when AO $\parallel$ CD and OB is the transversal)
$\angle ECD + \angle CDB = 180^\circ$ (consecutive interior angles are supplementary, DB $\parallel$ CE and *CD* is *the* transversal)
$\angle ECD = 180^\circ - 50^\circ = 130^\circ$

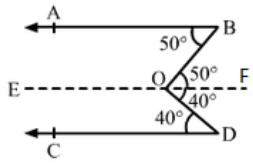


Q8

Answer :

Given : AB $\parallel$ CD
$\angle ABO = 50^\circ$
$\angle CDO = 40^\circ$
Construction : Through O, draw EOF $\parallel$ AB.
$\angle ABO = \angle BOF = 50^\circ$ (alternate angles, when AB $\parallel$ EF and OB is a transversal)
$\angle FOD = \angle ODC = 40^\circ$ (alternate angles, when CD $\parallel$ EF and OD is a transversal)

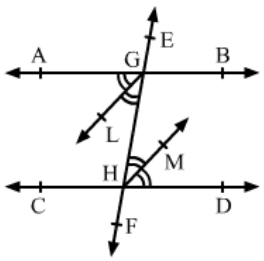
$\angle BOD = \angle BOF + \angle FOD$
$\angle BOD = 50^\circ + 40^\circ = 90^\circ$



Q9

Answer :

Given : AB $\parallel$ CD
GL and HM are angle bisectors of $\angle AGH$ and $\angle GHD$, respectively.
$\angle AGH = \angle GHD$ (alternate angles)
or $\frac{1}{2}\angle AGH = \frac{1}{2}\angle GHD$
or $\angle LGH = \angle GHM$ (given)
Therefore, GL $\parallel$ HM as we know that if the angles of any pair of alternate interior angles are equal, then the lines are parallel.



Q10

Answer :

Given : AB $\parallel$ CD

$\angle ABE = 120^\circ$

$\angle ECD = 100^\circ$

$\angle BEC = x^\circ$

Construction : FEG $\parallel$ AB

Now, since $AB \parallel FEG$ and $AB \parallel CD$, $FEG \parallel CD$

$\therefore EFG \parallel AB \parallel CD$

$\angle ABE = \angle BEG = 120^\circ$ (alternate angles)

or $x^\circ + y^\circ = 120^\circ$ ....(i)

$\angle DCE = \angle CEF = 100^\circ$ (alternate angles)

or $x^\circ + z^\circ = 100^\circ$ .....(ii)

Also, $x^\circ + y^\circ + z^\circ = 180^\circ$ (FEG is a straight line) ...(iii)

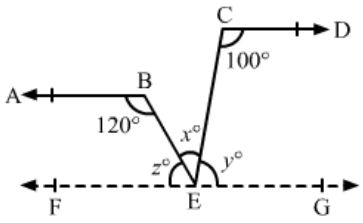
Adding (i) and (ii) :

$2x^\circ + y^\circ + z^\circ = 220^\circ$

or, $x^\circ + 180^\circ = 220^\circ$ (substituting (iii))

$x^\circ = 40^\circ$

$\therefore x = 40$



Q11

Answer :

Given : AB $\parallel$ CD

AD $\parallel$ BC

$\angle 1 + \angle 2 = 180^\circ$ (AB $\parallel$ CD and AD is the transversal) ...(i)

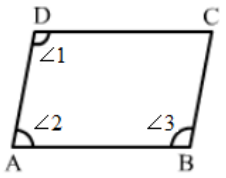
$\angle 2 + \angle 3 = 180^\circ$ (AD $\parallel$ BC and AB is the transversal) ...(ii)

From (i) and (ii) :

$\angle 1 + \angle 2 = 180^\circ = \angle 2 + \angle 3$

$\angle 1 = \angle 3$

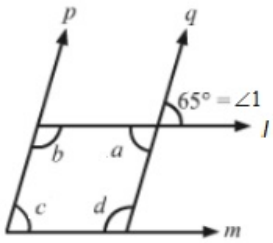
$\angle ADC = \angle ABC$



Q12

Answer :

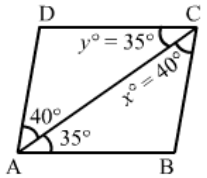
Given :
$l \parallel m$
$p \parallel q$
$\angle 1 = 65^\circ$
$\therefore \angle 1 = \angle a = 65^\circ$ (vertically opposite angles)
$\angle a + \angle d = 180^\circ$ (consecutive interior angles on the same side of a transversal are supplementary)
or $\angle d = 180^\circ - 65^\circ = 115^\circ$
$\angle c + \angle d = 180^\circ$ (consecutive interior angles on the same side of a transversal are supplementary)
or $\angle c = 180^\circ - 115^\circ = 65^\circ$
$\angle c + \angle b = 180^\circ$ (consecutive interior angles on the same side of a transversal are supplementary)
or $\angle b = 180^\circ - 65^\circ = 115^\circ$
$\therefore \angle a = 65^\circ$
$\angle b = 115^\circ$
$\angle c = 65^\circ$
$\angle d = 115^\circ$



Q13

Answer :

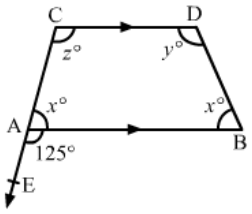
Given :
$AB \parallel DC$
$AD \parallel BC$
$\angle BAC = 35^\circ$
$\angle CAD = 40^\circ$
$\therefore \angle BAC = y = 35^\circ$ (alternate angles when $AB \parallel DC$)
$\angle CAD = x = 40^\circ$ (alternate angles when $AD \parallel BC$)
$\therefore x = 40$
$y = 35$



Q14

Answer :

Given :
$AB \parallel CD$
$\angle BAE = 125^\circ$
$\angle CAB + \angle BAE = 180^\circ$
or $125^\circ + x^\circ = 180^\circ$
or $x = 55$
$x + z = 180^\circ$ (consecutive interior angles on the same side of transversal are supplementary)
$z = 180 - x = 180 - 55 = 125$
$y + x = 180^\circ$ (consecutive interior angles on the same side of transversal are supplementary)
$y = 180 - x = 180 - 55 = 125$



Q15

Answer :

(i) $\angle 1 + \angle 2 = 180$ (linear pair)
or $130^\circ + \angle 2 = 180^\circ$
or $\angle 2 = 50^\circ \neq 40^\circ = \angle 3$
$\therefore l \nparallel m$
(ii) $\angle 2 + \angle 3 = 180^\circ$ (linear pair)
$35^\circ + \angle 3 = 180^\circ$
$\angle 3 = 145^\circ = 145^\circ = \angle 1$
$\therefore l \parallel m$
(iii) $\angle 2 + \angle 3 = 180$ (linear pair)
$\angle 3 = 180^\circ - 125^\circ = 55^\circ$
$\angle 3 = 55^\circ \neq 60^\circ = \angle 1$
$\therefore l \nparallel m$