Exercise 9A

1. If the mean of 5 observation $x, x+2, x+4, x+6$ and $x+8$, find the value of x.

Sol:

Mean of given observations $= \frac{sum\ of\ given\ observations}{total\ number\ of\ observations}$

$\therefore 11 = \frac{x + (x+2)+(x+4)+(x+6)+(x+8)}{5}$

$\Rightarrow 55 = 5x + 20$

$\Rightarrow 5x = 55 - 20$

$\Rightarrow 5x = 35$

$\Rightarrow x = \frac{35}{5}$

$\Rightarrow x = 7$

Hence, the value of x is 7.

2. If the mean of 25 observations is 27 and each observation is decreased by 7, what will be new mean?

Sol:

Mean of given observations $= \frac{sum\ of\ given\ observations}{total\ number\ of\ observations}$

Mean of 25 observations = 27

$\therefore$ Sum of 25 observations $= 27 \times 25 = 675$

If 7 is subtracted from every number, then the sum $= 675 - (25 \times 7)$

$= 675 - 175$

$= 500$

Then, new mean $= \frac{500}{25} = 20$

Thus, the new mean will be 20.

3. Compute the mean for following data:

Class	1 – 3	3 – 5	5 - 7	7 – 9
Frequency	12	22	27	19

Sol:

The given data is shown as follows:

Class	Frequency (f_i)	Class mark (x_i)	$f_i x_i$
1 – 3	12	2	24
3 – 5	22	4	88
5 – 7	27	6	162
7 – 9	19	8	152
Total	$\Sigma f_i = 80$		$\Sigma f_i x_i = 426$

The mean of given data is given by

$$\bar{x} = \frac{\sum_i f_i x_i}{\sum_i f_i}$$

$$= \frac{426}{80}$$

$$= 5.325$$

Thus, the mean of the following data is 5.325.

4. Find the mean using direct method:

Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Frequency	7	5	6	12	8	2

Sol:

Class	Frequency (f_i)	Mid values (x_i)	$f_i \times x_i$
0 – 10	7	5	35
10 – 20	5	15	75
20 – 30	6	25	150
30 – 40	12	35	420
40 – 50	8	45	360
50 – 60	2	55	110
	Σ f_i = 40		$\sum (f_i \times x_i) = 1150$

$$\therefore \text{Mean}, \bar{x} = \frac{\sum (f_i \times x_i)}{\sum f_i}$$

$$= \frac{1150}{40}$$

$$= 28.75$$

$$\therefore \bar{x} = 28.75$$

5. Find the mean of the following data, using direct method:

Class	25 – 35	35 – 45	45 – 55	55 – 65	65 – 75
Frequency	6	10	8	12	4

Sol:

Class	Frequency (f_i)	Mid values (x_i)	$(f_i \times x_i)$
25 – 35	6	30	180
35 – 45	10	40	400
45 – 55	8	50	400
55 – 65	12	60	720
65 – 75	4	70	280
	Σ f_i = 40		$\sum (f_i \times x_i) = 1980$

$$\therefore \text{Mean}, \overline{x} = \frac{\sum(f_i \times x_i)}{\sum f_i}$$

$$= \frac{1980}{40}$$

$$= 49.5$$

$$\therefore \overline{x} = 49.5$$

6. Find the mean of the following data, using direct method:

Class	0 – 100	100 – 200	200 – 300	300 – 400	400 – 500
Frequency	6	9	15	12	8

Sol:

Class	Frequency (f_i)	Mid values (x_i)	($f_i \times x_i$)
0 - 100	6	50	300
100 – 200	9	150	1350
200 – 300	15	250	3750
300 – 400	12	350	4200
400 – 500	8	450	3600
	$\Sigma f_i = 50$		$\sum(f_i \times x_i) = 13200$

$$\therefore \text{Mean}, \overline{x} = \frac{\sum(f_i \times x_i)}{\sum f_i}$$

$$= \frac{13200}{50}$$

$$= 264$$

$$\therefore \overline{x} = 264$$

7. Using an appropriate method, find the mean of the following frequency distribution:

Class	84 – 90	90 – 96	96 – 102	102 – 108	108 – 114	114 – 120
Frequency	8	10	16	23	12	11

Which method did you use, and why?

Sol:

Class	Frequency (f_i)	Mid values (x_i)	($f_i x_i$)
84 – 90	8	87	696
90 – 96	10	93	930
96 – 102	16	99	1584
102 – 108	23	105	2415
108 – 114	12	111	1332
114 – 120	11	117	1287
Total	$\Sigma f_i = 80$		$\Sigma f_i x_i = 8244$

The mean of the data is given by,

$$\overline{x} = \frac{\sum\limits_i f_i x_i}{\sum\limits_i f_i}$$

$$= \frac{8244}{80}$$

$$= 103.05$$

Thus, the mean of the following data is 103.05.

8. If the mean of the following frequency distribution is 24, find the value of p.

Class	0 – 10	10 – 20	20 – 30	30 – 40	40 - 50
Frequency	3	4	P	3	2

Sol:

The given data is shown as follows:

Class	Frequency (f_i)	Mid values (x_i)	($f_i x_i$)
0 – 10	3	5	15
10 – 20	4	15	60
20 – 30	p	25	25p
30 – 40	3	35	105
40 – 50	2	45	90
Total	$\Sigma\ f_i = 12 + p$		$\Sigma\ f_i\ x_i = 270 + 25p$

The mean of the given data is given by,

$$\overline{x} = \frac{\sum\limits_i f_i x_i}{\sum\limits_i f_i}$$

$$\Rightarrow 24 = \frac{270 + 25p}{12 + p}$$

$$\Rightarrow 24\ (12 + p) = 270 + 25p$$

$$\Rightarrow 288 + 24p = 270 + 25p$$

$$\Rightarrow 25p - 24p = 288 - 270$$

$$\Rightarrow p = 18$$

Hence, the value of p is 18.

9. The following distribution shows the daily pocket allowance of children of a locality. If the mean pocket allowance is ₹ 18 , find the missing frequency f.

Daily pocket allowance (in ₹)	11 – 13	13 – 15	15 – 17	17 – 19	19 - 21	21 – 23	23 – 25

Number of children	7	6	9	13	f	5	4

Sol:

The given data is shown as follows:

Daily pocket allowance (in ₹)	Number of children (f_i)	Class mark (x_i)	$f_i x_i$
11 – 13	7	12	84
13 – 15	6	14	84
15 – 17	9	16	144
17 – 19	13	18	234
19 – 21	f	20	20f
21 – 23	5	22	110
23 – 25	4	24	96
Total	$\Sigma f_i = 44 + f$		$\Sigma f_i x_i = 752 + 20f$

The mean of the given data is given by,

$$\overline{x} = \frac{\sum\limits_i f_i x_i}{\sum\limits_i f_i}$$

$\Rightarrow 18 = \frac{750 + 20f}{44 + f}$

$\Rightarrow 18\,(44 + f) = 752 + 20f$

$\Rightarrow 792 + 18f = 752 + 20f$

$\Rightarrow 20f - 18f = 792 - 752$

$\Rightarrow 2f = 40$

$\Rightarrow f = 20$

Hence, the value of f is 20.

10. The mean of following frequency distribution is 54. Find the value of p.

Class	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100
Frequency	7	p	10	9	13

Sol:

The given data is shown as follows:

Class	Frequency (f_i)	Class mark (x_i)	$f_i x_i$
0 – 20	7	10	70
20 – 40	p	30	30p
40 – 60	10	50	500
60 – 80	9	70	630
80 – 100	13	90	1170
Total	$\Sigma f_i = 39 + p$		$\Sigma f_i x_i = 2370 + 30p$

The mean of the given data is given by,

$$\overline{x} = \frac{\sum\limits_i f_i x_i}{\sum\limits_i f_i}$$

$\Rightarrow 54 = \frac{2370 + 30p}{39 + p}$

$\Rightarrow 54 \, (39 + p) = 2370 + 30p$

$\Rightarrow 2106 + 54p = 2370 - 2106$

$\Rightarrow 24p = 264$

$\Rightarrow p = 11$

Hence, the value of p is 11.

11. The mean of the following frequency data is 42, Find the missing frequencies x and y if the sum of frequencies is 100.

Class interval	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
Frequency	7	10	x	13	y	10	14	9

Find x and y.

Sol:

The given data is shown as follows:

Class interval	Frequency (f_i)	Class mark (x_i)	$f_i x_i$
0 – 10	7	5	35
10 – 20	10	15	150
20 – 30	x	25	25x
30 – 40	13	35	455
40 – 50	y	45	45y
50 – 60	10	55	550
60 – 70	14	65	910
70 – 80	9	75	675
Total	$\Sigma f_i = 63 + x + y$		$\Sigma f_i x_i = 2775 + 25x + 45y$

Sum of the frequencies = 100

$\Rightarrow \Sigma_i f_i = 100$

$\Rightarrow 63 + x + y = 100$

$\Rightarrow x + y = 100 - 63$

$\Rightarrow x + y = 37$

$\Rightarrow y = 37 - x$ ……..(1)

Now, the mean of the given data is given by,

$$\bar{x} = \frac{\sum_i f_i x_i}{\sum_i f_i}$$

$\Rightarrow 42 = \frac{2775 + 25x + 45y}{100}$

$\Rightarrow 4200 = 2775 + 25x + 45y$

$\Rightarrow 4200 - 2775 = 25x + 45y$

$\Rightarrow 1425 = 25x + 45(37 - x)$　　　　[from (1)]

$\Rightarrow 1425 = 25x + 1665 - 45x$

$\Rightarrow 20x = 1665 - 1425$

$\Rightarrow 20x = 240$

$\Rightarrow x = 12$

If x = 12, then y = 37 – 12 = 25

Thus, the value of x is 12 and y is 25.

12. The daily expenditure of 100 families are given below. Calculate f_1 and f_2 if the mean daily expenditure is ₹ 188.

Expenditure (in ₹)	140 – 160	160 – 180	180 – 200	200 – 220	220 – 240
Number of families	5	25	f_1	f_2	5

Sol:

The given data is shown as follows:

Expenditure (in ₹)	Number of families (f_i)	Class mark (x_i)	$f_i x_i$
140 – 160	5	150	750
160 – 180	25	170	4250
180 – 200	f_1	190	$190 f_1$
200 – 220	f_2	210	$210 f_2$
220 – 240	5	230	1150
Total	$\Sigma f_i = 35 + f_1 + f_2$		$\Sigma f_i x_i = 6150 + 190 f_1 + 210 f_2$

Sum of the frequencies = 100

$\Rightarrow \Sigma_i f_i = 100$

$\Rightarrow 35 + f_1 + f_2 = 100$

$\Rightarrow f_1 + f_2 = 100 - 35$

$\Rightarrow f_1 + f_2 = 65$

$\Rightarrow f_2 = 65 - f_1$ ……..(1)

Now, the mean of the given data is given by,

$$\overline{x} = \frac{\sum_i f_i x_i}{\sum_i f_i}$$

$\Rightarrow 188 = \frac{6150 + 190 f_1 + 210 f_2}{100}$

$\Rightarrow 18800 = 6150 + 190 f_1 + 210 f_2$

$\Rightarrow 18800 - 6150 = 190 f_1 + 210 f_2$

$\Rightarrow 12650 = 190 f_1 + 210(65 - f_1)$ [from (1)]

$\Rightarrow 12650 = 190 f_1 - 210 f_1 + 13650$

$\Rightarrow 20 f_1 = 13650 - 12650$

$\Rightarrow 20 f_1 = 1000$

$\Rightarrow f_1 = 50$

If $f_1 = 50$, then $f_2 = 65 - 50 = 15$

Thus, the value of f_1 is 50 and f_2 is 15.

13. Find the mean of the following frequency distribution is 57.6 and the total number of observation is 50.

Class	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	100 – 120
Frequency	7	f_1	12	f_2	8	5

Sol:

Class	Frequency (f_i)	Mid values (x_i)	($f_i \times x_i$)
0 – 20	7	10	70
20 – 40	f_1	30	$30 f_1$
40 – 60	12	50	600
60 – 80	18 - f_1	70	1260 - 70f_1
80 – 100	8	90	720
100 – 120	5	110	550
Total	$\Sigma f_i = 50$		$\Sigma (f_i \times x_i) = 3200 - 40 f_1$

We have:

$7 + f_1 + 12 + f_2 + 8 + 5 = 50$

$\Rightarrow f_1 + f_2 = 18$

$\Rightarrow f_2 = 18 - f_1$

$\therefore$ Mean, $\overline{x} = \frac{\Sigma_i (f_i \times x_i)}{\Sigma_i f_i}$

$\Rightarrow 57.6 = \frac{3200 - 40 f_1}{50}$

$\Rightarrow 40f_1 = 320$

$\therefore f_1 = 8$

And $f_2 = 18 - 8$

$\Rightarrow f_2 = 10$

$\therefore$ The missing frequencies are $f_1 = 8$ and $f_2 = 10$.

14. During a medical check-up, the number of heartbeats per minute of 30 patients were recorded and summarized as follows:

Number of heartbeats per minute	65 – 68	68 – 71	71 – 74	74 – 77	77 – 80	80 – 83	83 - 86
Number of patients	2	4	3	8	7	4	2

Find the mean heartbeats per minute for these patients, choosing a suitable method.

Sol:

Using Direct method, the given data is shown as follows:

Number of heartbeats per minute	Number of patients (f_i)	Class mark (x_i)	$f_i x_i$
65 – 68	2	66.5	133
68 – 71	4	69.5	278
71 – 74	3	72.5	217.5
74 – 77	8	75.5	604
77 – 80	7	78.5	549.5
80 – 83	4	81.5	326
83 – 86	2	84.5	169
Total	$\Sigma f_i = 30$		$\Sigma f_i x_i = 2277$

The mean of the data is given by,

$$\overline{x} = \frac{\sum_i f_i x_i}{\sum_i f_i}$$

$$= \frac{2277}{30}$$

$$= 75.9$$

Thus, the mean heartbeats per minute for these patients is 75.9.

15. Find the mean marks per student, using assumed-mean method:

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Number of Students	12	18	27	20	17	6

Sol:

Class	Frequency (f_i)	Mid values (x_i)	Deviation (d_i) $d_i = (x_i - 25)$	($f_i \times d_i$)
0 – 10	12	5	-20	-240
10 – 20	18	15	-10	-180
20 – 30	27	25 = A	0	0
30 – 40	20	35	10	200
40 – 50	17	45	20	340
50 – 60	6	55	30	180
Total	$\Sigma f_i = 100$			$\Sigma (f_i \times d_i) = 300$

Let A = 25 be the assumed mean. Then we have:

Mean, $\overline{x} = A + \dfrac{\sum(f_i \times d_i)}{\sum f_i}$

$= 25 + \dfrac{300}{100}$

$= 28$

$\therefore \overline{x} = 28$

16. Find the mean of the following frequency distribution, using the assumed-mean method:

Class	100 – 120	120 – 140	140 – 160	160 – 180	180 – 200
Frequency	10	20	30	15	5

Sol:

Class	Frequency (f_i)	Mid values (x_i)	Deviation (d_i) $d_i = (x_i - 150)$	($f_i \times d_i$)
100 – 120	10	110	-40	-400
120 – 140	20	130	-20	-400
140 – 160	30	150 = A	0	0
160 – 180	15	170	20	300
180 – 200	5	190	40	200
	$\Sigma f_i = 80$			$\Sigma (f_i \times d_i) = -300$

Let A = 150 be the assumed mean. Then we have:

Mean, $\overline{x} = A + \dfrac{\sum(f_i \times d_i)}{\sum f_i}$

$= 150 - \dfrac{300}{80}$

$= 150 - 3.75$

$\therefore \overline{x} = 146.25$

17. Find the mean of the following data, using assumed-mean method:

Class	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	100 - 120
Frequency	20	35	52	44	38	31

Sol:

Class	Frequency (f_i)	Mid values (x_i)	Deviation (d_i) $d_i = (x_i - 50)$	($f_i \times d_i$)
0 –20	20	10	-40	-800
20 – 40	35	30	-20	-700
40 –60	52	50 = A	0	0
60 – 80	44	70	20	880
80 – 100	38	90	40	1520
100 – 120	31	110	60	1860
	$\Sigma f_i = 220$			$\Sigma (f_i \times d_i) = 2760$

Let A = 50 be the assumed mean. Then we have:

Mean, $\bar{x} = A + \dfrac{\sum(f_i \times d_i)}{\sum f_i}$

$= 50 + \dfrac{2760}{220}$

$= 50 + 12.55$

$\therefore \bar{x} = 62.55$

18. The following table gives the literacy rate (in percentage) in 40 cities. Find the mean literacy rate, choosing a suitable method.

Literacy rate(%)	45 – 55	55 – 65	65 – 75	75 – 85	85 – 95
Number of cities	4	11	12	9	4

Sol:

Using Direct method, the given data is shown as follows:

Literacy rate (%)	Number of cities (f_i)	Class mark (x_i)	($f_i x_i$)
45 – 55	4	50	200
55 – 65	11	60	660
65 – 75	12	70	840
75 – 85	9	80	720
85 – 95	4	90	360
Total	$\Sigma f_i = 40$		$\Sigma f_i x_i = 2780$

The mean of the data is given by,

$\bar{x} = \dfrac{\sum\limits_{i} f_i x_i}{\sum\limits_{i} f_i}$ s

$= \frac{2780}{40}$

$= 69.5$

Thus, the mean literacy rate is 69.5%.

19. Find the mean of the following frequency distribution using step-deviation method.

Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	7	10	15	8	10

Sol:

Let us choose a = 25, h = 10, then $d_i = x_i - 25$ and $u_i = \frac{x_i - 25}{10}$

Using step-deviation method, the given data is shown as follows:

Class	Frequency (f_i)	Class mark (x_i)	$d_i = x_i - 25$	$u_i = \frac{x_i - 25}{10}$	($f_i u_i$)
0 – 10	7	5	-20	-2	-14
10 – 20	10	15	-10	-1	-10
20 – 30	15	25	0	0	0
30 – 40	8	35	10	1	8
40 – 50	10	45	20	2	20
Total	$\Sigma f_i = 50$				$\Sigma f_i u_i = 4$

The mean of the data is given by,

$\overline{x}$ = a + $\left(\frac{\Sigma_i f_i u_i}{\Sigma_i f_i} \right)$ ×h

$= 25 + \frac{4}{50} \times 10$

$= 25 + \frac{4}{5}$

$= \frac{125 + 4}{5}$

$= \frac{129}{5}$

$= 25.8$

Thus, the mean is 25.8.

20. Find the mean of the following data, using step-deviation method:

Class	5 – 15	15 – 25	25 – 35	35 – 45	45 – 55	55 – 65	65 - 75
Frequency	6	10	16	15	24	8	7

Sol:

Let us choose a = 40, h = 10, then $d_i = x_i - 40$ and $u_i = \frac{x_i - 40}{10}$

Using step-deviation method, the given data is shown as follows:

Class	Frequency (f_i)	Class mark (x_i)	$d_i = x_i - 40$	$u_i = \frac{x_i - 40}{10}$	($f_i u_i$)
5 – 15	6	10	-30	-3	-18
15 – 25	10	20	-20	-2	-20
25 – 35	16	30	-10	-1	-16

35 – 45	15	40	0	0	0
45 – 55	24	50	10	1	24
55 – 65	8	60	20	2	16
65 – 75	7	70	30	3	21
Total	$\Sigma f_i = 86$				$\Sigma f_i u_i = 7$

The mean of the data is given by,

$$\overline{x} = a + \left(\frac{\sum_i f_i u_i}{\sum_i f_i} \right) \times h$$

$= 40 + \dfrac{7}{86} \times 10$

$= 40 + \dfrac{70}{86}$

$= 40 + 0.81$

$= 40.81$

21. The weights of tea in 70 packets are shown in the following table:

Weight	200 – 201	201 – 202	202 – 203	203 – 204	204 – 205	205 – 206
Number of packets	13	27	18	10	1	1

Find the mean weight of packets using step deviation method.

Sol:

Let us choose a = 202.5, h = 1, then $d_i = x_i - 202.5$ and $u_i = \dfrac{x_i - 202.5}{1}$

Using step-deviation method, the given data is shown as follows:

Weight	Number of packets (f_i)	Class mark (x_i)	$d_i = x_i - 202.5$	$u_i = \dfrac{x_i - 202.5}{1}$	($f_i u_i$)
200 - 201	13	200.5	-2	-2	-26
201 – 202	27	201.5	-1	-1	-27
202 – 203	18	202.5	0	0	0
203 – 204	10	203.5	1	1	10
204 – 205	1	204.5	2	2	2
205 – 206	1	205.5	3	3	3
Total	$\Sigma f_i = 70$				$\Sigma f_i u_i = -38$

The mean of the given data is given by,

$$\overline{x} = a + \left(\frac{\sum_i f_i u_i}{\sum_i f_i} \right) \times h$$

$= 202.5 + \left(\dfrac{-38}{70} \right) \times 1$

$= 202.5 - 0.542$

$= 201.96$

Hence, the mean is 201.96 g.

22. Find the mean of the following frequency distribution table using a suitable method:

Class	20 – 30	30 – 40	40 – 50	50 – 60	60 - 70
Frequency	25	40	42	33	10

Sol:

Let us choose a = 45, h = 10, then $d_i = x_i - 45$ and $u_i = \frac{x_i - 45}{10}$

Using step-deviation method, the given data is shown as follows:

Weight	Number of packets (f_i)	Class mark (x_i)	$d_i = x_i - 45$	$u_i = \frac{x_i - 45}{10}$	($f_i u_i$)
20 – 30	25	35	-20	-2	-50
30 – 40	40	35	-10	-1	-40
40 – 50	42	45	0	0	0
50 – 60	33	55	10	1	33
60 – 70	10	65	20	2	20
Total	$\Sigma f_i = 150$				$\Sigma f_i u_i$ = -37

The mean of the given data is given by,

$$\overline{x} \ a + \left(\frac{\sum_i f_i u_i}{\sum_i f_i} \right) \times h$$

$= 45 - \left(\frac{37}{150} \right) \times 10$

$= 45 - \frac{37}{15}$

$= 45 - 2.466$

$= 42.534$

Hence, the mean is 42.534.

23. In an annual examination, marks (out of 90) obtained by students of Class X in mathematics are given below:

Marks Obtained	0 – 15	15 – 30	30 – 45	45 – 60	60 – 75	75 – 90
Number of students	2	4	5	20	9	10

Find the mean marks.

Sol:

Let us choose a = 52.5, h = 15, then $d_i = x_i - 52.5$ and $u_i = \frac{x_i - 52.5}{15}$

Using step-deviation method, the given data is shown as follows:

Weight	Number of students (f_i)	Class mark (x_i)	$d_i = x_i - 37.5$	$u_i = \frac{x_i - 52.5}{15}$	($f_i u_i$)
0 – 15	2	7.5	-45	-3	-6
15 – 30	4	22.5	-30	-2	-8
30 – 45	5	37.5	-15	-1	-5

45 – 60	20	52.5	0	0	0
60 – 75	9	67.5	15	1	9
75 – 90	10	82.5	30	2	20
Total	Σ f$_i$ = 50				Σ f$_i$ u$_i$ = 10

The mean of the given data is given by,

$$\bar{x} = a + \left(\frac{\sum_i f_i u_i}{\sum_i f_i} \right) \times h$$

$$= 52.5 + \left(\frac{10}{50} \right) \times 15$$

$$= 52.5 + 3$$

$$= 55.5$$

Thus, the mean is 55.5.

24. Find the arithmetic mean of the following frequency distribution using step-deviation method:

Age (in years)	18 – 24	24 – 30	30 – 36	36 – 42	42 – 48	48 – 54
Number of workers	6	8	12	8	4	2

Sol:

Class	Frequency (f$_i$)	Mid values (x$_i$)	$u_i = \frac{(x_i - A)}{h} = \frac{(x_i - 33)}{6}$	(f$_i$ × u$_i$)
18 – 24	6	21	-2	-12
24 – 30	8	27	-1	-8
30 – 36	12	33 = A	0	0
36 – 42	8	39	1	8
42 – 48	4	45	2	8
48 – 54	2	51	3	6
Total	Σ f$_i$ = 40			Σ (f$_i$ × u$_i$) = 2

Now, A = 33, h = 6, Σ f$_i$ = 40 and Σ (f$_i$ × u$_i$) = 2

$$\therefore \text{Mean, } \bar{x} = A + \left\{ h \times \frac{\sum (f_i \times u_i)}{\sum f_i} \right\}$$

$$= 33 + \left\{ 6 \times \frac{2}{40} \right\}$$

$$= 33 + 0.3$$

$$= 33.3$$

$$\therefore \bar{x} = 33.3 \text{ years}$$

25. Find the mean of the following data using step-deviation method:

Class	500 – 520	520 – 540	540 – 560	560 – 580	580 – 600	600 – 620
Frequency	14	9	5	4	3	5

Sol:

Class	Frequency (f_i)	Mid values (x_i)	$u_i = \frac{(x_i - A)}{h}$ $= \frac{(x_i - 550)}{20}$	$(f_i \times u_i)$
500 – 520	14	510	-2	-28
520 – 540	9	530	-1	-9
540 – 560	5	550 = A	0	0
560 – 580	4	570	1	4
580 – 600	3	590	2	6
600 – 620	5	610	3	15
	$\Sigma f_i = 40$			$\Sigma (f_i \times u_i) = -12$

Now, A = 550, h = 20, $\Sigma f_i = 40$ and $\Sigma (f_i \times u_i) = -12$

$$\therefore \text{Mean, } \overline{x} = A + \left\{ h \times \frac{\Sigma (f_i \times u_i)}{\Sigma f_i} \right\}$$

$$= 550 + \left\{ 20 \times \frac{(-12)}{40} \right\}$$

$$= 550 - 6$$

$$= 544$$

$$\therefore \overline{x} = 544$$

26. Find the mean age from the following frequency distribution:

Age (in years)	25 – 29	30 – 34	35 – 39	40 – 44	45 – 49	50 – 54	55 – 59
Number of persons	4	14	22	16	6	5	3

Sol:

Class	Frequency (f_i)	Mid values (x_i)	$u_i = \frac{(x_i - A)}{h}$ $= \frac{(x_i - 42)}{5}$	$(f_i \times u_i)$
24.5 – 29.5	4	27	-3	-12
29.5 – 34.5	14	32	-2	-28
34.5 – 39.5	22	37	-1	-22
39.5 – 44.5	16	42 = A	0	0
44.5 – 49.5	6	47	1	6
49.5 – 54.5	5	52	2	10
54.5 – 59.5	3	57	3	9
	$\Sigma f_i = 70$			$\Sigma (f_i \times u_i) = -37$

Now, A = 42, h = 5, $\Sigma f_i = 70$ and $\Sigma (f_i \times u_i) = -37$

$$\therefore \text{Mean}, \bar{x} = A + \left\{ h \times \frac{\sum(f_i \times u_i)}{\sum f_i} \right\}$$

$$= 42 + \left\{ 5 \times \frac{(-37)}{70} \right\}$$

$$= 42 - 2.64$$

$$= 39.36$$

$$\therefore \bar{x} = 39.36$$

$\therefore$ Mean age = 39.36 years.

27. The following table shows the age distribution of patients of malaria in a village during a particular month:

Age (in years)	5 – 14	15 – 24	25 – 34	35 – 44	45 – 54	55 - 64
No. of cases	6	11	21	23	14	5

Find the average age of the patients.

Sol:

Class	Frequency (f_i)	Mid values (x_i)	$u_i = \frac{(x_i - A)}{h} = \frac{(x_i - 29.5)}{10}$	$(f_i \times u_i)$
4.5 – 14.5	6	9.5	-2	-12
14.5 – 24.5	11	19.5	-1	-11
24.5 – 34.5	21	29.5 = A	0	0
34.5 – 44.5	23	39.5	1	23
44.5 – 54.5	14	49.5	2	28
54.5 – 64.5	5	59.5	3	15
	$\sum f_i = 80$			$\sum (f_i \times u_i) = 43$

Now, A = 29.5, h = 10, $\sum f_i = 80$ and $\sum (f_i \times u_i) = 43$

$$\therefore \text{Mean}, \bar{x} = A + \left\{ h \times \frac{\sum(f_i \times u_i)}{\sum f_i} \right\}$$

$$= 29.5 + \left\{ 10 \times \frac{43}{80} \right\}$$

$$= 29.5 + 5.375$$

$$= 34.875$$

$$\therefore \bar{x} = 34.875$$

$\therefore$ The average age of the patients is 34.87 years.

28. Weight of 60 eggs were recorded as given below:

Weight (in grams)	75 – 79	80 – 84	85 – 89	90 – 94	95 – 99	100 - 104	105 - 109
No. of eggs	4	9	13	17	12	3	2

Calculate their mean weight to the nearest gram.

Sol:

Let us choose a = 92, h = 5, then $d_i = x_i - 92$ and $u_i = \frac{x_i - 92}{5}$

Using step-deviation method, the given data is shown as follows:

Weight (in grams)	Number of eggs (f_i)	Class mark (x_i)	$d_i = x_i - 92$	$u_i = \frac{x_i - 92}{5}$	($f_i\, u_i$)
74.5 – 79.5	4	77	-15	-3	-12
79.5 – 84.5	9	82	-10	-2	-18
84.5 – 89.5	13	87	-5	-1	-13
89.5 – 94.5	17	92	0	0	0
94.5 – 99.5	12	97	5	1	12
99.5 – 104.5	3	102	10	2	6
104.5 – 109.5	2	107	15	3	6
Total	$\Sigma\, f_i = 60$				$\Sigma\, f_i\, u_i$ = -19

The mean of the given data is given by,

$$\bar{x} = a + \left(\frac{\sum_i f_i u_i}{\sum_i f_i} \right) \times h$$

$$= 92 + \left(\frac{-19}{60} \right) \times 5$$

$$= 92 - 1.58$$

$$= 90.42$$

$$\approx 90$$

Thus, the mean weight to the nearest gram is 90 g.

29. The following table shows the marks scored by 80 students in an examination:

Marks	0 – 5	5 – 10	10 – 15	15 – 20	20 – 25	25 – 30	30 – 35	35 – 40
No. of students	3	10	25	49	65	73	78	80

Sol:

Let us choose a = 17.5, h = 5, then $d_i = x_i - 17.5$ and $u_i = \frac{x_i - 17.5}{5}$

Using step-deviation method, the given data is shown as follows:

Marks	Number of students (cf)	Frequency (f_i)	Class mark (x_i)	$d_i = x_i - 17.5$	$u_i = \frac{x_i - 17.5}{5}$	($f_i\, u_i$)
0 – 5	3	3	2.5	-15	-3	-9
5 – 10	10	7	7.5	-10	-2	-14
10 – 15	25	15	12.5	-5	-1	-15
15 – 20	49	24	17.5	0	0	0
20 – 25	65	16	22.5	5	1	16
25 – 30	73	8	27.5	10	2	16
30 – 35	78	5	32.5	15	3	15
35 – 40	80	2	37.5	20	4	8

Total		$\Sigma f_i = 80$				$\Sigma f_i u_i = 17$

The mean of the given data is given by,

$$\overline{x} = a + \left(\frac{\sum\limits_i f_i u_i}{\sum\limits_i f_i} \right) \times h$$

$$= 17.5 + \left(\frac{17}{80} \right) \times 5$$

$$= 17.5 + 1.06$$

$$= 18.56$$

Thus, the mean marks correct to 2 decimal places is 18.56.

Exercise 9B

1. In a hospital, the ages of diabetic patients were recorded as follows. Find the median age.

Age (in years)	0 – 15	15 – 30	30 – 45	45 – 60	60 - 75
No. of patients	5	20	40	50	25

Sol:

We prepare the cumulative frequency table, as shown below:

Age (in years)	Number of patients (f_i)	Cumulative Frequency (cf)
0 – 15	5	5
15 – 30	20	25
30 – 45	40	65
45 – 60	50	115
60 – 75	25	140
Total	$N = \Sigma f_i = 140$	

Now, $N = 140 \Rightarrow \frac{N}{2} = 70$.

The cumulative frequency just greater than 70 is 115 and the corresponding class is 45 – 60.

Thus, the median class is 45 – 60.

$\therefore l = 45$, h = 15, f = 50, N = 140 and cf = 65.

Now,

$$\text{Median} = l + \left(\frac{\frac{N}{2} - cf}{f} \right) \times h$$

$$= 45 + \left(\frac{\frac{140}{2} - 65}{50} \right) \times 15$$

$$= 45 + \left(\frac{70 - 65}{50} \right) \times 15$$

$$= 45 + 1.5$$

$$= 46.5$$

Hence, the median age is 46.5 years.

2. Compute mean from the following data:

Marks	$0-7$	$7-14$	$14-21$	$21-28$	$28-35$	$35-42$	$42-49$
Number of Students	3	4	7	11	0	16	9

Sol:

Class	Frequency (f)	Cumulative Frequency (cf)
$0-7$	3	3
$7-14$	4	7
$14-21$	7	14
$21-28$	11	25
$28-35$	0	25
$35-42$	16	41
$42-49$	9	50
	$N = \Sigma f = 50$	

Now, $N = 50 \Rightarrow \frac{N}{2} = 25$.

The cumulative frequency just greater than 25 is 41 and the corresponding class is $35 - 42$.

Thus, the median class is $35 - 42$.

$\therefore l = 35$, h = 7, f = 16, cf = c.f. of preceding class = 25 and $\frac{N}{2} = 25$.

Now,

$$\text{Median} = l + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h$$

$$= 35 + 7 \times \left(\frac{25 - 25}{16}\right)$$

$$= 35 + 0$$

$$= 35$$

Hence, the median age is 46.5 years.

3. The following table shows the daily wages of workers in a factory:

Daily wages in (₹)	$0-100$	$100-200$	$200-300$	$300-400$	$400-500$
Number of workers	40	32	48	22	8

Find the median daily wage income of the workers.

Sol:

Class	Frequency (f)	Cumulative Frequency (cf)
$0-100$	40	40
$100-200$	32	72
$200-300$	48	120
$300-400$	22	142
$400-500$	8	150
	$N = \Sigma f = 150$	

Now, $N = 150$

$\Rightarrow \frac{N}{2} = 75.$

The cumulative frequency just greater than 75 is 120 and the corresponding class is 200 – 300.

Thus, the median class is 200 – 300.

$\therefore l = 200, h = 100, f = 48, cf = $ c.f. of preceding class $= 72$ and $\frac{N}{2} = 75.$

Now,

Median, $M = l + \left\{ h \times \left(\frac{\frac{N}{2} - cf}{f} \right) \right\}$

$= 200 + \left\{ 100 \times \left(\frac{75 - 72}{48} \right) \right\}$

$= 200 + 6.25$

$= 206.25$

Hence, the median daily wage income of the workers is Rs 206.25.

4.   Calculate the median from the following frequency distribution table:

Class	5 – 10	10 – 15	15 – 20	20 – 25	25 – 30	30 – 35	35 – 40	40 – 45
Frequency	5	6	15	10	5	4	2	2

Sol:

Class	Frequency (f)	Cumulative Frequency (cf)
5 – 10	5	5
10 – 15	6	11
15 – 20	15	26
20 – 25	10	36
25 – 30	5	41
30 – 35	4	45
35 – 40	2	47
40 – 45	2	49
	$N = \sum f = 49$	

Now, $N = 49$

$\Rightarrow \frac{N}{2} = 24.5.$

The cumulative frequency just greater than 24.5 is 26 and the corresponding class is 15 - 20.

Thus, the median class is 15 – 20.

$\therefore l = 15, h = 5, f = 15, cf = $ c.f. of preceding class $= 11$ and $\frac{N}{2} = 24.5.$

Now,

Median, $M = l + \left\{ h \times \left(\frac{\frac{N}{2} - cf}{f} \right) \right\}$

$= 15 + \left\{ 5 \times \left(\frac{24.5 - 11}{15} \right) \right\}$

$= 15 + 4.5$

$= 19.5$

Hence, the median = 19.5.

5.   Given below is the number of units of electricity consumed in a week in a certain locality:

Class	65 – 85	85 – 105	105 – 125	125 – 145	145 – 165	165 – 185	185 – 200
Frequency	4	5	13	20	14	7	4

Calculate the median.

Sol:

Class	Frequency (f)	Cumulative Frequency (cf)
65- 85	4	4
85 – 105	5	9
105 – 125	13	22
125 – 145	20	42
145 – 165	14	56
165 – 185	7	63
185 – 205	4	67
	$N = \Sigma f = 67$	

Now, N = 67

$\Rightarrow \frac{N}{2} = 33.5.$

The cumulative frequency just greater than 33.5 is 42 and the corresponding class is 125 - 145.

Thus, the median class is 125 – 145.

$\therefore l = 125$, h = 20, f = 20, cf = c.f. of preceding class = 22 and $\frac{N}{2} = 33.5.$

Now,

Median, $M = l + \left\{ h \times \left(\frac{\frac{N}{2} - cf}{f} \right) \right\}$

$= 125 + \left\{ 20 \times \left(\frac{33.5 - 22}{20} \right) \right\}$

$= 125 + 11.5$

$= 136.5$

Hence, the median = 136.5.

6.   Calculate the median from the following data:

Height(in cm)	135 - 140	140 - 145	145 - 150	150 - 155	155 - 160	160 - 165	165 - 170	170 - 175
Frequency	6	10	18	22	20	15	6	3

Sol:

Class	Frequency (f)	Cumulative Frequency (cf)
135 – 140	6	6
140 – 145	10	16
145 – 150	18	34
150 – 155	22	56
155 – 160	20	76
160 – 165	15	91
165 – 170	6	97
170 – 175	3	100
	$N = \Sigma f = 100$	

Now, N = 100

$$\Rightarrow \frac{N}{2} = 50.$$

The cumulative frequency just greater than 50 is 56 and the corresponding class is 150 - 155.

Thus, the median class is 150 – 155.

$\therefore l = 150$, h = 5, f = 22, cf = c.f. of preceding class = 34 and $\frac{N}{2} = 50$.

Now,

$$\text{Median, M} = l + \left\{ h \times \left(\frac{\frac{N}{2} - cf}{f} \right) \right\}$$

$$= 150 + \left\{ 5 \times \left(\frac{50 - 34}{22} \right) \right\}$$

$$= 150 + 3.64$$

$$= 153.64$$

Hence, the median = 153.64.

7. Calculate the missing frequency from the following distribution, it being given that the median of distribution is 24.

Class	0 – 10	10 – 20	20 – 30	30 – 40	40 - 50
Frequency	5	25	?	18	7

Sol:

Class	Frequency (f_i)	Cumulative Frequency (cf)
0 – 10	5	5
10 – 20	25	30
20 – 30	x	x + 30
30 – 40	18	x + 48
40 – 50	7	x + 55

Median is 24 which lies in 20 – 30

$\therefore$ Median class = 20 – 30

Let the unknown frequency be x.

Here, $l = 20, \frac{n}{2} = \frac{x+55}{2},$ c.f. of the preceding class = c.f = 30, f = x, h = 10

Now,

Median, $M = l + \frac{\frac{n}{2}-cf}{f} \times h$

$\Rightarrow 24 = 20 + \frac{\frac{x+55}{2} - 30}{x} \times 10$

$\Rightarrow 24 = 20 + \frac{\frac{x+55-60}{2}}{x} \times 10$

$\Rightarrow 24 = 20 + \frac{x-5}{2x} \times 10$

$\Rightarrow 24 = 20 + \frac{5x-25}{x}$

$\Rightarrow 24 = \frac{20 + 5x-25}{x}$

$\Rightarrow 24x = 25x - 25$

$\Rightarrow -x = -25$

$\Rightarrow x = 25$

Hence, the unknown frequency is 25.

8.   The median of the following data is 16. Find the missing frequencies a and b if the total of frequencies is 70.

Class	0 – 5	5 – 10	10 – 15	15 – 20	20 – 25	25 – 30	30 – 35	35 – 40
Frequency	12	a	12	15	b	6	6	4

Sol:

Class	Frequency (f)	Cumulative Frequency (cf)
0 – 5	12	12
5 – 10	a	12 + a
10 – 15	12	24 + a
15 – 20	15	39 + a
20 – 25	b	39 + a + b
25 – 30	6	45 + a + b
30 – 35	6	51 + a + b
35 – 40	4	55 + a + b
Total	$N = \sum f_i = 70$	

Let a and b be the missing frequencies of class intervals 5 – 10 and 20 – 25 respectively.

Then, $55 + a + b = 70 \Rightarrow a + b = 15$ …..(1)

Median is 16, which lies in 15 – 20. So, the median class is 15 – 20.

$\therefore l = 15, h = 5, N = 70, f = 15$ and $cf = 24 + a$

Now,

Median, $M = l + \left(\frac{\frac{N}{2}-cf}{f}\right) \times h$

$$\Rightarrow 16 = 15 + \left(\frac{\frac{70}{2} - (24 + a)}{15} \right) \times 5$$

$$\Rightarrow 16 = 15 + \left(\frac{35 - 24 - a}{3} \right)$$

$$\Rightarrow 16 = 15 + \left(\frac{11 - a}{3} \right)$$

$$\Rightarrow 16 - 15 = \frac{11 - a}{3}$$

$$\Rightarrow 1 \times 3 = 11 - a$$

$$\Rightarrow a = 11 - 3$$

$$\Rightarrow a = 8$$

$\therefore b = 15 - a$ [From (1)]

$\Rightarrow b = 15 - 8$

$\Rightarrow b = 7$

Hence, $a = 8$ and $b = 7$.

9. In the following data the median of the runs scored by 60 top batsmen of the world in one-day international cricket matches is 5000. Find the missing frequencies x and y

Runs scored	2500 – 3500	3500 – 4500	4500 – 5500	5500 – 6500	6500 – 7500	7500 - 8500
Number of batsman	5	x	y	12	6	2

Sol:

We prepare the cumulative frequency table, as shown below:

Runs scored	Number of batsman (f_i)	Cumulative Frequency (cf)
2500 – 3500	5	5
3500 – 4500	x	5 + x
4500 – 5500	y	5 + x + y
5500 – 6500	12	17 + x + y
6500 – 7500	6	23 + x + y
7500 – 8500	2	25 + x + y
Total	$N = \sum f_i = 60$	

Let x and y be the missing frequencies of class intervals 3500 – 4500 respectively. Then,

$25 + x + y = 60 \Rightarrow x + y = 35$ ……(1)

Median is 5000, which lies in 4500 – 5500. So, the median class is 4500 – 5500.

$\therefore l = 4500, h = 1000, N = 60, f = y$ and $cf = 5 + x$

Now,

Median, $M = l + \left(\frac{\frac{N}{2} - cf}{f} \right) \times h$

$$\Rightarrow 5000 = 4500 + \left(\frac{\frac{60}{2} - (5+x)}{y}\right) \times 1000$$

$$\Rightarrow 5000 - 4500 = \left(\frac{30 - 5 - x}{y}\right) \times 1000$$

$$\Rightarrow 500 = \left(\frac{25 - x}{y}\right) \times 1000$$

$$\Rightarrow y = 50 - 2x$$

$$\Rightarrow 35 - x = 50 - 2x \qquad [\text{From (1)}]$$

$$\Rightarrow 2x - x = 50 - 35$$

$$\Rightarrow x = 15$$

$$\therefore y = 35 - x \qquad [\text{From (1)}]$$

$$\Rightarrow y = 35 - 15$$

$$\Rightarrow y = 20$$

Hence, x = 15 and y = 20.

10. If the median of the following frequency distribution is 32.5, find the values of f_1 and f_2.

Class	0 – 10	10 – 20	20 – 30	30 -40	40 – 50	50 – 60	60 – 70	Total
Frequency	f_1	5	9	12	f_2	3	2	40

Sol:

Class	Frequency (f)	Cumulative Frequency (cf)
0 – 10	f_1	f_1
10 – 20	5	$f_1 + 5$
20 – 30	9	$f_1 + 14$
30 – 40	12	$f_1 + 26$
40 – 50	f_2	$f_1 + f_2 + 26$
50 – 60	3	$f_1 + f_2 + 29$
60 – 70	2	$f_1 + f_2 + 31$
	$N = \Sigma f = 40$	

Now, $f_1 + f_2 + 31 = 40$

$\Rightarrow f_1 + f_2 = 9$

$\Rightarrow f_2 = 9 - f_1$

The median is 32.5 which lies in 30 – 40.

Hence, median class = 30 – 40

Here, $l = 30, \frac{N}{2} = \frac{40}{2} = 20$, f = 12 and cf = 14 + f_1

Now, median = 32.5

$$\Rightarrow l + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h = 32.5$$

$\Rightarrow 30 + \left(\frac{20 - (14 + f_1)}{12}\right) \times 10 = 32.5$

$\Rightarrow \frac{6 - f_1}{12} \times 10 = 2.5$

$\Rightarrow \frac{60 - 10f_1}{12} = 2.5$

$\Rightarrow 60 - 10f_1 = 30$

$\Rightarrow 10f_1 = 30$

$\Rightarrow f_1 = 3$

From equation (i), we have:

$f_2 = 9 - 3$

$\Rightarrow f_2 = 6$

11. Calculate the median for the following data:

Class	19 – 25	26 – 32	33 – 39	40 – 46	47 – 53	54 - 60
Frequency	35	96	68	102	35	4

Sol: First, we will convert the data into exclusive form.

Class	Frequency (f)	Cumulative Frequency (cf)
18.5 – 25.5	35	35
25.5 – 32.5	96	131
32.5 – 39.5	68	199
39.5 – 46.5	102	301
46.5 – 53.5	35	336
53.5 – 60.5	4	340
	$N = \Sigma f = 340$	

Now, N = 340

$\Rightarrow \frac{N}{2} = 70.$

The cumulative frequency just greater than 170 is 199 and the corresponding class is 32.5 – 39.5.

Thus, the median class is 32.5 – 39.5.

$\therefore l = 32.5$, h = 7, f = 68, cf = c.f. of preceding class = 131 and $\frac{N}{2} = 170$.

$\therefore$ Median, $M = l + \left\{ h \times \left(\frac{\frac{N}{2} - cf}{f} \right) \right\}$

$= 32.5 + \left\{ 7 \times \left(\frac{170 - 131}{68} \right) \right\}$

$= 32.5 + 4.01$

$= 36.51$

Hence, the median = 36.51.

12. Find the median wages for the following frequency distribution:

Wages per day (in ₹)	61 – 70	71 – 80	81 – 90	91 – 100	101 – 110	111 – 120
No. of women workers	5	15	20	30	20	8

Sol:

Class	Frequency (f)	Cumulative Frequency (cf)
60.5 – 70.5	5	5
70.5 – 80.5	15	20
80.5 – 90.5	20	40
90.5 – 100.5	30	70
100.5 – 110.5	20	90
110.5 – 120.5	8	98
	$N = \Sigma f = 98$	

Now, N = 98

$\Rightarrow \frac{N}{2} = 49.$

The cumulative frequency just greater than 49 is 70 and the corresponding class is 90.5 – 100.5.

Thus, the median class is 90.5 – 100.5.

Now, $l = 90.5$, h = 10, f = 30, cf = c.f. of preceding class = 40 and $\frac{N}{2} = 49$.

$\therefore$ Median, $M = l + \left\{ h \times \left(\frac{\frac{N}{2} - cf}{f} \right) \right\}$

$= 90.5 + \left\{ 10 \times \left(\frac{49 - 40}{30} \right) \right\}$

$= 90.5 + 3$

$= 93.5$

Hence, median wages = Rs. 93.50.

13. Find the median from the following data:

Class	1 – 5	6 – 10	11 – 15	16 – 20	21 – 25	26 – 30	31 – 35	35 – 40	40 – 45
Frequency	7	10	16	32	24	16	11	5	2

Sol:

Converting into exclusive form, we get:

Class	Frequency (f)	Cumulative Frequency (cf)
0.5 – 5.5	7	7
5.5 – 10.5	10	17
10.5 – 15.5	16	33
15.5 – 20.5	32	65
20.5 – 25.5	24	89
25.5 – 30.5	16	105
30.5 – 35.5	11	116

35.5 – 40.5	5	121
40.5 – 45.5	2	123
	$N = \sum f = 123$	

Now, N = 123

$\Rightarrow \frac{N}{2} = 61.5$.

The cumulative frequency just greater than 61.5 is 65 and the corresponding class is 15.5 – 20.5.

Thus, the median class is 15.5 – 20.5.

$\therefore l = 15.5$, h = 5, f = 32, cf = c.f. of preceding class = 33 and $\frac{N}{2} = 61.5$.

$\therefore$ Median, $M = l + \left\{ h \times \left(\frac{\frac{N}{2} - cf}{f} \right) \right\}$

$\qquad = 15.5 + \left\{ 5 \times \left(\frac{61.5 - 33}{32} \right) \right\}$

$\qquad = 15.5 + 4.45$

$\qquad = 19.95$

Hence, median = 19.95.

14. Find the median from the following data:

Marks	No of students
Below 10	12
Below 20	32
Below 30	57
Below 40	80
Below 50	92
Below 60	116
Below 70	164
Below 80	200

Sol:

Class	Cumulative frequency (cf)	Frequency (f)
0 – 10	12	12
10 – 20	32	20
20 – 30	57	25
30 – 40	80	23
40 – 50	92	12
50 – 60	116	24
60 – 70	164	48
70 – 80	200	36
		$N = \sum f = 200$

Now, N = 200

$\Rightarrow \frac{N}{2} = 100$.

The cumulative frequency just greater than 100 is 116 and the corresponding class is 50 – 60.

Thus, the median class is 50 – 60.

$\therefore l = 50$, h = 10, f = 24, cf = c.f. of preceding class = 92 and $\frac{N}{2} = 100$.

$\therefore$ Median, $M = l + \left\{ h \times \left(\frac{\frac{N}{2} - cf}{f} \right) \right\}$

$= 50 + \left\{ 10 \times \left(\frac{100 - 92}{24} \right) \right\}$

$= 50 + 3.33$

$= 53.33$

Hence, median = 53.33.

Exercise 9C

1.   Find the mode of the following distribution:

Marks	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Frequency	12	35	45	25	13

Sol:

Here, the maximum class frequency is 45, and the class corresponding to this frequency is 30 – 40. So, the modal class is 30- 40.

Now,

Modal class = 30 – 40, lower limit (l) of modal class = 30, class size (h) = 10,

frequency (f_1) of the modal class = 45,

frequency (f_0) of class preceding the modal class = 35,

frequency (f_2) of class succeeding the modal class = 25

Now, let us substitute these values in the formula:

$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$

$= 30 + \left(\frac{45 - 35}{90 - 35 - 45} \right) \times 10$

$= 30 + \left(\frac{10}{30} \right) \times 10$

$= 30 + 3.33$

$= 33.33$

Hence, the mode is 33.33.

2.   Compute the mode of the following data:

Class	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100
Frequency	25	16	28	20	5

Sol:

Here, the maximum class frequency is 28, and the class corresponding to this frequency is 40 – 60. So, the modal class is 40 – 60.

Now,

Modal class = 40 – 60, lower limit (l) of modal class = 40, class size (h) = 20,

frequency (f_1) of the modal class = 28,

frequency (f_0) of class preceding the modal class = 16,

frequency (f_2) of class succeeding the modal class = 20

Now, let us substitute these values in the formula:

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 40 + \left(\frac{28-16}{56-16-20}\right) \times 20$$

$$= 40 + \left(\frac{12}{20}\right) \times 20$$

$$= 40 + 12$$

$$= 52$$

Hence, the mode is 52.

3. Heights of students of class X are givee in the flowing frequency distribution

Height (in cm)	150 – 155	155 – 160	160 – 165	165 – 170	170 - 175
Number of students	15	8	20	12	5

Find the modal height.

Also, find the mean height. Compared and interpret the two measures of central tendency.

Sol:

Here, the maximum class frequency is 20, and the class corresponding to this frequency is 160 – 165. So, the modal class is 160 – 165.

Now,

Modal class = 160 – 165, lower limit (l) of modal class = 160, class size (h) = 5,

frequency (f_1) of the modal class = 20,

frequency (f_0) of class preceding the modal class = 8,

frequency (f_2) of class succeeding the modal class = 12

Now, let us substitute these values in the formula:

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 160 + \left(\frac{20-8}{40-8-12}\right) \times 5$$

$$= 160 + \left(\frac{12}{20}\right) \times 5$$

$$= 160 + 3$$

$$= 163$$

Hence, the mode is 163.

It represents that the height of maximum number of students is 163cm.

Now, to find the mean let us put the data in the table given below:

Height (in cm)	Number of students (f_i)	Class mark (x_i)	$f_i x_i$
150 – 155	15	152.5	2287.5
155 – 160	8	157.5	1260
160 – 165	20	162.5	3250
165 – 170	12	167.5	2010
170 – 175	5	172.5	862.5
Total	$\Sigma f_i = 60$		$\Sigma f_i x_i = 9670$

$$\text{Mean} = \frac{\sum_i f_i x_i}{\sum_i f_i}$$

$$= \frac{9670}{60}$$

$$= 161.17$$

Thus, mean of the given data is 161.17.

It represents that on an average, the height of a student is 161.17cm.

4. Find the mode of the following distribution:

Class interval	10 – 14	14 – 18	18 – 22	22 – 26	26 – 30	30 – 34	34 – 38	38 – 42
Frequency	8	6	11	20	25	22	10	4

Sol:

As the class 26 – 30 has the maximum frequency, it is the modal class.

Now, $x_k = 26$, $h = 4$, $f_k = 25$, $f_{k-1} = 20$, $f_{k+1} = 22$

$\therefore$ Mode, $M_0 = x_k + \left\{ h \times \frac{(f_k - f_{k-1})}{(2f_k - f_{k-1} - f_{k+1})} \right\}$

$= 26 + \left\{ 4 \times \frac{(25-20)}{(2 \times 25 - 20 - 22)} \right\}$

$= 26 + \left\{ 4 \times \frac{5}{8} \right\}$

$= (26 + 2.5)$

$= 28.5$

5. Given below is the distribution of total household expenditure of 200 manual workers in a city:

Expenditure (in Rs)	Number of manual workers
1000 – 1500	24
1500 – 2000	40
2000 – 2500	31
2500 – 3000	28
3000 – 3500	32
3500 – 4000	23
4000 – 4500	17
4500 – 5000	5

Find the average expenditure done by maximum number of manual workers.

Sol:

As the class 1500-2000 has the maximum frequency, it is the modal class.

Now, $x_k = 1500$, $h = 500$, $f_k = 40$, $f_{k-1} = 24$, $f_{k+1} = 31$

$\therefore$ Mode, $M_0 = x_k + \left\{ h \times \dfrac{(f_k - f_{k-1})}{(2f_k - f_{k-1} - f_{k+1})} \right\}$

$= 1500 + \left\{ 500 \times \dfrac{(40-24)}{(2 \times 40 - 24 - 31)} \right\}$

$= 1500 + \left\{ 500 \times \dfrac{16}{25} \right\}$

$= (1500 + 320)$

$= 1820$

Hence, mode = Rs 1820

6. Calculate the mode from the following data:

Monthly salary (in Rs)	No of employees
0 – 5000	90
5000 – 10000	150
10000 – 15000	100
15000 – 20000	80
20000 – 25000	70
25000 – 30000	10

Sol:

As the class 5000-10000 has the maximum frequency, it is the modal class.

Now, $x_k = 5000$, $h = 5000$, $f_k = 150$, $f_{k-1} = 90$, $f_{k+1} = 100$

$\therefore$ Mode, $M_0 = x_k + \left\{ h \times \dfrac{(f_k - f_{k-1})}{(2f_k - f_{k-1} - f_{k+1})} \right\}$

$= 5000 + \left\{ 5000 \times \dfrac{(150-90)}{(2 \times 150 - 90 - 100)} \right\}$

$= 5000 + \left\{ 5000 \times \dfrac{60}{110} \right\}$

$= (5000 + 2727.27)$

$= 7727.27$

Hence, mode = Rs 7727.27

7. Compute the mode from the following data:

Age (in years)	0 – 5	5 – 10	10 – 15	15 – 20	20 – 25	25 – 30	30 - 35
No of patients	6	11	18	24	17	13	5

Sol:

As the class 15 – 20 has the maximum frequency, it is the modal class.

Now, $x_k = 15$, $h = 5$, $f_k = 24$, $f_{k-1} = 18$, $f_{k+1} = 17$

$\therefore$ Mode, $M_0 = x_k + \left\{ h \times \dfrac{(f_k - f_{k-1})}{(2f_k - f_{k-1} - f_{k+1})} \right\}$

$= 15 + \left\{ 5 \times \dfrac{(24-18)}{(2\times 24 - 18 - 17)} \right\}$

$= 15 + \left\{ 5 \times \dfrac{6}{13} \right\}$

$= (15 + 2.3)$

$= 17.3$

Hence, mode = 17.3 years

8.   Compute the mode from the following series:

Size	45 – 55	55 – 65	65 – 75	75 – 85	85 – 95	95 – 105	105 - 115
Frequency	7	12	17	30	32	6	10

Sol:

As the class 85 – 95 has the maximum frequency, it is the modal class.

Now, $x_k = 85$, $h = 10$, $f_k = 32$, $f_{k-1} = 30$, $f_{k+1} = 6$

$\therefore$ Mode, $M_0 = x_k + \left\{ h \times \dfrac{(f_k - f_{k-1})}{(2f_k - f_{k-1} - f_{k+1})} \right\}$

$= 85 + \left\{ 10 \times \dfrac{(32-30)}{(2\times 32 - 30 - 6)} \right\}$

$= 85 + \left\{ 10 \times \dfrac{2}{28} \right\}$

$= (85 + 0.71)$

$= 85.71$

Hence, mode = 85.71

9.   Compute the mode from the following data:

Class interval	1 – 5	6 – 10	11 – 15	16 – 20	21 – 25	26 – 30	31 – 35	36 – 40	41 – 45	46 – 50
Frequency	3	8	13	18	28	20	13	8	6	4

Sol:

Clearly, we have to find the mode of the data. The given data is an inclusive series. So, we will convert it to an exclusive form as given below:

Class interval	0.5 – 5.5	5.5 – 10.5	10.5 – 15.5	15.5 – 20.5	20.5 – 25.5	25.5 – 30.5	30.5 – 35.5	35.5 – 40.5	40.5 – 45.5	45.5 – 50.5
Frequency	3	8	13	18	28	20	13	8	6	4

As the class 20.5 – 25.5 has the maximum frequency, it is the modal class.

Now, $x_k = 20.5$, $h = 5$, $f_k = 28$, $f_{k-1} = 18$, $f_{k+1} = 20$

$\therefore$ Mode, $M_0 = x_k + \left\{ h \times \dfrac{(f_k - f_{k-1})}{(2f_k - f_{k-1} - f_{k+1})} \right\}$

$= 20.5 + \left\{ 5 \times \dfrac{(28-18)}{(2\times 28 - 18 - 20)} \right\}$

$= 20.5 + \left\{ 5 \times \dfrac{10}{18} \right\}$

$= (20.5 + 2.78)$

= 23.28

Hence, mode = 23.28

10. The agewise participation of students in the annual function of a school is shown in the following distribution.

Age (in years)	5 - 7	7 - 9	9 - 11	11 – 13	13 – 15	15 – 17	17 – 19
Number of students	x	15	18	30	50	48	x

Find the missing frequencies when the sum of frequencies is 181. Also find the mode of the data.

Sol:

It is given that the sum of frequencies is 181.

$\therefore$ x + 15 + 18 + 30 + 50 + 48 + x = 181

$\Rightarrow$ 2x + 161 = 181

$\Rightarrow$ 2x = 181 – 161

$\Rightarrow$ 2x = 20

$\Rightarrow$ x = 10

Thus, x = 10

Here, the maximum class frequency is 50, and the class corresponding to this frequency is 13 – 15. So, the modal class is 13 – 15.

Now,

Modal class = 13 – 15, lower limit (l) of modal class = 13, class size (h) = 2,

frequency (f_1) of the modal class = 50,

frequency (f_0) of class preceding the modal class = 30,

frequency (f_2) of class succeeding the modal class = 48

Now, let us substitute these values in the formula:

Mode = $l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$= 13 + \left(\frac{50-30}{100-30-48}\right) \times 2$

$= 13 + \left(\frac{20}{22}\right) \times 2$

$= 13 + 1.82$

$= 14.82$

Hence, the mode is 14.82.

Exercise 9D

1. Find the mean, median and mode of the following data:

Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70
Frequency	4	4	7	10	12	8	5

Sol:

To find the mean let us put the data in the table given below:

Class	Frequency (f_i)	Class mark (x_i)	$f_i x_i$
0 – 10	4	5	20
10 – 20	4	15	60
20 – 30	7	25	175
30 – 40	10	35	350
40 – 50	12	45	540
50 – 60	8	55	440
60 – 70	5	65	325
Total	$\Sigma f_i = 50$		$\Sigma f_i x_i = 1910$

$$\text{Mean} = \frac{\sum\limits_i f_i x_i}{\sum\limits_i f_i}$$

$$= \frac{1910}{50}$$

$$= 38.2$$

Thus, the mean of the given data is 38.2.

Now, to find the median let us put the data in the table given below:

Class	Frequency (f_i)	Cumulative Frequency (cf)
0 – 10	4	4
10 – 20	4	8
20 – 30	7	15
30 – 40	10	25
40 – 50	12	37
50 – 60	8	45
60 – 70	5	50
Total	$N = \Sigma f_i = 50$	

Now, $N = 50 \Rightarrow \frac{N}{2} = 25$.

The cumulative frequency just greater than 25 is 37 and the corresponding class is 40 – 50.

Thus, the median class is 40 – 50.

$\therefore l = 40$, h = 10, N = 50, f = 12 and cf = 25.

Now,

$$\text{Median} = l + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h$$

$$= 40 + \left(\frac{25 - 25}{12}\right) \times 10$$

$$= 40$$

Thus, the median is 40.

We know that,

Mode = 3(median) – 2(mean)

$$= 3 \times 40 - 2 \times 38.2$$

$$= 120 - 76.4$$

$$= 43.6$$

Hence, Mean = 38.2, Median = 40 and Mode = 43.6

2. Find the mean, median and mode of the following data:

Class	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	100 – 120	120 – 140
Frequency	6	8	10	12	6	5	3

Sol:

To find the mean let us put the data in the table given below:

Class	Frequency (f_i)	Class mark (x_i)	$f_i x_i$
0 – 20	6	10	60
20 – 40	8	30	240
40 – 60	10	50	500
60 – 80	12	70	840
80 – 100	6	90	540
100 – 120	5	110	550
120 – 140	3	130	390
Total	$\Sigma f_i = 50$		$\Sigma f_i x_i = 3120$

$$\text{Mean} = \frac{\sum_i f_i x_i}{\sum_i f_i}$$

$$= \frac{3120}{50}$$

$$= 62.4$$

Thus, the mean of the given data is 62.4.

Now, to find the median let us put the data in the table given below:

Class	Frequency (f_i)	Cumulative Frequency (cf)
0 – 20	6	6
20 – 40	8	14
40 – 60	10	24
60 – 80	12	36
80 – 100	6	42
100 – 120	5	47
120 – 140	3	50
Total	$N = \Sigma f_i = 50$	

Now, $N = 50 \Rightarrow \frac{N}{2} = 25$.

The cumulative frequency just greater than 25 is 36 and the corresponding class is 60 – 80.

Thus, the median class is 60 – 80.

$\therefore l = 60$, h = 20, N = 50, f = 12 and cf = 24.

Now,

$$\text{Median} = l + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h$$

$$= 60 + \left(\frac{25 - 24}{12}\right) \times 20$$

$$= 60 + 1.67$$

$$= 61.67$$

Thus, the median is 61.67.

We know that,

Mode = 3(median) – 2(mean)

$$= 3 \times 61.67 - 2 \times 62.4$$

$$= 185.01 - 124.8$$

$$= 60.21$$

Hence, Mean = 62.4, Median = 61.67 and Mode = 60.21

3. Find the mean, median and mode of the following data:

Class	0 – 50	50 – 100	100 – 150	150 – 200	200 – 250	250 – 300	300 - 350
Frequency	2	3	5	6	5	3	1

Sol:

To find the mean let us put the data in the table given below:

Class	Frequency (f_i)	Class mark (x_i)	$f_i x_i$
0 – 50	2	25	50
50 – 100	3	75	225
100 – 150	5	125	625
150 – 200	6	175	1050
200 – 250	5	225	1125
250 – 300	3	275	825
300 – 350	1	325	325
Total	$\Sigma f_i = 25$		$\Sigma f_i x_i = 4225$

$$\text{Mean} = \frac{\sum_i f_i x_i}{\sum_i f_i}$$

$$= \frac{4225}{25}$$

$$= 169$$

Thus, mean of the given data is 169.

Now, to find the median let us put the data in the table given below:

Class	Frequency (f$_i$)	Cumulative Frequency (cf)
0 – 50	2	2
50 – 100	3	5
100 – 150	5	10
150 – 200	6	16
200 – 250	5	21
250 – 300	3	24
300 – 350	1	25
Total	N = $\sum f_i$ = 25	

Now, N = 25 $\Rightarrow \frac{N}{2} = 12.5$.

The cumulative frequency just greater than 12.5 is 16 and the corresponding class is 150 – 200.

Thus, the median class is 150 – 200.

∴ l = 150, h = 50, N = 25, f = 6 and cf = 10.

Now,

Median = $l + \left(\frac{\frac{N}{2}-cf}{f}\right) \times$ h

$\qquad = 150 + \left(\frac{12.5 - 10}{6}\right) \times 50$

$\qquad = 150 + 20.83$

$\qquad = 170.83$

Thus, the median is 170.83.

We know that,

Mode = 3(median) – 2(mean)

$\qquad = 3 \times 170.83 - 2 \times 169$

$\qquad = 512.49 - 338$

$\qquad = 174.49$

Hence, Mean = 169, Median = 170.83 and Mode = 174.49

4. Find the mean, median and mode of the following data:

Marks obtained	25 - 35	35 – 45	45 – 55	55 – 65	65 – 75	75 - 85
No. of students	7	31	33	17	11	1

Sol:

To find the mean let us put the data in the table given below:

Marks obtained	Number of students (f$_i$)	Class mark (x$_i$)	f$_i$ x$_i$
25 – 35	7	30	210
35 – 45	31	40	1240

45 – 55	33	50	1650
55 – 65	17	60	1020
65 – 75	11	70	770
75 – 85	1	80	80
Total	$\Sigma\, f_i = 100$		$\Sigma\, f_i\, x_i = 4970$

$$\text{Mean} = \frac{\sum_i f_i x_i}{\sum_i f_i}$$

$$= \frac{4970}{100}$$

$$= 49.7$$

Thus, mean of the given data is 49.7.

Now, to find the median let us put the data in the table given below:

Class	Frequency (f_i)	Cumulative Frequency (cf)
25 – 35	7	7
35 – 45	31	38
45 – 55	33	71
55 – 65	17	88
65 – 75	11	99
75 – 85	1	100
Total	$N = \Sigma\, f_i = 100$	

Now, $N = 100 \Rightarrow \frac{N}{2} = 50$.

The cumulative frequency just greater than 50 is 71 and the corresponding class is 45 – 55.
Thus, the median class is 45 – 55.

$\therefore l = 45$, h = 10, N = 100, f = 33 and cf = 38.

Now,

$$\text{Median} = l + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h$$

$$= 45 + \left(\frac{50 - 38}{33}\right) \times 10$$

$$= 45 + 3.64$$

$$= 48.64$$

Thus, the median is 48.64.

We know that,

Mode = 3(median) – 2(mean)

$$= 3 \times 48.64 - 2 \times 49.70$$

$$= 145.92 - 99.4$$

$$= 46.52$$

Hence, Mean = 49.70, Median = 48.64 and Mode = 46.52

5. A survey regarding the heights (in cm) of 50 girls of a class was conducted and the following data was obtained:

Height in cm	120 – 130	130 – 140	140 – 150	150 – 160	160 – 170
No. of girls	2	8	12	20	8

Find the mean, median and mode of the above data.

Sol: We have the following

Height in cm	Mid value (x_i)	Frequency (f_i)	Cumulative frequency	($f_i \times x_i$)
120 – 130	125	2	2	250
130 – 140	135	8	10	1080
140 – 150	145	12	22	1740
150 – 160	155	20	42	3100
160 – 170	165	8	50	1320
		$\Sigma f_i = 50$		$\Sigma f_i \times x_i = 7490$

Mean, $\bar{x} = \dfrac{\sum(f_i \times x_i)}{\sum f_i}$

$= \dfrac{7490}{50}$

$= 149.8$

Now, N = 50

$\Rightarrow \dfrac{N}{2} = 25.$

The cumulative frequency just greater than 25 is 42 and the corresponding class is 150 – 160.

Thus, the median class is 150 – 160.

$\therefore l = 150$, h = 10, f = 20, c = cf of preceding class = 22 and $\dfrac{N}{2} = 25$

Now,

Median, $M_e = l + \left\{ h \times \left(\dfrac{\frac{N}{2} - c}{f} \right) \right\}$

$= 150 + \left\{ 10 \times \left(\dfrac{25 - 22}{20} \right) \right\}$

$= \left(150 + 10 \times \dfrac{3}{20} \right)$

$= 151.5$

Mode = 3(median) – 2(mean)

$= 3 \times 151.5 - 2 \times 149.8$

$= 154.9$

6. The following table gives the daily income of 50 workers of a factory:

Daily income (in Rs)	100 – 120	120 – 140	140 – 160	160 – 180	180 – 200
No. of workers	12	14	8	6	10

Find the mean, median and mode of the above data.

Sol:

We have the following:

Daily income	Mid value (x_i)	Frequency (f_i)	Cumulative frequency	($f_i \times x_i$)
100 – 120	110	12	12	1320
120 – 140	130	14	26	1820
140 – 160	150	8	34	1200
160 – 180	170	6	40	1020
180 – 200	190	10	50	1900
		$\Sigma f_i = 50$		$\Sigma f_i \times x_i = 7260$

Mean, $\bar{x} = \dfrac{\Sigma_i f_i \times x_i}{\Sigma_i f_i}$

$= \dfrac{7260}{50}$

$= 145.2$

Now, N = 50

$\Rightarrow \dfrac{N}{2} = 25.$

The cumulative frequency just greater than 25 is 26 and the corresponding class is 120 – 140.

Thus, the median class is 120 – 140.

$\therefore l = 120$, h = 20, f = 14, c = cf of preceding class = 12 and $\dfrac{N}{2} = 25$

Now,

Median, $M_e = l + \left\{ h \times \left(\dfrac{\frac{N}{2} - c}{f} \right) \right\}$

$= 120 + \left\{ 20 \times \left(\dfrac{25 - 12}{14} \right) \right\}$

$= \left(120 + 20 \times \dfrac{13}{14} \right)$

$= 138.57$

Mode = 3(median) – 2(mean)

$= 3 \times 138.57 - 2 \times 145.2$

$= 125.31$

7. The table below shows the daily expenditure on food of 30 households in a locality:

Daily expenditure (in Rs)	Number of households
100 – 150	6
150 – 200	7
200 – 250	12
250 – 300	3
300 – 350	2

Find the mean and median daily expenditure on food.

Sol:

We have the following:

Daily expenditure (in Rs)	Mid value (x_i)	Frequency (f_i)	Cumulative frequency	$(f_i \times x_i)$
100 – 150	125	6	6	750
150 – 200	175	7	13	1225
200 – 250	225	12	25	2700
250 – 300	275	3	28	825
300 – 350	325	2	30	650
		$\Sigma\, f_i = 30$		$\Sigma\, f_i \times x_i = 6150$

Mean, $\overline{x} = \dfrac{\Sigma_i f_i \times x_i}{\Sigma_i f_i}$

$= \dfrac{6150}{30}$

$= 205$

Now, N = 30

$\Rightarrow \dfrac{N}{2} = 15.$

The cumulative frequency just greater than 15 is 25 and the corresponding class is 200 – 250.

Thus, the median class is 200 – 250.

$\therefore l = 200$, h = 50, f = 12, c = cf of preceding class = 13 and $\dfrac{N}{2} = 15$

Now,

Median, $M_e = l + \left\{ h \times \left(\dfrac{\frac{N}{2} - c}{f} \right) \right\}$

$= 200 + \left\{ 50 \times \left(\dfrac{15 - 13}{12} \right) \right\}$

$= \left(200 + 50 \times \dfrac{2}{12} \right)$

$= 200 + 8.33$

$= 208.33$

Exercise 9E

30. Find the median of the following data by making a 'less than ogive'.

Marks	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80	80 - 90	90 - 100
Number of Students	5	3	4	3	3	4	7	9	7	8

Sol:

The frequency distribution table of less than type is given as follows:

Marks (upper class limits)	Cumulative frequency (cf)
Less than 10	5
Less than 20	5 + 3 = 8
Less than 30	8 + 4 = 12
Less than 40	12 + 3 = 15
Less than 50	15 + 3 = 18
Less than 60	18 + 4 = 22
Less than 70	22 + 7 = 29
Less than 80	29 + 9 = 38
Less than 90	38 + 7 = 45
Less than 100	45 + 8 = 53
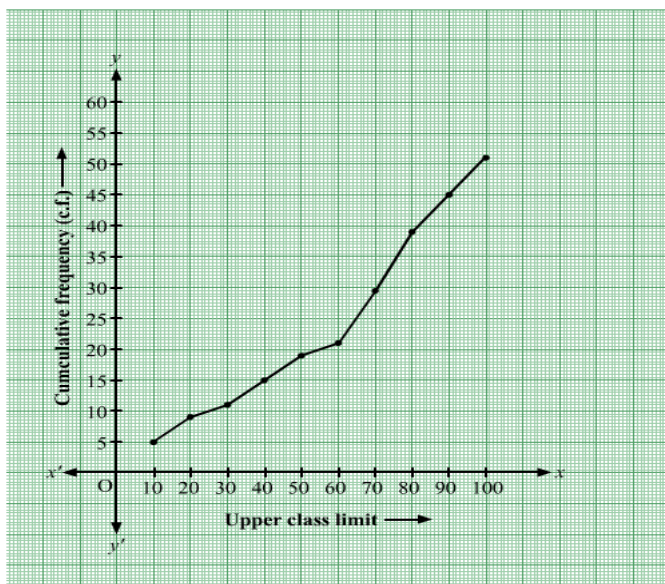
Taking upper class limits of class intervals on x-axis and their respective frequencies on y-axis, its ogive can be drawn as follows:

Here, $N = 53 \Rightarrow \frac{N}{2} = 26.5$.

Mark the point A whose ordinate is 26.5 and its x-coordinate is 66.4.
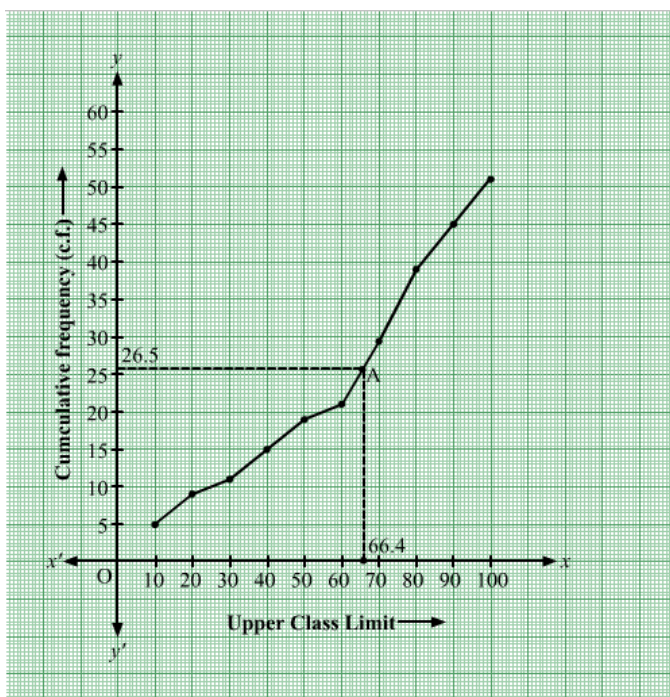
Thus, median of the data is 66.4.

31. The given distribution shows the number of wickets taken by the bowlers in one-day international cricket matches:

Number of Wickets	Less than 15	Less than 30	Less than 45	Less than 60	Less than 75	Less than 90	Less than 105	Less than 120
Number of bowlers	2	5	9	17	39	54	70	80

Draw a 'less than type' ogive from the above data. Find the median.
Sol:
Taking upper class limits of class intervals on x-axis and their respective frequencies on y-axis, its ogive can be drawn as follows:
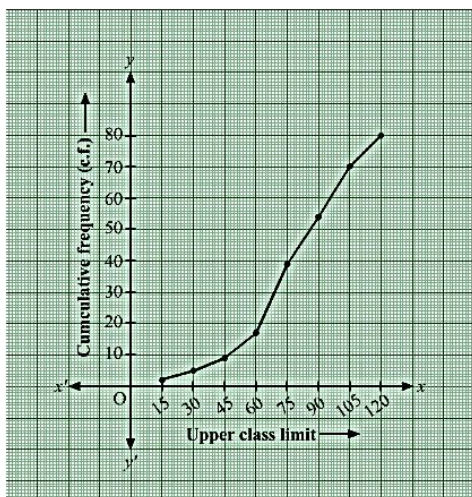
Here, N = 80 ⇒ $\frac{N}{2}$ = 40.

Mark the point A whose ordinate is 40 and
its x-coordinate is 76.
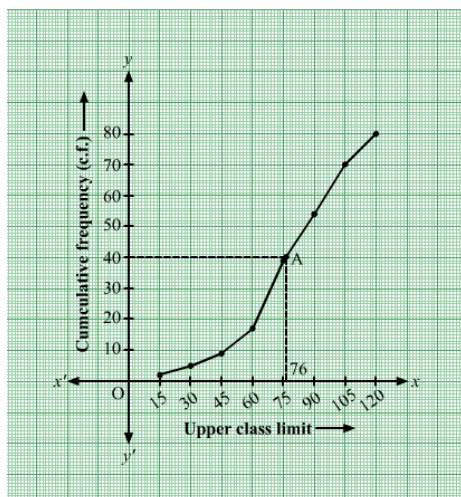
Thus, median of the data is 76.

32. Draw a 'more than' ogive for the data given below which gives the marks of 100 students.

Marks	0 – 10	10 – 20	20 – 30	30 - 40	40 – 50	50 – 60	60 – 70	70 – 80
No of Students	4	6	10	10	25	22	18	5

Sol:

The frequency distribution table of more than type is as follows:

Marks (upper class limits)	Cumulative frequency (cf)
More than 0	96 + 4 = 100
More than 10	90 + 6 = 96
More than 20	80 + 10 = 90
More than 30	70 + 10 = 80
More than 40	45 + 25 = 70
More than 50	23 + 22 = 45
More than 60	18 + 5 = 23
More than 70	5

Taking lower class limits of on x-axis and their respective cumulative frequencies on y-axis,
its ogive can be drawn as follows:
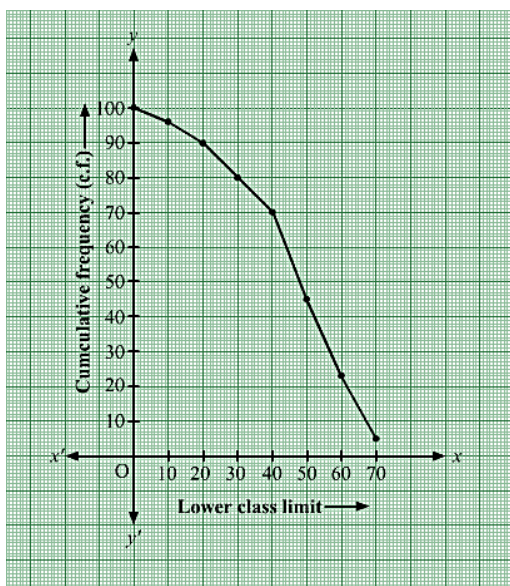
33. The heights of 50 girls of Class X of a school are recorded as follows:

Height (in cm)	135 - 140	140 – 145	145 – 150	150 – 155	155 – 160	160 – 165
No of Students	5	8	9	12	14	2

Draw a 'more than type' ogive for the above data.
Sol:
The frequency distribution table of more than type is as follows:

Height (in cm) (lower class limit)	Cumulative frequency (cf)
More than 135	$5 + 45 = 50$
More than 140	$8 + 37 = 45$
More than 145	$9 + 28 = 37$
More than 150	$12 + 16 = 28$
More than 155	$14 + 2 = 16$
More than 160	2

Taking lower class limits of on x-axis and their respective cumulative frequencies on y-axis, its ogive can be drawn as follows:
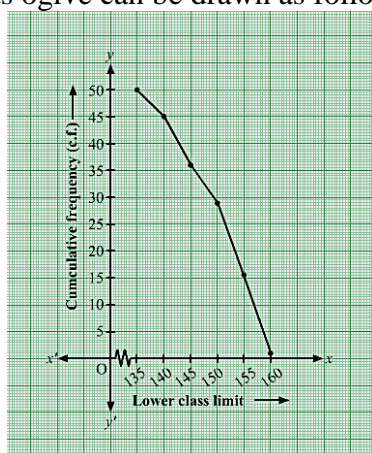
34. The monthly consumption of electricity (in units) of some families of a locality is given in the following frequency distribution:

Monthly Consumption (in units)	140 – 160	160 – 180	180 – 200	200 – 220	220 – 240	240 – 260	260 - 280
Number of Families	3	8	15	40	50	30	10

Prepare a 'more than type' ogive for the given frequency distribution.

Sol:

The frequency distribution table of more than type is as follows:

Height (in cm) (lower class limit)	Cumulative frequency (cf)
More than 140	3 + 153 = 156
More than 160	8 + 145 = 153
More than 180	15 + 130 = 145
More than 200	40 + 90 = 130
More than 220	50 + 40 = 90
More than 240	30 + 10 = 40
More than 260	10

Taking the lower class limits of on x-axis and their respective cumulative frequencies on y-axis, its ogive can be drawn as follows:
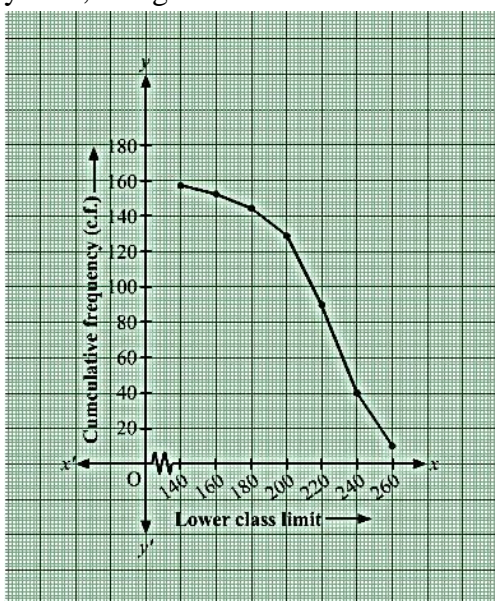
35. The following table gives the production yield per hectare of wheat of 100 farms of a village.

| Production Yield (kg/ha)	50 – 55	55 – 60	60 – 65	65- 70	70 – 75	75 – 80
Number of farms	2	8	12	24	238	16

Change the distribution to a 'more than type' distribution and draw its ogive. Using ogive, find the median of the given data.

Sol:

The frequency distribution table of more than type is as follows:

Production yield (kg/ha) (lower class limits)	Cumulative frequency (cf)
More than 50	$2 + 98 = 100$
More than 55	$8 + 90 = 98$
More than 60	$12 + 78 = 90$
More than 65	$24 + 54 = 78$
More than 70	$38 + 16 = 54$
More than 75	16

Taking the lower class limits on x-axis and their respective cumulative on y-axis, its ogive can be drawn as follows:
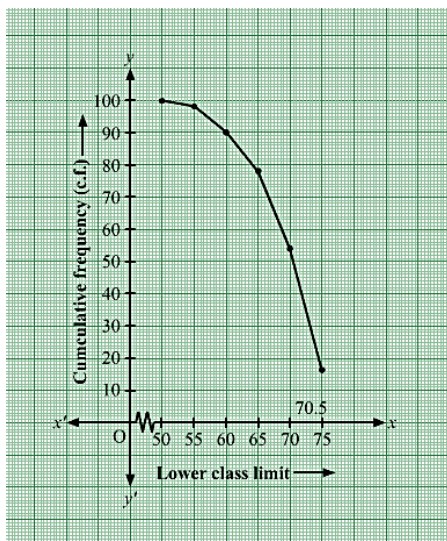
Here, $N = 100 \Rightarrow \dfrac{N}{2} = 50$.

Mark the point A whose ordinate is 50 and its x-coordinate is 70.5.
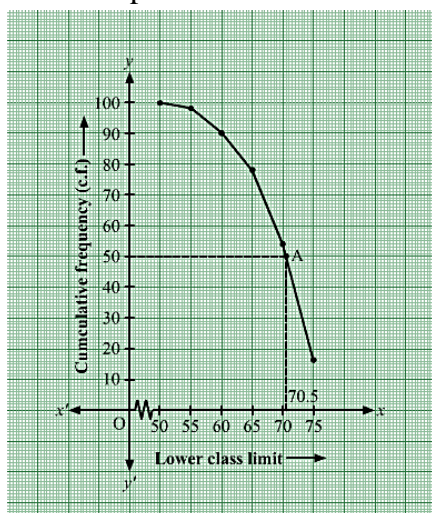
Thus, median of the data is 70.5.

36. The table given below shows the weekly expenditures on food of some households in a locality

Weekly expenditure (in ₹)	Number of house holds
100 – 200	5
200- 300	6
300 – 400	11
400 – 500	13
500 – 600	5
600 – 700	4
700 – 800	3
800 – 900	2

Draw a 'less than type ogive' and a 'more than type ogive' for this distribution.

Sol:

The frequency distribution table of less than type is as follows:

Weekly expenditure (in ₹) (upper class limits)	Cumulative frequency (cf)
Less than 200	5
Less than 300	5 + 6 = 11
Less than 400	11 + 11 = 22
Less than 500	22 + 13 = 35
Less than 600	35 + 5 = 40
Less than 700	40 + 4 = 44
Less than 800	44 + 3 = 47
Less than 900	47 + 2 = 49

Taking the lower class limits on x-axis and their respective cumulative frequencies on y-axis, its ogive can be obtained as follows
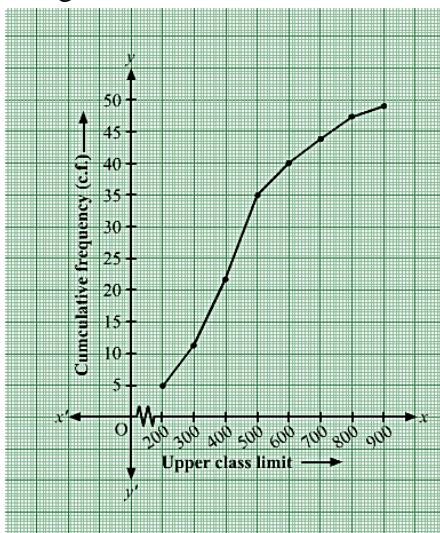

Now,

The frequency distribution table of more than type is as follows:

Weekly expenditure (in ₹) (lower class limits)	Cumulative frequency (cf)
More than 100	44 + 5 = 49
More than 200	38 + 6 = 44
More than 300	27 + 11 = 38
More than 400	14 + 13 = 27
More than 500	9 + 5 = 14
More than 600	5 + 4 = 9
More than 700	2 + 3 = 5
More than 800	2

Taking the lower class limits
on x-axis and their respective
cumulative frequencies on y-axis,
its ogive can be obtained as follows:
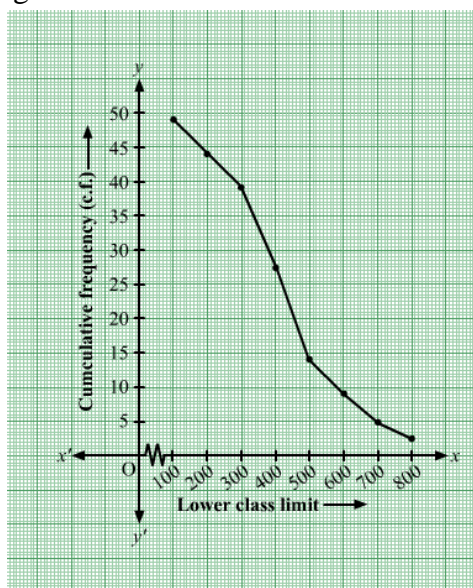
37. From the following frequency, prepare the 'more than' ogive.

| Score	Number of candidates
400 – 450	20
450 – 500	35
500 – 550	40
550 – 600	32
600 – 650	24
650 – 700	27
700 – 750	18
750 – 800	34
Total	230

Also, find the median.

Sol:

From the given table, we may prepare than 'more than' frequency table as shown below:

Score	Number of candidates
More than 750	34
More than 700	52
More than 650	79
More than 600	103
More than 550	135
More than 500	175
More than 450	210
More than 400	230

We plot the points A(750, 34), B(700,52),
C(650, 79), D(600, 103), E(550, 135), F(500, 175),
G(450, 210) and H(400, 230).
Join AB, BC, CD, DE, EF, FG, GH and HA with
a free hand to get the curve representing the
'more than type' series.
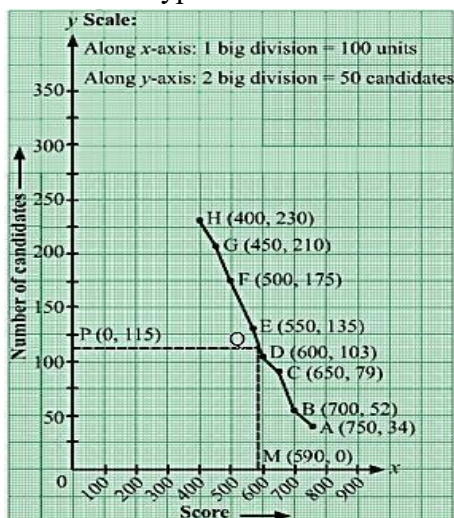
Here, N = 230

$$\Rightarrow \frac{N}{2} = 115$$

From P (0, 115), draw PQ meeting the curve at Q. Draw QM meeting at M.
Clearly, OM = 590 units
Hence, median = 590 units.

38. The marks obtained by 100 students of a class in an examination are given below:

Marks	Number of students
0 – 5	2
5 – 10	5
10 – 15	6
15 – 20	8
20 – 25	10

25 – 30	25
30 – 35	20
35 – 40	18
40 – 45	4
45 – 50	2

Draw cumulative frequency curves by using (i) 'less than' series and (ii) 'more than' series. Hence, find the median.

Sol:

(i) From the given table, we may prepare the 'less than' frequency table as shown below:

Marks	Number of students
Less than 5	2
Less than 10	7
Less than 15	13
Less than 20	21
Less than 25	31
Less than 30	56
Less than 35	76
Less than 40	94
Less than 45	98
Less than 50	100

We plot the points A(5, 2), B(10, 7), C(15, 13), D(20, 21), E(25, 31), F(30, 56), G(35, 76) and H(40, 94), I(45, 98) and J(50, 100).

Join AB, BC, CD, DE, EF, FG, GH, HI, IJ and JA with a free hand to get the curve representing the 'less than type' series.

(ii) More than series:

Marks	Number of students
More than 0	100
More than 5	98
More than 10	93
More than 15	87
More than 20	79
More than 25	69
More than 30	44
More than 35	24
More than 40	6
More than 45	2

Now, on the same graph paper, we plot the points (0, 100), (5, 98), (10, 94), (15, 76), (20, 56), (25, 31), (30, 21), (35, 13), (40, 6) and (45, 2).

Join with a free hand to get the 'more than type' series.
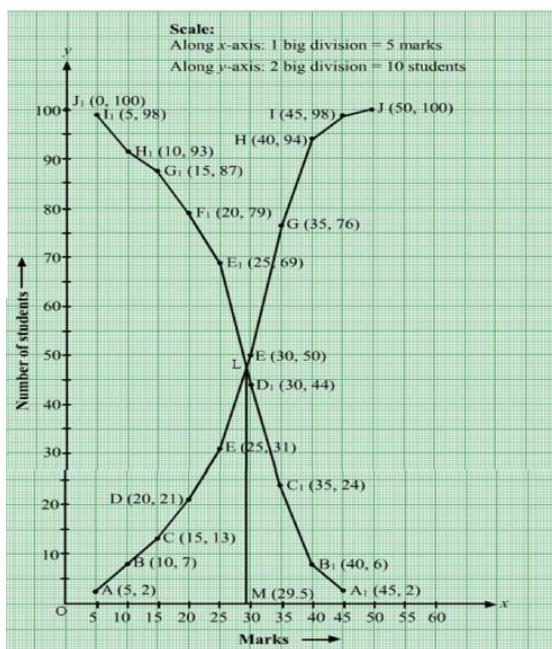
The two curves intersect at point L. Draw $LM \perp OX$ cutting the x-axis at M.

Clearly, M = 29.5

Hence, Median = 29.5

39. From the following data, draw the two types of cumulative frequency curves and determine the median:

Marks	Frequency
140 – 144	3
144 – 148	9
148 – 152	24
152 – 156	31
156 – 160	42
160 – 164	64
164 – 168	75
168 – 172	82
172 – 176	86
176 – 180	34

Sol:

(i) Less than series:

Marks	Number of students
Less than 144	3
Less than 148	12
Less than 152	36
Less than 156	67
Less than 160	109
Less than 164	173
Less than 168	248

Less than 172	230
Less than 176	416
Less than 180	450

We plot the points A(144, 3), B(148, 12), C(152, 36), D(156, 67), E(160, 109), F(164, 173), G(168, 248) and H(172, 330), I(176, 416) and J(180, 450).

Join AB, BC, CD, DE, EF, FG, GH, HI, IJ and JA with a free hand to get the curve representing the 'less than type' series.

(ii) More than series:

Marks	Number of students
More than 140	450
More than 144	447
More than 148	438
More than 152	414
More than 156	383
More than 160	341
More than 164	277
More than 168	202
More than 172	120
More than 176	34

Now, on the same graph paper, we plot the points A_1(140, 450), B_1(144, 447), C_1(148, 438), D_1(152, 414), E_1(156, 383), F_1(160, 277), H_1(168, 202), I_1(172, 120) and J_1(176, 34).

Join A_1B_1, B_1C_1, C_1D_1, D_1E_1, E_1F_1, F_1G_1, G_1H_1, H_1I_1 and I_1J_1 with a free hand to get the 'more than type' series.
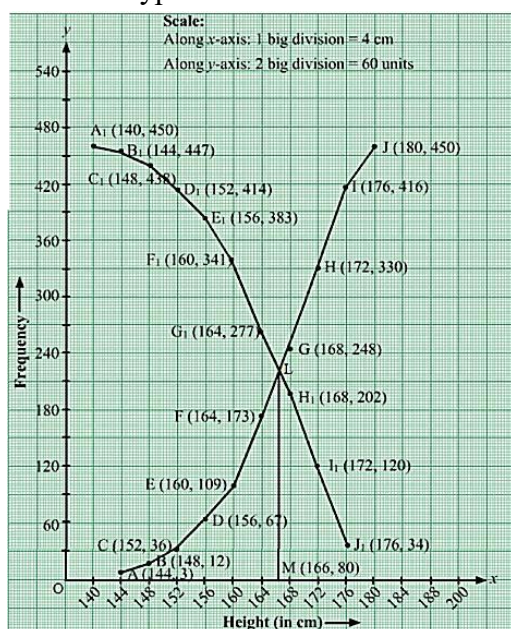
The two curves intersect at point L. Draw LM ⊥ OX cutting the x-axis at M. Clearly, M = 166cm

Hence, median = 166cm

Exercise 9F

1. Write the median class of the following distribution:

Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70
Frequency	4	4	8	10	12	8	4

Sol:

To find median let us put the data in the table given below:

Class	Frequency (f_i)	Cumulative frequency (cf)
0-10	4	4
10-20	4	8
20-30	8	16
30-40	10	26
40-50	12	38
50-60	8	46
60-70	4	50
Total	N=Σf_i = 50	

Now, $N = 50 \Rightarrow \frac{N}{2} = 25$

The cumulative frequency just greater than 25 is 26, and the corresponding class is 30-40.

Thus, the median class is 30-40.

2. What is the lower limit of the modal class of the following frequency distribution?

Age (in years)	0 - 10	10- 20	20 -30	30 – 40	40 – 50	50 – 60
Number of patients	16	13	6	11	27	18

Sol:

Here the maximum class frequency is 27, and the class corresponding to this frequency is 40-50 So the modal class is 40 -50.

Now,

Modal class = 40-50, lower limit (/) of modal class = 40.

Thus, lower limit (/) of modal class is 40

3. The monthly pocket money of 50 students of a class are given in the following distribution:

Monthly pocket money (in ₹)	0 - 50	50 – 100	100 – 150	150 -200	200 – 250	250 - 300
Number of Students	2	7	8	30	12	1

Find the modal class and give class mark of the modal class.

Sol:

Here the maximum class frequency is 30, and the class corresponding to the frequency is 150-200. So, the modal class is 150-200.

Also, class mark of the modal class is $\frac{150+200}{2} = 175$.

4. A data has 25 observations arranged in a descending order. Which observation represents the median?

Sol:

If the number of observations is odd, then the median is $\left(\frac{n+1}{2}\right) th$ observation.

Thus, $\left(\frac{25+1}{2}\right) = 13th$ observation represents the median.

5. For a certain distribution, mode and median were found to be 1000 and 1250 respectively. Find mean for this distribution using an empirical relation.

Sol:

There is an empirical relationship between the three measures of central tendency:

3median = mode + 2Mean

$\Rightarrow Mean = \frac{3Median - Mode}{2}$

$= \frac{3(1250) - 1000}{2}$

$= 1375$

Thus, the mean is 1375.

6. In a class test, 50 students obtained marks as follows:

Marks obtained	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100
Number of Students	4	6	25	10	5

Find the modal class and the median class.

Sol:

Here the maximum class frequency is 25, and the class corresponding to this frequency is 40-60.

So, the modal class is 40-60.

Now, to find the median class let us put the data in the table given below:

Marks Obtained	Number of students (f_i)	Cumulative frequency (cf)
0-20	4	4
20-40	6	10
40-60	25	35
60-80	10	45
80-100	5	50
Total	N=$\Sigma f_i = 50$	

Now, $N = 50 \Rightarrow \frac{N}{2} = 25$.

The cumulative frequency just greater than 25 is 35, and the corresponding class is 40-60.

Thus, the median class is 40-60.

7. Find the class marks of classes 10 -25 and 35 – 55.
   Sol:
   Class mark = $\frac{Upper\ limit + Lower\ limit}{2}$
   ∴ class mark of 10-25 = $\frac{10+25}{2}$
   $= 17.5$
   And class mark of 35-55 = $\frac{35+55}{2}$
   $= 45$

8. While calculating the mean of a given data by the assumed-mean method, the following values were obtained.
   A=25, $\sum f_i d_i = 110$, $\sum f_i = 50$
   Find the mean.
   Sol:
   According to assumed-mean method,
   $\bar{x} = A + \frac{\Sigma_i f_i d_i}{\Sigma_i f_i}$
   $= 25 + \frac{110}{50}$
   $= 25 + 2.2$
   $= 27.2$
   Thus, mean is 27.2.

9. The distribution X and Y with total number of observations 36 and 64, and mean 4 and 3 respectively are combined. What is the mean of the resulting distribution X + Y?
   Sol:
   According to the question,
   $4 = \frac{X}{36}$ and $3 = \frac{Y}{64}$
   $\Rightarrow X = 4 \times 36$ and $Y = 3 \times 64$
   $\Rightarrow X = 144$ and $Y = 192$
   $Now, X + Y = 144 + 192 = 336$
   And total number of observations $= 36 + 64 = 100$
   Thus, mean $= \frac{336}{100} = 3.36$.

10. In a frequency distribution table with 12 classes, the class-width is 2.5 and the lowest class boundary is 8.1, then what is the upper class boundary of the highest class?
    Sol:
    Upper class boundary = Lowest class boundary + width × number of classes
    $= 8.1 + 2.5 \times 12$
    $= 8.1 + 30$
    $= 38.1$
    Thus, upper class boundary of the highest class is 38.1.

11. The observation 29, 32, 48, 50, x, x+2, 72, 78, 84, 95 are arranged in ascending order. What is the value of x if the median of the data is 63?

Sol:

If number of observations is even, then the median will be the average of $\left(\frac{n}{2}\right)$th and the $\left(\frac{n}{2}+1\right)$th observations.

In the given case, $n = 10 \Rightarrow \left(\frac{n}{2}\right) th = 5 \; th \; and \; \left(\frac{n}{2}+1\right) th = 6th \; observation.$

Thus, $63 = \frac{x+(x+2)}{2}$

$\Rightarrow 126 = 2x + 2$

$\Rightarrow 124 = 2x$

$\Rightarrow x = 62$

Thus, the value of x is 62.

12. The median of 19 observations is 30. Two more observation are made and the values of these are 8 and 32. Find the median of the 21 observations taken together.

Hint Since 8 is less than 30 and 32 is more than 30, so the value of median (middle value) remains unchanged.

Sol:

Since, 8 is less than 30 and 32 is more than 30, so the middle value remains unchanged

Thus, the median of 21 observations taken together is 30.

13. If the median of $\frac{x}{5},\frac{x}{4},\frac{x}{2},x$ and $\frac{x}{3}$, where x > 0, is 8, find the value of x.

Hint Arranging the observations in ascending order, we have $\frac{x}{5},\frac{x}{4},\frac{x}{3},\frac{x}{2},x$ Median$=\frac{x}{3} = 8$.

Sol:

Arranging the observations in ascending order, we have

$\frac{x}{5},\frac{x}{4},\frac{x}{3},\frac{x}{2},x$

$Thus, the \; median \; is \frac{x}{3}$

$\Rightarrow \frac{x}{3} = 8$

$\Rightarrow x = 3 \times 8$

$\Rightarrow x = 24$

Thus, the value of x is 24.

14. What is the cumulative frequency of the modal class of the following distribution?

Class	$3-6$	$6-9$	$9-12$	$12-15$	$15-18$	$18-21$	$21-24$
Frequency	7	13	10	23	54	21	16

Sol:

Here the maximum class frequency is 23, and the class corresponding to this frequency is 12-15.

So, the modal class is 12.15.

Now, to find the cumulative frequency let us put the data in the table given below:

Class	Frequency (f_i)	Cumulative frequency (cf)
3-6	7	7
6-9	13	20
9-12	10	30
12-15	23	53
15-18	4	57
18-21	21	78
21-24	16	94
Total	$N = \Sigma f_i = 94$	

Thus, the cumulative frequency of the modal class is 53.

15. Find the mode of the given data:

Class Interval	$0-20$	$20-40$	$40-60$	$60-80$
Frequency	15	6	18	10

Sol:

Here the maximum class frequency is 18, and the class corresponding to this frequency is 40-60.

So, the modal class is 40-60.

Now,

Modal class = 40-60, lower limit (/) of modal class – 40, class size (h)=20,

Frequency (f_1) of the modal class =18,

Frequency (f_0) of class preceding the modal class =6,

Frequency (f_2) of class succeeding the modal class = 10.

Now, let us substitute these values in the formula:

$Mode = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$= 40 + \left(\frac{18-6}{36-6-10}\right) \times 20$

$= 40 + \left(\frac{12}{20}\right) \times 20$

$= 40 + 12$

$= 52$

Hence, the mode is 52.

16. The following are the ages of 300 patients getting medical treatment in a hospital on a particular day:

Age (in years)	$10-20$	$20-30$	$30-40$	$40-50$	$50-60$	$60\text{ -}70$
Number of patients	6	42	55	70	53	20

Form a 'less than type' cumulative frequency distribution.

Sol:

A 'less than type' cumulative frequency distribution table is given below:

Age (in years)	Cumulative frequency (cf)
Less than 20	60
Less than 30	102
Less than 40	157
Less than 50	227
Less than 60	280
Less than 70	300

17. In the following data, find the values of p and q. Also, find the median class and modal class.

Class	Frequency (f)	Cumulative frequency (cf)
100 – 200	11	11
200 – 300	12	P
300 – 400	10	33
400- 500	Q	46
500 – 600	20	66
600 – 700	14	80

Sol:

Here, $p = 11 + 12 = 23$

And $33 + q = 46$

$\Rightarrow q = 46 - 33$

$= 13$

Thus, p is 23 and q is 13.

Now,

Here the maximum class frequency is 20, and the class corresponding to this frequency is 500-600.

So, the modal class is 500-600.

Also, $\Sigma f = N = 80$

$\Rightarrow \frac{N}{2} = 40$.

The cumulative frequency just greater than 40 is 46, and the corresponding class is 400-500.

Thus, the median class is 400-500.

18. The following frequency distribution gives the monthly consumption of electricity ofr 64 consumers of locality.

Monthly consumption (in units)	65 – 85	85 – 105	105 – 125	125 – 145	145 – 165	165 – 185
Number of consumers	4	5	13	20	14	8

Form a ' more than type' cumulative frequency distribution.

Sol:

The cumulative frequency distribution table of more than type is as follows:

Monthly consumption (in units) (lower class limits)	Cumulative frequency (cf)
More than 65	60 + 4 =64
More than 85	55 + 5 = 60
More than 105	42 + 13 = 55
More than 125	22 + 20 = 42
More than 145	8 + 14 = 22
More than 165	8

19. The following table gives the life-time (in days) of 100 electric bulbs of a certain brand.

Life-tine (in days)	Less than 50	Less than 100	Less than 150	Less than 200	Less than 250	Less than 300
Number of Bulbs	7	21	52	79	91	100

Sol:

The frequency distribution is as follows:

Life-time (in days)	Frequency (f)
0-50	7
50-100	14
100-150	31
150-200	27
200-250	12
250-300	9

20. The following table, construct the frequency distribution of the percentage of marks obtained by 2300 students in a competitive examination.

Marks obtained (in percent)	11 – 20	21 – 30	31 – 40	41 – 50	51 – 60	61 – 70	71 – 80
Number of Students	141	221	439	529	495	322	153

(a) Convert the given frequency distribution into the continuous form.

(b) Find the median class and write its class mark.

(c) Find the modal class and write its cumulative frequency.

Sol:

(a) The frequency distribution into the continuous form is as follows:

Marks obtained (in per cent)	Number of students (f)
10.5-20.5	141
20.5-30.5	221
30.5-40.5	439
40.5-50.5	529

50.5-60.5	495
60.5-70.5	322
70.5-80.5	153

(b)Now, to find the median class let us put the data in the tale given below:

Marks obtained (in percent)	Number of students (f)	Cumulative frequency (cf)
10.5-20.5	141	141
20.5-30.5	221	362
30.5-40.5	439	801
40.5-50.5	529	1330
50.5-60.5	495	1825
60.5-70.5	322	2147
70.5-80.5	153	2300

Now, $N = 2300$

$\Rightarrow \frac{N}{2} = 1150$

The cumulative frequency just greater than 1150 is 1330, and the corresponding class is 40.5-50.5.

Thus, the median class is 40.5-50.5

Now, class mark $= \frac{upper\ class\ limit + lower\ class\ limit}{2}$

$\frac{40.5 + 50.5}{2} = \frac{91}{2} = 45.5$

Thus, class mark of the median class is 45.5

(c)Here the maximum class frequency is 529, and the class corresponding to this frequency is 40.5-50.5.

So, the modal class is 40.5-50.5 and its cumulative frequency is 1330.

21. If the mean of the following distribution is 27, find the value of p.

Class	$0 - 10$	$10 - 20$	$20 - 30$	$30 - 40$	$40 - 50$
Frequency	8	P	12	13	10

Sol: The given data is shown as follows:

Class	Frequency (f)	Class mark (x_i)	$f_i x_i$
$0 - 10$	8	5	40
$10 - 20$	P	15	15p
$20 - 30$	12	25	300
$30 - 40$	13	35	455
$40 - 50$	10	45	450
Total	$\sum f_i = 43 + p$		$\sum f_i x_i = 1245 + 15p$

The mean of given data is given by $\bar{x} = \dfrac{\sum_i f_i x_i}{\sum_i f_i}$

$$\Rightarrow 27 = \frac{1245 + 15p}{43 + p}$$

$$\Rightarrow 1161 + 27p = 1245 + 15p$$

$$\Rightarrow 27p - 15p = 1245 - 1161$$

$$\Rightarrow 12p = 84$$

$$\Rightarrow p = 7$$

Thus, the value of p is 7.

22. Calculate the missing frequency form the following distribution, it being given that the median of the distribution is 24.

Age (in years)	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Number of persons	5	25	?	18	7

Sol:

Let the missing frequency be x.

To find the median let us put data in the table given below:

Age (in years)	Number of persons (f)	Cumulative frequency (cf)
0-10	5	5
10-20	25	30
20-30	X	30 + x
30-40	18	48 + x
40-50	7	55 + x

The given median is 24,

∴ the median class is 20-30.

$$\therefore l = 20, h = 10, N = 55 + x, f = x \text{ and } cf = 30$$

$$Median = l + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h$$

$$\Rightarrow 24 = 20 + \left(\frac{\frac{55+x}{2} - 30}{x}\right) \times 10$$

$$\Rightarrow 24 - 20 = \left(\frac{55 + x - 60}{2x}\right) \times 10$$

$$\Rightarrow 4 = \left(\frac{x - 5}{2x}\right) \times 10$$

$$\Rightarrow 8x = 10x - 50$$

$$\Rightarrow 2x = 50$$

$$\Rightarrow x = 25$$

Thus, the missing frequency is 25.

Multiple choice questions

1. Which of the following is not a measure of central tendency?
   (a) Mean   (b) Mode
   (c) Median   (d) Standard Deviation
   Answer: (d) Standard Deviation
   Sol:
   The standard deviation is a measure of dispersion. It is the action or process of distributing thing over a wide area (nothing about central location).

2. Which of the following cannot be determined graphically?
   (a) Mean   (b) Median
   (c) Mode   (d) None of these
   Answer: (a) Mean
   Sol:
   The mean cannot be determined graphically because the values cannot be summed.

3. Which of the following measures of central tendency is influence by extreme values?
   (a) Mean   (b) Median
   (c) Mode   (d) None of these
   Answer: (a) Mean
   Sol:
   Mean is influenced by extreme values.

4. The mode of frequency distribution is obtained graphically from
   (a) a frequency curve   (b) a frequency polygon
   (c) a histogram   (d) an ogive
   Answer: (c) a histogram
   Sol:
   The mode of a frequency distribution can be obtained graphically from a histogram.

5. The medium of a frequency distribution is found graphically with the help of
   (a) a histogram   (b) a frequency curve
   (c) a frequency polygon   (d) ogives
   Answer: (d) ogives
   Sol:
   This because median of a frequency distribution is found graphically with the help of ogives.

6.   The cumulative frequency table is useful is determining the
     (a) Mean   (b) Median
     (c) Mode   (d) all of these
     Answer: (b) Median
     Sol:
     The cumulative frequency table is useful in determining the median.

7.   The abscissa of the point of intersection of the Less Than Type and of the More Than Type
     cumulative frequency curves of a grouped data gives its
     (a) Mean   (b) Median
     (c) Mode   (d)None of these
     Answer: (b) Median
     Sol:
     The abscissa of the point of intersection of the 'less than type' and that of the 'more than
     type' cumulative frequency curves of a grouped data gives its median.

8.   If x_i's are the midpoints of the class intervals of a grouped data, f_i's are the corresponding

     frequencies and $\bar{x}$ is the mean then $\sum f_i \left(x_i - \bar{x} \right) = ?$

     (a) 1   (b)0
     (c) -1   (d)2
     Answer: (b)0
     Sol:
     We know that $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$
     $\Rightarrow \bar{x} \, \Sigma \, f_i = \Sigma \, f_i x_i \quad \dots (i)$
     $Now, \Sigma \, f_i(x_i - \bar{x}) = \Sigma \, f_i x_i - \bar{x} \, \Sigma \, f_i$
     $\Rightarrow \Sigma \, f_i(x_i - \bar{x}) = \Sigma \, f_i x_i - \Sigma \, f_i x_i \quad [Using \ (i)]$
     $\Rightarrow \Sigma \, f_i \, (x_i - \bar{x}) = 0$

9.   For the finding the mean by using the formula, $\bar{x} = A + h \left(\dfrac{\sum f_i u_i}{\sum f_i} \right)$, we have $u_i = ?$

     (a) $\dfrac{(A - x_i)}{h}$ (b) $\dfrac{(x_i - A)}{h}$

     (c) $\dfrac{(A + x_i)}{h}$ (d) $h(x_i - A)$

     Answer: (b) $u_i = \dfrac{(x_i - A)}{h}$
     Sol:

$$u_i = \frac{(x_i - A)}{h}$$

10. In the formula, $\bar{x} = \left\{ A + \dfrac{\sum f_i d_i}{\sum f_i} \right\}$ for the following the mean of the grouped data, the d_i's are

the deviations from A of

(a) lower limits of the classes   (b) upper limits of the classes

(c) midpoints of the classes   (d) none of these

Answer: (c) midpoints of the classes

Sol:

The $d_i's$ are the deviations from A of midpoints of the classes.

11. While computing the mean of the groue data, we assume that the frequencies are

(a) evenly distributed over the classes

(b) centred at the class marks of the classes

(c) centred at the lower limits of the classes

(d) centred at the upper limits of the classes

Answer: (b) centred at the class marks of the classes

Sol:

While computing the mean of the group data, we assume that the frequencies are centred at the class marks of the classes.

12. The relation between mean, mode and median is

(a) mode=(3 × mean) – (2 × median)

(b) mode=(3 × median) – (2 ×mean)

(c) median=(3 × mean) – (2 × mode)

(d) mean=(3 × median) – (2 ×mode)

Answer: (b) mode=(3 × median) – (2 ×mean)

Sol:

mode=(3 × median) – (2 ×mean)

13. If the 'less than type' ogive and 'more than type' ogive intersect each other at (20.5, 15.5) then the median of the given data is

(a) 5.5   (b) 15.5

(c) 20.5   (d) 36.0

Answer: (c) 20.5

  Sol:

The x- coordinate represents the median of the given data.

Thus, median of the given data is 20.5.

14. Consider the frequency distribution of the heights of 60 students of a class

Height (in cm)	No. of students	Cumulative frequency
150 – 155	16	16
155 – 160	12	28
160 – 165	9	37
165 – 170	7	44
170 – 175	10	54
175 – 180	6	60

The sum of the lower limit of the modal class and the upper limit of the median class is

(a) 310   (b) 315   (c) 320   (d) 330

Answer: (b) 315

Sol:

The class having the maximum frequency is the modal class.

So, the modal class is 150 – 155 and its lower limit is 150.

Also, N = 60

$\Rightarrow \dfrac{N}{2} = 30$

The cumulative frequency just more than 30 is 37 and its class is 160 – 165, whose upper limit is 165.

∴ Required sum = (150 + 165) = 315

15. Consider the following frequency distribution

Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Frequency	3	9	15	30	18	5

The modal class is

(a) 10 – 20   (b) 20 – 30   (c) 30 – 40   (d) 50 – 60

Answer: (c) 30 – 40

Sol:

The class 30 – 40 has the maximum frequency, i.e., 30.

So, the modal class is 30 – 40.

16. Mode = ?

(a) $x_k + h \left\{ \dfrac{f_{k-1} - f_k}{2f_k - f_{k-1} - f_{k+1}} \right\}$

(b) $x_k + h \left\{ \dfrac{f_k - f_{k-1}}{2f_k - f_{k-1} - f_{k+1}} \right\}$

(c) $x_k + h \left\{ \dfrac{f_k - f_{k-1}}{f_k - 2f_{k-1} - f_{k+1}} \right\}$

(d) $x_k + h \left\{ \dfrac{f_k - f_{k-1}}{f_k - f_{k-1} - 2f_{k+1}} \right\}$

Answer: (b) $x_k + h \left\{ \dfrac{f_k - f_{k-1}}{2f_k - f_{k-1} - f_{k+1}} \right\}$

Sol:

$x_k + h \left\{ \dfrac{f_k - f_{k-1}}{2f_k - f_{k-1} - f_{k+1}} \right\}$

17. Median =?

(a) $l + \left\{ h \times \frac{\left(\frac{N}{2} - cf\right)}{f} \right\}$

(b) $l + \left\{ h \times \frac{\left(cf - \frac{N}{2}\right)}{f} \right\}$

(c) $l - \left\{ h \times \frac{\left(\frac{N}{2} - cf\right)}{f} \right\}$

(d) None of these

Answer: (a) $l + \left\{ h \times \frac{\left(\frac{N}{2} - cf\right)}{f} \right\}$

Sol:

$l \times \left\{ h \times \frac{\left(\frac{N}{2} - cf\right)}{f} \right\}$

18. If the mean and median of a set of numbers are 8.9 and 9 respectively, then the mode will be

(a) 7.2   (b) 8.2   (c) 9.2   (d) 10.2

Answer: (c) 9.2

Sol:

It is given that the mean and median are 8.9 and 9, respectively,

$\therefore$ Mode = (3 × Median) − (2 × Mean)

$\Rightarrow$ Mode = (3 × 9) − (2 × 8.9)

= 27 − 17.8

= 9.2

19. Look at the frequency distribution table given below:

Class interval	35 – 45	45 – 55	55 – 65	65 – 75
Frequency	8	12	20	10

The median of the above distribution is

(a) 56.5   (b) 57.5   (c) 58.5   (d) 59

Answer: (b) 57.5

Sol:

Class interval	35 – 45	45 – 55	55 – 65	65 – 75
Frequency	8	12	20	10
Cumulative frequency	8	20	40	50

Here, N = 50

$\Rightarrow \frac{N}{2} = 25$, which lies in the class interval of 55 – 65.

Now, cf = 55, f = 20 and l = 50

$\therefore$ Median = $l + \left\{ h \times \frac{\left(\frac{N}{2} - cf\right)}{f} \right\}$

$= 50 + \frac{65 - 55}{20} \times (25 - 20)$

$= 57.5$

20. Consider the following table:

Class interval	10 – 14	14 – 18	18 – 22	22 – 26	26 – 30
Frequency	5	11	16	25	19

The mode of the above data is

(a) 23. 5   (b) 24   (c) 24.4   (d) 25

Answer: (c) 24.4

Sol:

The maximum frequency is 25 and the modal class is 22 - 26.

Now, $x_k = 22$, $f_k = 25$, $f_{k-1} = 16$, $f_{k+1} = 19$ and $h = 4$

$\therefore$ Mode $= x_k + h \left\{ \dfrac{f_k - f_{k-1}}{2f_k - f_{k-1} - f_{k+1}} \right\}$

$= 22 + 4 \times \dfrac{(25-16)}{(2 \times 25 - 16 - 19)}$

$= 22 + 4 \times \dfrac{(25-16)}{(50 - 16 - 19)}$

$= 22 + 4 \times \dfrac{9}{15}$

$= 22 + \dfrac{12}{5}$

$= 22 + 2.4$

$= 24.4$

21. The mean and mode of a frequency distribution are 28 and 16 respectively. The median is

(a) 22   (b) 23.5   (c) 24   (d) 24.5

Answer: (c) 24

Sol:

Mode $= (3 \times$ median$) - (2 \times$ mean$)$

$\Rightarrow (3 \times$ median$) = ($mode $+ 2$ mean$)$

$\Rightarrow (3 \times$ median$) = 16 + 56$

$\Rightarrow (3 \times$ median$) = 72$

$\Rightarrow$ Median $= \dfrac{72}{3}$

$\therefore$ Median $= 24$

22. The median and mode of a frequency distribution are 26 and 29 respectively. Then, the mean is

(a) 27.5   (b) 24.5   (c) 28.4   (d) 25.8

Answer: (b) 24.5

Sol:

Mode $= (3 \times$ median$) - (2 \times$ mean$)$

$\Rightarrow (2 \times$ mean$) = (3 \times$ median$) -$ mode

$\Rightarrow (2 \times$ mean$) = 3 \times 26 - 29$

$\Rightarrow (2 \times$ mean$) = 49$

$\Rightarrow$ Mean $= \frac{49}{2}$

$\therefore$ Mean $= 24.5$

23. For a symmetrical frequency distribution, we have:

(a) mean < mode < median   (b) mean > mode > median

(c) mean = mode = median   (d) mode $= \frac{1}{2}$ (mean + median)

Answer: (c) mean = mode = median

Sol:

A symmetric distribution is one where the left and right hand sides of the distribution are roughly equally balanced around the mean.

24. Look at the cumulative frequency distribution table given below:

Monthly income	No. of families
More than ₹ 10000	100
More than ₹ 14000	85
More than ₹ 18000	69
More than ₹ 20000	50
More than ₹ 25000	37
More than ₹ 30000	15

Number of families having income range 20000 to 25000 is

(a) 19   (b) 16   (c) 13   (d) 22

Answer: (c) 13

Sol:

Converting the given data into a frequency table, we get:

Monthly income	No. of families	Frequency
30,000 and above	15	15
25,000 – 30,000	37	(37 – 15) = 22
20,000 – 25,000	50	(50 – 37) = 13
18,000 – 20,000	69	(69 – 50) = 19
14,000 – 18,000	85	(85 – 69) = 16
10,000 – 14,000	100	(100 – 85) = 15

Hence, the number of families having an income range of Rs. 20,000 – Rs. 25,000 is 13.

The correct option is (c).

25. The median of the first 8 prime numbers is

(a) 7 (b) 9 (c) 11 (d) 13

Answer: (b) 9

Sol:

First 8 prime numbers are 2, 3, 5, 7, 11, 13, 17 and 19.

Median of 8 numbers is average of 4th and 5th terms.

i.e., average of 7 and 11

Thus, the median is 9.

26.   The mean of 20 numbers is 0. OF them, at the most, how many may be greater than zero?

(a) 0 (b) 1 (c) 10 (d) 19

Answer: (d) 19

Sol:

It is given that mean of 20 numbers is zero.

i.e., average of 20 numbers is zero.

i.e., sum of 20 numbers is zero.

Thus, at most, there can be 19 positive numbers.

(such that if sum of 19 positive numbers is x, 20th number will be $-x$

27. If the median of the data 4, 7, x-1, x-3, 16, 25, written in ascending order, is 13 then x is equal to

(a) 13 (b) 14 (c) 15 (d) 16

Answer: (c) 15

Sol:

Median of 6 numbers is the average of 3rd and 4th term.

$\therefore 13 = \frac{(x-1)+(x-3)}{2}$

$\Rightarrow 26 = 2x - 4$

$\Rightarrow 2x = 30$

$\Rightarrow x = 15$

Thus, x is equal to 15.

28.   The mean of 2, 7, 6 and x is 15 and mean of 18, 1, 6, x and y is 10. What is the value of y?

(a) 5 (b) 10 (c) -20 (d) 30

Answer: (c) -20

Sol:

$\text{Mean} = \frac{sum\ of\ observations}{number\ of\ observations}$

$\Rightarrow 15 = \frac{2+7+6+x}{4}$

$\Rightarrow 60 = 15 + x$

$\Rightarrow x = 45 \qquad \dots\dots(1)$

Now,

$\text{Mean} = \frac{sum\ of\ observations}{number\ of\ observations}$

$\Rightarrow 10 = \frac{18+1+6+x+y}{5}$

$\Rightarrow 50 = 25 + x + y$

$\Rightarrow y = 25 - x$

$\Rightarrow y = 25 - 45$ [From (1)]

$\Rightarrow y = -20$

29.   Match the following:

Column I	Column II
(a) The most frequent value in a data is known as ………	(p) standard deviation
(b) which of the following cannot be determined graphically out of mean, mode and median?	(q) median
(c) An ogive is used to determine ……	(r) mean
(d) out of mean, mode, median and standard deviation, which is not a measure of tendency?	(s) mode

Sol:

Column I	Column II
(a) The most frequent value in a data is known as ………	(s) mode
(b) which of the following cannot be determined graphically out of mean, mode and median?	(r) mean
(c) An ogive is used to determine ……	(q) median
(d) out of mean, mode, median and standard deviation, which is not a measure of tendency?	(p) standard deviation

30.   Question consists of two statements, namely, Assertion (A) and Reason (R). For selecting the correct answer, use the following code:

(a) Both Assertion (A) and reason (R) are true and Reason (R) is a correct explanation of Assertion (A).

(b) Both Assertion (A) and reason (R) are true and Reason (R) is not a correct explanation of Assertion (A).

(c) Assertion (A) is true and Reason (R) is false.

(d) Assertion (A) is false and Reason (R) is true.

Assertion (A)	Reason (R)
If the median and mode of a frequency distribution are 150 and 154 respectively, then its mean is 148.	Mean, median and mode of a frequency distribution are related as: mode = (3 median) – (2 mean)

The correct answer is: (a) / (b) / (c) / (d)

Sol:

(a) Both Assertion (A) and reason (R) are true and Reason (R) is a correct explanation of Assertion (A).

Clearly, reason (R) is true.

Using the relation in reason (R), we have:

2 mean = (3 × median) – mode = (3 × 150) – 154 = 450 – 154 = 296

⇒ Mean = 148, which is true.

∴ This assertion (A) and reason (R) are both true and reason (R) is the correct explanation of assertion (A).

31. Question consists of two statements, namely, Assertion (A) and Reason (R). For selecting the correct answer, use the following code:

(a) Both Assertion (A) and reason (R) are true and Reason (R) is a correct explanation of Assertion (A).

(b) Both Assertion (A) and reason (R) are true and Reason (R) is not a correct explanation of Assertion (A).

(c) Assertion (A) is true and Reason (R) is false.

(c) Assertion (A) is false and Reason (R) is true.

Assertion (A)							Reason (R)
Consider the following frequency distribution:							The value of the variable which occurs most often is the mode.
Class interval	3 – 6	6 – 9	9 – 12	12 – 15	15 – 18	18 – 21	
Frequency	2	5	21	23	10	12	
The mode of the above data is 12.4.							

The correct answer is: (a) / (b) / (c) / (d)

Sol:

(b) Both assertion (A) and reason (R) are true, but reason (R) is not a correct explanation of assertion (A).

Clearly, reason (R) is true.

The maximum frequency is 23 and the modal class is 12 – 15.

Now, $x_k = 12$, $f_k = 23$, $f_{k-1} = 21$, $f_{k+1} = 23$ and h = 3

∴ Mode = $\left\{ 12 + 3 \times \dfrac{(23-21)}{(2 \times 23 - 21 - 10)} \right\}$

$= \left(12 + 3 \times \dfrac{2}{15} \right)$

= (12 + 0.4)

= 12

∴ Assertion (A) is true.

However, reason (R) is not a correct explanation of assertion (A).