# Angles, Linesand Triangles

## Exercise 4A

**Question 1:**

(i) Angle: Two rays having a common end point form an angle.

(ii) Interior of an angle: The interior of ∠AOB is the set of all points in its plane, which lie on the same side of OA as B and also on same side of OB as A.

(iii) Obtuse angle: An angle whose measure is more than 90° but less than 180°, is called an obtuse angle.

(iv) Reflex angle: An angle whose measure is more than 180° but less than 360° is called a reflex angle.

(v) Complementary angles: Two angles are said to be complementary, if the sum of their measures is 90o.

(vi) Supplementary angles: Two angles are said to be supplementary, if the sum of their measures is 180°.

**Question 2:**

∠A = 36° 27′ 46″ and ∠B = 28° 43′ 39″

∴ Their sum = (36° 27′ 46″) + (28° 43′ 39″)

| | Deg | Min | Sec | |
|---|---|---|---|---|
| | 36° | 27′ | 46″ | |
| + | 28° | 43′ | 39″ | [1° = 60′; 1′ = 60″] |
| | 65° | 11′ | 25″ | |

Therefore, the sum ∠A + ∠B = 65° 11′ 25″

**Question 3:**

Let ∠A = 36° and ∠B = 24° 28′ 30″

Their difference = 36° – 24° 28′ 30″

| | Deg | Min | Sec | |
|---|---|---|---|---|
| | 36° | 0′ | 0″ | |
| − | 24° | 28′ | 30″ | [1°=60′; 1′=60″] |
| | 11° | 31′ | 30″ | |

Thus the difference between two angles is ∠A – ∠B = 11° 31′ 30″

### Question 4:

(i) Complement of 58° = 90° – 58° = 32°

(ii) Complement of 16° = 90 – 16° = 74°

(iii) $\frac{2}{3}$ of a right angle = $\frac{2}{3}$ × 90° = 60°

Complement of 60° = 90° – 60° = 30°

(iv) 1° = 60′

⇒ 90° = 89° 60′

| | Deg | Min |
|---|---|---|
| | 89° | 60′ |
| | ~~90°~~ | ~~0′~~ |
| − | 46° | 30′ |
| | 43° | 30′ |

Complement of 46° 30′ = 90° – 46° 30′ = 43° 30′

(v) 90° = 89° 59′ 60″

| | Deg | Min | Sec |
|---|---|---|---|
| | 89° | 59′ | 60″ |
| | ~~90°~~ | ~~0′~~ | ~~0″~~ |
| − | 52° | 43′ | 20″ |
| | 37° | 16′ | 40″ |

Complement of 52° 43′ 20″ = 90° – 52° 43′ 20″

= 37° 16′ 40″

(vi) 90° = 89° 59′ 60″

| | Deg | Min | Sec |
|---|---|---|---|
| | 89° | 59′ | 60″ |
| | ~~90°~~ | ~~0′~~ | ~~0″~~ |
| − | 68° | 35′ | 45″ |
| | 21° | 24′ | 15″ |

∴ Complement of (68° 35′ 45″)

= 90° – (68° 35′ 45″)

= 89° 59′ 60″ – (68° 35′ 45″)

= 21° 24′ 15″

### Question 5:

(i) Supplement of 63° = 180° – 63° = 117°

(ii) Supplement of 138° = 180° – 138° = 42°

(iii) $\frac{3}{5}$ of a right angle = $\frac{3}{5}$ × 90° = 54°

∴ Supplement of 54° = 180° – 54° = 126°

(iv) 1° = 60′

⇒ 180° = 179° 60′

$$\begin{array}{lcc} & \text{Deg} & \text{Min} \\ & 179^\circ & 60' \\ & \cancel{180^\circ} & \cancel{0'} \\ - & 75^\circ & 36' \\ \hline & 104^\circ & 24' \end{array}$$

Supplement of 75° 36′ = 180° – 75° 36′ = 104° 24′

(v) 1° = 60′, 1′ = 60″

⇒ 180° = 179° 59′ 60″

$$\begin{array}{lccc} & \text{Deg} & \text{Min} & \text{Sec} \\ & 179^\circ & 59' & 60'' \\ & \cancel{180^\circ} & \cancel{0'} & \cancel{0''} \\ - & 124^\circ & 20' & 40'' \\ \hline & 55^\circ & 39' & 20'' \end{array}$$

Supplement of 124° 20′ 40″ = 180° – 124° 20′ 40″

= 55° 39′ 20″

(vi) 1° = 60′, 1′ = 60″

⇒ 180° = 179° 59′ 60″

$$\begin{array}{lccc} & \text{Deg} & \text{Min} & \text{Sec} \\ & 179^\circ & 59' & 60'' \\ & \cancel{180^\circ} & \cancel{0'} & \cancel{0''} \\ - & 108^\circ & 48' & 32'' \\ \hline & 71^\circ & 11' & 28'' \end{array}$$

∴ Supplement of 108° 48′ 32″ = 180° – 108° 48′ 32″

= 71° 11′ 28″.

### Question 6:

(i) Let the required angle be x°

Then, its complement = 90° – x°

$$\begin{aligned} \therefore \quad x^\circ &= 90^\circ - x^\circ \\ \Rightarrow \quad x + x &= 90 \\ \Rightarrow \quad 2x &= 90 \\ \Rightarrow \quad x &= \frac{90}{2} = 45 \end{aligned}$$

∴ The measure of an angle which is equal to its complement is 45°.

(ii) Let the required angle be x°

Then, its supplement = 180° – x°

$$\begin{aligned} \therefore \quad x^\circ &= 180^\circ - x^\circ \\ \Rightarrow \quad x + x &= 180 \\ \Rightarrow \quad 2x &= 180 \\ \Rightarrow \quad x &= \frac{180}{2} = 90 \end{aligned}$$

∴ The measure of an angle which is equal to its supplement is 90°.

### Question 7:

Let the required angle be x°

Then its complement is 90° – x°

$$\begin{aligned} \Rightarrow \quad x^\circ &= \left(90^\circ - x^\circ\right) + 36^\circ \\ \Rightarrow \quad x^\circ + x^\circ &= 90^\circ + 36^\circ \\ \Rightarrow \quad 2x^\circ &= 126^\circ \\ \Rightarrow \quad x &= \frac{126}{2} = 63 \end{aligned}$$

∴ The measure of an angle which is 36° more than its complement is 63°.

### Question 8:

Let the required angle be x°

Then its supplement is 180° – x°

$$\begin{aligned} \Rightarrow \quad x^\circ &= \left(180^\circ - x^\circ\right) - 25^\circ \\ \Rightarrow \quad x^\circ + x^\circ &= 180^\circ - 25^\circ \\ \Rightarrow \quad 2x &= 155 \\ \Rightarrow \quad x &= \frac{155}{2} = 77\frac{1}{2} \end{aligned}$$

$\therefore$ The measure of an angle which is $25^{o}$ less than its supplement is $77\frac{1}{2}^{o} = 77.5^{o}$.

**Question 9:**

Let the required angle be $x^{o}$

Then, its complement = $90^{o} - x^{o}$

$$\Rightarrow \quad x^{o} = 4\left(90^{o} - x^{o}\right)$$
$$\Rightarrow \quad x^{o} = 360^{o} - 4x^{o}$$
$$\Rightarrow \quad 5x = 360$$
$$\Rightarrow \quad x = \frac{360}{5} = 72$$

$\therefore$ The required angle is $72^{o}$.

**Question 10:**

Let the required angle be $x^{o}$

Then, its supplement is $180^{o} - x^{o}$

$$\Rightarrow \quad x^{o} = 5\left(180^{o} - x^{o}\right)$$
$$\Rightarrow \quad x^{o} = 900^{o} - 5x^{o}$$
$$\Rightarrow \quad x + 5x = 900$$
$$\Rightarrow \quad 6x = 900$$
$$\Rightarrow \quad x = \frac{900}{6} = 150.$$

$\therefore$ The required angle is $150^{o}$.

**Question 11:**

Let the required angle be $x^{o}$

Then, its complement is $90^{o} - x^{o}$ and its supplement is $180^{o} - x^{o}$

That is we have,

$$180^\circ - x^\circ = 4(90^\circ - x^\circ)$$
$$180^\circ - x^\circ = 360^\circ - 4x^\circ$$
$$4x^\circ - x^\circ = 360^\circ - 180^\circ$$
$$3x = 180$$
$$x = \frac{180}{3} = 60^\circ$$

$\therefore$ The required angle is $60^{o}$.

**Question 12:**

Let the required angle be $x^{o}$

Then, its complement is $90^{o} - x^{o}$ and its supplement is $180^{o} - x^{o}$

$$\therefore \quad 90^{o} - x^{o} = \frac{1}{3}\left(180^{o} - x^{o}\right)$$
$$\Rightarrow \quad 90 - x = 60 - \frac{1}{3}x$$
$$\Rightarrow \quad x - \frac{1}{3}x = 90 - 60$$
$$\Rightarrow \quad \frac{2}{3}x = 30$$
$$\Rightarrow \quad x = \frac{30 \times 3}{2} = 45$$

$\therefore$ The required angle is $45^{o}$.

**Question 13:**

Let the two required angles be $x^{o}$ and $180^{o} - x^{o}$.

Then,

$$\frac{x^{o}}{180^{o} - x^{o}} = \frac{3}{2}$$

$\Rightarrow 2x = 3(180 - x)$

$\Rightarrow 2x = 540 - 3x$

$\Rightarrow 3x + 2x = 540$

$\Rightarrow 5x = 540$

$\Rightarrow x = 108$

Thus, the required angles are $108^\circ$ and $180^\circ - x^\circ = 180^\circ - 108^\circ = 72^\circ$.

**Question 14:**

Let the two required angles be $x^\circ$ and $90^\circ - x^\circ$.

Then

$$\frac{x^\circ}{90^\circ - x^\circ} = \frac{4}{5}$$

$\Rightarrow 5x = 4(90 - x)$

$\Rightarrow 5x = 360 - 4x$

$\Rightarrow 5x + 4x = 360$

$\Rightarrow 9x = 360$

$\Rightarrow x = \frac{360}{9} = 40$

Thus, the required angles are $40^\circ$ and $90^\circ - x^\circ = 90^\circ - 40^\circ = 50^\circ$.

**Question 15:**

Let the required angle be $x^\circ$.

Then, its complementary and supplementary angles are $(90^\circ - x)$ and $(180^\circ - x)$ respectively.

Then, $7(90^\circ - x) = 3(180^\circ - x) - 10^\circ$

$\Rightarrow 630^\circ - 7x = 540^\circ - 3x - 10^\circ$

$\Rightarrow 7x - 3x = 630^\circ - 530^\circ$

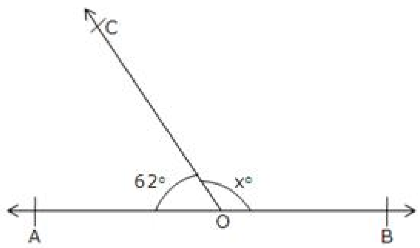
$\Rightarrow 4x = 100^\circ$

$\Rightarrow x = 25^\circ$

Thus, the required angle is $25^\circ$.

## Exercise 4B

**Question 1:**




Since $\angle BOC$ and $\angle COA$ form a linear pair of angles, we have

$\angle BOC + \angle COA = 180^\circ$

$\Rightarrow x^\circ + 62^\circ = 180^\circ$

$\Rightarrow x = 180 - 62$

$\therefore x = 118^\circ$

**Question 2:**

Since, $\angle BOD$ and $\angle DOA$ form a linear pair.

$\angle BOD + \angle DOA = 180^\circ$

$\therefore \angle BOD + \angle DOC + \angle COA = 180^\circ$

$\Rightarrow (x + 20)^\circ + 55^\circ + (3x - 5)^\circ = 180^\circ$

$\Rightarrow x + 20 + 55 + 3x - 5 = 180$

$\Rightarrow 4x + 70 = 180$

$\Rightarrow 4x = 180 - 70 = 110$

$\Rightarrow x = \frac{110}{4} = 27.5$

$\therefore \angle AOC = (3 \times 27.5 - 5)^\circ = 82.5\text{-}5 = 77.5^\circ$

And, $\angle BOD = (x + 20)^\circ = 27.5^\circ + 20^\circ = 47.5^\circ$.

**Question 3:**

Since ∠BOD and ∠DOA from a linear pair of angles.

⇒ ∠BOD + ∠DOA = $180^o$

⇒ ∠BOD + ∠DOC + ∠COA = $180^o$

⇒ $x^o + (2x - 19)^o + (3x + 7)^o = 180^o$

⇒ $6x - 12 = 180$

⇒ $6x = 180 + 12 = 192$

⇒ $x = \frac{192}{6} = 32$

⇒ $x = 32$

⇒ ∠AOC = $(3x + 7)^o = (3\ 32 + 7)^o = 103^o$

⇒ ∠COD = $(2x - 19)^o = (2\ 32 - 19)^o = 45^o$

and ∠BOD = $x^o = 32^o$

**Question 4:**

x: y: z = 5: 4: 6

The sum of their ratios = 5 + 4 + 6 = 15

But $x + y + z = 180^o$

[Since, XOY is a straight line]

So, if the total sum of the measures is 15, then the measure of x is 5.

If the sum of angles is $180^o$, then, measure of $x = \frac{5}{15} \times 180 = 60$

And, if the total sum of the measures is 15, then the measure of y is 4.

If the sum of the angles is $180^o$, then, measure of $y = \frac{4}{15} \times 180 = 48$

And ∠z = $180^o$ – ∠x – ∠y

= $180^o - 60^o - 48^o$

= $180^o - 108^o = 72^o$

∴ x = 60, y = 48 and z = 72.

**Question 5:**

AOB will be a straight line, if two adjacent angles form a linear pair.

∴ ∠BOC + ∠AOC = $180^o$

⇒ $(4x - 36)^o + (3x + 20)^o = 180^o$

⇒ $4x - 36 + 3x + 20 = 180$

⇒ $7x - 16 = 180^o$

⇒ $7x = 180 + 16 = 196$

⇒ $x = \frac{196}{7} = 28$

∴ The value of x = 28.

**Question 6:**

Since ∠AOC and ∠AOD form a linear pair.

∴ ∠AOC + ∠AOD = $180^o$

⇒ $50^o$ + ∠AOD = $180^o$

⇒ ∠AOD = $180^o - 50^o = 130^o$

∠AOD and ∠BOC are vertically opposite angles.

∠AOD = ∠BOC

⇒ ∠BOC = $130^o$

∠BOD and ∠AOC are vertically opposite angles.

∴ ∠BOD = ∠AOC

⇒ ∠BOD = $50^o$

**Question 7:**

Since ∠COE and ∠DOF are vertically opposite angles, we have,

∠COE = ∠DOF

⇒ ∠z = 50°
Also ∠BOD and ∠COA are vertically opposite angles.
So, ∠BOD = ∠COA
⇒ ∠t = 90°
As ∠COA and ∠AOD form a linear pair,
∠COA + ∠AOD = 180°
⇒ ∠COA + ∠AOF + ∠FOD = 180° [∠t = 90°]
⇒ t + x + 50° = 180°
⇒ 90° + x° + 50° = 180°
⇒ x + 140 = 180
⇒ x = 180 – 140 = 40
Since ∠EOB and ∠AOF are vertically opposite angles
So, ∠EOB = ∠AOF
⇒ y = x = 40
Thus, x = 40 = y = 40, z = 50 and t = 90

**Question 8:**

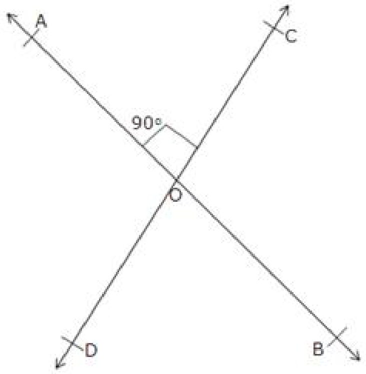
Since ∠COE and ∠EOD form a linear pair of angles.
⇒ ∠COE + ∠EOD = 180°
⇒ ∠COE + ∠EOA + ∠AOD = 180°
⇒ 5x + ∠EOA + 2x = 180
⇒ 5x + ∠BOF + 2x = 180
[∴ ∠EOA and BOF are vertically opposite angles so, ∠EOA = ∠BOF]
⇒ 5x + 3x + 2x = 180
⇒ 10x = 180
⇒ x = 18
Now ∠AOD = 2x° = 2 × 18° = 36°
∠COE = 5x° = 5 × 18° = 90°
and, ∠EOA = ∠BOF = 3x° = 3 × 18° = 54°

**Question 9:**

Let the two adjacent angles be 5x and 4x.
Now, since these angles form a linear pair.
So, 5x + 4x = 180°
⇒ 9x = 180°
⇒ x = $\frac{180}{9}$ = 20
∴ The required angles are 5x = 5x = 5 20° = 100°
and 4x = 4 × 20° = 80°

**Question 10:**

Let two straight lines AB and CD intersect at O and let ∠AOC = 90°.




Now, ∠AOC = ∠BOD [Vertically opposite angles]

$\Rightarrow \angle BOD = 90^o$

Also, as $\angle AOC$ and $\angle AOD$ form a linear pair.

$\Rightarrow 90^o + \angle AOD = 180^o$

$\Rightarrow \angle AOD = 180^o - 90^o = 90^o$

Since, $\angle BOC = \angle AOD$ [Verticallty opposite angles]

$\Rightarrow \angle BOC = 90^o$

Thus, each of the remaining angles is $90^o$.

**Question 11:**

Since, $\angle AOD$ and $\angle BOC$ are vertically opposite angles.

$\therefore \angle AOD = \angle BOC$

Now, $\angle AOD + \angle BOC = 280^o$ [Given]

$\Rightarrow \angle AOD + \angle AOD = 280^o$

$\Rightarrow 2\angle AOD = 280^o$

$\Rightarrow \angle AOD = \frac{280}{2} = 140^o$

$\Rightarrow \angle BOC = \angle AOD = 140^o$

As, $\angle AOC$ and $\angle AOD$ form a linear pair.

So, $\angle AOC + \angle AOD = 180^o$

$\Rightarrow \angle AOC + 140^o = 180^o$

$\Rightarrow \angle AOC = 180^o - 140^o = 40^o$

Since, $\angle AOC$ and $\angle BOD$ are vertically opposite angles.

$\therefore \angle AOC = \angle BOD$

$\Rightarrow \angle BOD = 40^o$

$\therefore \angle BOC = 140^o$, $\angle AOC = 40^o$ , $\angle AOD = 140^o$ and $\angle BOD = 40^o$.

**Question 12:**

Since $\angle COB$ and $\angle BOD$ form a linear pair

So, $\angle COB + \angle BOD = 180^o$

$\Rightarrow \angle BOD = 180^o - \angle COB$ .... (1)

Also, as $\angle COA$ and $\angle AOD$ form a linear pair.

So, $\angle COA + \angle AOD = 180^o$

$\Rightarrow \angle AOD = 180^o - \angle COA$

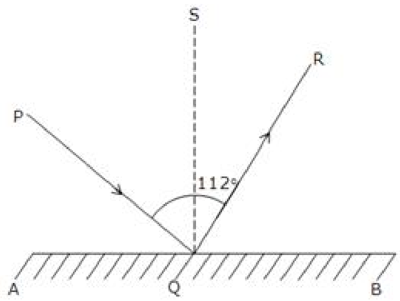
$\Rightarrow \angle AOD = 180^o - \angle COB$ .... (2)

[Since, OC is the bisector of $\angle AOB$, $\angle BOC = \angle AOC$]

From (1) and (2), we get,

$\angle AOD = \angle BOD$ (Proved)

**Question 13:**




Let QS be a perpendicular to AB.

Now, $\angle PQS = \angle SQR$

Because angle of incident = angle of reflection

$\Rightarrow \angle PQS = \angle SQR = \frac{112}{2} = 56^o$

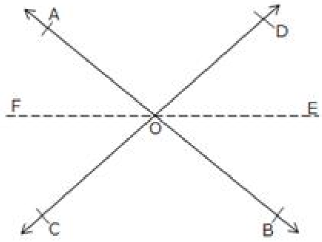
Since QS is perpendicular to AB, $\angle PQA$ and $\angle PQS$ are complementary angles.

Thus, $\angle PQA + \angle PQS = 90^o$

⇒ ∠PQA + 56° = 90°
⇒ ∠PQA = 90° – 56° = 34°

### Question 14:

Given : AB and CD are two lines which are intersecting at O. OE is a ray bisecting the ∠BOD. OF is a ray opposite to ray OE.



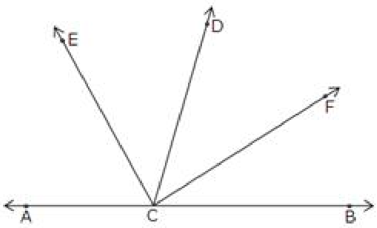

To Prove: ∠AOF = ∠COF

Proof : Since $\overrightarrow{OE}$ and $\overrightarrow{OF}$ are two opposite rays, $\overleftrightarrow{EF}$ is a straight line passing through O.

∴ ∠AOF = ∠BOE
and ∠COF = ∠DOE
[Vertically opposite angles]
But ∠BOE = ∠DOE (Given)
∴ ∠AOF = ∠COF
Hence, proved.

### Question 15:

Given: $\overrightarrow{CF}$ is the bisector of ∠BCD and $\overrightarrow{CE}$ is the bisector of ∠ACD.
To Prove: ∠ECF = 90°
Proof: Since ∠ACD and ∠BCD forms a linear pair.
∠ACD + ∠BCD = 180°




∠ACE + ∠ECD + ∠DCF + ∠FCB = 180°
∠ECD + ∠ECD + ∠DCF + ∠DCF = 180°
because ∠ACE = ∠ECD
and ∠DCF = ∠FCB
2(∠ECD) + 2 (∠CDF) = 180°
2(∠ECD + ∠DCF) = 180°
∠ECD + ∠DCF = $\frac{180}{2}$ = 90°
∠ECF = 90° (Proved)

## Exercise 4C

### Question 1:

Since AB and CD are given to be parallel lines and t is a transversal.
So, ∠5 = ∠1 = 70° [Corresponding angles are equal]
∠3 = ∠1 = 70° [Vertically opp. Angles]
∠3 + ∠6 = 180° [Co-interior angles on same side]
∴ ∠6 = 180° – ∠3
= 180° – 70° = 110°

$\angle 6 = \angle 8$ [Vertically opp. Angles]

$\Rightarrow \angle 8 = 110^o$

$\Rightarrow \angle 4 + \angle 5 = 180^o$ [Co-interior angles on same side]

$\angle 4 = 180^o - 70^o = 110^o$

$\angle 2 = \angle 4 = 110^o$ [ Vertically opposite angles]

$\angle 5 = \angle 7$ [Vertically opposite angles]

So, $\angle 7 = 70^o$

$\therefore \angle 2 = 110^o, \angle 3 = 70^o, \angle 4 = 110^o, \angle 5 = 70^o, \angle 6 = 110^o, \angle 7 = 70^o$ and $\angle 8 = 110^o$.

**Question 2:**

Since $\angle 2 : \angle 1 = 5 : 4$.

Let $\angle 2$ and $\angle 1$ be 5x and 4x respectively.

Now, $\angle 2 + \angle 1 = 180^o$, because $\angle 2$ and $\angle 1$ form a linear pair.

So, $5x + 4x = 180^o$

$\Rightarrow 9x = 180^o$

$\Rightarrow x = 20^o$

$\therefore \angle 1 = 4x = 4 \times 20^o = 80^o$

And $\angle 2 = 5x = 5 \times 20^o = 100^o$

$\angle 3 = \angle 1 = 80^o$[Vertically opposite angles]

And $\angle 4 = \angle 2 = 100^o$ [Vertically opposite angles]

$\angle 1 = \angle 5$ and $\angle 2 = \angle 6$ [Corresponding angles]

So, $\angle 5 = 80^o$ and $\angle 6 = 100^o$

$\angle 8 = \angle 6 = 100^o$[Vertically opposite angles]

And $\angle 7 = \angle 5 = 80^o$ [Vertically opposite angles]

Thus, $\angle 1 = 80^o, \angle 2 = 100^o, \angle 3 = \angle 80^o, \angle 4 = 100^o, \angle 5 = 80^o, \angle 6 = 100^o, \angle 7 = 80^o$ and $\angle 8 = 100^o$.

**Question 3:**

Given: AB || CD and AD || BC

To Prove: $\angle ADC = \angle ABC$

Proof: Since AB || CD and AD is a transversal. So sum of consecutive interior angles is $180^o$.

$\Rightarrow \angle BAD + \angle ADC = 180^o$ ....(i)

Also, AD || BC and AB is transversal.

So, $\angle BAD + \angle ABC = 180^o$ ....(ii)

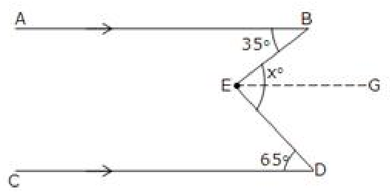
From (i) and (ii) we get:

$\angle BAD + \angle ADC = \angle BAD + \angle ABC$

$\Rightarrow \angle ADC = \angle ABC$ (Proved)

**Question 4:**

(i) Through E draw EG || CD. Now since EG||CD and ED is a transversal.




So, $\angle GED = \angle EDC = 65^o$ [Alternate interior angles]

Since EG || CD and AB || CD,

EG||AB and EB is transversal.

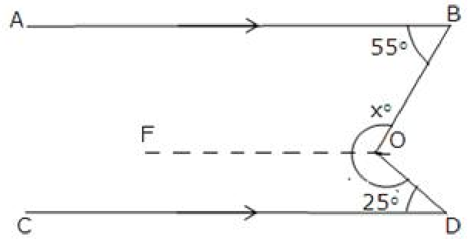
So, $\angle BEG = \angle ABE = 35^o$ [Alternate interior angles]

So, $\angle DEB = x^o$

⇒ ∠BEG + ∠GED = 35° + 65° = 100°.

Hence, x = 100.

(ii) Through O draw OF||CD.




Now since OF || CD and OD is transversal.

∠CDO + ∠FOD = 180°

[sum of consecutive interior angles is 180°]

⇒ 25° + ∠FOD = 180°

⇒ ∠FOD = 180° – 25° = 155°

As OF || CD and AB || CD [Given]

Thus, OF || AB and OB is a transversal.

So, ∠ABO + ∠FOB = 180° [sum of consecutive interior angles is 180°]

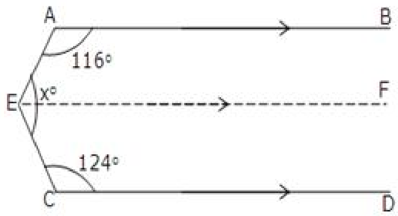
⇒ 55° + ∠FOB = 180°

⇒ ∠FOB = 180° – 55° = 125°

Now, x° = ∠FOB + ∠FOD = 125° + 155° = 280°.

Hence, x = 280.

(iii) Through E, draw EF || CD.




Now since EF || CD and EC is transversal.

∠FEC + ∠ECD = 180°

[sum of consecutive interior angles is 180°]

⇒ ∠FEC + 124° = 180°

⇒ ∠FEC = 180° – 124° = 56°

Since EF || CD and AB ||CD

So, EF || AB and AE is a trasveral.

So, ∠BAE + ∠FEA = 180°

[sum of consecutive interior angles is 180°]

∴ 116° + ∠FEA = 180°

⇒ ∠FEA = 180° – 116° = 64°

Thus, x° = ∠FEA + ∠FEC

= 64° + 56° = 120°.

Hence, x = 120.

**Question 5:**

Since AB || CD and BC is a transversal.

So, ∠ABC = ∠BCD [alternate interior angles]

⇒ 70° = x° + ∠ECD ....(i)

Now, CD || EF and CE is transversal.

So, ∠ECD + ∠CEF = 180° [sum of consecutive interior angles is 180°]

∴ ∠ECD + 130° = 180°

⇒ ∠ECD = 180° – 130° = 50°

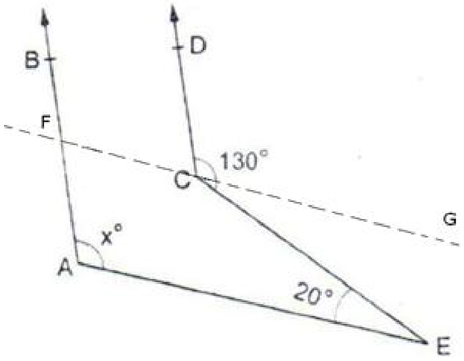
Putting $\angle ECD = 50^\circ$ in (i) we get,

$70^\circ = x^\circ + 50^\circ$

$\Rightarrow x = 70 - 50 = 20$

**Question 6:**

Through C draw FG || AE




Now, since CG || BE and CE is a transversal.

So, $\angle GCE = \angle CEA = 20^\circ$ [Alternate angles]

$\therefore \angle DCG = 130^\circ - \angle GCE$

$= 130^\circ - 20^\circ = 110^\circ$

Also, we have AB || CD and FG is a transversal.

So, $\angle BFC = \angle DCG = 110^\circ$ [Corresponding angles]

As, FG || AE, AF is a transversal.

$\angle BFG = \angle FAE$ [Corresponding angles]

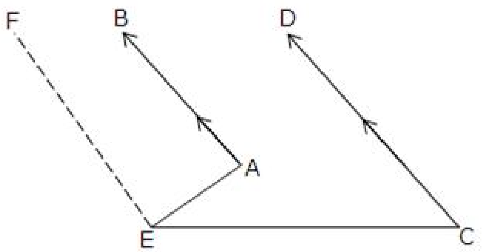
$\therefore x^\circ = \angle FAE = 110^\circ$.

Hence, x = 110

**Question 7:**

Given: AB || CD

To Prove: $\angle BAE - \angle DCE = \angle AEC$




Construction : Through E draw EF || AB

Proof : Since EF || AB, AE is a transversal.

So, $\angle BAE + \angle AEF = 180^\circ$ ....(i)

[sum of consecutive interior angles is $180^\circ$]

As EF || AB and AB || CD [Given]

So, EF || CD and EC is a transversal.

So, $\angle FEC + \angle DCE = 180^\circ$ ....(ii)

[sum of consecutive interior angles is $180^\circ$]

From (i) and (ii) we get,

$\angle BAE + \angle AEF = \angle FEC + \angle DCE$

$\Rightarrow \angle BAE - \angle DCE = \angle FEC - \angle AEF = \angle AEC$ [Proved]

**Question 8:**

Since AB || CD and BC is a transversal.

So, $\angle BCD = \angle ABC = x^\circ$ [Alternate angles]

As BC || ED and CD is a transversal.

$\angle BCD + \angle EDC = 180^o$

$\Rightarrow \angle BCD + 75^o = 180^o$

$\Rightarrow \angle BCD = 180^o - 75^o = 105^o$

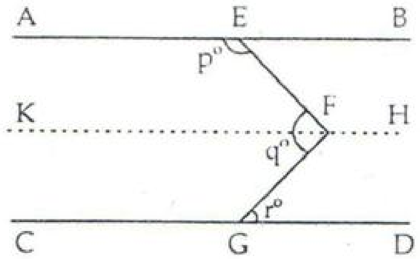
$\angle ABC = 105^o$ [since $\angle BCD = \angle ABC$]

$\therefore x^o = \angle ABC = 105^o$

Hence, x = 105.

**Question 9:**

Through F, draw KH || AB || CD




Now, KF || CD and FG is a transversal.

$\Rightarrow \angle KFG = \angle FGD = r^o$ .... (i)

[alternate angles]

Again AE || KF, and EF is a transversal.

So, $\angle AEF + \angle KFE = 180^o$

$\angle KFE = 180^o - p^o$ .... (ii)

Adding (i) and (ii) we get,

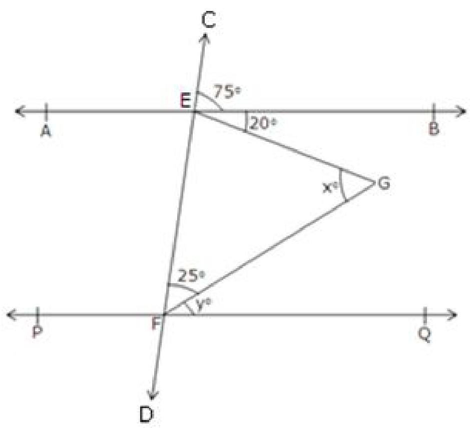
$\angle KFG + \angle KFE = 180 - p + r$

$\Rightarrow \angle EFG = 180 - p + r$

$\Rightarrow q = 180 - p + r$

i.e., $p + q - r = 180$

**Question 10:**




Since AB || PQ and EF is a transversal.

So, $\angle CEB = \angle EFQ$ [Corresponding angles]

$\Rightarrow \angle EFQ = 75^o$

$\Rightarrow \angle EFG + \angle GFQ = 75^o$

$\Rightarrow 25^o + y^o = 75^o$

$\Rightarrow y = 75 - 25 = 50$

Also, $\angle BEF + \angle EFQ = 180^o$ [sum of consecutive interior angles is $180^o$]

$\angle BEF = 180^o - \angle EFQ$

$= 180^o - 75^o$

$\angle BEF = 105^o$

$\therefore \angle FEG + \angle GEB = \angle BEF = 105^o$

$\Rightarrow \angle FEG = 105^o - \angle GEB = 105^o - 20^o = 85^o$

In $\Delta EFG$ we have,

$x^o + 25^o + \angle FEG = 180^o$

$\Rightarrow x° + 25° + 85° = 180°$

$\Rightarrow x° + 110° = 180°$

$\Rightarrow x° = 180° - 110°$

$\Rightarrow x° = 70°$

Hence, x = 70.

**Question 11:**

Since AB || CD and AC is a transversal.

So, $\angle BAC + \angle ACD = 180^o$ [sum of consecutive interior angles is $180^o$]

$\Rightarrow \angle ACD = 180^o - \angle BAC$

$= 180^o - 75^o = 105^o$

$\Rightarrow \angle ECF = \angle ACD$ [Vertically opposite angles]

$\angle ECF = 105^o$

Now in $\Delta CEF$,

$\angle ECF + \angle CEF + \angle EFC = 180^o$

$\Rightarrow 105^o + x^o + 30^o = 180^o$

$\Rightarrow x = 180 - 30 - 105 = 45$

Hence, x = 45.

**Question 12:**

Since AB || CD and PQ a transversal.

So, $\angle PEF = \angle EGH$ [Corresponding angles]

$\Rightarrow \angle EGH = 85^o$

$\angle EGH$ and $\angle QGH$ form a linear pair.

So, $\angle EGH + \angle QGH = 180^o$

$\Rightarrow \angle QGH = 180^o - 85^o = 95^o$

Similarly, $\angle GHQ + 115^o = 180^o$

$\Rightarrow \angle GHQ = 180^o - 115^o = 65^o$

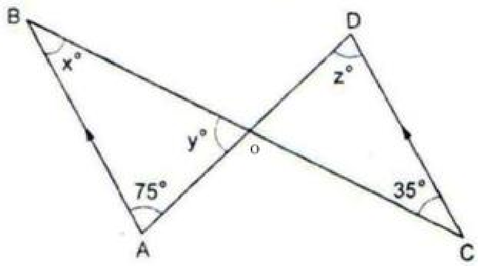
In $\Delta GHQ$, we have,

$x^o + 65^o + 95^o = 180^o$

$\Rightarrow x = 180 - 65 - 95 = 180 - 160$

$\therefore x = 20$

**Question 13:**




Since AB || CD and BC is a transversal.

So, $\angle ABC = \angle BCD$

$\Rightarrow x = 35$

Also, AB || CD and AD is a transversal.

So, $\angle BAD = \angle ADC$

$\Rightarrow z = 75$

In $\Delta ABO$, we have,

$\angle AOB + \angle BAO + \angle BOA = 180^o$

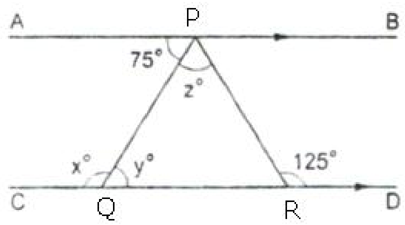
$\Rightarrow x^o + 75^o + y^o = 180^o$

$\Rightarrow 35 + 75 + y = 180$

$\Rightarrow y = 180 - 110 = 70$

∴ x = 35, y = 70 and z = 75.

**Question 14:**




Since AB || CD and PQ is a transversal.

So, y = 75 [Alternate angle]

Since PQ is a transversal and AB || CD, so x + APQ = $180^o$

[Sum of consecutive interior angles]

⇒ $x^o = 180^o$ – APQ

⇒ x = 180 – 75 = 105

Also, AB || CD and PR is a transversal.

So, ∠APR = ∠PRD [Alternate angle]

⇒ ∠APQ + ∠QPR = ∠PRD [Since ∠APR = ∠APQ + ∠QPR]

⇒ $75^o + z^o = 125^o$

⇒ z = 125 – 75 = 50

∴ x = 105, y = 75 and z = 50.

**Question 15:**

∠PRQ = $x^o = 60^o$ [vertically opposite angles]

Since EF || GH, and RQ is a transversal.

So, ∠x = ∠y [Alternate angles]

⇒ y = 60

AB || CD and PR is a transversal.

So, ∠PRD = ∠APR [Alternate angles]

⇒ ∠PRQ + ∠QRD = ∠APR [since ∠PRD = ∠PRQ + ∠QRD]

⇒ x + ∠QRD = $110^o$

⇒ ∠QRD = $110^o - 60^o = 50^o$

In ΔQRS, we have,

∠QRD + $t^o + y^o = 180^o$

⇒ 50 + t + 60 = 180

⇒ t = 180 – 110 = 70

Since, AB || CD and GH is a transversal

So, $z^o = t^o = 70^o$ [Alternate angles]

∴ x = 60 , y = 60, z = 70 and t = 70

**Question 16:**

(i) Lines l and m will be parallel if 3x – 20 = 2x + 10

[Since, if corresponding angles are equal, lines are parallel]

⇒ 3x – 2x = 10 + 20

⇒ x = 30

(ii) Lines will be parallel if $(3x + 5)^o + 4x^o = 180^o$

[if sum of pairs of consecutive interior angles is $180^o$, the lines are parallel]

So, (3x + 5) + 4x = 180

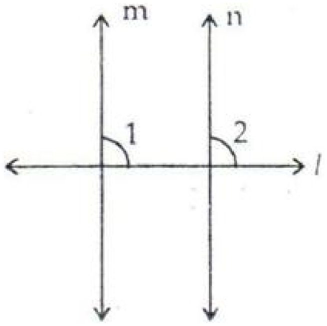
⇒ 3x + 5 + 4x = 180

⇒ 7x = 180 – 5 = 175

⇒ x = $\frac{175}{7}$ = 25

**Question 17:**

Given: Two lines m and n are perpendicular to a given line l.




To Prove: m || n

Proof : Since m ⊥ l

So, ∠1 = 90°

Again, since n ⊥ l

∠2 = 90°

∴ ∠1 = ∠2 = 90°

But ∠1 and ∠2 are the corresponding angles made by the transversal l with lines m and n and they are proved to be equal.

Thus, m || n.

## Exercise 4D

**Question 1:**

Since, sum of the angles of a triangle is 180°

∠A + ∠B + ∠C = 180°

⇒ ∠A + 76° + 48° = 180°

⇒ ∠A = 180° – 124° = 56°

∴ ∠A = 56°

**Question 2:**

Let the measures of the angles of a triangle are (2x)°, (3x)° and (4x)°.

Then, 2x + 3x + 4x = 180     [sum of the angles of a triangle is 180° ]

⇒ 9x = 180

⇒ x = $\frac{180}{9}$ = 20

∴ The measures of the required angles are:

2x = (2 × 20)° = 40°

3x = (3 × 20)° = 60°

4x = (4 × 20)° = 80°

**Question 3:**

Let 3∠A = 4∠B = 6∠C = x (say)

Then, 3∠A = x

⇒ ∠A = $\frac{x}{3}$

4∠B = x

⇒ ∠B = $\frac{x}{4}$

and 6∠C = x

⇒ ∠C = $\frac{x}{6}$

As ∠A + ∠B + ∠C = 180°

$\Rightarrow \quad \frac{x}{3} + \frac{x}{4} + \frac{x}{6} = 180$

$\Rightarrow \frac{4x + 3x + 2x}{12} = 180$

$\Rightarrow \quad 9x = 180 \times 12$

$\Rightarrow \quad x = \frac{180 \times 12}{9} = 240$

$\therefore \angle A = \frac{x}{3} = \frac{240}{3} = 80^o$

$\angle B = \frac{x}{4} = \frac{240}{4} = 60^o$

$\angle C = \frac{x}{6} = \frac{240}{6} = 40^o$

**Question 4:**

∠A + ∠B = 108$^o$ [Given]

But as ∠A, ∠B and ∠C are the angles of a triangle,

∠A + ∠B + ∠C = 180$^o$

⇒ 108$^o$ + ∠C = 180$^o$

⇒ C = 180$^o$ – 108$^o$ = 72$^o$

Also, ∠B + ∠C = 130$^o$ [Given]

⇒ ∠B + 72$^o$ = 130$^o$

⇒ ∠B = 130$^o$ – 72$^o$ = 58$^o$

Now as, ∠A + ∠B = 108$^o$

⇒ ∠A + 58$^o$ = 108$^o$

⇒ ∠A = 108$^o$ – 58$^o$ = 50$^o$

∴ ∠A = 50$^o$, ∠B = 58$^o$ and ∠C = 72$^o$.

**Question 5:**

Since. ∠A , ∠B and ∠C are the angles of a triangle .

So, ∠A + ∠B + ∠C = 180$^o$

Now, ∠A + ∠B = 125$^o$ [Given]

∴ 125$^o$ + ∠C = 180$^o$

⇒ ∠C = 180$^o$ – 125$^o$ = 55$^o$

Also, ∠A + ∠C = 113$^o$ [Given]

⇒ ∠A + 55$^o$ = 113$^o$

⇒ ∠A = 113$^o$ – 55$^o$ = 58$^o$

Now as ∠A + ∠B = 125$^o$

⇒ 58$^o$ + ∠B = 125$^o$

⇒ ∠B = 125$^o$ – 58$^o$ = 67$^o$

∴ ∠A = 58$^o$, ∠B = 67$^o$ and ∠C = 55$^o$.

**Question 6:**

Since, ∠P, ∠Q and ∠R are the angles of a triangle.

So, ∠P + ∠Q + ∠R = 180$^o$ ....(i)

Now, ∠P – ∠Q = 42$^o$ [Given]

⇒ ∠P = 42$^o$ + ∠Q ....(ii)

and ∠Q – ∠R = 21$^o$ [Given]

⇒ ∠R = ∠Q – 21$^o$ ....(iii)

Substituting the value of ∠P and ∠R from (ii) and (iii) in (i), we get,

⇒ 42$^o$ + ∠Q + ∠Q + ∠Q – 21$^o$ = 180$^o$

⇒ 3∠Q + 21$^o$ = 180$^o$

⇒ 3∠Q = 180$^o$ – 21$^o$ = 159$^o$

∠Q = $\frac{159}{3}$ = 53$^o$

$\therefore \angle P = 42^o + \angle Q$

$= 42^o + 53^o = 95^o$

$\angle R = \angle Q - 21^o$

$= 53^o - 21^o = 32^o$

$\therefore \angle P = 95^o$, $\angle Q = 53^o$ and $\angle R = 32^o$.

**Question 7:**

Given that the sum of the angles A and B of a ABC is $116^o$, i.e., $\angle A + \angle B = 116^o$.

Since, $\angle A + \angle B + \angle C = 180^o$

So, $116^o + \angle C = 180^o$

$\Rightarrow \angle C = 180^o - 116^o = 64^o$

Also, it is given that:

$\angle A - \angle B = 24^o$

$\Rightarrow \angle A = 24^o + \angle B$

Putting, $\angle A = 24^o + \angle B$ in $\angle A + \angle B = 116^o$, we get,

$\Rightarrow 24^o + \angle B + \angle B = 116^o$

$\Rightarrow 2\angle B + 24^o = 116^o$

$\Rightarrow 2\angle B = 116^o - 24^o = 92^o$

$\angle B = \frac{92}{2} = 46^o$

Therefore, $\angle A = 24^o + 46^o = 70^o$

$\therefore \angle A = 70^o$, $\angle B = 46^o$ and $\angle C = 64^o$.

**Question 8:**

Let the two equal angles, A and B, of the triangle be $x^o$ each.

We know,

$\angle A + \angle B + \angle C = 180^o$

$\Rightarrow x^o + x^o + \angle C = 180^o$

$\Rightarrow 2x^o + \angle C = 180^o$ ....(i)

Also, it is given that,

$\angle C = x^o + 18^o$....(ii)

Substituting $\angle C$ from (ii) in (i), we get,

$\Rightarrow 2x^o + x^o + 18^o = 180^o$

$\Rightarrow 3x^o = 180^o - 18^o = 162^o$

$x = \frac{162}{3} = 54^o$

Thus, the required angles of the triangle are $54^o$, $54^o$ and $x^o + 18^o = 54^o + 18^o = 72^o$.

**Question 9:**

Let $\angle C$ be the smallest angle of ABC.

Then, $\angle A = 2\angle C$ and $B = 3\angle C$

Also, $\angle A + \angle B + \angle C = 180^o$

$\Rightarrow 2\angle C + 3\angle C + \angle C = 180^o$

$\Rightarrow 6\angle C = 180^o$

$\Rightarrow \angle C = 30^o$

So, $\angle A = 2\angle C = 2\,(30^o) = 60^o$

$\angle B = 3\angle C = 3\,(30^o) = 90^o$

$\therefore$ The required angles of the triangle are $60^o$, $90^o$, $30^o$.

**Question 10:**

Let ABC be a right angled triangle and $\angle C = 90^o$

Since, $\angle A + \angle B + \angle C = 180^o$

$\Rightarrow \angle A + \angle B = 180^o - \angle C = 180^o - 90^o = 90^o$

Suppose $\angle A = 53^o$

Then, $53^o + \angle B = 90^o$

$\Rightarrow \angle B = 90^o - 53^o = 37^o$

$\therefore$ The required angles are $53^o$, $37^o$ and $90^o$.

**Question 11:**

Let ABC be a triangle.

Given, $\angle A + \angle B = \angle C$

We know, $\angle A + \angle B + \angle C = 180^o$

$\Rightarrow \angle C + \angle C = 180^o$

$\Rightarrow 2\angle C = 180^o$

$\Rightarrow \angle C = \frac{180}{2} = 90^o$

So, we find that ABC is a right triangle, right angled at C.

**Question 12:**

Given : $\Delta$ABC in which $\angle A = 90^o$, AL $\perp$ BC

To Prove: $\angle BAL = \angle ACB$

Proof :

In right triangle $\Delta$ABC,

$\Rightarrow \angle ABC + \angle BAC + \angle ACB = 180^o$

$\Rightarrow \angle ABC + 90^o + \angle ACB = 180^o$

$\Rightarrow \angle ABC + \angle ACB = 180^o - 90^o$

$\therefore \angle ABC + \angle ACB = 90^o$

$\Rightarrow \angle ACB = 90^o - \angle ABC$ ....(1)

Similarly since $\Delta$ABL is a right triangle, we find that,

$\angle BAL = 90^o - \angle ABC$ ...(2)

Thus from (1) and (2), we have

$\therefore \angle BAL = \angle ACB$ (Proved)

**Question 13:**

Let ABC be a triangle.

So, $\angle A < \angle B + \angle C$

Adding A to both sides of the inequality,

$\Rightarrow 2\angle A < \angle A + \angle B + \angle C$

$\Rightarrow 2\angle A < 180^o$ [Since $\angle A + \angle B + \angle C = 180^o$]

$\Rightarrow \angle A < \frac{180}{2} = 90^o$

Similarly, $\angle B < \angle A + \angle C$

$\Rightarrow \angle B < 90^o$

and $\angle C < \angle A + \angle B$

$\Rightarrow \angle C < 90^o$

$\Delta$ABC is an acute angled triangle.

**Question 14:**

Let ABC be a triangle and $\angle B > \angle A + \angle C$

Since, $\angle A + \angle B + \angle C = 180^o$

$\Rightarrow \angle A + \angle C = 180^o - \angle B$

Therefore, we get

$\angle B > 180^o - \angle B$

Adding $\angle B$ on both sides of the inequality, we get,

$\Rightarrow \angle B + \angle B > 180^o - \angle B + \angle B$

$\Rightarrow 2\angle B > 180^o$

$\Rightarrow \angle B > \frac{180}{2} = 90^o$

i.e., $\angle B > 90^o$ which means $\angle B$ is an obtuse angle.

$\Delta$ABC is an obtuse angled triangle.

**Question 15:**

Since $\angle$ACB and $\angle$ACD form a linear pair.

So, $\angle ACB + \angle ACD = 180^o$

$\Rightarrow \angle ACB + 128^o = 180^o$

$\Rightarrow \angle ACB = 180^o - 128 = 52^o$

Also, $\angle ABC + \angle ACB + \angle BAC = 180^o$

$\Rightarrow 43^o + 52^o + \angle BAC = 180^o$

$\Rightarrow 95^o + \angle BAC = 180^o$

$\Rightarrow \angle BAC = 180^o - 95^o = 85^o$

$\therefore \angle ACB = 52^o$ and $\angle BAC = 85^o$.

**Question 16:**

As $\angle$DBA and $\angle$ABC form a linear pair.

So, $\angle DBA + \angle ABC = 180^o$

$\Rightarrow 106^o + \angle ABC = 180^o$

$\Rightarrow \angle ABC = 180^o - 106^o = 74^o$

Also, $\angle$ACB and $\angle$ACE form a linear pair.

So, $\angle ACB + \angle ACE = 180^o$

$\Rightarrow \angle ACB + 118^o = 180^o$

$\Rightarrow \angle ACB = 180^o - 118^o = 62^o$

In $\angle$ABC, we have,

$\angle ABC + \angle ACB + \angle BAC = 180^o$

$74^o + 62^o + \angle BAC = 180^o$

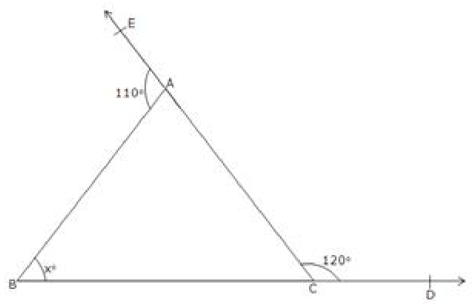
$\Rightarrow 136^o + \angle BAC = 180^o$

$\Rightarrow \angle BAC = 180^o - 136^o = 44^o$

$\therefore$ In triangle ABC, $\angle A = 44^o$, $\angle B = 74^o$ and $\angle C = 62^o$

**Question 17:**

(i) $\angle EAB + \angle BAC = 180^o$ [Linear pair angles]




$110^o + \angle BAC = 180^o$

$\Rightarrow \angle BAC = 180^o - 110^o = 70^o$

Again, $\angle BCA + \angle ACD = 180^o$ [Linear pair angles]

$\Rightarrow \angle BCA + 120^o = 180^o$

$\Rightarrow \angle BCA = 180^o - 120^o = 60^o$

Now, in $\Delta$ABC,

$\angle ABC + \angle BAC + \angle ACB = 180^o$

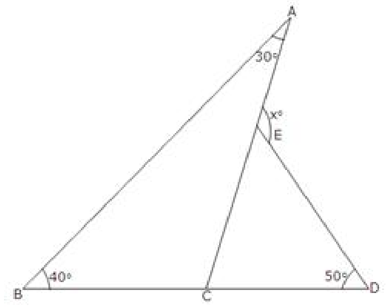
$x^o + 70^o + 60^o = 180^o$

$\Rightarrow x + 130^o = 180^o$

$\Rightarrow x = 180^o - 130^o = 50^o$

$\therefore x = 50$

(ii)




In $\Delta$ABC,

$\angle A + \angle B + \angle C = 180^o$

$\Rightarrow 30^o + 40^o + \angle C = 180^o$

$\Rightarrow 70^o + \angle C = 180^o$

$\Rightarrow \angle C = 180^o - 70^o = 110^o$

Now $\angle BCA + \angle ACD = 180^o$ [Linear pair]

$\Rightarrow 110^o + \angle ACD = 180^o$

$\Rightarrow \angle ACD = 180^o - 110^o = 70^o$

In $\Delta$ECD,

$\Rightarrow \angle ECD + \angle CDE + \angle CED = 180^o$

$\Rightarrow 70^o + 50^o + \angle CED = 180^o$

$\Rightarrow 120^o + \angle CED = 180^o$

$\angle CED = 180^o - 120^o = 60^o$

Since $\angle$AED and $\angle$CED from a linear pair

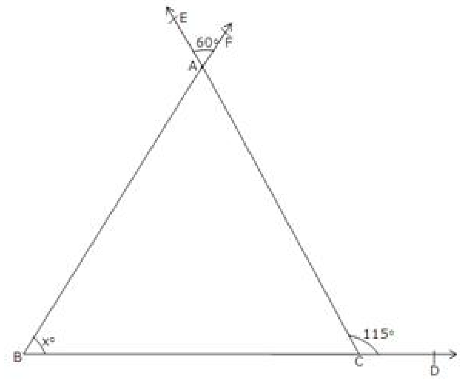
So, $\angle AED + \angle CED = 180^o$

$\Rightarrow x^o + 60^o = 180^o$

$\Rightarrow x^o = 180^o - 60^o = 120^o$

$\therefore x = 120$

(iii)




$\angle EAF = \angle BAC$ [Vertically opposite angles]

$\Rightarrow \angle BAC = 60^o$

In $\Delta$ABC, exterior $\angle$ACD is equal to the sum of two opposite interior angles.

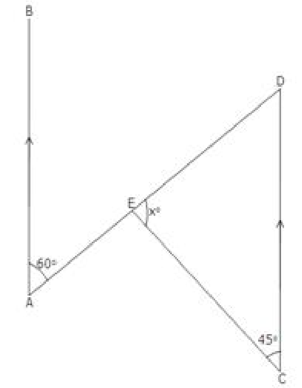
So, $\angle ACD = \angle BAC + \angle ABC$

$\Rightarrow 115^o = 60^o + x^o$

$\Rightarrow x^o = 115^o - 60^o = 55^o$

$\therefore x = 55$

(iv)




Since AB || CD and AD is a transversal.

So, ∠BAD = ∠ADC

⇒ ∠ADC = $60^{o}$

In ∠ECD, we have,

∠E + ∠C + ∠D = $180^{o}$

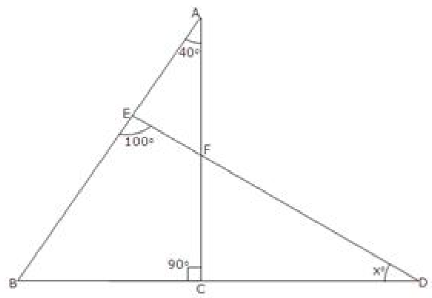
⇒ $x^{o} + 45^{o} + 60^{o} = 180^{o}$

⇒ $x^{o} + 105^{o} = 180^{o}$

⇒ $x^{o} = 180^{o} - 105^{o} = 75^{o}$

∴ x = 75

(v)




In ΔAEF,

Exterior ∠BED = ∠EAF + ∠EFA

⇒ $100^{o} = 40^{o}$ + ∠EFA

⇒ ∠EFA = $100^{o} - 40^{o} = 60^{o}$

Also, ∠CFD = ∠EFA [Vertically Opposite angles]

⇒ ∠CFD = $60^{o}$

Now in ΔFCD,

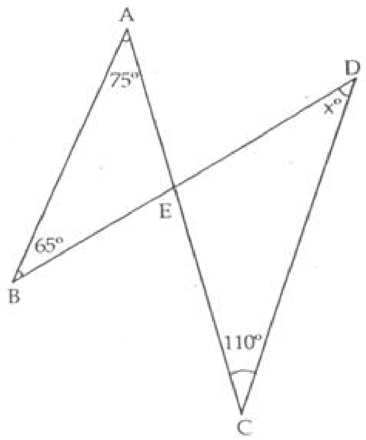
Exterior ∠BCF = ∠CFD + ∠CDF

⇒ $90^{o} = 60^{o} + x^{o}$

⇒ $x^{o} = 90^{o} - 60^{o} = 30^{o}$

∴ x = 30

(vi)




In $\Delta$ABE, we have,

$\angle A + \angle B + \angle E = 180^\circ$

$\Rightarrow 75^\circ + 65^\circ + \angle E = 180^\circ$

$\Rightarrow 140^\circ + \angle E = 180^\circ$

$\Rightarrow \angle E = 180^\circ - 140^\circ = 40^\circ$

Now, $\angle CED = \angle AEB$ [Vertically opposite angles]

$\Rightarrow \angle CED = 40^\circ$

Now, in $\Delta$CED, we have,

$\angle C + \angle E + \angle D = 180^\circ$

$\Rightarrow 110^\circ + 40^\circ + x^\circ = 180^\circ$

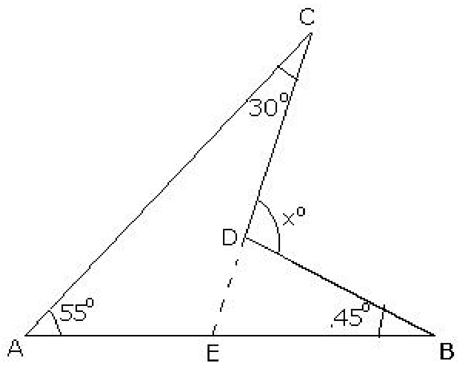
$\Rightarrow 150^\circ + x^\circ = 180^\circ$

$\Rightarrow x^\circ = 180^\circ - 150^\circ = 30^\circ$

$\therefore x = 30$

**Question 18:**

Produce CD to cut AB at E.




Now, in $\Delta$BDE, we have,

Exterior $\angle CDB = \angle CEB + \angle DBE$

$\Rightarrow x^\circ = \angle CEB + 45^\circ$ .....(i)

In $\Delta$AEC, we have,

Exterior $\angle CEB = \angle CAB + \angle ACE$

$= 55^\circ + 30^\circ = 85^\circ$

Putting $\angle CEB = 85^\circ$ in (i), we get,

$x^\circ = 85^\circ + 45^\circ = 130^\circ$

$\therefore x = 130$

**Question 19:**

The angle $\angle BAC$ is divided by AD in the ratio 1 : 3.

Let $\angle BAD$ and $\angle DAC$ be y and 3y, respectively.

As BAE is a straight line,

$\angle BAC + \angle CAE = 180^\circ$ [linear pair]

$\Rightarrow \angle BAD + \angle DAC + \angle CAE = 180^o$
$\Rightarrow y + 3y + 108^o = 180^o$
$\Rightarrow 4y = 180^o - 108^o = 72^o$
$\Rightarrow y = \frac{72}{4} = 18^o$
Now, in $\Delta ABC$,
$\angle ABC + \angle BCA + \angle BAC = 180^o$
$y + x + 4y = 180^o$
[Since, $\angle ABC = \angle BAD$ (given AD = DB) and $\angle BAC = y + 3y = 4y$]
$\Rightarrow 5y + x = 180$
$\Rightarrow 5 \times 18 + x = 180$
$\Rightarrow 90 + x = 180$
$\therefore x = 180 - 90 = 90$

**Question 20:**

Given : A $\Delta ABC$ in which BC, CA and AB are produced to D, E and F respectively.
To prove : Exterior $\angle DCA$ + Exterior $\angle BAE$ + Exterior $\angle FBD = 360^o$
Proof : Exterior $\angle DCA = \angle A + \angle B$ ....(i)
Exterior $\angle FAE = \angle B + \angle C$ ....(ii)
Exterior $\angle FBD = \angle A + \angle C$ ....(iii)
Adding (i), (ii) and (iii), we get,
Ext. $\angle DCA$ + Ext. $\angle FAE$ + Ext. $\angle FBD$
$= \angle A + \angle B + \angle B + \angle C + \angle A + \angle C$
$= 2\angle A + 2\angle B + 2\angle C$
$= 2 (\angle A + \angle B + \angle C)$
$= 2 \times 180^o$
[Since, in triangle the sum of all three angle is $180^o$]
$= 360^o$
Hence, proved.

**Question 21:**

In $\Delta ACE$, we have,
$\angle A + \angle C + \angle E = 180^o$ ....(i)
In $\Delta BDF$, we have,
$\angle B + \angle D + \angle F = 180^o$ ....(ii)
Adding both sides of (i) and (ii), we get,
$\angle A + \angle C + \angle E + \angle B + \angle D + \angle F = 180^o + 180^o$
$\Rightarrow \angle A + \angle B + \angle C + \angle D + \angle E + \angle F = 360^o$.

**Question 22:**

Given : In $\Delta ABC$, bisectors of $\angle B$ and $\angle C$ meet at O and $\angle A = 70^o$
In $\Delta BOC$, we have,

$\Rightarrow \angle BOC + \frac{1}{2}\angle B + \frac{1}{2}\angle C = 180^\circ$

$\Rightarrow \angle BOC = 180^\circ - \frac{1}{2}\angle B - \frac{1}{2}\angle C$

$= 180^\circ - \frac{1}{2}(\angle B + \angle C)$

$= 180^\circ - \frac{1}{2}\left[180^\circ - \angle A\right]$

$\left[\because \angle A + \angle B + \angle C = 180^\circ\right]$

$= 180^\circ - \frac{1}{2}\left[180^\circ - 70^\circ\right]$
$= 180^\circ - \frac{1}{2} \times 110^\circ$

∠BOC + ∠OBC + ∠OCB = $180^\circ$
= $180^\circ$ – $55^\circ$ = $125^\circ$
∴ ∠BOC = $125^\circ$.

### Question 23:

We have a ΔABC whose sides AB and AC have been procued to D and E. A = $40^\circ$ and bisectors of ∠CBD and ∠BCE meet at O.
In ΔABC, we have,
Exterior ∠CBD = C + $40^\circ$

$$\Rightarrow \quad \angle CBO = \frac{1}{2}\text{ Ext. } \angle CBD = \frac{1}{2}\left(\angle C + 40^\circ\right) = \frac{1}{2}\angle C + 20^\circ$$

And exterior ∠BCE = B + $40^\circ$

$$\Rightarrow \quad \angle BCO = \frac{1}{2}\text{ Ext. } \angle BCE = \frac{1}{2}\left(\angle B + 40^\circ\right) = \frac{1}{2}\angle B + 20^\circ.$$

Now, in ΔBCO, we have,
$\angle BOC = 180^\circ - \angle CBO - \angle BCO$
$= 180^\circ - \frac{1}{2}\angle C - 20^\circ - \frac{1}{2}\angle B - 20^\circ$
$= 180^\circ - \frac{1}{2}\angle C - \frac{1}{2}\angle B - 20^\circ - 20^\circ$
$= 180^\circ - \frac{1}{2}(\angle B + \angle C) - 40^\circ$
$= 140^\circ - \frac{1}{2}(\angle B + \angle C)$
$= 140^\circ - \frac{1}{2}\left[180^\circ - \angle A\right]$

$= 140^\circ - 90^\circ + \frac{1}{2}\angle A$

$= 50^\circ + \frac{1}{2}\angle A$

$= 50^\circ + \frac{1}{2} \times 40^\circ$

= $50^\circ$ + $20^\circ$
= $70^\circ$
Thus, ∠BOC = $70^\circ$

### Question 24:

In the given ΔABC, we have,
∠A : ∠B : ∠C = 3 : 2 : 1
Let ∠A = 3x, ∠B = 2x, ∠C = x. Then,
∠A + ∠B + ∠C = $180^\circ$
⇒ 3x + 2x + x = $180^\circ$
⇒ 6x = $180^\circ$

$\Rightarrow x = 30^o$

$\angle A = 3x = 3\ 30^o = 90^o$

$\angle B = 2x = 2\ 30^o = 60^o$

and, $\angle C = x = 30^o$

Now, in $\Delta ABC$, we have,

Ext $\angle ACE = \angle A + \angle B = 90^o + 60^o = 150^o$

$\angle ACD + \angle ECD = 150^o$

$\Rightarrow \angle ECD = 150^o - \angle ACD$

$\Rightarrow \angle ECD = 150^o - 90^o$ [since , $AD \perp CD$, $\angle ACD = 90^o$]

$\Rightarrow \angle ECD = 60^o$

## Question 25:

In $\Delta ABC$, AN is the bisector of $\angle A$ and $AM \perp BC$.

Now in $\Delta ABC$ we have;

$\angle A = 180^o - \angle B - \angle C$

$\Rightarrow \angle A = 180^o - 65^o - 30^o$

$= 180^o - 95^o$

$= 85^o$

Now, in $\Delta ANC$ we have;

Ext. $\angle MNA = \angle NAC + 30^o$

$= \frac{1}{2}\angle A + 30°$

$= \frac{85°}{2} + 30°$

$= \frac{85° + 60°}{2}$

$= \frac{145°}{2}$

Therefore, $\angle MNA = \frac{145°}{2}$

In $\Delta$ MAN. we have;

$\angle MAN = 180^o - \angle AMN - \angle MNA$

$= 180^o - 90^o - \angle MNA$ [since $AM \perp BC$, $\angle AMN = 90°$ ]

$= 90° - \frac{145°}{2}$ [since $\angle MNA = \frac{145°}{2}$]

$= \frac{180° - 145°}{2}$

$= \frac{35°}{2}$

$= 17.5°$

Thus, $\angle MAN =$

## Question 26:

(i) False (ii) True (iii) False (iv) False (v) True (vi) True.