# **Graph Retrieval Augmented Trustworthiness Reasoning**

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#### Abstract

Trustworthiness reasoning is crucial in multiplayer games with incomplete information, enabling agents to identify potential allies and adversaries, thereby enhancing reasoning and decision-making processes. Traditional approaches relying on pre-trained models necessitate extensive domainspecific data and considerable reward feedback, with their lack of real-time adaptability hindering their effectiveness in dynamic environments. In this paper, we introduce the Graph Retrieval Augmented Reasoning (GRATR) framework, leveraging the Retrieval-Augmented Generation (RAG) technique to bolster trustworthiness reasoning in agents. GRATR constructs a dynamic trustworthiness graph, updating it in realtime with evidential information, and retrieves relevant trust data to augment the reasoning capabilities of Large Language Models (LLMs). We validate our approach through experiments on the multiplayer game "Werewolf," comparing GRATR against baseline LLM and LLM enhanced with Native RAG and Rerank RAG. Our results demonstrate that GRATR surpasses the baseline methods by over 30% in winning rate, with superior reasoning performance. Moreover, GRATR effectively mitigates LLM hallucinations, such as identity and objective amnesia, and crucially, it renders the reasoning process more transparent and traceable through the use of the trustworthiness graph.

#### Introduction

In multiplayer games with incomplete information, reasoning is crucial for assessing the trustworthiness of players who may conceal their intentions based on their actions, dialogue, and other observable information. Autonomous agents must analyze and evaluate the reliability of available information sources to determine player trust and cooperation (Fig. 1). Currently, key methods supporting such reasoning include symbolic reasoning (Nye et al. 2021; Lu et al. 2021) and evidential theory (Liu et al. 2021), Bayesian reasoning (Wojtowicz and DeDeo 2020; Sohn and Narain 2021; Wan and Du 2021), and reinforcement learning (RL) (Wan et al. 2021; Wang, Hao, and Cao 2020; Tiwari, Zhu, and Pandey 2021).

Large language model (LLM) is a promising approach for trustworthiness reasoning in such games due to their robust

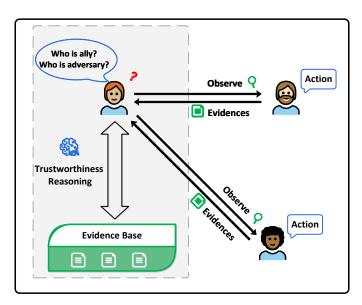


Figure 1: The player observe the action of other players during the observation phase to gain an evidence base, and then use the evidence base to do a trustworthiness reasoning (determine who his allies and adversaries are).

natural language understanding and generation capabilities. They are well-suited for interpreting complex dialogues, inferring different intentions, and detecting hidden motives from context. However, LLMs often face potential risks such as hallucinations and knowledge obsolescence, since their reasoning is based on pre-existing training data. To mitigate these issues, methods such as supervised fine-tuning (SFT), and RL, have been proposed to improve reasoning performance, they often require substantial historical data and reward feedback, which may be scarce in real-world scenarios.

Retrieval augmented generation (RAG)(Gao et al. 2024; Zhao et al. 2024) is an alternative paradigm for addressing LLM limitations by incorporating an external retrieval mechanism. In the RAG pipeline, a retriever first indexes and retrieves relevant data chunks, which are then used in conjunction with an input query to enhance the generation process. Optimization techniques such as step-by-step retrieval using a Chain of Thought (CoT) (Wang et al. 2024)

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decomposition of the query, fine-tuning embedding models to capture semantically similar content in vector space, and combining various retrieval methods to improve the robustness of the retriever. This approach can help mitigate issues of knowledge obsolescence and hallucinations by integrating up-to-date and contextually relevant information.

Trustworthiness reasoning in multiplayer games requires real-time collection and analysis of statements and behaviors exhibited by players to assess their trustworthiness. If a summary and reasoning is performed based on the entire evidence base, this is a very time-consuming process when there are many players. Therefore, we only need to perform reasoning on a specific player, that is, use RAG to retrieve evidence for that player from the evidence base. Current RAG methods are limited in handling temporal-dependence evidence and dynamic changes in player behavior over time. These methods typically focus on retrieving and integrating static external data, which does not effectively capture the evolving nature of interactions and trust relationships in real-time scenarios.

To address these limitations, this paper proposes a novel method, graph retrieval augmented trustworthiness reasoning (GRATR). GRATR leverages RAG technology by introducing a trust relationship graph that dynamically updates based on real-time evidence. This method constructs a graph where nodes represent players, and edges represent trust relationships updated through recursive evidence searches. By calculating and updating each node's trustworthiness based on accumulated evidence, GRATR enhances the ability to reason about player trustworthiness effectively. The contributions of this paper are as follows:

- This paper presents a GRATR framework that constructs a dynamic trust relationship graph. During the player observation phase, evidence is collected to update the nodes and edges of the graph in real time. During the player's turn, the relevant chain of evidence and the trustworthiness of the player's action object are retrieved to improve reasoning and decision making.
- We validate our approach through experiments on the multiplayer game "Werewolf," comparing GRATR against baseline LLMs and LLMs enhanced with Native RAG and Rerank RAG. Our results demonstrate that GRATR outperforms the baseline methods by over 30% in win rate, with superior reasoning performance. Moreover, when historical evidence is used as a way of contextual learning for reasoning by large language models, there is a 50% chance of the illusion of forgetting one's own role. Our algorithm can improve this accuracy to 80%, which to some extent alleviates this problem. At the same time, the previous method is untraceable. Our method: to some extent, the process can be made traceable and visualised through temporal evidence and evidence chains.

# **Preliminaries**

#### **Multiplayer Game with Incomplete Information**

In a multi-player game with incomplete information, the game can be described by the following components:

- Players:  $P = \{p_1, p_2, \dots, p_n\}$ , where  $p_i$  represents the i-th player.
- **Types**: Each player  $p_i$  has a private type  $\theta_i \in \Theta_i$ , where  $\Theta_i$  is the set of possible types for player  $p_i$ .
- Actions: In each round t, player  $p_i$  chooses an action  $a_i^t \in A_i$ , where  $A_i$  is the set of available actions for player  $p_i$ .
- Observations: After all players choose their actions, each player  $p_i$  receives an observation  $o_i^t \in O_i$ , where  $O_i$  is the set of possible observations for  $p_i$ . The observation  $o_i^t$  depends on the joint actions  $\mathbf{a}^t = (a_1^t, a_2^t, \dots, a_n^t)$  and possibly other public or private signals.
- **Beliefs**: Each player  $p_i$  maintains a belief  $\sigma_i^t(\theta_{-i} \mid h_i^t)$ , where  $\theta_{-i}$  denotes the types of all other players, and  $h_i^t$  is the history of observations and actions up to round t. The belief  $\sigma_i^t$  is updated after each round based on Bayes' rule.
- Strategies: A strategy for player p<sub>i</sub> is a function s<sub>i</sub>: H<sub>i</sub>× Θ<sub>i</sub> → A<sub>i</sub>, where H<sub>i</sub> is the set of possible histories for player p<sub>i</sub>. The strategy s<sub>i</sub>(h<sub>i</sub><sup>t</sup>, θ<sub>i</sub>) dictates the action a<sub>i</sub><sup>t</sup> player p<sub>i</sub> should take given their history h<sub>i</sub><sup>t</sup> and type θ<sub>i</sub>.
- **Objective**: Each player  $p_i$  aims to maximize their expected utility  $\mathbb{E}_{\sigma_i}[u_i(\mathbf{a}, \theta)]$ , where the expectation is taken over the belief distribution  $\sigma_i$ .
- Game Dynamics: The game proceeds as follows:
  - At the beginning of each round t, each player  $p_i$  observes  $h_i^t$  and selects an action  $a_i^t = s_i(h_i^t, \theta_i)$ .
  - After all actions a<sup>t</sup> are chosen, players receive observations o<sup>t</sup><sub>i</sub>.
  - Players update their beliefs  $\sigma_i^{t+1}(\theta_{-i} \mid h_i^{t+1})$  based on the new history  $h_i^{t+1}$  that includes  $o_i^t$  and  $a_i^t$ .
  - The game continues for a fixed number of rounds T, or until a stopping condition is met.

# **Reasoning Task**

In incomplete information games, players typically need to reason to get more potential information based on the observations in order to make better decisions. Especially in multiplayer games, players need to observe and analyze other players' behavior and historical data in real time. Typically, there is a large amount of data to be processed, and some of it contains misleading information. Hence, the reasoning ability of agents is of crucial importance (Wu et al. 2024; Zhang et al. 2024; Cheng et al. 2024; Qin et al. 2024; Costarelli et al. 2024).

In traditional game theoretic algorithms, the game is usually played through Bayesian method (Zamir 2020) and evolutionary game theory (Deng et al. 2015). In addition, there are machine learning methods such as Monte Carlo tree search (Cowling, Powley, and Whitehouse 2012) and RL (Heinrich and Silver 2016). With the introduction of deep

networks, RL methods have been widely used in multiplayer games due to their excellent inference ability.

While RL offer significant advantages in reasoning, it require a large amount of domain-specific training data and their inference capabilities are limited to that domain, making them unsuitable for a wide range of reasoning tasks. LLM-based reasoning provide new ideas for completing reasoning tasks due to their extensive knowledge and powerful language capabilities. Xu et.al. (Xu et al. 2023) proposed a framework combining LLMs and RL is proposed to develop strategic language agents that can overcome inherent biases and perform well in complex decision-making tasks by allowing RL strategies to choose the optimal decision from a diverse set of action candidates generated by LLMs.

However, LLMs not only consumes a lot of training cost, but also prevents it from updating data in real time, which is not suitable for real-time updating of information in multiplayer games. In addition, the illusion of LLMs can seriously affect reasoning.

RAG presents a promising paradigm to address this challenges and thus promises to go further in assisting LLMs in solving reasoning problems.

## **Retrieval Augmented Generation**

The traditional Retrieval-Augmented Generation (RAG) process involves three main steps. First, the documents are divided into chunks, which are then encoded into vectors and stored in a vector database. Next, the chunks most relevant to the query are retrieved based on semantic similarity. Finally, a prompt is constructed using the original question and the retrieved chunks, which serves as input to a generator built on a LLM to produce the final answer.

However, not all directly retrieved document chunks are highly relevant to the query. Reranking algorithms (Sun et al. 2023) address this by reordering the retrieved chunks, allowing the generator to make better use of the most relevant information and generate responses that are more accurate and pertinent to the query.

The Graph RAG method (Edge et al. 2024) further improves this process by constructing a graph model from the retrieved document chunks, representing entities and relationships as nodes and directed edges. Subsequently, knowledge and relationships related to the query are retrieved based on this graph, thus assisting the LLM in generating a more relevant response.

Retrieval-Augmented Reasoning (RAR) enhances generative models by integrating information retrieval with reasoning, allowing the model to use up-to-date external knowledge for more accurate and timely responses. While RAR improves reasoning and maintains output quality, it is limited to simple reasoning and struggles with complex tasks like trustworthiness reasoning.

# Methodology

In the context of multiplayer games with incomplete information, players often need to make decisions based on partial knowledge about the game state and the intentions of

other players. LLMs can generate actions directly from historical interactions, but this approach often leads to hallucinations, where the model produces outputs that are inconsistent or not grounded in the actual game history. Additionally, the reasoning process of LLMs in this context is typically opaque and difficult to trace.

To enhance the effectiveness of LLM reasoning, especially in environments where trust and strategic interactions are crucial, it is essential to retrieve the most relevant evidence from historical data. This motivated us to develop a framework where the information observed by agents is structured into a graph-based evidence base. By maintaining this evidence graph, we can retrieve related evidence chains, augment LLM reasoning, and mitigate the issues of hallucination and opacity. This methodology forms the foundation of our proposed GRATR system, which dynamically evaluates trustworthiness among players by leveraging the graph structure to organize and retrieve evidence, ultimately improving decision-making in multiplayer games.

# **Initialization of the Evidence Graph**

A directed graph  $G_i^t$  is initialized as a dynamic evidence base, storing the history of observations and actions  $h_i^t$  up to round t. This graph  $G_i^t$  serves as a foundation for the LLM's reasoning process, allowing for the retrieval and utilization of real-time evidence. The graph consists of two fundamental components, i.e., nodes and edges.

**Nodes** Each node in the graph  $G_i^t$  represents a player  $p_j$  and stores three parameters:

- Trustworthiness  $T_i^t(p_j)$ : The perceived trustworthiness of player  $p_j$  by player  $p_i$  at time t.
- Role Classification  $\mathcal{R}_i^t(p_j)$ : The classification of  $p_j$ 's role as perceived by  $p_i$ , determined by  $T_i^t(p_j)$ , which is as Table 1, where  $\epsilon$  is the tolerance threshold for neutral judgment. Player  $p_i$  has an inherent trustworthiness of 1 toward themselves, i.e.,  $T_i^t(p_i) = 1$ .

Table 1: Role Classification Based on Trustworthiness Value

<b>Trustworthiness Value</b>	Role Classification	$\mathcal{R}_i^t(p_j)$
$T_i^t(p_j) > \epsilon$	Ally	1
$T_i^t(p_j) > \epsilon$ $-\epsilon \le T_i^t(p_j) \le \epsilon$ $T_i^t(p_j) < -\epsilon$	Indifferent	0
$T_i^t(p_j) < -\epsilon$	Adversary	-1

• Historical Observations  $h_i^t(p_j)$ : The history of observations and actions gathered by player  $p_i$  about player  $p_j$  up to round t.

Noted that trustworthiness  $T_i^t(p_j)$  differs from the beliefs  $\sigma_i^t(\theta_{-i} \mid h_i^t)$ . While beliefs represent player  $p_i$ 's probabilistic assessment of other players' types, trustworthiness  $T_i^t(p_j)$  in this graph is a measure of the confidence player  $p_i$  has in player  $p_j$ 's reliability and intentions, directly influencing the role classification  $\mathcal{R}_i^t(p_j)$ .

**Edges** In the graph  $G_i^t$ , each directed edge  $E_i^t(p_j,p_k)$  connects player node  $p_j$  to player node  $p_k$ . This edge encapsulates two parameters:

- Evidence List  $h_i^t(p_i, p_k)$ : This list contains a set of evidence items  $e_i^t(p_i, p_k)$  that record the actions of player  $p_i$  towards player  $p_k$  as observed by player  $p_i$ . Each evidence item comprises the specific action taken and its associated weight  $w_i^t(p_i, p_k)$ , indicating the significance of this action in assessing trustworthiness.
- Edge Weight  $\tau_i^t(p_i, p_k)$ : This weight reflects the level of trustworthiness that player  $p_i$  attributes to player  $p_i$  concerning their actions towards player  $p_k$ . It is determined based on the accumulated evidence in the evidence list.

At the initial time t = 0, the edge weight is set to zero, i.e.,  $\tau_i^0(p_i, p_k) = 0$ , and the evidence list  $h_i^0(p_i, p_k)$  is empty, indicating no prior observations or assessments.

# **Update of the Evidence Graph**

When player  $p_i$  receives a new observation  $o_i^t(p_j)$  following an action by player  $p_i$ , the evidence graph  $G_i^t$  must be updated to incorporate this new information. This ensures that  $G_i^t$  accurately represents the current state of trustworthiness among the players at time t.

Player  $p_i$  uses the LLM to extract evidence items  $e_i^t(p_j, p_k)$  and their corresponding weights  $w_i^t(p_j, p_k)$  from the observation  $o_i^t(p_i)$  (the related prompts used for LLM interactions are provided in Appendix I). For each directed edge  $E_i^t(p_j, p_k)$  in the graph, the evidence list  $h_i^{t+1}(p_j, p_k)$ associated with the edge  $E_i^t(p_j, p_k)$  is updated by adding the new intention  $e_i^t(p_i, p_k)$ :

$$h_i^{t+1}(p_i, p_k) = h_i^t(p_i, p_k) \cup \{e_i^t(p_i, p_k)\}.$$
 (1)

The sign of  $w_i^t(p_i, p_k)$  indicates the nature of  $p_i$ 's intention towards  $p_k$ : negative for hostility and positive for support, with  $|w_i^t(p_i, p_k)|$  reflecting its strength. Noted that the evidence list  $h_i^t(p_j, p_k)$  is updated with the new observation, and the edge weight  $\tau_i^t(p_i, p_k)$  is adjusted accordingly during retrieval to maintain an accurate representation of trustworthiness.

Then, player  $p_i$  use the LLM to process this observation and update the trustworthiness of  $p_k$ . The update depends on several factors: the perceived trustworthiness of  $p_j$ , the strength  $|w_i^t(p_j, p_k)|$  of the  $e_i^t(p_j, p_k)$ , and  $p_j$ 's confidence  $c_i^t(p_j, p_k)$  of the  $p_k$ 's current role classification  $\mathcal{R}_i^t(p_k)$  within the interval [-1,1]. The updated trustworthiness  $T_i^t(p_k)$  is computed as follows:

$$u_i^t(p_k) = T_i^t(p_j) \cdot |w_i^t(p_j, p_k)| \cdot c_i^t(p_j, p_k)$$
 (2)

$$T_i^{t+1}(p_k) = \begin{cases} T_i^t(p_k) \text{ if } |u_i^t(p_k)| \le |T_i^t(p_k)| \\ u_i^t(p_k) \text{ if } |u_i^t(p_k)| > |T_i^t(p_k)| \end{cases}$$
(3)

where  $u_i^t(p_k)$  represents the inference of  $p_k$ 's trustworthiness through the observation  $o_i^t(p_i)$ .

# **Graph Retrieval Augmented Reasoning**

During a player  $p_i$ 's turn, particularly when deciding on an action involving player  $p_o$ , the reasoning process is augmented by retrieving and leveraging relevant evidence from the evidence graph  $G_i^t$ . This graph-based retrieval augments

Algorithm 1: Graph Update Process

**Input**: Graph  $G_i^t$ , observations  $o_i^t$ 

- 1: Query the LLM to extract  $\{\mathcal{R}_i^t(p_j, p_k), c_i^t(p_j, p_k), c_i^t(p_j, p_k), w_i^t(p_j, p_k)\}$  from the observation  $o_i^t$ ;

  2: **for** each intention  $e_i^t(p_j, p_k)$  **do**
- Update the evidence list  $h_i^{t+1}(p_j, p_k)$  using Eq. (1);
- 4: end for
- 5: for each player  $p_k$  connected by an edge  $E_i^t(p_i, p_k)$  do
- Update the trustworthiness  $T_i^t(p_k)$  using the Eqs.
- Store the new evidence  $o_i^t(p_k)$  in the graph  $G_i^t$ ; 7:
- 8: end for

the player's trustworthiness assessment by incorporating historical evidence into the reasoning process. The retrieval process is divided into three key phases: evidence merging, forward retrieval, and backward update, and reasoning.

**Evidence Merging** In this phase, the objective is to aggregate and evaluate the various pieces of evidence collected by player  $p_i$  over time, specifically related to the interactions between players  $p_i$  and  $p_k$ . Assume that player  $p_i$  has n pieces of evidence  $e_i^t(p_j, p_k)$  towards player  $p_k$  in the evidence list  $h_i^t(p_i, p_k)$  associated with the directed edge  $E_i^t(p_j, p_k)$ . The evidence is sorted in chronological order, with each piece of evidence having an associated weight  $w_i^t(p_j, p_k)$  and a temporal importance factor  $\rho$ . The updated edge weight  $\tau_i^{t+1}(p_j, p_k)$  is computed as follows:

$$\tau_i^{t+1}(p_j, p_k) = \tanh\left(\sum_{k=1}^n \rho^{n-k} \cdot w_i^t(p_j, p_k)\right)$$
(4)

where the impact of evidence decreases over time, with more recent evidence having greater influence. The tanh function is used to constrain the edge weight  $\tau_i^{t+1}(p_i, p_k)$ within the interval [-1, 1], providing a bounded measure of the trustworthiness between players.

Forward Retrieval Given that player  $p_i$  holds a trustworthiness value  $T_i^t(p_1)$  towards player  $p_1$ , if there exists a evidence chain  $\mathcal{C}_n: p_o \to p_{o-1} \to \cdots \to p_1$ , the value  $V_{\mathcal{C}_n}$  of this evidence chain and the cumulative trustworthiness update  $u_i^t(p_o)$  towards player  $p_o$  are computed as follows:

$$V_{\mathcal{C}_n} = \sum_{k=1}^{o-1} T_i^t(p_{k+1}) \cdot \tau_i^t(p_{k+1}, p_k)$$
 (5)

$$u_i^t(p_o) = T_i^t(p_1) \cdot \prod_{k=1}^{o-1} \tau_i^t(p_{k+1}, p_k)$$
 (6)

The uncertainty associated with the chain  $C_n$  is defined

$$H(\mathcal{C}_n) = -u_i^t(p_o)\log_2 u_i^t(p_o) \tag{7}$$

For the player  $p_o$  with m related evidence chains  $\mathcal{C}_1, \mathcal{C}_2, \dots, \mathcal{C}_m$ , the updated trustworthiness  $T_i^t(p_o)$  is given

$$T_i^{t+1}(p_o) = \frac{\sum_{n=1}^{m} (V_{\mathcal{C}_n} - H(\mathcal{C}_n)) \cdot u_i^t(p_o)}{\sum_{n=1}^{m} (V_{\mathcal{C}_n} - H(\mathcal{C}_n))}$$
(8)

where the trustworthiness update is a weighted sum of the relevant evidence chains, where each chain's weight is determined by its value and associated uncertainty.

**Backward Update** Once  $T_i^t(p_o)$  is updated, the edge weights associated with the relevant evidence chains need to be updated in reverse:

$$\tau_i^{t+1}(p_o, p_{o-1}) = \gamma \cdot \frac{T_i^t(p_o)}{T_i^t(p_{o-1})} + T_i^t(p_{o-1})$$
 (9)

Here,  $\gamma$  represents the learning rate for the backward update, and  $p_{j-1}$  is the preceding player in the evidence chain  $C_i$ .

**Reasoning** After updating the trustworthiness of player  $p_i$  towards  $p_o$ , A summary and reasoning is made based on the trustworthiness of player  $p_o$  and the relevant evidence chains retrieved. Specifically, the trustworthiness of player  $p_i$  towards  $p_o$  and the evidence chains are combined into a prompt that is sent to LLM, which ultimately returns the summary and reasoning of the player  $p_o$ . The prompt used are shown in Appendix 3.

# **Experiment**

In this section, we choose Xu's algorithm (Xu et al. 2024) as the baseline LLM. We validate our approach through experiments on the multiplayer game "Werewolf", comparing GRATR with baseline LLM and LLMs enhanced with Native RAG and Rerank RAG in terms of win rates, performance, and reasoning ability. The source code of the proposed GRATR is available at https://github.com/EvoNexusX/Graph-Retrieval-Augmented-Trustworthiness-Reasoning.

#### Setup

We implemented our GRATR method using the classic multi-player game "Werewolf". We used the framework of the Werewolf environment implemented by Ref (Xu et al. 2024). We set the number of players to 8 in the Werewolf game, with three leaders: the witch, the guard, and the seer, three werewolves, and two villagers. The history message window size K is set to 15. We use the gpt-3.5-turbo model as the backend LLM, and its temperature is set to 0.3.

We added Native RAG and Rerank RAG to the baseline LLM which help to enhance reasoning by retrieve history messages to generate two new compared algorithms. We tested GRATR with baseline LLM and LLM enhanced with Native RAG and Rerank RAG on the Werewolf framework and each algorithm was compared in 50 games.

In each game, there are four players on each algorithm, three of whom are randomly assigned to the leader and werewolf sides and the remaining one is assigned to the village side. When one side wins, the algorithm corresponding to that side is considered to have won.

# Algorithm 2: Graph Retrieval Augmented Reasoning

**Input**: Number of the selected top trustworthiness nodes w, the target player  $p_o$ ;

**Initialization**: Players  $p_1, \ldots, p_n$ ; Nodes N in  $G_i^t$ ; Evidence chains list  $C:[\mathcal{C}_1,\mathcal{C}_2,\ldots,\mathcal{C}_w]$  for player  $p_o$  (initially empty); Priority Queue  $Q=\emptyset$ ;

```
1: N \leftarrow \text{Sort}(N, T_i^t(n)) //Sort N by trustworthiness. 2: \{n_1, n_2, \dots, n_w\} = \text{Top-}w(N) // Select the top w
 3: Q \leftarrow \{n_1, n_2, \dots, n_w\}
4: for j = 1, 2, \dots, w do
5: \mathcal{C}_j \leftarrow \{n_j\}
 7: while Q \neq \emptyset do
         n_c, \mathcal{C}_c \leftarrow \operatorname{argmax}_{n \in Q} T_i^t(n)
          Q \leftarrow Q \setminus \{n_c\}
          for each n_k \in \text{Neighbors}(n_c) do
10:
              Merge evidence e_i^t(p_k, p_c) to update \tau_i^t(p_k, p_c)
11:
              based on the equation. // p_k, p_c are the players cor-
              responding to the nodes n_k, n_c.
              if player p_k \neq \text{player } p_o then
12:
13:
                  Update T_i^t(p_k) based on the equations.
14:
15:
                  Update u_i^t(p_o) based on the equations.
16:
              end if
17:
          end for
          n_{k^*} \leftarrow \operatorname{argmax}_{n_k \in \operatorname{Neighbors}(n_c)} T_i^t(p_k)
18:
         C_c \leftarrow C_c \cup \{n_{k^*}\}\
Q \leftarrow Q \cup \{n_{k^*}\}\
19:
20:
21: end while
22: Use the C to update T_i^t(p_o) based on the equations.
23: Summarize and reason based on T_i^t(p_o) and the evi-
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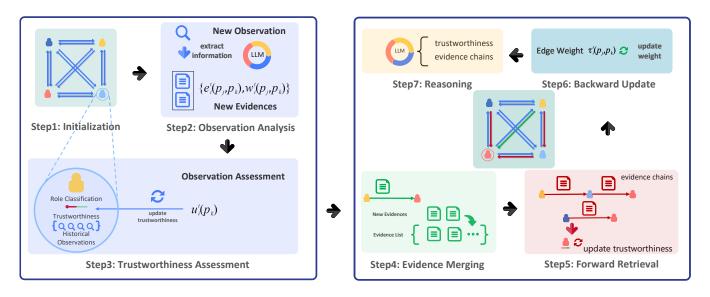
# **Experiment Results**

dence chains retrieved C.

**Win Rate Analysis** The Table 2 shows the comparison of the winning rates of GRATR, baseline LLM and LLM enhanced with Native RAG and Rerank RAG in the same game

In group 1, the GRATR algorithm significantly outperforms the baseline LLMs across all evaluated metrics. Specifically, the total win rate achieved by GRATR is 76.0%, markedly higher than the 24.0% attained by the baseline LLM. Moreover, GRATR demonstrates a substantial advantage in the Werewolf's win rate, achieving 72.4% compared to 27.6% for the baseline LLM. Additionally, the GRATR algorithm leads to a notably higher win rate for the leaders (81.0%), in contrast to the 19.0% observed with the baseline LLM. These results highlight the superior effectiveness of the GRATR algorithm in optimizing game outcomes across different roles.

In group 2, the win rate of the GRATR algorithm is compared against baseline LLM and LLM enhanced with Native RAG and Rerank RAG in the Werewolf game. The GRATR algorithm demonstrates a clear advantage, achieving a total win rate of 66.5%, which is nearly double that of the LLM enchanced with Native RAG approach at 33.5%. Further-



#### **Observation Stage**

#### **Reasoning Stage**

Figure 2: The overall framework of GRATR: Step 1. The process begins by initializing a graph-based evidence base  $G_i$  for each player  $p_i$ . Step 2. In each round of the game, player  $p_i$  observes their private history  $h_i^t$  and selects an action  $a_i^t$  based on a strategy function  $s_i(h_i^t,\theta_i)$ . Step 3. After executing all players' actions  $\mathbf{a}^t$ , player  $p_i$  receives new observations  $o_i^t$ . The evidence graph  $G_i$  is updated with new observations. Step 4,5,6. Trustworthiness values and combined evidence are retrieved and updated from the graph  $G_i$  using the retrieval and update algorithm (Algorithm 2). Step 7. Finally, player  $p_i$  summarizes and reasons based on the updated trustworthiness and evidence to guide their further actions. The selected action  $a_i^t$  is then executed.

Table 2: Comparison of win Rates among algorithms in the werewolf game.

Group	p Method	TWR	WWR	LWR
1	GRATR Baseline	<b>0.760</b> 0.240	<b>0.724</b> 0.276	<b>0.810</b> 0.190
2	GRATR Baseline+Native RAG	<b>0.665</b> 0.335	<b>0.720</b> 0.280	<b>0.720</b> 0.280
3	GRATR Baseline+Rerank RAG	<b>0.837</b> 0.335	<b>0.835</b> 0.280	<b>1.000</b> 0.280

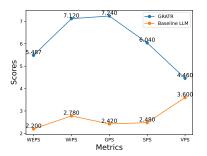
more, the GRATR algorithm significantly boosts the win rate of the Werewolf identity to 72.0%, compared to the 28.0% recorded for the other. The Leaders' (witch, guard, seer) win rate under GRATR also reflects this trend, standing at 72.0% as opposed to the 28.0% seen with LLM enchanced with Native RAG. This comparison illustrates the effectiveness of the GRATR algorithm in consistently enhancing the win rates across different player roles within the game.

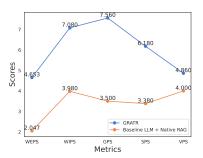
Group 3 provides a comparison between the GRATR algorithm and the combination of LLM enhanced Rerank RAG in the Werewolf game. The results clearly indicate the superior performance of the GRATR algorithm across all metrics. Specifically, GRATR achieves a total win rate of 83.7%, significantly higher than the 16.3% recorded by the LLM enhanced Rerank RAG. Similarly, GRATR's Were-

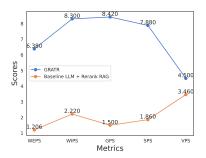
wolf win rate is 88.5%, while the LLM enhanced Rerank RAG manages only 11.5%. The disparity is most pronounced in the Leaders' win rate, where GRATR reaches a perfect 100%, compared to the 0.0% of the LLM enhanced Rerank RAG. These data suggest that, although Rerank RAG attempts to account for the correlations among evidence, this approach appears insufficient for effective reasoning in this context. In fact, the integration of Rerank RAG might even hinder the reasoning process, as evidenced by the significantly lower win rates across all roles when using the LLM enhanced Rerank RAG.

Game performance In this section, we will evaluate and compare the performance of GRATR and other algorithms in Werewolf games. Specifically, the winning party will receive 5 victory point, the vote weight of the werewolf is 0.5 point, the vote weight of the villager is 1 point, and the vote weight of the leader is 1.5 point. During the day, each round of voting correctly votes for the enemy to get the vote point, and votes for teammates to deduct the corresponding vote weight points. The performance score calculation table is as follows:

In Fig. 3a, the GRATR algorithm consistently outperforms the original algorithm across all roles in the Werewolf game. "WEPS", "WIPS", "GPS", "SPS" and "VPS" represent the average performance scores of the werewolf, witch, guard, seer and villager identities, respectively. GRATR excels most in the guard role, scoring 7.240, which is 4.820 points higher than the original algorithm, highlighting its su-







(a) Performance comparison GRATR and the baseline LLM.

between (b) Performance comparison between (c) Performance comparison between GRATR and LLM enhanced with Native GRATR and LLM enhanced with Native RAG.

Figure 3: Overall performance comparison of GRATR against the baseline LLM and LLM enhanced with Native RAG and Rerank RAG.

Table 3: "WS" stands for the score earned by winning, "CVS" stands for the score earned by correctly voting for the enemy during the day, and "WVS" stands for the score earned by incorrectly voting for a teammate during the day.

Identity	WS	CVS	WVS
Werewolf	5.0	0.5	-0.5
Witch	5.0	1.5	-1.5
Guard	5.0	1.5	-1.5
Seer	5.0	1.5	-1.5
Villager	5.0	1.0	-1.0

perior defensive strategy. In the witch role, GRATR scores 7.120, surpassing the original by 4.340 points, demonstrating stronger decision-making abilities. For the werewolf role, GRATR achieves a score of 5.487, 3.287 points above the original, indicating better deception and manipulation. The prophet role sees GRATR scoring 6.040, 3.560 points higher, reflecting improved identification and revelation skills. Overall, GRATR demonstrates superior strategic execution and adaptability across all roles.

In Fig. 3b, GRATR shows clear advantages across all roles in the Werewolf game. For the werewolf role, GRATR achieves a score of 4.653, which is 2.606 points higher than the 2.047 score of the original algorithm with Native RAG, indicating superior performance in deception and strategy. In the Witch role, GRATR scores 7.080, outperforming the original by 3.100 points, reflecting enhanced decision-making and potion use. The guard role demonstrates GRATR's notable strength, with a score of 7.560, exceeding the original by 4.060 points, highlighting its superior defensive capabilities. The seer role also sees significant improvement, with GRATR scoring 6.180, 2.800 points higher than the original's 3.380, showcasing better identification and revelation skills. Finally, in the villager role, GRATR scores 4.860, surpassing the original by 0.860 points, indicating a more consistent and reliable performance. Overall, GRATR's enhanced scores across these

roles suggest a more effective and adaptable strategy compared to the original algorithm with Native RAG.

In Fig. 3c, the GRATR algorithm demonstrates a marked improvement over the original algorithm assisted by Rerank RAG across all roles within the Werewolf game. Specifically, GRATR achieves a 5.174 point higher performance in the Werewolf role (6.380 compared to 1.206), a 6.080 point increase in the Witch role (8.300 versus 2.220), and a 6.920 point enhancement in the Guard role (8.420 as opposed to 1.500). These substantial improvements across diverse roles underscore the superior strategic execution, enhanced defensive mechanisms, and overall effectiveness of the GRATR algorithm.

Native RAG and Rerank RAG enhance Werewolf performance by improving character decision-making and information processing. However, their impact on overall game strategy is limited. GRATR offers broader strategic optimization through graph retrieval and trustworthiness reasoning, leading to superior performance in complex interactions, defense, deception, and decision-making.

#### Conclusion

This paper has presented a novel framework called Graph Retrieval Augmented Trustworthiness Reasoning (GRATR). GRATR employs a dynamic trustworthiness relationship graph, updated in real-time with new evidence, to enhance the accuracy and reliability of trustworthiness assessments among players. The framework consists of two main phases: During the player observation phase, evidence is collected to update the nodes and edges of the graph in real-time. During the player's turn, the relevant evidence chain and the trustworthiness of the player's action object are retrieved to improve reasoning and decision-making. The effectiveness of GRATR is demonstrated through experiments in the multiplayer game 'Werewolf', where it outperforms existing methods in key metrics such as game winning rate, overall performance, and reasoning ability. It also effectively solves the illusion of large language models, and enables the traceability and visualization of the reasoning process through time evidence and evidence chains.

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# **Appendix**

# Reasoning ability

To verify the effectiveness of our algorithm in trustworthiness reasoning, we compared our algorithm with the original algorithm, the original algorithm assisted by Native RAG, and the original algorithm assisted by Rerank RAG in trustworthiness reasoning test on Werewolf.

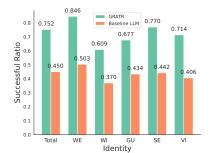
Specifically, we counted and compared the ratio of each algorithm that successfully reasoned about other identities under different identities.

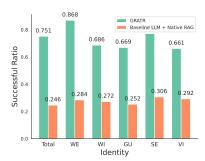
In Fig. 4a, "WE" means werewolf, "WI" means witch, "GU" means guard, "SE" means seer, "VI" means villager. The GRATR algorithm demonstrates a marked improvement in reasoning capabilities over the original algorithm across all roles within the Werewolf game. GRATR achieves an overall success ratio of 0.752, significantly surpassing the original algorithm's 0.450. Notably, GRATR exhibits exceptional performance in the Werewolf role, with a success ratio of 0.846, and in the Seer role, with a success ratio of 0.770, indicating a superior ability to both conceal its identity and accurately infer the identities of other players. Even in the Witch role, where GRATR's success ratio is relatively lower at 0.609, it still substantially outperforms the original algorithm. These findings underscore the enhanced strategic inference capabilities of GRATR across diverse roles in the game.

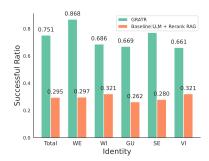
In Fig. 4b, compared to the original algorithm supported by Native RAG, the GRATR algorithm shows a significant improvement in reasoning success in all roles. Specifically, in the werewolf role, GRATR achieves a success rate of 0.868, which is significantly higher than the 0.284 achieved by the original algorithm. This indicates GRATR's superior ability to minimize false inferences, often referred to as 'phantoms', thereby allowing more accurate inferences to be made about other identities. For the villager role, GRATR achieved a success rate of 0.661, compared to 0.292 for the original algorithm. Despite the inherent limitations of the villager role and the lack of special abilities, GRATR's advanced reasoning capabilities allow it to draw more accurate conclusions from minimal information, demonstrating the algorithm's effectiveness in making the most of limited data.

In Fig. 4c, the GRATR algorithm shows a significant advantage over the original algorithm augmented with Rerank RAG in reasoning about other identities. GRATR achieves an overall success rate of 0.751, significantly higher than the 0.295 achieved by the original algorithm. This improved performance is evident across all roles: in the werewolf role, GRATR's success rate is 0.868, significantly higher than the original algorithm's 0.297. In the witch role, GRATR records a success rate of 0.686, compared to the original's 0.321, indicating superior effectiveness in exploiting the witch's unique abilities. Similarly, GRATR has a success rate of 0.669 in the Guardian role and 0.771 in the Seer role, both of which are significantly higher than the original algorithm's corresponding success rates of 0.262 and 0.280. In addition, GRATR's success rate of 0.661 in the villager role exceeds the original's 0.321, demonstrating its effectiveness in making accurate inferences from limited information. These results underscore GRATR's overall superiority in accurate role identification and strategic reasoning across different roles.

The GRATR algorithm exhibits a significant enhancement in the werewolf role by effectively minimizing the occurrence of incorrect inferences, often referred to as "phantoms," within the reasoning process. This reduction in erroneous reasoning allows the Werewolf to maintain its deceptive role more effectively and make more accurate deductions about other players' identities. In the villager role, GRATR demonstrates an impressive ability to derive strong reasoning outcomes using minimal information. Despite the villager's inherent limitations and lack of special abilities, GRATR's advanced inference capabilities enable it to make accurate conclusions, showcasing the algorithm's efficiency in maximizing the utility of limited data.







(a) The successful ratio comparison between (b) The successful ratio comparison between GRATR and the original algorithm.

GRATR and LLM enhanced with Native RAG.

(c) The successful ratio comparison between GRATR and LLM enhanced with Rerank RAG.

Figure 4: Overall successful ratio comparison of GRATR against the original algorithm and its variants assisted with Native RAG and Rerank RAG.

#### Appendix 1. Extract identity judgments from the evidence

There were [number of players] players in the game: [player's information].

Given the one player's statement. Through the statement, think step by step, you need to infer the identity of some players with a confidence level. Make sure the inference is reasonable. If you are the Seer, then inferring that player's identity cannot be the Seer, because there is only one Seer on the field. If you are a Witch, then it is impossible to infer that player's identity as a Witch, since there is only one Witch on the field. If you are a Guard, then it is inferred that the player's identity cannot be a Guard, since there is only one Guard on the field.

Note that you need to determine if the statement is spurious based on the players's identities you know and the identity of the speaking player. Of course, the moderator's statement must be correct. For example, if you are witch or a werewolf and the player speaks claiming to be a witch, you can infer that he might be a witch. However if you are a witch and the player speaks claiming to be a witch, you can infer that he is lying and thus infer that he is probably a werewolf.

Infer and choose from the option:[options fo identities]. The confidence level is greater than or equal to 0 and less than 10, the more certain and more evidence there is about the player's identity, the higher the confidence level, and vice versa the lower the confidence level. When the confidence level is lower than 5, it is a bold inference, and conversely it is a more confident inference.). Please return this strictly in the format [Player][identity][confidence][analysis] with no additional output. If still unable to determine the identity, still view it as unsure.

Please note that the statement might address multiple players simultaneously. In such cases, list each relevant result separately instead of in one line!!!.

Here are some examples:

Examples:

1.Statement: (Moderator):(1-th daytime) Player 2, Player 5 are villager, witch. Answer:[Player 2][villager][10][moderator's statement is right.]

[Player 5][witch][10][moderator's statement is right.]

2.Statement: (Player 3):(1-th daytime) I'm the seer. Last night I checked out Player 4. He's a werewolf. Answer:[Player 3][seer][8][I'm not a seer, and no one before Player 3 has declared him to be a seer, so it can be inferred that it might be a seer with a confidence level of 8.]

[Player 4][werewolf][8][Player 3 is inferred to be the seer, and he checked Player 4 as a werewolf last night, so it can be inferred that Player 4 is a werewolf.]

[inference]. Now given the statement:

Statement:[statement]

#### Appendix 2. Extract intent from the evidence

Werewolf is a role-playing deduction game where players use their statements to attack, defend, or deceive other players.

You are an expert in analyzing language within the context of the game Werewolf."

Your task is to analyze a given player's statement and determine its type."

Based on the statement provided, determine which of the following types it belongs to:

Attack: The player attempts to question or accuse another character, suggesting they might be suspicious, or provide evidence against another character, suggesting they might be a werewolf..

Defend: The player tries to defend a character, suggesting they are not suspicious. Note that character A and character B must are the members of [Player 1, Player 2,...] instead of their game role, and might be the same, meaning the statement might be self-defense.

Deceive: The player attempts to mislead other players with false information.

Additionally, provide a score indicating the strength or certainty of the statement's intent on a scale of 0 to 10 where 0 is very weak/uncertain and 10 is very strong/certain.

Carefully read the following statement and determine its type based on its content and tone: [Player's statement], Please choose the appropriate type and briefly explain your reasoning in the following format: [Role 1][Type][Role 2][Reason][Score]. Please note that the statement might address multiple players simultaneously. In such cases, list each relevant result separately instead of in one line!!!.

Here are some examples:

#### Example:

1.Statement: [(Player 1): I think Player 2's behavior was very strange. He kept avoiding important discussions. I believe Player 4 is innocent because he has been helping us.]

Answer: [Player 1][Attack][Player 2][The Player 1 is questioning Player 2's behavior, implying they might be suspicious.][6][Player 1][Defend][Player 2][The Player 1 is defending Player 4, suggesting they are not suspicious.][7]

2.Statement: [(Player 4):I observed that Player 3 was identified as a werewolf by the moderator. I believe we should carefully consider the roles of the remaining players and gather more information before making any decisions.]

Answer: [Player 4][Attack][Player 3][The current player indirectly accuses Player 4 of being a werewolf by mentioning the moderator's identification, influencing others' perceptions.][9]

3. Statement: [(Player 7): I believe Player 4 is innocent. He has been helping us analyze the situation.]

Answer: [Player 7] [Defind] [Player 4] [The Player 7 is defending Player 4, suggesting they are not suspicious.] [7]

4. Statement: [(Player 1):I choose to eliminate Player 3.]

Answer: [Player 1] [Attack] [Player 3] [The Player 1 is strongly attacting Player 7.] [10]

5. Statement: [(Player 2):I choose to protect Player 3.]

Answer: [Player 2] [Defend] [Player 3] [The Player 1 is strongly protecting Player 7.] [10]

(End of Example)

Now given the statement:

Statement:[statement]

#### **Appendix 3. Summary and Augment Reasoning**

```
Player 1 is inferred to be a [identity], my [judge]. My level of trust in him is [confidence][evidences].

Player 2 is inferred to be a [identity], my [judge]. My level of trust in him is [confidence][evidences].

Player 3 is inferred to be a [identity], my [judge]. My level of trust in him is [confidence][evidences].

Player 4 is inferred to be a [identity], my [judge]. My level of trust in him is [confidence][evidences].

Player 5 is inferred to be a [identity], my [judge]. My level of trust in him is [confidence][evidences].

Player 6 is inferred to be a [identity], my [judge]. My level of trust in him is [confidence][evidences].

Player 7 is inferred to be a [identity], my [judge]. My level of trust in him is [confidence][evidences].
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