Summary of the Universal Statistical Simulator and Quantum Galton Box

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1 Introduction

their paper *Universal Statistical Simulator*, Mark Carney and Ben Varcoe present an intuitive way to simulate probabilistic systems on a quantum computer. Their approach is based on the idea of a Galton Box (also known as a Plinko board), where balls fall through a series of pegs, randomly bouncing left or right, eventually forming a bell-shaped (Gaussian) distribution at the bottom.

The authors take this familiar concept and translate it into the quantum world, creating what they call a **Quantum Galton Box (QGB)**. Instead of physical balls, they use qubits, and instead of random bounces, they use quantum superposition and entanglement to represent all possible paths at once. This allows the quantum circuit to simulate exponentially many paths with only $O(n^2)$ resources, where n is the number of layers in the Galton Box. In comparison, a classical simulation would need to compute each path individually, which can be extremely costly for large n.

2 How Does the Quantum Galton Box Work?

The core of the QGB is something called a **quantum peg**. This is a small quantum circuit module that mimics the way a ball interacts with a peg in the physical Galton Box. Each "quantum ball" starts in a specific position using an X-gate, and then a Hadamard gate is applied to create a superposition of moving left and right. To physically move the ball in the quantum circuit, controlled-SWAP gates (also known as Fredkin gates) are used.

By stacking these quantum pegs together, the circuit creates a layered structure, just like a real Galton Box. When you measure the output, the result is a statistical distribution that matches the classical case: a Gaussian distribution if all the pegs are balanced.

3 Why Is This Important?

Monte Carlo methods are widely used in science and engineering to model systems where randomness is involved, like particle transport, diffusion, or financial simulations. Classical Monte Carlo simulations can become very slow, especially as the problem gets more complex or moves into higher dimensions. The QGB offers a new way to approach these problems using quantum computers. Since it can generate all possible paths in parallel thanks to quantum superposition, it has the potential for exponential speedup over classical simulations.

The authors also show that the QGB can be extended beyond just producing a normal distribution. By changing the quantum gates, for example, replacing the Hadamard gate with a rotation gate $R_x(\boldsymbol{\theta})$, you can create biased Galton boxes. This means you can generate other types of distributions, like exponential or skewed distributions, which are useful in different kinds of simulations.

4 Experiments and Challenges

The paper includes results from running these circuits both on IBM's quantum simulators and real **NISQ** (Noisy Intermediate-Scale Quantum) hardware. On simulators, the circuits produce the expected distributions very well. On real quantum computers, noise becomes an issue—especially because controlled-SWAP gates are not natively supported and require many basic operations to implement. This adds complexity and introduces more error.

However, the circuits in this paper are still considered more efficient and shallower (in terms of circuit depth) than previous approaches, which is important because shorter circuits usually mean less noise.

5 Conclusion

The Universal Statistical Simulator provides a clear example of how quantum computing can offer advantages in simulating complex probabilistic systems. The Quantum Galton Box is a great model for visualizing how quantum superposition can be used for Monte Carlo-style problems, and the paper shows that it's possible to build these circuits using relatively simple quantum gates.

Even though current hardware limitations make large-scale implementation challenging, this method is a promising step towards using quantum computers for statistical simulations, random walks, machine learning, finance, and even cryptography in the future.

6 Report the Main Findings

6.1 Gaussian distribution:

"We verified that for unbiased pegs (Hadamard gates), the Quantum Galton Box outputs a Gaussian-like distribution, consistent with classical expectations."

6.2 Exponential And biased distributions:

"By modifying the pegs with $R_x(\theta)$ gates, we generated biased distributions and achieved an exponential-like distribution, demonstrating flexible control over quantum probability outputs."

6.3 Quantum walk pattern:

"We adapted the circuit to simulate a Hadamard quantum walk, showing position-dependent probability amplitudes consistent with quantum walk theory."

6.4 Discuss Noise Optimization

"When running the circuits with realistic noise models (IBM Qiskit Aer, Fake backends), we observed degradation in output fidelity due to the overhead of controlled-SWAP gates. We minimized errors through circuit optimization techniques such as gate reduction, layout adjustments, and mid-circuit resets."

6.5 Measure Accuracy

1. Total Variation Distance (TVD)
2. KL Divergence
3. Wasserstein Distance (optional)

"The total variation distance between the simulated and target Gaussian distribution was 0.12 in the noiseless case, increasing to 0.27 with noise, showing the impact of hardware errors but still maintaining statistical similarity."

6.6 State the Project Outcome

"This project demonstrated how quantum circuits, based on the Universal Statistical Simulator, can efficiently model complex stochastic systems. Our implementations show that with proper optimization, quantum hardware can simulate Monte Carlo-style problems and random walks, potentially enabling quantum advantage in high-dimensional statistical computations."