Multilayer linear regression solved example

numerically solved example of linear regression with two features calculating MSE on a dataset with input features, weight matrix and output in detail with every step solved

Example:

Suppose we have the following dataset:

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```
| X1  | X2  | Y  |
|-----|-----|----|
| 2   | 3   | 8  |
| 1   | 4   | 5  |
| 3   | 1   | 12 |
```

Linear Regression Model:

Our linear regression model is given by:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2$$

Initialization:

Let's initialize the weights:

$$\beta_0 = 1,\ \beta_1 = 0.5,\ \beta_2 = 0.5$$

Predictions:

Now, let's calculate the predicted values for each instance in the dataset:

1. For the first row (X1=2, X2=3):
   $$\hat{Y} = 1 + 0.5 \cdot 2 + 0.5 \cdot 3 = 1 + 1 + 1.5 = 3.5$$
2. For the second row (X1=1, X2=4):
   $$\hat{Y} = 1 + 0.5 \cdot 1 + 0.5 \cdot 4 = 1 + 0.5 + 2 = 3.5$$
3. For the third row (X1=3, X2=1):
   $$\hat{Y} = 1 + 0.5 \cdot 3 + 0.5 \cdot 1 = 1 + 1.5 + 0.5 = 3$$

Actual vs. Predicted:

lua

X1	X2	Y	Predicted Y
2	3	8	3.5
1	4	5	3.5
3	1	12	3

Mean Squared Error Calculation:

Mean Squared Error (MSE) is calculated as the average of the squared differences between actual and predicted values.

$$\text{MSE} = \frac{1}{n} \sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2$$

Let's calculate MSE:

$$\text{MSE} = \frac{(8-3.5)^2 + (5-3.5)^2 + (12-3)^2}{3} = \frac{17.25 + 2.25 + 81}{3} = \frac{100.5}{3} \approx 33.5$$

So, the Mean Squared Error for our linear regression model on this dataset is approximately 33.5.