

What is likelihood function?



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The likelihood function is a fundamental concept in statistics and probability theory. It plays a key role in the process of estimating the parameters of a statistical model based on observed data. The likelihood function is a measure of how well the parameters explain the observed data, and it is used in maximum likelihood estimation (MLE).

Given a statistical model with parameters θ and a set of observed data x , the likelihood function, denoted as $L(\theta; x)$, is a function that describes the probability of observing the given data under different values of the parameters θ . In other words, it quantifies the likelihood of different parameter values given the observed data.

The likelihood function is defined as the joint probability density function (or mass function) of the observed data, viewed as a function of the parameters. Mathematically, it is expressed as:

$$L(\theta; x) = f(x; \theta)$$

where:



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where:


- $L(\theta; x)$ is the likelihood function.
- θ is the parameter or vector of parameters.
- x is the observed data.
- $f(x; \theta)$ is the probability density function (PDF) or probability mass function (PMF) of the data given the parameters θ .

The idea behind maximum likelihood estimation is to find the values of the parameters that maximize the likelihood function. This can be expressed as:

$$\hat{\theta}_{\text{MLE}} = \arg \max_{\theta} L(\theta; x)$$

In practice, it is often more convenient to work with the log-likelihood function, denoted as $\ell(\theta; x)$, which is the natural logarithm of the likelihood function:

$$\ell(\theta; x) = \log L(\theta; x)$$

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Taking the logarithm simplifies calculations, and the logarithm does not change the location of the maximum point, which is what matters for finding the maximum likelihood estimates.

The likelihood function is a powerful tool in statistical inference, providing a formal way to make inferences about unknown parameters based on observed data. It is widely used in various fields, including biology, economics, engineering, and many others.