numerically solved example of binary classification calculating accuracy on a dataset with input features, weight matrix and output

Certainly! Let's walk through a numerical example of binary classification using a simple logistic regression model, including input features, weight matrix, and output.

Example:

Suppose we have the following dataset with two features (X1 and X2) and a binary target variable (Y):

lua Copy code

```
| X1  | X2  | Y  |
|-----|-----|----|
| 2   | 3   | 0  |
| 1   | 4   | 1  |
| 3   | 1   | 0  |
```

Logistic Regression Model:

Assume our logistic regression model is given by:

$$P(Y=1) = \frac{1}{1+e^{-(\beta_0+\beta_1 X1+\beta_2 X2)}}$$

We initialize our weight matrix as $\beta_0 = -1,\ \beta_1 = 0.5,\ \beta_2 = 1.$

Predictions:

Now, let's calculate the output for each instance in the dataset:

1. For the first row (X1=2, X2=3):
   $$Z = -1 + 0.5 \cdot 2 + 1 \cdot 3 = 3.5$$
   $$P(Y=1) = \frac{1}{1+e^{-3.5}} \approx 0.97$$
   Prediction: $\hat{Y} = 1$ (spam)
2. For the second row (X1=1, X2=4):
   $$Z = -1 + 0.5 \cdot 1 + 1 \cdot 4 = 3.5$$
   $$P(Y=1) = \frac{1}{1+e^{-3.5}} \approx 0.97$$
   $\hat{}$

$Z = -1 + 0.5 \cdot 1 + 1 \cdot 4 = 3.5$

$P(Y = 1) = \frac{1}{1+e^{-3.5}} \approx 0.97$

Prediction: $\hat{Y} = 1$ (spam)

3. For the third row (X1=3, X2=1):

$Z = -1 + 0.5 \cdot 3 + 1 \cdot 1 = 0.5$

$P(Y = 1) = \frac{1}{1+e^{-0.5}} \approx 0.62$

Prediction: $\hat{Y} = 1$ (not spam)

Actual vs. Predicted:

lua Copy code

```
| X1  | X2  | Y  | Predicted Y |
|-----|-----|----|-------------|
| 2   | 3   | 0  | 1           |
| 1   | 4   | 1  | 1           |
| 3   | 1   | 0  | 0           |
```

Accuracy Calculation:

In this example, there are 2 correct predictions out of 3 instances.

$$\text{Accuracy} = \frac{\text{Number of Correct Predictions}}{\text{Total Number of Instances}} = \frac{2}{3} \approx 0.67$$

So, the accuracy of our binary classificatio del on this dataset is approximately 67%.