A Gaussian distribution, also known as a normal distribution or bell curve, is a continuous probability distribution that is symmetric around its mean, meaning that the probability of observing a value x is the same as the probability of observing -x. The shape of the distribution is bell-shaped, and it is characterized by two parameters: the mean (μ) and the standard deviation (σ).

The probability density function (PDF) of a Gaussian distribution is given by the formula:

�(�;�,�)=1�2��−(�−�)22�2*f*(*x*;*μ*,*σ*)=*σ*2*π*1*e*−2*σ*2(*x*−*μ*)2

where:

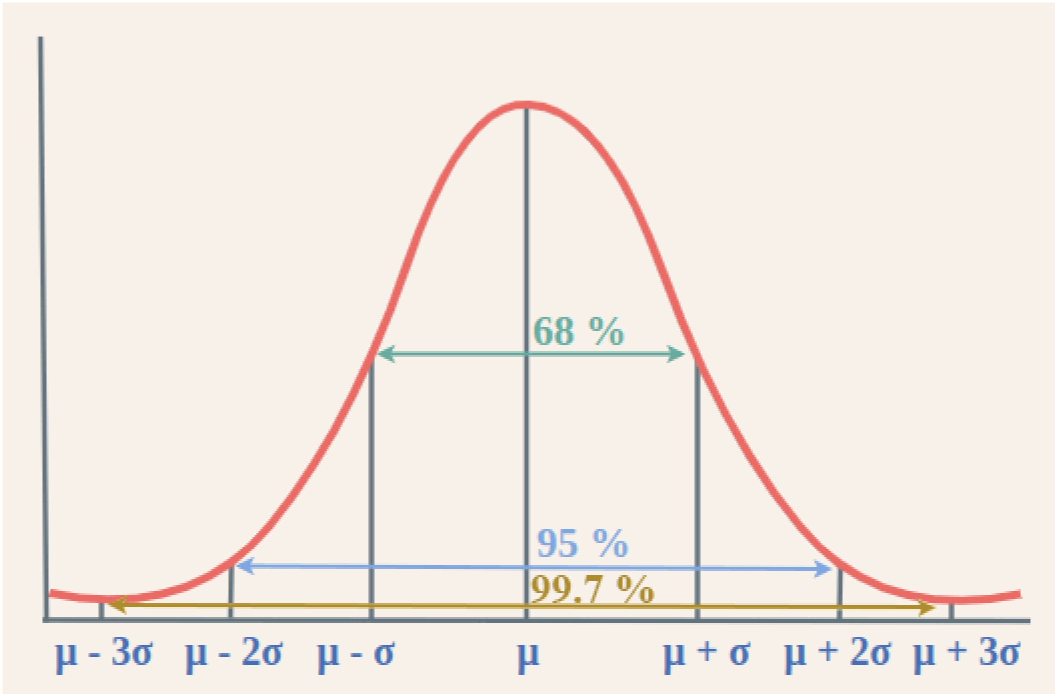
- �*x* is a random variable.
- �*μ* is the mean of the distribution.
- �*σ* is the standard deviation of the distribution.
- �*π* is the mathematical constant (approximately 3.14159).
- �*e* is the base of the natural logarithm (approximately 2.71828).

The graph of the Gaussian distribution is symmetric and bell-shaped, with the peak at the mean. The spread of the distribution is determined by the standard deviation: a larger standard deviation results in a wider, flatter curve, while a smaller standard deviation produces a narrower, taller curve.

Properties of the Gaussian distribution:

1. **68-95-99.7 Rule (Empirical Rule):** In a normal distribution, approximately 68% of the data falls within one standard deviation of the mean, 95% falls within two standard deviations, and 99.7% falls within three standard deviations.
2. **Central Limit Theorem:** The sum (or average) of a large number of independent, identically distributed random variables, each with any distribution, will be approximately normally distributed.
3. **Characterized by Mean and Standard Deviation:** The mean and standard deviation uniquely determine the shape and location of a Gaussian distribution.
4. **Standard Normal Distribution:** If �*X* is a normally distributed random variable with mean �*μ* and standard deviation �*σ*, then the standardized variable �*Z* given by �=�−��*Z*=*σX*−*μ* follows a standard normal distribution (mean = 0, standard deviation = 1).

The Gaussian distribution has widespread applications in statistics, probability theory, and various fields such as physics, finance, and engineering due to its simplicity and the ubiquity of natural processes that exhibit this distribution.


68 %
95 %
99.7 %
μ - 3σ
μ - 2σ
μ - σ
μ
μ + σ
μ + 2σ
μ + 3σ