The Poisson distribution is a discrete probability distribution that expresses the probability of a given number of events occurring in a fixed interval of time or space. It is named after the French mathematician Siméon Denis Poisson, who introduced the distribution in the early 19th century. The Poisson distribution is particularly useful in situations where events happen independently and at a constant rate.

The probability mass function (PMF) of the Poisson distribution is given by the formula:

�(�=�)=�−�·���!*P*(*X*=*k*)=*k*!*e*−*λ*·*λk*

where:

- �*X* is a random variable representing the number of events.
- �*k* is a non-negative integer (0, 1, 2, ...) representing the number of events.
- �*λ* is a positive real number representing the average rate of events occurring in the given interval.
- �*e* is the base of the natural logarithm (approximately 2.71828).

Key characteristics and properties of the Poisson distribution:

1. **Events are Independent:** The Poisson distribution assumes that events occur independently of each other. This means that the occurrence of one event does not affect the occurrence of another.
2. **Constant Rate:** The events occur at a constant average rate �*λ* over the specified interval.
3. **Discrete:** The Poisson distribution is discrete, meaning that it deals with countable outcomes (whole numbers) rather than continuous outcomes.
4. **Memoryless Property:** The Poisson distribution has the memoryless property, which means that the probability of a certain number of events occurring in the future is not influenced by past events. This property is useful in modeling scenarios where events happen randomly and independently.
5. **Parameter �*λ*:** The parameter �*λ* represents the average rate of events occurring in the given interval. It is also equal to the mean and variance of the distribution.

Applications of the Poisson distribution include modeling the number of phone calls received at a call center in a given time period, the number of emails received in an hour, or the number of accidents at a traffic intersection in a day.

If �*X* follows a Poisson distribution with parameter �*λ*, we write �∼Poisson(�)*X*∼Poisson(*λ*). The expected value (mean) and variance of a Poisson-distributed random variable are both equal to �*λ*.

Where,

- ***P(X = x)*** *is the probability that an event will occur x times,*

- ***X*** *is a random variable following a Poisson distribution,*
- ***λ*** *is the average number of times an event occurs,*
- ***x*** *is the number of times an event occurs, and*
- ***e*** is *Euler's constant (≈2.718).*