SARDAR PATEL MAHAVIDYALAYA, CHANDRAPUR(MH)

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CLASS SEMINAR

BY-

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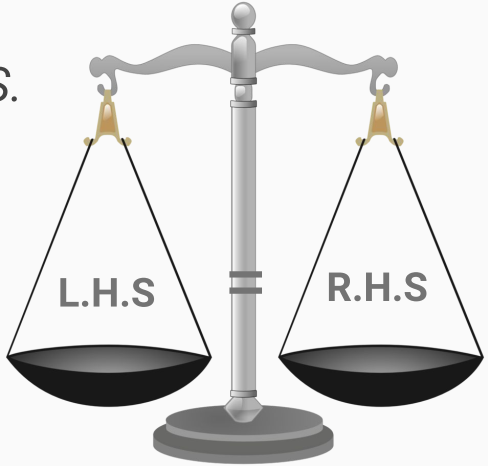
THEORY OF EQUATIONS

WHAT IS AN EQUATION?

An equation is like a balance scale.

Everything must be equal on both the sides i.e. L.H.S = R.H.S.



FACTOR THEOREM-

If a polynomial $f_n(x)$ is divisible by $(x-\propto)$, then α is a root of the equation $f_n(x) = 0$

Conversely, if α is a root of the equation $f_n(x) = 0$, then the polynomial $f_n(x)$ is divisible by $(x-\propto)$.

Now, let $\propto_1, \propto_2, \propto_3, \ldots\ldots$ be the given roots, then the required equation is:

$a_0(x-\propto_1)(x-\propto_2)(x-\propto_3)\ldots\ldots = 0$

Where, a_0 is constant.

EXAMPLE 1] *FORM AN EQUATION WHOSE ROOTS ARE 1,2 AND 3.*

SOLUTION:

As 1,2 and 3 are the roots of the required equation,

$\therefore$ Factors of the required equation will be (x-1), (x-2) and (x-3).

Hence, the required equation will be:

$(x-1)(x-2)(x-3)=0.$

$x^3-6x^2+11x-6=0.$

$\therefore$ $x^3-6x^2+11x-6=0$ is the required equation.

EXAMPLE 2] ***TWO ROOTS OF THE EQUATION $x^4-6x^3+18x^2-30x+25=0$ ARE OF THE FORM $\propto+\beta i$ AND $\beta+\propto i$, WHERE $\propto$ AND β ARE REAL. SOLVE IT COMPLETELY.***

SOLUTION:

$x^4-6x^3+18x^2-30x+25=0$ is fourth degree polynomial equation with real coefficients, so it has four roots. Two complex roots of it are given as $\propto+\beta i$ and $\beta+\propto i$.

Hence, $\propto-\beta i$ and $\beta-\propto i$ are the remaining roots.

The equation corresponding to the roots $\propto\pm\beta i$ and $\beta\pm\propto i$ will be:

$[x-(\propto+\beta i)][x-(\propto-\beta i)][x-(\beta+\propto i)][x-(\beta-\propto i)]=0$

On solving, we get:

$x^4-2(\propto+\beta)x^3+2(\propto^2+\beta^2+2\propto\beta)x^2-2(\propto+\beta)(\propto^2+\beta^2)x+(\propto^2+\beta^2)^2=0$

On comparing with the given equation, we get :

$\propto+\beta=3 \Rightarrow \beta=3-\propto$................................ I

$\propto^2+\beta^2+2\propto\beta=9$.................................... II

$(\propto+\beta)(\propto^2+\beta^2)=15$............................ III

$(\propto^2+\beta^2)^2=25 \Rightarrow (\propto^2+\beta^2)=5$............ IV

From II $\Rightarrow 5+2\propto\beta=9 \Rightarrow \propto\beta=2 \Rightarrow \propto(3-\propto)=2 \Rightarrow \propto^2-3\propto+2=0 \Rightarrow \propto=1,2$.

If $\propto=1$ then $\beta=2$ and if $\propto=2$ then $\beta=1$.

Hence, the roots are $1\pm2i$, $2\pm1i$.

THANK YOU!