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① a) $ty^2y' + t^3 = y^3$ Eq diferencial de Bernoulli

$y' + p(x)y = q(x)y^n$

EDO Bernoulli

$$\frac{\cancel{t}\cancel{y^2}y'}{\cancel{t}\cancel{y^2}} + \frac{t^3}{\cancel{t}y^2} = \frac{y^3}{ty^2} \Rightarrow y' + \frac{t^2}{y^2} = \frac{y}{t} \Rightarrow y' + \frac{t^2}{y^2} - \frac{t^2}{y^2} = \frac{y}{t} - \frac{t^2}{y^2} \Rightarrow$$

$$\Rightarrow y' = \frac{y}{t} - \frac{t^2}{y^2} \Rightarrow y' - \frac{y}{t} = \frac{y}{t} - \frac{t^2}{y^2} - \frac{y}{t} \Rightarrow y' - \frac{y}{t} = -\frac{t^2}{y^2}$$

Reescrever forma geral $\Rightarrow y' - \frac{y}{t} = -t^2 y^{-2}$ $\quad p(t) = \frac{-1}{t}$ $\quad q(t) = -t^2$ $\quad n = -2$

$$v(t) = e^{\int (1-n)\, p(t)\, dt} \Rightarrow \int (1-(-2)) - \frac{1}{t}\, dt = \int 3 - \frac{1}{t}\, dt = \int \frac{3t-1}{t}\, dt = 3t - \ln t + C$$

FATOR INTEGRANTE

$$v(t) = e^{3t - \ln t} = t^{-1} e^{3t} = \frac{e^{3t}}{t} \qquad y(t) = \frac{\int [v(t)(1-n)q(t)]\, dt + c}{v(t)}$$

$$y(t) = \frac{\int \left[\frac{e^{3t}}{t} \cdot 3 \cdot (-t^2)\right] dt}{\frac{e^{3t}}{t}} \Rightarrow \frac{\int (-3te^{3t})\, dt^{*} + C}{\frac{e^{3t}}{t}} = \boxed{\frac{-e^{3t}t + \frac{1}{3}e^{3t} + C}{\frac{e^{3t}}{t}}}$$

$y(t)$

$$\int (-3te^{3t})\, dt = -3\int te^{3t}\, dt = -3\int \frac{e^u u\, du}{9} = -3 \cdot \frac{1}{9}\left(e^u u - \int e^u du\right) \Rightarrow \frac{-3}{9}\left(e^{3t} \cdot 3t - e^{3t}\right)$$

$u = 3$ $\qquad u = 3t$

*

$-e^{3t}t + \frac{1}{3}e^{3t} + C$

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EDO Bernoulli

1 b) $y^2 + 2t + 2t\,y y' = 0$

$y' + p(x)\,y = g(x)\,y^n$ FORMA GER

$$\frac{y^2}{2ty} + \frac{2t}{2ty} + \cancel{\frac{2tyy'}{2ty}} = \frac{0}{2ty} \Rightarrow \frac{y}{2t} + \frac{1}{y} + y' = 0 \Rightarrow \frac{y}{2t} + y' = -\frac{1}{y} \Rightarrow \boxed{y' + \frac{y}{2t} = -y^{-}}$$

$$u(t) = e^{\int (1-n)\,p(t)\,dt}$$

$p(t) = \frac{1}{2t}$ $g(t) = -1$ $n = -1$

$$\int 2 \cdot \frac{1}{2t}\,dt = \ln t + C$$

$$u(t) = e^{\ln t} = \boxed{t}$$

FATOR INTEGRANTE

$$y(t) = \frac{\int (t \cdot 2 \cdot (-1))\,dt}{t} + \frac{C}{t} = \frac{\int -2t\,dt}{t} + \frac{C}{t} = \boxed{\frac{-t^2 + C}{t}}$$

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(1) c) $(e^{2y} - y\cos(ty))dt + (2te^{2y} - t\cos(ty) + 2y)dy = 0$

EDO exata $M(x,y) + N(x,y)y' = 0$

$e^{2y} - y\cos(ty) + (2te^{2y} - t\cos(ty) + 2y)\frac{dy}{dt} = 0$ y a variável dependente

$\psi_x(x,y) = M(x,y) = e^{2f_1(t)} - f_1(t)\cos(tf_1(1))$

$\psi_y(x,y) = N(x,y) = 2te^{2f_1(t)} - t\cos(tf_1(t)) + 2f_1(t))$

$\exists\ \psi(x,y) \mid \psi_x(x,y) = M(x,y)$ e $\psi_y(x,y) = N(x,y)$

$\psi_x + \psi_y \cdot y' = \frac{d\psi(x,y)}{dx} = 0 \Rightarrow \psi(x,y) = C$

$\frac{\partial M(x,y)}{\partial y} \text{ (I)} = \frac{\partial N(x,y)}{dx} \text{ (II)}$ = verdadeiro

$\frac{\partial M}{\partial f_1(t)} = 2e^{2f_1(t)} - \cos(tf_1(1))' = (e^{2f_1(t)})' - (f_1(t)\cos(tf_1(t)))' = e^{2f_1(t)} \cdot 2 - (\cos(tf_1(t)) - tf_1(t)\ sen(tf_1(t)))$

$\Rightarrow \boxed{2e^{2f_1(t)} - \cos(tf_1(t)) + tf_1(t)\,sen(tf_1(t)) = \frac{\partial M}{\partial f_1(t)}}$ (I) = (II)

$\frac{\partial N}{\partial t} = 2e^{2f_1(t)} - \cos(tf_1(t)) + tf_1(t)\,sen(tf_1(t)) \Rightarrow \underset{\text{Constante}}{f_1(t)} \Rightarrow (2te^{2f_1(t)})' - (t\cos(tf_1(t)))' + (2f_1(t))'$

$\Rightarrow (2te^{2f_1(t)})' = 2e^{2f_1(t)} \mid (2f_1(t))' = 0 \Rightarrow 2e^{2}f_1(t) - (\cos(f_1(t))t) - f_1(t)t\,sen(f_1(t)t) + 0$

$\Rightarrow \boxed{2e^{2f_1(t)} - \cos(tf_1(t)) + tf_1(t)\,sen(tf_1(t)) = \frac{\partial N}{\partial t}}$ (II) = (I)

continua prox página $\Rightarrow$

$$\Psi(x,y) = \Psi(t, f_1) = f_1^2 - sen(tf_1) + te^{2f_1} + C_1$$

$$\int N\,df_1 = \underbrace{\int 2f_1\,df_1}\ - \underbrace{\int \cos(tf_1)\,t\,df_1}\ + \underbrace{\int 2t\,e^{2f_1}\,df_1}$$

$$\Psi(t, f_1) = \underbrace{f_1^2 - sen(tf_1) + te^{2f_1} + C_1}$$

$$C^2 = \Psi(t, f_1)$$

$$C_1 = \eta(t)$$

$$\eta(t) = \int 0\,dt = C_1$$

$$\eta(t) = C_1$$

combinar as constantes

$$\boxed{f_1^2 - sen(tf_1) + te^{2f_1} = C_1}$$

$$f_1^2 - sen(tf_1) + te^{2f_1} + C_1 = C2$$

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ED de segunda ordem não homogênea de EULER

(2) $t^2y'' - 3ty' + 4y = t^2 \ln t$

$ax^2y'' + bxy' + cy = g(x)$

$t^2y'' - 3ty' + 4y = 0 \Rightarrow \boxed{y = t^r} \Rightarrow t^2((t^r))'' - 3t((t^r))' + 4t^r = 0$

$(t^r)'' = rt^{r-2}(r-1)$

$(t^r)' = rt^{r-1}$

$t^2 r t^{r-2}(r-1) - 3t(t^r)' + 4t^r = 0$

$t^2 r t^{r-2}(r-1) - 3t r t^{r-1} + 4t^r = 0$

$r^2t^r - 4rt^r + 4t^r = 0$

$t^r(r^2 - 4r + 4) = 0$

Resolver $t^r(r^2 - 4r + 4) = 0$

$\boxed{r = 2}$ $a = 1 \quad b = -4 \quad c = 4$

$\Delta = 0$ $r_1 = 2 = \dfrac{-(-4) \pm \sqrt{0}}{2}$

uma raiz $\boxed{r = 2}$ multiplicidade 2

$y = c_1t^r + c_2\ln(t)t^r \Rightarrow c_1t^2 + c_2\ln(t)t^2$

y_p: $t^2y'' - 3ty' + 4y = t^2\ln(t)$

$y_1' = 2t$

$y_2' = 2t\ln(t) + t$

$W(y_1, y_2) = t^2(2t\ln(t) + t) - 2t t^2 \ln(t)$

$W(y_1, y_2) = t^3$

$y_p = v_1y_1 + v_2y_2$

$v_1 = \int -\dfrac{t^2\ln(t)\ln(t)}{t^3} dt = \int \dfrac{-y_2 g(t)}{W(y_1, y_2)} dt$

$v_1 = -\dfrac{\ln^{2+1}(t)}{2+1} = -\dfrac{1}{3}\ln^3(t) + c$

$v_2 = \int \dfrac{t^2\ln(t)}{t^3} dt = \dfrac{1}{2}\ln^2(t)$

$y_p = \left(-\dfrac{1}{3}\ln^3(t)\right)t^2 + \dfrac{1}{2}\ln^2(t)t^2\ln(t)$

$\boxed{y_p = \dfrac{1}{6}t^2\ln^3(t)}$ solução particular

geral $y = y_h + y_p$

$y = c_1t^2 + c_2\ln(t)t^2 + \dfrac{1}{6}t^2\ln^3(t)$