TEOREMA de BAYES

$$P(A/B) = \frac{P(B/A)\,P(A)}{P(B)}$$

5 urnas — 1 bola de osso

2C1 3BB

2C2 2BB

1C3 6BB

$P(C_3/BB) = \frac{1 \cdot 1/5}{8/15} = 3/8$

urnas $\frac{2}{5}$ $\frac{3}{6}$ + ... bolas

Ñ são eventos independentes

Variável aleatória DISCRETA

/ obtenha a distribuição da variável

valores finitos $\{0,1,2\}$ EX3

X	P(x)
x_i	p_i
x_i	p_i
x_i	p_i

valor médio ou esperança matemática

$$E(x) = \sum_{i=1}^{n} x_i p_i$$

$E(x) = \mu(x)$

$Var(x) = \sigma^2(x)$

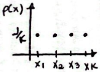
Variância

$$Var(x) = \sum_{i=1}^{n} [x_i - E(x)]^2 \cdot p_i$$

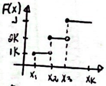
Modelos Probabilísticos

Distribuição Uniforme discreta

X	1	2	3	4	5	6	Total
P(x)	1/6	1/6	1/6	1/6	1/6	1/6	1,0



f de probabilidade



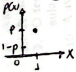
f de distribuição

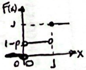
Distribuição de Bernoulli

$X=0$ fracasso $\boxed{0 < p < 1}$ $X=1$ sucesso

$E(x) = p$ $\quad Var(x) = p(1-p)$

$$F(x) = \begin{cases} 0, & se\ x<0 \\ 1-p, & se\ 0 \le x < 1 \\ 1, & se\ x \ge 1 \end{cases}$$





$P(x=1) = 1/6$

$P(x=0) = 5/6$

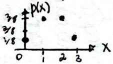
fracasso ≠ 1

Distribuição Binomial

$q = 1 - p = P(F)$ $\quad P(S) = q$

$p = 1/2$ $\quad n=3$ $\quad 2^3$

Nº sucessos	Proba.	
0 — 1	q^3	1/8
1 — 3	pq^2	3/8
2 — 3	p^2q	3/8
3 — 1	p^3	1/8



Distribuição de Poisson

se n for grande e $p = r/n$ pequeno

$\frac{P}{N} = p$ $\quad np \le 7$

$$P(x=K) = \frac{e^{-np}(np)^K}{K!} = \frac{e^{-\lambda}\lambda^K}{K!}$$

cinco chamados por minuto

Distribuição Hipergeométrica

extrações casuais feitas sem reposição de uma população dividida segundo 2 atributos r - elem com o atributo n

N objetos n - atributo

K - elem. de um grupo de n elementos

$$P_K = \frac{\binom{r}{K}\binom{N-r}{n-K}}{\binom{N}{n}}$$

Lote de 100 peças

10 são defeituosas

Escolhemos 5 peças

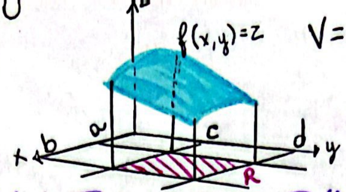
(Probabilidade de não haverem peças defeituosas = K)?

$$P(x=0) = \frac{\binom{10}{0}\binom{100-10}{5-0}}{\binom{100}{5}} = 0{,}584$$

Região retangular e TEOREMA de FUBINI

$f(x,y) = z > 0$

$$\int_c^d \left[\int_a^b f(x,y)\,dx\right] dy$$



$V = \iint_R f(x,y)\,dx\,dy$ ou

$V = \iint_R f(x,y)\,dA$ → área de R

Propriedades de Integrais Duplas

- $\iint_R c_1 f(x,y) \pm c_2 g(x,y)\,dA = c_1 \iint_R f(x,y)\,dA \pm c_2 \iint_R g(x,y)\,dA$
- se $f > g$

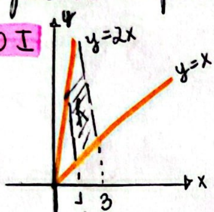
$\iint_R f(x,y)\,dA > \iint_R g(x,y)\,dA$

- $\iint_D 1\,dA \Rightarrow$ Área (D)

$\int dx = x$

Integrais duplas regiões **GERAIS**

TIPO I



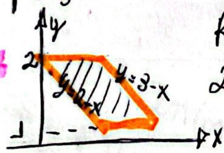
$R: 1 \leq x \leq 3$ → varia entre números (integral de fora)

$\text{BASE} \leq y \leq \text{TETO}$

$x \leq y \leq 2x$ → varia entre funções (integral de dentro)

$$\int_1^3 \int_x^{2x} f(x,y)\,\boxed{dy\,dx}$$

TIPO II



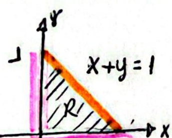
$R: 1 \leq y \leq 2$

$2-y \leq x \leq 3-y$

lateral esq. $\leq x \leq$ lateral direita

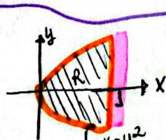
$$\int_1^2 \int_{2-y}^{3-y} y^2\,\boxed{dx\,dy}$$

$\int e^t = e^t$



TIPO I dydx: $0 \leq x \leq 1$, $0 \leq y \leq 1-x$

TIPO II dxdy: $0 \leq x \leq 1-y$, $0 \leq y \leq 1$

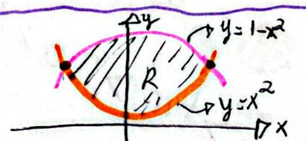


$x = 1$, $y = \pm 1$

TIPO II dxdy

$y^2 \leq x \leq 1$

$-1 \leq y \leq 1$



TIPO I dydx: $-\frac{\sqrt{2}}{2} \leq x \leq \frac{\sqrt{2}}{2}$, $x^2 \leq y \leq 1-x^2$

- $x^2 = 1 - x^2$

$x^2 = \frac{1}{2}$

$x = \pm \sqrt{\frac{1}{2}}$

$x = \pm \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$x = \pm \frac{\sqrt{2}}{2}$

$e^{0^x} = 1$

TIPO I

$a \leq x \leq b$

$f(x) \leq y \leq g(x)$

dydx

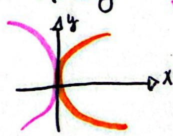
TIPO II

$g(y) \leq x \leq f(y)$

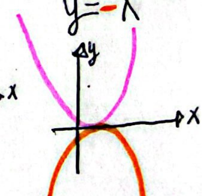
$c \leq y \leq d$

dxdy

$x = \pm y^2$

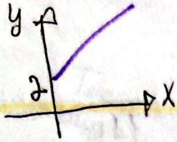


$y = \pm x^2$

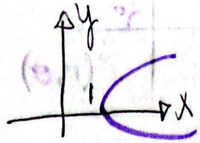


Parábolas

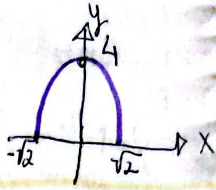
Retas



$y = x + 2$



$x = y^2 + 1$

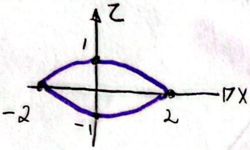


$y = 4 - x^2$

raiz $y = 0$

$x^2 = 4 = \pm\sqrt{2}$

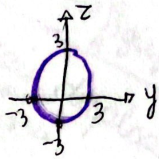
ELIPSE / CIRCUNFERÊNCIA



$$\frac{x^2}{\underset{a}{4}} + \frac{z^2}{\underset{b}{1}} = 1$$

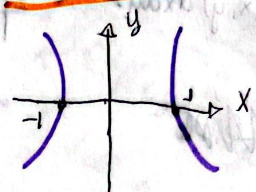
$\sqrt{4}$

± 2



$$y^2 + z^2 = 9$$

HIPÉRBOLE

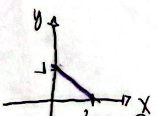


$$x - \frac{y^2}{4} = 1$$

~~xma~~ $y = 0$

$x = \pm 1$

EX: $x + 2y + z = 2$

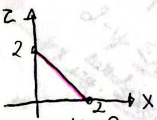


$z = 0$

$x + 2y = 2$

$y = 0 \quad x = 2$

$x = 0 \quad y = 1$

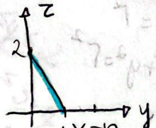


$y = 0$

$x + z = 2$

$x = 0 \quad z = 2$

$z = 0 \quad x = 2$

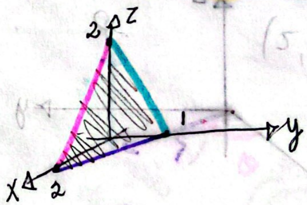


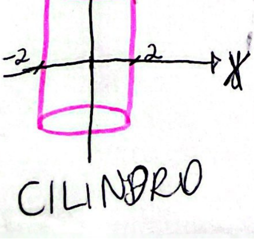
$x = 0$

$2y + z = 2$

$y = 0 \quad z = 2$

$z = 0 \quad y = 1$





$x^2 + y^2 = 4$

CILINDRO

Coordenadas polares

$0 \leq \theta \leq 2\pi$

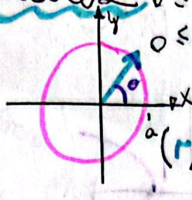
$0 \leq r \leq a$

$x = r\cos\theta$

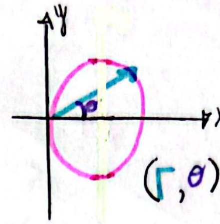
$y = r \operatorname{sen}\theta$

$J = r$

$x^2 + y^2 = r^2$



(r, θ)

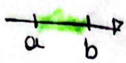


(r, θ)

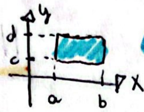
$0 \leq r \leq \text{BORDA}$

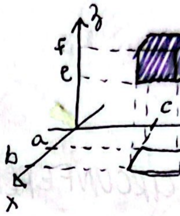
$-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$

$\int_a^b f(x)\,dx$ → 1D



$\int_c^d \int_a^b f(x,y)\,dx\,dy$ → 2D





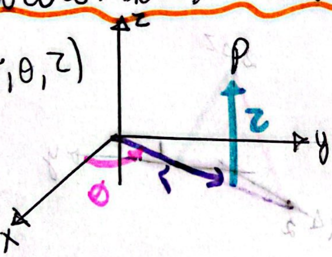
3D

$\int_e^f \int_c^d \int_a^b f(x,y,z)\,dx\,dy\,dz$

~~PARABOLOIDE~~

Coordenadas cilíndricas

$P(r, \theta, z)$



$x = r\cos\theta$

$y = r\operatorname{sen}\theta$

$z = z$

$J = r$

$x^2 + y^2 = r^2$

Coordenadas esféricas

$x^2 + y^2 + z^2 \neq J$

RO téta fi

$P(\rho, \theta, \varphi)$

$x = \rho \cos\theta \operatorname{sen}\varphi$

$y = \rho \operatorname{sen}\theta \operatorname{sen}\varphi$

$z = \rho\cos\varphi$

$x^2 + y^2 + z^2 = \rho^2$

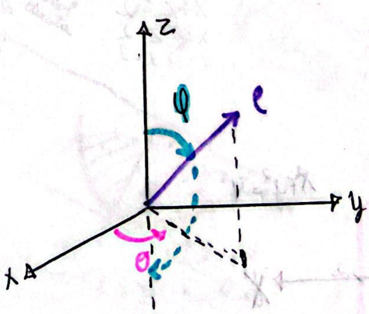
$J = \rho^2 \operatorname{sen}\varphi$

$0 \leq \rho \leq 1$

$0 \leq \theta \leq 2\pi$

$0 \leq \varphi \leq \pi$

semirrotação



Série geométrica $\sum a_n = \frac{a}{1-r}$ soma constante

$|r| < 1$ converge

$|r| \geq 1$ diverge

forma $\sum a(c_2)^n$ ↑ r

Pág 80

Se $\sum a_n$ converge $\rightarrow \lim a_n = 0$ TEOREMA

Pág 83

Se $\lim a_n \neq 0$ ou $\lim a_n \nexists$ $\rightarrow \sum a_n$ diverge série

CORO

$\sum a_n$ e $\sum b_n$ convergentes TEO

$\left.\begin{array}{l}\sum(a_n \pm b_n) = \sum a_n \pm \sum b_n \\ \sum c a_n = c \sum a_n\end{array}\right\}$ CONVERGENTES

Pág 84 ver pág 85

CORO

$\sum a_n$ diverge $\rightarrow c\sum a_n$ diverge $c \neq 0$

$\left.\begin{array}{l}\sum a_n \text{ converge} \\ \sum b_n \text{ diverge}\end{array}\right\} \sum(a_n \pm b_n)$ diverge

série harmônica $\frac{1}{n}$ diverge Pág 89

Pág 91 Teste INTEGRAL

P-série $\sum \frac{1}{n^p}$

$p > 1$ converge

$p \leq 1$ diverge

Pág 94 Testes da comparação

1º Teste da divergência

$L \neq 0$ ou $\nexists$ diverge

2º Série Geométrica

? $|r| < 1$ converge

3º Teste da Comparação

$a_n \leq b_n$ | con b_n

$a_n \geq b_n$ | div b_n

Série Harmônica

$\sum \frac{1}{n}$ diverge

série p-séries

$\sum \frac{1}{n^p}$ $p > 1$ converge

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Sequência $\exists$ lim -> convergente
$\nexists$ lim -> divergente

Se $\lim (a_n) = 0$ $-|a_n| \leq a_n \leq |a_n|$

Propriedades limites de SEQUÊNCIA

- $\lim_{n\to\infty} (a_n \pm_{\times} b_n) = (\lim a_n) \pm_{\times} (\lim b_n)$
- $\lim (c a_n) = c \cdot (\lim a_n)$
- $\lim \left(\frac{a_n}{b_n}\right) = \frac{\lim a_n}{\lim b_n}$ $b_n \neq 0$

$\lim f(a_n) = f(\lim a_n)$
lim vai p/ dentro da função (contínua)

Teorema Sequência Monótona (cresc ou decres) é convergente ($\exists$ lim) ~~sse~~ SSS for limitada

$\boxed{\text{Séries}} = \sum_{n=1}^{\infty} (a_n)$

① $\lim_{n\to\infty} \frac{\ln(n)}{n^{c}} = 0$ c→qualquer

② $\lim \sqrt[n]{c} = 1$ $c>0$

③ $\lim \sqrt[n]{n} = 1$ $n^{\frac{1}{n}}$

④ $\lim c^n = 0$ $c<1$, $c>-1$

⑤ $\lim c^n = \infty$ $c>1$

⑥ $\lim c^n = \nexists$ $c<-1$

⑦ $\lim \left(\frac{n+c}{n}\right)^n = e^c$ $\left(1+\frac{c}{n}\right)^n$

⑧ $\lim \frac{n^{c_1}}{c_2^{\,n}} = 0$

⑨ $\lim \frac{c^n}{n!} = 0$

⑩ $\lim \frac{n!}{n^n} = 0$

$\lim \left(\frac{n+1}{n}\right)^n = e$

$\ln(n) < n < n^x < c^n < n! < n^n$

$\lim_{n\to\infty} (-1)^{n+1} \nexists$

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