Boolean Algebra

CS 47

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1. Definition

A mathematical system dealing with binary variables and logic operations

2. Origin

Developed by George Boole in the mid-19th century

3. Importance

Fundamental to digital logic design and circuit analysis

4. Boolean Variables

A boolean variable assumes two values

- FALSE (0)
- TRUE (1)

4.1. Basic Operations

- AND (.) or conjunction ($\wedge$)
- OR (+) or disjunction ($\vee$)
- NOT (‘) or negation ($\neg$)

4.2. Logic Gates

4.2.1. Definition

Electronic devices that perform logical operations on one or more binary inputs to produce a single binary output

4.2.2. Types

4.2.2.1. AND

Ensures all inputs must be true for an output to be true

4.2.2.2. OR

Ensures any input can be true for an output to be true

4.2.2.3. NOT

Ensures any input is inverted for an output to be the inversion of an input

4.2.2.4. NAND

Ensures only one input can be true for an output to be true

4.2.2.5. NOR

Ensures no inputs can be true for an output to be true

4.2.2.6. XOR

Ensures one input must be true for an output to be true

4.2.2.7. XNOR

Ensures both inputs must be identical for an output to be true

4.2.3. Applications in Computing

4.2.3.1. CPUs

- Fundamental building blocks of CPU architecture
- Used extensively in various parts of the CPU

4.2.3.2. ALU

- Consists of a network of logic gates that manipulate binary data to perform tasks like addition, subtraction, AND, OR, and NOT operations

4.2.3.3. Registers

- Each register in the register file is implemented using a group of flip-flops and logic gates to store and manipulate data
- **Flip Flops** A group of gates arranged such that they have memory of previous inputs

4.2.3.4. Control Unit

- It uses logic gates to decode instruction opcodes, determine the

sequence of operations, and control the flow of data within the CPU

4.2.4. Importance of Logic Gate Design

- **Speed** Logic gates enable fast computation and processing, contributing to the overall performance of the laptop
- **Efficiency** Compact and optimized logic gate designs help conserve space and power, essential for portable devices like laptops
- **Versatility** Logic gates allow for the implementation of a wide range of functions and operations, making laptops capable of handling diverse tasks efficiently.
- **Low Power Consumption** Modern logic gate technologies, such as CMOS (Complementary Metal-Oxide-Semiconductor), offer low power consumption characteristics, extending the battery life of laptops and improving energy efficiency

4.2.5. Universal Gate

4.2.5.1. Definition

A gate that can be used to implement any other gate

4.2.6. Primary Gates

- NAND (AND followed by NOT)
- NOR (OR followed by NOT)

5. Functions and Terms

5.1. Example(s)

$$F(X, Y, Z) = XY' + Z'$$

- This is a boolean expression - function list of variables
- Each part of the right hand side of the equation is called a ‘term’

6. Identities

1. $X + 0 = X$
2. $X * 1 = X$
3. $X + 1 = X$
4. $X * 0 = 0$
5. $X + X = X$
6. $X * X = X$
7. $X + \overline{X} = 1$
8. $X * \overline{X} = 0$
9. $\overline{\overline{X}} = X$

7. Laws

1. $X + Y = Y + X$
2. $XY = YX$
3. $X + (Y + Z) = (X + Y) + Z$
4. $X(YZ) = (XY)Z$
5. $X(Y + Z = XY + XZ)$
6. $X + YZ = (X + Y)(X + Z)$
7. $\overline{X + Y} = \overline{X} * \overline{Y}$
8. $\overline{X * Y} = \overline{X} + \overline{Y}$

8. Consensus Theorem

8.1. Example(s)

$$XY + \overline{X}Z + YZ = XY + \overline{X}Z$$

9. Complement of a function

9.1. Definition

Apply DeMorgan's theorem as many times as needed to obtain the complement

9.2. Example(s)

$$F_1 = \overline{X}Y\overline{Z} + \overline{XY}Z$$

$$\overline{\overline{X}Y\overline{Z} + \overline{XY}Z} = \left(\overline{\overline{X}Y\overline{Z}}\right) * \left(\overline{XY}Z\right) = \left(X + \overline{Y} + Z\right)\left(X + Y + \overline{Z}\right)$$

10. Sum of Products (SOP)

Minterm A product term in which all variables appear exactly one (either complemented or uncomplemented)

10.1. SOP of Minterms

X	Y	Z	Product Term	Symbol
0	0	0	X'Y'Z'	m_0
0	0	1	X'Y'Z	m_1
0	1	0	X'YZ'	m_2
0	1	1	X'YZ	m_3
1	0	0	XY'Z'	m_4
1	0	1	XY'Z	m_5
1	1	0	XYZ'	m_6
1	1	1	XYZ	m_7

11. Product of Sums (POS)

Maxterm A sum term in which all variables appear exactly one (either complemented or uncomplemented)

11.1. POS of Maxterms

X	Y	Z	Sum Term	Symbol
0	0	0	X+Y+Z	M_0
0	0	1	X+Y+Z'	M_1
0	1	0	X+Y'+Z	M_2
0	1	1	X+Y'+Z'	M_3
1	0	0	X'+Y+Z	M_4
1	0	1	X'+Y+Z'	M_5
1	1	0	X'+Y'+Z	M_6
1	1	1	X'+Y'+Z'	M_7