

Dot Products and Orthogonality 1

Find the coordinates of the vector [...] with respect to both the elementary basis and the basis [...]

Write as augmented matrix

Reduce to RREF

The dot product of two vectors

Properties of dot product

$$x \cdot y = y \cdot x$$

$$x \cdot (y + z) = x \cdot y + x \cdot z$$

$$(ax) \cdot y = ax \cdot y = x \cdot (ay)$$

$$\|x\|^2 = x \cdot x$$

$$\|x\| \geq 0$$

$$\|x\| = 0 \text{ if and only if } x = 0$$

$$\|ax\| = |a| \|x\|$$

Assuming Euclidean Geometry, we can define the angle between two vectors by

$$x \cdot y = \|x\| \|y\| \cos(\theta)$$

Two vectors in the real number space are orthogonal if $x \cdot y = 0$

Distance between two vectors in \mathbb{R}^n is $d(x,y) = \|x - y\|$

Cauchy Inequality

$$|x \cdot y| \leq \|x\| \|y\|$$

Triangular inequality

$$\|x + y\| \leq \|x\| + \|y\|$$

A set of nonzero vectors is orthogonal if $x_i \cdot x_j = 0$ for all $i \neq j$

An orthogonal set of vectors is orthonormal if $x_i \cdot x_i = 1$ for each x_i ($\|x_i\| = 1$)

Every set of orthogonal vectors in \mathbb{R}^n is linearly independent

Example

Show that the vectors

$$\langle 3, 0, -4 \rangle$$

$$\langle 0, 1, 0 \rangle$$

$$\langle 4, 0, 3 \rangle$$

are orthogonal and then scale them to form an orthonormal set

$$\langle 3, 0, -4 \rangle \cdot \langle 0, 1, 0 \rangle = 0 + 0 + 0 = 0$$

$$\langle 3, 0, -4 \rangle \cdot \langle 4, 0, 3 \rangle = 12 + 0 + -12 = 0$$

$$\langle 0, 1, 0 \rangle \cdot \langle 4, 0, 3 \rangle = 0 + 0 + 0 = 0$$

This is an orthogonal set

To get the orthonormal set

First, for each vector you get a value by adding the squares of each value then square rooting it (this is the magnitude of the vector)

Then...

$$v_1 = x_1 / \|x\| = 1/5 (x_1) = \langle 3/5, 0, -4/5 \rangle$$

$$v_2 = \dots$$

$$v_3 = \dots$$

This is the orthonormal set

How would you find all vectors orthogonal to a given set of vectors

Find a basis for the space of vectors orthogonal to $\langle 1, 2, 3, 4 \rangle$ and $\langle 5, 6, 7, 8 \rangle$

Solve as a system (find RREF of matrix augmented with 0 in both rows)

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