Homework 4

CS 152 | Spring 2025 | Dr. Sengupta | Naya Singhania | 017192682

Question 1

Reduce the following lambda terms

a) $(\lambda x.(x+y))3$
$= [x := 3](x+y)$

$= 3+y$

b) $(\lambda x.(\lambda y.yx)(\lambda z.xz))(\lambda y.yy)$
$= [x := \lambda y.yy](\lambda y.yx)(\lambda z.xz)$

$= (\lambda y.y(\lambda y.yy))(\lambda z.(\lambda y.yy)z)$

$= [y := \lambda z.(\lambda y.yy)z](\lambda y.y(\lambda y.yy))$

$= (\lambda z.(\lambda y.yy)z)(\lambda y.yy)$

$= [z := \lambda y.yy]((\lambda z.(\lambda y.yy)z))$

$= (\lambda y.yy)(\lambda y.yy)$

$= \Omega$ combinator (because of the nonterminating recursion)

Question 2

Prove the following

$+21 = 3$

$+21 = 2+1$

$+21 = M+N$

$+21 = \lambda x.\lambda y.(Mx)((Nx)y)$

$+21 = \lambda x.\lambda y.(2x)((1x)y)$

Question 3

Use beta reduction to compute the following expression

$(\lambda x(\lambda x.+(-x1))x3)9$

$= (\lambda x(\lambda z.+(-z1))x3)9$

$= (\lambda z.+(-z1))93$

$= (-91)3$

$= +83$

$= 3+8$

$= 11$

Question 4

Write a Scheme function named elements which counts the number of elements in a list

- See attached file `q4.scm`

Example Input

```
(elements '(1 (2 (3) 4) 5 6))
```

Example Output

```
Welcome to DrRacket, version 8.16 [cs].
Language: Pretty Big; memory limit: 128 MB.
6
> |
```

Question 5

a)

Write a scheme function that calculates the inner product of two vectors

- See attached file `q5a.scm`

Example Input

```
(inner_product '(1 2 3) '(4 5 6))
```

Example Output

```
Welcome to DrRacket, version 8.16 [cs].
Language: Pretty Big; memory limit: 128 MB.
32
>
```

b)

Implement function interleave in scheme, which expects as arguments two lists xs and ys, and returns a single list obtained by choosing elements alternately, first from xs and then from ys. When either xs or ys runs out, interleave takes the remaining elements from the other list, so that the elements of the result are exactly the elements of the two argument lists taken together.

- See attached file `q5b.scm`

Example Input

```
(interleave '(1 2 3) '(a b c))
(interleave '(1 2 3) '(a b c d e f))
(interleave '(1 2 3 4 5 6) '(a b c))
```

Example Output

Welcome to DrRacket, version 8.16 [cs].
Language: Pretty Big; memory limit: 128 MB.

```
(1 a 2 b 3 c)
(1 a 2 b 3 c d e f)
(1 a 2 b 3 c 4 5 6)
>
```