Dot Products and Orthogonality

Find the coordinates of the vector [...] with respect to both the elementary basis and the basis [...]

Write as augmented matrix

Reduce to RREF

The dot product of two vectors

Properties of dot product

$x . y = y . x$

$x . (y + z) = x .y + x . z$

$(ax) . y = ax . y = x . (ay)$

$|| x ||^2 = x . x$

$|| x || >= 0$

$|| x || = 0$ if and only if $x = 0$

$|| a x || = | a | || x ||$

Assuming Euclidean Geometry, we can define the angle between two vectors by

$x . y = || x || || y || cos(theta)$

Two vectors in the real number space are orthogonal if $x . y = 0$

Distance between two vectors in IR is $d(x,y) = || x - y ||$

Cauchy Inequality

$| x . y | <= || x || || y ||$

Triangular inequality

$|| x + y || <= || x || + || y ||$

A set of nonzero vectors is orthogonal if $x_i . x_j = 0$ for all $i = j$

An orthogonal set of vectors is orthonormal if $x_i . x_i = 1$ for each x_i ($|| x || = 1$)

Every set of orthogonal vectors in IR is linearly independent

Example

Shoe that the vectors

<3,0,-4>

<0,1,0>

<4,0,3>

are orthogonal and then scale them to form an orthonormal set

<3,0,-4> . <0,1,0> = 0 + 0 + 0 = 0

<3,0,-4> . <4,0,3> = 12 + 0 + -12 = 0

<0,1,0> . <4,0,3> = 0 + 0 + 0 = 0

This is an orthogonal set

To get the orthonormal set

First, for each vector you get a value by adding the squares of each value then square rooting it (this is the magnitude of the vector

Then…

v_1 = x_1 / || x || = 1 / 5 (x_1) = < 3 / 5, 0, -4 /5 >

v_2 = …

v_3 = …

This is the orthonormal set

How would you find all vectors orthogonal to a given set of vectors

Find a basis for the space of vectors orthogonal to <1,2,3,4> and <5,6,7,8>

Solve as a system (find RREF of matrix augmented with 0 in both rows)

**