

Maximizers and Minimizers

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Let A be an $m \times n$ matrix

Let b be a vector in \mathbb{R}^m

Consider the system $Ax = b$

1. Any solution x to the normal equations $(A^T A)z = Ab$ is a best approximation to $Ax = b$ in the sense that $\|Az - b\| \leq \|Ax - b\|$ for all x in \mathbb{R}^n
2. If the columns of A are linearly independent then $A^T A$ is invertible and z is the unique solution $z = (A^T A)^{-1} A^T b$

If you have an inconsistent system A

Do $A^T A = \dots$ (multiply the matrices) = \dots

The $A^T A$ matrix is symmetric (same if transposed or not)

Do $A^T b$ (multiply A^T by the vector b)

Now do $z = (A^T A)^{-1} A^T b$

Augment $A^T A$ with b

A minimizer is the constants from each vector value

BEST FIT LINES

- Minimize the sum of the squares of the errors

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