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#### 1. Definition

A mathematical system dealing with binary variables and logic operations

### 2. Origin

Developed by George Boole in the mid-19th century

### 3. Importance

Fundamental to digital logic design and circuit analysis

### 4. Boolean Variables

A boolean variable assumes two values

- FALSE (0)
- TRUE (1)

### 4.1. Basic Operations

- AND (.) or conjunction ( $\land$ )
- OR (+) or disjunction  $(\lor)$
- NOT (') or negation (¬)

#### 4.2. Logic Gates

#### 4.2.1. Definition

Electronic devices that perform logical operations on one or more binary inputs to produce a single binary output

#### 4.2.2. Types

#### 4.2.2.1. AND

Ensures all inputs must be true for an output to be true

#### 4.2.2.2. OR

Ensures any input can be true for an output to be true

#### 4.2.2.3. NOT

Ensures any input is inverted for an output to be the inversion of an input

#### 4.2.2.4. NAND

Ensures only one input can be true for an output to be true

#### 4.2.2.5. NOR

Ensures no inputs can be true for an output to be true

#### 4.2.2.6. XOR

Ensures one input must be true for an output to be true

#### 4.2.2.7. XNOR

Ensures both inputs must be identical for an output to be true

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#### 4.2.3. Applications in Computing

#### 4.2.3.1. CPUs

- Fundamental building blocks of CPU architecture
- Used extensively in various parts of the CPU

#### 4.2.3.2. ALU

• Consists of a network of logic gates that manipulate binary data to perform tasks like addition, subtraction, AND, OR, and NOT operations

#### 4.2.3.3. Registers

- Each register in the register file is implemented using a group of flip-flops and logic gates to store and manipulate data
- Flip Flops A group of gates arranged such that they have memory of previous inputs

#### 4.2.3.4. Control Unit

• It uses logic gates to decode instruction opcodes, determine the

sequence of operations, and control the flow of data within the CPU

#### 4.2.4. Importance of Logic Gate Design

- **Speed** Logic gates enable fast computation and processing, contributing to the overall performance of the laptop
- **Efficiency** Compact and optimized logic gate designs help conserve space and power, essential for portable devices like laptops
- **Versatility** Logic gates allow for the implementation of a wide range of functions and operations, making laptops capable of handling diverse tasks efficiently.
- Low Power Consumption Modern logic gate technologies, such as CMOS (Complementary Metal-Oxide-Semiconductor), offer low power consumption characteristics, extending the battery life of laptops and improving energy efficiency

#### 4.2.5. Universal Gate

#### **4.2.5.1. Definition**

A gate that can be used to implement any other gate

#### 4.2.6. Primary Gates

- NAND (AND followed by NOT)
- NOR (OR followed by NOT)

#### 5. Functions and Terms

#### 5.1. Example(s)

$$F(X, Y, Z) = XY' + Z'$$

- This is a boolean expression function list of variables
- Each part of the right hand side of the equation is called a 'term'

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### 6. Identities

1. X + 0 = X

2. X \* 1 = X

3. X + 1 = X

4. X \* 0 = 0

5. X + X = X

6. 
$$X * X = X$$

7.  $X + \overline{X} = 1$ 

8.  $X * \overline{X} = 0$ 

9.  $\overline{X} = X$ 

### 7. Laws

1. 
$$X + Y = Y + X$$

2. XY = YX

3. X + (Y + Z) = (X + Y) + Z

4. X(YZ) = (XY)Z

5. 
$$X(Y + Z = XY + XZ)$$

6. X + YZ = (X + Y)(X + Z)

7.  $\overline{X+Y} = \overline{X} * \overline{Y}$ 

8.  $\overline{X * Y} = \overline{X} + \overline{Y}$ 

### 8. Consensus Theorem

### 8.1. Example(s)

$$XY + \overline{X}Z + YZ = XY + \overline{X}Z$$

# 9. Complement of a function

#### 9.1. Definition

Apply DeMorgan's theorem as many times as needed to obtain the complement

### 9.2. Example(s)

$$F_1 = \overline{X}Y\overline{Z} + \overline{X}\overline{Y}Z$$

$$\overline{\overline{X}Y\overline{Z} + \overline{X}\overline{Y}Z} = \left(\overline{\overline{X}Y\overline{Z}}\right) * \left(\overline{X}\overline{Y}Z\right) = \left(X + \overline{Y} + Z\right)\left(X + Y + \overline{Z}\right)$$

## 10. Sum of Products (SOP)

**Minterm** A product term in which all variables appear exactly one (either complemented or uncomplemented)

#### 10.1. SOP of Minterms

X	Y	Z	Product Term	Symbol
0	0	0	X'Y'Z'	$m_0$
0	0	1	X'Y'Z	$m_1$
0	1	0	X'YZ'	$m_2$
0	1	1	X'YZ	$m_3$
1	0	0	XY'Z'	$m_4$
1	0	1	XY'Z	$m_5$
1	1	0	XYZ'	$m_6$
1	1	1	XYZ	$m_7$

# 11. Product of Sums (POS)

**Maxterm** A sum term in which all variables appear exactly one (either complemented or uncomplemented)

### 11.1. POS of Maxterms

X	Y	Z	Sum Term	Symbol
0	0	0	X+Y+Z	$M_0$
0	0	1	X+Y+Z'	$M_1$
0	1	0	X+Y'+Z	$M_2$
0	1	1	X+Y'+Z'	$M_3$
1	0	0	X'+Y+Z	$M_4$
1	0	1	X'+Y+Z'	$M_5$
1	1	0	X'+Y'+Z	$M_6$
1	1	1	X'+Y'+Z'	$M_7$

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