

# Null Space 1

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The null space of a matrix  $A$  is the subspace  $\text{null}(A) = \{x \text{ such that } Ax = 0\}$

The solutions to the homogeneous system

The row space of an  $m \times n$  matrix  $A$  is the subspace of  $\mathbb{R}^n$  spanned by the rows of  $A$   
 $\text{row}(A) = \text{span}(R_1, R_2, \dots, R_m)$

The column space of an  $m \times n$  matrix  $A$  is the subspace of  $\mathbb{R}^m$  spanned by the columns of  $A$   
 $\text{col}(A) = \text{span}(c_1, c_2, \dots, c_n)$

The image or range of a linear transformation is the subspace  $\text{im}(A)$  or  $\text{range}(A)$  given by  $\{Ax \text{ where } x \in \mathbb{R}^n\}$

How do we find all possible outputs

If  $A \rightarrow B$  by elementary row operations then  $\text{row}(A) = \text{row}(B)$

If  $R$  is a row-echelon matrix

1. The nonzero rows of  $R$  are a basis for  $\text{row}(R)$
2. The columns of  $R$  that contain a leading 1 are a basis for  $\text{col}(R)$

The rank of a matrix is the number of leading 1s in row echelon form

$$\text{rank}(A) = \dim(\text{row}(A)) = \dim(\text{col}(A))$$

Let  $A$  be a  $m \times n$  matrix with rank  $r$ . If  $A$  is row-reduced to REF  $R$  then

1. The nonzero rows of  $R$  are a basis for  $\text{row}(A)$
2. The  $r$  columns of  $A$  that contain a leading 1 in REF are a basis for  $\text{col}(A)$

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