

# Final Review

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Implication negation

$P \rightarrow q$

Negate it

NOT  $p \vee q$

Injective - B can't have multiple A (one-to-one)

Surjective - every B has some A (many-to-few)

Prove by implication ( $p \rightarrow q$ )

Prove by contrapositive ( $\text{not } q \rightarrow \text{not } p$ )

Prove by contradiction (suppose p, suppose not q  $\rightarrow$  will lead to not p, contradicts with p)

Prove by exhaustion

Prove by cases

$\mathbb{N} = \{0, 1, 2, 3, \dots\}$

$\mathbb{Z} = \{\dots, -3, -2, -1, 1, 2, 3, \dots\}$

$\mathbb{Z}^+ = \{1, 2, 3, \dots\}$

$\mathbb{R}$  = real numbers

$\mathbb{R}^+$  = positive real numbers

$\mathbb{C}$  = complex numbers

Cartesian product

Use parentheses inside brackets

Empty item  $\{\}$  goes in as  $\{\}$  inside the parentheses

Union ( $\cup$ ) - contains all the elements in either set or both sets (all of venn diagram)

Intersection ( $\cap$ ) - contains only the elements in both sets (intersection of venn diagram)

Inclusion Exclusion -  $|A \cup B| = |A| + |B| - |A \cap B|$

Difference -  $A - B$  contains the elements that are in  $A$  but not in  $B$

Power set -  $P(A) = \{ \{\}, \{1\}, \{2\}, \{1,2\} \}$

$$|A| = 2$$

$$|P(A)| = 2^{|A|} = 2^2 = 4$$



Convert base 10 to other base

Divide number by base

Keep dividing

The remainder column gives digits from bottom (largest) to top

Convert base to base  $\rightarrow$  use 10 for middle

GCD

Break each number into smallest primes, use smallest prime power for each number

LCM

Break each number into smallest primes, use largest prime power for each number

Strong induction

Same basis step as induction

Inductive step: We show that  $P(1), P(2), \dots, P(k)$  are true imply that  $P(k+1)$  is true (using multiple for inductive step to make multiple inductive statements, ex.  $P(k+1), P(k-3)$ , 4 and 5 cent problem are good examples)

Functions from  $A$  to  $B$  is

$$B^A$$

Injective functions from  $A$  to  $B$

$$B! / A!$$

Generalized pigeonhole principle

If  $n$  objects are placed in  $k$  boxes, then at least 1 box has at least  $\lceil n/k \rceil$  (ceiling brackets) objects



Find coefficient of  $x^5 y^7$  in expansion of  $(x + y)^{12}$

$$\binom{12}{7} x^5 y^7$$

$$p(E|F) = p(E \cap F) / p(F)$$

Bernoulli Trials

Successes and failures

Example

Flip coin

Success: H

Failure: T

$$p = 1/2$$

$$q = 1/2$$

Bayes Theorem



Reflexive



Symmetric



Transitive

$$(a, b), (b, c), (a, c)$$

???

Anti-symmetric

$$A = \{1, 2, 3, 4\}$$

$$R = \{(1, 1), (2, 2), (3, 3), (4, 4)\}$$

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