

Null Space

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The null space of a matrix A is the subspace $\text{null}(A) = \{x \text{ such that } Ax = 0\}$

The solutions to the homogeneous system

The row space of an $m \times n$ matrix A is the subspace of \mathbb{R}^n spanned by the rows of A
 $\text{row}(A) = \text{span}(R_1, R_2, \dots, R_m)$

The column space of an $m \times n$ matrix A is the subspace of \mathbb{R}^m spanned by the columns of A
 $\text{col}(A) = \text{span}(c_1, c_2, \dots, c_n)$

The image or range of a linear transformation is the subspace $\text{im}(A)$ or $\text{range}(A)$ given by $\{Ax \text{ where } x \in \mathbb{R}^n\}$

How do we find all possible outputs

If $A \rightarrow B$ by elementary row operations then $\text{row}(A) = \text{row}(B)$

If R is a row-echelon matrix

1. The nonzero rows of R are a basis for $\text{row}(R)$
2. The columns of R that contain a leading 1 are a basis for $\text{col}(R)$

The rank of a matrix is the number of leading 1s in row echelon form

$$\text{rank}(A) = \dim(\text{row}(A)) = \dim(\text{col}(A))$$

Let A be a $m \times n$ matrix with rank r . If A is row-reduced to REF R then

1. The nonzero rows of R are a basis for $\text{row}(A)$
2. The r columns of A that contain a leading 1 in REF are a basis for $\text{col}(A)$

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