Chapter 8

8.1 - Hypotheses and Test Procedures

Statistical Hypothesis

Definition

A claim about a single population parameter, about values of several population parameters, or about the form of a probability distribution

Test Procedure

Definition

A rule, based on sample data, for deciding whether H_0 should be rejected

Example

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A hypothesis test consists of

- Stating the hypothesis
- Choosing a test statistic
- Finding a $P\text{-}value$
- Drawing the final conclusion

Test Hypotheses

Definition

States two contradictory statements known as the null hypothesis (denoted by H_0) an the alternative hypothesis (denoted by H_a)

- The null hypothesis is the claim that is initially assumed to be true
- The alternative hypothesis is the claim that is contradictory to the null hypothesis

Test Statistic

- Many options

Right-tailed (Upper-tailed) Test

$H_0 : \mu = \mu_0$
$H_a : \mu > \mu_0$
$Z = \frac{\overline{X} - \mu_0}{\frac{S}{\sqrt{n}}} \sim approx\ N(0,1)$

Left-tailed (Lower-tailed) Test

$H_0 : \mu = \mu_0$
$H_a : \mu < \mu_0$
$Z = \frac{\overline{X} - \mu_0}{\frac{S}{\sqrt{n}}} \sim approx\ N(0,1)$

Two-tailed Test

$H_0 : \mu = \mu_0$
$H_a : \mu \neq \mu_0$
$Z = \frac{\overline{X} - \mu_0}{\frac{S}{\sqrt{n}}} \sim approx\ N(0,1)$
The $P\text{-}value$ is twice the area in the tail beyond the test statistic value

$P\text{-}value$

> **Definition**
>
> The probability, calculated assuming H_0 is true, of obtaining a value of the test statistic at least as contradictory to H_0 as the value calculated from the available sample data

Errors in Hypothesis Testing

Type I Error

> **Definition**
>
> Rejecting the null hypothesis when it is true

- α (significance level) is used to represent the probability of a type I error
- $\alpha = P(reject\ H_0 | H_0\ is\ true)$
- Common values of α
    - 0.05

- 0.01
- 0.10

Type II Error

Definition

Failing to reject the null hypothesis when it is false

- β is used to represent the probability of a type II error
- $\beta = P(fail\,to\,reject\ H_0|H_0\ is\ false)$
- Power of the test
    - $1-\beta = P(reject\ H_0|H_0\ is\ false)$

Which Error is More Important to Control?

- H_0: the person accused of a crime is innocent
- H_a: the person accused of a crime is guilty

Type I Error

Type II Error

Sample Size, Significant Level, and the Power of Test

Tip

If the size of a sample is fixed, then decrease of α leads to increase in β and, as a result, the power of the test decreases

Tip

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Decision Criterion

Reject H_0 if $P\text{-}value \le \alpha$
Fail to reject H_0 if $P\text{-}value > \alpha$

8.2 - z Tests for Hypotheses about a Population Mean

Case 1 - Normal Population

Assumptions

- $X_1, X_2, \ldots, X_n$ are $iid\ rvs$
- $X \sim N(\mu, \sigma^2)$
- μ is unknown, σ is provided

Test Statistic (TS)

$$Z = \frac{\overline{X} - \mu_0}{\sigma} * \sqrt{n}$$

where $H_0 : \mu = \mu_0$

Upper-tailed Test

H_0 contains the inequality >

Lower-tailed Test

H_0 contains the inequality <

Two-tailed Test

H_0 contains the inequality $\neq$

Example 18

Given

- $H_0 : \mu = 75$
- $H_a : \mu < 75$
- $\sigma = 9$
- Normal distribution
- $n = 25$
- $\overline{x} = 72.3$
- $\alpha = .002$

Find conclusion

1. $H_0 : \mu = 75,\ H_a : \mu < 75$ (left tailed test)
2. TS value $z = \frac{\overline{x} - 75}{9} * 5 = -1.5$
3. $P\text{-}value = \phi(-1.5) = P(Z \leq -1.5) = .0668$
4. Since $.0668 > \alpha = .002 \Rightarrow$ fail to reject H_0

5. There is not enough evidence that the true average drying time of the paint with the additive is less than 75 minutes

Sample Size Calculations

Given

- $H_0 : \mu = \mu_0$
- $H_a : mu < \mu_0$
- α
- β
- $\mu = \mu^1 \neq \mu_0$
  Find n
- $\beta(\mu^1 = 1 - \phi(\frac{\mu_0 - \mu^1}{\sigma} * \sqrt{n} - Z_a)) \Rightarrow 1 - \beta = \phi(\frac{\mu_0 - \mu^1}{\sigma} * \sqrt{n} - Z_a)) \Rightarrow \phi(Z_\beta)$
- $\frac{\mu_0 - \mu^1}{\sigma} * \sqrt{n} - Z_\alpha = Z_\beta$
- $n = (\sqrt{n})^2 = (\frac{(Z_\beta + Z_\alpha)\sigma}{\mu_0 - \mu^1})$

Example 18

Given

- $\alpha = .002$
- $\beta(70) = .01$
- $\mu^1 = 70$
- $\mu_0 = 75$

Find n

1. $Z_\alpha = 2.88$
2. $Z_\beta = Z_{.01} = 2.33$
3. $n = \ldots = 87.95 \Rightarrow n = 88$

Case 2 - A Large Sample ($n > 30$)

From unknown distribution (σ is unknown)

Test Statistic (TS)

$$Z = \frac{\overline{X} - \mu_0}{S} * \sqrt{n},\ H_0 : \mu = \mu_0$$

If H_0 is true, $Z \sim\ approx\ N(0, 1)$

Example 24

1. Hypothesis
   - $H_0 : \mu = 153, H_a : \mu > 150$
   - Right tailed test
2. TS value
   - $z = \frac{\bar{x}-\mu_0}{S} * \sqrt{n} = \frac{191-153}{89}$
   - $z \approx 3.25$
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