1.5

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1.5 nested quantifiers

D: IR

before: A x (x^2 > 0)

ex. assume domain is IR

Ax Ay (x+y = y+ x)

means for all pairs of real numbers x and y

(commutative law of addition)

ex. Ax E y (x+y=0)

D: IR

for every real number x, there is a real number y s.t. x + y = 0

pick x = 1. If you pick y=-1, then 1 + -1 = 0

True

(each real number has a additive inverse)

ex. ExAy (x+y = 0)

there exists some real number x s.t. for every real number y, x+y= 0

False

(one x for all y's)

order matters!

ExEy s.t. (x+y=0)

there exists a pair of real numbers x and y s.t. x+y=0

Ex(Ey(x+y) = 0)

Ey((1+y)=0)

AxAyAz ((x+y) + x = x+(y+z))

for all real numbers x, y, and z, (x+y) + z = x + (y+z)

(associative law of addition)

statement when true when false table (look it up)

easier way:

Ax Ey (xy=1)

negation

NOT (AxEy (xy=1))

Ex NOT(Ey) (xy=1)

ExAy NOT(xy=1)

ExAy (xy not = 1)

rule: keep moving negation inside and change identifier to the other one

AxAy is true when NOT(AxAy) is false

ex. “The sun of two real positive numbers is always positive.” can be written as

AxAy ((x>0) ^ (y>0) -> (x+y>0))

D: positive real numbers

AxAy (x+y>0)

domain already specifies “positive real numbers”

ex. “Every real number except 0 has a multiplicative inverse”

D: IR

Ax ((x not = 0) -> Ey (xy=1))

ex. c(x) “x has a computer”

F(x,y) is “x and y are friends”

The domain is the set of all students in your school

Ax c(x) v Ey ((c(y) ^ F(x,y))

says “every student at your school has computer or has a friend who does”

ex. F(a,b) is “a and b are friends”

Domain is set of students in your school

ExAyAz ((F(x,y) ^ F(x,z) ^ (y not = z) -> NOT F (y,z))

There is a student at our school none of whose friends are friends with each other

1.6

rules of inference

p -> q   -> if all premises are true, it must follow that the conclusion is true

p

three dots q

so if both p-> q and p are true, it follows that q must be true

tables for this in the textbook

ex. If i study, then i will do well on an exam. If i do well on an exam, then I will get an A in the class.

Therefore, if i study, then I will get an A in the class

rule of inference

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