

# Maximizers and Minimizers 1

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Let  $A$  be an  $m \times n$  matrix

Let  $b$  be a vector in  $\mathbb{R}^m$

Consider the system  $Ax = b$

1. Any solution  $x$  to the normal equations  $(A^T A)z = Ab$  is a best approximation to  $Ax = b$  in the sense that  $\|Az - b\| \leq \|Ax - b\|$  for all  $x$  in  $\mathbb{R}^n$
2. If the columns of  $A$  are linearly independent then  $A^T A$  is invertible and  $z$  is the unique solution  $z = (A^T A)^{-1} A^T b$

If you have an inconsistent system  $A$

Do  $A^T A = \dots$  (multiply the matrices)  $= \dots$

The  $A^T A$  matrix is symmetric (same if transposed or not)

Do  $A^T b$  (multiply  $A^T$  by the vector  $b$ )

Now do  $z = (A^T A)^{-1} A^T b$

Augment  $A^T A$  with  $b$

A minimizer is the constants from each vector value

## BEST FIT LINES

- Minimize the sum of the squares of the errors

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