Number Systems

History

- Earliest written form of numeral systems date back to at least 300 BCE (Devanagari numerals)

Bases

- A number base is the number of digits or combination of digits that a system of counting uses to represent numbers
- A base can be any whole number greater than 0
- The most commonly used number system is the decimal system, commonly known as base 10
- Mathematically, base is written as a subscript after the number
- $V_k, 10_{16}, 10_{10}, 10_8, 10_2$

Base 2

- Computers use binary to reflect a high and low states

Negative numbers

- Sign magnitude representation doesn't work

Twos Complement

1. Convert the absolute value of the negative number into its binary representation
2. Invert all bits (one's complement)
3. Add 1 to the one's complement

> **Tip**
> Instead of subtracting two binary numbers, convert second number to two's complement (flip + 1) then add

Overflow

- Occurs in binary arithmetic when the result of an operation exceeds the maximum or minimum value
- Ex. adding 7 + 1 would not be -8 in a 4 bit signed situation
    - It should be 8

Detection

- If MSB changes from 0 to 1

Conversion between bases

Binary

Binary to decimal

- Multiply starting from least significant bit
- Example
  - $K = 2$
  - $2^3 = 8$
  - $2^2 = 4$
  - $2^1 = 2$
  - $2^0 = 1$

Decimal to binary

- Most significant bit (tiniest when dividing) $\rightarrow$ least significant bit (largest when dividing)
- Prepending 0s will not change the value
  - Useful when a certain number of bits is required but the number has less bits
- Example
  - Convert 91_{10} to 8-bit binary number
    - $91/2 = 45$ remainder 1
    - $45/2 = 22$ remainder 1
    - $22/2 = 11$ remainder 0
    - $11/2 = 5$ remainder 1
    - $5/2 = 2$ remainder 1
    - $2/2 = 0$ remainder 0
    - $1/2 = 0$ remainder 1
    - $\rightarrow 1011011 \rightarrow 01011011_2$

Hexadecimal

Hexadecimal to decimal

Decimal to hexadecimal

Hexadecimal to binary

- 4 bit binary $\rightarrow$ 1 digit hexadecimal