Lambda Calculus

- See the lambda calculus tutorial
- See the lambda calculus handout

Definition

The smallest universal programming language of the world

Common Functions

Identity

Definition

$$id = \lambda x.\, x$$

Example

$$(\lambda x.\, x)M = M$$

Selection

Definition

$$fst = \lambda x.\, \lambda y.\, x$$

$$snd = \lambda x.\, \lambda y.\, y$$

Example

$$(\lambda x.\, \lambda y.\, x)MN = (\lambda x.\, M)N = M$$

$$(\lambda x.\, \lambda y.\, y)MN = (\lambda x.\, N)M = N$$

Substitutions

Arithmetic

zero
suc(zero) = 1
suc(suc(zero)) = 2 = suc(1)

Definition

$$0 \equiv \lambda s.\, (\lambda z.\, z) \equiv \lambda sz.\, z$$
$$1 \equiv \lambda sz.\, s(z)$$
$$2 \equiv \lambda sz.\, s(s(z))$$

Addition

- e.g. $2S3 \equiv SS3 \equiv S4 \equiv 5$
  - You can keep the z in to simplify the substituting

Multiplication

Definition

$$(\lambda xyz.\, x(yz))$$

Example

$$(\lambda xyz.\, x(yz))22 \equiv (\lambda z.2(2z)) \equiv 4$$

Conditionals

Definition

$$T \equiv \lambda xy.\, x$$
$$F \equiv \lambda xy.\, y$$

Logical Operations

Definition

$$AND :\equiv \Lambda xy.\, y(\lambda uv.\, v) \equiv \lambda xyxyF$$
$$OR :\equiv \Lambda xy.\, x(\lambda uv.\, u) \equiv \lambda xyxTy$$
$$NOT :\equiv \lambda x.\, x(\lambda uv.\, v)(\lambda ab.\, a) = \lambda x.\, xFT$$

Examples

$z \equiv \lambda x.\, xF\neg F$
f applied to a 0 times: $0fa \equiv (\lambda sz.\, z)fa \equiv a$
F applied to a: $Fa \equiv (\lambda xy.\, y)a \equiv \Lambda y.\, y \equiv 1$