propositional logic

proposition = declarative sentence that's either true or false

(T) ① Sacramento is CA's state capital

(T) ② 1+1=2

(F) 3>8

non-examples

① what time is it

② $x+1=3$

③ $x+y=8$

propositional variables (statement variables)
like p, q, r, s, ... to represent propositions

if proposition is true, its truth value is T, & F for false

logical operators are used to form new propositions from old ones

Negation: let p be a proposition. The negation of p ($\neg p$), is the statement "It is not the case that p". The truth value of $\neg p$ is the opposite of p

truth table

p	$\neg p$
T	F
F	T

proposition: I like math

negation: I don't like math

conjunction (And)

Let p and q be propositions. The conjunction of p and q, denoted $p \wedge q$ is the proposition "p and q". The conjunction $p \wedge q$ is true when both p and q are true, otherwise false

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

$2^2=4$

If a statement or proposition involves k variables you need 2^k rows

Disjunction of p and q, denoted $p \vee q$

false when both are false, true otherwise

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

$\vee$ (or) is inclusive or

exclusive or $\oplus$

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

conditional statement / If p then q / Implication

Let p and q be propositions. The conditional statement $p \to q$ is the proposition "If p then q". $p \to q$ is false when p is true & q is false (otherwise true)

p is the hypothesis / antecedent / premise and q is called the conclusion / consequence

p	q	$p \to q$	$q \to p$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

different way to say $p \to q$

① if p then q

② p implies q

③ p only if q

④ A necessary condition for p is q

⑤ A sufficient condition for q is p

$p \to q$

converse: $q \to p$

contrapositive: $\neg q \to \neg p$

inverse: $\neg p \to \neg q$

when ~~each~~ two propositions have the same truth tables they are said to be equivalent

$p \to q$ and $\neg q \to \neg p$ are equivalent

p	q	$p \to q$	$\neg p$	$\neg q$	$\neg q \to \neg p$
T	T	T	F	F	T
T	F	F	F	T	F
F	T	T	T	F	T
F	F	T	T	T	T

$p \to q$: If a number is an integer, it is real

p is "a number is an integer"

q is "a number is real"

contrapositive: If a number is not real, then it is not an integer

converse: If a number is real, then it is an integer

inverse: If a number is not an integer then it is not real

Let p and q be propositions. The biconditional statement $p \leftrightarrow q$ is the proposition "p if and only if q". $p \leftrightarrow q$ is true when p & q have the same truth values (false otherwise)

also called bi-implications

$p \leftrightarrow q$ and $(p \to q) \wedge (q \to p)$ are equivalent

p	q	$p \leftrightarrow q$	$p \to q$	$q \to p$	$(p \to q) \wedge (q \to p)$
T	T	T	T	T	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T

other ways to say $p \leftrightarrow q$:

① p is necessary and sufficient for q

② p iff q (if and only if)

$p \leftrightarrow q$ is "A number is an integer iff it is real"

Precedence of logical operators

$\neg$	1	$\neg p \wedge q$
$\wedge$	2	$(\neg p) \wedge q$
$\vee$	3	
$\rightarrow$	4	
$\leftrightarrow$	5	

Logic bit operations

A bit is a symbol that is either 0 or 1

1-T A boolean variable is either true or

0-F false

A boolean variable can be represented using a bit

x	y	$x \vee y$	$x \wedge y$	$x \oplus y$
1	1	1	1	0
1	0	1	0	1
0	1	1	0	1
0	0	0	0	0

bit operator

$\wedge$ AND

$\vee$ OR

$\oplus$ XOR

bit string

0110110110

1100011101

1110111111 bitwise OR

0100010100 bitwise AND

1010101011 bitwise XOR

1.2 applications of propositional logic

Translate english sentences into logical statements

ex "You cannot ride the roller coaster if you are under 4ft tall unless you are older than 16 years"

Let q be "you can ride a roller coaster"

r be "you are under 4ft tall"

s be "you are older than 16 years"

$(r \wedge \neg s) \rightarrow q$

Logic Puzzles

Knights and Knaves puzzle

ex. A says "B is a knight"

B says "The two of us are opposite types"

solution

if A is night

then B is a knight

but then B's statement is false (contradiction)

if A is a knave

then B is a knave

1.3 Propositional Equivalences

A compound proposition that is always true is called a Tautology. A compound proposition that is always false is called a contradiction. A compound prop that is neither is called a contingency

$p \vee \neg p$ is tautology

p	$\neg p$	$p \vee \neg p$
T	F	T
F	T	T

$p \wedge \neg p$ is contradiction

p	$\neg p$	$p \wedge \neg p$
T	F	F
F	T	F

$p \wedge q$ is contingency

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

logically equivalent if truth table same

alternative definition of two compound propositions being equivalent

compound props p and q are called logically equivalent if $p \leftrightarrow q$ is a tautology

p	q	r	s
		T	T
		T	T
		F	F

$r \leftrightarrow s$ tautology

both are true or both are false

if p and q are log. eq. we denote it as $p \equiv q$

Let T denote a compound proposition that is always

logical equivalences

equivalence			
identity laws	$p \wedge T \equiv p$	Absorption Laws	$p \vee (p \wedge q) \equiv p$
	$p \vee F \equiv p$		$p \wedge (p \vee q) \equiv p$
domination laws	$p \vee T \equiv T$	negation laws	$p \vee \neg p \equiv T$
	$p \wedge F \equiv F$		$p \wedge \neg p \equiv T$
idempotent laws	$p \vee p \equiv p$	demorgan's laws	$\neg(p \wedge q) \equiv \neg p \vee \neg q$
	$p \wedge p \equiv p$		$\neg(p \vee q) \equiv \neg p \wedge \neg q$
double negation law	$\neg(\neg p) \equiv p$		
Commutative laws	$p \vee q \equiv q \vee p$		
	$p \wedge q \equiv q \wedge p$		
Associative Laws	$(p \vee q) \vee r \equiv p \vee (q \vee r)$		
	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$		
Distributive Laws	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$		
	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$		

logical equivalences involving conditional statements

$\boxed{p \rightarrow q \equiv \neg p \vee q}$

$p \rightarrow q \equiv \neg q \rightarrow \neg p$ contrapositive

$p \vee q \equiv \neg p \rightarrow q$

$p \wedge q \equiv \neg(p \rightarrow \neg p)$

$\neg(p \rightarrow q) \equiv p \wedge \neg q$

$(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$

$(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$

$(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r)$

$(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$

equivalences involving biconditionals

$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$

$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

show $\neg(p \wedge q) \equiv \neg p \vee \neg q$

p	q	$\neg p$	$\neg q$	$p \wedge q$	$\neg p \vee \neg q$	$\neg(p \wedge q)$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology

Pf

$\underbrace{(p \wedge q)}_{A} \rightarrow \underbrace{(p \vee q)}_{B} \equiv \neg(p \wedge q) \vee (p \vee q)$

$\equiv (\neg p \vee \neg q) \vee (p \vee q)$

$\equiv (\neg p \vee p) \vee (\neg q \vee q)$

$T \vee T \equiv T$

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

$p \wedge q$ is satisfiable

p is true and q is true is a solution to the satisfiability of $p \wedge q$