Unit 9

9.1
Relations and Their Properties

Theorem

Definition
Let A and B be sets
A binary relation from A to B is a subset of AxB
If (a, b) is in R, we say a is related to b. We denote this by aRb.
If (a, b) is not in R, we say a is not related to b. We denote this by aRb (R with slash through it)

Theorem

Definition
A relation on a set A is a relation from A to A

Theorem

Definition
A relation R on a set A is called reflexive if $(a, b) \in R$ for every element $a \in A$
A relation R on a set A is called symmetric if $(a, b) \in R \Rightarrow (b, a) \in R$
A relation R on a set A is called transitive if $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$
A relation R on a set A is called anti-symmetric if $(a, b) \in R$ and $(b, a) \in R \Rightarrow a = b$

Theorem

Definition
Let R be a relation from A to B, S a relation from B to C
The composite of the relations R and S, denoted $S \circ R$, has elements (a, c) for all $(a, b) \in R$ and $(b, c) \in S$

Theorem

Definition
Let R be a relation on A
$R^{n+1} = R^n \circ R$
$R^1 = R$
$R^2 = R^1 \circ R = R \circ R$
$R^3 = R^2 \circ R = (R \circ R) \circ R$
$R^4 = R^3 \circ R = ((R \circ R) \circ R) \circ R$