Matrixes

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Transpose

If $A = A^T$ we say A is symmetric

The Identity Matrix is the square matrix with 1s on the main diagonal and zeroes everywhere else
$IA = AI = A$ for appropriately sized I
The Identity Matrix is in RREF
The Identity Matrix is symmetric
The Identity Matrix is diagonal

Inverse Matrices

$\frac{1}{A} = A^{-1}$
$\frac{1}{A}$ is bad notation

If A is a square matrix, B is the inverse of A if and only if $AB = I$ and $BA = I$ we say A is invertible and $B = A$

The matrix inverse is unique if B and C are inverses of A then $B = C = A^{-1}$

If A is not invertible we say A is noninvertible or A is singular

To find the inverse of a matrix, row-reduce the augmented matrix A I

If you get a zero row then A is not invertible because it is inconsistent
Treat the A and I as one entire row

Properties of Inverses

- I is invertible and $I^{-1} = I$
- If A is invertible then $A^{-1^{-1}} = A$
- If A, B are invertible then $(AB)^{-1} = B^{-1}A^{-1}$

- If A is invertible then $(kA)^{-1} = \frac{1}{k}A^{-1}$
- If A is invertible so is A^T, $(A^T)^{-1} = (A^{-1})^T$

Invertible Matrix Theorem

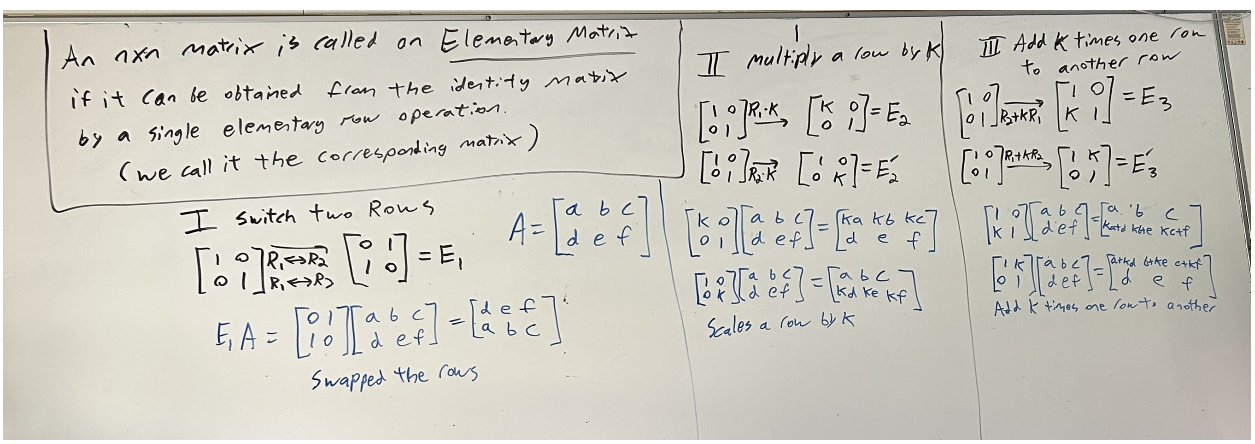
The following are equivalent for m by n matrix

1. A is invertible
2. The homogeneous system $Ax = 0$ has only the trivial solution
3. A is row-equivalent to the identity
4. $Ax = b$ has at least one solution for all b
5. There exists a matrix C such that $AC = I$

Elementary Matrices

An n x n matrix is called an elementary matrix if it can be obtained from the identity matrix by a single elementary matrix.
(we call it the corresponding matrix)

1. Switch two rows
2. Multiply a row by k
3. Add K times one row to another row



If an elementary row operation is performed on an m x n matrix A, the result is EA where E is the elementary matrix corresponding to the operation

Every elementary matrix is invertible, and E^{-1} is an elementary matrix of the same type

$[A : I] \rightarrow [I : A]$
$E_6E_5E_4E_3E_2E_1A = I$
$E_6E_5E_4E_3E_2E_1I = A^{-1} = E_6E_5E_4E_3E_2E_1$