week of 10/02

2.2

Inclusion/Exclusion
$|A \cup B| = |A| + |B| - |A \cap B|$
$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$

Example
A = {1,2} B = {1,3} C = {1,4}
$A \cup B \cup C = \{1, 2, 3, 4\}$
$A \cap B \cap C = \{1\}$
$A_1 \cup A_2 \cup ... \cup A_n = U_{i=1}^{n} = A_i$ (n goes above U, i = 1 goes below U)
for infinite, use infinity symbol

2.3: Functions

Let A and B be nonempty sets. A function f from A to B is assignment of exactly one element of B to each element of A. We write f(a) = b if b is the element assigned to the element a of A. If f is a function from A to B, we write f : A → B.
Functions are also called mappings or transformations.
A function can be defined as a relation.
f : A → B where f(a) = b can be expressed as the set of ordered pairs (a,b)

Example: Algebra
f(x) = x^2 f : IR → IR
f(0) = 0^2 = 0 (0,0)
f(10 = 1^2 = 1 (1,1)
f(-1) = $(-1)^2$ = 1 (-1,1)
You can plot the points

Definition: Let f be a function from A to B. A is the domain. B is the codomain. IF f(a) = b, we say b is the image of a and a is the preimage of b. The range/image of f is the set of all images under f. IF f is a function from A to B, we say f maps A to B.
f : A → B (A is the domain and B is the codomain)
Two functions are equal if they have the same domain, codomain, and assignments of elements in the domain

Example
A = {0,1} B = {0,1}
f : A → B
f(0) = 1 f(1) = 0
This is negation
f(F) = T f(T) = F
A = {F,T} B = {F,T}

Example
A = {0,1} → {F,T}
f : A x A → A
A x A =

{ (0,0), (0,1) }
{ (1,0), (1,1) }
f((0,0)) = 0
f((0,1)) = 1
f((1,0)) = 1
f((1,1)) = 1

A function is called a real value if its codomain is IR.
i i i i integer-valued if its codomain is Z

Example f : Z → Z f(x) = x + 1
f(0) = 0 + 1 = 1 (0,1)
f(1) = 1 + 1 = (1,2)
f(-1) = -1 + 1 = 0 (-1,0)

Let f_1 and f_2 be functions from A to IR. Then,
$(f_1, f_2) = f_1(x) + f_2(x)$https://www.overleaf.com/project/651c4dd40edf1cbc1a53c26f
$(f_1 f_2)(x) = f_1(x) f_2(x)$

Example
$f_1 : IR \rightarrow$ IR
$f_1(x) = x$
$f_2 : IR \rightarrow IR$
$f_2(x) = x + 1$
$(f_1 + f_2)(x) = (x) + (x + 1) = 2x + 1$
$(f_1 + f_2)(x) = 2x + 1$
$(f_1 f_2)(x) = (x)(x + 1) = x^2 + x$
$(f_1 f_2)(x) = x^2 + x$

Definition: Let f be a function from A to B and S is a subset of A. f(S), the image of S under f, is the subset of B consisting of images of elements of S under
$f(s) = \{t | \exists s \in S(t = f(s))\} = \{f(s) | s \in S\}$
f(S) = {b,c}

Definition: A function is one-to-one, if and only if f(a) = f(b) implies a = b for all a and b in the domain of f
A 1-1 function is also called an injective function or an injection
f(a) = f(b) ⇒ a = b
p → q
$\neg q \rightarrow \neg p$
$a \neq b \Rightarrow f(a) \neq f(b)$

Definition: A function f from A to B is onto if and only if for every element b ∈ B, there is some element a ∈ A s.t. f(a) = b.
An onto function is also called a subjective function or a subjection Basically, two different As could map onto one B

Definition: A function that is both 1-1 and onto is called a 1-1 correspondence or a bijection, or a bijective function

Its 1-1 and nothing is left without a mapping in B

Definition: the identity function on A maps each a $\in$ A to itself
Identity functions are bijections
$i_A : A \to A$, $i_A(x) = x$
f(x) = x
f : IR $\to$ IR
f(1) = 1
f(0) = 0

Definition: let f be a bijection from A to B. The inverse function of f is the function from B to A s.t. f^-1(b) = a if f(a) = b

(f^-1 reverses direction of all arrows in f)
The mapping arrows go from B to A instead of the usual A to B
The domain and codomain switch

Definition: A function is invertible if it has an inverse
only bijective functions are invertable

Example
y = f(x) = x^3
If you graph it, you get a normal cubed function
f : IR $\to$ IR
1-1 and onto (bijection)
Inverse
solve x in terms of y
$x^3 = y$
$x = y^{1/3}$
switch x and y
$y = x^{1/3}$
$g(x) = x^{1/3}$

Surjection/Injection Proof

To show f is injective
Show that $f(x) = f(y) \Rightarrow x = y$
$x \neq y \Rightarrow f(x) \neq f(y)$
Find $x \neq y$ s.t. f(x) = f(y)

To show f is surjective
Show that for any y $\in$ B we can find some x $\in$ A s.t. f(x) = b

To show f is not surjective
Find some y $\in$ B s.t. f(x) $\neq$ y for all x $\in$ A

ex. f(x) = x^3 f : IR $\to$ IR
Is f bijective? Yes
PF: Injection

Suppose $a \in A$ $b \in A$
f(a) = f(b)
$\Rightarrow a^3 = b^3 \Rightarrow a = b$
And so f is injective
PF: surjection
Suppose b is any real number. Then
Background work:
y = f(x) = x^3
y = x^3
$x^3 = y$
$x^{3^{1/3}} = y^{1/3}$
back to proof
f($b^{1/3}$) = ($b^{1/3^3}$) = b
So, f is a surjection

Find inverse
1. switch x and y
2. put y in front by doing operation

Definition: Suppose we have g : A → B and f : B → C. then (f ∘ g)(a) = f(g(a)).
f ∘ g is called the composition of f and g. The domain of f ∘ g is A, and the codomain is C

Definition: the Floor function assigns to a real number X the largest integer less than or equal to x f(x) = [x] [1] = 1 [0] = 0
[0.5] = 0
[1.5] = 1

Definition: the ceiling function assigns to a real number x the smallest integer greater than or equal to x
f(x) = [x]
[1] = 1 [0] = 0
[0.5] = 1 [1.5] = 2
only top bracket for ceil, only bottom bracket for floor
graph is the funny one with the lines like steps. floor: shaded on left dot, ceiling: shaded on right dot

factorial function
f(n) = n!

2.4: Sequences and Summations

ex. 1,2,3,4,5,...
$a_1, a_2, a_3, a_4, a_5, ...$
these are terms of a sequence
ex. 2,4,6,8,10

Definition: A geometric progression is a sequence of the form $a, ar, ar^2, ...$
a is the initial term and r is the common ratio

ex. 1,2,4,8,16,...
a = 1 r = 2
ex. 2,6,18,54,162
a = 2 r = 3

Def: an arithmetic progression is a sequence of the form a, a+d, a+2d, a+3d, ...
a is the intial term and d is the common difference
ex. 1,3,5,7,9,...
a = 1 d = 2
ex. 2,5,8,11,14,...
a = 2 d = 3

A finite sequence is called a string
1,2,3,4 is a string with length 5
Empty string has length 0

f(n) - 1 + 2n n = 1, 2, 3, ...
f(1) = 1 + 2 * 1 = 3
f(2) = 1 + 2 * 2 = 5
f(3) = 1 + 2 * 3 = 7

f(n) = $3 * 2^n$ n = 1, 2, 3, 4, ...
f(1) = ... = 6
f(2) = ... = 12
f(3) = ... = 24
f(4) = ... = 48
a = 6 r = 2
6, 12, 24, 48, ...

Another way to generate a sequence is by using recurrence relation
$a_1, a_2, a_3, a_4, ... = \{a_n\}$
Define $\{a_n\}$ by $a_0 = 1$ $a_n = a_{n-1} + 2$ n = 1, 2, 3, 4
You get 1, 3, 5, 7, ...

ex. $a_0 = 1$ $a_n = 2 * a_{n-1}$ for n = 1, 2, 3, ...
$a_1 = 2 * a_{1-1} = 2 * a_0 = 2 * 1 = 2$
$a_2 = 2 * a_{2-1} = 2 * a_1 = 2 * 2 = 4$
1,2,4,8,16

Fibonacci Sequence
$a_0 = 1$ $a_1 = 1$ $a_n = a_{n-1} + a_{n-2}$ $n = 2, 3, 4, 5, ...$

Closed formula
IF a closed formula exists, its called a solution of a recurrence relation
$a_0 = 1, a_n = 2 * a_{n-1}$ n = 1,2,3
1,2,4,8,16
f(n) = 2^n this is a closed formula/solution

Summations (adding up terms of a sequence)

ex. $\sum_{i=1}^{4} a_i = a_1 + a_2 + a_3 + a_4$
i is the index 0 is the lower limit
4 is the upper limit
ex. $\sum_{i=0}^{4} 1 = 1 + 1 + 1 + 1 + 1$
ex. $\sum_{i=1}^{4} i = 1 + 2 + 3 + 4 = 10$

Summation formula
NEED TO MEMORIZE*
$\sum_{k=0}^{n} a = (h + 1) * a$
$\sum_{k=0}^{n} ar^k = \frac{(ar^{n+1} - a)}{r-1} = \frac{a(r^{n+1} - 1)}{r-1} = \frac{a(1 - r^{n+1})}{1-r}$, r $\neq 1$
k formulas, look them up
other formulas from calculus