week of 9/18

1.8: Proof methods and strategy

Proof by exhaustion
ex. show that $(n+1)^3 \geq 3^n$ if n is a positive integer with $n \leq (n = 1, 2, 3, 4)$
Pf
n can only be 1, 2, 3, 4
$(1+1)^3 = 2^3 = 8$ $3^1 = 3$
$(2+1)^3 = 3^3 = 27$ $3^2 = 9$
$(3+1)^3 = 4^3 = 81$ $3^3 = 27$
$(4+1)^3 = 5^3 = 125$ $3^4 = 81$
therefore, (n+1)$^3 \geq 3^n$

Proof by cases
show that $n^2 \geq$ n for any integer n.
case 1 n = 0 $0^2 \geq 0 \rightarrow 0 \geq 0$ true
case 2: $n \geq 1$
$n \geq 1 \rightarrow n * n \geq n * 1 \rightarrow n^2 \geq n$
case 3
$n \leq -1$
we know $n^2 \geq 1$
also, $n \leq -1$
therefore, $n^2 \geq n$
$(-1)^2 = 1$
$(-2)^2 = 4$
$(-3)^2 = 9$

without loss of generality (WLOG)
this term means some cases of the proof are very similar to already proven cases. You say WLOG to shorten the proof statement
PF
case 1 case 3
case 2 case 4
similarly,

existence proof
1. constructive: giving specific examples
2. nonconstructive: you prove an example exists without showing a specific example

constructive ex. show that there exists a positive integer that can be written as the sum of two cubes
PF
$10 \geq 9 = 10^3 + 9^3 = 12^3 + 1^3$

nonconstructive ex. show that there exists irrational numbers x and y s.t. x^y is rational
PF
we know $\sqrt{2}$ is irrational. consider the number $\sqrt{2}^{\sqrt{2}}$
if $\sqrt{2}^{\sqrt{2}}$ is rational, then $x = \sqrt{2}$ and $y = \sqrt{2}$ are the pair we are looking for
if $\sqrt{2}^{\sqrt{2}}$ is irrational, then $\sqrt{2}^{\sqrt{2}^{\sqrt{2}}} = \sqrt{2}^{\sqrt{2}} = 2$

and so $x = \sqrt{2}^{\sqrt{2}}$ and $y = \sqrt{2}$ are the pair we are looking for. We conclude that one of the two pairs works (we're unsure which pair works)