

2.1

$$\begin{aligned} A &= \{1,2\} \quad B = \{a,b\} \\ A \times B &= \{(1,a), (1,b)\} \\ &\quad (2,a), (2,b) \end{aligned}$$

if $A \times B$ is empty

$$A \times B = B \times A$$

$$A \times B = \{\}$$

$$A = \{1,2\} \quad B = \{\}$$

$$A \times B = \{\}$$

if $A = B$, then $A \times B \rightarrow A \times A$

A subset of $A \times B$ is a relation from A to B. A relation to itself is called a relation on A

D is the domain The truth set of a statement $P(x)$ is the set of elements x in D s.t. $P(x)$ is true
 $\{x \in D | P(x) = x \in D | P(x) \text{ is true} \}$

2.2: Set Operations

Def: The union of the sets A and B, denoted $A \cup B$ contains the elements that are in A or B (or both)
 its the entire venn diagram (at least one section)

$$\text{ex. } A = \{1,2,3\} \quad B = \{2,3,4\} \quad A \cup B = \{1,2,3,4\}$$

Def: The intersection of the sets A and B denoted $A \cap B = \{x | x \in A \text{ and } x \in B\}$

Two sets are called disjoint if their intersection is empty (is the empty set)

$$\text{ex } A = \{1,2\} \quad B = \{3,4\}$$

A and B are disjoint

Inclusion - Exclusion

$$|A \cup B| = |A| + |B| - |A \cap B|$$

Def: the difference of sets A and B denoted by $A - B$, contains the elements that are in A, but not in B.
 $A - B$ is also called the complement of B with respect to A.

($A - B$ same as $A \cap B^c$)

$$A - B = \{x | x \in A \wedge x \notin B\} = A - B = \{x | (x \in A) \wedge \neg(x \in B)\}$$

ex.

$$A = \{1,2,3\} \quad B = \{3,4,5\}$$

$$A - B = \{1,2\} \quad B - A = \{4,5\}$$

Def: the complement of the set A denoted \overline{A} , is the complement of A with respect to u. So $\overline{A} = u - A$.

$$\overline{A} = \{x | x \in u \wedge x \notin A\}$$

$$= \{x | x \notin A\}$$

$$A - B = \{x | x \in A \wedge x \notin B\}$$

Fact: $A - B = A \cap \overline{B}$

$$A - B = \{x | x \in A \wedge x \notin B\}$$

Power set

$$P(A) = \{ \{\}, \{1\}, \{2\}, \{1,2\} \}$$

$$|A| = 2$$

$$|P(A)| = 2^{|A|} = 2^2 = 4$$

$$|P(5)| = 2^{|5|}$$

Rosen page 130 Table 1

compare to Rosen page 27 table 6

$$\wedge \rightarrow \cap \vee \rightarrow \cup$$

$$T \rightarrow u \quad F \rightarrow \emptyset$$

ex. Show that $A \cap u = A$ Pf.

$$A \cap u = \{x | x \in A \wedge x \in u\}$$

$$= \{x | x \in A\}$$

$$= A$$

ex. Show that $\overline{(\overline{A})} = A$

Pf:

$$\overline{A} = \{x | \neg(x \in A)\}$$

$$\overline{(\overline{A})} = \{x | \neg(\neg(x \in A))\} = \{x | x \in A\} = A$$