astrea

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Cubesat Constellation
Astrea

ANNEX I: Orbit Design

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1 | Orbit Geometry

Throughout this chapter, the bases of orbital geometry will be explained in order to correctly understand the parameters that will later be exposed when dealing with the constellation orbits (or the position of the satellites in them). However, long theoretical explanations will be avoided so as not to distract the reader from the main objective of the project.

To understand the movement in space is enough to apply the Newton's laws. These, however, need an inertial non-rotating frame to be correctly described. When dealing with Earth-orbiting, one usually chooses a reference system called *geocentric-equatorial system* which is shown in the figure 1.0.1 As can be seen, the XY plane coincides with the plane Equatorial with the X axis pointing in the direction of the vernal equinox [1]. The Z axis correspond the axis of rotation of the earth and points to the north (following the right-hand rule).

[1] an imaginary line found by drawing a line from the Earth to the Sun on the first day of spring

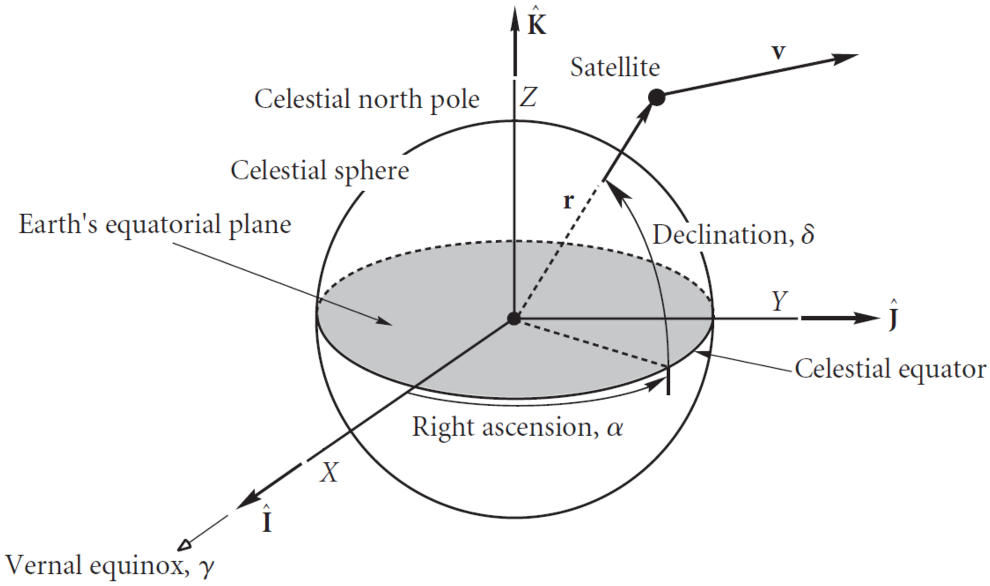

Figure 1.0.1: Geocentric-equatorial frame. Extracted from [2]

By defining this system, any point in the space can be depicted by its position vector r and we can study its movement by the velocity vector $\dot{r}$. These elements are useful especially for computational work but they nearly do not provide information about the orbit. For these reason, the orbital elements were developed.

1.1 Keplerian Geometry

The *Classical Orbital elements*, also known as the *Keplerian elements* as an attribution to Johannes Kepler, are six independent quantities which re sufficient to describe the size, shape and orientation of an orbit. This set of elements are shown in the figure 4.3.6 and are defined as follows:

- **Semi-major axis** (a): It is related to the size of the orbit and its defined by the sum of the apogee (furthest point) and the perigee (closest point) divided by two.
- **Eccentricity** (e): It defines the shape of the orbit with respect to that of a circle. Thus, the eccentricity of a circular orbit is null while hyperbolic orbits have an eccentricity greater than one.

Circular	$e = 1$
Elliptical	$0 < e < 1$
Parabolic	$e = 1$
Hyperbolic	$e > 1$

Table 1.1.1: Eccentricity values depending on the shape of the orbit

- **Inclination** (i): the inclination is the angle between the positive Z axis and the angular momentum vector (**h**) which is perpendicular to the orbital plane. The inclination of the orbit can take a value from $0\,\mathrm{deg}$ to $180\,\mathrm{deg}$. For $0\,\mathrm{deg} \leq i \leq 90\,\mathrm{deg}$ the motion *posigrade* and for $90\,\mathrm{deg} \leq i \leq 180\,\mathrm{deg}$ the motion is *retrogade*.

- **Right ascension of the ascending node - RAAN** (Ω): This parameter, along with the inclination define the orientation of the orbital plane. It is the angle between the positive X axis and the intersection of the orbital plane with the equatorial plane XY in counterclockwise direction. The intersection mentioned is called the node line and the point where the orbit passes through the node line (from south to north) is the ascension node ($0\,\mathrm{deg} \leq \Omega \leq 360\,\mathrm{deg}$).

- **Argument of perigee** (ω): Is defined as the angle between the ascending node and the perigee. It describes the orientation of the ellipse with respect to the frame ($0\,\mathrm{deg} \leq \omega \leq 360\,\mathrm{deg}$).

- **True Anomaly** (ϕ): This last quantity is used to describe the satellite's instantaneous position with respect to the perigee. Is the angle, measured clockwise, between the perigee and the satellite position. From all the orbital elements, the true anomaly is the only that changes continuously. Sometimes, true anomaly is substituted by the mean anomaly, which can be calculated using another auxiliary angle called the eccentric anomaly.

$$\begin{gathered} \cos E = \frac{e + \cos\theta}{1 + e\cos\theta} \\ M = E - e\sin E \end{gathered} \tag{1.1.1}$$

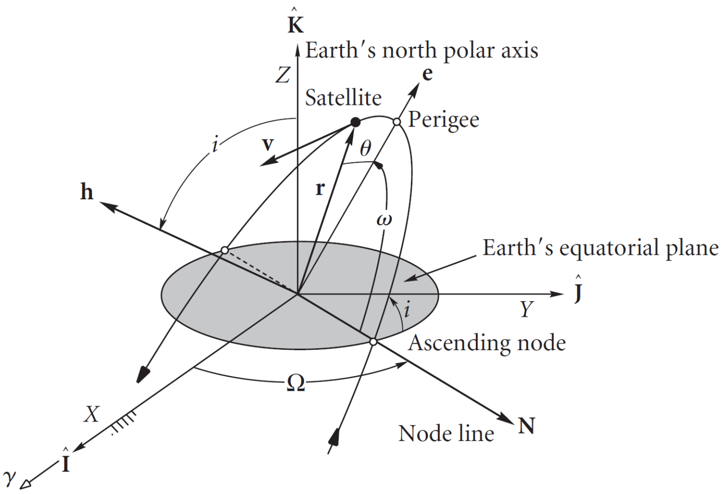

Figure 1.1.1: Geocentric-equatorial frame and the Classical Orbital Elements. Extracted from [2]

1.2 Dynamic equations

As aforementioned, the motion of an object in the space can be described using the Newton's laws. The basic idea developed by Newton is to study the Cubesat and the Earth as a spherical bodies in mutual gravitational attraction and neglect the gravitational forces caused by other objects (this is called the *two body* problem). The forces balance is simple since we only have the Earth gravitational attraction, which must compensate the centripetal acceleration of the satellite. Thus, using the law of universal gravitation,

$$-G\frac{M_E m_{sat}}{r^3}\vec{r} = m_{sat}\vec{a}_{sat} \tag{1.2.1}$$

Where G is the gravitational constant and r represents the distance between the satellite and the Earth. From the last equation, we only want to obtain the acceleration, therefore:

$$-G\frac{M_E}{r^3}\vec{r} = \vec{a}_{sat} = \frac{d^2\vec{r}}{dt^2} \tag{1.2.2}$$

For simplicity, it usual to denote $\mu = GM_{earth}$ resulting in the following equation:

$$-\frac{\mu}{r^3}\vec{r} = \frac{d^2\vec{r}}{dt^2} \tag{1.2.3}$$

This expression is a second order equation that models the motion of the Cubesat relative to the Earth and it can be analytically solved. The only problem is that several hypotheses have been applied that make the case different from reality. The formulation should be modified to take into account the effects due to:

- More bodies attracting the satellite (Sun, Moon, Venus, etc.)
- The existence of more forces like the drag, the solar radiation pressure, etc.
- The earth is not an spherical body.

The corrections for considering these things are called perturbations and they are explained in the Chapter 4 of this part of the report.

2 | Orbital Coverage

2.1 Satellite Footprint

2.1.1 Introduction

The first step to build a satellite network with global coverage is to compute a single satellite footprint.

The footprint of a satellite is defined as the region of Earth where a single satellite can be seen. This Earth coverage surface provided is spherical and depens on some orbital paramaeters such as:

- **Height**. When increasing height the footprint of a satellite grows.
- **Elevation angle**. When increasing the elevation angle, which is the angle between the satellite and the horizontal plane of an arbitrari point of the Eartg, the surface seen by the satellites descreases. (This parameter will be later studied in detail)

2.1.2 Footprint Computation

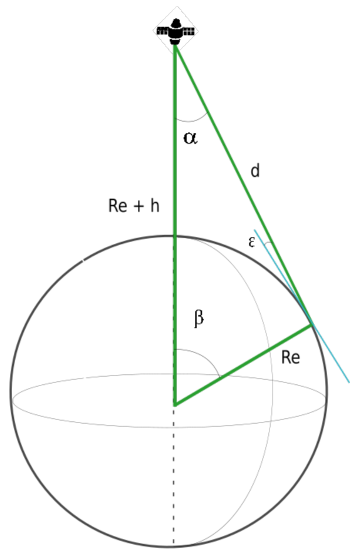

Figure 2.1.1: Single satellite coverage geometry

In order to compute the coverage area we must solve the triangle depicted in figure 2.1.1 where the basic geometry of a satellite footprint is shown. The MATLAB routine that does this calculation is found in [REF TO ANNEX VII. Satellite Footprint]

The most needed parameters are the distance from a random point on Earth (where we can suppose our ground station to be) to the satellite denoted by d and the central angle, denoted with a β .

Applying cosines law to the triangle shown in figure 2.1.1, we obtain the following expression:

$$r^2 = R_{earth}^2 + d^2 - cos(90 + \epsilon) \tag{2.1.1}$$

Isolating d from the equation above and changing $r = R_{earth} + h$, where h is the actual height of the satellite regarding the Earth surface, we arrive at:

$$d = R_{earth} \left[\sqrt{\left(\frac{h + R_{earth}}{R_{earth}} \right)^2 - cos^2\epsilon} - sin\epsilon \right] \tag{2.1.2}$$

From the figure 2.1.1 we can also extract a relation between the central angle, the distance d and the elevation angle. This relation together with the equation 2.1.2 allow us to find β.

$$\begin{aligned} d cos\epsilon &= (R_{earth} + h)\, sin\beta \\ \beta &= \frac{1}{R_{earth} + h} \arcsin\left[d(\epsilon) cos\epsilon\right] \end{aligned} \tag{2.1.3}$$

Once the central angle β has been computed we are able to obtain the footprint satellite's are using the equetion below:

$$S = 2\pi R^2_{earth}(1 - cos\beta) \tag{2.1.4}$$

The size of the footprint will determine the level of coverage our constellation provides, therefore when deciding the value of the orbital parameters it has to be a factor to consider.

2.2 Elevation Angle

The angle of elevation is essential to calculate the geometry of our constellation. As discussed previously, our aim in this project report is to justify how global coverage will be fulfilled. First, we define for a given groundstation the angle between its beam pointing right to the satellite and the horizontal local plane as the elevation angle. Secondly, a study is conducted in order to relate the height of the satellite, the elevation angle and the coverage of the Earth. Finally, we complete our orbital design by configuring a constellation that will securely define a global coverage fulfillment. Next, we will be defining how these parameters are related.

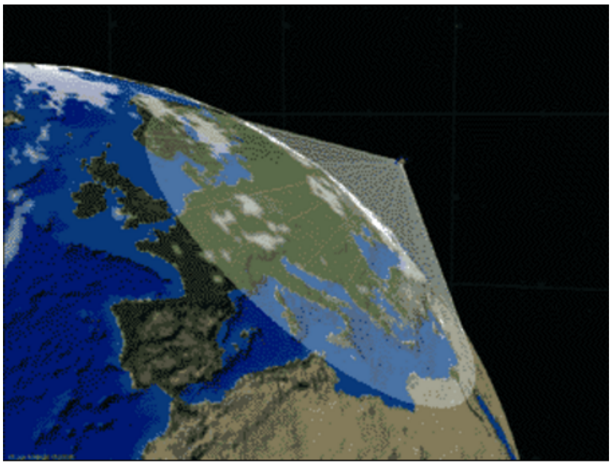

Figure 2.2.1: Elevation angle cone. Source: NOAA

2.2.1 Elevation angle cone

Global coverage will be discussed considering the elevation angle and its resulting footprint on Earth. The elevation angle is described by the angular orientation of the antennas in the ground station. However, this angle is also perceived by the satellite in a similar way - it will vary depending on the orientation of the satellite and the angle between horizontal local planes. In order to describe the footprints we must define a cone which vertex is set at the antennas of the satellite, pointing down to Earth, and which generatrix is given by the angle of elevation. This elevation angle based cone is the description of the paths that our communications can take place. In other words, the generatrix of this cone is setting the limits in which the antenna will operate as function of the elevation angle. This implies that our satellite will be able to communicate to all the points contained in the cone. Finally, this cone will be describing a circular surface on top of the Earth which we will call the footprint of the satellite. Additionally, this footprint is the coverage that a single satellite can generate, hence we will be distributing satellites all around the Earth in order to fulfill global coverage.

2.2.2 Atmospheric restrictive conditions

In order to obtain the final restrictive angle of elevation needed to contact the ground stations some considerations have to be made. First, a description of the different atmospheric conditions will be defined. Then, we will relate these to our bandwidth in order to analyse if they must be taken into account when communicating with ground stations. [elec2013cantero]

Atmospheric gases Water vapour and oxygen absorptions; important when frequencies are above 3 GHz. More information [64] and [07328546]

Precipitations and Clouds These conditions are relevant for signals above 10GHz.

Cross Polarization Discrimination Direct consequence of both terrestrial links and rain. Related to non-spherical rain drops which have a polarization rotated towards the component of the major axis, and hence may attenuate a signal wave.

Scintillation Is a rapid fluctuation in signal amplitude at low elevations.

Radio Refracting Index For elevation angles below 3 degrees (especially those below 1 degree) and depending on the latitude of our satellite we may find big signal losses due to the resulting differential ray bending.

Ionosphere layers

D layers 60-90km. Considerable signal absorptions for 10 MHz and below, with progressively less absorption at hihger frequencies and oblique incidencies.

E layers 90-150 km. Absorptions relevant for frequencies lower than 10 MHz, although for sporadic E propagations this value may be increased to 50MHz.

Sporadic E layers Reflections of radio waves in this thin-cloud small layer may reach to frequencies up to 225MHz. These layers are usually formed following the E layers altitudes.

F layers 150-500 km and higher. No absorptions or reflections for these layers. The F2 region allows the longest communication paths, above 210km of altitude.

By means of these physical phenomena we can substract the elevation angle as function of the latitude. However, we must take into account that these physical conditions give a value for the elevation angle which may not be the most restrictive. Global coverage conditions, bandwidths, inclination and the final distribution of our constellation will be considering this elevation angle and viceversa, iteratively.

The ASTREA CONSTELLATION was designed and optimized in order to fulfill global coverage for a constant elevation angle - respect to the latitude - of 20 degrees.

Our constellation will be operating at S-band for telemetry and X-band for data relay. Therefore, the satellites need to be operating up to 10 GHz. This directly implies that physical conditions such as atmospheric gases, precipitations and clouds must be studied when determining the elevation angle needed.

The minimum elevation angle is applied in low latitude regions for constellations based on polar orbits whereas this value is also applied out of the low-latitude region for inclined orbit constellations [a general evaluation criterion]. The minimum elevation angle is a specific value which is equivalent to the maximum elevation angle needed to fulfill coverage at a given latitude, considering that the distance between planes is maximum at the equator and that it is reduced for higher latitude positions.

This elevation angle is maximum in a Walker Delta constellation when the latitude is equal to the orbital inclination angle[a general evaluation criterion]. This means that the limiting restrictive elevation angle that we need in order to fulfill global coverage is defined at latitude equal to 72 degrees, which is the inclination of our constellation. Otherwise, we can define a constant elevation angle that will apply to the equator, which will then be, for this model, the restrictive condition.

Accordingly, the approach considered is that of a constant elevation angle to fulfill global coverage at 20 degrees. This implies that our constellation is configured and distributed in order to optimize coverage both at the equator and at the maximum elevation angle latitude. This value has been contrasted and discussed considering the atmospheric conditions and analysing experimental data, which contemplates also the rotation of the Earth among others.

This constant elevation angle model will be very useful in order to analyse and calcute the distribution of the constellation. Nevertheless, we need to describe in an accurate way the minimum elevation angle respect to the latitude. This is why a different model must be approached.

Thus, we need to describe the elevation angle respect to the latitude of our constellation taking into account all considerations above. First, for a latitude of 0 degrees the value of the minimum elevation angle will be of 10 degrees. In our model we have considered that this value was of constant 20 degrees, so in fact we have redundant global coverage. At latitudes between the equator and 45 degrees our second model increases linearly to 15 degrees. From 45 to 60 degrees the elevation angle also increases linearly to 22 degrees. Then, from 60 to 70 tha value increases highly reaching a peak at 70 degrees, where the elevation angle will be of 30 degrees. Finally, from 70 to 80 degrees this model is reduced linearly to 15 degrees, and from 80 to the north and south poles it falls to 0 degrees. This is a simple model that will guarantee global coverage, especially at the latitudes of our ground stations.

For the distribution of ground stations we need to guarantee that these will be covered either by one satellite or two at any given moment. As discussed before, the model used was based on a constant 20 degree constant elevation angle. However, for this last model that we have described - which is more realistic - we obtain more coverage than for the constant model except for those regions next to the peak. The most restrictive latitude is now 60 degrees - where all the ground stations are set - and has a 22 degree restriction of the angle of elevation, which is higher than the constant model described previously. These facts imply the following:

- At low latitudes (between 0 and 30 degrees) the constellation fulfills global coverage generously.
- At ground station latitude (60 degrees) the constellation is covering the station succesfully. As discussed before, our first model considered a constant 20 degree elevation angle instead of the 22 degrees that now must be corrected. For the previous model coverage was well established with margin. For the latter, the margin has decreased but coverage is still complete. Note: each orbit could be reduced by a number of satellites per plane, but this would endanger the correct and stationary working of the constellation. In this case we would not be able to control possible incidencies such as unoperative

satellites with enough margin.

- The ground stations are covered at all time for at least one satellite.

2.2.3 Elevation angle of other current constellations

Analysing the minimum elevation angle needed in order to fulfill global coverage requieres, as mentioned before, the understanding first of the restrictive conditions of the atmosphere and how these will alter it. As a consequence of the different physical conditions given before we will be able to determine a relation between latitude and elevation angle. All the same, the elevation angle depends on the bandwith in which the satellites operate, hence different distributions of this angle respect to the latitude will be described depending on the bandwidths used.

- Celestri: 18.8 to 20.2 GHz at 48 degree inclination.
- GlobalStar: 2.4 GHz at 52 degree inclination.
- Iridium: 20 to 30 GHz at 90 degree inclination - polar orbits.

Comparing our configuration to other present constellations some clarifications can be made:

- The minimum elevation angle peak is proportional to the bandwidth at which the satellite is communicating with Earth. For instance, Iridium's peak of elevation angle is the highest relative to the other configurations since it is also working with the highest frequency signals.
- The latitude position of the peaks is related to the inclination of the constellation. Iridium, - a polar orbit based configuration - describes a peak at 90 degrees of latitude whereas Celestri and GlobalStar are near 40 to 50 degrees.

With these tendencies our model can be confirmed as function of the frequencies of the signals and related to the inclination of the orbits.

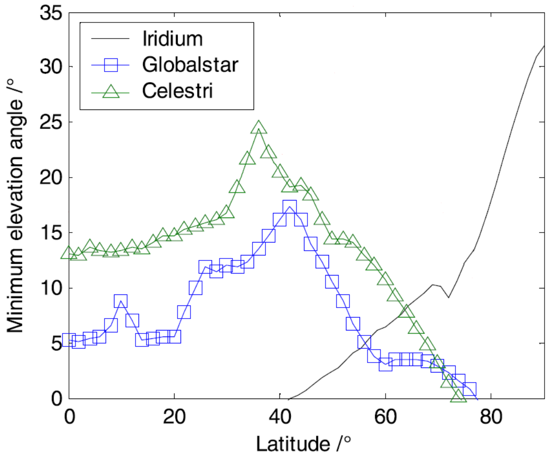

Figure 2.2.2: Minimum elevation angle as function of latitude. Source: [a general evaluation criterion]

2.3 Minimum Plane Inclination

As it has been pointed before, there are several factors to take into account in order to design a constellation that provides global coverage on Earth. In this section the minimum inclination to achieve that purpose is asessed. Using the theory previously developed in the MATLAB code [REF TO ANNEX VII. Minimum Plane Inclination], we can observe the following results:

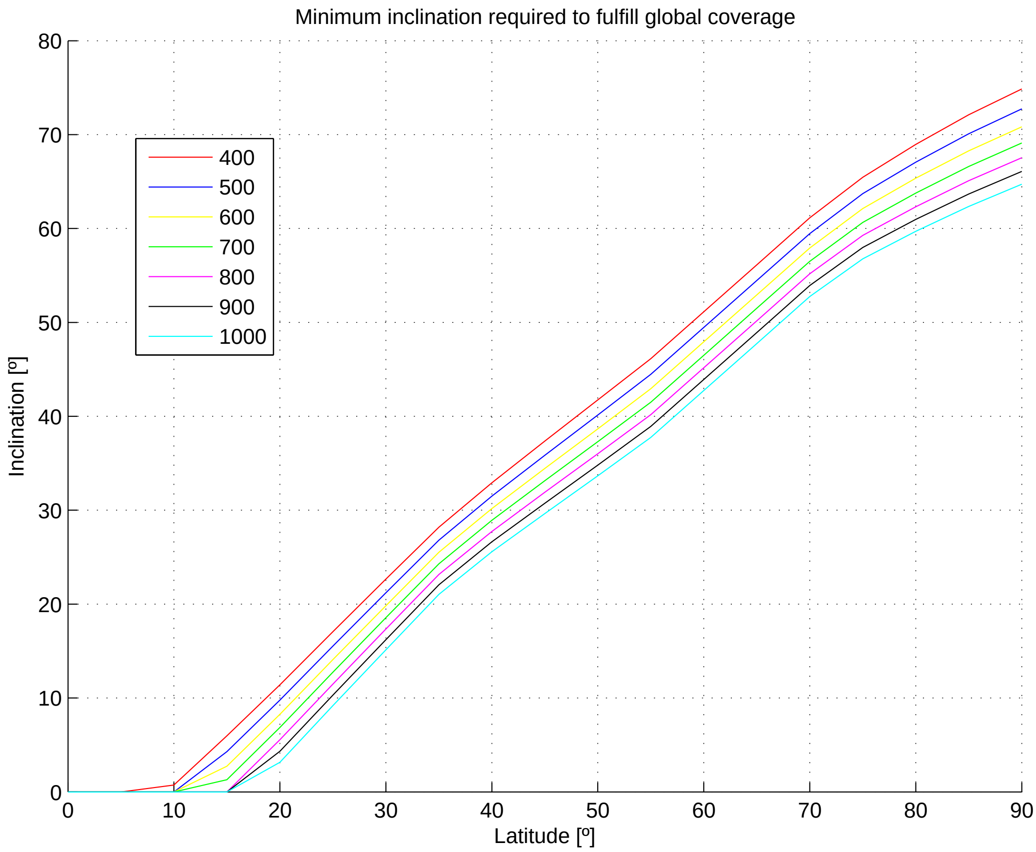

Figure 2.3.1: Minimum Inclination to provide coverage at different latitude for different orbit apogees.

As it can be observed, if the goal of the design is to provide full global coverage, the distribution of elevation angles with latitude is not significant, since the inclination is required to be higher than aproximately 75°. In the other cases, the change of minimum elevation angle distribution causes changes of tendency in the distribution of inclination required.

Conclusion

The main point is that there is a limit inclination for a Walker-Delta constellation configuration in order to provide global coverage at the desired latitude. With this study, this limits in the design algorithms can be set.

2.4 Satellite to Satellite Visibility

One of the restrictive conditions that we must take into account is the visibility between satellites. Communications among different satellites is they key point of our constellation.

Therefore, this has to be guaranteed considering a model which will represent the conditions of the atmosphere for LEO communications.

In order to fulfill communications among satellites we must consider that a straight beam can be described between two consecutive satellites, which will then communicate with others. These two satellites will need to be at a distance such that the Earth itself doesn't interfere in this straight beam. Depending on the bandwidth of our constellation we will also have to consider that this communication beam will not interfere with a given element of the atmosphere such as the upper layers of the ionosphere. Thus, a model will be developed in order to limit the minimum altitude at which this beam is guaranteed to pass through safely.

This model is a restrictive condition that we need to satisfy when designing our constellation.The highest restrictive conditions are the upper layers of the ionosphere, specifically the E layers at 150 km above the surface of the Earth. Reflections and absorptions can occur for both E layers and sporadic E layers. E layers may reflect signals of frequencies below 10 MHz whereas Sporadic E layers can be a problem up to 225 MHz. Working for S bands and X bands implies that neither of these layers will alter the signals of our constellation.

Operating and computing with these conditions a maximum distance is a obtained which defines how far these satellites can be from each other. A simple equation is used to calculate this distance considering the height of the satellites and the height of the E layers in the atmosphere.

$$d = 2\sqrt{(R + h_{sat})^2 - (R + h_{atm})^2}$$

$$h_{sat} = 550\ km$$

$$h_{atm} = 150\ km$$

$$R = 6371\ km$$

The final expression for the distance between two satellites indicates that distance between two satellites has to be smaller than 4640 km approximately. For this result we conclude that this restrictive condition is actually less restrictive than the 9 planes needed for our constellation. Thus, satellite to satellite visibility is a parameter which will not affect the design of our constellation after all.

3 | Constellation Configuration

3.1 EXAMPLE: The Global Positioning System Example

The GPS Constellation: An example of irregular distributed orbits [3]

The GPS is a constellation property of the U.S. It provides positioning, navigation and timing. The constellation was designed with a 24-slot arrangement to ensure a visibility of at least four satellites from any point on the planet. Nowadays the constellation has expanded to a total operative number of 27-slot since June 2011. Some characteristic parameters of the satellites are the following:

- Orbit: Almost Circular
- Height = 20,200 km (MEO);
- Lifetime = 12.5 years;
- Satellite Cost = 166 million USD;
- Inclination = 55°;
- Number of planes = 6;
- Phasing: 30°-105°-120°-105°;

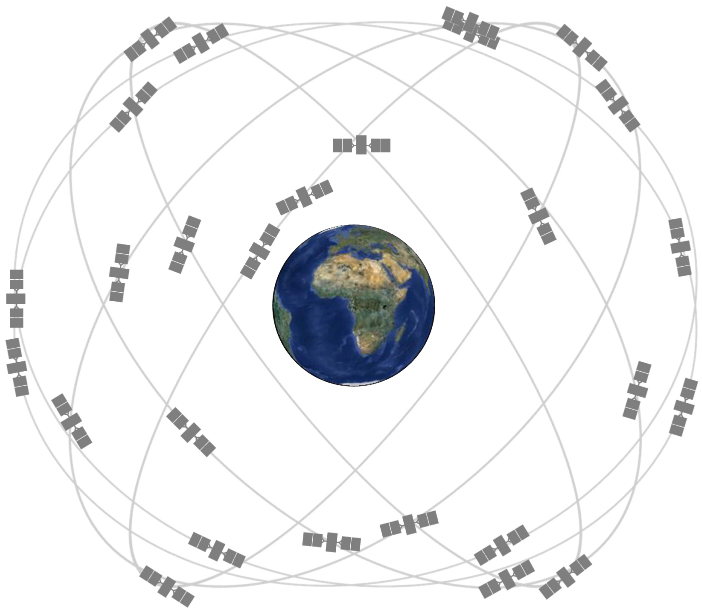

Figure 3.1.1: Distribution of the expanded 24-slot GPS constellation. [3]

3.2 EXAMPLE: The Iridium Constellation

The Iridium Constellation: An example of near polar orbits [7]

The Iridium constellation is a private constellation. It provides voice and data coverage to satellite phones among other services. The constellation was designed with 77 satellites, giving name to the constellation by the chemical element. The constellation was reduced to a number of 66. Sadly, Dysprosium is not such a good commercial name. Some characteristic parameters of the satellites are the following:

- Orbit: Almost Circular
- Height = 781 km (LEO);
- Satellite Cost = 5 million USD;
- Inclination = 86.4°;
- Number of planes = 11;
- Phasing: Regular;

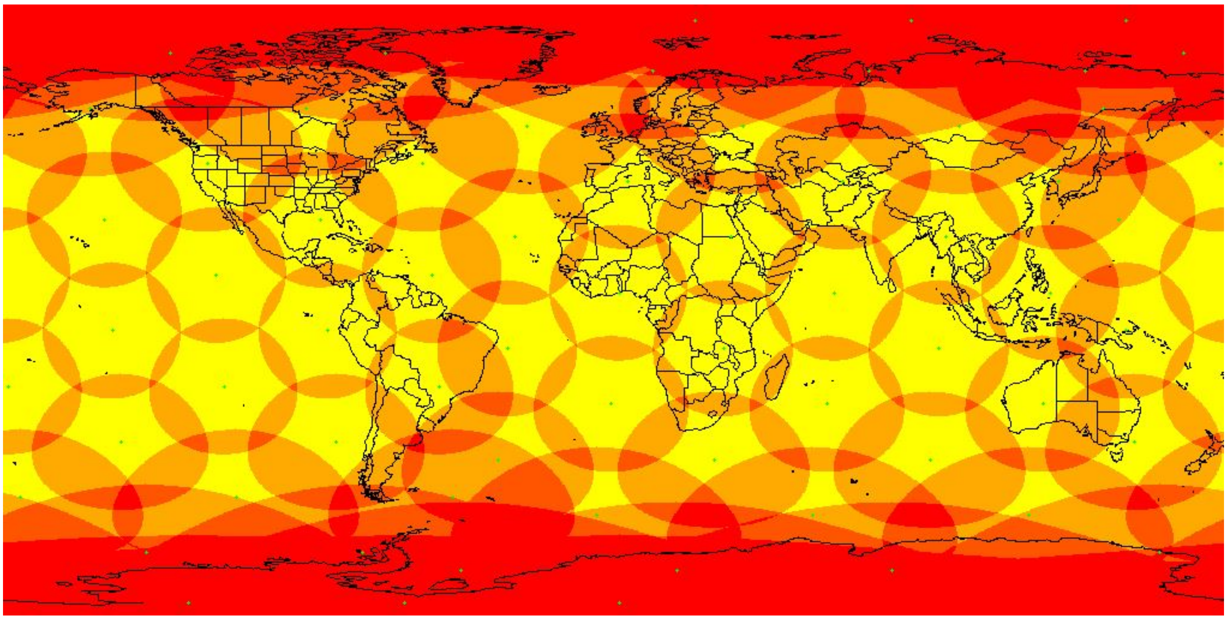

Figure 3.2.1: Distribution of the 66 Iridium constellation satellites. Generated using [4]

3.3 The Streets of Coverage Method

This Street of Coverage Method is obtained from [6]. As you can see in the figure below, the relations between angles seen from different satellites can be easily computed. The main variables are the following:

Streets of Coverage Method Variables	
N	Number of Satellites
n_p	Number of Planes
N_{pp}	Number of Satellites per plane
S	Separation between satellites of the same plane
D	General space between planes [º]
D_0	Space between antiparallel planes [º]
ε	Elevation angle [º]
λ_{street}	Street of coverage Width [º]
λ_{max}	Maximum footprint Radius [º]

Table 3.3.1: Streets of Coverage Method main variables

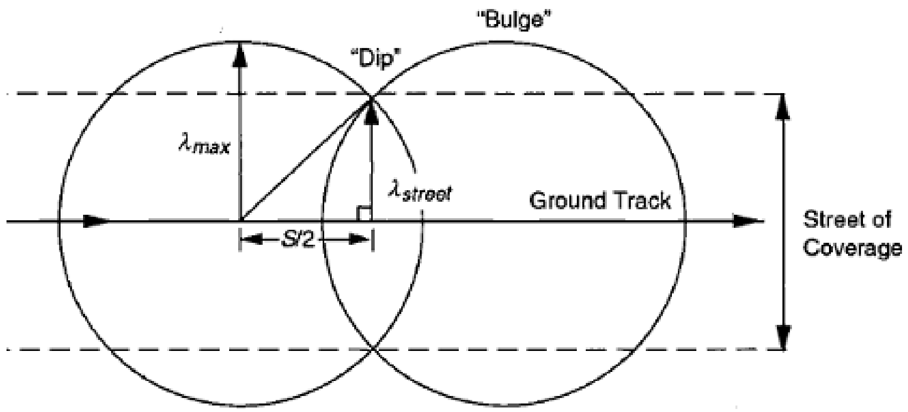

Figure 3.3.1: Single plain street of coverage. The footprints of the satellites superpose leading to a street. [5]

From the figure it can be inferred:

$$S < 2\lambda_{max}$$

$$cos(\lambda_{street}) = cos(\lambda_{street})/cos(S/2)$$

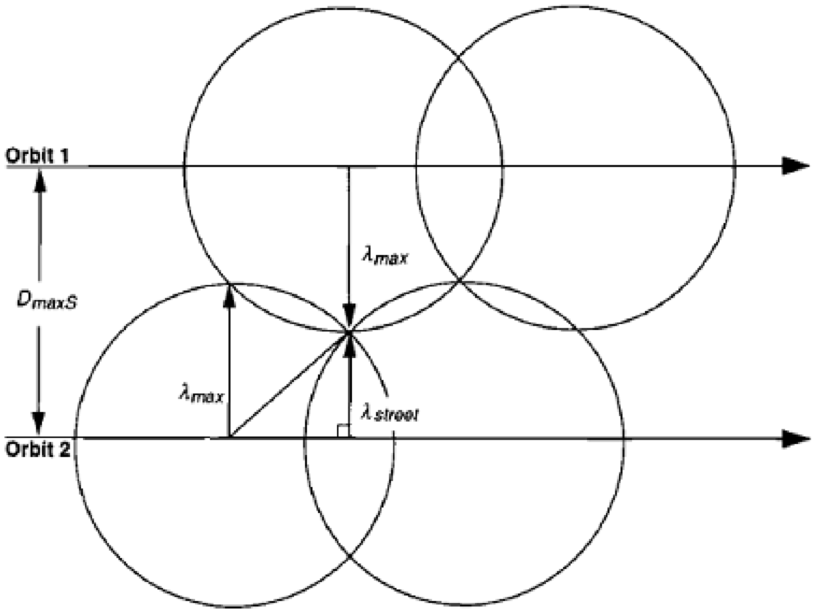

Figure 3.3.2: Two plains streets of coverage. An optimum phasing needs to be obtained. [5]

From this point of view, in general:

$$D = \lambda_{street} + \lambda_{max}$$

n For the antiparallel planes:

$$D_0 = 2\lambda_{street}$$

And the overall relationship between planes sums:

$$180 = (n_p - 1)D + D_0$$

The algorithm for computing the Streets of Coverage Results is defined in the following way:

Inputs: Height, elevation, inclination... $\rightarrow \lambda_{max} \rightarrow N_{pp} = \left\lceil \frac{360}{2\lambda_{max}} \right\rceil \rightarrow$
$S = 360/N_{pp} \rightarrow \lambda_{street} \rightarrow n_p \rightarrow N = N_{pp} * n_p$

3.3.1 Results of Streets of Coverage

A MATLAB routine has been designed to compute the previously described algorithm. In this conceptual design phase, different heights are computed in order to see the evolution of the number of satellites.

General Solution

The program in runned in a broad range of parameters to see the evolution of the number of satellites. As it can be predicted, as the height increases the number of satellites is reduced. The reason is that the footprint of the satellites increases with the height. In addition, as the minimum elevation over the horizon to contact the satellites is reduced, the number of satellites is also reduced for the same reason.

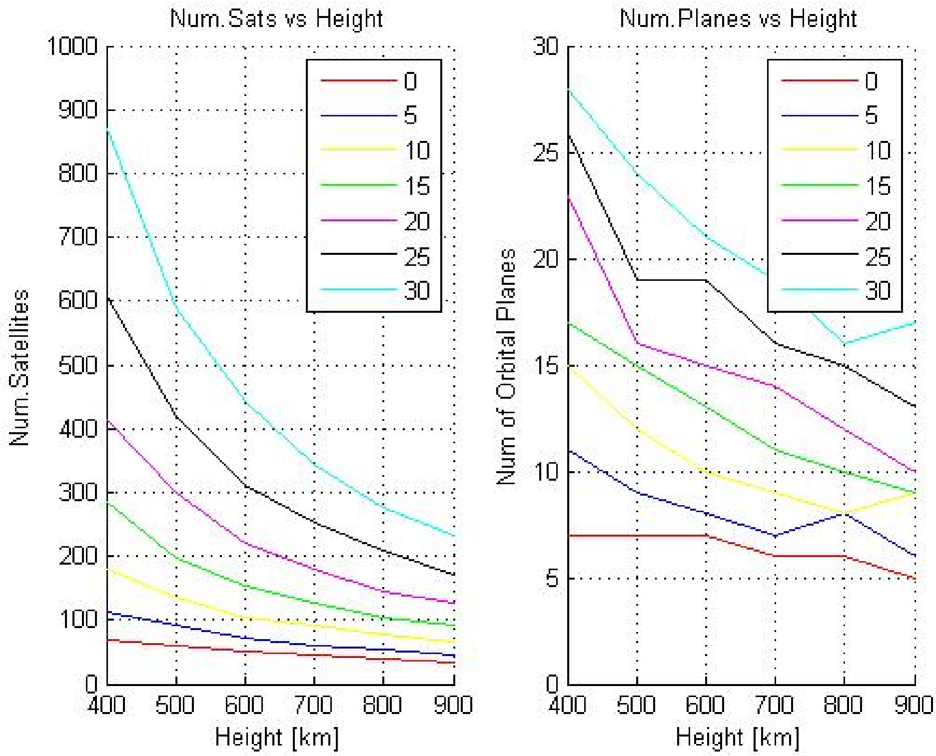

Figure 3.3.3: Variation of number of satellites for different heights and elevation angles

Detailed Solution

Given the previously justified assumptions, the same simulation is computed for a more reasonable range of results. In this case, the elevation is set as:

$$\varepsilon = 20^{\circ}$$

.

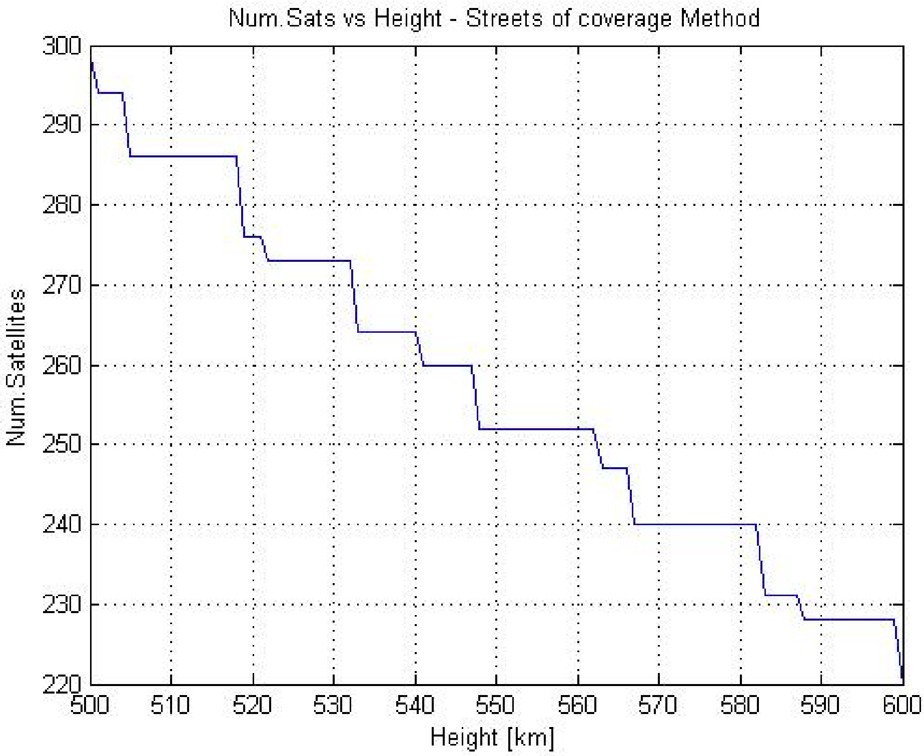

Figure 3.3.4: Variation of number of satellites for different heights between 500 and 600km.

Conclusion

The computation and the design of this constellation requires small computational and conceptual effort. However, the number of satellites and planes is greater than expected. Even though the technical complexity can be reduced, the availability of small launchers to reach this particularly inclined orbit is also small. In conclusion, more constellation configurations need to be assessed to compare and select the most feasible one.

3.4 Walker-Delta Constellation

3.4.1 Full Walker-Delta Constellation

3.4.1.1 Notation

J.G. Walker developed a notation to define these constellations with only 4 parameters [8]:

$$i : T/P/F$$

- i: inclination of the orbit in degrees
- T: total number of satellites
- P: number of planes
- F: relative phase difference between satellites from adjacent planes

Since all satellites are placed at the same altitude, with these notation the shape of the pattern is completely determined. However, to determine all the orbital parameters it is necessary to know the radius of the orbits.

3.4.1.2 Coverage

The previous section has shown that in polar orbits the coverage of the constellation could be determined with the streets of coverage method. On the other hand, in delta patterns it is necessary to study each configuration to verify its coverage. J.G. Walker determined that delta patterns gave better coverage than polar orbits, but not substantially better in the case of single coverage. This kind of patterns are more useful for double or triple coverage constellations, as it can be seen in Figure 3.4.1. However, his calculations were for a low number of satellites, so it is necessary to compute new results for the number of satellites of the Astrea constellation.

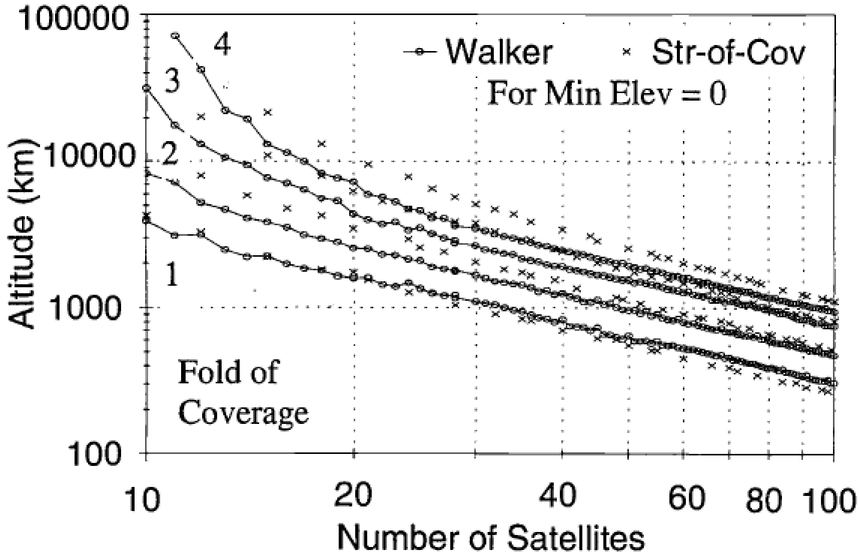

Figure 3.4.1: Minimum altitude for continuous global coverage. Comparison between polar patterns and Walker delta patterns. Extracted from [6]

3.4.2 Semi Walker-Delta Constellation

As discussed in the report, Semi Walker Delta Constellations (SWDC) are a type of constellations in which the ascending nodes of each orbital plane are evenly spaced across 180 degrees.

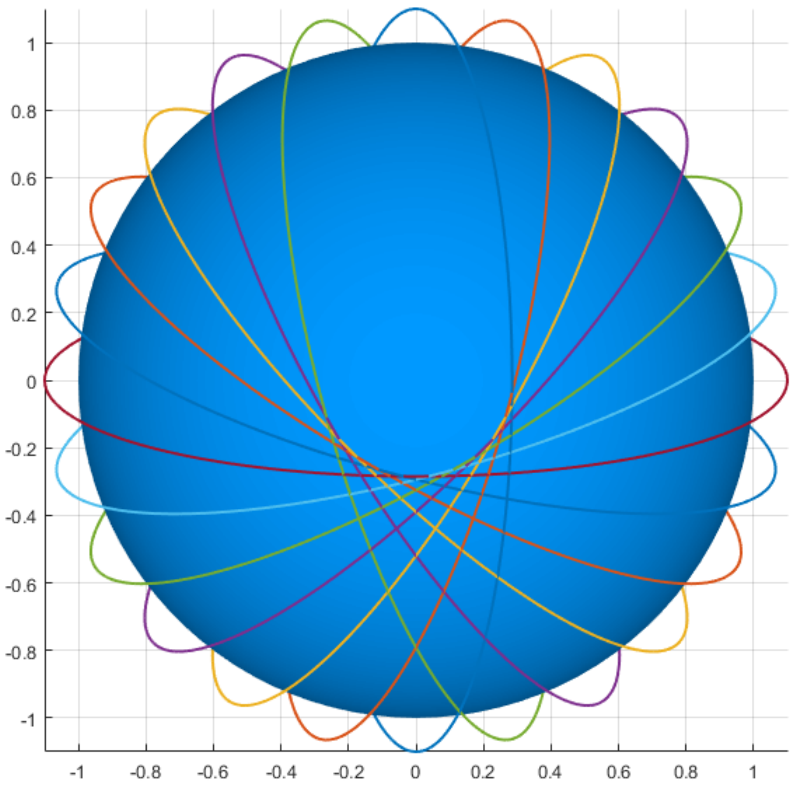

Figure 3.4.2: 12 plane SWDC. Note the gap and the equidistant planes

3.4.2.1 Advantages

- **Distance between planes reduced.** With the SWDC constellation the redundant orbits are directly corrected, thus the distance between planes is reduced to half, as results from the geometry itself.
- **Less number of planes needed.** This means that in order to approach global coverage fewer planes will be requiered due to the decrease in distance between planes.
- **Satellites following the same direction - sense.** With the SWDC constellation the orbits have no interaction with each other, thus the satellites for each orbit can be set

following the same direction. This will significantly improve the communications among satellites from different planes; also, we will be avoiding the Doppler Effect.

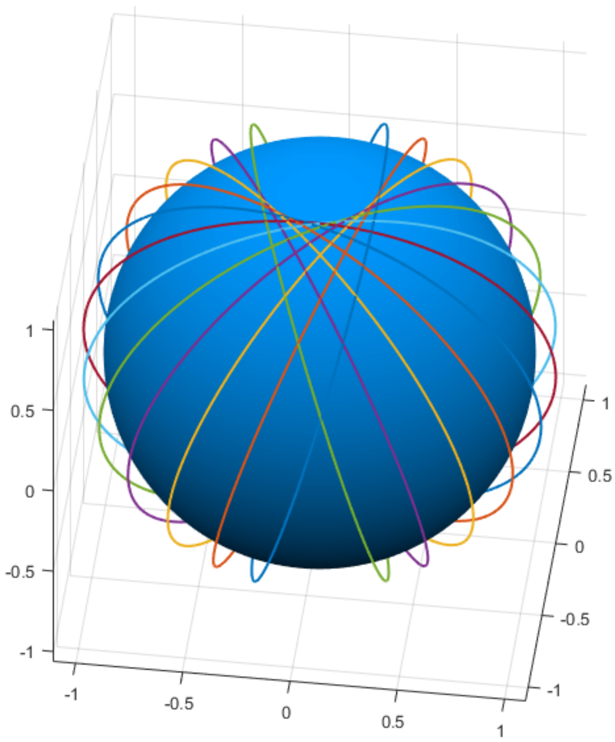

Figure 3.4.3: This geometry distribution induces a large anti-symmetric gap

3.4.2.2 Disadvantages

– **Gap configuration.** With the SWDC constellation, the main problem is the gap that results from configuring the constellation at a given inclination and describing equidistant orbits. In order to fulfill global coverage this gap will have to be covered by means of auxiliar orbits.

3.4.2.3 SWDC including an additional polar orbit.

As we have discussed for the SWDC, the main disadvantage respect to the Walker Delta Configuration is the fact that a gap is obtained, thus a global coverage network cannot be described. In order to cover the entire Earth we have analysed some ways of covering the gap with auxiliar orbits.

The most simple way would be to add a polar orbital plane. This polar orbit would be set directly on top of the gap described by the SWDC. The main issue with polar orbits, as discussed before in this report, is the complex reorientation and decay in inclination that takes place.

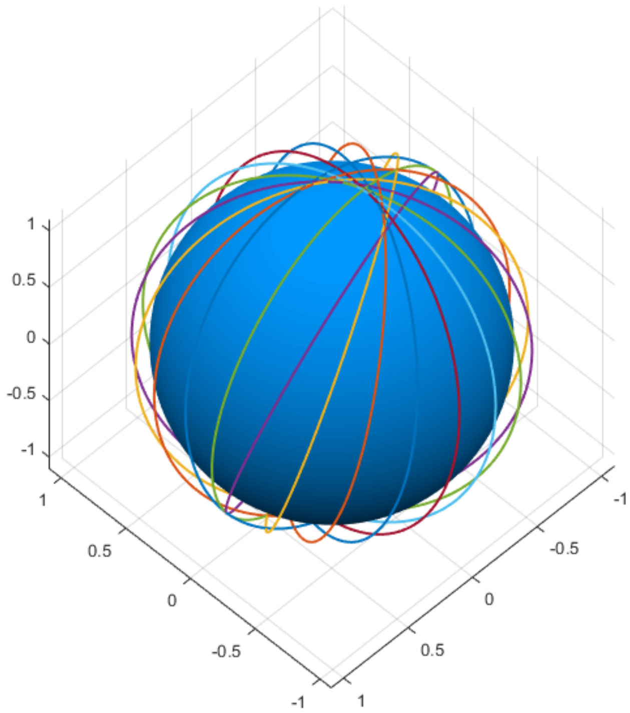

Figure 3.4.4: Added polar orbit to the 11 plane based SWDC

Moreover, in a Walker Delta Constellation, since all orbits have the same inclination, same height, and the planes are evenly spaced, the orbit perturbations are equal on each orbit and the constellation keeps its original shape. If a polar orbit is inserted in the constellation, the perturbations will be different than on the other orbital planes and the constellation will lose its initial configuration. Therefore, this solution is not suitable for our constellation and is discarded.

3.5 Testing Method

In this part of the Attachment the method to compute optimum constellation configuration given some requirements will be discussed. The method applied is based in some conditions that must be fulfilled to asses that global coverage is obtained.

The variables that this method take as inputs have been enumerated in the report. they can also be found in the table 3.5.1

3.5.0.1 Global Coverage Conditions

Same plane condition

In order to fulfill the desired coverage, the distance between two satellites on the same plane must not be more than two times the central angle β. This condition is visually represented in Figure 3.5.1 .

Different plane condition

To accomplish the coverage requirements, the distance between two satellites on different planes must not be more than the central anglep β. This condition is visually represented in Figure 3.5.1

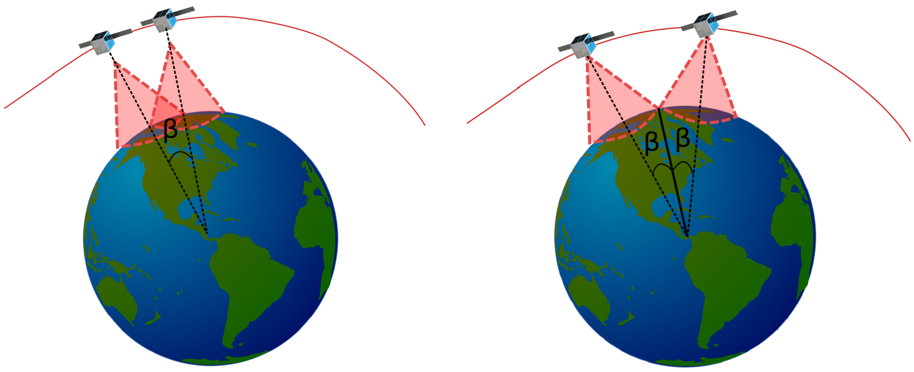

Figure 3.5.1: Geometrical conditions needed to fulfill global coverage. On the left: Condition between satellites of different planes. On the right: Condition between satellites of the same plane

3.5.0.2 Procedure of Testing Method

A MATLAB routine has been designed to compute the described algorithm. This routine can be found in the ANNEX VI 1.4. In this phase different values of all the variables have been computed in order to found the most suitable solution. The values tested are the following:

Coverage Testing Method Variables	
typeC	[180 210 225 240 360] [°]
ε	[20] [°]
h	[540-550] [km]
in	[70-80] [°]
n_p	[5-12]
N_{pp}	[10-24]

Table 3.5.1: Testing Values for the Coverage Testing Method

General Solution

The program has been run for all the range specified above in order to find the best constellations options. As it can be deduced, both the number of planes and satellites decreases when increasing height because as explained before the footprint of the satellites gets incremented with height. If height is left as a constant, a less intuitive results are obtained. We have now different configurations in terms of number of satellites an planes due to the variation of the inclination angle of the planes. In the Figure 3.5.5 the results obtained for one of the analysed configurations are shown.

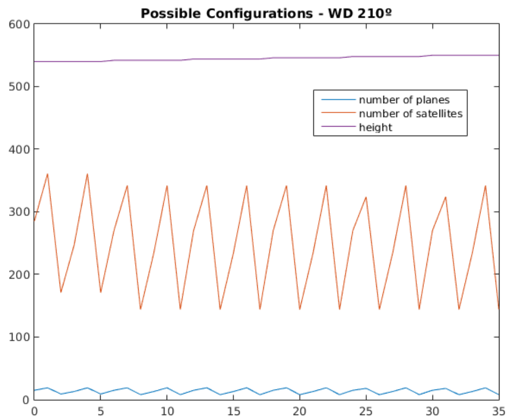

Figure 3.5.2: Possible satellite configurations for a 210° Walker Delta configuration

As mentioned in the Report, once the results have been obtained, three of them have been plotted to visually verify if the coverage obtained is the desired. Below this paragraph the ground track and a conceptual view of the earth can be seen for these three configurations.

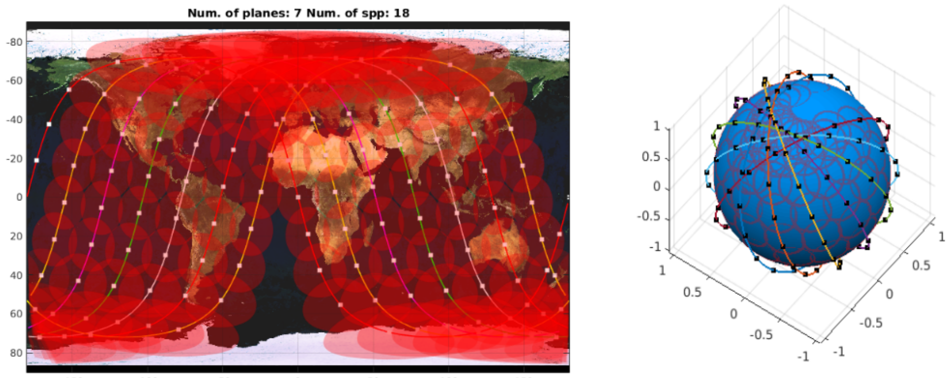

Figure 3.5.3: Ground track and spherical representation for a 180° Walker Delta configuration

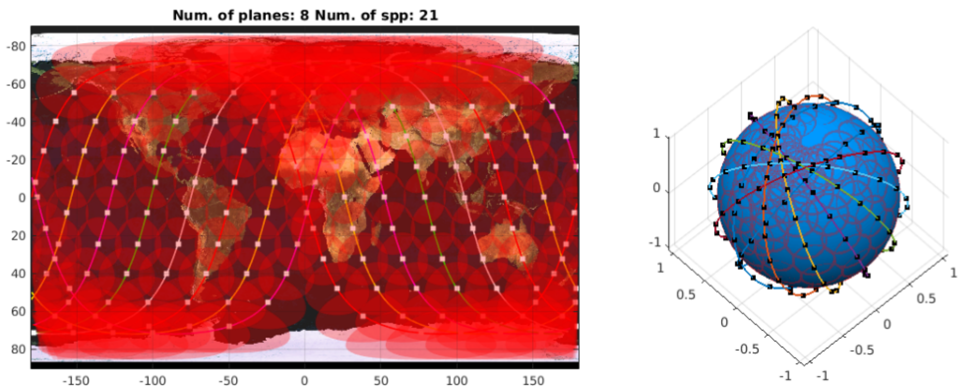

Figure 3.5.4: Ground track and spherical representation for a 210° Walker Delta configuration

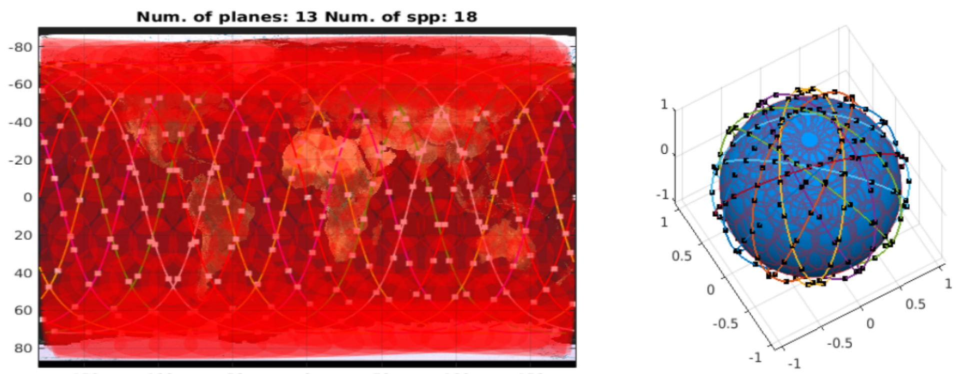

Figure 3.5.5: Ground track and spherical representation for a 360° Walker Delta configuration

To realize these graphs has been necessary to develop a some codes that, again, can be found in the ANNEX VI, in the section 1.6.

4 | Orbit Perturbations

4.1 Sources of Perturbation

4.1.1 Introduction to Orbit Perturbations [1]

All the perturbations can be deeply studied. Consequently, analytical solutions are very hard to find, and even they were found, they do not show clealy a meaning or are not really useful. Instead, there are two mainly used approaches:

- Special Perturbacion: Step-by-step numerical integration of the motion equations with perturbation.
- General Perturbation: Through analytical expansion and integration of the equations of variation of orbit parameters.

The Approach of the Perturbations Study

For the purposes of these study the different approaches will be assessed. The first analysis will discuss which of the peturbations are the most significant to the study. This analysis will be done considering General Perturbation Techniques. In a deeper second analysis, the two approaches for the perturbations will be assessed and compared considering only the most significant perturbation sources.

4.1.2 Gravity Potential of Earth

Earth's aspherical shape can be modelled as a sum of terms corresponding to the Legendre polynomials. These polynomials can be empirically measured and consider radial symmetry. If one would like to compute also variations in longitude, then should use associated Legendre polynomials.

$$V(r,\delta,\lambda) = -\frac{\mu}{r}\left[\sum_{n=1}^{\infty}\left(\frac{R_e}{r}\right)^n \sum_{m=0}^{n} P_{nm}cos(\delta)(C_{nm}cosm\lambda + S_{nm}sinm\lambda)\right] \quad (4.1.1)$$

General Legendre associated polynomials developed Gravitational Potential

$$V(r,\delta) = -\frac{\mu}{r}\left[1 - \sum_{n=2}^{\infty} J_n \left(\frac{R_e}{r}\right)^n P_n(sin\delta)\right] \quad (4.1.2)$$

General Legendre polynomials developed Gravitational Potential

For Earth, the J_n coefficients are the following:

$J_2 = 0.00108263$ $J_3 = -0.00000254$ $J_4 = -0.00000161$

Given this distribution, the only sognificant term J_2.

$$V(r,\delta) = -\frac{\mu}{r}\left[1 - \frac{1}{2}J_2\left(\frac{R_e}{r}\right)^2(1 - 3sin^2\delta\right] \quad (4.1.3)$$

Aproximated Gravitational Potential

If we integrate the force that derives from this potential we can afterwards compute the effect of J_2 On the different orbtial elements:

- $\Delta a = 0$
- $\Delta e = 0$
- $\Delta i = 0$
- $$\Delta\Omega = -3\pi\frac{J_2R_e^2}{p^2}cos\, i\; [rad/orbit] \quad (4.1.4)$$
- $$\Delta\omega = \frac{3}{2}\pi\frac{J_2R_e^2}{p^2}(4 - 5sin^2\, i)\; [rad/orbit] \quad (4.1.5)$$

4.1.3 Atmospheric Drag

In order to compute the effect of the remaining atmosphere we use the typical definition of atmospheric drag knowing a drag coefficient:

$$\vec{a}_{drag} = \frac{1}{2}\frac{C_d A}{m}\rho v_{rel}^2 \frac{\vec{v}_{rel}}{|\vec{v}_{rel}|} \tag{4.1.6}$$

The **ballistic coefficient** B_c is defined as $\frac{m}{C_d A}$, characterizing the behaviour of the satellite against atmospheric drag.

Modelling the Atmosphere

There are several modells for the atmosphere. For instance, the most commonly used, the exponential model:

$$\rho = \rho_0 e^{-\frac{h-h_0}{H}} \tag{4.1.7}$$

$$H = \frac{kT}{Mg} \tag{4.1.8}$$

Where:

Exponential Atmosphere Variables	
ρ	Density at given height
ρ_0	Density at a reference height
h	Height over the ellipsoid
h_0	Reference height
H	Scale Height
k	Boltzmann Constant
T	Temperature
M	Molecular Weight
g	Gravity

Table 4.1.1: Exponential Atmosphere Model main Variables

In addition, other models for the exospheric temperature and the molecular weight need to be used. For this study the ones proposed by The Australian Weather Space Agency are used.

In addition, it is important to note that the following phenomena interfere with the previsions:

- Diurnal Variations
- 27-day solar-rotation cycle

- 11-year cycle of Sun spots
- Semi-annual/Seasonal variations
- Rotating atmosphere
- Winds
- Magnetic Storm Variations
- Others: Tides, Winds,...

Again, if we integrate this force in a period of time, considering the orbit nearly circular, we obtain:

$$\Delta r = -2\pi\rho r^2/B \ [/orbit] \tag{4.1.9}$$

4.1.4 3rd Body Perturbations

The effects of this extra bodies in the system can be computed considering the motion equations. However, some aproximations can be found in the reference as:

$$\dot{\Omega} = \frac{A_m + A_s}{n} cos\, i \ [^\circ/day] \tag{4.1.10}$$

$$\dot{\omega} = \frac{B_m + B_s}{n} (4 - 5sin^2\, i) \ [^\circ/day] \tag{4.1.11}$$

Where n stands for the rate of rotation in orbits/day. In that case, the A_m, A_s, B_m and B_s coefficients take as values:

	$A_m + A_s$	$B_m + B_s$
Moon	-0.00338	0.00169
Sun	-0.00154	0.00077

Table 4.1.2: Third Body Perturbations Coefficients

4.1.5 Other Perturbations

In this bag the following low-intensity can be classified:

- Solar Radiation Pressure

- Solid-Earth and Ocean Tides
- Magnetic Field
- South Atlantic Anomaly

4.2 Significant Perturbations

Propagation Algorithm

Given the definitions and aproxmations to compute perturbations described in the previous section, a propagation in time for the change in orbital parameters is solved. The results are plotted in the graph below:

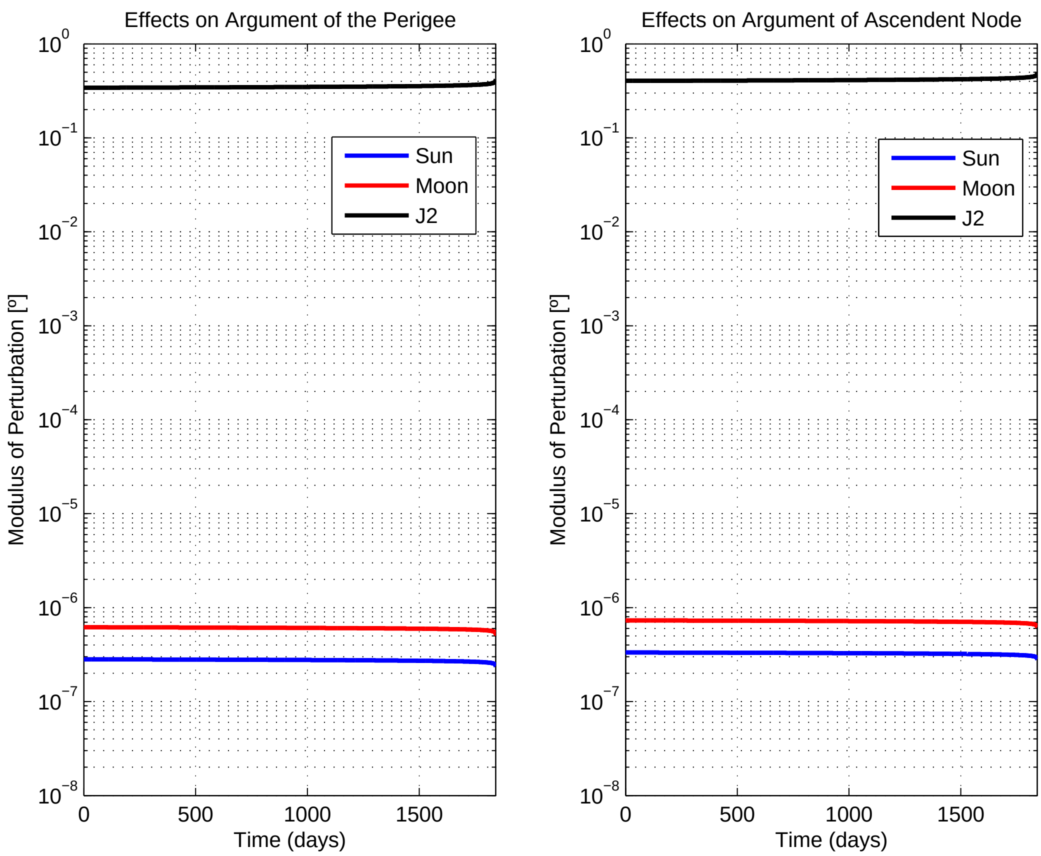

Figure 4.2.1: Logaritmic plot of the modulus of the increases in Angular Arguments of the orbit

As it can be seen, the perturbations caused by 3rd bodies are several orders of magnitude below the order of magnitude of the variation caused by Earth's oblateness. It is also remarkable that the moon has a higher effect than the sun given the relative distance to Earth, even if the sun is way more massive.

Another important obsevation is that given the very low eccentricity we are considering, the deviation of the argument of the perigee does not affect the performance of the constellation. In other words, since the orbits are considered almost circular there is not a defined Perigee for the orbit.

In conclusion

The effects of the Moon and the Sun are neglected in comparison with the effects of J2 for the Argument of the ascendent node as well as for the argument of the Perigee.

4.3 Orbit Decay

In this chapter the effects of the main perturbations are deeply studied. Firstly, an introduction on the effects of Earth's oblateness on the orbital parameters. Secondly and in more detail, the effects of Atmospheric drag. This is significant because it deviates the power and mass budget to engines and propellant.

4.3.1 Effects on the Ascention Node

4.3.1.1 Introduction

Due to the non sphericity of the Earth, two deviations exist in terms of perigee and ascendent node. This perturtions are related to the J2 effect descrived before. Both effects are related to the orbital planes inclination angle, so depending in which inclination they are positionated, the perturbation will be more or less significant.

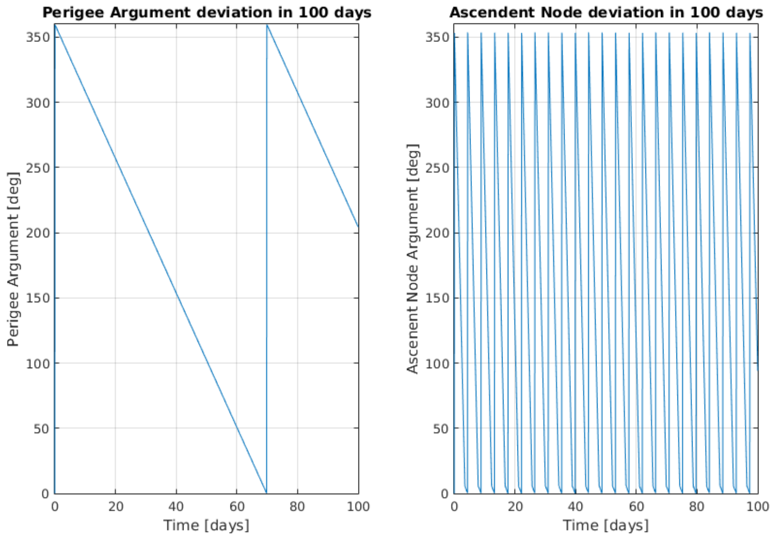

Figure 4.3.1: Ascention node perturbation. On the left: Perigee deviation in terms of time. On the right: Ascending node deviation in terms of time

4.3.1.2 Perigee Effect

The Perigee effect is the responsible of the rotation of the orbit regarding the Earth and is found inside the orbital plane itself. Therefore the perigee of an eliptical orbit is not static in an Earth's point but moves around it.

This effect is noticed when having eliptical orbits. Consequently Astrea constellation will not be affected because the satellites describe almost circular orbits.

4.3.1.3 Ascention Node

In this case the perturbation affects the rotation of the orbital plane. So the plan longitude variates with time. That means, that if we had just one orbital plane it would not cover always the same fraction of Earth.

This effect is noticed when having planes with different inclinitations. That is not Astrea's constellation case since all its planes are positioned in the same inclination angle.

4.3.1.4 Conclusion

As explained, both perturbations do not affect Astrea's constellation so they will not be considered as atctive agents on the orbit decay proces

The Figure 4.3.1 shows the propagation in time of both effects which are periodic due to the constant velocity of orbits.

4.3.2 Effects of the Solar Cicle

It is important to consider many parameters when calculating the orbital decay of a satellite. The most important of these parameters for LEO based constellations is drag. As discussed in other chapters, the drag of a satellite depends on the coefficient of drag, its surface, the density of the air and the velocity at which operates. Solar cycles will directly affect the density of the upper atmosphere. This phenomena is relevant when calculating the drag of the satellite and therefore is essential to compute the orbital decay.

Solar cycles are periodic changes in the Sun's activity of approximately 11 years. In each period a solar maximum and minimum can be determined, referring to the amount of periods of sunspot counts. The intensities for these periods vary from cycle to cycle.

Different studies have been made throughout the 20th century cycles. In order to understand the change density of the air changes as consequence of these solar cycles we considered the result data of an old study regarding the 19th solar cycle, which had a duration of 10.5 years between 1958 and 1968. This solar cycle had the highest maximum smoothed sunspot number ever recorded (since 1755), which was of 201.3. This maximum value was recorded in March 1958. This value is high in comparison to other cycles, especially when comparing it to the current 24th solar cycle. In this chapter an analysis will be developed in order to study the influence of the solar cycles on the drag of our satellites.

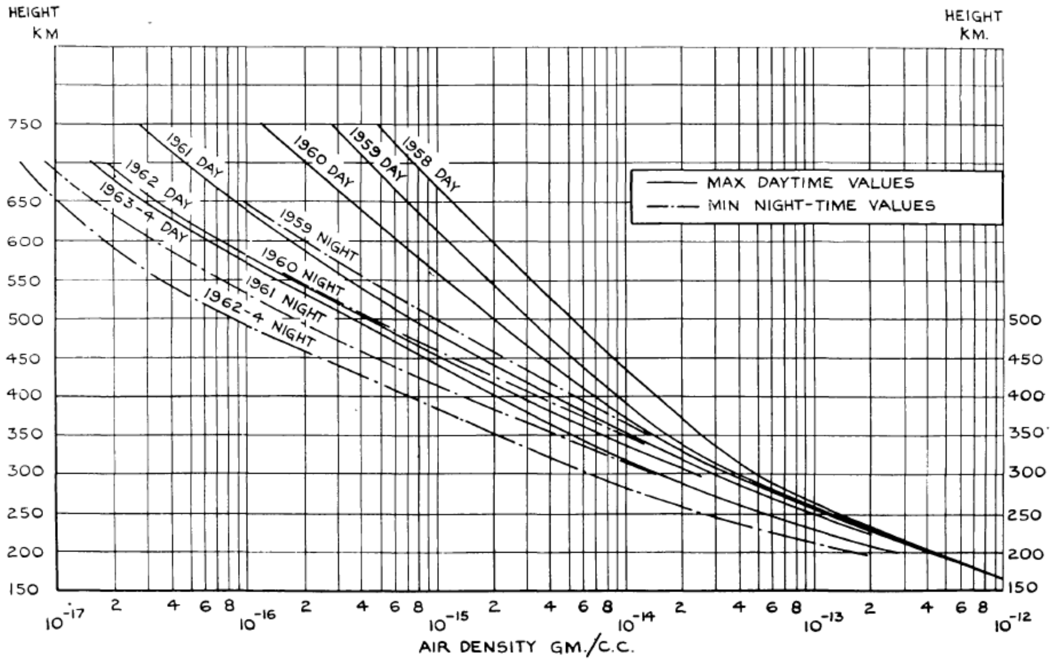

Figure 4.3.2: Deviation of densities in the upper atmosphere due to the 19th solar cycle

At 550 km:

Year	D/N	Density at 550km [g/cc]
1958	Day	3.2E-14
1958	Night	5.0E-15
1964	Day	1.35E-15
1964	Night	3.35E-16

These values referring to day and night are the densities of the upper atmosphere at 550 km of altitude respect to the surface of the Earth. The upper atmosphere densities rise during the day following the increase of temperature caused by the radiation of the Sun whereas these values are reduced at night. The orbital decay is on the order of several years whereas these deviations appear every few hours. Thus, in order to compute the orbital decay we will not be taking into account these daily deviations but rather a main value. Therefore the mean density for 1958 will be of 1.85E-14 g/cc and the solar minimum's density of 1964 will be of 8.4E-16 g/cc.

In order to analyse how these values may apply to our constellation we first must adjust these - which belong to the 19th solar cycle - to those of the current 24th cycle, which is noticeable less intense. A way of operating this adjustment is comparing the mean solar maximum achieved by each cycle. The maximum monthly smoothed sunspot number of the 19th cycle had a value of 201.3 and a minimum of 9.6 whereas the current 24th ranges between 11.7 and 81.9 approximately. This means that for the 19th cycle a total deviation of 191.7 was measured whilst for the 24th cycle this deviation was only of 70.2. This is crucial if we want to analyse the solar maximum densities.

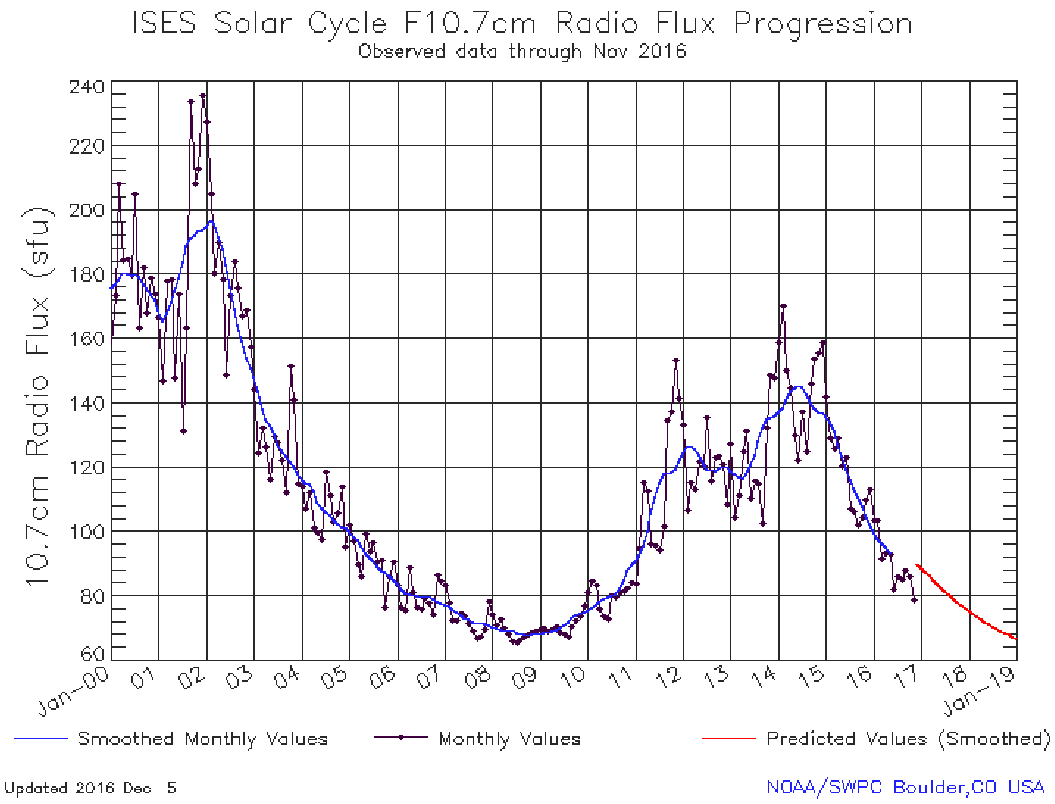

Figure 4.3.3: Measured intensities of the 23rd and 24th solar cycles. Source: NOAA

We must now adjust the mean constant density defined initially to the conditions that this 24th cycle imposes. It is important to note that our satellites will be launched in 2017, and that the 24th cycle is currently decreasing its intensity. Thus, our calculations will be near the conditions of solar minimum, meaning that the drag of our satellite will be smaller than first considered.

Our new approach to the density of the atmosphere at 550 km is near the first approximation, but will consider that we are now entering the solar minimum which will remain more or less constant until 2022. As discussed before, the solar minimum represents a singularity with a minimum density of 8.4E-16 g/cc. The approximation taken will be the resulting constant value which represents the mean smoothed densities between 2017 and 2022.

The final density at 550 km considering the solar minimum during 2017 to 2022 will be of 2.0E-15 g/cc.

4.3.3 Orbital Decay Propagation Results

4.3.3.1 Introduction

In this section a first aproach of the drag computation have been done in order to determine the orbit decay and consequently compute how much time a satellite last untill it reenters the Earth atmosphere.

4.3.3.2 Drag Computation Algorithm

Given the definitions to calculate orbital perturbations described in 4.1.1 a computation of the atmosphere drag has been done together with the computation of the other main perturbations that have been discussed in previous sections.

As explained in the last section the atmospheric drag depens on the drag's coefficient and it surface, that are constant values, on the velocity at which the satellite operates and on the air density.

So in order to see the effects of variations in air density the orbit decay has been estimated and plotted for several F10 Radio Flux values corresponding to different moments of a solar cycle. (This data has been extracted from the figure 4.3.3).

The data selected and the results obtained are shown in 4.3.1 and 4.3.4 respectively.

Selected Values	
Year	F10 Radio Flux
2002	195
2004	115
2009	70
2013	120
2016	100

Table 4.3.1: Selected data to compute orbit decay extracted from figure 4.3.3

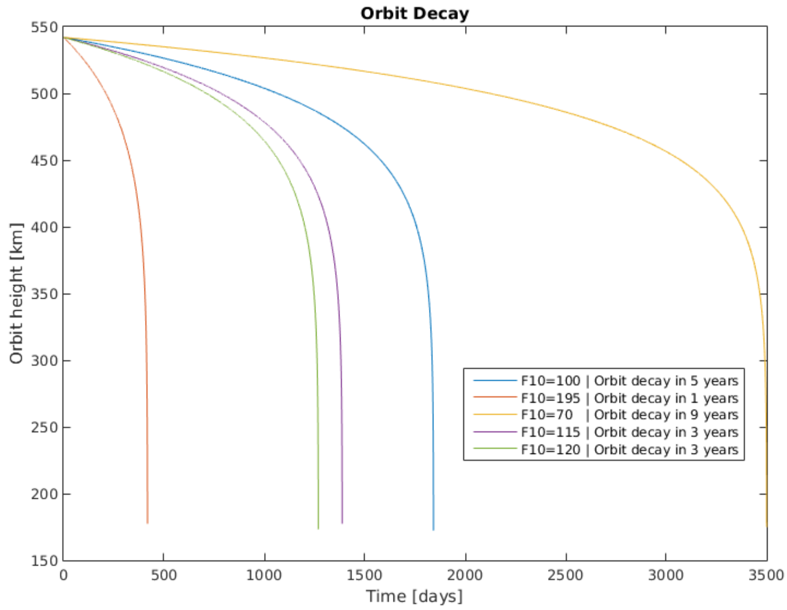

Figure 4.3.4: Orbit Decay computed for several values of F10 Radio Flux

As it can be seen, the orbit decay strongly depends on the positioning in time of a solar cycle. (In 7 years the difference in lasting time of the satellite is reduced in 4 years).

Conclusion

The lasting time in orbit of satellites is affected by period of the solar cycle. According to the data then Astrea's satellites will have an approximated orbit decay of 5 years.

4.3.4 Dynamic Orbit Decay Computation

4.3.4.1 Introduction

In this part of the chapter the orbital is studied using the model of special perturbations, which as previously defined, is the one that uses a numerical step-by-step integration. There are three manly used methods to study the dynamic propagation of an orbit, which are:

Cowell's method This is the simplest method since it does not require any assumption or approximation. It is based on quantifying the accelerations produced by the perturbations and adding them to the dynamic equation of a Keplerian orbit (see Orbit Design: Chapter 1 equation 1.2.3) leading to:

$$\frac{d^2\vec{r}}{dt^2} = -\frac{\mu}{r^3}\vec{r} + \vec{a}_p \tag{4.3.1}$$

Where $\vec{a}_p$ is the acceleration produced by the perturbations. This second order differential equation is the one that must be integrated in order to propagate the orbit. Although the formulas and application of this method are simple, this does not imply that it lacks robustness or precision. Its results are as good as any of the following two methods but the major drawback of *Cowell's method* is that it requires smaller time-steps being therefore slower (in terms of computation speed).

Encke's Method: This method is based on correcting the defects of the previous method. Encke uses a schema based on what is called *predictor-corrector*. First, it evaluates the orbit as if it were a Keplerian orbit (i.e. without perturbations) and then it integrates only the perturbations to correct the deviation caused by considering the unperturbed orbit. Its advantage over Cowell's method is clear, since it only integrates perturbations, and since these vary less over time than the position itself, we can relax the integration by increasing the time step. In short, this scheme is faster but also more complex to program than the one proposed by Cowell.

Variation of the parameters: This method, developed by Lagrange, is based on considering the orbit as a succession of Keplerian orbits, each of them being tangent to the satellite orbit at a certain point. Thus we can obtain differential equations that model the variation of the orbital parameters as a function of time.

The formulations and schemes followed by each of these methods can be found in any reference dealing with orbital mechanics. For example, the reader can refer to [1] or the chapter 20 of [9] to obtain more detailed information about these methods.

For the purposes of this study, implementing the simplest method is enough. As it has already mentioned, it is based on adding the perturbations (discussed at the beginning of this chapter) to the dynamics equation. A *Matlab* routine has been developed that follows the next scheme:

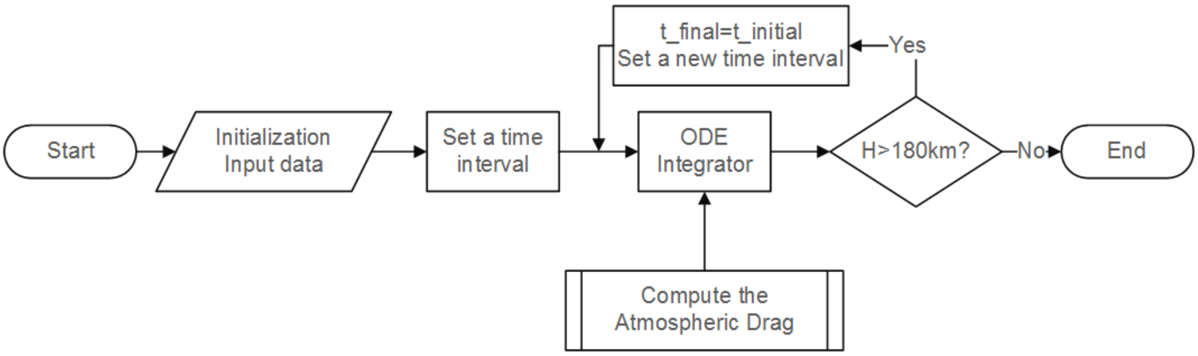

Figure 4.3.5: Algorithm of resolution used to solve the orbital propagation.

The disturbances that the routine includes are:

- The potential field of the earth.
- The Atmospheric drag.
- The influences of 3 bodies.
- Solar Radiation Pressure.

As it has been seen in previous sections, the only truly significant perturbation for the orbital decay at the altitude in which the constellation is located is the one caused by the atmospheric drag. Thus, other contributions have been deactivated to speed up the calculation. Therefore, explaining the formulation used to obtain the accelerations caused by these perturbations is not of interest for the development of the study. However, the following are the sources from which they were obtained:

- The calculation of the Earth gravity Potential uses the equation 4.1.1. Following the indications of [9] both the Legendre polynomials and the parameters C_{nm} and S_{nm} can be obtained.
- The equations present in [10] have been used to compute the perturbations due to other bodies,
- For Solar Radiation pressure the formulation used is the one presented in [11] including a 'shadow factor' (if the earth is between the sun and the satellite, the latter will not receive direct radiation from the Sun) modeled by a normal statistical distribution.
- For the calculation of Drag, the equation 4.1.6 and the atmosphere model presented in the same section have been used.

To be able to integrate the system we must take into account that, in fact, as we work in Cartesian coordinates, it is a system of three equations. Moreover, since it is a second-order equation we must rewrite it as a first-order system. Let $x_1 = r = (x, y, z)$ and $x_2 = \dot{x}_1 = \dot{r} = (vx, vy, vz)$. Therefore:

$$X = \begin{pmatrix} x \\ y \\ z \\ v_x \\ v_y \\ v_z \end{pmatrix} \Rightarrow \dot{X} = \begin{pmatrix} v_x \\ v_y \\ v_z \\ \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{pmatrix} = \begin{pmatrix} v_x \\ v_y \\ v_z \\ a_{p,x} - \frac{\mu}{r^3}x \\ a_{p,y} - \frac{\mu}{r^3}y \\ a_{p,z} - \frac{\mu}{r^3}z \end{pmatrix} \tag{4.3.2}$$

To integrate this system, you can use the *Matlab* built-in function **ode45**, which is a runge-kutta 4-5 with a variable step control that basically modifies the time step if the error is too large. Also, the **juliandate.m** function (included in the Matlab Aerospace module) have been used. It calculates the Julian Date,that is the number of days since noon Universal Time on January 1, 4713 ECB (On the Julian calendar).

4.3.4.2 Results

A simulation has been executed with the same parameters as in the previous section. After 932 seconds of computation, the results obtained are shown below:

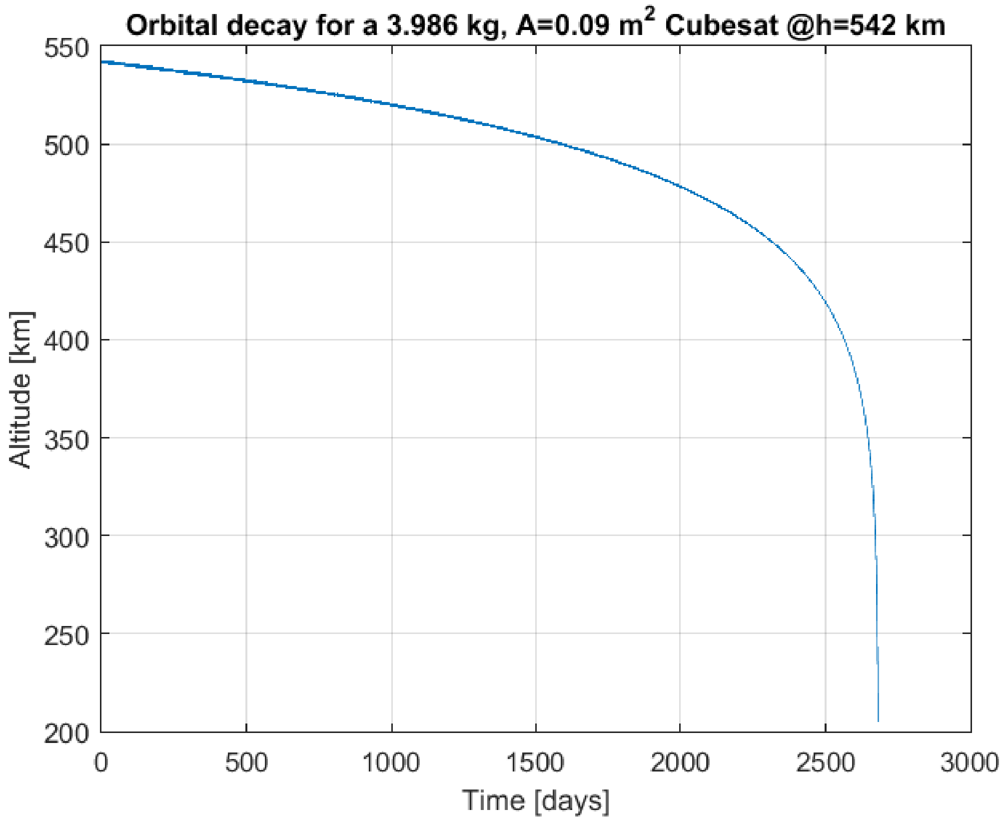

Figure 4.3.6: Orbital decay of the satellite.

As it can be seen, the estimated orbital decay for a satellite like Astrea's Cubesat is about 2700 days or, what is the same, 7.4 years. This estimation and the temporal evolution of the altitude is in agreement with the results obtained by the semi-analytic method. It is therefore verified that for a preliminary analysis and the respective modifications that it can present (i.e. changes in weight, changes in area, initial height, geometry of the orbit) it is enough with the results obtained by the semi-analytic study, which do not require almost computation time (only a few seconds), avoiding the expense of computing resources that would produce a dynamic simulation for every modification.

4.4 Orbital Station-Keeping

We will study:

- Increased height
- Thrusters

4.4.1 Raising the orbit height to increase Lifetime

The key to understand this solution is to see from another point of view the atmospheric drag phenomena. Once we have designed the constellation to provide certain coverage to specific points of the globe, the action of increasing the height of the orbit has the effect of increasing the footprint area on the surface of the earth. As the constellation is set, the time that take the satellites to reach the design height is extra lifetime.

From this point of view, the atmospheric drag phenomena can be recomputed and plotted it in this new way:

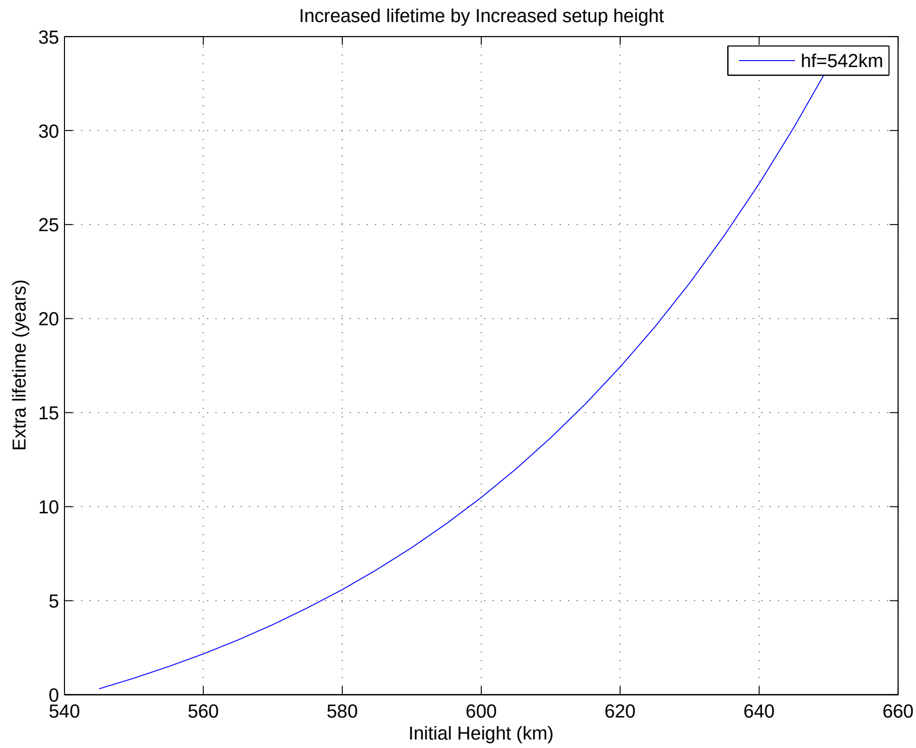

Figure 4.4.1: Increase in the Lifetime obtained by setting the constellation in a higher orbit

As it can be seen, the lifetime increases radically with time. However, this is a dangerous solution, since the coupling with another design parameters is compromised. To list the complications that can lead to:

- **Clients:** With the current technology, the satellites currently in orbit are set to point towards Earth. This means, if the contellation's satellites are at a higher orbit, the contact is impossible. As the market study reveals, it is important to place the satellites as low as possible.

- **Spacecraft Subsystems:** A higher orbit means a higher gain for the antennas and therefore an increase in the required power.

- **Constellation Reconfiguration:** The overall time to reconfigurate the constellation increases with height, since the period of the transition orbits is higher.

Conclusion

This tool is a very powerful option to deal with the orbit decay, even though it is not exactly an operation of Station Keeping itself. Given the high correlation it shows with another subsystems, the possibility of using it needs to be considered while the other design decisions are taken.

4.4.2 Using Thrusters to increase Lifetime

As mentioned in the report, the calculations in the variation of height are made considering that a Hohmann maneuver is applied. The main characteristics of this maneuver are explained on the following sections.

4.4.2.1 Energy equation

The deduction of the equations needed to solve the Hohmann maneuver begins with the energy equation:

$$\frac{V^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a} \tag{4.4.1}$$

where V is the orbital velocity of the satellite, r is the distance from the focus, a the semimajor axis of the orbit and μ the gravitational constant of the attracting body, in this case, the Earth. This expression shows that the total energy of the satellite equals the sum of its kinetic and potential energy (per mass unit).

This equation can be arranged to obtain the velocity of the satellite. In the case of a circular orbit, the radius is constant, and equal to the semimajor axis. Replacing $a = r$ in the energy equation and after some operations, the expression of the velocity of a circular orbit is obtained:

$$V_c = \sqrt{\frac{\mu}{r}} \tag{4.4.2}$$

As it can be deduced from the energy equation, a change in orbital velocity leads to a change in the value of the semimajor axis. This property is used in satellites to change their orbit through a velocity increment ΔV. This process is called an orbital maneuver.

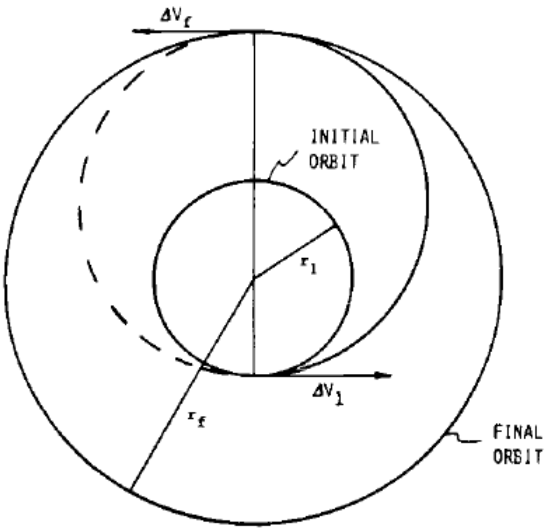

Figure 4.4.2: Hohmann transfer. Extracted from [6]

4.4.2.2 Delta-V

If the velocity increment ΔV is done instantaneously, the maneuver is called an impulsive maneuver. The Hohmann transfer is a two-impulse transfer between coplanar circular orbits. From an inicial circular orbit, a tangential velocity increment ΔV_1 is applied to change the orbit to an ellipse. This ellipse is the transfer orbit, in which the perigee radius is the radius of the initial circular orbit and the apogee radius equals the radius of the final circular orbit. When the satellite reaches the apogee, a second velocity increment ΔV_2 is applied, so that the satellite reaches the final circular orbit with the apogee radius. If this second velocity is not applied, the satellite will remain in the elliptic orbit.

With the energy equation defined above, it is easy to determine the velocity of the satellite in each orbit. The first orbit and the final ones are circular:

$$V_1 = \sqrt{\frac{\mu}{r_1}} \tag{4.4.3}$$

$$V_f = \sqrt{\frac{\mu}{r_f}} \tag{4.4.4}$$

The velocity in the transfer orbit can be easily calculated with the energy equation applying the definition of the semimajor axis of an ellipse:

$$a = \frac{r_1 + r_f}{2} \tag{4.4.5}$$

The velocities in the perigee and apogee are:

$$V_p = \sqrt{\frac{2\mu r_f}{r_1(r_1 + r_f)}} \tag{4.4.6}$$

$$V_a = \sqrt{\frac{2\mu r_1}{r_f(r_1 + r_f)}} \tag{4.4.7}$$

Therefore the velocity increments are:

$$\Delta V_1 = V_p - V_1 = \sqrt{\frac{2\mu r_f}{r_1(r_1 + r_f)}} - \sqrt{\frac{\mu}{r_1}} \tag{4.4.8}$$

$$\Delta V_2 = V_f - V_a = \sqrt{\frac{\mu}{r_f}} - \sqrt{\frac{2\mu r_1}{r_f(r_1 + r_f)}} \tag{4.4.9}$$

4.4.2.3 Time

It is also necessary to know the time needed to do the maneuver. This time is equal to half of the period of the transfer ellipse:

$$t = \frac{T}{2} = \frac{1}{2}\sqrt{\frac{4\pi^2 a^3}{\mu}} \tag{4.4.10}$$

4.4.2.4 Propellant

In order to know the mass of propellant needed in the maneuver, the Tsiolkovsky rocket equation is applied:

$$\Delta V = g_0 I_{sp} \ln \frac{m_1}{m_f} = g_0 I_{sp} \ln \frac{m_1}{m_1 - m_{prop}} \tag{4.4.11}$$

where $\Delta V = \Delta V_1 + \Delta V_2$ is the total velocity increment of the maneuver, g_0 is the Earth's gravity, I_{sp} is the specific impulse of the thruster used, m_1 is the initial mass of the satellite, m_f is its final mass and m_{prop} is the mass of propellant used in the maneuver.

$$m_{prop} = m_1 \left(1 - \exp\left(-\frac{\Delta V}{g_0 I_{sp}}\right)\right) \tag{4.4.12}$$

5 | Constellation Design Decision

5.1 Considered Designs

5.1.1 Introduction

In this chapter it is seen how the final constellation decision is made. To do that an analysis of weighted weights will be performed.

The constellations candidates selected to their later evaluation are the following:

5.1.2 Candidate 1: Polar - Global Coverage

This polar constellation (Figure 5.1.1) came from the street coverage method explained in 3.3. It is a network of polar orbits that provides global coverage. Its characteristics orbit parameters are the following:

- Height: $560\ km$
- Inclination of the planes: 90 °
- Number of planes: 20
- Number of satellites per plane: 21
- Total number of satellites: 420
- Range of argument of ascending node: 360 °

5.1.3 Candidate 2: Polar - GS Coverage

The second candidate that will be compared is a polar orbit extracted from the coverage method explained in 3.3(Figure 5.1.2). This constellation provides total coverage to the Astrea's team ground stations. The network orbits parameters are:

- Height: $550\ km$
- Inclination of the planes: 90 °
- Number of planes: 18
- Number of satellites per plane: 16
- Total number of satellites: 288
- Range of argument of ascending node: 360 °

5.1.4 Candidate 3 and 4: Walker-Delta GS Coverage

Two Walker-Delta constellation configurations have been also chosen due to their reduced number of planes and satellites while being able of providing total coverage on the lattitudes where the ground stations are located.(Figures 5.1.3 and 5.1.4).

This constellations have been obtained with the algorithm explained in 3.5

Candidate 3

- Height: $542\ km$
- Inclination of the planes: 72 °
- Number of planes: 8
- Number of satellites per plane: 21
- Total number of satellites: 168
- Range of argument of ascending node: 210 °

Candidate 4

- Height: $542\ km$
- Inclination of the planes: 72 °
- Number of planes: 9
- Number of satellites per plane: 17
- Total number of satellites: 153
- Range of argument of ascending node: 225 °

5.1.5 Candidate 5: Walker-Delta Lat: 0-58

Another Walker-Delta constellation has been selected with the criteria of total coverage of a range of lattitudes going from 0 to 58 (Figure 5.1.5). Therefore the parameters needed to fulfill this particular condition of the constellation obtained from 5.2 are the following:

- Height: $560\ km$
- Inclination of the planes: 72 °
- Number of planes: 14
- Number of satellites per plane: 19
- Total number of satellites: 226
- Range of argument of ascending node: 210 °

5.1.6 Candidate 6: Polar - Walker-Delta J2 + Rotació

With the goal of providing constant coverage at the Ground Stations we can design a constellation that takes profit of the rotation of the Earth. If we also consider Earth's oblateness that causes another Ω derivative with time, we can exacty compute the longitudinal position of a plane after an orbit has passed. Now, if we design the constellation in a way that this deviation after an orbit matches the separation between planes, a line of satellites will always be on the GS. (Figure 5.1.6)

- Height: $560\ km$

- Inclination of the planes: $72\ °$
- Number of planes: 14
- Number of satellites per plane: 19
- Total number of satellites: 226
- Range of argument of ascending node: $210\ °$

5.1.7 Candidate 7: Walker-Delta GS Coverage 3

The last configuration to be studied is a Walker-Delta constellation configuration designed to provide total coverage to the ground stations (Figure 5.1.7). It came up from candidate 3 constellation adding one more plane in order to increase its global coverage and minimize the gaps. As can be seen below, its parameters are the same as candidate 3 adding a single plane.

- Height: $542\ km$
- Inclination of the planes: $72\ °$
- Number of planes: 9
- Number of satellites per plane: 21
- Total number of satellites: 189
- Range of argument of ascending node: $225\ °$

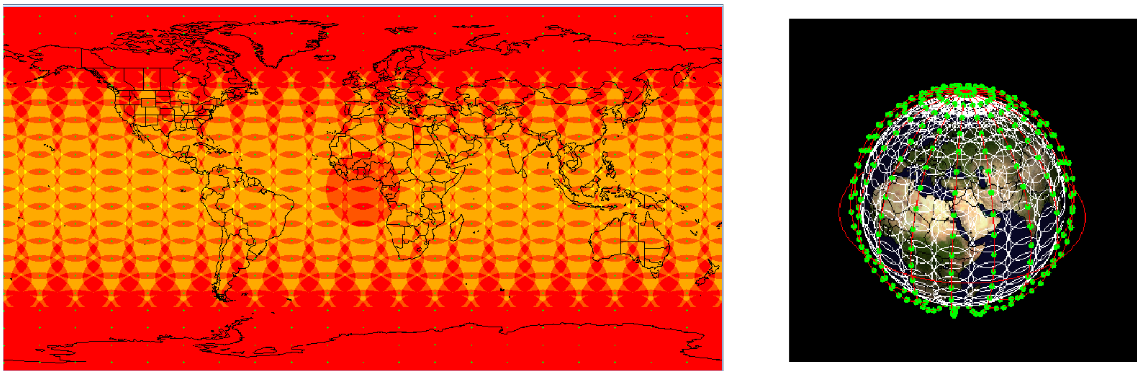

Figure 5.1.1: Candidate 1. Full Polar constellation with global coverage. h= 560km; Np=20; Npp=21; Tsat=420

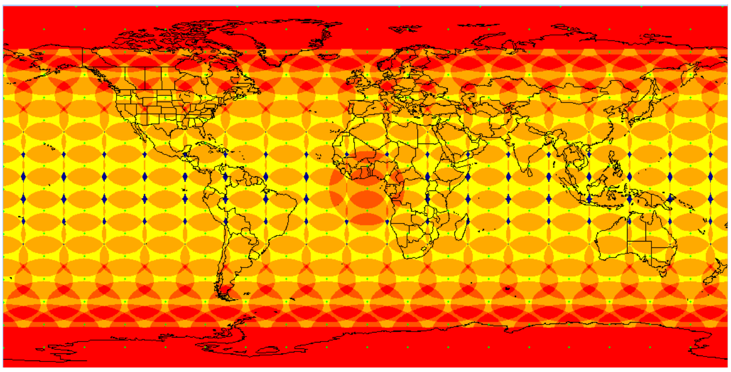
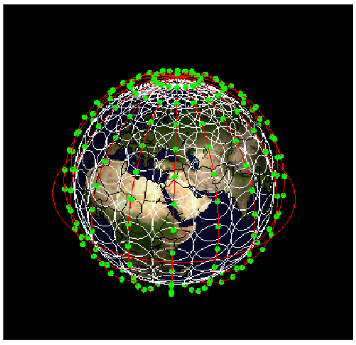

Figure 5.1.2: Candidate 2. Full Polar constellation with total ground station coverage. h= 550km; Np=18; Npp=20; Tsat=288

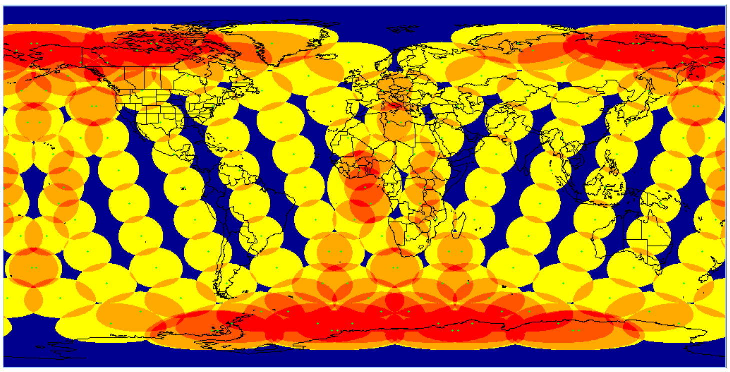
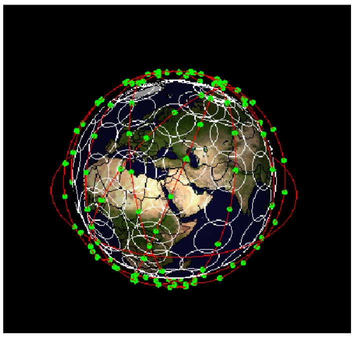

Figure 5.1.3: Candidate 3. 210° Walker-Delta constellation configuration. h= 542km; in=72; Np=8; Npp=21; Tsat=168

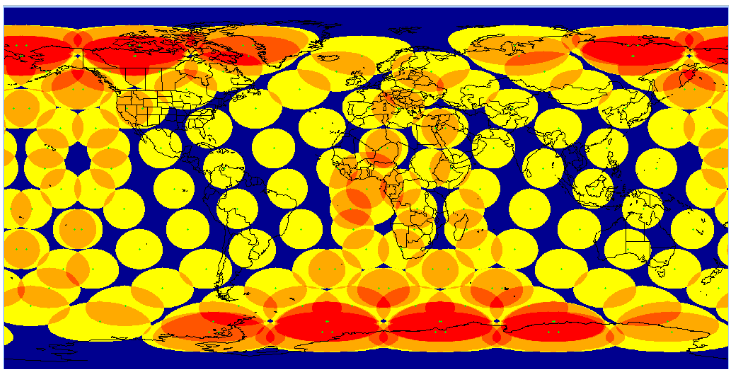
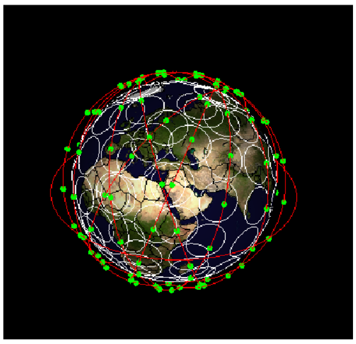

Figure 5.1.4: Candidate 4. 225° Walker-Delta constellation configuration. h= 542km; in=72; Np=9; Npp=17; Tsat= 153

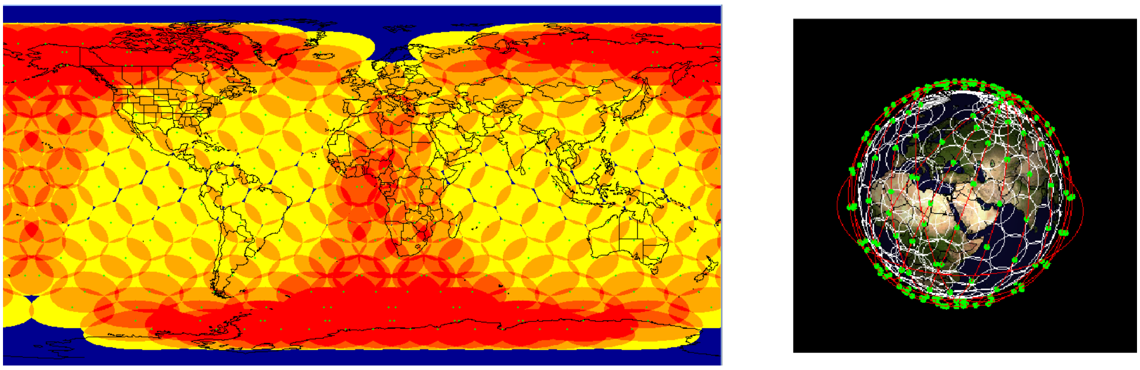

Figure 5.1.5: Candidate 5. 210° Walker-Delta constellation configuration with total coverage of the lattitudes from 0 to 52 degrees. h= 560km; in=72; Np=9; Npp=17; Tsat= 153

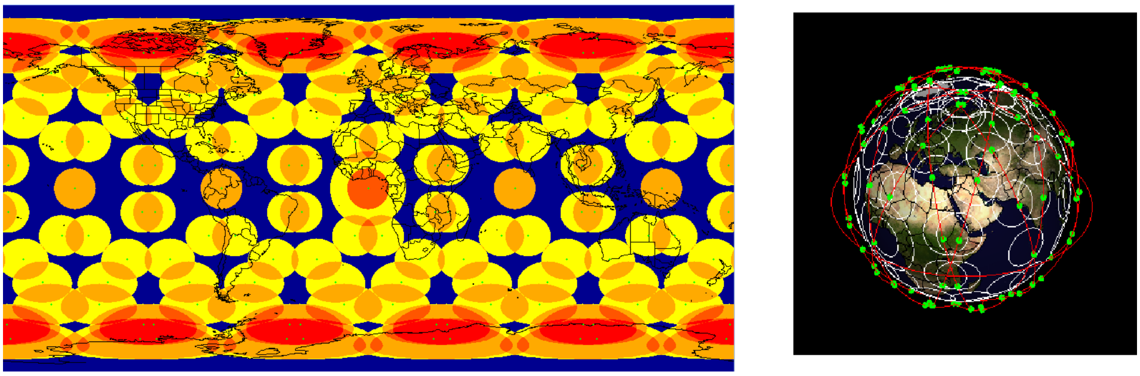

Figure 5.1.6: Candidate 6. 225° Walker-Delta constellation configuration. h= 542km; in=72; Np=9; Npp=21; Tsat= 189

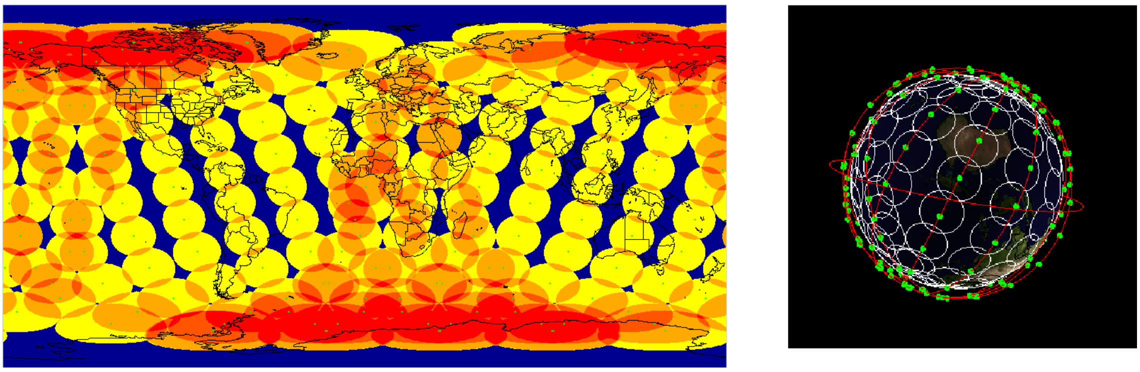

Figure 5.1.7: Candidate 7. Full Walker-Delta constellation configuration.

5.2 Constellation Performance Analysis

Even though the design requirements are included in the computation of the different configurations, it is necessary to evaluate how does the constellation perform when deployed. With this purpose, another MATLAB routine was developed.

Time factor It is important to remark that the design methods used so far did not consider coverage in a certain period of time, but the coverage at a given instant. This section summarizes a method to compute this variation.

Quality Time Another factor that was not considered in the design process was the pass times of the satellites. If a pass is too short the contact with the satellite cannot be produced.

5.2.1 Performance Evaluation

In order to determine if the performance of the Constellation is good enough and to compare different constellations, we define the following parameters that are to be used in the weighted ordered average decision5.3.1.

Simulation parameters important to clarify:

- Simulation time: 25h. This time is enough to observe the motion of the whole constellation on Earth considering its rotation and the rotation of the plains due to the Earth's oblateness.
- Minimum contact time: 3 minutes. Time enough to download data, tracking and Telecommanding the satellite.
- Time precision: 10 seconds. It is empirically observed to be precise enough.

The computed parameters:

- Fraction of time with flybys on the GS: Ratio between the time in which there is any satellite in the field of view of the Ground Station and the total simulation time. (Referred in table 5.3.1 as % Coverage)
- Mean number of links with the satellite
- Fraction of time with flybys longer than 3 minutes: In this case the ratio is with the time in which there is a satellite doing a useful pass, since a full contact can be done. (Referred in table 5.3.1 as %Quality Time)
- Mean pass time: This parameter is used to guarantee a minimum of quality and to compare different configurations. (Referred in table 5.3.1 as Average Pass Time)
- Number of gaps: Gaps are in this chapter defined as periods of time without a pass that is lasting/will last more than 3 minutes. (Referred in table 5.3.1 as Num Gaps)

- Maximum gap time: At high latitudes all the Walker-Delta configurations show a characteristic gap that can last even for hours, which is not admissible. This parameter will tell us if we exceed a maximum defined as 3 minutes for this study. (Referred in table 5.3.1 as Max Gap Time)
- Mean gap time: As it is obvious, a minimum or a 0 is desired.

You can find below an example of the analysis, for a constellation in a Semi Walker-Delta configuration.

Constellation	Full WD
Number of Planes	$p = 8$
Satellites per plane	$spp = 18$
Inclination	$i = 75^o$
GS Latitude	$\lambda = 80^o$
GS Longitude	$\phi = 0^o$

Table 5.2.1: Constellation parameters for the Example Constellation

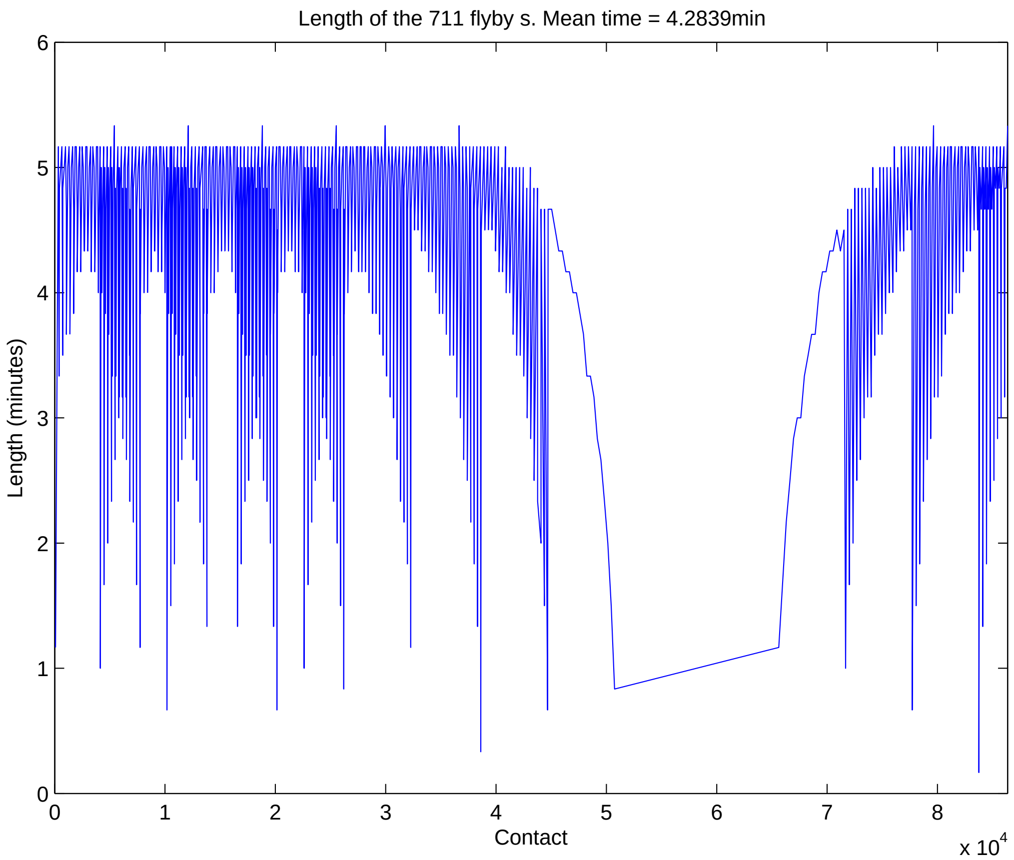

Figure 5.2.1: Length of the passes on the example GS.

Pass Time Ratio	77.53%
Quality Time Ratio	75.77%
Mean Pass Time	$4.28min$
Number of gaps	37
Maximum Gap Time	$314.33min$

Table 5.2.2: Performance Parameters for the Example Constellation

Given the high latitude of the Ground Station plus the Semi Walker-Delta Configuration there is an enormous gap. In addition, between planes some gaps are also observed.

5.3 Ordered Weighting Average based Decision

The Described Constellations are weighted and averaged in the table below. The detailed explanation of the parameters can be found in 5.2.1:

		Candidates						
Criteria	W	1	2	3	4	5	6	7
Price	15	1	2.35	5	4.94	3.21	3.92	4.67
% Coverage	4	5	4.77	2.94	2.14	4.43	1	3.86
Max Gap Time	3	3.12	3.62	1	2.88	3.51	5	4.75
%Quality time	5	4.91	4.49	4.05	1	3.19	5	4.98
Average Pass Time	5	1.21	1.14	1.14	1	1.90	5	4.72
Num Gaps	2	4.73	4.44	4.23	1	3.03	4.99	5
% Sats above	6	1	1	5	5	1	5	5
SUM (p*g)	40	90.42	108.17	154.19	133.29	113.94	167.71	188.21
OWA		0.452	0.541	0.771	0.666	0.570	0.838	0.941

Table 5.3.1: Constellation Configuration OWA Decision

With this comparison table, the optimum Constellation is option number 7:

The Astrea Constellation

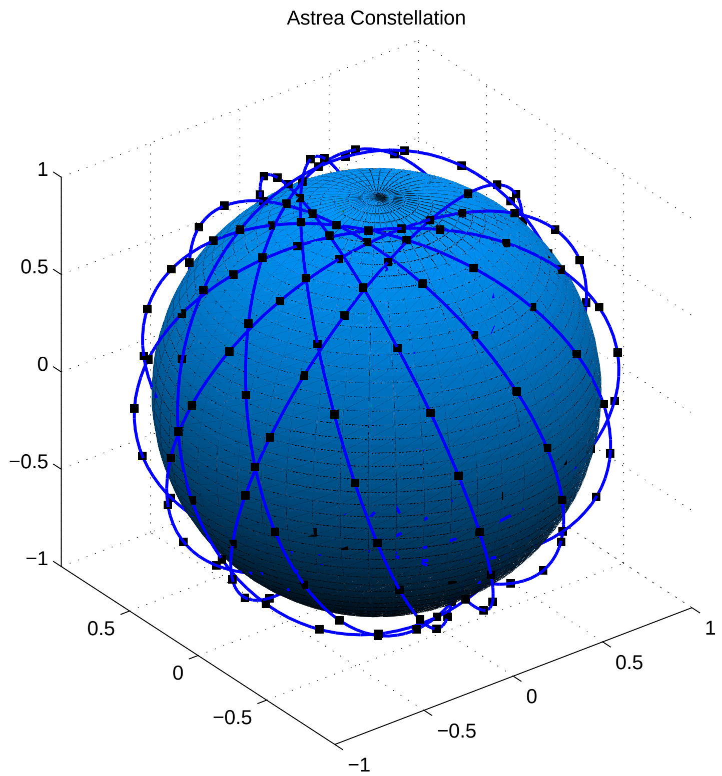

Figure 5.3.1: Astrea Constellation Final Configuration.

6 | Bibliography

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