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Astrea

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Contents

List of Tables iv

List of Figures v

I Orbit Design 1

1 Orbit Geometry 2

. . . 3

1.1 Keplerian Geometry . . . 4

. . . 4

1.2 Dynamic equations . . . 6

. . . 6

2 Orbital Coverage 7

2.1 Satellite Footprint . . . 8

2.1.1 Introduction . . . 8

2.1.2 Second Subsection . . . 8

2.2 Elevation Angle . . . 9

2.2.1 Introduction . . . 9

2.2.2 Second Subsection . . . 9

2.3 Minimum Plane Inclination . . . 10

In conclusion

. . . 10

2.4 Market Study: Current Nanosatellites in Orbit . . . 11

2.4.1 Criteria for the orbital height of the satellites . . . 11

New Space: Satellites in Orbit

. . . 11

The most interesting potential clients

. . . 11

2.4.2 New Space: Adapting to new society needs . . . 12

3 Constellation Configuration 13

3.1 Introduction: The Global Positioning System Example . . . 14

3.2 Polar Orbit Constellation 15
3.2.1 Introduction 15
3.2.2 General Configuration 16
3.2.2.1 The Streets of Coverage Method 16
3.2.3 Results of Streets of Coverage 19
3.3 Walker-Delta Constellation 21
3.3.1 Full Walker-Delta Constellation 21
3.3.1.1 Characteristics 21
. . . . 22
3.3.1.2 Notation 22
3.3.1.3 Coverage 22
3.3.2 Semi Walker-Delta Constellation 23
3.3.2.1 Introduction 23
3.3.2.2 Second Subsection 23
3.3.3 Custom Walker-Delta Constellation 23
3.3.3.1 Introduction 23
3.3.3.2 Second Subsection 24
3.4 Testing Method 25
3.4.1 Introduction 25
3.4.2 Second Subsection 25

4 Orbit Perturbations **26**
4.1 Sources of Perturbation 27
4.1.1 Introduction to Orbit Perturbations 27
The Approach of the Perturbations Study 28
4.1.2 Gravity Potential of Earth 28
4.1.3 Atmospheric Drag 29
4.1.4 3rd Body Perturbations 30
4.1.5 Other Perturbations 30
4.2 Significant Perturbations 31
Propagation Algorithm
. . . . 31
In conclusion
. . . . 31
4.3 Orbit Decay 32
4.3.1 Effects on the Ascention Node 32
4.3.1.1 Introduction 32
4.3.1.2 Second Subsection 32
4.3.2 Linearized Orbit Decay Computation 32
4.3.2.1 Introduction 32
4.3.2.2 Second Subsection 32

4.3.3 Dynamic Orbit Decay Computation 32
4.3.3.1 Introduction 32
4.3.3.2 Second Subsection 32
4.4 Orbital Station-Keeping 33
4.4.1 Raising the orbit height to increase Lifetime 33
4.4.2 Using Thrusters to increase Lifetime 34
4.4.2.1 Energy equation 34
. . . . 34
. . . . 35
4.4.2.2 Delta-V 35
4.4.2.3 Time 36
4.4.2.4 Propellant 36
4.4.2.5 Orbit maintenance 36
4.4.2.6 Results 37
. . . . 38

List of Tables

1.1.1 Eccentricity values depending on the shape of the orbit 4

List of Figures

1.0.1 Geocentric-equatorial frame. Extracted from [?]. 3
1.1.1 Geocentric-equatorial frame and the Classical Orbital Elements. Extracted from [?]. 5

2.3.1 Minimum Inclination to provide coverage at different latitude for different orbit apogees. 10
2.4.1 Distribution of the currently in orbit nanosatellites. 11

3.1.1 Distribution of the expanded 24-slot GPS constellation. 14
3.2.1 Distribution of the 66 Iridium constellation satellites. 15
3.2.2 Distribution of the planes for Polar Orbits design. 16
3.2.3 Single plain street of coverage. The footprints of the satellites superpose leading to a street. 17
3.2.4 Two plains streets of coverage. An optimum phasing needs to be obtained. 17
3.2.5 Variation of number of satellites for different heights and elevation angles 19
3.2.6 Variation of number of satellites for different heights between 500 and 600km. 20
3.3.1 Definition of the inclination δ. Extracted from [?] 21
3.3.2 Delta pattern as seen from the North Pole. Extracted from [?] 22
3.3.3 Delta pattern 65°: 30/6/1 23
3.3.4 Minimum altitude for continuous global coverage. Comparison between polar patterns and Walker delta patterns. Extracted from [?] 24

4.2.1 Logaritmic plot of the modulus of the increases in Angular Arguments of the orbit 31
4.4.1 Increase in the Lifetime obtained by setting the constellation in a higher orbit 33
4.4.2 Hohmann transfer. Extracted from [?] 35
4.4.3 Height variation of the satellite 38
4.4.4 Height variation of the satellite with a more restrictive minimum height 39

Part I

Orbit Design

Chapter 1

Orbit Geometry

Throughout this chapter, the bases of orbital geometry will be explained in order to correctly understand the parameters that will later be exposed when dealing with the constellation orbits (or the position of the satellites in them). However, long theoretical explanations will be avoided so as not to distract the reader from the main objective of the project.

To understand the movement in space is enough to apply the Newton's laws. These, however, need an inertial non-rotating frame to be correctly described. When dealing with Earth-orbiting, one usually chooses a reference system called *geocentric-equatorial system* which is shown in the figure 1.0.1 As can be seen, the XY plane coincides with the plane Equatorial with the X axis pointing in the direction of the vernal equinox [1]. The Z axis correspond the axis of rotation of the earth and points to the north (following the right-hand rule).

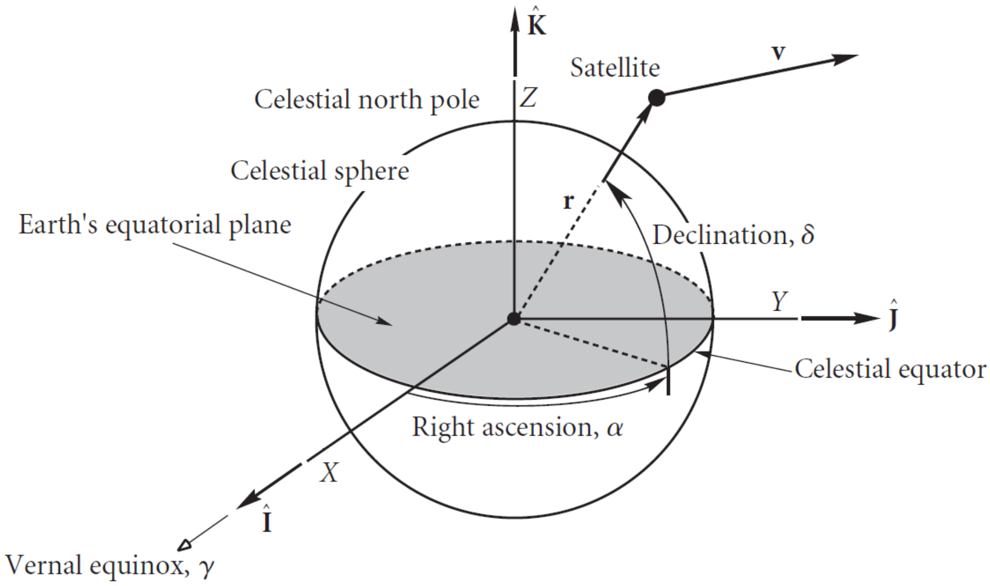

Figure 1.0.1: Geocentric-equatorial frame. Extracted from [?].

By defining this system, any point in the space can be depicted by its position vector r and we can study its movement by the velocity vector $\dot{r}$. These elements are useful especially for computational work but they nearly do not provide information about the orbit. For these reason, the orbital elements were developed.

[1] an imaginary line found by drawing a line from the Earth to the Sun on the first day of spring

1.1 Keplerian Geometry

The *Classical Orbital elements*, also known as the *Keplerian elements* as an attribution to Johannes Kepler, are six independent quantities which re sufficient to describe the size, shape and orientation of an orbit. This set of elements are shown in the figure 1.1.1 and are defined as follows:

- **Semi-major axis** (a): It is related to the size of the orbit and its defined by the sum of the apogee (furthest point) and the perigee (closest point) divided by two.

- **Eccentricity** (e): It defines the shape of the orbit with respect to that of a circle. Thus, the eccentricity of a circular orbit is null while hyperbolic orbits have an eccentricity greater than one.

Circular	$e = 1$
Elliptical	$0 < e < 1$
Parabolic	$e = 1$
Hyperbolic	$e > 1$

Table 1.1.1: Eccentricity values depending on the shape of the orbit

- **Inclination** (i): the inclination is the angle between the positive Z axis and the angular momentum vector ($\mathbf{h}$) which is perpendicular to the orbital plane. The inclination of the orbit can take a value from 0 deg to 180 deg. For $0\,\text{deg} \leq i \leq 90\,\text{deg}$ the motion *posigrade* and for $90\,\text{deg} \leq i \leq 180\,\text{deg}$ the motion is *retrogade*.

- **Right ascension of the ascending node - RAAN** (Ω): This parameter, along with the inclination define the orientation of the orbital plane. It is the angle between the positive X axis and the intersection of the orbital plane with the equatorial plane XY in counterclockwise direction. The intersection mentioned is called the node line and the point where the orbit passes through the node line (from south to north) is the ascension node ($0\,\text{deg} \leq \Omega \leq 360\,\text{deg}$).

- **Argument of perigee** (ω): Is defined as the angle between the ascending node and the perigee. It describes the orientation of the ellipse with respect to the frame ($0\,\text{deg} \leq \omega \leq 360\,\text{deg}$).

- **True Anomaly** (ϕ): This last quantity is used to describe the satellite's instantaneous position with respect to the perigee. Is the angle, measured clockwise, between the perigee and the satellite position. From all the orbital elements, the true anomaly is the only that changes continuously. Sometimes, true anomaly is substituted by the mean anomaly, which can be calculated using another auxiliary angle called the eccentric anomaly.

$$\begin{aligned} \cos E &= \frac{e + \cos\theta}{1 + e\cos\theta} \\ M &= E - e\sin E \end{aligned} \tag{1.1.1}$$

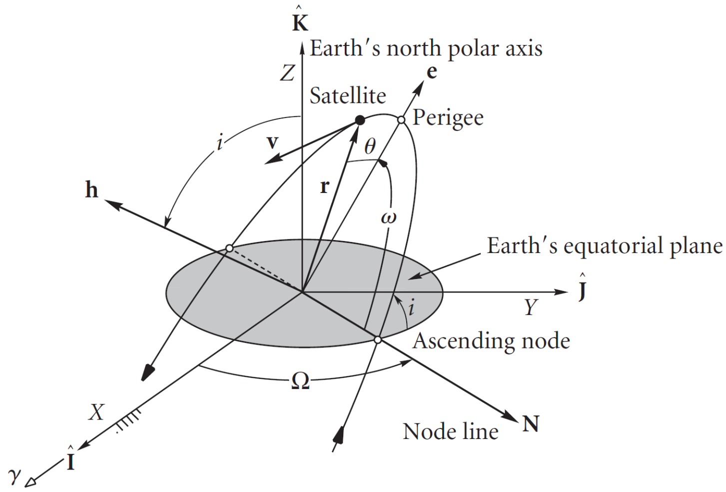

Figure 1.1.1: Geocentric-equatorial frame and the Classical Orbital Elements. Extracted from [?].

1.2 Dynamic equations

As aforementioned, the motion of an object in the space can be described using the Newton's laws. The basic idea developed by Newton is to study the Cubesat and the Earth as a spherical bodies in mutual gravitational attraction and neglect the gravitational forces caused by other objects (this is called the *two body* problem). The forces balance is simple since we only have the Earth gravitational attraction, which must compensate the centripetal acceleration of the satellite. Thus, using the law of universal gravitation,

$$-G\frac{M_E m_{sat}}{r^3}\vec{r} = m_{sat}\vec{a}_{sat} \tag{1.2.1}$$

Where G is the gravitational constant and r represents the distance between the satellite and the Earth. From the last equation, we only want to obtain the acceleration, therefore:

$$-G\frac{M_E}{r^3}\vec{r} = \vec{a}_{sat} = \frac{d^2\vec{r}}{dt^2} \tag{1.2.2}$$

For simplicity, it usual to denote $\mu = GM_{earth}$ resulting in the following equation:

$$-\frac{\mu}{r^3}\vec{r} = \frac{d^2\vec{r}}{dt^2} \tag{1.2.3}$$

This expression is a second order equation that models the motion of the Cubesat relative to the Earth and it can be analytically solved. The only problem is that several hypotheses have been applied that make the case different from reality. The formulation should be modified to take into account the effects due to:

- More bodies attracting the satellite (Sun, Moon, Venus, etc.)
- The existence of more forces like the drag, the solar radiation pressure, etc.
- The earth is not an spherical body.

The corrections for considering these things are called perturbations and they are explained in the Chapter **??** of this part of the report.

Chapter 2

Orbital Coverage

2.1 Satellite Footprint

2.1.1 Introduction

- Primera cosa
- Segunda cosa
- Tercera cosa

2.1.2 Second Subsection

2.2 Elevation Angle

2.2.1 Introduction

- Primera cosa
- Segunda cosa
- Tercera cosa

2.2.2 Second Subsection

2.3 Minimum Plane Inclination

As it has been pointed before, there are several factors to take into account in order to design a constellation that provides global coverage on Earth. In this section the minimum inclination to achieve that purpose is asessed. Using the theory previously developed, we can observe the following results:

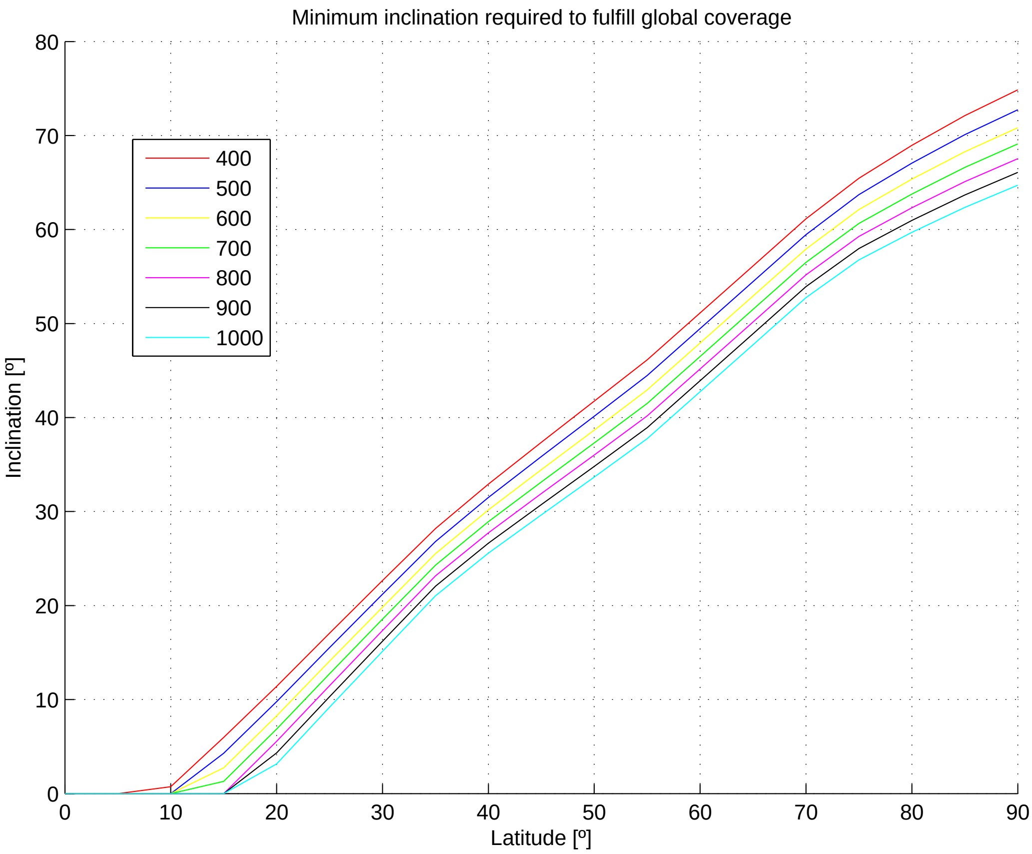

Figure 2.3.1: Minimum Inclination to provide coverage at different latitude for different orbit apogees.

As it can be observed, if the goal of the design is to provide full global coverage, the distribution of elevation angles with latitude is not significant, since the inclination is required to be higher than aproximately 75º. In the other cases, the change of minimum elevation angle distribution causes changes of tendency in the distribution of inclination required.

In conclusion

The main point is that there is a limit inclination for a Walker-Delta constellation configuration in order to provide global coverage at the desired latitude. With this study, this limits in the design algorithms can be set.

2.4 Market Study: Current Nanosatellites in Orbit

2.4.1 Criteria for the orbital height of the satellites

Satellites currently in Orbit

If only geometric considerations were to be applied in the design of a satellite constellation, it is clear that the higher the orbit the broader is the footprint in the surface leading to a smaller number of satellites. However, if the service of communications is to be offered, the satellites currently in orbit or in design phases need to be at higher orbit than the one of the constellation. The purpose of that requirement is to intersect the field of view of the satellites that nowadays point to Earth.
From source [?] we can study how the currently on orbit satellites are launched and specially, in which orbits. The results of the study of this source is presented below. All of them are in Low Earth Orbits, and half of them above 550km. In total, there are 203 operational satellites.

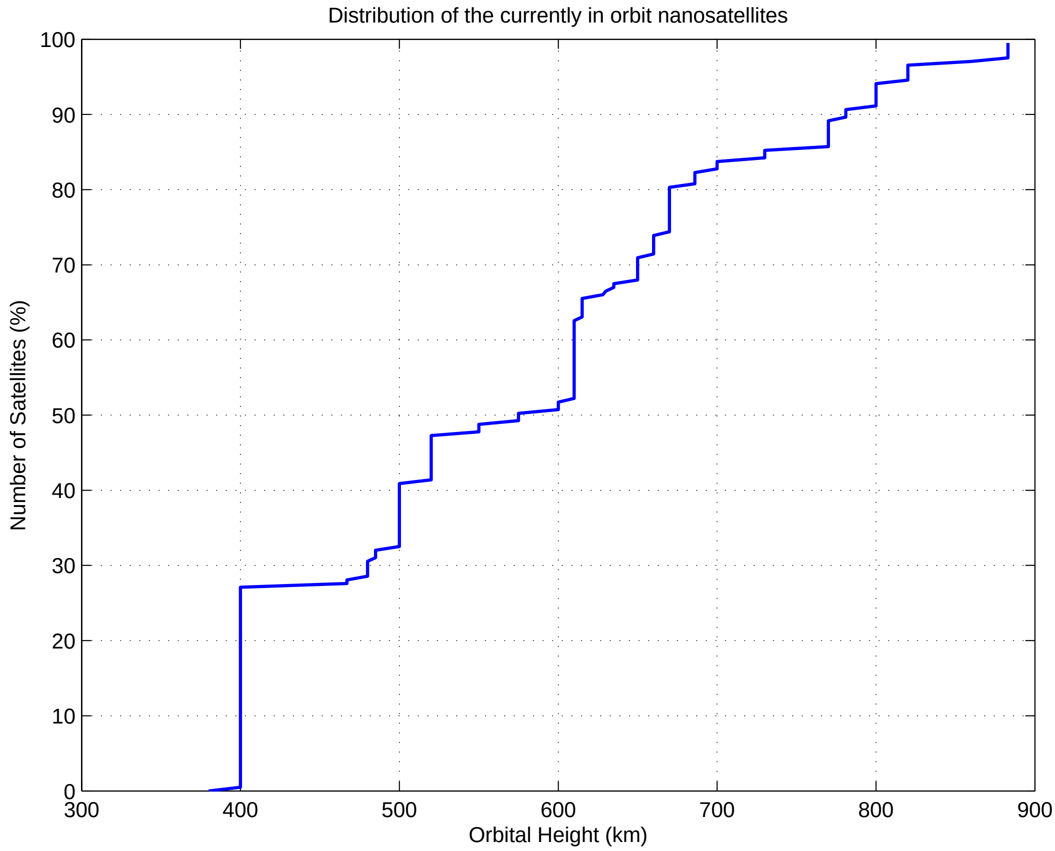

Figure 2.4.1: Distribution of the currently in orbit nanosatellites.

The most interesting potential clients

Lots of satellites are orbiting at heights lower than 500km, mainly because one of the most feasible way of launching a small satellite is from the International Space Station.

However, this very low LEOs are related to very high speeds and specially to low lifetimes, since drag affects them in a more significant way. To the interest of the constellation, the satellites at higher altitudes are a better commercial target, since they are going to be in orbit for longer missions. In addition, the same orbit decay problems are avoided for the constellation satellites.

2.4.2 New Space: Adapting to new society needs

Nowadays new satellites willing to provide services to Earth are being positioned closer than ever. Where closer can be applied in many points of view. Physically, the satellites are placed every time at lower orbits, since the energetic requirement is lower. Technically, the space certified materials and hardware are becoming more feasible, and new launchers are smaller. In the end, everything comes down to an economic approach, launching satellites is becoming cheaper every time and this means closer to the private pocket.

In the future, the possibility of using the Astrea constellation to contact Earth can reduce the requirements for the antennas and AOCSs to communicate with ground, leading to a whole new level of resources for the satellite payload. For instance, by communicating to the constellation pointing to outter space instead of pointing down to Earth. That is just a way in which Astrea is in the New Space Generation.

Chapter 3

Constellation Configuration

"Our two greatest problems are gravity and paperwork. We can lick gravity, but sometimes the paperwork is overwhelming."

Werner von Braun, 1958

3.1 Introduction: The Global Positioning System Example

Depending on the application the Space Segment of a mission can vary in an infinite number of ways. Probably the most famous and widely used satellite constellation is the the Global Positioning System satellite network. In this case, it uses an irregular geometry.

The GPS Constellation: An example of irregular distributed orbits

The GPS is a constellation property of the U.S. It provides positioning, navigation and timing. The constellation was designed with a 24-slot arrangement to ensure a visibility of at least four satellites from any point on the planet. Nowadays the constellation has expanded to a total operative number of 27-slot since June 2011. Some characteristic parameters of the satellites are the following:

- Orbit: Almost Circular
- Height = 20,200 km (MEO);
- Lifetime = 12.5 years;
- Satellite Cost = 166 million USD;
- Inclination = 55º;
- Number of planes = 6;
- Phasing: 30º-105º-120º-105º;

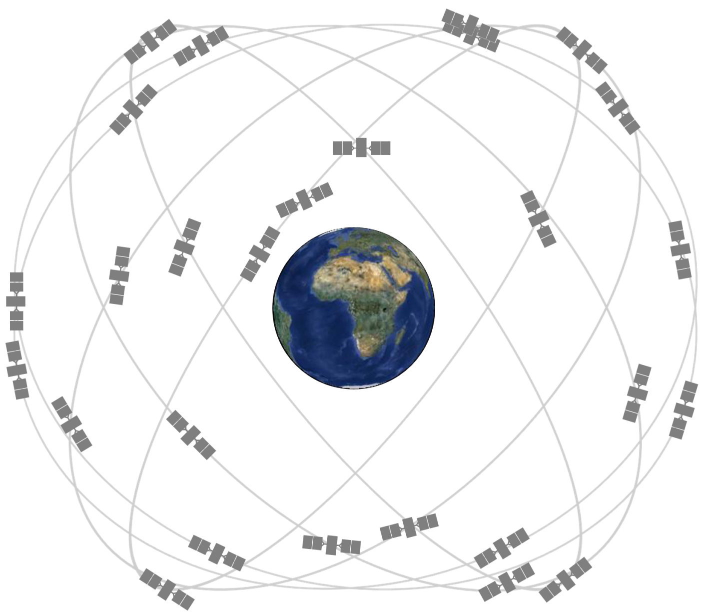

Figure 3.1.1: Distribution of the expanded 24-slot GPS constellation.

3.2 Polar Orbit Constellation

3.2.1 Introduction

Polar Orbits are probably the simplest way to configure an evenly spaced constellation. As we will see in the section **Orbit Perturbations** when the inclination is the same for all the planes, the deviations tend to be the same for all the satellites. In addition, the computation of the number of satellites required is also easier.

The Iridium Constellation: An example of near polar orbits

The Iridium constellation is a private constellation. It provides voice and data coverage to satellite phones among other services. The constellation was designed with 77 satellites, giving name to the constellation by the chemical element. The constellation was reduced to a number of 66. Sadly, Dysprosium is not such a good commercial name. Some characteristic parameters of the satellites are the following:

- Orbit: Almost Circular
- Height = 781 km (LEO);
- Satellite Cost = 5 million USD;
- Inclination = 86.4º;
- Number of planes = 11;
- Phasing: Regular;

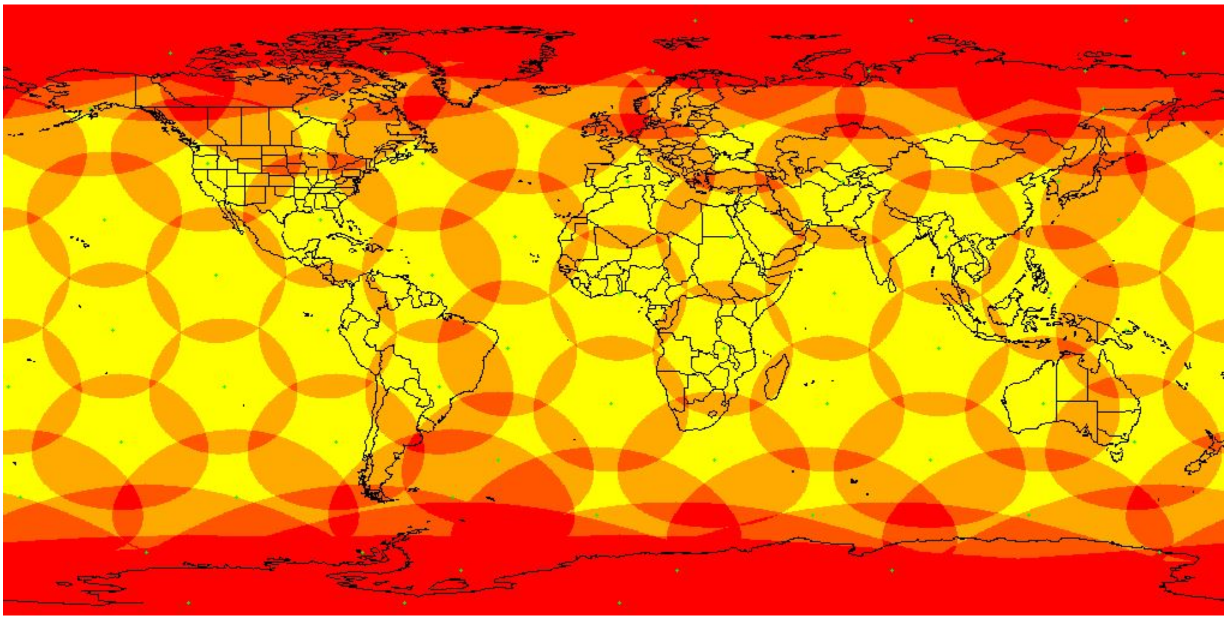

Figure 3.2.1: Distribution of the 66 Iridium constellation satellites.

3.2.2 General Configuration

The Polar Orbits configuration consists in the distribution of plains with inclination equal to 90 degrees. Note that the satellites will be travelling parallel to the satellites of the next plain except for the communications between the first and the last plane.

The communications between satellites in antiparallel directions require less space between plains to be fulfilled. In order to solve this isconvenience the separation between the first and the last plain is reduced.
The plains are splitted in the following pattern:

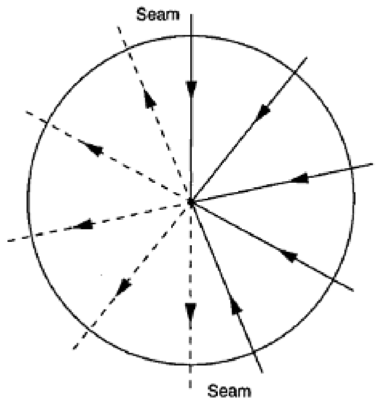

Figure 3.2.2: Distribution of the planes for Polar Orbits design.

3.2.2.1 The Streets of Coverage Method

This Street of Coverage Method is obtained from [?]. As you can see in the figure below, the relations between angles seen from different satellites can be easily computed. The main variables are the following:

Streets of Coverage Method Variables	
N	Number of Satellites
n_p	Number of Planes
N_{pp}	Number of Satellites per plane
S	Separation between satellites of the same plane
D	General space between planes [º]
D_0	Space between antiparallel planes [º]
ε	Elevation angle [º]
λ_{street}	Street of coverage Width [º]
λ_{max}	Maximum footprint Radius [º]

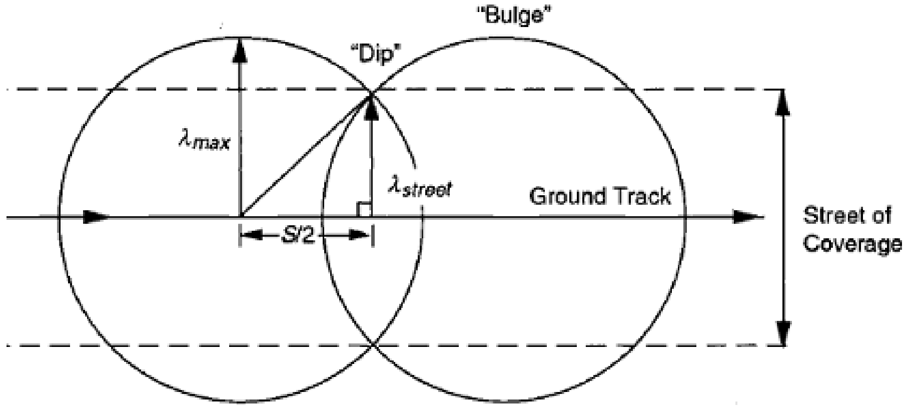

Figure 3.2.3: Single plain street of coverage. The footprints of the satellites superpose leading to a street.

From the figure it can be inferred:

$$S < 2\lambda_{max}$$

$$cos(\lambda_{street}) = cos(\lambda_{street})/cos(S/2)$$

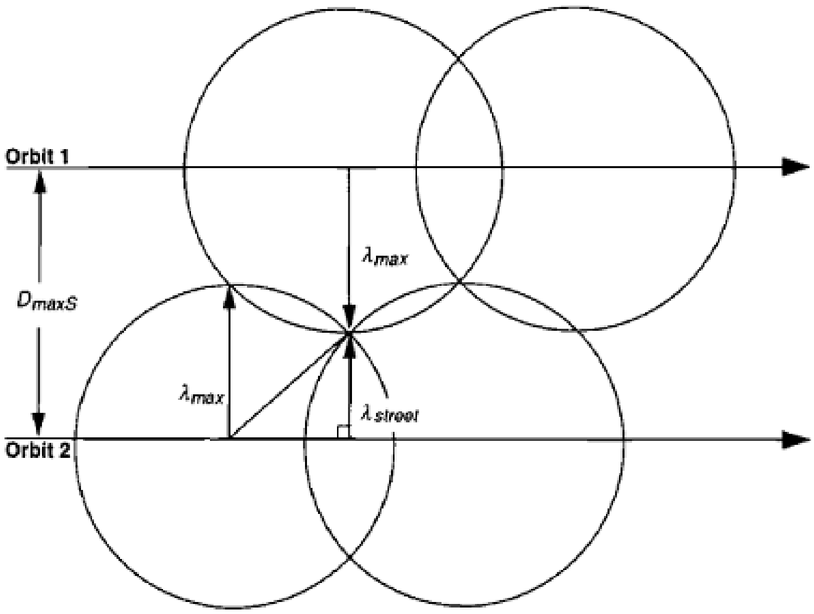

Figure 3.2.4: Two plains streets of coverage. An optimum phasing needs to be obtained.

From this point of view, in general:

$$D = \lambda_{street} + \lambda_{max}$$

n For the antiparallel planes:

$$D_0 = 2\lambda_{street}$$

And the overall relationship between planes sums:

$$180 = (n_p - 1)D + D_0$$

The algorithm for computing the Streets of Coverage Results is defined in the following way:

$$\text{Inputs: Height, elevation, inclination...} \rightarrow \lambda_{max} \rightarrow N_{pp} = \left\lceil \frac{360}{2\lambda_{max}} \right\rceil \rightarrow$$
$$S = 360/N_{pp} \rightarrow \lambda_{street} \rightarrow n_p \rightarrow N = N_{pp} * n_p$$

3.2.3 Results of Streets of Coverage

A MATLAB routine has been designed to compute the previously described algorithm. In this conceptual design phase, different heights are computed in order to see the evolution of the number of satellites.

General Solution

The program in runned in a broad range of parameters to see the evolution of the number of satellites. As it can be predicted, as the height increases the number of satellites is reduced. The reason is that the footprint of the satellites increases with the height. In addition, as the minimum elevation over the horizon to contact the satellites is reduced, the number of satellites is also reduced for the same reason.

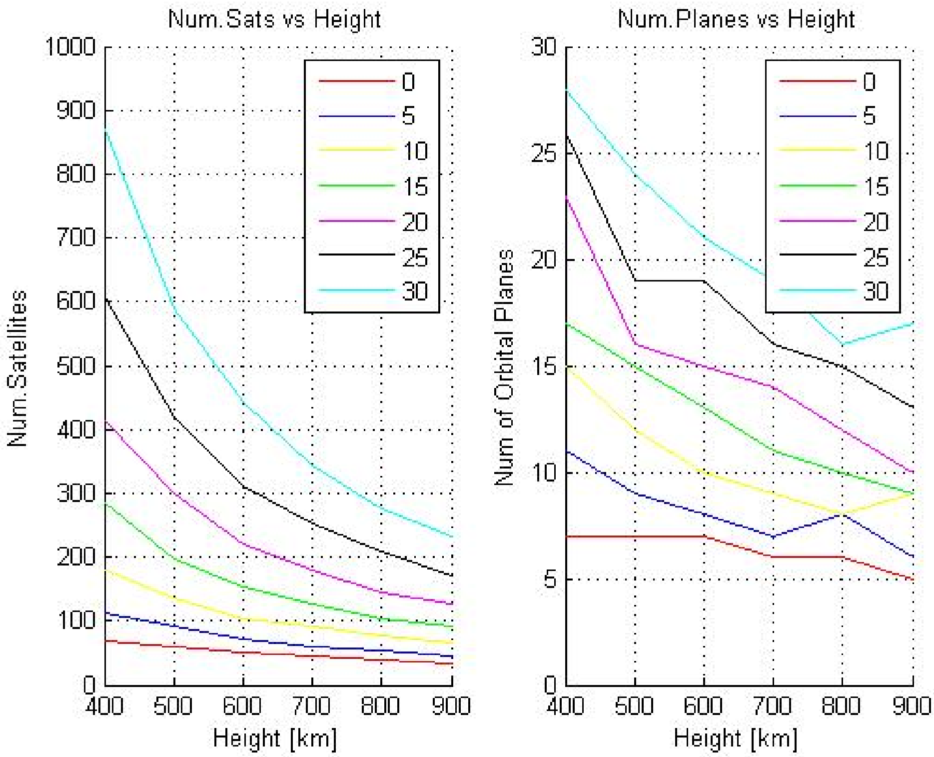

Figure 3.2.5: Variation of number of satellites for different heights and elevation angles

Detailed Solution

Given the previously justified assumptions, the same simulation is computed for a more reasonable range of results. In this case, the elevation is set as:

$$\varepsilon = 20^o$$

.

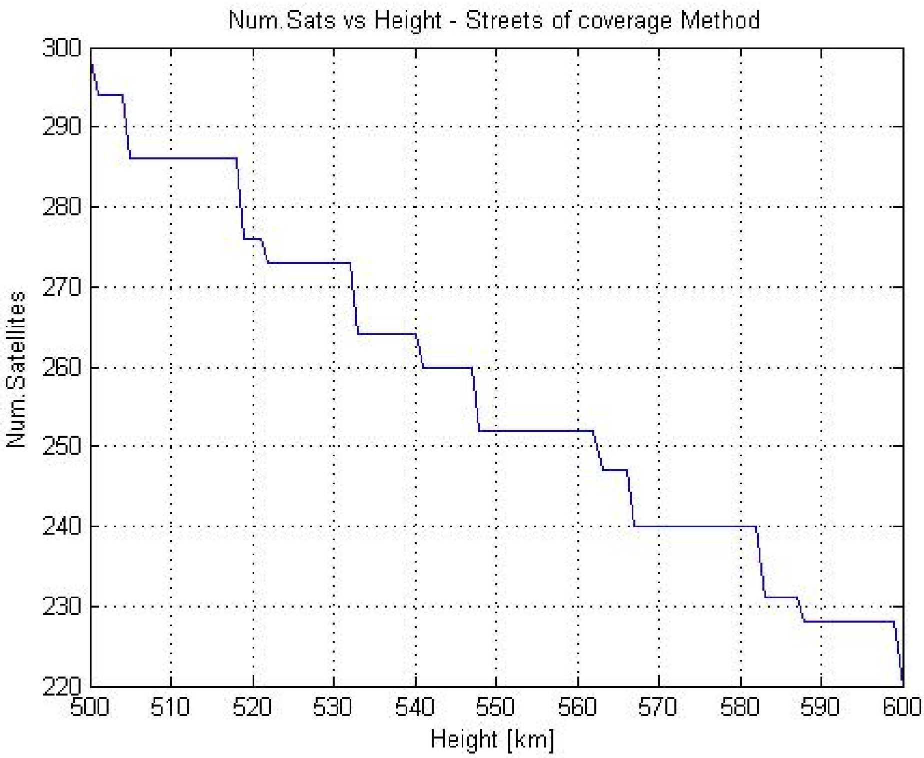

Figure 3.2.6: Variation of number of satellites for different heights between 500 and 600km.

Conclusion

The computation and the design of this constellation requires small computational and conceptual effort. However, the number of satellites and planes is greater than expected. Even though the technical complexity can be reduced, the availability of small launchers to reach this particularly inclined orbit is also small. In conclusion, more constellation configurations need to be assessed to compare and select the most feasible one.

3.3 Walker-Delta Constellation

Walker Delta Pattern constellations are a type of symmetric, inclined constellation made of equal-radius circular orbits, with an equal number of satellites each one. There are several ways to construct a Walker-Delta Constellation:

- Full Walker-Delta Configuration
- Semi Walker-Delta Configuration
- Custom Walker-Delta Configuration

3.3.1 Full Walker-Delta Constellation

3.3.1.1 Characteristics

A typical delta pattern has the following characteristics:

- The constellation contains a total of T satellites evenly spaced in each of the P orbital planes. All planes have the same number of satellites, defined as S, equally distributed. Thus:

$$T = SP \tag{3.3.1}$$

$$\Delta\varphi = \frac{2\pi}{S} \tag{3.3.2}$$

  Where $\Delta\varphi$ is the angle between satellites in the same plane.

- All orbits have equal inclinations δ to a reference plane. If this plane is the Equator (it usually is), then the inclination δ equals the orbital parameter inclination i [?].

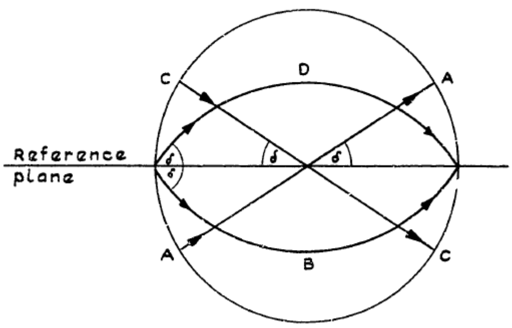

Figure 3.3.1: Definition of the inclination δ. Extracted from [?]

- The ascending nodes of the orbits are equally spaced across the full 2π (360° of longitude) at intervals of:

$$\Delta\Omega = \frac{2\pi}{P} \tag{3.3.3}$$

- The position of the satellites in different orbital planes is measured through the factor F. When a satellite is at its ascending node, a satellite in the most easterly adjacent plane has covered a relative phase difference F. The real phase difference is defined as:
$$\Delta\Phi = F\frac{2\pi}{P} \tag{3.3.4}$$
In order to have the same phase difference between all orbital planes, F is defined as an integer, which may have any value from 0 to (P-1).

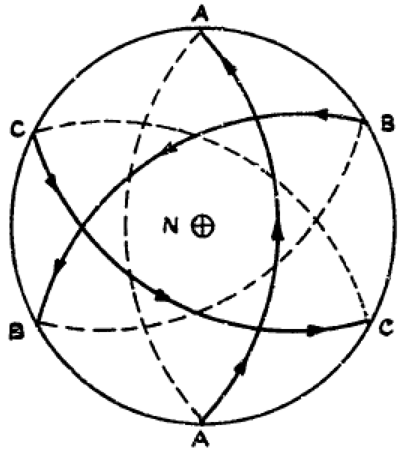

Figure 3.3.2: Delta pattern as seen from the North Pole. Extracted from [?]

With these characteristics, delta constellations are more complex than polar constellations. Because of the inclination of the orbits, the ascending and descending planes and the coverage of the satellites continuously overlap. This characteristic is a constraint on intersatellite networking because the relative velocities between satellites in different orbital planes are larger than in a polar constellation. Consequently, tracking requirements and Doppler shift are increased [?].

3.3.1.2 Notation

J.G. Walker developed a notation to define this constellations with only 4 parameters [?]:

$$i : T/P/F$$

Since all satellites are placed at the same altitude, with these notation the shape of the pattern is completely determined. However, to determine all the orbital parameters it is necessary to know the radius of the orbits.

3.3.1.3 Coverage

The previous section has shown that in polar orbits the coverage of the constellation could be determined with the streets of coverage method. On the other hand, in delta patterns

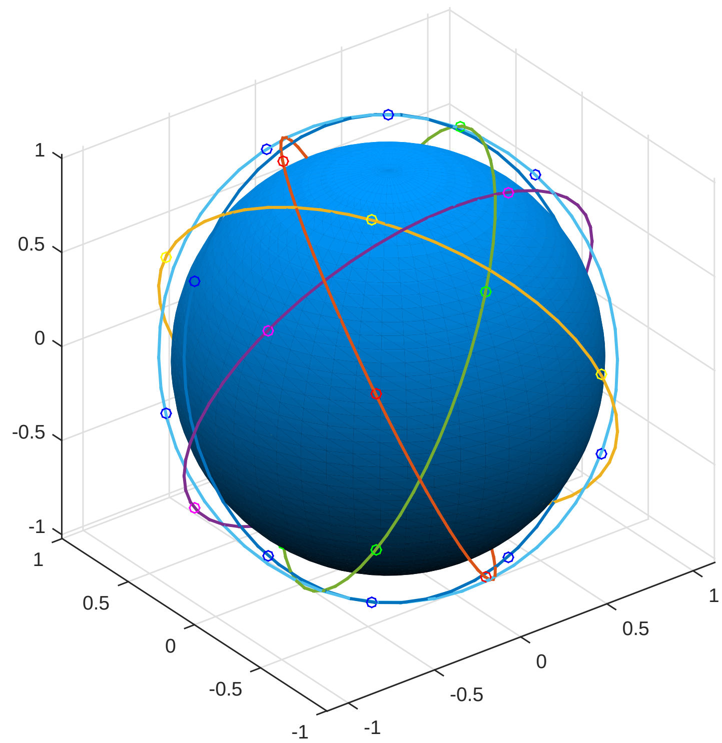

Figure 3.3.3: Delta pattern 65°: 30/6/1

it is necessary to study each configuration to verify its coverage. J.G. Walker determined that delta patterns gave better coverage than polar orbits, but not substantially better in the case of single coverage. This kind of patterns are more useful for double or triple coverage constellations, as it can be seen in Figure 3.3.4. However, his calculations were for a low number of satellites, so it is necessary to compute new results for the number of satellites of the Astrea constellation.

3.3.2 Semi Walker-Delta Constellation

3.3.2.1 Introduction

- Primera cosa
- Segunda cosa
- Tercera cosa

3.3.2.2 Second Subsection

3.3.3 Custom Walker-Delta Constellation

3.3.3.1 Introduction

- Primera cosa

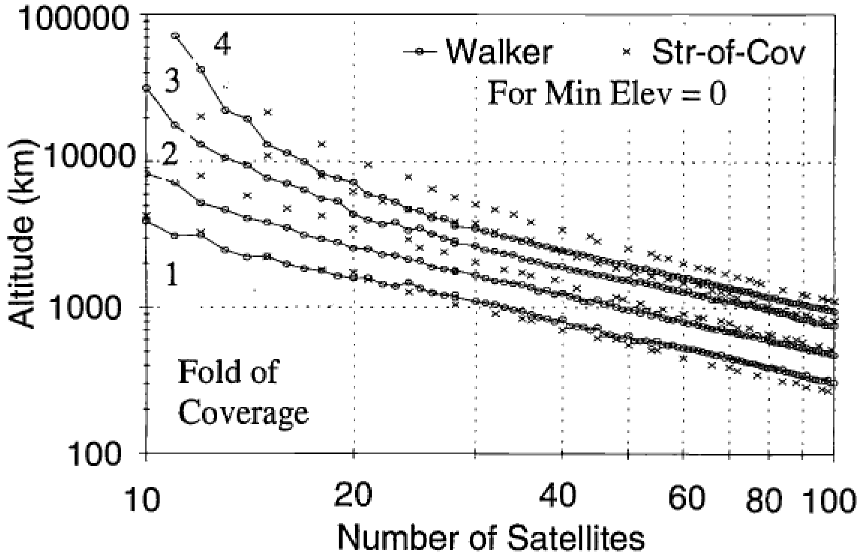

Figure 3.3.4: Minimum altitude for continuous global coverage. Comparison between polar patterns and Walker delta patterns. Extracted from [?]

- Segunda cosa
- Tercera cosa

3.3.3.2 Second Subsection

3.4 Testing Method

3.4.1 Introduction

- Primera cosa
- Segunda cosa
- Tercera cosa

3.4.2 Second Subsection

Chapter 4

Orbit Perturbations

4.1 Sources of Perturbation

4.1.1 Introduction to Orbit Perturbations

In this chapter it is seen how the designed orbit configuration varies in time due to external perturbation sources. While some of them can be neglected, there are other of major importance to the future of the constellation. For instance, atmospheric drag determines in plenty of cases the lifetime of the constellation. A first classification of perturbations depending on the time in which their effects are present is the following:

- Secular terms (Sec): They depend on the semimajor axis, the excentricity and the inclination.
- Short Period terms (SP): They depend on the anomalies, this leads to a strong variation in each period.
- Long Period terms (LP): They depend on the argument of the periapsis or the ascendent node.

Even though most of the outter space is vacuum, there ideal models need to consider some factors that escape the typical two body problem. For instance, we can no longer consider Earth as a punctual mass, neither the atmospheric density equal to 0. To enumerate, here is a typical list of the main perturbation sources:
Sources of perturbation:

- Gravity Field of the Central Body
- Atmospheric Drag
- Third Body perturbations
- Solar-Radiation Pressure
- Other Perturbations

All the perturbations can be deeply studied. Consequently, analytical solutions are very hard to find, and even they were found, they do not show clealy a meaning or are not really useful. Instead, there are two mainly used approaches:

- Special Perturbacion: Step-by-step numerical integration of the motion equations with perturbation.
- General Perturbation: Through analytical expansion and integration of the equations of variation of orbit parameters.

The Approach of the Perturbations Study For the purposes of these study the different approaches will be assessed. The first analysis will discuss which of the peturbations are the most significant to the study. This analysis will be done considering General Perturbation Techniques. In a deeper second analysis, the two approaches for the perturbations will be assessed and compared considering only the most significant perturbation sources.

4.1.2 Gravity Potential of Earth

Earth's aspherical shape can be modelled as a sum of terms corresponding to the Legendre polynomials. These polynomials can be empirically measured and consider radial symmetry. If one would like to compute also variations in longitude, then should use associated Legendre polynomials.

$$V(r,\delta,\lambda) = -\frac{\mu}{r}\left[\sum_{n=1}^{\infty}\left(\frac{R_e}{r}\right)^n \sum_{m=0}^{n} P_{nm}cos(\delta)(C_{nm}cosm\lambda + S_{nm}sinm\lambda)\right] \tag{4.1.1}$$

General Legendre associated polynomials developed Gravitational Potential

$$V(r,\delta) = -\frac{\mu}{r}\left[1 - \sum_{n=2}^{\infty} J_n \left(\frac{R_e}{r}\right)^n P_n(sin\delta)\right] \tag{4.1.2}$$

General Legendre polynomials developed Gravitational Potential
For Earth, the J_n coefficients are the following:
$J_2 = 0.00108263$ $J_3 = -0.00000254$ $J_4 = -0.00000161$
Given this distribution, the only sognificant term J_2.

$$V(r,\delta) = -\frac{\mu}{r}\left[1 - \frac{1}{2}J_2\left(\frac{R_e}{r}\right)^2(1 - 3sin^2\delta\right] \tag{4.1.3}$$

Aproximated Gravitational Potential
If we integrate the force that derives from this potential we can afterwards compute the effect of J_2 On the different orbtial elements:

- $\Delta a = 0$
- $\Delta e = 0$
- $\Delta i = 0$
- $$\Delta\Omega = -3\pi\frac{J_2 R_e^2}{p^2}cos\, i\ [rad/orbit] \tag{4.1.4}$$
- $$\Delta\omega = \frac{3}{2}\pi\frac{J_2 R_e^2}{p^2}(4 - 5sin^2\, i)\ [rad/orbit] \tag{4.1.5}$$

4.1.3 Atmospheric Drag

In order to compute the effect of the remaining atmosphere we use the typical definition of atmospheric drag knowing a drag coefficient:

$$\vec{a}_{drag} = \frac{1}{2}\frac{C_d A}{m}\rho v_{rel}^2 \frac{\vec{v}_{rel}}{|\vec{v}_{rel}|} \tag{4.1.6}$$

The **ballistic coefficient** B_c is defined as $\frac{m}{C_d A}$, characterizing the behaviour of the satellite against atmospheric drag.

Modelling the Atmosphere

There are several modells for the atmosphere. For instance, the most commonly used, the exponential model:

$$\rho = \rho_0 e^{-\frac{h-h_0}{H}} \tag{4.1.7}$$

$$H = \frac{kT}{Mg} \tag{4.1.8}$$

Where:

Exponential Atmosphere Variables	
ρ	Density at given height
ρ_0	Density at a reference height
h	Height over the ellipsoid
h_0	Reference height
H	Scale Height
k	Boltzmann Constant
T	Temperature
M	Molecular Weight
g	Gravity

In addition, other models for the exospheric temperature and the molecular weight need to be used. For this study the ones proposed by The Australian Weather Space Agency are used.
In addition, it is important to note that the following phenomena interfere with the previsions:

- Diurnal Variations
- 27-day solar-rotation cycle
- 11-year cycle of Sun spots
- Semi-annual/Seasonal variations

- Rotating atmosphere
- Winds
- Magnetic Storm Variations
- Others: Tides, Winds,...

Again, if we integrate this force in a period of time, considering the orbit nearly circular, we obtain:

$$\Delta r = -2\pi\rho r^2/B \ [/orbit] \tag{4.1.9}$$

4.1.4 3rd Body Perturbations

The effects of this extra bodies in the system can be computed considering the motion equations. However, some aproximations can be found in the reference as:

$$\dot{\Omega} = \frac{A_m + A_s}{n} cos\, i \ [^{o}/day] \tag{4.1.10}$$

$$\dot{\omega} = \frac{B_m + B_s}{n}(4 - 5sin^2\, i) \ [^{o}/day] \tag{4.1.11}$$

Where n stands for the rate of rotation in orbits/day. In that case, the A_m, A_s, B_m and B_s coefficients take as values:

	$A_m + A_s$	$B_m + B_s$
Moon	-0.00338	0.00169
Sun	-0.00154	0.00077

4.1.5 Other Perturbations

In this bag the following low-intensity can be classified:

- Solar Radiation Pressure
- Solid-Earth and Ocean Tides
- Magnetic Field
- South Atlantic Anomaly

4.2 Significant Perturbations

Propagation Algorithm

Given the definitions and aproxmations to compute perturbations described in the previous section, a propagation in time for the change in orbital parameters is solved. The results are plotted in the graph below:

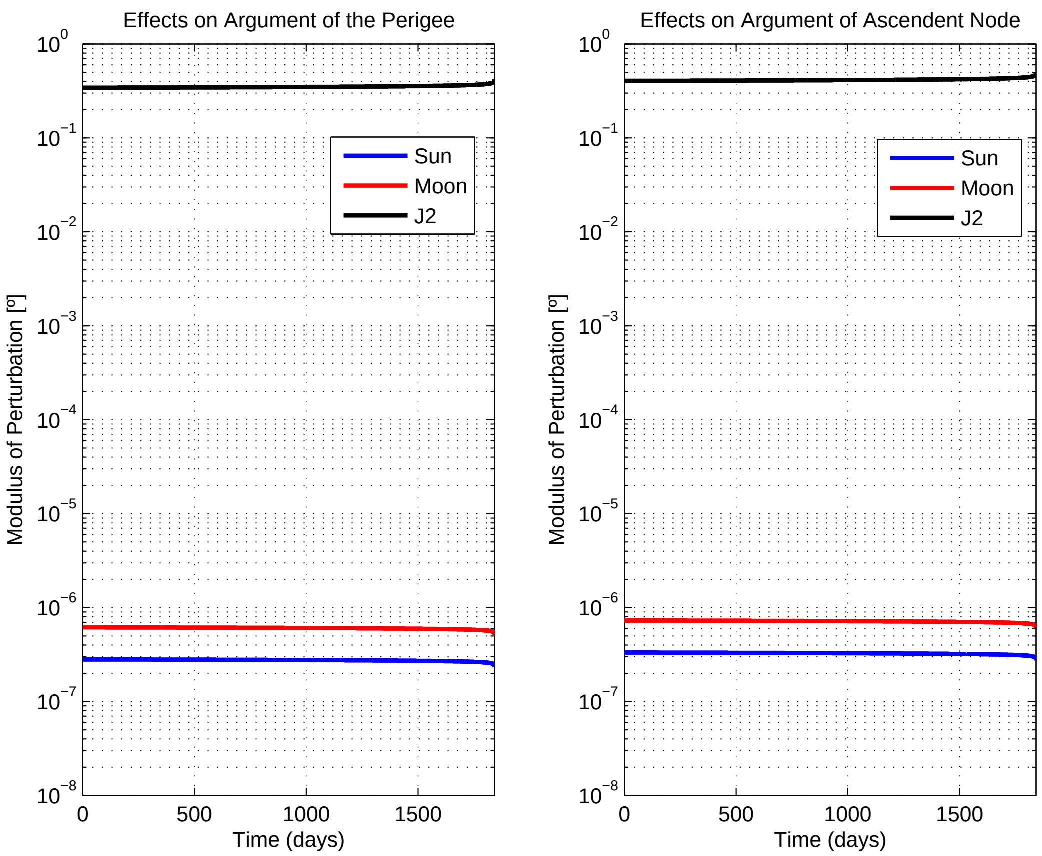

Figure 4.2.1: Logaritmic plot of the modulus of the increases in Angular Arguments of the orbit

As it can be seen, the perturbations caused by 3rd bodies are several orders of magnitude below the order of magnitude of the variation caused by Earth's oblateness. It is also remarkable that the moon has a higher effect than the sun given the relative distance to Earth, even if the sun is way more massive.

Another important obsevation is that given the very low eccentricity we are considering, the deviation of the argument of the perigee does not affect the performance of the constellation. In other words, since the orbits are considered almost circular there is not a defined Perigee for the orbit.

In conclusion

The effects of the Moon and the Sun are neglected in comparison with the effects of J2 for the Argument of the ascendent node as well as for the argument of the Perigee.

4.3 Orbit Decay

We will study:

- Inclination related
- Decay simple
- Decay dinamic

4.3.1 Effects on the Ascention Node

4.3.1.1 Introduction

- Primera cosa
- Segunda cosa
- Tercera cosa

4.3.1.2 Second Subsection

4.3.2 Linearized Orbit Decay Computation

4.3.2.1 Introduction

- Primera cosa
- Segunda cosa
- Tercera cosa

4.3.2.2 Second Subsection

4.3.3 Dynamic Orbit Decay Computation

4.3.3.1 Introduction

- Primera cosa
- Segunda cosa
- Tercera cosa

4.3.3.2 Second Subsection

4.4 Orbital Station-Keeping

We will study:

- Increased height
- Thrusters

4.4.1 Raising the orbit height to increase Lifetime

The key to understand this solution is to see from another point of view the atmospheric drag phenomena. Once we have designed the constellation to provide certain coverage to specific points of the globe, the action of increasing the height of the orbit has the effect of increasing the footprint area on the surface of the earth. As the constellation is set, the time that take the satellites to reach the design height is extra lifetime.
From this point of view, the atmospheric drag phenomena can be recomputed and plotted it in this new way:

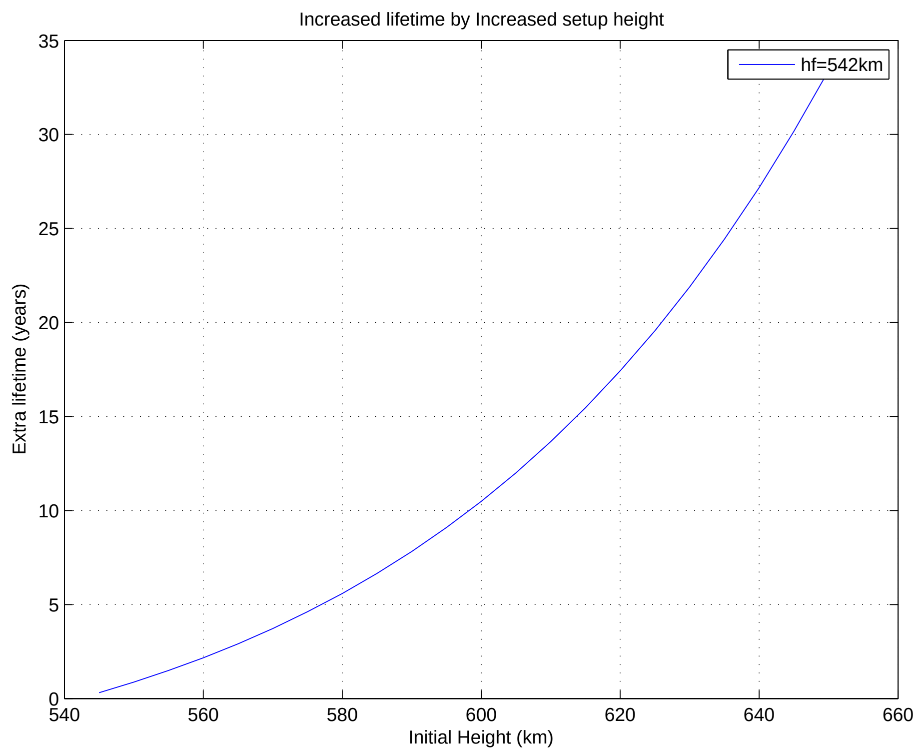

Figure 4.4.1: Increase in the Lifetime obtained by setting the constellation in a higher orbit

As it can be seen, the lifetime increases radically with time. However, this is a dangerous solution, since the coupling with another design parameters is compromised. To list the complications that can lead to:

- **Clients:** With the current technology, the satellites currently in orbit are set to point towards Earth. This means, if the contellation's satellites are at a higher orbit, the contact is impossible. As the market study reveals, it is important to place the satellites as low as possible.

- **Spacecraft Subsystems:** A higher orbit means a higher gain for the antennas and therefore an increase in the required power.
- **Constellation Reconfiguration:** The overall time to reconfigurate the constellation increases with height, since the period of the transition orbits is higher.

In conclusion
This tool is a very powerful option to deal with the orbit decay, even though it is not exactly an operation of Station Keeping itself. Given the high correlation it shows with another subsystems, the possibility of using it needs to be considered while the other design decisions are taken.

4.4.2 Using Thrusters to increase Lifetime

In order to maintain the configuration of the constellation for a longer time, a thruster is installed in each satellite to correct the decrease in altitude due to the orbit decay. The most optimal way to maintain the altitude is through a low-thrust maneuver. However, since this is a preliminar study, the calculations will be computed for a Hohmann transfer maneuver, which is simpler and more effective, but requires more propellant and greater increases of velocity. That is, by computing the velocity and propellant needed for a Hohmann maneuver, the results will be safe for a low-trust maneuver, because the late one requires less energy.

4.4.2.1 Energy equation

The deduction of the equations needed to solve the Hohmann maneuver begins with the energy equation:

$$\frac{V^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a} \tag{4.4.1}$$

where V is the orbital velocity of the satellite, r is the distance from the focus, a the semimajor axis of the orbit and μ the gravitational constant of the attracting body, in this case, the Earth. This expression shows that the total energy of the satellite equals the sum of its kinetic and potential energy (per mass unit).

This equation can be arranged to obtain the velocity of the satellite. In the case of a circular orbit, the radius is constant, and equal to the semimajor axis. Replacing $a = r$ in the energy equation and after some operations, the expression of the velocity of a circular orbit is obtained:

$$V_c = \sqrt{\frac{\mu}{r}} \tag{4.4.2}$$

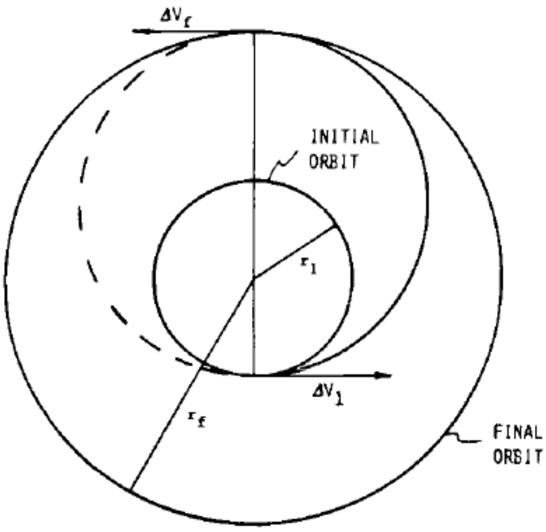

Figure 4.4.2: Hohmann transfer. Extracted from [?]

As it can be deduced from the energy equation, a change in orbital velocity leads to a change in the value of the semimajor axis. This property is used in satellites to change their orbit through a velocity increment ΔV. This process is called an orbital maneuver.

4.4.2.2 Delta-V

If the velocity increment ΔV is done instantaneously, the maneuver is called an impulsive maneuver. The Hohmann transfer is a two-impulse transfer between coplanar circular orbits. From an inicial circular orbit, a tangential velocity increment ΔV_1 is applied to change the orbit to an ellipse. This ellipse is the transfer orbit, in which the perigee radius is the radius of the initial circular orbit and the apogee radius equals the radius of the final circular orbit. When the satellite reaches the apogee, a second velocity increment ΔV_2 is applied, so that the satellite reaches the final circular orbit with the apogee radius. If this second velocity is not applied, the satellite will remain in the elliptic orbit.
With the energy equation defined above, it is easy to determine the velocity of the satellite in each orbit. The first orbit and the final ones are circular:

$$V_1 = \sqrt{\frac{\mu}{r_1}} \tag{4.4.3}$$

$$V_f = \sqrt{\frac{\mu}{r_f}} \tag{4.4.4}$$

The velocity in the transfer orbit can be easily calculated with the energy equation applying the definition of the semimajor axis of an ellipse:

$$a = \frac{r_1 + r_f}{2} \tag{4.4.5}$$

The velocities in the perigee and apogee are:

$$V_p = \sqrt{\frac{2\mu r_f}{r_1(r_1 + r_f)}} \tag{4.4.6}$$

$$V_a = \sqrt{\frac{2\mu r_1}{r_f(r_1 + r_f)}} \tag{4.4.7}$$

Therefore the velocity increments are:

$$\Delta V_1 = V_p - V_1 = \sqrt{\frac{2\mu r_f}{r_1(r_1 + r_f)}} - \sqrt{\frac{\mu}{r_1}} \tag{4.4.8}$$

$$\Delta V_2 = V_f - V_a = \sqrt{\frac{\mu}{r_f}} - \sqrt{\frac{2\mu r_1}{r_f(r_1 + r_f)}} \tag{4.4.9}$$

4.4.2.3 Time

It is also necessary to know the time needed to do the maneuver. This time is equal to half of the period of the transfer ellipse:

$$t = \frac{T}{2} = \frac{1}{2}\sqrt{\frac{4\pi^2 a^3}{\mu}} \tag{4.4.10}$$

4.4.2.4 Propellant

In order to know the mass of propellant needed in the maneuver, the Tsiolkovsky rocket equation is applied:

$$\Delta V = g_0 I_{sp} \ln \frac{m_1}{m_f} = g_0 I_{sp} \ln \frac{m_1}{m_1 - m_{prop}} \tag{4.4.11}$$

where $\Delta V = \Delta V_1 + \Delta V_2$ is the total velocity increment of the maneuver, g_0 is the Earth's gravity, I_{sp} is the specific impulse of the thruster used, m_1 is the initial mass of the satellite, m_f is its final mass and m_{prop} is the mass of propellant used in the maneuver.

$$m_{prop} = m_1 \left(1 - \exp\left(-\frac{\Delta V}{g_0 I_{sp}}\right)\right) \tag{4.4.12}$$

4.4.2.5 Orbit maintenance

As explained at the beggining of the section, the orbital maneuvers exposed are intented to maintain the altitude of the satellite for a longer time and, consequently, lengthen its life. The method proposed begins when the satellite is deployed at a given height. This height will decrease due to the orbit decay, reaching a critical value, the limit altitude in which the constellation provides global coverage or another given height. Once this critical altitude is achieved, the satellite is put once again at its initial height through a Hohmann maneuver. The process is repeated several times until the satellite runs out of propellant

or until it reaches its desired lifetime.
In reality the satellite will perform a low-thrust maneuver, which is more practical for an electric thruster. In this non-impulsive maneuvers, the thruster is constantly providing a velocity increment to the satellite, but it is so small that the whole transfer maneuver requires a lot of time. This means that it is not necessary to wait until the satellite reaches the critical altitude. The maneuver will start when the satellite is deployed or when it reaches a given altitude (higher than the critical altitude) so that it counteracts the effect of the orbital decay.

4.4.2.6 Results

The results are computed for a 3U CubeSat with an ion thruster. The characteristics of the thruster are the following ones (for more characteristics of the thruster refer to the section **??**.):

Thrust	100 μN
Specific Impulse	2150 s

The first parameters to be defined are the maximum and minimum height of the orbit, mesured from the surface of the Earth. The maximum height is the altitude at which the satellite is deployed, and minimum height is the altitude at which the Hohmann transfer maneuver is applied. The satellite has to be above the minimum height to be functional. Figure 4.4.3 is an example of the height variation of the satellite using the Hohmann maneuver to reach the maximum height once the satellite is in the minimum height. The results of this maneuver are:

Maximum height	545 km
Minimum height	542 km
Number of Hohmann Maneuvers	32
Maximum ΔV_1	0.8237 m/s
Maximum ΔV_2	0.8236 m/s
Total ΔV Budget	52.7116 m/s
Propellant mass	10 g
Lifetime of the satellite	33.3288 years

Since the thruster used is an ion thruster, the specific impulse is big, and the mass propellant is very low. In this case, the variation of height due to the orbit decay is approximately 3 km per year, so the thruster needs to do a Hohmann maneuver per year. With only 10 g of propellant, the lifetime of the satellite is over 30 years.
Figure 4.4.4 is another example of the Hohmann maneuver with the same amount of propellant but with a more restrictive range of operational heights, only 80 m. It should have the same shape as Figure 4.4.3, but since a lot of maneuvers are applied, the lines have overlapped. The characteristics of this maneuver are:

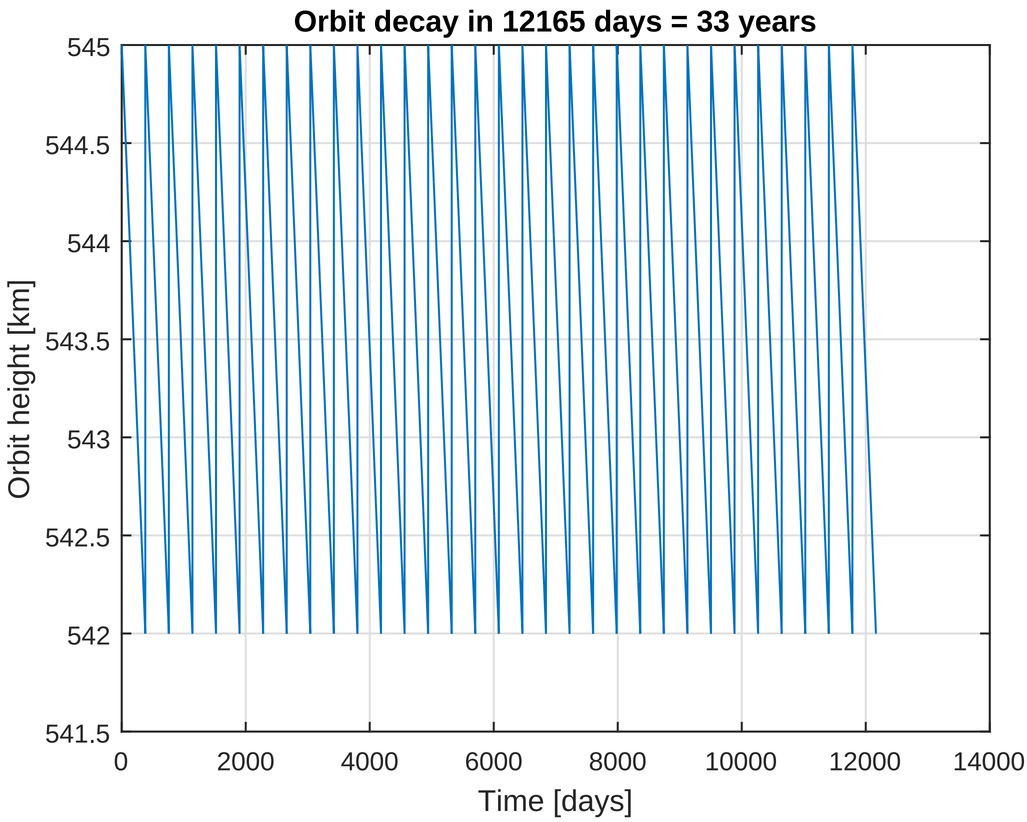

Figure 4.4.3: Height variation of the satellite

Maximum height	545 km
Minimum height	544.92 km
Number of Hohmann Maneuvers	1200
Maximum ΔV_1	0.0221 m/s
Maximum ΔV_2	0.0221 m/s
Total ΔV Budget	52.7570 m/s
Propellant mass	10 g
Lifetime of the satellite	34.5726 years

Comparing these results with the previous ones, it can be seen that with a more restrictive range of heights, the lifetime of the satellite is practically the same. The velocity increments are lower because the difference in the heights is extremely low, but at the same time, the satellite reaches before the minimum height and the maneuvers needed to maintain the satellite in this range are many more than on the other case. Since the ΔV budget is practically the same in both cases, it can be assured that the only difference between them is the number of maneuvers computed.

As mentioned earlier, the results obtained are for a Hohmann maneuver when in reality the satellite will compute a low-thrust maneuver, that requires less velocity increments

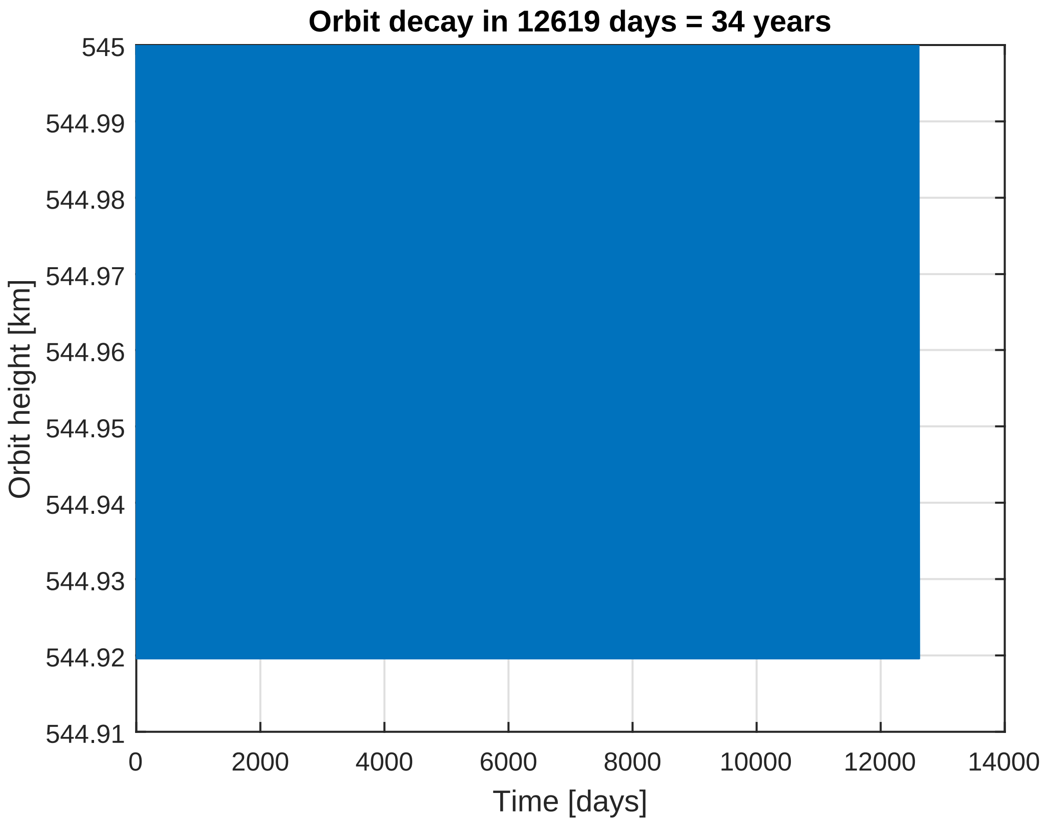

Figure 4.4.4: Height variation of the satellite with a more restrictive minimum height

and less propellant. In conclusion, taking into account these results, it can be stated that the lifetime of the satellite will not be determined by its orbit decay but for the failure of its systems or other external causes. It can also be assured that the satellite is capable of carrying enough propellant to maintain its altitude and to compute other maneuvers if necessary.