Orbital Propagation and Rapid Orbital Decay of a CubeSat

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Outline

1. Orbital Propagation
2. Rapid Orbital Decay
3. Conclusion

PART I - Orbital Propagation

Orbital Propagation

Objective of study

Develop an orbital propagator for a CubeSat's on-board computer(OBC).

Steps Involved

- Begin with the equation of motion for the two-body problem with perturbations

$$\bar{a} = -\frac{\mu}{r^3}\bar{r} + \bar{a}_{oblate} + \bar{a}_{drag} + \bar{a}_{3-body} + \bar{a}_{SRP} \qquad (1)$$

- Use the special perturbation technique called the Cowell's Method represented by:

$$\dot{\mathbf{r}} = \bar{v}$$
$$\dot{\mathbf{v}} = -\frac{\mu}{r^3}\bar{r} + \bar{a}_p \qquad (2)$$

Orbital Propagation

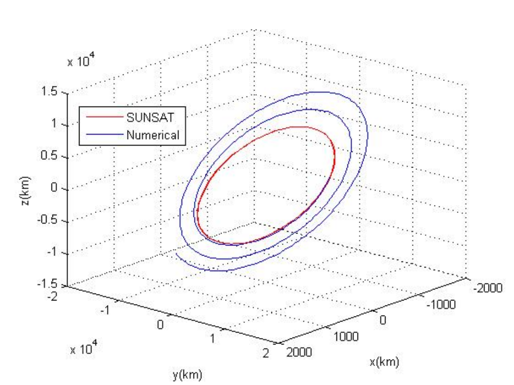
- Numerically integrate eq.(2) using the Runge-Kutta-Fehlberg method:

$$
\begin{aligned}
k1 &= f(t_n, y_n) \\
k2 &= f(t_n + c_2 h, y_n + a_{21} k1) \\
k3 &= f(t_n + c_3 h, y_n + a_{31} k1 + a_{32} k2) \\
&. \\
k_s &= f(t_n + c_s h, y_n + a_{s1} k1 + a_{s2} k2 + \ldots + a_{s,s-1} k_{s-1})
\end{aligned}
$$

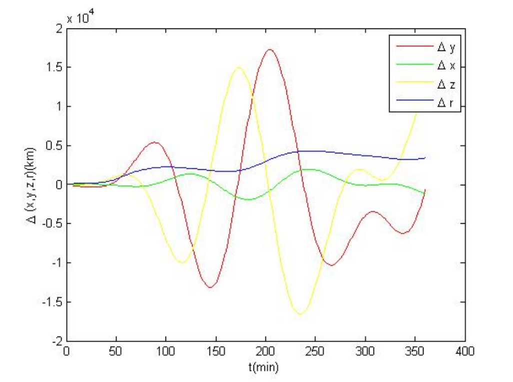
$$
y_{n+1} = y_n + h \sum_{i=1}^{s} b_i k_i
$$

- Use C++ to develop the propagator.
- CubeSat's OBC will run Linux FreeRTOs(Free real-time operating system)

Two-body Orbit vs SLR-Measured Orbit of SUNSAT



Error in position vector and radius between SLR-measured(SUNSAT) and Numerical Method(Two-Body)



Orbital Propagation

Earth Oblateness

- The Earth is not spherically symmetric.
- Has equatorial bulge, a slight pear shape and a flattening at the poles.
- This oblateness results in unsymmetrical gravity potential which is given by:

$$\Phi = \frac{\mu}{r}\left[1 + \sum_{n=2}^{\infty}\sum_{m=0}^{n}\left(\frac{R_e}{r}\right)^n P_{nm}[sin\phi_{sat}]C_{nm}cosm\lambda_{sat} + S_{nm}sinm\lambda_{sat}\right]$$

- To get the equation which describes the acceleration of a satellite due to this oblateness the gradient of the potential function has to be found.

$$\bar{a}_{aspherical} = \nabla\Phi$$

Orbital Propagation

Earth's Oblateness

$$\mathbf{a}_I = \left\{ \frac{1}{r}\frac{\partial \Phi}{\partial r} - \frac{r_K}{r^2\sqrt{r_I^2 + r_J^2}}\frac{\partial \Phi}{\partial \phi_{sat}} \right\} r_I - \left\{ \frac{1}{r_I^2 + r_J^2}\frac{\partial \Phi}{\partial \lambda_{sat}} \right\} r_J$$

$$\mathbf{a}_J = \left\{ \frac{1}{r}\frac{\partial \Phi}{\partial r} - \frac{r_K}{r^2\sqrt{r_I^2 + r_J^2}}\frac{\partial \Phi}{\partial \phi_{sat}} \right\} r_J + \left\{ \frac{1}{r_I^2 + r_J^2}\frac{\partial \Phi}{\partial \lambda_{sat}} \right\} r_I$$

$$\mathbf{a}_K = \frac{1}{r}\frac{\partial \Phi}{\partial r} r_K + \frac{\sqrt{r_I^2 + r_J^2}}{r^2}\frac{\partial \Phi}{\partial \lambda_{sat}} \tag{3}$$

- Pertubation due to Earth oblateness is the dominant of all perturbations.

Orbital Propagation

Three-body

- Result from another body other than the main attracting body exerting gravity on a satellite.
- From the N-Body problem we find that the acceleration experienced by a satellite due to the presence of a third attracting body is given by:

$$\mathbf{a}_{3-body} = -\frac{Gm_{\oplus}\mathbf{r}_{\oplus sat}}{r^3_{\oplus sat}} + Gm_3\left(\frac{\mathbf{r}_{sat3}}{r^3_{sat3}} - \frac{\mathbf{r}_{\oplus 3}}{r^3_{\oplus 3}}\right) \quad (4)$$

Orbital Propagation

Atmospheric Drag

- Atmospheric forces represent the largest non-gravitational pertubations acting on LEO satellites.
- Its actually the main cause of LEO satellites falling back to the Earth and the most dominant force during the final stages of the satellite.

$$\mathbf{a}_{drag} = -\frac{1}{2}\frac{C_D A}{m}\rho r_{rel}^2 \frac{\dot{\mathbf{r}}_{rel}}{|\dot{\mathbf{r}}_{rel}|}$$

- 1976 Standard Atmosphere method will be used to determine the atmospheric density(ρ) in time.

Orbital Propagation

Solar Radiation Pressure

- Non-conservative force exerted on a satellite by the momentum flux from the sun.
- Its effects exceed that of drag above 600KM.
- Represented by:

$$\mathbf{a}_{SR} = -\frac{P_{SR} c_R A_{\odot}}{m} \frac{\mathbf{r}_{\odot sat}}{|\mathbf{r}_{\odot sat}|} \tag{5}$$

Part II - Rapid Orbital Decay

M. Afful

Rapid Orbital Decay

Study Objectives

- Evaluate the semi-analytic Liu theory(SALT) for predicting satellite orbital lifetime by comparing predicted orbital element evolution with observed Two Line Element(TLE) derived orbital elements.
- Investigate the effect of a de-orbit sail on satellites lifetime to illustrate the technology as a a possible space debris mitigation mechanism.

Rapid Orbital Decay

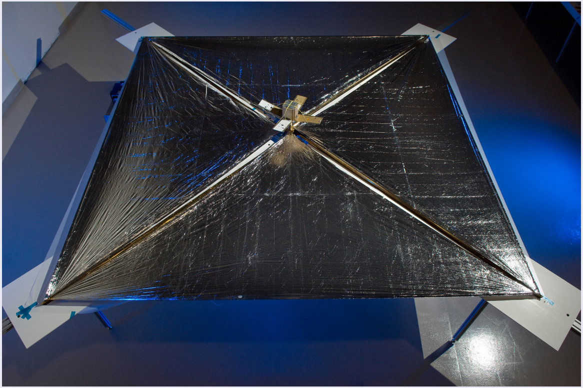


Figure: NanoSail-D with a de-orbit sail

Picture by: nasa.gov

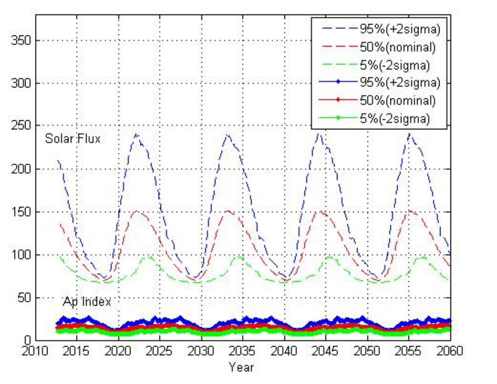
Rapid Orbital Decay

Liu Theory

- LEO satellite orbit decay results from the pertubative influence of Earth Oblateness and atmospheric drag.
- The Earth oblateness influence on orbital elements can be modelled analytically.
- Though atmospheric drag is well understood, uncertainties in atmospheric density values for long-term prediction arise.
- Refer to long-term predicted F10.7cm and Ap magnetic index values used for calculating atmospheric density.

Rapid Orbital Decay

Predicted F10.7 solar flux and Ap magnetic index



Ref: *http : //solarscience.msfc.nasa.gov/images/ssn_predict.txt*

Rapid Orbital Decay

Orbital elements variation due to Earth oblateness

$$
\begin{aligned}
(\dot{e}_m)_{ob} = &-\frac{3}{2} n J_2^2 \left(\frac{R_e}{p}\right)^4 \sin^2 i \,.\, (14 - 15\sin^2 i) \,.\, e \,.\, (1 - e^2) \,.\, \sin 2\omega \\
&-\frac{3}{8} n J_3 \left(\frac{R_e}{p}\right)^3 \sin i \,.\, (4 - 5\sin^2 i) \,.\, (1 - e) \,.\, \cos\omega \\
&-\frac{15}{32} n J_4 \left(\frac{R_e}{p}\right)^4 \sin^2 i \,.\, (6 - 7\sin^2 i) \,.\, e \,.\, (1 - e^2) \,.\, \sin 2\omega
\end{aligned}
$$

$$
\begin{aligned}
(\dot{i}_m)_{ob} = &\frac{3}{64} n J_2^2 \left(\frac{R_e}{p}\right)^4 \sin 2i \,.\, (14 - 15\sin^2 i) \,.\, e^2 \,.\, \sin 2\omega \\
&+\frac{3}{8} n J_3 \left(\frac{R_e}{p}\right)^3 \cos i \,.\, (4 - 5\sin^2 i) \,.\, e \,.\, \cos\omega \\
&+\frac{15}{64} n J_4 \left(\frac{R_e}{p}\right)^4 \sin 2i \,.\, (6 - 7\sin^2 i) \,.\, e^2 \,.\, \sin 2\omega
\end{aligned}
$$

The values of J2, J3 and J4 as used in SALT are 1.0826310-3; 2.54e-6 and -1.58 e-6 respectively.

Rapid Orbital Decay

Orbital elements variation due to Earth oblateness

$$
\begin{aligned}
(\dot{\omega}_m)_{ob} = {} & \frac{3}{4} n J_2 \left(\frac{R_e}{p}\right)^2 \left(4 - 5\sin^2 i\right) \\
& + \frac{3}{4} n J_2^2 \left(\frac{R_e}{p}\right)^4 \left[12 - \frac{103}{4}\sin^2 i + \frac{215}{16}\sin^4 i + \left(\frac{7}{4} - \frac{9}{8}\sin^2 i - \frac{45}{32}\sin^4 i\right) . e^2 \right. \\
& \qquad \left. + \frac{3}{2}\left(1 - \frac{3}{2}\sin^2 i\right) . \left(4 - 5\sin^2 i\right)\sqrt{1-e^2}\right] \\
& - \frac{15}{32} n J_4 \left(\frac{R_e}{p}\right)^4 \left[\left(16 - 62\sin^2 i + 49\sin^4 i\right) + \frac{3}{4}\left(24 - 84\sin^2 i + 63\sin^4 i\right) . e^2\right] \\
& + \frac{3}{64} n J_2^2 \left(\frac{R_e}{p}\right)^4 \left[-2\left(14 - 15\sin^2 i\right)\sin^2 i + \left(28 - 158\sin^2 i + 135\sin^4 i\right) . e^2\right] \cos 2\omega \\
& + \frac{3}{8} n J_3 \left(\frac{R_e}{p}\right)^3 \frac{1}{e\sin i}\left[\left(4 - 5\sin^2 i\right)\left(\sin^2 i - e^2\cos^2 i\right) + 2\sin^2 i\left(13 - 15\sin^2 i\right) . e^2\right] \sin\omega \\
& - \frac{6}{32} n J_4 \left(\frac{R_e}{p}\right)^4 \left[3\sin^2 i\left(6 - 7\sin^2 i\right) + \frac{1}{2}\left(-36 + 210\sin^2 i - 189\sin^4 i\right) . e^2\right] \cos 2\omega
\end{aligned}
$$

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Rapid Orbital Decay

Orbital elements variation due to atmospheric drag

$$(\dot{a}_m)_d = -\frac{1}{2\pi}\int_0^{2\pi} B\rho V \frac{a}{1-e^2}\left[1+e^2+2e\cos\nu - \omega_a \cos i\sqrt{\frac{a^3\left(1-e^2\right)^3}{\mu}}\right] dM$$

$$(\dot{e}_m)_d = -\frac{1}{2\pi}\int_0^{2\pi} B\rho V\left\{e+\cos\nu - \frac{\omega_a}{2}\frac{r^2\cos i}{\mu a(1-e^2)}\left[2(e+\cos\nu) - e\sin^2\nu\right]\right\} dM$$

$B =$ Inverse ballistic coefficient $= \frac{c_D A}{m_s}$, m_s is the satellite mass.

$\rho =$ Atmospheric density.

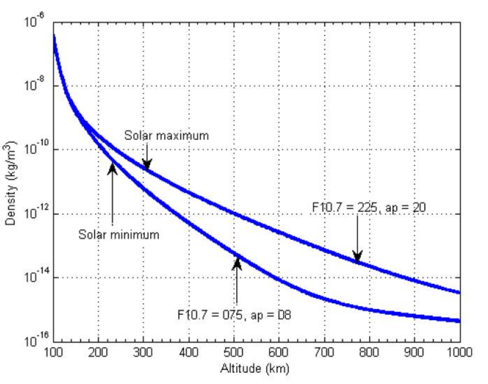
$\omega_a =$ Earth rotational speed.

$V =$ Magnitude of satellite's velocity, explicitly given as

$$V = \sqrt{\frac{\mu}{p}}\left(1+e^2+2e\cos\nu\right)^{\frac{1}{2}}\left[1 - \frac{(1-e^2)^{\frac{3}{2}}}{1+e^2+2e\cos\nu}\frac{\omega_a}{n}\cos i\right]$$

Rapid Orbital Decay

Atmospheric Density

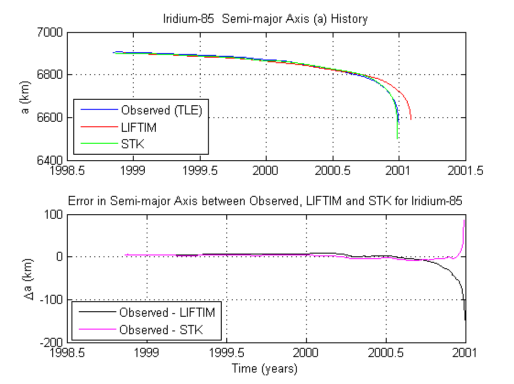


Rapid Orbital Decay

- The SALT as implemented in LITFIM utilizes the simplified Jacchia 70 model to calculate the atmospheric density.
- The model provides for density variations due to solar activity and the semi-annual variation.
- Also used Satellite Toolkit (STKs) lifetime estimation program.

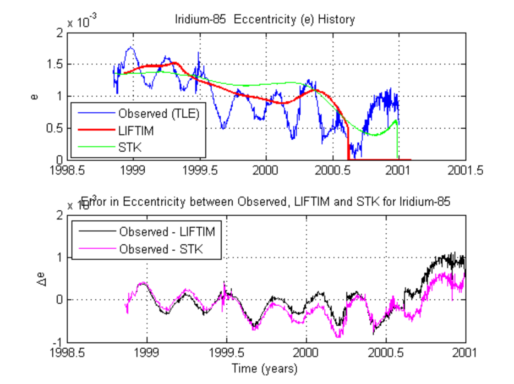
Rapid Orbit Decay

Variation in the Semi-major axis of Iridium-85 from launch to decay



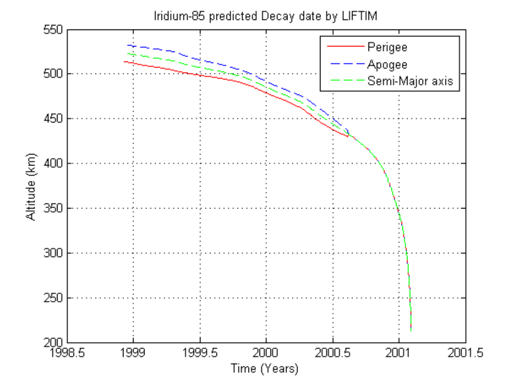
Rapid Orbit Decay

Variation in Eccentricity of Iridium-85 satellite from launch to decay date



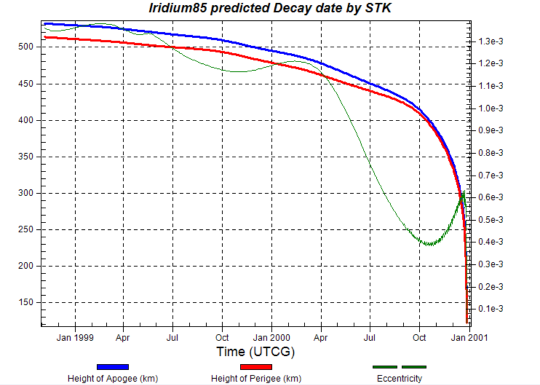
Rapid Orbit Decay

Iridium-85 Predicted Decay Date by LIFTIM



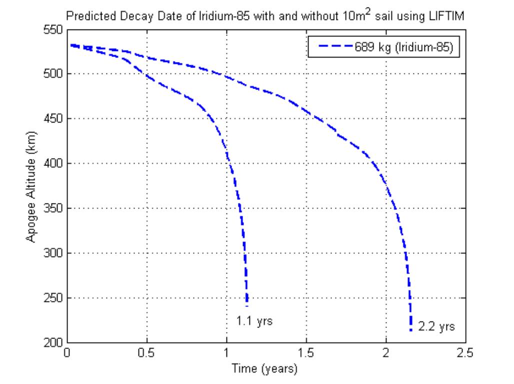
Rapid Orbit Decay

Iridium-85 Predicted Decay Date by STK



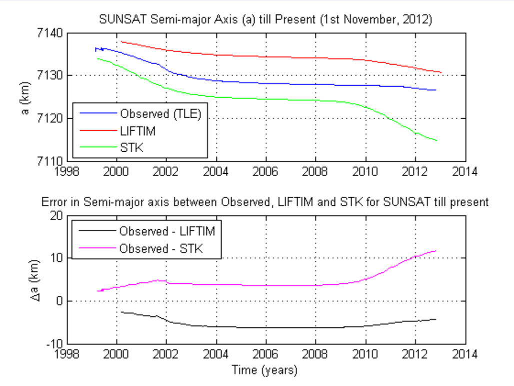
Rapid Orbit Decay

Predicted Decay Date of Iridium-85 with and without $10m^2$ sail using LIFTIM



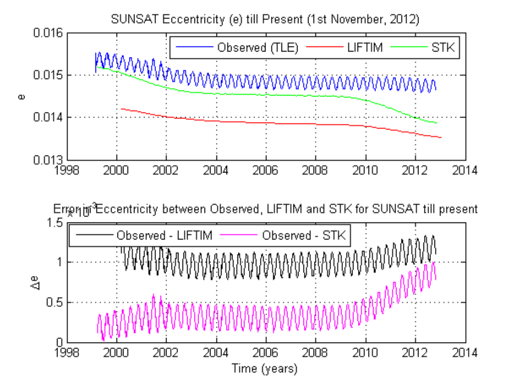
Rapid Orbit Decay

Variation in the Semi-major axis of SUNSAT from launch till Present (23 Feb 1999 - Nov 2012)



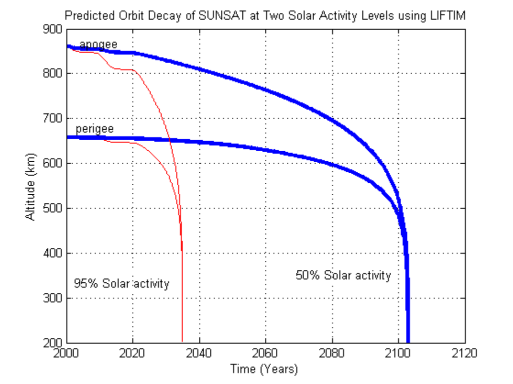
Rapid Orbit Decay

Variation in Eccentricity of SUNSAT from launch date till present (23 Feb 1999 - Nov 2012)



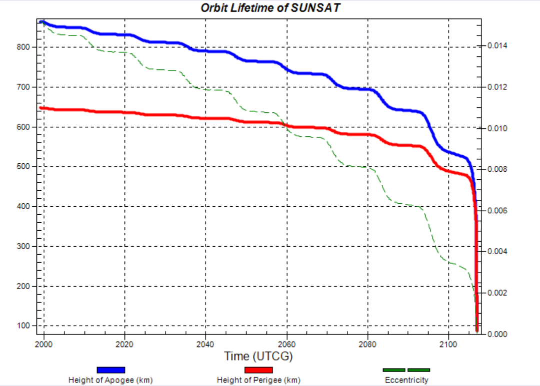
Rapid Orbit Decay

Predicted Orbit Decay of SUNSAT at Two Solar Activity Levels Using LIFTIM



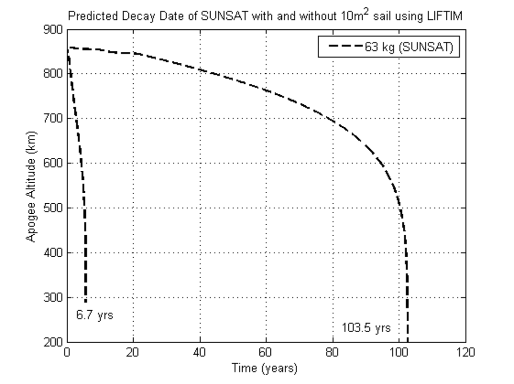
Rapid Orbit Decay

Orbit Lifetime of SUNSAT



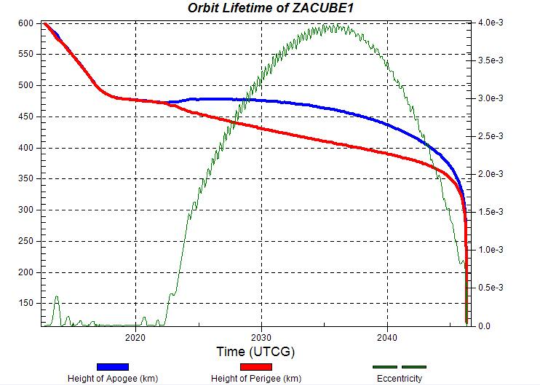
Rapid Orbit Decay

Predicted Decay Date of SUNSAT with and without $10m^2$ sail using LIFTIM



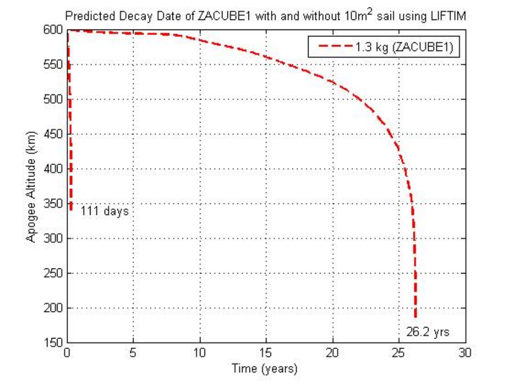
Rapid Orbit Decay

Orbit lifetime of ZACUBE1



Rapid Orbit Decay

Predicted decay date of ZACUBE1 with and without $10m^2$ sail using LIFTIM



Conclusion

- LEO Orbital propagator is still underway.
- Its hard to make advanced prediction of the atmospheric density.
- The deorbit sail decreases the lifetime of a LEO satellite rapidly.
- The deorbit sail is a promising technology to reduce space debris.

Thank You
Ndo Livhuwa
Gracious
Děkujeme
Danke