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Cubesat Constellation
Astrea

**Contents:** ANNEX VI: Matlab Codes

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# 1 Matlab Codes

## 1.1 Satellite Footprint

```
%----ASTREA CONSTELLATION----
%PROJECTS - 220028
%Aerospace Engineering Barchelor's Degree
%ESEIAAT - UPC
%Autumn 2016-2017

% ORBIT DESIGN TEAM
% COMPUTATION OF A SATELLITE FOOTPRINT

clc
clear all
close all

%% PHISICAL CONSTANTS AND PARAMETERS

Re = 6378;            %Earth's Radius [Km]
Se = 4*pi*Re^2;       %Earth's Surface [Km^2]
h = 500:100:1000;     %Satellte height [Km]
eo = 0:5:40;          %Elevation angle [deg]

%% SOLVER

for i = 1:length(h)
    for j = 1:length(eo)

        d(i,j)= Re*(sqrt(((h(i)+Re)/Re)^2-(cosd(eo(j)))^2 )-sind(eo(j)));
        Bo(i,j) = asind( d(i,j)*cosd(eo(j)) / (h(i)+Re) );

        S(i,j) = 2 * pi * Re^2 * ( 1-cosd(Bo(i,j)) );

        R(i,j) = sqrt( d(i,j)^2 - h(i)^2 );

        %Satellite coverage expressed as a fraction of Earth's Area (%)
```

```
        Cov(i,j) = ( S(i,j) / Se )*100;

        %Number of satellites needed
        nsat(i,j) = ( Se / S(i,j) );

    end
end

%% PLOTS

figure
for k = 1:length(h)
    plot ( eo , Cov(k,:) )
    hold on
end

title('Coverage vs Elevation')
xlabel('Elevation [deg]') % x-axis label
ylabel('Coverage [%]') % y-axis label
legend(num2str(h(1)),num2str(h(2)),num2str(h(3)),...
    num2str(h(4)),num2str(h(5)),num2str(h(6)))

figure
for k = 1:length(h)
    plot ( eo , nsat(k,:) )
    legend ('h(j)')
    hold on
end

title('Num.Satellites vs Elevation')
xlabel('Elevation [deg]') % x-axis label
ylabel('Num.Satellites') % y-axis label
legend(num2str(h(1)),num2str(h(2)),num2str(h(3)),...
    num2str(h(4)),num2str(h(5)),num2str(h(6)))

figure
for k = 1:length(h)
    plot ( eo , 2*Bo(k,:) )
    hold on
end

title('Angle vs Elevation')
xlabel('Elevation [deg]') % x-axis label
ylabel('Angle between satellites[deg]') % y-axis label
legend(num2str(h(1)),num2str(h(2)),num2str(h(3)),...
    num2str(h(4)),num2str(h(5)),num2str(h(6)))
```

## 1.2 Minimum Plane Inclination

```
%----ASTREA CONSTELLATION----
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%ESEIAAT - UPC
%Autumn 2016-2017

% ORBIT DESIGN TEAM
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% MINIMUM INCLINATION - To provide Full coverage %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%PROBLEM:
% Given: - Minimum Elevation over the horizon to contact the satellite
%         - Height of the orbit
% Compute the minimum elevation to ensure visual contact
clear; clc;

%% Key variable
min_elevation_pole=5; %Initially set by @lfore as 32;

%% Previous Calculation: Visibility(latitude)

R = 6371;
N = 180;
deg = 180;
x = linspace(0,90,N);

%%%% Minimum angle of elevation due to atmospheric conditions at given
%%%% latitudes. 0 km represents the poles whereas 6371 km represents the
%%%% Earth's equator taking as reference point the south pole.

elvlat = zeros (1,deg);
for i = 1:deg/5
    elvlat(i) = min_elevation_pole-(min_elevation_pole-15)/(2*(90-75))*i;
end
for i = deg/5+1:4*deg/10
    elvlat(i) = elvlat(deg/5);
end
for i = 4*deg/10+1:6*deg/10
    elvlat(i) = elvlat(2*deg/5)+3/15*(i-4*deg/10);
end
for i = 6*deg/10+1:8*deg/10
    elvlat(i) = elvlat(6*deg/10)-3/15*(i-6*deg/10);
end
for i = 8*deg/10+1:10*deg/10
    elvlat(i) = elvlat(8*deg/10)-1/15*(i-8*deg/10);
```

```
end
elvlat=fliplr(elvlat); %Sets the first value to the value in the equator
                        %The first value matches 0 latitude

%% Minimum i computation

h = 400:100:1000;    %Satellte height [Km]
L = 0:5:90;          %Latitude angle [deg]
e = (interp1q(x',elvlat',L'))'; %Minimum elevations interpolation [deg]

figure(1)
subplot(1,2,1)
plot(x,elvlat)
subplot(1,2,2)
title('Interpolation Verification')
plot(L,e)

inc=zeros(length(h),length(L)); %Inclinations preallocation

for i = 1:length(h)
    for j = 1:length(L)

        A=cosd(e(j))/(1+h(i)/R);
        theta=acosd(A)-e(j); %Earth Central Angle
        inc(i,j)=L(j)-theta;
        if inc(i,j)<0
            inc(i,j)=0;
        end
    end
end

%% 3. Results Plotting

figure(2)
colors=['r','b','y','g','m','k','c'];
for k = 1:length(h)
    hold on
    plot ( L , inc(k,:),colors(k))
end

grid on
title('Minimum inclination required to fulfill global coverage')
xlabel('Latitude [deg]')
ylabel('Inclination [deg]')
legend(num2str(h(1)),num2str(h(2)),num2str(h(3)),...
    num2str(h(4)),num2str(h(5)),num2str(h(6)),num2str(h(7)))

%% Bonus!!!

L=90;
```

```
e=5.33;
h=500:1:600;
inc=zeros(1,length(h));
for i = 1:length(h)
        A=cosd(e)/(1+h(i)/R);
        theta=acosd(A)-e; %Earth Central Angle
        inc(i)=L-theta;
        if inc(i)<0
            inc(i)=0;
        end
end

figure(3)
plot(h,inc)
xlabel('Heights')
ylabel('Inclination')
grid on
axis([min(h) max(h) 45 90])
```

## 1.3 Satellite Number Computation for Polar Orbits

```
%----ASTREA CONSTELLATION----
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%Autumn 2016-2017

% ORBIT DESIGN TEAM
% POLAR ORBITS Number of satellites computation

%PROBLEM:
% Given: - Elevation over the horizon to contact the satellite
%        - Height of the orbit
% Compute the final number of satellites
% Through the "Streets of coverage" method.

clear; clc;

%% 1. Input data
% PHISICAL CONSTANTS AND PARAMETERS

Re = 6378;          %Earth's Radius [Km]
Se = 4*pi*Re^2;     %Earth's Surface [Km^2]
h = 500:20:600;   %Satellte height [Km]
eo = 20;        %Elevation angle [deg]

```

```
%% 2. Number of satellites computation

d=zeros(length(h),length(eo));
nsats_opt=d;

for i = 1:length(h)
    for j = 1:length(eo)

        d(i,j)= Re*(sqrt(((h(i)+Re)/Re)^2-(cosd(eo(j)))^2)-sind(eo(j)));
        Bo = asind( d(i,j)*cosd(eo(j)) / (h(i)+Re) );
        Sup = 2 * pi * Re^2 * ( 1-cosd(Bo) );

        Nspp_min=ceil(360/(2*Bo));          %Number of satellites per plain
        Nsatspp=Nspp_min:1:Nspp_min+50;     %Array of sats/plain to optimize

        S=360./Nsatspp;

        Lstreet=acosd(cosd(Bo)./cosd(S/2));

        DmaxS=Lstreet+Bo;
        DmaxO=2*Lstreet;

        Nplains=ceil(1+((180-DmaxO)./DmaxS));
        Nsats=Nplains.*Nsatspp;
        [row,col]=find(min(Nplains));
        Nplains_opt(i,j)=Nplains(row,col);
        nsats_opt(i,j)=Nsats(row,col);

        %Number of satellites needed
        nsat(i,j) = ( Se / Sup );

    end
end

%% 3. Results Plotting

figure(2)
colors=['r','b','y','g','m','k','c'];
for k = 1:length(eo)
    subplot(1,2,1)
    hold on
    semilogy ( h , nsats_opt(:,k),colors(k))
    subplot(1,2,2)
    hold on
    semilogy ( h , Nplains_opt(:,k),colors(k))
end

subplot(1,2,1)
title('Num.Sats vs Height')
xlabel('Height [km]') % x-axis label
```

```
ylabel('Num.Satellites') % y-axis label
%legend(num2str(eo(1)),num2str(eo(2)),num2str(eo(3)),...
%    num2str(eo(4)),num2str(eo(5)),num2str(eo(6)),num2str(eo(7)))
axis([400 900 0 1000])
grid on

subplot(1,2,2)
title('Num.Planes vs Height')
xlabel('Height [km]') % x-axis label
ylabel('Num of Orbital Planes') % y-axis label
% legend(num2str(eo(1)),num2str(eo(2)),num2str(eo(3)),...
%     num2str(eo(4)),num2str(eo(5)),num2str(eo(6)),num2str(eo(7)))
axis([400 900 0 30])
grid on

%% Single detailed analysis

figure(1)
plot(h,nsats_opt)
title('Num.Sats vs Height - Streets of coverage Method')
xlabel('Height [km]') % x-axis label
ylabel('Num.Satellites') % y-axis label
grid on

%% Multianalysis variation with height

figure(3)
colors=['r','b','y','g','m','k'];
for k = 1:length(h)
    subplot(1,2,1)
    hold on
    plot ( eo , nsats_opt(k,:),colors(k))
    subplot(1,2,2)
    hold on
    plot ( eo , Nplains_opt(k,:),colors(k))
end

subplot(1,2,1)
title('Num.Sats vs Elevation - Streets of coverage Method')
xlabel('Elevation [deg]') % x-axis label
ylabel('Num.Satellites') % y-axis label
legend(num2str(h(1)),num2str(h(2)),num2str(h(3)),...
    num2str(h(4)),num2str(h(5)),num2str(h(6)))
axis([0 40 0 700])
grid on

subplot(1,2,2)
title('Num.Planes vs Elevation - Surfaces Method')
xlabel('Elevation [deg]') % x-axis label
ylabel('Num.Orbital Planes') % y-axis label
```

```
legend(num2str(h(1)),num2str(h(2)),num2str(h(3)),...
    num2str(h(4)),num2str(h(5)),num2str(h(6)))
axis([0 40 0 30])
grid on
```

## 1.4 Orbit Parameters

```
%----ASTREA CONSTELLATION----
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%ESEIAAT - UPC
%Autumn 2016-2017

% ORBIT DESIGN TEAM
% ORBIT PARAMETERS

% This routine calculates the minimum number of plans and satellites
% to guarantee global coverage.
% ¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬
% _v2 --> Computes angles with arccosine
% _v3 --> Also iterates in f

clc
clear all
close all

tic

%% DATA INPUT

Re = 6371;              %Earth's Radius [Km]
Se = 4*pi*Re^2;         %Earth's Surface [Km^2]
h = 500:5:600;          %Height vector [km]
in = 75:3:90;           %Inclination vector [deg]
pmin = 5; p=12;         %Minimum number of plans
pmax = 20;              %Maximum number of plans
sppmin = 10; spp=10;    %Minimum number of satellites each plane
sppmax = 24;            %Maximum number of satellites each plane
typeWD = 2;             %Defines:
                        % 1- Semi-Walker-Delta
                        % 2- 3/4 of Walker-Delta
                        % 3- Full Walker-Delta
plaunch = 5.76e6;       %Launch price
psat = 250e3;           %Price per satellite
eo = 20;                %Vison angle
q=0;                    %Variable used as counter
```

```
nplanes_win=0;
%Boolean, Indicates a succesful combination with that number of planes

%% COMPUTATIONS
%result=zeros(6,100);

for j=1:length(h)
    fprintf('\n\nH = %g km',h(j))
    Bo = visibleangle(h(j),eo);  %Footprint angle

    for k=1:length(in)
        fprintf('\ni=%g:  ',in(k))

        for p=pmin:pmax
            F=0:1:p-1; nF=length(F); anglesp_f=zeros(1,nF);

            for spp=sppmin:sppmax
                angles = 360/spp; %Angle between satellites of same plane

              for kf=1:nF
               % We look for the optimum f for this spp,npp,inc,h
               anglesp_f(kf)=distance_opt_f(in(k),h(j),p,spp,typeWD,F(kf));
               %Angle between satellites of different planes
              end

                [anglesp,kf_min]=min(anglesp_f);

                 if (angles ≤ 2*Bo) && (anglesp ≤ Bo)

                    q=q+1;
                    result(:,q)=[h(j) in(k) spp p spp*p F(kf_min)];
                    fprintf(['Solution %g: h=%gkm - i=%g - '...
                       '%gspp - %gplanes - %g sats - f=%g'],q,result(:,q))
                    nplanes_win=1;
                    break %Stop iteration in satellites per plain

                 end

            end
            if nplanes_win     %If a combination with that num of planes
                nplanes_win=0; %works, then jump to the next inclination
                break
            end
        end
    end
end

%% Optimum Results
% Minimum number of cubesats
numsat = result (5,1:q);
```

```
[minsat,index] = min(numsat);

disp(sprintf('\nMINIMUM SATELLITES\nheight: %d', result(1,index)))
disp(sprintf('inclination: %d',result(2,index)))
disp(sprintf('number of satellites in each plane: %d',result(3,index)))
disp(sprintf('number of planes: %d',result(4,index)))
disp(sprintf('minimum number of satellites: %d',result(5,index)))

% Minimum number of planes
numplane = result(4,1:q);
[minplane,index]= min(numplane);

disp(sprintf('\nMINIMUM PLANES\nheight: %d', result(1,index)))
disp(sprintf('inclination: %d',result(2,index)))
disp(sprintf('number of satellites in each plane: %d',result(3,index)))
disp(sprintf('number of planes: %d',result(4,index)))
disp(sprintf('minimum number of satellites: %d',result(5,index)))

% Price Optimization
TCost = plaunch*result(4,:) + psat*result(5,:); TCost = TCost/(1e6);
[CostOpt,index] = min(TCost);
height = result(1,:);

disp(sprintf('\nMINIMUM PRICE\nprice[M]: %d', TCost(index)))
disp(sprintf('height: %d', result(1,index)))
disp(sprintf('inclination: %d',result(2,index)))
disp(sprintf('number of satellites in each plane: %d',result(3,index)))
disp(sprintf('number of planes: %d',result(4,index)))
disp(sprintf('minimum number of satellites: %d',result(5,index)))

fprintf('\n\n MORE THAN ONE SOLUTION COULD HAVE THE SAME MIN NUMBER\n')

%% Plots

x=0:q-1;
plot(x,numplane,x,numsat,x,TCost,x,height)
strValues = strtrim(cellstr(num2str([x(:) numplane(:)],'(%d, %d)')));
text(x,numplane,strValues,'VerticalAlignment','bottom');
strValues = strtrim(cellstr(num2str([x(:) numsat(:)],'(%d, %d)')));
text(x,numsat,strValues,'VerticalAlignment','bottom');
strValues = strtrim(cellstr(num2str([x(:) TCost(:)],'(%d, %.1f)')));
text(x,TCost,strValues,'VerticalAlignment','bottom');
strValues = strtrim(cellstr(num2str([x(:) height(:)],'(%d, %d)')));
text(x,height,strValues,'VerticalAlignment','bottom');
legend('number of planes','number of satellites','Total Cost (M)')




toc
```

```
function [Bo] = visibleangle (h,eo)

d=zeros(length(h),length(eo));
nsats_opt=d;
Re = 6371;

for i = 1:length(h)
    for j = 1:length(eo)

        d(i,j)= Re * ( sqrt( ( (h(i)+Re)/Re )^2 - ( cosd(eo(j)) )^2 ) - sind(eo(j)) );
        Bo = asind( d(i,j)*cosd(eo(j)) / (h(i)+Re) );

    end
end

end
```

```
function [anglesp] = distance_opt_f (i,a,p,s,typeWD,F)
%% Data input

% i: Inclination angle [deg]
% a: Height of the orbit [km]
% p: Number of planes
% s: Number of satelites per plane

%% Walker Delta type of geometry
WD=[180 225 360];
degreegenerate = WD(typeWD);
% Walker Delta 360 deg, SemiWalker Delta 180 deg,
% other constellations range between 180 and 360

m = degreegenerate/180;
% generates Walker Delta Constellation
%  (m=2 -> 2*180 generated constellation),
% Semi Walker (m=1 -> 1*180) or other 1<m<2.

% Phasing between adjacent planes
%f = F*p*cosd(i); % parameter defined graphically
 f = F; % Whereas F is a number 0 < f < (p-1)
        % According to Astrodynamics notes

REarth = 6371; % [km]
h = (REarth+a)/REarth; %radius of the orbit
%(in terms of the sphere radius) R=h*r;

%% Distribution of Coordinates of the satellites
Omega = zeros(1,p);
nu = zeros(s,p);
```

```
angle = zeros(1,p);
X = zeros(3,s,p);

for j = 1:p
    Omega(j) = m*pi*(j-1)/p;
    angle(j) = f*2*pi*(j-1)/(s*p); %Phasing due to f between planes
    for k = 1:s
        % True anomalies of the s satellites
        nu(k,j) = 2*pi*(k-1)/s+angle(j);
        X(:,k,j) = cartesian(h,0,i,Omega(j),0,nu(k,j));
        % h[adim] --> X[adim]
    end
end

%% Fast comprovation
% Does the adjacent phasing exceed pi
% THIS MEANS
% The minimum might be between


%% Angle between different satellites

c=zeros(s,p); %General aproach. Where is the minimum angle?)

for j=2:p
  for i=1:s-1

    % We need to assess the angles between the last and first plane
    % Specially in Semi-Walker configurations. That's why we add this
    % auxiliar variables.

      p1=j;
      if j==1
          p2=p;
      else
          p2=j-1;
      end

    % Angles a2 and a3 could be used to increase de reliability of the
    % constellation, allowing greater possibilities of communication
    % between planes. (not only between (i,p1) and (i,p2)

    % Angle between satellite (i,p1) and satellite (i,p2)
    a1=acosd((X(:,i,p1)'*X(:,i,p2))/(norm(X(:,i,p1))*norm(X(:,i,p2))));

    % Angle between satellite (i,j) and satellite (i+1,j-1)
    a2=acosd((X(:,i,p1)'*X(:,i+1,p2))/(norm(X(:,i,p1))*norm(X(:,i+1,p2))));

    % Angle between satellite (i,j) and satellite (i-1,j-1)
    a3=acosd((X(:,i,p1)'*X(:,i-1,p2))/(norm(X(:,i,p1))*norm(X(:,i-1,p2))));
```

```

    % Among the computed angles, choose the minimum
    [angle(i,j),c(i,j)]=min([a1 a2]);

    % Why minimum? Then we know that at least it will be able to
    % cover the angle with one of the two adjacent close nodes.

  end
end

anglesp = max(max(angle));
%Maximum angle between satellites of different planes
```

## 1.5 Walker Delta Testing

```
%----ASTREA CONSTELLATION----
%PROJECTS - 220028
%Aerospace Engineering Barchelor's Degree
%ESEIAAT - UPC
%Autumn 2016-2017

% ORBIT DESIGN TEAM
% ORBIT SMART WALKER-DELTA

% This routine compute the WD possible configurations that give global
% coverage by having the distance between planes matches the planes
% rotation

p=[12 15 18];
S=360./p; S=S*pi/180;

re=6378.01e3;
u=3.986012e14;
J2=1.0826e-3;

we=2*pi/(24*3600);

syms a;
inc=75; inc=inc*pi/180;
a_sol=zeros(1,length(inc));

for i=1:length(inc)
    dOmega=-1.5*J2*(re/a)^2*sqrt(u/a^3)*cos(inc(i));
    P0=2*pi*sqrt(a^3/u);
    Pn=P0*(1-1.5*J2*(re/a)^2-0.75*J2*(4-5*sin(inc(i))^2)*(re/a)^2);
    eq=S(2)-Pn*(we-dOmega);
```

```
    sol=vpasolve(eq,re*1.15);
    if imag(sol)==0
        a_sol(i)=sol;
    end
end

plot(inc*180/pi,(a_sol-re)/1000)
h=(a_sol-re)/1000

%% Satellites computation

emin=20*pi/180;
Lstreet=5*pi/180;

rho=re/(re+h*1000);
etha=asin(sin(rho)*cos(emin));
Lmax=pi/2-emin-etha;
S=2*acos(cos(Lmax)/cos(Lstreet));
N=ceil(2*pi/S)
```

## 1.6 Orbit Plotter

```
%----ASTREA CONSTELLATION----
%PROJECTS - 220028
%Aerospace Engineering Barchelor's Degree
%ESEIAAT - UPC
%Autumn 2016-2017

% ORBIT DESIGN TEAM
% ORBIT PLOTTER - Influential phenomena computation

% This matlab routine plots the position of the satellites of a Walker
% Delta / SemiWalker Delta or other generated configurations.

clc, clear, close all;
%-------------------------------------------------------------------------
%% 0. Data Input
%add the subfolder programs to the current path
addpath(genpath('./auxiliar'));

%_____Physical input__________

global h, h = 542;  % [km]
degreegen = 225;    % Walker Delta 360 deg, SemiWalker Delta 180 deg,
                    % other constellations range between 180 and 360 deg
in = degtorad(72);  % inclination of the planes
```

```
s = 21;              % satellites per plane
p = 9;               % number of planes
f=8;                 % f = 1.25*p*cos(i); % parameter defined graphically
eo=degtorad(20);     % elevation angle [rad]

%____Numerical Input_________
r = 1;     %radius of the sphere
n = 50;    %number of cells of the sphere generated
N = 50;    %number of points used to print the orbit
nn = 100; %number of points used to compute the footprint

%____Physical Constants_____
global Re, Re = 6371;  %Earth's Radius [Km]
%% 1. Preprocessing

global m, m = degreegen/180;
            % generates Walker Delta Constellation (m=2 -> 2*180)
            % generated constellation), Semi Walker (m=1 -> 1*180)
            % or other 1<m<2.

global a, a = (Re+h)/Re;
            %radius of the orbit (in terms of the sphere radius) R=h*r;

%% 2. Main Process

%compute the sat RAAN and some sat. parameters (Laura)
Omega = zeros(1,p);
nu = zeros(s,p);      %true anomaly of the satellites
angle = zeros(1,p);   %angle between satellites
for i = 1:p
    Omega(i) = m*pi*(i-1)/p;    %orbits

    angle(i) = f*2*pi*(i-1)/(s*p); %sats
    for k = 1:s
        nu(k,i) = 2*pi*(k-1)/s+angle(i);
    end
end

% Compute the satellite coordinates (Laura)
X = zeros(3,s,p);
for i = 1:p
    for k = 1:s
        X(:,k,i) = cartesian(a,0,in,Omega(i),0,nu(k,i));
    end
end
sCOOR=reshape(X,3,p*s)'; %put the data in a more friendly structure

%compute the radius of the footprint
%SATELLITE_FOOTPRINT_COMPUTATION.m
d=Re*((((h+Re)/Re)^2-cos(eo)^2)^.5-sin(eo));
```

```
Bo=asin(d*cos(eo)/(h+Re));
S=2*pi*Re^2*(1-cos(Bo));
R=(S/pi)^.5;

%create the footprint of each satellite
SATfp=[];
for i=1:s*p
    %unitary vector pointing to the sat from the earth's center
    u=-sCOOR(i,:)/norm(sCOOR(i,:));
    %Generate the circle coordinates (2D)
    [satfp(:,1),satfp(:,2),satfp(:,3)]=circle3D(sCOOR(i,:),u, R/Re, nn);
    satfp=satfp+repmat(u*h/Re,nn,1); %move the circle to be tg to Earth

    for j=1:size(satfp,1)
        v=sCOOR(i,:)-satfp(j,:); v=v/norm(v);
        %unitary vector sat->footprint

        %verify if the footprint intersect with the earth's surface
        tol=satfp(j,1)^2+satfp(j,2)^2+satfp(j,3)^2-r^2;
        while tol>1e-2
            satfp(j,:)=satfp(j,:)-0.01*v;
            tol=satfp(j,1)^2+satfp(j,2)^2+satfp(j,3)^2-r^2;
        end
    end
    %store the footprint
    SATfp=[SATfp; satfp];
end


%% 3. Plots

%3D plot
[COOR]=Orbitplot3D( in,r, X,n, SATfp, nn );
%Ground track
x=zeros(size(COOR,1),size(COOR,2)/3); y=x; z=x;
for i=1:size(COOR,2)/3
    x(:,i)=COOR(:,3*(i-1)+1);
    y(:,i)=COOR(:,3*(i-1)+2);
    z(:,i)=COOR(:,3*(i-1)+3);
end
GroundTrackPlot( x, y, z, sCOOR, SATfp, nn);
```

```
function [x,y,z]=circle3D(center,normal,radius,n)
%This matlab function generates the points of a circle normal to a 3D
%vector
%In:
%   center: center of the circle (x, y, z)
%   normal: vector normal to the circle
%   radius: radius of the circle
```

```
%   n: number of points (small number can make the circle not smooth)
%Out:
%   x, y z: vectors cintaining the coord.
%¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬

theta=linspace(0,2*pi,n);
v=null(normal);
points=repmat(center',1,size(theta,2))+...
    radius*(v(:,1)*cos(theta)+v(:,2)*sin(theta));
x=points(1,:); y=points(2,:); z=points(3,:);
end
```

```
function [COOR] = Orbitplot3D( in,r, X,n, sCOOR, N )
%Orbitplot3D is a function that creates a 3D representation of a
%constellation and plots the satellites. It can be also configured to plot
%their footprint
%In:
%   in: inclinaton angle [rad]
%   r: sphere radius [-]
%   X: Coordinates of the sats (3,spp,p)
%   n: number of points used to draw thhe sphere
%   ¬¬o¬¬
%   COOR: coordinates of the footprints
%   N: number of points used to compute the footprint
%¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬
p=size(X,3);
s=size(X,2);
global a m;

% Earth
%Create the earth instance as an sphere
[x,y,z]=sphere(n); %Generate the sphere
surf(r*x,r*y,r*z,'FaceColor','interp','EdgeColor','none'); %plot the sphere
colormap(linspace(0,1,n)'*[0 0.6 1]); %custom colormap (blue tones);
axis equal;
box off;
hold on;
rotate3d on;

% Orbits
%Generate the circle coordinates (2D)
coor=zeros(N+1,3);
for i=1:N
    coor(i,:)=[a*r*cos(2*i*pi/N), a*r*sin(2*i*pi/N), 0];
end
coor(N+1,:)=coor(1,:);

%Rotation Matrices
Ry=[ cos(in) 0 sin(in)
```

```
        0      1    0
    -sin(in) 0 cos(in)];

Rz= @(Omega) [-sin(Omega) cos(Omega) 0
              cos(Omega)  sin(Omega) 0
                  0           0      1]; % he modificat aquesta matriu

coor=coor*Ry; %Rotate the circle in the y axis (inclination)

%rotate the plane to get the other planes
COOR=zeros(N+1,3*p);
for i=1:p
    COOR(:,(1+(i-1)*3):(3+(i-1)*3))=coor*Rz(m*(i-1)*pi/p);
end
for i=1:p
    plot3(COOR(:,1+(i-1)*3),COOR(:,2+(i-1)*3),COOR(:,3+(i-1)*3),...
        'linewidth',1.5);
end

% Satellites (Laura)
for i = 1:p
    for k = 1:s
      scatter3(X(1,k,i),X(2,k,i),X(3,k,i),25,'square','k','filled');
      hold on;
    end
end

%Plot also the footprint if the number of input variables is bigger than 4
if nargin > 4
    for i=1:p*s
        b=1+N*(i-1); c=N+N*(i-1);
        fill3(sCOOR(b:c,1),sCOOR(b:c,2),sCOOR(b:c,3),[1 0 0],...
            'FaceAlpha','.3','LineStyle','none');
    end
end
end
```

```
function [] = GroundTrackPlot( x, y, z, coor, fp_coor, N)

%GroundTrackPlot is used to plot the ground
%track of the constellation orbits
%given their coordinates (cartesian).It has two variants, only introducing
%the first 3 inputs will plot the orbits and if you introduce all the
%inputs it plots the satellites and their footprint.
%In:
%   x: matrix containing the x coordinate of the orbits to plot
%   y: matrix containing the y coordinate of the orbits to plot
%   z: matrix containing the z coordinate of the orbits to plot
%   ¬¬o¬¬
```

```
%   coor: satellite coordinates (nx3)
%   fp_coor: satellite footprint coordinates ((n*nn)x3)
%   N: number of points used to compute the footprint
%--------------------------------------------------------------------------
p=size(x,2); %n of planes
s=size(coor,1)/size(x,2); %n of spp

%% Set the background image for the ground track plot
figure('name','Constellation Ground Track');
tit=['Num. of planes: ' num2str(p) ' Num. of spp: ' num2str(s)];
ground='World-satellite-map.png';
im=imread(ground);
imagesc([-180,180],[-90,90],im);
grid on;
axis([-180 180 -90 90]);
title(tit);
hold on;
%set the plot lines color
c = ['r','y','m','g','w']; C=[];
for i=1:ceil(size(x,2)/5)
    C=[C c];
end

%% Main process
for i=1:size(x,2)
    %convert the cartesian coordinates to spherical
    [lon,lat]=cart2sph(x(:,i),y(:,i),z(:,i));
    %get the coordinates of the orbit in spherical coord.
    lon=radtodeg(lon); %pass to degrees
    lat=radtodeg(lat);
    Lon=[lon(end); lon(1:end-1)];

    %Plot the sat ground track
    [¬,j]=max(abs(lon-Lon));
    plot(lon(1:j-1),lat(1:j-1),C(i),'linewidth',1.2);
    plot(lon(j:end),lat(j:end),C(i),'linewidth',1.2);
end

if nargin > 3
    [slon,slat]=cart2sph(coor(:,1),coor(:,2),coor(:,3));
    %get the coordinates of the sats in spherical coord.
    slon=radtodeg(slon); %pass to degrees
    slat=radtodeg(slat);

    %plot the sats
    scatter(slon,slat,30,'square','w','filled');

    %get the coordinates of the sats footprint in spherical coord.
    [fplon,fplat]=cart2sph(fp_coor(:,1),fp_coor(:,2),fp_coor(:,3));
    fplon=radtodeg(fplon); %pass to degrees
```

```
    fplat=radtodeg(fplat);

    %Plot their footprint
    for i=1:p*s
        b=1+N*(i-1); c=N+N*(i-1);
        fpLon=[fplon(c); fplon(b:c-1)];

        %get the maximum values (plotting purposes)
        [sorted, I]=sort(abs(fplon(b:c)-fpLon));
        M=sorted(end-1:end);
        j=I(end-1:end); j(2)=j(2)-1;

        if M(1) > 300
            if j(1) < j(2)
                auxlon=fplon(b+j(1):b+j(2)-1); %negative lon. part
                auxlat=fplat(b+j(1):b+j(2)-1);

                auxlon2=[fplon(b+j(2):c); fplon(b:b+j(1)-2)]; %+ lon. part
                auxlat2=[fplat(b+j(2):c); fplat(b:b+j(1)-2)];
            else
                auxlon2=fplon(b+j(2):b+j(1)-2); %negative lon. part
                auxlat2=fplat(b+j(2):b+j(1)-2);

                auxlon=[fplon(b+j(1):c); fplon(b:b+j(2)-1)];
                %positive lon. part
                auxlat=[fplat(b+j(1):c); fplat(b:b+j(2)-1)];
            end

         fill(auxlon,auxlat,[1 0 0],'FaceAlpha','.2','LineStyle','none');
         fill(auxlon2,auxlat2,[1 0 0],'FaceAlpha','.2','LineStyle','none');
        else
            fill(fplon(b:c),fplat(b:c),[1 0 0],...
                'FaceAlpha','.3','LineStyle','none');

        %%%%%%%%%%%%
        pause(.05); %
        %%%%%%%%%%%%
        end


    end
end
end
```

```
% This function converts kepler orbit elements to cartesian coordinates

% a: semimajor axis
% e: eccentricity
% i: inclination [rad]
```

```
% Omega: longitude of the ascending node [rad]
% w: Argument of periapsis [rad]
% nu: True anomaly [rad]

function [X] = cartesian(a,e,i,Omega,w,nu)

% Position in cylindrical coordinates
r = a*(1-e^2)/(1+e*cos(nu));

% Position components
X = [r*(cos(Omega)*cos(w+nu)-sin(Omega)*sin(w+nu)*cos(i));
    r*(sin(Omega)*cos(w+nu)+cos(Omega)*sin(w+nu)*cos(i));
    r*sin(i)*sin(w+nu)];

end
```

## 1.7 Perturbations

```
%----ASTREA CONSTELLATION----
%PROJECTS - 220028
%Aerospace Engineering Barchelor's Degree
%ESEIAAT - UPC
%Autumn 2016-2017

% ORBIT DESIGN TEAM
% ORBIT PERTURBATIONS - Influential phenomena computation

%PROBLEM:
% Given: - Initial orbital parameters
% Compute the final orbtial parameters after each orbit
%clear; clc;

%% 1.A Input data physical constants
% PHISICAL CONSTANTS AND PARAMETERS
%Physical data
RE=6.378e6; %Earth Radius [m]
u=3.986e14; %GM Earth
J2=1082.6e-5; %J2 coefficient;

%% 1. Input data Orbit parameters
% SPACECRAFT DATA
m=4;          % Satellite mass [kg]
Cd=2.2;       % Drag Coefficient
A=0.1;   % Satellite surface [m^2] --> 3U pointing to Earth
Bc=m/(Cd*A);% Ballistic coefficient of the satellite;

```

```
% ORBIT DATA
H0=542e3;      % Altura inicial [m]
e0=0.0001;      % Excentricity
i0=72*pi/180;        % Inclination [deg]
a0=RE+H0;

% INITIAL PARAMETERS
P=2*pi*(a0^3/u)^.5; % Orbit Period [s]
v=sqrt(u/a0);        %satellite velocity [m/s]
n=(2*pi/P)*86400;   %number of revolutions/day


% SIMULATION PARAMETERS
N=10000000;

%% 2. Perturbations propagation
%Outputs de la funcio Perturbation:
%   Pert: matrix (4x5) with the perturbations causes at the rows (J2,Drag,
%   Moon,Sun) and the orbital elements in the columns (a,e,i,omega,Omega)
%     a  e  i  w  Omega
%   [                   ]J2
%   [                   ]Drag
%   [                   ]Moon
%   [                   ]Sun as 3rd Body



t=zeros(1,N); t(1)=0;
a=t; a(1)=a0;
e=t; e(1)=e0;
i=t; i(1)=i0;

w_sun=zeros(1,N-1); w_moon=w_sun; w_J2=w_sun;
Om_sun=w_sun; Om_moon=w_sun; Om_J2=w_sun;

w0=0; Omega0=0;
w=t; w(1)=w0;
Omega=t; Omega(1)=Omega0;

tic
for n=2:N

    a0=a(n-1);
    e0=e(n-1);
    i0=i(n-1);
    w0=w(n-1);
    Omega0=Omega(n-1);

    P=2*pi*(a0^3/u)^.5; % Orbit Period [s]

```

```
    [Perti rho]=Perturbation(a0,e0,i0,Bc);

    a(n)=a0+sum(Perti(:,1));
    e(n)=e0+sum(Perti(:,2));
    i(n)=i0+sum(Perti(:,3));
    w(n)=w0+sum(Perti(:,4));
    Omega(n)=Omega0+sum(Perti(:,5));

    w_sun(n-1)=Perti(4,4);
    w_moon(n-1)=Perti(3,4);
    w_J2(n-1)=Perti(1,4);

    Om_sun(n-1)=Perti(4,5);
    Om_moon(n-1)=Perti(3,5);
    Om_J2(n-1)=Perti(1,5);

    % We don't want angles bigger than 360!
    w(n)=w(n)-360*floor(w(n)/360);
    Omega(n)=Omega(n)-360*floor(Omega(n)/360);

    if Omega(n)>2*pi
        Omega(n)=Omega(n)-2*pi;
    end

    t(n)=t(n-1)+P;

    if a(n)<(RE+180e3)
     fprintf('Your satellite succesfully burned in the atmosphere.\n\n')
      break
    end

end

%% Post processing and results plotting

time=toc;
fprintf('Time to do %g iterations = %g s.\n ',n,time)

figure(1)
plot(t(1:n)/(3600*24),(a(1:n)-RE)/1000)
ylim([100 500])
grid on
ylabel('Orbit height [km]')
xlabel('Time [days]')
tfin=floor(t(n)/(3600*24));
days = num2str(floor(tfin/365));
titulaso=['Orbit decay in ' num2str(tfin) ' days = ' days 'years'];
title(titulaso)
```

```
figure(2)
subplot(1,2,1)
plot(t(1:n)/(3600*24),w(1:n))
grid on
ylabel('Perigee Argument [deg]')
xlabel('Time [days]')
title('Perigee Argument deviation in 100 days')
axis([0 100 0 360])

subplot(1,2,2)
plot(t(1:n)/(3600*24),Omega(1:n))
grid on
ylabel('Ascenent Node Argument [deg]')
xlabel('Time [days]')
tfin=t(n)/(3600*24);
title('Ascendent Node deviation in 100 days')
axis([0 100 0 360])

%% Modulus Analysis

figure(3)

subplot(1,2,1)
semilogy(t(2:n+1)/(3600*24),abs(w_sun(1:n)),'b','LineWidth',2);
hold on
semilogy(t(2:n+1)/(3600*24),abs(w_moon(1:n)),'r','LineWidth',2)
semilogy(t(2:n+1)/(3600*24),abs(w_J2(1:n)),'k','LineWidth',2)
legend('Sun','Moon','J2')
xlabel('Time (days)');
ylabel('Modulus of Perturbation []');
title('Effects on Argument of the Perigee');
grid on
axis([0 t(n)/(3600*24) 1e-8 1])

subplot(1,2,2)
semilogy(t(2:n+1)/(3600*24),abs(Om_sun(1:n)),'b','LineWidth',2);
hold on
semilogy(t(2:n+1)/(3600*24),abs(Om_moon(1:n)),'r','LineWidth',2)
semilogy(t(2:n+1)/(3600*24),abs(Om_J2(1:n)),'k','LineWidth',2)
legend('Sun','Moon','J2')
xlabel('Time (days)');
ylabel('Modulus of Perturbation []');
title('Effects on Argument of Ascendent Node');
axis([0 t(n)/(3600*24) 1e-8 1])
grid on

print -depsc ModulusAngulars
```

```
function [pert rho] = Perturbation( a,e,i,Bc)
```

```
%% Perturbations
% This matlab is used to compte the perturbaons in the classical orbital
% elements due to:
%  J2 (non spherical earth)
%  Atmospheric drag
%  Third body (Moon and Sun)
%Inputs:
%   a: semi-major axis [m]
%   e: eccentricity [-]
%   i: inclination [rad]
%   m: mass of the satellite [kg]
%   Cd: drag coefficient [-]
%   A: Area of the satellite  [m^2]
%Outputs:
%   Pert: matrix (4x5) with the perturbations causes at the rows (J2,Drag,
%   Moon,Sun) and the orbital elements in the columns (a,e,i,omega,Omega)
%   incP/revolution
%   incv/revolution
%TO DO:
%Les perturbacions degudes a J2 i 3rd body no se si son en pert/dia o no,
%cal mirar-ho per tal de tenir totes les pertrubacions escalades igual
% ------------------------------------------------------------------------
%% Previous calculations and PreAllocation
%Physical data
RE=6.378e6; %Earth Radius [m]
u=3.986e14; %GM Earth
J2=1082.6e-5; %J2 coefficient;

P=2*pi*(a^3/u)^.5; % Orbit Period [s]
v=sqrt(u/a); %satellite velocity [m/s]
n=(2*pi/P)*86400;  %number of revolutions/day


% ATMOSPHERIC MODEL
%get the density from MSISE model (extracted as a list from
%http://omniweb.gsfc.nasa.gov/vitmo/msis_vitmo.html
%    h[km]    rho[kg/m^3]
% filename1='msis_26371.lst'; filename2='msis_2226.lst';
% formatSpec='%f %f';
% size=[2 Inf];
% fileID=fopen(filename1,'r');
% rho_h=fscanf(fileID,formatSpec,size);
% fclose(fileID);
% fileID=fopen(filename2,'r');
% rho_h=[rho_h fscanf(fileID,formatSpec,size)];
% fclose(fileID);
% rho=interp1(rho_h(1,:),rho_h(2,:)',(a-RE)/1000);

    H = (a - RE)*1e-3;
    % Compute exospheric temperature [K]
```

```
    T = 900 + 2.5*(120-70);
    % Compute effective atmospheric molecular mass [km/K], valid 180<H<500
    M = 27 - 0.012 * (H - 200);
    % Compute atmospheric scale height [km]
    SH = T / M;
    % Compute atmospheric density [kg/m3]
    rho = 6E-10 * exp(-(H - 175) / SH);


pert=zeros(4,5);

%% Computation

%J2 pertubation________________ PER REVOLUCIO
j=1;
pert(j,5)=-1.5*n*J2*(RE/a)^2*cos(i)*(1-e^2)^-2;
pert(j,4)=0.75*n*J2*(RE/a)^2*(4-5*(sin(i))^2)*(1-e^2)^-2;

%Atmospheric Drag______________ PER REVOLUCIO
%Perturbations/revolution
j=2;
pert(j,1)=-2*pi*rho*a^2/Bc;

% incP=-6*pi^2*a^2*rho/(v*Bc);
% incv=pi*a*rho*v/Bc;

%Moon Perturbation_____________ /DIA --> /REV
j=3;
pert(j,5)=-0.00338*cos(i)/n;
pert(j,4)=0.00169*(4-5*(sin(i))^2)/n;

% Fins aqui son /dia,
% Canviem unitats + apliquem periode actual
pert(j,5)=pert(j,5)*P/(24*3600);
pert(j,4)=pert(j,4)*P/(24*3600);

%Sun Perturbation_____________ /DIA --> /REV
j=4;
pert(j,5)=-0.00154*cos(i)/n;
pert(j,4)=0.00077*(4-5*(sin(i))^2)/n;

% Fins aqui son /dia,
% Canviem unitats + apliquem periode actual
pert(j,5)=pert(j,5)*P/(24*3600);
pert(j,4)=pert(j,4)*P/(24*3600);


end
```

## 1.8 Orbit Decay

```
%----ASTREA CONSTELLATION----
%PROJECTS - 220028
%Aerospace Engineering Barchelor's Degree
%ESEIAAT - UPC
%Autumn 2016-2017

% ORBIT DESIGN TEAM
% ORBIT DECAY

% This routine computes the orbital decay of an spacecraft with time
% using the cowell's method.
% The differential equations are integrated using "ode45"(Runge-Kutta 4th,
% 5th order)


%-----------------------------------------------------------------------------
%%
clear all;
clc;
close all;

%% Initialization %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
global mu mmu smu re rs rm omega J2;

%_______Physical Variables____________________________
mu=3.986004418e14; % earth gravitational constant [m^3/s^2]
mmu=4902.800076e9; %moon gravitational constant [m^3/s^2]
smu=132712440040.944e9; %sun gravitational constant [m^3/s^2]

re=6378136.3; %earth radius [m]
rs=696e6; %sun radius [m]
rm=1730e3; %moon radius [m]

omega=7.292115e-5; %earth angular velocity [rad/s]
J2=0.001081874; %earth oblateness gravity coeff []

au=149597870691; %astronomical unit [m]
c=2.99792458e8; %[m/s] speed of light

%____Numerical variables_______
hlim=180; %minimum height for considering the decay [km]
dt=15*60; %timestep of the simulation [s]
t0=0; %initial time of the sim
tf=dt;

%% User Input %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
%__Orbital elements_______________
a=re+542e3; %[m] semimajor axis
e=0.0001; % eccentricity
i=degtorad(72); %[rad] inclination
w=0; %[rad] argumetn f periapsis
Omega=0; %[rad] RAAN
nu=0; %true anomaly

date=datetime(2017,2,4); %datetime array [Y, M, D]

%__Oblateness parameters__________
znls=0; %zonals [0-18]
tssrls=0; %tesserals [0-18]

%__Drag perturbation inputs______
Cd=2.2; %Drag coefficient (Typical for sats, various ref.)
Adrag=.09; %[m^2] Wet area for the drag
m=4.1; %[kg] Cubesat mass

Bc=Adrag*Cd/m; %Ballistic coeff.

%__SRP perturbation inputs______
Cr=2; %reflectivity ct.
Asrp=.5; %[m^2] Area for the SRP

% Ws (@Dap) = 1361/(1+0.0334*cos(2*pi*Dap/365);
Ws=1361; %W/m^2 approximation for the solar irradiance

srp=Ws/c*Cr*Asrp/m*au^2; %Solar radiation pressure ct.

%% Main Process %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
display('Computing...');
f=@(q) q*((3+3*q+q^2)/(1+(1+q)^3/2)); %richard Battin's function

aobl=zeros(3,1);
adrag=aobl; asun=aobl; amoon=aobl; asrp=aobl;


y=cartesian2(a,e,i,Omega,w,nu); %obtain the state vector (r,V)
jdate0=juliandate(date); %compute the julian date

tic
iter=0; tt=[]; xx=[];

h=(norm(y(1:3))-re)*1e-3;
while h>hlim
    jdate=jdate0+t0/86400;
%     GMST=JD2GMST(jdate);
%
%     %Acceleration due to oblateness
```

```
%     aobl=Oblat((t0+tf)/2,y,znls,tssrls,GMST);
%
%     %Acceleration due to the 3rd body pert.
%     rsun=planetEphemeris(jdate,'Earth','Sun');
      %geocentric pos of the Sun
%     rmoon=planetEphemeris(jdate,'Earth','Moon');
      %geocentric pos of the Moon
%     rsun=rsun'; rmoon=rmoon';
%
%     rs2s=y(1:3)-rsun; %vector sun-sat
%     rm2s=y(1:3)-rmoon; %vector moon-sat
%
%     qs=dot(y(1:3),(y(1:3)-2*rsun))/dot(rsun,rsun);
%     qm=dot(y(1:3),(y(1:3)-2*rmoon))/dot(rmoon,rmoon);
%
%     asun=-smu/norm(rs2s)^3*(y(1:3)+f(qs)*rsun);
%     amoon=-mmu/norm(rm2s)^3*(y(1:3)+f(qm)*rmoon);
%
%     %Acceleration due to the solar radiation presure.
%     usun=rsun/norm(rsun); %unit vector of the pos of the sun
%     us=y(1:3)/norm(y(1:3)); %unit vector of the pos of the sat
%
%     shadow=random('Normal',0.5,.1); %shadow factor
%
%     asrp=srp*rs2s/norm(rs2s)^3;

    %Acceleration due to the drag force
    % Compute exospheric temperature [K]
    T = 900 + 2.5*(100-70);
    % Compute effective atmospheric molecular mass [km/K], valid 180<H<500
    M = 27 - 0.012 * (h - 200);
    % Compute atmospheric scale height [km]
    SH = T / M;
    % Compute atmospheric density [kg/m3]
    rho = 6E-10 * exp(-(h - 175) / SH);

    atmosV=y(4:6)+omega*[y(2); -y(1); 0];
    %relative vel between sat and atmos

    adrag=-.5*rho*Bc*atmosV*norm(atmosV);

    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    acc=aobl+asun+amoon+asrp+adrag;

    ode=@(t,y)[y(4);
            y(5);
            y(6);
            acc(1)-mu/norm(y(1:3))^3*y(1);
            acc(2)-mu/norm(y(1:3))^3*y(2);
            acc(3)-mu/norm(y(1:3))^3*y(3)
```

```
              ]; %function to integrate (6eqn)

    [t,x]=ode45(ode,[t0 tf], y);

    y=x(size(x,1),:);
    %save one value each 12 hours
    if mod(t(length(t)),12*3600)==0
        tt=[tt; t(length(t))]; xx=[xx; y];
    end
    y=y';

    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    %End of the timestep
    t0=tf;
    tf=tf+dt;

    h=(norm(y(1:3))-re)*1e-3;

    iter=iter+1;
    if mod(iter,300)==0
        fprintf('Days elapsed: %0.1f \nH=%0.3f km\n',(jdate-jdate0),h);
    end
end
toc
%% Post Process %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
fprintf('The satellite decays in %0.0f days',tf/86400);

h=zeros(size(xx,1),1);
for i=1:size(xx,1)
    h(i)=(norm(xx(i,1:3))-re)*1e-3;
end

plot(tt/86400,h);
```

```
% This function converts kepler orbit elements to cartesian coordinates

% a: semimajor axis
% e: eccentricity
% i: inclination [rad]
% Omega: longitude of the ascending node [rad]
% w: Argument of periapsis [rad]
% nu: True anomaly [rad]

function [y] = cartesian2(a,e,i,Omega,w,nu)
global mu;
% Position in cylindrical coordinates
r = a*(1-e^2)/(1+e*cos(nu));

% Position components
```

```
X = [r*(cos(Omega)*cos(w+nu)-sin(Omega)*sin(w+nu)*cos(i));
     r*(sin(Omega)*cos(w+nu)+cos(Omega)*sin(w+nu)*cos(i));
     r*sin(i)*sin(w+nu)];

A = sin(Omega)*cos(i)*(e*cos(w)+cos(w+nu));
v(1)=-sqrt(mu/( a*(1-e^2)))*(cos(Omega)*(e*sin(w)+sin(w+nu))+A);

B = cos(Omega)*cos(i)*(e*cos(w)+cos(w+nu));
v(2)=-sqrt(mu/( a*(1-e^2)))*(sin(Omega)*(e*sin(w)+sin(w+nu))-B);

v(3)=sqrt(mu/( a*(1-e^2)))*(e*cos(w)+cos(w+nu))*sin(i);

y=[X;v'];
end
```

## 1.9 Performance Evaluator

```
%----ASTREA CONSTELLATION----
%PROJECTS - 220028
%Aerospace Engineering Barchelor's Degree
%ESEIAAT - UPC
%Autumn 2016-2017

% ORBIT DESIGN TEAM
% PERFORMANCE EVALUATOR

clear all;

%% Input Data

a=6378.01e3;
u=3.986012e14;
h=(a+546.5101e3)/a;
ap=h*a;
J2=1.0826e-3;% J2=0;

ws=sqrt(u/ap^3); % Satellite mean motion
we=2*pi/(24*3600); % Earth mean motion
dOmega=-1.5*J2*(a/ap)^2*sqrt(u/ap^3)*cosd(75);

At=1/6; % [min]          % Every 10 seconds
t=0:At:60*24; t=t*60; % Time array in seconds

i=72*pi/180;

p=9;
```

```
spp=21;
N=p*spp;
%f=floor(0.25*spp);
f=0;
fprintf('\ni=%gdeg, p=%g, spp=%g, N=%g\n',i*180/pi,p,spp,N);

latGS=57.5*pi/180;
longGS=(17.73-60)*pi/180; longGS=0;

fprintf('GS Coordinates: Lat=%g, Long=%g \n',latGS*180/pi, longGS*180/pi)

f1=pi/2-latGS; f2=longGS;
R1=[cos(f1) 0 -sin(f1);0 1 0;sin(f1) 0 cos(f1)];
R2=[cos(f2) sin(f2) 0; -sin(f2) cos(f2) 0; 0 0 1];
RotGS=R1*R2;

% Rotation Matrix:
% From ECEF - To SEZ
% Where:
% ECEF = Earth Centered Earth Fix - Coordinate System
% SEZ = Topocentric Horizon - Coordinate System

emin=20*pi/180;

%% Le simulasion
%Omega = 0:30:240; Omega=Omega*pi/180;
Omega = 0:225/(p-1):225; Omega=Omega*pi/180;
%Omega=0:360/p:360-360/p; Omega=Omega*pi/180;

nu = zeros(spp,p);
angle = zeros(1,p);
X = zeros(3,spp*p);

flight_time=zeros(1,1000);     % Length of the flyby of a satellite
links_at_time=zeros(1,1000);   % Number of links when one sat has finished
time_end_flyby=zeros(1,1000); % When did this flyby end?

contact=zeros(1,length(t)); % Number of links

quality_time=zeros(1,length(t));
%If a flyby lasts longer than 3 minutes,
% then the previous 3 minutes were successfuly covered.

time_record=zeros(1,N);      % Accumulates Timesteps being on the GS
before_tracking=zeros(1,N);
% Boolean to know which ones are already passing by
now_tracking=zeros(1,N);    % Boolean to know which ones are now passing by
Nflyby=0;                   % Number of flight paths computed

for n=1:length(t)
```

```

    % J2 deviation
    Omegat=Omega+dOmega*t(n);

    % Ground Station Coordinates
    f1=we*t(n)+longGS;
    f2=latGS;
    RGS=cos(f2);
    XGS=[RGS*cos(f1);RGS*sin(f1);sin(f2)];

    R2=[cos(f1) sin(f1) 0; -sin(f1) cos(f1) 0; 0 0 1];
    RotGS=R1*R2;

    % Constellation Coordinates
    nu_t=ws*t(n);   % True anomaly due to time passing by
    for j = 1:p
        angle(j) = f*2*pi*(j-1)/(spp*p); %Phasing due to f between planes
        for k = 1:spp
            % True anomalies of the s satellites
            A = spp*(j-1)+k; % Number of the satellite
            nu(k,j) = 2*pi*(k-1)/spp+angle(j);
            X(:,A) = cartesian(h,0,i,Omegat(j),0,nu(k,j)+nu_t);
        end
    end

    % Contact Evaluation
    win=0;  % Counter to know number of links
    now_tracking=zeros(1,N);
    for sat=1:N
        X_hor=RotGS*(X(:,sat)-XGS);
%  fprintf('%g,%g,%g --> |r|=%g\n',X_hor(1),X_hor(2),X_hor(3),norm(X_hor))
        ang=asin(X_hor(3)/norm(X_hor));
        if ang ≥ emin
            win=win+1;
            time_record(sat)=time_record(sat)+1;
            now_tracking(sat)=1;
        end

        % Flight time computation and reset of this satellite flyby
        if before_tracking(sat)==1 && now_tracking(sat)==0
            Nflyby=Nflyby+1;
            flight_time(Nflyby)=At*time_record(sat);

            links_at_time(Nflyby)=contact(n-1);
            time_end_flyby(Nflyby)=t(n);

            % THE KEY QUESTION
            % Was this flyby useful?
            if flight_time(Nflyby)>3
                start=n-time_record(sat);
```

```
                finish=n;
                quality_time(start:finish)=quality_time(start:finish)+1;
            end
            time_record(sat)=0;
        end
    end
    before_tracking=now_tracking;
    contact(n)=win;
end

%% Post-Processing
% PLOT 1: NUMER OF LINKS
figure(1)
title('Links vs Time')
plot(t/(3600),contact)
axis([min(t)/(3600) max(t)/(3600) 0 max(contact)+1]);
 fails=length(find(contact==0));
 ratio=fails/length(contact);
 fprintf('Links on GS          : %g percent of the time\n',(1-ratio)*100)

% PLOT 2: LENGTH OF THE LINKS
figure(2)
plot(flight_time(1:Nflyby));
titola=['Length of the ' num2str(Nflyby) ' flyby s. Mean time = '...
    num2str(mean(flight_time(1:Nflyby))) 'min'];
title(titola)
ylabel('Length (minutes)')
xlabel('Contact')
%xlim([1 Nflyby])

% PLOT 3: ANALYSIS OF THE FLYBYS
figure(3)
index=1:Nflyby; % X variable to the following plots
plot(time_end_flyby(index)/3600,flight_time(index),...
     time_end_flyby(index)/3600,links_at_time(index),...
     t/3600,quality_time);

titola='Flybys Analysis';
title(titola)

legend('Length of flybys','Links by end of flyby',...
    'Num of sats @flybys longer than 3 min')
xlabel('Time (h)')

epic_wins=length(find(quality_time≥1));
ratio_covered=epic_wins/length(t);
fprintf('Quality flybys on GS: %g percent of the time\n',ratio_covered*100)
fprintf('Mean flyby time = %g minutes\n',mean(flight_time(1:Nflyby)))
```

```
%% MAXIMUM GAP SEARCH
gap=0;
at_gap=0;
gap_record=zeros(1,100);
ngap=0;

for n=1:length(t)
    if quality_time(n)==0
        at_gap=1;
        gap=gap+1;
    end

    if quality_time(n)>0 && at_gap==1
        at_gap=0;
        ngap=ngap+1;
        gap_record(ngap)=gap*At;
        gap=0;
    end
end

max_gap=max(gap_record);
fprintf('Maximum gap = %g minutes\n',max_gap)
fprintf('Number of gaps = %g\n\n',ngap)
```

## 1.10 Thrust

```
%----ASTREA CONSTELLATION----
%PROJECTS - 220028
%Aerospace Engineering Barchelor's Degree
%ESEIAAT - UPC
%Autumn 2016-2017

% ORBIT DESIGN TEAM
% THRUST

% This function computes the ΔV and propellant mass necessary to
% maintain an orbit between to heights

%function [mp,ΔV,tHoh] = thrust(hmax,hmin,ms,Isp,Thr)

% This function computes the ΔV and propellant mass necessary to
% maintain an orbit between to heights

% Input variables:
% - hmax: maximum height [m]
% - hmin: minimum height [m]
```

```
% - ms: dry mass of the spacecraft [kg]
% - Isp: specific impulse of the spacecraft [s]
% - Thr: thrust of the spacecraft [N]

% Output variables:
% - mp: array of propellant mass necessary for every Hohmann transfer [kg]
% - ΔV: array of ΔVs necessary for every Hohmann transfer [m/s]
% - tHoh: array of time necessary to do a Hohmann transfer [s]

% Proposed values:
 hmax = 550e3;
 hmin = hmax-8;
 ms = 3.95;        % Dry mass [kg]
 Isp = 2150;
 Thr = 100e-6;

%% Data

% PHYSICAL CONSTANTS AND PARAMETERS
RE = 6.378e6; %Earth Radius [m]
mu = 3.986e14; %GM Earth
g0 = 9.81;

% SPACECRAFT DATA
m = 4;          % Satellite mass [kg]
Cd = 2.2;       % Drag Coefficient
A = 0.1*0.3;    % Satellite surface [m^2] --> 3U pointing to Earth

% ORBIT DATA
H0 = hmax;      % Altura inicial [m]
E0 = 0.01;      % Excentricity
I0 = 80*pi/180;        % Inclination [deg]
A0 = RE+H0;

% SIMULATION PARAMETERS
N=100000;
M = 10000;

%% 2. Perturbations propagation

% Creation of the matrices
temp=zeros(M,N);
temp(1,1)=0;
H = zeros(M,N);
H(1,1) = H0;
% ΔV = zeros(2,N);

% Asign initial values
W0=0; OMEGA0=0;
a=zeros(1,N); a(1)=A0;
```

```
e=a; e(1)=E0;
i=a; i(1)=I0;
t=a; t(1)=0;
w=a; w(1)=W0;
Omega=a; Omega(1)=OMEGA0;

for j = 1:M

    n = 1;
    while a(n)≥(RE+hmin)

        n = n+1;

        a0=a(n-1);
        e0=e(n-1);
        i0=i(n-1);
        w0=w(n-1);
        Omega0=Omega(n-1);

        P=2*pi*(a0^3/mu)^.5; % Orbit Period [s]

        Bc = m/(Cd*A);% Ballistic coefficient of the satellite
        Perti=Perturbation(a0,e0,i0,Bc);

        a(n)=a0+sum(Perti(:,1));
        e(n)=e0+sum(Perti(:,2));
        i(n)=i0+sum(Perti(:,3));
        w(n)=w0+sum(Perti(:,4));
        Omega(n)=Omega0+sum(Perti(:,5));

        % We don't want angles bigger than 360!
        w(n)=w(n)-360*floor(w(n)/360);
        Omega(n)=Omega(n)-360*floor(Omega(n)/360);

        if Omega(n)>2*pi
            Omega(n)=Omega(n)-2*pi;
        end

        t(n)=t(n-1)+P;

    end

    % Hohmann
    [ΔV1,ΔV2,mpit,tHohit] = Hohmann(a(n)-RE,hmax,m,Isp);
    m = m-mpit;

    if m≤ms
        break
    end

```

```
    ΔV(1,j) = ΔV1; % first row of the column -> ΔV1
    ΔV(2,j) = ΔV2; % second row of the column -> ΔV2
    mp(j) = mpit;
    tHoh(j) = tHohit;

    a(1)=A0;
    e(1)=E0;
    i(1)=I0;
    w(1)=W0;
    Omega(1)=OMEGA0;
    altura = a-RE;

    H(j,:) = altura;
    temp(j,:) = t;

end

%% Post process

mfr = Thr/(g0*Isp);

for i = 1:N
    if H(1,i)≤0
        break
    end
end
for j = 1:M
    if H(j,1)≤0
        break
    end
end
H = H(1:(j-1),1:(i-1));

for i = 2:N
    if temp(1,i)≤0
        break
    end
end
for j = 2:M
    if temp(j,2)≤0
        break
    end
end
temp = temp(1:(j-1),1:(i-1));
for j = 2:size(temp,1)
    temp(j,:) = temp(j,:)+tHoh(j-1)+temp(j-1,size(temp,2));
end

% Convert matrix to vector
H = reshape(H.',1,size(H,1)*size(H,2));
```

```
temp = reshape(temp.',1,size(temp,1)*size(temp,2));

figure(1)
plot(temp/(3600*24),H/1000)
grid on
ylabel('Orbit height [km]')
xlabel('Time [days]')
tfin=floor(temp(length(temp))/(3600*24));
days=num2str(floor(tfin/365));
titulaso=['Orbit decay in ' num2str(tfin) ' days = ' days ' years'];
title(titulaso)

%% Final calculations

% % Perimeter of the ellipse
% r1 = RE+hmax; % semimajor axis
% r2 = RE+hmin; % semiminor axis
% h = (r1-r2)^2/(r1+r2)^2;
% % Ramanujan approximation
% C = pi*(r1+r2)*(1+3*h/(10+sqrt(4-3*h)));
%
% % Check if ΔV is possible for the given thruster
% OK = true;
% for j = 1:length(mp)
%     for i = 1:2
%         mfrnecessary = ΔV(i,j)*mp(j)/(C/2);
%         if mfrnecessary>mfr
%             OK = false;
%         end
%     end
% end
%
% if OK ==true
%     fprintf('The trajectory is possible\n\n');
% else
%     fprintf('Trajectory not possible\n\n');
% end
if length(mp)<M
    fprintf('There is still propellant left ._.\n\n')
end
```

```
function [ΔV1,ΔV2,mp,t] = Hohmann(hinicial,hfinal,m,Isp)

% Hohmann transfer orbit between two circular orbits
% - hinicial: height of the first orbit [m]
% - hfinal: height of the second orbit [m]
% - m: mass of the satellite (total) [kg]
% - Isp: Specific impulse [s]
% - mp: fuel mass [kg]
```

```
% - t: time needed to do the maneuver [s]

mu = 3.986004418e14; % Standard gravitational parameter (Earth)
REarth = 6.371e6; % [m]
g0 = 9.81; % Earth's gravity [m/s^2]

% First orbit
r1 = REarth+hinicial; % [m]
v1 = sqrt(mu/r1); % [m/s]

% Second orbit
r2 = REarth+hfinal; % [m]
v2 = sqrt(mu/r2); % [m/s]

% Transfer orbit (ellipse)
vp = sqrt(2*mu*r2/(r1*(r1+r2))); % [m/s]
va = sqrt(2*mu*r1/(r2*(r1+r2))); % [m/s]
a = (r1+r2)/2;
T = sqrt(2*pi^2*a^3/mu);

% ΔV 1 -> p(transfer orbit)
ΔV1 = vp-v1;
% ΔV a(transfer orbit) -> 2
ΔV2 = v2-va;
% Total
ΔV = ΔV1+ΔV2;

% Fuel mass
mp = m*(1-exp(-ΔV/(g0*Isp)));

% Time
t = T/2;

end
```

## 1.11 Satellites Datasheet

```
%----ASTREA CONSTELLATION----
%PROJECTS - 220028
%Aerospace Engineering Barchelor's Degree
%ESEIAAT - UPC
%Autumn 2016-2017

% ORBIT DESIGN TEAM
% SATELLITES DATASHHET

```

```
% Writing the constellation caracteristics

a=6378.01e3;
u=3.986012e14;
h=(a+542e3)/a;
ap=h*a;
J2=1.0826e-3;% J2=0;

ws=sqrt(u/ap^3); % Satellite mean motion
i=72*pi/180;

p=9;
spp=21;
N=p*spp;
%f=floor(0.25*spp);
f=0;
fprintf('\ni=%gdeg, p=%g, spp=%g, N=%g\n',i*180/pi,p,spp,N);


%Omega = 0:30:240; Omega=Omega*pi/180;
Omega = 0:225/(p-1):225; %Omega=Omega*pi/180;
%Omega=0:360/p:360-360/p; Omega=Omega*pi/180;

P=2*pi/ws/60;

%% Excel IMPORT

D=zeros(N,9);
for sat=1:N

    Om=ceil(sat/spp); ID_p=sat-(Om-1)*spp;
    ph=(ID_p-1)*360/spp;
   %[ID Plane h P i e Omega phase per]
    D(sat,1:9)=[sat Om (ap-a)/1000 P i*180/pi 0 Omega(Om) ph 0];

end
NameCells=['B2:I' num2str(N)];
xlswrite('SatsDatasheet.xls',D,NameCells)

%% Latex IMPORT

for sat=1:N

    Om=ceil(sat/spp); ID_p=sat-(Om-1)*spp;
    ph=(ID_p-1)*360/spp;
   %[ID Plane h P i e Omega phase per]
    D(sat,1:9)=[sat Om (ap-a)/1000 P i*180/pi 0 Omega(Om) ph 0];

end

```

```
[rows,cols]=size(D);
file=fopen('Autotable.txt','w');

for i=1:rows
    Text=['AstreaSAT ' num2str(i) ' $ '];
    for j=2:cols
        if j≠cols
            Text=[Text num2str(D(i,j-1)) ' $ '];
        else
            Text=[Text num2str(D(i,j-1)) ' \\ '];
        end
    end
    fprintf(file,[Text '\n']);
end

fclose(file);
```

## 1.12 Ground Station Localization

```
%----ASTREA CONSTELLATION----
%PROJECTS - 220028
%Aerospace Engineering Barchelor's Degree
%ESEIAAT - UPC
%Autumn 2016-2017

% COMMUNICATION TEAM
% GROUND STATIONS LOCALIZATION

clc; clear;

%% Input Data
N_sat=21;   %number of sats in a plane
N_planes=9; %number of orbital planes
h=542;      %high of the sats in km
I=72;       %inclination of the orbits in degrees
phase=210/(N_planes-1); %Angle between planes in the equator
At=0.1;       %time step in minutes
T=48;       %time to simulate in hours
t=0:At:T*60;    %array of time in minutes
e_min=7.5;        %minumum elevation in degrees
lambda=57.5; %latitudes to simulate in degrees
lat=length(lambda);
mu=0;           %longitudes to simulate in degrees
long=length(mu);
l=length(t);

```

```
%% Solver

Xs=Orbitalposition(N_sat,N_planes,h,I,phase,t);
L=zeros(lat,l);

for i=1:lat
    Xg=Groundposition(lambda(i),mu,t);
    [L(i,:)]=Links(Xs,Xg,e_min);
end

%% Plots

figure;plot(t/60,L);legend 0 30 60 90;xlabel('time (h)');ylabel('links');
```

```
function [X] = Orbital_position(N_sat,N_planes,h,I,mu,t)
%N_sat: number of sats per plane
%N_planes: number of planes
%h: Orbit high in km
%I: inclination of the orbital plane in degrees
%mu: Angle between two adjacent orbital planes at the equatiorial plane in
%degrees
%t: array of time in minutes

I=I*pi/180;    %I in rad
mu=mu*pi/180; %mu in rad
Omega=0:mu:(N_planes-1)*mu;
%longitude of the ascending node respect the inertial X axe [rad]

%for defining the orbits are defined a system of
%local axes X_o in which the y_o is the rotation axe and x_o is
%contained in the equatiorial plane. Thetransformation for the inertial
%system X_I to the local system 2 rotations are defined.
%FIRST ROTATION: it defines a intermediate system X_1. The inertial system
%rotates arround z_I an Omega angle.
%SECOND ROTAtion: The X_1 system is rotated (90deg-I) arround x_1

L_1o=[1 0 0; 0 sin(I) cos(I); 0 -cos(I) sin(I)]; %1st rotation matrix.
%It transform a vector from local coordinates X_o to intemediate
%coordinates X_1. This matrix is the same for every orbital plane
%since all orbits have the same inclination

Re=6371;  %Earth radius km.
%The distances are going to be refered to the earth radius
h=h/Re;      %high in earth radius
GM=6.474e-11*5.972e24*3600/Re^3;
%Gravitational constant*earth mass [(Re^3)/min^2]
Ro=1+h;      %orbital radius in earth radius

w=sqrt(GM/(Ro^3));  %sat's angular speed [rad/min]
```

```
phi=0:2*pi/N_sat:(N_sat-1)*2*pi/N_sat;
%relative angle between every sat of a plane respect the first one
f=8;
%factor to module the relative desfase of the first sat of one plane
%to the first sat of the next plane
l=length(t);

for p=1:N_planes    %going through each plane

    phi_0(p)=f*2*pi*(p-1)/(N_planes*N_sat);
    %relative desfase of the first sat of one plane to the
    %first sat of the next plane

    One = cos(Omega(p));
    Two = sin(Omega(p));
    L_I1=[One -Two 0; Two One 0; 0 0 1];
    %2nd rotation matrix. It transforms a vector from
    %intermediate coordinates X_1
    %to inertial coordinates X_I. This matrix is characteristic of every
    %plane since evey plane has a diferent longitude of the ascending node
    %(Omega)

    L_eo=L_I1*L_1o;
    %Global rotation matrix form local coordinates to nertial coordinates

    for s=1:N_sat   %going through every sat of the plane

        for k=1:l   %going through every time step

            First = Ro*cos(w*t(k)+phi(s)+phi_0(p));
            Second = -Ro*sin(w*t(k)+phi(s)+phi_0(p));
            X_loc=[First;0;Second];
            %Coordinates of the sat at the given time in
            %local coordinates X_o.
            %It defines a circumference at the plane x_o-z_o with a angular
            %velocity w, a initial phase respect the first sat and a
            %initial phase respect the first sat of the previous orbit
            %first sat.

            X(:,(p-1)*N_sat+s,k)=L_eo*X_loc;
            %Coordinates of the sat in inertial system
            %(Coordinate [x y z], Sat number, instant(time)]
        end
    end
end
end
```

```
function [X] = Ground_position(lambda,long,t)
%Gorund_position calculates the positon of a place of the earth, with a
```

```
%wiven longitude and latitude, in an inertial system of coordiantes X_I in
%a given period of time

%lamda: is the latitude of the place in degrees[-90,90]
%long: is the longitude of the place iin degrees [0,360)
%t: is a array of time in minutes

long=long*pi/180;   %longin radians
lambda=lambda*pi/180;   %lambda in radians

Rg=cos(lambda);
%distance between the Earth point to the rotation axis of the Earth [0,1]
hg=sin(lambda);
%distance between th Earth point to the equatorial plane [0,1]
%this 2 distances are measured in earth radius.

w=2*pi/(24*60); %angular velocity of the Earth [rad/min]

X=[Rg*cos(w*t+long);Rg*sin(w*t+long);hg*ones(1,length(t))];
%position of the point in X_I
%for the given time. The point descrives a cicumference
%with a Rg radius in the horizontal
%plane x_I-y_I with an angular velocity w and a initial phase long. The
%coordinate z_I is allways hg. It is expresed in earth radius.
%X(cooridinates, instant (time)

end
```

# 2 | Bibliography