

CSE 204: Data Structures and Algorithms

Assignment - 8: Divide and Conquer

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Course No: CSE 204

Complexity analysis:

The recursive function is :

```
public static List<Distance> secondClosestPoint(List<Point> pointsX, int start, int end)
```

Here `List<Point> pointsX` represents the points sorted by x

The cost is $T(n)$

The base cases:

```
    if (size == 2) {
        List<Distance> distances = new ArrayList<>();
        distances.add(new Distance(pointsX.get(start),
pointsX.get(end)));
        Point temp = pointsX.get(start);
        if (pointsX.get(end).y < pointsX.get(start).y) {
            pointsX.set(start, pointsX.get(end));
            pointsX.set(end, temp);
        }
        return distances;
    }
    if (size == 3) {
        List<Distance> distances = new ArrayList<>();
        distances.add(new Distance(pointsX.get(start),
pointsX.get(start + 1)));
        distances.add(new Distance(pointsX.get(start + 1),
pointsX.get(end)));
        distances.add(new Distance(pointsX.get(start),
pointsX.get(end)));
        Collections.sort(distances, (d1, d2) -> {
            double dist = d1.distance - d2.distance;
            if (dist < 0)
                return -1;
            if (dist > 0)
                return 1;
            return 0;
        });
    }
```

```

List<Point> temp = new ArrayList<>();
temp.add(pointsX.get(start));
temp.add(pointsX.get(start + 1));
temp.add(pointsX.get(end));
Collections.sort(temp, (d1, d2) -> {
    double dist = d1.y - d2.y;
    if (dist < 0)
        return -1;
    if (dist > 0)
        return 1;
    return 0;
});
for (int i = 0; i < 3; i++) {
    pointsX.set(start + i, temp.get(i));
}
if (distances.get(0).distance == distances.get(1).distance) {
    distances.remove(0);
} else
    distances.remove(2);
return distances;
}

```

In the base cases, the cost is $O(1)$

Divide step:

```

int mid = (start + end) / 2;
List<Distance> distances1 = secondClosestPoint(pointsX, start, mid);
List<Distance> distances2 = secondClosestPoint(pointsX, mid + 1, end);

```

The total cost of the divide step is $2T(n/2)$

Conquer step:

Step 1:

```

List<Distance> minDistance = new ArrayList<>(distances1);
for (int i = 0; i < 2; i++) {

```

```

        if (distances1.size() >= 1) {
            if (distances1.get(0).distance ==
distances2.get(0).distance)
                distances1.remove(0);
            if (distances2.size() == 2 && distances1.size() >= 1) {
                if (distances1.get(0).distance ==
distances2.get(1).distance)
                    distances1.remove(0);
            }
        }
    }
    minDistance.addAll(distances2);
    Collections.sort(minDistance, (d1, d2) -> {
        double dist = d1.distance - d2.distance;
        if (dist < 0)
            return -1;
        if (dist > 0)
            return 1;
        return 0;
    });
    int minSize = minDistance.size();
    if (minSize == 4) {
        minDistance.remove(3);
        minDistance.remove(2);
    }
    if (minSize == 3)
        minDistance.remove(2);

```

The cost of step-1 is $O(1)$

Step 2:

```
merge(pointsX, start, mid, end);
```

Here's the merge() function,

```

private static void merge(List<Point> points, int low, int mid, int high)
{
    int i = low, j = mid + 1;
    for (int k = low; k <= high; k++)
        Main.temp[k - low] = points.get(k);
    for (int k = low; k <= high; k++) {
        Point temp1;
        if (i > mid)

```

```

        temp1 = Main.temp[j++ - low];
    else if (j > high)
        temp1 = Main.temp[i++ - low];
    else if (Main.temp[j - low].y < Main.temp[i - low].y)
        temp1 = Main.temp[j++ - low];
    else
        temp1 = Main.temp[i++ - low];
    points.set(k, temp1);
}
}

```

So, the cost of step -2 is $O(n)$

Step-3:

```

List<Point> midPoints = new ArrayList<>();

for (int i = start; i <= end; i++) {
    if (pointsX.get(i).x >= pointsX.get(mid).x -
minDistance.get(0).distance
        && pointsX.get(i).x <= pointsX.get(mid).x +
minDistance.get(0).distance) {
        midPoints.add(pointsX.get(i));
    }
}

```

The cost of step-3 is $O(n)$

Step-4:

```

int midPointSize = midPoints.size();

for (int i = 0; i < midPointSize - 1; i++) {
    for (int j = 1; j <= 7; j++) {
        Distance midPointDistance;
        if (i + j < midPointSize) {

            midPointDistance = new Distance(midPoints.get(i),
midPoints.get(i + j));
            if (midPointDistance.distance !=
minDistance.get(0).distance
                && midPointDistance.distance !=
minDistance.get(1).distance) {
                minDistance.add(midPointDistance);
            }
        }
    }
}

```

```

        Collections.sort(minDistance, (d1, d2) -> {
            double dist = d1.distance - d2.distance;
            if (dist < 0)
                return -1;
            if (dist > 0)
                return 1;
            return 0;
        });
        minDistance.remove(2);
    }
} else
    break;
}
}
return minDistance;

```

The cost of step-4 is $O(n)$

So the total cost of conquer step is $3*O(n) + O(1) = O(n)$

So, $T(n) = 2*T(n/2) + O(n)$

According to Master theorem, $T(n) = a*T(n/b) + f(n)$

Here, $a=2, b=2, f(n)=O(n)$

Now, $\log_2 2 = 1$

So, $f(n)$ is polynomially equal and follows the 2nd condition of Master's Theorem.

So, $T(n) = \Theta(n^1 \log n) = \Theta(n \log n)$