University of Girona

Spain

Visual Perception
Lab 3 - Epipolar Geometry

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1 Introduction

The visual perception lab 3 is focused on epipolar geometry. Epipolar Geometry is the fundamental relationship between two perspective cameras or the intrinsic projection between two different views. The commonly used nomenclature in epipolar geometry is epipole, epipolar line and epipolar plane. Epipole is the point of intersection for the baseline with the image plane. Epipolar line is the straight line of intersection of the epipolar plane with the image plane and they intersect at epipole. Epipolar plane is the plane defined by a 3D point and the optical centres. For the implementation, we try to compute the fundamental matrix by using both 8-point method and SVD. We have discussed some comparisons when we increase the noise level and number of 3D points in the following sections. We have divided the work into three different parts and the parts are explained respectively in the below sections. All the implementation is resulted in MATLAB.

2 Part 1

2.1 Step 1 and 2

At first step we have to define intrinsic (I) and extrinsic (E) parameters for camera 1. Camera 1 frame is set to world frame, the rotation matrix is identity and translation matrix is equal to zero. In Second step we have to define the parameters for camera 2. The parameters in Matlab are defined as below figure.

```
%step 1 Camera 1 is set to world frame
au1 = 100; av1 = 120; uo1 = 128; vo1 = 128;
height1 = 256;
width1 = 256; %Image size: 256 x 256
R1 = eye(3,3);
T1 = zeros (3,1);

%step 2 Camera 2
au2 = 90; av2 = 110; uo2 = 128; vo2 = 128;
ax = 0.1; by = pi/4 ; cz = 0.2 ;
tx = -1000; ty = 190 ; tz = 230 ;
height2 = 256;
width2 = 256; %Image size: 256 x 256
```

Figure 1: Camera 1 and 2 parameters

2.2 Step 3

Get the intrinsic transformation matrices of both the cameras. The total rotation matrix is the multiplication of the rotation matrices in x,y and z directions. The intrinsic matrix can be written as:

$$I = \begin{pmatrix} \alpha_u & 0 & u_0 & 0 \\ 0 & \alpha_v & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

```
%step 3  intrinsic transformation matrices
I1 = [au1, 0, uo1;0, av1, vo1;0, 0, 1];
I2 = [au2,0,uo1;0,av2,vo2;0,0,1];
Rotx = [1,0,0;0, cos(ax), -sin(ax);0, sin(ax),  cos(ax)];
Roty = [cos(by), 0, sin(by);0, 1, 0;-sin(by), 0, cos(by)];
Rotz = [cos(cz),-sin(cz),0;sin(cz), cos(cz),0;0,0,1];
R2 = Rotx*Roty* Rotz;
T2 = [tx;ty;tz];
```

Figure 2: Intrinsic transformation matrices for camera 1 and 2

2.3 Step 4

We have to get the fundamental matrix (F) and it is formulated as below formula where A and $A^{'}$ are the intrinsic matrices of the camera 1 and 2 and $[t]_x$ is the translation in antisymmetrix matrix:

$$F = A^{'} R^t [t]_x A^{-1}$$

$$[t]_x = \begin{pmatrix} 0 & -t3 & t2 \\ t3 & 0 & -t1 \\ -t2 & t1 & 0 \end{pmatrix}$$

```
%step 4 Fundamental Matrix
T_cross = [0,-T2(3),T2(2);
            T2(3),0,-T2(1);
            -T2(2), T2(1),0];
F= inv(I2') * R2' * T_cross * inv(I1);
F = F./F(9);
```

Figure 3: Fundamental Matrix

2.4 Step 5, 6 and 7

For step 5 we will define the set of object points as mentioned in the given document. For the next consecutive step we have to compute the $2D$ points for two image planes and plot them as per the below figures.

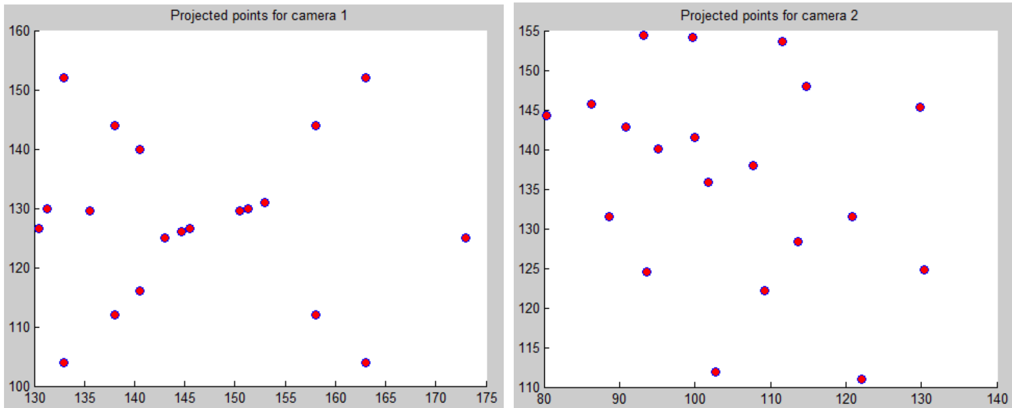


Figure 4: Projected points for camera 1 and 2

2.5 Steps 8 and 9

This step explains about computing fundamental matrix by 8-point method and least squares, It is given by equation as below:

$$m^{'t}Fm = 0$$

where $m^{'}$ and m are the $2D$ points already acquired for the fundamental matrix. formula is modulated into a linear equation form as:

$$U_n f = 0$$

where U_n is composed of the multiplication of m^1 and m elements and f is considered as fundamental matrix. When we divide f by the last element of f , only 8 unknown elements remain and that is why we need at least 8 points to solve the problem. Finally, we can use the least mean square method to solve the linear equation. Step 9 explains to compare the fundamental matrix obtained in before steps with the one we computed with 8-point method. The results of the both matrices should be same.

2.6 Step 10

In step 10 we will plot epipoles and epipolar lines. We will get the cross product of two vectors in the π plane, whose final form can be written as:

$$l^{'}_m = Fm = [u1, u2, u3]^t$$

Any point that lies on the corresponding epipolar line should have $[x, y, 1][u1, u2, u3]^t = 0$, where we can find m and d from $y = mx + d$.All the epipoles are plotted in the below figure.

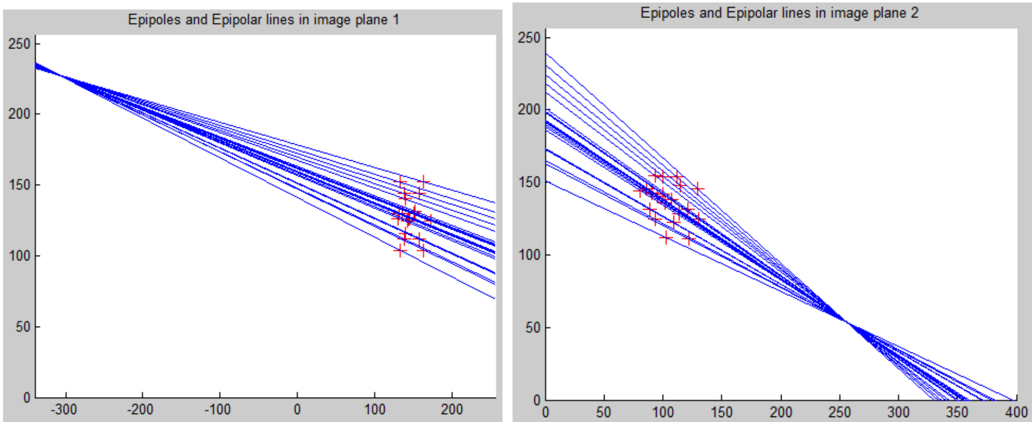


Figure 5: Projection of epipoles and epipolar plane for both the cameras

2.7 Step 11,12 and 13

In this steps we will add the gaussian noise as 0.5 to the both 2D image planes and we will repeat all the above steps again to plot the epipoles with respect to noise. From the below figure we have observed that all the points are not on epipolar lines and the mean distance increase as gaussian noise level is increased.

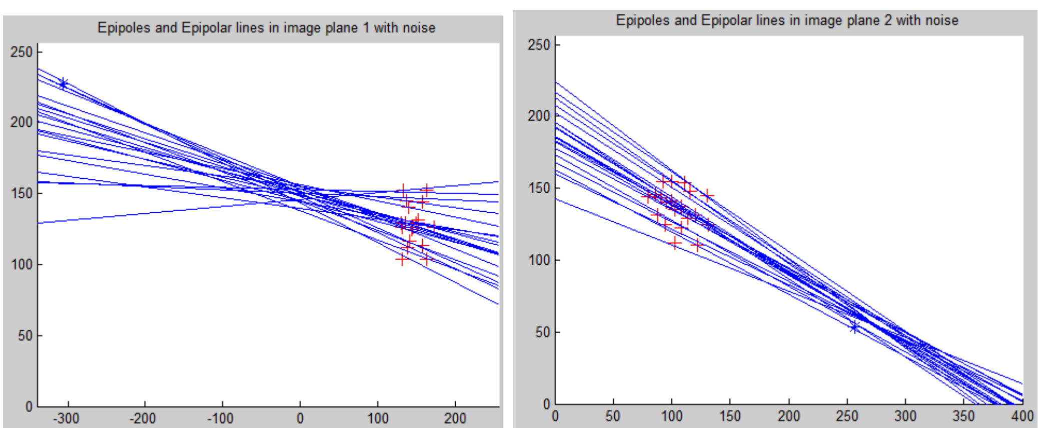


Figure 6: Projection of epipoles and epipolar plane for both the cameras with noise=0.5

For the next step we will increase the gaussian noise to 1 and the results are plotted in below figure:

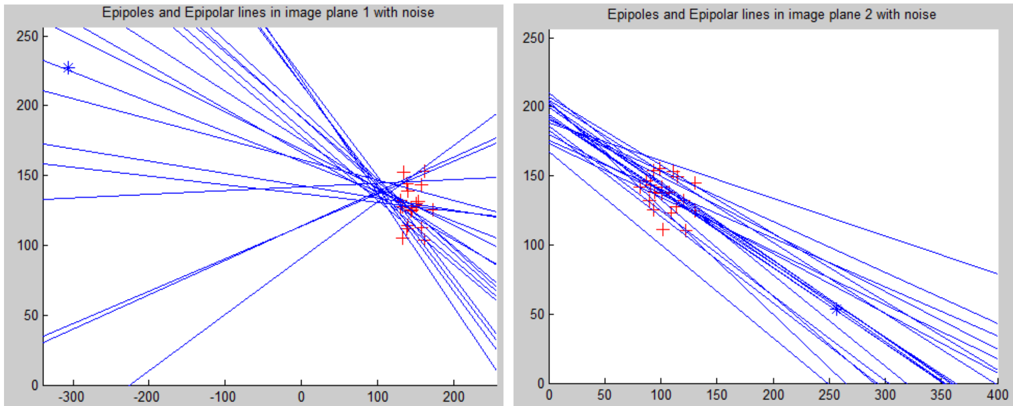
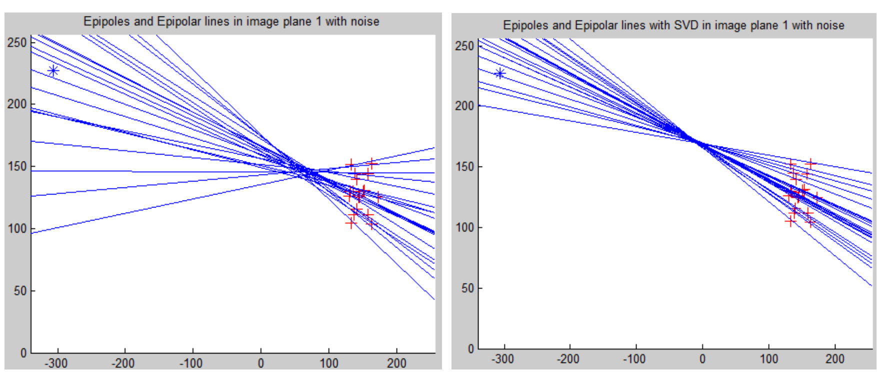


Figure 7: Projection of epipoles and epipolar plane for both the cameras with noise=1

3 Part 2

Part 2 describes to find the fundamental matrix with SVD. As we did in the previous steps we will model it as $U_n f = 0$ but instead of dividing f by the last element, we directly get the nullspace of U_n using SVD. We have defined the last column of U_n as 1, the rank should be 8 and the nullspace is the last column of V from SVD(U_n). When we compare the two matrices from this step and from the step 8 it will be the same. For the next step we have to perform the same steps from step 10 but with respect to the SVD matrix we have obtained. We will compare the distance between this two matrices to find out which will have minimizes the distance between points and epipolar lines. And we have analysed that there will be a difference in SVD, Which makes SVD to outperforms with the other matrices we have obtained in previous steps.The results are labelled in below diagram:



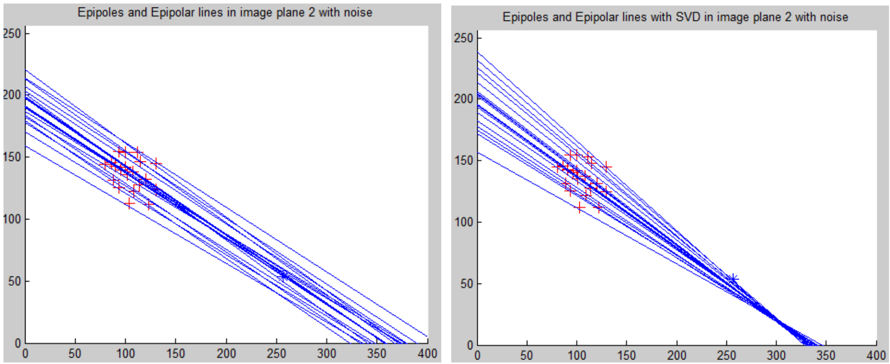


Figure 8: Projection of epipoles and epipolar plane for both the cameras with noise=0.5 with LS and SVD matrices

4 Part 3

In final step we have to simulate the whole two cameras epipolar geometry in 3D world coordinate system. The focal length is set to be 80mm for both cameras.

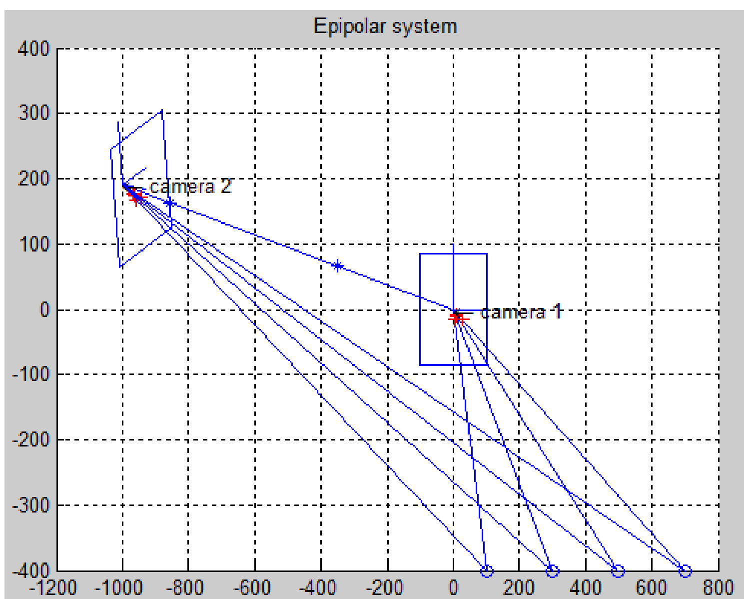


Figure 9

5 Conclusion

In this lab we have learnt about epipolar geometry for two cameras. We have analized with different methods like LS and SVD by adding gaussian noise and we have compared the results of both and concluded SVD is better. At the last part we have drawn the 2D points of two cameras into 3D world coordinate.