Fourth Year B.S. (Honors) 2023-2024

Department of Applied Mathematics, University of Dhaka

Course Title: Math Lab IV, Course No.: AMTH 450

Assignment 03

Name: **Roll:** **Group:**

Use Python to solve each of the following problems.

1. Solve the following initial value problems using `odeint` and `solve_ivp` commands and compare the actual error at each step. Also, plot the solutions with exact solutions on the same set of axes.

   i) $y' = te^{3t} - 2y;\ 0 \le t \le 1,\ y(0) = 0$, [exact solution: $y(t) = \frac{1}{5}te^{3t} - \frac{1}{25}e^{3t} + \frac{1}{25}e^{-2t}$]

   ii) $y' = 1 + (t - y)^2;\ 2 \le t \le 3, y(2) = 1$, [exact solution: $y(t) = t + \dfrac{1}{1-t}$]

2. Consider the Lotka-Volterra predator-prey model defined by

$$\frac{dx}{dt} = -0.1x + 0.02xy$$
$$\frac{dy}{dt} = 0.2y - 0.025xy$$

   where the populations $x(t)$ (predators) and $y(t)$ (prey) are measured in thousands. Suppose $x(0) = 6$ and $y(0) = 6$. Solve the system to find $x(t)$ and $y(t)$, and use the graphs to approximate the time $t > 0$ when the two populations are first equal.

3. Consider the competition model defined by

$$\frac{dx}{dt} = x(2 - 0.4x - 0.3y)$$
$$\frac{dy}{dt} = y(1 - 0.1y - 0.3x)$$

   where the populations $x(t)$ and $y(t)$ are measured in thousands and t in years. Analyze the populations over a long period of time for each of the following cases:
   a) $x(0) = 1.5,\ y(0) = 3.5$
   b) $x(0) = 1,\ y(0) = 1$
   c) $x(0) = 2,\ y(0) = 7$
   d) $x(0) = 4.5,\ y(0) = 0.5$

4. The motion of a swinging pendulum under certain simplifying assumptions is described by the second-order differential equation

$$\frac{d^2\theta}{dt^2} + \frac{g}{L}sin\theta = 0$$

   Suppose the pendulum is 2 feet long and $g = 32.17ft/s^2$. Find the values of θ for $0 \le t \le 2$ and initial conditions $\theta(0) = \frac{\pi}{6}, \theta'(0) = 0$, taking increment of $0.1s$.

5. Solve the following system of ODEs

$$x_2''' = -x_1''^3 + x_2' + x_1 + sin(t)$$
$$x_1''' = -2x_2'^2 + x_2$$

(*hint: reduce it to a system of first order ODES*)

6. Use the **Linear Shooting Algorithm** to approximate the solution of $y = e^{-10x}$ to the boundary value problem

$$y'' = 100y,\ 0 \le x \le 1,\ y(0) = 1,\ y(1) = e^{-}10$$

Use h = 0.1 and 0.05. Also use `solve_bvp` to find the solution of the same problem.