**1. Explain the term with examples:**

a. Basic properties of probability

1. *Probability (Event)=(Number of favorable outcomes of an event) / (Total Number of possible outcomes)*
2. Probability of a sure/certain event is 1.
3. Probability of an impossible event is zero (0).
4. Probability of an event always lies between 0 and 1. It is always a positive.

*0 <= Probability(Event) <= 1*

1. *If A and B are 2 events that are said to be mutually exclusive events then*

*P(AUB) = P(A) + P(B).*

1. *Sum of probabilities of complementary events is 1.*

*P(A)+P(A’)=1*

**Example:**

1. **What is the probability of getting a Tail when a coin is tossed?**

**Solution:**

Number of Favorable Outcomes- {Tail} = 1

Total Number of possible outcomes- {Head, Tail} - 2

Probability of getting Tail= 1/2 = 0.5

b. Sum of probability

Sum Rule:

The sum rule is given by P(A + B) = P(A) + P(B). Explain that A and B are each events that could occur, but cannot occur at the same time.

Example:

1. The probability that the next person walking into class will be a student and the probability that the next person will be a teacher. If the probability of the person being a student is 0.8 and the probability of the person being a teacher is 0.1, then the probability of the person being either a teacher or student is 0.8 + 0.1 = 0.9.
2. The probability that the next flip of a coin is heads or that the next person walking into the class is a student. If the probability of heads is 0.5 and the probability of the next person being a student is 0.8, then the sum is 0.5 + 0.8 = 1.3; but probabilities must all be between 0 and 1.

Product Rule:

 The product rule is P( E​*F) = P(E)*​P(F) where E and F are events that are independent. Explain that independence means that one event occurring has no effect on the probability of the other event occurring.

Example:

1. One example: When picking cards from a deck of 52 cards, the probability of getting an ace is 4/52 = 1/13, because there are 4 aces among the 52 cards (this should have been explained in an earlier lesson). The probability of picking a heart is 13/52 = 1/4. The probability of picking the ace of hearts is 1/4\*1/13 =1/52.

c. Complement rule

The complement rule is stated as "the sum of the probability of an event and the probability of its complement is equal to 1," as expressed by the following equation:

P(AC) = 1 – P(A)

Example

1. A young man goes to buy a new phone model which has a 10 different color collection, but he does not like two of those colors, if when he buys the phone they give him a random color ¿What are the chances of him getting a color that he actually likes?

Solution:

First we find p(c) being c getting one of the unwished colors

p(c) = 2 / 10

p(c) = 0.2

Then we find p( c )

p( c ) = 1 - 0.2

p( c ) = 0.8

p( c ) = 0.8 \* 100%

p( c ) = 80%

d. Probability of involving multiple events

We are often interested in finding the probability that one of multiple events occurs. Suppose we are playing a card game, and we will win if the next card drawn is either a heart or a king. We would be interested in finding the probability of the next card being a heart or a king. The union of two events E and F,written E∪FE and F,written E∪F, is the event that occurs if either or both events occur.

P(E∪F)=P(E)+P(F)−P(E∩F)

**Example problem:** The probability of getting a job you applied for is 45% and the probability of you getting the apartment you applied for is 75%. What is the probability of getting both the new job *and*the new car?

**Step 1:** [*Convert your percentages*](https://www.statisticshowto.com/statistics-basics/calculate-percentages/)*of the two events to decimals.* In the above example:

* 45% = **.45.**
* 75% = **.75.**

**Step 2:** *Multiply the decimals from step 1 together:*

.45 x .75 = **.3375** or 33.75 percent.

e. Addition for joint and disjoint rule

**2. Explain Conditional probability with example**

**ANSWER:**

A conditional probability is the probability of one event if another event occurred. In the “die-toss” example, the probability of event A, three dots showing, is P(A) = 1 6 on a single toss. But what if we know that event B, at least three dots showing, occurred? Then there are only four possible outcomes, one of which is A. The probability of A = {3} is 1 4 , given that B = {3, 4, 5, 6} occurred. The conditional probability of A given B is written P(A|B).

P(A|B) = P(A · B) P(B)

I roll a fair die twice and obtain two numbers X1=X1= result of the first roll and X2=X2= result of the second roll. Given that I know X1+X2=7X1+X2=7, what is the probability that X1=4X1=4 or X2=4X2=4?

**Solution**

Let AA be the event that X1=4X1=4 or X2=4X2=4 and BB be the event that X1+X2=7X1+X2=7. We are interested in P(A|B)P(A|B), so we can use

P(A|B)=P(A∩B)P(B)P(A|B)=P(A∩B)P(B)

We note that

A={(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),(1,4),(2,4),(3,4),(5,4),(6,4)},A={(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),(1,4),(2,4),(3,4),(5,4),(6,4)},

B={(6,1),(5,2),(4,3),(3,4),(2,5),(1,6)},B={(6,1),(5,2),(4,3),(3,4),(2,5),(1,6)},

A∩B={(4,3),(3,4)}.A∩B={(4,3),(3,4)}.

We conclude

P(A|B)=P(A∩B)P(B)P(A|B)=P(A∩B)P(B)

=236636=236636

=13.

**3. Explain Bayes rule with example**

**Answer:**

Bayes Formula. Let the event of interest A happens under any of hypotheses Hi with a known (conditional) probability P(A|Hi). Assume, in addition, that the probabilities of hypotheses H1, . . . , Hn are known (prior probabilities). Then the conditional (posterior) probability of the hypothesis Hi , i = 1, 2, . . . , n, given that event A happened, is

P(Hi |A) = P(A|Hi)P(Hi) P(A) , where P(A) = P(A|H1)P(H1) + · · · + P(A|Hn)P(Hn).

Imagine you are a financial analyst at an investment bank. According to your research of [publicly-traded companies](https://corporatefinanceinstitute.com/resources/knowledge/finance/private-vs-public-company/), 60% of the companies that increased their share price by more than 5% in the last three years replaced their [CEOs](https://corporatefinanceinstitute.com/resources/careers/jobs/what-is-a-ceo-chief-executive-officer/) during the period.

At the same time, only 35% of the companies that did not increase their share price by more than 5% in the same period replaced their CEOs. Knowing that the probability that the stock prices grow by more than 5% is 4%, find the probability that the shares of a company that fires its CEO will increase by more than 5%.

Before finding the probabilities, you must first define the notation of the probabilities.

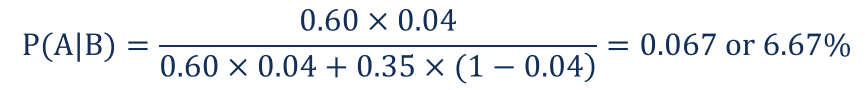
P(A) – the probability that the stock price increases by 5%

P(B) – the probability that the CEO is replaced

P(A|B) – the probability of the stock price increases by 5% given that the CEO has been replaced

P(B|A) – the probability of the CEO replacement given the stock price has increased by 5%.

Using the Bayes’ theorem, we can find the required probability:



Thus, the probability that the shares of a company that replaces its CEO will grow by more than 5% is 6.67%.

**4. Describe normal distribution, Bernoulli distribution, binomial distribution with example**

**Answer:**

Normal Distribution: The normal distribution, also known as the Gaussian distribution, is the most important probability distribution in statistics for independent, random variables. Most people recognize its familiar bell-shaped curve in statistical reports.

The normal distribution is a continuous probability distribution that is symmetrical around its mean, most of the observations cluster around the central peak, and the probabilities for values further away from the mean taper off equally in both directions. Extreme values in both tails of the distribution are similarly unlikely. While the normal distribution is symmetrical, not all symmetrical distributions are normal. For example, the Student’s t, Cauchy, and logistic distributions are symmetric.

As with any probability distribution, the normal distribution describes how the values of a variable are distributed. It is the most important probability distribution in statistics because it accurately describes the distribution of values for many natural phenomena. Characteristics that are the sum of many independent processes frequently follow normal distributions. For example, heights, blood pressure, measurement error, and IQ scores follow the normal distribution.

In this blog post, learn how to use the normal distribution, about its permiters, the Empirical Rule, and how to calculate Z-scores to standardize your data and find probabilities.

For the same above scenario, now find the probability of a randomly selected employee earning more than $85,000 a year.

**Solution**:

So, in this question, we need to find out the shaded area from 85 to right tail using the same formula.

Given:

Mean (µ) = $60,000

Standard deviation (σ) = $15000

Random Variable (X) = $85,000

Transformation (z) = (85000 – 60000 /15000)

Transformation (z) = 1.67

As per the Z-table, the equivalent value of 1.67 is 0.9525 or 95.25%, which shows that the probability of randomly selecting an employee earning less than $85,000 per annum is 95.25%.

But as per the question, we need to determine the probability of random employees earning more than $85,000 a year, so we need to subtract the calculated value from 100.

Random Variable (X) = 100% – 95.25%

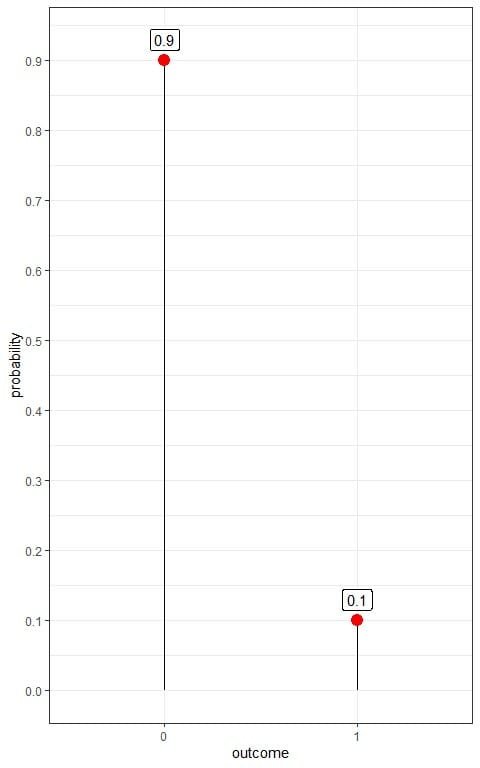
Random Variable (X) = 4.75%

So, the probability that employees earn more than $85,000 per year is 4.75%.

Bernoulli distribution

The prevalence of a certain disease in the general population is 10%.

If we randomly select a person from this population, we can have only two possible outcomes (diseased or healthy person). We call one of these outcomes (diseased person) success and the other (healthy person), a failure.

The probability of success (p) or diseased person is 10% or 0.1. So, the probability of failure (q) or healthy person = 1-p = 1-0.1 = 0.9.*If we denote diseased person as 1 and healthy person as 0, we can plot this Bernoulli distribution as follows:* 

*We have two outcomes:*

A healthy person or 0 with a probability of 0.9.

A diseased person or 1 with a probability of 0.1.

Binomial distribution

A **probability distribution** is a function or rule that assigns probabilities of occurrence to each possible outcome of a random event. Probability distributions give us a visual representation of all possible outcomes of some event and the likelihood of obtaining one outcome relative to the other possible outcomes.

A **binomial distribution** is a specific probability distribution. It is used to model the probability of obtaining one of two outcomes, a certain number of times (*k*), out of fixed number of trials (*N*) of a **discrete random event**.

A binomial distribution has only two outcomes: the expected outcome is called a success and any other outcome is a failure. The probability of a successful outcome is *p* and the probability of a failure is 1 - *p*.

A successful outcome doesn't mean that it's a favorable outcome, but just the outcome being counted. Let's say a discrete random event was the number of persons shot by firearms last year. We'd be looking for the probability of obtaining some number of victims out of the pool of shootings. Being shot is neither a favorable nor a successful outcome for the victim, yet it is the outcome we are counting for this discrete variable.

Example of Binomial Distribution

Suppose, according to the latest police reports, 80% of all petty crimes are unresolved, and in your town, at least three of such petty crimes are committed. The three crimes are all independent of each other. From the given data, what is the probability that one of the three crimes will be resolved?

Solution

The first step in finding the binomial probability is to verify that the situation satisfies the four rules of binomial distribution:

Number of fixed trials (n): 3 (Number of petty crimes)

Number of mutually exclusive outcomes: 2 (solved and unsolved)

The probability of success (p): 0.2 (20% of cases are solved)

Independent trials: Yes

Next:

We find the probability that one of the crimes will be solved in the three independent trials. It is shown as follows:

Trial 1 = Solved 1st, unsolved 2nd, and unsolved 3rd

= 0.2 x 0. 8 x 0.8

= 0.128

Trial 2 = Unsolved 1st, solved 2nd, and unsolved 3rd

= 0.8 x 0.2 x 0.8 = 0.128

Trial 3 = Unsolved 1st, unsolved 2nd, and solved 3rd

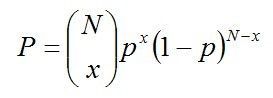
= 0.8 x 0.8 x 0.2

= 0.128

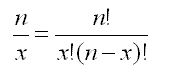
Total (for the three trials):

= 0.128 + 0.128 + 0.128 = 0.384

Alternatively, we can apply the information in the binomial probability formula, as follows:



Where:



 In the equation, x = 1 and n = 3. The equation gives a probability of 0.384.

**5. Describe simple random sampling, systematic sampling, stratified random sampling, cluster sampling with example.**

**Answer:**

**Simple Random Sampling:**

Definition: Simple random sampling is defined as a sampling technique where every item in the population has an even chance and likelihood of being selected in the sample. Here the selection of items entirely depends on luck or probability, and therefore this sampling technique is also sometimes known as a method of chances.

Simple random sampling is a fundamental sampling method and can easily be a component of a more complex sampling method. The main attribute of this sampling method is that every sample has the same probability of being chosen.

The sample size in this sampling method should ideally be more than a few hundred so that simple random sampling can be applied appropriately. They say that this method is theoretically simple to understand but difficult to implement practically. Working with large sample size isn’t an easy task, and it can sometimes be a challenge to finding a realistic sampling frame.

**Example of simple random sampling**

Follow these steps to extract a simple random sample of 100 employees out of 500.

Make a list of all the employees working in the organization. (as mentioned above there are 500 employees in the organization, the record must contain 500 names).

Assign a sequential number to each employee (1,2,3…n). This is your sampling frame (the list from which you draw your simple random sample).

Figure out what your sample size is going to be. (In this case, the sample size is 100).

Use a random number generator to select the sample, using your sampling frame (population size) from Step 2 and your sample size from Step 3. For example, if your sample size is 100 and your population is 500, generate 100 random numbers between 1 and 500.

**Systematic Sampling**

Systematic sampling is a statistical method that researchers use to zero down on the desired population they want to research. Researchers calculate the sampling interval by dividing the entire population size by the desired sample size. Systematic sampling is an extended implementation of probability sampling in which each member of the group is selected at regular periods to form a sample.

Example

A pen factory performs quality control checks on a sample of its pens to ensure they're free of defects. The factory consistently produces 100,000 ballpoint pens every day. Because it remains cost-effective to check a larger sample of the pens, the sample size is 15% of all the pens produced, which amounts to 15,000 pens. Dividing the total output by the sample size equals approximately 6.66, so the sampling interval is six. The quality control team randomly starts at the twelfth pen and performs a quality control check on every sixth pen thereafter.

**6. Describe with example:**

**a. Convenience sample**

**Answer:**

**b. Purposive sampling**

**c. Snow ball sampling**