EE403 Digital Communications
DGIST, 2022 Spring

April 11, 2022
Due: 11:59am, April 27

# Problem Set #3

Discussions are allowed and encouraged, but please write your own answers.

1. (5 pts each) Let $\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$ be the pdf of the standard Gaussian distribution.

   (a) Prove that $Q(x) < \frac{\phi(x)}{x}$ for $x > 0$. [Hint: use $1 < u/x$ if $u > x$.]

   (b) Prove that $\left(\frac{x}{1+x^2}\right)\phi(x) < Q(x)$. [Hint: start from $(1 + x^{-2})Q(x)$ and use $\frac{1}{x^2} > \frac{1}{u^2}$ if $u > x$. You may need $(-\frac{\phi(u)}{u})' = (1 + u^{-2})\phi(u)$.]

   (c) Let $R(x) = \frac{Q(x)}{\phi(x)}$. It is called the Mills ratio and appears in many literature. Prove that $R(x) < \frac{1}{x}$.

2. (10 pts) Consider a half-width rectangular pulse $p(t) = \prod(2t/T)$, e.g., the pulse used in unipolar RZ. In the case of differential coding, the line coding signal $X(t) = \sum_{k=-\infty}^{\infty} a_k p(t-k), a_k \in \{-1, +1\}$ is constructed as follows. If the bit $b_k = 1$, the pulse that is identical to the previous one is sent, i.e., $a_k = a_{k-1}$. If the bit $b_k = 0$, the pulse that flips the previous one is sent, i.e., $a_k \neq a_{k-1}$. See the figure below. For this line coding, derive the power spectral density.

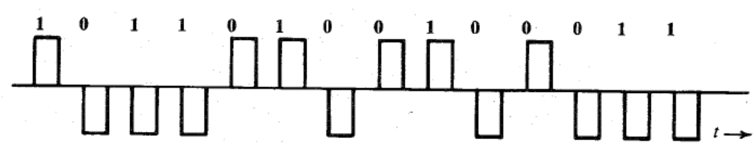


Figure 1: Differential coding

3. (5, 5, 10, 5 pts) Consider a binary communication system utilizing the two signal waveforms shown below. The noise in the channel $n(t)$ is zero-mean white Gaussian noise with power spectral density $\frac{N_0}{2}$. Let $T = 1$ for simplicity.

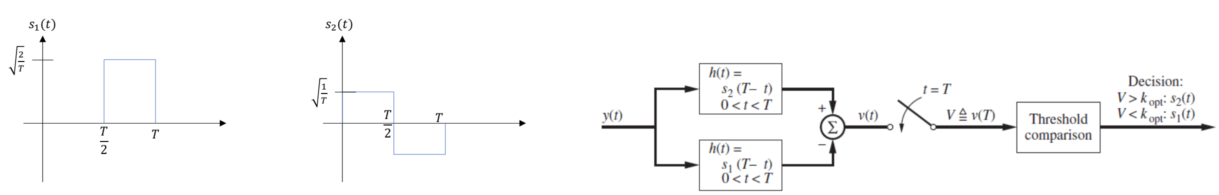


Figure 2: Left: Signal waveforms, right: matched filter-based receiver

   (a) What are the matched filters corresponding to $s_1(t), s_2(t)$? Sketch the waveforms.

   (b) Compute two probability distributions of output of the matched filter $V \triangleq v(T)$ when $s_1(t), s_2(t)$ were sent. Recall the noise is additive.

(c) Compute the smallest error probability and specify its decision rule when the receiver in Fig. 2 right is used. Assume bits are equally likely.

(d) Obtain the smallest error probability using Eq. (9.59) of the textbook. Compare the result with (c).