EE403 Digital Communications | April 6, 2022
DGIST, 2022 Spring | Due: 11:59am, April 13

Problem Set #2

Discussions are allowed and encouraged, but please write your own answers.

1. (5pts each) Two random processes are given by

$$X(t) = A\cos(2\pi f_0 t + \theta)$$
$$Y(t) = A\sin(2\pi f_0 t + \theta),$$

where A, f_0 are given constants and θ is a uniform random variable over $[-\pi, \pi)$.

(a) Find the autocorrelation functions of $X(t)$ and $Y(t)$.

(b) Are $X(t)$ and $Y(t)$ wide-sense stationary? Justify your answer.

(c) Find the power spectral densities of $X(t)$ and $Y(t)$. Are they identical? Why?

2. (5 pts each) A band-limited analog signal is sampled at its Nyquist rate $f_s = 1/T_s$ and then quantized into L distinct levels. The L levels are binarized in the end. The obtained binary stream will be sent in real-time using a line coding scheme that we have learned in class.

(a) Show that one bit symbol duration can be at most $T \leq \frac{T_s}{\log_2 L}$.

(b) Specify when the above equality holds, i.e., $T = \frac{T_s}{\log_2 L}$.

3. (5 pts each) A compact disc (CD) stores audio signals digitally. Assume that the audio signal bandwidth equals 15kHz.

(a) If the Nyquist samples are uniformly quantized into $L = 65536$ levels and then binarized, determine the number of binary digits required to encode a sample.

(b) Determine the number of binary digits per second(bit/s) required to encode the audio signal.

(c) For practical reasons, signals are sampled at a rate well above the Nyquist rate. Practical CDs use 44.1k samples per second. If $L = 65536$, determine the number of bits per second required to encode the signal.

4. (5 pts each) Consider the unipolar RZ coding. See Lecture note 5, pp. 11–16. Suppose the symbol period is T.

(a) Determine values of a_k, their probability, and $p(t)$ for unipolar RZ.

(b) Compute correlation coefficients R_m.

(c) Compute the power spectrum of unipolar RZ coding. Why does it have discrete components?

5. (10 pts) We want to design a transmission system that sends data at 10$k$bps over a channel of bandwidth 8kHz usign raised-cosine pulses. What is the maximum value of the roll-off factor β we can use?

6. (5 pts each) Assume the following channel pulse response samples.

$p_c(-3T) = 0.001 \quad p_c(-2T) = -0.01 \quad p_c(-T) = 0.1 \quad p_c(0) = 1.0 \quad p_c(T) = 0.2 \quad p_c(2T) = -0.02 \quad p_c(3T) = 0.005.$

(a) Find the tap coefficients for a three-tap zero forcing equalizer.

(b) Find the output samples for $mT = -2T, -T, 0, T, 2T$.