Homework #1

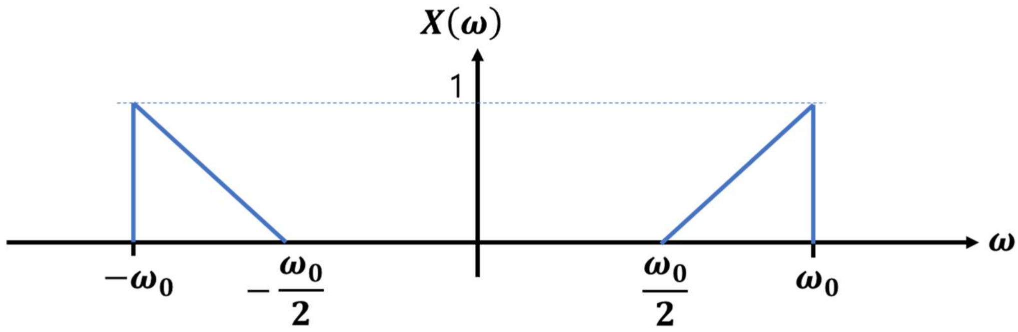
2022 EE401
Digital Signal Processing

1. From Euler's formula,

$$\mathrm{f(t)} = \cos(\omega_0 \mathrm{t}) = \frac{1}{2}\left(e^{j\omega_0 \mathrm{t}} + e^{-j\omega_0 \mathrm{t}}\right).$$

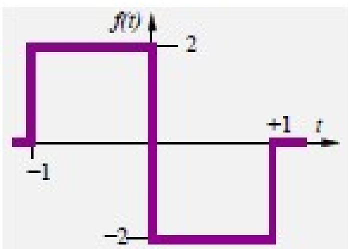
   Prove this relationship using the complex form of the Fourier series. (30 pts)

2. (50 pts) A continuous-time signal $x(t)$, with Fourier transform $X(\omega)$ shown in the figure below, is sampled with a sampling period T to form the sequence $\mathrm{x[n]} = x(nT)$.



   a) What is the Nyquist rate? Sketch the Fourier transform of the sequence $\mathrm{x[n]}$ sampled at the sample rate. Indicate the important parameters and values. (20 pts)
   b) If $\mathrm{T} = 2\pi/\omega_0$, sketch the Fourier transform of the sequence $\mathrm{x[n]}$ and Indicate the important parameters and values. Is this spectrum unique? Explain. (20 pts)
   c) In the case of b), can you recover $\mathrm{x(t)}$ from the sequence $\mathrm{x[n]}$? Provide the rationale behind your answer. (10 pts)

3. (120 pts)
   a) Find the Fourier Transform of $\mathrm{f(t)}$ below. (20 pts)



   b) If the original signal $\mathrm{f(t)}$ is repeated with a 2 period, now called $\mathrm{g(t)}$, how does the spectrum of $\mathrm{g(t)}$ become? Sketch and explain the spectrum. In your sketch, you must label magnitude levels at every significant location. (20 pts)

c) Write a MATLAB program to plot the complex form of the Fourier series of $g(t)$ in the range of $-N \leq n \leq N$ where N=1, 4, 10, 100. (30 pts)

d) Using the MATLAB program, find the value of N when the mean difference between the signal $g(t)$ and the waveform obtained from the Fourier series is less than 1%. Also, plot $g(t)$ and the waveform obtained in the case of the value of N. (20 pts)

e) When you digitize $f(t)$, aliasing inevitably occurs. Explain why. (10 pts)

f) You can digitize $f(t)$ with a minimal error (or aliasing). How to do that? What would be the sampling rate for this? Explain rationale behind what you select the sampling rate. (hint: you can use the results of Question (d)). (20 pts)