Midterm Exam

Name:

Student ID:

Problem	Score
#1	/10
#2	/10
#3	/10
#4	/10
#5	/25
#6	/10
#7	/10
Total	/85

1. (2 pts each) These are 'True/False' questions. Mark your answer. You do not need to justify your answer. You will get 2 points if your answer is correct, and 0 if the answer is wrong or you do not answer.

(a) **(True / False)** Let $x(t)$ be a periodic square wave with fundamental period T. Suppose that we have its Fourier series coefficients $\{X_k\}$. Then, for reconstructed signal $x'(t) = \sum_{k=-\infty}^{\infty} X_k e^{jnw_0 t}$ is identical to $x(t)$ on continuous intervals.

(b) **(True / False)** Among all possible signals, we can design a signal $x(t), t \in \mathbb{R}$ that is nonzero only on a finite interval $t \in [0, T]$ and has finite bandwidth in frequency.

(c) **(True / False)** A system represented by the operation $y(t) = \mathcal{L}\{x(t)\} = \int_0^T x(t)dt$ is an LTI system. Here, T is a given constant.

(d) **(True / False)** The envelop detector will work correctly for DSB-SC (DSB suppressed carrier) if the message signal of the modulator is always positive.

(e) **(True / False)** Super-heterodyne receiver varies the intermediate frequency by controlling local oscillator's frequency so that manufacturing cost is inexpensive.

2. (3, 7 pts) Consider the following signal $a_c(t)$ in Figure 1, where $c > 0$. Let $b(t) = \lim_{c\to\infty} a_c(t)$.

(a) **(Yes / No)** Is $b(t)$ a unit delta function?

(b) Describe the reason for your answer.

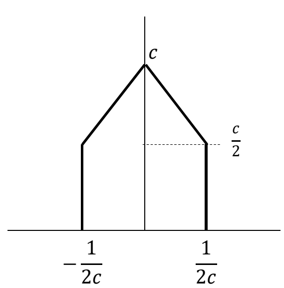

Figure 1: $a_c(t)$

3. (10 pts) Prove that the convolution of two unit sinc functions is also sinc, i.e.,

$$\int_{-\infty}^{\infty} \mathrm{sinc}(t-\tau)\mathrm{sinc}(\tau)d\tau = \mathrm{sinc}(t).$$

4. (5 pts each) Consider a message signal $m(t) = 2\cos(2\pi f_m t) + \cos(2\pi(2f_m)t) + 2\cos(2\pi(3f_m)t)$. Assume the carrier frequency is $f_c \gg f_m$.

(a) Sketch the spectrum of DSB-SC modulated signal. Assume the unit scaling constant.

(b) Sketch the spectrum of lower-sideband SSB modulated signal. Assume the unit scaling constant.

5. (5,5,10,5 pts) Assume the message signal $m(t) = \cos(2\pi f_m t)$ and carrier signal $\cos(2\pi f_c t)$, where $f_m \ll f_c$. Suppose the scaling constant is A_c.

(a) Note that $m(t)$ is already normalized, that is, $|m(t)| \leq 1$. Write down the expression for the AM modulated signal $x_c(t)$ with modulation index α.

(b) Draw the frequency spectrum of $x_c(t)$.

(c) Obtain the message signal up to a constant factor via **coherent** demodulation, that is, the accurate carrier $\cos(2\pi f_c t)$ is available.

(d) Calculate the power efficiency E_{ff} in terms of α.

6. (5 pts each) Assume that a message signal is given by $m(t) = \cos(2\pi f_m t)$, where $f_m < f_c$.

(a) Write down and simplify the following SSB expression using trigonometric identity. Here, $\hat{m}(t)$ is the Hilbert transform of $m(t)$. You do not need to show the derivation of $\hat{m}(t)$.

$$x_c(t) = m(t)\cos(2\pi f_c t) + \hat{m}(t)\sin(2\pi f_c t).$$

(b) Sketch the amplitude spectrum of $x_c(t)$ and specify whether it is lower-sideband SSB or upper-sideband SSB.

7. (10 pts) Suppose that a superheterodyne receiver uses an IF frequency of 455 kHz. The receiver is tuned to a transmitter having a carrier frequency of 1100 kHz. Give two permissible frequencies of the local oscillator and the image frequency for each.

F.2 TRIGONOMETRIC IDENTITIES

$$\cos(u) = \frac{e^{ju} + e^{-ju}}{2}$$

$$\sin(u) = \frac{e^{ju} - e^{-ju}}{2j}$$

$$\cos^2(u) + \sin^2(u) = 1$$

$$\cos^2(u) - \sin^2(u) = \cos(2u)$$

$$2\sin(u)\cos(u) = \sin(2u)$$

$$\cos(u)\cos(v) = \frac{1}{2}\cos(u - v) + \frac{1}{2}\cos(u + v)$$

$$\sin(u)\cos(v) = \frac{1}{2}\sin(u - v) + \frac{1}{2}\sin(u + v)$$

$$\sin(u)\sin(v) = \frac{1}{2}\cos(u - v) - \frac{1}{2}\cos(u + v)$$

$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$

$$\cos^2(u) = \frac{1}{2} + \frac{1}{2}\cos(2u)$$

$$\cos^{2n}(u) = \frac{1}{2^{2n}}\left[\sum_{k=0}^{n-1} 2\binom{2n}{k}\cos 2(n-k)u + \binom{2n}{n}\right]$$

$$\cos^{2n-1}(u) = \frac{1}{2^{2n-2}}\left[\sum_{k=0}^{n-1} 2\binom{2n-1}{k}\cos(2n-2k-1)u\right]$$

$$\sin^2(u) = \frac{1}{2} - \frac{1}{2}\cos(2u)$$

$$\sin^{2n}(u) = \frac{1}{2^{2n}}\left[\sum_{k=0}^{n-1} (-1)^{n-k}\, 2\binom{2n}{k}\cos 2(n-k)u + \binom{2n}{n}\right]$$

$$\sin^{2n-1}(u) = \frac{1}{2^{2n-2}}\left[\sum_{k=0}^{n-1} (-1)^{n+k-1}\, 2\binom{2n-1}{k}\sin(2n-2k-1)u\right]$$

F.3 SERIES EXPANSIONS

$$(u + v)^n = \sum_{k=0}^{n}\binom{n}{k}u^{n-k}v^k, \quad \binom{n}{k} = \frac{n!}{(n-k)!k!}$$

Letting $u = 1$ and $v = x$ where $|x| \ll 1$ results in the approximations:

$$(1 + x)^n \cong 1 + nx; \quad (1 - x)^n \cong 1 - nx; \quad (1 + x)^{1/2} \cong 1 + \frac{1}{2}x$$

$$\log_a u = \log_e u \log_a e; \quad \log_e u = \ln u = \log_e a \log_a u$$

$$e^u = \sum_{k=0}^{\infty} u^k/k! \cong 1 + u, \quad |u| \ll 1$$

F.5 FOURIER-TRANSFORM PAIRS

Signal	Fourier transform
$\Pi(t/\tau) = \begin{cases} 1, & \lvert t\rvert \le \tau/2 \\ 0, & \text{otherwise} \end{cases}$	$\tau \,\mathrm{sinc}(f\tau) = \tau \dfrac{\sin(\pi f \tau)}{\pi f \tau}$
$2W\,\mathrm{sinc}(2Wt)$	$\Pi(f/2W)$
$\Lambda(t/\tau) = \begin{cases} 1 - \lvert t\rvert/\tau, & \lvert t\rvert \le \tau \\ 0, & \text{otherwise} \end{cases}$	$\tau\,\mathrm{sinc}^2(f\tau)$
$W\,\mathrm{sinc}^2(Wt)$	$\Lambda(f/W)$
$\exp(-\alpha t)\,u(t),\ \ \alpha > 0$	$1/(\alpha + j2\pi f)$
$t\exp(-\alpha t)\,u(t),\ \ \alpha > 0$	$1/(\alpha + j2\pi f)^2$
$\exp(-\alpha\lvert t\rvert),\ \ \alpha > 0$	$2\alpha/\left[\alpha^2 + (2\pi f)^2\right]$
$\exp\left[-\pi(t/\tau)^2\right]$	$\tau\exp\left[-\pi(\tau f)^2\right]$
$\delta(t)$	1
1	$\delta(f)$
$\cos(2\pi f_0 t)$	$\frac{1}{2}\delta(f - f_0) + \frac{1}{2}\delta(f + f_0)$
$\sin(2\pi f_0 t)$	$\frac{1}{2j}\delta(f - f_0) - \frac{1}{2j}\delta(f + f_0)$
$u(t)$	$\dfrac{1}{j2\pi f} + \frac{1}{2}\delta(f)$
$1/(\pi t)$	$-j\,\mathrm{sgn}(f);\ \mathrm{sgn}(f) = \begin{cases} 1, & f > 0 \\ -1, & f < 0 \end{cases}$
$\sum_{m=-\infty}^{\infty}\delta(t - mT_s)$	$f_s\sum_{n=-\infty}^{\infty}\delta(f - nf_s);\ \ f_s = 1/T_s$

F.6 FOURIER-TRANSFORM THEOREMS

Name	Time-domain operation (signals assumed real)	Frequency-domain operation
Superposition	$a_1x_1(t) + a_2x_2(t)$	$a_1X_1(f) + a_2X_2(f)$
Time delay	$x(t - t_0)$	$X(f)\exp(-j2\pi t_0 f)$
Scale change	$x(at)$	$\lvert a\rvert^{-1}X(f/a)$
Time reversal	$x(-t)$	$X(-f) = X^*(f)$
Duality	$X(t)$	$x(-f)$
Frequency translation	$x(t)\exp(j2\pi f_0 t)$	$X(f - f_0)$
Modulation	$x(t)\cos(2\pi f_0 t)$	$\frac{1}{2}X(f - f_0) + \frac{1}{2}X(f + f_0)$
Convolution[4]	$x_1(t) * x_2(t)$	$X_1(f)X_2(f)$
Multiplication	$x_1(t)x_2(t)$	$X_1(f) * X_2(f)$
Differentiation	$\dfrac{d^n x(t)}{dt^n}$	$(j2\pi f)^n X(f)$
Integration	$\int_{-\infty}^{t} x(\lambda)\,d\lambda$	$X(f)/(j2\pi f) + \frac{1}{2}X(0)\delta(f)$

[4] $x_1(t) * x_2(t) \triangleq \int_{-\infty}^{\infty} x_1(\lambda)x_2(t - \lambda)\,d\lambda$.