Homework #2

2022 EE401
Digital Signal Processing

1. (110 pts)
   a) The signal $\mathrm{x}_1(t)$ is passed through the LTI system whose impulse response $\mathrm{h}(t)$ where
   $$\mathrm{x}_1(\mathrm{t}) = sinc^2\left(\frac{\omega_0 t}{2\pi}\right), \qquad \mathrm{h}(\mathrm{t}) = sinc\left(\frac{\omega_0 t}{\pi}\right)$$
   Find the output $y_1(t)$ of the LTI system (30 pts).

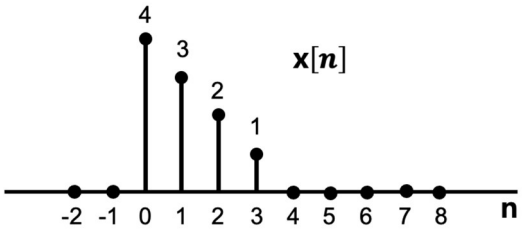
   b) If the signal
   $$\mathrm{x}_2(\mathrm{t}) = sinc^2\left(\frac{\omega_0 t}{\pi}\right)$$
   is passed through the LTI system with $\mathrm{h}(t)$ above, find the output signal $y_2(t)$ (30 pts).

   c) Draw the magnitude of the Fourier transform of $y_2(t)$. You should provide magnitude values and frequency at each important location on the plot (10 pts).

   d) If you digitize $y_2(t)$ at a sampling period of $1/3\omega_0$, draw the magnitude of the spectrum of $y_2[n]$. You should provide indices at each important location on the plot (20 pts).

   e) If you use 30 samples of $y_2[n]$ (n=0, 1, …, 29) to obtain the magnitude of the spectrum of $y_2[n]$ by using a computer, draw the magnitude of the spectrum expected to display on a monitor. You should provide indices at each important location on the plot (20 pts).

2. (60 pts) Consider the real finite-length sequence $x[n]$ shown in the figure below



   a) (20 pts) Sketch the finite-length sequence $y[n]$ whose six-point DFT is
   $$Y[k] = W_6^{4k} X[k],$$
   where $X[k]$ is the six-point DFT of $x[n]$.

   b) (20 pts) Sketch the finite-length sequence $w[n]$ whose six-point DFT is
   $$W[k] = Re\{X[k]\}.$$

   c) (20 pts) Sketch the finite-length sequence $q[n]$ whose three-point DFT is
   $$Q[k] = X[2k], \quad k = 0, 1, 2.$$

3. (40 pts) The following figures shows two finite-length sequences. Sketch their N-point circular convolution for N=6 and for N=10, respectively.