Problem Set #1

Discussions are allowed and encouraged, but please write your own answers.

1. (5 pts each) Using the property of the unit impulse, calculate the following.

   (a) $\int_{-\infty}^{\infty} \cos(6t)\delta(t-3)dt$

   (b) $\int_{-\infty}^{\infty} \frac{10\delta(t-3)}{1+t} dt$

2. (5 pts each) Prove the following three properties of the Fourier series.

   (a) If $x(t)$ is real, $X_n^* = X_{-n}$

   (b) If $x(t)$ is real and even, that is $x(t) = x(-t)$, X_n is purely real and even

   (c) If $x(t)$ is real and odd, that is $x(t) = -x(-t)$, X_n is purely imaginary and odd

3. (5 pts each) Let $y_s(t) = \sum_{m=-\infty}^{\infty} \delta(t - mT_s)$.

   (a) Prove that $y_s(t) = f_s \sum_{n=-\infty}^{\infty} e^{j2\pi n f_s t}$.

   (b) Prove that $y_s(t) \longleftrightarrow Y_s(f) = \sum_{m=-\infty}^{\infty} e^{j2\pi m T_s f} = f_s \sum_{n=-\infty}^{\infty} \delta(f - nf_s)$

4. (5 pts each) Find the Fourier transforms of the signals below. Assume $A, \tau > 0$.

   (a) $x_1(t) = A\exp(-t/\tau)u(t)$

   (b) $x_2(t) = A\exp(t/\tau)u(-t)$

   (c) $x_3(t) = x_1(t) - x_2(t)$

   (d) $x_4(t) = x_1(t) + x_2(t)$

5. (5 pts each) Find the inverse Fourier transforms of the spectra below. For (a), do not use the Fourier transform table. You may use the table for (b) and (c).

   (a) $X_1(f) = \prod(f/2B)$

   (b) $X_2(f) = 2\cos(2\pi f)\prod(f)\exp(-j4\pi f)$

   (c) $X_3(f) = \left[\prod\left(\frac{f+4}{2}\right) + \prod\left(\frac{f-4}{2}\right)\right]\exp(-j8\pi f)$

6. (5 pts each)

   (a) Show that the Fourier transform of $x(t) * y(t) * z(t)$ is $X(f)Y(f)Z(f)$, where $X(f), Y(f), Z(f)$ are Fourier transforms of $x(t), y(t), z(t)$, respectively. Do not use Fourier transform tables.

   (b) What is the Fourier transform of $x(at+b), a \neq 0$? Represent your answer in terms of $X(f)$. Do not use Fourier transform tables.