EE403: Digital Communications

Lecture 5: Baseband Digital Communications

Daewon Seo

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Outline

- Baseband bit representation
  - How to represent 0 and 1 by voltage levels
  - Power spectra (frequency domain)
- Intersymbol interference
  - Pulse shaping
  - Zero-forcing equalizer
- Eye diagram: performance visualization
- Synchronization & modulation

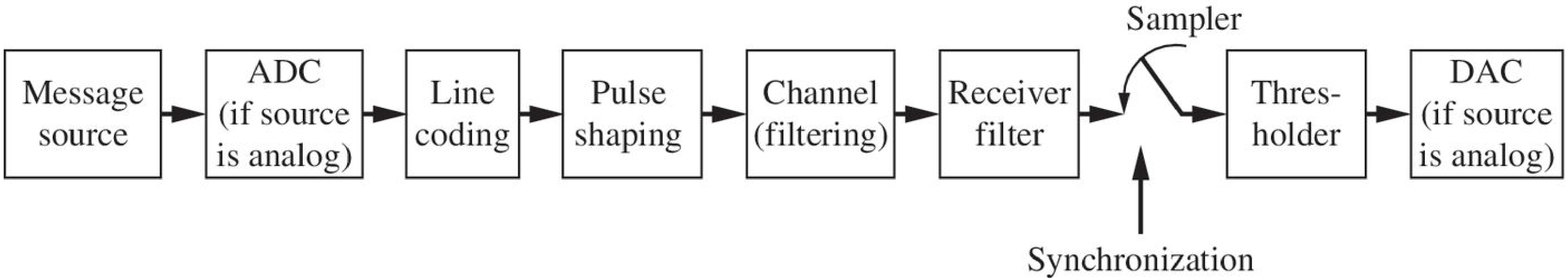
Digital Baseband Signaling

- What is digital?
  - Only a finite number of values during each transmission interval
  - The format may be the result of sampling and quantizing an analog signal

- Benefits of digital representation
  - Robust to noise, easy to synchronize
  - **Error correction** can be used ➔ No quality degradation after reproduction
  - High spectrum efficiency, etc

- This chapter assumes **no modulation**
  - The signal is in baseband, e.g., common in wired communications
  - Also assumes **no noise,** too
  - With noise/modulation will be covered later

ADC / DAC

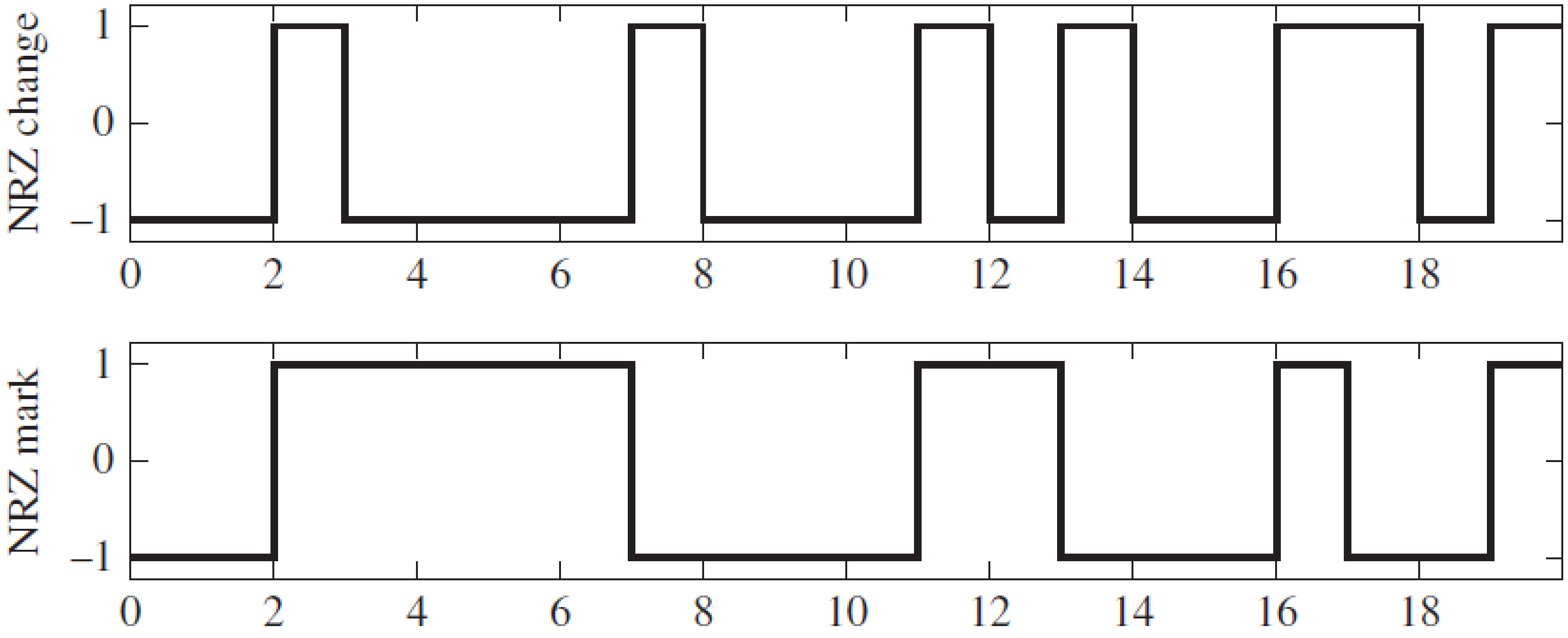


- ADC (if source is analog)
  - Quantization: quantize and binarize
  - Sampling: BW of the source < $\frac{1}{2} f_s$ (sampling rate)
  - What if the source is not band-limited?
    ➔ Aliasing in spectrum and cannot recover the source perfectly

- DAC converts to its analog form (e.g., via low-pass filtering)

Line Coding

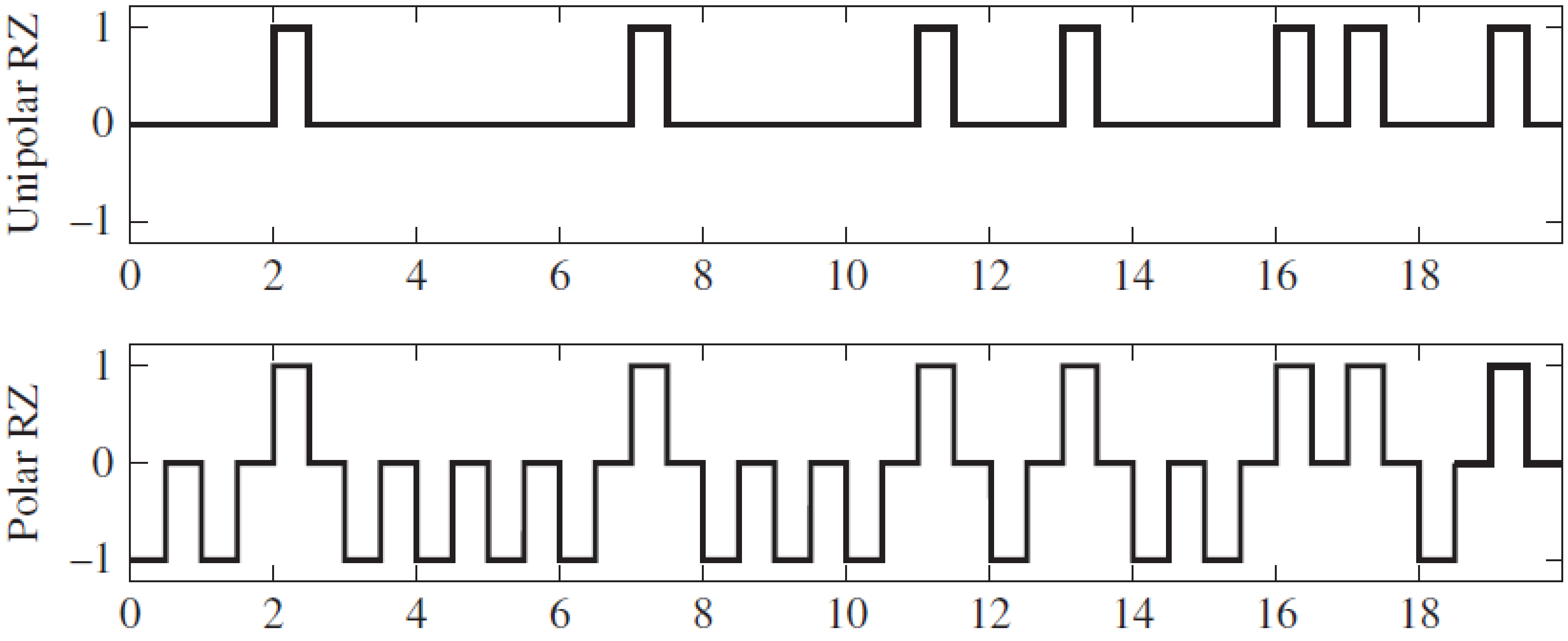
0 0 1 0 / 0 0 0 1 / 0 0 0 1 / 0 1 0 0 / 1 1 0 1



- Nonreturn-to-zero (NRZ) change, or simply NRZ
  - High=1, Low=0
- NRZ mark: toggle=1

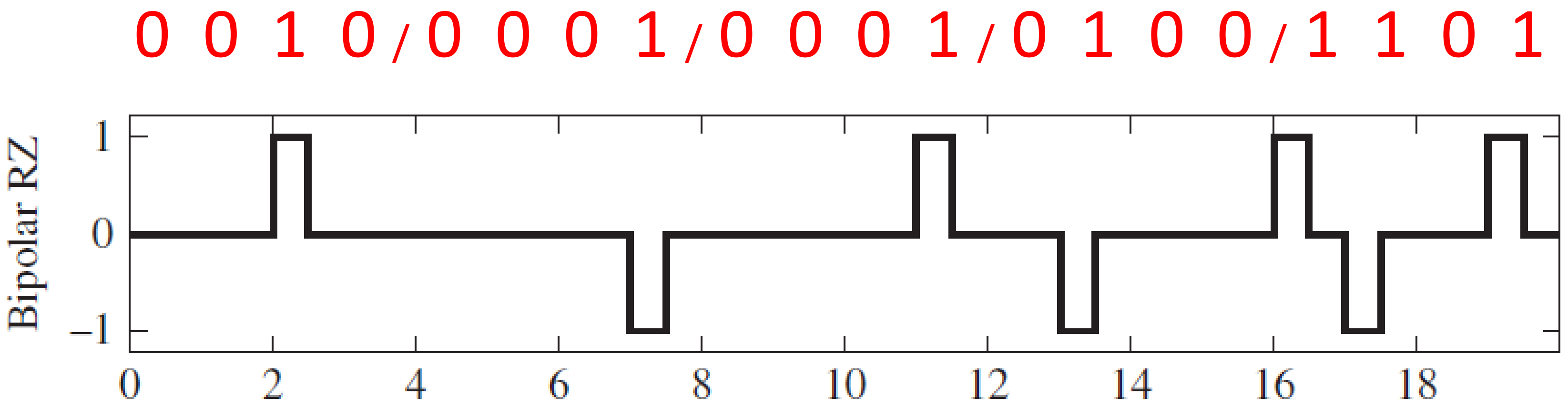
Line Coding

0 0 1 0 / 0 0 0 1 / 0 0 0 1 / 0 1 0 0 / 1 1 0 1



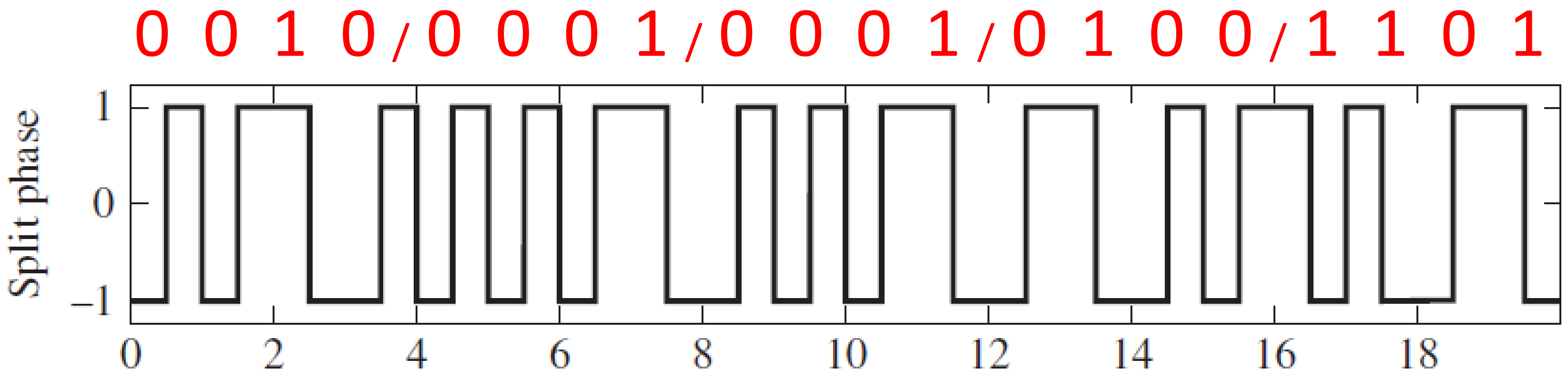
- Unipolar RZ
  - 1/2–width pulse = 1, No pulse=0
- Polar RZ
  - Positive ½-width pulse=1, negative ½-width pulse=0

Line Coding



- Bipolar RZ
  - Zero level=0
  - ½-width pulse with alternate sign=1

Line Coding



- Split phase (Manchester)
  - Switching at the ½ symbol period
  - Low->High=0
  - High->Low=1

Line Coding

- Criteria
  - Self-synchronization
  - Power of signaling
  - Power spectrum of channel: e.g., the channel may not pass low frequencies
  - Intersymbol interference
  - Transmission bandwidth
  - Error performance (in the presence of noise)

Ergodicity

- A sequence of random variables, $X_1, X_2, X_3, \ldots$ is called a **random process**
- Time average$= \frac{1}{N}\sum_{t=1}^{N} X_t$
- Ensemble average at time $t = \mathbb{E}[X_t]$
- A WSS (wide-sense stationary) process $\{X_t\}$ is called **ergodic** if
  - ensemble average does not depend on t
  - ensemble average $=$ time average

(Note: this slide is very informal)

Power Spectra

- The transmitted signal is a pulse train

$$X(t) = \sum_{k=-\infty}^{\infty} a_k p(t - kT)$$

  - a_k: random coefficient, $p(t)$: pulse waveform
  - For instance of NRZ, a_k: each bit (± 1), $p(t)$: width-1 pulse

- Trick:

$$X(t) = \sum_{k=-\infty}^{\infty} a_k p(t - kT) = \underbrace{\left(\sum_{k=-\infty}^{\infty} a_k \delta(t - kT) \right)}_{=:X'(t)} * p(t)$$

Power Spectra

- Autocorrelation if discrete-time and coefficients are deterministic,

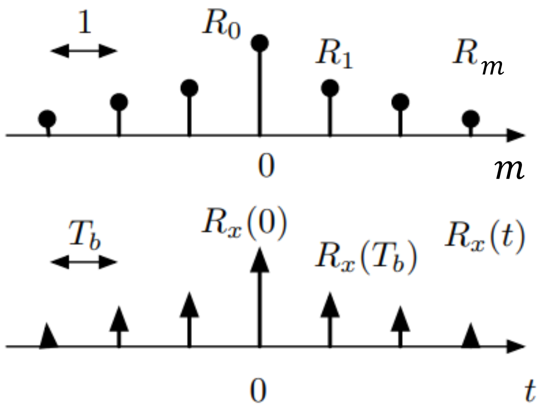
$$X'[t] = \sum_{i=-\infty}^{\infty} a_i \delta[t-i]$$

$$\Rightarrow R_{X'}[m] = \lim_{N\to\infty} \frac{1}{2N} \sum_{k=-N}^{N} X'[k]X'[k-m] = \lim_{N\to\infty} \frac{1}{2N} \sum_{k=-N}^{N} a_k a_{k-m} =: R_m$$

$X'[k]$	$\dots a_1$	a_2	a_3	a_4	a_5 $\dots$					
$X'[k-3]$				$\dots a_1$	a_2	a_3	a_4	a_5 $\dots$		

Autocorrelation R_m = symbol-wise multiplication and then time average

Power Spectra



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Assume time ave = ensemble ave (i.e., ergodic)

- If continuous-time and coefficients are random, letting $\boxed{R_m = \mathbb{E}[a_k a_{k-m}]}$

$$R_{X'}(t) = \frac{1}{T} \sum_{m=-\infty}^{\infty} R_m \delta(t-mT) \Rightarrow S_{X'}(f) = \mathfrak{F}[R_{X'}(t)] = \frac{1}{T} \sum_{m=-\infty}^{\infty} R_m e^{-j2\pi mTf}$$

Power Spectra

- Since $X(t) = X'(t) * p(t)$,

$$S_X(f) = |P(f)|^2 S_{X'}(f)$$

$$\Rightarrow S_X(f) = \frac{1}{T}|P(f)|^2 \sum_{m=-\infty}^{\infty} R_m e^{-j2\pi mTf}$$

Power Spectra

- Example of NRZ
  - The pulse is square,

$$p(t) = \sqcap(t/T) \Rightarrow P(f) = T\mathrm{sinc}(Tf)$$

  - Assuming i.i.d. bit stream,

$$R_m = \begin{cases} \frac{1}{2}A^2 + \frac{1}{2}(-A)^2 = A^2 & m = 0 \\ \frac{1}{4}A(A) + \frac{1}{4}A(-A) + \frac{1}{4}(-A)A + \frac{1}{4}(-A)(-A) = 0 & m \neq 0 \end{cases}$$

  - Therefore, $S_{NRZ}(f) = A^2T\mathrm{sinc}^2(Tf)$

Power Spectra

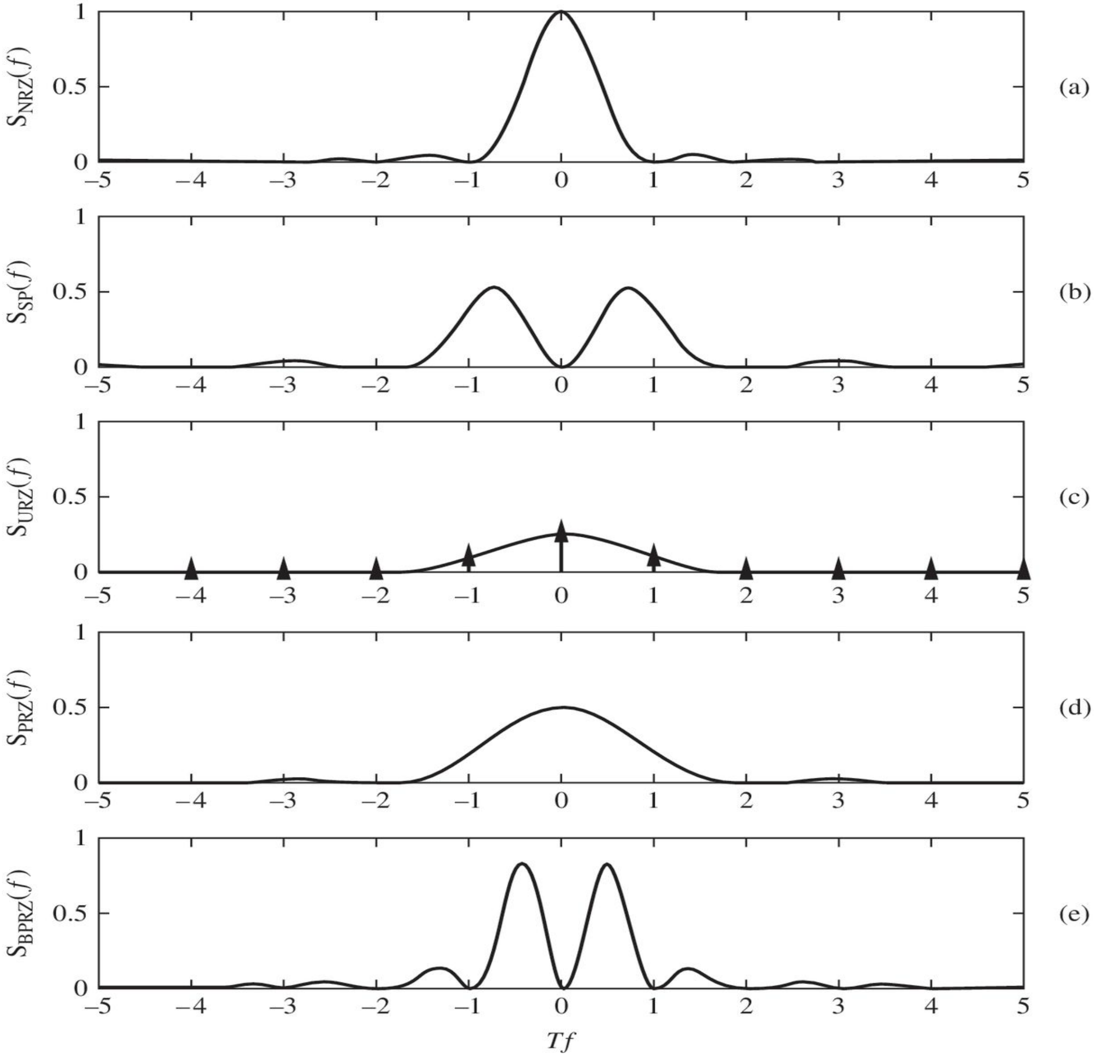
nonreturn-to-zero

Split phase

Unipolar RZ

Polar RZ

Bipolar RZ



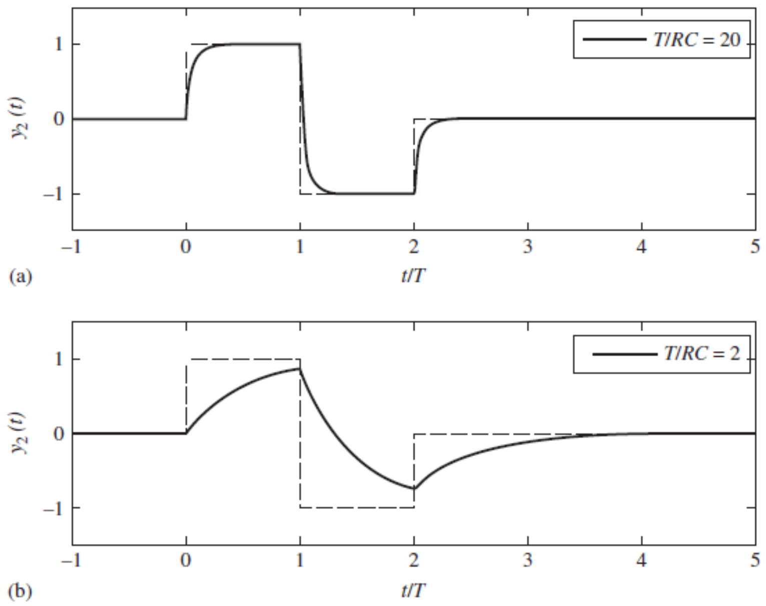
URZ has discrete components as well. Why?

Hint: compute R_m and use the fact that
$\sum e^{-j2\pi mTf} = \frac{1}{T}\sum\delta\left(f - \frac{n}{T}\right)$

Inter-symbol Interference (ISI)

- Ideally, each symbol (or waveform) must be completely separable
- However, ISI occurs typically when bandwidth of the channel is not large enough

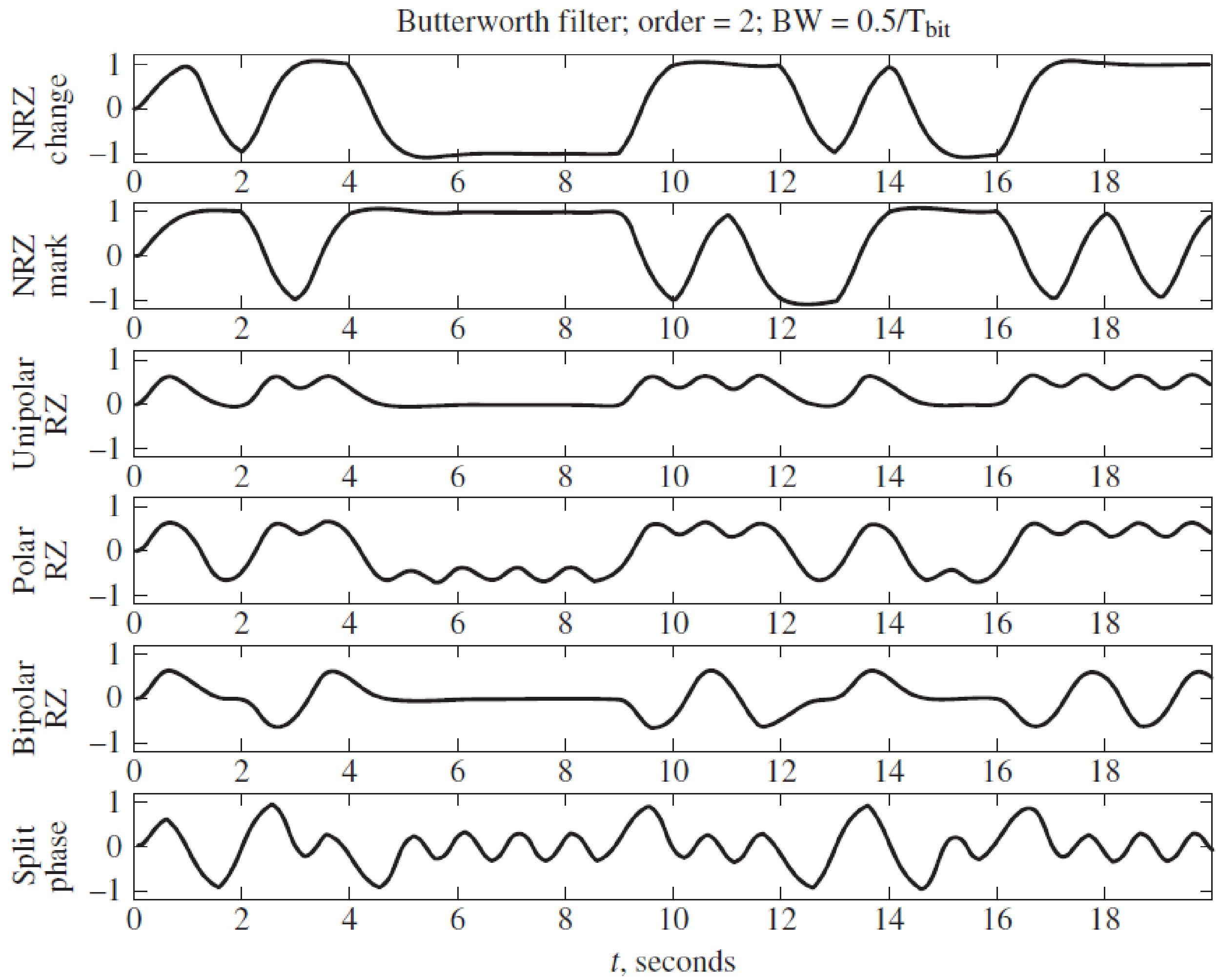
- Example: Rectangular waveform may interfere the next waveform



The channel is indeed an LPF!

1) BW is large enough: no or small ISI
2) BW is not large enough: effect of a previous symbol still survive and interferes the current one

Inter-symbol Interference (ISI)



so so…

indiscernible

Inter-symbol Interference (ISI)

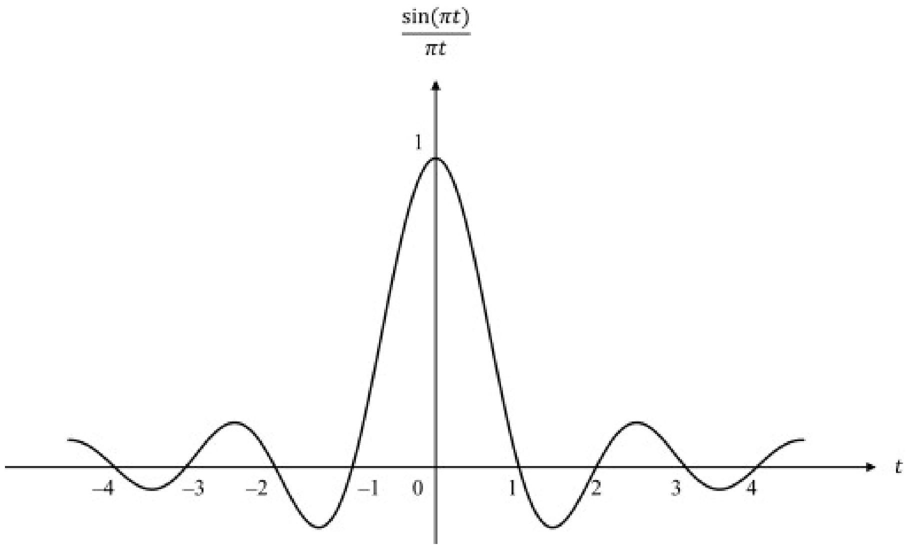
- Two solutions
  - Pulse shaping: Design Tx/Rx filters to ideally eliminate interference between adjacent pulses
  - Equalization: If the channel is non-ideal, an equalizer helps in reducing ISI

ISI Reduction: Pulse Shaping

- Recall that a previous pulse interferes the next symbol's pulse. This is a nature of physical channels (e.g., humidity, walls, hardware devices, ...), which we cannot control

- However, we wish to recover **digital bits** correctly, not exact pulse shape

- Digital bits = values after sampling $\Rightarrow$ interfering next symbols is allowed as long as the **values at sampling points** are unaffected



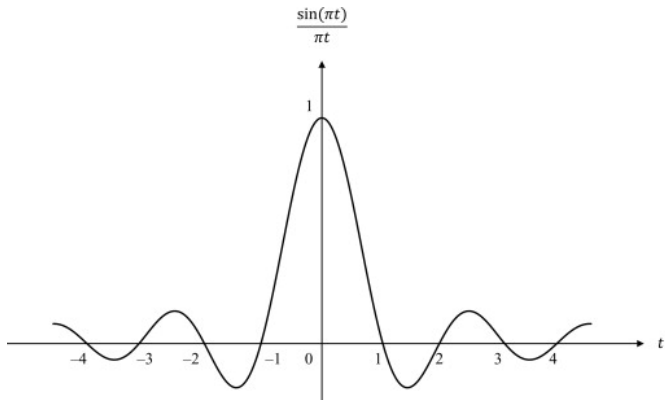
ISI Reduction: Pulse Shaping

- Consider $2W$ independent samples per sec are sent through an ideal LPF channel with bandwidth WHz. The channel output is

$$y(t) = \sum_{n=-\infty}^{\infty} y_n(t) = \sum_{n=-\infty}^{\infty} a_n \text{sinc}\left[2W\left(t - \frac{n}{2W}\right)\right]$$

- If the output is sampled every $t_m = m/2W$, the values are exactly a_m since

$$\text{sinc}(m-n) = \begin{cases} 1 & m = n \\ 0 & m \neq n \end{cases}$$



- Is sinc a unique function satisfying this property? **NO**

ISI Reduction: Pulse Shaping

- One of what people found is **Raised Cosine**, whose time response is

$$p_{RC}(t) = \frac{\cos(\pi\beta t/T)}{1-(2\beta t/T)^2} \cdot \mathrm{sinc}(t/T)$$

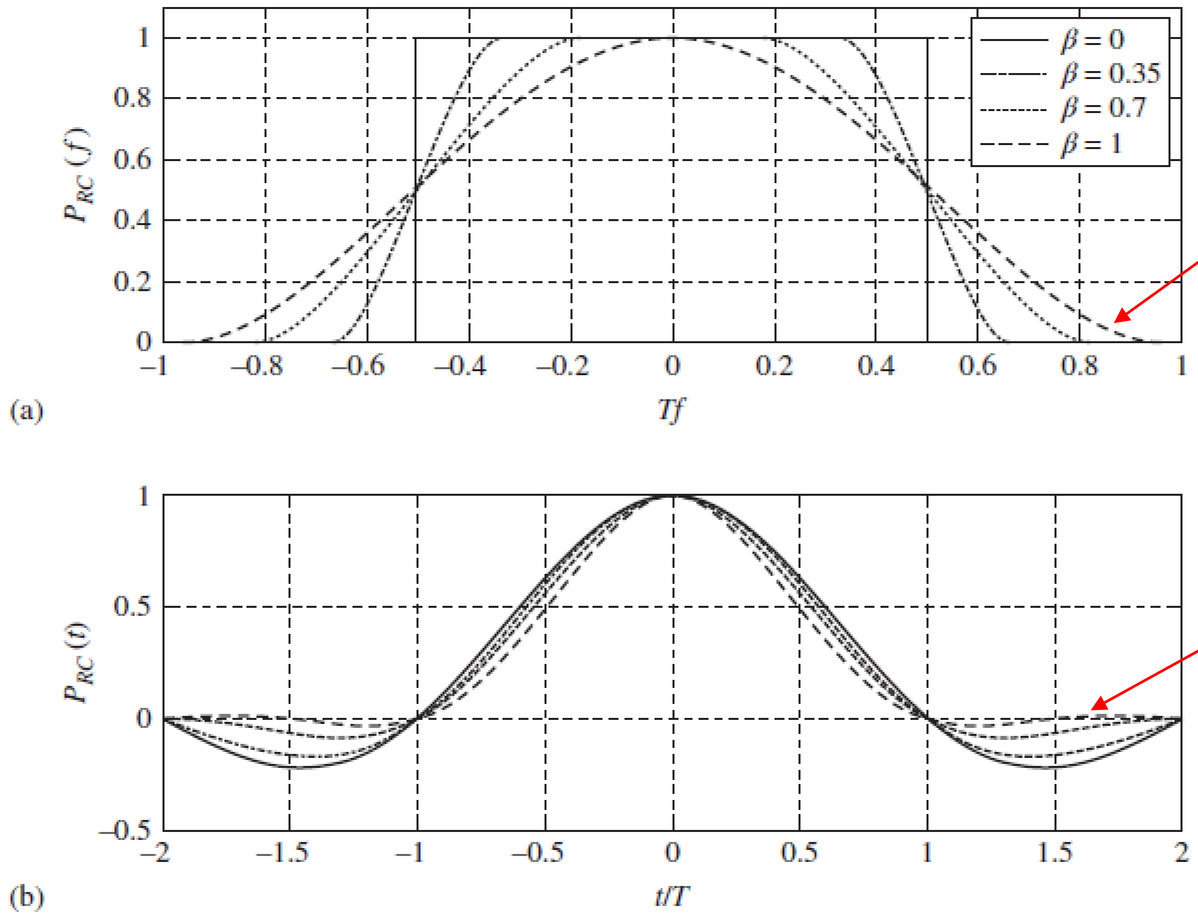
where $\beta \in [0,1]$ is the roll-off factor (smoothness parameter)

- Its frequency response:

$$P_{RC}(f) = \begin{cases} T, & |f| \le \frac{1-\beta}{2T} \\ \frac{T}{2}\left\{1+\cos\left(\frac{\pi T}{\beta}\left[|f|-\frac{1-\beta}{2T}\right]\right)\right\}, & \frac{1-\beta}{2T} < |f| \le \frac{1+\beta}{2T} \\ 0, & |f| > \frac{1+\beta}{2T} \end{cases}$$

  - $\beta = 0$: ideal LPF
  - $\beta = 1$: cosine in frequency

ISI Reduction: Pulse Shaping



$\beta = 1$,cosine

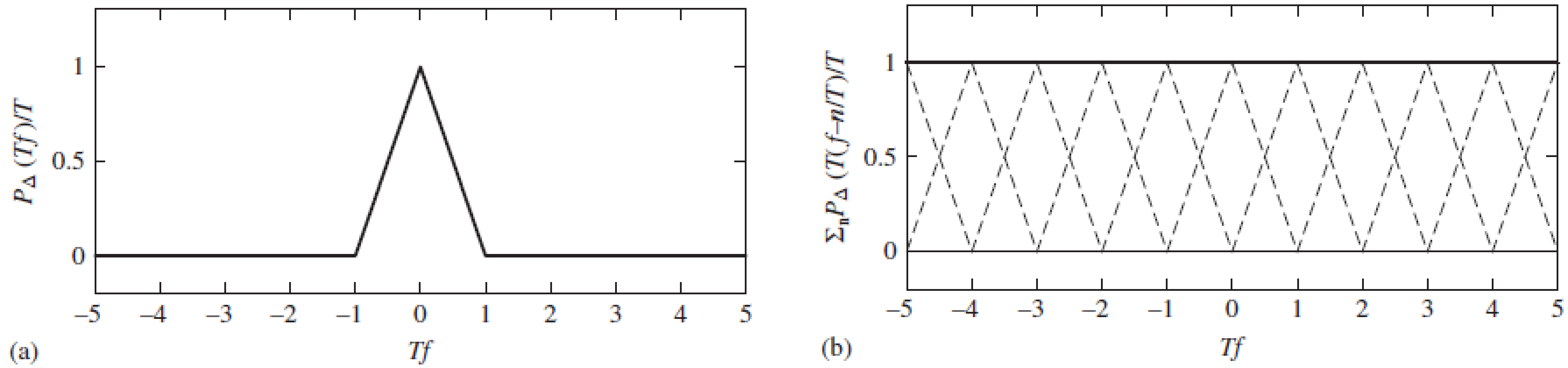
$\beta = 1$, very small side-lobe
➔ Small interference even the timing is in error slightly

ISI Reduction: Pulse Shaping

- Nyquist's pulse shaping criterion: If a pulse shape $p(t)$ satisfies the FT spectrum

$$\sum_{k=-\infty}^{\infty} P(f + k/T) = T, \quad |f| \leq \frac{1}{2T},$$

then, its sampled value $p(nT) = 1$ only when $n = 0$. Otherwise, $p(nT) = 0$



ISI Reduction: Pulse Shaping

(The textbook is wrong)

- Proof: Since $p(t) = \int P(f)e^{j2\pi ft}df$,

$$p(nT) = \int_{-\infty}^{\infty} P(f)e^{j2\pi fnT}df = \sum_{k=-\infty}^{\infty} \int_{\frac{2k-1}{2T}}^{\frac{2k+1}{2T}} P(f)e^{j2\pi fnT}df$$

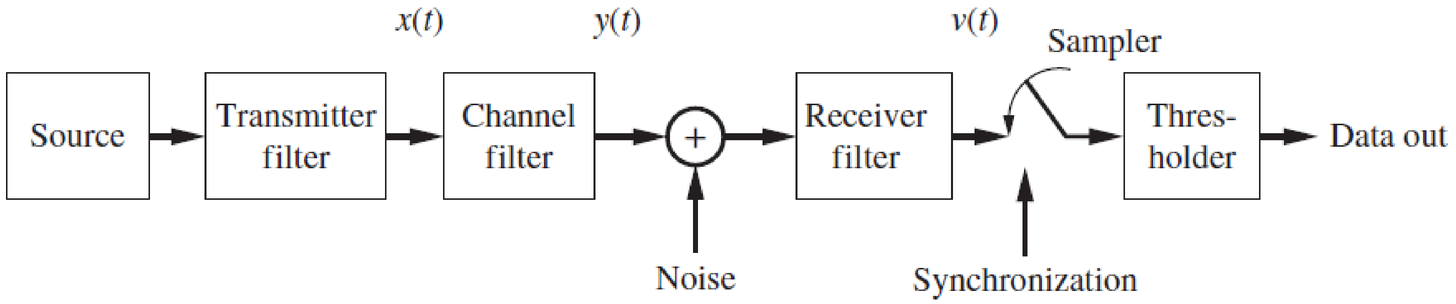
$$= \int_{-\frac{1}{2T}}^{\frac{1}{2T}} \sum_{k=-\infty}^{\infty} P(u+k/T)e^{j2\pi unT}du = \int_{-\frac{1}{2T}}^{\frac{1}{2T}} \left(\sum_{k=-\infty}^{\infty} P(u+k/T) \right) e^{j2\pi unT}du$$

by the change of variable $u = f - k/T$ and interchanging $\sum$ and $\int$.

- Also, $(\cdot)$ is T by the assumption. Hence,

$$p(nT) = T\int_{-\frac{1}{2T}}^{\frac{1}{2T}} e^{j2\pi unT}du = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0. \end{cases}$$

ISI Reduction: Pulse Shaping



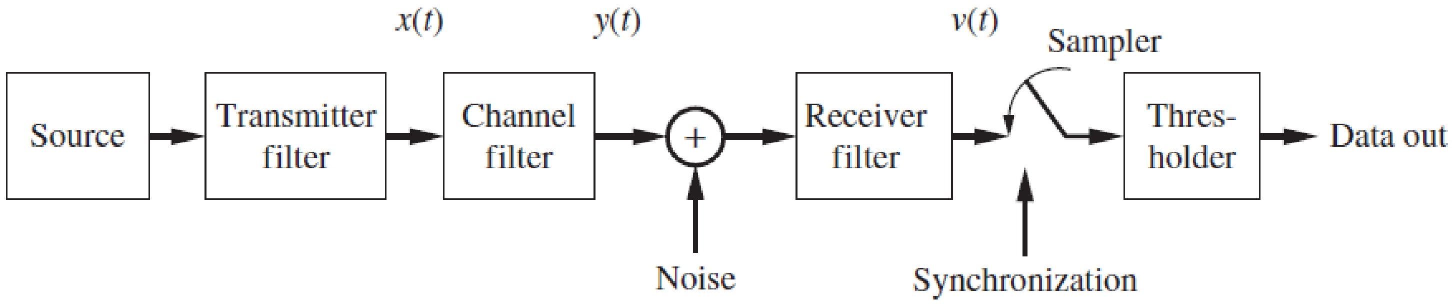
- Suppose the channel input (after Tx filter) is

$$x(t) = \sum_{k=-\infty}^{\infty} a_k \delta(t - kT) * h_T(t) = \sum_{k=-\infty}^{\infty} a_k h_T(t - kT)$$

- After the channel, $y(t) = x(t) * h_C(t)$
- After Rx filter,

$$v(t) = y(t) * h_R(t) = \sum_{k=-\infty}^{\infty} a_k \delta(t - kT) * h_T(t) * h_C(t) * h_R(t)$$

ISI Reduction: Pulse Shaping



- For $v(t)$ to have zero-ISI property,

$$v(t) = \sum_{k=-\infty}^{\infty} a_k \delta(t - kT) * h_T(t) * h_C(t) * h_R(t) = \sum_{k=-\infty}^{\infty} a_k A \cdot p_{RC}(t - kT)$$

- In other words, $A \cdot P_{RC}(f) = H_T(f) H_C(f) H_R(f)$
- Assume the same Tx, Rx filters, $H_T(f) = H_R(f)$. As the channel is fixed,

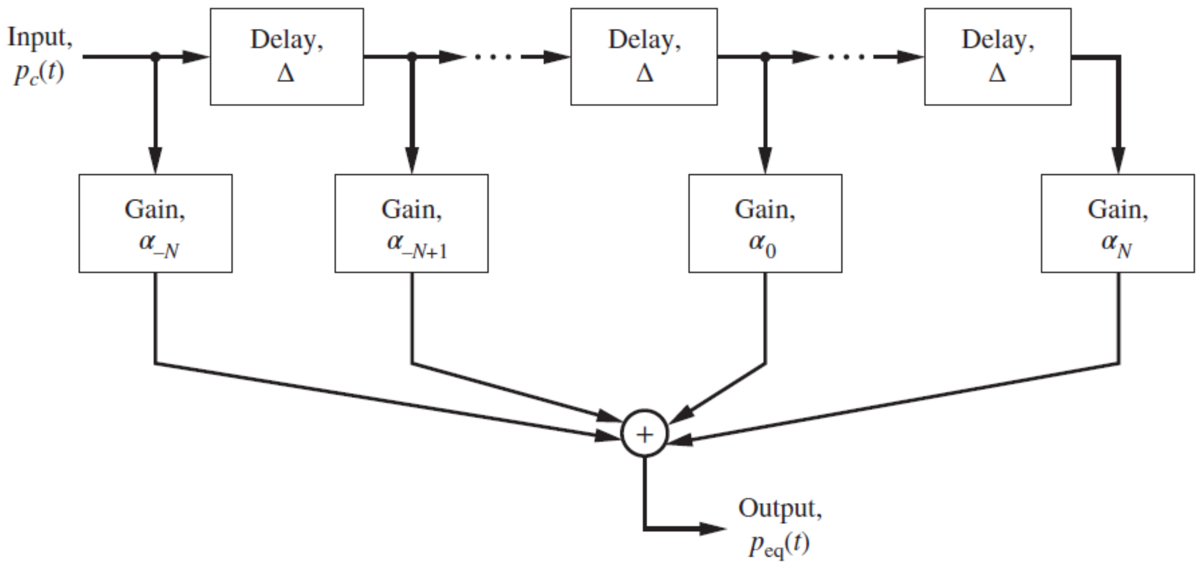
$$|H_T(f)|^2 = |H_R(f)|^2 = \frac{\boxed{AP_{RC}(f)}}{|H_C(f)|}$$

Can be any zero-ISI shapes

ISI Reduction: ZF Equalization

- Pulse-shaping designs Tx, Rx filters so that the overall signal is ISI-free. However, the channel that each device sees could be different. In this case, the signal still has ISI

- For such a case, we can still eliminate ISI by digital signal processing, called zero-forcing equalization



ISI Reduction: ZF Equalization

- Letting $p_c(t)$ is the channel's impulse response, the equalizer's impulse response is

$$p_{eq}(t) = \sum_{n=-N}^{N} a_n p_c(t - n\Delta)$$

- Δ is called "tap" space, usually $\Delta = T$
- The zero-ISI condition says

$$p_{eq}(mT) = \sum_{n=-N}^{N} a_n p_c((m-n)T) = \begin{cases} 1, & m = 0 \\ 0, & m \neq 0 \end{cases}$$

ISI Reduction: ZF Equalization

- Rewriting in matrix-vector form,

$$[P_{eq}] = [P_c][A]$$

where

$$[P_{\text{eq}}] = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \begin{matrix} \left.\vphantom{\begin{matrix}0\\0\\\vdots\\0\end{matrix}}\right\} N \text{ zeros} \\ \vphantom{1} \\ \left.\vphantom{\begin{matrix}0\\0\\\vdots\\0\end{matrix}}\right\} N \text{ zeros} \end{matrix}$$

$$[P_c] = \begin{bmatrix} p_c(0) & p_c(-T) & \cdots & p_c(-2NT) \\ p_c(T) & p_c(0) & \cdots & p_c[(-2N+1)T] \\ \vdots & & & \vdots \\ p_c(2NT) & & & p_c(0) \end{bmatrix} \qquad [A] = \begin{bmatrix} \alpha_{-N} \\ \alpha_{-N+1} \\ \vdots \\ \alpha_N \end{bmatrix}$$

ISI Reduction: ZF Equalization

- Multiplying both sides by $[P_c]^{-1}$,

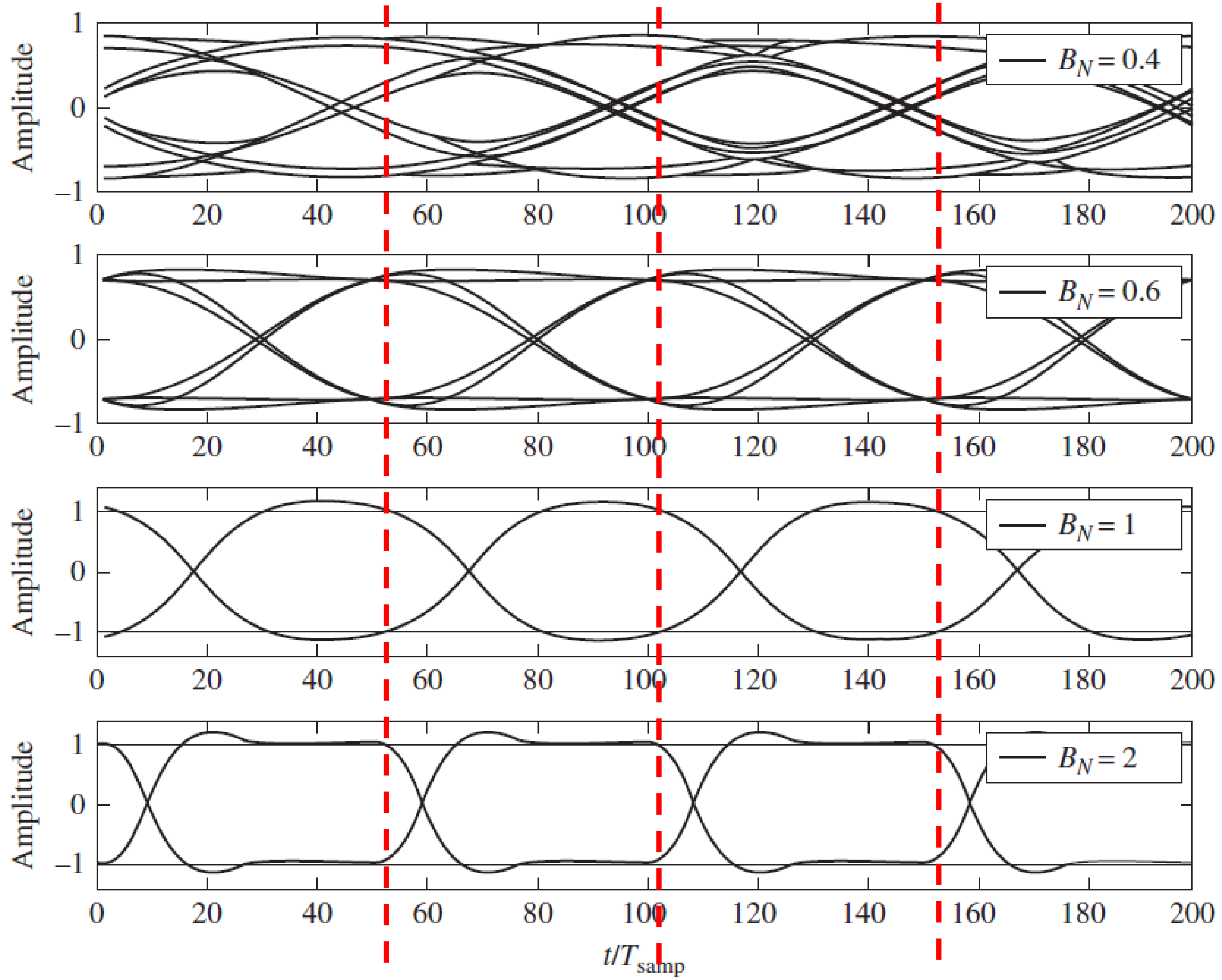
$$[P_c]^{-1}[P_{eq}] = [A].$$

Note that $[P_{eq}]$ only has 1 in the middle, which means $[A] =$ the middle column of $[P_c]^{-1}$

Eye Diagram

- Eye diagram: Constructed by overlapping a number of segments of the baseband signals
- If the signal is accurate, “eye” must be displayed
- A qualitative measure of the system performance.

Eye Diagram



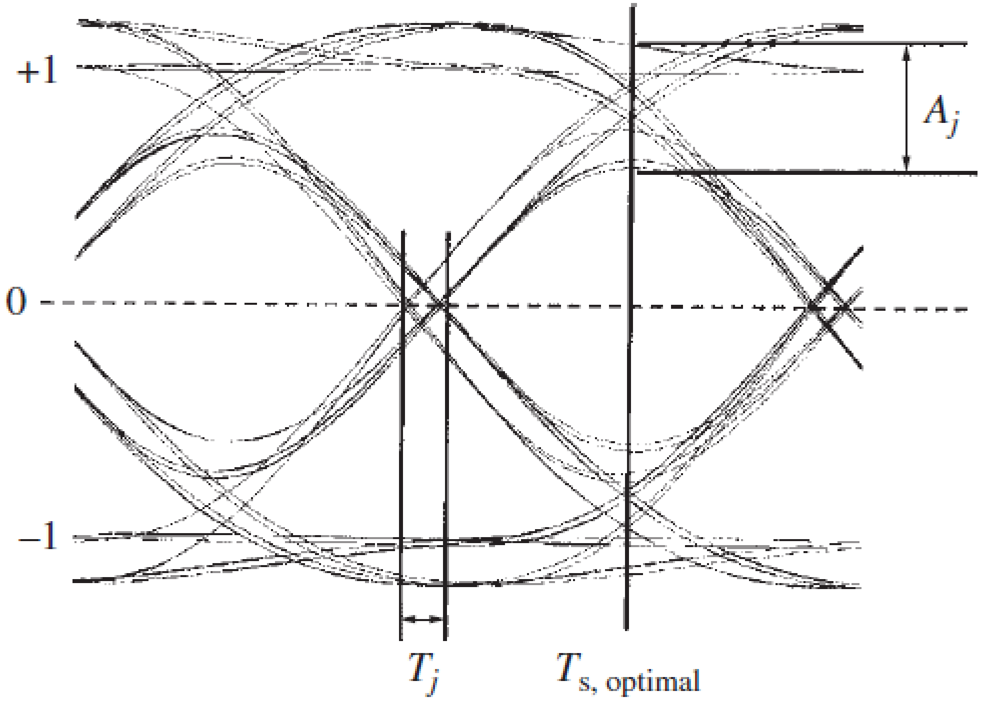
Large ISI due to small BW

Almost no ISI due to large BW (but it accepts large noise if noise is present)

Eye diagram of NRZ baseband signal

Eye Diagram

- Two symbol eye diagram



Synchronization

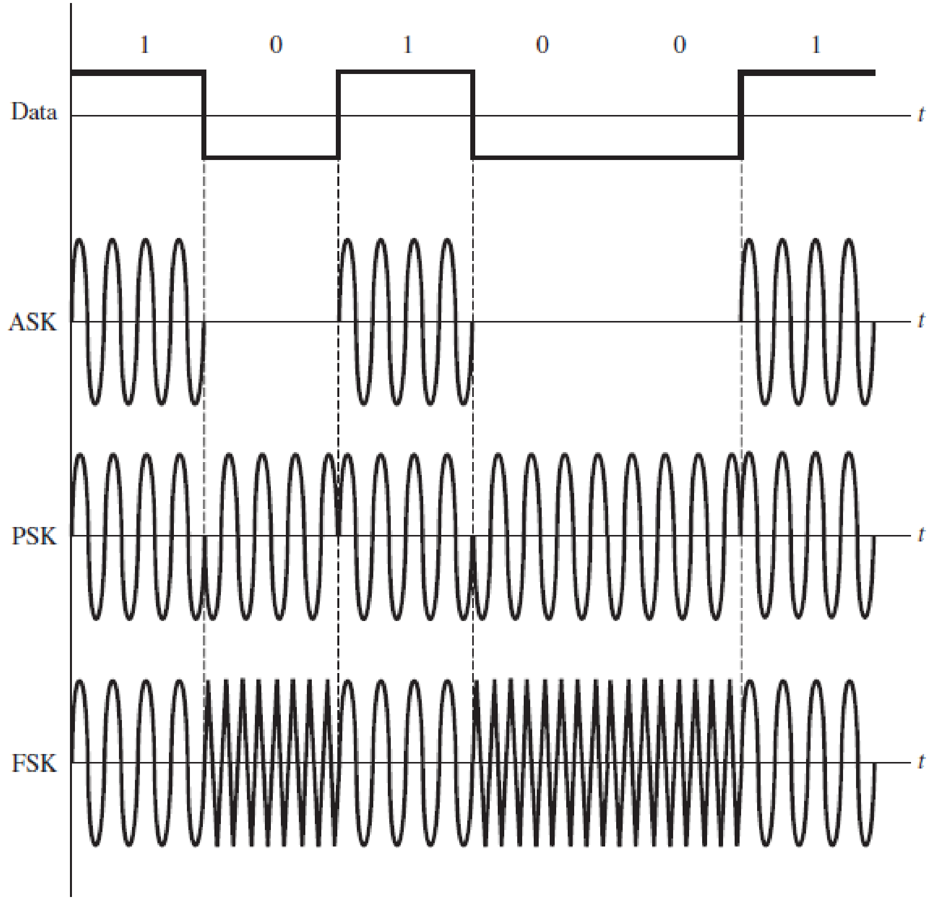
- In analog communication, timing information is needed for coherent detection. For instance, via a Costas PLL

- In digital communication, there are several levels of synchronization
  - Symbol (our focus), codeword, frame

- For symbol synchronization,
  1. External timing source
  2. Send a separate pilot signal
  3. Self-synchronization
     - Polar RZ and split phase always have transition during each symbol
     - For other formats, PLL can be used to track symbol timing (perhaps with nonlinear processing)

Modulation

- Bit streams are typically up-converted using RF carrier modulation
- Example: Assume NRZ data



Amplitude-shift keying (ASK):

$$x_{ASK}(t) = A[1 + d(t)]\cos(2\pi f_c t)$$

Phase-shift keying (PSK):

$$x_{PSK}(t) = A\cos\left(2\pi f_c t + \frac{\pi}{2} d(t)\right)$$

Frequency-shift keying(FSK)

$$x_{FSK}(t) = A\cos\left(2\pi f_c t + k_f \int d(t)\right)$$