Homework #3

2022 EE401
Digital Signal Processing

1. (40 pts)
The system function of a causal linear time-invariant system is

$$H(z) = \frac{1 - z^{-1}}{1 + \frac{3}{4}z^{-1}}$$

The input to this system is

$$\mathrm{x[n]} = \left(\frac{1}{3}\right)^{n} u[n] + u[-n-1]$$

a) Find the impulse response of the system h[n]. (15 pts)
b) Find the output y[n]. (15 pts)
c) Is the system stable? That is, is h[n] absolutely summable? (10 pts)

2. (20 pts)
An LTI system is characterized by the system function

$$\mathrm{H(z)} = \frac{\left(1 - \frac{1}{2}z^{-2}\right)}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 - \frac{1}{4}z^{-1}\right)}, \quad |z| > \frac{1}{2}$$

a) Determine the impulse response of the system. (10 pts)
b) Determine the difference equation relating the system input x[n] and the system output y[n]. (10 pts)

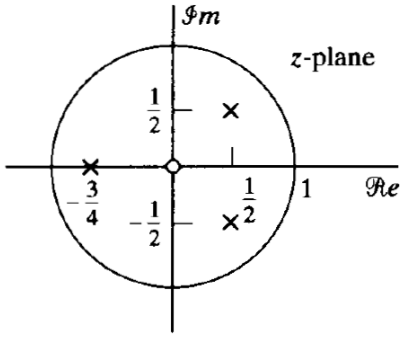
3. (30 pts)
Determine the inverse z-transform for each of the following:

a) $\mathrm{X(z)} = \frac{1}{\left(1+\frac{1}{2}z^{-1}\right)^{2}(1-2z^{-1})(1-3z^{-1})}$, stable sequence. (10 pts)

b) $\mathrm{X(z)} = \frac{3z^{-3}}{\left(1-\frac{1}{4}z^{-1}\right)^{2}}$, x[n] left sequence. (10 pts)

c) $\mathrm{X(z)} = \sin(z)$, ROC includes |z|=1. (10 pts)

4. (30 pts)
The following pole-zero diagram corresponds to the z-transform X(z) of a causal sequence x[n].



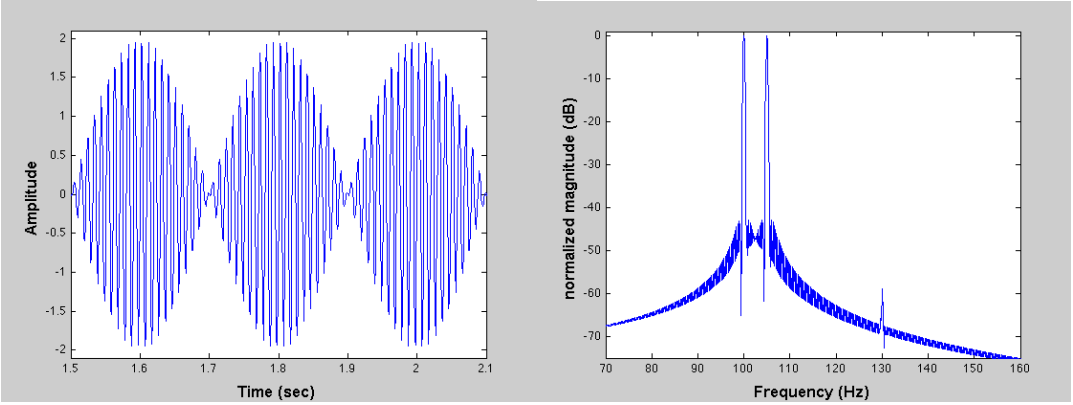
a) Find X(z). (10 pts)
b) Sketch the pole-zero diagram of Y(z), where y[n]=x[-n+3]. (10 pts)
c) Specify the region of convergence for Y(z). (10 pts)

5. (50 pts)
A causal linear time-invariant system has the system function

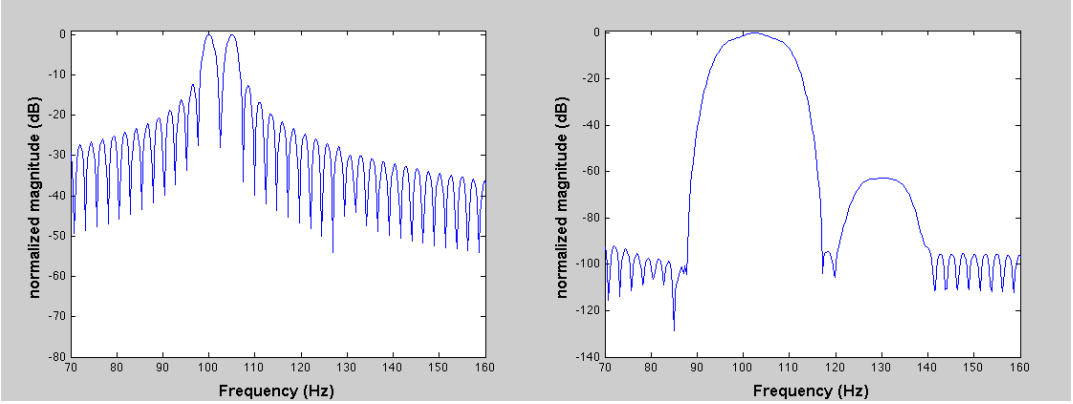
$$\mathrm{H(z)} = \frac{(1 - 1.5z^{-1} - z^{-2})(1 + 0.9z^{-1})}{(1 - z^{-1})(1 + j0.7z^{-1})(1 - j0.7z^{-1})}$$

a) Write the different equation that is satisfied by the input and the output of the system. (10 pts)
b) Plot the pole-zero diagram and indicate the region of convergence for the system function. (10 pts)
c) Sketch $\left|\mathrm{H}\left(\mathrm{e}^{\mathrm{j}\omega}\right)\right|$. (10 pts)
d) State whether the following are true or false about the system: (each 5 pts)
   A. The system is stable.
   B. The impulse response approaches a constant for large n.
   C. The magnitude of the frequency response has a peak at approximately $\omega = \pm\frac{\pi}{4}$.
   D. The system has a stable and causal inverse.

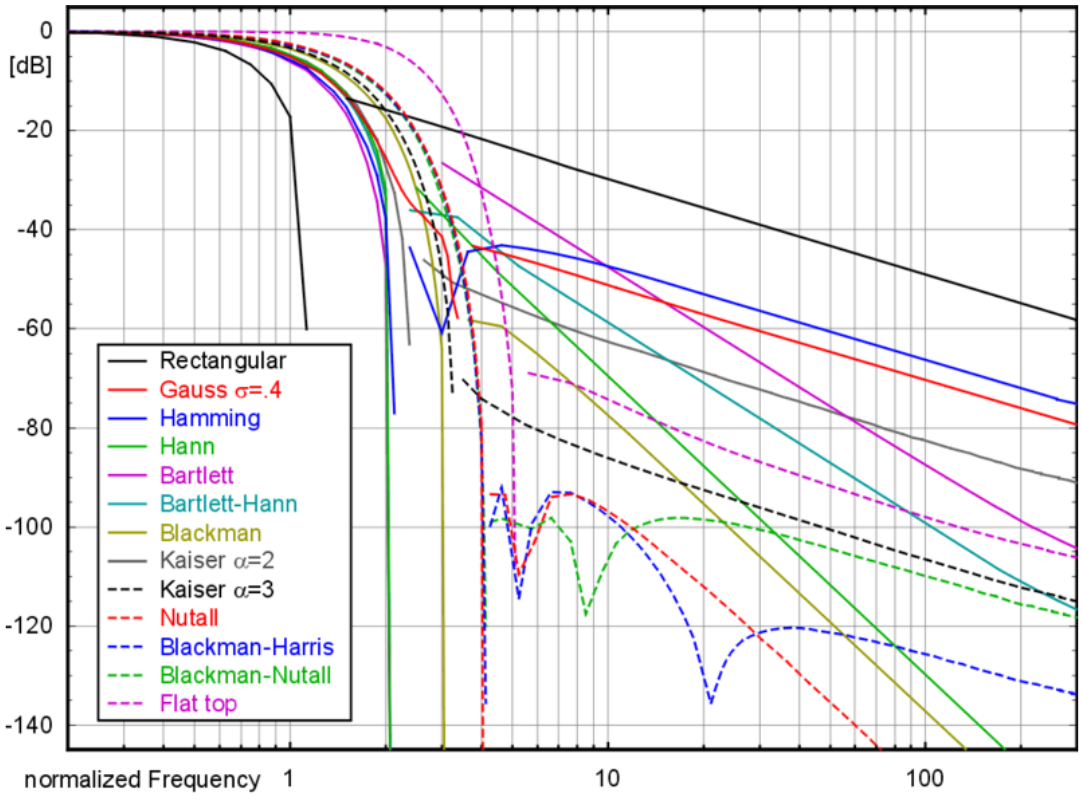
6. (30 pts)
The time signal contains three frequency components at 100, 105, and 130 Hz. Energy of the component at 100 Hz is equal to that at 105 Hz, but it is 1000 times higher than that of the component at 130 Hz. Two different data windows (i.e., A and B windows) were applied to the original time data acquired by a measurement system to analyze its spectral density. The results are shown in the following figures.



<Original Time Signal> < Spectrum of the Time Signal>



<Spectrum of the Signal Windowed by A> <Spectrum of the Signal Windowed by B >



<Frequency was normalized by 5 Hz>

a) Explain theoretically why the two results obtained by the two windows are different (15 pts).
b) By referring to the spectra of various windows, provide all possible names of data windows that may lead to the same result as the window B, i.e., distinguishing the frequency component at 130 Hz from others. You must explain the rationale behind your choice (15 pts).