Backpropagation 1

수업 목표

이번 수업의 핵심:

- Computational graph의 개념
- Backpropagation 적용 예시
- 여러가지 Gradient flow pattern 확인

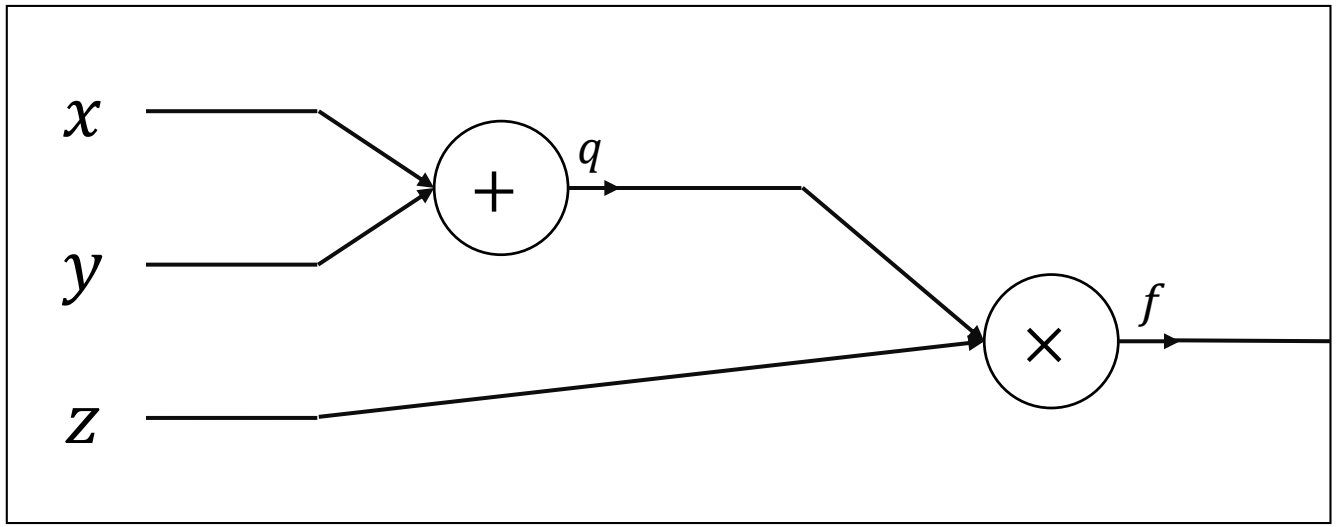
핵심 개념

- Computational graph
- Backpropagation
- Gradient flow pattern

Computational Graph

Computational Graph: 간단한 예시

$$f(x, y, z) = (x + y)z$$

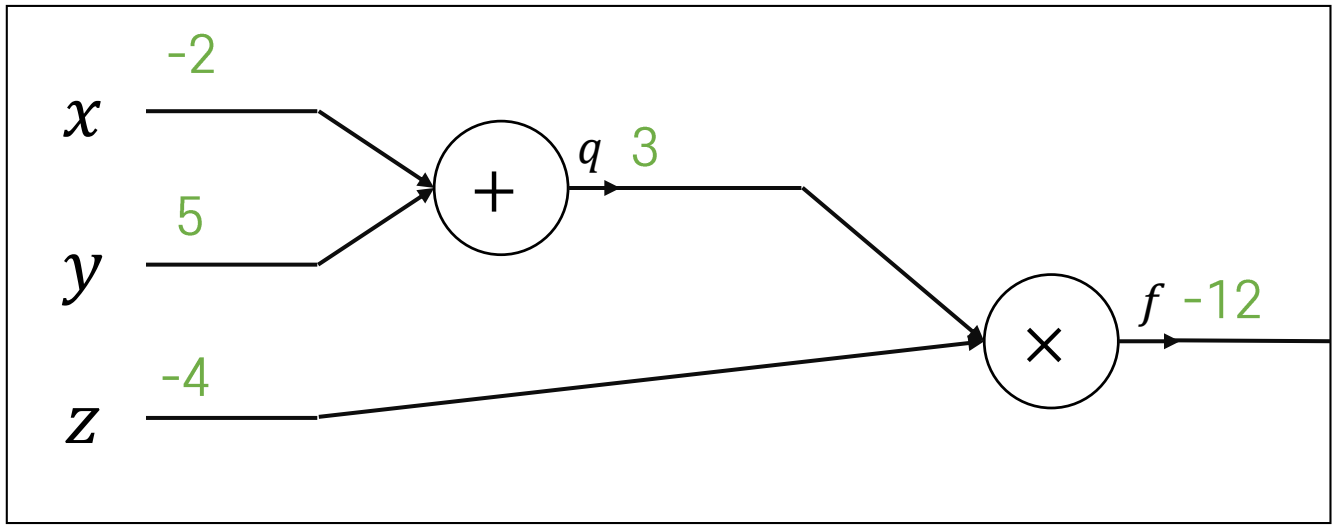


Forward Propagation

Computational Graph: 간단한 예시

$$f(x, y, z) = (x + y)z$$

e.g., $x = -2,\ y = 5,\ z = -4$



1. Forward pass: Output 계산

$$q = x + y, \qquad f = qz$$

Backpropagation

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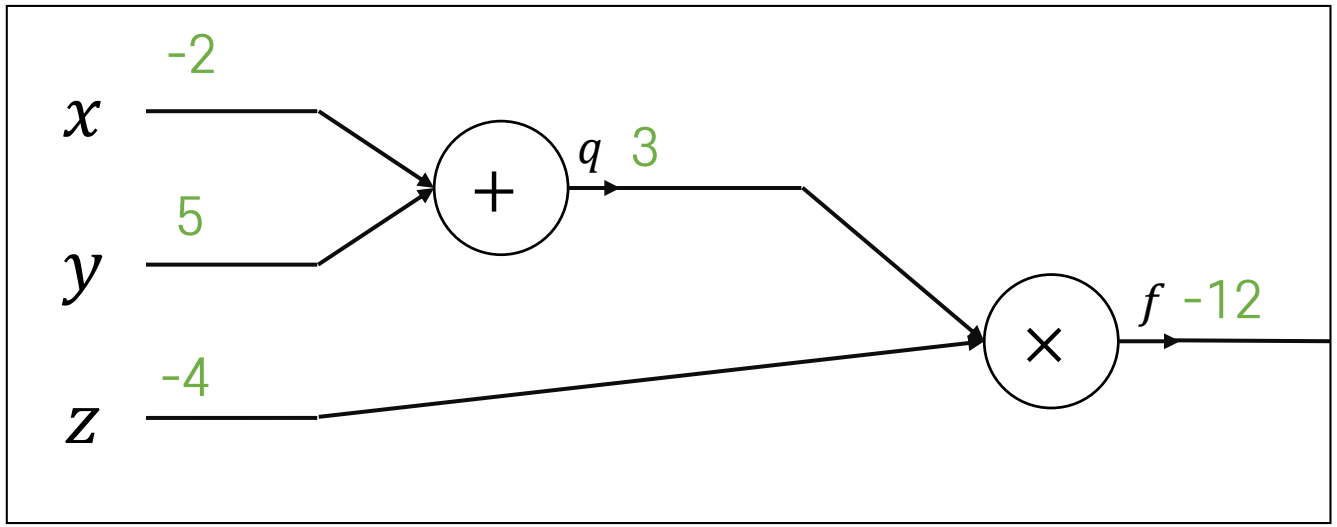

1. Forward pass: Output 계산

$$q = x + y, \qquad f = qz$$

2. Backward pass: Gradient 계산

목표: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$

Backpropagation

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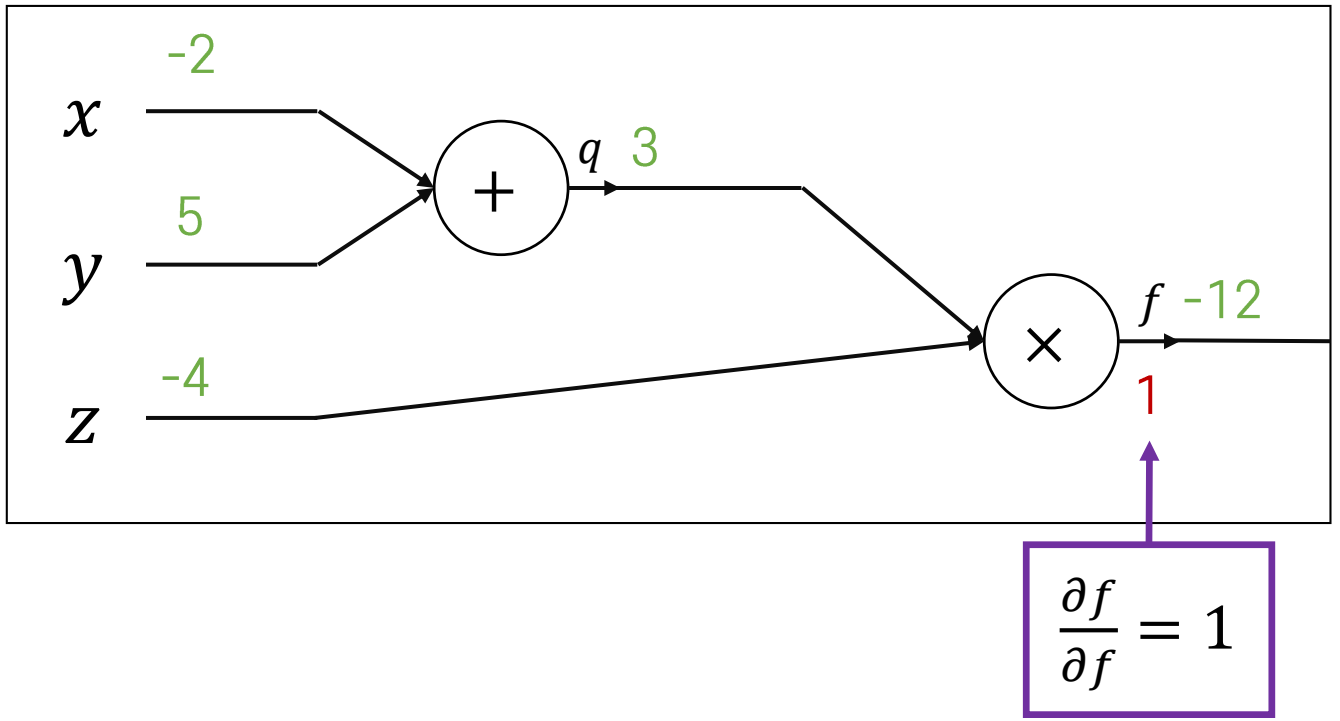

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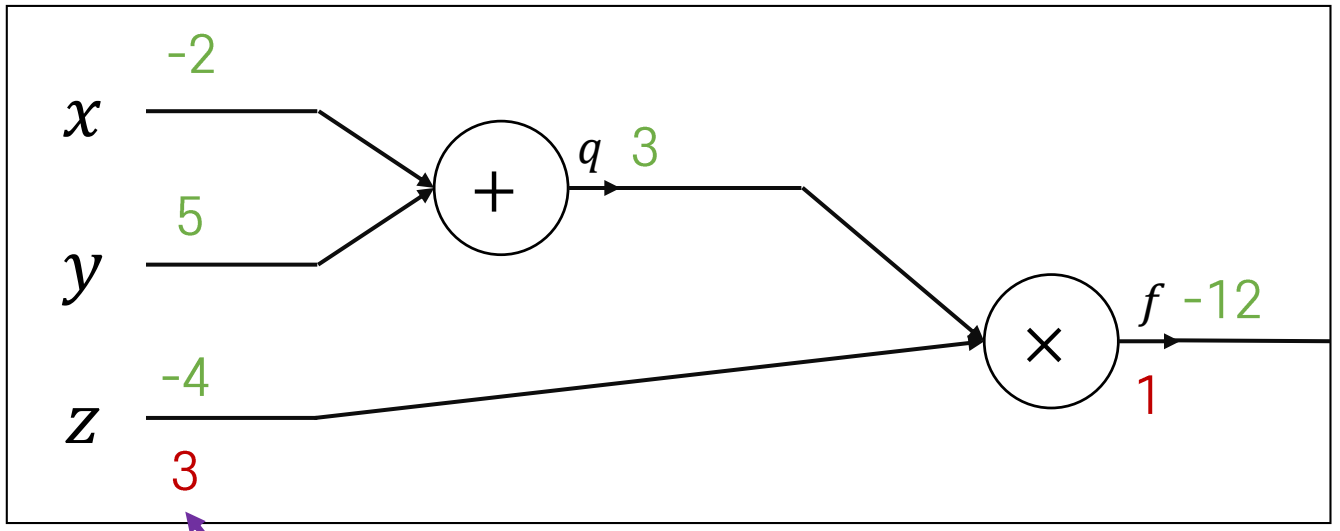

1. Forward pass: Output 계산

$$q = x + y, \qquad f = qz$$

2. Backward pass: Gradient 계산

$$\frac{\partial f}{\partial z} = q$$

목표: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$

Backpropagation

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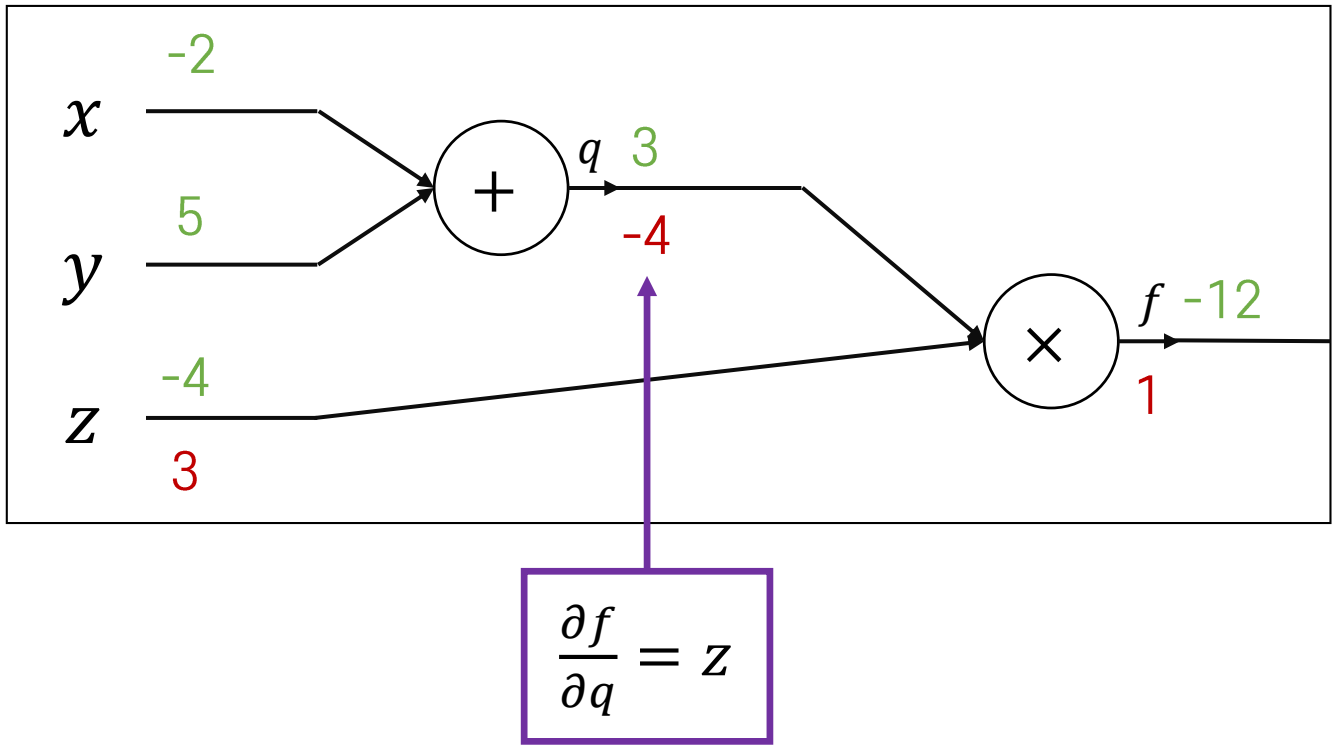

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Backpropagation

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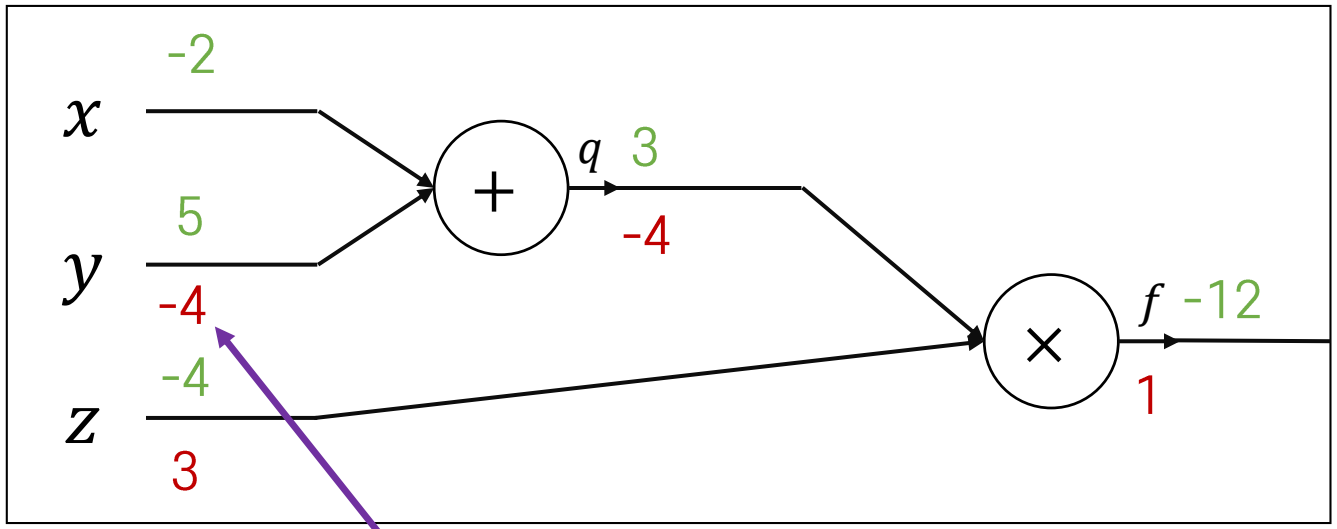

1. Forward pass: Output 계산

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2. Backward pass: Gradient 계산

목표: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$

Chain Rule $\frac{\partial f}{\partial y} = \frac{\partial q}{\partial y}\frac{\partial f}{\partial q}$

$\frac{\partial q}{\partial y} = 1$

Upstream Gradient

Local Gradient

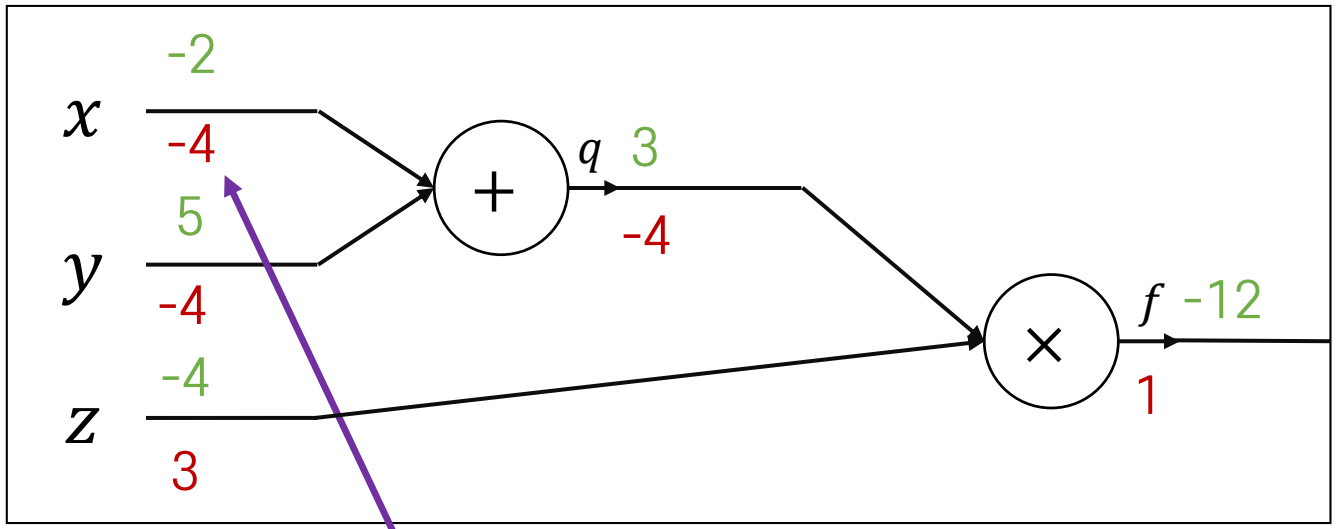
Downstream Gradient

Backpropagation

Computational Graph: 간단한 예시

$$f(x, y, z) = (x + y)z$$

e.g., $x = -2,\ y = 5,\ z = -4$



1. Forward pass: Output 계산

$$q = x + y, \qquad f = qz$$

2. Backward pass: Gradient 계산

목표: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$

Chain Rule $\frac{\partial f}{\partial x} = \frac{\partial q}{\partial x}\frac{\partial f}{\partial q}$

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Upstream Gradient

Local Gradient

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Backpropagation

Computational Graph: 간단한 예시

$$f(x, y, z) = (x + y)z$$

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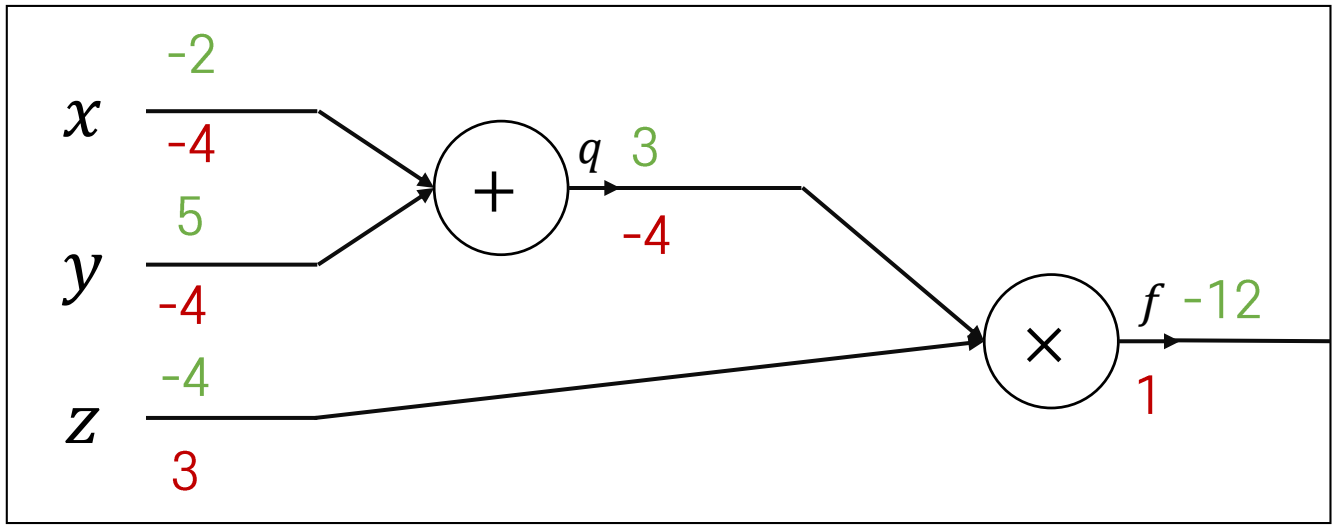


1. Forward pass: Output 계산

$$q = x + y, \qquad f = qz$$

2. Backward pass: Gradient 계산

$$\frac{\partial f}{\partial x} = -4, \frac{\partial f}{\partial y} = -4, \frac{\partial f}{\partial z} = 3$$

만약 Learning rate $\alpha = 0.10$이라면:

$$x := x - \alpha \frac{\partial f}{\partial x} = -2 - 0.1 \times (-4) = -1.6$$

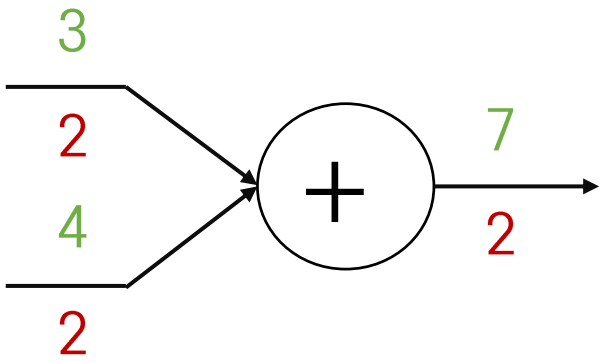
$$y := y - \alpha \frac{\partial f}{\partial y} = 5 - 0.1 \times (-4) = 5.4$$

$$z := z - \alpha \frac{\partial f}{\partial z} = -4 - 0.1 \times 3 = -4.3$$

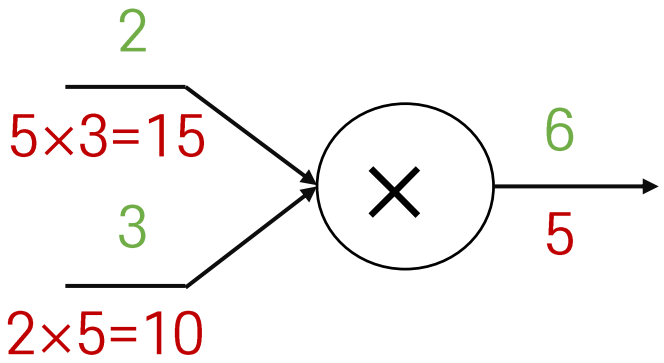
업데이트 이후: $(x + y)z = -16.34$

Gradient Flow의 Pattern

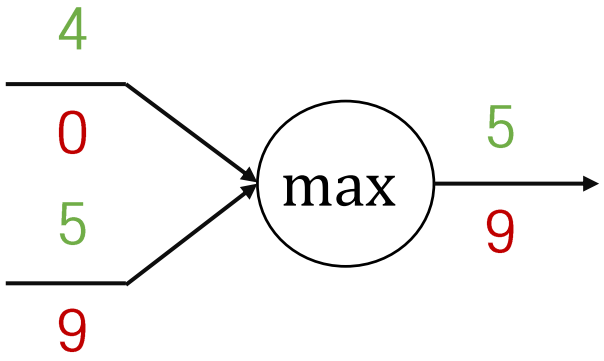
add gate: gradient distributor



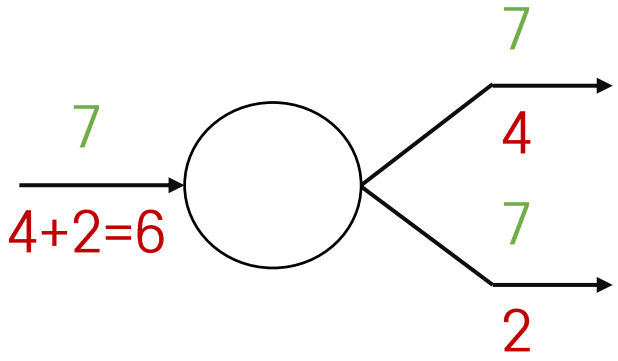
mul gate: swap multiplier



max gate: gradient router



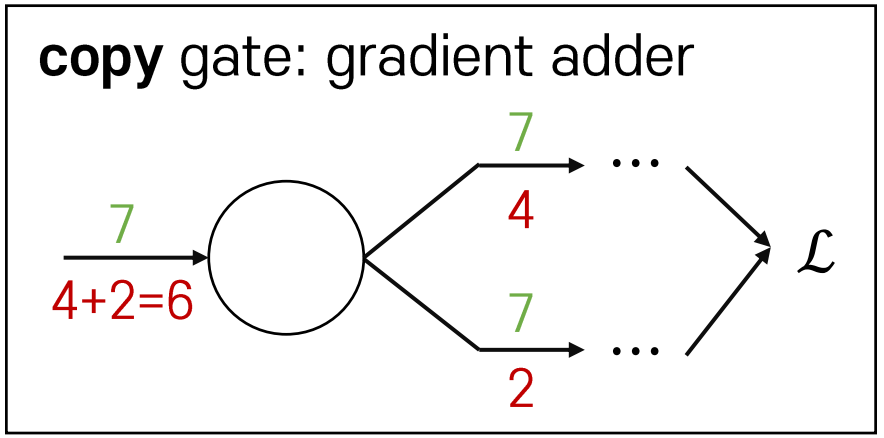
copy gate: gradient adder



Copy Gate

다변수 함수 $h(f(x), g(x))$의 미분:
(증명은 아래 영상 참고)

$$\frac{\partial h}{\partial x} = \frac{\partial h}{\partial f}\frac{\partial f}{\partial x} + \frac{\partial h}{\partial g}\frac{\partial g}{\partial x}$$



Copy gate는 값을 복사하여 이후 Node에 분배

→ 분배된 값은 최종적으로 Loss $\mathcal{L}$에 영향: $\mathcal{L} = h(f(x), g(x))$

이때, 단순 복사해서 전달하므로 $f(x) = g(x) = x$, $\frac{\partial f}{\partial x} = \frac{\partial g}{\partial x} = 1$

$$\frac{d\mathcal{L}}{dx} = \frac{\partial \mathcal{L}}{\partial f}\frac{\partial f}{\partial x} + \frac{\partial \mathcal{L}}{\partial g}\frac{\partial g}{\partial x} = \frac{\partial \mathcal{L}}{\partial f} + \frac{\partial \mathcal{L}}{\partial g}$$

참고 영상:
https://youtu.be/hFvBZf-Jx28
https://youtu.be/NO3AqAaAE6o
https://youtu.be/5mMLaK1ByZc
https://youtu.be/RjwznuzQAOo?t=12m14s (22m 44s까지)

요약

- Computational graph의 구성 요소 및 간단한 예시
- 간단한 Computational graph에서의 Backpropagation 적용
- Copy gate를 포함한 다양한 Gradient flow의 Pattern 확인