① players : Alice, Bob, celiene

Strategy sets : {facebook, twitter, G+} for all players

we could formulate payoffs and make tables like this

player 3 = celiene → {facebook}

(player 2) Bob

(player 1) Alice	facebook	twitter	G+
facebook	$u_{111}, u_{211}, u_{311}$		
twitter			
G+			

player 3 = celiene ⇒ {twitter}

player 3 = celiene ⇒ {G+}

Similar tables to the one above

② players : country_1, country_2

Strategy sets : {attack, remain peaceful} for all players

		q attack (2)	1−q peaceful
p	attack (1)	−10, −10	−13, −15
1−p	peaceful	−15, −13	0, 0

$Eu_1(\text{attack}, q) = -10q + (-13)(1-q)$

$= 3q - 13$

$Eu_1(\text{peaceful}, q) = -15q$

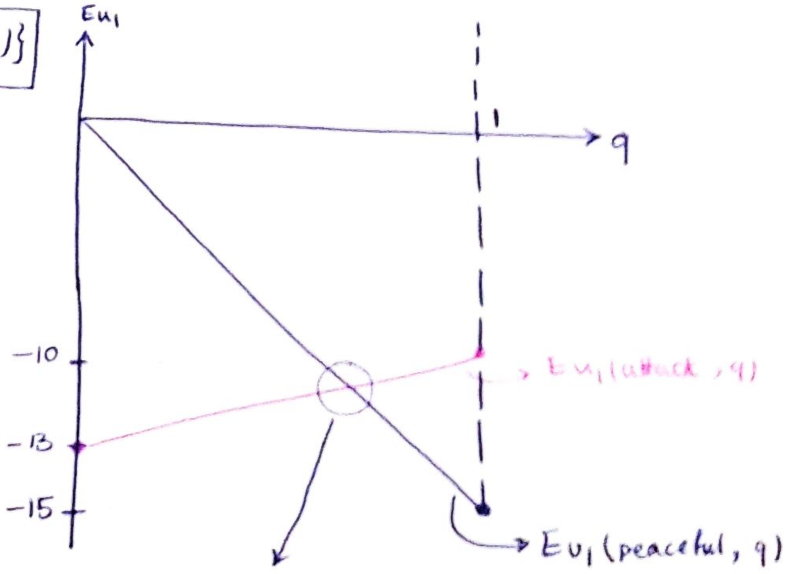
No strictly or weakly dominated strategies

$\boxed{\text{(Nash) NE} = \{(\text{attack}, \text{attack}), (\text{peaceful}, \text{peaceful})\}}$

$Eu_2(\text{attack}, p) = -10p - 13(1-p) = 3q - 13$

$Eu_2(\text{peaceful}, p) = -15p$

So similar plot for Eu_2 and p



$pr_1(\text{attack})$ $\quad pr_2(\text{attack})$

mixed Nash Equilibrium $= \left\{\left(\left(\frac{13}{18}, \frac{5}{18}\right), \left(\frac{13}{18}, \frac{5}{18}\right)\right)\right\}$

$3q - 13 = -15q$

$\boxed{q = \frac{13}{18}}$

③

(1) \ (2)	a	b
A	2,1	1,1
B	1,1	1,3

For player (1):

$u_1(A,a) = 2 \;>\; u_1(B,a) = 1$

$u_1(A,b) = 1 \;\geq\; u_1(B,b) = 1$

$\Rightarrow$ A weakly dominates B

for player (2):

$u_2(A,a) = 1 \;\leq\; u_2(A,b) = 1$

$u_2(B,a) = 1 \;<\; u_2(B,b) = 3$

$\Rightarrow$ a is weakly dominated by b

remove B

For player (1)

(1) \ (2)	a	b
A	2,1	1,1

now a is no longer weakly dominated by b and player (2) is indifferent so this is as far as we can go

Let's find Best responses

		q	1−q
		a	b
p	A	2,1	1,1
1−p	B	1,1	1,3

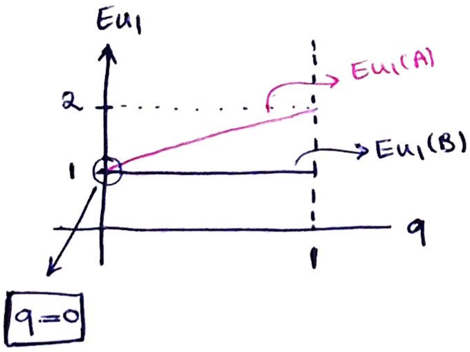
Nash Equilibrium $= \{(A,a), (A,b), (B,b)\}$

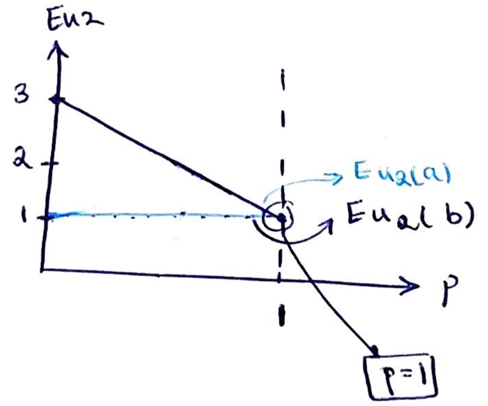
$Eu_1(A,q) = 2q + (1-q) = q+1$

$Eu_1(B,q) = q + 1 - q = 1$

$Eu_2(a,p) = p + 1 - p = 1$

$Eu_2(b,p) = p + 3(1-p) = 3 - 2p$





▶ So there are no mixed Nash equilibriums.

(4)

(1) \ (2)	e	f	g	h
E	2,11	1,9	3,10	17,22
F	27,0	3,1	1,1	1,0
G	4,2	6,10	7,12	18,0

for player (1), G strictly dominats E. for player (2) g weakly dominates f.

No player playes his/her strictly dominated strategy.

remove E for player (1)

(1) \ (2)	e	f	g	h
F	27,0	3,1	1,1	1,0
G	4,2	6,10	7,12	18,0

for player (2) e is strictly dominated by f and f is weakly dominated by g. h is also strictly dominated by f and g.

remove, player(2)'s e and h strategies

(1) \ (2)	f	g
F	3,1	1,1
G	6,10	7,12

for player (1) G strictly dominates F.
for player (2) g weakly dominates f.

remove F for player (1)

(1) \ (2)	f	g
G	6,10	7,12

for player (2) g strictly dominates f.

remove f for player (2)

	g
G	7,12

So the profile strategy resulted by iteratively removing dominated strategies is (G, g)

(5)

		q	1−q
		e	h
p	E	3,−3	−1,1
1−p	H	2,1	7,−6

there are no dominated strategies so we can't solve this game like that.
Let's find Best Responses :

$BR_1(e) = E$ $\quad BR_2(E) = h$

$BR_1(h) = H$ $\quad BR_2(H) = e$

→ No Nash Equilibrium

$Eu_1(E,q) = 3q - 1 + q = 4q - 1$

$Eu_1(H,q) = 2q + 7(1-q) = 7 - 5q$

$\Rightarrow 4q - 1 = 7 - 5q$

$\boxed{q = \frac{8}{9}}$

$Eu_2(e,p) = -3p + 1 - p = 1 - 4p$

$Eu_2(h,p) = p - 6 + 6p = 7p - 6$

$1 - 4p = 7p - 6$

$\boxed{p = \frac{7}{11}}$

mixed NE $= \{((7/11, 4/11), (8/9, 1/9))\}$

(6)

(1) \ (2)	A	B	C
a	3, 2	3, 1	2, 3
b	2, 2	1, 3	3, 2

for player (2), C weakly dominates A.

$\xrightarrow[\text{for player(2)}]{\text{remove } A}$

	B	C
a	3, 1	2, 3
b	1, 3	3, 2

we can't go further.

let's find Best Responses here

		q B	$1-q$ C
p	a	3, 1	2, 2
$1-p$	b	1, 3	3, 2

$$\left.\begin{aligned} Eu_1(a, q) &= 3q + 2(1-q) = q+2 \\ Eu_1(b, q) &= q + 3(1-q) = 3 - 2q \end{aligned}\right\} \xrightarrow{=} q+2 = 3-2q \rightarrow 3q = 1 \rightarrow \boxed{q = 1/3}$$

$$\left.\begin{aligned} Eu_2(B, p) &= p + 3(1-p) = 3 - 2p \\ Eu_2(C, p) &= 3p + 2(1-p) = 2 + p \end{aligned}\right\} \rightarrow \boxed{p = 1/3}$$

so one mixed nash equilibrium is

$$\{((\underbrace{1/3}_{pr_1(a)}, 2/3), (\underbrace{1/3}_{pr_2(B)}, \underbrace{2/3}_{pr_2(C)}))\}$$

But since we removed the weakly dominated strategy that might remove NEs

so we needed to do this calculation on the initial table

		q_a A	q_b B	$1-q_a-q_b$ C
(p)	a	3, 2	3, 1	2, 3
$(1-p)$	b	2, 2	1, 3	3, 2

$$\left.\begin{aligned} Eu_1(a, (q_a, q_b)) &= 3q_a + 3q_b + 2(1-q_a-q_b) = 2 + q_a + q_b \\ Eu_1(b, (q_a, q_b)) &= 2q_a + q_b + 3(1-q_a-q_b) = 3 - q_a - 2q_b \end{aligned}\right\} \xrightarrow{=} 2 + q_a + q_b = 3 - q_a - 2q_b$$

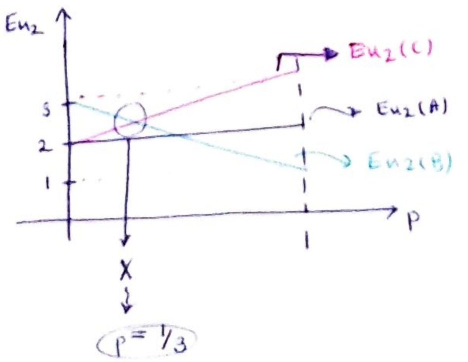
$$\boxed{2q_a + 3q_b = 1} \; (*)$$

$$Eu_2(A, p) = 2p + 2(1-p) = 2$$
$$Eu_2(B, p) = p + 3(1-p) = 3 - 2p$$
$$Eu_2(C, p) = 3p + 2(1-p) = 2 + p$$

$2 = 3 - 2p \rightarrow p = 1/2$

$2 = 2 + p \rightarrow p = 0$ ✗

$3 - 2p = 2 + p \rightarrow \boxed{p = 1/3}$



A is never a Best response so it can not be in the Nash so the mixed Nash is at X and also

$\{q_a = 0\}$ and putting that together with $(*)$

$(q_b = 1/3)$ and $(q_c = \frac{2}{3})$.

$$\text{Nash Equilibrium} = \{((1/3, 2/3), (\underbrace{0}_{pr_2(A)}, \underbrace{1/3}_{pr_2(B)}, \underbrace{2/3}_{pr_2(C)}))\}$$

it seems we get to the same result

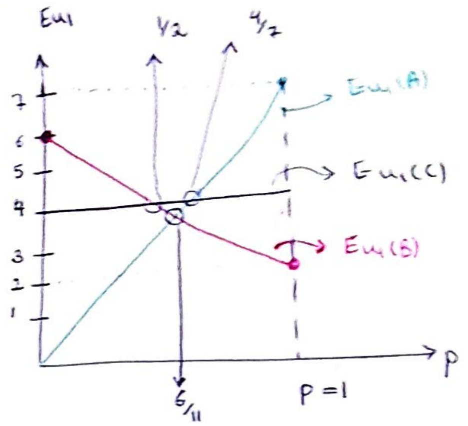
(7)

		p e	$1-p$ h
q_a	A	7,3	0,2
q_b	B	2,1	6,1
$q_c = 1-q_a-q_b$	C	4,0	4,2

$BR_1(e) = A \qquad BR_1(h) = B$

$BR_2(A) = e \qquad BR_2(B) = \{e, h\} \qquad BR_2(C) = h$

$NE = \{(A, e), (B, h)\}$



$Eu_1(A|p) = 7p \longrightarrow p = 6/11$

$Eu_1(B|p) = 2p + 6(1-p) = 6-4p \longrightarrow p = 4/7$

$Eu_1(C|P) = 4p + 4(1-p) = 4 \longrightarrow 4 = 6-4p \rightarrow p = 1/2$

Best responses considering different Ps.

$$BR_1 \begin{cases} B & p < 1/2 \\ C & 1/2 < p < 6/11 \\ C & 6/11 < p < 4/7 \\ A & p > 4/7 \end{cases}$$

So they are all best responses at some point so we can not remove and so we have to consider all points.

$Eu_2(e, (q_a, q_b)) = 3q_a + q_b$

$Eu_2(h, (q_a, q_b)) = 2q_a + q_b + 2(1-q_a-q_b) = 2 - q_b$

$\Longrightarrow 3q_a + q_b = 2 - q_b$

$\boxed{3q_a + 2q_b = 2}$

(8)

(1) \ (2)	a	b	c
A	5,3	70,-1	4,2
B	6,7	71,2	2,1

pure strategy $NE = \{(B, a)\}$

(9)

(1) \ (2)	a	b	c
A	6,7	2,1	4,6
B	0,4	3,8	2,3
C	1,2	1,5	1,1

pure strategy $NE = \{(A, a), (B, b)\}$

(10)

(1) \ (2)	a	b
E	π, e	$\boxed{1-\pi}$, $\sqrt{e}$
H	$\boxed{\sqrt{2}}$, $\frac{1}{e}$	2, 1

assume $e > 1$ → pure $NE = \{(E, a), (H, b)\}$

assume $0 < e < 1$ → pure $NE = \{(H, a), (E, b)\}$

assume $e < 0$: not possible

if $\pi > 1.4$

if $1 < \pi < 1.4$

if $0 < \pi < 1$

if $-1 < \pi < 0$

if $\pi < -1$

blues → pure $NE = \{(H, a)\}$

blues → pure $NE = \{(H, b)\}$

(11)

(1) \ (2)	a	b
A	3 , 5	$2-\alpha , \alpha$
B	$4\alpha , 6$	α , α^2

▶ (A, a) is pure NE if: $3 > 4\alpha \longrightarrow 3/4 > \alpha$, $5 > \alpha$ $\Rightarrow$ $\alpha < 3/4$

So now if $\alpha < 3/4$: $2-\alpha > 2-3/4 \Rightarrow 2-\alpha > 5/4$ then there is no other NE.

▶ (A, b) is pure NE if:

$2-\alpha > \alpha \longrightarrow 2 > 2\alpha \longrightarrow \alpha < 1$

$\alpha > 5$

$\longrightarrow$ X

▶ (B, a) is a pure NE if:

$4\alpha > 3 \longrightarrow \alpha > 3/4$

$6 > \alpha^2 \longrightarrow \alpha < \sqrt{6}$

$\longrightarrow 3/4 < \alpha < \sqrt{6}$

▶ (B, b) is a pure NE if:

$\alpha^2 > 6 \longrightarrow \alpha > \sqrt{6}$

$\alpha > 2-\alpha \longrightarrow \alpha > 1$

$\longrightarrow \alpha > \sqrt{6}$

So here we go:

$$\begin{cases} \alpha > \sqrt{6} & (B, b) \\ 3/4 < \alpha < \sqrt{6} & (B, a) \\ \alpha < 3/4 & (A, a) \end{cases}$$

(12)

(1) \ (2)	e	h
E	1, 1	1, 1
H	0, 2	2, 0

for player (2) e weakly dominates h

$\xrightarrow[\text{remove } h]{\text{player (2)}}$

(1)	e
E	1, 1
H	0, 2

now for player (1) E strictly dominates H

remove H for player (1)

	e
E	1, 1

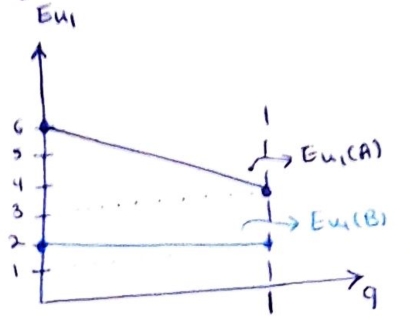
So the strategy profile is (E, e)

(13)

		a (q)	b ($1-q$)
p	A	3, 2	6, 5
$1-p$	B	1, 4	2, 3

(rows: player (1); columns: player (2))

pure NE = $\{(A, b)\}$



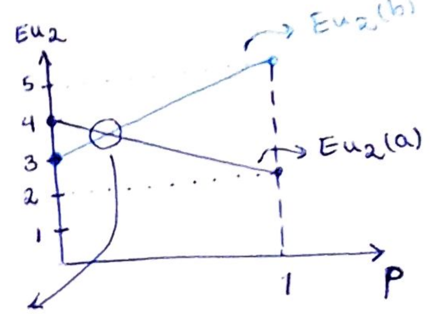
B is never a best response

$Eu_1(A, q) = 3q + 6(1-q) = 6 - 3q$

$Eu_1(B, q) = q + 2(1-q) = 2 - q$

No mixed nash strategies

A strictly dominates B (for player (1))



$Eu_2(a, p) = 2p + 4(1-p) = 4 - 2p$

$Eu_2(b, p) = 5p + 3(1-p) = 3 + 2p$

$3 + 2p = 4 - 2p$

$4p = 1$ $\boxed{p = 1/4}$

so if $p = 1/4$ we would have a mixed strategy but since A dominates B then we know $p = 1$ and thus there are no mixed strategies.

(14) Strictly dominated strategy: for player i, strategy s_i if strictly dominated if there exists another strategy s'_i such that $(s_i \neq s'_i)$: $u_i(s_i, s_{-i}) < u_i(s'_i, s_{-i})$ for all s_{-i}

(15)

		a	b
(1)	A	3, 2	0, 0
	B	1, 1	2, 3

(columns: player (2))

there are no dominated (weakly or strongly) strategies.

$BR_1(a) = A$ $BR_1(b) = B$

$BR_2(A) = a$ $BR_2(B) = b$

pure Nash equilibrium = $\{(A, a), (B, b)\}$

(16) weakly dominated strategy: for player i, s_i is a weakly dominated strategy if there exists a strategy s'_i for player i ($s'_i \in S_i$) if : $\forall s_{-i}$ $u_i(s_i, s_{-i}) \leq u_i(s'_i, s_{-i})$

(17) Normal Form Game includes

+ a finite number of players
+ a strategy set for each player
+ payoff functions for the players : $u_i : S_1 \times S_2 \times \dots \times S_N \to \mathbb{R}$

(18)

(1) \ (2)	C	D
C	-1, -1	-4, 0
D	0, -4	-3, -3

For player (1) and (2) strategy D strictly dominates C

remove C →

	D
D	-3, -3

$BR_1(C) = D(D)$ $BR_1(D) = D(D)$ $BR_2(C) = D$ $BR_2(D) = D$

pure NE = {(D, D)}

(19)

(1) \ (2)	L	C	R
T	5, -1	11, 3	0, 0
B	6, 4	0, 2	2, 0

for player (2) R is strictly dominated by C

remove R

(1) \ (2)	L	C
T	5, -1	11, 3
B	6, 4	0, 2

pure NE = {(B, L), (T, C)}

(20)

(1) \ (2)	C	D
C	-1, -1	-4, 0
D	0, -4	-3, -3
F	-2, -3	-5, 1

for player (1) D strictly dominates C and F.

for player (2) D strictly dominates C.

delete everything other than D →

	D
D	-3, -3

(21)

(1) \ (2)	e	h
E	1, 1	3, 1
H	0, 2	2, 0

for player (1) E strictly dominates H.

remove H →

	e	h
E	1, 1	3, 1

now play (2) is indifferent.

for player (2) e weakly dominates h

(22)

(1) \ (2)	e	h
E	1, 1	2, 1
H	0, 2	2, 0

for player (1) E weakly dominates H

and for player (2) e weakly dominates h.

(23)

(2)

	c_1	c_2	c_3
(1) r_1	4,3	5,1	6,2
r_2	2,1	8,4	3,6
r_3	3,0	9,6	2,8

for player (1) r_1 strictly dominates r_2.

for player (2) c_3 strictly dominates c_2.

remove c_2 and r_2.

(2)

	c_1	c_3
(1) r_1	4,3	6,2
r_3	3,0	2,8

now for player (1) r_1 strictly dominates r_3.

remove r_3

	c_1	c_3
r_1	4,3	6,2

for player (2) c_1 strictly dominates c_3

remove c_3

	c_1
r_1	4,3

the resulting strategy profile : (r_1, c_1)

(24) ok so if we think that everybody will pick 100 then the average would be 100 making (2/3 avg) 66. so any strategy above 66 is strictly dominated by strategies belloco 66.

So now if everybody picks 66. (2/3 avg) would be 44. so any strategy above 44 is dominated (strictly) and thus will not be played.

So now the max. the average can get is 44 making (2/3 avg), $29,\bar{3}$

and if we keep going like that we will see that the only remaining strategy is 1

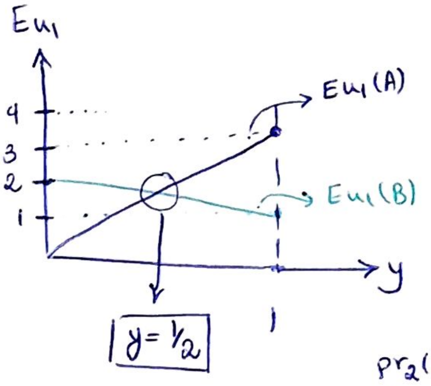
(25) Best response: strateg s^* is a best response for player i in response to strategy profile s_{-i} if and only if

$$u_i(s^*, s_{-i}) \geq u_i(s, s_{-i}) \text{ for all } s \in S_i$$

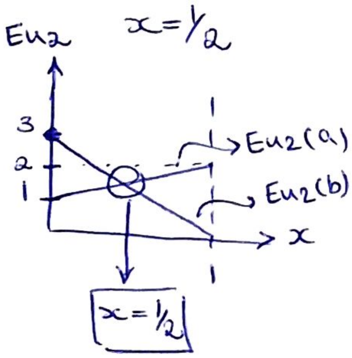
26

		y (2) a	$1-y$ b
x (1)	A	3,2	0,0
$1-x$	B	1,1	2,3

$$Eu_1(A,y) = 3y + 0(1-y) = 3y$$
$$Eu_1(B,y) = y + 2(1-y) = 2-y$$
$$\xrightarrow{=} 3y = 2-y$$
$$y = 2/4 = 1/2$$



$$Eu_2(a,x) = 2x + 1 - x = 1 + x$$
$$Eu_2(b,x) = 3(1-x) = 3 - 3x$$
$$\xrightarrow{=} 3 - 3x = 1 + x$$
$$x = 1/2$$



$$\text{mix NE} = \{((\underbrace{1/2}_{pr_1(A)}, \underbrace{1/2}_{pr_1(B)}), (\overbrace{1/2}^{pr_2(a)}, \overbrace{1/2}^{pr_2(b)}))\}$$

27

	c_1	c_2	c_3	c_4
r_1	5,4	3,3	5,2	3,2
r_2	3,2	9,3	5,2	7,1
r_3	4,5	2,1	5,4	5,6
r_4	2,2	4,0	5,0	8,1

pure NE = $\{(r_1, c_1), (r_2, c_2)\}$

28

	L	C	R
T	1,1	2,2	4,1
M	2,2	5,5	3,6
B	1,4	6,3	0,0

No pure nash equilibrium

29

	L	C	R
T	9,9	2,2	4,1
M	2,2	5,5	3,6
B	1,4	6,3	0,0

pure NE = $\{(T, L)\}$

30

	L	C	R
T	+1,-1	-1,-1	-1,1
M	-1,-1	-1,1	1,-1
B	0,0	0,0	0,0

pure NE = $\{(B, C)\}$

(31)

	L	C	R
T	1, -1	-1, -1	-1, 1
M	-1, -1	-1, 1	1, -1
B	-1, -1	1, -1	-1, 1

no pure nash equilibrium

(32)

1 2 3 4 5 6 7 8 9 10

$u_1(1,1) = 50\% < u_1(2,1) = 90\%$
$u_1(1,2) = 10\% < u_1(2,2) = 50\%$
$u_1(1,3) = 15\% < u_1(2,3) = 20\%$
$u_1(1,4) = 20\% < u_1(2,4) = 25\%$
$u_1(1,5) = 25\% < u_1(2,5) = 30\%$
$u_1(1,6) = 30\% < u_1(2,6) = 35\%$
$u_1(1,7) = 35\% < u_1(2,7) = 40\%$
$u_1(1,8) = 40\% < u_1(2,8) = 45\%$
$u_1(1,9) = 45\% < u_1(2,9) = 50\%$
$u_1(1,10) = 50\% < u_1(2,10) = 55\%$

→ so strategy (1) is strictly dominated by strategy (2).

we can show that 9 is strictly dominated by 10 the same way.

So we will remove the strictly dominated strategies because they will not be played

2 3 4 5 6 7 8 9

$u_1(2,2) = 50\%$ $u_1(3,2) = 90\%$
$u_1(2,3) = 20\%$ $u_1(3,3) = 50\%$
$u_1(2,4) = 25\%$ $u_1(3,4) = 30\%$
$u_1(2,5) = 30\%$ $u_1(3,5) = 35\%$
$u_1(2,6) = 35\%$ $u_1(3,6) = 40\%$
$u_1(2,7) = 40\%$ $u_1(3,7) = 45\%$
$u_1(2,8) = 45\%$ $u_1(3,8) = 50\%$
$u_1(2,9) = 50\%$ $u_1(3,9) = 55\%$

→ so 3 strictly dominates 2 and in the same way 8 strictly dominates 9.

and if we keep going there will only be 5 and 6 left.

(33)

	w	t
W	2, 1	0, 0
T	0, 0	1, 2

pure NE $= \{(W, w), (T, t)\}$

(34)

		p (2) $1-p$	
		l	r
(1)	U	5,1	0,2
	M	1,3	4,1
	D	4,2	2,3

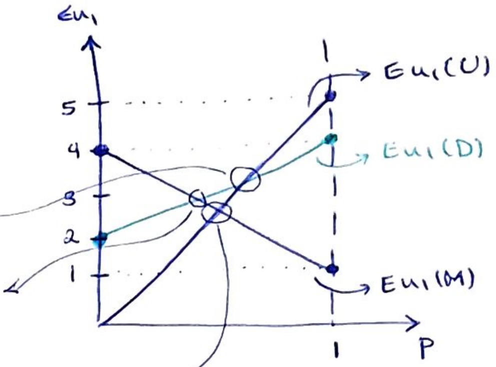
$Eu_1(U, p) = 5p + 0(1-p) = 5p$

$Eu_1(M, p) = p + 4(1-p) = 4 - 3p$

$Eu_1(D, p) = 4p + 2(1-p) = 2p + 2$

$2p + 2 = 5p \Rightarrow \boxed{p = 2/3}$

$4 - 3p = 2p + 2 \rightarrow \boxed{p = \frac{2}{5}}$



$4 - 3p = 5p \Rightarrow \boxed{p = 1/2}$

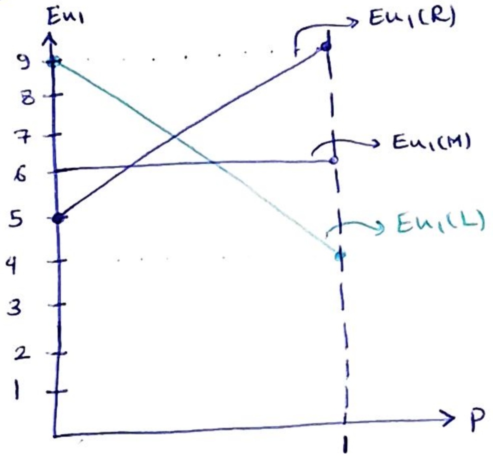
(35)

		(2)	
		p	$1-p$
		l	r
	L	4,−4	9,−9
(1)	M	6,−6	6,−6
	R	9,−9	4,−4

$Eu_1(L, p) = 4p + 9(1-p) = 9 - 5p$

$Eu_1(M, p) = 6p + 6(1-p) = 6$

$Eu_1(R, p) = 9p + 4(1-p) = 5p + 4$



(36)

		(2)	
		l	r
(1)	U	1,1	0,0
	D	0,0	0,0

pure NE = $\{(U, l), (D, r)\}$

(37) to solve this we will first guess the nash equilibrium and then show that it/they are NEs and then show that other possibilities are not nash equilibriums.

NE 1 ⟶ no one invests : no one would want to change because if they did they would lose money

NE 2 ⟶ everybody invests : no one wishes to change because then they wouldn't make any profit

all the cases in between, if above 90% then the ones who didn't invest want to invest and if less than 90% then investors would like to not invest.

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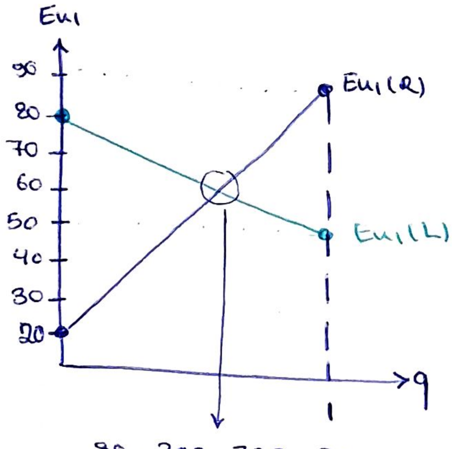
		q (2) l	$1-q$ r
p (1)	L	50,50	80,20
$1-p$	R	90,10	20,80

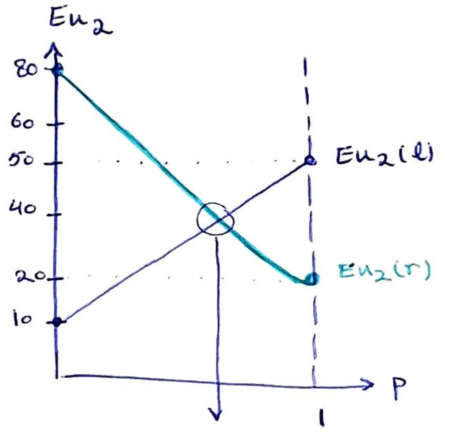
$Eu_1(L,q) = 50q + 80(1-q) = 80 - 30q$

$Eu_1(R,q) = 90q + 20(1-q) = 70q + 20$

$Eu_2(l,p) = 50p + 10(1-p) = 10 + 40p$

$Eu_2(r,p) = 20p + 80(1-p) = 80 - 60p$





$80 - 30q = 70q + 20$

$60 = 100q \longrightarrow \boxed{q = 6/10}$

$10 + 40p = 80 - 60p$

$100p = 70 \longrightarrow \boxed{p = \frac{7}{10}}$

mixed nash equilibrium $= \left\{ \left((pr_1(L) = 7/10, pr_1(R) = 3/10), (pr_2(l) = 6/10, pr_2(r) = 4/10) \right) \right\}$

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	I	R
U	12,2	3,9
D	5,8	4,2

no pure nash equilibrium.

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Player 3: D

(1) \ (2)	C	D
C	5,5,5	3,6,3
D	6,3,3	4,4,1

D → player 3

Player 3: C

(1) \ (2)	C	D
C	3,3,6	1,4,4
D	4,1,4	2,2,2

C - player 3

$BR_1(C, C) = D$ $\quad BR_1(C, D) = D$

$BR_1(D, C) = D$ $\quad BR_1(D, D) = D$

also D strictly dominates C for player (1)
and D strictly dominates C for player (2)
and C strictly dominates D for player (3)

strategy profile $= \{(D, D, C)\}$ with iterative dominated strategy removed

$NE = \{(D, D, C)\}$

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no-rain (probability of no rain: p)

(1) \ (2)		
	-1,-1	-1,0
	0,-1	0,0

rain (probability of rain: $1-p$)

(1) \ (2)		
	-2,-2	-2,0
	0,-2	-10,-10

⟹

(1) \ (2)		
	$-p-2(1-p), -p-2(1-p)$	$-p-2(1-p), 0$
	$0, -p-2(1-p)$	$-10(1-p), -10(1-p)$

	a	b
A	$-p-2(1-p)\ ,\ -p-2(1-p)$	$-p-2(1-p)\ ,\ 0$
B	$0\ ,\ -p-2(1-p)$	$-10(1-p)\ ,\ -10(1-p)$

if $\quad -p-2(1-p) > -10(1-p) \Rightarrow 8-8p-p>0 \Rightarrow p< 8/9$

$$\begin{cases} p< 8/9 & : NE = \{(B,a),\ (A,b)\} \\ p = 8/9 & : NE = \{(B,a),\ (B,b),\ (A,b)\} \\ p> 8/9 & : NE = \{(B,b)\} \end{cases}$$

(42) profit $u_1(q_1, q_2) = Pq_1 - cq_1$

$p = a - b(q_1 + q_2)$

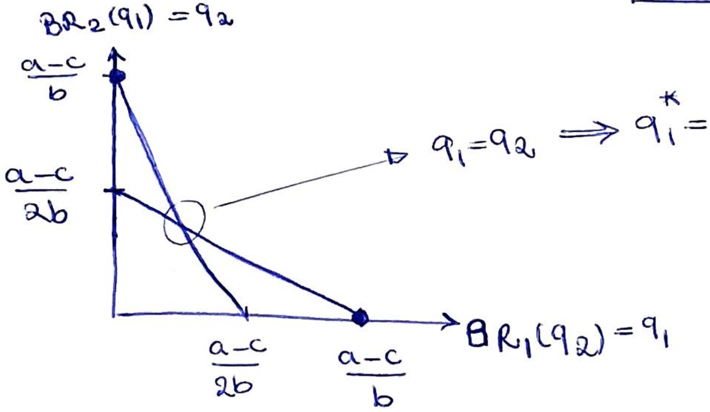
$BR_1(q_2) = \max_{q_1} pq_1 - cq_1 = \max_{q_1} (a - b(q_1+q_2))q_1 - cq_1 = \max_{q_1} aq_1 - bq_1^2 - bq_1q_2 - cq_1$

FOC $\frac{d BR_1(q_2)}{dq_1} = 0 \longrightarrow a - 2bq_1 - bq_2 - c = 0 \longrightarrow \frac{a - c - bq_2}{2b} = q_1$

$$\boxed{BR_1(q_2) = q_1 = \frac{a-c}{2b} - \frac{q_2}{2}}$$

SOC $\frac{d^2 BR_1(q_2)}{dq_1^2} = -2b < 0$ ✓

similarly $\boxed{BR_2(q_1) = q_2 = \frac{a-c}{2b} - \frac{q_1}{2}}$



$q_1 = q_2 \Longrightarrow q_1^* = \frac{a-c}{2b} - \frac{q_1^*}{2} \Rightarrow 3/2\, q_1^* = \frac{a-c}{2b} \Longrightarrow q_1^* = \frac{a-c}{3b}$

$$\boxed{q_1^* = q_2^* = \frac{a-c}{3b}}$$

* in case of monopoly

$Q = q_1 + q_2 \qquad q_1 = q_2$

$\max_{Q} p(q_1+q_2) - c(q_1+q_2) = \max_{Q} pQ - cQ = \max_{Q} (a - bQ)Q - cQ$

$= \max_{Q} aQ - bQ^2 - cQ$

$\frac{d \max_Q aQ - bQ^2 - cQ}{dQ} = a - 2bQ - c = 0 \longrightarrow Q = \frac{a-c}{2b}$

$\frac{d^2 \sim}{dQ^2} = -2b < 0$ ✓

$$\boxed{q_1 = q_2 = \frac{a-c}{4b}}$$

(43) (1) $u_1(s_1, s_2) = \begin{cases} s_1 & s_1+s_2 \leq 10 \\ 0 & s_1+s_2 > 10 \end{cases}$

So the pure NE is (s_1, s_2) where $s_1+s_2=10$ and $s_i \geq 0$

(2)

$u_1(s_1, s_2) = \begin{cases} s_1 & s_1+s_2>10 \text{ and } s_1 < s_2 \\ 10-s_2 & s_1+s_2>10 \text{ and } s_1>s_2 \\ 5 & s_1+s_2>10 \text{ and } s_1=s_2 \\ s_1 & s_1+s_2 \leq 10 \end{cases}$

$NE = \{(5,5)\}$

(3) $NE = \{(5,5), (6,5), (5,6), (6,6)\}$

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(1) $BR_1(\delta_2) = \delta_1 = \max\limits_{\delta_1} \; 2(\delta_1 + \delta_2 + b\delta_1\delta_2) - \delta_1^2$

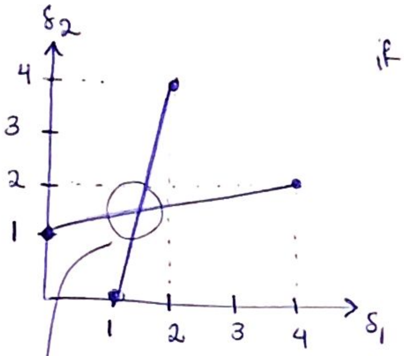
$$\frac{d\,BR_1(\delta_2)}{d\delta_1} = 2 + 2b\delta_2 - 2\delta_1 = 0 \longrightarrow \boxed{\delta_1 = 1 + b\delta_2}$$

$$\frac{d^2 BR_1(\delta_2)}{d\delta_1^2} = -2 < 0 \checkmark$$

(2) Similarly for player (2) $\boxed{BR_2(\delta_1) = \delta_2 = 1 + b\delta_1}$

(3)

if $b = 1/4$



(4)

Nash Equilibrium

$$\left.\begin{aligned} 1 + b\delta_1 &= 1 + b\delta_2 \\ \delta_1 &= \delta_2 \end{aligned}\right\} \Rightarrow \quad 1 + b\delta_1 = \delta_1 \qquad \delta_1 = \frac{1}{1-b}$$

$$\boxed{\delta_1^* = \delta_2^* = \frac{1}{1-b}}$$

(5)

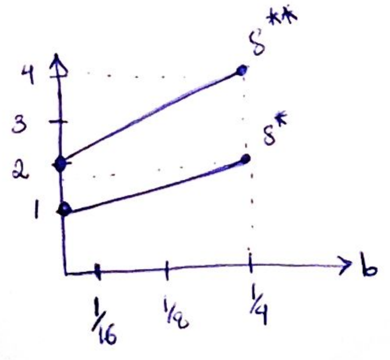
$$\max \; 4(\delta_1 + \delta_2 + b\delta_1\delta_2) - \delta_1^2 - \delta_2^2 \overset{\delta_1 = \delta_2 = \delta'}{=\!=\!=} \max \; 4(2\delta' + b\delta'^2) - 2\delta'^2$$

$$= \max \; 8\delta' + \delta'^2(4b - 2)$$

$$\frac{d\max\sim}{d\delta'} = 8 + 2\delta'(4b-2) = 0 \implies \delta' = \frac{-8}{2(4b-2)} = \frac{-4}{4b-2} = \frac{-2}{2b-1}$$

$$\frac{d^2\max}{d\delta'} = 8b - 2 < 0 \rightarrow b < 1/4 \checkmark$$

$$\boxed{\delta_1^{**} = \delta_2^{**} = \frac{-2}{2b-1}} = \frac{1}{\frac{1}{2} - b}$$



So the δ_1^{**} and δ_2^{**} values are bigger than the nash equilibrium (δ_1^*, δ_2^*) values.