Machine Learning: Exercise Sheet 2

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Exercise 1

(a) $p(x)=\int_0^{+\infty}\lambda\eta e^{-\lambda x-\eta y}dy=\lambda\eta e^{-\lambda x}(-\frac{1}{\eta}e^{-\eta y})\Big|_0^{+\infty}$

$=\lambda e^{-\lambda x}$

$p(y)=\int_0^{+\infty}\lambda\eta e^{-\lambda x-\eta y}dx=\eta e^{-\eta y}$

$\because p(x)p(y)=\lambda\eta e^{-\lambda x-\eta y}=p(x,y)$

$\therefore$ x and y are independent.

(b) $l(\lambda)=\ln p(D|\lambda)=\ln\prod_{i=1}^{N}p(x_i,y_i|\lambda)=\sum_{i=1}^{N}\ln p(x_i,y_i|\lambda)$

$=\sum_{i=1}^{N}\ln(\lambda\eta e^{-\lambda x_i-\eta y_i})=N\ln(\lambda\eta)-\sum_{i=1}^{N}(\lambda x_i+\eta y_i)$

$=N\ln(\lambda\eta)-N\lambda\bar{x}-N\eta\bar{y}$

$\frac{\partial l(\lambda)}{\partial\lambda}=N\frac{1}{\lambda\eta}\cdot\eta-N\bar{x}=0\quad\Rightarrow\quad\hat{\lambda}=\frac{1}{\bar{x}}$

(c) when $\eta=1/\lambda$, we have $p(x,y)=\lambda\cdot\frac{1}{\lambda}e^{-\lambda x-\frac{1}{\lambda}y}=e^{-\lambda x-\frac{y}{\lambda}}$

$l(\lambda)=\ln\prod_{i=1}^{N}e^{-\lambda x_i-\frac{y_i}{\lambda}}=\sum_{i=1}^{N}(-\lambda x_i-\frac{y_i}{\lambda})=-\lambda N\bar{x}-\frac{N\bar{y}}{\lambda}$

$\frac{\partial l(\lambda)}{\partial\lambda}=-N\bar{x}+\frac{N\bar{y}}{\lambda^2}=0\quad\Rightarrow\quad\hat{\lambda}=\sqrt{\frac{\bar{y}}{\bar{x}}}$