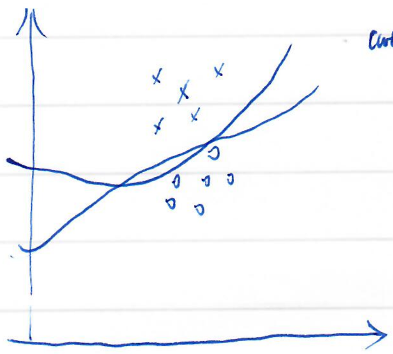
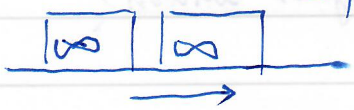


category.

ω_1 ω_2

{salmon, seabass}



$p(\omega_1) = 0.7$

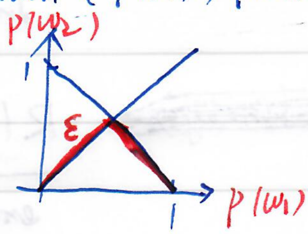
$p(\omega_2) = 0.3$

decide $\begin{cases} \omega_1 & \text{if } p(\omega_1) > p(\omega_2) \\ \omega_2 & \text{else} \end{cases}$

error of decision

$\varepsilon = \min(p(\omega_1), p(\omega_2))$

$\varepsilon = \begin{cases} \omega_1 > \omega_2 : p(\omega_2) \\ \omega_1 < \omega_2 : p(\omega_1) \end{cases}$



length $x \in \mathbb{R}$.

likelihood $p(x|\omega_1)$

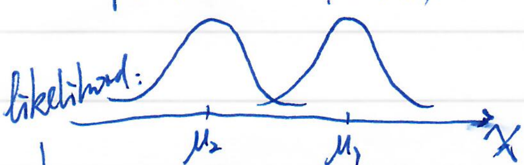
$p(x|\omega_2)$.

decide ω_1 if $p(\omega_1|x) > p(\omega_2|x)$

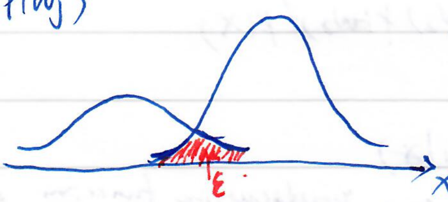
prior → posterior

$$p(\omega_j|x) = \frac{p(x|\omega_j)P(\omega_j)}{\sum_j p(x|\omega_j)P(\omega_j)} = \frac{\text{likelihood} \times \text{prior}}{\text{evidence}}$$

Assume Gaussian Dist.

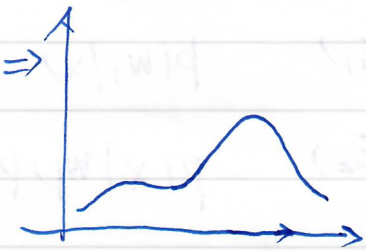


$\times P(\omega_j)$

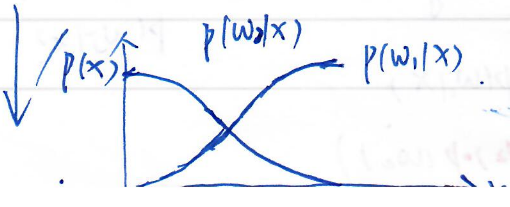


ω_1 is more likely than ω_2.

$p(x) \Rightarrow$



error function.



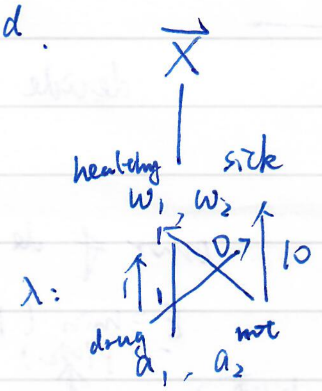
$\varepsilon|x = \min(p(\omega_1|x), p(\omega_2|x))$.

$= \int_x \min(p(\omega_1, x), p(\omega_2, x))dx$

$\varepsilon = \int_x \min(p(\omega_1|x), p(\omega_2|x)) \cdot p(x)\,dx$

$x = (\text{length}, \text{weight}, \text{width})$

$\vec{X} \in \mathbb{R}^d$.



(λ denotes the number).

$$\underbrace{R(a_i|\vec{x})}_{\text{expected loss.}} = \sum_{d} \lambda(a_i|w_j) \cdot p(w_j|\vec{x})$$

Risk function:

$$R(a_1|\vec{x}) = \lambda_{11} p(w_1|\vec{x}) + \lambda_{12} p(w_2|\vec{x})$$

$$R(a_2|\vec{x}) = \lambda_{21} p(w_1|\vec{x}) + \lambda_{22} p(w_2|\vec{x})$$

decision rule: decide a_1 if $R(a_1|\vec{x}) < R(a_2|\vec{x})$

i.e. $(\lambda_{21} - \lambda_{11}) p(w_1|\vec{x}) >$ $(\lambda_{12} - \lambda_{22}) p(w_2|\vec{x})$

$$R = \int_x \min(R(a_1|x), R(a_2|x))\, p(x)\, dx.$$

$$p(x) = \frac{1}{(2\pi)^{d/2} |\Sigma|^{1/2}} \cdot \exp\left[-\frac{1}{2}(x-\mu)^T \Sigma^{-1} (x-\mu)\right]$$

$w_1(\mu_1, \Sigma_1)$ $\quad p(w_1|x) > p(w_2|x)$

$w_2(\mu_2, \Sigma_2)$ $\quad p(x|w_1)p(w_1)/p(x) > p(x|w_2)p(w_2)/p(x)$

$g_1 . g_2$ $\quad g_1(x) > g_2(x)$. $\quad g_1(x) = 2p(w_1|x)$

any nondecreasing function of $p(w_j|x)$

esp. $g_1(x) = p(x) \cdot p(w_1|x)$

$\Rightarrow \log(p(x|w_1)\, p(w_1)) > \log(p(x|w_2) \cdot p(w_2))$

$$-\cancel{\frac{d}{2}\log 2\pi} - \log|\Sigma_1^{1/2}| - \frac{1}{2}(x-\mu_1)^T\Sigma_1^{-1}(x-\mu_1) + \log p(w_1) > -\cancel{\frac{d}{2}\log 2\pi}$$

$$-\log|\Sigma_2^{1/2}| - \frac{1}{2}(x-\mu_2)^T\Sigma_2^{-1}(x-\mu_2) + \log p(w_2) \quad (*)$$

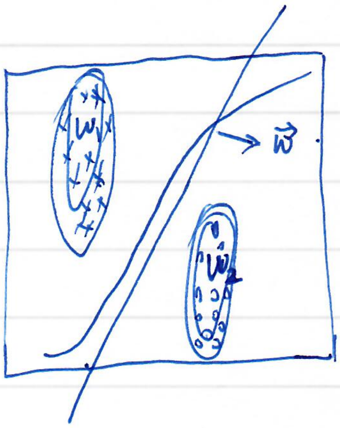
if $\Sigma_1 = \Sigma_2$. then $(*) \Rightarrow$

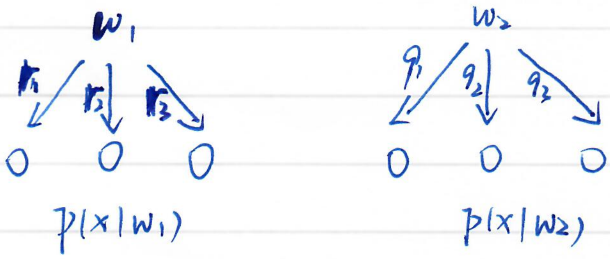
$$-\cancel{x^T\Sigma^{-1}x} + x^T\Sigma^{-1}\mu_1 - \frac{1}{2}\mu_1^T\Sigma^{-1}\mu_1 + \log p(w_1)$$

$$> -\cancel{\frac{1}{2}x^T\Sigma^{-1}x} + x^T\Sigma^{-1}\mu_2 - \frac{1}{2}\mu_2^T\Sigma^{-1}\mu_2 + \log p(w_2).$$

$$x^T(\Sigma^{-1}(\mu_1-\mu_2)) > \frac{1}{2}\mu_1^T\Sigma^{-1}\mu_1 - \frac{1}{2}\mu_2^T\Sigma^{-1}\mu_2 + \log p(w_2) - \log p(w_1).$$

$$x^T\mathbf{w} > b.$$





$X = \{0, 1\}^d.$

$\forall_{j=1}^{d}$ $r_j = p(x_j = 1 | w_1)$

$q_j = p(x_j = 1 | w_2)$

$$p(x|w_1) = \prod_{i=1}^{d}\left[(1-r_i)\mathbb{1}_{x_i=0} + r_i\cdot\mathbb{1}_{x_i=1}\right] = \prod_{i=1}^{d} r_i^{x_i}(1-r_i)^{1-x_i}$$

decision rule: w_1 if $p(w_1|\mathbf{x}) > p(w_2|\mathbf{x})$

$$p(x|w_1)\cdot p(w_1) > p(x|w_2)\cdot p(w_2)$$

$$\log p(x|w_1) + \log p(w_1) > \log p(x|w_2) + \log p(w_2)$$

$$\Rightarrow \sum_{i=1}^{d} x_i \log r_i + (1-x_i)\log(1-r_i) + \log p(w_1)$$

$$> \sum_{i=1}^{d} x_i \log q_i + (1-x_i)\log(1-q_i) + \log p(w_2)$$

$$\Rightarrow x^T(\log(r) - \log(1-r)) + const_1$$

$$> x^T(\log(q) - \log(1-q)) + const_2$$