Machine Learning 1
Exercise sheet 09

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Exercise 1.

(a) $\text{Bias}(\hat{\mu}) = E[\hat{\mu}-\mu] = E[\frac{1}{N}\Sigma_{i=1}^N X_i - \mu] = \frac{1}{N}E[\Sigma_{i=1}^N X_i] - \mu$

$= \frac{1}{N}\Sigma_{i=1}^N E[X_i] - \mu$

$= \frac{1}{N}\cdot N \cdot \mu - \mu$

$= 0$

$\text{Var}(\hat{\mu}) = E[(\hat{\mu} - E(\hat{\mu}))^2] = E[(\frac{1}{N}\Sigma_{i=1}^N X_i - \mu)^2]$

$= \frac{1}{N^2}E[(\Sigma_{i=1}^N X_i - N\mu)^2] = \frac{1}{N^2}E[\Sigma_{i=1}^N (X_i-\mu)^2 + \sum_{i\neq j}(X_i-\mu)(X_j-\mu)]$

$= \frac{1}{N^2}\sum_{i=1}^N E(X_i-\mu)^2 + \frac{1}{N^2}\sum_{i\neq j} E(X_i-\mu)(X_j-\mu)$

↓ because of independence

$= \frac{1}{N^2}\cdot N \cdot \sigma^2 + \frac{1}{N^2}\sum_{i\neq j} E(X_i-\mu)\cdot E(X_j-\mu)$

$= \frac{1}{N}\sigma^2 + 0 = \frac{\sigma^2}{N}$

$\text{Error}(\hat{\mu}) = \text{Bias}(\hat{\mu})^2 + \text{Var}(\hat{\mu}) = 0^2 + \frac{\sigma^2}{N} = \frac{\sigma^2}{N}$

(b). $\text{Bias}(\hat{\mu}) = E(\hat{\mu} - \mu) = E(0-\mu) = -\mu.$

$\text{Var}(\hat{\mu}) = E[(\hat{\mu} - E(\hat{\mu}))^2] = E[(0-E(0))^2] = 0.$

$\text{Error}(\hat{\mu}) = \text{Bias}(\hat{\mu})^2 + \text{Var}(\hat{\mu}) = \mu^2$