

Exercise Sheet 11

Exercise 1: The Dual SVM (10+20+10 P)

The primal program for the linear hard margin SVM is

$$\begin{aligned} \min_{\mathbf{w}, b} \quad & \|\mathbf{w}\|^2 \\ \text{subject to} \quad & y_i(\mathbf{w}^\top \mathbf{x}_i + \theta) \geq 1, \quad \text{for } 1 \leq i \leq n, \end{aligned}$$

where $\|\cdot\|$ denotes the Euclidean norm, and the minimization is performed in $\mathbf{w} \in \mathbb{R}^d, \theta \in \mathbb{R}$, while the data $\mathbf{x}_i \in \mathbb{R}^d, y_i \in \{-1, 1\}$ are regarded as fixed constants.

- (a) Write down the Lagrange function $\Lambda(\mathbf{w}, \theta, \boldsymbol{\alpha})$ for the hard margin SVM, where $\boldsymbol{\alpha} \in \mathbb{R}^n$ is the vector of Lagrange multipliers.
- (b) Derive the Lagrange dual of the program above. Describe how the solution for the primal program can be obtained from a solution of the dual program. (Hint: The Lagrange dual is a maximization problem in $\boldsymbol{\alpha}$.)
- (c) Write down the kernelized versions of the primal program and of the dual program.

Exercise 2: SVMs and Quadratic Programming (10 P)

We consider the CVXOPT Python software for convex optimization. The method `cvxopt.solvers.qp` solves the quadratic optimization problem given in matrix form as:

$$\begin{aligned} \min_x \quad & \frac{1}{2} \mathbf{x}^\top P \mathbf{x} + \mathbf{q}^\top \mathbf{x} \\ \text{subject to} \quad & G \mathbf{x} \preceq \mathbf{h} \\ \text{and} \quad & A \mathbf{x} = \mathbf{b}. \end{aligned}$$

Here, \preceq denotes the element-wise inequality: $(\mathbf{h} \preceq \mathbf{h}') \Leftrightarrow (\forall_i : h_i \leq h'_i)$. Note that the meaning of the variable \mathbf{x} is different from that of the same variable in the previous exercise.

- (a) Express the matrices and vectors $P, \mathbf{q}, G, \mathbf{h}, A, \mathbf{b}$ in terms of the variables of Exercise 1, such that the quadratic minimization problem above corresponds to the kernel dual SVM.

Exercise 3: Programming (50 P)

Download the programming files on ISIS and follow the instructions.