Exercise 10

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Exercise 1. Kernology

(a) (i) when coefficients $c_1, c_2, \cdots, c_n \in \mathbb{R}$, Then

$\sum_{i=1}^{n}\sum_{j=1}^{n} c_i c_j k(x, x') = \sum_{i=1}^{n}\sum_{j=1}^{n} c_i c_j a$ since $a \in \mathbb{R}^+$ $\sum_{i=1}^{n}\sum_{j=1}^{n} c_i c_j a \geq 0$ is satisfied

so $k(x, x') = a$, $a \in \mathbb{R}^+$ is a Mercer kernel.

(ii) Let $\varphi(x) = x$ then $\varphi(x') = x'$ be the feature map for x and x'

then $k(x, x') = \langle x, x' \rangle = \langle \varphi(x), \varphi(x') \rangle$ for all $x, x' \in \mathbb{R}^d$

so $k(x, x') = \langle x, x' \rangle$ is a Mercer kernel.

(iii) $k(x, x') = f(x) \cdot f(x')$ since there is just one feature ~~fo~~ defined by $f(\cdot)$

~~so~~ that $f(x) \cdot f(x') = \langle f(x), f(x') \rangle$ where $f: \mathbb{R}^d \to \mathbb{R}$ is an arbitrary continuous function.

Thus: $k(x, x') = \langle f(x), f(x') \rangle$ for all $x, x' \in \mathbb{R}^d$

so: $k(x, x') = f(x) \cdot f(x')$ is a Mercer kernels.

(b) (i) $k_1(x, x') = k_1$, $k_2(x, x') = k_2$, $k(x, x') = k$

since $k = k_1 + k_2$ and k_1, k_2 are two Mercer kernels

so $\forall$ ~~coeffi~~ coefficients $c_1, c_2, \cdots, c_n \in \mathbb{R}$ the inequality:

$\sum_{i=1}^{n}\sum_{j=1}^{n} c_i c_j k(x, x') = \sum_{i=1}^{n}\sum_{j=1}^{n} c_i c_j k_1(x, x') + \sum_{i=1}^{n}\sum_{j=1}^{n} c_i c_j k_2(x, x') \geq 0$ is satisfied

so $k(x, x') = k_1(x, x') + k_2(x, x')$ is a Mercer kernels.

(ii) Because $\sum_{i=1}^{n}\sum_{j=1}^{n} a_i a_j k_1(x, x') \geq 0$, $\sum_{i=1}^{n}\sum_{j=1}^{n} b_i b_j k_2(x, x') \geq 0$

so $\left[\sum_{i=1}^{n}\sum_{j=1}^{n} a_i a_j k_1(x, x')\right]\left[\sum_{i=1}^{n}\sum_{j=1}^{n} b_i b_j k_2(x, x')\right] = \cancel{\sum_{i=1}^{n}\sum_{j=1}^{n} c_i^2 c_j^2 k} \sum_{i=1}^{n}\sum_{j=1}^{n} a_i b_i a_j b_j \, k_1(x, x') k_2(x, x')] \geq 0$

Let $c_i = a_i b_i$ $c_j = a_j b_j$ then $\sum_{i=1}^{n}\sum_{j=1}^{n} c_i c_j k(x, x') \geq 0$ is satisfied so $k(x, x') = k_1(x, x') k_2(x, x')$ is a Mercer kernel

$k_1(x, x') k_2(x, x') = k(x, x')$