Global States in Partial Semantics

November 21, 2014

Preliminary for runtime verification in partial semantics

Composite Component in Global Semantics

$B = \Gamma(B_1, \ldots, B_n)$

- B_i is a tuple $(P_i, L_i, T_i, X_i, \{g_{i_\tau}\}_{\tau \in T_i}, \{f_{i_\tau}\}_{\tau \in T_i})$
  - (P_i, L_i, T_i) is an LTS over a set of ports P_i, control locations L_i and set of transitions $T_i \subseteq L_i \times P_i \times L_i$
  - X_i is a set of variables.
- Semantics of B_i is an LTS $(Q_i, P_i, \rightarrow_i)$
  - $Q_i = \{(l, v) \in L_i \times X_i\}$
  - $\rightarrow_i = \{((l', v'), p, (l, v)) \in Q_i \times P_i \times Q_i \mid \exists \tau \in T_i : g_{i_\tau}(v') \wedge v = f_{i_\tau}(v')\}$
- Γ is set of connectors
  A connector γ is a tuple $(\mathcal{P}_\gamma, G, F)$
  - $\mathcal{P}_\gamma = \{p_i[x_i] \mid p_i \in B_i.P\}_{i \in I}$ such that $\forall i \in I : \mathcal{P}_\gamma \cap B_i.P = \{p_i\}$
  - G is a Boolean expression over the set of variables $\cup_{i \in I}\, x_i$ (the guard)
  - F is an update function defined over the set of variables $\cup_{i \in I}\, x_i$
  - Interaction of γ in denoted with $\mathcal{I}_\gamma$

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Composite Component in Global Semantics

Semantics

Semantics of B is defined as a transition system $(Q, A, \longrightarrow)$

- $Q = B_1.Q \times \ldots \times B_n.Q$ is the set of global state, $Init \in Q$ as the initial state
- $A = \cup_{\gamma \in \Gamma}\{\mathcal{I}_\gamma\}$ is the set of all possible interactions,
- $\longrightarrow$ is the least set of transitions satisfying rule:

$$\frac{\exists(a = \{p_i\}_{i \in I}) \in A \qquad G_a(v(X)) \\ \forall i \in I : \; q_i \xrightarrow{p_i}_i q_i' \wedge v_i = F_{a_i}(v(X)) \qquad \forall i \notin I : \; q_i = q_i'}{(q_1, \ldots, q_n) \xrightarrow{a} (q_1', \ldots, q_n')}$$

Set of all global Run

$R(B) = \{(q_0 \cdot q_1 \cdots q_s) \mid q_0 = Init, \forall i \in [1, s] : q_i \in Q \wedge \exists a \in A, q_{i-1} \xrightarrow{a} q_i\}$

Set of all global Trace

$Tr(B) = \{(q_0 \cdot a_1 \cdot q_1 \cdots a_s \cdot q_s) \mid q_0 = Init, \forall i \in [1, s] : q_i \in Q \wedge a_i \in A, q_{i-1} \xrightarrow{a_i} q_i\}$

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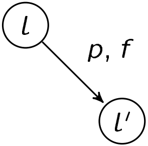
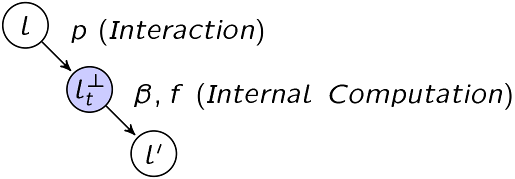
Composite Component in Partial Semantics

$B^{\perp} = \Gamma^{\perp}(B_1^{\perp}, \ldots, B_n^{\perp})$

- $B_i^{\perp} = (P_i \cup \{\beta_i\}, L_i \cup L_i^{\perp}, T_i^{\perp}, X_i, \{g_{i_\tau}\}_{\tau \in T_i^{\perp}}, \{f_{i_\tau}\}_{\tau \in T_i^{\perp}})$ is the partial state model of B_i where :
  - $L_i^{\perp} = \{l_t^{\perp} \mid t \in T_i\}$ such that $L_i^{\perp} \cap L_i = \emptyset$
  - $\beta_i \notin P_i$ is a new port
  - $T_i^{\perp} \subseteq (L_i \cup L_i^{\perp}) \times P_i \cup \{\beta_i\} \times (L_i \cup L_i^{\perp})$ such that $T_i^{\perp} = \{(l, p, l_t^{\perp}), (l_t^{\perp}, \beta, l') \mid \exists (l, p, l') \in T_i\}$
- Semantics of B_i is an LTS $(Q_i \cup Q_i^{\perp}, P_i \cup \{\beta_i\}, \rightsquigarrow_i)$
  - $Q_i^{\perp} = \{(l, v) \in L_i^{\perp} \times \{\perp\}\}$
  - $\rightsquigarrow_i = \{((l, v), p, (l_t^{\perp}, \perp)), ((l_t^{\perp}, \perp), \beta_i, (l', v')) \mid (l, p, l_t^{\perp}), (l_t^{\perp}, \beta_i, l') \in T_i^{\perp}, (l, v), (l', v') \in Q_i, (l_t^{\perp}, \perp) \in Q_i^{\perp} \wedge \exists \tau \in T_i : g_{i_\tau}(v) \wedge v' = f_{i_\tau}(v)\}$





- $\Gamma^{\perp} = \Gamma$

Composite Component in Partial Semantics

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Semantics of $B^{\perp}$ is defined as a transition system $(Q^{\perp}, A^{\perp}, \rightsquigarrow)$

- $Q^{\perp} = \bigotimes_{i=1}^{n}(Q_i \cup Q_i^{\perp})$, $Q \subset Q^{\perp}$
- $A^{\perp} = A \cup \{\beta_i\}_{i=1}^{n}$, where $\{\beta_i\}_{i=1}^{n}$ is the set of busy transitions
- The transition relation $\rightsquigarrow$ can be defined by the following rules

$$\frac{\exists(a = \{p_i\}_{i \in I}) \in A \qquad G_a(v(X)) \qquad \forall i \in I: \; q_i \overset{p_i}{\rightsquigarrow}_i q_i' \wedge v_i = F_{a_i}(v(X)) \qquad \forall i \not\in I: \; q_i = q_i'}{(q_1, \ldots, q_n) \overset{a}{\rightsquigarrow} (q_1', \ldots, q_n')}$$

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Set of all partial Run

$R(B^{\perp}) = \{(q_0 \cdot q_1 \cdots q_m) \mid q_0 = Init, \forall i \in [1, m] : q_i \in Q^{\perp} \wedge \exists a \in A^{\perp}, q_{i-1} \overset{a}{\rightsquigarrow} q_i\}$

Set of all partial Trace

$Tr(B^{\perp}) = \{(q_0 \cdot a_1 \cdot q_1 \cdots a_m \cdot q_m) \mid q_0 = Init, \forall i \in [1, m] : q_i \in Q^{\perp} \wedge a_i \in A^{\perp}, q_{i-1} \overset{a_i}{\rightsquigarrow} q_i^{p}\}$

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Definition (Bisimulation)

A bisimulation over B and $B^\perp$ is a relation $R \subseteq Q \times Q^\perp$ such that

- $\forall (q, r) \in R,\ a \in A : q \xrightarrow{a} q' \implies \exists r' : (q', r') \in R \wedge r \overset{\beta^* a}{\rightsquigarrow} r'$
- $\forall (q, r) \in R,\ a \in A : r \overset{a}{\rightsquigarrow} r' \implies \exists q' : (q', r') \in R \wedge q \xrightarrow{a} q'$
- $\forall (q, r) \in R,\ \beta \in \{\beta_i\}_{i=1}^{n} : r \overset{\beta}{\rightsquigarrow} r' \implies (q, r') \in R$

We say that B and $B^\perp$ are bisimilar.

Theorem

Let $B^\perp = \Gamma^\perp(B_1^\perp, \cdots, B_n^\perp)$ be the corresponding partial state semantic model obtained from $B = \Gamma(B_1, \cdots, B_n)$. B and $B^\perp$ are bisimilar.

Proof.

We define the relation $R' = \{(q, r) \in Q \times Q^\perp \mid r \overset{\beta^*}{\rightsquigarrow} q\}$ and prove that R' satisfies all the bisimulation properties.

□

Definition (Witness)

Given a bisimulation R between B and $B^{\perp}$, the witness relation $W \subseteq Tr(B) \times Tr(B^{\perp})$ is the smallest set that contains $(Init, Init)$ and satisfies the following rules

For $(\sigma_1, \sigma_2) \in W$:

- $(\sigma_1, \sigma_2 \cdot \beta \cdot q_2) \in W$ if $(last(\sigma_1), q_2) \in R$
- $(\sigma_1 \cdot a \cdot q_1, \sigma_2 \cdot a \cdot q_2) \in W$ if $a \in A$ and $(q_1, q_2) \in R$

If $(\sigma_1, \sigma_2) \in W$, we say that σ_1 is a witness of σ_2

Property

- For all $(\sigma_1, \sigma_2) \in W$, Sequence of interactions of σ_2 and its witness trace σ_1 are equal
- For any trace $\sigma_2 \in Tr(B^{\perp})$ there <u>exists</u> an <u>unique</u> witness trace $\sigma_1 \in Tr(B)$

Proof.

$\square$

Question

How to find the unique witness global trace of a partial trace in online fashion?

Reconstruction of the global trace from the partial trace

Definition (Accumulator function)

$Acc : Tr(B^{\perp}) \longrightarrow Q \times (A \times Q)^{*} \times (A \times Q^{\perp})^{\leq n}$

- $Acc(Init) = Init$
- $Acc(\sigma \cdot a \cdot q) = Acc(\sigma) \cdot a \cdot q$ for $a \in A$
- $Acc(\sigma \cdot \beta_j \cdot q) = \text{Map } [upd] \ (q, Acc(\sigma))$ for $\beta_j \in \{\beta_i\}_{i=1}^{n}$, where

let $q = (q''_1, \cdots, q''_n)$ in

$upd : Q^{\perp} \times (Q^{\perp} \cup A) \longrightarrow Q^{\perp} \cup A$

- $upd(a) = a$,for $a \in A$
- $upd\Big((q_1, \cdots, q_n)\Big) = (q'_1, \cdots, q'_n)$ where, $\forall k \in [1, n]$,

$$q'_k = \begin{cases} q''_k & \text{if } (j = k) \wedge (q_k \in Q_k^{\perp}) \\ q_k & \text{otherwise} \end{cases}$$

Reconstruction of the global trace from the partial trace

Definition (Discriminant function)

$\mathcal{D} = Q \times (A \times Q)^* \times (A \times Q^{\perp})^{\leq n} \longrightarrow Tr(B)$

- $\mathcal{D}(Init) = Init$
- $$\mathcal{D}((Acc(\sigma) \cdot a \cdot q) = \begin{cases} \mathcal{D}(Acc(\sigma)) \cdot a & \text{if } (a \in A) \wedge (q \notin Q) \\ \mathcal{D}(Acc(\sigma)) \cdot q & \text{if } (a \notin A) \wedge (q \in Q) \\ \mathcal{D}(Acc(\sigma)) & \text{if } (a \notin A) \wedge (q \notin Q) \\ n/a^* & \text{otherwise} \end{cases}$$

$*$: The next state after an intercation is always a partial state, therefore $(a \in A) \wedge (q \in Q)$ never occurs

Theorem

$\forall \sigma \in Tr(B^{\perp}) : (\mathcal{D}(Acc(\sigma)), \sigma) \in W$