



EMT212 PROJECT 1 SEM II 2019/2020

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Table of Contents

List of Figures.....	1
Figure 1: Cross-sectional of Brick Wall With Cooling Duct.....	2
Figure 2: Boundary Conditions	3
Figure 3: Solution's Plot Contour.....	11
Figure 3.1: Plot Contour Satisfied All The Boundaries.....	11
List of Tables.....	1
Table 1: Color Indicator.....	4
Problems description	2
Geometric Parameter	2
Boundary Conditions	3
The Governing Equation	4
Calculation.....	5
Central Different Approximation Scheme.....	5
The Equations Set in Matrix Form	8
Matlab Code	9
Plot of The Solution	11
References.....	12

Problems Description

The problem given is an industrial cooling system is used to provide cold air within the working area. A typical design of a cooling duct is of rectangular shape. As illustrated in Figure 1, the cross section of the cooling duct pass through a brick wall is shown. The temperature distribution of the brick wall measuring 7 units by 7 units is governed by:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

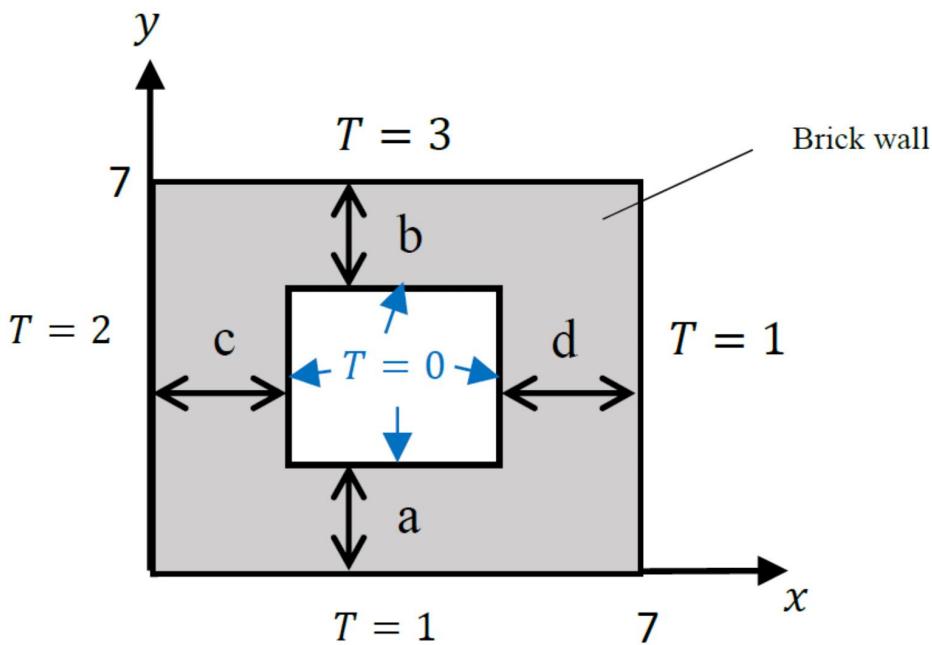


Figure 1: Cross section of a brick wall with cooling duct inside the wall.

Geometric Parameters

Individual students are assigned to different geometric parameters. The spacing a , b , c and d are different from each student. For mine,

$$a = 3$$

$$b = 2$$

$$c = 4$$

$$d = 2$$

Boundary Conditions

The heat conduction problem is subjected to boundary conditions as illustrated on the Figure 1 which can be summarized as,

At $x = 0 ; T = 2$

At $x = 7 ; T = 1$

At $y = 0 ; T = 1$

At $y = 7 ; T = 3$

It should be also noted that the cooling duct boundary is subjected to $T=0$.

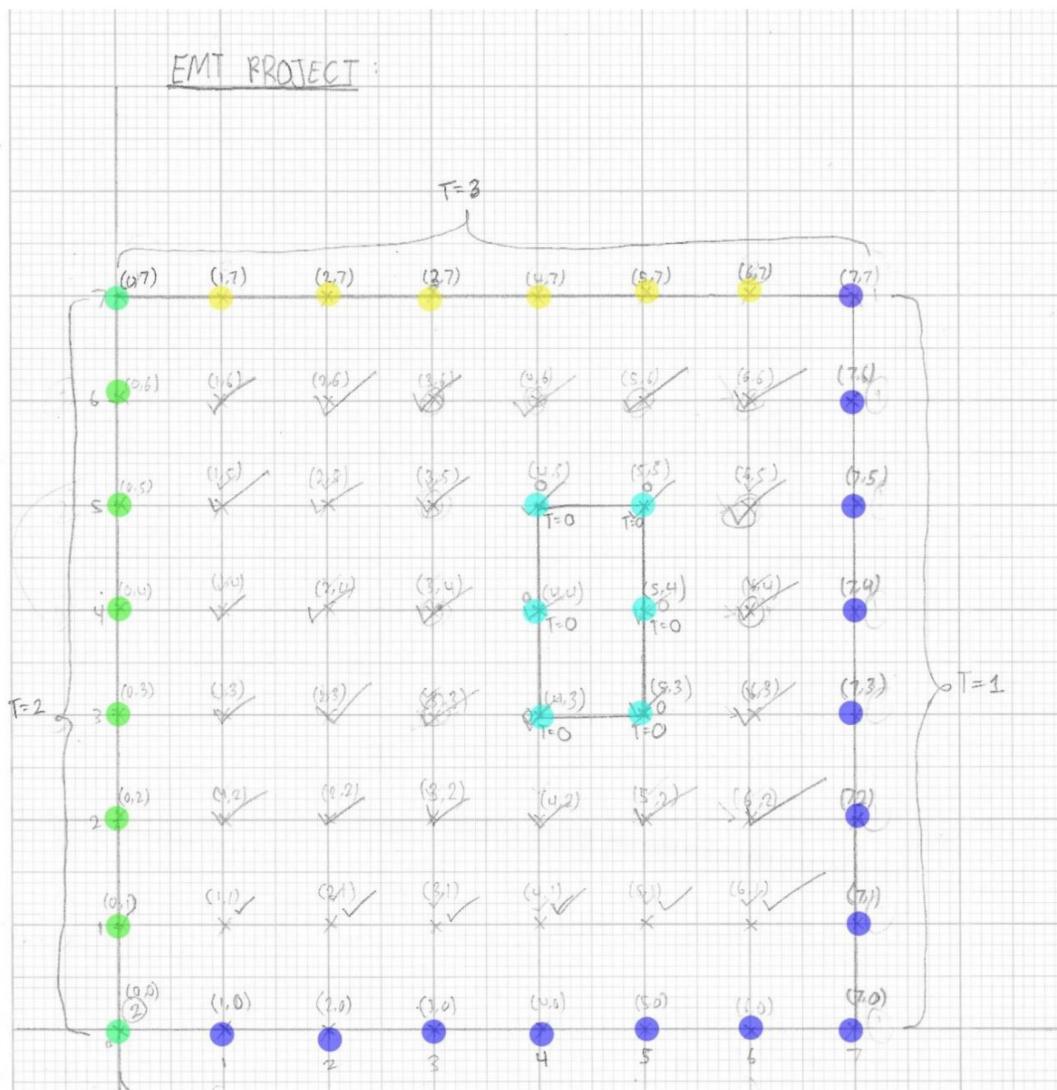


Figure 2: boundary Conditions

The colored point in Figure 2 indicate that it has known value represent the boundary conditions with different color lead to different value. Here are the indicators:

Color	Temperature, T
Green	2
Yellow	3
Dark Blue	1
cyan	0

Table 1: Color Indicator

The governing equation

Consider 2D Poisson equation on a square plate:

Transient 2D heat Conduction – T (x,y,t)

$$\frac{\partial T}{\partial t} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)$$

Where α = Thermal Diffusivity

From question, the governing equation given is:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

Which indicate that $\frac{\partial T}{\partial t} = 0$ and $\alpha = 1$.

Calculation

Consider the use of central difference approximation scheme with respect to x and y :

$$\frac{\partial^2 T}{\partial x} = \frac{T_{i+1} - 2T_{i+1} + T_{i-1}}{(\Delta x)^2}$$

$$\frac{\partial^2 T}{\partial y} = \frac{T_{i+1} - 2T_{i+1} + T_{i-1}}{(\Delta y)^2}$$

$$\frac{T_{i+1} - 2T_{i+1} + T_{i-1}}{(\Delta x)^2} + \frac{T_{i+1} - 2T_{i+1} + T_{i-1}}{(\Delta y)^2} = 0$$

To save time and make it less complicated, I assume the spacing of $\Delta x=1$ and $\Delta y=1$. So here we will have 8 nodes for both x and y directions like shown in figure 2.

Perform the central differencing approximation on the Poisson equation:

At x_1 and y_1

$$\frac{T_{2,1} - 2T_{1,1} + 2}{(1)^2} + \frac{T_{1,2} - 2T_{1,1} + 1}{(1)^2} = 0$$

Simplified:

$$-4T_{1,1} + T_{1,2} + T_{2,1} = -3$$

Employ the same steps on other nodes:

At x_2 and y_1

$$T_{1,1} - 4T_{2,1} + T_{2,2} + T_{3,1} = -1$$

At x_3 and y_1

$$T_{2,1} - 4T_{3,1} + T_{3,2} + T_{4,1} = -1$$

At x_4 and y_1

$$T_{3,1} - 4T_{4,1} + T_{4,2} + T_{5,1} = -1$$

At x_5 and y_1

$$T_{4,1} - 4T_{5,1} + T_{5,2} + T_{6,1} = -1$$

At x_6 and y_1

$$T_{5,1} - 4T_{6,1} + T_{6,2} = -2$$

At x_1 *and* y_2

$$T_{1,1} - 4T_{1,2} + T_{1,3} + T_{2,2} = 0$$

At x_2 *and* y_2

$$T_{1,2} + T_{2,1} - 4T_{2,2} + T_{2,3} + T_{3,2} = 0$$

At x_3 *and* y_2

$$T_{2,2} + T_{3,1} - 4T_{3,2} + T_{3,3} + T_{4,2} = 0$$

At x_4 *and* y_2

$$T_{3,2} + T_{4,1} - 4T_{4,2} + T_{5,2} = 0$$

At x_5 *and* y_2

$$T_{4,2} + T_{5,1} - 4T_{5,2} + T_{6,2} = 0$$

At x_6 *and* y_2

$$T_{5,2} + T_{6,1} - 4T_{6,2} + T_{6,3} = -1$$

At x_1 *and* y_3

$$T_{1,2} - 4T_{1,3} + T_{1,4} + T_{2,3} = -2$$

At x_2 *and* y_3

$$T_{1,3} + T_{2,2} - 4T_{2,3} + T_{2,4} + T_{3,3} = 0$$

At x_3 *and* y_3

$$T_{2,3} + T_{3,2} - 4T_{3,3} + T_{3,4} = 0$$

At x_4 *and* y_3

$$T_{4,3} = 0$$

At x_5 *and* y_3

$$T_{5,3} = 0$$

At x_6 *and* y_3

$$T_{6,2} - 4T_{6,3} + T_{6,4} = -1$$

At x_1 *and* y_4

$$T_{1,3} - 4T_{1,4} + T_{1,5} + T_{2,4} = -2$$

At x_2 *and* y_4

$$T_{1,4} + T_{2,3} - 4T_{2,4} + T_{2,5} + T_{3,4} = 0$$

At x_3 *and* y_4

$$T_{2,4} + T_{3,3} - 4T_{3,4} + T_{3,5} = 0$$

At x_4 and y_4

$$T_{4,4} = 0$$

At x_5 and y_4

$$T_{5,4} = 0$$

At x_6 and y_4

$$T_{6,3} - 4T_{6,4} + T_{6,5} = -1$$

At x_1 and y_5

$$T_{1,4} - 4T_{1,5} + T_{1,6} + T_{2,5} = -2$$

At x_2 and y_5

$$T_{1,5} + T_{2,4} - 4T_{2,5} + T_{2,6} + T_{3,5} = 0$$

At x_3 and y_5

$$T_{2,5} + T_{3,4} - 4T_{3,5} + T_{3,6} = 0$$

At x_4 and y_5

$$T_{4,5} = 0$$

At x_5 and y_5

$$T_{5,5} = 0$$

At x_6 and y_5

$$T_{6,4} - 4T_{6,5} + T_{6,6} = -1$$

At x_1 and y_6

$$T_{1,5} - 4T_{1,6} + T_{2,6} = -5$$

At x_2 and y_6

$$T_{1,6} + T_{2,5} - 4T_{2,6} + T_{3,6} = -3$$

At x_3 and y_6

$$T_{2,6} + T_{2,5} - 4T_{3,6} + T_{4,6} = -3$$

At x_4 and y_6

$$T_{3,6} - 4T_{4,6} + T_{5,6} = -3$$

At x_5 and y_6

$$T_{4,6} - 4T_{4,6} + T_{5,6} = -3$$

At x_6 and y_6

$$T_{5,6} + T_{6,5} - 4T_{6,6} = -4$$

The Equations Set in Matrix Form


```

y =[0*ones(1,8); 1*ones(1,8); 2*ones(1,8); 3*ones(1,8); 4*ones(1,8);
    5*ones(1,8); 6*ones(1,8); 7*ones(1,8)];
```

```

T=zeros(n);
T(n,1:n)=3;
T(1:n,n)=1;
T(1,2:7)= t(1:1);
T(1,1:n)=1;
T(2,2:7)= t(1:6);
T(3,2:7)= t(7:12);
T(4,2:7)= t(13:18);
T(5,2:7)= t(19:24);
T(6,2:7)= t(25:30);
T(7,2:7)= t(31:36);
T(1:n,1)=2;
```

```

contourf (x,y,T)
```

Plot for The Solution

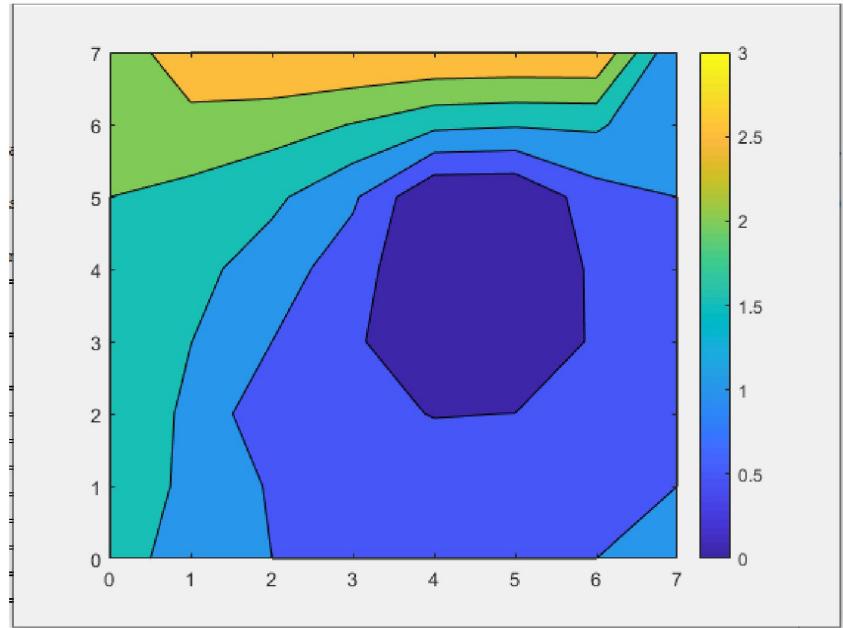


Figure 3: Solution's Plot Contour

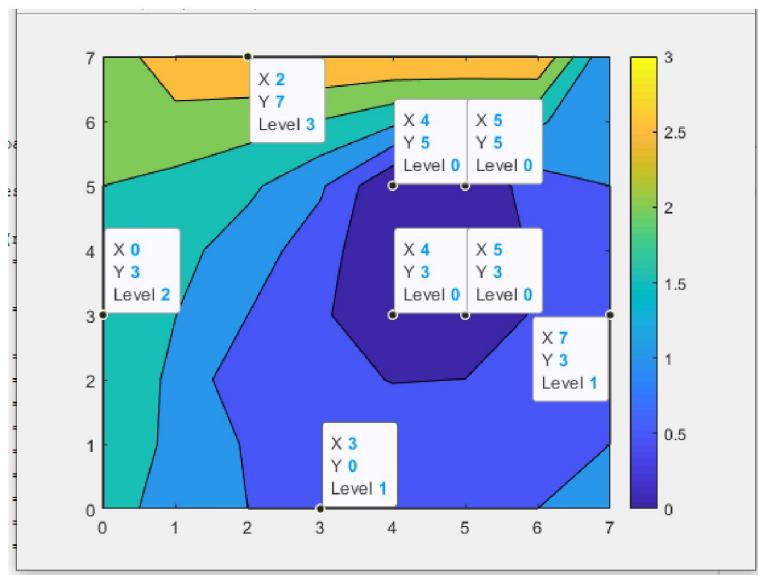


Figure 3.1: Contour Satisfied All The Boundaries

We can see in Figure 3.1 that all the boundary conditions are satisfied.

References

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