

Target Localization Using Quantized Observations in Delay-Sensitive Networks

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I. APPENDIX

Here, we derive the squared position error bound (SPEB) of \mathbf{p}^+ in Proposition 1. The unknown parameters are collected as $\boldsymbol{\theta} = [(\mathbf{p}^+)^T, \mathbf{p}^T]^T$ and the log-likelihood function is

$$\ln f(\mathbf{z}, \boldsymbol{\theta}) = \ln f(\mathbf{z}|\mathbf{p}) + \ln f(\mathbf{p}^+|\mathbf{p}) + \ln f(\mathbf{p}), \quad (1)$$

where $f(\mathbf{x})$ denotes the probability density function (PDF) $f_{\mathbf{x}}(\mathbf{x})$ of the random vector \mathbf{x} unless specified otherwise. Then, the Fisher information matrix (FIM) of $\boldsymbol{\theta}$ can be decomposed into three parts, as follows [1]:

$$\mathbf{J}_{\boldsymbol{\theta}}(\mathbf{b}) = \mathbf{J}_{\text{ob}}(\mathbf{b}) + \mathbf{J}_{\text{M}}(\mathbf{b}) + \mathbf{J}_{\text{P}} \quad (2)$$

where $\mathbf{J}_{\text{ob}}(\mathbf{b})$, $\mathbf{J}_{\text{M}}(\mathbf{b})$, and \mathbf{J}_{P} represent FIMs from quantized observations, target motions, and a priori knowledge of the target, respectively. The expression of $\mathbf{J}_{\text{ob}}(\mathbf{b})$ is

$$\mathbf{J}_{\text{ob}}(\mathbf{b}) = -\mathbb{E}_{\mathbf{z}, \mathbf{p}} \{ \Delta_{\boldsymbol{\theta}}^2 \ln f(\mathbf{z}|\mathbf{p}) \} \quad (3)$$

where $\Delta_{\boldsymbol{\theta}}^2 \ln f(\mathbf{z}|\mathbf{p})$ denotes $\partial^2 \ln f(\mathbf{z}|\mathbf{p}) / \partial \boldsymbol{\theta} \partial \boldsymbol{\theta}^T$ and (3) is rewritten since the observation is not associated with \mathbf{p}^+ as

$$\mathbf{J}_{\text{ob}}(\mathbf{b}) = \begin{bmatrix} \mathbf{O} & \mathbf{O} \\ \mathbf{O} & \mathbb{E}_{\mathbf{p}} \{ \mathbf{J}_{\text{Q}}(\mathbf{b}) \} \end{bmatrix} \quad (4)$$

where \mathbf{O} denotes the 2×2 zero matrix and

$$\mathbf{J}_{\text{Q}}(\mathbf{b}) = -\mathbb{E}_{\mathbf{z}|\mathbf{p}} \{ \Delta_{\mathbf{p}}^2 \ln f(\mathbf{z}|\mathbf{p}) \}. \quad (5)$$

The expression for $\mathbf{J}_{\text{Q}}(\mathbf{b})$ is computed similarly to previous studies analyzing performance limits for unknown parameters from quantized observations [2]. It is provided as follows:

$$\mathbf{J}_{\text{Q}}(\mathbf{b}) = \sum_{n=1}^{N_a} \sum_{k=1}^K \gamma_n \rho_{n,k}(b_n) \frac{\partial s_k(\tau_n)}{\partial \mathbf{p}} \frac{\partial s_k(\tau_n)}{\partial \mathbf{p}^T}, \quad (6)$$

where $\gamma_n = |\beta_n|^2 / \sigma_n^2$ represents the signal-to-noise ratio (SNR) at n -th agent, $s_k(\tau_n)$ refers to the k -th element of the signal vector $\mathbf{s}(f_s; \tau_n)$, and $\rho_{n,k}(b_n)$ represents the quantized coefficient at k -th quantizer of n -th agent, calculated by

$$\rho_{n,k}(b_n) = \frac{1}{2\pi} \sum_{l=1}^{L_n} \frac{(e^{-r_{l+1}^2/2} - e^{-r_l^2/2})^2}{\Phi(r_{l+1}) - \Phi(r_l)}, \quad (7)$$

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$$r_l = [u_l - \beta_n s_k(\tau_n)] / \sigma_n, \quad l = 1, 2, \dots, L_n - 1 \quad (8)$$

where $\Phi(x)$ denotes the tail distribution function of the standard norm distribution with $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{t^2}{2}} dt$ [3]. Subsequently, the matrix $\mathbf{J}_{\text{M}}(\mathbf{b})$ can be computed as follows:

$$\mathbf{J}_{\text{M}}(\mathbf{b}) = -\mathbb{E}_{\mathbf{p}, \mathbf{p}^+} \{ \Delta_{\boldsymbol{\theta}}^2 \ln f(\mathbf{p}^+|\mathbf{p}) \}, \quad (9)$$

which can be calculated as

$$\mathbf{J}_{\text{M}}(\mathbf{b}) = \frac{1}{\alpha \Delta t(\mathbf{b})} \begin{bmatrix} \mathbf{I}_2 & -\mathbf{I}_2 \\ -\mathbf{I}_2 & \mathbf{I}_2 \end{bmatrix}. \quad (10)$$

Furthermore, the FIM from the a priori knowledge of the target is independent of the bit-depth, which is given by

$$\mathbf{J}_{\text{P}} = -\mathbb{E}_{\mathbf{p}} \{ \Delta_{\mathbf{p}}^2 \ln f(\mathbf{p}) \}. \quad (11)$$

Due to the a priori knowledge only relies on \mathbf{p} , we can obtain

$$\mathbf{J}_{\text{P}} = \begin{bmatrix} \mathbf{O} & \mathbf{O} \\ \mathbf{O} & \sigma_0^{-2} \mathbf{I}_2 \end{bmatrix} \quad (12)$$

Substituting (4), (10), and (12) into (2), we can obtain the partition of the FIM $\mathbf{J}_{\boldsymbol{\theta}}(\mathbf{b})$ is given by

$$\mathbf{J}_{\boldsymbol{\theta}}(\mathbf{b}) = \begin{bmatrix} \mathbf{A} & -\mathbf{A} \\ -\mathbf{A}^T & \mathbf{A} + \mathbf{W} \end{bmatrix} \quad (13)$$

where $\mathbf{A} = 1 / [\alpha \Delta t(\mathbf{b})] \mathbf{I}_2$ and $\mathbf{W} = \mathbb{E} \{ \mathbf{J}_{\text{Q}}(\mathbf{b}) \} + \sigma_0^{-2} \mathbf{I}_2$. Then, the equivalent Fisher information matrix (EFIM) for estimating \mathbf{p}^+ is given by [4]

$$\mathbf{J}_{+,+}^{\text{e}}(\mathbf{b}) = \mathbf{A} - \mathbf{A}(\mathbf{A} + \mathbf{W})^{-1} \mathbf{A}^T. \quad (14)$$

According to the matrix inverse lemma, we can obtain the closed-form EFIM for estimating \mathbf{p}^+ , which is given by

$$\mathbf{J}_{+,+}^{\text{e}}(\mathbf{b}) = [\mathbf{W}^{-1} + \alpha \Delta t(\mathbf{b}) \mathbf{I}_2]^{-1} \quad (15)$$

Finally, the closed-form SPEB for estimating \mathbf{p}^+ is given as

$$\mathcal{P}(\mathbf{p}^+; \mathbf{b}) = \text{tr} \left\{ (\mathbb{E}_{\mathbf{p}} \{ \mathbf{J}_{\text{Q}}(\mathbf{b}) \} + \sigma_0^{-2} \mathbf{I}_2)^{-1} \right\} + 2\alpha \Delta t(\mathbf{b}), \quad (16)$$

which completes the proof of Proposition 1.

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