Applied AI

Lecture 3

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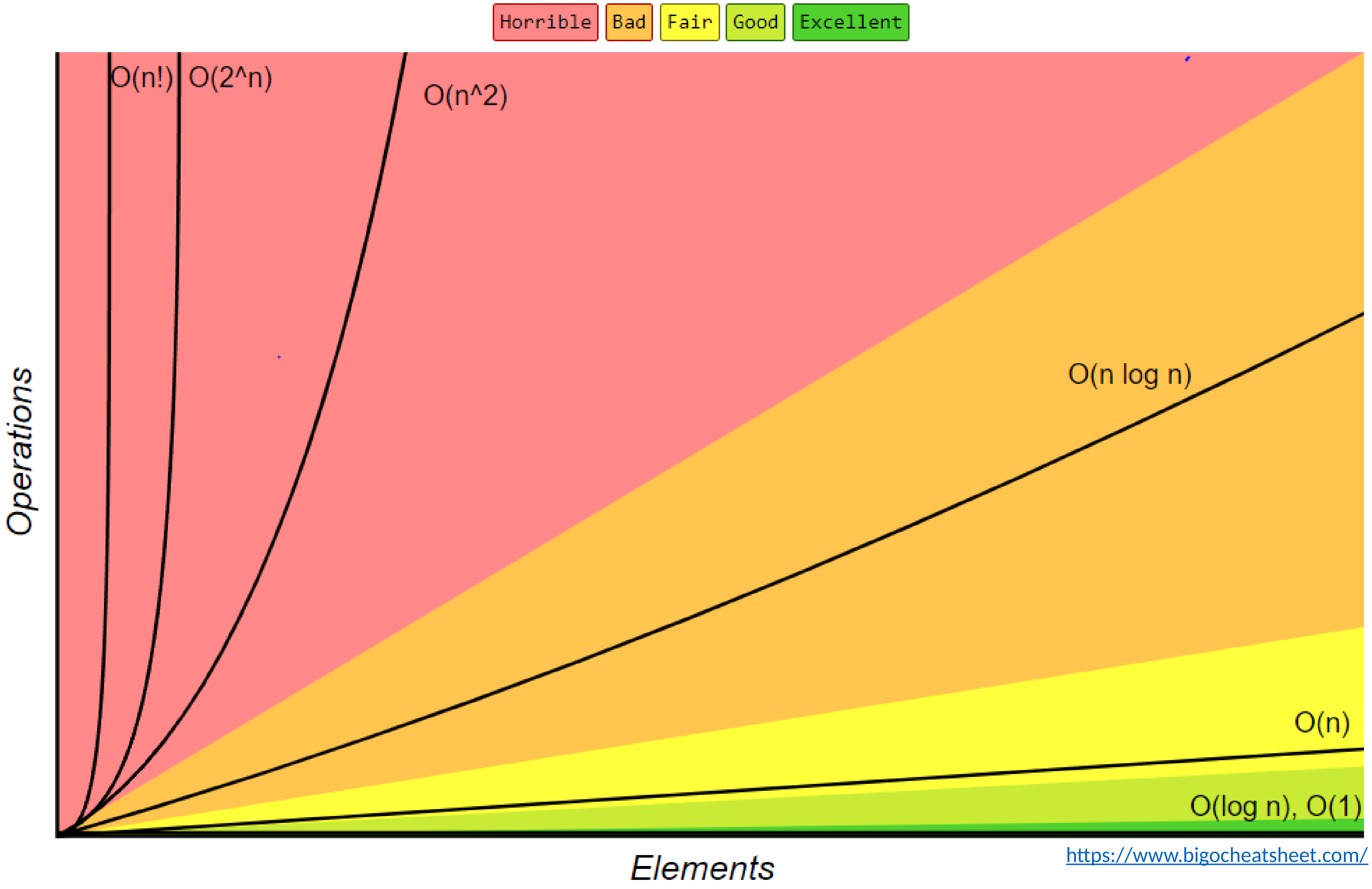
[slides adapted from Artificial Intelligence: A Modern Approach, Russel and Norvig]

Agenda

- Search continued
- Big Oh Notation
- Comparison of uninformed search algorithms
- Inform the search algorithms
- Comparison informed search algorithms

Big-O Complexity Chart



https://www.bigocheatsheet.com/

Big-O for some common algorithms

Data Structure Operation

	Time Complexity								Space Complexity
	Average				Worst				Worst
	Access	Search	Insertion	Deletion	Access	Search	Insertion	Deletion	
Array	Θ(1)	Θ(n)	Θ(n)	Θ(n)	Θ(1)	Θ(n)	Θ(n)	Θ(n)	Θ(n)
Stack	Θ(n)	Θ(n)	Θ(1)	Θ(1)	Θ(n)	Θ(n)	Θ(1)	Θ(1)	Θ(n)
Queue	Θ(n)	Θ(n)	Θ(1)	Θ(1)	Θ(n)	Θ(n)	Θ(1)	Θ(1)	Θ(n)
Singly-Linked List	Θ(n)	Θ(n)	Θ(1)	Θ(1)	Θ(n)	Θ(n)	Θ(1)	Θ(1)	Θ(n)
Doubly-Linked List	Θ(n)	Θ(n)	Θ(1)	Θ(1)	Θ(n)	Θ(n)	Θ(1)	Θ(1)	Θ(n)

Array Sorting Algorithms

	Time Complexity			Space Complexity
	Best	Average	Worst	Worst
Quicksort	Ω(n log(n))	Θ(n log(n))	O(n^2)	O(n log(n))
Mergesort	Ω(n log(n))	Θ(n log(n))	O(n log(n))	O(n log(n))
Timsort	Ω(n)	Θ(n log(n))	O(n log(n))	Θ(n)
Heapsort	Ω(n log(n))	Θ(n log(n))	O(n log(n))	O(n log(n))
Bubble Sort	Ω(n)	Θ(n^2)	O(n^2)	Θ(n)

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Properties of breadth-first search

Complete: Yes (if b is finite)

Time: $1 + b + b^2 + b^3 + \ldots + b^d + b(b^d - 1) = O(b^{d+1})$, i.e., exp. in d

Space: $O(b^{d+1})$ (keeps every node in memory)

Optimal: Yes (if cost = 1 per step); not optimal in general

Space is the big problem; can easily generate nodes at 100MB/sec so 24hrs = 8640GB.

Properties of depth-first search

Complete: No: fails in infinite-depth spaces, spaces with loops

Modify to avoid repeated states along path

⇒ complete in finite spaces

Time: $O(b^m)$: terrible if m is much larger than d

but if solutions are dense, may be much faster than breadth-first

Space: $O(bm)$, i.e., linear space!

Optimal: No

How do we deal with these problems?

- Limit depth.
- Iterative deepening.
- Think about using heuristics.
- What are heuristics – we will look at this later.

"Uses domain-specific hints about the location of goals"

Depth-limited research

= depth-first search with depth limit *l*, i.e., nodes at depth *l* have no successors

Recursive implementation:

function Depth-Limited-Search(*problem*, *limit*) **returns** soln/fail/cutoff
Recursive-DLS(Make-Node(Initial-State[*problem*]), *problem*, *limit*)

function Recursive-DLS(*node*, *problem*, *limit*) **returns** soln/fail/cutoff
  cutoff-occurred? ← false
  if Goal-Test(*problem*, State[*node*]) **then return** *node*
  else if Depth[*node*] = *limit* **then return** *cutoff*
  else for each *successor* **in** Expand(*node*, *problem*)
    do *result* ← Recursive-DLS(*successor*, *problem*, *limit*) **if** *result* = *cutoff* **then** *cutoff-occurred?* ← true

Iterative deepening search

```
function Iterative-Deepening-Search( problem) returns a solution
    inputs: problem, a problem
    for depth ← 0 to ∞ do
        result ← Depth-Limited-Search( problem, depth)
        if result /= cutoff then return result
    end
```

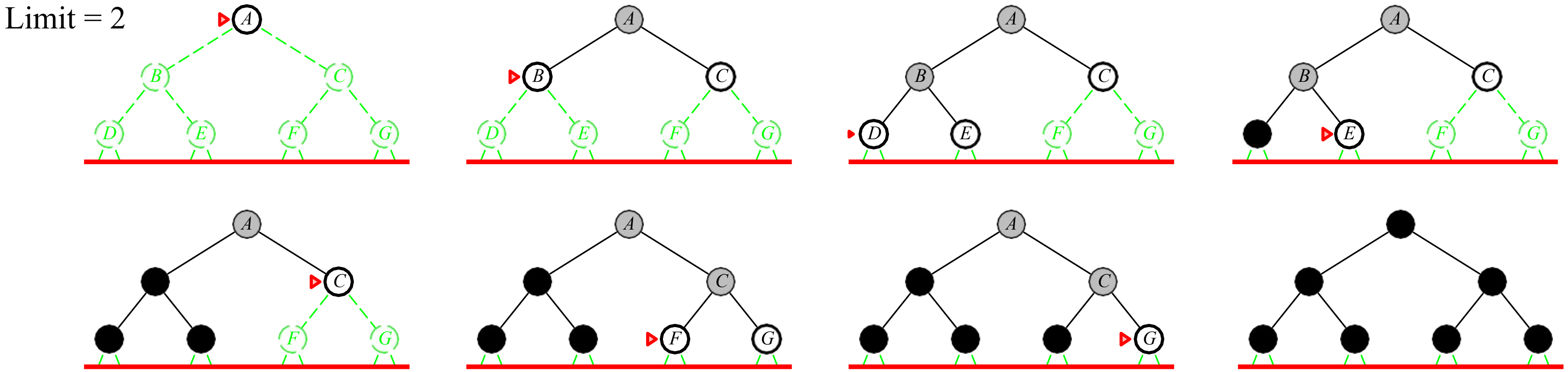
Iterative deepening search $l = 0$

Limit = 0

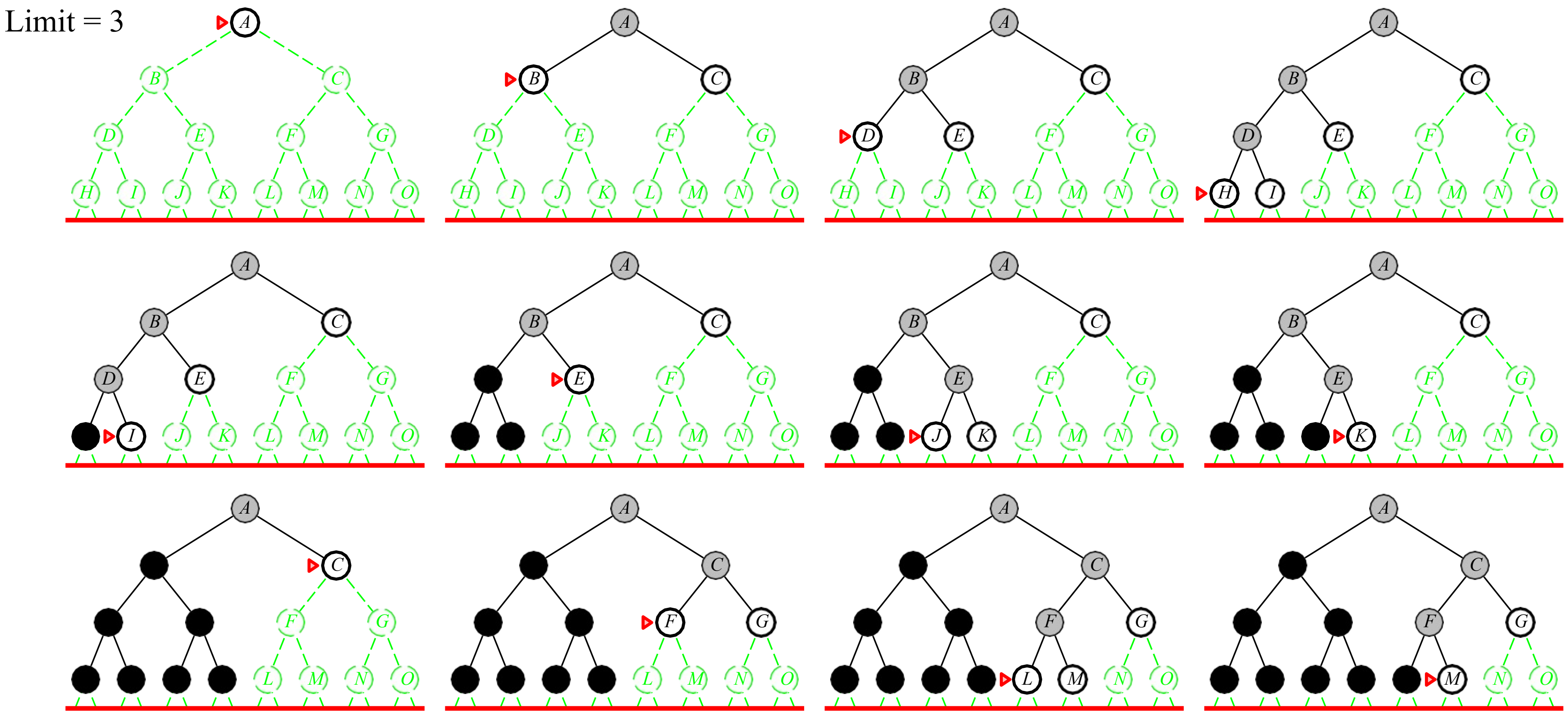
Iterative deepening search $l = 1$

Limit = 1

Iterative deepening search $l = 2$



Iterative deepening search $l = 3$



Properties of iterative deepening search

Complete: Yes

Time: $(d + 1)b^0 + db^1 + (d - 1)b^2 + \ldots + b^d = O(b^d)$

Space: $O(bd)$

Optimal: Yes, if step cost = 1

Can be modified to explore uniform-cost tree

Numerical comparison for $b = 10$ and $d = 5$, solution at far right leaf:

$N(\text{IDS}) = 50 + 400 + 3{,}000 + 20{,}000 + 100{,}000 = 123{,}450$

$N(\text{BFS}) = 10 + 100 + 1{,}000 + 10{,}000 + 100{,}000 + 999{,}990 = 1{,}111{,}100$

IDS does better because other nodes at depth d are not

Comparison of algorithms

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening
Complete?	Yes*	Yes*	No	Yes, if $l \geq d$	Yes
Time	b^{d+1}	$b^{fC*/cl}$	b^m	b^l	b^d
Space	b^{d+1}	$b^{fC*/cl}$	bm	bl	bd
Optimal?	Yes*	Yes	No	No	Yes*

Graph search

```
function Graph-Search( problem, fringe) returns a solution, or
failure
   closed ← an empty set
   fringe ← Insert(Make-Node(Initial-State[problem]), fringe)
   loop do
       if fringe is empty then return failure
       node ← Remove-Front(fringe)
       if Goal-Test(problem, State[node]) then return node
       if State[node] is not in closed then
           add State[node] to closed
           fringe ← InsertAll(Expand(node, problem), fringe)
   end
```

Uninformed Search Summary

Problem formulation usually requires abstracting away real-world details to define a state space that can feasibly be explored

Variety of uninformed search strategies

Iterative deepening search uses only linear space
and not much more time than other uninformed algorithms

Graph search can be exponentially more efficient than tree search

Informed search algorithms

Best-first search

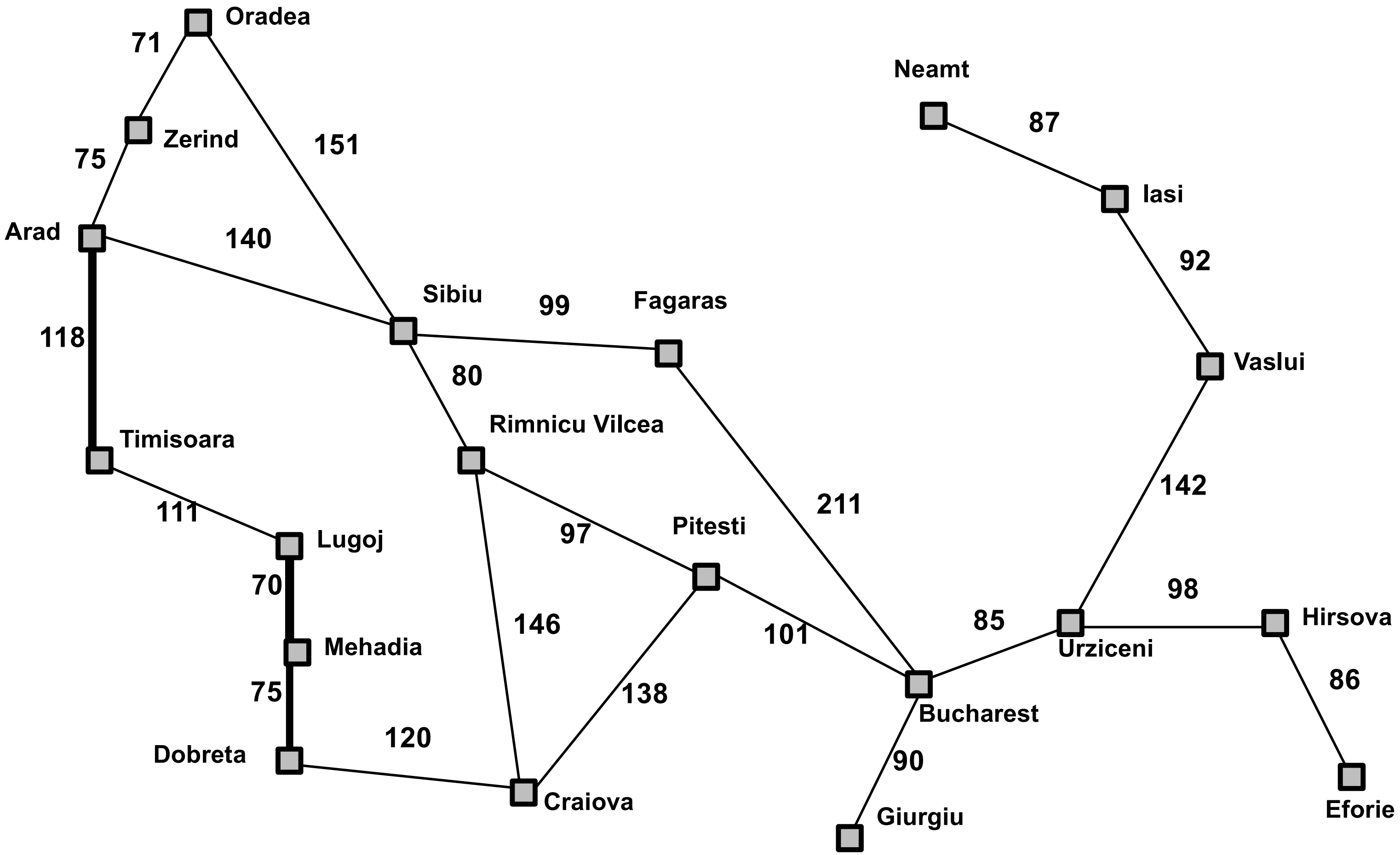
Idea: use an evaluation function for each node
– estimate of "desirability"

⇒ Expand most desirable unexpanded node

Implementation:
fringe is a queue sorted in decreasing order of desirability

Special cases:
greedy search
$A*$ search

Romania graph with step costs in kilometres



Straight−line distance
to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

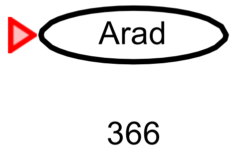
Greedy search

Evaluation function $h(n)$ (heuristic)
$= $ estimate of cost from n to the closest goal

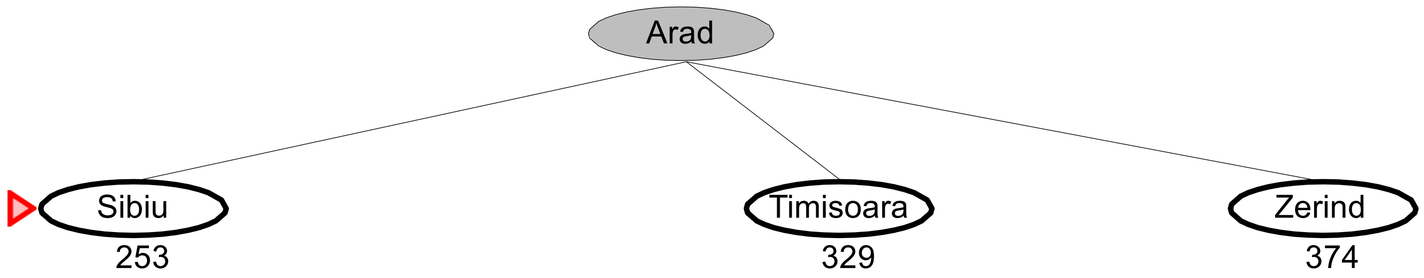
E.g., $h_{\mathrm{SLD}}(n) =$ straight-line distance from n to Bucharest

Greedy search expands the node that appears to be closest to goal

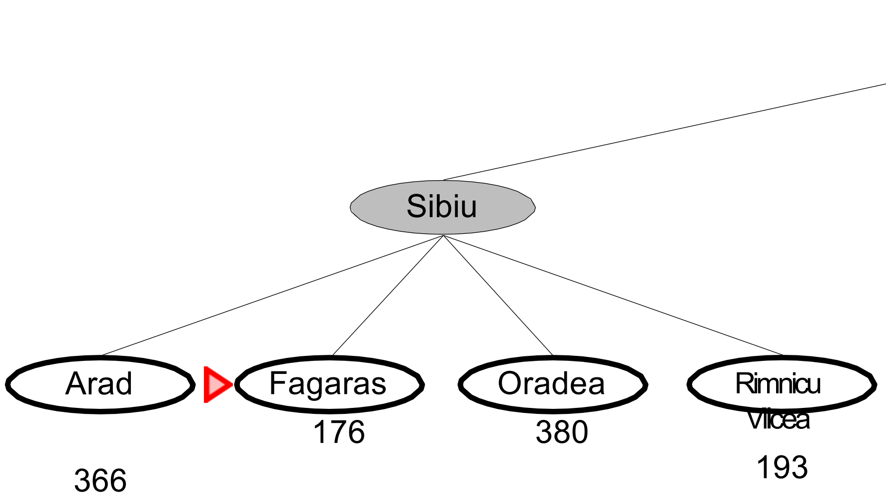
Greedy search example



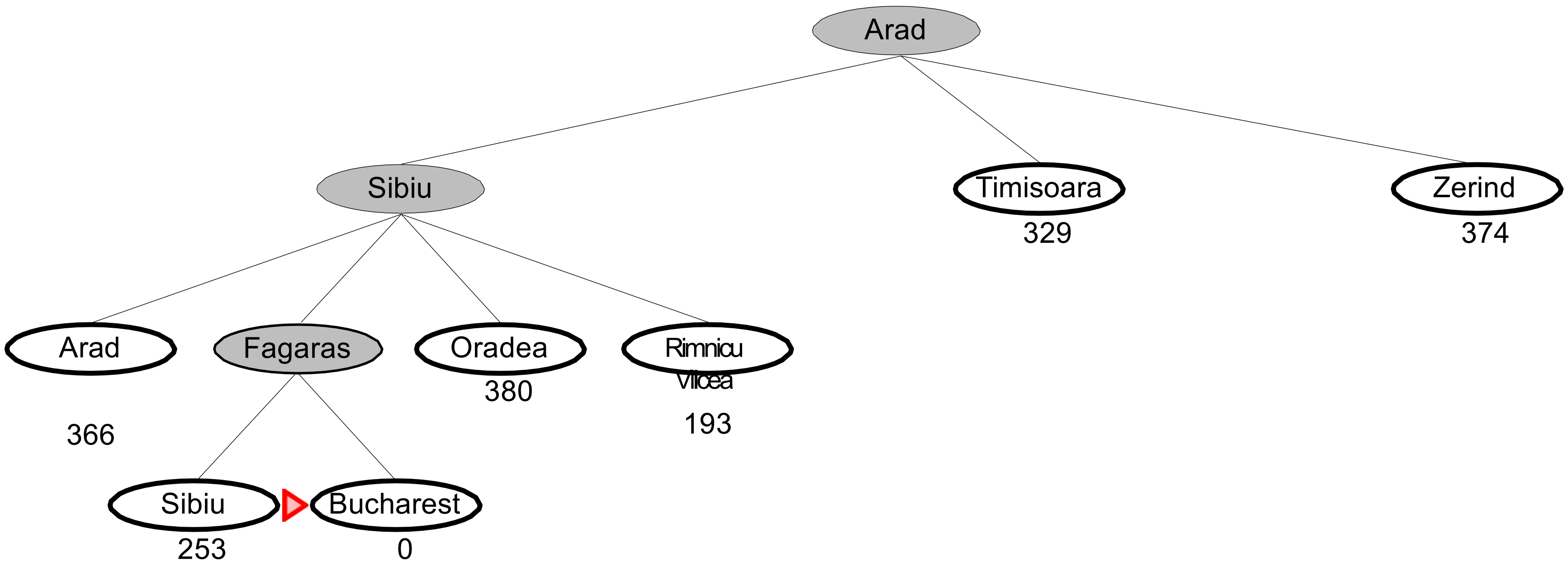
Greedy search example



Greedy search example



Greedy search example



Properties of greedy search

Complete: No–can get stuck in loops,
e.g., Iasi → Neamt → Iasi → Neamt →
Complete in finite space with repeated-state checking

Time: $O(b^m)$, but a good heuristic can give dramatic improvement

Space: $O(b^m)$—keeps all nodes in memory

Optimal: No

A* search

Idea: avoid expanding paths that are already expensive

Evaluation function $f(n) = g(n) + h(n)$ $g(n)$ = cost so far to

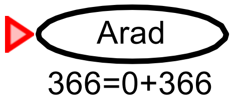
reach n
$h(n)$ = estimated cost to goal from n
$f(n)$ = estimated total cost of path through n to goal

A∗ search uses an admissible heuristic
i.e., $h(n) \leq h^*(n)$ where $h^*(n)$ is the true cost from n.
(Also require $h(n) \geq 0$, so $h(G) = 0$ for any goal G.)

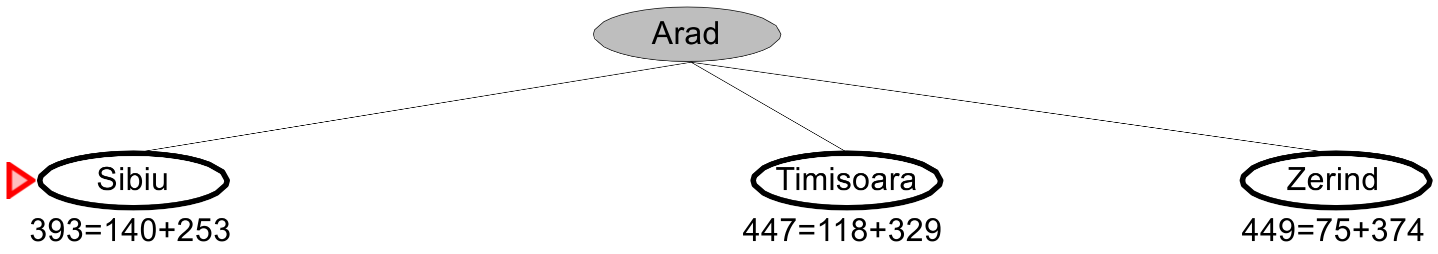
E.g., $h_{\mathrm{SLD}}(n)$ never overestimates the actual road distance

Theorem: A∗ search is optimal

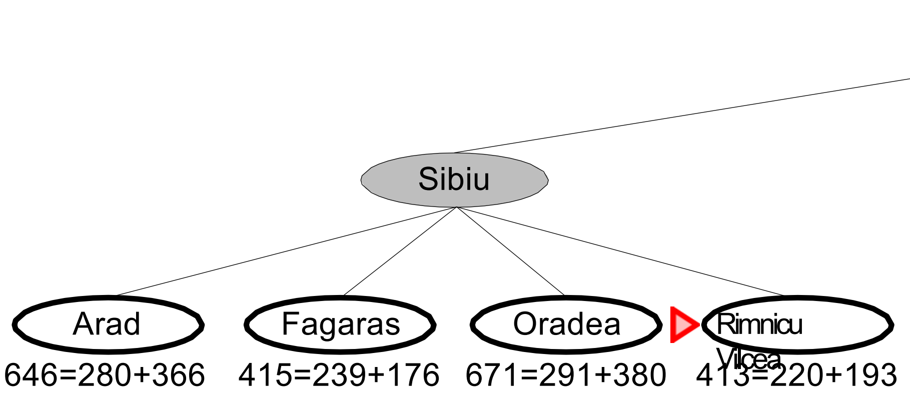
A* search example



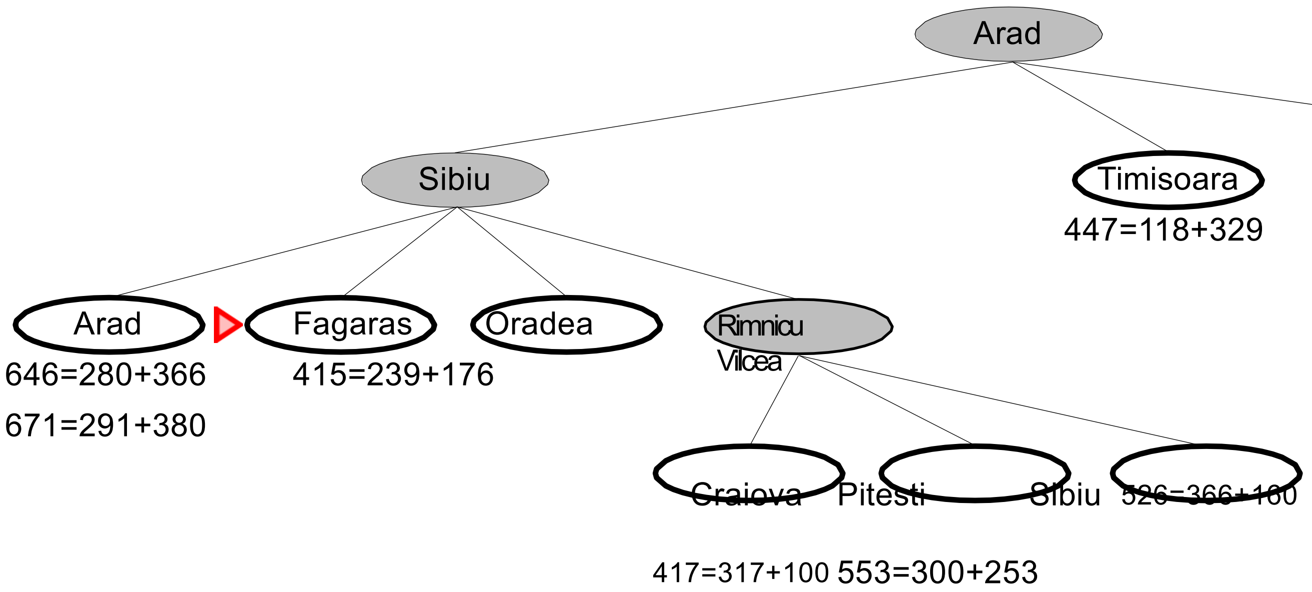
A* search example



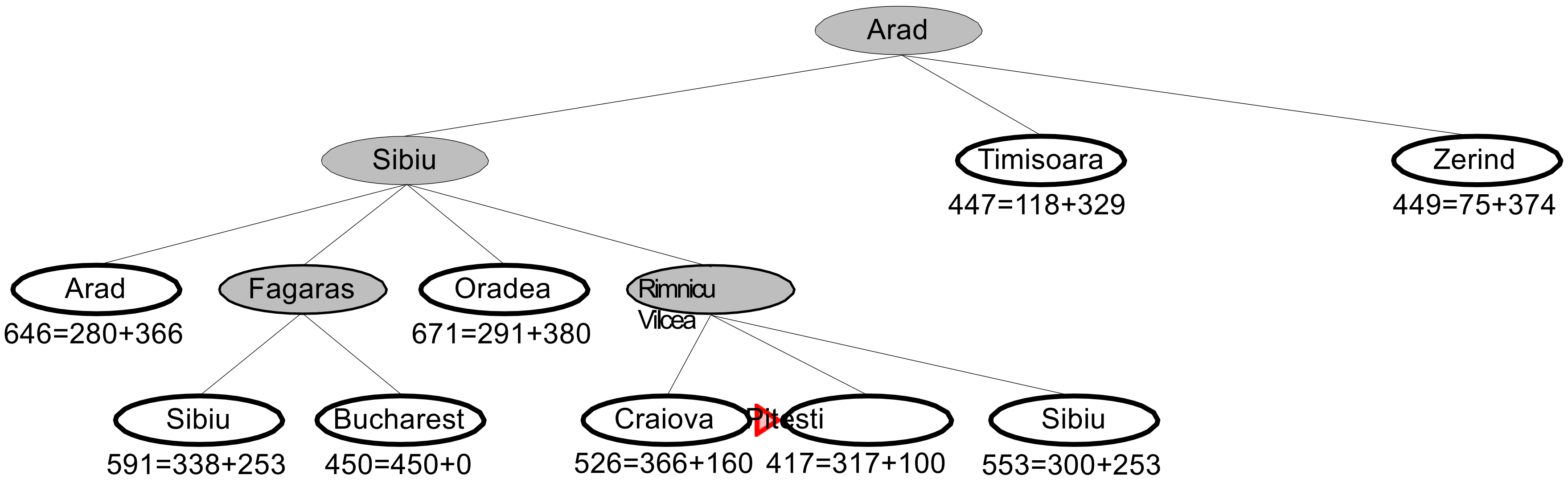
A* search example



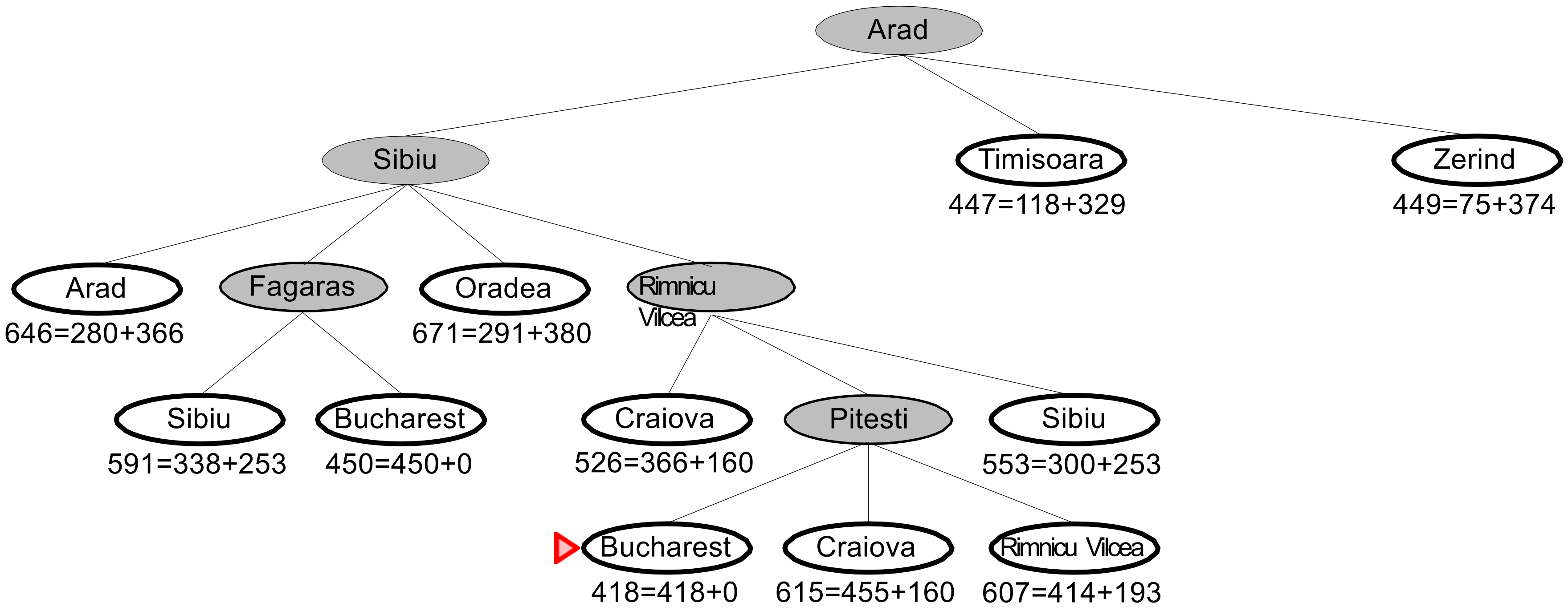
A* search example



A* search example



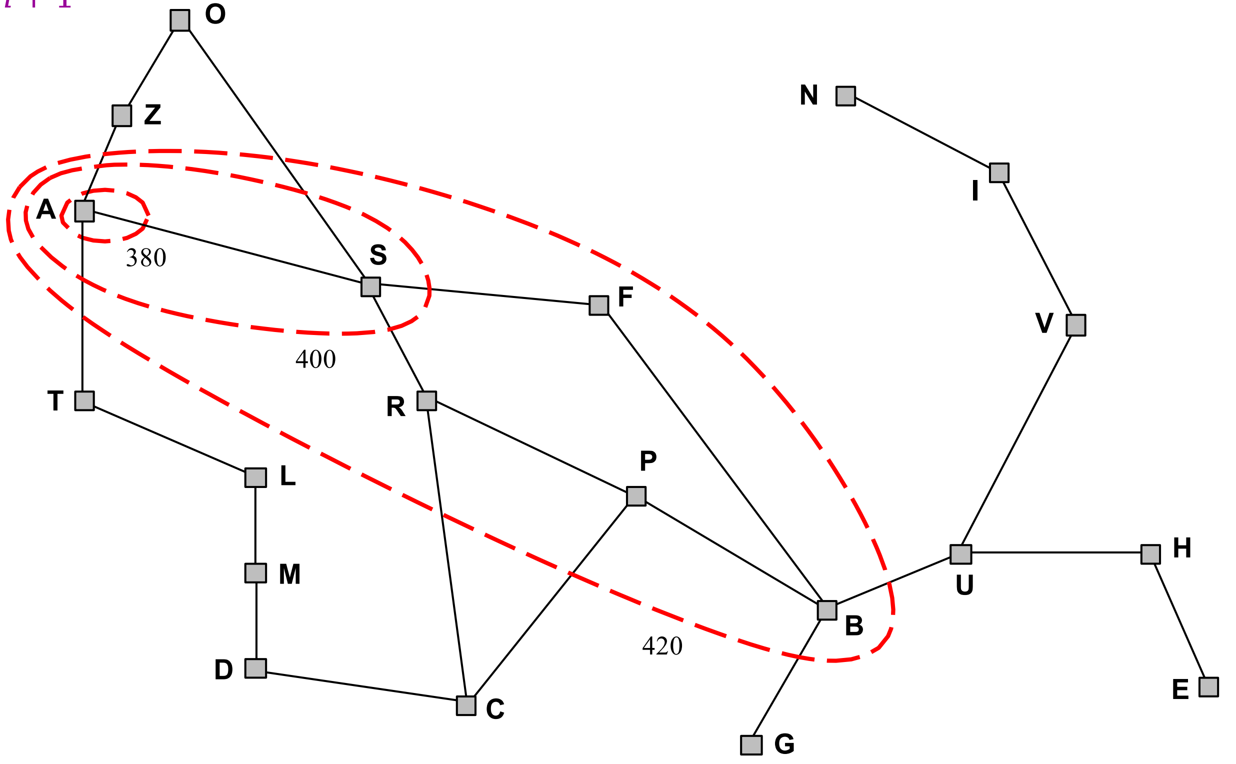
A* search example



Optimality of A*

A* expands nodes in order of increasing f value*

Gradually adds "f-contours" of nodes (cf. breadth-first adds layers) Contour i has all nodes with $f = f_i$, where $f_i < f_{i+1}$



Properties of A* algorithm

Complete: Yes, unless there are infinitely many nodes with $f \leq f(G)$

Time: Exponential in [relative error in h × length of soln.]

Space: Keeps all nodes in memory

Optimal: Yes—cannot expand f_{i+1} until f_i is finished

A* expands all nodes with $f(n) < C^*$
A* expands some nodes with $f(n) = C^*$
A* expands no nodes with $f(n) > C^*$

Summary of informed search algorithms

Heuristic functions estimate costs of shortest paths

Good heuristics can dramatically reduce search cost

Greedy best-first search expands lowest h
- incomplete and not always optimal

A$*$ search expands lowest $g + h$
- complete and optimal
- also optimally efficient (up to tie-breaks, for forward search)

A$*$ is widely used

End

- Any questions?