



### Overview

- Heuristic Search Strategies
- The A\* Algorithm (for Tree Search)
- Properties of A\*
- Graph Search with A\*
- (Good) Heuristics











## **Heuristic Functions**

- heuristic function h: state space → ℝ
- h(n) = estimated cost of the cheapest path from node
   n to a goal node
- if n is a goal node then h(n) must be 0
- heuristic function encodes problem-specific knowledge in a problem-independent way

#### Best-First Search

- an instance of the general tree search (or graph) search algorithm
  - strategy: select next node based on an <u>evaluation</u> function f: state space → R
  - select node with lowest value f(n)
- implementation: selectFrom(fringe, strategy)
  - priority queue: maintains fringe in ascending order of f-values



## Greedy Best-First Search

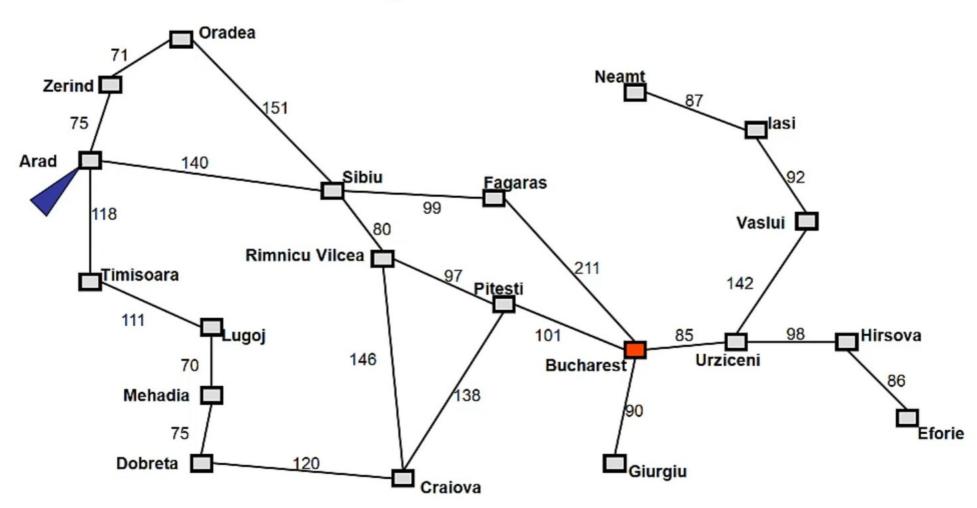
use heuristic function as evaluation function:

$$f(n) = h(n)$$

- always expands the node that is closest to the goal node
- eats the largest chunk out of the remaining distance, hence, "greedy"



# Real-World Problem: Touring in Romania

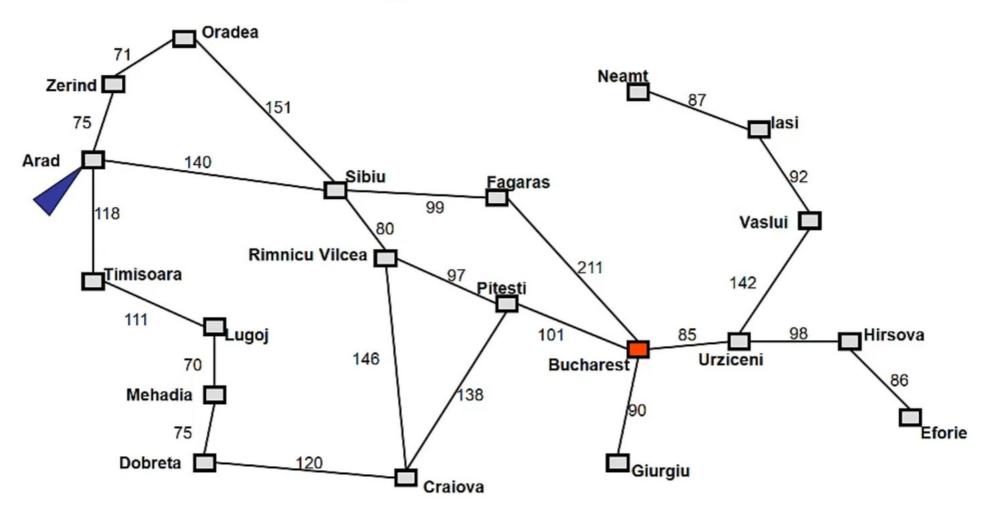


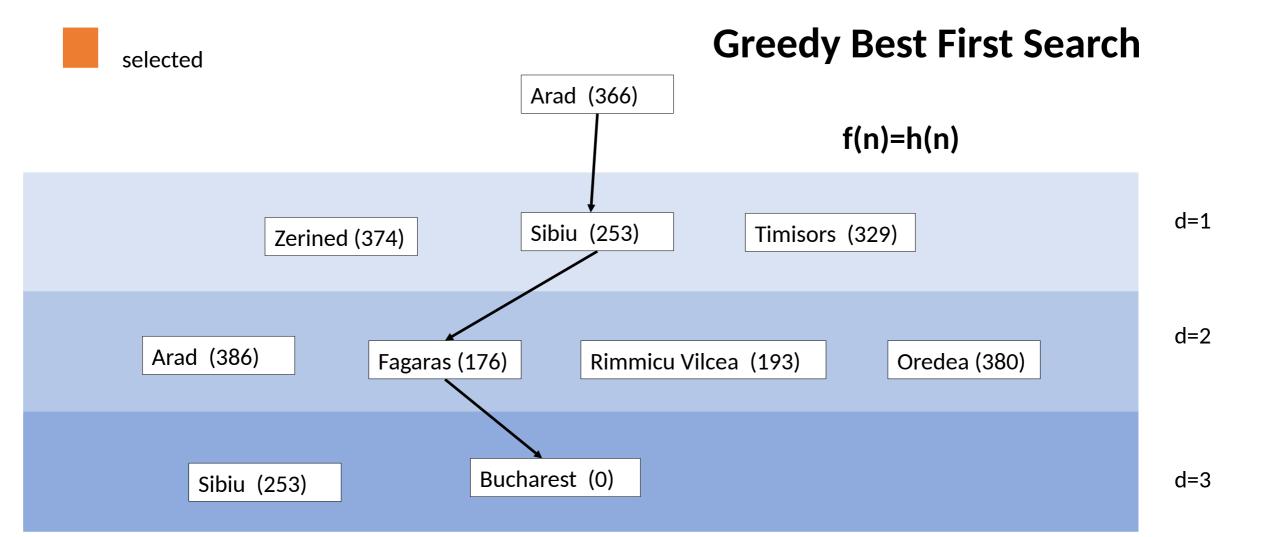
#### **Touring Rumania: Heuristics**

 $h_{SLD}(n) = Straight line distance to Bucharest$ 

Hirsova 151	Rimnicu Vilcea 193
Lasi 226	Craiova 160
Sibiu 253	Dobreta 242
Timisoara 329	Eforie 161
Urziceni 80	Fagaras 176
Vaslui 199	Giurgiu 77
Zerind 374	
	Lasi 226 Sibiu 253 Timisoara 329 Urziceni 80 Vaslui 199

# Real-World Problem: Touring in Romania





Total Distance = 450

### A\* Search

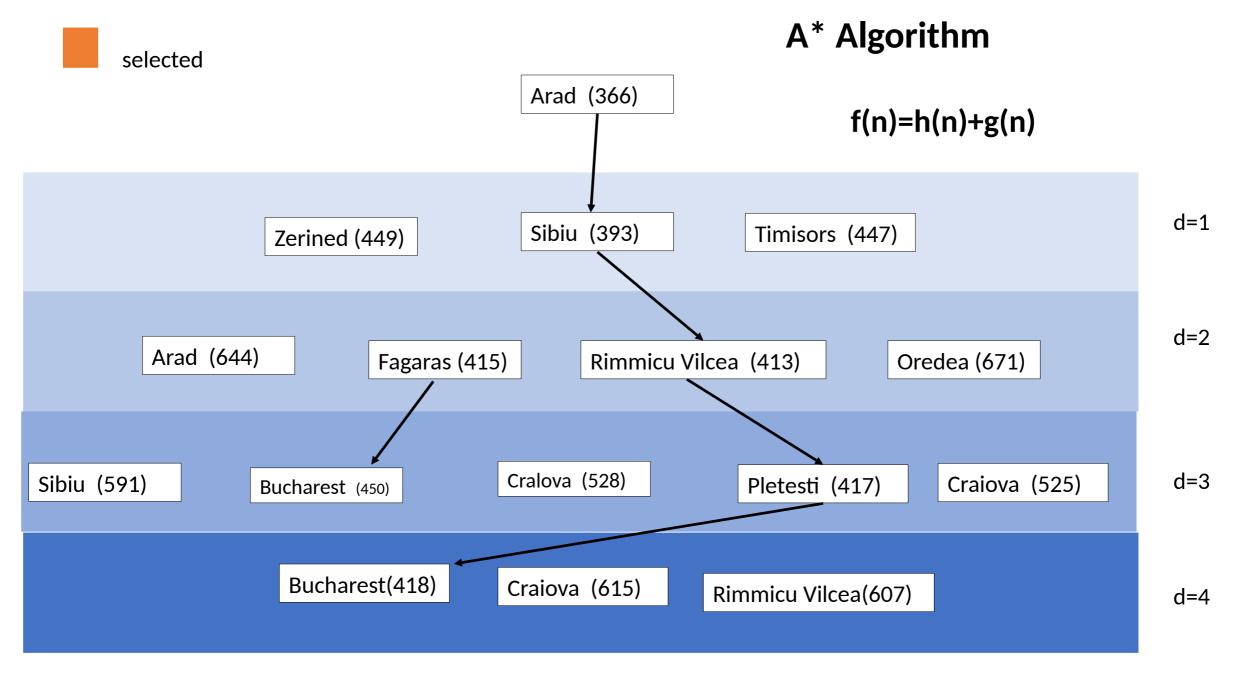
best-first search where

$$f(n) = h(n) + g(n)$$

- -h(n) the heuristic function (as before)
- -g(n) the cost to reach the node n
- evaluation function:

f(n) = estimated cost of the cheapest solution through n

A\* is optimal if h(n) is admissible



Total Distance = 418

# 8-Tile Puzzle

The tiles can be moved to the empty space horizontal and vertical directions only.

•Exercise: Find the plan to solve this problem using A\* algorithm.

- g(n) =Depth of the search tree
- •(i) h(n) = Number of misplaced tiles
- •(ii) h(n) = Manhattan distance to the goal.

**Initial State** 

**Goal State** 

1	2	3
	4	6
7	5	8

1	2	3
4	5	6
7	8	

### Admissible Heuristics

A heuristic h(n) is admissible if it *never overestimates* the distance from n to the nearest goal node.

- example: h<sub>SLD</sub>
- A\* search: If h(n) is admissible then f(n) never overestimates the true cost of a solution through n.

h(n)≤ actual distance to the goal from n

### Completeness of A\*: Contours

- contours: sets of states that can be reached within a certain cost
  - prerequisite for drawing contours: f-values along a path are nondecreasing
- A\* fans out from the start node, adding nodes in concentric bands (contours) of increasing f-values
- A\* is complete: it must reach a contour that includes a goal node