STATISTICAL COMPUTATION

WEEK 7 – RESAMPLING

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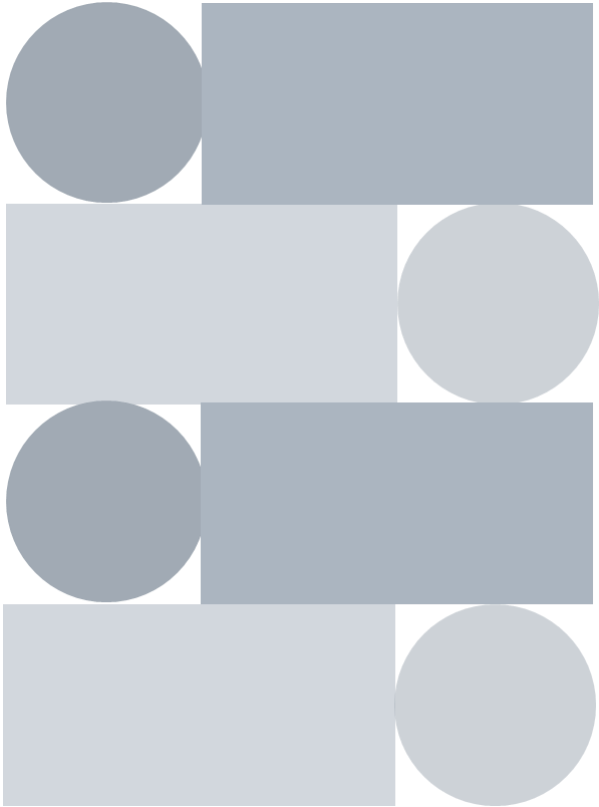

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MATERIALS

- Bootstrap
- Jackknife

01

BOOTSTRAP

ALGORITHM

- Generate a large number of “bootstrap” samples by resampling (with replacement) from the dataset
- Resample with the same structure (dependence, sample sizes) as used in the original sample
- Compute your estimator, $\hat{\theta}$, (here, $\hat{\theta} = \bar{X}$), for each of the bootstrap samples
- Compute the the estimator and “standard deviation” from the statistics calculated above.

$$\hat{\theta}_{bootstrap} = \hat{\theta}_b = \frac{1}{REP}\sum_{i=1}^{REP}\hat{\theta}_{b,i} \qquad \widehat{se}_{boot}\left[\bar{X}\right] = \sqrt{\hat{\sigma}_b^2} \qquad \hat{\sigma}_b^2 = \frac{\sum_{j=1}^{B}(\bar{X}^{(j)} - \bar{X}^{(\cdot)})^2}{B-1}$$

For other estimators, simply replace $\bar{X}$ with the $\hat{\theta}$ of your choice.

ALGORITHM

Bootstrap sample	Bootstrap estimates
1: $(X_1^{(1)}, X_2^{(1)}, \ldots, X_n^{(1)})$	$\rightarrow \hat{\theta}(X_1^{(1)}, X_2^{(1)}, \ldots, X_n^{(1)}) = \overline{X}^{(1)}$
2: $(X_1^{(2)}, X_2^{(2)}, \ldots, X_n^{(2)})$	$\rightarrow \hat{\theta}(X_1^{(2)}, X_2^{(2)}, \ldots, X_n^{(2)}) = \overline{X}^{(2)}$
⋮	⋮
B: $(X_1^{(B)}, X_2^{(B)}, \ldots, X_n^{(B)})$	$\rightarrow \hat{\theta}(X_1^{(B)}, X_2^{(B)}, \ldots, X_n^{(B)}) = \overline{X}^{(B)}$

$$\hat{\theta}_{bootstrap} = \hat{\theta}_b = \frac{1}{REP}\sum_{i=1}^{REP}\hat{\theta}_{b,i}$$

$$\overline{\overline{X}}^{(.)} = \frac{1}{B}\sum_{j=1}^{B}\overline{X}^{(j)}$$

$$\widehat{se}_{boot}\left[\overline{X}\right] = \sqrt{\hat{\sigma}_b^2}$$

$$\hat{\sigma}_b^2 = \frac{\sum_{j=1}^{B}(\overline{X}^{(j)} - \overline{\overline{X}}^{(.)})^2}{B-1}$$

02

JACKKNIFE

ALGORITHM

- Resampling by removing the ith sample elements, i = 1, 2, … , n, so we get the ith resample (i = 1, 2, . . . , n)
- Compute your estimator, $\hat{\theta}$, (here, $\hat{\theta} = \overline{X}$), for each of the jackknife samples
- Compute the the estimator and “standard deviation” from the statistics calculated above.

$$\hat{\theta}_{jackknife} = \hat{\theta}_j = \frac{1}{n}\sum_{i=1}^{n}\hat{\theta}_{j,i} \qquad \widehat{se}_{jack} = \sqrt{\frac{n-1}{n}\sum_{i=1}^{n}(\hat{\theta}_{(i)} - \bar{\hat{\theta}}_{(.)})^2}$$

For other estimators, simply replace $\overline{X}$ with the $\hat{\theta}$ of your choice.

ALGORITHM

~~x_1~~	x_2	x_3	x_4	x_5	x_6	x_7	x_8	...	x_N
x_1	~~x_2~~	x_3	x_4	x_5	x_6	x_7	x_8	...	x_N
x_1	x_2	~~x_3~~	x_4	x_5	x_6	x_7	x_8	...	x_N
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋱	⋮
x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	...	~~x_N~~

$$\hat{\theta}_{jackknife} = \hat{\theta}_j = \frac{1}{n}\sum_{i=1}^{n}\hat{\theta}_{j,i}$$

$$\widehat{se}_{jack} = \sqrt{\frac{n-1}{n}\sum_{i=1}^{n}(\hat{\theta}_{(i)} - \bar{\hat{\theta}}_{(.)})^2}$$

THANKS

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