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Lagrange Interpolation Polynomial

Let $x_0, x_1, \dots, x_n$ denote n distinct real numbers and $f_0, f_1, \dots, f_n$ be arbitrary real numbers. The points (x_0, f_0), (x_1, f_1), .. (x_n, f_n) can be imagined to be data values connected by a curve. Any function $P(x)$ satisfying the condition

$$P(x_k) = f_k \text{ for } k = 0, 1, \dots \dots \dots, n$$

is called interpolation function. Let us consider a second order polynomial of the form,

$$P_2(x) = b_0 (x - x_1)(x - x_2) + b_1 (x - x_0)(x - x_2) + b_2 (x - x_0)(x - x_1) \dots \dots \dots (i)$$

If (x_0, f_0), (x_1, f_1) and (x_2, f_2) are three interpoling points then we have,

$$P_2(x_0) = f_0 = b_0 (x_0 - x_1)(x_0 - x_2)$$

$$\therefore b_0 = \frac{f_0}{(x_0 - x_1)(x_0 - x_2)}$$

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$$P_2(x_1) = f_1 = b_1 (x_1 - x_0)(x_1 - x_2)$$

$$\therefore b_1 = \frac{f_1}{(x_1 - x_0)(x_1 - x_2)}$$

$$P_2(x_2) = f_2 = b_2 (x_2 - x_0)(x_2 - x_1)$$

$$\therefore b_2 = \frac{f_2}{(x_2 - x_0)(x_2 - x_1)}$$

① $\Rightarrow$

$$P_2(x) = f_0 \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)} + f_1 \frac{(x - x_2)(x - x_0)}{(x_1 - x_0)(x_1 - x_2)} + f_2 \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)}$$

$$= f_0 l_0(x) + \cancel{f_1 f} \, f_1 l_1(x) + f_2 l_2(x)$$

$$= \sum_{i=0}^{2} f_i l_i(x) \quad \cdots\cdots\cdots \; (2)$$

Where, $l_i(x) = \prod_{j=0, j \neq i}^{n} \frac{x - x_j}{x_i - x_j} \quad \cdots\cdots\cdots (3)$

$$P_n(x) = \sum_{i=0}^{n} b_i \prod_{j=0, j \neq i}^{n} \frac{(x - x_j)}{\cancel{(x_i - x_j)}} \rightarrow$$ n degree polynomial

$P_n(x) = \sum_{i=0}^{n} f_i l_i(x)$ in general $\cdots \cdots \cdots (4)$

where $l_i(x) = \prod_{j=0,\, j\neq i}^{n} \dfrac{x - x_j}{x_i - x_j}$ .... (5)

Eq^n (4) is called Lagrange Interpolation polynomial

Eq^n (5) is called Lagrange Basis polynomial.

For $n=1$

$$P_1(x) = f_0 l_0(x) + f_1 l_1(x)$$

$$= f_0 \frac{x - x_1}{x_0 - x_1} + f_1 \frac{x - x_0}{x_1 - x_0}$$

$$l_0(x) = \frac{x - x_1}{x_0 - x_1}$$

$$l_1(x) = \frac{x - x_0}{x_1 - x_0}$$

$$= f_0 + \frac{f_1 - f_0}{x_1 - x_0}(x - x_0) \;\ldots\ldots\ldots\; (6)$$

Eq^n (6) is called Linear Interpolation Formula.

$$= \frac{f_1 x - f_1 x_0 - f_0 x + f_0 x_1}{x_1 - x_0}$$

$$= \frac{f_1 x - f_1 x_0 - f_0 x + f_0 x_1 \overbrace{- f_0 x_0 + f_0 x_0}}{x_1 - x_0}$$

→ $f_0 x_0$ একবার যোগ ও বিয়োগ করেছে (extra)

$$= \frac{(x_1 - x_0) f_0 + x(f_1 - f_0) + x_0(f_0 - f_1)}{x_1 - x_0}$$

$$= f_0 + \frac{(f_1 - f_0)(x - x_0)}{x_1 - x_0}$$

$$= f_0 + \frac{f_1 - f_0}{x_1 - x_0}(x - x_0)$$

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Eg - 9.3

→ well defined fn করার sqr root এর table. আমরা জানি যার experimental ও হতে পারে

x	1	2	3	4	5
$f(x)$	1	1.4142	1.7321	2	2.2361

Determine the square root of 2.5 using linear interpolation polynomial.

Soln:

$x_1 = 2, \quad f(x_1) = 1.4142$

$x_2 = 3, \quad f(x_2) = 1.7321$

$$\therefore f(x) = f(x_1) + (x - x_1) * \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$ //straight line eqn এটা $y = mx + c$ আকারের

$$\therefore f(2.5) = 1.4142 + (2.5 - 2) * \frac{1.7321 - 1.4142}{3 - 2}$$

$$= 1.5732$$

Correct Answer 1.5811 //exactly এটাই আসবে

error calculate করতে বললে just actual টা থেকে পাওয়া টা বিয়োগ করবো। আর % এ প্রকাশ করতে বললে 100 দিয়ে গুন করে % লিখবো। আবার relative error calc. করলে original টা দিয়ে ভাগ করতে হবে।

→ interval এর gap বেশি হলে accuracy কমবে।

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তাই error কমানোর জন্য বাড়তি কিছু point নেয়া হয়। অথবা higher order এর interpolation use করা হয়।

// 1° polynomial → Linear → order → 1 so $1+1 = 2$টি point লাগবে।

// যদি order না বলে দেয়, তাহলে যতগুলো point থাকবে তার থেকে 1 কম order use করবে

Eg 9.4 □ <same dataset>

Determine the square root of 2.5 using <u>second order</u> Lagrange interpolation polynomial

→ এর মানে হলো যেহেতু 5টি datapoint তাই মোট মানে $P_0 - P_4$ পর্যন্ত দিবো; 0th থেকে $(n-1)^{th}$ পর্যন্ত

<u>Sol^n:</u>

$$P_n = \sum_{i=0}^{n} y_i l_i(x) \longrightarrow$$ // Lag. interpolation polynomial (মূল formula)

where $$l_i(x) = \prod_{j=0, j\neq i}^{n} \left(\frac{x - x_j}{x_i - x_j}\right)$$ / Lag. basis polynomial

<u>Sol^n:</u> $$P_2(x) = \sum_{i=0}^{2} y_i l_i(x)$$

$$= y_0 l_0(x) + y_1 l_1(x) + y_2 l_2(x) \quad \ldots \ldots \ldots \ldots \ldots (i)$$

$$l_0(x) = \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)}$$

$$l_0(2.5) = \frac{(2.5-3)(2.5-4)}{(2-3)(2-4)} = 0.3750$$

যে point এর জন্য value বের করতে হবে ঐ point এর জন্য
~~এম~~ এমন তিনটি (যেহেতু এখানে order = 2 তাই 2+1 তিনটি)
পরপর point choose করবো (data table থেকে x এর value) যাতে
তাদের মধ্যে range এর মধ্যে ঐ point (2.5) থাকে।
এখন ঐ তিনটি point এর প্রথমটি x_0 ~~(2)~~, পরেরটি x_1
ও তার পরেরটি x_2। এখানে $x_0 = 2, x_1 = 3, x_2 = 4$
ও 2.5 এদের ভেতরে যেকোনো একটি বিন্দু। এবার formula তে
বসাবো।

Now $x_1 = 2 \quad f(x_1) = 1.4142$

$x_2 = 4 \quad f(x_2) = 2$

Then $f(x) = f(x_1) + (x - x_1) * \dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$

$$\therefore f(2.5) = 1.4142 + (2.5 - 2) * \left(\frac{2 - 1.4142}{4 - 2}\right)$$

$$= 1.5607$$

$$l_1(x) = \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)}$$

$$\therefore l_1(2.5) = \frac{(2.5 - 5)(2.5 - 4)}{(3 - 2)(3 - 4)}$$

$$= 0.7500$$

$$l_2(x) = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$$

$$\therefore l_2(2.5) = \frac{(2.5-2)(2.5-3)}{(4-2)(4-3)}$$

$$= -0.125$$

(1) $\Rightarrow$

$$P_2(2.5) = 1.4142 \times 0.3750 + 1.7321 * 0.7500 + 2 * (-0.125)$$

$$= 1.5794$$

Eg 9.5

i	0	1	2	3
x_i	0	1	2	3
$e^{x_i}-1$	0	1.7138	6.3891	19.0855

এটাও well defined function (e^x)

find $e^{1.5} = ?$

Soln:

$$P_3(x) = f_0 l_0(x) + f_1 l_1(x) + f_2 l_2(x) + f_3 l_3(x) \quad \cdots\cdots\cdots \text{(i)}$$

$$l_0(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}$$

$$= \frac{(x-1)(x-2)(x-3)}{(0-1)(0-2)(0-3)}$$

$$= \frac{x^3-6x^2+11x-6}{-3}$$

$$= \cancel{x^3}$$

$$l_1(x) = \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}$$

$$= \frac{(x-0)(x-2)(x-3)}{(1-0)(1-2)(1-3)}$$

$$= \frac{x^3-5x^2+6x}{2}$$

$$l_2(x) = \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}$$

→ ২ এর জন্য হিসাব করলে x এর index বাদে বাকিগুলো লবে x থেকে বিয়োগ হবে ও হরে x এর index থেকে বিয়োগ হবে এবং উভয়ক্ষেত্রেই term গুলো গুণ আকারে থাকবে

$$= \frac{(x-0)(x-1)(x-3)}{(2-0)(2-1)(2-3)}$$

$$= \frac{x^3-4x^2+3x}{-2}$$

$$l_3(x) = \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}$$

$$= \frac{(x-0)(x-1)(x-2)}{(3-0)(3-1)(3-2)}$$

$$= \frac{x^3-3x^2+2x}{6}$$

① ⇒

প্রথমটি 0 এসেছে (dataset এ 0 নিয়ে হিসাব করলে function এর value 0 আসছে)

$$P_3(x) = 0 + \frac{1.7138(x^3-5x^2+6x)}{2} + \frac{6.3891(x^3-4x^2+3x)}{-2} + \frac{19.0856(x^3-3x^2+2x)}{}$$

$$= \frac{5.0732x^3 - 6.3621x^2 + 11.5987x}{6}$$

$$= 0.8455x^3 - 1.0604x^2 + 1.9331x$$

$\therefore P_3(1.5) = 3.3677$

$\Rightarrow e^{1.5} - 1 = 3.3677$

$\therefore e^{1.5} = 4.3677$

Limitations/Drawbacks:

→ অনেক বেশি multiplication ও division

→ নতুন একটি point incorporate করলে আগের value use করা যাবে না কারণ সবগুলোর এদের উপর base করে change হয়ে যাবে। আবার

প্রথম থেকে calculation করে আসছি।

→ এ সমস্যার জন্য Newton interpolation use হয়।

Oct 25, 2017
Rocky sir

Newton Interpolation Polynomial

The Newton form of polynomial is —

$$P_n(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1) + \cdots\cdots + a_n(x-x_0)(x-x_1) + \cdots\cdots (x-x_{n-1}) \quad \cdots\cdots\cdots (i)$$

// এটি polynomial এর shifted power form.

To constant the interpolation polynomial, we need to determine the coefficient $a_0, a_1, \ldots, a_n$.

Let us assume that $(x_0, f_0), (x_1, f_1), \ldots\ldots\ldots, (x_{n-1}, f_{n-1})$ are interpolating points. That is,

$$P_n(x_k) = f_k, \quad k = 0, 1, \ldots\ldots, n-1 \quad \cdots\cdots\cdots (ii)$$

// ↳ interpolating function.

Now at $x = x_0$, we have from ~~an~~ equation (i),

$$P_n(x_0) = a_0 = f_0 \quad \cdots\cdots\cdots (iii)$$

// ↳ x এর পরিবর্তে x_0 বসাচ্ছি (i) নং এ

Similarly at $x = x_1$,

$$P_n(x_1) = a_1 + a_1(x_1 - x_0) = f_1$$

// ↳ x এর পরিবর্তে (i) নং এ x_1 বসাচ্ছি

// আমার target: in terms of functional value
// co-efficient $a_0, a_1, a_2, \dots$ এগুলোর value বের করা

$$\therefore f_0 + a_1(x_1 - x_0) = f_1$$

$$\therefore a_1 = \frac{f_1 - f_0}{x_1 - x_0} \quad \dots \dots \dots (iv)$$

At $x = x_2$

$$P_n(x_2) = a_0 + a_1(x_2 - x_0) + a_2(x_2 - x_0)(x_2 - x_1) = f_2 \quad \dots\dots (v)$$

substituting the value of a_0 and a_1 in eq^n (v) we get,

$$f_0 + \left(\frac{f_1 - f_0}{x_1 - x_0}\right)(x_2 - x_0) + a_2(x_2 - x_0)(x_2 - x_1) = f_2$$

$$\therefore a_2 = \frac{[(f_2 - f_1)(x_2 - x_1)] - [(f_1 - f_0)/(x_1 - x_0)]}{x_2 - x_1} \quad \dots \dots \dots (vi)$$

// $a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots$ // polynomial
// এর power form

Let us define a notation

$$f[x_k] = f_k$$

$$f[x_k, x_{k+1}] = \frac{f[x_{k+1}] - f[x_k]}{x_{k+1} - x_k}$$

$$f[x_k, x_{k+1}, x_{k+2}] = \frac{f[x_{k+1}, x_{k+2}] - f[x_k, x_{k+1}]}{x_{k+2} - x_k}$$

(প্রথমটি বাদে বাকিগুলো আছে তার f^nval – শেষেরটা বাদে বাকিগুলো আছে তাদের f^nval)/শেষেরটা – প্রথমটা

$$f[x_k, x_{k+1}, \ldots, x_i, x_{i+1}] = \frac{f[x_{k+1}, \ldots, x_{i+1}] - f[x_k, \ldots, x_i]}{x_{i+1} - x_k}$$

These quantities are called divided differences. Now we can express the co-efficients a_i in terms of these divided differences.

$$a_0 = f_0 = f[x_0]$$

$$a_1 = \frac{f_1 - f_0}{x_1 - x_0} = f[x_0, x_1]$$

$$a_2 = \frac{\left(\frac{f_2 - f_1}{x_2 - x_1}\right) - \left(\frac{f_1 - f_0}{x_1 - x_2}\right)}{x_2 - x_1} = \frac{f[x_1, x_2] - f[x_1, x_0]}{x_2 - x_0}$$

$$= f[x_0, x_1, x_2]$$

Thus, $a_n = f[x_0, x_1, \ldots, x_n]$

① ⇒

$$P_n(x) = f[x_0] + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x_1 - x_0)(x - x_1) + \cdots + f[x_0, x_1, \ldots, x_n](x - x_0)(x - x_1) \cdots (x - x_{n-1}) \quad \cdots\cdots (vii)$$

order

// 2nd order বললে 3টি data point নিব

// 4th " " 4 " " " "

// কিছু না বললে সব data point নিবে

In compact form,

$$P_n(x) = \sum_{i=0}^{n} f[x_0, \dots, x_i] \prod_{j=0}^{i-1} (x-x_j)$$

<u>eg. 9.6</u> : 2^{nd} order New. Int. pol. use করে.

i	0	1	2	3
x_i	1	2	3	4
$\log x_i$	0	0.3010	0.4771	0.6021

$\log 2.5 = ?$ using 2^{nd} order Newton Interpolation polynomial

<u>Soln</u>: $P_2(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1)$

$a_0 = f[x_0] = 0$

$a_1 = \cancel{f[x_0] = 0}\ f[x_0, x_1] = \dfrac{f_1 - f_0}{x - x_0}$

$$= \frac{0.3010 - 0}{2-1}$$

$$= 0.3010$$

$$a_2 = f[x_0, x_1, x_2] = \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0}$$

$$= \frac{f[x_1, x_2] - 0.3010}{3-0}$$

$$f[x_1, x_2] = \frac{f_2 - f_1}{x_2 - x_1} = \frac{0.4771 - 0.3010}{3-2} = 0.1761$$

$$a_2 = \frac{0.1761 - 0.3010}{3-1}$$

$$= -0.06245$$

$$P_2(x) = 0 + 0.3010(x-1) - 0.06245(x-1)(x-2)$$

$$P(2.5) = 0 + 0.3010(2.5-1) - 0.06245(2.5-1)(2.5-2)$$

$$= 0.4047$$

/ Exercise দেখবো ।

Oct 29, 2017
Moon Mam

Jacobi Iterative Method

x, y, z

$\underline{n}$

$$x_1 = \frac{b_1 - (a_{12}x_2 \cdots a_{1n}x_n)}{a_{11}}$$

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n = 0$$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = 0$$

$$\vdots$$

$$a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = 0$$

$\rightarrow$ 1st iteration এ একটি value ধরে নিয়ে x_1 calc. করবো

$E = 0.01$

$$|x_1^{(3)} - x_1^{(2)}| \;\&\; |x_2^{(3)} - x_2^{(2)}| \ldots \longrightarrow x_1, x_2$$

সবগুলোতে যদি satisfy করবে

Oct 30, 2017
Mahmud Sir

Newton ~~Raphson~~ Interpolation Polynomial:

$$P_n(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1) + \cdots\cdots +$$

shifted power form $a_n(x-x_0)(x-x_1) * \cdots\cdots * (x-x_{n-1})$

$$= f[x_0] + f[x_0, x_1](x-x_0) + f[x_0, x_1, x_2]$$
$$(x-x_0)(x-x_1) + \cdots\cdots + f[x_0, x_1, \cdots, x_n]$$
$$(x-x_0)(x-x_1)\cdots\cdots(x-x_{n-1})$$

Divided Difference Table:

$$a_2 = f[x_0, x_1, x_2]$$
$$= \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0}$$

$$f[x_1, x_2] = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f_2 - f_1}{x_2 - x_1}$$

$$f[x_0, x_1] = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{f_1 - f_0}{x_1 - x_0}$$

similarly $f[x_1, x_2, x_3, x_4] = \dfrac{f[x_2, x_3, x_4] - f[x_1, x_2, x_3]}{x_4 - x_1}$

$$\therefore f[x_2, x_3, x_4] = \frac{f[x_3, x_4] - f[x_2, x_3]}{x_4 - x_2}$$

$$f[x_3, x_4] = \frac{f_4 - f_3}{x_4 - x_3}$$

$$f[x_2, x_3] = \frac{f_3 - f_2}{x_3 - x_2}$$

$$f[x_1, x_2, x_3] = \frac{f[x_2, x_3] - f[x_1, x_2]}{x_3 - x_1}$$

$$f[x_2, x_3] = \frac{f_3 - f_2}{x_3 - x_2}$$

$$f[x_1, x_2] = \frac{f_2 - f_1}{x_2 - x_1}$$

i	x_i	$f[x_i]$	First Difference	Second Difference	Third Difference	Forth Difference
0	x_0	$f[x_0] \to a_0$				
			$f[x_0, x_1] \to a_1$			
1	x_1	$f[x_1]$		$f[x_0, x_1, x_2] \to a_2$		
			$f[x_1, x_2]$		$f[x_0, x_1, x_2, x_3] \to a_3$	
2	x_2	$f[x_2]$		$f[x_1, x_2, x_3]$		$f[x_0, x_1, x_2, x_3, x_4] \to a_4$
			$f[x_2, x_3]$		$f[x_1, x_2, x_3, x_4]$	
3	x_3	$f[x_3]$		$f[x_2, x_3, x_4]$		
			$f[x_3, x_4]$			
4	x_4	$f[x_4]$				

5টি point এর জন্য P(4) হবে 0-4 5টি point

Divided difference table

আগের step এর calculation use করা যায় তাই calculation effort কম লাগে।

//বইতে ভুল আছে।

eg — 9.7

i :	0	1	2	3	4
x_i	1	2	3	4	5
$f(x_i)$	0	7	26	63	124

$f(x) = x^3 - 3$

$f'(x) = 3x^2$

$f''(x) = 6x$

$f'''(x) = 6$

$f^{iv}(x) = 0$

$\downarrow$

$= f[x_0, x_1, x_2, x_3 x_4]$
এটা 0 আসবে

$f(1.5) = ?$

here,

$f[x_0] = 0$

$f[x_1] = 7$

$$f[x_0, x_1] = \frac{\overset{f_1 - f_0}{\cancel{f(x_1) - f(x_0)}}}{x_1 - x_0} = \frac{7 - 0}{2 - 1} = 7$$

$$f[x_1, x_2] = \frac{f_2 - f_1}{x_2 - x_1} = \frac{26 - 7}{3 - 2} = 19$$

$$f[x_0, x_1, x_2] = \frac{19 - 7}{3 - 1} = \frac{12}{2} = 6$$

$$f[x_2, x_3] = \frac{63 - 26}{4 - 3} = 37$$

$$f[x_1, x_2, x_3] = \frac{f[x_2, x_3] - f[x_1, x_2]}{x_3 - x_1}$$

$$= \frac{37 - 19}{4 - 2}$$

$$= 9$$

$$f[x_0, x_1, x_2, x_3] = \frac{9 - 6}{4 - 0} = 0.75$$

$\cancel{f[x_1, x_2, x_3, x_4]} =$

$\cancel{f[x_2, x_3, x_4]} =$

Exercise থেকে ৩ টিকে form এ linear, Newton, Lagrange এর data point use করে বের করে দেখতে হবে। Xm একটা থাকবে, like Lagrange এ

Newton দিয়ে হিসাব করে compare। comments কর। Std. func. দিয়ে error=?

$$f[x_2, x_3] = \frac{63-26}{4-3} = 37$$

$$f[x_3, x_4] = \frac{124-63}{5-4} = 61$$

$$\therefore f[x_2, x_3, x_4] = \frac{61-37}{5-3} = 12$$

$$\therefore f[x_1, x_2, x_3, x_4] = \frac{12-9}{124-7} = 0.025641$$

$\therefore$ ~~$P_4(x\ 2.5)$~~

$$\therefore P_4(x) = f[x_0] + f[x_0, x_1](x-x_0) + f[x_0, x_1, x_2](x-x_0)(x-x_1) + f[x_0, x_1, x_2, x_3](x-x_0)(x-x_1)(x-x_2) + f[x_0, x_1, x_2, x_3, x_4](x-x_0)(x-x_1)(x-x_2)(x-x_3)$$

$$P_4(1.5) = 0 + 7(1.5-1) + 6(1.5-1)(1.5-2) + 0.75(1.5-1)(1.5-2)(1.5-3) + 0.025641(1.5-1)(1.5-2)(1.5-3)(1.5-4)$$

$$= 2.375$$

Oct 31, 2017
Mahmud Sir

INTERPOLATION WITH EQUIDISTANT POINTS

আগের গুলোর মতো এগুলোও equidistant হবে এখন x এর value গুলো একটি factor এ change হবে like $x = 10, 20, 30, \ldots\ldots$; মানে 10 করে যেকোনো সংখ্যক করে বাড়লে, Interpolation with equidistant points হবে। এটাকে Newton Gragory Interpolation ও বলে।

Let us assume that,

$$x_k = x_0 + kh$$

, where x_0 is the reference point and h is the step size. The integer k may take either positive/negative values.

আমরা method এর divided difference quantities কে simple difference এ নিয়ে আসবো। এজন্য কিছু notation define করা হলো—

The first forward difference Δf_1 is defined as,

$$\Delta f_i = f_{i+1} - f_i$$ // base case | recursive way তে solve করার সময় বার-বার নিচে approach করবে

The second forward difference is defined as,

$$\Delta^2 f_i = \Delta f_{i+1} - \Delta f_i$$

In general $\Delta^j f_i = \Delta^{j-1} f_{i+1} - \Delta^{j-1} f_i$

We know that $f[x_0, x_1] = \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}$

$$= \frac{f_1 - f_0}{h}$$

$\therefore f_1 - f_0 = h f[x_0, x_1]$

$\Rightarrow \Delta f_0 = h f[x_0, x_1]$

Similarly,

$\Delta f_1 = h f[x_1, x_2]$

Now, $\Delta^2 f_0 = \Delta f_1 - \Delta f_0$

$= \{f[x_1, x_2] - f[x_0, x_1]\}$

$\therefore \Delta^2 f_0 = h \cdot 2h \cdot f[x_0, x_1, x_2]$

$f[x_0, x_1, x_2] = \dfrac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0}$ ← equidistant ও দুটি point এর মাঝের দূরত্ব $h+h=2h$

$= \dfrac{f[x_1, x_2] - f[x_0, x_1]}{2h}$

$\therefore f[x_1, x_2] - f[x_0, x_1] = 2h f[x_0, x_1, x_2]$

<u>In general</u> by induction,

$\Delta^j f_i = j!\, h^j f[x_i, x_{i+1}, \dots\dots\dots, x_{i+j}]$

$\therefore \Delta^j f_0 = j!\, h^j f[x_0, x_1, \dots\dots, x_j]$

// i এর value 0 ধরলাম কারণ 1st, 2nd, 3rd, ... difference এর প্রথম সব value লাগবে যা প্রতি column এর topmost value টা নিয়ে (আগেরটার মতো) ।.... এখানে f_0 এর মূল্য লাগবে।

$\Rightarrow f[x_0, x_1, \dots\dots, x_j] = \dfrac{\Delta^j f_0}{j!\, h^j}$

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From Newton interpolation,

$$P_n(x) = \sum_{j=0}^{n} f[x_0, x_1, \ldots\ldots, x_j] \prod_{k=0}^{j-1} (x - x_k) \quad \ldots\ldots\ldots (i)$$

(আগের class এর চেয়ে মতোই।

$$(i) \Rightarrow P_n(x) = \sum_{j=0}^{n} \frac{\Delta^j f_0}{j!\, h^j} \prod_{k=0}^{j-1} (x - x_k) \quad \text{// putting value} \quad \ldots\ldots\ldots (ii)$$

Let us set

$x = x_0 + sh$ and ~~$P_n(s)$~~ $P_n(s) = P_n(x)$

We know that, $x_k = x_0 + kh$ // initially বলে এসেছি

$$\therefore x - x_k = (s - k)h$$

Putting this in Eq^n (ii),

$$(ii) \Rightarrow P_n(s) = \sum_{j=0}^{n} \frac{\Delta^j f_0}{j!\, h^j} \prod_{k=0}^{j-1} (s-k)h$$

$$= \sum_{j=0}^{n} \frac{\Delta^j f_0}{j!\, \cancel{h^j}} \left[s(s-1)(s-2)\cdots\cdots(s-j+1)\right] \cancel{h^j}$$

0 – (j-1) → j সংখ্যক term এ h গুণ ~~আছে~~ আকারে আছে তাই h^j

$$= \sum_{j=0}^{n} \frac{s(s-1)(s-2)\cdots\cdots(s-j+1)}{j!} \Delta^j f_0$$

$$= \sum_{j=0}^{n} \binom{s}{j} \Delta^j f_0 \quad \ldots\ldots\ldots (iii)$$

$${}^sC_j = \frac{s!}{(s-j)!\, j!}$$

where $\binom{s}{j} = \dfrac{s(s-1)(s-2)\cdots(s-j+1)}{j!}$

Equations (III) and (II) are known as Gragory-Newton forward difference formula (or forward difference interpolation/ interpolation with equidistance point).

Forward difference table:

general case টা ছিল $\Delta^j f_i = \Delta^{j-1} f_{i+1} - \Delta^{j-1} f_i$

x	f	Δf	$\Delta^2 f$
x_0	f_0		
		Δf_0	
x_1	f_1		$\Delta^2 f_0$
		Δf_1	
x_2	f_2		

- (NOT) দিয়ে যেগুলো আমার just ~~...~~ এগুলো লাগবে

1st difference (Δ এর উপর pow 1)

2nd diff.

divided difference এ (last − first) point দিয়ে আবার ভাগ করতে হবে । এখানে তা লাগবে না

→ Backward difference table → self study
(এটাই just reversely calculate করতে হয়)

Example 9.8

θ	10	20	30	40	50
$\sin\theta$	0.1736	0.3420	0.5000	0.6428	0.7660

// যদি Gragory interpolation বলে নাও দেয় প্রশ্নে data table
// থেকে বুঝতে হবে যে এটা Grag-Newton দিয়ে করতে হবে কারণ
// θ এর value factor আকারে বাড়ছে। এখানে factor = 10
// মানে 1 এর বেশি value তে change হলেই grag-newt.
// এ করতে হবে। But 1 করে করে change হলে Newton Int. use করলেই হবে।

$\theta = 25^\circ = ?$

Sol^n:

$x_0 = \theta_0 = 10$

$h = 10$

$s = \dfrac{x - x_0}{h} = \dfrac{25 - 10}{10} = 1.5$

$$P_4(s) = \sum_{j=0}^{4} \frac{\Delta^j f_0}{j!}\,[s(s-1)(s-2)\cdots(s-j+1)]$$

$$= f_0 + \frac{\Delta f_0\, s}{\lfloor 1} + \frac{\Delta^2 f_0}{\lfloor 2} s(s-1) + \frac{\Delta^3 f_0}{\lfloor 3} s(s-1)(s-2) + \frac{\Delta^4 f_0}{\lfloor 4} s(s-1)(s-2)(s-3)$$

এই value গুলো equidistant table থেকে পাবো। এই গুলো বুঝতে সমস্যা হবে।

x	f	Δf	Δf^2	Δf^3	$\Delta^4 f$
x_0	f_0				
		Δf_0			
x_1		~~Δf~~			
⋮					
x_4 পর্যন্ত					

// just simple বিয়োগ করো করো

// আমার just not suffix এর term গুলো লাগবে (top most গুলো)

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