

1. $-17 \bmod 23$

We know that,

$$a \bmod m = (a+m) \bmod m \text{ if } a < 0;$$

$$\begin{aligned} \therefore -17 \bmod 23 &= (-17+23) \bmod 23 \\ &= 6 \bmod 23 \\ &= 6 \end{aligned}$$

(Ans:)

2. Multiplicative inverse of $-13 \bmod 23$.

We want a number x such that

$$(-13)x \equiv 1 \bmod 23$$

Let's convert $-13 \bmod 23$ into its positive equivalent

$$-13 \bmod 23 = 10 \bmod 23$$

$$\therefore 10 \cdot x \equiv 1 \bmod 23$$

Using Extended Euclidean algorithm

$$23 = 2 \times 10 + 3$$

$$10 = 3 \times 3 + 1$$

$$3 = 3 \times 1 + 0$$

Back substitute:

$$1 = 10 - 3 \times 3$$

$$\Rightarrow 1 = 10 - 3 \times (23 - 2 \times 10)$$

$$= 10 - 3 \times 23 + 6 \times 10$$

$$= 7 \times 10 - 3 \times 23$$

$$\Rightarrow 10^{-1} \bmod 23 = 7$$

$$\therefore (-13)^{-1} \bmod 23 = 7 \quad (\text{Ans:})$$