QUADRATICS

Function

$f(x) = x+3$

$f(2) = 2+3$

$f(5) = 5+3$

equation

$x^2+3x+5 = 2$ $x = ?$

vs

$f(x) = x^2+3x+5$ } Not a function.

expression.

Quadratic expression.

Linear Expressions: $mx+b$

Quadratic " : ax^2+bx+c

Factorised Form

$(x+m)(x-m)$

E.g $(x+3)(x-5)$

Completed Square

$a(x-b)^2+c$

E.g $2(x-3)^2+4$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Express $2x^2+8x+12$ in the form $p(x+q)^2+r$

completing the square

$2(x^2+4x+6)$

$2\left(x^2+\underbrace{2\times x\times 2}_{2ab}+2^2+2^2+6\right)$

$2\left[(x+2)^2-2^2+6\right]$

$2\left[(x+2)^2+2\right]$

$2(x+2)^2+4$

$(a+b)^2=a^2+2ab+b^2$

$x^2+2\cdot x\cdot 3+3^2$

$=(x+3)^2$

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ax^2+bx+c

$2x^2+8x+12$

Step 1: Divide every term by a: and then multiply everything by a.

$2(x^2+4x+6)$

Half $4\rightarrow 2$

Add 2^2, Subtract 2^2

Step 2:

$2\left(x^2+4x+2^2-2^2+6\right)$

Step 3:

$2\left((x+2)^2-2^2+6\right)$

$2\left[(x+2)^2+2\right]$

$2(x+2)^2+4$

$3x^2 - 18x + 30 \rightarrow$ Complete square

$3(x^2 - 6x + 10) \rightarrow$ Step 1

$3(x^2 - 6x + 3^2 - 3^2 + 10) \rightarrow$ Step 2

$3((x-3)^2 - 3^2 + 10)$

$3((x-3)^2 + 1)$

$3(x-3)^2 + 3$

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1) Take 'a' common

2) Add middle term's half's square and subtract it again.

3) form $(\ )^2$ by looking at the terms.

$2x^2 - 10x + 5 \rightarrow$ Completing the square.

$2(x^2 - 5x + \frac{5}{2})$

$2(x^2 - 5x + (\frac{5}{2})^2 - (\frac{5}{2})^2 + \frac{5}{2})$

$2((x - \frac{5}{2})^2 - (\frac{5}{2})^2 + \frac{5}{2})$

$2[(x - \frac{5}{2})^2 - \frac{25}{4} + \frac{5}{2}]$ $= -\frac{15}{4}$

$2[(x - \frac{5}{2})^2 - \frac{15}{4}]$

$\boxed{2(x - \frac{5}{2})^2 - \frac{15}{2}}$

Homework:

Complete the squares for the following:

i) $x^2 - 6x + 6$

ii) $2x^2 - 16x - 4$

① $2[x^2 - 8x - 2]$

② $2[(x^2 - 8x + 4^2 - 4^2 - 2)]$

$2[(x-4)^2 - 16 - 2]$

$= 2[(x-4)^2 - 18]$

$= 2(x-4)^2 - 36$ ✓

Factorising Quadratic Expression:

Factorising: $4x + 10 \Rightarrow 2(2x+5)$

$a^2 - b^2 = (a+b)(a-b)$

factor $\rightarrow x^2 - 16 = (x)^2 - (4)^2$ ($a^2 - b$)

$= (x-4)(x+4)$

factor: $x^2 - 5$ ($5 \rightarrow (\sqrt{5})^2$)

$x^2 - (\sqrt{5})^2$

$\hookrightarrow (x+\sqrt{5})(x-\sqrt{5})$

$(\sqrt{5})^2 = 5$

Factorise: $x^2+6x-16$ } 3 terms
↳ Middle term break

Step 1: Multiply the coefficent of x^2 and the constant number. (ignore sign)

Step 2: Write all the factorisation of 16 →

16

		Add	Sub
①	1 × 16	17	15
②	2 × 8	10	6
③	4 × 4	8	0

Step 3: Break the middle term.

$x^2 + 8x - 2x - 16$

Step 4: Group the terms into 2 pieces

$x(x+8) - 2(x+8)$

Step 4: Transform into factors.

$(x-2)(x+8)$

$x^2 - 2x - 24$: Factorise.

$x^2 + 4x - 6x - 24$

$x(x+4) - 6(x+4)$

$\Rightarrow (x+4)(x-6)$

24

1 × 24
2 × 12
3 × 8
4 × 6
↳ 4 − 6

$2x^2 + 29x - 15$ } Factorise.

① $2 \times 15 = 30$

② $2x^2 + 30x - 1x - 15$

③ $2x(x+15) - 1(x+15)$

④ $(2x-1)(x+15)$

Ans

30

$1 \times 30 = 30$

2×15

3×10

5×6

a) $4x^2 - 32x + 40$ } Complete the square form.

↳ $a(x+b)^2 + c$

① $4(x^2 - 8x + 10)$

② $4\left(x^2 - 8x + 4^2 - 4^2 + 10\right)$

③ $4\left((x-4)^2 - 4^2 + 10\right)$ $\quad -6$

$= 4\left((x-4)^2 - 6\right)$

$= \boxed{4(x-4)^2 - 24}$

$x + 5 = 3$

$\boxed{x = -2}$

b) Solve $1u^2 - 5u - 14 = 0$ } factorise this.

To solve any quadratic equation you need to factorise it.

14
1 × 14
2 × 7

$u^2 - 7u + 2u - 14$

$u(u-7) + 2(u-7)$

$(u-7)(u+2) = 0$

$u - 7 = 0$ or $u + 2 = 0$

$\boxed{u = 7 \text{ or } u = -2}$

c) Solve

$$\frac{w^2-10}{w+2} + w - 4 = w - 3$$

$$\frac{w^2-10}{w+2} = \frac{1}{1}$$

$$w^2 - 10 = w + 2$$

$$1w^2 - w - 12 = 0$$

$$w^2 + 3w - 4w - 12 = 0$$

$$w(w+3) - 4(w+3) = 0$$

$$(w+3)(w-4) = 0$$

$w+3=0$ or $w-4=0$

$w=-3$ or $w=4$

12
1×12
2×6
3×4

$3, -4$

Ans: $w=-3$ or $w=4$

$$2x^2 + 8x + 8$$

$$2x^2 + 4x + 4x + 8$$

} Factorise

16	
1×16	
2×8	
4×4	$\rightarrow 8$

$$2x(x+2) + 4(x+2)$$

$$(x+2)(2x+4)$$

$\hookrightarrow 2(x+2)$

$$\Rightarrow (x+2) \times 2(x+2)$$

$$\Rightarrow 2(x+2)^2$$

Sketching Quadratic functions.

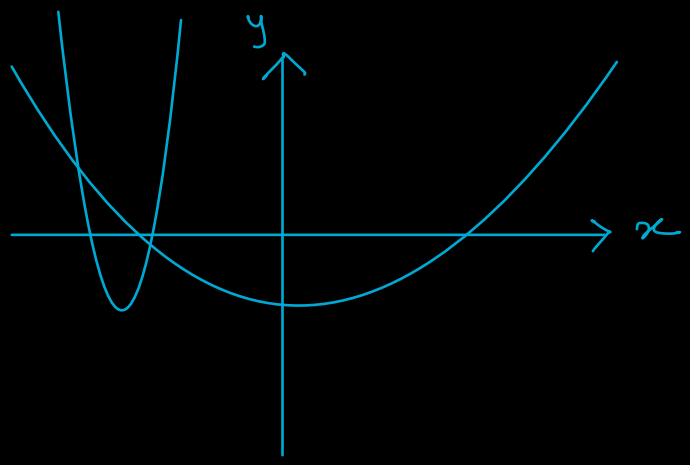
① ax^2+bx+c

if a is +ve → happy face

if a is -ve → sad face

② Completing the square form:

$$y = a(x-b)^2 + c$$



Thursday : 1.5 hrs

~~Sunday~~ : 1.5 hrs