QUADRATICS

Function

$f(x) = x+3$

$f(2) = 2+3$

$f(5) = 5+3$

equation

$x^2+3x+5 = 2$ $\quad x = ?$

vs

$f(x) = \underbrace{x^2+3x+5}_{\text{expression.}}$ } Not a function.

input → Machine → output/resut

Quadratic expression.

Linear Expressions: $mx+b$

Quadratic ": ax^2+bx+c

Factorised Form

$(x+m)(x-m)$

E.g $(x+3)(x-5)$

Completed Square

$a(x-b)^2+c$

E.g $2(x-3)^2+4$

Quadric Formula

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

Express $\underbrace{2x^2+8x+12}$ in the form $p(x+q)^2+r$ → completing the square

$2(x^2+4x+6)$

$2(\underbrace{x^2+\underset{2\ a\ b}{2\times x\times 2}+2^2}-2^2+6)$

$(a+b)^2 = a^2+2ab+b^2$

$a^2+2ab+b^2$

$\boxed{x^2+2\cdot x\cdot 3+3^2}$

$= (x+3)^2$

$2\left[(x+2)^2-2^2+6\right]$

$2\left[(x+2)^2+2\right]$

$2(x+2)^2+4$

$a x^2 + bx + c$

$2x^2+8x+12$

Step 1: Divide every term by a and then multiply everything by a.

$2(x^2+4x+6)$

Half $4 \to 2$
Add 2^2, Subtract 2^2

Step 2:

$2(\underbrace{x^2+4x+2^2}-2^2+6)$

Step 3:

$2((x+2)^2-2^2+6)$

$2\left[(x+2)^2+2\right]$

$2(x+2)^2+4$

$3x^2 - 18x + 30 \rightarrow$ Complete square

$3(x^2 - 6x + 10) \rightarrow$ Step 1

$3(\underbrace{x^2 - 6x + 3^2} - 3^2 + 10) \rightarrow$ Step 2

$3((x-3)^2 - 3^2 + 10)$

$3((x-3)^2 + 1)$

$3(x-3)^2 + 3$

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1) Take 'a' common
2) Add middle term's half's square and subtract it again.
3) form $(\quad)^2$ by looking at the terms.

$2x^2 - 10x + 5 \rightarrow$ Completing the square.

$2(x^2 - 5x + \frac{5}{2})$

$2(\underbrace{x^2 - 5x + (\frac{5}{2})^2} - (\frac{5}{2})^2 + \frac{5}{2})$

$2((x - \frac{5}{2})^2 - \underbrace{(\frac{5}{2})^2 + \frac{5}{2}})$

$2[(x - \frac{5}{2})^2 - \underbrace{\frac{25}{4} + \frac{5}{2}}]$

$= -\frac{15}{4}$

$2[(x - \frac{5}{2})^2 - \frac{15}{4}]$

$$\boxed{2(x - \frac{5}{2})^2 - \frac{15}{2}}$$

Homework:

Complete the squares for the following:

i) $x^2 - 6x + 6$

ii) $\underbrace{2x^2 - 16x - 4}$

① $2[x^2 - 8x - 2]$

② $2[(x^2 - 8x + 4^2 - 4^2 - 2)]$

$2[(x - 4)^2 - 16 - 2]$

$= 2[(x-4)^2 - 18]$

$= 2(x-4)^2 - 36$

Factorising Quadratic Expression.

Factorising: $4x + 10 \Rightarrow 2(2x+5)$

$a^2 - b^2 = (a+b)(a-b)$

factor $\rightarrow x^2 - 16 = x^2 - 4^2$ $\rightarrow a^2 - b$

$= (x-4)(x+4)$

factor: $x^2 - 5 \rightarrow (\sqrt{5})^2$

$x^2 - (\sqrt{5})^2$

$\hookrightarrow (x+\sqrt{5})(x-\sqrt{5})$

$(\sqrt{5})^2 = 5$

Factorise: $1x^2+6x-16$ } 3 terms
↳ Middle term break

Step 1: Multiply the coefficent of x^2 and the constant number. (ignore sign)

Step 2: Write all the factorisation of 16

Step 3: Break the middle term.

16	Add	sub
① 1×16	17	15
② 2×8	10	6
③ 4×4	8	0

$$x^2+8x\underbrace{-2x-16}$$

Step 4: Group the terms into 2 pieces

$$x(x+8)-2(x+8)$$

Step 4: Transform into factors.

$$\boxed{(x-2)(x+8)}$$

$x^2-2x-24$: Factorise.

$$\boxed{x^2+4x}\boxed{-6x-24}$$

$$x(x+4)-6(x+4)$$

$$\Rightarrow (x+4)(x-6)$$

24

1×24

2×12

3×8

4×6 ↳ $4-6$

$2x^2+29x-15$ } Factorise.

30

$1\times30=30$

2×15

3×10

5×6

1. $2\times15=30$

2. $\underbrace{2x^2+30x}-1x-15$

3. $2x(x+15)-1(x+15)$

4. $(2x-1)(x+15)$

Ans

a) $4x^2 - 32x + 40$ } Complete the square form.

↳ $\underbrace{a(x+b)^2 + c}$

① $4(x^2 - 8x + 10)$

② $4(x^2 - 8x + 4^2 - 4^2 + 10)$

③ $4((x-4)^2 \underbrace{-4^2 + 10}_{-6})$

$= 4((x-4)^2 - 6)$

$= \boxed{4(x-4)^2 - 24}$

$x + 5 = 3$

$\boxed{x = -2}$

b) Solve $1u^2 - 5u - 14 = 0$ } factorise this.

To solve any quadratic equation you need to factorise it.

14	
1x14	
2x7	

$u^2 - 7u + 2u - 14$

$u(u-7) + 2(u-7)$

$(u-7)(u+2) = 0$

$u - 7 = 0$ or $u + 2 = 0$

$\boxed{u = 7 \quad \text{or} \quad u = -2}$

c) Solve

$$\frac{w^2-10}{w+2} + \cancel{w} - 4 = \cancel{w} - 3$$

$$\frac{w^2-10}{w+2} = \frac{1}{1}$$

$$w^2 - 10 = w + 2$$

$$1w^2 - w - 12 = 0$$

12
1×12
2×6
3×4
$3, -4$

$$w^2 + 3w - 4w - 12 = 0$$

$$w(w+3) - 4(w+3) = 0$$

$$(w+3)(w-4) = 0$$

$w+3=0$ or $w-4=0$

$w=-3$ or $w=4$

Ans: $w=-3$ or $w=4$

$$2x^2 + 8x + 8$$

$$2x^2 + 4x + 4x + 8$$ } Factorise

16
1×16
2×8
$4 \times 4 \rightarrow 8$

$$2x(x+2) + 4(x+2)$$

$$(x+2)(2x+4) \rightarrow 2(x+2)$$

$$\Rightarrow (x+2) \times 2(x+2)$$

$$\Rightarrow 2(x+2)^2$$

Sketching Quadratic functions.

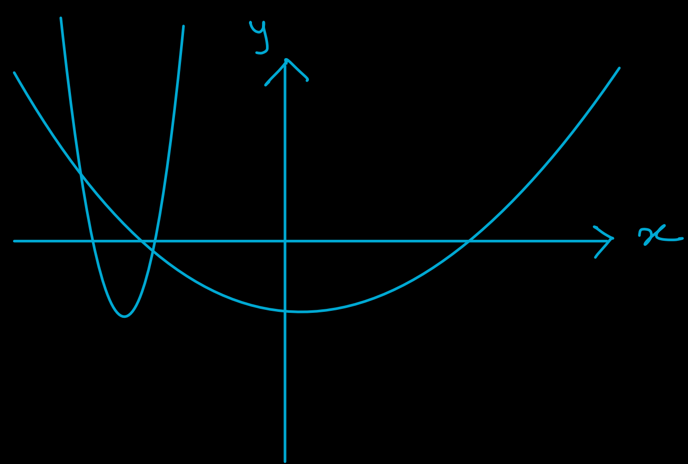
① ax^2+bx+c

if a is +ve → happy face ◡

if a is −ve → sad face ⌒

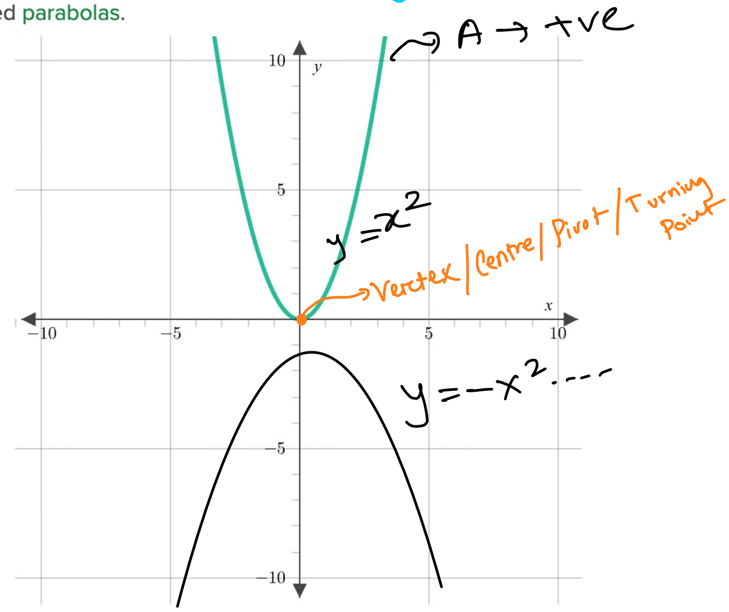
② Completing the square form.

$$y = a(x-b)^2 + c$$



→ y-intercept

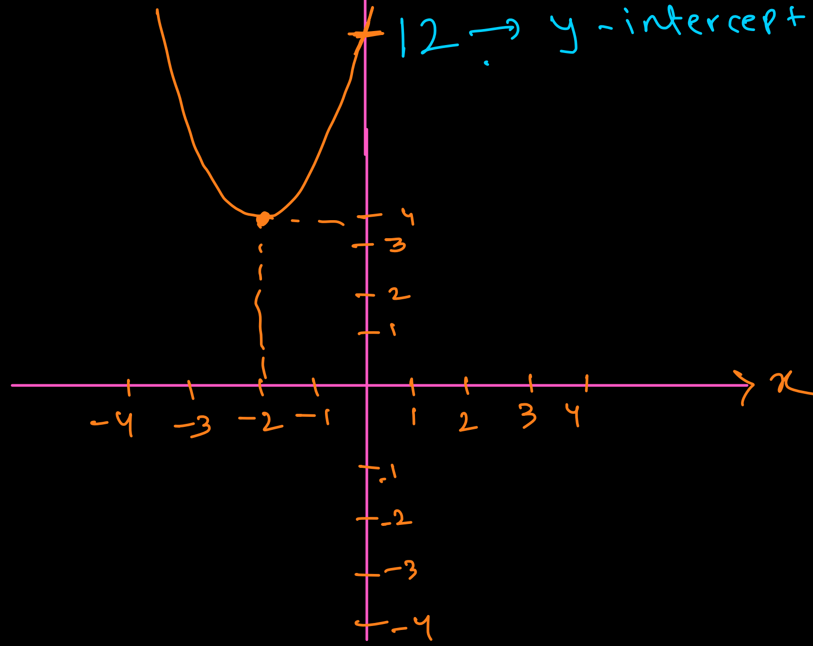
Graphs of quadratic equations of the form $y = Ax^2 + Bx + C$ (where A, B, and C are any number and $A \neq 0$) are called parabolas.



The parabola defined by $y = x^2$

$2x^2+8x+12$

$12 \rightarrow$ y-intercept



To sketch a graph

Steps

① Complete the square -

$2(x^2+4x+6)$

$2(x^2+4x+(2)^2-2^2+6)$

$2((x+2)^2-2^2+6)$

$2((x+2)^2+2)$

$\boxed{2(x+2)^2+4}$

Centre/Vertex : $(-2,4)$

Shape : ∪

To find the vertex

x-coordinate

y-coordinate

Centre $\rightarrow (-2,4)$

$y=\underbrace{x^2-2x-24}$

factorise

$x^2-2x-24$: Factorise.

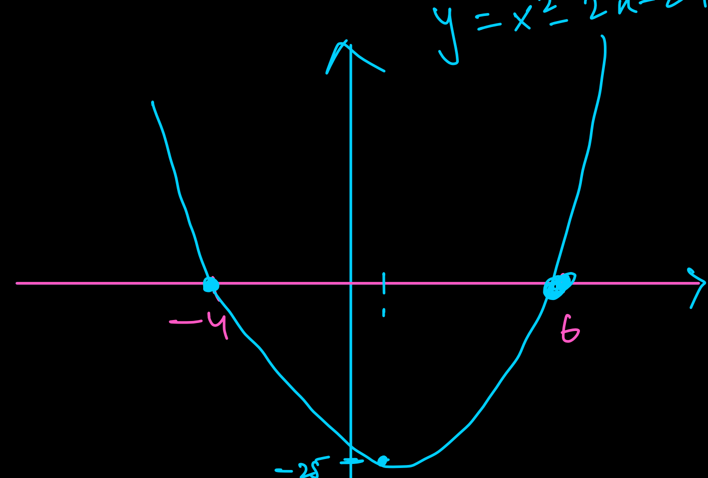
$\boxed{x^2+4x}\boxed{-6x-24}$

$x(x+4)-6(x+4)$

$\Rightarrow (x+4)(x-6)=0$

$x=-4, \quad x=+6$

24
1×24
2×12
3×8
4×6
$4 - 6$



Complete the Square:

$x^2-2x-24$

$\underbrace{x^2-2x+(1)^2}-\underbrace{(1)^2-24}$

$(x-1)^2\underline{-25}$

Centre: $(1,-25)$

$-x^2 - 2x - 24 \rightarrow$ Try factorising
you cannot

Completing the square.

$-1(x^2 + 2x + 24)$

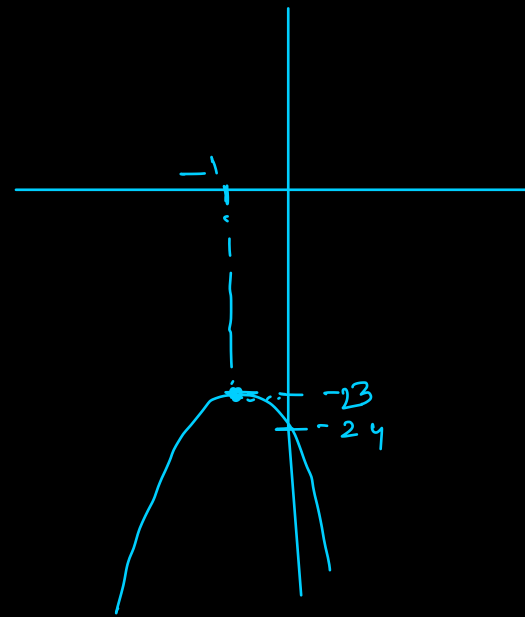
$-1\left(x^2 + 2x + 1^2 - 1^2 + 24\right)$

$-1\left[(x+1)^2 - 1 + 24\right]$

$= -1\left[(x+1)^2 + 23\right]$

$= -(x+1)^2 - 23$

Centre: $(-1, -23)$



y-intercept pt?
$= -24$

Features of parabolas

Like lines, parabolas will always have a y-intercept. This is the point on the graph which touches the y-axis. We can find this by setting $x = 0$ and finding the value of y.

Similarly, we can look for x-intercepts by setting $y = 0$ and then solving for x. Because this is a quadratic equation, there could be 0, 1, or 2 solutions, and there will be the same number of x-intercepts.

Parabolas have an axis of symmetry which is the vertical line $x = -\frac{B}{2A}$. This is also the midpoint of the x-intercepts if they exist.

The point on the parabola which intersects the axis of symmetry is called the vertex of the parabola. The x-value of the vertex will be the axis of symmetry, and we can find the y-value by substituting this x-value into the equation.

Finally, parabolas have a concavity. If the vertex is the minimum point on the graph then the parabola is concave up and if the vertex is the maximum point on the graph then the parabola is concave down.