

QUADRATICS

Function

$$f(x) = x + 3$$

$$f(2) = 2 + 3$$

$$f(5) = 5 + 3$$

equation

$$x^2 + 3x + 5 = 2 \quad x = ?$$

vs

$$f(x) = x^2 + 3x + 5$$

expression.

} Not a function.



Quadratic expression.

Linear Expressions: $mx + b$

Quadratic " : $ax^2 + bx + c$

Factorised Form

$$(x+m)(x-m)$$

E.g. $(x+3)(x-5)$

Completed Square

$$a(x-b)^2 + c$$

E.g. $2(x-3)^2 + 4$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Express $2x^2 + 8x + 12$ in the form $p(x+q)^2 + r$

$$\begin{aligned}
 & 2(x^2 + 4x + 6) \\
 & 2\left(\underbrace{x^2 + 2 \cdot x \cdot 2 + 2^2}_{(x+2)^2} + 2^2 + 6\right) \\
 & 2\left[(x+2)^2 - 2^2 + 6\right] \\
 & 2\left[(x+2)^2 + 2\right] \\
 & 2(x+2)^2 + 4
 \end{aligned}$$

completing the square

$$\begin{aligned}
 (a+b)^2 &= a^2 + 2ab + b^2 \\
 \downarrow \quad \downarrow \quad \downarrow \\
 a^2 + 2ab + b^2 &= \boxed{x^2 + 2 \cdot x \cdot 2 + 2^2} \\
 &= (x+2)^2
 \end{aligned}$$

$$ax^2 + bx + c$$

$$2x^2 + 8x + 12$$

Step 1: Divide every term by a : and then multiply everything by a .

$$2(x^2 + 4x + 6)$$

Half 4 \rightarrow 2
Add 2^2 , Subtract 2^2

Step 2:

$$2(x^2 + 4x + 2^2 - 2^2 + 6)$$

Step 3:

$$2((x+2)^2 - 2^2 + 6)$$

$$2[(x+2)^2 + 2]$$

$$2(x+2)^2 + 4$$

$$3x^2 - 18x + 30 \rightarrow \text{Complete square}$$

$$3(x^2 - 6x + 10) \rightarrow \text{Step 1}$$

$$3\left(\underbrace{x^2 - 6x + 3^2}_{(x-3)^2} - \underbrace{3^2}_{+9} + 10\right) \rightarrow \text{Step 2}$$

$$3\left((x-3)^2 - 3^2 + 10\right)$$

$$3\left((x-3)^2 + 1\right)$$

$$3(x-3)^2 + 3$$

1) Take 'a' common

2) Add middle term's half's square and subtract it again.

3) form $(\quad)^2$ by looking at the terms.

$$2x^2 - 10x + 5 \rightarrow \text{Completing the square.}$$

$$2\left(x^2 - 5x + \frac{5}{2}\right)$$

$$2\left(x^2 - 5x + \left(\frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 + \frac{5}{2}\right)$$

$$2\left(\left(x - \frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 + \frac{5}{2}\right)$$

$$2\left[\left(x - \frac{5}{2}\right)^2 - \frac{25}{4} + \frac{5}{2}\right]$$

$$2\left[\left(x - \frac{5}{2}\right)^2 - \frac{15}{4}\right]$$

$$= -\frac{15}{4}$$

$$2\left(x - \frac{5}{2}\right)^2 - \frac{15}{2}$$

Homework:

Complete the squares for the following:

i) $x^2 - 6x + 6$

ii) $2x^2 - 16x - 4$