

QUADRATICS

Function

$$f(x) = x + 3$$

$$f(2) = 2 + 3$$

$$f(5) = 5 + 3$$

equation

$$x^2 + 3x + 5 = 2 \quad x = ?$$

vs

$$f(x) = x^2 + 3x + 5$$

expression.

} Not a function.



Quadratic expression.

Linear Expressions: $mx + b$

Quadratic " : $ax^2 + bx + c$

✓ Factorised Form

$$(x+m)(x-m)$$

$$\text{E.g. } (x+3)(x-5)$$

✓ Completed Square

$$a(x-b)^2 + c$$

$$\text{E.g. } 2(x-3)^2 + 4$$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Express $2x^2 + 8x + 12$ in the form $p(x+q)^2 + r$

$$\begin{aligned}
 & 2(x^2 + 4x + 6) \\
 & 2\left(\underbrace{x^2 + 2 \cdot x \cdot 2 + 2^2}_{(x+2)^2} + 2^2 + 6\right) \\
 & 2\left[(x+2)^2 - 2^2 + 6\right] \\
 & 2\left[(x+2)^2 + 2\right] \\
 & 2(x+2)^2 + 4
 \end{aligned}$$

completing the square

$$\begin{aligned}
 (a+b)^2 &= a^2 + 2ab + b^2 \\
 \downarrow \quad \downarrow \quad \downarrow \\
 a^2 + 2ab + b^2 &= \boxed{x^2 + 2 \cdot x \cdot 2 + 2^2} \\
 &= (x+2)^2
 \end{aligned}$$

$$ax^2 + bx + c$$

$$2x^2 + 8x + 12$$

Step 1: Divide every term by a : and then multiply everything by a .

$$2(x^2 + 4x + 6)$$

Half 4 \rightarrow 2
Add 2^2 , Subtract 2^2

Step 2:

$$2(x^2 + 4x + 2^2 - 2^2 + 6)$$

Step 3:

$$2((x+2)^2 - 2^2 + 6)$$

$$2[(x+2)^2 + 2]$$

$$2(x+2)^2 + 4$$

$$3x^2 - 18x + 30 \rightarrow \text{Complete square}$$

$$3(x^2 - 6x + 10) \rightarrow \text{Step 1}$$

$$3\left(\underbrace{x^2 - 6x + 3^2}_{(x-3)^2} - \underbrace{3^2}_{-9} + 10\right) \rightarrow \text{Step 2}$$

$$3\left((x-3)^2 - 3^2 + 10\right)$$

$$3\left((x-3)^2 + 1\right)$$

$$3(x-3)^2 + 3$$

1) Take 'a' common

2) Add middle term's half's square and subtract it again.

3) form $(\quad)^2$ by looking at the terms.

$$2x^2 - 10x + 5 \rightarrow \text{Completing the square.}$$

$$2\left(x^2 - 5x + \frac{5}{2}\right)$$

$$2\left(x^2 - 5x + \left(\frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 + \frac{5}{2}\right)$$

$$2\left(\left(x - \frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 + \frac{5}{2}\right)$$

$$2\left[\left(x - \frac{5}{2}\right)^2 - \frac{25}{4} + \frac{5}{2}\right]$$

$$2\left[\left(x - \frac{5}{2}\right)^2 - \frac{15}{4}\right]$$

$$= -\frac{15}{4}$$

$$2\left(x - \frac{5}{2}\right)^2 - \frac{15}{2}$$

Homework:

Complete the squares for the following:

i) $x^2 - 6x + 6$

ii) $2x^2 - 16x - 4$

① $2[x^2 - 8x - 2]$

② $2[(x^2 - 8x + 4^2 - 4^2 - 2)]$

$2[(x - 4)^2 - 16 - 2]$

$= 2[(x - 4)^2 - 18]$

$= 2(x - 4)^2 - 36$ ✓

Factorising Quadratic Expression.

Factorising: $4x + 10 \Rightarrow 2(2x + 5)$

$a^2 - b^2 = (a + b)(a - b)$

factor $\rightarrow x^2 - 16 = (x^2 - 4^2) \rightarrow a^2 - b^2$
 $= (x - 4)(x + 4)$

factor: $x^2 - 5 \rightarrow (\sqrt{5})^2$

$x^2 - (\sqrt{5})^2$
 $\rightarrow (x + \sqrt{5})(x - \sqrt{5})$

$(\sqrt{5})^2 = 5$

Factorise: $x^2 + 6x - 16$ } 3 terms
 ↳ Middle term break

Step 1: Multiply the coefficient of x^2 and the constant number. (ignore sign)

Step 2: Write all the factorisation of 16

| | 16 | Add | Sub |
|---|---------------|-----|-----|
| ① | 1×16 | 17 | 15 |
| ② | 2×8 | 10 | 6 |
| ③ | 4×4 | 8 | 0 |

Step 3: Break the middle term.

$$x^2 + 8x - 2x - 16$$

Step 4: Group the terms into 2 pieces

$$\rightarrow (x)(x+8) - 2(x+8)$$

Step 4: Transform into factors.

$$(x-2)(x+8)$$

$x^2 - 2x - 24$: Factorise.

$$x^2 + 4x - 6x - 24$$

$$x(x+4) - 6(x+4)$$

$$\Rightarrow (x+4)(x-6)$$

$$\begin{array}{r} 24 \\ \underline{1 \times 24} \\ 2 \times 12 \\ 3 \times 8 \\ 4 \times 6 \\ \hookrightarrow 4-6 \end{array}$$

$$2x^2 + 29x - 15 \quad \} \text{Factorise.}$$

① $2 \times 15 = 30$

② $2x^2 + 30x - 1x - 15$

③ $2x(x+15) - 1(x+15)$

④ $(2x-1)(x+15)$
Ans

$$30$$

$$1 \times 30 = 30$$

$$2 \times 15$$

$$3 \times 10$$

$$5 \times 6$$

Sketching Quadratic functions.

① $ax^2 + bx + c$

if a is +ve \rightarrow happy face \smile

if a is -ve \rightarrow sad face \frown

② Completing the square form.

$$y = a(x-b)^2 + c$$



