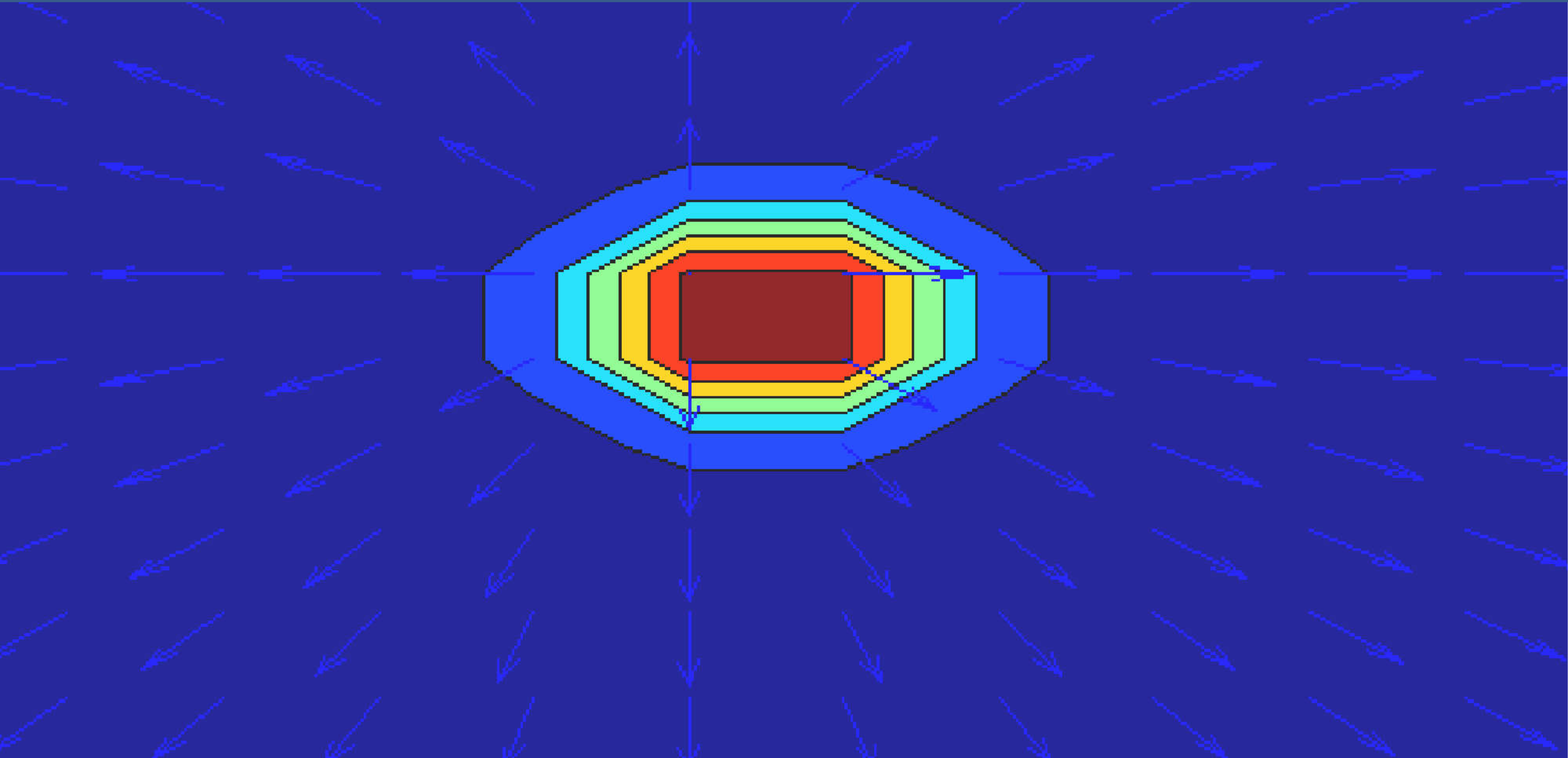
Electric Field for Stationary Charge Distribution:

Solving Poisson’s Equation in 2D and 3D



What is Poisson’s Equation and How to Solve it Numerically?

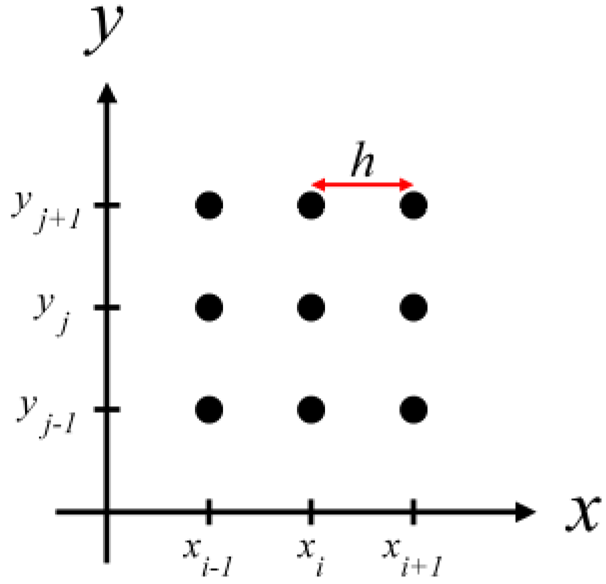
Poisson’s Equation is a second-order PDE relating electric potential and charge distribution in space.

$$\nabla^2 V = -\frac{\rho}{\epsilon}$$

We have solved it using **FDM (Finite Difference Method)** in the required domain. In this process, the partial derivatives are replaced with **Finite Difference equation** and the total equation becomes a linear equation which is then solved by **Iterative Method** to get Potential at discrete number of points

Step 1: Discretizing space and selecting the solution domain

We solved the equation in a **square domain** in XY plane (2D) and **cubic domain** in XYZ space. The co-ordinate of the corner of the region is given as **input**. Then number of points in each direction-> ‘N’ is taken. After that, we divide the domain into N*N (N*N*N for 3D) grids. These discretized space is then represented by X, Y (X, Y, Z) matrix .



Step 2: Setting charge distribution

A charge density matrix **rho** of N*N*N order is set up. Charge density function is given as input. Then the rho values at the sample points are calculated.

Step 3: Setting the boundary conditions

We used **Dirichlet Boundary condition,** which means we have specified electrical potential values which we are going to solve at the boundary points. Having the boundary conditions we can start solving for the poisson's equation.

Step 4: Solving the finite difference equation by iteration

Finite difference equation for V is ->

For 2D; $$V(i,j) = \frac{1}{4}\left(V(i-1,j) + V(i+1,j) + V(i,j-1) + V(i,j+1) + \frac{h^2\rho}{\epsilon}\right)$$

For 3D; $$V(i,j,k) = \frac{1}{6}\left(V(i-1,j,k) + V(i+1,j,k) + V(i,j-1,k) + V(i,j+1,k) + V(i,j,k-1) + V(i,j,k+1) + \frac{h^2\rho}{\epsilon}\right)$$

We set the potential matrix **V** to have all zero entry. Then We iteratively solve for V(I,j,k) using the previous values of V. The iteration is guaranteed to converge

Step 5: Solving electric field from scalar potential and normalising electric field

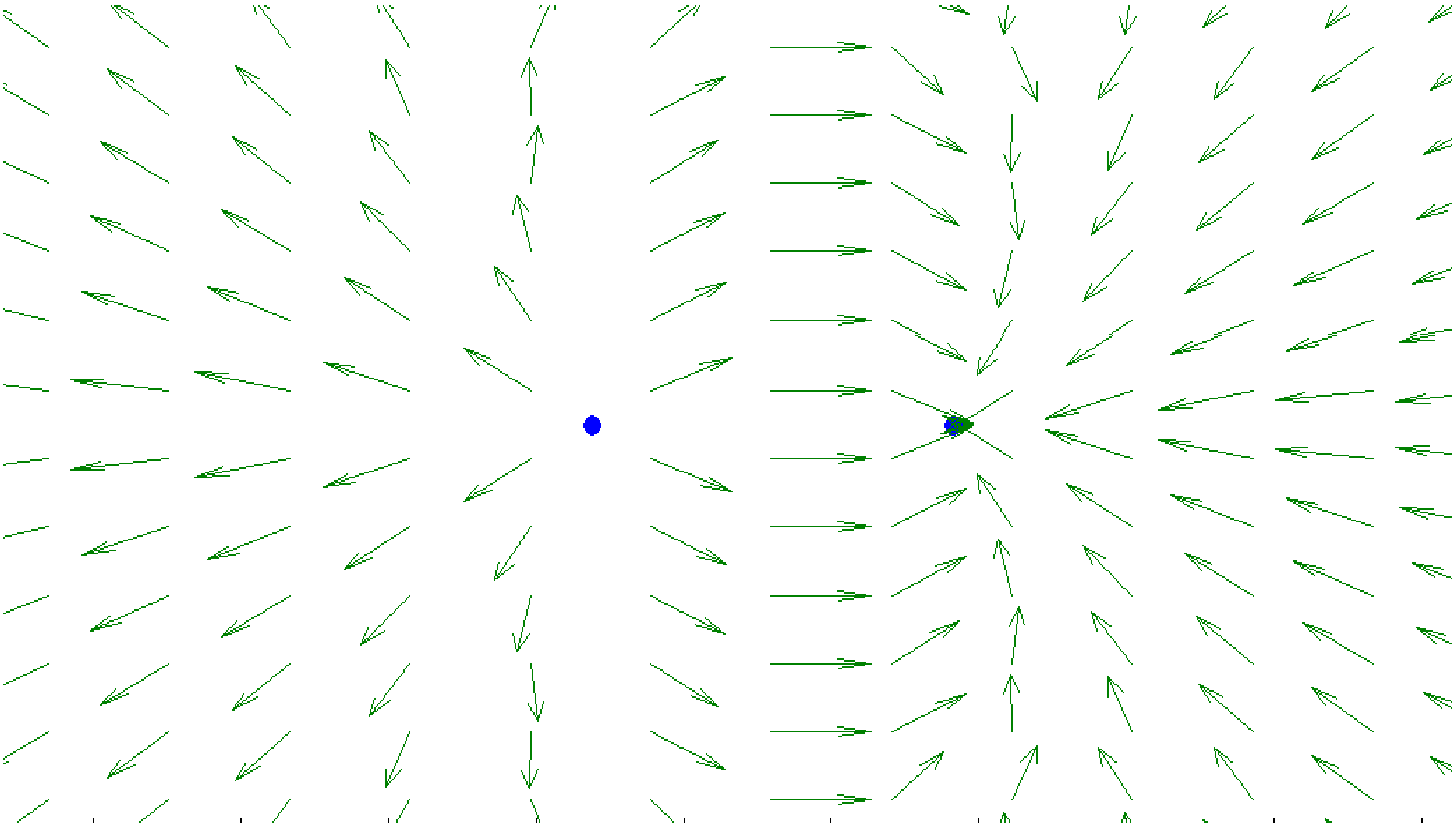
We know that, electric field is the negative gradient of potential. After solving for potential at every point, we take partial derivatives and get electric field component Ex, Ey, Ez. Then we normalize E field by diving each component by the electric field value.

Step 6: Solution visualization

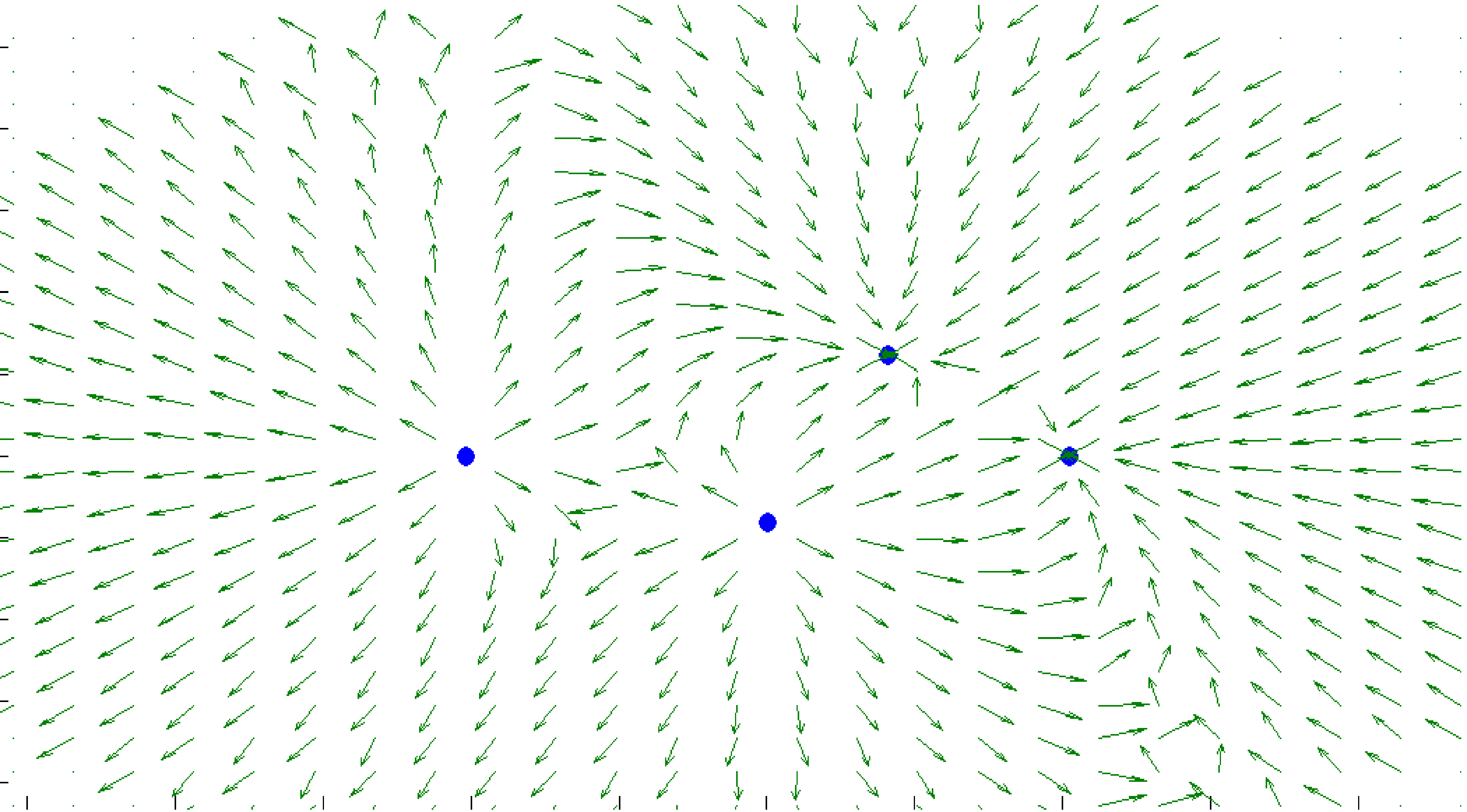
For 2D, we can visualize E-field by vector field diagram and potential by surface plot and contour curves.
For 3D, we can visualize E-field by vector field diagram and potential by contour surface.

Applications

*Dipole electric field (2D and 3D)



*Field for Arbitrary point charge distribution (2D and 3D)



*Parallel plate capacitor